FUSION OF MULTI-SPECTRAL INFORMATION USING INVARIANT ALGEBRA

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ABSTRACT

The very large varieties of the multi-sensor scene signatures that needs to be considered for building a robust machine-based scene analysis system requires that one explores the possibility of deriving multi-sensor representations that are at the least invariant to scale, translation, and rotation with respect to the observer.

In this paper we develop methodologies for multi-sensor scene analysis based on the *theory of invariant algebra*. Two cases of similar and dissimilar sensor types are considered. The fused invariant for the case of similar sensor types are unchanged under material as well as under affine transformations. For the case of dissimilar sensors the approach leads to the derivation of the multi-sensor invariant expressions that remain unchanged under scale, translation and rotation with respect to the observer. Examples of the application of the approach on real multi-sensor data are presented.

1. INTRODUCTION

The large varieties of the multi-sensor scene signatures that needs to be considered for building a robust machine-based scene analysis system requires that one explores the possibility of deriving multi-sensor representations that are at the least invariant to scale, translation, and rotation with respect to the observer.

In this paper we develop methodologies for multi-sensor scene analysis based on the *theory of invariant algebra*. Two cases of similar and dissimilar sensor types are considered. The fused invariant for the case of similar sensor types are unchanged under material as well as under affine transformations. For the case of dissimilar sensors the approach leads to the derivation of the multi-sensor invariant expressions that remain unchanged under scale, translation and rotation with respect to the observer.

In the Case I the sensor types are assumed to be similar, correlated, co-located and registered. Hyperspectral images are good example of this Case. Case II deals with dissimilar sensor types where the imagery can be assumed to be uncorrelated. Laser range, passive IR , and Doppler imaging sensors are examples of this second case.

2. CASE I : SIMILAR SENSORS

From the Planck Law one has the following relationship between the emissivity, temperature, wavelength, and the spectral radiant emittance W_λ.

$$W_\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{\varepsilon_\lambda}{e^{\frac{ch}{\lambda KT}} - 1} \qquad (1)$$

Where T is temperature in degree Kelvin, ε_λ is spectral emissivity, h is the Planck's constant, and λ is the wavelength· In (1) only emissivity is material dependent.

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (FROM - TO)
01-01-1998	Conference Proceedings	xx-xx-1998 to xx-xx-1998

4. TITLE AND SUBTITLE
Fusion of Multi-Spectral Information Using Invariant Algebra
Unclassified

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Sadjadi, Firooz ;

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME AND ADDRESS
Lockheed Martin Corporation
Eagan, MN55121

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME AND ADDRESS
Director, CECOM RDEC
Night Vision and Electronic Sensors Directorate, Security Team
10221 Burbeck Road
Ft. Belvoir, VA22060-5806

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
APUBLIC RELEASE
,

13. SUPPLEMENTARY NOTES
See Also ADM201041, 1998 IRIS Proceedings on CD-ROM.

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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19. NAME OF RESPONSIBLE PERSON
			Public Release	7	Fenster, Lynn lfenster@dtic.mil
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER International Area Code Area Code Telephone Number 703767-9007 DSN 427-9007

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39.18

The scene radiation, $R_n(j,k)$ where n is the index indicating a particular spectral band is obtained by integrating (1) over different bands of frequencies. From a wide range of frequency bands (different n values) different spectral images corresponding to the same scene are obtained.

When the effects of the radiation reflectance are negligible or ignored and we assume that the scene is in thermal equilibrium the radiation varies only with ε_λ (and frequency) in a particular scene. The thermal equilibrium assumption is useful because target detection and classification, under this assumption cannot be performed only by exploiting the temperature differences between targets and background. The variations of ε_λ with frequency for different materials and paints are well documented [1]. Table I shows typical emissivity values of several material averaged over all wavelengths at 20° C.

TABLE I. Typical Emissivity of Various Material

Material	Emissivity
Water	0.96
Wet Soil	0.95
Concrete	0.92
Steel	016
Wood	0.90
Glass	0.94

When the effects of radiation reflectance can not be ignored the thermal equilibrium assumption is no longer useful for scene analysis.

