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13. ABSTRACT *(Maximum 200 Words)*

In this report we summarize our recent research on the development of a systematic methodology for the design of feedback laws achieving stabilization and regulation of nonlinear control systems. We consdier the stabilization and control of both lumped nonlinear systems and nonlinear distributed parameter systems. The principal control objective is output regulation, the ability of the output of a system to track a desired signal while rejecting signals produced by a known exogenous system. This objective must also be achieved in a manner that is robust with respect to variation in unkown plant parameters; i.e., we develop a mehodology for output regulation that is robust against real parametric uncertainty.

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Contents

AFOSR FINAL REPORT

December 1, 1997 – November 30, 2000

NONLINEAR CONTROL SYSTEMS

AFOSR Grant No. – F49620 - 98 - 1 - 0170

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1 Executive Summary

The principal goal of the proposed research program is to develop a systematic methodology for the design of feedback control schemes capable of shaping the response of complex dynamical systems. In particular, the ability to systematically control, or take advantage of, dominant nonlinear effects in the evolution of complex dynamical systems is an important research goal, with applications in several existing and emerging DOD research and development programs. Notable examples, widely appreciated within the aerospace industry, include the development of flight controllers for increasing the high angle-of-attack or high agility capabilities of existing or future generations of aircraft and missiles. Indeed, the importance of including the nonlinear behavior of aerodynamic parameters, such as the coefficient of lift, as a function of the angle-of-attack has long been recognized. In addition, the couplings between the pitch, roll and yaw moments at high angles-of-attack exhibit nonlinear effects which cannot be ignored. For example, better control of the yaw moment at high-angles-of attack would lead to improvements in weapon pointing systems and underscores the need for a systematic theory of robust nonlinear control design.

The incorporation of linear and nonlinear distributed parameter effects also present an opportunity in the control of complex dynamical systems, for example, the earlier design of the F/A-18 included aeroelastic wings, which would result in less weight and an increased opportunity to take advantage of flexible effects. However, understanding the stability characteristics of the loaded flexible wing still presents a challenge, a challenge which is currently being addressed through the Active Aeroelastic Wing program. Another example of the potential impact of nonlinear control of distributed systems is in problems of flow control, such as in the control of instabilities in the unsteady separated shear layer, which have been experimentally shown to greatly influence stall and lift behavior and is known to result in damage or in reduction of the life cycle of aircraft parts. Indeed, active control of flutter and of buffeting would increase life-cycle of aircraft through suppressing buffet loads on fighter aircraft with vertical tails, such as on the F/A-18, as well as on the tails of commercial and transport aircraft. Moreover, distributed effects also offer the opportunity to produce large displacements with relatively small control effort. For example, the development of a nonlinear distributed parameter control system that takes advantage of large variations in an unsteady flow would offer the promise of greatly increased aircraft maneuverability, survivability, and structural fatigue life.

A continuation of an ongoing research effort, the research program we are conducting is aimed at the development of a systematic control methodology for lumped and distributed parameter systems, similar in scope and applicability to classical automatic control design for lumped linear systems. The typical design objectives would involve designing feedback schemes which achieve one or more of the following: asymptotic tracking, stabilization, disturbance attenuation or rejection, robustness against uncertainty. We will also require the design of observers or filters which asymptotically produce either the system state or a functional of the state, such as a desired feedback law. The need to control systems in which the rigid aerodynamics are coupled with the effects of fluid flow and flexible structural modes underscores the desirability for a unified feedback design methodology, capable of producing controllers for both nonlinear lumped and distributed parameter systems.

2 Accomplishments/New Findings

2.a Feedback Design for Nonlinear Systems

As a starting point, we recall that one of the classical problems which involves shaping the response of a system is output regulation, in which the objective is that of controlling a plant in order to have its output track (or reject) exogenous commands or disturbances. The nonlinear regulator theory developed in [46] is valid in case a model for an "exosystem" which generates the exogenous signal to be tracked and the exogenous disturbance to be rejected is known and satisfies certain conditions. These conditions currently prohibit the incorporation of certain signal generators which also have application, such as tracking a ramp or a limit cycle. One of the goals of the present research effort is to extend the class of signal generators to which nonlinear regulator theory will apply. Among the additional research tasks we propose is to develop robust control schemes for output regulation in the large and for robust model following.

Even for lumped linear systems, the more refined task of developing a robust regulator theory involves still more challenging cases arising when either:

1. a signal generator, or reference model, is known but certain parameters in the system or in the disturbance channel are unknown (*structured uncertainty*); or
2. a signal generator for disturbances is not known (*unstructured uncertainty*).

Both of these problems are central focal points in the modern approach to robust control. Despite significant effort, for lumped, linear systems robust control for structured uncertainties is not nearly as well understood as robust control for unstructured uncertainties. In general, one can always augment the system dynamics by adding as new state variables the parameters accounting for the structured uncertainty. Of course, for linear systems this results in a nonlinear system, so that linear regulator theory cannot be directly applied and one might not expect a linear controller to exist. However, even for nonlinear systems there arises a technical problem in the application of nonlinear regulator theory as it is presently formulated; viz., the parameter values are not necessarily observable or detectable through error measurements. Our first research task makes some of these goals explicit.

Task 2.1: *The development of a separation principle for robust output regulation*

Perhaps our most significant discovery relating to Task 2.1 has been for systems which undergo a bifurcation as a plant parameter changes. In his thesis, "A nonequilibrium theory for robust output regulation for nonlinear systems," James Ramsey develops an approach to output regulation for systems undergoing a codimenison one bifurcation resulting in a stable attractor. The results (see appended manuscript) use the recently established existence of an observer for bifurcating systems and an extension of the Byrnes-Isisdori approach to regulation from the equilibrium case to the nonequilibrium case.

Prior to this thesis and the thesis of Andrea Serrani, most of the results obtained for robust output regulation have been local in nature. However, in several interesting simulations involving – for example, the robust output regulation of the controlled van der Pol oscillator – we have observed that these regulation schemes actually may continue to apply relatively far from the system equilibrium, suggesting that a global version of nonlinear regulator theory should be valid for a class of systems containing many examples of interest. We expect that a global version of output regulator theory, valid for example for larger amplitude limit cycles, will depend rather heavily on the development of a general feedback design methodology for stablization about attractors. Our next task focuses on nonlocal robust output regulation;

the problem of designing feedback control laws which would make robust output regulation possible for arbitrary choices of compact regions of the state spaces of both the system to be controlled and the exosystem. For example, we have been able to achieve semiglobal or global robust output regulation for various useful classes of control systems, such as those systems having a finite Volterra series.

Task 2.2: *The development of a feedback design methodology for solving semiglobal or global robust output regulation problems.*

It is well known that, for systems having uniform relative degree one and a globally asymptotically stable zero dynamics, even in the nonlinear case, high-gain output feedback can force a desirable asymptotic behavior. In particular, when the zero dynamics are globally asymptotically and locally exponentially stable, high-gain output feedback can asymptotically stabilize the equilibrium, while enlarging the basin of attraction so as to contain any pre-assigned compact set of initial data, a property known as semiglobal stabilizability. More generally, the same strategy applies to those systems for which "backstepping" methods would apply, or the class of passive systems, for which there is an underlying global stability mechanism.

On the other hand, if the zero dynamics are just asymptotically stable, the use of only a memoryless output feedback may not asymptotically stabilize the origin but, nevertheless, can help to steer the trajectory to an arbitrary small neighborhood of the origin (practical stabilizability).The question, however, of describing the asymptotic behavior of the closed loop system so obtained inside this small neighborhood has remained open. In our recent research, we have addressed this question. In particular, for a system having relative degree one, we have been able to describe explicitly, in terms of properties of the open-loop system, the structure of any compact attractor that might exist and to determine whether such an attractor is stable in the sense of Lyapunov. Recalling that the closed-loop trajectories converge to trajectories of the open-loop zero dynamics as the gain is increased, we have used the gain coefficient as a bifurcation parameter in a perturbation of the zero dynamics. Such a formalism is indeed possible in great generality, and enables us to describe the asymptotic behavior inside the small neighborhood of the origin as a bifurcation from the zero dynamics.

In another recent paper, we have proposed an approach to stabilization via output feedback, which uses a feedback law that takes explicit advantage of the observability of the state variables associated with the zero dynamics. In doing this, we could substantially remove the hypothesis of minimum-phaseness and have it replaced by a less restrictive hypothesis, which by the way happens to be even *necessary* in the case of linear systems. In the current research, we have shown how the method in question can be rendered robust versus modeling errors on the so-called "high-frequency gain" of the system. This is a particularly sensitive issue in the case of an unstable system possessing an unstable zero dynamics, where it is well-known that stabilization with arbitrary, lower and upper, gain margins in not possible.

The ability to design semiglobally, or globally, valid output regulation schemes will also impact another limitation of the existing theory of output regulation. The existing results on robust output regulation do give sufficient conditions for the local solution of the nonlinear regulator problem, provided the exosystem is "neutral stability". There are quite a few standard signals which can be tracked by such exosystems. For example, asymptotic tracking of a constant function, i.e. set-point control, is accommodated by a one-dimensional neutrally stable system. As another example, asymptotic tracking of any trigonometric polynomial can be approached by combining the exogeneous system corresponding to set-point control with a finite number of harmonic oscillators. Thus, approximate asymptotic tracking of a periodic saw-tooth waveform can be approached within the present context of output regulation. However, an important extension of the nonlinear regulator theory would be the design of controllers achieving asymptotic tracking and disturbance attenuation for signals generated either by unstable exogenous systems, which for example might generate "ramps," or by more complicated exogenous systems, capable of producing limit cycles or other stable nonlinear phenomena. This is the content of our next explicit research task:

Task 2.3: *Output regulation in the case of exosystems which are not necessarily "neutrally stable", including, for example, exosystems which have ramps, saw-tooth waves, unstable motions to stable limit cycles, invariant tori, etc.*

In this part of our research, we have begun by re-casting the problem of output regulation in very general terms, so as to render it suitable to deal with more general classes of exosystems. In this way, we have introduced the concepts of *Steady State Locus*, a special set in the combined state-space of the cascade plant-exosystem. This concept extends to a global setting the notion of zero-error manifold, which was used so far to establish important results in output regulation theory, but limited to the cases in which this manifold could be globally given the structure of a graph of a mapping. With this new concept, we are able to offset these limitations and we are ready to start with the development of a general theory of output regulation for genuinely nonlinear systems.

