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14. ABSTRACT

This is the final report for research supported under AFOSR Grant F49620-98-1-0026 during the period 01 October 1997 through 31 August 2001. The research focused on broadening the class of solvable robust control problems and on developing a firm information theoretic foundation for incorporating the real-time effects of evolving experimental data. Robust control theory concerns the design of control systems capable of robustly maintaining performance to within prescribed tolerances in the face of large-but-bounded modeling uncertainties and nonlinearities. Significant advances were achieved in nonlinear robustness analysis for systems having repeated monotone nonlinearities and in reliable data-driven adaptive control synthesis techniques based on unfalsified control theory. The theory enables design of nonlinear feedback control systems that learn, discover and evolve in order to robustly compensate for battle damage, equipment failures and other changing circumstances.

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Final Report:
ROBUST CONTROL FEEDBACK AND LEARNING
AFOSR Grant F49620-98-1-0026

October 1, 1997 – August 31, 2001

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November 30, 2002

Contents

1 Objectives

Feedback control systems for aerospace applications must maintain precise control despite uncertain operating conditions and unanticipated circumstances such as battle damage. These systems must be designed to perform robustly, despite uncertain design models and difficult to analyze nonlinear effects. They must also be capable of learning and adapting when accumulating data indicates that previous models must be abandoned and that existing control strategies must be changed. This three-year research program centered on developing and improving the engineering techniques for robust control design, with particular emphasis on the need for *data-driven* design methods well-suited to situations in which available mathematical models are poor or unreliable. The research results are expected to facilitate the design of control systems that learn, discover and evolve in order to compensate for the effects of battle damage, equipment failures and other changing circumstances. Potential applications include aircraft stability augmentation systems, highly maneuverable aircraft design, missile guidance systems, and precision pointing and tracking systems.

2 Executive Summary

Thirty-nine publications supported under AFOSR Grant F49620-98-1-0026 have either appeared, been submitted or are currently pending publication [1]–[39]. The subject matter of these AFOSR supported publications may be roughly grouped as follows:

- Robust LMI/BMI/IQC Multivariable Control Theory
  - Controller Synthesis [2, 15]
  - Uncertain Time-delays [9, 10, 21, 25, 29, 33, 35, 39]
  - Nonlinear Systems [4, 12, 13, 25, 30, 31, 34, 38]
- Unfalsified-Control, Learning, Adaptation & Controller Identification [5, 8, 11, 16, 20, 22, 23, 26, 27, 32, 36, 37]
- Control Design Applications [1, 11, 20, 21, 32]
- Editorial, Review & Survey Articles [3, 7, 6, 14, 17, 19, 24, 28]

Generally, the theoretical developments embodied in the above listed recent AFOSR-supported publications have been accompanied by software implementation and test case studies. The LMI /BMI/IQC theory plays a critical role in extending and generalizing the H_∞ robust controller design theory that has already proven its value in aircraft flight control applications [40, 41, 42]. It allows greater flexibility in handling structured uncertainties, controller complexity constraints and gain-scheduling requirements. The generalized Popov multiplier robustness analysis and synthesis techniques [43, 44, 45] developed in have led directly to improved approaches for the design of active vibration damping systems for flexible space structures [1]. The effective and rapid transition from theory to practice has been facilitated by my on-going non-AFOSR-supported involvement with Dr. R. Y. Chiang in upgrading the MATLAB ROBUST CONTROL TOOLBOX, a robust control design software product published by The MathWorks and in widespread use by government, university and aerospace engineering company labs [46].

2.1 Research Highlights

Nonlinear Robustness Analysis

A major breakthrough has been the development of optimally precise multiplier methods for analyzing robustness of systems having repeated monotone nonlinearities, such as saturations or relays [34, 13]. We have derived the largest class of bounded linear time-invariant operators that preserve positivity of repeated monotone nonlinearities. It reduces conservativeness of IQC stability analysis of systems having repeated monotone nonlinearities to the minimum possible. Utility of this characterization stems from the fact that such nonlinearities are typically encountered in stability of anti-reset windup schemes among other applications. We have also derived the time varying counterpart of this class of operators.

These results dovetail with earlier results giving a multiplier interpretation of robustness theory in general, including its newly popular LMI/BMI/IQC formulations. Our results enable accurate reliable analysis of stability robustness of systems like missiles, aircraft and robots that typically have control actuators or other components with position or rate limits (i.e.saturation) or on-off (i.e., relay) devices. These results are surprising because (1) they are a quantum leap forward in a once much-studied field that had lain dormant for most of the past 30 years and (2) they will have an very broad practical impact because will give significantly less conservative sufficient conditions for stability for the most commonly encountered sorts of nonlinearities. The results are the best possible for repeated monotone nonlinearities in the sense that there is no broader class of multiplier-type (or IQC) robustness conditions that can do better without further information about he nonlinearities.

Data-Driven Unfalsified Control

Perhaps the most significant new results pertain to our recent development of data-driven methods for robust and adaptive control design based on our *unfalsified control* theory [5, 8, 11, 16, 20, 22, 23, 26, 27, 32, 36, 37].Unfalsified control theory facilitates the representation of adaptive processes of control law discovery from *evolving* information flows and noisy data. In this paper, the theory of unfalsified adaptive control is examined from the behavioral perspective of Willems. An abstract, but parsimonious, min-max optimization problem formulation is developed that describes and unifies direct adaptive control, learning theory and system identification problems in a common behavioral setting based on the concept of controller/model unfalsification. Thus, adaptive control is seen to be firmly and directly linked to, and to conceptually unified with, the growing body of knowledge on behavioral approaches to model validation and unfalsified system identification. The results elucidate and underscore the fertile conceptual links that exist between adaptive control theory and the rich theory of system identification.

This unfalsified control theory is a precise, data-driven approach to adaptive controller synthesis based on evolving measurements of plant response. Unlike traditional control design methods where controller choices generally depend heavily of prior knowledge of plant models and error-bounds, the unfalsified control theory gives primary emphasis to the precise analysis of the implications of evolving experimental measurement data. A plant model, though useful, is not essential and common adaptive control pitfalls associated with unrealistic assumptions and excessive reliance on prior knowledge are circumvented.

2.2 Personnel

Personnel supported by AFOSR Grant F49620-98-1-0026 included the Michael G. Safonov (PI), and PhD students V. Kulkarni, M. Jun, S. Bohacek, A. Grishencko. The research supported by the grant resulted in two PhD theses, including V. Kulkarni [38] and M. Jun [39]. Three other fellowship students worked closely with the PI on the research, including H. Meng, P. Brugarolas and R. Mancera whose PhD theses expected to be completed in 2002.

