Final Grant Report:

Computational Strategies for Treatment of Transient Interfacial Behavior

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Abstract

This research had as its primary objective the development of rigorous physical descriptions and corresponding numerical strategies for the treatment of dynamically contacting solid and structural entities, and produced useful new algorithms for description of dynamic frictional response, coupled thermomechanical interfacial behaviors, elastic and inelastic impact interaction, and tribological complexity. The unifying goal of this work was to assure that problems involving transient impacts between solids could be numerically solved with the same level of robustness, accuracy, and physical insight as is currently brought to bear in other areas of nonlinear mechanics. The results obtained in this work have importance well beyond contact mechanics, extending also to other applications where computational models of neighboring, but distinct, domains must be coupled together in a numerically accurate manner.

Contents

1 Objectives and Accomplishments of this Research

This research had as its overarching goal the development of contact/impact algorithms for computational nonlinear solid mechanics, such that finite element simulation of interfacial phenomena could be accomplished with the same degree of robustness, accuracy, and reliability currently present in algorithmic treatments of such features as inelasticity, large deformation response, and dynamics. The effort has relevance not only to contact mechanics, but also to other applications (such as phase transformation, surface adhesion and debonding, fragmentation and fracture, and structural composite response) where distinct domains must be coupled together in a numerically accurate manner.

Specific achievements of this research, fitting under the aforementioned overall objective, included:

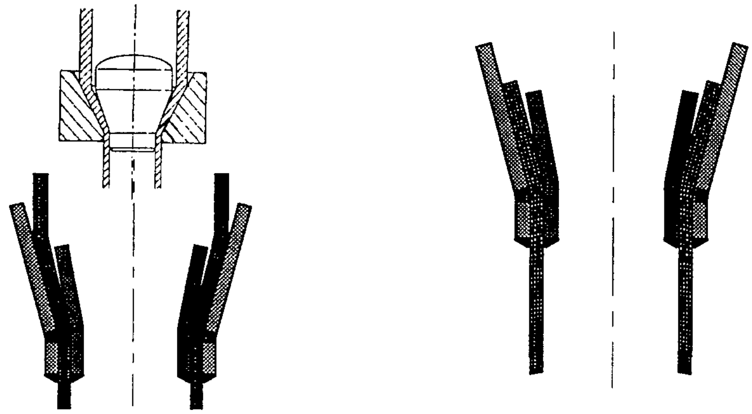
- Reliable treatments of dynamic frictional response, including new interface constitutive laws that describe slip instability, as well as corresponding numerical algorithms that treat such models in a stable manner
- Improved numerical description of coupled thermomechanical interfacial behavior, treating in particular frictional heating and thermal softening associated with friction and developing both monolithic and partitioned solution strategies for such problems;
- New finite element algorithms for impact interaction, considering the energy-momentum paradigm as a new framework in which unconditional stability of impact calculations could be assured and in which partitioning of energy and momentum during impact events (both elastic and inelastic) was accurately predicted; and
- New methods for spatial discretization of contact problems, developing new means by which stresses on interfaces could be accurately approximated and by which dissimilar domains can be reliably joined numerically.

Subsequent sections will describe advances made in each of these areas in somewhat more detail.

2 Technological Issues Considered in this Project

2.1 Computational Treatment of Stick-Slip Instabilities in Nonlinear Mechanics

It has been known for centuries that under conditions of constant pressure across an interface, the frictional force required to sustain kinetic frictional sliding is generally lower than that required to initiate it (see, e.g., [Eul48]). Indeed, it is also often observed that the apparent coefficient of friction prevailing on an interface is dependent on sliding velocity, although for many metals, over at least certain ranges of sliding velocity, this variation occurs slowly enough so that the "kinetic" coefficient of friction might be taken as a constant, in general lower than the "static" coefficient ([Rab65]). In general, the capability to introduce functional rate dependence of frictional stress is also desirable in many applications.



(a) (b)

Figure 1: Tube drawing simulation: (a) schematic representation of the drawing procedure and initial configuration of the mesh; (b) final configuration of the mesh.