Our next explicit research task addresses the design of feedback laws which can achieve robust model following. The starting point for this research effort is the observation that one could view several problems in parameter adaptive control as feedback design problems for nonlinear systems. From this point of view, one should expect adaptive control algorithms to be nonlinear, as indeed they often are. Based on preliminary calculations, in our research we will investigate the possibility of extending our method for robust semiglobal regulation to the case in which the exosystem is not just an autonomous linear system, but a linear *reference model* driven by time-varying reference command signals.

Task 2.4: *The development of a systematic feedback design methodology for robust model following for classes of linear and nonlinear systems.*

The main objective of this part of our research is the development of a control structure in which, using only sensed information from the error between actual and trajectory, accurate steady state tracking is achieved by means of an error-feedback control which incorporates an "internal model" of the "external" operational conditions (such as any trajectory requirement, or any dominant disturbance). It is well-known, in fact, that internal-model-based controller possesses invaluable robustness properties.

The main obstruction to widespread use of internal-model based control is the need for an exact mathematical model of the exosystem, which is supposed to generate the external trajectories and/or disturbances. To overcome this obstruction, we have concentrated our efforts on the study of the possibility of autonomous recognition of the natural modes of the exosystem. To this end, we are developing a methodology for autonomous tuning of the natural frequencies of the internal model. This is not a standard frequency estimator, but rather a two-layer internal model. The device does not estimate the frequency of the exogenous input, but rather estimates the control needed to secure a zero steady-state tracking error. Preliminary simulations show very promising results. From the standpoint of the design of control systems, is an important breakthrough. The need of an accurate model of the exogenous inputs, which was a recognized roadblock, is no longer an issue. Our newly developed control methodology incorporates a mechanism for autonomous tuning of the parameters of an exosystem of a fixed structure, a feature that none of the current control methodologies has.

In this way we have also contributed to a clarification of a major theoretical issue: in dealing with the control of uncertain systems, where is the boundary between robust control and adaptive control? In our case, all uncertainties related to system parameters are dealt with via techniques of robust stabilization, while self-tuning methods are used to estimate parameters of the exogenous inputs.

Thus far we have concentrated our research efforts on the problem of detecting unknown inputs (disturbances) in a nonlinear system. Using methods from differential geometry, we have been able to develop general conditions for the design of a filter capable of independently detecting the presence of unknown inputs. The results obtained in this way are far more advanced than those obtainable by means of the classical Beard-Jones detection filter for linear systems or, for that matter, any other published result about unknown-input detection for nonlinear systems. In particular our technique incorporates a procedure that minimizes the dimension of the filter needed to estimate the presence of such disturbance. This feature is particularly significant in the (realistic) situation in which the available observations are corrupted by noise.

In particular, we have compared our design methods with some recently published techniques that use game theory to select the filter parameters so as to achieve a prescribed attenuation of the effect of the noise on the signal generated by the filter. We have been able to show that the preliminary implementation of our reduction methods greatly simplify the design of the optimal filter and automatically guarantees the required observability properties needed for the design of the optimal filter in the case of infinite horizon.

The last decade witnessed an increasing interest in the study of the nonlinear equivalent of the so-called H_∞ suboptimal control problem of linear systems theory. A fundamental role in this analysis has been played by the notion of a dissipative system, introduced by J.C.Willems, which lends itself to a very expressive characterization of the so-called L_2 gain of a system (a concept which - for a nonlinear system - replaces the more popular notion of H_∞ norm of a transfer function), via an extension of the so-called "bounded real lemma" of linear system theory. An equally important role is played by the theory of differential games which, as illustrated in great detail by T.Basar and P.Bernhard in [6], shows that the problem of finding a feedback law which renders the L_2 gain of a linear system less than a prescribed number γ can be interpreted as a problem of achieving a nonpositive value function in a two-person, zero sum, linear differential game with quadratic cost.

Pioneering contributions towards the development of this theory (which, in the literature on nonlinear systems is more often referred to as theory of *disturbance attenuation* with internal stability) were those of J.A.Ball and J.W.Helton [3] and A.Van der Schaft [77]. In particular, [77] has shown that, in the case of *full information*, that is when the set of measured variables which are available for feedback includes the state of the controlled plant and the exogenous disturbance input, the solution of the problem can be determined from the solution of a Hamilton-Jacobi equation (or inequality, as in [78]), which is the nonlinear version of the Riccati equation considered in [68], [38], and [6] for the corresponding H_∞ suboptimal control problem of linear systems. Sufficient conditions for the solution of the problem of disturbance attenuation in the case of arbitrary *measurement feedback* were given in [44] and also in [2]. In particular, the paper [44] suggested the use of a "certainty equivalence" controller, whose adjustable parameters (namely, the "gain" of a "output injection" correction term) were to be calculated on the basis of the solution of another Hamilton-Jacobi inequality.

One alternative approach we are developing is based on a method for solving the model matching problem in H^∞ control that reduces the problem to a Nevanlinna-Pick interpolation problem with degree constraint, a problem for which we have developed a complete theory. As a special case, we obtain the sensitivity reduction problem. The main difference between our method and the existing H^∞ controller design methods is that we do not use the weighting functions to shape the frequency response of the sensitivity function. Instead, we will tune the spectral zeros of a positive real function related to the sensitivity function to obtain a desirable frequency response. If necessary, extra interpolation constraints can be introduced. A set of design rules are being developed.

We have previously shown that an arbitrary solution of the Nevanlinna-Pick interpolation problem with degree constraint can be determined by solving a convex optimization problem. Solving this optimization problem by Newton's method can lead to an ill-conditioned problem, especially when there are spectral zeros close to the unit circle. To overcome these problems a new approach based on a homotopy continuation method with predictor-corrector steps has been developed. This solver turns out to be quite efficient and numerically robust.

We have also developed a geometric approach to disturbance attenuation and fault detection. Thus far we have concentrated our research efforts on the problem of detecting unknown inputs (disturbances) in a nonlinear system. Using methods from differential geometry, we have been able to develop general conditions for the design of a filter capable of independently detecting the presence of unknown inputs. The results obtained in this way are far more advanced than those obtainable by means of the classical Beard-Jones detection filter for linear systems or, for that matter, any other published result about unknown-input detection for nonlinear systems. In particular our technique incorporates a procedure that minimizes the dimension of the filter needed to estimate the presence of such disturbance. This feature is particularly significant in the (realistic) situation in which the available observations are corrupted by noise.

In particular, we have compared our design methods with some recently published techniques that use game theory to select the filter parameters so as to achieve a prescribed attenuation of the effect of the noise on the signal generated by the filter. We have been able to show that the preliminary implementation of our reduction methods greatly simplify the design of the optimal filter and automatically guarantees the required observability properties needed for the design of the optimal filter in the case of infinite horizon.

Task 2.5: *The development of a feedback design methodology for achieving disturbance attenuation for systems in the* **critical** *case.*

The challenge to the design of control schemes which are robust against real parametric uncertainty is that, for many control systems involving real parametric uncertainty, the unobservability or nondetectabilty of the unknown parameters from the system error requires an alternative formulation of dynamic compensation schemes. Indeed, whenever the plant equilibrium is independent of the parameter and the output is function of just the system state, then the entire plant/exosystem state is not observable or detectable in the first approximation. However, in the case where the equilibrium varies with the parametric uncertainty the plant/exosystem state might actually be observable, even when the output function is a function of just the system state. This phenomenon actually occurs in some of the most generic (local codimension one) bifurcations, an observation which motivates the next task, largely solved in the appended thesis of James Ramsey.

Task 2.6: *Characterize those systems and outputs which undergo an observable or detectable codimension one bifurcation, and provide a systematic design methodology for constructing dynamic observers.*

The next explicit research task which we shall propose is to formalize our understanding of when such a construction is possible and, when it is possible, how one can design such a dynamical system.

Task 2.7: *Develop the underlying theory for the existence and the systematic design of asymptotic proxies for robust feedback control laws.*

There are also antecedents in various approaches to Kalman filtering for the asymptotic propagation of a functional, viz. the Kalman gain, rather than estimating the functional by estimating the entire state of a system and then applying the functional to the state estimate.More explicitly, it is well-known that the one-step predictor generated by the Kalman filter involves a gain which can be determined via a matrix Riccati equation. On the other hand, one could investigate whether it is possible to directly propagate the Kalman gain, rather than solving the matrix Riccati equation. While a closed system determining just the gain is unknown, it is possible to propagate an n-vector related to the gain, along with an n-dimensional co-state, in a closed system. In contrast to the typical state-costate representation of the matrix Riccati equation as a linear two-point boundary value problem, this dynamical system is a nonlinear initial-value problem which asymptotically computes the Kalman gain for positive real $v(z)$. Because this evolves in $2n$ variables rather than in $n(n+1)$variables, this dynamical system has been referred to as a fast-filtering algorithm. A phase-portrait for this nonlinear dynamical system has been developed in the single-input, single-output case and has led to not only a better understanding of its asymptotic properties but also to the unanticipated resolution of a long standing problem with applications in signal processing and speech synthesis. Indeed, in our recent paper [28] we presented a geometric duality between filtering and interpolation, tying together two fundamental classes of problems, rational covariance extension and the dynamics of fast filtering algorithms. Because of the unanticipated usefulness of this dynamical systems approach in a variety of problems involving systems and signals, we formulated the multivariable extension of this methodology as our next explicit research task.

