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Major accomplishments of this project included

o The development of efficient non-negatively constrained optimization algorithms for image deblurring. This includes a new pre-conditioner based on a. sparse approximation to the blurring operator. See [1, 2j for (details.

o The development of efficient pre-conditioners for the joint phase and object estimation problem in phase diversity. These pre-conditioners were based on the Hessian of the (quadratic) regularization terms. See [4]. This paper also contains a careful numerical study and comparison of trust region vs. limited memory BFGS methods for the numerical solution to optimization problems arising in phase diversity estimation.

Data for this study was obtained from the US Air Force Maui Spate Surveillance Complex in collaboration with Dr. David Tyler.

The development of obtained preconditioned conjugate gradient schemes for volume refractive index (turbulence) estimation. See [3, 5, 6]. These schemes make, efficient use of the layered structure of the atmospheric turbulence profiles. This layered structure gave rise to block-structured matrices. We employed a block analogue of symmetric Gauss-Seidel iteration as our multi-grid smoother.

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# FINAL REPORT

March 1, 1999 to August 31, 2002

# Computational Methods for Atmospheric Optics

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# Contents

# 1 Introduction

This project dealt with computational methods for inverse problems related to light propagation through the earth's atmosphere. In this section we describe three important applications:

1. Image deblurring.

2. Estimation of phase and object from phase diversity data.

3. Estimation of the volume refractive index profile of the atmosphere from wavefront sensor data.

Of fundamental importance in each application is the aperture-plane phase, or wavefront aberration, $\phi(x, y)$. This can be interpreted as the deviation from planarity of the wavefront of light, at planar incidence at the top of the earth's atmosphere, after it has propagated through the atmosphere [10]. From the phase, one can compute the point spread function (PSF) for a ground-based telescope. Assuming no imperfections in the optics of the telescope and an incoherent light source, the on-axis PSF takes the form

$$s = |\mathcal{F}(Ae^{i\phi})|^2, \tag{1}$$

where $\mathcal{F}$ denotes the two-dimensional Fourier transform, $i = \sqrt{-1}$, and $A$ denotes the aperture function. For an ideal telescope, $A = 1$ inside the telescope aperture, and $A = 0$ outside the aperture.

From the PSF, one can formulate a simple model for the blurred, noisy, pixelated image recorded by a charge coupled device (CCD) array attached to a telescope,

$$d_{ij} = \underbrace{\int \int s(x_i - x', y_j - y') f(x', y') \, dx' dy'}_{(Sf)(x_i, y_j)} + \eta_{ij}. \tag{2}$$

Here $f$ denotes the light source, or object, and $\eta_{ij}$ represents stochastic noise in the image formation process. The indices $i, j$ denote position in the pixel array.

We can now describe several important inverse problems arising in atmospheric optics. Perhaps the simplest is image deblurring, i.e., the estimation of the object $f$ in (2) given pixelated image data $d_{ij}$. Due to the ill-posedness

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of inverse of the underlying convolution integral operator $S$ defined in equation (2), the image deblurring problem is highly ill-conditioned. For this reason, regularization (see [11]) must be applied. This results in a large-scale, mildly ill-conditioned optimization problem. If prior information like nonnegativity of the object is incorporated, the problem becomes constrained and much more difficult to solve numerically.

In practice, the PSF $s$ in the model (2) as well as the object $f$ may be unknown. In this case, one may use phase diversity [7, 9] to simultaneously estimate both the phase $\phi$ and the object $f$. The key idea is to collect suitable additional data. Denote the data in model (2) by

$$\mathbf{d} = s[\phi] \star f + \eta. \tag{3}$$

Here $s = s[\phi]$ denotes the dependence of the PSF on the phase $\phi$ in equation (1), and $\star$ denotes convolution product. In two-channel, single frame phase diversity, one collects additional data

$$\mathbf{d}' = s[\phi + \theta] \star f + \eta'. \tag{4}$$

Here $\theta$ denotes a known phase aberration, for example, aperture plane defocus, imposed before the second image is formed. From the two images $\mathbf{d}$ and $\mathbf{d}'$, one can estimate both the phase $\phi$ and the object $f$. From (1) the dependence of PSF $s$ on phase $\phi$ is nonlinear. Hence, the phase diversity inverse problem is nonlinear. It is also ill-posed.

