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Directional Distributions and Mean Square Slopes of Surface Waves

Paul A. Hwang and David W. Wang

Oceanography Division, Naval Research Laboratory, Stennis Space Center, Mississippi

Field observations show that the crosswind component constitutes a significant portion of the ocean surface mean square slope. The average ratio between the crosswind and upwind mean square slope components is 0.88 in slick-covered ocean surfaces. This large crosswind slope component cannot be explained satisfactorily based on our present models of ocean wave directional distributions. Two-dimensional spectral analysis of 3D ocean surface topography reveals that bimodal directional distribution is a common feature for wave components shorter than the peak wavelength. The calculated result of the upwind and crosswind mean square slope components using a bimodal directional distribution function is in very good agreement with field measurements.

1. INTRODUCTION

Ocean surface waves are the roughness element of the air-water interface. The directional distribution of the ocean surface roughness is an important parameter in air-sea interaction studies and ocean remote sensing applications. Examples include magnitude and direction of energy and momentum fluxes between air and water, scatterometer wind velocity measurement, synthetic aperture radar (SAR) imaging of the ocean wave field, and optical detection of surface and subsurface features. Close correlation between the gas transfer velocity and surface wave parameters, especially the mean square slope, has been reported extensively in the literature (e.g., papers presented in the Air-Water Gas Transfer Symposia, *Brutsaert and Jirka*, 1984; *Wilhelms and Gulliver*, 1991; *Jahne and Monahan*, 1995; *Donelan et al.*, 2001).

The preferred representation of the ocean surface roughness properties is the directional wavenumber spectrum of the ocean waves. Such data become available only recently. The results on the roughness wavenumber spectrum come in small pieces because of difficulties in data acquisition and analysis. Several field measurements of wavenumber spectra of short waves (from several millimeters to several decimeters) obtained by scanning laser slope sensing have been reported [e.g., *Bock and Hara*, 1995; *Hara et al.*, 1994, 1998; *Hwang et al.*, 1996]. In principle, scanning slope data include two orthogonal components of ocean surface slopes measured in the space domain and detailed information on the directional distributions of individual spectral components of ocean waves are available, but only limited results on the directional properties of short waves over a small range of wind speeds have been reported [*Hwang*, 1995; *Klinke and Jahne*, 1995; *Hara et al.*, 1998].

Most ocean wave directional spectra are obtained from temporal measurements acquired by directional buoys [e.g., *Longuet-Higgins et al.*, 1963; *Mitsuyasu et al.*, 1975; *Hasselmann et al.*, 1973, 1980] or wave gauge arrays [e.g., *Donelan et al.*, 1985] and frequency spectra are derived. The transformation from frequency to wavenumber domain is carried out using the dispersion relationship. This approach is rather successful for long gravity waves. The application to shorter waves is not very satisfactory because of the large Doppler frequency shift caused by convection of short waves by surface currents. The large frequency shift introduces large uncertainties in the interpretation of the length scales of the measured (apparent) wave frequency [e.g., *Phillips*, 1985; *Hwang et al.*, 1996; *Donelan et al.*, 1999).

For surface roughness investigations, the surface slope data of Cox and Munk (1954) remain the most comprehensive in terms of the range of wind speeds encountered and the extent of their statistical analysis. Based on their measurements, the ocean surface roughness has a significant crosswind component. In most cases, the ratio of the

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crosswind and upwind mean square slope components is greater than 0.7. The large crosswind to upwind ratio is also found in later measurements of high frequency ocean wave spectra using laser slope gauges [e.g., *Hughes et al.*, 1977; *Tang and Shemdin*, 1983; *Hwang and Shemdin*, 1988]. This ratio is much larger than what's expected from applying unimodal directional distribution functions currently established in the ocean wave spectral models.

