

Nearshore airborne lidar observations

Alexandru Sheremet

8 Aug 2001

1 Introduction

Hwang et al. (1999a and 1999b) uses standard FFT procedures to estimate deep water 2-dimensional wavenumber spectra from lidar images. The characteristic wavelength analyzed is $O(100m)$, homogeneity scale is $O(10km)$ and wave evolution scale is $O(100km)$. Knowledge of the local bathymetry allows, via the linear dispersion relation, is needed for retrieving frequency-directional spectrum.

The classical Fourier transform approach (FT) fails in the nearshore region due to the strong spatial inhomogeneities of the wavefield, generated by lower order wave-bathymetry and quadratic wave-wave interaction. Shoaling waves evolve much faster than deep-water waves, over $O(1km)$ scales (Elgar and Guza, 1985).

2 Progress

Other approaches, such as wavelet transforms, have been occasionally used in ocean wave data analysis. The method is relatively new and its physical interpretation has yet to be set on a solid basis.

In this first period the research has been geared toward some very basic aim: assessing the potential of the wavelet transform approach, identifying the directions of research and the implementing the numerical tools necessary to pursue them.

I have performed a extensive survey of the relevant literature, partly to educate myself on the subject, but the main concern was to identify ways to generalize the classical harmonic analysis approach to the nearshore. A summary of the ideas is presented in Subsection 2.1. The information is vast, the wavelet research is booming, with new papers and monographs coming out each day. Unfortunately, in the field of ocean surface waves, the method has been applied sporadically and mechanically, mainly as a drop-in replacement of the classical Fourier approach. I talk more on this in Section 2.2. There are however some very interesting achievements (not in the ocean waves field of research, though).

A considerable amount of work was also done towards building the software basis needed for numerical simulations and analysis (Subsection 2.3). This involved identification available wavelet transform software, selection of the most suited package, familiarization with its architecture. The process of complementing/integrating it is ongoing, and will continue for a quite a while.

Section 3 describes work in progress and further directions of research.

2.1 Generalization of classical FT

The deep-water analysis method may be regarded as a windowed FT approach (Mallat 1998), where the window width is large enough to assume practically infinite resolution in wave number in the Fourier space. In the case of a windowed FT, the Fourier functions e^{ikx} are replaced by $e^{ikx}g(x-x_0)$, where g a constant width window function of compact support translated by x_0 . Its resolution in the wavenumber space depends on the choice of the window. The corresponding Heisenberg boxes satisfy the uncertainty inequality $\sigma_k\sigma_x \leq 1/2$ (σ is a measure of the resolution; classical FT corresponds to $\sigma_k = 0$ and $\sigma_x = \infty$).

The restriction of a constant width window makes the windowed FT a good way to use on signals that exhibit strong inhomogeneity across scales. Allowing the window to vary in size leads to the Gabor transform; generalizing the family of basis functions (to satisfy some quite general "admissibility" conditions) leads to a continuous wavelet transform (CWT). These transforms conserve energy and admit the definition of an energy density.

Unlike the classical FT, both the windowed FT and the CWT representations are highly redundant. The discrete wavelet transform (DWT), based on multi-resolution analysis eliminates the redundancy by extracting a complete orthogonal set, based on a multi-resolution analysis (MRA, Wojtaszczyk 1997). Assuming that a local Fourier spectrum may be defined, the the Fourier and wavelet spectra contain equivalent information and can be retrieved from each other. Moreover, for transient, short time series, the DWT spectral estimator seems to be more robust than the classical Fourier or CWT, the latter exhibiting scale mixing problems due to overcompleteness (Pando and Fang, 1998).

The above observations seem to indicate that, by smoothly modifying the window width and the basis functions, it is possible to devise a unified approach for the spectral analysis of lidar images all the way from the deep sea into the nearshore. Such an approach would facilitate physical interpretation and the reconstruction of traditional wave field parameters (frequency-directional spectra, mean direction of propagation, mean period etc).

2.2 Physical interpretation of wavelet transform

The use of the CWT/DWT raises several important questions related to the physical interpretation of the results.

Is there an optimal wavelet shape to be used for a given purpose? The literature is extremely rich in wavelet shapes, classes and methods to generate them. Owing to its simple analytic form, a Gaussian wave group, the Morlet wavelet (Grossmann and Morlet 1985) has been traditionally preferred in the description of progressive waves. There is no formal justification, though, to choose it over, say, a chirp wavelet (Holschneider 1998), which includes a wavenumber shift that can account for linear evolution. For two dimensional applications, Cauchy wavelets (Antoine et al. 1999) have minimal uncertainty and much better angular selectivity than Morlet wavelets. Different applications will require different wavelet features; for example, wave crest detection would use real wavelets such as the Hermitian class (Lewalle 1997). A study of shore reflection might require use of boundary wavelets (Monasse and Perrier 1998).