3 Major Accomplishments

3.1 All Multipliers for Repeated Monotone Nonlinearities

Introduction

In stability analysis, a given system S is often decomposed into two interconnected subsystems — a linear time invariant subsystem H in the feedforward path and an otherwise subsystem Δ in the feedback path. More often than not, a repeated monotone nonlinearity, say N, is encountered as the subsystem Δ (see, e.g., [47], [48], [49], [50], [51] and references therein). A key step in multiplier based stability analysis of such systems is to characterize a class of *multipliers*, i.e. a class of *convolution operators*, such that every element M of it *preserves positivity* of N in the sense that $N \geq 0$ implies $M^*N \geq 0$. Stability of the system is then deduced if there exists *at least one* such multiplier M such that $MH > 0$ and if, in addition, H has a finite gain (see [52, Theorem 2], [47], [4] and references therein for a detailed relevant discussion). Effectively, positivity preserving multipliers give an integral quadratic constraint (IQC) characterization of N (see [53] for IQC's — theory and applications). The *larger* the class of the positivity preserving multipliers the *better* it is, for the sharper is its IQC characterization and the lesser is the conservativeness in the stability analysis.

The best available class of positivity preserving multipliers so far for repeated SISO monotone nonlinearities is the one recently derived by D'Amato et al. [47]. Whether it is the best *possible* as well has been unclear. It turns out that they have stipulated an unnecessary condition on the multipliers. Identifying and relaxing this condition, in this note we have obtained a larger — indeed, the largest possible — class of positivity preserving multipliers for such nonlinearities. Specifically, we have characterized the largest possible classes of both linear time-invariant as well as *linear time varying* operators that preserve positivity of such nonlinearities. Essentially, our results generalize the non-repeated monotone nonlinearity results of Willems [54, Ch. 3] to the case of repeated monotone nonlinearities.

Saturation nonlinearities, dead zone nonlinearities, sigmoidal nonlinearities are some of the many examples of monotone nonlinearities. When input-output channels of Δ, or a sub block of it, feature the *same* such nonlinearity, an instance of repeated monotone nonlinearity is on hand. Computation of stability margin of anti windup schemes is one of the engineering applications in which repeated monotone nonlinearities appear (see, e.g., [47, 49, 55]). Reduction of conservatism in such stability margin estimates is thus a motivating application of this paper.

The paper is organized as follows. In Section 3.1, the necessary terminology is introduced and the problems are formally posed in Section 3.1. Background results are in Section 3.1. Our main results are presented in Section 3.1 and discussed in Section 3.1. The paper is concluded in Section 3.1.

Preliminaries

The notation used is summarized in Table 1. Capital letter symbols, e.g. F and G, denote operators whereas small letters, e.g. x and y, denote real signals which may possibly be vector-valued or matrix-valued. The vector space ℓ_2^n is generally referred to as ℓ_2. $\mathbb{Z}$ denotes the set of all integers. $(\cdot)^*$ denotes conjugate transpose of a vector or matrix $(\cdot)$; $(\cdot)^T$ denotes its transpose. A sequence $\{x(k)\}_{k=-\infty}^{\infty}$ is described simply as $\{x\}$. Statements of the form "A related to B" and "C related to D" are abbreviated as "A (C) related to B (D)". $D \in \mathcal{R}^{n\times n}$ is said to be *Hurwitz* if each of its eigenvalues has a strictly negative real part. Other terms not defined here may be found in [56] and [53].

Table 1: Notation

Symbol	Meaning
$\mathcal{R}$ ($\mathbb{C}$)	Set of all real (complex) numbers.
$\mathbb{Z}$	Set of all integers.
herm(m)	$= \frac{1}{2}(m + m^*)$, for $m \in \mathbb{C}^{n\times n}$ or $\mathcal{R}^{n\times n}$.
skew(m)	$= \frac{1}{2}(m - m^*)$, for $m \in \mathbb{C}^{n\times n}$ or $\mathcal{R}^{n\times n}$.
$\langle x, y\rangle$	$= \begin{cases} \sum_{k=-\infty}^{\infty} y(k)^T x(k) & \text{for discrete time signals;} \\ \int_{-\infty}^{\infty} y(t)^T x(t)\, dt & \text{for continuous time signals.} \end{cases}$
$\|\|x\|\|$	$= \sqrt{\langle x, x\rangle}$.
$\|\|x\|\|_1$	$= \begin{cases} \sum_{k=-\infty}^{\infty} \|x(k)\| & \text{if } x \text{ discrete time;} \\ \int_{-\infty}^{\infty} \|x(t)\|\, dt & \text{if } x \text{ continuous time.} \end{cases}$
ℓ_2	Space of discrete time signals x for which $\|\|x\|\|$ exists.
$\mathcal{L}_2$	Space of continuous time signals x for which $\|\|x\|\|$ exists.
$\widehat{x}(\cdot)$	Fourier transform of x, either discrete or continuous.
δ	$= \begin{cases} \text{Kronecker } \delta(k), & \text{if discrete time;} \\ \text{Dirac } \delta(t), & \text{if continuous time.} \end{cases}$
$\lambda_i(H)$	i-th eigenvalue of matrix H.
$\underline{\lambda}(D)$	Least eigenvalue of a Hermitian matrix D.
MIMO	Multi-Input-Multi-Output.
SISO	Single-Input-Single-Output.

Definition 1 [operator: positive, bounded]
An operator F mapping a space X into itself is said to be positive *if $\langle x, Fx\rangle \geq 0\ \forall x \in X$. A set S is said to be* bounded *if there exists a $\gamma \in \mathcal{R}^+$ such that $\|y\| < \gamma$ for all $y \in S$. An operator $F : X \to Y$ is said to be* bounded *if the image under F of every bounded subset of X is a bounded subset of Y.* □

Definition 2 [sequences: similarly ordered, unbiased]
The sequences $\{x\}$ and $\{y\}$ of real scalars are said to be similarly ordered *if $x(k) < x(l)$ implies $y(k) \leq y(l)$ for all $k, l \in \mathbb{Z}$. They are said to be* unbiased *if $x(k)y(k) \geq 0\ \forall k$. They are said to be* similarly ordered and symmetric *if they are unbiased and, in addition, the sequences $\{|x|\}$ and $\{|y|\}$ are similarly ordered.* □

