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A Procedure for Calibrating Magnetic Sensors

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ARL-MR-524 JANUARY 2002

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Aberdeen Proving Ground, MD 21005-5066

ARL-MR-524 January 2002

A Procedure for Calibrating Magnetic Sensors

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Abstract

The calibration of magnetometers is of interest to the U.S. Army Research Laboratory because these sensors are being investigated as components of navigation and instrumentation packages. Calibration is the process of estimating the coefficients of an analytical model of sensor output. In many cases, it is possible to represent the output of a sensor analytically. This representation embodies our knowledge of the sensor. For this process to proceed, an adequate model of the sensor and an evaluation procedure need to be implemented. These data sets indicate different scale factors for the same magnetometer. The fits for the peak-to-peak and average values do not agree. This should be investigated when time and money permit. The establishment of analytical formulae for measurement is central to the calibration process. These formulae, coupled with a least squares estimation, provide the framework for calibration; other methods should explain their omission of least squares.

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A PROCEDURE FOR CALIBRATING MAGNETIC SENSORS

1. Introduction

The calibration of magnetometers is of interest to the U.S. Army Research Laboratory (ARL) because these sensors are being investigated as components of navigation and instrumentation packages. Calibration is the process of estimating the coefficients of an analytic model of sensor output.

In many cases, it is possible to represent the output of a sensor analytically. This representation embodies our knowledge of the sensor. For this process to proceed, an adequate model of the sensor and an evaluation procedure need to be implemented.

2. Background

The "MAGSONDE" (MAGnetic SONDE), which employs fixed magnetic sensor(s) on a rotating body, estimates that body's orientation with respect to a stationary magnetic field. Harkins and Hepner **[1]** discuss the theory associated with this device and present models for the output of each magnetometer. Harkins and Hepner use zero crossings of the magnetometers to find the direction to the magnetic field. The first two formulae are taken directly from their report.

Sensor 1 is aligned orthogonally to the spin axis. The measurement from Sensor 1 is

$$M_{s1} = |M| \sin \sigma_m \sin \varphi_s , \tag{1}$$

in which σ_m is the angle between the spin axis and the magnetic field, and φ_s is the roll angle (about the spin axis). If Sensor 2 is at an angle of λ to the spin axis in the same plane as Sensor 1 and the spin axis, then the measurement from Sensor 2 is

$$M_{s2} = \cos \lambda |M| \cos \sigma_m + \sin \lambda |M| \sin \sigma_m \sin \varphi_s \; . \tag{2}$$

For each of these measurements, if the magnitude of the magnetic field is known, then each measurement can be scaled to be the inner product of the normalized magnetometer axis and the magnetic field direction.

The simplest models of output need to include bias terms and scale factors as part of the model. The bias term is the expected output when the input to the sensor is zero. The scale factor can be complicated but in the simplest representation is a scalar factor of the output. An enhancement of Equation 1 to include these additional variables is

$$M_{s1} = Sf\,|M|\sin\sigma_m \sin\varphi_s + b \tag{3}$$

in which Sf is the scale factor and b is the bias. The output for Sensor 2 can be modeled more realistically in a similar fashion; it becomes

$$M_{s2} = Sf\,(\cos\lambda|M|\cos\sigma_m + \sin\lambda|M|\sin\sigma_m \sin\varphi_s) + b \tag{4}$$

We can refine this equation by considering different scale factors for each term. If this is done, the calibration equation is

$$M_{s2} = Sf_1 \cos\lambda|M|\cos\sigma_m + Sf_2 \sin\lambda|M|\sin\sigma_m \sin\varphi_s + b \tag{5}$$

Equations 3, 4, and 5 are all candidates for use in calibration. Residual error will be used to indicate how well the estimation procedure fits the data. A large residual sum of squares will indicate either a poor model or poor technique in the collection of data. Information about using least squares for calibration is included in Thompson **[2]**.