Banner and Young [1994] emphasize that bimodality is a robust feature of the directional distribution of wind-generated waves. The dynamic process that produces a bimodal directional distribution is the nonlinear wave-wave interaction mechanism. Although bimodal directional distributions have been obtained from directional buoy data using analysis techniques such as maximum entropy method (MEM) or maximum likelihood method (MLM) [*Young*, 1994; *Young et al.*, 1995; *Ewans*, 1998; *Wang and Hwang*, 2000], there are continuous disagreements on the existence of directional bimodality, mainly because the quantitative results of bimodal directional distribution differ significantly depending on the processing method employed. *Hwang et al.* [2000a,b] report 2D wavenumber spectral analysis of 3D surface topography measured by an airborne topographic mapper (ATM, an airborne scanning lidar system). The directional resolution derived from 3D topographic data is excellent. The analysis presented in *Hwang et al.* [2000b] illustrates that the directional resolution for a wave field generated by a steady 10 m/s wind field is better than 10° for wavenumber components higher than the peak wavenumber. The directional spectra of the ATM data show clear directional bimodality [*Hwang et al.*, 2000b]. They also notice that cases of bimodal directional distributions are commonly found in the directional wavenumber spectra derived from similar 3D spatial measurements, such as those acquired by aerial stereo photography [*Phillips*, 1958; *Cote et al.*, 1960; *Holthuijsen*, 1983], airborne imaging radar (*Jackson et al.* 1985] and land-based imaging radar [*Wyatt*, 1995].

In this paper, we present computations of the upwind and crosswind mean square slope components using four different directional distribution models. These calculations are compared with the measurement of mean square slopes by *Cox and Munk* [1954]. Due to the fact that there remain major uncertainties on the spectral properties in the short wave regime, we limit our investigation to gravity wave components in the equilibrium and saturation ranges of the surface wave spectrum. Quantitative results of the ratio of the crosswind and upwind slope components calculated using four directional distribution functions are presented in Section 2. Further discussions on the spectral function and directional distributions are presented in Section 3. A summary of the study is given in Section 4.

2. DIRECTIONAL DISTRIBUTIONS AND MEAN SQUARE SLOPE COMPONENTS

2.1. Field Data

The mean square slope results reported in *Cox and Munk* [1954] are derived from analyzing the sun glitter patterns of the ocean surface obtained from an aircraft. The area of coverage for each image of glitter patterns is typically on the order of one-half square kilometer. The results, therefore, yield a high degree of statistical confidence. Based on these data, the total mean square slopes of the ocean surface increase linearly with wind speed, and the following two formulas are given:

$$s_{clean}{}^2 = 5.12\times10^{-3}U + (3\pm4)\times10^{-3}, \tag{1}$$

and

$$s_{slick}{}^2 = 1.56\times10^{-3}U + (8\pm4)\times10^{-3}. \tag{2}$$

In (1) and (2), the wind velocity, U, is measured at 12.5-m elevation. Combining their results with other later datasets and making corrections for cases that are affected by background swell, a slightly different formula in terms of U_{10}, the neutral wind speed at 10-m elevation, is given in *Hwang* [1997]

$$s_{clean}{}^2 = 5.12\times10^{-3}U_{10} + 1.25\times10^{-3}. \tag{3}$$

The data of special interest to this paper are those collected from slick covered surfaces. Altogether, *Cox and Munk* [1954] report 9 slick cases (one natural slick and 8 man-made slicks) with wind speeds ranging from 1.6 to 10.6 m/s. The ratio between crosswind and upwind components is quite large. For the 9 slick cases, the mean value with one standard derivation is 0.88±0.097, and none of the cases has a ratio less than 0.75. This large fraction of the crosswind mean square slope component has never been explained satisfactorily. As will be shown below, calculations using established unimodal directional distribution functions under-predict the crosswind component considerably.

2.2. Directional Distribution

For a given directional spectrum, $\chi(k,\theta)=\chi(k)D(k,\theta)$, the upwind and crosswind slope spectra can be expressed as

$$\chi_{1u}(k,\theta) = k_u^2\chi(k)D(k,\theta), \tag{4}$$

and

$$\chi_{1c}(k,\theta) = k_c^2\chi(k)D(k,\theta), \tag{5}$$

where k is wavenumber, subscripts u and c denote the upwind and crosswind components, $\chi_1(k)$ is the slope spectrum, relating to the displacement spectrum $\chi(k)$ by $\chi_1(k)=k^2\chi(k)$, and $D(k,\theta)$ is the directional distribution function (assuming wind direction is at $\theta = 0$, thus $k_u=k\cos\theta$, and $k_c=k\sin\theta$). In the following, the displacement spectral function is assumed to be

$$\chi(k)=\begin{cases} bu_* g^{-0.5}k^{-2.5}, & k\le k_i \\ Bk^{-3}, & k>k_i \end{cases}. \tag{6}$$

