PERFORMANCE PREDICTION OF REAL-TIME COMMAND, CONTROL, AND COMMUNICATIONS (C3) SYSTEMS

By Insub Shin

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14. ABSTRACT Predicting the performance of a C3 system consisting of sub-systems requires the integration of such sub-system models with the communication system models. When the system is used for a time critical mission, the network delay may play a decisive role in battle management. The architecture of a C3 system is represented by two layers: the functional layer and the physical layer. The synthetic execution model contains both architecture layers as separate executable models. Then, the layered executable models are combined to develop a performance prediction model. The executable functional model uses a Petri net to describe the logical behavior and the executable physical model uses a queuing net to represent the demand and/or contention of resources. The message-passing pattern is generated from the executable functional model using a state space analysis technique. The executable physical model processes these messages preserving the message-passing pattern. Once the network delay is measured in the executable physical model it is inserted into the functional model.
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Insub Shin

This final technical report is based on the unaltered thesis of Insub Shin, submitted to the School of Information Technology and Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Information Technology and Engineering at George Mason University in October 2000. The research was conducted at the System Architectures Laboratory of the C3I Center of George Mason University with support provided by the Air Force Office for Scientific Research under grant no. F49620-98-1-0179.

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ABSTRACT

Many of the Command, Control, and Communication (C3) systems in the real world are supported by a common network or a network of networks. Predicting the performance of a C3 system consisting of sub-systems requires the integration of such sub-system models with the communication system models. When the system is used for a time critical mission, the network delay may play a decisive role in battle management.

In this dissertation, the architecture of a C3 system is represented by two layers: the functional layer and the physical layer. The synthetic execution model contains both architecture layers as separate executable models. Then, the two layered executable models are combined to develop a performance prediction model.

The executable functional model uses a Petri net to describe the logical behavior and the executable physical model uses a queueing net to represent the demand and/or contention of resources. The message-passing pattern is generated from the executable functional model using a state space analysis technique. The executable physical model processes these messages preserving the message-passing pattern. Once the network delay is measured in the executable physical model, the delay value is inserted into the executable functional model for performance prediction.

Since the communication service demands are isolated from the executable functional model, the communications network can be specified in any preferred level of detail independently. This enables the executable functional model to be invariant with respect to the executable physical model resulting in flexibility for designing a large-scale C3 information system. This property, together with the synthesis technique, enables both formal and simulation methods to be used for system analysis: the state space analysis technique of Petri nets and the simulation technique of queueing nets.

A case study in this dissertation shows how a small network delay in a C3 system affects the outcome of a time critical mission. It also illustrates design choices and shows how to develop tactics to provide tolerance to network delays.
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CHAPTER 1. INTRODUCTION

1.1 Motivation

A Command, Control, and Communications (C3) system is typically a real-time distributed system consisting of multiple software and multiple hardware components.

In general, a real time system is characterized by its timely response to external stimuli (Tsai and Yang, 1995). A response consists of a series of task executions, and a task execution is usually characterized by its start time, execution time, and deadline (Stankovic et al., 1985). Analysis of the timing of each task and time management in a series of task executions are essential to guarantee that the system responds at the right time, or before but not after a due time.

A C3 system for a high-level military organization is a system-of-systems consisting of heterogeneous subsystems in operation under distributed computing environments. At the early stage of the system development cycle, however, there are significant uncertainties in the performance parameters, task execution time, and communication delays. These uncertainties are often due to incomplete system specifications and/or complexity of the problem. Furthermore, the message delays from each subsystem can change the order of task execution in an autonomous system and may cause a synchronization problem. Asynchronous execution at distributed portions of the system may threaten the principles of military doctrine.

For a C3 system dealing with time critical missions, the estimates of the elapsed time for each interacting subsystem and the message delay can play a critical role in developing combat engagement rules, establishing decision thresholds, and battle time management. A well-formed performance prediction model supported by an accurate timing estimate method is required.

1.2 Problem Statement

A number of researchers (Bucci et al., 1995; Li et al., 1998; Ostroff, 1989; Popova and Heiner, 1997; Toussaint et al., 1997; Tsai et al., 1995) have addressed the verification of the timing behavior, given timing constraints, instead of controlling the timing behavior of a real
time system. Timing constraints are expressed in the form of \([t_{\text{min}}, t_{\text{max}}]\), where \(t_{\text{min}}\) and \(t_{\text{max}}\) are the minimum and maximum time instants, respectively, elapsing between the enabling and execution of each task. The execution of a task is constrained by external stimuli such as mission requirements or by service capabilities of physical resources. The time interval can be interpreted in various ways. Cothier (1984) used the notion of time interval as "timeliness" (called "window of opportunity"). There are basically two types of windows: the window of capability which characterizes the system response capabilities, while the window of opportunity expresses the requirements of the mission which the system is expected to fulfill (Cothier, 1984). In terms of the mission requirements involving a decision making process, the time interval represents the windows of opportunity such that the decision must be made between \(t_{\text{min}}\) and \(t_{\text{max}}\) (after the arrival of its input). In this case, \(t_{\text{max}}\) is an upper-bound threshold such that, if a decision making organization issues commands in response to the input after the threshold, there will not be enough time to execute the response (Andreadakis, 1988). A decision-making organization can be seen as an information processing system that performs a series of tasks to accomplish its mission (Andreadakis, 1988; Levis, 1991). In terms of a physical resource’s performance capability for information processing in the decision-making organization, the time interval represents the windows of capability such that the information processing system is capable of supporting the decision-making process only within that interval; the decision making organization demands at least \(t_{\text{min}}\) time units and at most \(t_{\text{max}}\) time units to accomplish its decision making process using the information processing system.

Setting different decision thresholds within the windows of opportunity may generate different orderings of message passing among multiple tasks over a network. It may cause different connection delays\(^1\) between tasks resulting in different windows of capability. In turn, the different connection delays may cause different orderings of task executions and this may violate task synchronization in an autonomous system without specific synchronization measure. We encounter a "chicken and egg" problem. Thus, the performance prediction of a real-time system derived under an assumption of communication delays with a probabilistic distribution does not guarantee the realistic representation of the system’s behavior unless we solve the

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\(^1\) The term “connection delays” will be used in reference to the delay between two tasks in a functional architecture that is different from the data transmission delay between two nodes in a physical architecture.
“chicken and egg” problem. In the literature, the term “task” is sometimes used interchangeably with “process,” “grain,” “thread,” etc. (Dietz et al., 1997). In this report, the term “process” will be used as an analogue to the term “task” when the task represents information processing. The constraints of process synchronization include precedence constraints; a process, which produces data for another process, will complete before that data is required, and exclusion constraints: if either one of the two processes has started and is not yet finished, the other process cannot be started (Dong et al., 1997). Delays over interconnections must be considered to manage the complexity of meeting these synchronization constraints.

The properties and behavior of a system can be specified and expressed in various ways depending on the point of view: decomposition into sub-systems or layering by architectural levels. Predicting the performance of a real-time distributed system requires synthesis of the sub-systems or layered architectures together with the communication network. The traditional method is to represent the communication demand of each task and the contention of the demands as an embedded link in a single integrated model. For example, Link_{(k, k+1)} in Figure 1.1 is embedded in a single system.

![Integrated view of a system](image)

Figure 1.1 Integrated view of a System

This approach may have limitations in representing the demand and the contention of network resources that are shared among multiple processes in a large-scale real-time distributed system. The objective of this report is to present methods to synthesize architecture layers and to estimate the connection delays between tasks so that the performance of a C3 system that performs multiple tasks in a real-time distributed computing environment can be predicted.
1.3 Approach

Developing an executable model enables the prediction of the performance of a real time system. Operational formalisms such as Petri nets (Murata, 1989) and state machines are suitable to specify and express executable models. For example, in a system architecting cycle of analysis-synthesis-evaluation (Levis and Wagenhals, 2000), an executable model is developed using a Petri net consisting of "transitions" and "places." Each "transition" and "place" can represent tasks and resources respectively. Interactions between tasks are represented by "arcs". The timing behavior of a system is derived by introducing time to either "transitions" or "places".

In the case of the Real-Time Object Oriented Modeling approach (Gullekson and Selic, 1996; Selic, 1998), an executable model is represented by a network of collaborating state machines called "actors" (i.e., software modules). Interactions between actors are represented by message-passing through explicit interfaces called "ports". The receipt of a message by an actor triggers the appropriate state transitions of the actor's state machine (its behavior). The timing behavior of a system is handled by time-out events in its event-driven run-time "timing service". A task can have as attribute "duration of time" (i.e., deadline or maximum processing time) and can send a "time-out" event message to a timer at the desired instant when a task needs to initiate time-related activity.

In both cases, we can model network delays between tasks as another "transition" or "actor". If the physical communication link between tasks is simply an immediately adjacent transmission link, the modeling effort may be easy. When multiple connections share common network resources other than a single, immediate, adjacent transmission link, the cost of modeling may be very high. In some cases, modeling the contention of communication resources may be practically impossible. Also adding more "transitions" or "actors" can raise the problem of state space explosion.

In this report, the logical behavior of a system is specified in a functional architecture layer and the supporting resources are specified in a physical architecture layer. Both layers describe or specify the same system. In the paradigm adopted in this report, each architecture layers are modeled as a separate executable model and performance parameters are obtained by exchanging the necessary state information between the two models.
Suppose that the two models are developed as shown in Figure 1.2 in which the functional architecture model is a Petri net and the physical architecture model is a queueing net. The circle in the functional architecture model is a “place” and the bar is a “transition”. Transition \( T_j \) has a delay \( \tau_j \) other than network connection delays. The queueing net consists of a server and a queue depicted by a bar and a circle respectively. The server has a message processing rate \( B(t) \) at time \( t \). Let \( A(t) \) denotes an arrival time of a message. If a message arrives at time \( t \), depending on the processing rate, the message will be delivered after a delay \( \delta \). In Figure 1.2, the departure time of the message is denoted by \( A(t+\delta) \).

Figure 1.2 Two Separate Functional and Physical Architecture Models

Central to synthesizing the layered architectures is the concept of two state machines communicating with each other. Once those executable models are developed, both models may communicate with each other during “run-time.” Figure 1.3 depicts the run-time communication scheme between the two models. Once the executable functional model has a message at the place \( P_{i-1} \) at time \( t \), the transition \( T_j \) will produce a message at the place \( P_i \) at time \( (t + \tau_j) \) after the completion of the task. At the same time, this message arrives at the executable physical model. The executable physical model processes the message and produces the message after delay \( \delta_j \) at time \( (t + \tau_j + \delta_j) \). The message produced at the place \( P_i \) after firing the transition \( T_j \) in the functional architecture model is available at time \( (t + \tau_j + \delta_j) \) which is when transition \( T_{j+1} \) can use the message.
Instead of the run-time simulation of the two models, an “off-line” simulation technique is used in this report. Figure 1.4 depicts the “off-line” communication scheme between the two models. The executable functional model sends its whole expected state transition information into the executable physical model to obtain the connection delays between tasks. Then, the executable physical model uses this state information for its event scheduling for simulation and produces network delay values corresponding to each logical connection link between tasks. Finally these delay values are inserted into the executable functional model. The resulting executable functional model with the estimated delay values is used for predicting the performance of the system.

Figure 1.3 Run-Time Communication between Functional and Physical Architecture Models.
Figure 1.4 Off-Line Communication between Functional and Physical Architecture Models.

1.4 Related Work

The concept of viewing a system as a layered architecture was proposed by Rolia and Sevcik (1995) as a performance prediction model of software design architecture in which software processes share both hardware and software resources. Processes that can be said to have statistically identical behavior form a group or class of processes. Groups that request service at the higher level are considered as customers, and groups that provide service at the lower level are represented as servers. In such systems, software processes can act as both customers and servers while sharing hardware resources. The requests for service among processes and for devices are described as a software process architecture. Their “method of layers” estimates contention for resources and process throughputs by introducing a “Layered Queueing Model,” which is an extension of a queueing model consisting of two complementary models: a software contention model and a device contention model. The software contention model describes the relationships in the software process architecture and is used to predict software contention delays. The device contention model is used to determine queueing delays at devices. A similar concept has been proposed by Woodside (1995) and Woodside et al. (1995).
This "method of layers" has been explored for performance prediction of real-time software design architecture by the ‘ObjecTime’ research group (Hrischuk et al., 1998; Woodside et al., 1998). The central idea of their approach is the use of an abstract performance model of software execution that captures the demands and contention of software resources (e.g., codes, modules, components) and hardware devices (e.g., CPU, storage, I/O devices, etc.). In their work, however, their approach does not view the software process architecture as communicating state machines even though they used a layered architecture. There is no communication between a higher layer (i.e., software contention model) and a lower layer (i.e., device contention model). Instead, they modeled the customer-server relationships between the higher layer and lower layers as one queueing network model. Furthermore, the measurement of communication delays between tasks is not addressed. Instead, it is assumed that communication is reliable, point-to-point, dynamically established at execution, having finite but unpredictable delay. In this report, the communication delay is measured rather than assumed via the communication between the two layered architecture models.

1.5 Contributions

The major contributions of this work are:

- A synthetic execution model, via "off-line" communication between the executable functional and physical models of the two layered architectures, has been developed for performance prediction of a real-time distributed system.
- A method to transform a Petri net into a simulation-based queueing net has been developed.
- A method to estimate network delays with realistic representation of communications has been developed via dynamic scheduling of messages using the realistic traffic captured from the logical behavior of a system.
- An analysis method has been introduced that combines the strengths of both the formal approach (i.e., states space analysis) and the simulation approach (i.e., simulation in any preferred level of detail).
• An "off-line" simulation technique has been introduced to avoid the inefficiency of the "run-time" distributed simulation technique for a distributed computing system with different time scales.

1.6 Overview

The rest of this report is organized as follows. The next chapter provides the precise problem definition. It begins with the problem faced by a real-time Command Control and Communications system that is required to execute missions in time-pressured and uncertain battle environments. The timeliness of a response with the desired quality is the major concern. Following the definition, the objectives of the report are then addressed, specifically how to verify a system response within the windows of opportunity and how to measure the windows of capability. Chapter 3 presents the theoretical background (formalisms, theories, techniques) needed to understand the problem solving approach taken in this report. Chapter 4 begins with the premises of the report. It summarizes the core properties of the formalisms, theories, and techniques introduced in Chapter 3. This chapter presents the six-step methodology to develop the performance prediction model of a real-time distributed system. This chapter describes the transformation method from Petri nets into queueing nets and various algorithms in each step. Chapter 5 presents a brief overview of the synthetic execution model generator implemented for performance prediction of a real-time information system. It illustrates how the performance prediction model can be used for system analysis using both formal and simulation-based approach. Chapter 6 presents the application of the methodology and the results of a case study model of a real-time C3 system to carry out missions in time-pressured and uncertain battle environments. Chapter 7 presents the overall conclusions including future research directions.
2.1 Constraints of a Real-Time C3 System

As stated in Chapter 1, the execution of a task is constrained by external stimuli such as mission requirements or by the capabilities of physical resources. Suppose that an Anti-Air Warfare (AAW) system (Figure 2.1) carries out a mission of negating an incoming threat.

![Operational Concept Diagram](image)

The threat must be intercepted if the flight is hostile and if the flight moves into a specified area (firing zone). To carry out the mission, the AAW system is decomposed into three tasks; sense, command, and act. Each task is carried out by assigned assets; sensor, commander, and shooter, respectively, and the assets are connected through a network. Each asset has its own information processing devices to carry out the assigned task.

From the mission needs, we can derive the timing constraints of the AAW system as follows. The system must intercept the threat when the threat is located within the interval \([L_{\text{min}}, L_{\text{max}}]\). Let \(V\) and \(L_0\) be the velocity and the initial location of the flight respectively. Then the system must respond to the external stimuli (i.e., intercept the threat) within the time interval \([t_{\text{min}}, t_{\text{max}}]\), where
This is the timing constraint of the AAW system. The AAW system must respond to the threat after \( t_{\text{min}} \) but before \( t_{\text{max}} \). This response must match the desired goal of the mission. In Andreadakis (1988), the degree to which the actual response matches the desired or ideal response was defined as the “accuracy” of the response. The accuracy of the response is another constraint on the AAW system. In the example scenario, the mission needs statement “the threat must be negated if the flight is hostile” specifies the required accuracy of the response.

### 2.2 Measuring the Performance of a Real-Time C3 System

A C3 system is not just an information system but also a man-machine system. The performance of a C3 system cannot be measured by the behavior of the physical components only. We also have to consider the behavior of the humans. In Miller (1969) and Sheridan and Ferrel (1974), human decision makers are limited in their capacity to process information and make decisions (quoted from Andreadakis, 1988). This limitation is called bounded rationality (Simon, 1976). If we consider this cognitive processing as another type of information processing\(^1\), then information theoretic measures can be used. The performance of the information system can be measured by the data processing delay, the distribution delay of the messages, and the cognitive processing delay of the decision maker (i.e., task executor). Both data processing and distribution delays are related with the magnitude of the data, while the cognitive processing time is related to the semantics of the information or symbols. Levis (1995) expressed the bounded rationality of human decision maker quantitatively by the rationality threshold \( F_0 \). \( F_0 \) is the maximum processing rate of the human decision maker, and is expressed in \textit{bits/sec}. The processing time \( t \) required by a process, whose total activity is \( G \text{ bits/symbol} \), is computed by dividing the workload by the processing rate, \( F \), of the human decision maker.

\[
t = \frac{G}{F} \tag{2.2}
\]

and has units of \textit{sec/symbol}. The minimum processing time, \( t_0 \), corresponds to the maximum processing rate \( F_0 \).

\(^1\)A decision-making organization can be seen as an information processing system that performs a series of tasks to accomplish its mission (Andreadakis, 1988; Levis, 1991).
To verify whether the system meets the constraints, we need to be able to measure the performance of the information system in the perspectives of both time and accuracy. The accuracy of the response is associated with the semantics of the information (e.g., the content of reports, the quality of data, credibility of source, etc.) in the information system, while the timing of the response is associated with complexity of the calculation and the magnitude of the information (e.g., the number of messages, the size of data, etc.) in the information system. The objective of this dissertation is to estimate the network connection delay. Hence the size of data is considered because the transmission delay in a communication network depends on the size of the messages to be transmitted.

In general, the analysis of a system specification can be done in two phases: the functional analysis and the timing analysis phases. Functional analysis is performed first to ascertain the correctness of the logical behavior of the system. Timing constraints are then postulated between the external stimuli and the response. By specifying the logical behavior as the computation of the semantics (or symbolic computation) in its functional architecture, the functional architecture can provide a means to measure the correctness of the logical behavior. In other words, the accuracy of the response can be measured by the logical correctness of the mappings between the semantics (or symbols) of the information. In the rule of the example scenario in section 2.1, the mapping between the input conditions \{threat = hostile or neutral\} and the output actions \{order = fire or not-fire\} deals with the semantics of information. Similarly, the timing of the response can be measured by the physics of the system in the physical architecture. For example, the timing of the response of an information system can be measured by the mappings between the workload of the physical systems and the data processing rate of the computing systems and data distribution rate of the communication systems.

Once the information system receives physical messages, the messages are pre-processed to provide task executors with the semantics of information. This pre-processing delay of task \(i\) is defined as \(x(i)\). After the task is completed, the information system post-processes the symbolic information resulting from the completion of the task in the form of a physical message to be sent over the network. This post-processing delay of task \(i\) is defined as \(y(i)\). The network delays of messages between task \(i\) and task \(j\) are defined as \(\delta(i, j)\), where \(i\) is the source and \(j\) is the sink of the information.
If the task executor is a physical component such as a weapon, the task execution time may be derived easily from the physical characteristics of weapon systems. The time required to carry out the task $i$ (constrained by its own physical capability of the asset other than information processing and distribution capabilities) is defined as an action time $\tau(i)$. A mechanism to derive this type of time from the physics of weapon systems was well illustrated in Cothier (1984). If the task executor is a human decision maker, the action can be derived from the rationality threshold introduced above (Equation 2.2).

Suppose the mission of the AAW system is carried out as a sequence of task executions as shown in Figure 2.2. Let the tasks 'sense', 'command', and 'act' be represented by tasks $i$, $j$, and $k$ respectively. The total response time ($T_r$) can be calculated as,

$$T_r = t(i) + t(j) + t(k) + \delta_{(0,i)} + \delta_{(i,j)} + \delta_{(j,k)} + \delta_{(k,0)}$$  \hspace{1cm} (2.3)

where

- $t(n)$ is the elapsed time at node $n$ calculated by
  $$t(n) = x(n) + t(n) + y(n),$$  \hspace{1cm} (2.4)
- $\delta_{(0,i)}$ is the initiating time of task $i$ from the time when the external stimulus takes place,
- and
- $\delta_{(k,0)}$ is the impact time of task $k$ on the external environment.

Suppose that a surveillance radar (an instance of the asset sensor) has a scanning cycle of 2 seconds, and reports the result to the fire direction commander (an instance of the asset commander). The fire direction commander issues a firing order to the missile launcher (an instance of the asset shooter) immediately after the target is reported. The shooter can either take-off immediately at time $t$ (when it is armed at the arrival time of the firing order), or at time $(t + 5)$ (assuming a loading of 5 time units). The probability of target detection is assumed to be 1.0 when the target exists in the surveillance area. The intercept time of the missile (i.e., the arrival time of the missile, after being launched from its initial location, at the intercept point in the firing zone) is assumed to be between the time interval $[1, 2]$. Then we have:

- $\tau(s) = [0, 2]$, $\tau(c) = [0, 0]$, $\tau(a) = [0, 5]$, $\delta(0, s) = [0, 0]$, and $\delta(a, 0) = [1, 2]$. 

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Figure 2.2 Operational Sequence Diagram of AAW System

Now, if we can predict the data processing time (x and y) and the network delay of the information system (δ), we can predict the total response time interval $T_r$ (the window of capability), and verify whether the system responds within the allowed time interval $T_a$ (the window of opportunity).

$T_r$ can be defined as the time interval:

$$T_r = \delta(0, s) + \pi(s) + \Delta(s, c) + \tau(c) + \Delta(c, a) + \pi(a) + \Delta(a, 0)$$

where

$\Delta(s, c) = [x(s) + y(s) + \delta(s, c)]$,

$\Delta(c, a) = [x(c) + y(c) + \delta(c, a)]$, and

$\Delta(a, 0) = [x(a) + y(a) + \delta(a, 0)]$. 
From the operational concept of Figure 2.1 and Equation 2.1, $T_a$ is defined as the time interval:

$$T_a = [t_{min}, t_{max}], \text{ where } t_{min} = (L_{min} - L_0)/V \text{ and } t_{max} = (L_{max} - L_0)/V$$

Once we can predict the connection delay $\Delta(i, j)$ between functional task $i$ and task $j$ from the post-processing time $\eta$ at resource $i$, the pre-processing time $\xi$ at resource $j$, and network delay $\delta(i, j)$ between physical resource $i$ and resource $j$ (as constrained by the capabilities of the physical systems), we can verify whether the system meets the timing constraints. (i.e., whether the window of capability overlaps with the window of opportunity).

There are two types of decisions in a C3 system; command decision and situation assessment. A command decision is a mapping from an input condition to an output action. The rule of “if the threat is hostile then fire, otherwise ...” specifies the command decision. This rule maps the symbol “hostile” to the symbol “fire”. A situation assessment is an informational judgment about the value of information. It may be associated with the credibility of the information sources. Making a judgment about whether the threat is hostile or not requires a decision threshold. The desired level of quality is determined by the decision threshold used to specify the rule or mapping. The quality of the current information can be measured by the degree to which the input information meets the threshold. Figure 2.3 shows the two measurement issues of a typical real-time C3 system.

![Figure 2.3 Measurement Issues for Performance Prediction of a Typical C3 System](image)

Clearly, the timing of the response can be measured from the time at which the system initiates processing (due to the threat arrival: the external stimulus) to the impact time (at which the system’s response affects the environment). The elapsed time from the start to the end is categorized in two types, $\tau$ and $\Delta$. Action time $\tau$ is the time necessary for the actor to take action,
and delay time $\Delta$ is the time necessary for the supporting information systems to process and distribute the information.

The accuracy of the response can be measured by the degree of the state change toward the desired state at the response time. When the C3 system is defined as an information system, the response time of the C3 system can be measured by the timeliness of the information arrival and the final state of the information system. Defining the level of the desired quality of information, the degree of state change may be measured by the difference between the desired quality of information and the current quality of information produced by the information system.

If the information processing is a deterministic process, we can verify the accuracy and timing of the response separately. One of the robust techniques for verification of the timing behavior has been well studied in Zaidi’s (1999) work. The TL/PN (Temporal Logic and Petri Net) approach transforms the system's specifications given by temporal statements into a graph structure. Once the system is verified for correctness, the “Temporal Inference Engine” infers temporal relations among the system’s intervals, identifies the windows of interest to the user, calculates lengths of intervals and windows, and infers actual time of occurrence of events.

### 2.3 Research Problem and Objective

Commanders in the battlefield make a decision in the presence of uncertainty as well as timing constraints. Levis and Athans (1988) defined the uncertainty as the difference between what one needs to know and what one knows.

$$U = K_N - K_A$$

where $K_N$ represent the knowledge needed to carry out a mission, or solve a problem, or make a decision effectively, and $K_A$ represent the knowledge that a decision making entity has at the point in time and place that a choice needs to be made.

Uncertainty generally reaches an acceptable level after the required action time is past (Van Trees, 1989) as shown in Figure 2.4. If the level of uncertainty reaches an acceptable level (i.e., decision threshold), the decision-maker makes a decision using this information. Meanwhile, the allowed time (i.e., window of opportunity) to make a decision may close before the uncertainty reaches an acceptable level.
Consider a decision making process that is state dependent and adaptive to the tempo of operations. If the tempo of operations becomes higher, the number of messages in a given time interval increases and the arrival rate of messages to the communication network becomes higher. Depending on the tempo of operations, the node processing delay and network transmission delay of the supporting information system will vary.