3. Data

The data were collected by Davis and Harkins of ARL in March 2001 **[3]**. Two MAGSONDES were used; since each contained two magnetometers, four sets of data were generated. The MAGSONDE was attached to a transit, and then with the MAGSONDE spinning, the transit was adjusted until the output from the orthogonal sensor was invariant with respect to spin. At this point, the spin axis was assumed to be aligned with the magnetic field vector. Next, the transit was rotated from this position, and data were collected with the spin axis at a fixed angle from the aligned position. Knowledge of the angle between the spin axis and magnetic field vector, coupled with knowledge of the strength of the magnetic field, allows calibration with Equations 3 through 5 and least squares.

Under each set, the average output and the peak-to-peak variation were calculated as dependant variables measured in bits. Equations 3 and 5 were used as models for the data. The independent variable was the angle between the spin axis and the magnetic field. With these measurements and the formulae for the output of the signal of the MAGSONDE calibration, I used least squares to find the scale factor and bias. The average output was calculated as the sum of the magnetometer output over a spin cycle. I found the peak-to-peak output by subtracting the smallest output from the largest (over a spin cycle) and then dividing by 2. The peak-to-peak output is then the measured amplitude of the sinusoidal component. When the calibration formulae are used, the magnetic roll angle can be ignored because it is averaged or its effects are reported as amplitude.

4. Analysis

4.1 Data Set 1

The first data set corresponds to the sensor placed orthogonally to the spin axis. The formula for this sensor shows that the average output corresponds to its zero point. Since the signal is a purely sinusoidal over an integral number of cycles, its sum will be 0; thus, the output is attributable to the bias term. Table 1 presents the angle of the spin axis to the magnetic field, the average output over a spin cycle, and the peak-to-peak variation or amplitude.

Table 1. Data Set 1

1 Angle of the Spin Axis to the Magnetic Field	2 Average Output Over a Spin Cycle	3 Peak-to-Peak Variation or Amplitude
180.0	83	0
0.0	84	0
343.7	83.5	4.5
17.4	83.5	3.5
238.8	84	11
256.6	83	11
190.1	83	2
196.8	83.5	3.5
214.1	83.5	6.5
14.6	84	3

The average value for Column 2 is 83.5; this corresponds to the bias for this sensor. Peak-to-peak variation is always a positive value. One can find the scale factor by using the absolute value of the sine of Column 1 as the independent variable and Column 3 as the dependent variable. A simple regression yields $y = 11.83x + 0.14$ as the line of best fit. The bias term for this equation, 0.14, is not easily explained. This value may be small enough to attribute to error in the calculation of peak-to-peak data; however, the data collection procedure needs to be observed in detail to explain why the term is not zero. The slope of the line is a combination of the scale factor and the magnetic field strength. The magnetic field strength is 0.46 so a simple division yields 25.72 as the scale factor. The residuals are plotted in Figure 1. Observation 3 can be identified as an outlier statistically or visually.

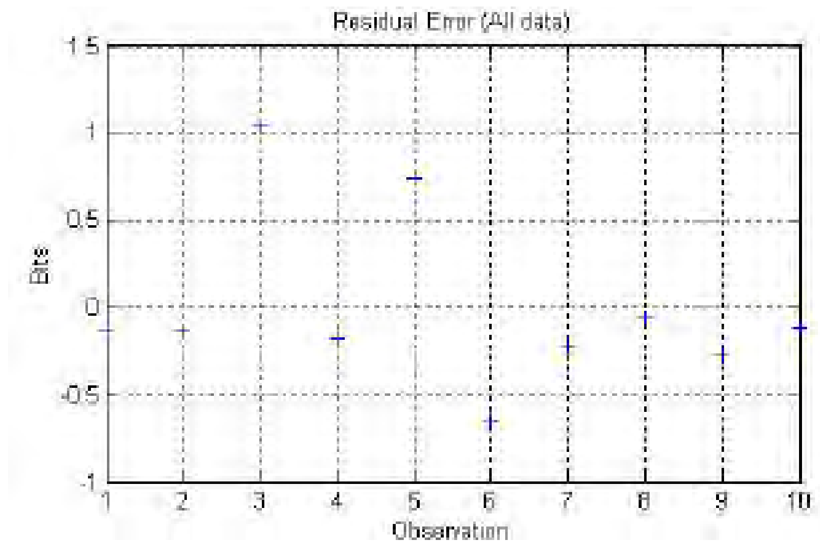

Figure 1. Residuals for Data Set 1.