where B=4.6×10^{-3}, b=5.2×10^{-2}, and k_i is the matching wavenumber separating the equilibrium and the saturation spectra; the magnitude of k_i is in the neighborhood of 6.5k_p [e.g., *Phillips*, 1977, 1985; *Hwang et al.*, 2000a; *Hwang and Wang* 2000]. The ratio of crosswind and upwind mean square slope components integrated to an upper limit wavenumber, k_I, can be written as

$$r(k_I)=\frac{\int_{k_p}^{k_I} k^2\chi(k)\left[\int_{-\pi}^{\pi}\sin^2\theta D(k,\theta)d\theta\right]dk}{\int_{k_p}^{k_I} k^2\chi(k)\left[\int_{-\pi}^{\pi}\cos^2\theta D(k,\theta)d\theta\right]dk}. \tag{7}$$

For individual wavenumber components, the ratio is

$$d(k)=\frac{\int_{-\pi}^{\pi}\sin^2\theta D(k,\theta)d\theta}{\int_{-\pi}^{\pi}\cos^2\theta D(k,\theta)d\theta}. \tag{8}$$

Clearly, the specific functional form of the directional distribution plays an important role on the magnitudes of $r(k_I)$ and $d(k)$. Four different directional distribution models reported in the literature are investigated here. The first three models [*Mitsuyasu et al.*, 1975; *Hasselmann et al.*, 1980; *Donelan et al.*, 1985, with modification by *Banner*, 1990] are unimodal. The key features of these models include: (i) The dominant propagation directions of all wave components are in the wind direction. (ii) The directional beamwidth is narrowest near the spectral peak and becomes broader as the wavelength of an individual spectral component increases or decreases from the peak wavelength. Of these three unimodal directional distribution models, the *Donelan et al.* distribution is narrower, and the *Hasselmann et al.* distribution is broader, than the *Mitsuyasu et al.* distribution. The directional distributions of shorter wave components ($k\geq1.3k_p$) of the fourth model [*Hwang et al.*, 2000b, Appendix A] are bimodal. As stated earlier, bimodality is a robust feature of nonlinear wave-wave interaction [*Banner and Young*, 1994]. The beamwidth (an integrated property of the directional distribution) predicted by all four models are comparable [*Hwang et al.*, 2000b].

Figures 1a and b show the computed $d(k)$ and $r(k_I)$ for a wave field generated by 10 m/s wind. The ratio between crosswind and upwind slope components, $d(k)$ calculated using the bimodal function is usually larger than those calculated using unimodal directional distributions, especially in the lower range of k/k_p. The integrated ratio, $r(k_I)$, based on the bimodal function is much closer to the observed value. Following the calculation that waves shorter then 0.3 m are suppressed by slicks [*Cox and Munk*, 1954; *Phillips*, 1977], the corresponding wavenumber is k_s=2π/0.3 rad/m. For this wind speed, k_s=214k_p, and $r(k_s)\approx$0.8. In contrast, $r(k_s)$ is approximately 0.7 based on the models of *Mitsuyasu et al.* [1975] and *Hasselmann et al.* [1980], and is less than 0.6 based on the model of *Donelan et al.* [1985]. *Hwang and Wang* [2000] examine the effect of cutoff wavenumber on the integrated mean square slope, and show that the effect is minor. Sensitivity tests on other spectral parameters are also described in *Hwang and Wang* [2000].

The ratio $r(k_s)$ as a function of wind speed is plotted in Fig. 1c. For comparison, the field data of *Cox and Munk* [1954] are also shown. Of the four directional models, the results of bimodal directional distribution are in best agreement with the field data, but still underestimate the crosswind to upwind ratio by approximately 14%, the other three (unimodal) models underestimate the ratio by 26 to 39%.

3. DISCUSSIONS

As noted in Section 2, the crosswind to upwind ratio of mean square slopes calculated using a bimodal directional distribution is still 14% lower than the field data. Several factors may contribute to the observed larger crosswind surface slope component in the ocean. Firstly, the computation is based on the assumption of steady wind forcing. Fluctuations in the wind field, especially the wind direction, may contribute significantly to a higher magnitude of $r(k_s)$. Secondly, the derivation of the bimodal directional function is based on data from a mature wave field where the dominant wave component aligns with the wind vector. In a young sea, the dominant wave may propagate in oblique angles with respect to the wind vector in order for the wave field to maintain in resonant condition with the forcing wind field. According to *Phillips* [1957] resonance mechanism of wind-wave generation, two wave systems propagating at oblique angles symmetric to the wind are generated. Spatial measurements of 3D ocean surface topography using airborne scanning radar [*Walsh et al.*, 1985, 1989] and