Task 2.8: *Study the geometry of the fast filtering algorithm in the multivariable case.*

To this end, preliminary studies have been undertaken to investigate the use of the power method and the theory of Grassmanian manifolds to describe the geometry of the phase-portrait of the fast algorithms in the multivariable case. Our interest in these dynamical systems lies in their relationship to methods for asymptotically producing "proxies" for a state feedback law evaluated as an estimate of the current state. However, there are other applications of these ideas which have importance to current DOD missions, particularly those involving digital, wireless communications. The connection to these applications is the rational covariance extension problem, which was formulated by Kalman as an interpolation problem and which plays an important role in signal processing, spectral analysis, and speech processing.In our work we described a fundamental geometric duality between filtering and interpolation. This duality has several corollaries which provide solutions and insight into some very interesting and intensely researched problems. The solution of the rational covariance extension problem leads to the ability to design "notch" filters which match the "notches" in a segment of a digitized speech waveform, an ability which addresses a fundamental current limitation in speech synthesis. In order to extend this analysis to image processing, such as studying time series of images, one would need to obtain the corresponding multivariate results. This motivates the following research task.

Task 2.9: *Solve the rational covariance extension problem in the multivariate case.*

As a first step to solve the interpolation problem in the multivariate case, we have studied a duality between a problem of finding a solution to a system of a nonlinear equations $f(x) = y$ and a corresponding variational problem, under which the former problem is well-posed in the sense of Hadamard and the variational problem has an unique minimum which is an interior point, provided f is proper and the variational problem is known to have at most one minimizing point, as is the case for a convex problem. This theory is a finite-dimensional analogue of the Dirichlet Principle which links the solutions of certain nonlinear PDE's to solutions of certain variational problems in the Calculus of Variations.We have also made progress on Tasks 2.8 and 2.9 with our recent development of an abstract theory of analytic interpolation with complexity constraints in a Banach algebra setting.

2.b Feedback Design for Distributed Parameter Systems

Our main long term goal in this research area is concerned with the systematic development of design methodologies capable of shaping the response of systems whose dynamics is governed by nonlinear distributed parameter systems. For example, for such systems we are interested in developing the theory of output regulation, including tracking and disturbance rejection.

As in the nonlinear lumped case there are three ingredients to achieve output regulation:

1. the stability or stabilizability of the plant,

2. the existence of a steady-state response, and

3. the ability to design a feedback system to shape the steady-state response.

As part of this research effort we have made significant progress in this area. In order to describe our results we begin with the case of bounded actuators and sensors. In particular we present our first main result for the problem of output regulation using state feedback given in [16] (with the addition of a detectability condition on the class of systems the same results hold for the error feedback case). First we introduce some notation. Consider a plant described by an abstract distributed parameter control system in Hilbert space:

$$\dot{z}(t) = Az(t) + Bu(t) + d(t), \tag{1}$$

$$y(t) = Cz(t), \text{ (measured output)} \tag{2}$$

$$z(0) = z_0,$$

where $z \in Z$ is the state of the system, Z is a separable Hilbert space (state space), $u \in U$ is an input , $y \in Y$ is the measured output, U and Y are Hilbert control and output spaces, respectively. A is assumed to be the infinitesimal generator of a strongly continuous semigroup on a Hilbert space Z, $B \in \mathcal{L}(U, Z)$ and $C \in \mathcal{L}(Z, Y)$. Here we use the notation $\mathcal{L}(W_1, W_2)$ to denote the set of all bounded linear operators from a Hilbert space W_1 to a Hilbert space W_2. The term $d(t)$ represents a disturbance.

In addition we will assume that there exists a finite dimensional linear system, referred to as the exogenous system (or exosystem), that produces a reference output $y_r(t)$ and which is also used to model the disturbance $d(t)$:

$$\dot{w}(t) = Sw(t) \tag{3}$$

$$y_r(t) = Qw(t) \tag{4}$$

$$d(t) = Pw(t) \tag{5}$$

$$w(0) = w_0.$$

Here $S \in \mathcal{L}(W)$, W is a finite dimensional vector space, $Q \in \mathcal{L}(W, Y)$ and $P \in \mathcal{L}(W, Z)$.

We refer to the difference between the measured and reference outputs as the error

$$e(t) = y(t) - y_r(t) = Cz(t) - Qw(t). \tag{6}$$

Problem 2.1. ***State Feedback Regulator Problem:***
Find a feedback control law in the form

$$u(t) = Kz(t) + Lw(t)$$

such that $K \in \mathcal{L}(Z, U)$, $L \in \mathcal{L}(W, U)$ *and*

(1.a) *the system* $\dot{z}(t) = (A + BK)z(t)$ *is stable, i.e.* $(A + BK)$ *is the infinitesimal generator of an exponentially stable* C_0 *semigroup, and*

(1.b) *for the closed loop system*

$$\begin{aligned} \dot{z}(t) &= (A + BK)z(t) + (BL + P)w(t), \\ \dot{w}(t) &= Sw(t), \end{aligned} \tag{7}$$

the error

$$e(t) = Cz(t) - Qw(t) \longrightarrow 0 \quad \text{as } t \to \infty,$$

for any initial conditions $z_0 \in Z$ *in (??) and* $w_0 \in W$ *in (3).*

Problem 2.2. ***Error Feedback Regulator Problem:***
Find an error feedback controller of the form

$$\begin{aligned} \dot{X}(t) &= FX(t) + Ge(t), \\ u(t) &= HX(t) \end{aligned} \tag{8}$$

where $X(t) \in \mathfrak{X}$ *for* $t \geq 0$, $\mathfrak{X}$ *is a Hilbert space,* $G \in \mathcal{L}(Y, \mathfrak{X})$, $H \in \mathcal{L}(\mathfrak{X}, U)$ *and* F *is the infinitesimal generator of a* C_0 *semigroup on* $\mathfrak{X}$ *with the properties that*

(2.a) *the system*

$$\begin{aligned} \dot{z}(t) &= Az(t) + BHX(t), \\ \dot{X}(t) &= FX(t) + Ge(t) \end{aligned} \tag{9}$$

is exponentially stable when $w \equiv 0$, *i.e.* $\begin{bmatrix} A & BH \\ GC & F \end{bmatrix}$ *is the infinitesimal generator of an exponentially stable* C_0 *semigroup, and*

(2.b) *for the closed loop system*

$$\dot{z}(t) = Az(t) + BHX(t) + Pw(t), \qquad (10)$$
$$\dot{X}(t) = GCz(t) + FX(t) - GQw(t),$$
$$\dot{w}(t) = Sw(t)$$

the error

$$e(t) = Cz(t) - Qw(t) \longrightarrow 0 \quad as\ t \to \infty,$$

for any initial conditions $z_0 \in Z$ in (??), $X(0) \in \mathfrak{X}$ and and $w_0 \in W$ in (3).

As in [?], we impose the following standard assumptions.

Assumption 1. **H1** *The exosystem is neutrally stable, as in [?]. In the linear case, this is equivalent to the origin being Lyapunov stable forward and backward in time. This implies that $\sigma(S) \subset j\mathbb{R}$ (the imaginary axis) and S has no nontrivial Jordan blocks. Here and below we use the notation $\sigma(T)$ for the spectrum of an operator T. Also, by $\rho(T)$ we will denote the resolvent set of T.*

H2 *The pair (A, B) is exponentially stabilizable, i.e., there exists $K \in \mathcal{L}(Z, U)$ such that $A + BK$ is the infinitesimal generator of an exponentially stable C_0 semigroup.*

H3 *The pair*

$$\left(\begin{bmatrix} A & P \\ 0 & S \end{bmatrix}, \begin{bmatrix} C & -Q \end{bmatrix} \right)$$

is exponentially detectable, i.e., there exists $G \in \mathcal{L}(Y, Z \times W)$, with

$$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \ G_1 \in \mathcal{L}(Y, Z), \ G_2 \in \mathcal{L}(Y, W)$$

such that

$$\begin{bmatrix} A & P \\ 0 & S \end{bmatrix} - G \begin{bmatrix} C & -Q \end{bmatrix} \qquad (11)$$

is the infinitesimal generator of an exponentially stable C_0 semigroup.

The first main results from [16] characterizing the solvability of the regulator problems for linear distributed parameter systems are given in Theorem 1.

One of the main results of [16] given in Theorem 1 provide necessary and sufficient conditions for the solvability of the state feedback regulator problems.

Theorem 1. *Let* **H1** *and* **H2** *hold. The linear state feedback regulator problem is solvable if and only if there exist mappings* $\Pi \in \mathcal{L}(W,Z)$ *with* $Ran\ (\Pi) \subset \mathcal{D}(A)$ *and* $\Gamma \in \mathcal{L}(W,U)$ *satisfying the "regulator equations,"*

$$\Pi S = A\Pi + B\Gamma + P, \tag{12}$$

$$C\Pi = Q. \tag{13}$$

In this case a feedback law solving the state feedback regulator problem is given by

$$u(t) = Kz(t) + (\Gamma - K\Pi)w(t), \tag{14}$$

where K *is any stabilizing feedback for* (A,B).

As we have commented above, under an additional detectability assumption on the plants, solvability of the Error Feedback Regulator Problem, is also characterized in terms of solvability of the same regulator equations given in (7).