Removing this point and re-running the regression yields y = 11.94x - 0.01 as the line of best fit. Removal of the outlier reduces the bias term to an acceptable level addressing the previously mentioned concern. This yields a scale factor of 25.96. The residuals associated with this model are shown in Figure 2.

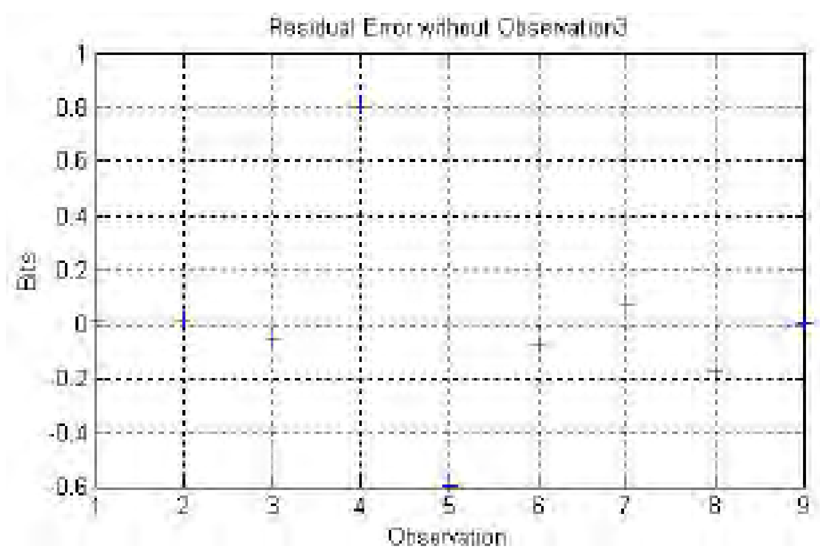

Figure 2. Residual Errors Without Observation 3.

In this figure, the current Observation 4 can be considered an outlier. Removing this observation and re-running the regression yields y = 11.33 + 0.08 as the line of best fit. The residual errors are shown in Figure 3.

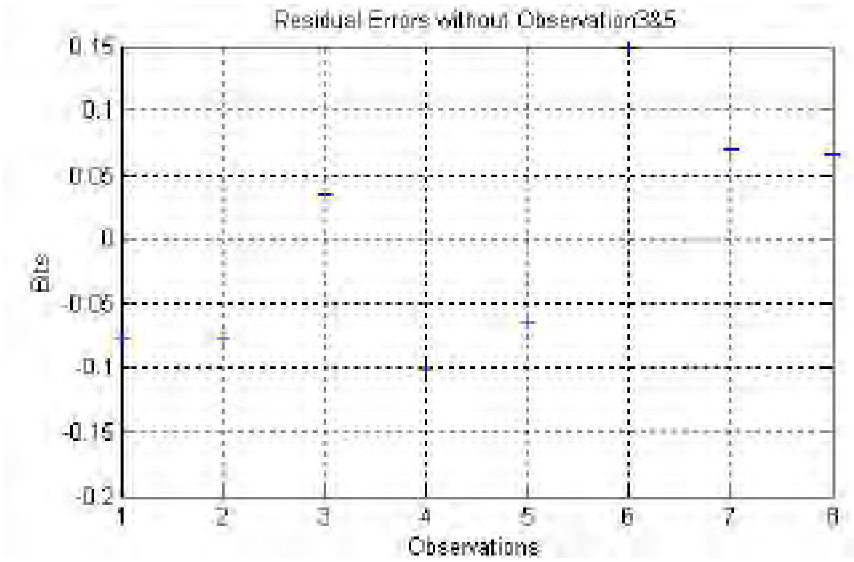

Figure 3. Residual Errors Without Observations 3 and 5.

Note that the magnitude of the residuals has decreased dramatically. This last model yields a scale factor of 24.63. The range of the dependent variable is 0 to 11 bits. The limited range could make quantization error a factor. Figure 4 displays the remaining data and the line of best fit. It appears that the line of best fit is a good model for the observations. With more data, it would be possible to compare a model based on positive sine values, with a model for negative sine values. The plots of the residuals are not shown for the next three data sets.