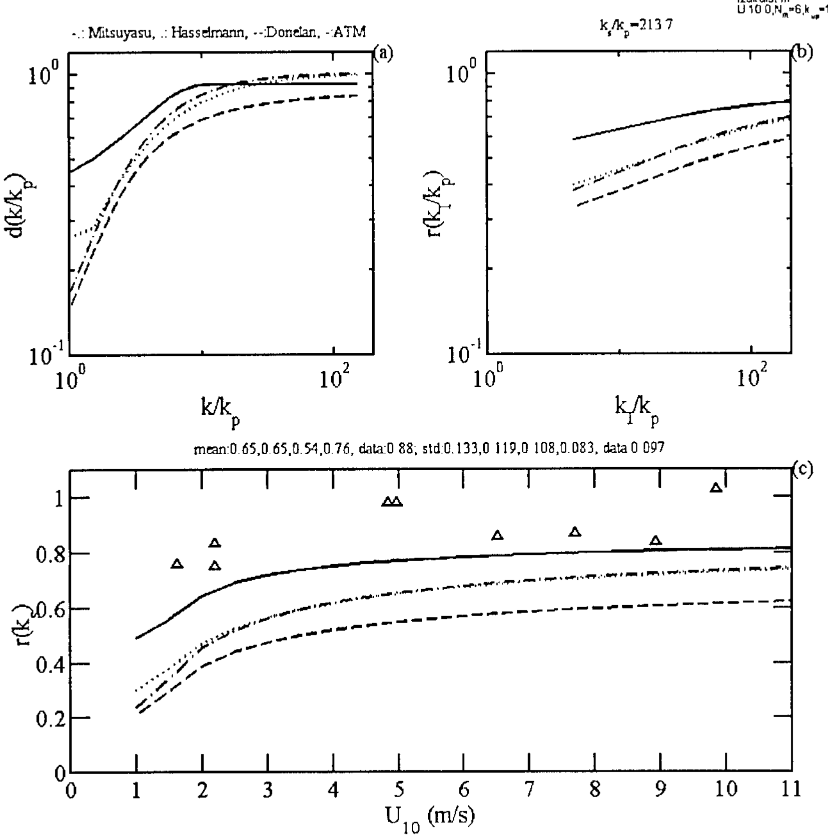

Figure 1. The ratios (a) $d(k)$ (Eq. 8), and (b) $r(k_l)$ (Eq. 7), calculated for a wave field generated by 10 m/s wind. (c) The ratio $r(k_s)$ as a function of wind speed, where the upper limit of integration wavenumber, k_s=2π/0.3 rad/m, representing the wavenumber above which the waves are damped by the slicks. The results of slick cases of *Cox and Munk* [1954] are also shown as triangles in the plot. Four different directional distribution models are used in the calculation: dashed-and-dotted curves: *Mitsuyasu et al.* [1975], dotted curves: *Hasselmann et al.* [1980], dashed curves: *Donelan et al.* [1985], and solid curves: *Hwang et al.* [2000b].

airborne scanning lidar [*Hwang and Wang*, 2000] have obtained examples of waves propagating in oblique angles with respect to the wind vector under steady wind forcing. In both cases presented by *Walsh et al.* [1985, 1989], most of the reported directional spectra have one wind-generated wave system. At shorter fetches (less than approximately 100 km for 10 m/s wind forcing), the direction of the wind-wave system deviates significantly from the wind vector. In the case reported by *Hwang and Wang* [2000], two wave systems straddling the wind vector exist. The swell condition in the dataset of *Hwang and Wang* [2000] is much milder than those encountered in *Walsh et al.* [1985, 1989], and may contribute to the observed differences. In any case, one expects a significant increase in the ratio between crosswind and upwind mean square slope components when waves are propagating at an oblique angle with respect to wind.