Theorem 2. *Let* **H1**, **H2** *and* **H3** *hold. The linear error feedback regulator problem is solvable if and only if there exist mappings* $\Pi \in \mathcal{L}(W,Z)$ *and* $\Gamma \in \mathcal{L}(W,U)$ *with* $Ran(\Pi) \subset \mathcal{D}(A)$, *such that*

$$\Pi S = A\Pi + B\Gamma + P \tag{15}$$

$$C\Pi = Q. \tag{16}$$

With this Π *and* Γ *a controller solving the error feedback regulator problem is given by*

$$\begin{aligned} \dot{X}(t) &= FX(t) + Ge(t), \\ u(t) &= HX(t). \end{aligned} \tag{17}$$

where $X \in \mathfrak{X} = Z \times W$,

$$G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}, \ H = \begin{bmatrix} K & (\Gamma - K\Pi) \end{bmatrix}, \tag{18}$$

$$F = \begin{bmatrix} (A + BK - G_1C) & (P + B(\Gamma - K\Pi) + G_1Q) \\ -G_2C & (S + G_2Q) \end{bmatrix}. \tag{19}$$

Here $K \in \mathcal{L}(Z,U)$ *is an exponentially stabilizing feedback for the pair* (A,B) *and* $\begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$ *is an exponentially stabilizing output injection (such* K *and* G *exist by* **H2** *and* **H3**).

Our first specific research task was concerned with extending the above results to the case of systems for which the input and outputs are given in terms of unbounded operators on the Hilbert state space. These controller designs would take advantage of truly distributed parameter effects which have no lumped counterpart, and therefore could not be designed on the basis of lumped approximations to the distributed parameter models.

Task 3.1: *Extend the theory of output regulation to include unbounded inputs and outputs.*

There are numerous technical obstacles that had to be overcome en route to carrying out Task 3.1. For example, for unbounded B, even if A generates an analytic semigroup it may happen that $(A+B)$ is not a generator. Further, for unbounded B and C (and even possibly K) expressions such as CB or BKC may make no sense. On the other hand there is considerable interest in the case of unbounded inputs and outputs that arise, for example, in the study of boundary control systems governed by partial differential equations. Typical applications include actuators and sensors supported at isolated points or on lower dimensional hypersurfaces in, or on the boundary of, a spatial domain.

After a considerable effort our work focused on extending our geometric approach to the class of regular linear systems. A system is called regular provided the system is *Well Posed* and satisfies the *Regularity Condition.* For complete details of what these concepts involve we refer the reader to [84]-[83] and suffice to present a short overview. We consider regular linear systems

$$\begin{aligned} \dot{z} &= Az + Bu \\ y &= C_\Lambda z + Du \end{aligned}, \tag{20}$$

where C_Λ is the Λ-extension of the observation operator C (see [85])) defined for $z \in \mathcal{D}(C_\Lambda)$ by

$$C_\Lambda z \equiv \lim_{\lambda \to +\infty} C\lambda(\lambda I - A)^{-1} z \ \text{ exists.}$$

1. *Well Posedness*: A system (20) is well posed provided that B and C are *Admissible*, and there exists a *Transfer Function* $G(s) = C_\Lambda(sI - A)^{-1}B$ for some (hence, for every) $s \in \rho(A)$ (this means that

$$(sI - A)^{-1}BU \subset \mathcal{D}(C_\Lambda).$$

2. *Regularity*: A well posed system is called regular provided there exists a feed-through term $D \in \mathcal{L}(U, V)$, such that

$$\lim_{\substack{s \to +\infty \\ s \in \mathbb{R}}} G(s)\varphi = D\varphi, \ \ \forall \ \ \varphi \in U$$

While we will not go into detail concerning the questions of admissibility, etc., we still need to introduce certain terminologies. Let us define the space

$$Z_1 = \mathcal{D}(A) \subset Z \text{ with } \|z\|_1 = \|(\beta I - A)z\| \ , \ \beta \in \rho(A),$$

and the space

$$Z_{-1} \text{ the completion of } Z \text{ with respect to } \|z\|_{-1} = \|(\beta I - A)^{-1}z\|.$$

Then there are the dense embeddings

$$Z_1 \hookrightarrow Z \hookrightarrow Z_{-1}.$$

Assumption 2. *1. We assume that $B \in \mathcal{L}(U, Z_{-1})$, $C \in \mathcal{L}(Z_1, Y)$ are admissible*

2. $G(s) = C_\Lambda(sI - A)^{-1}B$ exists for $s \in \rho(A)$.

3. (A, B) stabilizable: There exists $K \in \mathcal{L}(Z_1, U)$ so that $(A + K_\Lambda B)$ is a stable generator.

Within this setting we have extended the solvability results for both the state and error feedback regulator problems and, for example, after several important adjustments due to unbounded B and C, we have obtained the following analog of Theorem 1.

Problem 2.3. ***State Feedback Regulator Problem for Regular Systems:***
Find a feedback control law in the form

$$u(t) = K_\Lambda z(t) + Lw(t)$$

such that $K \in \mathcal{L}(Z, U)$, $L \in \mathcal{L}(W, U)$ and

(1.a) *the system $\dot{z}(t) = (A + BK_\Lambda)z(t)$ is stable, i.e. $(A + BK_\Lambda)$ is the infinitesimal generator of an exponentially stable C_0 semigroup, and*

(1.b) *for the closed loop system*

$$\begin{aligned} \dot{z}(t) &= (A + BK_\Lambda)z(t) + (BL + P)w(t), \\ \dot{w}(t) &= Sw(t), \end{aligned} \tag{21}$$

the error

$$e(t) = C_\Lambda z(t) - Qw(t) \in L^2_\alpha(0, \infty)$$

where for some $\alpha < 0$

$$L^2_\alpha(0, \infty) = \left\{ \phi \;\middle|\; \int_0^\infty |\phi(t)|^2 \, e^{-\alpha t} \, dt < \infty \right\}.$$

We give the following result from [13].

Theorem 3. *Under the above assumptions, the state feedback regulator problem is solvable if and only if there exist mappings* $\Pi \in \mathcal{L}(W, \underset{\sim}{Z} \subset Z)$ *and* $\Gamma \in \mathcal{L}(W, U)$ *satisfying the "Regulator Equations"*

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + P \\ C_\Lambda \Pi - Q &= 0 \end{aligned} .$$

Here the space $\underset{\sim}{Z}$ *is given by*

$$\underset{\sim}{Z} = \mathcal{D}(A) + (\lambda I - A)^{-1} PW + (\lambda I - A)^{-1} BU, \quad \textit{for} \quad \lambda \in \rho(A).$$

If Π *and* Γ *satisfy the regulator equations then a feedback law solving the problem of output regulation is given by* $u = K_\Lambda z + (\Gamma - K_L \Pi) w$.

Our next research task was concerned with the design of compensators which solve the error feedback problem.

Task 3.2: *For the error feedback regulator problem a considerable effort will be devoted to the problem of designing infinite and finite dimensional compensators for a wide class of systems and to developing a useful theory of finite dimensional approximation of these observers. This will be applied in the case of bounded and unbounded inputs and outputs*

A system (1), (36) (or (20)) is said to satisfy the *Spectrum Decomposition Assumption at* δ if the spectrum of A (denoted $\sigma(A)$) decomposes into two parts

$$\sigma_+(A) = \{\lambda \in \sigma(A) \, : \, \mathrm{Re}(\lambda) \geq \delta\}, \quad \sigma_-(A) = \{\lambda \in \sigma(A) \, : \, \mathrm{Re}(\lambda) < delta\}$$

in such a way that $\sigma_+(A)$ consists of a finite number of eigenvalues of finite multiplicity which can be enclosed in the interior a simple closed rectifiable curve and with $\sigma_-(A)$ contained in the exterior.

An operator A that generates a C_0 semigroup is said to satisfy the *Spectrum Determined Growth Condition* provided the growth bound of the semigroup is equal to the upper bound of the real part of the spectrum of A. Operators that satisfy this condition include (a) bounded operators A,; (b) operators A for which the semigroup is differentiable in the strong operator topology; (c) operators A for which the semigroup is compact for some fixed value of time; (d) if A is a Reisz spectral operator. Fortunately this included most examples of interest in applications to partial differential equations. On the other hand this condition is not preserved under very simple perturbations (see [87]).

For plants whose dynamics are governed by operators A satisfying the *Spectrum Decomposition Assumption* for $\delta < 0$ and the *Spectrum Determined Growth Condition*, we have been able to show that our results lead to finite dimensional feedback laws that solve the state and error feedback regulator problems. These controller designs would take advantage of truly distributed parameter effects which have no lumped counterpart, and therefore could not be designed on the basis of lumped approximations to the distributed parameter models.

The fact that the solvability of the regulator problem is related to the system zeros is well known for finite dimensional systems. In [46] the solvability of the regulator problems for nonlinear finite dimensional systems was interpreted as a property of the zero dynamics of the composite system formed from the plant and the exogenous system. Namely, under certain assumptions it can be shown that the regulator problem is solvable if and only if the zero dynamics of the composite system can locally be decomposed into diffeomorphic copies of the exosystem and the plant's zero dynamics. We will develop analogous results for linear infinite dimensional systems with bounded control and observation operators. For this reason, it will be important to introduce the concept of the zero dynamics of a distributed parameter system and to relate it to the system transmission zeros.

