

EVALUATION OF SPASTICITY IN UPPER LIMBS OF HEMIPLEGIC SUBJECTS USING A MATHEMATICAL MODEL

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Abstract The purpose of this study is to propose a new technique for evaluating spasticity in the upper limbs of hemiplegic patients. Each subject lay on a bed, and his forearm was supported with a jig to measure the elbow joint angle. The subject was instructed to relax and not to resist the step-like load which was applied to extend the elbow joint. The elbow joint angle and electromyograms of the biceps muscle, triceps muscle and brachioradialis muscle were measured. The step-like response was approximated with a mathematical model which consisted of elastic components depending on both muscle activities and elbow joint angle. The response of hemiplegic subjects were approximated well with the model. The torque generated by the elastic component was estimated. The normalized elastic torque was approximated with a damped sinusoid by the least square method. The time constant of the elastic torque showed significant differences between the healthy subjects and the hemiplegic subjects and among the different grades of subjects. These results suggest that the time constant of the elastic torque can be a quantitative index of spasticity.

Key words — elasticity, viscosity, elbow, spasticity

I. INTRODUCTION

The Ashworth scale is widely used as a grade of spasticity. The measurement is simple, and no special equipment is needed. The grade is based on a relationship between range of motion and resistance force during extension/flexion of the joint. However, it is a qualitative index and sometimes fluctuates depending on a doctor's experience. Several reports have given quantitative evaluations of spasticity. Some have proposed range of motion applying a constant load. While a decrease in range of motion is one of characteristics of spasticity, it is not sufficient to evaluate spasticity. Velocity and force have also been measured under the various restriction of motion in others reports. However, it is necessary to use kinematic equipment for measurement under such special conditions as isovelocity extension/flexion.

The purpose of this study is to propose a new technique for evaluating spasticity quantitatively using the measurement of step-like response and a mathematical model.

II. METHOD

A. Subjects and Experimental Setup

The subjects included two male healthy subjects and four male hemiplegic subjects. All gave informed consent (Table 1).

Fig. 1 shows a schematic illustration of the experimental setup. Each subject lay on a bed with his forearm supported with a jig to measure the elbow joint angle. With

Table 1. Subjects and Ashworth scale

Subject	Age	Ashworth scale	Load (kgf)
A	22	—	0.5, 1.0, 1.5
B	24	—	0.4, 0.5, 0.7
C	47	2	0.5, 1.0, 1.5
D	53	3 or 4	2.0, 2.5, 3.0
E	63	3 or 4	0.5, 1.0, 1.5
F	57	3	0.5, 1.0, 1.5

to extend the elbow joint. The subject was instructed to relax and not to resist the external load. The initial elbow joint angle was about 100°, and the forearm was supported by an experimenter, which released the forearm at an arbitrary time.

Three different weights were used, which allowed observation of a sufficient amplitude of the step-like response without causing full extension (Table 1). The measurements were repeated three times for each weight. The elbow joint angle was measured with a potentiometer. For bipolar recording, electromyograms (EMG) were obtained with Ag-AgCl surface electrodes, 10 mm in diameter, taped to the skin 15 mm apart. EMGs were recorded from the biceps muscle, triceps muscle and brachioradialis muscle. EMG signals were full-wave rectified and then smoothed with a second order low-pass filter ($f_c = 2.6$ Hz) to obtain an integrated electromyogram (IEMG).

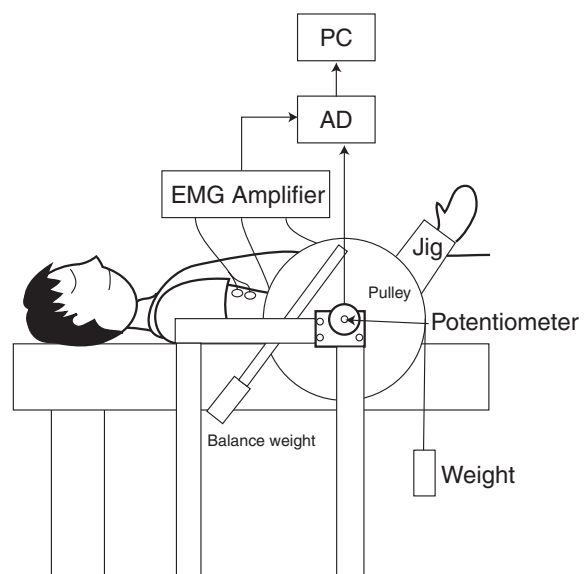


Fig. 1. Schematic illustration of experimental setup.