Definition 3 [associated matrix, kernel]
Given a bounded possibly time varying linear operator $M : \ell_2^p \to \ell_2^p$, $y = Mx$ is given as

$$y(k) \doteq \sum_{l=-\infty}^{\infty} \overline{m}_{k,l}\, x(l) \quad \forall k \in \mathbb{Z},$$

where $\overline{m}_{k,l} \in \mathcal{R}^{p\times p}$ $\forall k, l$; the associated matrix $\widetilde{M}$ *of M is defined as*

$$\widetilde{M} \doteq \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \overline{m}_{-1,-1} & \overline{m}_{-1,0} & \overline{m}_{-1,1} & \overline{m}_{-1,2} & \ddots & \ddots \\ \ddots & \overline{m}_{0,-1} & \overline{m}_{0,0} & \overline{m}_{0,1} & \overline{m}_{0,2} & \ddots & \ddots \\ \ddots & \overline{m}_{1,-1} & \overline{m}_{1,0} & \overline{m}_{1,1} & \overline{m}_{1,2} & \overline{m}_{1,3} & \ddots \\ \ddots & \overline{m}_{2,-1} & \overline{m}_{2,0} & \overline{m}_{2,1} & \overline{m}_{2,2} & \overline{m}_{2,3} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}.$$

The symbol m_{ij}, $i, j \in \mathbb{Z}$ denotes the (i,j)-th scalar element of the matrix $\widetilde{M}$; for example, m_{00} denotes the upper left entry in the $p \times p$ matrix $\overline{m}_{0,0}$ and $m_{-p,0}$ denotes the upper left entry in the $p \times p$ matrix $\overline{m}_{-1,0}$. If $\overline{m}_{k,l} = \overline{m}_{k+n,l+n}$ $\forall k, l, n \in \mathbb{Z}$ then $\widetilde{M}$ is said to be block Toeplitz *and M is said to be a* time invariant operator *or, alternatively, a* convolution operator. *For a bounded possibly time varying continuous time linear operator $M : \mathcal{L}_2 \to \mathcal{L}_2$*

$$y(t) = \int_{-\infty}^{\infty} \overline{m}(t,\tau)x(\tau)\, d\tau \quad \forall t \in \mathcal{R}.$$

the kernel $\overline{m}(t,\tau) \in \mathcal{R}^{p\times p}$ *is the counterpart of $\overline{m}_{k,l}$. In the continuous time case, M is called a* time invariant operator *or, alternatively, a* convolution operator *if $\overline{m}(t,\tau) = \overline{m}(t+\nu, \tau+\nu)$ $\forall t, \tau, \nu \in \mathcal{R}$. For a convolution operator M, a shorthand notation for $\overline{m}(t,\tau)$ and $\overline{m}_{i,j}$ is $\overline{m}(t-\tau)$ and $\overline{m}(i-j)$, respectively with $\overline{m}(t)$ and $\overline{m}(k)$ denoting the respective* impulse response. □

Definition 4 [hyperdominance, dominance]
An operator $M : \ell_2 \to \ell_2$ is said to be doubly dominant *if the elements m_{ij} of its associated matrix have the following properties.*

$$m_{ii} \geq \sum_{j=-\infty, j\neq i}^{\infty} |m_{ij}|, \quad m_{ii} \geq \sum_{j=-\infty, j\neq i}^{\infty} |m_{ji}| \quad \forall i$$

If, in addition, it also holds that

$$m_{ij} \leq 0, \ \forall i \neq j$$

then M said to be doubly hyperdominant. *For an operator $M : \mathcal{L}_2 \to \mathcal{L}_2$, these notions are defined in terms of its kernel in an analogous manner with integrals suitably replacing sums.* □

Definition 5 [monotone nonlinearity]
The class $\mathcal{N}_M$ of MIMO monotone *nonlinearities consists of all memoryless mappings $N : \mathcal{R}^p \to \mathcal{R}^p$ such that $N(x)$ is the gradient of some convex function $P : \mathcal{R}^p \to \mathcal{R}$ and $\exists C \in \mathcal{R}^+ \ni \|N(x)\| \leq C\|x\|$.*
$\mathcal{N} \doteq \{N \in \mathcal{N}_M | N(0) = 0\}$, $\mathcal{N}_{odd} \doteq \{N \in \mathcal{N} | N(x) = -N(-x)\ \forall x\}$. □

Definition 6 [repeated SISO monotone nonlinearity]
The class of repeated SISO monotone *nonlinearities is the subclass $\mathcal{N}^{RS}$ of $\mathcal{N}$ with element $N \in \mathcal{N}^{RS}$ of the form*

$$N(\zeta) \doteq [\phi(\zeta_1)\ \phi(\zeta_2)\ \ldots \phi(\zeta_p)]^T \quad \forall \zeta \in \mathcal{R}^p \tag{1}$$

where $\phi \in \mathcal{N}$, ϕ SISO. A shorthand notation for (1) is $N = diag(\phi)$. The class $\mathcal{N}^{RS}_{odd}$ is defined by replacing $\mathcal{N}$ in the definition of $\mathcal{N}^{RS}$ by $\mathcal{N}_{odd}$. □

Definition 7 [multipliers]
$\mathcal{M}^{RS}_{odd}$ denotes the class of MIMO convolution operators, either continuous or discrete, such that the impulse response of an $M \in \mathcal{M}^{RS}_{odd}$ is of the form

$$m = g\ \delta - h \tag{2}$$

where $g, h(\cdot) \in \mathcal{R}^{p \times p}$ satisfy

$$g_{ii} \geq \sum_{i=1, i \neq j}^{n} |g_{ij}| + \sum_{i=1}^{n} \|h_{ij}\|_1 \qquad \forall i = 1, 2, \ldots, n \tag{3}$$

$$g_{ii} \geq \sum_{j=1, j \neq i}^{n} |g_{ij}| + \sum_{j=1}^{n} \|h_{ij}\|_1 \qquad \forall i = 1, 2, \ldots, n. \tag{4}$$

The subclass $\mathcal{M}^{RS}$ is obtained by further stipulating

$$g_{ij} \leq 0\ \forall i \neq j, \quad h_{ij}(\cdot) \geq 0\ \forall i, j. \tag{5}$$

Under the restriction

$$g,\ h \text{ are Hermitian matrices}, \tag{6}$$

the subclass $\mathcal{M}^D$ ($\mathcal{M}^D_{odd}$) is derived from $\mathcal{M}^{RS}$ ($\mathcal{M}^{RS}_{odd}$). □

Remark 1 D'Amato et al. [47] showed that $\mathcal{M}^D$ ($\mathcal{M}^D_{odd}$) preserves positivity of $\mathcal{N}^{RS}$ ($\mathcal{N}^{RS}_{odd}$). ■

Problem Formulation

Problem 1 *Find the largest class of bounded linear operators and the largest class of bounded convolution operators that preserve positivity of* every *nonlinearity in $\mathcal{N}^{RS}$ ($\mathcal{N}^{RS}_{odd}$).* □

Background Results

Paraphrased for notational ease, the main result of D'Amato et al., viz. [47, Theorem 1], is as follows.