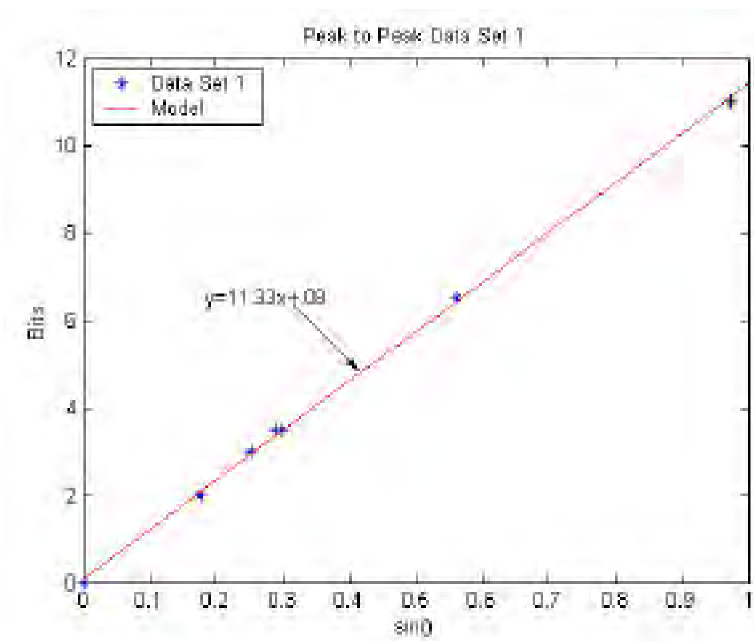

Figure 4. Line of Best Fit With Outliers Removed.

4.2 Data Set 2

For the second data set, the sensor is in the plane between the spin axis and the orthogonal sensor at an angle of 75 degrees to the spin axis. The average value for this data set depends on the cosine of the angle between the magnetic field and the spin axis; the peak-to-peak variation can be analyzed as in the previous case. Table 2 shows the data.

Table 2. Data Set 2

1 Angle of the Spin Axis to the Magnetic Field	2 Average Output Over a Spin Cycle	3 Peak-to-Peak Variation or Amplitude
180.0	79	0
0.0	99	0
180.0	82	0
0.0	104	0
343.7	98	10
17.4	98	9
238.8	84.5	26.5
256.6	86	30
190.1	79	6
196.8	79.5	9.5
214.1	80.5	17.5
14.6	98	7

Two points (the third and fourth) were identified as outliers. These were removed from the data set, and a line was fit to the data. The first data set used the cosine of the angle as the independent variable and the average value as the dependent variable. The line of best fit is $y = 9.76x + 88.77$. This indicates that the bias is 88.77. The scale factor is the slope divided by the signal strength and the cosine of the offset from the spin axis or 81.98. Figure 5 shows the data and the model.

With the peak-to-peak data as the dependent variable and the absolute value of the sine of the angle as the independent variable, a best fit line was $y = 30.87x + 0.14$. The scale factor in this case is the slope divided by the magnitude of the signal and the sine of the angle to the spin axis or 69.48. This contrasts with the previous result of 81.98. One possible explanation may be that the sensor does not react to fluctuations quickly enough, or the band pass of the magnetometer was too low to accurately acquire the peak-to-peak information. This information is illustrated in Figure 6.