When imaging over relatively wide fields of view, the situation is even more complicated in that the PSF $s$ may depend on the viewing direction $\theta$ (the model (1)-(2) is accurate only for narrow fields of view). The aperture plane phase in the direction $\theta$ can be described as a line integral

$$\phi(x, y; \theta) = \int_{z=0}^{H} n(x(z; \theta), y(z; \theta), z)\, dz. \tag{5}$$

Here $n = n(x, y, z)$ denotes the refractive index of the atmosphere, and $(x(z; \theta), y(z; \theta), z)$ denotes a light ray path from the top of the atmosphere $(z = H)$ to the bottom $(z = 0)$. A device called a wavefront sensor measures the gradient of $\phi$. The inverse problem is to estimate the volume refractive index profile $n(x, y, z)$ from wavefront sensor measurements in several directions $\theta$. This is a limited angle tomography problem, and is ill-posed. Prior information can make this problem much more tractable. The atmospheric refractive index profile is highly correlated with atmospheric turbulence. Atmospheric turbulence is concentrated in a few thin layers, and it can be accurately modeled using Kolmogorov statistics. See [8].

# 2   Objectives

The broad objective of this project was the development of efficient computational algorithms to solve inverse problems that arise in atmospheric optics. We focused on the three specific application problems enumerated in the introduction.

These three problems share several features. Each is ill-posed, so regularization is required to obtain accurate solutions. Each problem has solutions which are functions of two or three spatial variables. When discretized, these yield ill-conditioned systems with very large numbers of unknowns. In the case of phase diversity, these systems are nonlinear, and in the case of image deblurring, the incorporation of prior knowledge makes these systems nonnegatively constrained. In summary, we needed to numerically solve large, ill-conditioned, possibly constrained, and possibly nonlinear systems of equations.

To compute solutions to these systems, there are several fairly general "tricks of the trade", e.g., quasi-Newton methods for nonlinear systems, projected Newton methods for nonnegatively constrained systems, and the conjugate gradient (CG) method for large linear systems. To gain robustness and efficiency, we were forced to develop very special preconditioners. Preconditioners can be viewed as transformations that take advantage of special problem structure. Our preconditioners are described in the next section.

# 3   Major Accomplishments and New Findings

Major accomplishments of this project included

- The development of efficient nonnegatively constrained optimization algorithms for image deblurring. This includes a new preconditioner based on a sparse approximation to the blurring operator. See [1, 2] for details.

- The development of efficient preconditioners for the joint phase and object estimation problem in phase diversity. These preconditioners were based on the Hessian of the (quadratic) regularization terms. See [4]. This paper also contains a careful numerical study and comparison of trust region vs. limited memory BFGS methods for the numerical solution to optimization problems arising in phase diversity estimation.

Data for this study was obtained from the US Air Force Maui Space Surveillance Complex in collaboration with Dr. David Tyler.

- The development of multigrid preconditioned conjugate gradient schemes for volume refractive index (turbulence) estimation. See [3, 5, 6]. These schemes make efficient use of the layered structure of the atmospheric turbulence profiles. This layered structure gave rise to block-structured matrices. We employed a block analogue of symmetric Gauss-Seidel iteration as our multigrid smoother.

- The publication of a research monograph entitled "Computational Methods for Inverse Problems" [11]. This publication presents both the general theory and specific algorithms for the solution of inverse problems.

# References

[1] J. M. Bardsley, *Constrained Optimization Techniques for Image Reconstruction*, Ph.D Thesis, Department of Mathematical Sciences, Montana State University, May 2002.

[2] J. M. Bardsley and C.R. Vogel, *A Nonnnegatively Constrained Convex Programming Method for Image Reconstruction*, submitted to SIAM Journal on Scientific Computing, July 2002.

[3] B. L. Ellerbroek, Luc Gilles, and C.R. Vogel, *A Computationally Efficient Wavefront Reconstructor for Simulation of Multi-Conjugate Adaptive Optics on Giant Telescopes*, Proc. SPIE **4839-116**, Adaptive Optics System Technologies II (2002).

[4] Luc Gilles, C.R. Vogel, and J.M. Bardsley, *Computational Methods for a Large-Scale Inverse Problem Arising in Atmospheric Optics*, Inverse Problems, **18** (2002), pp. 237-252.

[5] Luc Gilles, C.R. Vogel, and Brent Ellerbroek, *A Multigrid Preconditioned Conjugate Gradient Method for Large Scale Wavefront Reconstruction*, Journal of the Optical Society of America A, **19** (2002), pp. 1817-1822.

[6] Luc Gilles, B. L. Ellerbroek, and C.R. Vogel, *Layer-Oriented Multigrid Wavefront Reconstruction Algorithms for Multi-Conjugate Adaptive Optics*, Proc. SPIE **4839-118**, Adaptive Optics System Technologies II (2002).