Lemma 1 `[47, D'Amato et al.]`
$\mathcal{M}^D$ ($\mathcal{M}^D_{odd}$) is positivity preserving for $\mathcal{N}^{RS}$ ($\mathcal{N}^{RS}_{odd}$). □

The above result is stated a sufficiency condition and it is not made clear if there exists a larger class of bounded convolution operators that preserves positivity of $\mathcal{N}^{RS}$ ($\mathcal{N}^{RS}_{odd}$). In this regard, it is worthwhile to note the following interesting SISO case result of Willems (see [54, Theorem 3.11, pp. 63]). For easy reading, its statement is slightly modified.

Lemma 2 `[54, Willems]`
Let $M : \ell_2 \to \ell_2$ be a bounded linear operator. Then, $\langle x, My\rangle$ is nonnegative for all similarly ordered unbiased (similarly ordered symmetric unbiased) sequences $\{x\}, \{y\} \in \ell_2$ if and only if M is doubly hyperdominant (doubly dominant). □

Main Result

Theorem 1 [Solution to Problem 1]
A bounded linear operator M mapping ℓ_2^p into ℓ_2^p [or $\mathcal{L}_2$ into $\mathcal{L}_2$] preserves positivity of every $N \in \mathcal{N}^{RS}$ ($N \in \mathcal{N}^{RS}_{odd}$) if and only if its associated matrix [kernel] is doubly hyperdominant (doubly dominant). Furthermore, a bounded convolution operator M mapping $\mathcal{L}_2$ into $\mathcal{L}_2$, or mapping ℓ_2^p into ℓ_2^p, preserves positivity of every $N \in \mathcal{N}^{RS}$ ($N \in \mathcal{N}^{RS}_{odd}$) if and only if $M \in \mathcal{M}^{RS}$ ($M \in \mathcal{M}^{RS}_{odd}$). □

Proof: We shall prove the result for $\mathcal{N}^{RS}$. The case for $\mathcal{N}^{RS}_{odd}$ follows on similar lines. First, the result will be proved for the discrete time case.

An $N \in \mathcal{N}^{RS}$ can be expressed as $N = \mathrm{diag}(\phi)$ where $\phi \in \mathcal{N}$, ϕ SISO. Given sequences $\{x_i\}$ of real valued scalars, define $y_i = \phi(x_i)$ $i = 1, 2, \ldots, p$. Note that the sequences $\{x_i\}$ and $\{y_i\}$ are similarly ordered and unbiased for all i since $N \in \mathcal{N}$. Define $\widetilde{x}(k) \doteq [x_1(k)\ x_2(k)\ \ldots\ x_p(k)]^T$, $\widetilde{y}(k) \doteq [y_1(k)\ y_2(k)\ \ldots\ y_p(k)]^T$ for all $k \in \mathbb{Z}$. Note that the sequences $\{\widetilde{x}\}$ and $\{\widetilde{y}\}$ are similarly ordered and unbiased. Observe that $\langle x, My\rangle = \left\langle \widetilde{x}, \widetilde{M}\widetilde{y}\right\rangle$ where the sequences $\{x\}$ and $\{y\}$ are defined by

$$x(k) \doteq [x_1(k)\ x_2(k)\ \ldots\ x_p(k)]^T,\ y(k) \doteq [y_1(k)\ y_2(k)\ \ldots\ y_p(k)]^T\ \ \forall k$$

and $\widetilde{M} : \ell_2 \to \ell_2$ with its associated matrix same as the associated matrix $\widetilde{M}$ of M. Since M is bounded, $\widetilde{M}$ is a bounded operator as well. By Lemma 2, $\left\langle \widetilde{x}, \widetilde{M}\widetilde{y}\right\rangle$ is nonnegative if and only if $\widetilde{M}$ is doubly hyperdominant. This is proves the result for bounded linear operators. To prove the result for bounded convolution operators, note that the associated matrix $\widetilde{M}$ of a bounded convolution operator M is block Toeplitz. Since $\widetilde{M}$ is block Toeplitz, the double hyperdominance conditions need only be checked on a block of p columns and on a block of p rows. The conditions are precisely the ones given by (3)-(5).

To prove the result in the continuous time case, note that continuous time signals x_c, y_c can be sampled with sampling interval ϵ to produce discrete time signals in ℓ_2, say $x_{d,\epsilon}, y_{d,\epsilon}$ such that $x_{d,\epsilon}(k) = \sqrt{\epsilon}\ x_c(k\epsilon)$, $y_{d,\epsilon}(k) = \sqrt{\epsilon}\ y_c(k\epsilon)$. Likewise, given a continuous time linear operator $M_c : \mathcal{L}_2 \to \mathcal{L}_2$,

$$z_c(t) \doteq \int_{-\infty}^{\infty} \overline{m}_c(t,\tau) y(\tau)\, d\tau\ \forall t,$$

may be discretized as $z_{d,\epsilon} \doteq M_{d,\epsilon} y_{d,\epsilon}$ i.e. as

$$z_{d,\epsilon}(k) \doteq \sum_{l=-\infty}^{\infty} \overline{m}_{k,l} y_{d,\epsilon}(l) \quad \forall k$$

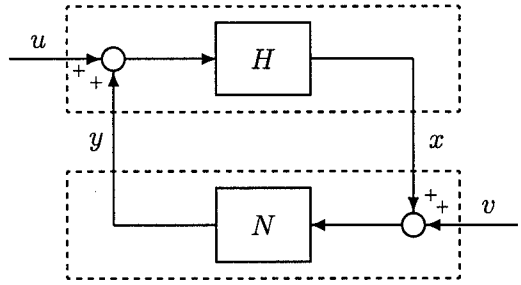

Figure 1: The feedback system $\mathcal{S}$. H is stable, causal and linear time invariant; $N \in \mathcal{N}_{odd}^{RS}$.

where, for all k and l,

$$\overline{m}_{kl} \doteq \frac{1}{\epsilon} \int_{t\in((k-1)\epsilon,k\epsilon]} \int_{\tau\in((l-1)\epsilon,l\epsilon]} \overline{m}_c(t,\tau)\ d\tau\ dt.$$

With this discretization, the continuous time inner-product $\langle x_c, M_c y_c\rangle$ is then recoverable as the limit

$$\langle x_c, M_c y_c\rangle = \lim_{\epsilon\to 0} \langle x_{d,\epsilon}, M_{d,\epsilon} y_{d,\epsilon}\rangle\,.$$

Taking the limit as $\epsilon \to 0$, the continuous time case proof then follows using the discrete time case arguments. QED.