The out put of a focal plane array (FPA), in general, is a linear function of the incidence photons [6]:

$$N_{ij} = K_{ij} \int_{\lambda_1}^{\lambda_2} \zeta_{ij}(\lambda)\{\varepsilon_\lambda(i,j)W_\lambda(T_{ij}) + [1-\varepsilon_\lambda(i,j)]W_\lambda(T_b)\}d\lambda + N_{ij}^d \qquad (2)$$

Where N_{ij} is the total number of accumulated electrons at pixel ij, K_{ij} is a coefficient that is dependent on the active pixel area, optical transmission,, frame time, and pixel angular displacement from the optical axis, and the f-number of the optics. The quantum efficiency of the ij pixel is denoted by $\zeta_{ij}(\lambda)$, and background radiant reflectance is shown by $W_\lambda(T_b)$. T_b is the background temperature in degrees Kelvin, and λ_1 and λ_2 define the spectral band of the sensor. Finally, the dark charge for the pixel ij is denoted by N_{ij}^d. The $\varepsilon_\lambda(i,j)$ indicates the spectral emissivity at the pixel location ij.

The effect of the spectral emissivity- and its variations with frequency for different materials-can be seen in the equation (2).

For each pixel $\varepsilon(i,j)$, an n-dimensional vector (n being the number of frequencies used), consider the probability of it being from a material π_K, k being the number of different materials in the scene, be denoted as $p(\varepsilon|\pi_k)$ and the probability of material occurrence π_K as $p(\pi_K)$. Then according to the Bayes Decision rule one has to select the following:

$$\max_k \{p(\pi_k|\varepsilon) = \frac{p(\varepsilon|\pi_k)p(\pi_k)}{p(\varepsilon)}; \forall_k\} \qquad (3)$$

$p(\varepsilon|\pi_k)$ is assumed to be known for each frequency k at a range of temperatures of interest. This assumption is not restrictive since for different material (and paints) the emissivity as function of frequency and temperature has been documented [1]. The $p(\varepsilon)$ is obtained from the following:

$$p(\varepsilon) = \sum_{\pi_k} p(\varepsilon|\pi_k)p(\pi_k) \tag{4}$$

Once for each pixel a material label has been chosen a new image is formed. This image is formed by replacing the value of each pixel wit its most likely emissivity label (iron=1, water=2, etc.). Denoting each pixel as π_k (i,j) the information content of the image varies by the frequency of occurrence of the emissivity label in the image.

3. MOMENT GENERATING FUNCTION AND INVARIANT ALGEBRA

If the joint characteristic function of random variables x and y has a Taylor-series expansion valid in some region about the origin, it is uniquely determined in this region by the moments of the random variables [2] :

$$M_{xy}(u,v) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} E(x^n y^m) \frac{(u)^n (v)^m}{n!m!} \tag{5}$$

Hence knowing $M_{xy}(u,v)$ would imply knowing $\pi_k(x,y)$, the joint probability density function of the x and y random variables. This implies that one can represent the probability density function denoting an image in terms of a bi-variate homogenous polynomial referred to as a binary form in terms of the image statistical moments. The changes in the orientation, scale or position in the field of view of the objects in this new image, or in other words the change of the coordinate system will transform the probability density function $\pi_k(x,y)$ accordingly.

What remains unchanged under these transformations fall under the domain of *theory of invariant algebra*. The objectives pursued under this branch of algebra, developed in the 19th Century by Cayley and Silvester, is the study of the algebraic expressions that remain unchanged under linear transformation of the coordinate systems [3,4].

If a binary p-ic (polynomial of two variables and order p) has an invariant

$$f(a'_{p0},...,a'_{0p})=\Delta^{\omega}f(a_{p0},...,a_{0p}) \tag{6}$$

then the moments of order p has an algebraic invariant

$$f(\mu'_{p0},...,\mu'_{0p})=|J|\Delta^{\omega}f(\mu_{p0},...,\mu_{0p}) \tag{7}$$

where J is the Jacobian of the transformation Δ and ω are constants.

Using the tools of invariant algebra the following seven expressions that are invariant under size, rotation and translation can be obtained [5].