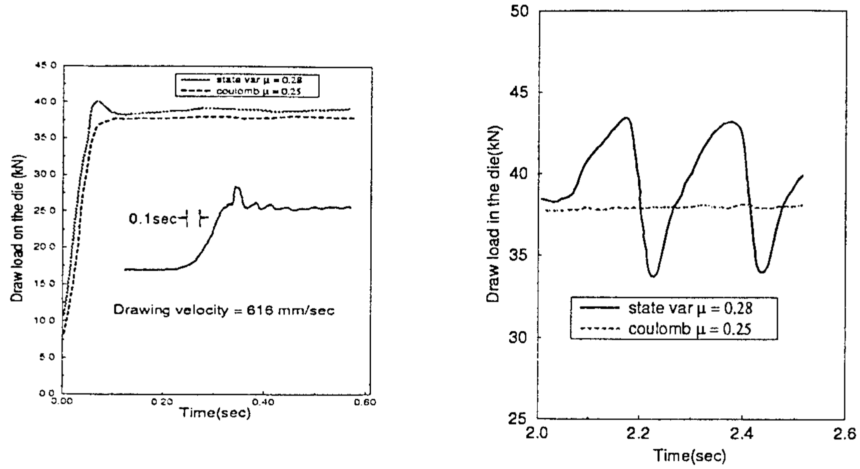
Complicating the picture somewhat, however, is the experimentally observed fact that a simple one-to-one relation between friction coefficient and instantaneous sliding rate is not appropriate for many interface combinations. In particular, as examples presented in [OM85] show, the apparent friction coefficient in a dynamic setting is often not a reversible function of the sliding velocity, but displays hysteretic behavior instead.

Our work has considered the use of *rate and state dependent* friction laws, of the sort pioneered by [Die79] and [Rui83] in the geophysics community, for engineering applications of nonlinear mechanics. The models developed here have the following two characteristics:

1. the apparent coefficient of friction at a given steady state velocity is a single-valued (generally decreasing) function of that velocity through the state variable definition; and

2. responses to more sudden changes in slip velocity occur on a length scale related to typical asperity spacings on the surface, with the relationship between friction stress and slip rate being of a viscous nature over short time scales

A typical application of our methodology is shown in Figures 1 and 2, which collectively depict a tube drawing example. Of particular note is the inability of a traditional Coulomb description to describe intermittent "stick-slip" frictional behavior, which manifests itself in this case in experimentally observed chatter of the drawn part against the machine tools. The rate and state dependent friction model developed in this research reproduces the experimentally observed behavior, and in the presence of large deformations and significant inelasticity.



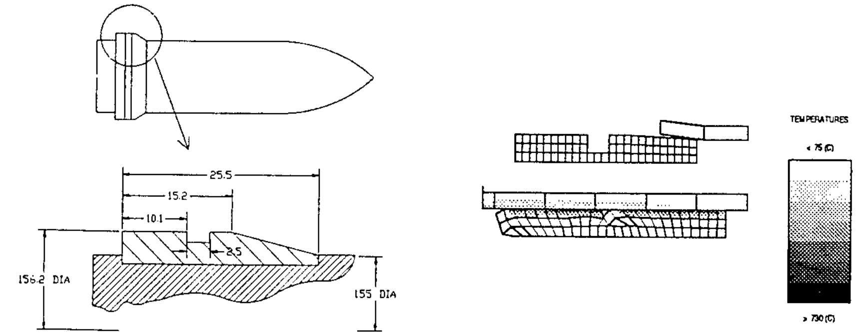
(a) (b)

Figure 2: Tube drawing simulation: (a) computed force on the die during early deformation, with experimental data of [Coo81] shown in inset; (b) computed drawing force on the tube, after some length has already been drawn.

