A THEORY OF INFORMATIONAL EXCHANGES
- RANDOM SET FORMALISM -

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ABSTRACT

This paper describes a theory of data fusion in random-set formalism. Data fusion problems are defined as problems for estimating random sets of *targets*, i.e., an unknown number of objects whose states are to be estimated, based on information given in terms of random sets, i.e., a collection of sets of unknown numbers of observables with unknown origins. In this theory, information, i.e., a state of knowledge, is described, both *a priori* and *a posteriori*, in terms of random-set probability density functions, sometimes known as Janossy densities. Using this formalism, this paper considers an abstract distributed information processing system consisting of multiple information processing agents that, in addition to processing local information obtained through local information gathering sources, exchanges information with each other to achieve a globally optimal informational state collectively.

1. Introduction

Efforts to formalize distributed data fusion processing by multiple processing nodes (or data processing agents) were made in the 1980's under the DARPA-sponsored Distributed Sensor Networks Projects, as reported in Refs. [1] to [8]. In those efforts, not only network connectivity among processing nodes but also the precise timing of "who talks what to whom" is mathematically described, to model information flows in a given communication network. In short, these efforts were to describe information exchanges as precisely as possible, by describing temporal and spatial information changes as a directed graph, called an *information graph* ([8]).

In this theory of information exchange, an underlying estimation problem was formulated as a general abstract estimation problem on an appropriate abstract state space. However, the research was clearly motivated by multiple-sensor, multiple-target, tracking problems, or multiple-source, multiple-object, dynamic state estimation problems. Hence, in parallel to these efforts to establish a general theory of information exchanges, attempts were made to build a general theory of multi-target tracking, as a natural extension of a general nonlinear filtering theory, which we may view as a theory of abstract tracking with a known number of targets and a known origin for each unit of measurements.

These efforts to establish a general framework for multi-sensor, multi-target problems resulted in a general theory that expands the multi-hypothesis filtering algorithm described in the paper [9] by D. B. Reid. This general theory of multi-target tracking was reported in [10] and [11], and is

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14. ABSTRACT
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recently referred to as the MCTW filter in [12]. In this framework, targets, or objects to be estimated, are modeled as a *random finite sequence* in an abstract state space. This formalism was incorporated into the general theory of multi-agent estimation problems, mentioned above, and resulted in a general, distributed data fusion theory, as summarized in [8]. Subsequently, treatment of non-deterministic cases which had been left more or less ambiguous in [8] was explored in [13].

Recently, this general multi-target tracking theory based on the random finite sequence formalism was re-examined in view of a surge of interest in *random set formalism*, as seen in [14], [15], [16], and [17]. These attempts at re-examination resulted in the rewriting of the general multi-target tracking theory explicitly using the random-set formalism as established in [18] and [19], and they are described in [20], [21], and [22]. The *Point-process formalism* as seen in [14] and [23] is equivalent to the random-set formalism, as shown in [24]. At this point, however, despite several claims to providing new tracking algorithm development, [16] and [17], the practical benefits of using the random-set formalism remain, in my opinion, unknown. As a theory, however, the random-set formalism gives us a more consistent picture of various complicate problems, although it seems to some a mere rewriting of the same story in a different language.

The purpose of this paper is to re-examine the theory of information exchanges, as described in [8], using the random-set formalism established in [20], exploring the possibility of some advantage in using the random-set formalism in describing a general data fusion theory. In the next two sections, Sections 2 and 3, we will review the distributed data processing theory in [8], and the random-set formalism [20] of a general class of multi-target tracking problems, which is followed by Section 4 which re-writes the main result of [8] in the random-set formalism.

2. A Theory of Distributed Information Processing

This section reviews the general theory of data processing as described in [8]. Let us consider a finite set of data processing nodes, called *agents*. We call them agents rather than nodes mainly not to confuse them with nodes in the information graph described later. Each data processing agent has its own set of *sensors* and processes information gathered by those local sensors more or less autonomously. However, agents are connected by a communication network, and exchange information with each other, under a given set of rules.