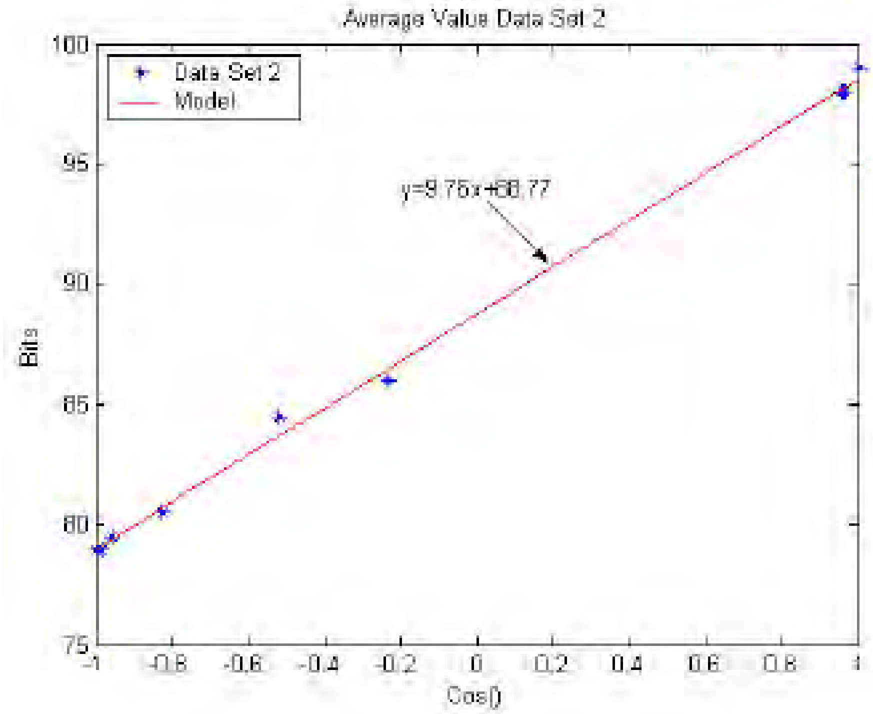

Figure 5. Line of Best Fit for Average Output.

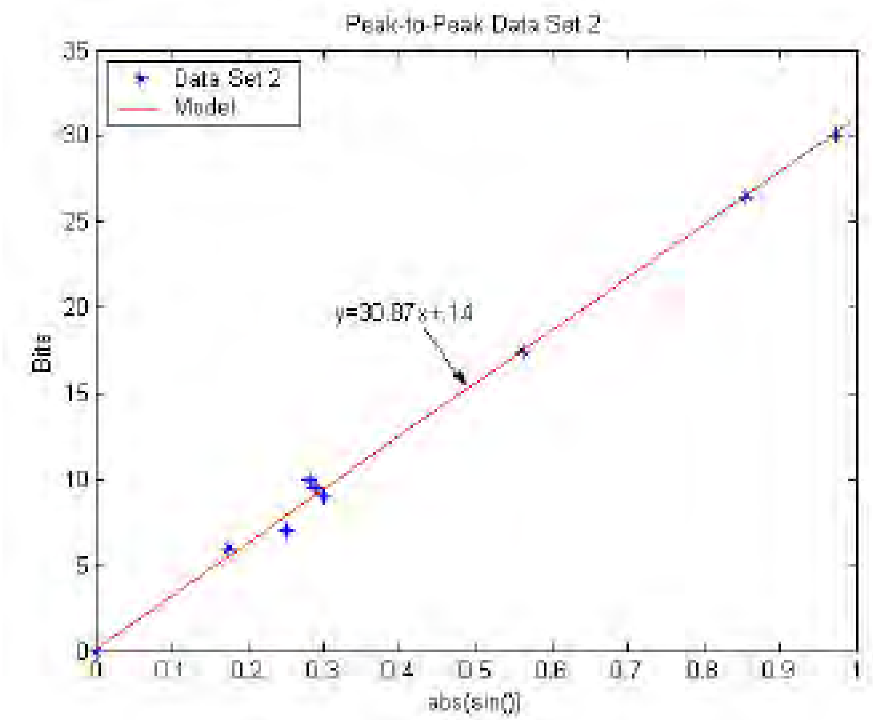

Figure 6. Line of Best Fit for Peak to Peak.

4.3 Data Set 3

As with the first set, the third set of data is from a sensor orthogonal to the spin axis; thus, there is no offset term. By finding the mean of the average output, one can find the bias term: 124.67. The data are shown in Table 3.

Table 3. Data Set 3

1 Angle of the Spin Axis to the Magnetic Field	2 Average Output Over a Spin Cycle	3 Peak-to-Peak Variation or Amplitude
180.0	125	0
0.0	124	0
345.9	126	3
14.6	124.5	2.5
238.8	124.5	8.5
256.6	124	10
190.1	125	2
196.8	124.5	3.5
214.1	124.5	5.5

To find the scale factor, I used the third column of peak-to-peak data set with the absolute value of the sine of the first column. The line of best fit is y = 9.95x + 0.19. The resulting scale factor is 22.39. Figure 7 shows the fit of the model to the data.