2.2 Robust Algorithmic Strategies for Coupled Thermomechanical Friction on Interfaces

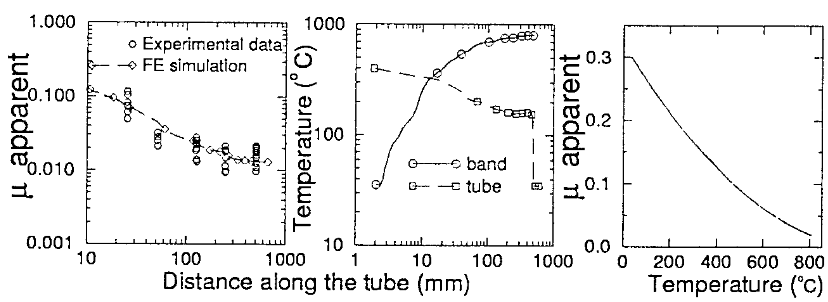
Another product of this research was been the extension of computational methods to encompass thermomechanical effects on interfaces, such that frictional heating, thermal frictional softening, and coupling effects are explicitly considered by the formulation. A full treatment of both thermal and mechanical effects was included in our modeling approach, such that balance of momentum and balance of energy were enforced throughout the contacting continua.

A key factor in the approach followed in this research was the development of discrete numerical representations obeying underlying thermodynamical restrictions. In particular, *a priori* stability estimates were used to guide the development of integrators for the frictional equations of evolution, and to extend such estimates to the thermally coupled case so that unconditionally stable staggered algorithms for thermomechanical friction problems resulted. The overall algorithmic approach was similar to that utilized by [AS93] in their study of thermoplasticity in bulk materials, but required development of the appropriate interface kinematics and thermodynamical restrictions to enable new stability estimates to be constructed and numerically approximated (see [Lau99]). An example benefiting from the technology developed is shown in Figures 3 and 4, which depict a finite element simulation of a Howitzer shell firing. Here, the frictional heating and subsequent thermal softening of the shell/barrel interface is a key factor in the performance of the system during firing.



(a) (b)

Figure 3: Frictional heating in a 155 mm Howitzer: (a) schematic representation of projectile and dimensions of the band; (b) undeformed initial mesh for the band and tube (upper) and deformed configuration and temperature map after 50 mm of sliding.



(a) (b) (c)

Figure 4: Projectile-barrel problem: comparison with available experimental data. (a) Evolution of the average apparent coefficient of friction as the projectile advances along the tube: finite element simulation and experimental data compiled from [Mon76]; (b) average temperature evolution at the surface of the band as sliding proceeds along the tube, showing also the temperature distribution for the tube after 500 mm of sliding; (c) average apparent coefficient of friction as a function of temperature.

2.3 New Energy Consistent Algorithms for Inelastic Impact Simulation

The new paradigm we have developed for inelastic impact simulation has its origins in the energy-momentum method for elastodynamics, as originally proposed by Simo and Tarnow [ST92]. As originally conceived, the method was intended to ensure unconditional stability in the fully nonlinear regime, while also providing for exact conservation of linear and angular momentum for autonomous dynamical systems. In the work done under this contract, the paradigm has been extended not only to provide for unconditionally stable treatment of impact interactions, but also to allow for inelastic dissipation (giving rise to energy consistency, rather than conservation) both on contact interfaces and in the bulk continua (see [ML02]. Furthermore, introduction of temporal discontinuities in the velocity-displacement updates utilized on contact interfaces, in a manner consistent with the physics being simulated and the conservation laws being enforced, has produced a new *velocity update algorithm* [LL01] which provides arbitrarily accurate enforcement of contact constraints, giving an unprecedented combination of stability and accuracy.

In impact mechanics, each local contact event introduces a jump in the contact velocities, with the magnitude of the jump being related to the material properties of the contacting entities (in an elastic analysis, these would be the impedances of the bodies in question). Our work has shown that algorithmic counterparts of these jumps can be introduced into the velocity-displacement relationship on interfaces, with the magnitude of the jumps being found from the global conservation conditions. The algorithm is applicable to any given constitutive description, and is designed for fully nonlinear applications in large deformations. It can enforce the impenetrability conditions associated with contact arbitrarily closely, while also retaining all conservation properties. An example is given in the two ring impact example in Figure 5, which depicts also the accurate enforcement of the impenetrability conditions in a typical time step (inset). The evolution of the total system energy and angular momentum is shown in Figure 6.