Is then the choice of the wavelet critical for the results of the analysis? General statements about wavelets behaviour can also be made. Statistical and spectral properties are well understood and seem to be relatively insensitive to the type of wavelet used in the analysis (Nason et al. 1997, Masry 1998, Perrier et al. 1995, Pando and Fang 1998, Brillinger 1996).

The concern here is not the "mechanical" decomposition/reconstruction of the lidar images, but the retrieval of meaningful physical quantities, and their definitions stem directly from the governing equations of the physical processes. Wavelet use with partial differential equations suffers from the fact that they are not eigenfunctions of space/time-invariant operators (eg. ∇, ∂_t). The decomposition yields infinite system of equations, making analytical approaches difficult. Studies have been restricted mostly to numerical solutions algorithms (Bacry et al. 1991, Konno and Lomdahl 1994, Beylkin and Coult 1998, Monasse and Perrier 1998). This argument is, however, somewhat deceptive, since the wavelet decompositions are tools best fitted to transient/inhomogeneous problems, which are not really described by invariant operators. Very simple and intriguing results obtained by Lewalle 1997/1998, using Hermitian wavelets on the Poisson and the diffusion equation indicate, that they can be very effective tools if used not just as drop-in replacement of the classical Fourier analysis.

2.3 Numerical implementation

Generic implementations for the different wavelet transforms are available as free downloadable packages on the Internet, a few of them designed to be used in the Matlab environment. A short list comprises the *W-transform Toolbox*, which provides a class of discrete transforms that treats signal endpoints differently than usual and allows signals of any length to be handled efficiently; the *Uvi-Wave Toolbox*, developed by the Signal Theory Group, University of Vigo, and provides filter generation, wavelet/wavelet packet

transforms and MRA; WaveLab from Stanford University. With over one thousand and Matlab files, datasets, and demonstration scripts, the latter is the most comprehensive package. WaveLab is a collection of Matlab functions that have been used by the authors and collaborators to implement a variety of computational algorithms related to wavelet analysis. A partial list of the techniques include orthogonal/biorthogonal wavelet transforms, translation-invariant wavelets, interpolating wavelet transforms, cosine packets, wavelet packets, matching pursuit. One nice feature is that some computationally expensive routines have been implemented as Matlab MEX functions.

The WaveLab package is being used at present as the basis for the development of a number of complementary numerical routines (eg. routines for generating different type of wavelets, such as Cauchy, chirp, boundary wavelets). A certain amount of time was and will continue to be spent for the familiarization with the architecture of the package and for its integration with the rest of the software under development (data retrieval routines, deep-water classical 2D Fourier analysis, etc).

3 Present and further directions of research

Nearshore lidar images contain traces of numerous complex processes, from random directionally spread waves undergoing nonlinear shoaling, to wave breaking, shear currents, over unknown bathymetry.

In this phase, the stress of the study is set on the mathematical formulation of the first order (linear) problem and implementation of the necessary numerical tool, which has to be done in a tightly controlled environment. The research is therefore limited at present to simulated unidirectional random wave fields (normal incidence to the shoreline), over a very mildly sloping bottom (known bathymetry, the wave field can be approximated locally by a superposition of plane waves to a very good degree of accuracy). The analysis domain does not extend into the surf zone.

The immediate goal is to build an algorithm capable of reconstructing the local frequency spectra from simulated unidirectional lidar images.

A direct lidar data analysis approach being pursued at present is outlined in Subsection 2.1. The classical FT approach is being compared with the DWT based Fourier spectral analysis in the deep water domain in order to establish the capabilities of the latter method. The DWT approach can be extended into shallow water; assuming the results are comparable, the study will create a platform for testing the performance of the method in the nearshore and evaluating (from a data analysis/synthesis point of view) the impact of using different wavelet shapes.

The retrieval of frequency spectra from wavelet analysis requires the knowledge of the relationship between the wavelet scale and the frequency (similar to the Fourier dispersion relation). The search for such a relation follows the ideas outlined in Subsection 2.2. This is essentially an attempt to derive evolution equations for wavelet coefficients starting from the governing equations, using an approach similar to the one described in Agnon (1999). Using operational calculus, Agnon was able to derive a general mild slope equation that accounts for all the terms that are linear in the derivatives (to any order) of the depth, and reduces, under additional assumptions, to well known forms given by Berkhof (1972), Kirby (1986) and Chamberlain and Porter (1995). The formal solution to the governing equations does not use the explicitly a "local" plane wave structure for the wave, which makes it suitable for a wavelet decomposition approach.