Task 3.3: *The development of a complete understanding of the relationship between zero dynamics and transmission zeros for a large class of linear distributed parameter systems, including systems with unbounded control and sensing.*

We have made some progress in this direction in [16], [11]. For simplicity we consider the special case in which the finite dimensional Hilbert input space U and output space Y satisfy

$$\dim(U) = \dim(Y) = m \ .$$

We first recall that for SISO systems transmission zeroes are defined as the zeroes of the transfer function. In the MIMO case the transfer function is an $m \times m$ matrix given by

$$G(s) = C(sI - A)^{-1}B. \tag{22}$$

We shall assume $\det G(s) \not\equiv 0$. In this case, we make the following definition.

Definition 1. *$s_0 \in \mathbb{C}$ is a transmission zero of the system (1) if* $\det G(s_0) = 0$.

It is also useful to introduce the concept of an invariant zero.

Definition 2. *$s_0 \in \mathbb{C}$ is an invariant zero of the system (1) if the system*

$$\begin{bmatrix} (A - s_0 I) & B \\ C & 0 \end{bmatrix} \begin{bmatrix} z_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{23}$$

has a solution

$$\begin{bmatrix} z_0 \\ u_0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

In the SISO case it is straightforward to show that for $s_0 \in \rho_\infty(A)$ (the connected component of the resolvent set of A containing a right half plane, see [37], page 70), the concepts of transmission zeros and invariant zeros coincide (see, e.g., [89]). We include a short proof of this result in the MIMO case for completeness.

Lemma 1. *If $s_0 \in \rho(A)$, then s_0 is a transmission zero if and only if it is an invariant zero.*

Using a similar argument, or appealing directly to the lemma above, one can show the following.

Lemma 2. *If $s_0 \in \rho(A)$ and $s_0 \in \rho(A + BK)$ then s_0 is a transmission zero of G if and only if it is a transmission zero of G_K where $G_K(s) = C(sI - A_K)^{-1}B$.*

In the classical automatic control of lumped SISO systems, it is well-known that the regulator problem is solvable provided no eigenvalue of S is a transmission zero, i.e., $\lambda \in \sigma(S)$ implies $G(\lambda) \neq 0$. For distributed parameter systems, it is not immediate that G would be defined at λ. Since we assume that A is the infinitesimal generator of a C_0 semigroup and B and C are bounded, it is known that the transfer function G exists and is defined on $\rho_\infty(A)$, where $\rho_\infty(A)$ is the connected component of the resolvent which contains infinity and intersects the positive real axis, cf. [37]. Even for a fixed system, we are of course interested in solving output regulation problems for a variety of exosystems, so that we should regard λ as being an arbitrary point in the closed right-half plane. This observation is the basis for our first nonresonance result.

Remark 1. *We note that it follows immediately from Theorems 1 and 2 that under the hypotheses* **H1, H2, H3** *the state feedback regulator problem is solvable if and only if the error feedback regulator problem is solvable. Thus in providing necessary and sufficient conditions for solvability of these problems we need not distinguish between the two cases. For this reason, from now on we will only refer to the state feedback case.*

Task 3.4: *Develop a complete understanding of the role of zero dynamics in solvability of the regulator equations for distributed parameter systems. We plan to carry out this research task by addressing the problem from the point of view of geometry and invariance as well as from the more functional theoretic viewpoint of invariant and transmission zeros. The latter will require a further treatment of transfer functions for distributed parameter systems.*

Theorem 4. *For the system (1) with exosystem (1)-(3) satisfying hypotheses* **H1** *and* **H2** *and the assumption that $\sigma(S) \subset \rho_\infty(A)$, the regulator equations (12) and (13) are solvable, and the output regulation via state-feedback is achievable, provided no eigenvalue of S is a transmission zero, i.e., $\lambda \in \sigma(S)$ implies $\det G(\lambda) \neq 0$.*

Corollary 1. *Under the same hypotheses as the theorem, the regulator equations (12) and (13) are solvable, and the output regulation via state-feedback is achievable, for every choice of P and Q if, and only if, $\det G(j\alpha_i) \neq 0$ for $i = 1, \cdots, k$.*

We next relax the condition relating the spectrum of the exogenous system with the component of the resolvent which contains infinity.

Corollary 2. *Assume that A satisfies the spectrum decomposition assumption with respect to the closed right half plane and that the system (1) with exosystem (1)-(3) satisfies hypotheses* **H1** *and* **H2** *of the basic Assumption 1. The regulator equations (15) and (16) are then solvable, and the output regulation via state-feedback is achievable, for every choice of P and Q if and only if no eigenvalue of S is a transmission zero, i.e., $\lambda \in \sigma(S)$ implies* $\det G(\lambda) = C(\lambda I - A)^{-1}B \neq 0$.

Our final nonresonance result will remove the condition $\sigma(S) \subset \rho(A)$. This begins with the observation that one can also express the basic nonresonance condition in terms of a generalized Hautus test involving invariant zeros.

Corollary 3. *For the system (1) with exosystem (1)-(3) satisfying hypotheses* **H1** *and* **H2**, *the regulator equations (15) and (16) are solvable for every choice of P and Q if and only if no eigenvalue of S is an invariant zero; i.e., if and only if*

$$\ker \begin{bmatrix} (A - \lambda I) & B \\ C & 0 \end{bmatrix} = \{0\}, \ \textit{for all} \ \ \lambda \in \sigma(S) \ .$$

The results presented so far give necessary and sufficient conditions for solvability of the regulator equations for every choice of P and Q. The analysis for a particular choice of P and Q is of course more difficult. We have obtained such an analysis for the SISO case. In this case we need to formulate an additional resonance condition for the plant and exosystem, which is a consequence of hypothesis **H3**. In order to draw attention to the fact that we are in the SISO case we now denote the transfer functions using lowercase letters.

Corollary 4. *Suppose the plant and exosystem satisfies hypotheses* **H1**-**H3**. *The regulator equations are solvable, and output regulation by error feedback can be achieved, if and only if no natural frequency of the exosystem is a transmission zero of the plant, i.e., $g(\lambda) \neq 0$ for all $\lambda \in \sigma(S)$.*

Remark 2. *This corollary imposes additional restrictions on P and Q, viz., hypothesis* **H3** *in order to obtain necessity of the resonance condition and to be able to design error feedback control schemes.*

Certainly one of most important long range research objectives in our research in distributed parameter systems is to study output regulation for nonlinear distributed parameter systems. We expect that through a considerable research effort, we will be able to design feedback laws capable of shaping the response of nonlinear distributed parameter systems. This research will include an effort to extend the geometric theory developed in [46] to provide necessary and sufficient conditions for solvability of both the state and error feedback problems. A key point in our extension of the regulator theory to nonlinear distributed parameter systems is the requirement that the closed loop composite systems governed by nonlinear evolution equations in a Hilbert (or, more generally a Banach) space has a local center manifold.

In this direction we have made some progress in our preliminary study of the following three tasks.

Task 3.5: *For special classes of nonlinear distributed parameter systems, develop a theory of output regulation based on center manifold theory analogous to our development of linear distributed parameter systems. For these classes of systems we will first consider the case of bounded input and output operators as discussed above for the output regulator problem for linear distributed parameter systems.*

Task 3.6: *One of our long range goals will be to develop a theory of output regulation for nonlinear distributed parameter systems with bounded inputs and outputs and for systems containing convective nonlinearities based on center manifold theory as described above in Task 3.5. Part of this work will include the development of an appropriate center manifold theory applicable for these problems.*

Task 3.7: *Extend the theory of output regulation to general convective reaction diffusion equations with unbounded inputs and outputs.*

In general terms, we consider a plant, exosystem and feedback law:

$$\begin{aligned} \dot{z} &= \Phi(z, u, w) \\ \dot{w} &= s(w) \\ u &= \alpha(z, w) \end{aligned} \tag{24}$$

where $z \in Z$, $u \in U$, Z and U are Hilbert spaces, and $w \in W$.

Since we are first interested in the local theory, we assume without loss of generality that the point $(0,0,0)$ is an equilibrium and that Φ, s, α are such that

$$\Phi(0,0,0) = 0, \quad s(0) = 0, \quad \alpha(0,0) = 0.$$

And, moreover, assume that they can be represented in a neighborhood of $(0,0,0,) \in Z \times U \times \mathbb{R}^k$ via first order approximations so that the system (24) can be written as

$$\begin{aligned} \dot{z} &= Az + Bu + Pw + \chi(z,u,w) \\ \dot{w} &= Sw + \psi(w) \\ u &= Kz + Lw + \beta(z,w). \end{aligned} \tag{25}$$

We also impose the following assumptions

1. A is assumed to be the infinitesimal generator of a strongly continuous semigroup on a Hilbert space Z, $B \in \mathcal{L}(U,Z)$ and $C \in \mathcal{L}(Z,Y)$.

2. The nonlinear terms χ, ψ, β satisfy:

   (a) χ, ψ, β are of class C^2 in the sense of Frechet.

   (b) The following conditions hold

$$\begin{aligned} &\text{i. } \chi(0,0,0) = 0 \ , \ \frac{\partial}{\partial w}\chi(0,0,0) = 0, \ \frac{\partial}{\partial z}\chi(0,0,0) = 0, \\ &\quad \frac{\partial}{\partial u}\chi(0,0,0) = 0, \\ &\text{ii. } \psi(0) = 0, \ \ \psi'(0) = 0, \\ &\text{iii. } \beta(0,0) = 0, \ \frac{\partial}{\partial z}\beta(0,0) = 0, \ \frac{\partial}{\partial w}\beta(0,0) = 0. \end{aligned} \tag{26}$$

   (c) The nonlinear terms are good enough so that the initial value problem for the uncontrolled and closed loop systems have unique global in time strong solutions in the appropriate Hilbert spaces. Namely, this should be true for the systems

$$
\begin{aligned}
&\text{i.} \quad \dot{z} = Az + \chi(z,0,0), \\
&\text{ii.} \quad \begin{aligned} \dot{z} &= Az + Pw + \chi(z,0,w), \\ \dot{w} &= s(w) \end{aligned} \\
&\text{iii.} \quad \begin{aligned} &\dot{z} = (A+BK)z + (P+BL)w + \phi(z,w), \\ &\dot{w} = Sw + \psi(w), \\ &\text{where} \ \ \phi(z,w) = \chi(z,\alpha(z,w),w) + B\beta(z,w). \end{aligned}
\end{aligned}
\tag{27}
$$

Our preliminary calculations suggest that under these general assumptions, we expect that the development of a theory of output regulation for nonlinear distributed parameter systems can be based on center manifold theory analogous to our development of linear distributed parameter systems. Namely, for examples governed by partial differential equations under which the above conditions hold we have developed some analytical and numerical results that demonstrate the feasibility of obtaining a theory of output regulation for systems governed by nonlinear partial differential equations. For example our preliminary calculations include examples from the following class of systems.