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[9] R. Paxman, T. Schultz, and J. Fineup, "Joint estimation of object and aberrations by using phase diversity", *J. Optical Soc. Amer. A*, **9** (1992), pp. 1072-1085.

[10] M. C. Roggemann and B. Welsh, *Imaging Through Turbulence*, CRC Press, Boca Raton, 1996.

[11] C. R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.

# 4   Personnel Supported by Grant

- The PI: Curtis R. Vogel, Professor of Mathematics, Montana State University.

- 2 Graduate Research Assistants: Johnathan Bardsley and Scott Hyde. Bardsley completed his Ph.D. in Mathematics in May 2002 under the direction of the PI. He recently accepted a postdoctoral research position with the Statistical and Applied Mathematical Sciences Institute (SAMSI) in North Carolina. Scott Hyde is a Ph.D. student in Statistics, also at Montana State University.

# 5   Publications

1. C. R. Vogel, *Negative results for multilevel preconditioners in image deblurring*, in *Scale-Space Theories in Computer Vision*, Edited by M. Nielsen, P. Johansen, O.F. Olsen, and J. Weickert, Springer-Verlag, 1999.

2. C. R. Vogel, *A limited memory BFGS method for an inverse problem in atmospheric imaging*, in *Methods and Applications of Inversion*, Edited by P. C. Hansen, B. H. Jacobsen, and K. Mosegaard, Lecture Notes in Earth Sciences, **92** (2000), pp. 292-304, Springer-Verlag.

3. M. Hanke, J. Nagy, and C. R. Vogel, *A Quasi-Newton approach to nonnegative image restoration*, *Linear Algebra and Its Applications*, **316** (2000), pp. 223-236.

4. Luc Gilles and P. Tran, *Optical Switching in Nonlinear Chiral Distributed Bragg Reflectors with Defect Layers*, Journal of the American Optical Society B, **19** (2002), pp. 630-639.

5. Luc Gilles, C.R. Vogel, and J.M. Bardsley, *Computational Methods for a Large-Scale Inverse Problem Arising in Atmospheric Optics*, Inverse Problems, **18** (2002), pp. 237-252.

6. Luc Gilles, C.R. Vogel, and Brent Ellerbroek, *A Multigrid Preconditioned Conjugate Gradient Method for Large Scale Wavefront Reconstruction*, Journal of the Optical Society of America A, **19** (2002), pp. 1817-1822.

7. J. M. Bardsley, *Constrained Optimization Techniques for Image Reconstruction*, Ph.D Thesis, Department of Mathematical Sciences, Montana State University, May 2002.

8. C. R. Vogel, *Computational Methods for Inverse Problems*, SIAM, Philadelpha, 2002.

9. J. M. Bardsley and C.R. Vogel, *A Nonnnegatively Constrained Convex Programming Method for Image Reconstruction*, submitted to SIAM Journal on Scientific Computing, July 2002.

10. B. L. Ellerbroek, Luc Gilles, and C.R. Vogel, *A Computationally Efficient Wavefront Reconstructor for Simulation or Multi-Conjugate Adaptive Optics on Giant Telescopes*, Proc. SPIE **4839-116**, Adaptive Optics System Technologies II (2002).

11. Luc Gilles, B. L. Ellerbroek, and C.R. Vogel, *Layer-Oriented Multigrid Wavefront Reconstruction Algorithms for Multi-Conjugate Adaptive Optics*, Proc. SPIE **4839-118**, Adaptive Optics System Technologies II (2002).

# 6   Presentations

- Sept. 26-30, 1999. Santa Clara, California. Annual Meeting of the Optical Society of America. Attended and presented contributed paper entitled "Fast algorithms for nonnegatively constrained image deblurring".

- March 7-8, 2000. Maui, Hawaii. Workshop on Post-Detection Processing and Inverse Problems in Ground-Based Imaging at the Maui High Performance Computing Center. Attended and presented talk entitled "Computational techniques for inverse problems".

- June 28-July 1, 2000. Lake St. Wolfgang, Austria. Conference on Inverse Problems. Attended and presented paper "Incorporating statistical information in the regularized solution of ill-posed inverse problems".

- July 10-12, 2000. San Juan, Puerto Rico. SIAM 2000 Annual Meeting. Attended and gave 2 minisymposium talks: (i) "Inverse problems in atmospheric imaging I: Mathematical models"; and (ii) "Regularization parameter selection from noise statistics".

- Oct. 16-20, 2000. Minneapolis, Minnesota. Participated in IMA Workshop "Image Processing and Low Level Vision".