We assume that information flow into this system of data processing agents is through the sensors, each of which is owned undisputedly by one of the agents. Thus the basic unit of information in this system is a triple (y,s,t) of a *data set* y which is an element in a measurable space[2] E_s associated with sensor s, the sensor index itself s, and a time index t. Then we call

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[2] In this paper, by a measurable space we mean a locally compact Hausdorff space satisfying the second axiom of countability. Every locally compact linear space is finite-dimensional. On the other hand, a locally compact Hausdorff space is a regular topological space, and if it satisfies the second axiom of countability, it is metrizable. Hence, in practice, we can consider such a space as a so-called *hybrid space*, i.e., a direct-product space of a Euclidean space (or its subset … to represent "continuous" states or observations) and a finite (or at most countable) set (to represent discrete components of states or observations). It is interesting to see that this topological property is necessary to develop a general theory of random sets ([18]) as well as to define conditional probabilities ([25]).

any finite set of data sets an *information set*. Each information set represents an accumulation of information gathered by sensors in the system. Thus we can describe information possessed by a data processing agent at a given time by a particular information set.

In order to describe temporal and spatial changes of information sets, it is convenient to use a directed graph called an *information graph*. Each element, called an *information graph node*, or simply a node, of a information graph represents an event in which some change in informational state, i.e., information sets in each data processing agent takes place, and is partially ordered by time and informational flow. For example, Fig. 1 shows a three-agent system in which agents A_1 and A_2 act as sensor processing agents, and agent A_3 is the higher-level data fusion agent. In this figure, the inputs from each sensor are represented by squares, while the informational nodes are depicted by circles, labeled as i_{jk}, where *j* refers to each agent and *k* is the ordering indices.

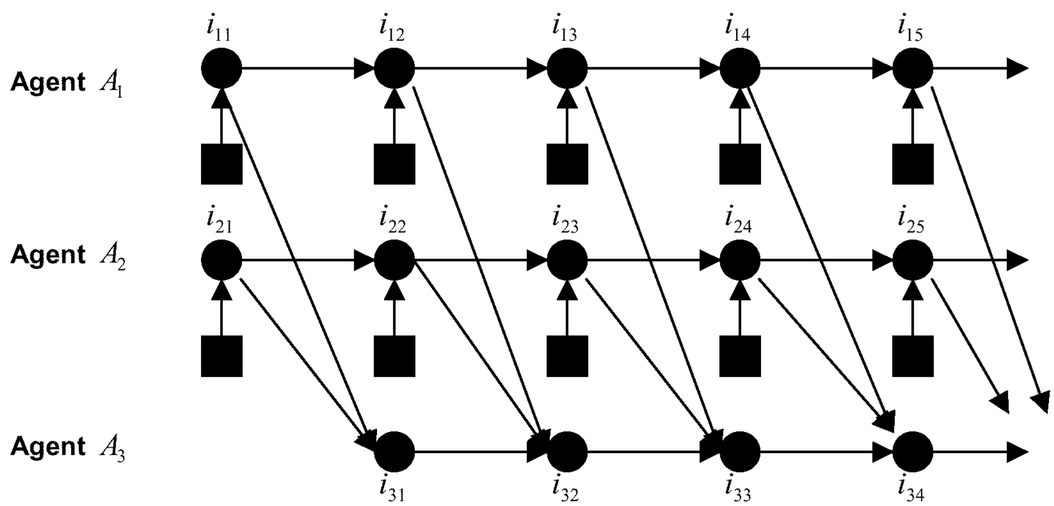

Fig. 1: Hierarchical Data Processing System

The horizontal lines (arcs) represent an accumulation of, and lack of loss of, information within each agent. The downward lines represent the informational flow from the subordinate agents to the superior agent. For each information node i_{jk}, let Z_{jk} be the information set accumulated at that node. Then, apparently, we have $Z_{3k} = Z_{1k} \cup Z_{2k}$.

Figs. 2 and 3 show examples in which agents are more or less equal to each other, but with different information exchange patterns, i.e., broadcasting and cyclic. Unlike Fig. 1, Figs. 2 and 3 have extra information nodes to represent information increases due to information received from the other nodes. In the broadcasting case of Fig. 2, each agent exchanges information with each other agent periodically so that, after each exchange, the information set at each agent becomes identical, i.e., $Z_{1k} = Z_{2k} = Z_{3k}$, for each $k = 2,4,...$ On the other hand, changes in the information sets possessed by each agent become much more complex when an asymmetric information pattern such as the cyclic information pattern shown in Fig. 3 is used.