3.1. Second and Third Order Invariants

Defining the central moment of order p+q as:

$$\mu_{pq} = \sum_{x} \sum_{y} (x - \bar{x})^p (y - \bar{y})^q \pi_k(x,y) \tag{8}$$

The normalized central moments are:

$$\eta_{pq}(\pi_k) = \frac{\mu_{pq}}{\mu_{00}^{\frac{p+q}{2}} + 1} \tag{9}$$

The first and the second Order Invariants are then

$$\phi_1(\pi_k) = \eta_{20} + \eta_{02} \tag{10}$$

$$\phi_2(\pi_\kappa) = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2) \tag{11}$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} - \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03)})$$

$$[3(\eta_{30} + \eta_{12})^2 + (\eta_{21} - \eta_{03})^2]$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})^2$$

$$\phi_7(\pi_k) = (3\eta_{21} - \eta_{30})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2]$$
$$+ (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \tag{12}$$

4. INVARIANCY TO MATERIAL CHANGES

For the Case I, the above functions are 1)themselves functions of the material that the targets and scene are made of and 2) are k-ary forms. Consequently for any linear transformation in π_k (changes in material) there exist a set of invariant expressions that will remain unchanged. These Second Order Invariants will be Invariants under scene rotation, scale, translation and material transformations. The expressions $\phi_1(\pi_k)$ to $\phi_7(\pi_k)$ are polynomials of order 1 to 4 in terms of π_k for various k. Each of different k values indicate a different material. Any linear transformation of π_k indicates a change of material in the scene such as changing the paint on a target, or having the objects on a dry land verses wet land, or for a target being on a grass verses being on a concrete background.

5. CASE II: DISSIMILAR SENSOR

For this case the output of the sensors can be assumed to be non-correlated. This is valid for the case of colocated, registered passive IR , Laser range, and Doppler imaging sensors.

One can assume that in this case each image represents a conditional density function $p(f(x,y)|\lambda)$. Where λ represents a particular imaging sensor output. Then the effect of using all of the sensory outputs is equivalent to the use of the total probability function. The total probability density function can be obtained by:

$$p(f(x,y)) = \sum_\lambda p(f(x,y)|\lambda)p(\lambda) \tag{13}$$

The fused invariant expressions then can be extracted from this total probability function.

Figures 1-3 show the co-located, and registered outputs of a particular laser, Doppler and passive IR imaging sensors obtained from the US Army's Night Vision and Electro-Optical Center. The Table II shows their corresponding seven invariant expressions. These invariants can be seen to be different for different sensory outputs.
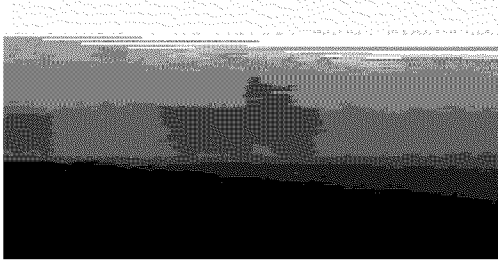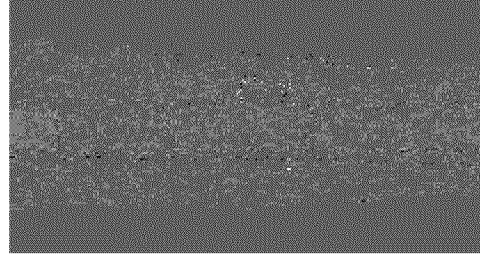
Figure 1. Laser Range Image
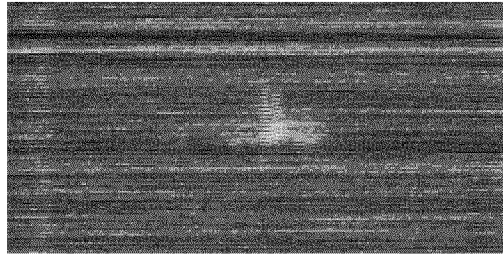

Figure 2. Doppler Image

Figure 3. Passive IR Image

Figures 4-7 show the total probability image at four different orientations. Their corresponding invariant expressions are summarized in the Table III. As can be seen the fused invariants remain as expected unchanged under the orientation changes.