- Dec. 5-7, 2000. Maui, Hawaii. Workshop on Post-Detection Processing and Inverse Problems in Ground-Based Imaging at the Maui High Performance Computing Center. Attended and presented talk entitled "Computational Methods for Nonnegatively Constrained Image Reconstruction". Postdoc Luc Gilles also attended and presented a talk on joint work, "Numerical Issues Arising in Phase Diversity Estimation".

- Dec 14-16, 2000, Hong Kong. Workshop on Mathematics in Image Processing. Attended and presented invited talk entitled "Statistically Based Regularization Parameter Selection Schemes for Image Deblurring".

- June 18-22, 2001. Conference on Applied Inverse Problems in Montecatini, Italy. Attended and gave invited talk entitled "Statistically-Based Regularization Parameter Selection Methods for Regularized Ill-Posed Problems".

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- Nov. 5-7, 2001. Albuquerque, New Mexico. Attended the Optical Society of America's Conference on Signal Recovery and Synthesis and presented talk "A comparison of computational methods for phase-diverse object and phase reconstruction".

- Jan. 15-17, 2002. Maui, Hawaii. Attended AFOSR-sponsored Workshop on Advanced Topics in Space Situational Awareness and presented talk entitled "Some thoughts on nonnegatively constrained image reconstruction". Postdoc Luc Gilles also attended and presented a talk on joint work, "Fast Algorithms for Multiconjugate Adaptive Optics".

- July 10-12, 2002. Philadelphia, Pennsylvania. Attended SIAM Annual Meeting, organized minisymposium "Computational Methods for Large-Scale Inverse Problems", and presented talk "Multiconjugate Adaptive Optics".

- Aug. 22-23, 2002. Maui, Hawaii. Presented short course "Multiconjugate Adaptive Optics" at the Maui Scientific Research Center.

- Aug. 24-26, 2002. Waikaloa, Hawaii. Attended SPIE Conference on Astronomical Telescopes and Instrumentation and presented talk "Multigrid wavefront reconstruction algorithms for extreme and multiconjugate adaptive optics".

# 7 Collaborative Research and Transactions at US Air Force Laboratories

During the course of this project, the PI visited Air Force research facilities on the island of Maui, Hawaii, four times. Three of these visits were to attend annual workshops and meet with fellow researchers at the Maui High Performance Computing Center (MHPCC). The fourth visit was to present a short course on multiconjugate adaptive optics at the Maui Scientific Research Center (MSRC), which is next door to the MHPCC. The PI's MSRC contact was Dr. Stuart Jeffries.

The PI also collaborated with Dr. David Tyler in the analysis of phase diversity data. This data was collected by Dr. Tyler at the US Air Force Maui Space Surveillance Complex on Mt. Halleakala. Results are described in publication [5] above.

# 8   Inventions or Patent Disclosures

None.

# 9   Summary

This project dealt with computational methods for inverse problems related to light propagation through the earth's atmosphere. It dealt specifically with the following applications: (1) image deblurring; (2) the simultaneous recovery of object and phase from phase diversity data; and (3) The estimation of the volume refractive index profile of the atmosphere from wavefront sensor data.

Each of these three application problems was ill-posed, and regularization was required for their accurate solution. In each case, the implementation of regularization gave rise to optimization problems which were large and ill-conditioned. In addition, the image deblurring problem was nonnegatively constrained, and the phase diversity problem was nonlinear. These properties made the optimization problems very difficult to solve.

Several fairly general numerical techniques were available for these problems. These included trust region and limited memory BFGS nonlinear optimization methods and conjugate gradient iteration to solve the third problem (volume refractive index estimation) and to solve linear subproblems arising in the other two applications. The key to obtaining *robust, efficient solutions* was to employ an approach called preconditioning. The PI and his collaborators developed very effective special-purpose preconditioners for each of the three specific applications.

The PI collaborated with several AFOSR-sponsored researchers on the island of Maui, Hawaii during this project. The PI applied the computational techniques that he developed to image data collected at the US Air Force Maui Space Surveillance Complex by Dr. David Tyler. The PI also presented a short course on the topic of multiconjugate adaptive optics at the Maui Scientific Research Center.

A total of 9 peer-reviewed scientific journal articles were prepared under this project. A PhD thesis was also written by a student directed by the PI and supported under this project. In addition, a research monograph entitled "Computational Methods for Inverse Problems" was written by the PI. This monograph was recently published by the Society for Industrial and Applied

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Mathematics.

Preprints and reprints of papers prepared under this project can be downloaded directly from the PI's web site at

http://www.math.montana.edu/~vogel/