Another key achievement pertaining to this portion of the project was the development of an energy consistent algorithm for finite strain plasticity, such that bulk inelasticity could be simulated within the energy momentum framework. Indeed, our previous work on frictional contact algorithms had shown that dissipative phenomena could be treated within the energy momentum framework by forcing a discrete version of an a priori stability estimate to be algorithmically obeyed. In this research, we succeeded in implementing this idea for two classes of inelastic solids: an additively decomposed hyperelastic-plastic constitutive law based on a plastic Green's strain, and a model based on the multiplicative decomposition of the deformation gradient into elastic and plastic components. An inelastic impact simulation benefiting from this methodology is shown in Figures 7 and 8; here, it is seen that a linearly stable time integrator can produce unstable results in the fully nonlinear regime even in the presence of potentially significant inelastic dissipation. The energy consistent algorithms developed in this work are seen to eliminate this possibility, producing dissipation in a manner consistent with the material and interface laws in use.

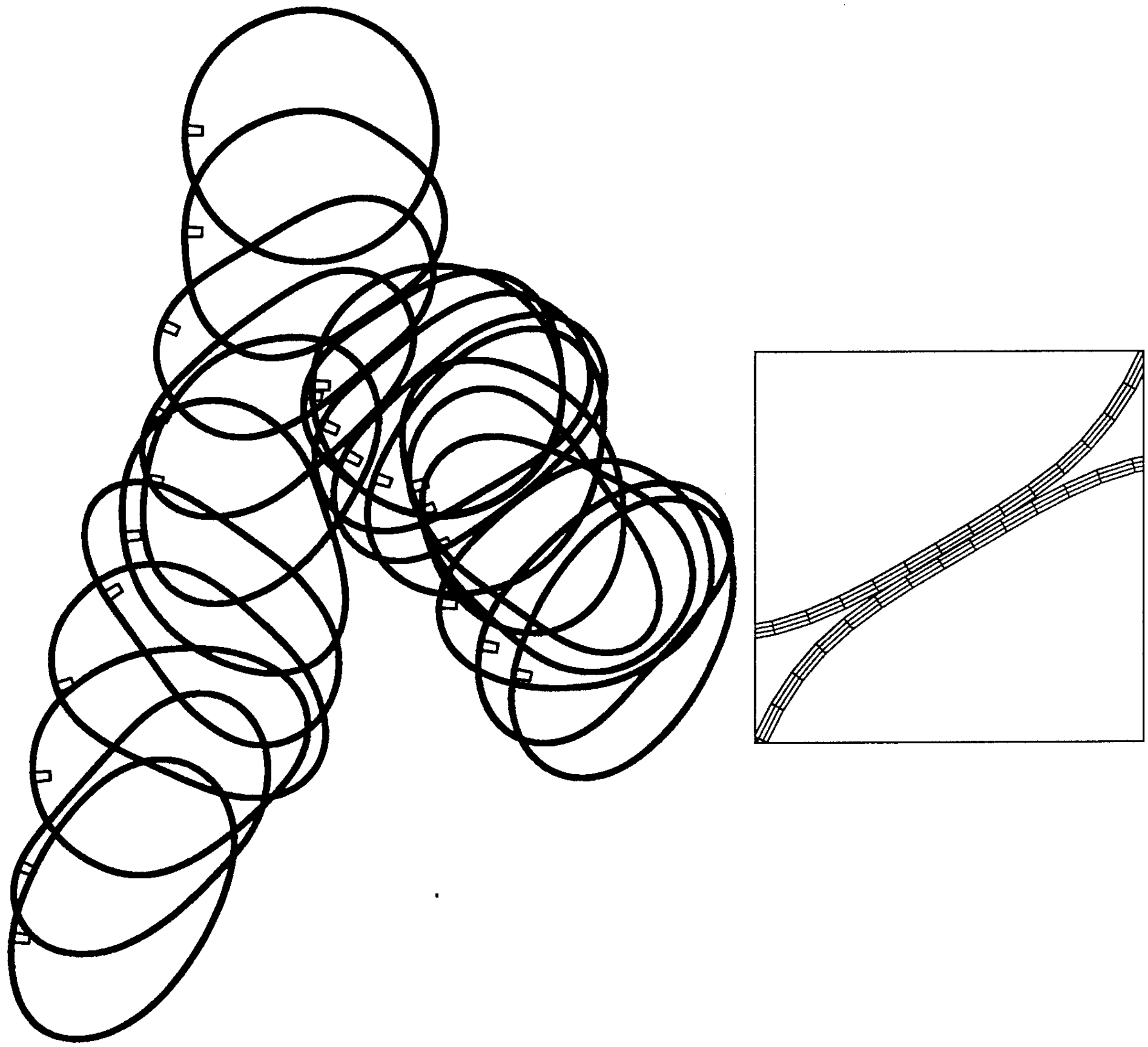


Figure 5: Two ring impact problem, dynamic configurations, $t = [0, 2, 4, ..., 20]$. The inset shows a zoom of the contact interface at a typical time step. The velocity update algorithm is seen to produce stable results and to facilitate accurate constraint enforcement.

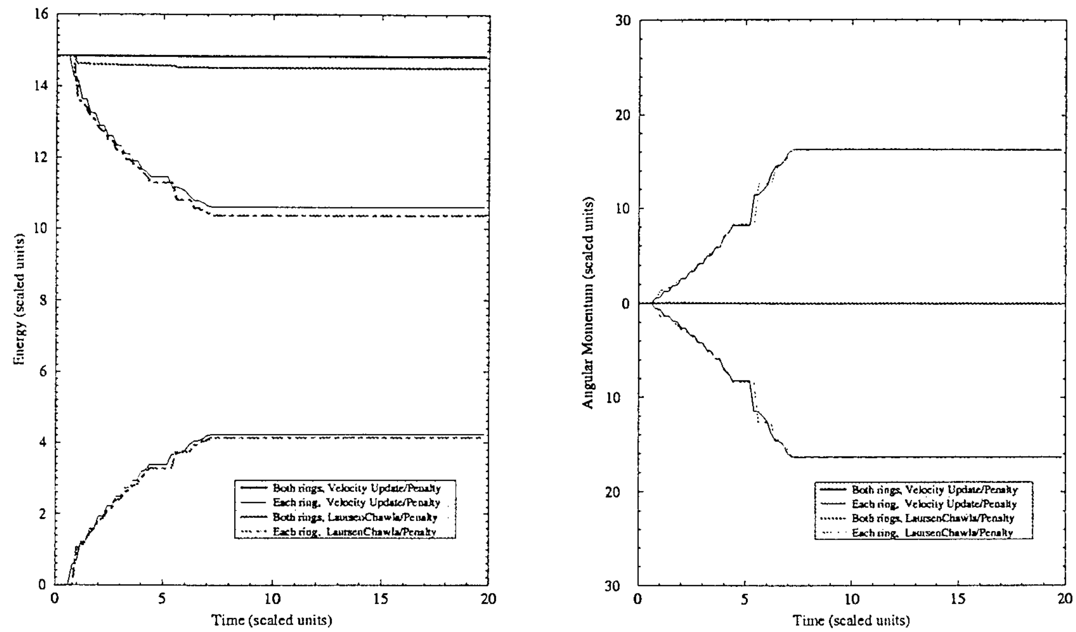


Figure 6: Two ring impact problem: (a) energy evolution, (b) angular momentum evolution.