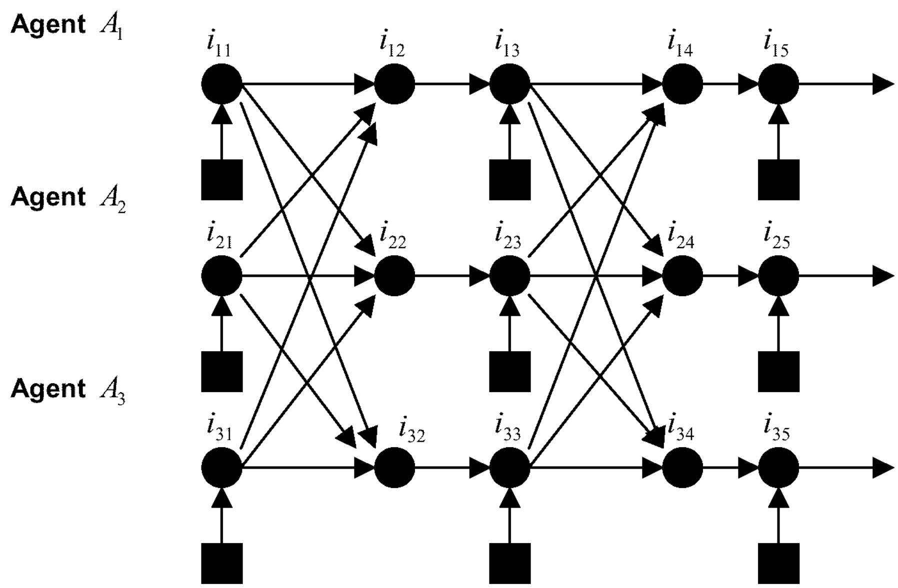

Figure 2: Broadcasting Communication

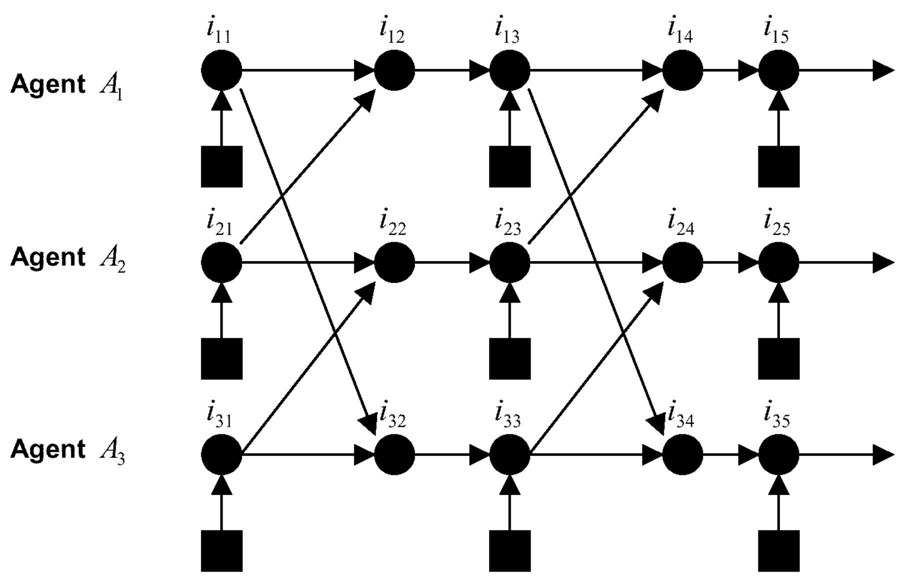

Figure 3: Cyclic Communication

Our problem is to obtain an efficient estimation method exploiting given information exchange patterns. Let us assume that the goal of the system as a whole, as well as the individual agents, is to estimate a random system state x in a *measurable state space*[3] E. We assume that this state

[3] Footnote 1 is applied to this space also.

space E, as well as each sensor measurement space Y_s, has an appropriate measure[4], and both system state distribution and each sensor data set distribution has a well-defined density function with respect to the given measure. We can assume that the system state can represent anything but the state is static for the moment. Moreover, all the data sets are conditionally independent, i.e.,

$$p(Z|x) = \prod_{(y,s,t)\in Z} p(y|x) \tag{1}$$

for any information set Z.