Maintaining functionally correct end-to-end values may involve a large set of interacting components. To ensure that the end-to-end constraints are satisfied, each of these components will, in turn, be subject to its own intermediate timing constraints (Gerber et al., 1994). For example, multiple sensors can be geographically dispersed to cover the broad range of the battlefield and configured with various data fusion algorithms for different functional purposes. Figure 2.5 is an example of a data fusion net for a target identification function and target tracking function.

If task 4 (i.e., the target identification task) and task 6 (i.e., the target tracking task) in the figure require all information from the preceding tasks, task 6 cannot be started before task 4 and task 5 are completed. Depending on the data fusion rule, task 4 may need only partial information and can be started before all of the tasks in the set \{1,2,3\} are completed. Depending on the designed logical behavior of the system, the order of task execution varies. The functional property of the system generates different orders of message-passing among multiple tasks over a network and may cause different connection delays between tasks and vice versa.
Verifying that the window of capability meets the window of opportunity requires estimation of the bounded values of the "connection delays" between tasks over a network for the different tempos of operations, the different orders of task executions, and the different decision thresholds (depending on the decision threshold of the acceptable level of uncertainty, the decision making time varies and results in different message generation times).

If the physical communication link between tasks is simply an immediately adjacent transmission link, the modeling effort may be easy. In Cothier (1984), the network delay was composed of a set of link delays. If arbitrary values are assigned to each link delay, the minimum and maximum network delay can be computed from the sum of the link delays. The operating scheme of each link was represented as a decision tree with associated probabilities. Probabilities of reliability and survivability were used to represent the availability of each link at the operating time; the network delay was computed from the reliability and survivability functions in the decision tree. Suppose that the network is operating using specific protocols and priorities, and the information has its own data types for different information systems (such as image processing and message handling), and multiple connections share common network resources other than single, immediate, adjacent transmission links. The cost of modeling would be very high and, in some cases, modeling the contention of communication resources may be practically impossible.
The objective of this thesis is to present an effective method for estimating the connection delays between tasks so that the performance of a real-time distributed system (both in terms of timing and accuracy of response) that performs multiple tasks can be predicted.
CHAPTER 3. THEORETICAL BACKGROUND

3.1 System Architecting

Systems architecting is the process of creating (conceptualizing, designing, and building) unprecedented, complex systems (Rechtin, 1991). Levis and Wagenhals (2000) describe a three-phase system architecting cycle: Analysis-Synthesis-Evaluation. Figure 3.1 shows the system architecting process.

![Figure 3.1 A Three-Phase System Architecting](image)

An architecture is defined as the structure of components, their relationships, and the principles and guidelines governing their design and evolution over time (IEEE STD 610.12). The architecture development process starts with an Operational Concept. The Operational Concept specifies what the mission is, the type of tasks it will carry out, and how it is to be
carried out. The mission and the operational concept induce functional requirements as well as some physical system and organizational requirements.

At the analysis phase, both static functional and physical architectures are specified. The functional architecture is a set of activities or functions arranged in a specified (partial) order that, when activated, achieves a defined goal. The physical architecture is a set of physical resources (nodes) that constitute the system and their connectivity (links) (Levis and Wagenhals, 2000). At the synthesis phase, both static functional and physical architectures are combined for a given operational concept and an executable model is developed. Finally, behavior analysis and performance evaluation can be carried out using scenarios consistent with the operational concept. A scenario is a sequence of expected events following from a certain input stimulus (or set of stimuli), and a certain initial state.

The first step in the development of a functional architecture is *Functional Decomposition*. This decomposition is a partitioning process and is carried out until each function can be allocated to a single physical resource (Levis, 1991). Functions necessary for the execution of the mission as prescribed by the operational concept are identified and further decomposed into sub-functions that need to be performed. The set of sub-functions has to be mutually exclusive and possibly exhaustive.

A well-established model for representing functional architectures is the *Activity Model* while IDEF0 (i.e., Integrated Computer Aided Manufacturing (ICAM) DEFINition language 0; FIPS 183) is an appropriate language for describing the model. The activity model is derived by specifying the data exchanged between sub-functions. It shows the flow of information through the system from inputs from the environment to outputs to the environment. The syntax of the IDEF0 diagram is shown in Figure 3.2.

In IDEF0, a box represents an activity or a function, and arrows or arcs represent flows of materials, flows of information, interconnections, interfaces, and feedback. There are four types of arcs: Input arcs, Control or Constraint arcs, Output arcs, and Mechanism arcs. Consequently, the IDEF0 diagram represents up to four relations between functions. In developing the functional architecture, only inputs, outputs, and controls are modeled; mechanisms are obtained from the physical architecture.
Figure 3.2 Syntax of IDEF0 Diagram

Figure 3.3 is an example of the architectural specifications of a typical C4ISR system in the abstract and static level. The static functional architecture shows three tasks and their logical interactions. The static physical architecture shows the assets and their physical interconnections.

Figure 3.3 An Example of Architecture Specifications
But the performance prediction model requires the synthesis of both functional and physical models. The process model of IDEF0 lacks timing and triggering information which is required for the performance model. As shown in Figure 3.1, an executable model for performance prediction requires the synthesis of the data model (specifying the relations between data), the rule model (specifying the conditions under which functions are performed), and the dynamics model (specifying the order in which they are performed). It also needs the allocation of resources (i.e., assets) to functions, in addition to the flow of data.

The properties of the executable model from the functional architecture can be used to verify the logical behavior (i.e., accuracy of the response). The properties of the executable model from the physical architecture can be used to verify the timing behavior (i.e., timing of the response). The assets play the role of interface between the two models.

In a military command and control system, there are three types of measurements for a system evaluation: (Levis and Wagenhals, 2000)

- Measures of Performance (MOPs): quantities that measure attributes of system behavior
- Measures of Effectiveness (MOEs): quantities that measure how (well) the system performs its function
- Measures of Force Effectiveness (MOFEs): quantities that measures how well the force of which the system is a part performs the mission

MOPs are specified within the boundary of the system and the MOEs and MOFEs are specified outside the boundary of the system. To measure effectiveness, one must establish requirements. The requirements must be expressed in a form that is commensurate with the MOPs. Then MOEs are obtained by comparing the MOP values to the requirements.

$$MOE = f(MOPs, Requirements)$$

Alternatively, the MOE may have an implicit standard (e.g., the MOE as an index) embedded in its definition that reflects the requirements. Suppose that the current variance of estimated location at time $t$ is $S_t$ and a launched missile can negate the target if the variance is less than $S_a$. Given the requirements $S_a$, MOPs of the target tracking system can be expressed as $S_t$ and MOEs of the target tracking system can be measured by
\[ MOE = \begin{cases} 1 & \text{if } S_1 < S_2 \\ 0 & \text{otherwise} \end{cases} \]

3.2 Petri Net Theory

As stated, a mission is structured as a set of tasks. The set of tasks or functions may be performed sequentially or concurrently in a distributed computing environment over a network. Petri nets are a mathematical modeling tool for describing and analyzing discrete event systems that are characterized as being concurrent, asynchronous, distributed, parallel, non-deterministic, and/or stochastic (Murata, 1989). Since information processing in distributed discrete event systems exhibits such properties, Petri nets have been used for their modeling (Tabak and Levis, 1985). The graphical nature of the Petri net helps to easily visualize the complexity of a system, and the executability allows the system to be simulated to observe its complex behavior and to measure its performance.

3.2.1 Basic Definitions

**Multi-Set**

A multi-set \( m \), over a non-empty set \( S \), is a function \( m \in [S \rightarrow \mathbb{N}] \). The non-negative integer \( m(s) \in \mathbb{N} \) is the number of appearances of the element \( s \) in the multi-set \( m \). We usually represent the multi-set \( m \) by a formal sum:

\[
\sum_{s \in S} m(s)s.
\]

By \( S_{MS} \) we denote the set of all multi-sets over \( S \). The non-negative integers \( \{m(s) \mid s \in S\} \) are called the coefficients of the multi-set \( m \), and \( m(s) \) is called the coefficient of \( s \).

**Multi-Set Operations**

Some operations of multi-set are defined as:

- **Addition:**
  \[
m_1 + m_2 = \sum_{s \in S} (m_1(s) + m_2(s))s.
  \]

- **Comparison:**
  \[
m_1 \neq m_2 \quad \exists s \in S : m_1(s) \neq m_2(s)
  \]
\[ m_1 \leq m_2 \quad \forall s \in S : m_1(s) \leq m_2(s). \]

- **Subtraction:**
  \[ m_1 - m_2 = \sum_{s \in S} (m_1(s) - m_2(s))s, \text{ when } m_2 \leq m_1. \]

**Petri net**

A Petri net is a bipartite directed graph represented by \( PN = (P, T, I, O) \), where

- \( P = \{ p_1, p_2, \ldots, p_n \} \) is a finite set of places.
- \( T = \{ t_1, t_2, \ldots, t_m \} \) is a finite set of transitions.
- \( I \) is a mapping from \( P \times T \) to \( \mathbb{N} \), where \( \mathbb{N} \) is the set of non-negative integers, corresponding to the set of directed arcs from places to transitions. \( I(p, t) = 1 \) means that the transition \( t \) is connected the place \( p \), in the sense that there exists a directed arc from \( p \) to \( t \).
- \( O \) is a mapping from \( T \times P \) to \( \mathbb{N} \), corresponding to the set of directed arcs from transitions to places. \( O(t, p) = 1 \) means that there exists a directed arc from \( t \) to \( p \).

**Ordinary Petri net**

A Petri net is said to be *ordinary* if and only if the mappings \( I \) and \( O \) take their values in the set \( \{0, 1\} \).

**Pure Petri net**

A Petri net is said to be *pure* if and only if it has no self-loop (i.e., no place can be both an input and an output of the same transition).

An example of a pure ordinary Petri net is shown in Figure 3.4. A place is depicted by a circle, a transition by a filled bar, and an arc by a directed line segment. The structure of the net as defined by the quadruple is:

- \( P = \{ p_1, p_2, p_3, p_4, p_5 \} \).
- \( T = \{ t_1, t_2, t_3, t_4, t_5 \} \).
- \( I(p_1, t_2) = 1; I(p_2, t_4) = 1; I(p_3, t_5) = 1; I(p_4, t_3) = 1; I(p_5, t_2) = 1; \text{ all other } I = 0. \)
- \( O(t_1, p_1) = 1; O(t_2, p_2) = 1; O(t_2, p_3) = 1; O(t_3, p_1) = 1; O(t_4, p_5) = 1; O(t_5, p_4) = 1; \text{ all other } O = 0. \)
Marking of a Petri net

The distribution of tokens in the places of a Petri net is called the marking of the Petri net. The marking is a mapping from $P$ to $N$ which assigns a non-negative number of tokens to each place of the net. So a marking is a multi-set over $P$. This mapping is represented by a $n$-dimensional vector $M$ of non-negative integers, where $n$ is the number of places and the $i^{th}$ element of $M$, denoted $M(p_i)$, is the number of tokens in the $i^{th}$ place. On a Petri net graph, tokens are represented by dots, $\bullet$, in the circles which represent the places of a Petri net. Figure 3.5 depicts a Petri net with the following marking:

$$M(p_1) = 0; M(p_2) = 1; M(p_3) = 0; M(p_4) = 0; M(p_5) = 0; M(p_6) = 1.$$  

The marking of a Petri net characterizes the state of the Petri net. The initial marking is denoted by $M_0$. The state of the system in Figure 3.5 is the initial condition, $M_0 = [0 1 0 0 0 1]^T$. [Figure 3.4 Example of a Pure Ordinary Petri net]
Enabling of Transitions

A Petri net executes by firing transitions. In order to fire, a transition must be enabled. A transition is said to be enabled for a marking \( M \), if and only if, for each place of the net:

\[
M(p_i) \geq I(p_i, t_j) \text{ for all } p_i \in I(p_i, t_j)
\]

Firing of Transitions

Any change in the marking and, therefore, any change in the state, is controlled by transitions. When the transition is enabled, it can fire. When it fires, tokens are removed from each of its input places and tokens are put in each of its output places. Firing of the transition \( t_j \) results in a new marking \( M' \) where:

\[
M'(p_i) = M(p_i) - I(p_i, t_j) + O(t_j, p_i) \text{ for all } p_i \in I(p_i, t_j) \text{ and for all } p_i \in O(t_j, p_i)
\]

Figure 3.6 shows an example of the firing of a transition. Transition \( t_1 \) is enabled because there is at least one token in each of its input places, \( p_1, p_2, \) and \( p_3 \). When \( t_1 \) fires, one token is removed from each of the input places of \( t_1 \) and tokens are added to each of the output places, \( p_4 \) and \( p_5 \). The firing of transitions causes the marking of the net to change from \([1 1 1 0 0]^T\) to \([0 0 0 1 1]^T\) and therefore introduces a change in the state of the system.
Conflict net

The condition of conflict can occur in a Petri net if the structure has any places that are input to more than one transition. Unless such a place is in a self loop with each of the input transitions, if a token appears in such a place, and two or more of the input transitions can become enabled, a conflict occurs. If one transition fires, others become disabled. Figure 3.7 shows a Petri net with conflict. Note that both t1 and t2 are enabled, but if one fires the other cannot fire. In general, it is undesirable to have conflict in models, because they represent ambiguity in the model and denote contention for resources. There is no information provided in the Petri net that can be used to select which transition to fire, and arbitrarily selecting a transition results in a different set of states. In the case where there are multiple transitions that are concurrently enable but not in conflict, it is possible to get to a common state regardless of the sequence in which the concurrently enabled transitions fire.

Figure 3.6 Firing of a Transition

Figure 3.7 Example of Conflict net
3.2.2 Algebraic Representation

**Incidence Matrix**

The structure of Petri nets can also be represented algebraically by an integer matrix $C$, called an *incidence* matrix. $C$ is an $n \times m$ matrix whose $m$ columns correspond to the transitions and whose $n$ rows correspond to the places of the net. The elements $C_{ij}$ of the incidence matrix $C$ are defined as follows:

$$C_{ij} = O(t_j, p_i) - I(p_i, t_j) \quad \text{for} \quad 1 \leq i \leq n, 1 \leq j \leq m.$$

The incidence matrix of the Petri net in Figure 3.4 is given below. Note that the rows and columns of the matrix have been annotated with the corresponding places and transitions for clarity.

$$C(PN) = \begin{bmatrix}
1 & -1 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & 1 \\
0 & -1 & 0 & 1 & 0 \\
-1 & 0 & 0 & 0 & 0
\end{bmatrix}$$

The value attributed to each cell describes the relation between the corresponding places and transition. In the case of ordinary Petri nets, $C_{ij}$ takes values in $\{-1, 0, 1\}$ only:

- $-1$ indicates that the place is an input place to the transition,
- $1$ indicates that the place is an output place of the transition, and
- $0$ indicates that there is no arc connecting the place to the transition.

It should be noted that self-loops cannot be accounted in this representation. If $t_i$ and $p_j$ form a self-loop, then we have $I(p_j, t_i) = 1$ and $O(t_i, p_j) = 1$, and therefore $C_{ij} = 0$. Consequently, the incidence matrix represents uniquely only pure Petri nets.

While it is possible to illustrate the dynamics of a Petri net by graphically showing the change in the marking as transitions fire, it is also possible to mathematically calculate the change of the marking of a Petri net using the incidence matrix and linear algebra. To see this, two notions are needed, the firing sequence and the firing vector.

**Firing Sequence**

A *firing sequence* defines a sequence of ordered firings. It is represented by
\[ \sigma_s = t_1 \cdot t_2 \cdot t_3 \ldots \]

This means that \( t_1 \) fires first, then \( t_2 \), then \( t_3 \). The firing sequence is simply a list of transitions in the order in which they fire.

**Firing Vector**

A firing vector, \( \mathbf{N}_s \), can be associated with a firing sequence \( \sigma_s \). \( \mathbf{N}_s \) is a column vector whose dimension is equal to the number of transitions, \( m \). The \( i \)-th element of \( \mathbf{N}_s \) denotes the number of occurrences of transition \( t_i \) in the firing sequence \( \sigma_s \). Note that a firing sequence can always be used to generate a firing vector, but the converse is not true, because the firing vector does not have any indication of the sequence in which the firings take place.

Given a marking \( M \), the incidence matrix \( C \), and a firing vector \( \mathbf{N}_s \) that corresponds to a firing sequence \( \sigma_s \), the resultant marking \( M' \) of the firing sequence can be calculated algebraically by:

\[ M' = M + C \mathbf{N}_s \]

**Reachable Marking**

Given an initial marking \( M_0 \) of the net, a reachable marking is any possible marking of the net which can be obtained from the initial marking. In other words, given a net and initial marking \( M_0 \) for this net, \( M \) is a reachable marking if and only if there exists a firing sequence \( \sigma \) such that:

\[ M_0 \xrightarrow{\sigma} M \]

Or there exists an \( \mathbf{N}_s \) corresponding to the firing sequence \( \sigma \) such that:

\[ M = M_0 + C \mathbf{N}_s \]

For example, suppose that the initial marking of the net of Figure 3.5 is,

\[ M_0 = [0 1 0 0 0 1]^T. \]

A valid firing sequence is to fire each of the transitions in the sequence

\[ \sigma_s = t_1 \cdot t_4 \cdot t_2 \cdot t_5 \cdot t_3. \]

The corresponding firing vector is: \( \mathbf{N}_s = [1 1 1 1 1]^T \)

The marking \( M \) reached by this firing sequence is obtained by:
This equation indicates that after the firing sequence $N_s$, the resulting marking has one token each in places $p_1$ and $p_2$ while the other places have no tokens.

However, this approach should be used with caution in order to avoid its two main pitfalls:

- As noted previously, self-loops cannot be accounted in the matrix representation of the net
- The firing vector must correspond to a feasible firing sequence.

**Reachability Set**

Given an initial marking $M_0$ of a Petri net, the *reachability set* - denoted $R(M_0)$ - is the set of all possible reachable markings. In other words, a marking $M$ belongs to $R(M_0)$ if there exists a firing sequence $\sigma$ leading from $M_0$ to $M$.

**3.2.3 Occurrence Graph**

The *occurrence graph* is another means of depicting reachability. In this graph, the nodes represent the state (marking) of the net and the directed arcs indicate the firing of an enabled transition in a state causing the state to change. These directed arcs are called occurrences. Each occurrence in the occurrence graph represents the firing of a single transition. Thus an occurrence graph is a state transition diagram.

An example of an occurrence graph is shown in Figure 3.8. This graph was generated from the Petri net of Figure 3.5 with the initial marking $M_0 = [0 1 0 0 1]^T$.

The occurrence graph shows all the reachable markings *from the given initial state*. Thus, it contains the reachability set of the net for the given initial marking. Different occurrence graphs will be generated for different initial markings. It also shows all of the legitimate firing sequences. Each firing sequence is a path through the graph. For example, the firing sequence $\sigma_s$,
= t₁ • t₄ • t₂ • t₅ • t₃ is shown as a path on the right hand side of the graph: M₀ - M₂ - M₃ - M₄ - M₆ - M₂.

Figure 3.8 Example of Occurrence Graph

### 3.2.4 Timed Petri Nets

In ordinary Petri nets, time is not included: transitions fire either in sequence or concurrently, but nothing says when these firings are taking place. An extension of ordinary Petri nets is Timed Petri nets where time is introduced to model delays associated with processes or buffers. Two approaches are possible: the place model and the transition model. In timed Petri nets, the time can be introduced either at a “transition” or at “places”. It can be proved that assigning processing times to places is equivalent to assigning processing times to transitions (Sifakis, 1980).
Place Model of Time

In the place model, a delay is associated with each place and corresponds to the delay associated with buffers. Tokens are in one of two states: available or unavailable. If $M_a$ denotes the marking of available tokens in a place and $M_u$ the marking of unavailable tokens, the total marking $M$ of a place is: $M = M_a + M_u$. A transition is enabled only by available tokens. When a transition is enabled, it fires instantly: the enabling available tokens are removed from the input places and unavailable tokens are generated in the output places. These tokens remain unavailable for a time equal to the delay associated with the place in which they reside.

Transition Model of Time

In the transition model, a delay is associated with each transition and corresponds to the delays of processes. Tokens are in one of two states: reserved or unreserved. If $M_{ur}$ denotes the marking of unreserved tokens in a place and $M_r$ the marking of reserved tokens in the same place, the total marking $M$ is: $M = M_r + M_{ur}$. A transition is enabled only by unreserved tokens present in the input places. When a transition is enabled, firing is initiated and the enabling tokens become reserved during a time equal to the delay associated with the transition. When the firing terminates, the reserved tokens are removed from the input places and unreserved tokens are generated in the output places.

Timed Occurrence Graph

An ordinary Petri net has no notion of time. Neither does it contain any information about the order in which concurrently enabled transitions will fire. Because this information is not contained in the Petri net, the occurrence graph shows all of the possible states that the system could encounter. In a Timed Petri net, the state of a system can be represented by a $n$-dimensional state vector in which a state is a pair $(M, r)$ where $M$ is a marking and $r$ is a time value. The initial state is the pair $S_0 = (M_0, r_0)$ and the sets of all states are denoted by $S$. In the timed occurrence graph of a Timed Petri net, each node contains a time value in a timed marking.
3.3 Execution of Distributed Systems

3.3.1 Causality vs. Time

If the execution of an event $A$ causes or effects the execution of event $B$, then the execution of $A$ and $B$ must be scheduled in real time so that $A$ is completed before $B$ starts. Violation of this constraint is referred to as a causality error (Unger et al., 1993). A distributed simulation system must explicitly coordinate the advance of time in order to maintain temporal consistency between the components. Human beings use the concept of causality to plan, schedule, and execute an enterprise, or to determine a plan’s feasibility. The notion of time is basic to capturing the causality between events.

In daily life, we use global physical time to deduce causality. However, distributed computation systems have no built-in global physical time. In the execution of distributed computation systems, the notion of time can be viewed in terms of virtual time and a logical time. The virtual time is a counterpart of a physical clock such as a wall-clock. The logical time is derived to capture the causal precedence relation between events. The logical time is defined at the level of temporal semantics in terms of event ordering (Carroll and Borschchev, 1996).

A logical clock is a data structure coupled with a function (i.e., protocol) in which the function is used to update the clock (i.e., the data structure) when an event is executed (Kalantery, 1997). In a logical clock, every event is assigned a time stamp, by which a process can infer the causality relations between events. The time stamp can be defined as Scalar, Vector, or Matrix in its data structure.

In Raynal and Singhal (1996), the logical time system consists of a time domain $T$, and a logical clock $C$. Elements of $T$ form a partially ordered set over a relation $<$. This relation is usually called "happened before" or causal precedence. The logical clock $C$ is a function that maps an event $e$ in a distributed system to an element in the time domain $T$, denoted as $C(e)$ and called the time stamp of $e$. The clock $C$ is defined as:

$$C : H \alpha T$$

where $H$ is a set of events in a distributed system, to satisfy the following property:

$$e_i \rightarrow e_j \Rightarrow C(e_i) < C(e_j) \text{ for all } i \text{ and } j \text{, and } i \neq j.$$
The binary relation → defines a total order on events and expresses causal dependencies among the events. The time stamps assigned to events obey the fundamental monotonicity property. That is, an event a causally affects an event b, then the time stamp of a is smaller than the time stamp of b. This monotonicity is called the clock consistency condition. When $T$ and $C$ satisfy the following condition,

$$ e_i \rightarrow e_j \iff C(e_i) < C(e_j) \text{ for all } i \text{ and } j, \text{ and } i \neq j, $$

the clock is said to be strongly consistent.

The causal relationships in a distributed system yield only a partial order. Lamport (1978) proposed a scalar time representation for totally ordering events in a distributed system. In his scalar time representation, the time domain is a set of non-negative integers. The protocol to update the clock consists of two rules. Rules R1 and R2 update the clock as follows. (quoted from Raynal and Singhal, 1996)

- **R1.** Before executing an event, $p_i$ executes the following:

  $$ C_i := C_i + d \quad (d > 0) $$

  In general, every time R1 is executed, $d$ can have a different value, which can be application-dependent. However, $d$ is typically kept 1, since this allows a process to identify the time of each event uniquely at a process while minimizing $d$’s rate of increase.

- **R2.** Each message piggybacks the clock value of its sending time. When a process $p_i$ receives a message with the time stamp $C_{msg}$, it executes the following actions:

  Before executing an event, $p_i$ executes the following:

  1. $C_i := \max (C_i, C_{msg})$
  2. Execute R1
  3. Deliver the message.

The main problem in totally ordering the events is that two or more events at different processes can have the identical time stamp. In Figure 3.9, for example, event $c$ of process $p_1$ and event $j$ of process $p_2$ have the same time stamp; the same occurs for event $l$ of process $p_2$ and event $x$ of process $p_3$.

Genuinely, concurrent events have no influence on one another (Fidge, 1991). Since both events $c$ and $j$ are independent (i.e., not causally related), the system can order them using any criterion without violating the causality relation. We can order them using the process identifier.
Let the time stamp of an event be denoted by a tuple \((t, i)\), where \(t\) is its time of occurrence and \(i\) is the process at which it occurred. The total order relation \(\pi\) on two events \(x\) and \(y\) with time stamps \((h, i)\) and \((k, j)\), respectively, is
\[
x \pi y \Leftrightarrow (h < k \text{ or } (h = k \text{ and } i < j)).
\]
Typically, process identifiers are linearly ordered. So, when two events have the same time stamp, the event at the lower ordered process is ordered lower.

After Lamport (1978)'s scalar time system, many efforts have been made to capture the causality in distributed systems using Vector time (Fidge, 1991; Mattern, 1988; Schmuck, 1988) and Matrix time (Fischer and Michael, 1982). The comparison and efficient implementation of their logical time systems are found in the literature (Carroll and Borshchev, 1996; Garg and Tomlinson, 1994; Raynal and Singhal, 1996).