Report Documentation Page

Report Date 25OCT2001	Report Type N/A	Dates Covered (from... to) -
Title and Subtitle Evaluation of Spasticity in Upper Limbs of Hemiplegic Subjects Using a Mathematical Model	Contract Number	
	Grant Number	
	Program Element Number	
Author(s)	Project Number	
	Task Number	
	Work Unit Number	
Performing Organization Name(s) and Address(es) Dept. Applied Physics and Physico-Informatics, Fac. Science and Technology, Keio University Yokohama, Japan	Performing Organization Report Number	
Sponsoring/Monitoring Agency Name(s) and Address(es) US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500	Sponsor/Monitor's Acronym(s)	
	Sponsor/Monitor's Report Number(s)	
Distribution/Availability Statement Approved for public release, distribution unlimited		
Supplementary Notes Papers from the 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society, October 25-28, 2001, held in Istanbul, Turkey. See also ADM001351 for entire conference on cd-rom., The original document contains color images.		
Abstract		
Subject Terms		
Report Classification unclassified	Classification of this page unclassified	
Classification of Abstract unclassified	Limitation of Abstract UU	
Number of Pages 4		

B. Mathematical Model

The equation of motion around the elbow joint is (1).

$$I\ddot{\theta}(t) + \eta\dot{\theta}(t) + k\theta(t) = f(t) \quad (1)$$

where I is the inertia of the forearm and the jig, $\theta(t)$ is the relative elbow joint angle, η is the viscous coefficient and k is the elastic coefficients. $f(t)$ is the torque generated by the external load:

$$\begin{aligned} f(t) &= m(g - a)r \\ &= mgr - mr^2\ddot{\theta} \end{aligned} \quad (2)$$

where r is the radius of the pulley, m is the weight, g is the gravitational constant and a is the acceleration of the weight. The relative elbow joint angle $\theta(t)$ is zero at the onset of the extension.

Substituting (2) to (1), (3) is derived:

$$I'\ddot{\theta}(t) + \eta\dot{\theta}(t) + k\theta(t) = mgr \quad (3)$$

where I' is $I + mr^2$.

The inertia I' , viscous coefficient η and elastic coefficient k were calculated by the least square method. The relative elbow joint angle was estimated, using these calculated values, by the Runge-Kutta method and was compared with the observed elbow joint angle.

According to our pre-measurements for healthy subjects, the linear equation of motion (1) did not approximate the observed data. We modified the linear model as shown in (4):

$$k = k_0 + k_1\theta(t) \quad (4)$$

The elastic coefficient k depends on the relative elbow joint angle $\theta(t)$ in Model A.

In addition to the modification of (4), we assumed that the elastic coefficients were promotional to muscle activities (IEMG) in (5). This is model B:

$$k = k_0 + k_1\theta(t) + a_1e_1(t) + a_2e_2(t) + a_3e_3(t) \quad (5)$$

where $e_1(t)$, $e_2(t)$ and $e_3(t)$ are IEMGs of the biceps muscle, triceps muscle and brachioradialis muscle, respectively.

C. Normalized Elastic Torque

The torque caused by the elastic component was evaluated as follows. First, the elastic torque was calculated by subtracting torques caused by the inertia and viscosity from the total torque mgr . Then the elastic torque was normalized by the total torque as shown in (6):

$$g(t) = \frac{mgr - I'\ddot{\theta}(t) - \eta\dot{\theta}(t)}{mgr} \quad (6)$$

where $g(t)$ is the normalized elastic torque.

The normalized elastic torque was approximated with a damped sinusoid in (7):

$$g(t) = A \exp(-t/\tau) \sin(2\pi f_0 t + \phi) + 1.0 \quad (7)$$

where A is the relative magnitude, τ is a time constant of dumping, f_0 is the natural frequency and ϕ is the phase.