TABLE II. Invariants of the Conditional Probabilities

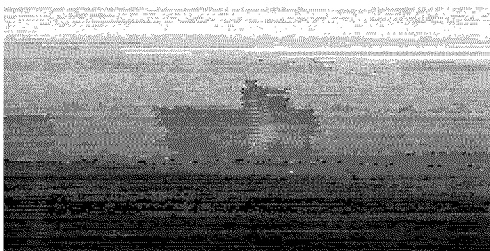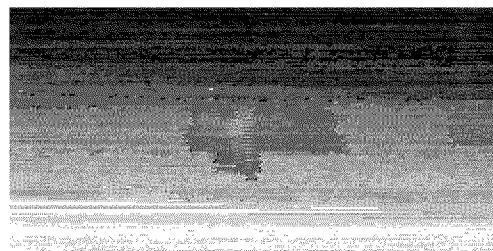| INVARIANTS	RANGE	DOPPLER	IR
$\Phi 1$	12381.447771	13568.759615	13354.107680
$\Phi 2$	89364967.289028	65854066.127031	62477079.926206
$\Phi 3$	1214.153333	0.287704	99.834628
$\Phi 4$	1679.326671	0.348603	50.435774
$\Phi 5$	-2175770.238461	0.085832	3499.153708
$\Phi 6$	863328.796422	2824.946900	348173.107680
$\Phi 7$	554695.407939	-0.095781	-812.440325

Figure 4. Total Probability Image

Figure 5. Total Probability Image
180 Degrees Rotated

Figure 6. Total Probability Image
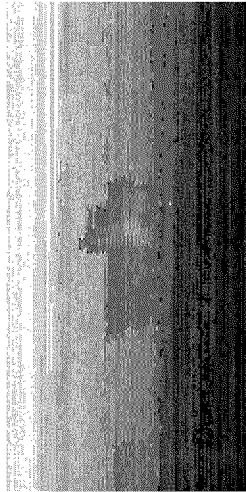Mirrored Around Vertical Axes

Figure 7. Total Probability Image 90 Degrees Rotated

TABLE III. Invariants of the Total Probability at Four Different Orientations

INVARIANTS	Original Position	90 Degrees Rotation	180 Degrees Rotation	Mirror around Vertical Axes
$\Phi 1$	12736.175825	12736.175825	12736.175825	12736.175825
$\Phi 2$	81187446.960661	81187446.960661	81187446.960661	81187446.960661
$\Phi 3$	1070.623457	1070.623457	1070.623457	1070.623457
$\Phi 4$	1162.027092	1162.027092	1162.027092	1162.027092
$\Phi 5$	−1204674.097088	−1204674.097088	−1204674.097088	−1204674.097088
$\Phi 6$	−2936904.647600	−2936904.647600	−2936904.647600	−2936904.647600
$\Phi 7$	531810.313835	531810.313835	531810.313835	531810.313835

The testing of the presented procedure to answer the issues of computational complexities verses storage requirements needed in general model/template techniques is currently underway. It is however an accepted fact that one can not store signatures at all conceivable variations in scene and target geometry and for every variations in their material contents. The procedures presented here are attempts to address the combinatorial explosive problem of signature storage without sacrificing the target classification performance requirements The number of Invariants needed for scene/target representation is a function of the target types and scene complexity. For example for separating targets such as tank versus truck the second and third order Invariants may be sufficient. However discriminating among similar type targets such as tank versus tank, requires higher order Invariants.

6. SUMMARY

In this paper a methodology for multi-sensor scene analysis, based on the *theory of invariant algebra* is presented.

Two different cases are considered. For the case of similar sensors such as hyperspectral imaging sensors the methodology leads to the representations that are Invariants to scene material s as well as scale, rotation, and translation with respect to the observer.

For the case of dissimilar imaging sensors, such as laser range, Doppler and passive IR the presented methodology leads to the derivation of sensor fused Invariants.

The use of these approaches can substantially reduce the number of frames/templates needed for multi-sensor/multi-spectral scene analysis, and target classification by their exploitation of the scene/ target invariant properties.

7. REFERENCES

[1]. R. Hudson, Jr. *Infrared System Engineering*, John Wiley & Sons, New York, 1969.

[2]. W. B. Davenport, Jr. , and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*, IEEE Press, 1987.

[3]. G. B. Gurevich, *Fundamentals of the Theory of Algebraic Invariants*, Noordhoff LTD, The Netherlands, 1964.

[4] .A. I. Borisenko, and I. E. Tarapov, *Vector and Tensor Analysis with Applications,* Dover Publications, New York, 1979.

[5]. F. Sadjadi, E. L. Hall, "Numerical Computations of Moment Invariants for Scene Analysis," *Proceedings of the IEEE Conference on Pattern Recognition and Image Processing,* 1978.

[6]. J. M. Mooney, F. D. Shephered, W. S. Ewing, and J. Silverman, " Responsivity Nonuniformity Limited Performance of Infrared Staring Camera," *Optical Engineering.* Vol.28 No. 11, November 1989.