Remark 2 By Theorem 1, $\mathcal{M}_{odd}^{RS}$ preserves positivity of the identity matrix. Thus, every element M of $\mathcal{M}_{odd}^{RS}$ has the property that $\text{herm}(\widehat{m}(\cdot)) \geq 0$. ■

Remark 3 From Theorem 1, it follows that every convolution operator that preserves positivity of $\mathcal{N}^{RS}$ ($\mathcal{N}_{odd}^{RS}$) and, *in addition*, has a Hermitian frequency response matrix is an element of $\mathcal{M}^{D}$ ($\mathcal{M}_{odd}^{D}$). However, $\mathcal{M}^{D}$ ($\mathcal{M}_{odd}^{D}$) is strictly a subset of $\mathcal{M}^{RS}$ ($\mathcal{M}_{odd}^{RS}$) because the condition (6) stipulated by D'Amato et al. [47] is unnecessary for the positivity preservation. ■

Discussion

By letting go of the unnecessary condition (6) stipulated by D'Amato et al. [47], we have obtained a larger — indeed, the largest possible — class $\mathcal{M}^{RS}$ ($\mathcal{M}_{odd}^{RS}$) of multipliers that preserve positivity of $\mathcal{N}^{RS}$ ($\mathcal{N}_{odd}^{RS}$). However, an incremental improvement can be claimed only if it can be demonstrated that use of this larger class of multipliers leads to a further reduction in conservativeness of IQC stability analysis. The following example demonstrates that the conservatism is indeed further reduced.

Example 1 Consider the feedback system $\mathcal{S}$ (see Fig. 1) in which $N \in \mathcal{N}^{RS}$ and H is the trivial stable memoryless linear operator having constant frequency response

$$\widehat{h}(\omega) = \begin{bmatrix} 0.5 & -1 \\ 2 & -0.6 \end{bmatrix} \quad \forall\omega.$$

The objective is to determine if the system is stable. Stability is established if (cf. [52, Theorem 2], [53, Theorem 1]) there exists a positivity preserving multiplier M such that the operator MH is positive, i.e.

$$\text{herm}\left(\widehat{m}(\omega)\widehat{h}(\omega)\right) > 0 \quad \forall\omega. \tag{7}$$

It may be checked that $-\widehat{h}(\omega) \in \mathcal{R}^{2\times 2}$ is not Hurwitz so that, by Lyapunov Criterion (see, e.g., [57, Theorem 2.6-1]), herm $\left(P\widehat{h}(\omega)\right) \not> 0$ for any $P = P^T > 0$. In view of the condition (7), it then follows that stability *cannot* be determined using $M \in \mathcal{M}^D$ since (see Remark 2) $\widehat{m}(\omega) = \widehat{m}(\omega)^T \geq 0$ for every $M \in \mathcal{M}^D$. On the other hand, choosing the *asymmetric* multiplier

$$M = \begin{bmatrix} 1 & 0 \\ -0.9 & 1 \end{bmatrix}, \tag{8}$$

which may be verified to be in $\mathcal{M}^{RS}$ (so that $M \in \mathcal{M}^{RS}_{odd}$ as well), the condition (7) is satisfied so that stability of the system *is* established using the class $\mathcal{M}^{RS}$ derived in this note. Even if the nonlinearity N were odd, using similar arguments it follows that stability cannot be determined using $M \in \mathcal{M}^D_{odd}$ whereas it can be determined using the multiplier given by (8). This demonstrates an example in which $\mathcal{M}^{RS}$ ($\mathcal{M}^{RS}_{odd}$) leads to a strictly less conservative stability analysis than $\mathcal{M}^D$ ($\mathcal{M}^D_{odd}$). ■

Remark 4 The above example shows that our main result incrementally improves upon the main result in [47]. Its utility can be seen via the same engineering application considered by D'Amato et al. [47, *Automatica* version]. ■

Summary: All Multipliers for Repeated Nonlinearities

We have derived the largest possible class of bounded MIMO linear operators that preserve positivity of repeated monotone nonlinearities. Also derived are the largest possible classes $\mathcal{M}^{RS}$ and $\mathcal{M}^{RS}_{odd}$ of bounded MIMO convolution operators that preserve positivity of repeated monotone and repeated odd monotone nonlinearities, respectively. It follows that (see Remark 2) *modulo the restriction* that multipliers have a Hermitian frequency response for all frequencies, D'Amato et al. [47] have actually derived the largest possible classes of multipliers that preserve positivity of such nonlinearities. The less restrictive nature of our multipliers produces less conservative stability results. We have demonstrated this conservativeness reduction via an example.

3.2 Unfalsified Control — A Behavioral Approach to Learning and Adaptation

Introduction

A fundamental activity in processes of system identification and, more generally, in all processes of scientific discovery is the fitting of models to data. Goals or cost functions are set based on beliefs about instrument accuracy or other model performance criteria, then and scientists attempt experimental validation, or unfalsification, of data against various parameterized classes of plausible models in the hope the one or more of the hypothesized plausible models proves have a superior fit to the data, relative to the specified modeling goals or cost functions. The challenges faced by the to the system identification specialist and the experimental scientist are the same. Moreover, these challenges are not dissimilar to those faced by adaptive control designers seeking to find a faithful mathematical representation of the control-decision-relevant information in evolving observational data.

One interesting development in recent years has been the advent of the unfalsified control paradigm [58]-[5] which has advanced the model validation/unfalsification paradigm of system identification theory to the control validation paradigm for understanding and analyzing adaptive control algorithms. In adaptive control and system identification, as in other scientific endeavors, a parsimonious mathematical representation of the essential issues is preferred. For adaptive control, one

such paradigm is provided by unfalsified control theory. The unfalsified control theory views the control problem as an identification problem in which the objective is that of directly identifying a control law or "action rule" that is consistent with traditional control performance goals, prior knowledge, and evolving observational data.

Unfalsified adaptive control software has been developed and design studies have been conducted. For example, the theory has recently been applied to the design of a robust adaptive controller for a two-link robot manipulator arm in [11] and to the design of an adaptive missile autopilot [20]. Unfalsified control theory provided the basis for a general purpose algorithm for automatic tuning of PID controller gains [22]. Recently, the unfalsified control approach has been applied in experimental settings ranging from the ACC benchmark control problem [62] to industrial process control [64, 65, 66]. These design studies seem to confirm theoretical expectations that adaptive controllers optimally designed via unfalsified control theory exhibit a precise, sure-footed response in the face of evolving uncertainties and parameter variations.