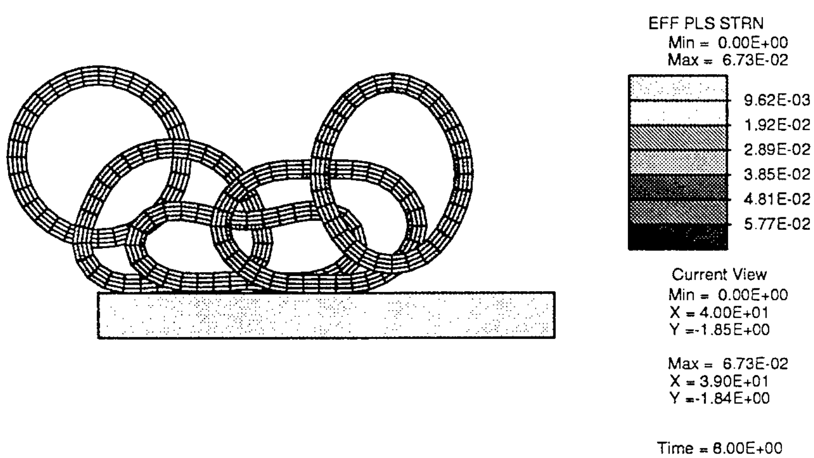
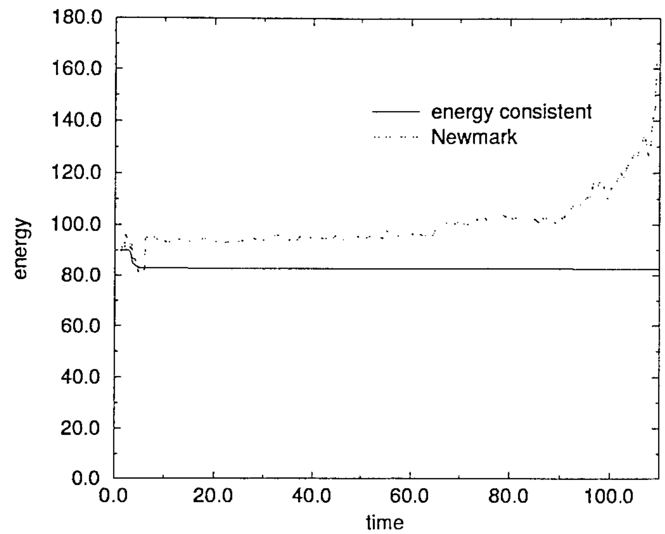
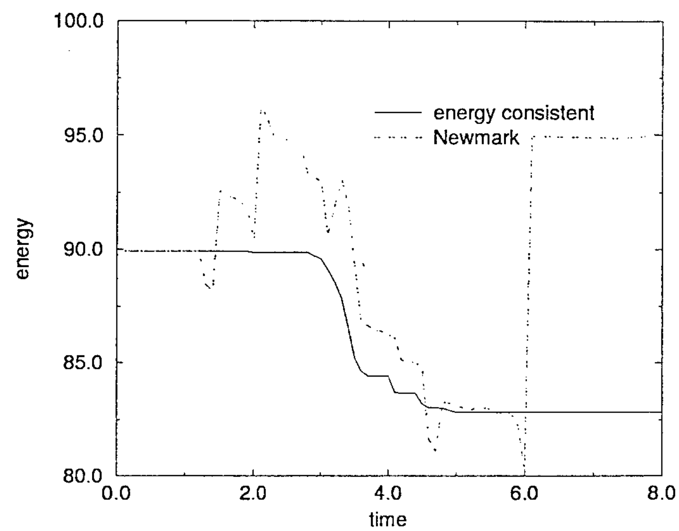


Figure 7: Deformed sequence corresponding to an elastoplastic ring impact calculation, using the energy consistent algorithm developed in this research.



(a)



(b)

Figure 8: Energy evolution in the elastoplastic ring impact problem, using energy consistent and Newmark algorithms: (a) the whole history of evolution; (b) enlarged plot showing energy evolution during impact.

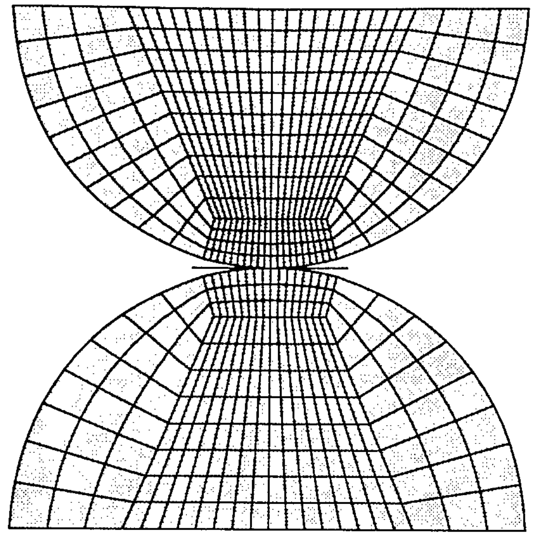


Figure 9: Discretization, cylinder on cylinder contact problem.