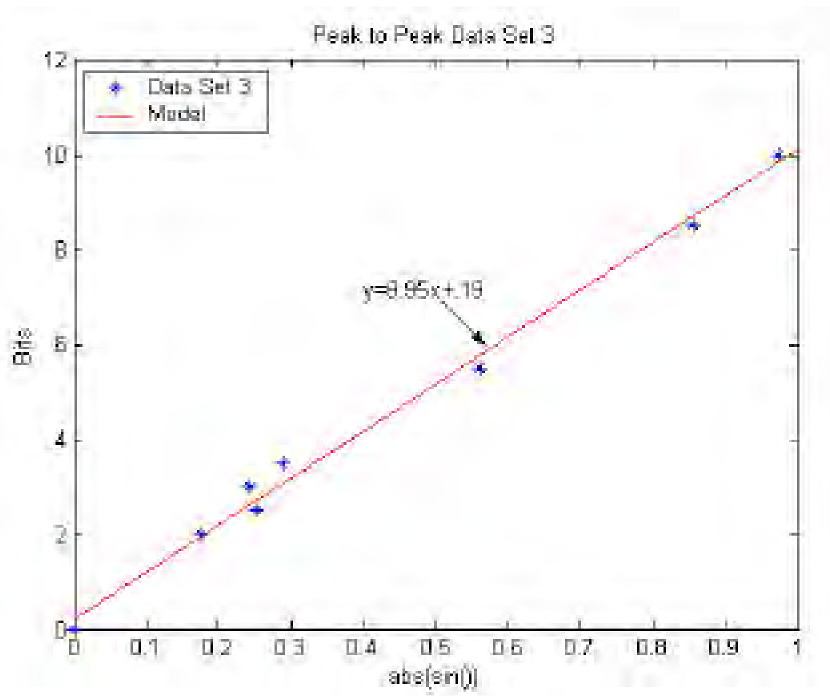

Figure 7. Line of Best Fit for Peak to Peak, Data Set 3.

4.4 Data Set 4

The fourth set of data is analyzed in a similar manner as the second set. The data are shown in Table 4.

Table 4. Data Set 4

1 Angle of the Spin Axis to the Magnetic Field	2 Average Output Over a Spin Cycle	3 Peak-to-Peak Variation or Amplitude
180.0	85	0
0.0	65	0
345.9	68	7
14.6	64.5	6.5
238.8	79	25
256.6	76.5	28.5
190.1	83.5	5.5
196.8	83.5	9.5
214.1	82.5	16.5

Observations 1 and 3 were considered to be outliers. The line fitted to the average values is y = -9.63x + 74.22 so the bias is 74.22 bits and the scale factor can be found from the slope as –80.89. Figure 8 shows the model for the average reading.

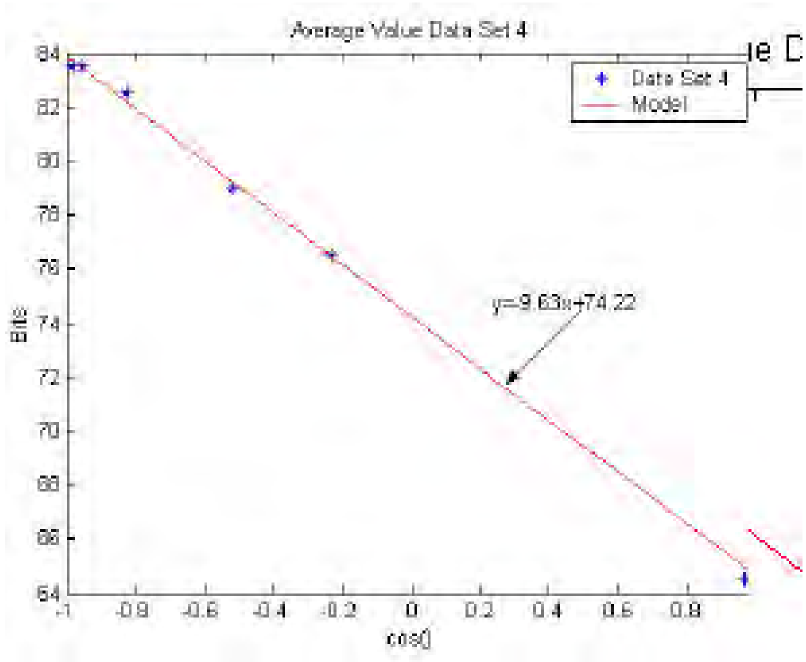

Figure 8. Line of Best Fit for Average Output, Data Set 4.