Finally, the time constant and the natural frequency were evaluated by the Student's t -test and compared with Ashworth scale.

III. RESULTS

A. Model A

Fig. 2 shows an example of a step-like response of the elbow joint angle (Subject A). The elbow joint angle was estimated with the Model A. The top panel (a) represents the elbow joint angle when the external load was 0.5 kgf. The solid red line is the estimated angle. The green broken-dotted line represents the observed angle. The estimated curve fits the observed one well. The blue dotted line represents the small residual.

The lower panel (b) represents the elbow joint angle when the external load was 1.0 kgf. The elbow joint angle was estimated well with the model A. The elbow joint angle of Subject B was also estimated with Model A (not shown).

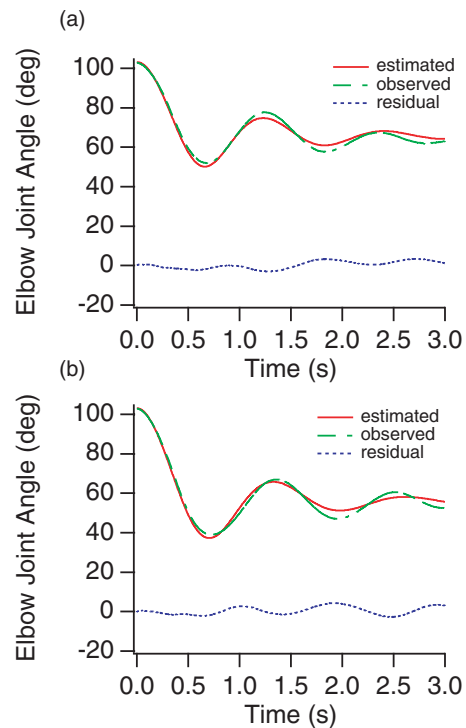


Fig. 2. Example of a step-like response of the elbow joint angle (subject A). (a) External load was 0.5 kgf. (b) External load was 1.0 kgf.

Fig. 3 shows the elbow joint angle of hemiplegic subjects. The top panel (a) represents the elbow joint angle of Subject E. The external load was 1.0 kgf. The observed angle shows less oscillation than that of the healthy subjects. The estimated angle was inconsistent with the observed one.

The bottom panel (b) shows the angle of Subject F. The external load was 1.0 kgf. The amplitude of the second peak was smaller than that of a healthy subjects. This

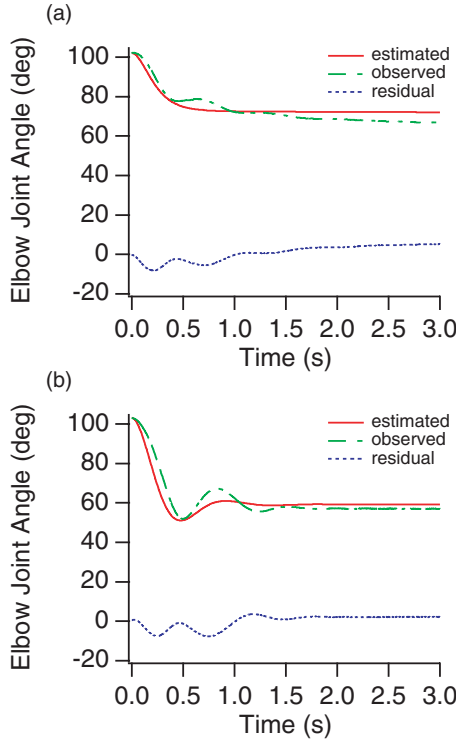


Fig. 3. Example of a step-like response of the elbow joint angle of hemiplegic subjects. (a) Subject E. (b) Subject F.

estimated angle also was inconsistent with the observed one.

B. Model B

Fig. 4 shows the elbow joint angle estimated with Model B. The third panel (c) represents the elbow joint angle estimated from the same data shown in Fig. 3 (a). The elbow joint angles were estimated better with Model B than with Model A.

The bottom panel (d) shows the angle estimated from the same data shown in Fig. 3 (b). The estimated angles provided better agreement with the observed ones than those estimated with Model A.