In this paper, we formulate the unfalsified adaptive control problem in the behavioral framework of Willems ([67],[68]). Our results are closely related to, but different from, the indirect adaptive method of Polderman [69] in which an unfalsified plant model is identified from within a prescribed model set. Unlike Polderman, we bypass the intermediate step of plant model identification and, also unlike Polderman, we make no assumptions about the 'true plant' lying in an assumed model set. We make no assumptions on the plant.

Background: Behavioral Theory

At the heart of the behavioral theory of Willems [67, 68] is the definition of a mathematical model. This definition is formulated according to the black box point of view, "in which we focus on how a system behaves, on the way it interacts with its environment, instead of trying to understand, in the tradition of physics, how it is put together and how its components work" ([67],[68]) . This definition of a mathematical model formalizes the black box point of view. Like Willems, we back off "from the usual input/output setting, from the processor point of view, in which systems are seen as influenced by inputs, acting as causes, and producing outputs through these inputs, the internal conditions, and the system dynamics."

Willems begins with the assumption that there is a phenomenon to be modeled. He then "casts the situation in the language of mathematics by assuming that the phenomenon produces elements in a set $\mathbf{Z}$" ([67],[68]), called the *universum.* The elements of $\mathbf{Z}$ are called the outcomes of the phenomenon. "A (deterministic) mathematical model for the phenomenon (viewed purely from the behavioral, the black box point of view) claims that certain outcomes are possible, while others are not. Hence a model recognizes a certain subset $\mathcal{B}$ of $\mathbf{Z}$. This subset will be called the behavior (of the model)." Formally,

Definition 3.1 *A mathematical model is a pair* $(\mathbf{Z}, \mathcal{B})$, *with* $\mathbf{Z}$ *the universum — its elements are called outcomes — and* $\mathcal{B} \subseteq \mathbf{Z}$ *the behavior.*

Definition 3.2 *A controller is a mathematical model.*

Regarding data and measurements, Willems [67] says: "We will now cast measurements in this setting. We will assume that we make certain measurements which we will call the data." "...we ...assume that the data consists of observed realizations of the phenomenon itself. Thus, a data set will be a nonempty subset $\mathcal{D}$ of $\mathbf{Z}$. "

Following Safonov and Tsao [58], we will work with data information that can evolve with time. Thus we will have a universum of time signals and a data set contained in a time varying projection of this universum.

Definition 3.3 *Given a vector space of time signals* $\mathbf{Z}$*, a model* $(\mathbf{Z}, \mathcal{B})$*, a mapping* $P_\tau : \mathbf{Z} \to \mathbf{Z}$ *and a data set* $\mathcal{D}_\tau \triangleq P_\tau \mathcal{D} \subset P_\tau(\mathbf{Z})$*, we say that the model* $(\mathbf{Z}, \mathcal{B})$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$\mathcal{D}_\tau \subset P_\tau(\mathcal{B}).$$

Typically $P_\tau(x)$ is the experimental observation time sampling operator, which returns values of $x(t)$ only for past time instants (or possibly time intervals) over which experimental observations of $x(t)$ have been recorded. In this setting, definition of controller falsification (cf. [58, 60]) becomes

Definition 3.4 *Given a vector space of time signals* $\mathbf{Z}$*, a controller* $(\mathbf{Z}, \mathcal{B}_c)$*, a desired closed loop behavior* $(\mathbf{Z}, \mathcal{B}_d)$*, a mapping* $P_\tau : \mathbf{Z} \to \mathbf{Z}$*, and a data set* $\mathcal{D}_\tau \subset P_\tau(\mathbf{Z})$*, we say that a controller* $(\mathbf{Z}, \mathcal{B}_c)$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$P_\tau((P_\tau^{-1}(\mathcal{D}_\tau)) \cap \mathcal{B}_c) \subset P_\tau(\mathcal{B}_d).$$

The data set $\mathcal{D}_\tau$ is a set of actual experimental observations of the plant behavior as observed through the time-sampler P_τ. Thus, $P_\tau^{-1}(\mathcal{D}_\tau)$ is the set of behaviors that interpolate the observed data. For example, if we have recorded experimental observations of the first component $x_1(t)$ of a vector-valued signal $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in L_2^n[0, \infty)$ during the time interval $t \in [0, 5]$, then $P_\tau^{-1}(x)$ is the set of signals $\{y \in L_2^n[0, \infty) \mid y_1(t) = x_1(t) \forall t \in [0, 5]\}$. The set $\mathcal{B}_c$ is the set of signals which satisfy the constraints imposed by the controller c, so definition 3.4 says roughly that a controller is defined to be unfalsified if the set of signals x that are consistent with the data and the controller is, at the past observation times, a subset of a given performance target set $P_\tau(\mathcal{B}_d)$.

A particularly useful projection operator for dealing with past time only information is the time truncation operator P_τ defined by

$$[P_\tau(x)](t) = \begin{cases} x(t), & \text{if } t \leq \tau \\ 0, & \text{if } t > \tau. \end{cases}$$

As explained by Willems ([67],[68]), the intersection of behaviors is "a way of formalizing that additional laws are imposed on a system." Thus, the role of a controller is to impose constraints on the plant behavior. On the other hand, our goal is to select, based on the data, the constraints imposed by the control law and the performance criterion, the best among the set of given controllers. In order to do that we introduce a *cost function*

$$J(z) : \mathbf{Z} \to \mathcal{R} \tag{9}$$

which may be used to sift controllers and to choose an optimal cost-minimizing controller having the least unfalsified cost based on the experimental evidence $\mathcal{D}_\tau$.

Direct Adaptive Control: Behavioral Formulation

We now explain how problems of adaptive control and learning theory may be parsimoniously and precisely embedded within the behavioral framework. At the crux is the observation that most such problems may be faithfully represented in terms of constraints on signals and other variables $z \in \mathbf{Z}$. The set $\mathbf{Z}$ is called the *universum*. Typically, the n-tuple z includes directly observable

manifest variables z_{manifest} (viz., control and sensor signals), command input signals z_{command}, and possibly additional *latent variables* z_{latent} such as disturbances, noise, state-variable trajectories, error signals and so forth. That is, $z = \{z_{\mathsf{manifest}}, z_{\mathsf{command}}, z_{\mathsf{latent}}\} \in \mathbf{Z}_{\mathsf{manifest}} \times \mathbf{Z}_{\mathsf{command}} \times \mathbf{Z}_{\mathsf{latent}} = \mathbf{Z}$. In stochastic settings, the latent variable n-tuple z_{latent} also includes conditional probability density functions describing some of the other latent variables [60, 18, 8].