In distributed algorithm design, the properties of a system of logical clocks and the causality relation helps in the analysis of a system's functional behavior such as liveness, fairness, deadlock, concurrency, and consistency check. A total order is generally used to ensure liveness properties in distributed algorithms (requests are time stamped and served according to the total order on these time stamps) (Ramport, 1978; quoted from Raynal and Singhal, 1996). Similar kinds of analysis can be done by temporal logic and Petri nets. In Levis (1998) and Zaidi (1999), for example, a system is initially specified by temporal logic (first order logic) and the properties of the system are verified using a Petri net whose structure is equivalently constructed from temporal logic specifications.
For the performance evaluation of a distributed system (especially the timing behavior of the system), we have to know the processing time of each event at each process. Once we can measure the processing time based on the capabilities of the processors used to carry out the processes, we can verify whether the system meets the timing constraints. (i.e., whether the windows of capability overlaps with the windows of opportunity). One of the robust techniques for verification of the timing behavior is well studied by Ma (1999) on the extension of Zaidi (1999)'s work.

3.3.2 A Model of Distributed Execution

In Raynal and Singhal (1996), a distributed program is composed of a set of $n$ asynchronous processes $\Psi_1, \Psi_2, ..., \Psi_n$ that communicate by message-passing over a communication network. The execution of $\Psi_i$ produces a sequence of events:

$e_i^0, e_i^1, ..., e_i^r, e_i^{r+1}, ..., $

denoted by $H_i$ where

$H_i = (h_i, \rightarrow_i)$.

The set of events produced by $\Psi_i$ is $h_i$. The binary relation $\rightarrow_i$ expresses causal dependencies among the events $h_i$ of process $\Psi_i$. The events are classified into three types: internal event, send-message event, and receive-message event. An internal event affects only the process at which it occurs, and the events at a process are ordered by their order of occurrence. So the binary relation $\rightarrow_i$ defines a total order on $h_i$. Events Send-message and receive-message signify the flow of information between processes and establish causal dependency between the sender process and the receiver process. Consequently, the execution of a distributed application results in a set of distributed events produced by the process. A relation $\rightarrow_{msg}$ is defined to represent the causal dependencies between the pairs of corresponding send-message and receive-message events. For every message $m$ exchanged between two processes, we have

send-message$(m) \rightarrow_{msg} receive-message(m)$.

Then the distributed execution of a set of processes is a partial order:

$H = (H, \rightarrow)$, where
For example, the distributed system of Figure 3.9 can be modeled:

\[ H_1 = (A_1, \rightarrow_1), \text{ where } A_1 = \{ a, b, c, d, e \} \text{ and } \rightarrow_1 = \{ (a, b), (b, c), (c, d), (d, e) \}, \]

\[ H_2 = (A_2, \rightarrow_2), \text{ where } A_2 = \{ i, j, k, l, m, n \} \text{ and } \rightarrow_2 = \{ (i, j), (j, k), (k, l), (l, m), (m, n) \}, \]

\[ H_3 = (A_3, \rightarrow_3), \text{ where } A_3 = \{ w, x, y, z \} \text{ and } \rightarrow_3 = \{ (w, x), (x, y), (y, z) \}, \text{ and } \]

\[ \rightarrow_{msg} = \{ (b, j), (k, x), (z, d), (e, m) \}. \]

Then the distributed execution of the system is a partial order:

\[ H = (H, \rightarrow), \text{ where } \]

\[ H = h_1 \cup h_2 \cup h_3 \]

\[ \rightarrow = \rightarrow_1 \cup \rightarrow_2 \cup \rightarrow_3 \cup \rightarrow_{msg}. \]

If the distributed system processes the events in a total order \( \pi \):

\[ \pi = \{ (a, i), (i, w), (w, b), (b, c), (c, f), (j, k), (k, l), (l, x), (x, y), (y, z), (z, d), (d, e), (e, m), (m, n) \} \]

it does not violate the causality relation.

### 3.3.3 Simulation of Distributed Discrete-Event Systems

For a non-distributed discrete event simulation, the notion of simulated time is explicitly available and the sequential uniprocessor simulation (time- or event-driven simulation) is a straightforward method because the entire system is centralized (i.e., centralized event queueing) and only one object can be simulated at any given time (i.e., single stepping clocks). This sequential, uniprocessor, discrete event simulation often relies on shared global memory, and its simulation time is implemented as a global variable (McAffer, 1990). The simulation clock records the occurrence time (time stamp) of the last input event and is used to calculate a time stamp for the occurrence of the next output event (Kalantery, 1997).

In the execution of Timed Petri nets, for example, the occurrence of an event relates to the firing of a transition. The simulation engine maintains an event list (scheduled transition firings). It repeatedly executes the first event in the list, simulating the behavior caused by the event (change the virtual time to the event time stamp, change the markings, enable new transitions and/or disable transitions that were already enabled), inserting (removing) corresponding new
events to be scheduled (old events to be descheduled) into (from) the event list, and finally removing the executed event from the list (Chiola and Ferscha, 1993).

In distributed event-driven simulation, the processes do not share global memory and communicate solely by passing messages. Unlike conventional sequential programs, the computations performed by distributed computing systems do not yield a linear sequence of events. The interrelationships between the events performed in a distributed system is inherently a partial ordering (Fidge, 1991). So a distributed system must explicitly coordinate the advance of time in order to maintain temporal consistency between the computations. The system must define how time is advanced when, according to models, it should be advanced. In real-time simulation of distributed discrete-event systems, the semantics of the simulation program rely on the simulation time. The simulation time progresses in a way that avoids violating the causality relations of the program.

Simulation is a cost-effective approach to performance evaluation of a complex distributed system. But the simulation of a distributed system must not violate the causality relation among events. This can be done by using a total ordering of events for the simulation. Clearly, scalar clocks satisfy monotonicity, and hence the consistency property. We can use the scalar clocks to totally order events since a total order is consistent with the causality relation as shown in the previous section.

The general techniques for distributed event-driven simulation guarantee the serializability of the behavior of a distributed simulation and therefore yield results equivalent to some sequential simulation (Ricciulli et al., 1996). Simulation techniques of this type are usually classified as either pessimistic or optimistic. Both of these techniques use the same notion of local and global virtual time. The local virtual time (LVT) of an object is defined as the time up to which that object has executed the simulation. The global virtual time (GVT) is the point in simulation time up to which all components have successfully executed. The GVT at a particular point in the simulation is equal to the minimum of all the LVTs in the system. If some component has LVT that is equal to the GVT, then it is said that the component defines the GVT (McAffer, 1990).
**Pessimistic Simulation**

Chandy and Misra (1981) introduced the pessimistic simulation technique, called *conservative* approach, because it assumes that components will communicate out of sequence, even though some events might have been independent of each other. It guarantees that a processor does not advance its simulation clock until it is certain that it will not bypass some simulation time at which another processor affects it. It uses synchronous message passing to coordinate the concurrent components.

The conservative approach is inefficient because all events simulating global state change must go through a central controller, therefore creating a serialization blocking. In other words, messages cannot be received until the receiver’s LVT equals to GVT. The message’s sender remains blocked until the receiver defines the GVT. In order to avoid deadlock and to achieve simulation efficiency, the conservative approach requires “lookahead” which is the ability of a processor to predict its future behavior, as regards when next (in simulation time) it may affect another processor’s submodel (Nicol and Roy, 1991).

**Optimistic Simulation**

Jefferson (1985) introduced an optimistic simulation technique, called *Time Warp*, which allows parts of a simulation to get ahead of themselves. In contrast to the pessimistic technique, objects using Time Warp simulate and communicate freely with other components until they receive a message. If the new message is from the receiver’s future (i.e., if the message time stamp is greater than the receiver’s LVT), then the receiver increases its LVT to the message’s time stamp value and processes the message. If the message is from the component’s past, then the receiver must undo, “rollback,” any processing it did or messages it sent between its current LVT and the new message’s time stamp value. The rollback procedure relies on the reconstructibility of past states which can be guaranteed by a systematic state saving policy and corresponding state saving reconstruction procedures, causing respective overheads (Ferscha and Richter, 1997).

State saving mechanisms are an essential part of the *Time Warp* approach for the efficiency of simulation. Various methods and their comparisons are described in Ronngren et al. (1996) and Radhakrishnan et al. (1996). Since all message passing is asynchronous, Time Warp is entirely free of simulation induced deadlock. Time Warp is efficient for loosely coupled, highly
asynchronous systems, but is inefficient when models have mixed time scales or diverse interaction behaviors (McAffer, 1990).
4.1 The Premises in the Dissertation

An executable model for performance prediction can be developed in various ways. A straight-forward approach is to develop a single integrated executable model in which network connection delays and the contention for resources are directly modeled at the functional architecture level using additional transitions and places as shown in Figure 4.2, as an extension of the executable functional model of Figure 4.1.

![Diagram of an Executable Functional Model as an Ordinary Petri net](image)

Figure 4.1 An Executable Functional Model as an Ordinary Petri net

In the synthetic execution approach, both functional and physical architectures are modeled as separate executable models. The central idea of this dissertation is to synthesize layered architectures using the concept of two state machines communicating with each other. From the perspective of distributed simulation systems, the performance prediction model of real-time distributed simulation consists of two processors: one for the C2 model and the other for the communication model. The proposed approach is not the typical run-time distributed simulation model because there is no message exchange between the two communicating state machines at run-time.
The motivation behind distributed simulation is to gain a speedup over traditional sequential simulation, while guaranteeing equivalent results to those of some sequential simulation. The two communicating state machines in the problem domain are tightly coupled. The number of logical communication channels between the two models can be as many as the number of arcs in the Petri net model. So none of those distributed simulation technique might be efficient. However, if the messages generated by one model can be totally ordered in the other model’s time domain to guarantee the serializability of events (i.e., total ordering of events), we can simulate the other model separately “off-line”, rather than using distributed simulation at “run-time”. In fact, Jefferson’s (1985) optimistic simulation is the converse approach in that that it specifies a total order on the events, and then provides a mechanism that causes the events of a computation to be executed in a partial order that conforms to the specified total order. (Bayerdorffer, 1995)
In the problem domain of this dissertation, the network delays are unknown. How can we capture the precedence relation with the unknown delays? In Petri nets, however, the precedence relations can be captured by the occurrence graph. Using the information contained in the occurrence graph, we can capture the traffic pattern and order the messages linearly on the communications network simulation time domain.

4.1.1 Capturing Precedence Relations from the Occurrence Graph

As defined in the previous chapter, given the current marking $M$ of a Petri net, firing of the transition $t$ results in a new marking $M'$ where:

$M'(p_i) = M(p_i) - I(p_i, t) + O(t, p_i)$, for $i = 1, ..., n$.

Suppose that the Petri net is an ordinary net. If we perform a subtraction operation of markings; $M'(p_i) - M(p_i)$, the computation results in $\{+1, 0, -1\}$ as shown in Table 4.1.

<table>
<thead>
<tr>
<th>$O(t, p_i)$</th>
<th>$-$</th>
<th>$I(p_i, t)$</th>
<th>$=$</th>
<th>$M'(p_i) - M(p_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>1</td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0</td>
<td></td>
<td>+1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

The "+" and "-" signs tell whether the token is a newly placed one or a removed one respectively. Note that this interpretation can be made only on pure Petri nets. If the Petri net is not pure (i.e., the structure has a self-loop), the "0" in the fourth row occurs. Clearly, one token is removed and one token is newly generated. But it is not distinguished from the "0" in the first row that represents no change after the firing of a transition. For example, place $p_6$ in Figure 4.1 has a self-loop and computes the result "0" at the place $p_6$ in Table 4.2.
Table 4.2 Subtraction of Marking Vectors from an Occurrence Graph

<table>
<thead>
<tr>
<th></th>
<th>M₀</th>
<th>M₁</th>
<th>M₂</th>
<th>M₃</th>
<th>M₁ - M₀</th>
<th>M₂ - M₁</th>
<th>M₃ - M₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(p₁)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M(p₂)</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>M(p₃)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>M(p₄)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>M(p₅)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>M(p₆)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M(p₇)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>M(p₈)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>

Note that this subtraction of two markings is different from the subtraction of multi-set. Recall that the subtraction of multi-set is defined as:

\[ m₁ - m₂ = \sum_{s \in S} (m₁(s) - m₂(s))s \text{, when } m₂ \leq m₁ \text{ and } S \text{ is a non-empty set.} \]

The operation has the pre-condition \( m₂ \leq m₁ \). When \( m₁ \leq m₂ \), the subtraction operation of multi-set cannot be conducted. By defining the intersection \( \cap \) of two multi-sets as:

\[ m₁ \cap m₂ = \sum_{s \in S} \{ \min(m₁(s), m₂(s))s \mid s \in m₁, s \in m₂ \} \quad (4.1) \]

the subtraction operation of two marking vectors \( M₁ \) and \( M₂ \) can be defined as:

\[ M₁ - M₂ = \sum_{p \in P} \{ (M₁(p) - [M₁(p) \cap M₂(p)]) \} \quad (4.2) \]

where \( M_j(p) \) is the marking of place \( p \) at the marking vector \( M_j \) and \( P \) is a set of places.

Remember that a marking is a multi-set over \( P \). Using Equations 4.1 and 4.2, given two subsequent marking vectors \( M \) and \( M' \) of pure Petri nets (not necessarily ordinary Petri nets), we can distinguish new tokens and removed tokens by subtraction of the two marking vectors by:

\[ \text{new token(s)} = M' - M \text{ and removed token(s)} = M - M' \quad (4.3) \]

If the two subsequent markings \( M \) and \( M' \) are the ones before and after firing one transition respectively (i.e., not concurrent firing of multiple transitions) of a pure Petri net, the removed tokens become the triggering tokens of the new tokens that have the precedence relations.
between those two sets of tokens. Suppose that Figure 4.1 is an executable functional model

corresponding to the architecture specifications of Figure 3.3. The occurrence graph with an
initial marking $M_0 = [1 1 0 0 1 1 0 0]^T$ is shown in Figure 4.3. Table 4.2 is the result of the
subtraction operation of the marking vectors.

\[
M_0 = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}^T \\
M_1 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}^T \\
M_2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}^T \\
M_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}^T
\]

Figure 4.3 Occurrence Graph of an Executable Functional Model

4.1.2 Total Ordering of Events

The occurrence graph shown in Figure 4.3 was generated by a firing sequence of transitions
\( \sigma = t_1 \cdot t_2 \cdot t_3 \) in the Petri net shown in Figure 4.2. The executable functional model represents a
mission consisting of a set of tasks \( \{t_1, t_2, t_3\} \). The sequence of tasks, how the mission is carried
out, is represented by the firing sequence of transitions. The set of events of the executable
functional model maps to the set of transitions \( \{t_1, t_2, t_3\} \), and the causality between events maps
to the firing sequence: \( t_1 \rightarrow t_2 \) and \( t_2 \rightarrow t_3 \). Thus the process of the executable functional model
\( (P_{c2}) \) has a total order:

\[ H_{c2} = (h_{c2}, \rightarrow_{c2}) \), where \( h_{c2} = \{t_1, t_2, t_3\} \) and \( \rightarrow_{c2} = \{(t_1, t_2), (t_2, t_3)\} \).

Let \( m_j(p_i) \) be the marking of place \( p_i \) at the marking vector \( M_j \). The firing of transitions \( t_1, t_2, \) and \( t_3 \) generates a set of new tokens \( \{m_1(p_3), m_1(p_4)\}, \{m_2(p_2), m_2(p_7)\}, \) and \( \{m_3(p_5), m_3(p_8)\} \),
respectively (identified by the “+” signs in Table 4.2). This set of new tokens plus the initial
tokens \( \{m_0(p_1), m_0(p_2), m_0(p_3), m_0(p_6)\} \) are the set of messages to be transmitted over the
communication network. The transitions \( t_1, t_2, \) and \( t_3 \) are conditioned on a set of input tokens
\( \{m_0(p_1), m_0(p_2)\}, \{m_1(p_3), m_1(p_5)\}, \) and \( \{m_2(p_4), m_2(p_7)\} \), respectively (identified by the “-“ signs
in Table 4.2). So the process of the communication network model has the following causality
between sets of messages:
{m_0(p_1), m_0(p_2)} \rightarrow \{m_1(p_3), m_1(p_4)\}

{m_1(p_3), m_1(p_5)} \rightarrow \{m_2(p_2), m_2(p_7)\}

{m_2(p_4), m_2(p_7)} \rightarrow \{m_3(p_5), m_3(p_8)\}.

However, m_1(p_5) and m_2(p_4) equal to m_0(p_5) and m_1(p_4), respectively, since the marking did not change (identified by “0” signs at each of the previous steps in Table 4.2). Thus the process of the communication model (P_{net}) has a total order:

H_{net} = (h_{net}, \rightarrow_{net}), where

h_{net} = \{m_0(p_1), m_0(p_2), m_0(p_5), m_0(p_6), m_1(p_3), m_1(p_4), m_2(p_2), m_2(p_7), m_3(p_5), m_3(p_8)\}

\rightarrow_{net} = \{\{m_0(p_1), m_0(p_2)\}, \{m_1(p_3), m_1(p_4)\},

(\{m_1(p_2), m_0(p_5)\}, \{m_2(p_2), m_2(p_7)\}),

(\{m_1(p_4), m_2(p_7)\}, \{m_3(p_5), m_3(p_8)\}\}.}

Figure 4.4 Evolution of Scalar Time in a Communications Network

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If we simulate the communication network model using the set of events $h_{\text{net}}$ (i.e., traffic), while not violating the precedence relation $\rightarrow_{\text{net}}$, we can measure the network delay as the state of the system evolves as designed in carrying out the mission. Figure 4.4 shows the evolution of logical time, using $d = 1$, at the communications network model.

Note that Figure 4.4 is not a model of distributed computation. It just illustrates the order of messages. In the figure, a message identification number ($m_i$) is assigned to each message; first, a sequential integer number is assigned to each initial token at the initial marking vector $M_0$, then the number is increased whenever a new token is identified at the next marking vector in the sequel. The arrow lines represent the precedence relation which defines a total order. Remember that the binary relation $\rightarrow_{\text{i}}$ defines a total order among the events ($h_i$) of a process ($P_i$) at which the events occur.

The example of how the logical time stamps are associated with the precedence relations for simulation will be demonstrated in the next section.

4.1.3 Simulation of Discrete-Event Systems

In a non-distributed system, a logical time system can be derived such that the sequential position of a message arising from anywhere in the running program is clearly identified by its time stamp and the time stamps provide a facility to determine the precedence relations between already received messages (Kalantery, 1997). During the running time of the network model, messages $m_1$, $m_2$, $m_3$, and $m_4$ in Figure 4.4 can be ordered on a single time line by comparing their simulation time stamps.

Let $V(m_i)$ be the virtual time stamp of message $m_i$. Then we can make an event list (i.e., scheduling traffic) by comparing their virtual time stamps. Suppose $V(m_1) = 4$, $V(m_2) = 3$, $V(m_3) = 2$, and $V(m_4) = 1$; then the scheduler makes an event list $[m_4, m_3, m_2, m_1]$. If any of these events is executed,

1) the scheduler deletes the event from the event list,
2) determines the next event, and
3) updates the event list (i.e., reorder the events comparing the time stamp of the new event and those of events on the event list), and finally
4) the virtual time advances to the time stamp of the first event in the event list.
For example, if the event \( m_4 \) is executed at time 1, the event \( m_4 \) is deleted on the event list. It is not supposed to trigger any events. So the virtual time advances to the time stamp 2 of event \( m_3 \) which becomes the first event on the event list after deletion of the event \( m_4 \). Step 2 and 3 are skipped. Because the virtual time has been advanced to the virtual time stamp of event \( m_3 \), the simulation engine executes the event \( m_3 \) at time 2.

After the event \( m_3 \) is executed, it is supposed to trigger events \( m_7 \) and \( m_8 \). But events \( m_7 \) and \( m_8 \) cannot be executed since the preceding events \( m_1 \) and \( m_2 \) have not been executed (the logical time stamps of the two events \( m_7 \) and \( m_8 \) are higher than the time stamps of events \( m_1 \) and \( m_2 \)). So the scheduler deletes event \( m_3 \) on the event list and the virtual time advances to the virtual time stamp 3 of event \( m_2 \) which becomes the first event on the event list after deletion of the event \( m_3 \).

After event \( m_2 \) is executed, it is supposed to trigger events \( m_5 \) and \( m_6 \). But events \( m_5 \) and \( m_6 \) cannot be executed since the preceding event \( m_1 \) has not been executed (the logical time stamps of the two events \( m_5 \) and \( m_6 \) are higher than the time stamps of the event \( m_1 \)). So the scheduler deletes event \( m_2 \) on the event list and the virtual time advances to the virtual time stamp 4 of event \( m_1 \) which becomes the first event on the event list after deletion of event \( m_2 \).

Once event \( m_1 \) is executed, it can trigger events \( m_3 \) and \( m_4 \) since every event preceding the two events has been executed. So the scheduler deletes event \( m_1 \) on the event list (step 1), declares events \( m_3 \) and \( m_4 \) as the new events and adds them on the event list (step 2), then reorders the events by comparing the time stamps of the added events to the ones of existing events, if other events exist (step 3). At this time, there is no existing event that has been scheduled. So the events \( m_3 \) and \( m_4 \) are the only events on the event list and they can be executed in any order since their logical time stamps are the same. As introduced in the previous chapter, the events with the same logical time stamps can be ordered using any criterion (e.g., FIFO or Priority) since the order does not violate the total order of the functional architecture. Thus, the events \( m_3 \) and \( m_4 \) can be executed in any order in the communication network. The virtual time advances to the virtual time stamp of the first event (either event \( m_3 \) or event \( m_4 \)) on the event list. The same procedure is conducted until there is no event to execute. Following this scheduling scheme guarantees avoiding the violation of the total order.
4.2 A Model of Synthetic Execution

In the model of synthetic execution, the architecture of a system is layered into two layers: a functional architecture layer and a physical architecture layer. The division of the two layers is based on a concept of layered architectural states as shown in Figure 4.5. The state of system is divided into the inner state (state of physical architecture) and the outer state (state of functional architecture) across the boundary of the physical architecture and functional architecture.

Figure 4.5 Concept of Layered Architectural States

In a traditional International Standard Reference Model of Open Systems Interconnection (OSI, 1983), the functional architecture may correspond to the highest layer (i.e., applications layer) and the physical architecture may correspond to all the combined lower layers. The functional architecture layer consists of a set of activities (or functions), their order, and the interactions among activities. The physical architecture layer consists of the assets (sensors, computing devices, weapon systems, etc.) that are attached to the network as well as the communications model. The functional architecture layer generates a list of events (e.g., traffic), and the physical architecture layer process the events (e.g., traffic). The response time of the system is determined by the processing time of the event by the physical resources.

In this approach, the executable model for performance prediction is also developed as two models: the executable functional model and the executable physical model. The executable
The functional model uses a Petri net to describe the logical behavior and the executable physical model uses a queueing net to represent the demand and/or contention of resources necessary to implement the logical behavior. From the executable functional model, the message-passing pattern is generated using a state space analysis technique. The executable physical model processes these messages preserving the message-passing pattern in a total order. The executable functional model is used for analysis of the system's logical behavior. The executable physical model is used for analysis of the system's timing behavior. Once the network delay is measured, the delay values extracted from the executable physical model are inserted into the executable functional model so that both the logical behavior and timing behavior can be analyzed.

One example of the layered architecture model is shown in Figure 4.6. The functional architecture has three tasks (or functions); $T_i$, $T_j$, and $T_k$. The physical architecture provides services to carry out the tasks of the functional architecture. Resources $R_i$, $R_j$, and $R_k$ can be interpreted as servers to carry out the tasks $T_i$, $T_j$, and $T_k$, respectively. The information flow $m(0, i)$, $m(i, j)$, $m(j, k)$, and $m(k, 0)$ generates a message passing pattern depending on the order of task executions. In turn, this message passing pattern generates traffic over the communication networks between source-destination node pairs $(N_0, N_i)$, $(N_i, N_j)$, $(N_j, N_k)$, and $(N_k, N_0)$, respectively, where $N_0$ is the environment that is external to the system boundary.

![Figure 4.6 Layered Architectures](image-url)
When Rj receives input messages \( m(., i) \), it takes pre-processing time \( x(i) \), allows a specified action time \( \tau(i) \) to elapse, and takes post-processing time \( y(i) \) for output messages \( m(i, .) \). Let \( B(i) \) be the message processing rate of Rj. Then we have

\[
\tau_{(i)} = x_{(i)} + \tau_{(i)} + y_{(i)}
\]

where,

\[
x(i) = \sum m(n, i) / B(i)
\]

\[
y(i) = \sum m(i, n) / B(i).
\]

Network performance determines the network delay \( \delta(., i) \) and \( \delta(i, .) \).

The performance prediction model can be specified as a queueing net consisting of four sub-systems as shown in Figure 4.7, where, \( A_i \) is the arrival rate of messages at the Resource Rj, \( A_0 \) is the background traffic other than the messages of the applications, and function \( D \) is a delay function for each message.

![Figure 4.7 Executable Physical Model as a Queueing Net](image)

When the system is complex, it is not easy to develop the mathematical model of the queueing net. One of the complexities is the message-passing pattern. In order to represent the order of task execution, we use the total order of events. Modeling the arrival rate of messages or the occurrence rate of events in probabilistic distribution forms does not guarantee the total order. Also, each message can have a different message size and priority. So, the executable physical
model is developed as a simulation-based queueing net so that the total order of messages can be preserved during the simulation.

4.3 Workflow for System Analysis

In this methodology, the state of an executable functional model is represented by the occurrence graph of the corresponding Petri net. State vector $[m(p_1), m(p_2), m(p_3), ...]^T$ in the executable functional model represents the logical state of a system. From the occurrence graph, the order of messages being passed is captured and generated in a traffic log file. The order of message passing $[\{a,b\} \rightarrow \{c,d\} \rightarrow \{e\}, ...]$ has information about the number of messages and the precedence relations between sets of messages. The communication network simulator processes each message in a sequential order over a single time line, and generates the delay values of each message. The result of the network simulation is the estimates of two processing delays (i.e., pre-processing delay $x(i)$ and post-processing delay $y(i)$) for each node and the network transmission delays $\delta(i)$ for links. Finally, all the delay values are mapped back to the functional model for system analysis. Figure 4.8 shows the workflow for the development of the performance prediction model for analysis.