The top panel (a) is the angle of Subject C. The second panel (b) is the angle of Subject D. They were also estimated well with Model B.

C. Normalized Elastic Torque

Fig. 5 represents the normalized elastic torque. The top panel (a) shows that of Subject A. The black solid line denotes the normalized elastic torque calculated by (6). The red dotted line denotes the torque approximated by (7). The elastic torque was approximated well.

The normalized elastic torque declined gradually and converged with the total torque. The time constant was 1.76 s. The frequency was 0.89 Hz.

The bottom plane (b) shows the normalized elastic torque of Subject E. The normalized elastic torque decreased more rapidly than that of Subject A did. The time constant was 0.40 s. The frequency, 1.67 Hz, was higher than that of Subject A.

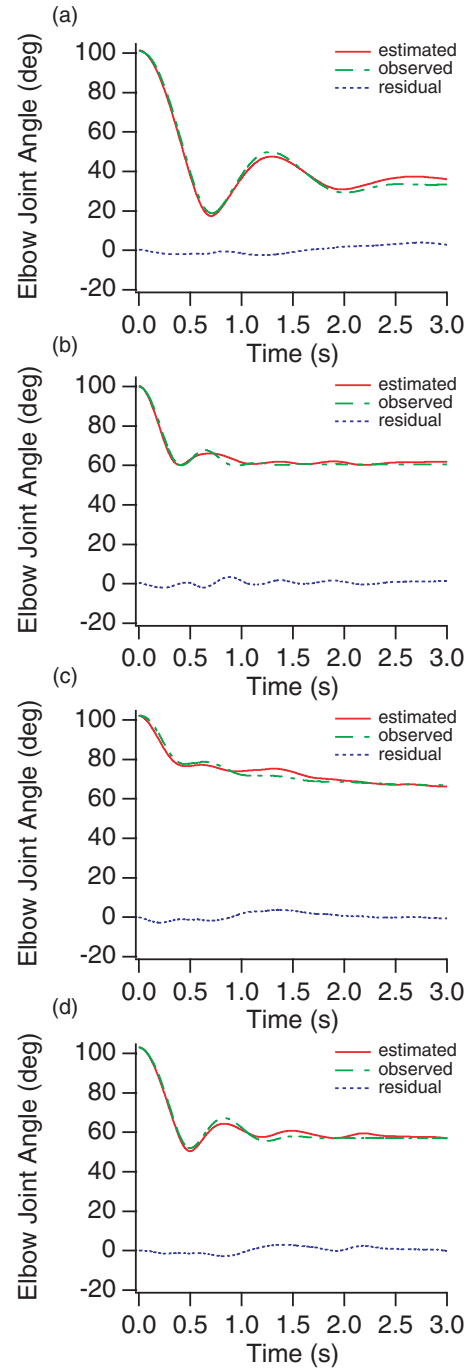


Fig. 4. Step-like response of the elbow joint angle of hemiplegic patients. (a) subject C (1.0 kgf), (b) subject D (2.5 kgf), (c) subject E (1.0 kgf), (d) subject F (1.0 kgf).

Table 2 represents the time constant τ and the natural frequency f_0 of the normalized elastic torque. The time constants of the healthy subjects were larger than those of hemiplegic subjects. Moreover, the higher Ashworth scale was, the shorter time constant was. Table 3 shows the results of the Student's t -test of time constants. There were significant differences between the healthy subjects and the hemiplegic subjects. There were no significant differences between the healthy subjects A and B or between

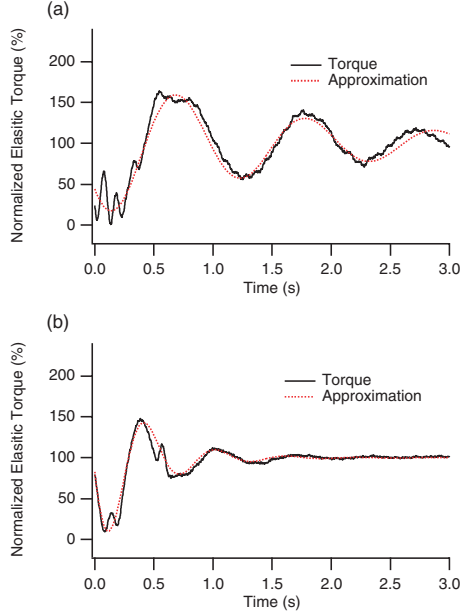


Fig. 5. Normalized elastic torque of a healthy subject and a hemiplegic patient. (a) subject A (1.0 kgf), (b) subject E (1.0 kgf)

same grade subjects D and E.