Consider systems with state space $Z = L^2(\Omega, \mathbb{R}^N)$ where $\Omega \subset \mathbb{R}^n$ is a bounded domain. The dynamics of the controlled composite system are governed by

$$
\begin{cases}
z_t &= \mathcal{F}[z] + b[z]u + p[z]w, \\
w_t &= s(w), \\
e(t) &= h[z(t)] - q(w(t))
\end{cases}
\tag{28}
$$

where

$$
\mathcal{F}[z] = Az + \sum_{i=1}^{n} f_i(z) z_{x_i} + g(z) \tag{29}
$$

with boundary conditions on the boundary $\partial\Omega$ of Ω of the form

$$
\mathcal{B}(z) \equiv \left[\sum_{i,j=1}^{n} a_{ij}(x) z_{x_i}(x,t) \eta_j(x) - k(x) z(x,t) \right] \Bigg|_{x \in \partial\Omega} = 0. \tag{30}
$$

The function $k(x) > 0$ defined on $\partial\Omega$ plays the role of the system gain.

The operator A is assumed to be a formally self-adjoint uniformly elliptic differential operator:

$$Az = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}(x)\frac{\partial z}{\partial x_i}\right). \tag{31}$$

The rest of the terms in $\mathcal{F}$ are assumed to satisfy

1. (Reaction Term)
   (a) $g : \mathbb{R}^N \to \mathbb{R}^N$ is a nonlinear function,
   (b) $g \in C^2, \quad g(0) = 0, \ g'(0) = 0$
   (c) $g(\xi) \cdot \xi \geq -C(|\xi|^2 + |\xi|^{r+1})$,
   (d) $|g(\xi)| \leq C(1 + |\xi|^r)$ with $r < 2 + 4/n$.
2. (Convective Term)
   (a) $\displaystyle\sum_{i=1}^{n} f_i(z)\frac{\partial z}{\partial x_i}$ is a convective term.
   (b) $f_i(z) \in C^2(\mathbb{R}^N, \mathbb{R})$ are nonlinear functions satisfying $f_i(0) = 0$,
   (c) $|f_i(\xi)| \leq C(|\xi|^2 + |\xi|)$.
3. (Input) $u(t) \in U$ is the control input $b : Z \to \mathcal{L}(U, Z) : z \mapsto b[z]$
4. (Output)
   (a) (Measured Output) $h : z(\cdot, t) \mapsto h[z(\cdot, t)] \in \mathbb{R}^p$ with $h(0) = 0$.
   (b) (Reference Output) $q : w(t) \in W \mapsto q(w(t)) \in \mathbb{R}^p$ with $q(0) = 0$.
5. (Exosystem)
   (a) $s(w)$ is a smooth vector field on W with $s(0) = 0$.
   (b) More specifically, we assume exosystem is Poisson stable. Namely, we assume that
   $$s(w) = Sw + s_1(w)$$
   is neutrally stable.

6. (Disturbance) p is a disturbance term $p : Z \to \mathcal{L}(W, Z) : z \mapsto p[z]$.
7. (Error) The error is given by $e(t) = h[z(t)] - q(w(t))$ where we assume that $h : Z \to \mathbb{R}^p$ and $q : W \to \mathbb{R}^p$ are continuous.

For the system described above the following hold:

1. For every initial condition z_0, there is a unique classical solution defined for all $t > 0$.
2. There are the estimates $\begin{cases} \|z(t)\| \leq M_0(\rho), \ t \in [0, \infty) \\ \|z(t)\|_{H^1(\Omega,\mathbb{R}^N)} \leq M_1(t_0, \rho), \ \ t \in [t_0, \infty) \end{cases}$

   where M_0 and M_1 are continuous monotone increasing functions.
3. Moreover, for any $R > 0$, there exists $K(R) > 0$, $\alpha(R) > 0$ and positive continuous functions $C_j(R) > 0$ such that, if
$$\|\varphi\| \leq R, \text{ and } \min_{x\in\partial\Omega} k(x) \geq K(R)$$

   Then $\begin{cases} \|z(t)\| \leq C_0(R)e^{-\alpha t}, \ \ R < C_0(R) < 2R, \ \ t \geq 0, \\ \|z(t)\|_{H^\ell(\Omega)} \leq C_1(R, t_0)e^{-\alpha t}, \ \ t \geq t_0 > 0, \ \ \ell = [n/2] + 1 \end{cases}$

Consider the
PROBLEM: Find a feedback law $u = \alpha[w] = Lw + \beta[w] : W \to U$
with $L \in \mathcal{L}(W, U)$, $\beta(0) = \beta_w(0) = 0$ so that:
For some neighborhood of $(0, 0)$ in $Z \times W$ there exists a global in time solution of the closed loop system
$$\begin{cases} z_t &= \mathcal{F}[z] + b[z]\alpha[w] + p[z]w, \\ w_t &= s(w) \end{cases}$$
and this solution satisfies $e(t) = h[z(t)] - q(w(t)) \xrightarrow{t\to\infty} 0$.

We have shown that
THEOREM 2: For every $\alpha[w]$ (as described above) the system
$$\begin{cases} z_t = \mathcal{F}[z] + b[z]\alpha[w] + p[z]w, \\ w_t = s(w) \end{cases} \tag{*}$$
plus boundary conditions
has a Local Center Manifold $M \subset Z \times W$, with $(0, 0) \in M$ and M can be expressed as the graph of a mapping $\Pi : W^0 \subset W \to Z$ (for some neighborhood W^0 of 0 in W).

The mapping Π has the following properties:

1. $\Pi \in C^2$, with $\Pi(0) = 0$ and $\Pi'(0) = 0$.

2. $M = \text{Graph}(\Pi) = \{(\Pi(w), w) \in Z \times W^0\}$ is invariant for $(*)$.

3. Invariance of M can be expressed through the mapping Π:
$$\frac{\partial \Pi}{\partial w} s(w) = \mathcal{F}[\Pi(w)] + b[\Pi(w)]\alpha[w] + p[\Pi(w)]w$$

THEOREM 3: Under all the above assumptions, a feedback law $\alpha[w]$ solves the State Feedback Regulator Problem (for our class of systems) if and only if the Center Manifold mapping Π (whose existence is guaranteed by Theorem 2) also satisfies the "error zeroing" condition
$$h[\Pi(w)] - q(w) = 0, \quad \forall \;\; w \in W^0.$$

THEOREM 4: Under our assumptions. The state feedback regulator problem is solvable if and only if there exists two mappings
$$\Pi : W^0 \subset W \to Z, \qquad \alpha : W^0 \subset W \to U$$
satisfying the "Regulator Equations":
$$\frac{\partial \Pi}{\partial w} s(w) = \mathcal{F}[\Pi(w)] + b[\Pi(w)]\alpha(w) + p[\Pi(w)]w$$
$$h[\Pi(w)] - q(w) = 0, \quad \forall \;\; w \in W^0.$$

If the regulator equations are satisfied a feedback control law achieving regulation is given by $u = \alpha(w)$.

As it can be seen by examples it is sometimes possible to obtain very simple and physically motivated boundary feedback stabilizing control laws for nonlinear systems. Our next research task concerns the problem of stabilization of distributed parameter systems and the investigation of the extent to which simple design methodologies can be used to stabilize boundary control systems.

Task 3.8 *We will investigate the design of boundary feedback stabilization schemes for linear and nonlinear distributed parameter systems.*

Task 3.9: *Further develop the notion of zero pole dynamics, relating high gain limits of trajectories and attractors to the concept of zero dynamics for nonlinear distributed parameter systems. Using this analysis we also propose the design of boundary feedback stabilization schemes based on the concepts of zero pole dynamics for minimum phase nonlinear distributed parameter systems.*

Task 3.10: *Investigate similar stability results for related feedback schemes and different geometric configurations of the spatial domains.*

As an important first step in the analysis of Tasks 8 – 10 we have made some interesting discoveries, reported in [18], concerning zero pole dynamics for a controlled viscous Burgers' equation and we have applied these results to prove both semiglobal and practical stabilization for our boundary feedback laws.

We now show that, with $u = 0$, this boundary controlled system is locally exponentially stable. In fact, we can be show the system is semiglobally stabilizable when the gains are tunable. To see this, consider the related Burgers' equation given by given by

$$w_t - \epsilon w_{xx} + w w_x = f(x,t), \tag{32}$$

$$w(x,0) = \varphi(x), \quad \varphi \in L^2(0,1), \tag{33}$$

where $x \in [0,1]$ and $t \in [0,\infty)$.