In Figure 4.8, $\tau$ is the action time and $\Delta(i, j)$ is the sum of node processing delays and transmission delays: $\Delta(i, j) = x(i) + y(i) + \delta(i, j)$. (see Equation 2.4 in Chapter 2)

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**Figure 4.8** Workflow for the development of the performance prediction model
4.4 Procedures to Develop Synthetic Executable Model

The methodology for developing a synthetic executable model for performance prediction of a real-time distributed system consists of six major steps:

1. Develop basic executable functional model,
2. Identify network service interfaces,
3. Create message dependency relations,
4. Simulate executable physical model,
5. Estimate node processing time and network delays, and
6. Introduce time to basic executable functional model.

4.4.1 Step 1: Develop basic executable functional model

The functional decomposition can be carried to any level of detail. It may depend on the level of abstraction and modeling purpose. In practical terms, it is carried out to the point that the lowest level tasks must be executed by a single resource (Levis, 1992). To estimate the network delays, we need to identify the physical resources in the communication network architecture. So, the decomposition process may stop at the level at which the physical resources can be identified at the communication network and its elemental performance parameters are known.

Suppose that the static functional architecture in Figure 3.3 describes a system to be used to clear threats in which a man-machine decision making system makes decisions to respond to the threats based on input sensor values. A corresponding Petri net structure can be depicted as shown in Figure 4.9.

As stated in the previous section, the performance prediction model requires the synthesis of both functional and physical models. An executable model for performance prediction needs the allocation of resources (i.e., assets) to functions. Once a set of tasks is represented by "transitions," the allocation of assets to perform the tasks is represented by tokens at "places." Places p11, p12, and p13 are used for assignment of assets to tasks. Resources R_i, R_j, and R_k are instances of the assets. These physical resources will have dual roles: one role as computing devices in the functional architecture and the other role as traffic generators in the physical architecture.
As a computing device, the physical resource can be interpreted as a server to carry out a task. Adding these physical resources to the executable functional model denotes that the task uses the service of the resource to execute the task. If those resources at places p11, p12, and p13, are used only once and consumed, those arcs will be uni-directional arcs from p11 to Ti, p12 to Tj, and p13 to Tk. Similarly, if the engagement rule at place p6 is used only once and consumed, the arc from p6 to Tj will be uni-directional. If the rule is used repeatedly and updated dynamically, the arc must express a self-loop. If the rule changes dynamically depending on the state, it must have a time stamp and hence the color set of place p6 must be timed.
4.4.2 Step 2: Identify network service interfaces

The communication network simulation model can be generalized as a network of a set of nodes, namely \( N_1, N_2, ..., N_n \), and a set of links which are node pairs \((i, j)\) between node \( N_i \) and node \( N_j \). Each link has a capacity \( C_{(i, j)} \), counted in bandwidth units. Links are undirected; \((i, j)\) and \((j, i)\) denote the same link. These links describe the duplex transmission links of the communications network. In order to describe the node processing of an information system, a special link with node pair \((x, x)\) is added as an extension to the transmission link. The capacity \( C_{(x, x)} \) of the special link, then, can be used to specify the node processing rate in bandwidth units.

In this executable physical model, the resources are modeled as part of the network nodes. As shown in Figure 4.10, the resource \( R_i, R_j, \) and \( R_k \) are added to the communication network as leaf nodes \( N_i, N_j, \) and \( N_k \), respectively. The physical resources allocated to the functional architecture model generate the application network traffic. The same physical resources connected to a communication network model can be used to identify the communication service access points in the network.

![Figure 4.10 Communication Network Access Points](image-url)
When resource $R_x$ is added to the communication network, the resource node is composed of two nodes $N_x$ and $N_{x'}$, where $N_x$ is the communication service access points and $N_{x'}$ is an extra terminal node added to model the special link for representing node processing rate. That is, the capacity of the special link for the node processing rate is represented by the capacity $C_{(x, x')}$ of the link $(x, x')$. In Figure 4.10, the capacities $C_{(i, i')}$, $C_{(j, j')}$, and $C_{(k, k')}$ represent the bandwidth of the information processing rate. In this methodology, the message traffic has seven attributes; message identification (ID) number, source node, destination node, message size, priority, protocol, and message dependency relation.

The appearance of tokens $m(i, j)$, $m(i, k)$, $m(j, k)$, $m(j, i)$, and $m(k, j)$ in places $p_3$, $p_4$, $p_7$, $p_2$, and $p_5$ in Figure 4.9, respectively, represents the message-passing among tasks $T_i, T_j,$ and $T_k$. The corresponding source-destination node pairs in the physical model are identified by the physical node pairs $(N_i, N_j)$, $(N_i, N_k)$, $(N_j, N_k)$, $(N_j, N_i)$, and $(N_k, N_j)$, respectively. The appearance of tokens in place $p_6$ in Figure 4.9 does not involve the network transmission delay. The message $m(j, j)$ involves only the node processing time in the information processing system. The source-destination nodes pair at the physical node is identified by the special link with node pair $(N_j, N_j)$. Practically, it is implemented by the link with node pair $(N_j, N_{j'})$ adding the additional leaf node $N_{j'}$.

4.4.3 Step 3: Create message dependency relations

When the firing sequence of transitions is known, we can determine the sequence of message passing. When the firing sequence is not known, we have to consider all possible sequences of transitions. All possible sequences are captured in the full occurrence graph of the Petri net. Generating a full occurrence graph and searching it exhaustively are not practically acceptable when the state space is large.

In a deterministic Petri net with a conflict free net structure, the firing of one transition does not disable other transitions. Once tokens in the input places of a transition fulfill the enabling conditions, the transition will definitely fire. So, even though multiple transitions in a Petri net are enabled concurrently, the final markings of the net are the same regardless of the firing order of the concurrently enabled transitions in a deterministic Petri net with a conflict free net structure. Also, the group of messages that are produced after firing of concurrently enabled transitions have no precedence relations within them. Message dependency relations occur only
between different sets of messages that are produced after subsequent firings other than concurrent firings. Thus, given a conflict free net with initial markings, one sequence of occurrence of markings in a partial occurrence graph has all the information necessary to count the number of messages in the system and their precedence relations. If we generate a partial occurrence graph using the depth-first search algorithm and there is no circuit in the net, it is not costly to capture the message dependency relations, and the methodology developed can be used effectively.

If a Petri net is not conflict free, however, the net may have multiple final markings. Even if the net has a single final marking, there may exist multiple sequences of occurrences of markings that have different precedence relations amongst different sets of messages. To handle those non-conflict free nets, the methodology may require repeated simulations of the communications network corresponding to the different paths in the full occurrence graph.

One alternative way to estimate network delays between all transitions is to set the initial marking so that all transitions are covered in each path of the full occurrence graph. Then, the methodology can estimate the network delay between tasks by selecting one path under the assumptions that the communications network has the following characteristics: (1) network delays are longer under heavier traffic loads than under lighter traffic loads, and (2) the degree of traffic loads is defined as the amount (e.g., number or size) of messages in a given time interval. By selecting the path that has the maximum number of messages, the methodology can estimate the upper bound of network delays given the initial marking. Similarly, by selecting the path that has the minimum number of messages, the methodology can estimate the lower bound of network delays given the initial marking. Instead of repeated simulations of a communications network (as many as the number of paths in a full occurrence graph), the alternative method may be more efficient for a worst-case timing analysis.

The newly generated tokens are identified by subtraction between subsequent markings. However, the occurrence graph of an ordinary Petri net cannot distinguish the new tokens in a self-loop. By using a Colored Petri net (see Appendix A) model and by adding a counter at the self-looped place, we can distinguish them. In a model of a real-time system, new tokens in a self-loop can be distinguished by their time stamps. So there are two ways for distinguishing new tokens in a self-loop (a) by adding a logical time stamp, and (b) by using a counter. If we
generate an occurrence graph after assigning time delays to transitions, the occurrence graph becomes a timed occurrence graph. The subtraction operation over the markings of a timed occurrence graph can distinguish them.

For example, by adding time delay "1" on the arc from transition Tj to place p6 in Figure 4.7, if necessary, we can distinguish the new tokens in the self-loop. Table 4.3 shows the message dependency relations that were generated from a timed occurrence graph. At each column of the right side of the table, messages with "+" sign are triggered by messages with the "-" sign.

The evolution of the state change at the place p6 can now be identified in Table 4.3 while it was non-identifiable in Table 4.2 (see the bolded cell in both figures). This is necessary depending on the applications. Suppose that the task Tj is a computerized algorithm and the place p6 is a rule base. The delay value associated with the self-looped arc between transition Tj and place p6 may represent the processing or the retrieving time of the rule from the database even though it has no message transmission delay.

Table 4.3 Subtraction of Marking Vectors from the Timed Occurrence Graph

<table>
<thead>
<tr>
<th></th>
<th>M0</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M1 - M0</th>
<th>M2 - M1</th>
<th>M3 - M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(p1)</td>
<td>(1,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>M(p2)</td>
<td>(1,0)</td>
<td>(0,0)</td>
<td>(1,2)</td>
<td>(1,2)</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>M(p3)</td>
<td>(0,0)</td>
<td>(1,1)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>M(p4)</td>
<td>(0,0)</td>
<td>(1,1)</td>
<td>(1,1)</td>
<td>(0,0)</td>
<td>+1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>M(p5)</td>
<td>(1,0)</td>
<td>(1,0)</td>
<td>(0,0)</td>
<td>(1,3)</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>M(p6)</td>
<td>(1,0)</td>
<td>(1,0)</td>
<td>(1,2)</td>
<td>(1,2)</td>
<td>0</td>
<td>±1</td>
<td>0</td>
</tr>
<tr>
<td>M(p7)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(1,2)</td>
<td>(0,0)</td>
<td>0</td>
<td>+1</td>
<td>-1</td>
</tr>
<tr>
<td>M(p8)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(0,0)</td>
<td>(1,3)</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>

Whether one uses a counter or a logical time stamp depends on the applications. The occurrence graph using the logical time stamp may generate a partial occurrence graph since a timed occurrence graph is a subset of an untimed occurrence graph (Jensen, 1997). Whether to use the counter or the logical time stamp depends on the trade-off between having the benefit of the full occurrence graph and the benefit of the efficiency in generating the partial occurrence.
graph respectively. In a conflict free net, the order of message passing does not change even though we use the logical time stamp. So in a conflict free net, we can extract the number of messages and their precedence relations without affecting the logical behavior.

### 4.4.4 Step 4: Simulate executable physical model

The role of message dependency (or precedence relations) is to determine the order of message passing to schedule every message over a single time line during the simulation. Figure 4.11 shows the message dependency graph derived from the timed occurrence graph based on Table 4.3. At this step, the message attributes: message identification (ID) number, source node, destination node, message size, priority, protocol, and message dependency relation, are designated using the information obtained in the previous step. Node ID 0 (N₀) in the graph means the external environment.

Table 4.4 is a traffic log in table format representing the message dependency relations. Instead of having a message generation time, which is usually specified by a probabilistic form such as Poisson or exponential distribution in a typical communication network simulation approach, each message is specified by its dependency relations.

Since the delays are not known at the logical model, the time stamp of messages from the logical model can not be used directly for scheduling. The time stamps of the initial messages are scheduled at the stamped time in the event list. Table 4.4 shows that messages 1, 2, 3, and 4 are the initial messages and have no preceding messages. So these four messages are scheduled at the stamped time on the event list. After that, the messages are ordered by dynamic scheduling as explained in section 4.1.3.

Figure 4.11 shows that two messages with IDs 5 and 6 will be triggered after two initial messages with IDs 1 and 2 are processed. If transition Tᵢ (see Figure 4.9) has an action delay (e.g., τ₁₁), however, messages with IDs 5 and 6 must be generated after the action delay. Similarly, three messages with IDs 7, 8, and 9 will be triggered after three preceding messages with IDs 3, 4, and 5 are processed. If transition Tⱼ has an action delay (e.g., τ₁₂), then, messages with IDs 7, 8, and 9 must be generated after the action delay. In turn, two messages with IDs 10 and 11 will be triggered after two preceding messages with IDs 6 and 9 are processed. If transition Tⱼ has an action delay (e.g., τ₁₃), then, messages with IDs 10 and 11 must be generated after the action delay.
Figure 4.11 Message Dependency Graph

Table 4.4 Message Dependency Table

<table>
<thead>
<tr>
<th>ID</th>
<th>Source</th>
<th>destination</th>
<th>size</th>
<th>priority</th>
<th>protocol</th>
<th>dependency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>m(p₁)</td>
<td>EXTERNAL</td>
<td>Nᵢ</td>
<td>1 byte</td>
<td>UDP</td>
<td>[]</td>
</tr>
<tr>
<td>2</td>
<td>m(p₂)</td>
<td>INITIAL</td>
<td>Nᵢ</td>
<td>1 byte</td>
<td>UDP</td>
<td>[]</td>
</tr>
<tr>
<td>3</td>
<td>m(p₅)</td>
<td>INITIAL</td>
<td>Nⱼ</td>
<td>1 byte</td>
<td>UDP</td>
<td>[]</td>
</tr>
<tr>
<td>4</td>
<td>m(p₆)</td>
<td>INITIAL</td>
<td>Nⱼ</td>
<td>1 byte</td>
<td>UDP</td>
<td>[]</td>
</tr>
<tr>
<td>5</td>
<td>m(p₃)</td>
<td>Nᵢ</td>
<td>Nⱼ</td>
<td>1000byte</td>
<td>TCP</td>
<td>[1,2]</td>
</tr>
<tr>
<td>6</td>
<td>m(p₄)</td>
<td>Nᵢ</td>
<td>Nᵦ</td>
<td>1000byte</td>
<td>TCP</td>
<td>[1,2]</td>
</tr>
<tr>
<td>7</td>
<td>m(p₂)</td>
<td>Nⱼ</td>
<td>Nᵢ</td>
<td>1000byte</td>
<td>TCP</td>
<td>[3,4,5]</td>
</tr>
<tr>
<td>8</td>
<td>m(p₆)</td>
<td>Nⱼ</td>
<td>Nⱼ</td>
<td>1000byte</td>
<td>UDP</td>
<td>[3,4,5]</td>
</tr>
<tr>
<td>9</td>
<td>m(p₇)</td>
<td>Nⱼ</td>
<td>Nⱦ</td>
<td>1000byte</td>
<td>TCP</td>
<td>[3,4,5]</td>
</tr>
<tr>
<td>10</td>
<td>m(p₅)</td>
<td>Nⱦ</td>
<td>Nⱼ</td>
<td>1000byte</td>
<td>TCP</td>
<td>[6,9]</td>
</tr>
<tr>
<td>11</td>
<td>m(p₈)</td>
<td>Nⱦ</td>
<td>EXTERNAL</td>
<td>1000byte</td>
<td>UDP</td>
<td>[6,9]</td>
</tr>
</tbody>
</table>
4.4.5 Step 5: Estimating node processing time and network delays

For each task node \( i \) of the executable functional model, the node processing times \( x(i) \) and \( y(i) \) are measured using the Equation 4.2. The message processing rate \( B(i) \) is the capacity \( C(i, i') \) of the special link with node pair \((N_i, N_{i'})\). The pre-processing time \( x(i) \) is measured by the processing time of the input messages \( \sum_n m(n, i) \), where \( n \) is the number of input resource nodes to the resource node \( i \), and the post-processing time \( y(i) \) is measured by the processing time of the output messages \( \sum_n m(i, n) \), where \( n \) is the number of output resource nodes from the resource node \( i \).

If multiple messages are competing to use the computing device of the resource \( i \) at the same time\(^1\), they can be ordered with respect to any criteria since the messages are scheduled dynamically preserving the total order. In a tactical communication system, the messages often have a priority or FIFO protocol is used. In Table 4.4, each message has its own priority for practice. So they will be processed with the designated priority first, then with FIFO protocol. The processing time will be measured from the time at which each message enters the queue of the special link to the time at which the message departs the special link after being processed. Using this mechanism, the contention of information processing device can be represented, when several tasks compete to use the same computing device of the resource \( i \).

Once the node processing of each message is completed, the network delay of each message will be measured from the time it departs the processing node (i.e., the time it enters the communications network) to the time at which it arrives at the destination node. If a message does not involve the network transmission delay identified by the same node IDs of the source node and the destination node, the network transmission delay becomes zero. For example, the place p6 in Figure 4.9 is in a self-loop having the identical source and destination nodes. So the message with ID 8 does not involve a network delay (i.e., \( \delta(i, i) = 0 \)). Similarly, the message with ID 11 has no network delay because its destination node is the external environment (i.e., \( \delta(k, 0) = 0 \)).

---

\(^1\) Note that Equation 4.2 produces multiple messages since new tokens are generated by multiple elements (places) of a marking vector and each element (place) can have multiple tokens.
The network connection delays can be estimated in two ways: the exact value estimation approach and the worst-case estimation approach. The exact value estimation approach is to generate all messages out of the occurrence graph and measure the delay of all messages in a system. This exact value estimation approach may not be efficient when the system has periodic information flow. Consider an AAW system with multiple threats. If a sensor observing a target periodically reports the status of the target, the total number of messages in the system, which are generated from the occurrence graph, may increase tremendously.

The worst-case estimation approach is to measure the network delay under the assumption that the communication network characteristics are: (1) network delays are longer under heavier traffic load than under lighter traffic load, and (2) the degree of traffic load is defined as the number of messages in a given simulation time such that the traffic load with the same number of messages in a short simulation time is higher than those in a long simulation time.

Consider that the messages are triggered by the occurrence of the threat. The number of threats does not change the message-passing pattern assuming that the system does not dynamically adapt as a function of load. If we figure out the message-passing pattern using a case with one threat, we can use this pattern repeatedly. We can represent multiple threats using the inter-arrival time of threats (messages). Suppose the scanning cycle of the sensor is 2 seconds. This generates a periodic message flow every 2 seconds. But the message pattern does not change. We can simulate the communication network using the message pattern from a single threat case with the inter-arrival time of 2 seconds.

Now suppose that, in section 4.4.4, the action times $\tau_{11}$ (scanning cycle of a sensor), $\tau_{12}$ (decision time of a decision maker), and $\tau_{13}$ (loading time of a missile) can have interval values depending on the characteristics of resources. Suppose $\tau_{11} = [0, 2], \tau_{12} = [5, 10], \tau_{13} = [1, 2]$. If we use the lower bound action times (i.e., $\tau_{11} = 0, \tau_{12} = 5, \tau_{13} = 1$) for simulation, the message inter-arrival times are reduced and the traffic becomes denser compared to the case that we use the upper bound action times (i.e., $\tau_{11} = 2, \tau_{12} = 10, \tau_{13} = 2$). If we set all the inter-arrival times to be 0, the resulting network delay, under the assumptions, will represent the maximum delay value, while the resulting network delay with the upper bound inter-arrival time will represent the minimum value. So the minimum and maximum action time of a task
execution time may be used to estimate the bounded network delays (the minimum and maximum network delays) under the assumptions.

After measuring the processing time and network delay while running the simulation, two types of delay information can be obtained. One is the statistical representation of the delay in a probabilistic form, and the other is a pair of bounded network delays. Assume that the communications network has the characteristics explained above. If the delays with light traffic load (lower bound network delays) are smaller than the delays with heavy traffic load (upper bound network delays), and if they are consistent for various degrees of traffic load, the two assumptions regarding the characteristics of the communications network may be considered reasonable. Then, we can determine the minimum and maximum network delay pairs for each message and use them to verify the end-to-end timing constraints.

4.4.6 Step 6: Introduce time to the basic executable functional model

Once we obtain the node processing time and network delays with either a probabilistic distribution form or bounded delays, these node processing time and network delay values are specified as arc expressions of a Petri net. This is the last step to develop the performance prediction model. The resulting Timed Petri net model has all the information to predict the performance of the system. If we care only about the end-to-end timing delay, this step is not necessary since the output data from the executable physical model has the same information. However, this step helps to analyze the logical behavior and timing behavior together using the resulting timed occurrence graph.

Suppose that the sub-total delays of the information system, as computed by Equation 2.5, are \( \Delta \), \( \Delta_{ij} \), \( \Delta_{ik} \), \( \Delta_{kj} \), \( \Delta_{ij,k} \), \( \Delta_{kj,k} \), and \( \Delta_{ij,k} \), associated with each place p1, p2, p3, p4, p5, p6, p7, and p8, respectively. The performance prediction model (Figure 4.12) can be completed by adding the time values to the basic executable functional model of Figure 4.9.
Figure 4.12 Performance Prediction Model

In the figure, $A_{(0,i)}$ is the initializing time of the system. $\tau_1$, $\tau_2$, and $\tau_3$ are the action times of tasks $T_i$, $T_j$, and $T_k$ (i.e., task execution time of the actor using resources $R_i$, $R_j$, and $R_k$ associated with each place $p_{11}$, $p_{12}$, and $p_{13}$, respectively).
CHAPTER 5. SYNTHETIC EXECUTABLE MODEL GENERATOR

A synthetic executable model generator (SYNE-X) for performance prediction of a real-time C3 system has been developed so that it can be used efficiently and effectively for various scenarios or design options. Design/CPN (1999) has been used for the description of the functional model and Network Simulator (1999) for the description of the physical model. The SYNE-X consists of 5 major software components:

- Reachability Tree Generator (RTG)
- Message Generator (MG)
- Performance Prediction Model Converter (PPMC)
- Network Topology Generator (NTG)
- Communication Network Simulation Script (CNSS)

The four components RTG, MG, PPMC and NTG were implemented using the ML (Meta Language) imbedded in Design/CPN. CNSS was implemented using the TCL (Tool Command Language) embedded in Network Simulator.

Figure 5.1 is the detailed workflow model implemented in the Design/CPN environment. As stated, this dissertation focuses on the synthesis phase of the system architecting cycle introduced in Section 3.1. The workflow begins with a functional architecture while a physical architecture is produced at the end of the analysis phase. Given the basic Petri net model shown in Figure 4.8, the SYNE-X generates the performance prediction model shown in Figure 4.11. Typically, the basic Petri net model will be an untimed one. It can be developed as a timed Petri net model, but with incomplete timing information. The performance prediction model is the complete timed Petri net model.
Figure 5.1 Detailed Workflow Model for System Analysis

To describe each step, the same AAW system shown in Figure 4.8 will be used. Figure 5.2 is the basic Petri net model developed in Design/CPN corresponding to Figure 4.8. Table 5.1 is the declaration of the variables (global declaration node). See Appendix B for an explanation of the net descriptions implemented by Design/CPN.
Figure 5.2 A Basic Petri Net Model

The operation concept of the model is as follows. The system senses threats using radar. It scans the battlefield periodically, 30 times per minute (e.g., the minimum action delay of sensing is 0 seconds and the maximum action delay is 2 seconds). It sends the tactical picture to its missile Fire Direction Center (FDC). Once the FDC receives information on a new threat, the FDC checks the availability of weapons, and issues an order to a Missile Launcher in concordance with the engagement rule, and issues a surveillance directive. This action is invoked automatically by embedded algorithms in the automated missile controller. It takes some time for data fusion, but the delay is assumed to depend only on the size of the tactical picture received and the bit processing power of the computing device. The missile launcher platform requires 5 seconds warming time to fire a missile (e.g., minimum delay of 0 seconds when it is already warm and maximum action delay of 5 seconds). All messages in the system are 1000 bytes (8000 bits) long.
Table 5.1 Global Declaration Node

(* simple AAW model Global Declaration Node *)

```plaintext
color RealCS = int;
color Identification = RealCS;
color Counter = int;
color TgtID = int timed;
color SensorID = int timed;
color WeaponID = int timed;
color Threat = product TgtID * Identification;
color Asset = union Sensor:SensorID + Weapon:WeaponID;
color TacticalPicture = product SensorID * Threat;
color ScanModes = with Default | TrackingOnlyFor | ScaningOnlyFor;
color Surveillance_Directive = product SensorID * ScanModes * TgtID;
color Rules_of_Engagement = product RealCS * Counter timed;
color Decision = with Fire | WithDraw | Wait;
color Order = product TgtID * Decision;
color Assessment = with Cleared | KillSelf | Misshit | Leak;
color Negated = product TgtID * Assessment * WeaponID;
color Report = Negated;

var dth: RealCS;
var counter: Counter;
var threat: Threat;
var sid: SensorID;
var wid: WeaponID;
var rule_s: Surveillance_Directive;
var task: Order;

fun ThreatIsHostile (threat: Threat, dth: RealCS) = #2(threat) > dth;
fun CorrectlyIdentified (threat: Threat) = #2(threat) = 100;
```

5.1 Step1: Generating network topology

The network topology is generated by Network Topology Generator (NTG). Figure 5.3 is an example of a network topology drawn by NTG. NTG provides the user with a GUI to enter the number of nodes and their attributes; the number of switching (with the circle symbol) and the number of terminal nodes (with the square box symbol) which are the physical resources connected to the net. It automatically lays out the nodes in a two-dimensional grid and connects them in a default network topology of tactical communication networks.

Figure 5.3 shows the nine switching nodes (X_0 to X_8) and four terminal nodes (S101, W101, FDC, and AWACS). The bandwidth of all links is 1.024 Mbps with the propagation delay of 10ms and the queue type of DropTail. The bandwidth between switching node and terminal...
node is 56Kbps with propagation delay of 10ms and queue type of DropTail. Also, the figure shows that every terminal node has bit processing power of 66Mbps. This is the bandwidth of the special link that represents the node processing rate of the information system.

Figure 5.3 An Example of Network Topology drawn by NTG

Once the physical architectures are specified, NTG generates a text file of the network topology in a specific format so that it can be used by the CNSS. Table 5.2 is an example of the traffic log file in TCL script format.