The natural frequency of the healthy subjects had a tendency to be lower than those of the hemiplegic subjects, although this tendency was not clear. Table 4 represents the Student's t -test of the natural frequency. There was a significant difference between the healthy subjects A and B.

Table 2. Time constant of normalized elastic torque.

Subject	AS	Time const. (s)	Freq. (Hz)
A	—	1.70 ± 0.49	0.88 ± 0.06
B	—	1.89 ± 0.28	0.63 ± 0.03
C	2	1.10 ± 0.17	0.71 ± 0.12
D	3 or 4	0.44 ± 0.10	1.71 ± 0.12
E	3 or 4	0.45 ± 0.06	1.62 ± 0.05
F	3	0.81 ± 0.13	1.40 ± 0.14

Table 3. Student's t -test of time constant.

	A	B	C	D	E
A	—				
B	ND	—			
C	< 0.01	< 0.001	—		
D	< 0.001	< 0.001	< 0.001	—	
E	< 0.001	< 0.001	< 0.001	ND	—
F	< 0.005	< 0.001	< 0.001	< 0.001	< 0.001

IV. DISCUSSION

A. Elastic Coefficient

The step-like response of the elbow joint was approximated well with Model B. However, it was impossible to

Table 4. Student's t -test of natural frequency.

	A	B	C	D	E
A	—				
B	< 0.001	—			
C	< 0.05	< 0.05	—		
D	< 0.001	< 0.001	< 0.001	—	
E	< 0.001	< 0.001	< 0.001	< 0.05	—
F	< 0.005	< 0.001	< 0.001	< 0.001	< 0.005

separate the IEMG-dependent elasticity (a_1 , a_2 and a_3) and the independent ones (k_0 and k_1) for evaluation of elastic characteristics. IEMGs of the healthy subjects had almost constant values close to zero, indicating that the constant elasticity was distributed to both k_0 and IEMG-dependent elasticity a_1 , a_2 and a_3 with an unknown ratio.

IEMGs of the hemiplegic subjects increased during extension of their forearms. However, the amplitude of some IEMGs did not show large changes. The activation pattern of each muscles related to those of other muscles, the elbow joint angle, angular velocity and so on. It is hard to assume the linear independence of the muscle activation. Therefore, the elastic coefficients were not evaluated directly in this study.

B. Time Constant of Normalized Elastic Torque

Considering a short time period, we can presume that the elastic coefficient k is a constant value. On this presumption, (3) is a linear second order equation and one of the solution $\theta(t)$ is (8):

$$\theta(t) = z \left\{ 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\sqrt{1-\zeta^2}\omega_0 t + \phi) \right\}$$

$$\tan \phi = \frac{\sqrt{1-\zeta^2}}{\zeta} \quad (8)$$

$$\omega_0 = \sqrt{\frac{k}{I'}}, \quad \zeta = \frac{\eta}{2\sqrt{kI'}}, \quad z = \frac{mgr}{k}$$

The normalized elastic torque is $k\theta(t)/mgr = \theta(t)/z$, which is similar to (7). The time constant τ is $1/\zeta\omega_0 = 2I'/\eta$, which is independent of k . However, we approximated the normalized elastic torque with the damped sinusoid instead of directly calculating $2I'/\eta$ because k is not constant.

V. CONCLUSION

We proposed a new technique for modeling the step-like response of hemiplegic subjects. The response was approximated with a model whose elastic coefficients depend on the elbow joint angle and muscle activity. The normalized elastic torque was estimated and approximated with a damped sinusoid. The time constant shows significant differences between the healthy subjects and the hemiplegic subjects and among the different grades of subjects. The results suggest that the time constant can be a quantitative index of spasticity.