4. SUMMARY

The mean square slope dataset of *Cox and Munk* [1954] is regarded a masterpiece in the study of ocean surface roughness. It has been used as a major calibration reference for many areas of research, ranging from air-sea interaction and wave dynamics, to acoustic and electromagnetic remote sensing applications. One aspect of the dataset remains quite puzzling over the last half century or so is the large value of the crosswind slope component. For slick covered cases, the ratio between crosswind and upwind mean square slope components ranges from 0.75 to 1.03. The mean value with one standard deviation is 0.88±0.097. The large crosswind slope component cannot be explained by the unimodal directional functions established in modern wave spectral models (Fig. 1c). The mean ratio from three unimodal directional distribution functions [*Donelan* *et al.*, 1985, with modification by *Banner,* 1990; *Hasselmann et al.*, 1980; *Mitsuyasu et al.*, 1975] are 0.54±0.11, 0.65±0.12 and 0.65±0.13, respectively.

Directional spectral analysis of 3D ocean surface topography collected by an airborne scanning lidar system shows that in a mature sea, unimodal directional distribution exists only in a narrow wavenumber range near the spectral peak. For wave components shorter than the dominant wavelength, bimodality is a robust feature of the directional distribution. Nonlinear wave-wave interaction is the mechanism that generates the bimodal feature [*Banner and Young*, 1994; *Hwang and Wang*, 2000]. Using the bimodal directional distribution function, the calculated average ratio between crosswind and upwind slope components is 0.76±0.083, which is in much better agreement with field measurements. This mean value is still about 14% lower than the field data. Possible factors contributing to the observed larger crosswind slope components in the field data include fluctuation in the wind field and less-than-mature stage of the wave field. In both situations, wave components traveling at oblique angles from the wind vector contribute to the observed larger magnitude of the average crosswind slope component.

APPENDIX. BIMODAL DIRECTIONAL DISTRIBUTION FUNCTION

Hwang et al. (2000b) acquire 3D ocean surface topography using an airborne scanning lidar system. Based on data obtained under a quasi-steady wind field the directional distribution function at each wavenumber of the measured 2D spectrum is expressed in Fourier series,

$$D(k,\theta)=\frac{1}{\pi}\left[1+\sum_{n=1}^{N}A_n(k)\cos 2n\theta\right],\quad -\pi/2\le\theta\le\pi/2 \qquad \text{(A1)}$$

Coefficients for the third order polynomial fitting to each of the first 9 Fourier components, A_n, n=1, 2, ..., 9, are tabulated for reconstructing the bimodal function. The database that establishes the polynomial coefficients is limited to $k \le 10\ k_p$ so extrapolation too far beyond 10 k_p may produce large excursions in the directional function. The computation presented in this paper is based on extrapolation of the directional coefficients to $12k_p$, and the coefficients at $12k_p$ are used for the remaining higher wavenumber components in the computation range. The coefficients of the third order polynomial fitting of the first 9 Fourier components are tabulated below.

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Donelan, M. A., W. M. Drennan, and E. A. Terray, Wavenumber spectra of wind waves in the range of 1-50 m, in *The wind-driven air-sea interface*, ed. M. L. Banner, ADFA Document Producton Center, Camberra. Australis, 35-42, 1999.

Table A1. Third-order polynomial fitting ($y = c_1x^3 + c_2x^2 + c_3x + c_4$, where y is A_1, A_2, ... A_9, and x is k/k_p) of the Fourier components, A_1, A_2, ... A_9, of the bimodal directional distribution (A1).

	c_1	c_2	c_3	c_4
A_1	-6.83×10^{-4}	2.20×10^{-2}	-2.42×10^{-1}	9.87×10^{-1}
A_2	-2.66×10^{-3}	5.32×10^{-2}	-3.82×10^{-1}	7.83×10^{-1}
A_3	-1.44×10^{-3}	3.29×10^{-2}	-2.08×10^{-1}	3.26×10^{-1}
A_4	-1.13×10^{-3}	2.15×10^{-2}	-1.01×10^{-1}	1.17×10^{-1}
A_5	-7.22×10^{-4}	1.09×10^{-2}	-4.70×10^{-2}	5.96×10^{-2}
A_6	-9.04×10^{-4}	1.21×10^{-2}	-4.92×10^{-2}	7.40×10^{-2}
A_7	5.92×10^{-4}	-8.34×10^{-3}	2.75×10^{-2}	-9.78×10^{-3}
A_8	-1.10×10^{-3}	1.57×10^{-2}	-7.13×10^{-2}	9.80×10^{-2}
A_9	4.33×10^{-4}	-5.93×10^{-3}	2.06×10^{-2}	-1.52×10^{-2}

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Paul A. Hwang and David W. Wang, Oceanography Division, Naval Research Laboratory, Bldg. 1009, Stennis Space Center, MS 39529-5004.