It is convenient to view the constraints on the n-tuple of signals $z \in \mathbf{Z}$ as arising from four distinct types of information, each possibly evolving with time:

1. Goal (cost function and/or design specification)
2. Belief (assumptions, prior knowledge, noise models, plant parameterizations, etc.)
3. Hypothesis (candidate control law, candidate plant/noise model)
4. Data (observations, samples of the signal z_{manifest} available at current time τ)

Each of these four types of information is representable as a mathematical constraint on z:

$$\forall z_{\mathsf{command}},\ J(z) \leq \gamma, \text{ (cost } J(z) \geq 0 \text{ no bigger than } \gamma \text{ for any command input)} \tag{10}$$

$$K(z) = 0, \quad \text{(hypothetical controller and/or model } K \in \mathsf{K}) \tag{11}$$

$$B(z) \leq 0, \quad \text{(fixed beliefs, assumptions \& prior knowledge)} \tag{12}$$

$$P_\tau(z_{\mathsf{manifest}}) = z_{\mathsf{data}}, \quad (z_{\mathsf{manifest}} \text{ must interpolate observed data } z_{\mathsf{data}}) \ . \tag{13}$$

In turn, the four constraints (10)-(13) define, respectively, four subsets of the universum $\mathbf{Z}$, viz.

$$\mathbf{Z}_{\mathsf{goal}}(\gamma), \mathbf{Z}_{\mathsf{hypothesis}}(K), \mathbf{Z}_{\mathsf{belief}}, \mathbf{Z}_{\mathsf{data}} \subset \mathbf{Z}. \tag{14}$$

Thus, the problem of direct adaptive control (or, controller identification from data), can be formulated in the Willems behavioral framework as follows.

Problem 1 (Behavioral Adaptive Control) *Given a class of controllers* $\mathcal{K} \triangleq \{(\mathbf{Z}, \mathcal{B}_c(\theta)) \mid \theta \in \Theta\}$, *where* Θ *is a set of parameter vectors, the performance (cost) index* $J(z)$ *the time truncation operator* P_τ, τ, *and a data set* $\mathcal{D}_\tau \subset P_\tau \mathbf{Z}$, *find the set of parameters* Θ^* *such that* $K \in \mathsf{K}$ *that minimizes the cost* γ *subject to the constraint (cf. [60, 18, 8]) that, for each*

$$\xi \in \mathbf{Z}_{\mathsf{hypothesis}}(K) \cap \mathbf{Z}_{\mathsf{belief}} \cap \mathbf{Z}_{\mathsf{data}}, \tag{15}$$

there is at least one z *such that*

$$z_{\mathsf{command}} = \xi_{\mathsf{command}} \tag{16}$$

and

$$z \in \mathbf{Z}_{\mathsf{goal}}(\gamma) \cap \mathbf{Z}_{\mathsf{hypothesis}}(K(\theta)) \cap \mathbf{Z}_{\mathsf{belief}} \cap \mathbf{Z}_{\mathsf{data}}. \tag{17}$$

Discussion

If the set (15) is empty for some K, then the currently available data z_{data} provides no information on this K, which is therefore trivially optimal with cost $\gamma = 0$; otherwise, the adaptive feedback control problem emerges as the following optimization: At each time τ, find a control law K which solves

$$\gamma_{\mathsf{opt}} := \min_K \max_\xi \min_z \gamma \tag{18}$$

subject to (15)-(17). In many practical cases (e.g., [58, 18, 20, 11]), the cost γ can be expressed directly in terms of z_{data} and K in which case (15)-(18) simplify to $\gamma_{\mathsf{opt}} := \min_K \gamma(z_{\mathsf{data}}, K)$.

Noteworthy are the symmetries revealed in the condition (17) with respect the information content of goal, belief, hypothesis and data. Set intersection is a commutative and associative operation; so all four types of information are logically equivalent in (17). For example, this means that the prejudice inherent in viewing one's data through a prism of belief $\mathbf{Z}_{\mathsf{belief}}$ is logically equivalent to assuming additional data "interpolation" constraints ($\mathbf{Z}_{\mathsf{data}} \leftarrow \mathbf{Z}_{\mathsf{data}} \cap \mathbf{Z}_{\mathsf{belief}}$). The standard *unfalsified control* problem considered in [58, 59, 60, 20, 11, 5] corresponds to the limiting case in which "the prism of belief" $\mathbf{Z}_{\mathsf{belief}}$ is the unconstraining "all-pass" filter $\mathbf{Z}$ (i.e., the universum).

Equations (15)–(17) underscore fertile conceptual links between adaptive control theory and the rich theory of system identification: The chief differences between identification and adaptive control arise from the precise forms of the cost functions $J(z)$ and of the admissible hypotheses $K(z)$. In system identification the admissible $K(z)$'s are typically noisy open-loop plant models and the cost function $J(z)$ measures probable modeling error deduced via a prism of beliefs about noise statistics. In adaptive control on the other hand, the admissible $K(z)$'s might typically be candidate controllers and the cost $J(z)$ could be a weighted sum of the sizes of tracking error and control signals.

Summary: Unfalsified Control — A Behavioral Approach to Learning and Adaptation

The main goal of unfalsified control theory has been to close the loop on the adaptive and robust control design processes by developing data-driven methods to complement traditional model-based methods for the design of robust control systems. The crux of the unfalsified control theory is the observation that adaptive control is from a behavioral theory perspective essentially equivalent to system identification. In this paper, we have developed a behavioral formulation of the problem of direct adaptive control, viz., the problem of identifying an optimal controller that is unfalsified by data available at each time τ with respect to the least value of a cost function. The main result is the formulation of direct adaptive control problems provided by Problem 1. This result establishes a firm theoretical link between Willems' behavioral framework and direct adaptive control theory, expanding known links to model validation, unfalsified system identification theory, and behavioral indirect adaptive control approach of Polderman [69].

4 Conclusions

With support from AFOSR Grant F49620-98-1-0026, significant progress has been made in theory for reliable computation of robust controllers, in the field of nonlinear robustness analysis, and in the development of the unfalsified control theory formulation of adaptation and learning problems. Our new results in nonlinear robustness analysis for systems with repeated nonlinearities are important breakthroughs of broad applicability to most all control designs. They will enable one to more accurately predict and correct stability problems that commonly occur with control actuators hit position or rate limits.