The following rather simple equation, a direct consequence of this assumption, is indeed the basis of the theory of information exchanges, as summarized in [8].

$$p(x|Z_1 \cup Z_2) = C^{-1} \frac{p(x|Z_1)p(x|Z_2)}{p(x|Z_1 \cap Z_2)} \qquad (2)^5$$

with the normalizing constant C, for any pair of information sets, Z_1 and Z_2. Eqn. (2) claims that when fusing two information sources, we first add the two but subtract the common information afterwards to avoid informational double-counting.

For every node i in a given information graph, we can define the information set Z_i at node i as the union of all the information sets at the nodes which precede node i. A node without any predecessor should be connected to a sensor input which becomes its information set. This definition assumes the perfect memory of each agent as well as perfect communication among agents. We can view eqn. (2) as an abstract binary *data fusion equation*, which can be generalized in the following form. For a given node $\hat{i}$ in an information graph, let I be the set of all the nodes preceding node $\hat{i}$. We can show ([6]) that there exists a set $\bar{I}$ of nodes preceding $\hat{i}$ and an integer-valued function α defined on $\bar{I}$ such that

$$p\left(x \middle| \bigcup_{i\in I} Z_i\right) = C_I^{-1} \prod_{\bar{i}\in\bar{I}} p(x|Z_{\bar{i}})^{\alpha(\bar{i})} \tag{3}$$

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[4] When the state space or any of the sensor measurement spaces are of "hybrid" type in the sense explained in Footnote 1, an appropriate measure is the direct measure of the Lebesgue measure on the continuous part and the counting measure on the discrete part.

[5] The conditioning by an information set may not be well defined without the conditional independence assumption (1). With this assumption, however, conditioning by $Z_1 \cup Z_2$ and $Z_1 \cap Z_2$ coincides with that by the upper bound $\sigma_1 \vee \sigma_2$ and the lower bound $\sigma_1 \wedge \sigma_2$ of the two σ-algebras generated by the information sets Z_1 and Z_2, respectively.

which we can use to fuse the information contained by the all the nodes in I to calculate the information at node $\hat{i}$.

The general fusion equation can be used for any communication pattern, or even for any kind of information exchange. This approach was tied to the general formalism of multi-target tracking using the random finite sequences as its basis ([1] – [8]), and applied to tracking of airborne targets by distributed network of acoustic sensors in collaboration with MIT Lincoln Laboratory's experimentation using "real" sensors and "real" targets, as described in [26] – [28].

3. Multi-Target Tracking in Random-Set Formalism

This section reviews the random-set formulation of a general class of multi-target tracking problems in an abstract form, as described in [20] – [22]. We will restrict ourselves to the Poisson-i.i.d. cases, i.e., a class of target models without *a priori* identification[6]. We redefine the system state as a random finite set X (rather than a point x) in a target state space E. A Poisson-i.i.d. model can then be written as

$$p(X) = e^{-\gamma(E)} \prod_{x \in X} \beta(x) \tag{4}$$

where γ is a finite measure on the state space E, called the *intensity measure*, having the density function β with respect to a σ-finite measure μ on the state space E. The real-valued function p defined on the space of all the finite sets in the target state space E through eqn. (4) is historically called the *Janossy density* [24]. In this section, as well as in the preceding and succeeding sections, we assume that the targets are static. Removal of this seemingly restrictive assumption will be discussed at the end of the next section. We will maintain this assumption to keep the arguments simple.

Each data set y introduced in the previous section is now redefined as a finite random set Y in the sensor measurement space E_s of a sensor s. Then each data set can be modeled as

$$p(Y|X) = \sum_{a \in A(X,Y)} \left(\prod_{x \in \mathrm{Dom}(a)} p_{st}^{M}(a(x)|x) p_{st}^{D}(x) \right) \left(\prod_{x \in X \setminus \mathrm{Dom}(a)} \left(1 - p_{st}^{D}(x)\right) \right) p_{st}^{FA}(Y \setminus \mathrm{Im}(a)) \qquad (5)^{7}$$

where $p_{st}^{M}(y|x)$ is the conditional probability density function of a measurement y when originating from a target at a state x, and $p_{st}^{D}(x)$ is the probability of a target at state x being

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[6] By targets without *a priori* identification we mean targets given as a random set, as opposed to the targets with *a priori* identification, which are modeled as a fixed, ordered array of individual target states. This distinction was first explicitly described in [11].