For this equation, we introduce controls through the "flux" at the endpoints of the interval [0,1]:

$$u(t) = \begin{pmatrix} -w_x(0,t) \\ w_x(1,t) \end{pmatrix} \in \mathbb{R}^2 \text{ for all } t \in [0,\infty). \tag{34}$$

We then take as output (observation) the difference between the "temperature" (or the "fluid velocity") at the same endpoints.

$$y(t) = \begin{pmatrix} w(0,t) - v_0(t) \\ w(1,t) - v_1(t) \end{pmatrix} \in \mathbb{R}^2 \text{ for all } t \in [0,\infty). \tag{35}$$

We comment that, under our assumptions, the uncontrolled $(u = 0)$ and undisturbed $(f = 0)$ open loop system (66), (67), (68) is not asymptotically stable since $w(x,t)$ =constant is a solution for initial data $\varphi(x)$ =constant. On the other hand, if we introduce a feedback mechanism by the law:

$$u(t) = -\begin{pmatrix} k_0 & 0 \\ 0 & k_1 \end{pmatrix} y(t), \;\; t \in [0,\infty), \tag{36}$$

where $k_0 > 0, k_1 > 0$ are the gain parameters. This feedback law (36) can be rewritten in the form of boundary conditions

$$w_x(0,t) - k_0\,(w(0,t) - v_0(t)) = 0 \tag{37}$$

$$w_x(1,t) + k_1\,(w(1,t) - v_1(t)) = 0.$$

In this formal sense, we can interpret (75), (33), (37) as the closed loop boundary control system for the open loop system (75), (33), with controls given by (34) and outputs given by (35). We will denote the solution of the closed loop system by $w^K(x,t)$.

The zero dynamics is defined by the condition that the output be constrained to be zero:

$$y(t) = 0 \ , \ \ t \in [0, \infty). \tag{38}$$

The condition (38) formally gives the nonhomogeneous Dirichlet boundary conditions

$$w(0,t) = v_0(t), \ \ w(1,t) = v_1(t) \ , \ \ t \in [0, \infty). \tag{39}$$

Thus, the zero dynamics consists of the initial boundary value problem (75), (33), (39) in the state space $L^2(0,1)$. The solution of this problem will be denoted by $w(x,t)$.

We are particularly interested in the more common situation for the zero dynamics in which $v_0 \equiv 0$, $v_1 \equiv 0$ corresponding to Burgers' equation with homogeneous Dirichlet boundary conditions

$$w(0,t) = 0, \ \ w(1,t) = 0 \ , \ \ t \in [0, \infty). \tag{40}$$

Intuitively, Dirichlet boundary conditions also correspond formally to letting k_0 and k_1 tend to ∞ in the closed loop boundary conditions (37). Thus, as in classical automatic control, we should obtain the zeros, or more properly the zero dynamics, as a high gain limit of the closed loop poles, or more properly the closed loop dynamics. More precisely, in [19, 18] it is shown that the solution $w^K(x,t)$ of the problem (75), (33), (37) converges in some sense to the solution $w(x,t)$ of the problem (75), (33), (40) as $k_0, k_1 \to \infty$, just as the closed loop trajectories converge locally to the trajectories of the zero dynamics in the lumped case [7]. In particular, it is shown in [18] that:

a) The feedback law (36) semiglobally exponentially stabilizes the open loop system (75), (40) with zero forcing term $f = 0$. This result is valid in the $L^2(0,1)$, $H^1(0,1)$ and $L^\infty(0,1)$-norms.

b) This result is robust in the sense of practical stability: if the disturbance $f \neq 0$ but is "sufficiently small" then our feedback law still provides a practical semiglobal stabilization of the open loop system dynamics. Moreover, this result remains valid if the boundary disturbances are also nonzero but are "small enough."

More specifically, let us introduce certain notation from [18]. By $\|\cdot\|$ we will denote the norm in the space $L^2(0,1)$. The principal linear part of Equation (75) is the linear operator $A_K = -\dfrac{d^2}{dx^2}$ with dense domain in $L^2(0,1)$ given by

$$D(A_K) = \{\psi \in H^2(0,1) : \psi'(0) - k_0\psi(0) = 0, \psi'(1) + k_1\psi'(1) = 0\}.$$

A_K is a positive self-adjoint operator (recall that $k_0, k_1 > 0$) whose inverse A_K^{-1} is compact. The domain $D(A_K^{1/2})$ of the operator $A_K^{1/2}$ coincides with the Sobolev space $H^1(0,1)$. In this space one can introduce a new norm

$$\|\psi\|_1^2 = \|A_K^{1/2}\psi\|^2 = \|\psi_x\|^2 + k_0|\psi(0)|^2 + k_1|\psi(1)|^2. \tag{41}$$

The norm (41) is equivalent to the usual H^1-norm $\|\psi\|^2_{H^1(0,1)} = \|\psi_x\|^2 + \|\psi\|^2$. However, it is important to keep in mind that, in contrast with the H^1-norm, the norm $\|\cdot\|_1$ depends on the gain parameters and $\|\psi\|_1 \to \infty$ as $k_0, k_1 \to \infty$ for a given $\psi \in H^1(0,1)$.

We use the following notations: $Q_T = [0,1] \times [0,T]$, where $T > 0$; $Q_{t_0,\,t_1} = [0,1] \times [t_0,\, t_1]$ for any $t_1 > t_0 > 0$ so that $Q_{0,T} \equiv Q_T$.

We introduce several function spaces which were used in [19]. The weak solution of our problem is an element of the Banach space

$$\mathcal{M}_T = C\left([0,T], L^2(0,1)\right) \cap L^2\left([0,T], H^1(0,1)\right)$$

The norm in this space is

$$|w|_{\mathcal{M}_T} = \max_{t\in[0,T]} \|w(t)\| + \left(\int_0^T \|w(t)\|^2_{H^1(0,1)}\, dt\right)^{1/2}. \tag{42}$$

(Here and below we denote by $w(t)$ the function $w(\cdot,t)$ considered as a function of x with t fixed.) Elements of $\mathcal{M}_T$ are continuous functions of $t \in [0,T]$ with respect to the norm of $L^2(0,1)$, which means that

$$\|w(t+\Delta t) - w(t)\| \to 0 \text{ as } \Delta t \downarrow 0 \text{ for any } t \in [0,T],\ \ w \in \mathcal{M}_T. \tag{43}$$

Denote by $H^{4,2}(Q_{t_0,T})$ the Hilbert space with the norm (see, e.g., [61]):

$$\begin{aligned}\|u\|^2_{H^{4,2}(Q_{t_0,T})} &= \int_{t_0}^T \left(\|u\|^2 + \|u_x\|^2 + \|u_{xx}\|^2 + \|u_t\|^2 + \right.\\ &\quad + \|u_{xxx}\|^2 + \|u_{xt}\|^2 + \|u_{xxxx}\|^2 + \\ &\quad \left. + \|u_{xxt}\|^2 + \|u_{tt}\|^2\right)\, dt.\end{aligned} \tag{44}$$

By $C^{2,1}(Q_{t_0,T})$ we denote the Banach space of functions $u(x,t)$ such that $u, u_x, u_{xx}, u_t \in C(Q_{t_0,T})$. The norm in this space is

$$\|u\|_{C^{2,1}(Q_{t_0,T})} = \max_{(x,t)\in Q_{t_0,T}} (|u| + |u_x| + |u_{xx}| + |u_t|). \tag{45}$$

We have the following embedding

$$H^{4,2}(Q_{t_0,T}) \subset C^{2,1}(Q_{t_0,T}). \tag{46}$$

The definition of a weak solution of the problem (75), (33), (37) is standard (see, e.g., [61], [62], [63]) and is given in equation (2.12) in [19].

Now we are in a position to formulate Theorem 5.

Theorem 5. *a) Let $T > 0$ be arbitrary and consider the problem (75), (33), (37) on the time interval $[0,T]$. Let $f \in L^2([0,T], L^2(0,1)) \equiv L^2(Q_T)$ and $v_0, v_1 \in H^1(0,T) \subset C[0,T]$. Then the problem (33), (33), (37) with arbitrary initial function $\varphi \in L^2(0,1)$ has a unique weak solution $w^K(x,t)$ such that $w^K \in \mathcal{M}_T$.*

b) Assume that, in addition to the assumptions in item a), $f \in H^{4,2}(Q_{t_0,T})$ for any $t_0 \in (0,T)$ and $v_0, v_1 \in H^2(0,T) \subset C^1[0,T]$. Then for $t > 0$ the weak solution is $C^{2,1}$-smooth and, therefore, satisfies Eq. (75) in the classical sense. More precisely, $w^K \in H^{4,2}(Q_{t_0,T})$ for any $t_0 \in (0,T)$.

c) Let the conditions of a) be satisfied. Let $\widetilde{k} > 0$ and assume that the gain parameters k_0 and k_1 satisfy the condition

$$k_0, k_1 \in [\tilde{k}, \infty), \tag{47}$$

(It will be convenient to choose $\widetilde{k} = \pi/2$. However, the value of $\widetilde{k}$ is immaterial for us because we will consider the limit $k_0, k_1 \to \infty$). Then the weak solution w^K satisfies the continuity condition (43) uniformly with respect to k_0 and k_1. In particular, the limit

$$\|w^K(t) - \varphi\| \to 0 \quad as \quad t \to 0 \tag{48}$$

is uniform with respect to k_0 and k_1.