Progress in *unfalsified control theory*, a new formulation of adaptive control problems developed with AFOSR support, gives sharp mathematical representation of the role of experimental data in identifying robust control laws and provides a practical technique for identifying robust controllers in real-time with little or no apriori information. This theory cuts to the heart of a long-standing problem faced by control engineers, which has been the need for a data-driven theory that provides a unified basis for precisely representing and exploiting *evolving* information flows from models, noisy data, and more. The theory has unveiled important links between robust control, adaptive control and artificial intelligence. It is a conceptual breakthrough because it distills the mathematical essence of control-oriented learning by focusing sharply on what is, and is not, knowable from experimental data and by challenging both the need and the appropriateness of a number of common assumptions. The results of our unfalsified control research have not only solidified the theoretical foundation of adaptive control. They have also permitted efficient design of feedback control systems endowed with an unprecedented ability to accurately assess and exploit evolving real-time information flows as they unfold, thereby endowing control systems with the intelligence to quickly and sure-footedly adapt to unfamiliar environments. The effectiveness of the unfalsified

approach to adaptive control design has recently been demonstrated with simulated design studies for missile autopilot [20], a robot manipulator arm [11], an industrial machine tool [66] and a general purpose adaptive PID controller that automatically discovers suitable controller gains, when they exist, based on real-time data without the need for plant model [22]. These research results will mean control designs that can more quickly and reliably compensate for uncertain effects of battle damage, equipment failures and other changing circumstances.

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Interactions & Coupling Activities

Gov't Panel: Michael G. Safonov
NSF Panel on Knowledge and Distributed Intelligence.
Learning and Intelligent Systems.
Arlington, VA, July 1998.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, Workshop for Michael Athans, Tampa, FL, December 19, 1998, 1998.
60th birthday celebration honoring M. Athans.

Talk: M. G. Safonov.
Synthesis of positive real feedback systems: A simple derivation.
Talk, UCSB, Santa, Barbara, CA, October 16, 1998.

Gov't Panel: NSF Panel. Knowledge and Distributed Intelligence: Learning and Intelligent Systems, Washington, DC, March 15–16, 1999.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, IFAC Intl. Workshop on Control of Uncertain Systems, Hong Kong, Univ. of Science & Technology, June 30 – July 2, 1999.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, Poster Presentation, AFOSR Workshop on Dynamics and Control, Dayton, OH, August 4–6, 1999.
`ftp://routh.usc.edu/pub/safonov/safo99h.ppt`

Talk: M. G. Safonov.
CACSD design process.
Talk, Panel Discussion on Perspectives on Computer Aided Control Systems Design, IEEE Symp. on Computer Aided Control System Design, Kohala Coast–Island of Hawaii, HI, August 22, 1999.
Plenary session.

Talk: M. G. Safonov.
Multiplier IQC's for uncertain time delays.
Talk, Tokyo University, Tokyo, Japan, May 22, 2000.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, SICE Conference, Kariya, Japan, May 24, 2000.
Invited plenary talk.

Talk: M. G. Safonov.
Robust control tutorial.
Talk, Tokyo University, Tokyo, Japan, May 31, 2000.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, Kyoto University, Kyoto, Japan, June 2, 2000.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, Osaka University, Osaka, Japan, June 6, 2000.

Talk: M. G. Safonov.
Unfalsified direct adaptive control of a two-link robot arm.
Talk, Titech, Tokyo, Japan, June 13, 2000.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, AFOSR Workshop on Dynamics and Control, Dayton, OH, August 21–23, 2000.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, University of California, Santa Barbara, CA, October 13, 2000, 2000.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, Caltech, Pasadena, CA, October 16, 2000, 2000.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, University of California, Santa Barbara, CA, November 17, 2000.

Talk: M. G. Safonov.
Robust control, feedback and learning.
Talk, IEEE Control Society, San Diego Chapter, La Jolla, CA, January 18, 2001.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, University of California San Diego, La Jolla, CA, January 18, 2001.

Talk: M. G. Safonov.
Zames-Falb multipliers for MIMO nonlinearities.
Talk, Workshop in honor of Boyd Pearson, Rice University, Houston, TX, March 9–10, 2001.

Talk: M. G. Safonov.
Data-driven behavioral formulation of the adaptive feedback control problem.
Talk, AFOSR Workshop on Dynamics and Control, Dayton, OH, July 30 –August 02, 2001.
`ftp://routh.usc.edu/pub/safonov/safo01o.pps`

Software: Effective and rapid transition from theory to practice has been facilitated by my ongoing non-AFOSR-supported involvement with Dr. R. Y. Chiang in upgrading the MATLAB ROBUST CONTROL TOOLBOX, a robust control design software product published by The MathWorks and in widespread use by government, university and aerospace engineering company labs.

Technology Transitions

- Semiconductor Manufacturing Process Control

Enabling Research:	Unfalsified Control Theory
Performer:	Dr. Robert Kosut, SC Solutions, Sunnyvale, CA 94085; (408) 617-4527
Customer:	Dr. Robert Kosut, SC Solutions, Sunnyvale, CA 94085; (408) 617-4527
Result:	Used unfalsified control technique to tune controller for semiconductor manufacturing process.
Application:	Higher quality semiconductors.

- Commercial Robust Control Software

Enabling Research:	Robust control theory, including LMI/BMI methods
Performer:	Michael G. Safonov/USC (213) 740-4455
Customer:	The MathWorks, Inc./Natick, MA Mr. John Little, (508) 653-1415
Result:	Improved and reorganized algorithms for robust control in various MATLAB software products, including the *Robust Control Toolbox*, *Mu-Synthesis Toolbox*, and *LMI Toolbox.*
Application:	Faster, cheaper, more reliable control design for advanced aerospace and other control systems.

New Discoveries, Inventions, or Patent Disclosures. None

Honors/Awards

- Honor/Award: Elected IEEE Fellow 1989
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: IEEE

- Honor/Award: Author of 4 papers selected for inclusion is IEEE Press anthology of key papers in control, "Robust Control" ed. P. Dorato 1986
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: IEEE

- Honor/Award: Author of 4 papers selected for inclusion is IEEE Press anthology of key papers in control, "Recent Advances in Robust Control" ed. P. Dorato and R. Yedavalli — 1990
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: IEEE

- Honor/Award: Awards Chair — 1993–1995
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: American Automatic Control Council

- Honor/Award: Editorial Board. Int. J. Robust and Nonlinear Control — 1990–2001
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: J. Wiley

- Honor/Award: Assoc. Editor, Systems and Control Letters — 1995–2001
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: Elsevier Science Publishers

- Honor/Award: Invited Plenary Speaker — May 2000
  Honor/Award Recipient(s): Michael G. Safonov
  Awarding Organization: Society of Instrumentation and Control Engineers (SICE, Japan).