[7] For any function a, we denote its domain and its image by $\mathrm{Dom}(a)$ and $\mathrm{Im}(a)$. For any pair (X,Y) of sets, $A(X,Y)$ is the set of all the one-to-one maps from X to Y, i.e., $A(X,Y) = \bigcup_{D \subseteq X} \{a \in Y^D | \#(\mathrm{Im}(a)) = \#(D)\}$, with $\#(X)$ being the cardinality of any set X.

detected by a sensor s at time t. p_{st}^{FA} is the density function of the false alarm random set, and using a Poisson-i.i.d. model, can be written as $p_{st}^{FA}(Y)=e^{-\gamma_{sr}(E_s)}\prod_{y\in Y}\beta_{st}(y)$, where γ_{st} is the intensity measure on the sensor measurement space E_s of sensor s at time t, having the density function β_{st} with respect to the σ-finite measure on E_s. Using this sensor model, we explicitly exclude any split or merged measurements.

Then we can define any information set Z as a collection of data sets, each of which is a triple (Y,s,t) including a random finite set Y in E_s taken by sensor s at time t. The straightforward application of the Bayes' rule, coupled with the conditional independence assumption, yields

$$p(X|Z)=\frac{p(Z|X)p(X)}{p(Z)}=\frac{\prod_{(Y_s,s,t)\in Z}p(Y_s|X)p(X)}{p(Z)} \tag{6}$$

Therefore, the goal of each body of theory of multi-target tracking using this random-set formalism is to show how far we can break down this expression into a form that may be useful for practical problems, or at least useful in providing us with some insights into each particular problem.

The following form of the solution uses the concepts of *tracks* and *(data-to-data association) hypotheses*, as first defined in [29]. Those concepts are used as the basis for multi-hypothesis target tracking as described in [9] – [11]. First let the collection of *tagged measurements* in a given information set Z be defined by $\bigcup_{(Y,s,t)\in Z}Y\times\{(s,t)\}$. Namely, every element in the collection of tagged measurements in a information set Z is a triple, (y,s,t), of a measurement y included in a data set Y taken by sensor s at time t. We call any subset τ of the collection of the tagged measurements in an information set Z a track on Z if it contains at most one measurement from each data set in it, and we denote the set of all the tracks on Z by $\mathcal{T}(Z)$. Then a data-to-data hypothesis or simply a hypothesis λ on Z is a collection of non-empty and non-overlapping tracks on Z, and we denote all the hypotheses on Z by $\Lambda(Z)$.

Then the general solution described in [10] – [11] can be re-written as

$$p(X|Z)=\sum_{\substack{\lambda\in\Lambda(Z)\\ \#(\lambda)\leq\#(X)}}\hat{p}(\lambda|Z)\sum_{a\in A(\lambda,X)}\hat{q}(X\setminus\mathrm{Im}(a)|Z)\prod_{\tau\in\lambda}\hat{f}_\tau(a(\tau)) \tag{7}$$

where $\hat{p}(\lambda|Z)$ is the evaluation of each hypothesis λ on Z, determined as

$$
\begin{aligned}
\hat{p}(\lambda|Z) &= C_Z^{-1}\left(\prod_{\tau\in\lambda}\int_E \prod_{(s,t)\in K(Z)} g_{st}(x\,;\tau)\beta(x)\mu(dx)\right)\cdot\left(\prod\left\{\beta_{st}^{FA}(y)\middle|(y,s,t)\notin\bigcup\lambda\right\}\right) \\
&= C_Z^{-1}\left(\prod_{\tau\in\lambda}\int_E \prod_{(s,t)\in K(Z)} g_{st}(x\,;\tau)\gamma(dx)\right)\cdot\left(\prod\left\{\beta_{st}^{FA}(y)\middle|(y,s,t)\notin\bigcup\lambda\right\}\right)
\end{aligned}
\tag{8}
$$

with C_Z being the normalizing constant so that we have $\sum_{\lambda\in\Lambda(Z)}\hat{p}(\lambda|Z)=1$, $K(Z)$ being the set of all the indices in *Z*, i.e., $K(Z)=\{(s,t)|(Y,s,t)\in Z\}$, and each g_{st} being defined as

$$
g_{st}(x\,;\tau)=\begin{cases} p_{st}^{M}(y|x)p_{st}^{D}(x) & \text{if } (y,s,t)\in\tau \\ 1-p_{st}^{D}(x) & \text{otherwise}\end{cases}
\tag{9}
$$

for every $(s,t)\in K(Z)$, every $x\in E$, and every $\tau\in\mathcal{T}(Z)$.