The peak-to-peak data are modeled by the line y = 29.25x + 0.06. This indicates a scale factor of 65.83. The model and data are shown in Figure 9.

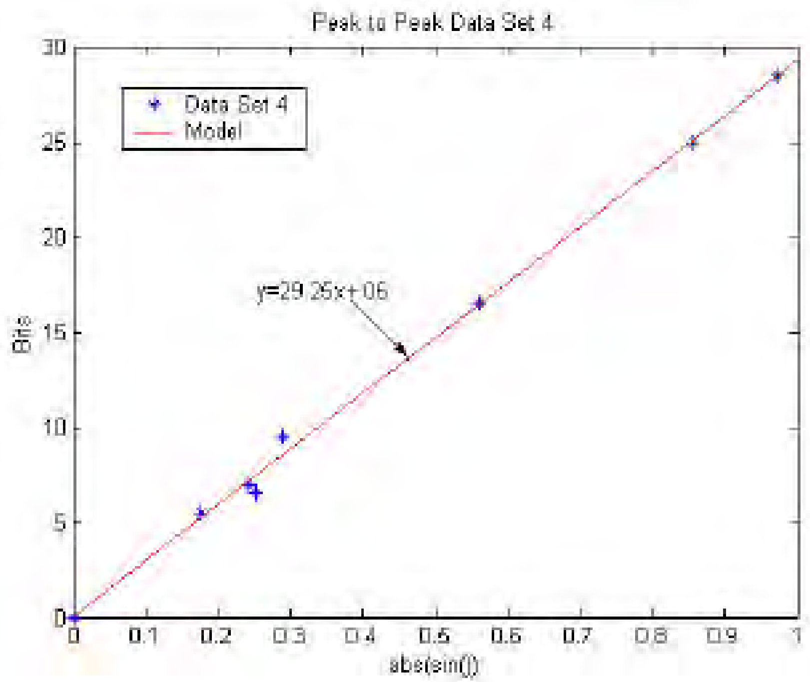

Figure 9. Line of Best Fit for Peak to Peak, Data Set 4.

5. Discussion

These data sets indicate different scale factors for the same magnetometer. The fits for the peak-to-peak values and the average values do not agree. This needs to be investigated. An experiment can be performed to determine if the peak-to-peak data are affected by frequency. When data are obtained from the magnetometer at different frequencies, they should be usable to determine if frequency is a factor.

When the observations are chosen to be in two equal groups at the extremes of the interval of interest, the estimates of slope and interception have the lowest possible standard deviation. It is usually prudent to check for curvature, and points on the interior of the region of interest are necessary to accomplish this goal. Balancing these two objectives will increase the effectiveness of a calibration.

The establishment of analytical formulae for measurement is central to the calibration process. These formulae, coupled with a least squares estimation, provide the framework for calibration; other methods should explain their omission of least squares.

References

1. Harkins, T.E., and D.J. Hepner, “MAGSONDE: A Device for Making Angular Measurements on Spinning Projectiles Via Magnetic Sensors,” ARL-TR-2310, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, December 2000.

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3. Davis, B.S., and T.E. Harkins, unpublished data, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, 2001.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
January 2002

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
A Procedure for Calibrating Magnetic Sensors

5. FUNDING NUMBERS
PR: AH80

6. AUTHOR(S)
Thompson, A.A. (ARL)

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
Weapons & Materials Research Directorate
Aberdeen Proving Ground, MD 21005-5066

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Laboratory
Weapons & Materials Research Directorate
Aberdeen Proving Ground, MD 21005-5066

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARL-MR-524

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

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14. SUBJECT TERMS
calibration
magnetometer

15. NUMBER OF PAGES
20

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT
Unclassified

20. LIMITATION OF ABSTRACT