5.2 Step2: Mapping physical resources to transitions

The process of selecting alternative resources is an issue of resource allocation or an issue of physical design to implement the logical functions and it depends on the scenarios. The mapping between transition and resource is done by NTG and MG. The main use of NTG is to allow the user to select the physical resources interactively.
Table 5.2 Network Topology File

```plaintext
# wan nodes list
set caesar_wan_nodes { AWACS W101 S101 FDC X_2 X_8 X_6 X_0 X_5 X_7 X_1 X_4 X_3 }
# lan nodes list
set caesar_lan_nodes { }
# links in simplex
set caesar_links {
  [{X_0 AWACS} 56Kb 10ms DropTail]
  [{AWACS X_0} 56Kb 10ms DropTail]
  [{X_3 W101} 56Kb 10ms DropTail]
  [{W101 X_3} 56Kb 10ms DropTail]
  [{W101 X_4} 56Kb 10ms DropTail]
  [{X_4 W101} 56Kb 10ms DropTail]
  [{X_4 S101} 56Kb 10ms DropTail]
  [{S101 X_4} 56Kb 10ms DropTail]
  [{S101 X_1} 56Kb 10ms DropTail]
  [{X_1 S101} 56Kb 10ms DropTail]
  [{FDC X_4} 56Kb 10ms DropTail]
  [{X_4 FDC} 56Kb 10ms DropTail]
  [{FDC X_8} 56Kb 10ms DropTail]
  [{X_8 FDC} 56Kb 10ms DropTail]
  [{X_0 X_3} 1.024Mb 10ms DropTail]
  [{X_3 X_6} 1.024Mb 10ms DropTail]
  [{X_6 X_7} 1.024Mb 10ms DropTail]
  [{X_7 X_8} 1.024Mb 10ms DropTail]
  [{X_8 X_5} 1.024Mb 10ms DropTail]
  [{X_5 X_2} 1.024Mb 10ms DropTail]
  [{X_2 X_1} 1.024Mb 10ms DropTail]
  [{X_1 X_0} 1.024Mb 10ms DropTail]
  [{X_0 X_1} 1.024Mb 10ms DropTail]
  [{X_1 X_2} 1.024Mb 10ms DropTail]
  [{X_2 X_5} 1.024Mb 10ms DropTail]
  [{X_5 X_8} 1.024Mb 10ms DropTail]
  [{X_8 X_7} 1.024Mb 10ms DropTail]
  [{X_7 X_6} 1.024Mb 10ms DropTail]
  [{X_6 X_3} 1.024Mb 10ms DropTail]
  [{X_3 X_0} 1.024Mb 10ms DropTail]
  [{X_3 X_4} 1.024Mb 10ms DropTail]
  [{X_4 X_5} 1.024Mb 10ms DropTail]
  [{X_5 X_4} 1.024Mb 10ms DropTail]
  [{X_4 X_7} 1.024Mb 10ms DropTail]
  [{X_7 X_4} 1.024Mb 10ms DropTail]
  [{X_4 X_1} 1.024Mb 10ms DropTail]
  [{X_1 X_4} 1.024Mb 10ms DropTail]
  [{X_4 X_3} 1.024Mb 10ms DropTail]
}
```
MG allows the user to allocate resources in order to generate the source and destination nodes of the communication network. NTG and MG provide the user with a Graphic User Interface (GUI) developed in the Design/CPN environment in order to obtain information necessary to produce the traffic log file. This GUI helps to select instances of resources that are connected to the communication network as leaf nodes. The specification of the physical resources has two attributes: the name of the asset and the action time when the actor executes the task (transition) using the asset. Figure 5.4 shows the specification at a dotted square box.

MG also allows the user to enter the attributes of actors. This action time is differentiated from the node processing delay and network transmission delays. The delay of an actor is associated with the delay of the task executor(s). If every task is an autonomous data processing task, the actor’s delay may be set to “0”. Once this attribute is obtained, it is maintained at a special region of each transition.

Figure 5.4 Mapping Resources to Transition

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5.3 Step 3: Checking the syntax of the basic Petri net model

This step is supported by RTG. The SYNE-X was designed to import any Petri net model once it is developed in Design/CPN. It uses the Occurrence Graph Analyzer, which is a component of Design/CPN, to capture the order of message-passing. Design/CPN has a built-in syntax checker. RTG allows the user to fix the imported model to be compliant with the syntax of OGA.

5.4 Step 4: Generating an Occurrence Graph

The RTG primarily uses the software function codes provided by the OGA of Design/CPN. However, this component was implemented to be interactive with users. It generates a reachability tree, searches final states, and calculates the shortest and longest paths from the initial state to final states. RTG generates a partial reachability tree until it finds at least one final state. Figure 5.5 is the full OG of the basic Petri net model generated by RTG.

If the net is conflict free, this partial reachability tree can be used to capture all the necessary state information. If the net is not conflict free, the user will be provided an option to search all final states and select one of the paths from the initial state to multiple states. The example model is a conflict free net. Any path in the occurrence graph can capture the order of events.

Table 5.3 is the list of newly generated tokens derived from the sequence of markings M1-M3-M5-M7-M10-M11. From Table 5.3, we can identify which tokens ("Message_ID") are generated from which transition ("Fired_Transition") to which place ("Place Instance"). Table 5.4 is the list of triggering tokens derived from the same sequence of markings.

The major role of RTG is to capture the precedence relations between tokens (i.e., messages). From the causal relation between tokens on the two tables, we can identify the total order of message-passing.

5.5 Step 5: Developing an intermediate Timed Petri net model

This step is supported by PPMC. The main purpose of the Performance Prediction Model Converter is to transform the basic Petri net into a Timed Petri net (i.e., Performance Prediction Model). An example of the transformed Timed Petri net model is shown in Figure 5.6.
Figure 5.5 Full Occurrence Graph from Basic Petri Net (Untimed)

Table 5.3 List of Newly Generated Tokens

<table>
<thead>
<tr>
<th>Marking_No</th>
<th>Fired_Transition Binding</th>
<th>Place Instance with List of Tapped (Msg_ID, Marking, Time_Stamp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mi_1</td>
<td>INITIAL INITIAL</td>
<td></td>
</tr>
<tr>
<td>IADS'Threat 1</td>
<td>(1,(1,100),0.0) (2,(2,0),0.0)</td>
<td></td>
</tr>
<tr>
<td>IADS'Rule 1</td>
<td>(3,(90,0),0.0)</td>
<td></td>
</tr>
<tr>
<td>IADS'Directive 1</td>
<td>(4,(101,Default,0),0.0)</td>
<td></td>
</tr>
<tr>
<td>IADS'Sensor 1</td>
<td>(5,(Sensor(101),0.0))</td>
<td></td>
</tr>
<tr>
<td>IADS'Shooter 1</td>
<td>(6,(Weapon(101),0.0))</td>
<td></td>
</tr>
<tr>
<td>Mi_3</td>
<td>IADS'Sense 1 IADS'Sense 1: {threat=(1,100),sid=101,rule_s=(101,Default,0)}</td>
<td></td>
</tr>
<tr>
<td>IADS'Identity 1</td>
<td>(7,(101,(1,100)),0.0)</td>
<td></td>
</tr>
<tr>
<td>IADS'Location 1</td>
<td>(8,(101,(1,100)),0.0)</td>
<td></td>
</tr>
<tr>
<td>Mi_5</td>
<td>IADS'Command 1 IADS'Command 1: {threat=(1,100),sid=101,dth=90,counter=0}</td>
<td></td>
</tr>
<tr>
<td>IADS'Order 1</td>
<td>(9,(1,Fire),0.0)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.4 List of Triggering Tokens

<table>
<thead>
<tr>
<th>Marking No</th>
<th>Fired_Transition</th>
<th>Binding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mi_1</td>
<td>INITIAL</td>
<td>INITIAL</td>
</tr>
<tr>
<td>Mi_3</td>
<td>IADS'Sense 1</td>
<td>IADS'Sense 1: {threat=(1,100),sid=101,rule_s=(101,Default,0)}</td>
</tr>
<tr>
<td></td>
<td>IADS'Threat 1</td>
<td>(1,(1,100),0.0)</td>
</tr>
<tr>
<td></td>
<td>IADS'Directive 1</td>
<td>(4,(101,Default,0),0.0)</td>
</tr>
<tr>
<td>Mi_5</td>
<td>IADS'Command 1</td>
<td>IADS'Command 1: {threat=(1,100),sid=101,dth=90,counter=0}</td>
</tr>
<tr>
<td></td>
<td>IADS'Rule 1</td>
<td>(3,(90,0),0.0)</td>
</tr>
<tr>
<td>Mi_7</td>
<td>IADS'Act 1</td>
<td>IADS'Act 1: {wid=101,threat=(1,100),task=(1,Fire),sid=101}</td>
</tr>
<tr>
<td></td>
<td>IADS'Location 1</td>
<td>(8,(101,(1,100)),0.0)</td>
</tr>
<tr>
<td></td>
<td>IADS'Order 1</td>
<td>(9,(1,Fire),0.0)</td>
</tr>
<tr>
<td>Mi_10</td>
<td>IADS'Sense 1</td>
<td>IADS'Sense 1: {threat=(2,0),sid=101,rule_s=(101,Default,0)}</td>
</tr>
<tr>
<td></td>
<td>IADS'Threat 1</td>
<td>(2,(2,0),0.0)</td>
</tr>
<tr>
<td></td>
<td>IADS'Directive 1</td>
<td>(11,(101,Default,0),0.0)</td>
</tr>
<tr>
<td>Mi_11</td>
<td>IADS'Command 1</td>
<td>IADS'Command 1: {threat=(2,0),sid=101,dth=90,counter=1}</td>
</tr>
<tr>
<td></td>
<td>IADS'Rule 1</td>
<td>(10,(90,1),0.0)</td>
</tr>
</tbody>
</table>
Figure 5.6 Intermediate Performance Prediction Model

Compare the transformed intermediate Timed Petri net model (Figure 5.6) to the basic Petri net model (Figure 5.1). Figure 5.6, additionally, has a local declaration node, a time delay arc expression, a code segment, and a temporary declaration node.

The local declaration node defines the variables that represent delay values. Each variable name specifies a logical connection link name between two tasks. It consists of a transition name (source) and a place name (sink). Table 5.5 is an example of the local declaration node.

<table>
<thead>
<tr>
<th>(* Local Declaration Node *)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pagerref IADS'Sense_1 = [7,14,8,15];</td>
</tr>
<tr>
<td>pagerref IADS'Sense_1'IADS'Identity = [7,14];</td>
</tr>
<tr>
<td>pagerref IADS'Sense_1'IADS'Location = [8,15];</td>
</tr>
<tr>
<td>pagerref IADS'Command_1 = [9,10,16,11,17];</td>
</tr>
<tr>
<td>pagerref IADS'Command_1'IADS'Order = [9];</td>
</tr>
</tbody>
</table>
The variable name "IADS'command_ IADS'order" means that the first instance of transition "command" at the "IADS" page sends messages to place "order" at the "IADS" page. Those integer values assigned to each variable are message identification numbers. So the total number of messages in a link represents the total number of newly created messages. For example, the transition "command" generated 5 tokens (i.e., message ID 9,10,16,11,and 17) and the place "order" received one token (message ID [9]), and the places "Rule" and "Directive" received two tokens (message IDs [10,16] and [11,17] respectively) each from the transition "command". The values of the delay variables at the local declaration node shall be replaced by the estimated ones at the final step.

The delay variables are automatically added at the tail of each arc expression with a time expression. The "fnDELTA" on an arc expression is the delay selection function. If a time delay expression appears on an arc, the arc’s ending place must be declared as a timed color set.

The delay variables of the local declaration node and the delay variables of arcs are mapped in the code segment. Table 5.6 is an example of the code segment.

Table 5.6 Code Segment

```
output (d3,d2,d1,b1);
action
let val
  d3 = getDelayNet MinOrMaxNet IADS'Command_ IADS'Directive and
  d2 = getDelayNet MinOrMaxNet IADS'Command_ IADS'Rule and
  d1 = getDelayNet MinOrMaxNet IADS'Command_ IADS'Order and
  b1 = getDelayBase MinOrMaxBase IADS'Command_1
in (d3,d2,d1,b1) end;
```

In the code segment, another two delay retrieving functions are used with names "getDelayBase" and "getDelayNet". These are used to retrieve the delay values from the local declaration node and send them to each arc. The function "getDelayBase" retrieves the pre-
processing time (plus the action time and the post processing time, depending on the application) while the function "getDelayNet" retrieves the transmission delay (and the post processing time, depending on the application). The base delay is the common delay of all arcs and the net delay is only applied to the specific arc. Depending on the application, the user may consider the post processing as the delay value of the arc. The difference of the two types of delay shall be considered when the delay values are introduced at the last step 9.

The functions associated with delays (i.e., getDelayBase, getDelayNet, and fnDELTA) are defined at the temporary declaration node (Table 5.7). These functions can be also defined as user’s preference. By default, they were defined to get the minimum delay, maximum delay, zero delay, and random delays. The user can select this option anytime during simulation of the Petri net.

Table 5.7 Temporary Declaration Node

(* Temporary Declaration Node *)

| exception TimeNotDetermined ;
| globref TIMESCALE = 1000000.0;
| globref MinOrMaxBase = "Max" ;
| globref MinOrMaxNet = "Max" ;

| var d3,d2,d1,b1 : TIME ;
| val DelayUnit = "real" ;
| val DELAY_IN_SEC = 1;

| fun D2Sec (d:int) = Intlnf.fromInt (d * DELAY_IN_SEC);
| fun R2Time (r:real) = Option.valOf (Intlnf.fromString (Real.toString (r * (!TIMESCALE))));
| fun I2Time (i:lntlnf.int) =
  let
    val r = (Option.valOf (Real.fromStiing (Intlnf.toString i)));
  in R2Time r end;

| fun Time2R (i:lntlnf.int) = (Option.valOf (Real.fromString (Intlnf.toString i))) /(!TIMESCALE);

| fun defineDelayUnit_InLocaDecNode () = if DelayUnit = "int" then
  usestring ["fun Delay2Time (ID:int) = Intlnf.fromString ID; val DELAY_UNIT = 0;"
else usestring ["val Delay2Time = R2Time; val DELAY_UNIT = 0.0;"

| defineDelayUnit_InLocaDecNode () ;

| fun ZeroDelay (xs) = DELAY_UNIT;

| fun AvgDelay (xs) =
  let fun sum nil = DELAY_UNIT
    | sum (x::xs) = x + sum(xs)
  in (sum xs) / (real (length xs)) end;

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fun MinDelay nil = DELAY_UNIT
| MinDelay (x::nil) = x
| MinDelay (x::xs) = if x < hd(xs) then MinDelay (x::tl(xs)) else MinDelay xs;

fun MaxDelay nil = DELAY_UNIT
| MaxDelay (x::nil) = x
| MaxDelay (x::xs) = if x > hd(xs) then MaxDelay (x::tl(xs)) else MaxDelay xs;

fun getMinOrMaxDelay delayindex = if delayindex = 1 then "Min"
else if delayindex = 2 then "Random"
else if delayindex = 3 then "Max"
else if delayindex = 4 then "Zero"
else if delayindex = 5 then "Avg"
else "Random"

fun getDelayBase xBase (xs) =
  let
    val delay= if (IxBase) = "Zero" then ZeroDelay (!xs)
      else if (IxBase) = "Min" then MinDelay (!xs)
      else if (IxBase) = "Max" then MaxDelay (!xs)
      else if (IxBase) = "Avg" then AvgDelay (!xs)
      else random (list_to_ms (!xs))
    in Delay2Time delay end;

fun getDelayNet xNet (xs) =
  let
    val delay=if (IxNet) = "Zero" then ZeroDelay (!xs)
      else if (IxNet) = "Min" then MinDelay (!xs)
      else if (IxNet) = "Max" then MaxDelay (!xs)
      else if (IxNet) = "Avg" then AvgDelay (!xs)
      else random (list_to_ms (!xs));
    in Delay2Time delay end;

fun fnDELTA (xs) = List.nth (xs,0);

infix plus;
fun x plus y = IntInf.+(x,y);

5.6 Step 6: Specifying the attributes of physical messages

This step is to specify the attributes of the physical message. This step is supported by MG.

The main feature of MG is to identify automatically the attributes of the messages using the information provided at all the previous steps. Table 5.8 shows the physical message dependency relations.
### Table 5.8 Message Dependency Relations

(* Traffic Specification *)

```plaintext
def LinkSpec =
    [link="INITIAL'SENSE_S", src="INITIAL", dest="S101", delay="[0,0]", proc="8",
    protocol="UDP", size="1", priority="0", msgs=[1,2,4]},
  {link="INITIAL'SENSE_S", src="INITIAL", dest="FDC", delay="[0,0]", proc="8",
    protocol="UDP", size="1", priority="0", msgs=[3]},
  {link="S101'SENSE_S',FDC',S101", src="S101", dest="FDC", delay="[0,2]", proc="66Mb",
    protocol="TCP", size="1000", priority="0", msgs=[7,14]},
  {link="S101'SENSE_S',FDC',S101", src="S101", dest="W101", delay="[0,2]", proc="66Mb",
    protocol="TCP", size="1000", priority="0", msgs=[8]},
  {link="S101'SENSE_S',EXTERNAL',S101", src="S101", dest="EXTERNAL", delay="[0,2]", proc="66Mb",
    protocol="UDP", size="1000", priority="0", msgs=[15]},
  {link="FDC'COMMAND_L',EXTERNAL',FDC", src="FDC", dest="W101", delay="[0,0]", proc="66Mb",
    protocol="TCP", size="1000", priority="0", msgs=[9]},
  {link="FDC'COMMAND_L',FDC',FDC", src="FDC", dest="FDC", delay="[0,0]",
    proc="66Mb", protocol="UDP", size="1000", priority="0", msgs=[10]},
  {link="FDC'COMMAND_L',FDC',FDC", src="FDC", dest="S101", delay="[0,0]", proc="66Mb",
    protocol="TCP", size="1000", priority="0", msgs=[11]},
  {link="FDC'EXTERNAL',FDC',FDC", src="FDC", dest="EXTERNAL", delay="[0,0]",
    proc="66Mb", protocol="UDP", size="1000", priority="0", msgs=[16,17]},
  {link="W101'EXTERNAL',W101',EXTERNAL", src="W101", dest="EXTERNAL", delay="[0,5]",
    proc="66Mb", protocol="UDP", size="1000", priority="0", msgs=[12,13]}
];
```

MG automatically identifies the logical link (transition to place pair), the physical link (source –destination node pair), the action time of the task execution, the node processing rate of the physical information system, and the dependency of messages. In addition to the automatic identification of the attributes, it allows the user to enter the protocol, size, and priority of the message for transmission.

### 5.7 Step 7: Generating the traffic log file

Once MG captures all the attributes of messages, it generates a traffic log file in a specific format so that it can be used by the CNSS. Table 5.9 is the example of the traffic log file in TCL script format.
### Table 5.9 Traffic Log File

<table>
<thead>
<tr>
<th>#ID</th>
<th>src</th>
<th>dest</th>
<th>size</th>
<th>Protocol</th>
<th>Priority</th>
<th>ProcRate</th>
<th>ActionDelay</th>
<th>TrgIDs</th>
<th>Tstamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INITIAL</td>
<td>S101</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>INITIAL</td>
<td>S101</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>INITIAL</td>
<td>FDC</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>INITIAL</td>
<td>FDC</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>INITIAL</td>
<td>EXTERNAL</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>INITIAL</td>
<td>EXTERNAL</td>
<td>1</td>
<td>Agent/UDP</td>
<td>0 8</td>
<td>{0 0}</td>
<td>{}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>S101</td>
<td>FDC</td>
<td>1000</td>
<td>Agent/TCP</td>
<td>0 66Mb</td>
<td>{0 2}</td>
<td>{1 4}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>S101</td>
<td>W101</td>
<td>1000</td>
<td>Agent/TCP</td>
<td>0 66Mb</td>
<td>{0 2}</td>
<td>{1 4}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>S101</td>
<td>EXTERNAL</td>
<td>1000</td>
<td>Agent/UDP</td>
<td>0 66Mb</td>
<td>{0 2}</td>
<td>{2 11}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>FDC</td>
<td>W101</td>
<td>1000</td>
<td>Agent/TCP</td>
<td>0 66Mb</td>
<td>{0 0}</td>
<td>{7 3}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>FDC</td>
<td>S101</td>
<td>1000</td>
<td>Agent/TCP</td>
<td>0 66Mb</td>
<td>{0 0}</td>
<td>{7 3}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>FDC</td>
<td>EXTERNAL</td>
<td>1000</td>
<td>Agent/UDP</td>
<td>0 66Mb</td>
<td>{0 0}</td>
<td>{14 10}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>FDC</td>
<td>EXTERNAL</td>
<td>1000</td>
<td>Agent/UDP</td>
<td>0 66Mb</td>
<td>{0 0}</td>
<td>{14 10}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>W101</td>
<td>EXTERNAL</td>
<td>1000</td>
<td>Agent/UDP</td>
<td>0 66Mb</td>
<td>{0 5}</td>
<td>{8 9}</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>W101</td>
<td>EXTERNAL</td>
<td>1000</td>
<td>Agent/UDP</td>
<td>0 66Mb</td>
<td>{0 5}</td>
<td>{8 9}</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

#### 5.8 Step 8: Simulating the communications network

This step is done by CNSS. NS provides an object-oriented, event-driven simulation engine with various protocol modules. Figure 5.7 is a communication network simulation model implemented by Network Simulator.

The nodes with the circle symbol in Figure 5.7 represent the communication nodes and those with the hexagon symbol represent the information processing nodes. The shaded bars on each link represent the message flow over the links for emulation.
The developed methodology and tools support description of 3 types of delay: Node processing delay (pre- and post- processing delays) of the computing device, action delay of the task executor, and transmission delay of the network.

The node processing time describes two types of data processing delays: pre-processing delay and post-processing delay. The pre-processing delay is a delay between the time at which all conditioned information arrives and the time at which the task starts execution (e.g., the decision making process starts). The processing node estimates the data processing time of incoming data. Assume that sensor A reports data in the form of a large image with the features of a detected object and sensor B reports data for a small signal regarding the location of the object. The time to display the information to the decision maker will vary. Some application
may not need the pre-processing. In this case, the pre-processing event shall not be measured at
the network simulation model (CNSS). In other words, no pre-processing event is scheduled and
the pre-processing time of every message will be zero.

The post-processing delay is the delay between the time at which the task is completed
(e.g., the decision is made) and the time at which the message (e.g., the order) begins to be
broadcast. To handle those pre- and post- processing delays, CNSS creates another terminal node
and attaches it to the corresponding network node. The bit processing power of the computing
devices is specified as the bandwidth of a special link between those two nodes (e.g., the node
pair (3, 13) in Figure 5.7). The node processing delay is measured by the special link delay. The
delay time includes the residual time of the message waiting in queue while the computing
device is in service for other messages.

Assume that a transition supported by the corresponding computing device has three input
places with one token at each place and that the three tokens are 1000, 2000, and 3000 bytes long
each. If the bit processing power of the computing device is 1000 bps, the pre-processing time
will be 48 seconds; 6000 (byte) * 8 (bit/byte) / 1000 (bit /second) = 48 seconds. If the same
transition has two output places, produces two tokens at each place, and the token at the first
place is 100 bytes long and the one at the second place is 1000 bytes long, then the post-
processing delay will be 17.6 seconds; {2 (token) *100 (byte/token) * 8 (bit/byte) + 2 (token) *
1000 (byte/token) * 8 (bit/byte) } / 1000 (bit/second) = 17.6 seconds.

NS produces a large amount of simulation data because it executes protocols at each layer
in detail. CNSS creates a monitoring agent and measures the network delays during the run-time,
and produces only that delay information. These monitored delays are also used for scheduling
the next messages in compliance with the message dependency relations. CNSS can simulate the
physical model repeatedly with the same order of message passing with various options and
generate multiple instances of delay values. Table 5.10 shows an example output of the network
simulation with the minimum and maximum action times.
Table 5.10 Delay Values of Messages (simulation with min and max action time)

<table>
<thead>
<tr>
<th>model name: /simpleAAW/</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Original message: 17</td>
</tr>
<tr>
<td>Number of Processed message: 17</td>
</tr>
<tr>
<td>variables: [&quot;msg_id&quot;, &quot;pre_proc_delay&quot;, &quot;action_time&quot;, &quot;post_proc_delay&quot;, &quot;net_delay&quot;]/</td>
</tr>
</tbody>
</table>

| (1) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (2) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (3) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (4) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (5) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (6) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (7) | [0.000000, 2.000000, 0.000145, 0.334274] |
| (8) | [0.000000, 2.000000, 0.000267, 0.477009] |
| (9) | [0.000000, 2.000000, 0.000267, 0.477009] |
| (10) | [0.000121, 0.000000, 0.000122, 0.311312] |
| (11) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (12) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (13) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (14) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (15) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (16) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (17) | [0.000121, 0.000000, 0.000243, 0.000000] |

| (1) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (2) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (3) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (4) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (5) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (6) | [0.000000, 0.000000, 0.000000, 0.000000] |
| (7) | [0.000000, 2.000000, 0.000145, 0.334274] |
| (8) | [0.000000, 2.000000, 0.000267, 0.477009] |
| (9) | [0.000000, 2.000000, 0.000267, 0.477009] |
| (10) | [0.000121, 0.000000, 0.000122, 0.311312] |
| (11) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (12) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (13) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (14) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (15) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (16) | [0.000121, 0.000000, 0.000243, 0.000000] |
| (17) | [0.000121, 0.000000, 0.000243, 0.000000] |
5.9 Step 9: Introducing time to the basic Petri net model

This step maps the delay variables of the local declaration node to the estimated delay values. Once the delay values are obtained from the physical model, the message IDs are replaced with their delay values. Figure 5.8 shows the final performance prediction model of the example. The delay values were imported into the Timed Petri net model as a list of values.