The function $\hat{f}_\tau$ for each track $\tau\in\lambda$ in the last factor of eqn. (7) is the density of the target state distribution of the target hypothesized by track τ, and defined by

$$
\hat{f}_\tau(x)=\frac{\prod_{(s,t)\in K(Z)} g_{st}(x\,;\tau)\beta(x)}{\int_E \prod_{(s,t)\in K(Z)} g_{st}(x'\,;\tau)\beta(x')\mu(dx')}
\tag{10}
$$

for every $x\in E$ and $\tau\in\mathcal{T}(Z)$. The function $\hat{q}(\cdot|Z)$ in (7) is the (Janossy) density of the random set of targets that have remained undetected in the data sets in *Z*, which, we can show, is a Poisson finite random set (or Poisson point process) with the intensity measure $\hat{\gamma}(\cdot|Z)$ having the density $\hat{\beta}(\cdot|Z)$, defined as

$$
\hat{\gamma}(dx|Z)=\hat{\beta}(x|Z)\mu(dx)=\prod_{(s,t)\in K(Z)}\left(1-p_{st}^{D}(x)\right)\beta(x)\mu(dx)=\prod_{(s,t)\in K(Z)}\left(1-p_{st}^{D}(x)\right)\gamma(dx)
\tag{11}
$$

4. Distributed Data Fusion Equations In Random-Set Formalism

The results shown in the previous section claim that a set of sufficient statistics for the multi-target tracking problem given an information set *Z* is a triple, $\Sigma(Z)=\left(\left(\hat{p}(\lambda|Z)\right)_{\lambda\in\Lambda(Z)},\left(\hat{f}_\tau(\cdot)\right)_{\tau\in\mathcal{T}(Z)},\hat{\beta}(\cdot|Z)\right)$. The purpose of this section is to show how to generate the sufficient statistics when the information set *Z* on a given node in an information graph $\hat{i}$ is given as the union of the information sets on nodes preceding $\hat{i}$. The probability density function, described as the Janossy density, can be formally obtained by eqn. (3) of Section 2. We would like to have a set of expressions to express the sufficient statistics $\Sigma(Z)$ through the sufficient statistics $\Sigma(Z_{\bar{i}})$ of predecessor nodes $\bar{i}\in\bar{I}$, just as we derived (7) in place of the formal expression (6).

For a given information graph node $\hat{i}$ and the set I of immediate predecessors of $\hat{i}$, the first step is to identify the set $\bar{I}$ of the predecessor nodes of $\hat{i}$, with an integer-valued function α defined on $\bar{I}$, such that eqn. (3) holds. In fact, there may be many pairs $(\bar{I},\alpha)$ that satisfy such a condition. We would like to have a pair which gives the smallest node set $\bar{I}$. An algorithm to achieve it is described in [8]. Then the problem becomes that of expressing the sufficient statistics $\Sigma(Z)$ from $(\Sigma(Z_{\bar{i}}))_{\bar{i}\in\bar{I}}$. The second step is to construct the set $\mathcal{T}(Z)$ of all the tracks on Z and the set $\Lambda(Z)$ of all the hypotheses on Z from the collection $(\mathcal{T}(Z_{\bar{i}}),\Lambda(Z_{\bar{i}}))_{\bar{i}\in\bar{I}}$ of tracks and hypotheses given on the predecessor nodes $\bar{i}$ in $\bar{I}$. A rather straightforward algorithm to do this is described in [8].

For each "global" track $\tau\in\mathcal{T}(Z)$ and each predecessor information graph node $\bar{i}\in\bar{I}$, there exists a "local" track $\tau_{\bar{i}}\in\mathcal{T}(Z_{\bar{i}})$ such that $\tau_{\bar{i}}$ is the intersection of τ and the collection of tagged measurements in $Z_{\bar{i}}$. Using this notion, the individual target state distribution can be calculated as

$$f_\tau(x)=\frac{\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}\neq\varnothing}} f_{\tau_{\bar{i}}}(x)^{\alpha(\bar{i})}\right)\cdot\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}=\varnothing}} \hat{\beta}(x|Z_{\bar{i}})^{\alpha(\bar{i})}\right)}{\int_E\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}\neq\varnothing}} f_{\tau_{\bar{i}}}(x')^{\alpha(\bar{i})}\right)\cdot\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}=\varnothing}} \hat{\beta}(x'|Z_{\bar{i}})^{\alpha(\bar{i})}\right)\mu(dx')} \tag{12}$$