2.4 New Spatial Discretization Strategies for Contact Interaction

The mortar element method, previously developed for domain decomposition problems, was extended in this project to provide a new technique for the spatially discretization of contact interactions. The method relies on an intermediate contact surface, over which all contact variables (kinematic and traction) are defined. These contact variables are related via a variational projection to their counterparts on the two contacting bodies, such that the contact constraints (enforced over the intermediate surface) are related in a nonlocal manner to the displacement and force quantities on the surfaces of the contacting bodies. This procedure results in an accurate and variationally consistent algorithm to recover contact pressures on contact surfaces, and ensures passage of contact patch tests in general. A frictional example depicting the effectiveness of the procedure is shown in Figures 9 and 10, where a frictional Hertzian example is considered. As can been seen, the contact tractions are very accurately represented using this new procedure, even though the meshes of the contacting bodies are nonconforming throughout the contact patch.

Another technological issue associated with spatial discretization of contact interaction is seen when tangential relative motion occurs between two contacting boundaries. In this instance, sliding must take place in general across nonsmooth regions created by the C^0 finite element discretization, which gives rise to a lack of numerical robustness and/or oscillations in the contact tractions. For any reasonable description of tribological complexity (e.g., wear, lubrication) to take place, these issues of spatial discretization should be resolved. This research succeeded in developing a new Hermitian smoothing method for contact interactions, relying on an underlying variational principle to consider alternative (geometrically smoothed) representations of contact geometry. The result is greatly enhanced numerical robustness, as well as a significant decrease in observed numerical oscillations. An example of the application of such ideas is given in Figures 11 and 12, which depict a drawbead clamping operation such as is commonly encountered in metal forming. As can be seen, the

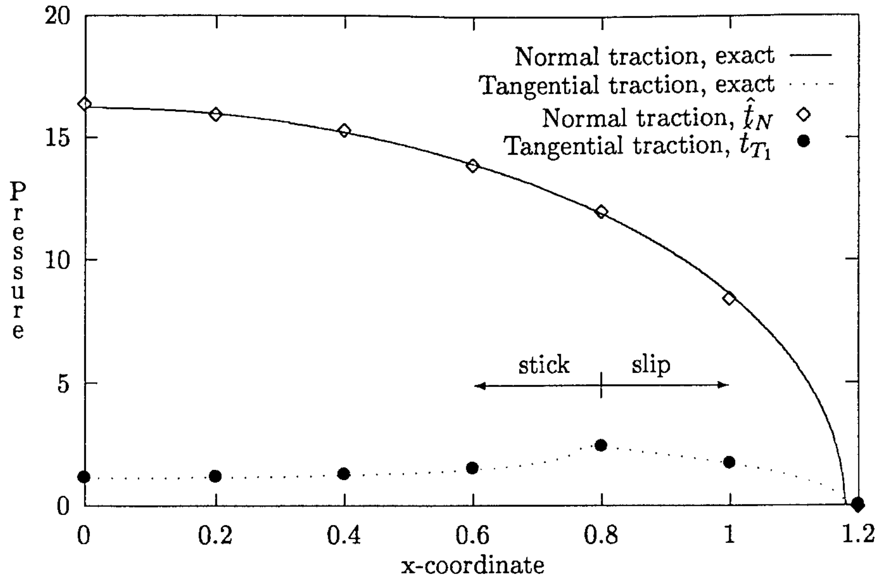


Figure 10: Computed mortar surface nodal tractions for frictional cylinder on cylinder contact problem.

contact smoothing procedure gives rise to a significant improvement in the numerical results obtained, giving an improvement also by about a factor of four in computational time.