![Figure 5.8 Final Performance Prediction Model](image)

Table 5.11 is the new local declaration node replaced at this step. The message ID of the local declaration node (Table 5.5) was replaced with the estimated delay values.
Table 5.11 New Local Declaration Node with Estimated Delay Values

<table>
<thead>
<tr>
<th>New Local Declaration Node with Min and Max task execution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>pageref IADS'Sense_1 = [0.0,0.0,0.0,0.0,0.000121,0.000121,0.000121,0.000121];</td>
</tr>
<tr>
<td>pageref IADS'Sense_1'IADS'Identity = [0.305957,0.311433,0.334419,0.334419];</td>
</tr>
<tr>
<td>pageref IADS'Sense_1'IADS'Location = [0.000243,0.000243,0.477276,0.477276];</td>
</tr>
<tr>
<td>pageref IADS'Command_1 = [0.000121,0.000121,0.000121,0.000121,0.000242,0.000242,0.000243,0.000243,0.000243];</td>
</tr>
<tr>
<td>pageref IADS'Command_1'IADS'Order = [0.311434,0.311434];</td>
</tr>
<tr>
<td>pageref IADS'Command_1'IADS'Rule = [0.000242,0.000243,0.000243,0.000243];</td>
</tr>
<tr>
<td>pageref IADS'Command_1'IADS'Directive = [0.000121,0.000122,0.454291,0.454291];</td>
</tr>
<tr>
<td>pageref IADS'ActJ = [0.000242,0.000242,0.000242,0.000242];</td>
</tr>
<tr>
<td>pageref IADS'ActJ'IADS'Negated = [0.000121,0.000121];</td>
</tr>
<tr>
<td>pageref IADS'ActJ'IADS'Report = [0.000243,0.000243];</td>
</tr>
</tbody>
</table>

In Table 5.5, the logical link "IADS'Act_1'IADS'Report" has one message (message ID 13). In Table 5.11 (last entry), the link shows two values. It is because the network model has been run two times with minimum action time and maximum action time. Multiple runs will produce multiple instances of the delay values.

As introduced in section 5.5, the function "getDelayBase" retrieves the pre-processing time, while the function "getDelayNet" retrieves the transmission delay. The base delay retrieved by the "getDelayBase" is the common delay of all arcs. The net delay retrieved by the "getDelayNet" is the transmission delay specific to each arc. For example, the variable name "IADS'Act_1" maps to this common delay of the message (ID 12) of the arc link "IADS'Act_1'IADS'Negated" and the message (ID 13) of the arc link "IADS'Act_1'IADS'Report". See the message dependency in Table 5.9. The messages with ID 12 and 13 are triggered by messages 8 and 9. The pre-processing times of messages 12 and 13 are the processing times of both the preceding messages 8 and 9 after they are received. Therefore, the pre-processing times are common to the messages 12 and 13. The post-processing times of messages 12 and 13 are the times of each message processing. The size of message 12 is equal to
that of message 13. But Table 5.10 shows different processing times (121\text{us} for message 12 and 243\text{us} for message 13). This occurs because the waiting time at the queue is different.

In Table 5.10, each message has four types of delays: "pre\_proc\_delay," "action\_time," "post\_proc\_delay," and "net\_delay." Typically, the base delay will be composed of "pre\_proc\_delay" and "action\_time," while the net delay will be composed of "post\_proc\_delay" and "net\_delay." Depending on the application, the base delay and net delay shall be computed (summation of the component delays) differently. If action time is already expressed as a transition delay at the "Time" region of a transition (or arc delay at the arc expression), the base delay will not count the "action\_time" in the delay of the base delay during the summation. If the post-processing delay is considered as the base delay, it will not be counted in the net delay.

In this example, the action time is expressed separately as shown in Figure 5.8. The action time of the task "sense" has the time expression @+I2Time(D2Sec 2) at the arc from the transition "sense" to the place "sensor." If this action time is the duration of the task execution, the duration time needs to be expressed at the time region of the transition so that the time delay is applied to all arcs. But in this scenario, the action time is to specify the scanning cycle of the sensor. The scanning delay is applied for the next scan. The delay of the first scan should be modeled as the initial time of the tokens at the place "sensor" for correct modeling. Similarly, the action time of the task "act" has the time expression @+I2Time(D2Sec 5) at the arc from the transition "act" to the place "shooter." So it must be expressed as an arc expression, too. The first loading time should be modeled as the initial time of the tokens at the place "shooter" for correct modeling. The transition "command" has no action time under the assumption that the firing order is issued by automated computerized algorithms. The human decision making time of the commander was not modeled.

5.10 Follow-on step: Performance evaluation

Once the time is introduced, two types of system analysis can be done. Remember that the response of a system has been characterized by accuracy and timing. One system analysis method is the examination of the final state vector of a timed occurrence graph. The accuracy of the response is analyzed by examining the markings of the final state vector. This determines the degree to which the state of the system evolves toward the desired state. The timing of the
response is analyzed by measuring the timestamps of the final state vector, i.e., by examining how the system meets the timing constraint (or the windows of opportunity).

Figure 5.9 is the timed occurrence graph generated with the maximum information delay. The reachable state of the system has been reduced from four paths in the untimed occurrence graph (Figure 5.5) to two paths in the timed occurrence graph (Figure 5.9).

Figure 5.9 Full Occurrence Graph from Timed Petri Net

In Figure 5.9, the ordering of the sequence of markings varies depending on the binding of the initial threats. If the non-hostile threat (i.e., the threat with target ID 2 and with the identity 0) is processed first by the sensor, the sequence of markings results in M1-M2-M4-M6-M9-M11.
Similarly, if the hostile threat (i.e., the threat with ID 1 and with the identity 100) is processed first, the sequence of markings results in M1-M3-M5-M7-M8-M10.

Tables 5.12 and 5.13 are the state vectors of the final state of the timed occurrence graph when it was generated with the maximum information delay. If we disregard the timestamps, both the final state vectors M11 and M10 are identical to the final state vector M11 of the untimed occurrence graph in Figure 5.5 (note that the timed occurrence graph is a subset of the untimed occurrence graph.) In the scenario of this example AAW model, the threat was expressed as tokens at the place “Threat.” The final state vector shows no tokens at that place, i.e., all threats are cleared. The result of the response was expressed as tokens at the place “Negated.” The final state vector shows that the threat with target ID 1 has been “Cleared” (i.e., not [“KillSelf” or “Leak.”]) So the system responded correctly toward the desired state.

The timestamp of the place “negated” has 0.65 seconds at the state vector M10 and 2.65 seconds at the state vector M11. The system responded against the threat within the time interval [0.65, 2.65] seconds. The set of the response time has been derived from all final state vectors (i.e., all possible sequence of markings; all possible sequence of transitions; all possible sequence of tasks). Thus this interval of the response time may represent the window of capability of the system for the worst case since the set of the response times was derived based on the maximum information delay. However, whether it is a worst case or not depends only on whether the system responds faster when the information delay is smaller compared to when the information delay is larger. There is no guarantee when the order of task executions generates different response times. Comparing this window of capability to the timing constraint, we can verify whether the system meets the windows of opportunity.

Both the accuracy and the timing of the response for the worst case have been analyzed using the final state vector and the timed occurrence graph. The same type of analysis can be done for the minimum and average information delays. Tables 14 through 17 are the final state vectors for those cases. The time intervals [0.62, 2.62] and [0.63, 2.63] represent the windows of capability of the system for minimum and average cases, respectively.
Table 5.12 Final State Vector from the Sequence of Markings M1-M2-M4-M6-M9-M11 with maximum information delay

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1'(90,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(101,(2,0))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2335026]</td>
<td></td>
<td></td>
<td>[4646217]</td>
<td>[2646702]</td>
<td>[646702]</td>
<td>[2789074]</td>
<td>[2646580]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[4000000]</td>
<td></td>
<td></td>
<td>[2646702]</td>
<td>[646702]</td>
<td>[2789074]</td>
<td>[2646580]</td>
<td>[2473797]</td>
</tr>
</tbody>
</table>

Table 5.13 Final State Vector from the Sequence of Markings M1-M3-M5-M7-M8-M10 with maximum information delay

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1'(90,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(101,(2,0))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2335026]</td>
<td></td>
<td></td>
<td>[4646217]</td>
<td>[2646702]</td>
<td>[646702]</td>
<td>[2789074]</td>
<td>[2646580]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[4000000]</td>
<td></td>
<td></td>
<td>[2646702]</td>
<td>[646702]</td>
<td>[2789074]</td>
<td>[2646580]</td>
<td>[2473797]</td>
</tr>
</tbody>
</table>

Table 5.14 Final State Vector from the Sequence of Markings M1-M2-M4-M6-M9-M11 with minimum information delay

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1'(90,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(101,(2,0))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2306320]</td>
<td></td>
<td></td>
<td>[4617512]</td>
<td>[2617997]</td>
<td>[2617997]</td>
<td>[2306199]</td>
<td>[2617875]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[4000000]</td>
<td></td>
<td></td>
<td>[2617997]</td>
<td>[2617997]</td>
<td>[2306199]</td>
<td>[2617875]</td>
<td>[243]</td>
</tr>
</tbody>
</table>
Table 5.15 Final State Vector from the Sequence of Markings M1-M3-M5-M7-M8-M10 with minimum information delay

<table>
<thead>
<tr>
<th></th>
<th>IADS'Identity 1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>IADS'Order 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Rule 1: 1'(90,2)@[2306320]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Thread 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Sensor 1: 1'Sensor(101)@[4000000]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Shooter 1: 1'Weapon(101)@[2617512]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Report 1: 1'(1,Cleared,101)@[617997]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Directive 1: 1'(101,Default,0)@[2306199]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Negated 1: 1'(1,Cleared,101)@[617875]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Location 1: 1'(101,(2,0))@[2000243]</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.16 Final State Vector from the Sequence of Markings M1-M2-M4-M6-M9-M11 with average information delay

<table>
<thead>
<tr>
<th></th>
<th>IADS'Identity 1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>IADS'Order 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Rule 1: 1'(90,2)@[2322028]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Thread 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Sensor 1: 1'Sensor(101)@[4000000]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Shooter 1: 1'Weapon(101)@[4633220]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Report 1: 1'(1,Cleared,101)@[2633705]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Directive 1: 1'(101,Default,0)@[2548992]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Negated 1: 1'(1,Cleared,101)@[2633582]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Location 1: 1'(101,(2,0))@[238819]</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.17 Final State Vector from the Sequence of Markings M1-M3-M5-M7-M8-M10 with average information delay

<table>
<thead>
<tr>
<th></th>
<th>IADS'Identity 1:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>IADS'Order 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Rule 1: 1'(90,2)@[2322028]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Thread 1:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Sensor 1: 1'Sensor(101)@[4000000]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Shooter 1: 1'Weapon(101)@[2633220]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Report 1: 1'(1,Cleared,101)@[2633705]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Directive 1: 1'(101,Default,0)@[2548992]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Negated 1: 1'(1,Cleared,101)@[2633583]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IADS'Location 1: 1'(101,(2,0))@[2238819]</td>
<td></td>
</tr>
</tbody>
</table>

The windows of capability of the system that were derived based on the minimum information delay may represent the optimistic case (see the lower and upper bounds of the
interval are minimum amongst the four cases.) Similar interpretation can be made for the average case. This type of system analysis technique is supported by the formalism of the Petri net. When the state space is large, this type of formal approach may be inefficient, since the exhaustive search of all the final states may take a long time. Simulation-based performance analysis is an alternative. SYNE-X provides a performance evaluation module in order to simulate the Timed Petri net repeatedly with various information delay options and automatically collect the final state information.

The formal approach described in this chapter can be applied when the information delays are random. However, the time interval does not represent the bounds of the windows of capability of the system until we test all the delay values. For random information delays, we need to collect the results repeatedly. Table 5.18 is an example of collected data with repeated simulations.

Table 5.18 Results of Repeated Simulations with random information delay

<table>
<thead>
<tr>
<th>TIMESCALE: 1000000.0 BaseDly: Random</th>
<th>NetDly: Random</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Marking of Negated</td>
</tr>
<tr>
<td>2.646458</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>2.646337</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>0.646579</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>2.646458</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>2.646337</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>2.623472</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>0.646337</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>2.617996</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>0.646459</td>
<td>(1,Cleared,101)</td>
</tr>
<tr>
<td>0.617996</td>
<td>(1,Cleared,101)</td>
</tr>
</tbody>
</table>

The lower values of the response time reside between [0.62, 0.65] and the upper values of the response time resizes between [2.62, 2.65]. The average of the delays is the interval [0.64, 2.64]. From the two types of analysis, we can draw the windows of capability as shown in Figure 5.10.
Figure 5.10 Windows of Capabilities (formal vs. simulation)

5.11 Summary

A synthetic executable model generator (SYNE-X) has been introduced. The SYNE-X consists of 5 major software components: Reachability Tree Generator (RTG), Message Generator (MG), Performance Prediction Model Converter (PPMC), Network Topology Generator (NTG), and Communication Network Simulation Script (CNSS). It develops a performance prediction model (complete Timed Petri net), given a basic Petri net model, through 9 steps:

- Step 1: Generate network topology
- Step 2: Map physical resources to transitions
- Step 3: Check syntax of the basic Petri net model
- Step 4: Generate an Occurrence Graph
- Step 5: Develop intermediate timed Petri net model
- Step 6: Specify the attributes of physical messages
- Step 7: Generate traffic log file
- Step 8: Simulate communications network
- Step 9: Introduce time to the basic Petri net model.

Once the Timed Petri net model is constructed, two types of performance evaluation can be done for the analysis of the functional properties and timing properties of a real-time distributed information system: the formal approach (state space analysis approach) and the simulation based approach.
CHAPTER 6. A CASE STUDY

Modern surveillance and tracking systems often use multiple sensors of different types to provide intelligence to a command organization. Multiple sensors or intelligence agents are connected through communications networks. The sensors are usually physically dispersed at geographic locations. What combination of the component systems has the best combat outcome in terms of the overall force effectiveness? This is a design issue for a system (or resource allocation) to carry out the mission. Suppose that the data fusion system provides probability values about the identity and the location of the target continuously. One of the warfighters' problem is how to judge that the object is a hostile threat given the probability values. Decision makers need to establish a decision threshold.

6.1 Problems in question

The first issue is what is the relationship among uncertainty, decision time, and the effect of decisions. Altukhov (1984) explained the relationship (Figure 6.1) as follows: there is a certain level of information for optimal decision when it is assumed that the effect of the decision is proportional to the level of information available.

When less information is available, it takes a long time to make a decision due to lack of information. When information is more than sufficient, it takes a long time to process the large mass of information and results in a delay. He stated that the optimal decision point is to make a decision when the information level is in the range of $c-d$ in the figure by deriving the required level of information from the acceptable level of decision effect.

Meanwhile, Van Trees (1989) illustrated the relationship between uncertainty and decision time using Figure 6.2. Uncertainty decreases over the elapsed time; a decision maker makes a decision at a certain time accepting a certain level of uncertainty. We can derive two types of decision constraints to make a decision: time and uncertainty. If the decision time does not affect the outcome of the decision, then the best decision making point in terms of time is the maximum time allowed, assuming that the quality of decision is proportional to the information available. If decision time affects the quality of the decision, there exist some trade-offs between
uncertainty and time. From the relations between decision time and uncertainty, we can derive the operating range (thresholds) of decision making as shown in Figure 6.2.

![Figure 6.1 Uncertainty, Time, and Decision](image)

If we can measure the performance of the information fusion system in terms of the level of information at a certain time and the quality of decision made at that time, we can use this performance measure as an operating factor to make a trade-off between time and uncertainty. If time and uncertainty have a balanced effect on the outcome of the decision, the command and
control process can be defined as a management function of the time-uncertainty factors in conducting a mission.

Under this definition, we can consider two types of problems in developing a decision strategy over a command and control timeline. If the problem to be solved has no system noise as in target identification (it has no system noise because the identity of the target never changes), then, making a decision at the last moment within the allowed time such as $T_{\text{max}}$ in Figure 6.2 is the best decision strategy. The sample graph (Figure 6.3) shows the probability profile of a target identity over the observation time.

![Target Identification Graph](image)

**Figure 6.3** Target Identification with Bayesian Rule

The graph was generated by a data fusion system which uses a Bayesian rule with prior information (probability of hostility 0.5), given that the likelihood ratio ($r$) of sensor type $i$
18 (probability of 0.9 for positive identification and 0.05 for false identification,) the ratio of sensor type \( t \) is 9 (probability of 0.9 for positive identification and 0.1 for false identification) with 2 seconds of the measurement cycle. The dotted line shows the probability profile obtained by fusing the three sensory data with a centralized data fusion algorithm with an assumption of conditional independence among the sensor data. The solid line is the decision threshold (probability of hostility 0.9), required to make a judgment about the identity of the object. In this system, waiting more time can reduce the uncertainty of the identity. If a decision maker makes a judgment about the hostility of the object too early, there is a possibility of wrong judgment. If he waits too long to be confident of the judgment, however, there is the possibility of closing the window of opportunity to intercept the object.

If there is system noise as in a target tracking system, waiting more time does not help the best decision quality once it reaches a certain level. The variance of estimated location can never be zero-variance when there is system noise even though we measure the track of a target an infinite number of times. Once the uncertainty reaches the level of the steady-state covariance (Chang, 1997), it stays constant at the value that corresponds to system noise.

Suppose that a target tracking system measures the location and velocity of target periodically and estimates the next location with a certain variance. The sample graph (Figure 6.4) shows the change of the variance of the estimated location over the observation time. The graph was generated by a Kalman filter with a system of one dimensional constant velocity model. A target is moving with the velocity 680 m/sec and with the initial variance of the estimated location of 100 m. The target has system noise with a Gaussian distribution with zero mean and the variance of 1 cm/sec. The tracking radar has the measurement noise of a Gaussian distribution with zero mean and variance of 10 m/sec. The tracking radar scans the target every 2 seconds. In this case, the uncertainty can never be resolved due to the system noise (1 cm/sec). So we have to establish an acceptable level of uncertainty such as \( U_{\text{max}} \) in Figure 6.2.
Given a time-uncertainty distribution function of a C3 system, we can derive two types of operating ranges of decision ($U_{\text{dec}}$ and $T_{\text{dec}}$ in Figure 6.2) from those two $T_{\text{max}}$ and $U_{\text{max}}$ constraints. Verifying whether the window of capability overlaps with the window of opportunity is to find the operating range given the time-uncertainty distribution function. Furthermore, after analyzing the performance of the system behavior within that operating range we can select the decision-making strategy to optimize the trade-offs between the accuracy and timeliness of the response.

6.2 Operational concept of an Anti-Air Warfare system

Suppose that there is an air defense system whose mission is to clear air threats when multiple unknown objects are approaching friendly forces with different velocities at different locations. If the object is a hostile one, it must be shot down before it arrives at the battle line.
This results in a timing constraint on the response. But neutral flights must not be attacked. This results in an accuracy constraint on the response.

To carry out the mission, an early warning system provides intelligence for the identification of the objects using a Bayesian rule. A tracking radar provides the estimated location of the objects using a Kalman filter. One air defense unit consists of one organic radar with the capability to track multiple targets with time-sharing protocols, and one missile launcher with the capability to arm multiple missiles.

An early warning system geographically distributed over the battlefield supports the air defense unit for identification of targets. Figure 6.5 is the command and control system with two information fusion systems.

![Figure 6.5 Command and Control of an Anti-Air Warfare System](image)

The variables of interest are:

- $U_{1jk}$: Uncertainty of target identity at time $k$,
- $t_1$: Decision Time to use $U_1$
- $U_{2jk}$: Uncertainty of target location at time $k$,
- $t_2$: Decision Time to use $U_2$
- $C(u,t)$: command order under uncertainty $U$ at time $t$

The identity of the object is either hostile (H) or neutral ($H^c$). The target identification system observes the features of targets recursively and calculates the probability of the identity $p_{I_{(L, e, k)}}$, and decides the identity $I_{(u_1, t_1)}$ with a decision thresholds (acceptable probability and
allowed time,) where $u_1$ is the uncertainty at the decision time $t_1$. Figure 6.6 represents this Bayesian fusion engine.

The motion of a target is specified by both location ($L$) and velocity ($V$). The target tracking system observes the movements of targets recursively and estimates the variance of estimation $S_{M(j,e,k)}$ and decides whether to re-localize or not with decision thresholds (acceptable variance and allowed time,) where $u_2$ is the uncertainty at the decision time $t_2$. Figure 6.7 represents this information fusion engine that uses a Kalman Filter for a one dimensional constant velocity model.

Figure 6.6 Data Fusion for Target Identification

Decision Threshold 1: $dth_1$, $t_{max1}$

Threat ID:
$I \rightarrow \{H, H^c\}$

Bayes Net $p_{j(e,k)}$ Identification $I_{(e,k)} \rightarrow \{H^*, H^c*\}$

Sensor Type 1 (IMINT)
Sensor Type 2 (ELINT)
Sensor Type 3 (COMINT)

Figure 6.6 Data Fusion for Target Identification

Figure 6.7 Data Fusion for Target Tracking

Decision Threshold 2: $dth_2$, $t_{max2}$

Threat Motion:
$M \rightarrow (L, V)$

Kalman Filter $S_{M(j,e,k)}$ Localization $M_{(e,k)} \rightarrow (L^*, V^*)$

Scan Cycle 1 (1/16 T)
Scan Cycle 2 (2/16 T)
Scan Cycle 3 (4/16 T)

Figure 6.7 Data Fusion for Target Tracking
Finally, the fire direction commander issues a command order \( C(u, t) \), whether to fire (F) or not (\( F^c \)) using the information from the data fusion systems as shown in Figure 6.8. The fire decision is governed by the decision threshold. An engagement rule forces the weapon system to not fire a missile against a neutral object.

![Engagement Rule Diagram](image)

**Figure 6.8 Command Decision**

Suppose that the number of missiles per target is limited due to the resource constraints. To carry out the mission successfully, we need to address the following questions:

- What kinds of design choices do we have?
- How do we develop decision thresholds?
- What do we do to resolve the tolerance of the network delays?

### 6.3 Experimental system design

From the mission statement in the previous section, the global inputs and outputs of the experimental system have been identified as shown in Figure 6.9.

The scenario is follows. The threats are characterized by their identity (e.g., hostile flight, friendly force flight, decoy, a flock of birds, etc.), their initial location, their velocity, and their number. Each threat has a different probability of detection according to the avenue of approach, such that the probability is low in mountainous areas and high in open areas. The interceptor of the AAW system is characterized by the location of its initial deployment, its velocity, firing range, and probability of kill. The AAW system should handle multiple threats. But there is a limited number of resources. The weapon platform (missile launcher) can load multiple missiles at the same time and it takes a certain time to reload after firing one.
The output of the combat operations is assessed as one of three combat results: kill enemy (H) if it is not detected and false alarm if it is detected; the results of kill or false alarm are affected by the response of the system and the uncertainty level at the decision time. The result of kill or miss depends on the probability of kill of the interceptor. It is assumed, for simplicity, that the probability of kill is determined by the accuracy of the estimated location of the target. If the variance of the accuracy is within the hit range of the interceptor, the target is killed. Once the result is identified as a miss, the interceptor fires repeatedly until the target is within the windows of opportunity.

The sensor is characterized by its type (i.e., identification and tracking), the likelihood ratio of identification (i.e., likelihood ratio of positive identification over false identification), the variance of estimated location and velocity, the surveillance range, scan cycles, and the number of sensors. Multiple sensors are configured for centralized data fusion.

Figure 6.10 is the deployment diagram of all the resources of the AAW system attached to the communication network, in which

- $X_n$: a node of the communication network. The bandwidth of a link between nodes is 1,024 Mbps; the electronic propagation delay of each link is 10ms.
• **Sn**: a surveillance radar that performs mainly identification of targets within the area of interest (i.e., global battle area); calculates locally the probability of target identification using Bayesian updates; and reports the probability of identity and trajectory data to centralized data fusion center (AWACS).

• **Rnnn**: a tracking radar that performs only the tracking of targets within the area of responsibility (i.e., local area assigned to the battery); and reports the trajectory data to the local fire direction center (FDC).

• **Wnnn**: a weapon that intercepts targets and launches missiles following the order from the local FDC.

![Deployment Diagram of an AAW system](image)

Figure 6.10 Deployment Diagram of an AAW system

• **FDC**: a computing device of the fire direction center of the local battery. It calculates the target trajectory using tracking data from its local tracking radar (Rnnn) and global
surveillance radar \( (Sn) \); displays the trajectory on the terminal \( (Display) \). 

\[ FDC \text{ and } Display \] are connected to a wireless local area network whose bandwidth is 10Mbps; the electronic propagation delay of the LAN links is 10ms.

- **AOC**: a computing device of the global Air Operations Center. It receives target information (identification and tracking data) from AWACS; allocates targets to each local battery \( (FDC) \); issues a firing order under centralized air control doctrine; maintains the status of the battlefield and displays the tactical picture on the terminal \( (Screen) \). 

\[ AOC \text{ and } Screen \] are connected to a wired local area network whose bandwidth is 10Mbps; the electronic propagation delay of the LAN links is 10ms.

- **AWACS**: a computing device of the early warning system. It calculates target identity using data from multiple sources and uses a centralized data fusion algorithm.

It is assumed and illustrated by the dotted square box in Figure 6.10 that the computing devices of all resources connected to the network have the data processing rate of 100 Mbs by which all incoming and outgoing data are processed bit by bit sequentially.

The problem is to choose a design option to resolve the tolerance of the network delays that may affect the result of combat, and to develop the decision threshold. The system will be used under different tactical operational environments. To represent the situation, two rules of interception have been established:

- **Aggressive Intercept**: when the object is identified as a threat (i.e., a hostile target), the system should intercept it regardless of the current location of the target before the object arrives at the battle line.

- **Defensive Intercept**: when the object is identified as a threat (i.e., a hostile target), the system should intercept it when the target is within a specified firing zone (e.g., within the arbitrary interval \([10\text{km}, 40\text{km}]\) from the battle line; see Figure 2.1).

An executable performance prediction model has been designed with seven input parameters, in addition to the rules of interception, and four output performance measures. The parameter variables are:

- **Number of Sensors**: one of three \{#1, #2, #3\} surveillance radars plus 1 tracking radar; each additional sensor has the next longer surveillance range from the set \{short, medium, long\}; for the experiment, ranges of \{40km, 70km, 150km\} were used (these are arbitrary
numbers: 40km has been chosen from the line-of-sight range on the sea surface, the others from the typical criterion to classify short and long range radar).