while the undetected target density is calculated as

$$\beta(x|Z)=\prod_{\bar{i}\in\bar{I}}\beta(x|Z_{\bar{i}}) \tag{13}$$

Finally using the normalizing constant that appears in eqn. (12),

$$L_{\bar{i}}(\tau)=\int_E\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}\neq\varnothing}} f_{\tau_{\bar{i}}}(x)^{\alpha(\bar{i})}\right)\cdot\left(\prod_{\substack{\bar{i}\in\bar{I}\\ \tau_{\bar{i}}=\varnothing}} \hat{\beta}(x|Z_{\bar{i}})^{\alpha(\bar{i})}\right)\mu(dx) \tag{14}$$

We can write the hypothesis evaluation equation as

$$\hat{p}(\lambda|Z)=C^{-1}\prod_{\bar{i}\in\bar{I}}\hat{p}(\lambda_{\bar{i}}|Z_{\bar{i}})^{\alpha(\bar{i})}\prod_{\tau\in\lambda}L_{\bar{i}}(\tau) \tag{15}$$

for each $\lambda\in\Lambda(Z)$, where, for each $\bar{i}\in\bar{I}$, $\lambda_{\bar{i}}$ is the "local" hypothesis on the information graph node $\bar{i}$, obtained by restricting the "global" hypothesis λ to $Z_{\bar{i}}$.

Up to this point, we consider the system state X (that is modeled by a random finite set) as stationary. In order to expand the results developed with this statics assumption into dynamic, and generally non-deterministic cases, we only need to replace the target state space E by the direct product space E^N by multiplying itself as many times as necessary. Namely, with $K(Z)$ being the set of all the data set indices, (s,t), pairs of sensor identifiers and observation times, by expanding the state space from E to $E^{K(Z)}$, we can treat any kind of target dynamics, at least theoretically. Needless to say, the direct implementation of this approach is impractical because of the potentially very high dimensionality of the targets state space. However, with a common assumption on Markovian target dynamics, the batch-processing type algorithm shown in the previous section can be reduced to a more familiar recursive form with an appropriate extrapolation step being inserted between two consecutive update processes.

As shown in [8], even with such a modification, the fusion equations shown in this section become no longer valid if the target dynamics are non-deterministic, except for very special cases of informational exchanges, such as immediate broadcasting-type information exchanges after each local observation as shown in [2]. A method for calculating the track-to-track likelihood function (14) in such non-deterministic cases is described in [13]. The algorithm described in [13] uses individual measurements stored in each track, thereby violating our "rule" that was set up in the beginning of this section, i.e., we should calculate the sufficient statistics of a given information set from those of a given set of predecessor information sets. In absence of better algorithms for non-deterministic cases, however, the track likelihood calculation described in [13] will remain useful in many cases where track-to-track association is more desirable than more traditional central report-to-track association (correlation) approaches.

5. Conclusion

A theory of information exchange, developed in the 1980's, [1] – [8], was revisited in view of the recent reformulation of multi-target tracking problems using random-set formalism. The random-set formalism provides us with deceptively simple formula both for usual Bayes updates and data fusion equations in distributed data processing systems. The simplicity in those equations is obtained by increasing dimensionality of the problems. For example, when the cardinality of the target space is M, the cardinality of the domain of the Janossy density for a random finite set is given by $\sum_{n=0}^{N} M^n / n!$ where N is the maximum cardinality of the random set, or the maximum number of targets.

These simple-looking equations are contrasted with significantly more complex expressions using the "traditional" concepts of tracks and hypotheses. Equivalence between a direct formula and its track-hypothesis representation can be shown but only through a long chain of derivation, which is omitted for this paper. The track-hypothesis approach used for multi-hypothesis algorithms have been often criticized for its fast growing complexity, and hence, processing requirements. The random-set formalism may solve this complexity growth problem by not using the combinatrics explicitly. At this point, however, it is not clear at all that the random-set formalism, as shown in this paper, actually provides an algorithm with less complexity than track-hypothesis-based multiple-hypothesis algorithms. To determine if the random-set formalism has any practical use besides some theoretical insights, further studies must be conducted, in particular, in complexity analysis and possibility of effective approximation techniques.

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