Further research will involve a slow relaxation of the initial assumptions, starting with discarding the very mild slope assumption and the unidirectionality of the waves. In the following stage, the work will focus on the derivation of an algorithm for the reconstruction of the directional frequency spectrum and associated parameters.

4 References

- Agnon, Y. Phys. Rev. E 59, 1999
- Antoine et al. , Appl. Comp. Harmonic Analysis 6, 1999
- Bacry et al. Technical Report, Courant Inst. of Mathematical Sciences 591, 1991
- Berkhoff, J. C., Proc. 13th Intl. Conf. Coastal Eng. ASCE 471, 1972
- Beylkin, G. and Coult, N. Appl. Comp. Harmonic Analysis 5, 1998

Brillinger, D., *Nonparametric Statistics* 6, 1996
Chamberlain P. G. and Porter, D., *J. Fluid Mech.* 291, 1995
Elgar, S. and Guza, R. T., *J. Fluid Mech.* 161, 1985.
Holschneider, M. *Wavelets: an analysis tool.* Clarendon Press, Oxford. 1998
Hwang et al., *J. Phys. Oceanogr.*, in press., 1999a.
Hwang et al., *J. Phys. Oceanogr.*, in press., 1999b.
Kirby, J. T. *J. Fluid Mech.* 162, 1986
Konno, H. and Lomdahl, P., *J. Phys. Soc. Japan*, 63(11), 1994
Lewalle, J., *Phys. Rev. E* 55(2), 1997
Lewalle, J., *J. Math. Phys.* 39(8), 1998
Mallat, S. *Academic Press*, 1998
Masry, E., *Signal Processing* 64, 1998
Monasse, P. and Perrier, V., *SIAM J. Math. Anal.* 29(4), 1998
Nason et al., *Technical Report 516*, Dept. of Statistics, Stanford University, 1997
Pando, J. and Fang, L-Z., *Phys. Rev. E*, 57(3), 1998
Perrier et al., *J. Math. Phys.*, 36(3), 1995
Wojtaszczyk, P. *A mathematical introduction to wavelets* Cambridge Univ. Press, 1997

REPORT DOCUMENTATION PAGE

Form Approved

OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average one hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY) 08-08-2001	2. REPORT TYPE Final (Technical) Report	3. DATES COVERED (FROM - TO) 19-08-1999 - 31-03-2001
---	--	---

4. TITLE AND SUBTITLE Nearshore Airborne Lidar Observations	5a. CONTRACT NUMBER --
	5b. GRANT NUMBER N00173-99-1-G905
	5c. PROGRAM ELEMENT NUMBER --

6. AUTHOR(S) Sheremet, Alexandru	5d. PROJECT NUMBER --
	5e. TASK NUMBER --
	5f. WORK UNIT NUMBER --

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES) University of California, San Diego Scripps Institution of Oceanography Office of Contract and Grant Administration Mail Code 0210 La Jolla, CA 92093-0210	8. PERFORMING ORGANIZATION REPORT NUMBER --
---	--

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research Resident Representative 4520 Executive Drive Suite 300 San Diego, CA 92121-3019	10. SPONSOR/MONITOR'S ACRONYM(S) NRL
	11. SPONSORING/MONITORING AGENCY REPORT NUMBER --

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release

13. SUPPLEMENTARY NOTES
--

14. ABSTRACT
Hwang et al. (1999a and 1999b) uses standard FFT procedures to estimate deep water 2-dimensional wavenumber spectra from lidar images. The characteristic wavelength analyzed is $O(100m)$, homogeneity scale is $O(10km)$ and wave evolution scale is $O(100km)$. Knowledge of the local bathymetry allows, via the linear dispersion relation, is needed for retrieving frequency-directional spectrum.
The classical Fourier transform approach (FT) fails in the nearshore region due to the strong spatial inhomogeneities of the wavefield, generated by lower order wave-bathymetry and quadratic wave-wave interaction. Shoaling waves evolve much faster than deep-water waves, over $O(1km)$ scales (Elgar and Guza, 1985).
Other approaches, such as wavelet transforms, have been occasionally used in ocean wave data analysis. The method is relatively new and its physical interpretation has yet to be set on a solid basis.

15. SUBJECT TERMS
nearshore, lidar, wavelet, ocean waves

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Unclassified Unlimited	18. NUMBER OF PAGES 5	19a. NAME OF RESPONSIBLE PERSON Sheremet, Alexandru
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) 858-534-4333