- **Number of Transponders**: One transponder(s) per sensor from the set \(\{\text{single, multi}\}\). It may be a design alternative for a multi-purpose radar since multi-transponders have the effect of reducing the scanning cycle and increasing the number of sensors. The scanning cycle may have critical impact to track multi-threats in a time sharing surveillance mode. In the experiment, eight transponders were used. Given that the scanning cycle of a sensor is 2 seconds (rotate 30 times per minute), the eight transponders will reduce the scanning cycle to 2/8 seconds assumed that all eight transponders are equally spaced.

- **Sensor Likelihood Ratio**: The probability of positive identification over the probability of false identification. It takes one of the values from the set \(\{6, 9, 18\}\).

- **Computation Rule for Fusion**: The computation for data fusion is *dynamic* or *fixed*. When some sensor data is unavailable due to any reason (by the sensor being out-of-service, by jamming of communication link, by non-detection of the target, or by the different surveillance ranges of sensors in operation, etc.), the computation for data fusion needs to have a rule or to wait until the remaining sensor data arrive; if more than one sensor is in operation and if the sensor data is received from only a partial number of sensors at a specific time, the *fixed* computation rule uses the partial sensor data only for fusion (a set of conditional probability matrices was used for this rule); meanwhile the *fixed* computation rule keeps each sensor data at the store and uses the stored data together with the partial information received.

- **Surveillance Mode**: Either of *active* or *passive* surveillance mode. If the uncertainty of a target does not meet the decision criterion, the *fixed* rule assigns priority to the use of specific sensor for the specific target until the uncertainty decreases so that a judgment can be made; the 'passive' rule scans the battlefield in routine manner without priority; when the target is a High Valued Target (HVT), the HVT will have priority so that the target is intensively searched and tracked. In the *passive* mode, the sensor scans the battlefield periodically every two seconds in the case of a sensor with a single transponder and every 2/8 seconds of a sensor with eight transponders. In the *active* mode, the sensor allocates all time slots to the specified target.
• **Information Delay:** It includes information processing time and the information distribution delay. It takes one of the values from the set \{zero, min, max, random\}.

• **Decision Threshold:** The decision criterion for making a judgment whether the object is a hostile target or not. If the probability of the target identity is larger than the threshold, the target is considered as a hostile target and a fire order is issued. It takes one of the values from the set \{0.7, 0.8, 0.9, 0.99\}.

The effectiveness of the system was measured by:

• **Mission Success:** the ratio of the number of threats cleared over the total number of threats within the allowed time. It measures performance in a \([0.0, 1.0]\) scale.

• **Decision Quality:** the ratio of the number of correct decisions over the total number of decisions. It measures performance in a \([0.0, 1.0]\) scale.

• **Weapon Efficiency:** the ratio of the number of threats cleared over the number of interceptors (i.e., missiles) used. It measures performance in a \([0.0, 1.0]\) scale.

• **Operational Readiness:** the time at which the system finishes all the tasks and switches the operation mode into the idle state. It measures performance in units of time.

The information delay and the speed of the interceptor may significantly affect the timeliness of the system. Similarly, the probability of kill may significantly affect the *weapon efficiency* of the system. In this experiment, the variance of track estimation was used to determine the probability of kill (see Appendix C for details). The speed of the interceptors was set at Mach 3 for every interceptor.

6.4 Synthetic executable model of the AAW system

To measure the performance of the AAW system, a performance prediction model has been developed. Figure 6.11 is the top-level diagram of the AAW system. It is an extension of Figure 5.2.

The model shows four values (0.7, 0.8, 0.9, and 0.99) for the decision threshold to be used as a criterion for making a judgment about the hostility of the objects (the initial marking of the place named *rules of engagement*). It also shows four threats with seven attributes (the initial marking of the place named *threat*): target nomination (threat ID), identity (P_H), velocity (V),
initial location \((L_0)\), initial variance of track parameter \((S_0)\), probability of detection \((P_D)\), and the event occurrence time of the threat.

\[
\begin{align*}
\text{Surveillance\_Directive} & \quad \text{rule}_s \\
\text{Rule} & \quad 1'(R2I 0.7),(R2I 0.8),(R2I 0.9),(R2I 0.99) \\
\text{Rules\_of\_Engagement} & \quad \text{rule}_s \\
\text{Tactical\_Picture} & \quad 1'(R2I 1.0),2.30,(R2I 0.9),(R2I 0.99),(D2Sec 0)++ \\
\text{rule}_s & \quad 1'(R2I 1.0),2.50,(R2I 0.9),(R2I 0.99),(D2Sec 0)++ \\
\text{rule}_s & \quad 1'(R2I 1.0),2.70,(R2I 0.9),(R2I 0.99),(D2Sec 0)++ \\
\text{rule}_s & \quad 1'(R2I 1.0),2.90,(R2I 0.9),(R2I 0.99),(D2Sec 0) \\
\text{Tactical\_Picture} & \quad \text{rule}_e \\
\text{Rule} & \quad \text{rule}_e \\
\text{Order} & \quad \text{order} \\
\text{Negated} & \quad \text{negated} \\
\end{align*}
\]

Figure 6.1 Top Level Diagram of an AAW system

The arbitrary characteristics of the threats that were used in this experiment are shown in Table 6.1. The life time of the threats (timing constraints or windows of opportunity) varies depending on the rule to intercept the threats: aggressive intercept or defensive intercept since a rule specifies the firing zone (see also section 2.2 in Chapter 2). Table 6.2 shows the timing constraints that the AAW system must meet as derived from the rules of interception and the arbitrary characteristics of the threats. For example, the threat with ID 4 must be intercepted in the time interval \([0.0, 132.35]\) seconds under the aggressive intercept rule and \([73.53, 117.65]\) seconds under the defensive intercept rule.
Table 6.1 Arbitrary Characteristics of Threats

<table>
<thead>
<tr>
<th>Threat ID</th>
<th>$P_H$</th>
<th>$V$</th>
<th>$L_0$</th>
<th>$S_0$</th>
<th>$P_D$</th>
<th>$t_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>Mach 2</td>
<td>30km</td>
<td>100m</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>Mach 2</td>
<td>50km</td>
<td>100m</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>Mach 2</td>
<td>70km</td>
<td>100m</td>
<td>0.9</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>Mach 2</td>
<td>90km</td>
<td>100m</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 6.2 Timing constraints of the AAW system (in seconds)

<table>
<thead>
<tr>
<th>Target ID</th>
<th>Aggressive Intercept</th>
<th>Defensive Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.0, 44.12]</td>
<td>[0.0, 29.41]</td>
</tr>
<tr>
<td>2</td>
<td>[0.0, 73.53]</td>
<td>[14.705, 58.82]</td>
</tr>
<tr>
<td>3</td>
<td>[0.0, 102.94]</td>
<td>[44.117, 88.24]</td>
</tr>
<tr>
<td>4</td>
<td>[0.0, 132.35]</td>
<td>[73.529, 117.65]</td>
</tr>
</tbody>
</table>

Suppose that a sensor periodically scans the battlefield and provides the FDC with sensory data about the identity and the location of the object as shown in Figure 6.4. If we choose the decision threshold 0.9, it takes 9 seconds with high and medium sensitivity (likelihood ratio 18 and 9) and 11 seconds with the low likelihood ratio (likelihood ratio 6) for the sensor to provide the required information. If we disregard the information delay, all threats except the threat with ID 1 under the defensive intercept rule can be cleared. Suppose the sensor has a single built-in transponder, tracks the object in the time-sharing protocol, and processes the ID 1 threat after the other three threats. Then it needs a minimum of 27 seconds to process the other three threats and only 2.41 seconds to process the threat ID 1. Clearly, one sensor with a single transponder cannot guarantee the success of the mission when all four threats are incoming at the same time. At least, we need to increase either the number of sensors with single transponder or the number of transponders of a single sensor. Then, how many sensors or transponders are needed to guarantee the successful completeness of the mission? To answer this type of question, multiple sensors were used to measure the MOEs. Figure 6.12 shows the centralized configuration of three sensors (see Figure 2.5).
The figure shows three surveillance radars (Tb1, Tb2, and Tb3) and one tracking radar (Tb5). The surveillance radar reports its sensory data (the local identification of the target and the location) to the central target identification system (Tc4). Tc4 reports the fused identity of the target to the decision maker and disseminates the data to the target tracking system (Tc6). Also, the tracking radar reports the trajectory of the target to the tracking system (Tc6). See Appendix B for details.
The threats trigger information flow between tasks and in turn generate the traffic in the communications network. How does the information delay affect the success of the mission? To estimate the node processing delays and network transmission delays, an executable physical model has been developed. Figure 6.13 is the executable physical model implemented using Network Simulator (1999).

The nodes with the circle symbol in Figure 6.13 represent the communication network nodes and those with the hexagon symbol represent the information processing nodes. A processing node estimates the data processing time of incoming and outgoing data. The delay time includes the residual time of a message waiting in queue while the computing device is in service for other messages. The directed bars on each link represent the message flow over the links for emulation.

The delay values have been estimated using the TCP protocol for the messages of command information (e.g., fire order) and the UDP protocol for the common tactical picture (e.g., sensory data messages). The network may be commonly used for other C4ISR systems. To
represent this, the simulation has also been run with background traffic: a traffic bias level between all network nodes with the UDP protocol, a message inter-arrival time of two seconds, and the lowest priority versus the highest priority messages of the AAW system.

During the simulation, the size of all message has been arbitrarily set to 1000 bytes. The size of message to be processed for data fusion at each sensor may be different according to the characteristics of the sensor. For example, if a sensor performs image processing, the size may depend on the resolution of image taken by the sensor. These messages sizes have also been arbitrarily set to 100 Kbytes for all types of sensors.

6.5 Findings

The results of the experiment are shown in Appendix C.

- **Number of Sensors and/or Transponders:** In general, more sensors or more transponders produce higher effectiveness. However, the relation was not true in the case of the *fixed* computation rule.

- **Computation Rule:** The *dynamic* computation for data fusion supports better timeliness (operational readiness) than the fixed computation rule, even though the decision criterion of the *dynamic* computation rule has a higher threshold than that of the *fixed* computation rule.

- **Number of Sensors vs. Computation Rule:** In the *dynamic* computation for data fusion, as the number of sensors increases, the system shows better timeliness (operational readiness). However, the opposite was true in the *fixed* computation rule.

- **Decision Threshold:** Clearly, there is a relationship between the decision criterion and the success of mission. If we focused on the timeliness of a response, the lower threshold for the decision criterion is preferred. However, if we focused on the accuracy of a response, the higher threshold is preferred.

- **Decision Threshold vs. Computation Rule:** In the *dynamic* computation rule for data fusion, the decision threshold needed to be set to the higher value for better performance. This is reasonable because this computation rule loses the benefit of multiple sensors since it uses a partial number of sensors, and so the higher threshold is required to get higher quality of information. Conversely, in the *fixed* computation rule for data fusion,
the decision threshold needed to be set to the lower value. This rule fuses data assuming there are multiple sets of sensory data. It takes a long time to reach the higher threshold when some sensory data is unavailable. So, it is reasonable that the lower threshold is required to respond quickly.

- **Information Delay:** Obviously the information delay should affect the timeliness of the response. But it was not true that the minimum information delay always guarantees the fastest response. It means that the information delay affects the behavior of the system not just in terms of the timeliness, but also the ordering of tasks in an autonomous system depending on the arrival time of information. It is conjectured that, once the information processing and distribution systems have the potential capability to carry out the mission successfully, the performance of the system (response time) may be more highly affected by the order of the information arrivals than by the delay of information.

Throughout the results, when a system consisting of a data fusion sub-system with a single transponder was used, the possible combinations of input parameters to meet the performance requirements were restricted. When multiple transponders were used, there were many more possible combinations of input parameters that met the performance requirements.

Based on this experiment, the fixed computation rule shows better weapon efficiency than the dynamic computation rule does. It can be interpreted that the estimation of the tracking system with the fixed computation rule provides a more accurate target trajectory. This is reasonable for this experiment, since the dynamic computation rule uses the partial set of local sensory data at the time of computation disregarding the old information stored at the central data fusion system, while the fixed rule uses the full sensory data stored at the central data fusion system. For more realistic analysis, the tracking algorithm may need to be implemented in more detail.

### 6.6 Answers to War-Fighter’s Question

In general, the number of transponders per sensor has a critical impact: the more transponders the better. Given the number of transponders, if there are multiple sensors and if the data fusion system is used in an autonomous firing mode by the computerized automated system,
• under an unstable system operational environment due to any reasons (poor survivability of sensors, jamming on the communications link, etc.),
• if the ranges of surveillance radars are different, or
• if the probability of target detection is low

then, the data fusion system must be designed using the dynamic computation rule for data fusion and the decision criterion for the judgment of threat identification needs to be established with the higher threshold. More sensors will increase system performance.

If the data fusion system is designed using the fixed computation rule, as the number of sensors increases, there is possibility that the system effectiveness may decrease. So the number of sensors needs to be chosen properly. More experiments are required using parameters under the stable system operational environment, identical surveillance ranges, and higher probability of target detection. In the scenario of this experiment, the lower decision criterion threshold increased performance. The decision threshold needs to be reexamined if the fixed computation rule is used under different scenarios.
CHAPTER 7. CONCLUSIONS

7.1 Summary

Many of the C3 systems in the real world are supported by a common network or a network of networks. Predicting the performance of a C3 system consisting of sub-systems requires the integration of such sub-system models with the communication system models. When the system is used for a time critical mission, the network delay may play a decisive role in battle management. So, the integrated model must be able to represent the network delay properly. Ensuring that the system responds within the windows of opportunity (i.e., meeting the timing constraints) requires estimation of the bounded values of the task connection delays over the network.

In the model of synthetic execution, the architecture of a system has been represented in two layers: the functional architecture layer and the physical architecture layer. The physical architecture consists of the communications model and the resources (sensors, computing devices, weapon systems, etc.) that are attached to the network. The approach presented in this thesis models both architecture layers as executable models. Then both layered executable models are combined for performance prediction. Total ordering of events is the core of the theoretical background of the approach. State space analysis techniques are used for the total ordering of events and a discrete event simulation technique is used for execution, preserving the total order.

The executable functional model uses a Petri net to describe the logical behavior and the executable physical model uses a queueing net to represent the demand and the contention of resources necessary to implement the logical behavior. The message-passing pattern is generated from the reachability tree of the executable functional model. The executable physical model processes those messages preserving the message-passing pattern.

Once the network delay is measured in the executable physical model, the delay value is inserted into the executable functional model. The resulting Timed Petri net
model allows both formal state space analysis and simulation for performance prediction of a real-time distributed system. By examining the timestamps of all the possible final states of the system, the windows of capability can be measured so that they can be used to verify whether the system responses are within the windows of opportunity. By examining the markings of all the possible final states of the system, the accuracy of the response can be measured. When the system is a large-scale system and suffers from state space explosion, the simulation technique can be used for efficient analysis.

7.2 Advantages and limitations of the approach

The major advantages of the synthetic execution approach for performance prediction of a real-time distributed system are as follows. In synthetic execution, the time values are estimated by a separate communication network model rather than directly modeling the contention of communication resources as a single integrated model. The communication service demands are isolated from the functional model. This enables the executable functional model to be invariant with respect to the executable physical model. Because of the invariant feature of the functional model from the physical model:

- Efficient collaborative work between teams is possible in designing the functional and physical architectures, and
- Existing communication models can be easily reused.

This may provide higher efficiency in developing large-scale system models through the development cycle from the early stage of system design to performance prediction. Because the Petri net formalism is used as the basis of the synthetic execution model for performance prediction:

- The strengths of both formal and simulation-based approaches for performance evaluation can be obtained;
- The synchronization problem, one of the major issues in military doctrine, is resolved by the model's net structure.

The use of the discrete event simulation technique rather than the probabilistic queueing model as the executable physical model, allows the following:

- Communications models can be specified in any preferred level of detail, and
• Realistic representation of communications is possible, since the messages are captured from the state space of the functional model and processed in total order in the physical model.

The limitation of the approach arises from the state space explosion problem of the occurrence graph of the Petri net. The method for capturing the total order from the occurrence graph of the Petri net model will be used efficiently for a conflict free net since one path of the occurrence graph contains all the required information needed for total ordering of events. A non-conflict free net may have multiple final states. Even if the net has a single final state, different paths can have different orderings of events resulting in different total orders. For theoretical soundness, the approach requires the repetition of the procedures from the first step to the last step as many times as the number of different total orders. Given the two assumptions about the characteristics of the communications network: (1) network delays are longer under heavier traffic loads than under lighter traffic loads, and (2) the degree of traffic load is defined as the amount (e.g., number or size) of messages in a given time interval, the total order of events based on the longest path from the initial state to final state may be used for efficiency. The approach needs to generate the full occurrence graph to find the longest path when the net is not conflict free. Unless we can model the system with the conflict free net structure\(^1\), the approach presented in this thesis does not address the inefficiency problem.

7.3 Future Research Directions

In conclusion, the major contributions of this dissertation are as follows.

• A model of synthetic execution of a real-time distributed system has been developed so that both logical and timing behavior of system can be predicted.

• A method for capturing a realistic traffic pattern based on the logical behavior of a system has been developed.

An “off-line” simulation technique using the synthetic execution model has been presented to avoid the inefficiency of the “run-time” distributed simulation protocol when the distributed models have different time scales. When the executable physical model is large, it may be

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\(^1\) The author believes that most information system models for C4ISR systems can be modeled with the conflict free net structure.
partitioned into several parts for engineering level efficiency in simulation. In this case, one may want to use the “run-time” distributed simulation technique on the engineering level. A comparison of “off-line” and “run-time” simulation techniques can be conducted.

Suppose that multiple heterogeneous systems use different network resources, but share some resources. The order of message passing between tasks from different functional architectures of the multiple heterogeneous systems may have the precedence relations of events depending on the interactions amongst the multiple systems. Suppose that there is a mission to be carried out by two systems: a combat service support system (CSS) and a force maneuvering control system (MCS). The message passing between tasks to carry out the mission represents the logistics flow and the force movement over the transportation network with limited terrain resources in addition to the information flows over the communication network for command and control. Across the boundaries of the multiple systems, some sets of messages may have multiple time lines. Currently the methodology has been developed for performance prediction of information systems. Also the simulation of the executable physical model (i.e., the queueing net model transformed from the Petri net model) schedules the events in total order over the single time line. The theoretical challenge for future research is to expand the synthesis method to deal with multiple functional architectures with non-monotonic time. The notion of vector and matrix time that are extensions of the scalar time introduced in Chapter 3 may offer insight to this extension. It is believed, however, that the methodology presented in this dissertation could be used by scaling up the boundary of the system. By aggregating component systems, we can scale up the boundary of the system. Then the total order of events can be captured from the scaled-up system. Finally the two heterogeneous physical network models can be executed while preserving the total order. The “run-time” distributed simulation of the two separate physical network models may be suitable for implementation.
A.1 Colored Petri Nets

While ordinary timed or untimed Petri nets offer a mathematical way of modeling complex concurrent and asynchronous processes, their usefulness is limited, in part, because the tokens are indistinguishable. Modeling even a simple system can result in very complex structures using ordinary Petri nets. Colored Petri nets are a full extension of ordinary Petri nets that provide a very compact way of modeling complex systems. Colored Petri nets use the same structure as ordinary Petri nets. They consist of places, arcs, transitions, and markings. In addition, in Colored Petri nets, tokens are no longer indistinguishable. They can take on attributes or values, sometimes referred to as colors. The values or colors for tokens are defined using color sets. A color set is a vector of attributes. These attributes can be any standard data type, integers, reals, characters, strings, enumerated, etc. Any particular token is characterized by the values assigned to each of the attributes in the vector that defines its color set. The term color was derived by an analogy to the color wheel, in which any color in the visible spectrum can be defined by the strength of each of the three colors (attributes), Red, Green, and Blue. By introducing these color sets, it is possible to reduce the complexity of the structure of an ordinary Petri net model, although rules governing the enablement and firing of transitions become more complex.
Design/CPN is a graphical computer tool that supports the construction, simulation, and logical, behavioral, and performance evaluation of CP net models. Thus it has three components, a graphical editor for construction, a simulator for both graphical and automatic simulation, and a State Space analyzer for evaluation. A CP net is constructed as a drawing. It consists of places, shown as ellipses, transitions, drawn as rectangles, and arcs connecting places to transitions and transitions to places. By convention, places and transitions are given names written inside the places or transitions. The names have no formal meaning but they can have a significant impact on the readability of the net.

Design/CPN allows the use of variables to represent, in a compact way, different enabling conditions of transitions. A token is an element of a given color set and can only take the values defined by the color set. The definition of the color sets and of the variables is done in the Global Declaration Node. Color sets in Design/CPN correspond to types in a programming language such as Pascal or C and can be of two types: atomic and complex. An atomic color set is a color set whose elements are of a simple type. Examples of atomic color sets are: enumerated, integer, real, string, etc. A complex color set is a color set obtained by combining different color sets. Examples of complex color sets are product, union, list, records, etc. Variables can be defined for the color sets to be used in the model.

A place can only contain tokens of a given color set. The color associated with a place is written in italics and must be declared in the Global Declaration Node. The marking of a place is defined as a multiset and is written:
\[ M(p_i) = \sum_{h=1}^{n} n_{ih}^{'a_{ih}}. \]

where \( n_{ih} \) is the number of tokens of color \( a_{ih} \) in place \( p_i \). The initial marking of a place, which is the number and type of tokens contained in the place at the beginning of the simulation, is underlined. For example the marking of a place \( P \) containing two blue tokens and three red tokens is written:

\[ M(P) = 2^{\text{blue}} + 3^{\text{red}} \]

Annotations on the arcs specify the number and the color of tokens that the arcs can carry and are written in the form of multiset. The number and the color of tokens can be specified as variables. Variables defined for color sets are handled like an element of this color set.

An arc expression can contain if-then-else or case statements to represent different possible combinations of tokens that can be carried on the arc. A transition is enabled when the tokens in the input places satisfy the arc expression connecting those places to the transition. When the transition fires, it removes from the input places as many tokens of the colors defined in the input arc expression and generates in the output places as many tokens of a color as specified by the arc expressions connecting the transition to the output places. Enablement of a transition results in the binding of color set variables to tokens contained in the input places. This binding of variables remains the same for the generation of tokens. An example is shown on Figure A.1.
Two color sets have been defined in the Global Declaration Node: \textit{Sample} is an enumerated color set and contains the elements \textit{blue}, \textit{yellow} and \textit{red}; \textit{Size} is also an enumerated color set and contains the elements \textit{big} and \textit{small}. A variable \textit{s} has been defined for the color set \textit{Sample}. Place P1 has the color set \textit{Size} associated with it and contains as initial marking 1 \textit{big} token and 1 \textit{small} token. Places P2 and P3 have the color set \textit{Sample} associated with them. Place P2 contains initially one \textit{red} token. The use of the variable \textit{s} on the arc expression allows to represent different combination of tokens for the enablement of transition t1. The variable \textit{s} is first bound to a color contained in place P2. According to the value taken by the variable \textit{s}, one \textit{big} token or one \textit{small} token contained in place P2 is used for the enabling of the transition. If the required token is not present in place P1, the transition is not enabled. When the transition fires, the enabling tokens are removed from places P1 and P2 and the tokens specified by the output arc
expression are generated in place P3: if \( s \) is bound to red then one blue token is produced, otherwise one red and one yellow tokens are produced in place P3. For the initial marking specified for places P1 and P2, the variable \( s \) is bound to red and the transition t1 is enabled because the required big token is present in place P1. When the transition t1 fires, the big token is removed from place P1 and the red token is removed from P2; one blue token is generated in place P3.

One can see from this example that the use of variables and if-then-else statements is a practical implementation of the matrices associated with the arcs in the formal Colored Petri Nets. The main advantage is that the use of variables and if-then-else statements can be used for large color sets while matrices on large color sets are hardly usable.

To facilitate the specification of different enablement of a transition, Design/CPN allows the use of a guard function associated with the transition. A guard function is a Boolean expression operating on the variables used in the input arc expression. A transition is thus enabled if the input places contain the tokens specified by the input arc expression and if the guard function associated with the transition evaluates to true.

Finally, in Design/CPN, the firing of a transition can trigger the execution of a code segment associated with the transition. A code segment is a piece of code written in ML (Meta Language) which is a functional programming language and that can be used either to generate the attributes of the tokens put in the output places or to perform any operations not directly related to the simulation of the Colored Petri Net such as storing
of data in a file or automatic updating of a chart. Figure A.2 displays a simple Colored
Petri Net which uses guard functions and code segments.

The Global Declaration Node defines three color sets: Order contains the elements
wait and fire, Threat contains the elements hostile and neutral, and Response contains the
elements Monitor and Engage. A variable for each of these color sets is defined. A color
set associated with each place defines the type of tokens that each place can contain. Each
arc is assigned an expression that specifies the number and type of tokens that needs to go
t through the arc. For example 1 threat means that one token of color set Threat can go
t through the arc. The expression threat = hostile (or "threat equal to hostile") is the guard
function of the transition. The guard function adds another condition that must be
fulfilled besides the presence of tokens in the input places. Finally, the code segment specifies how the output token is generated from the input token. For example, if a threat is hostile and the order is fire then the response is Engage; otherwise Monitor the threat.