3 Statement of Relevance to the Navy

Among the Navy applications requiring good interfacial descriptions are: weapons effect calculations, durability studies, predictions of various machining and materials processing operations, characterization of nonlinear vibrational behavior of various onboard systems, and even description of solid/fluid interaction, where the solid/fluid interface may be thought of as a contact interface. Without numerical algorithms in which temporal and spatial accuracy are assured, the predictive power of such calculations remains an open question. This research has made significant algorithmic and theoretical contributions which significantly enhance our ability to perform these types of calculations. In this sense, it seems that the goals of the project as stated in the original proposal have been achieved.

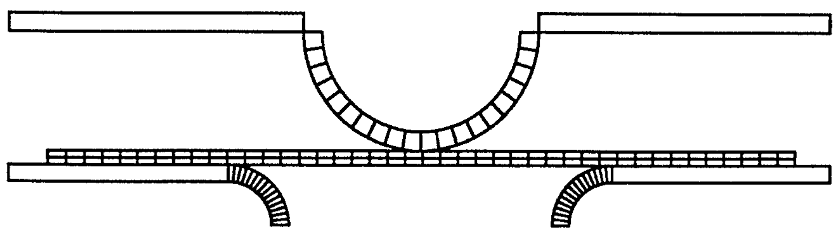


Figure 11: Undeformed configuration of the sheet-drawbead system.

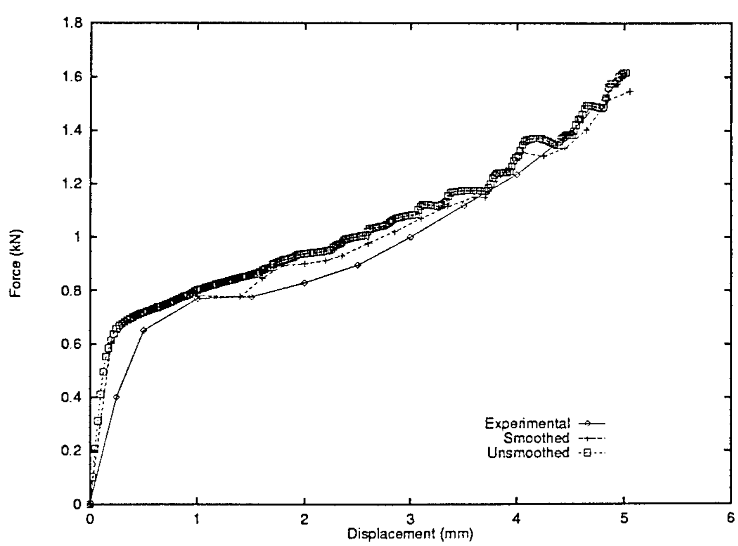


Figure 12: Force-displacement plots for the smoothed and unsmoothed drawbead problems.

4 Publications Resulting from this Award

Books Acknowledging ONR Support

Laursen, T.A. (2002), *Computational Contact and Impact Mechanics*, Springer-Verlag, Heidelberg (publication expected in early 2002).

Refereed Papers Acknowledging ONR Support

Meng, X.N. & T.A. Laursen (2001), "On Energy Consistency of Large Deformation Plasticity Models, with Application to the Design of Unconditionally Stable Time Integrators," *Finite Elements in Analysis and Design*, in press.

Meng, X.N. & T.A. Laursen (2001), "Energy Consistent Algorithms for Dynamic Finite Deformation Plasticity," *Computer Methods in Applied Mechanics and Engineering*, in press.

Laursen, T.A. & G.R. Love (2001), "Improved Implicit Integrators for Transient Impact Problems—Geometric Admissibility Within the Conserving Framework," *International Journal for Numerical Methods in Engineering*, in press.

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Padmanabhan, V. & T.A. Laursen (2001), "A Framework for Development of Surface Smoothing Procedures in Large Deformation Frictional Contact Analysis," *Finite Elements in Analysis and Design*, **37**, 173–198.

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Abstracts of Presentations Acknowledging This Support

Love, G.R. & T.A. Laursen (2001), "Adhering to Admissibility: The Application of Discrete Velocity Updates to Energy Consistent Dynamic Impact," in *USACM Sixth U.S. National Congress on Computational Mechanics: Abstracts*, Mechanical Engineering Department, University of Michigan, p. 471.

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