A.2 Timed Colored Petri Nets

Time is introduced into Colored Petri Nets as follows. The tokens that are sensitive to time must belong to a color set defined as timed in the Global Declaration Node. Each token belonging to a timed color set has a timestamp associated with it represented by the suffix of the type @[n] where n is the simulation time at which the token will be available: if n is less than the simulation time, the token can not enable any transition. If n is equal or larger than the simulation time, then the token is available. The timestamp of a token is updated in two ways: through the firing of a timed transition or a timed arc expression. A timed transition is a transition having a time region which contains an expression of the type @+(expression) where expression can be a finite value, a variable, or a function of variables returning an integer or a real value which will be the delay associated with the transition computed for the values of the variables. When a timed transition fires, timed tokens are put in the output places as defined by the output arc expressions with a timestamp equal to the current simulation time augmented by the value of the expression of the time region. If a transition has no time region, the timestamp associated with the timed tokens put in the output places is equal to the current simulation time. One can see that this is a variation of the transition model of time in
Ordinary Petri Nets. The only difference is that reserved tokens are generated in the output place instead of having enabling tokens become reserved in the input places. A timed output arc expression is an expression associated with an output arc of the type \( n'(\text{expression 1})@+(\text{expression 2}) \) where \( \text{expression 1} \) defines the tokens that need to be put in the output place, and \( \text{expression 2} \) defines the timestamp that needs to be associated with the tokens. A timed arc expression can only be used for arcs that connect a transition to a place whose color set has been defined as timed in the Global Declaration Node. When a transition fires, timed tokens are put in the output places as defined by the output arc expressions with a timestamp equal to the current simulation time augmented by the value of the time expression of the arc expression. One can see that the use of timed arc expression corresponds to the implementation of the place model of time in Ordinary Petri Nets.

The use of the variation of the transition model and of the place model provides flexibility in the use of time in CP nets. Sometimes it is more natural to associate a time delay with a process that is modeled by a transition, and other times a time delay associated with an arc that represents the transfer of the output of a process to another process is more appropriate. Both models can be used at the same time in a model. When a timed arc expression is used with a timed transition, the time expression of the arc expression is added to the transition timed region so that the output token has a timestamp equal to the current model time augmented by the sum of the two time regions.

Figure A.3 illustrates the handling of time in Colored Petri Nets. The transition process models a process that needs to be triggered and that takes for inputs \( il \) and
produces, after a processing time of 10 units of time, an output $o1$. Output $o1$ is retained locally and so there is no transmission delay to send it to other process. After the initiation of a process a report $rpt$ has to be sent and there is a delay of 2 time units for the transmission to other process. The status of the process is turned to $on$ just after initiation.

In the Global Declaration Node, the color sets Input, Output, Report and Trigger are declared as timed. During execution, the Design/CPN starts with the model time equal to zero. There are no enabled transitions at this model time. The simulator advances the model time to the earliest time that will enable any transition. In the left hand side of the figure, the transition is enabled only when the simulation time reaches 15 at which the token $1^{'}trg$ at the place Trigger is available. The marking of the net obtained after firing of the transition at time 15 is shown on the right hand side. The token $1^{'}rpt$ in the place of...
color set *Report* has a timestamp equal to 27 as the result of the time expression @ +10 at the time region of the transition plus the timed arc expression 1′rpt@ +2, while the token *ol* has a timestamp equal to 25 because of the time expression @ +10 at the time region of the transition. Finally, the token 1′on in the place of color set *Status* does not have a timestamp because this color set has not been declared as timed and therefore the token *on* is immediately available to enable other transitions.

### A.3 Hierarchical Colored Petri Nets

The modeling of a large-scale system leads to a Colored Petri Net representation which is very large and that cannot be easily looked at and analyzed. The notion of Hierarchical Colored Petri nets has been introduced in Design/CPN to deal with this problem. A set of transition and its interconnecting places can be replaced by a substitution transition, which then refers to a subpage containing the subnet which replaces this substitution transition. This substitution process preserves the logical consistency and the dynamical behavior of the system model. In Figure A.4, the subnet made of transition t2, t3 and t4 and their interconnecting places p4, p5, p6 has been replaced by the substitution transition st, this subnet being displayed on another page referred to as the Hierarchical Substitution (HS) region of the substitution transition. Places p2, p3 and p7 that represent the interface between the subnet and the remaining of the net are duplicated on the subpage. These places are called socket nodes in the upper page and port nodes in the lower level page. There is a one to one correspondence
between a socket node and a port node: they are virtually the same place and the marking of a socket node is the same as the marking of the port node.

An interesting feature of Hierarchical Colored Petri Nets is that different substitution transitions can refer to the same subnet. To each of these substitution transitions corresponds a different instance of the subnet that behaves independently of the other instances. The distinction between the different instances is made through the correspondence of socket nodes at the upper level and port nodes of the instance at the lower level. The development of Colored Petri Net model can then be done through the development of independent and reusable modules arranged in a hierarchical structure that looks like a lattice. Any change in a subpage will affect all its instances but will not affect the rest of the model as long as the correspondence between the socket nodes of the upper level page and the port nodes in the lower level page is maintained.
Figure A.4 Substitution Process
APPENDIX B. DESCRIPTION OF THE PETRI NET MODEL OF THE AAW SYSTEM

High level representation of sensor operation

The diagram in Figure B.1 is the substituted transition of sense in the top-level diagram (Figure 6.11). AWACS receives the surveillance directive from AOC. An intelligence collection unit operates their organic sensors and disseminates the intelligence (identity and location of targets) to the combat units in the battlefield.
(identification to AOC and location to FDC). By default, it collects data using the *active* scanning mode. In the *active* mode, it scans and tracks the specified target only with higher priority to collect more intensive data for the target. In the *passive* mode, it scans and tracks the targets periodically without the priority.

By changing the marking of the place *Sensors*, it represents the number of sensors in operation. The transition *Operate Sensors* is decomposed into the lower level in the hierarchy. The label *AWACS* in the dotted box is the name of the physical node (see Figure 6.10) assigned to carry out the logical task (i.e., transition *Update Directive*). The label [0,0] means the action time to carry out the task.

**Decomposition of sensor operation**

The diagram in Figure B.2 is the substituted transition of *Operate Sensors* in Figure B.1. The nodes *Sense1*, *Sense2*, and *Sense3* are the substitution transitions of surveillance radar *S1*, *S2*, and *S3* in the physical architecture (Figure 6.10), respectively. The node *Sense5* is the substitution transition of the tracking radar *R101* in the physical architecture. The nodes *Sense4* and *Sense6* are the substitution transitions of central data fusion nodes *AWACS* and *FDC* in the physical architecture, respectively. The label *EXTERNAL* in the dotted box is the name of the physical node assigned to carry out the logical task (i.e., transition *Generate*). Since this transition is an event generator for simulation, no physical resource is assigned.
Figure B.2 Petri Net Diagram: Operate Sensors
**Operation of a local sensor**

The diagram in Figure B.3 is the substituted transition of Sense1 in Figure B.2. The local sensor (Tb1) detects targets, calculates and maintains (at the place store) the probability of target identity and the variance of tracking data, and sends (by Td1) the new target information to central data fusion node. The identification algorithm has been implemented using Bayesian rule to update the new probability. The tracking algorithm has not been implemented using a full Kalman filter. For simplification, only the variance of track estimation was computed. The purpose of tracking data in the scenario is to
determine the probability of kill such that, if the variance of the estimated location of the
target is less than a certain criteria, the result is 'kill' otherwise 'miss'. This scenario requires the variance only. Thus the algorithm has been simplified and computes only the variance of the estimation.

The label \textit{RI01} in the dotted box is the name of the physical node (see Figure 6.10) assigned to carry out the logical task (i.e., transitions \textit{TbI} and \textit{TdI}). The label [0,2] and [0,0] mean the action times to carry out the tasks. The interval [0,2] means that the transition \textit{TbI} scans the battlefield periodically every 2 seconds. The time 0 represents the time that the target can be detected at any time of the cycle. It can be used to estimate the maximum processing time and the maximum network delay under assumptions about the characteristics of information system and communications network. The interval [0,0] means that the transition \textit{TdI} has no action time because it only disseminates the target information, and so it has only the network delay.

The parameter of the sensor is represented by the marking of the place \textit{RI}. From the left in the data structure of the marking \textit{(1,(D2Sec 2),(map R2I ([0.05,0.9,40.0])))}, \textit{1} represents the sensor ID, \textit{(D2Sec 2)} represents the default action time (the default scan cycle of 2 seconds), and \textit{[0.05,0.9,40.0]} represents [probability of false detection, probability of positive detection, surveillance range] in which that the sensitivity (likelihood ratio) is 18 and the surveillance range of the radar is 40.0 km.
Target identification: Sense 6 has the same structure

The diagram in Figure B.4 is the substituted transition of Sense4 in Figure B.2. This fusion node (Tc4) calculates and maintains the probability of target identity and the variance of tracking data from three sensors, and disseminates (by Td4) the new target information.
The label AWACS in the dotted box is the name of the physical node (see Figure 6.10) assigned to carry out the logical task (i.e., transitions $Tc4$ and $Td4$). The label $[0,0]$ means the action time to carry out the task.

The fusion rule is represented by the marking of the place $R4$. From the left in the data structure of the marking $(4, (D2Sec 0), 0, (map R2I ([0.0, 0.5, 0.5, 1.0])))$, 4 represents fusion node ID, $(D2Sec 0)$ represents no action time, 0 is the counter to be used for FIFO (First In First Out) processing of incoming sensory data, and $[0.0, 0.5, 0.5, 1.0]$ represents the conditional probabilities of the target identity for data fusion dealing with two sensory reports in order of [FF, FT, TF, TT] in which FF means the conditional probability when two sensors reports that the target is a non-hostile object and TT means the conditional probability when two sensors report the target is a hostile object.

**Command Function**

The diagram in Figure B.5 is the substituted transition diagram of *command* in the top-level diagram (Figure 6.11). The transition *Decide* make a judgment whether the target is hostile or not using the probability of target identity and issues the firing order if the probability meets the decision criteria (threshold). The results of the combat received from each platform is displayed on *Screen* and maintained in a database (place *Cleared*). If the target is assessed to be hostile, it issues the firing order again to any available interceptor. The decision is made by the automated computing algorithm. If the uncertainty (i.e., probability) does not meet the decision criteria, the algorithm issues the
new surveillance directive in order to switch the surveillance mode from default mode (Passive mode) to Active mode (i.e., scan or track only the specific target).

Figure B.5 Petri Net Diagram: Command

The label $AOC$ in the dotted box is the name of the physical node (see Figure 6.10) assigned to carry out the logical task (i.e., transitions Decide). The label $[0,0]$ represents the action time to carry out the task. It does not have the human decision making time due
to the automated algorithm. The label *Screen* is the physical device to display the tactical picture after the actions taken.

**Act function**

![Petri Net Diagram: Act](image)

Figure B.6 Petri Net Diagram: Act

This diagram in Figure B.6 is the substituted transition of *act* in the top-level diagram (Figure 6.11). The transition *update task* receives an air tasking order from AOC, update the current mission on *Display* of FDC, and sends the order to the platform (transition node *Intercept*). The transition *update track* receives the target track...
information for localization and update the current location on Display of FDC, and sends the track to the platform (transition node Intercept). The platform (transition Intercept) checks (the marking at the place named Weapon) the availability of a missile and launches it when available. The initial marking shows that the platform has 4 missiles armed. After elapsing the flight time that are required to intercept the threat, the platform reports the after action report (AAR).

The label S101 in the dotted box is the name of the physical node (see Figure 6.10) assigned to carry out the logical task (i.e., transition Intercept). The label [0,5] and [0,0] mean the action times to carry out the tasks. The interval [0,5] means that the platform (transition Intercept) needs 5 seconds to reload the missile after firing one. The time 0 represents the time that the target can be detected at any time of the cycle. It can be used to estimate the maximum processing time and the maximum network delay under assumptions about the characteristics of information system and communications network. The interval [0,0] means that two transitions update task and update track has no action time other than information processing to update and display. The label Display is the physical device to display the current mission and the current location of the target.

The availability of missiles are represented by the markings of the place weapon. The data structure of the marking represents (weapon ID, velocity, location). For example, 4'(1,3,0) represents a platform (ID 1) has 4 missiles with velocity Mach 3 and the initial deployment location 0.
APPENDIX C. PERFORMANCE MEASURES FOR THE AAW SYSTEM

C.1 Experimental Results

The titles of tables from Table C.1 through Table C.22 represent a set of input parameters that are listed in order of rule of intercept, criteria to assess combat result, number of transponders, surveillance mode, and information delay. For simplicity, the variance of track estimation was used to determine the probability of kill so that the probability can be used to assess the combat result. In this experiment, for simplification, the probability of kill was used either 1.0 or 0.0 (it was assumed if the variance of the track estimation is less than a criteria, the probability of kill equals 1.0, otherwise 0.0). The criteria were represented by the parameter \( j \) in the title of each table. \( j \) represents that the criteria was disregarded: the probability of kill is always 1.0 once the missile is launched and meets the timing constraint. \( j \) represents that the criteria was disregarded: the probability of kill is always 1.0 if the variance is less than 1.0 m, the probability of kill is always 1.0, otherwise 0.0.

For example, the \( \text{Aggressive}_1 \text{ Single } \text{Active } \text{Dynamics } \text{Zero} \) of Table C1, represents that the simulation has been conducted with the \( \text{aggress} \text{ve intercept, the probability of kill 1.0 (j } \text{S00} \text{Single } \text{Active } \text{Dynamics } \text{Zero} \text{), and the input parameters used in the experiment. As the column titles represent, they are snumber of sensor, Likelihood} \)
\( \text{Ratio}_i \pm \text{decision threshold}_i \pm \text{information delay}_i \pm \) Te parameter value of \( \text{information delay}_i \) may have 4 values in some tables: 1 for minimum delay, 2 for random delay, 3 for maximum delay, and 4 for zero delay.

The right 4 columns are the performance measures (MOPs). The analysis has been conducted with the requirements of MOPs: \( \text{mission success} = 1.0, \text{decision quality} = 1.0, \) and \( \text{weapon efficiency} = 1.0. \) So the data set only shows a subset of the results that meets the requirements. The results of the MOPs in every row represent the average of 50 runs and the value was represented as an index of MOEs.

Table C.1 MOEs: Aggressive_S00_Single_Active_Dynamics_Zero

<table>
<thead>
<tr>
<th>No of Sensor</th>
<th>Likelihood Ratio</th>
<th>Decision Threshold</th>
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<th>Mission Success</th>
<th>Decision Quality</th>
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Table C.2 MOEs: Aggressive_S00_Single_Active_Fixed_Zero

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Table C.3 MOEs: Defensive_S00_Single_Active_Dynamic_Zero

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149
Table C.4 MOEs: Defensive_S00_Single_Active_Fixed_Border

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Table C.5 MOEs: Defensive_S00_Multi_Active_Dynamics_Random

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Table C.6 MOEs: Defensive_S00_Multi_Active_Fixed_Random

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Table C.7 MOEs: Aggressive_S00_Single_Active_Dynamics_Random

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Table C.8 MOEs: Aggressive_S00_Single_Active_Fixed_Random

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Table C.9 MOEs: Aggressive_S00_Multi_Active_Dynamics_Random

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Table C.10 MOEs: Aggressive_S00_Multi_Active_Fixed_Random

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Table C.11 MOEs: Defensive_S00_Single_Active_Dynamic_Zero with a requirement

\(mission\ success = 1.0\)

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Table C.12 MOEs: Defensive_S00_Single_Active_Dynamic_Zero with a requirement

\(decision\ quality = 1.0\)

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Table C.13 MOEs: Defensive_S00_Single_Active_Fixed_Zero

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Table C.14 MOEs: Aggressive_S00_Single_Active_Dynamics_1024

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Table C.15 MOEs: Aggressive_S10_Single_Active_Dynamics_Random

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Table C.16 MOEs: Aggressive_S10_Single_Active_Fixed_Random

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Table C.17 MOEs: Aggressive_S10_Multi_Active_Dynamics_Random

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### Table C.18 MOEs: Aggressive_S10_Multi_Active_Fixed_Random

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### Table C.19 MOEs: Defensive_S10_Single_Active_Dynamic_Random

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### Table C.20 MOEs: Defensive_S10_Single_Active_Fixed_Random

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154
### Table C.21 MOEs: Defensive_S10_Multi_Active_Dynamics_Random

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C.2 Observations of system behavior

Before examining the result of the experiment, let us see the general characteristics of a sensor system. The characteristics of a sensor system are associated with the number of observations. To analyze the general characteristics of the system behavior, it may be reasonable and preferable to focus on the tactical operation environment with the aggressive intercept rule. The defensive intercept case does not respond, even if the uncertainty requirements are met, until the target moves into a specified zone.

If we make a decision at the last moment, it may reduce the operational readiness (the time to be ready for the next mission) resulting in the next threat leaking and so fail to carry out the mission successfully. If we make a decision early, it may reduce the quality of the decision resulting in killing friendly forces. This type of dilemma may occur in a time critical decision environment. So it may be reasonable and preferable to focus on the tactical operation environment with the defensive intercept rule since it has the smaller windows of opportunity.

The probability of kill and the information delay can be ignored for analysis of the pure sensor system’s performance.

**Number of Sensors and/or Transponders**

In general, more sensors or more transponders produce higher effectiveness.

In a tactical situation of aggressive interception, the system with only three sensors meets the requirements of mission success and decision quality, if the sensor has a single
transponder (see Table C.1 and Table C.2). When the window of opportunity is smaller (i.e., the system is operated under defensive intercept rule; see also Table 6.2), the system with even three sensors was not enough to meet the requirements if the sensors have a single transponder and if the dynamic computation rule for data fusion is applied (see Table C.3). In the fixed computation rule case, the system with three sensors with the high likelihood ratio meets the requirements (see Table C.4).

With the system with multi transponders, however, the system with even one sensor meets the requirements (see Table C.5 and Table C.6)

**Computation Rule vs. Timeliness**

The dynamic computation for data fusion supports better timeliness (operational readiness) than the fixed computation rule, even though the decision criterion of the dynamic computation rule has a higher threshold than that of the fixed computation rule.

In the defensive interception rule, the system does not respond, even if the decision is made, until the target moves into a specified zone. So the situation with the defensive intercept rule is not good sample space for the general analysis of the relationship between timeliness and computation rule. The analysis of this general relationship has been conducted with the situation with the aggressive interception rule.

Compare the operational readiness of Table C.1 and the one of Table C.2. For example, the operational readiness of the system of three sensors with the likelihood ratio 18 is 28.86 in the dynamic computation rule and 39.87 in the fixed computation rule. The dynamic computation for data fusion supports better timeliness (operational readiness)
than the fixed computation rule, even though the decision criterion of the *dynamic* computation rule is higher than that of the *fixed* computation rule. The same relationship has been observed in Table C.7 and Table C.8. For example, the operational readiness of the system with the likelihood ratio 6 is 38.99 for two sensors and 39.21 for three sensors in the *dynamic* computation rule. The corresponding ones in the *fixed* computation rule are 43.83 seconds and 81.37 seconds.

This relationship is clearly shown from the sensor system with multi-transponders. Compare the operational readiness of Table C.9 and Table C.10 in which the results were sorted by the *operational readiness*. Consider the system of three sensors with the likelihood ratio 18 at which the identical decision criterion 0.99 was used. In the dynamic computation rule, the operational readiness shows 8.97 seconds (minimum value) meanwhile the one in the fixed computation rule 79.38 seconds (maximum).

**Number of Sensors vs. Computation Rule**

In the *dynamic* computation for data fusion, as the number of sensors increases, the system shows better timeliness (operational readiness). However, the opposite was true in the *fixed* computation rule.

See the operational readiness of Table C.9 and the one of Table C.10. Consider the system of three sensors with the likelihood ratio 18 at which the identical decision criterion 0.99. In the *dynamic* computation rule case, the operational readiness of the system marks 11.66, 9.41, and 8.97 for one, two, and three sensors respectively. In the *fixed* computation, however, the corresponding operational readiness shows 11.57, 39.83
and 79.83. The same relationship exists in the system of the likelihood ratio 9 and 6. The operational readiness of the system with the likelihood ratio 9 in the dynamic computation shows in the decreasing order such as 14.74, 12.66, and 11.52 as the number of sensors increases. Meanwhile, the operational readiness of the system with the likelihood ratio 6 in the fixed computation marks in the increasing order such as marks 15.60, 41.94, and 79.36 as the number of sensors increases.

**Decision Threshold**

Clearly, there is a relationship between the decision criterion and the success of mission. If we focused on the timeliness of a response, the lower threshold for the decision criterion is preferred (see Table C.11 whose data set shows only those results meeting the requirement of *mission success*=1). However, if we focused on the accuracy of a response, the higher threshold is preferred (see Table C.12 whose data set shows only those results meeting the requirement of *decision quality*=1).

Table C.11 and Table C.12 are good examples since the system does not meet both requirements (*mission success*=1 and *decision quality*=1) at the same, and show the dilemma (due to the smaller window of opportunity from the *defensive* intercept rule) between the timeliness (mission success) and accuracy (decision quality) of the response. The results have been produced from the dynamic computation rule. It is also shown in the fixed computation rule for data fusion under the same dilemma due to the smaller windows of opportunity (see Table C.13).
For example, the system with the decision criterion 0.7 for one or two sensors system meets the timeliness (*mission success*), but fails to meet the accuracy (*decision quality*). The system with the decision criterion 0.99 for one or two sensors system meets the accuracy (*decision quality*), but fails to meet timeliness (*mission success*). By adding one more sensor, the system can meet both requirements.

**Decision Threshold vs. Computation Rule**

In the *dynamic* computation rule for data fusion (see Table C.1 and Table C.2), the decision threshold needed to be set to the higher value for better performance. This is reasonable because this computation rule loses the benefit of multiple sensors since it uses a partial number of sensors, and so the higher threshold is required to get higher quality of information.

Conversely, in the *fixed* computation rule for data fusion (see Table C.7 and Table C.8), the decision threshold needed to be set to the lower value. This rule fuses data assuming there are multiple sets of sensory data. It takes a long time to reach the higher threshold when some sensory data is unavailable. So it is reasonable that the lower threshold is required to respond quickly.

**Information Delay**

Obviously the information delay should affect the timeliness of the response. But it was not true that the minimum information delay always guarantees the fastest response.
Compare the operational readiness of Table C.1 (zero information delay) to the one of Table C.7 (random information delay) in which the results were generated in the tactical situation of the aggressive intercept with the dynamic computation rule. For example, consider the system of three sensors with the likelihood ratio 18 and 6 at which the decision criterion 0.99 was used. The operational readiness of the system of three sensors with the likelihood ratio 18 and 6 shows 29.07 seconds and 41.87 seconds each when there is no information delay (Table C.1). The corresponding ones when there is a random delay (Table C.7) show 28.86 seconds and 39.21 seconds. The timeliness in the case of random delay is smaller than the one in the case of no delay.

Now, compare the one of Table C.2 (zero information delay) to the one of Table C.8 (random information delay) in which the results were generated in the tactical situation of aggressive intercept with the fixed computation rule. The operational readiness of the system of three sensors with the likelihood ratio 6 and with the decision criterion 0.7 shows 39.33 seconds when there is no information delay (Table C.2). The corresponding one when there is a random delay (Table C.8) shows 45.19 seconds. The timeliness in the case of random delay is now larger than the one in the case of no delay.

The ones with no information delay do not show the better operational readiness in the dynamic computation rule compared to the ones with information delay. The one with no information delay does show the better operational readiness in the fixed computation rule compared to the ones with information delay. But there is no evidence that the information delay has a relationship with the computation rule. See Table C.14 in which the results were produced by various information delay options. The values of
information delay 1, 2, 3, and 4 represent minimum, random, maximum, and zero delay respectively. In the sensor with the likelihood ratio of 6 shows that the operational readiness produced from zero delay is larger than the one from maximum delay. But in the sensor with the likelihood ratio of 18 shows that the operational readiness produced from zero delay is smaller than the one from minimum delay.

It means that the information delay affects the behavior of the system not just in terms of the timeliness, but also the ordering of tasks in an autonomous system depending on the arrival time of information. It is the authors' conjecture that, once the information processing and distribution systems have the potential capability to carry out the mission successfully, the performance of the system (response time) may be more highly affected by the order of the information arrivals than by the delay of information. So the response time based on the state space analysis technique (introduced in section 5.10 and illustrated in Figure 5.10 in Chapter 5) using the maximum information delay does not guarantee the worst case response time.

**Dominant Variables**

Throughout the results, when a system consisting of a data fusion sub-system with a single transponder was used, the possible combinations of input parameters to meet the performance requirements (with both requirements of mission success = 1.0 and decision quality = 1.0) were restricted. When multiple transponders were used, there were many more possible combinations of input parameters that met the performance requirements.
For example, compare the results of Table C.19 (single transponder and single computation rule) to the ones of Table C.21 (multi-transponders and single computation rule). Also compare the results of Table C.20 (single transponder and dynamic computation rule) to the ones of Table C.22 (multi-transponders and dynamic computation rule). Clearly, the dominant variable affecting the performance of the system is the number of transponders per sensor.

Measurements of Effectiveness (MOEs)

For the scenario of the experiment with the component system specifications:

- The bandwidth of communications link 1.024Mbps,
- The bit processing capability of the information system 100Mbps, and
- The variance of tracking estimation required due to the accuracy of weapon system 1m,

the best combination of the AAW system is as follows:

In the tactical situation of the aggressive operation, the best combination is the one with three sensors, multi-transponders (eight transponders per sensor in this experiment), likelihood ratio of 18, dynamic computation rule for data fusion, and the decision criterion of 0.99. See Table C.15 through Table C.18. All combinations show the equal MOEs (Mission Success, Decision Quality, Weapon Efficiency) except the Operational Readiness. The bolded combination in Table C.17 shows the best effectiveness in the Operational Readiness.

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In the tactical situation of the *defensive* operation, the best combination is the one with three sensors, multi-transponders (eight transponders per sensor in this experiment), likelihood ratio of 18, *fixed* computation rule for data fusion, and the decision criterion of 0.8. See Table C.19 through Table C.22. All combinations show the equal MOEs on Mission Success and Decision Quality, but Weapon Efficiency is less than 1.0 in the *dynamic* computation rule while it is 1.0 in the *fixed* computation rule. The bolded combination in Table C.22 shows the best effectiveness in the *Operational Readiness*. 
LIST OF REFERENCES


Altukhov, P. K. (1984), Theory of Command and Control, Military Publishers, Moscow, USSR.


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