d) Select any function $W(x,t)$ which is C^∞-smooth in x and satisfies the conditions: $W(0,t) = v_0(t)$, $W(1,t) = v_1(t)$, $W_x(0,t) = 0$, $W_x(1,t) = 0$. Define

$$\widetilde{w}^K(x,t) = w^K(x,t) - W(x,t).$$

Under the assumptions of item a), the weak solution $w^K(t) \in H^1(0,1)$ for any $t \in (0,T]$ and there exists the weak derivative $w^K_{xx} \in L^2(Q_{t_0,T})$ for any $t_0 \in (0,T)$. The function $\widetilde{w}^K$, and hence the solution w^K, satisfies the following estimates:

$$\|\widetilde{w}^K(t)\|^2 + \epsilon \int_0^t \|\widetilde{w}^K(\tau)\|_1^2 d\tau \le M_0(\|\varphi\|, t). \tag{49}$$

$$\|\widetilde{w}^K(t)\|_1 \leq t^{-1/2} M_1(\|\varphi\|, t), \tag{50}$$

$$\int_{t_0}^{t} \|\widetilde{w}^K_{xx}(\tau)\|^2 d\tau \leq t_0^{-1} M_2(\|\varphi\|, t). \tag{51}$$

If, in addition, f and v_0, v_1 have the weak derivatives

$$f_t \in L^2(Q_{t_0,T}) \ \ \textit{and} \ \ v_0'', v_1'' \in L^2(t_0, T) \ \ \textit{for any} \ \ t_0 \in (0, T], \tag{52}$$

then w^K has the weak derivative

$$w_t^K \in L^2([t_0, T], H^1(0,1)), \ \ \textit{for any} \ \ t_0 \in (0, T),$$

and the following estimate holds

$$\int_{t_0}^{t} \|\widetilde{w}^K_\tau(\tau)\|_1^2 d\tau \leq t_0^{-2} M_3(\|\varphi\|, t). \tag{53}$$

Here all the functions $M_i(\xi, t)$ $(i = 0, 1, 2, 3)$ are positive continuous and monotone increasing in $\xi \in (0, \infty)$, $t \in [0, T]$. These functions do not depend on k_0 and k_1 if the condition (47) is satisfied. (They can only depend on $\widetilde{k}$.)

Theorem 6. *a) Assume that $\varphi \in L^2(0,1)$, $f \in L^2(Q_T)$ and $v_0, v_1 \in H^1(0,T)$ for any $T > 0$. Then there exists a unique weak solution $w(x,t)$ of the problem (75), (33), (39). For any $T > 0$ the solution $w \in \mathcal{M}_T$, where the space $\mathcal{M}_T$ is defined in (42). The solution $w(t) \in H^1(0,1)$ for any $t > 0$ and there exists the weak derivative $w_{xx} \in L^2(Q_{t_0,T})$ for arbitrary $t_0 \in (0, T]$, $T > 0$. For all $t > 0$ the solution w satisfies the following estimates:*

$$\|w(t)\|^2 + \epsilon \int_0^t \|w_x(\tau)\|^2 d\tau \leq N_0(\|\varphi\|, t), \tag{54}$$

$$\|w_x(t)\| \leq t^{-1/2} N_1(\|\varphi\|, t), \tag{55}$$

$$\int_{t_0}^{t} \|w_{xx}(\tau)\|^2 d\tau \leq t_0^{-1} N_2(\|\varphi\|, t). \tag{56}$$

where the functions $N_i(\xi, t)$ $(i = 0, 1, 2)$ have the same properties as the functions $M_i(\xi, t)$ from Theorem 5.

b) Statement b) of Theorem 5 holds for the solution w of problem (75), (33), (39): if $f \in H^{4,2}(Q_{t_0,T})$ and $v_0, v_1 \in H^2(0,T)$ for any $0 < t_0 < T$ then $w \in H^{4,2}(Q_{t_0,T}) \subset C^{2,1}(Q_{t_0,T})$ for any $0 < t_0 < T$.

Now we are in a position to formulate the first of the two main results of [18].

Theorem 7. *Let w^K and w be the solutions of the closed loop problem (75), (33), (37) and the zero-dynamics problem (75), (33), (39) correspondingly. Assume that φ, f, and v_0, v_1 in both problems are the same and satisfy the conditions of Theorem 5, statement a). Then*

$$\lim_{k_0,k_1\to\infty} \|w^K(t) - w(t)\| = 0 \tag{57}$$

for any $t \geq 0$. The convergence is uniform on any finite interval $[0,T]$.

Theorem 8. *Assume that all conditions of Theorem 7 are satisfied and, in addition, (52) holds for v_0, v_1 and f. Then*

$$\lim_{k_0,k_1\to\infty} \|w^K(t) - w(t)\|_{H^1(0,1)} = 0 \tag{58}$$

for any $t > 0$ and the convergence is uniform on any finite interval $[t_0, T]$ with $t_0 > 0$. Due to the embedding $H^1(0,1) \subset C[0,1]$, (58) implies that for $t_0 > 0$

$$\max_{t\in[t_0,T]} \max_{x\in[0,1]} |w^K(x,t) - w(x,t)| \to 0 \text{ as } k_0, k_1 \to \infty. \tag{59}$$

These results were used to prove the following stabilization results found in [18]

Theorem 9. *a) The undisturbed open loop system defined by equation (75) with $f = 0$ and boundary conditions (40) is semiglobally stabilizable in the $L^2(0,1)$, $H^1(0,1)$ and $L^\infty(0,1)$ norms by the feedback law (36) with zero boundary disturbances $v_0 = v_1 = 0$.*

More precisely, for any $R > 0$ there exists $K(R) > 0$ such that the following statement holds. If the initial function $\varphi \in L^2(0,1)$ satisfies

$$\|\varphi\| \leq R \tag{60}$$

and the gain parameters

$$k_0, k_1 \geq K(R), \tag{61}$$

then the solution w^K of the closed loop initial boundary problem (75), (33), (37) corresponding to $f = 0$ and $v_0 = v_1 = 0$ satisfies the estimates:

$$\|w^K(t)\| \le C_0(R)e^{-\alpha t}, \quad R < C_0(R) < 2R, \quad t \ge 0, \tag{62}$$

$$\|w^K(t)\|_{H^1(0,1)} \le C_1(R, t_0)e^{-\alpha t}, \quad t \ge t_0 > 0, \tag{63}$$

where $\alpha > 0$ can be taken as $\alpha = \epsilon\pi^2/4$, C_0 and C_1 are positive continuous functions of their arguments, and $\lim\limits_{t_0 \to 0} C_1(R, t_0) = \infty$. If $\varphi \in H^1(0,1)$ and $\|\varphi\|_{H^1(0,1)} \le R$, then (63) holds for $t \ge 0$ and $C_1 = C_1(R)$.

An estimate similar to (63) holds for the norm $\|w^K(t)\|_{L^\infty(0,1)}$.

b) The open loop system (75), (40) with a sufficiently small disturbance f is semiglobally practically stabilizable in the $L^2(0,1)$ norm by the feedback law (36) with sufficiently small boundary disturbances v_0 and v_1.

More precisely, for any $R > 0$ and $\delta > 0$ there exist $K(R, \delta) > 0$ and $\widehat{\sigma}(\delta) > 0$ such that the following statement holds. If the initial function φ satisfies (60), the gain parameters k_0 and k_1 satisfy (61) with $K(R)$ replaced by $K(R, \delta)$, and the disturbances v_0, v_1 and f satisfy

$$\operatorname{ess\,sup}_{t \in [0,\infty)} \{\|f(t)\|, |v_0'(t)|, |v_1'(t)|, |v_0(t)|, |v_1(t)|\} \le \sigma. \tag{64}$$

(where $\rho > 0$ and $\sigma > 0$ are sufficiently small constants, which are independent of k_0 and k_1) with $\sigma \le \widehat{\sigma}(\delta)$ then the solution w^K of the closed loop problem (75), (33), (37) satisfies the estimate

$$\|w^K(t)\| \le \delta \quad \text{for} \quad t \ge T(R, \delta), \tag{65}$$

where $T(R, \delta) = C_0 \ln(R/\delta)$, $C_0 > 0$.

In our recent preliminary report [24] we have made some progress toward establishing existence of global in time dynamics for the open loop, closed loop and zero dynamics systems generated by a class of systems governed by boundary controlled convection reaction diffusion equations (see (66)-(74) below). This work relates to our specific Tasks 3.13 and 3.14.

Task 3.13: *As a specific research task devoted to the study of the steady state response of nonlinear distributed parameter systems we intend to carry out an analysis parallel to that described above for a boundary controlled viscous Burgers' equation for this class of* *hydrodynamic systems in higher dimensions, [24, 25]. We propose to establish, among other things, the existence of long time dynamics and Lyapunov stability for all initial data and existence of a global attractor.*

Task 3.14: *Using techniques from functional analysis and partial differential equations we propose to employ a variety of feedback regularization methods to suppress possible instabilities or blowups in a subdomain Ω_1 of a domain $\Omega \subset \mathbb{R}^n$. For example, if Ω_1 is a thin layer near the boundary of Ω, we propose to investigate the suppression of the onset of turbulence in the boundary layer.*

Consider a class systems governed by higher dimensional convection reaction diffusion equations evolving on a bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ with C^2-boundary Γ.

In particular, we consider boundary control systems of the form

$$w_t - Lw + \operatorname{div} \vec{F}(w) + G(w) = h, \tag{66}$$

$$w(x,0) = \varphi(x),\ \varphi \in L^2(\Omega), \tag{67}$$

$$\mathcal{B}(w) \equiv \sum_{i,j=1}^{n} a_{ij}(x) w_{x_i}(x,t)\eta_j(x)\Big|_{x\in\Gamma} = u(t), \tag{68}$$

$$y(t) = w(x,t), \quad x \in \Gamma \tag{69}$$

where $x = (x_1, \cdots, x_n) \in \Omega$ and $t \geq 0$ and $\vec{\eta}(x)$ is the unit normal vector to Γ.

The operator L is a formally self-adjoint uniformly elliptic differential operator:

$$Lw = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}\frac{\partial w}{\partial x_i}\right),$$

Lw represents a diffusion term in non-isotropic, non-homogeneous media, where $w(x,t)$ is the state of system, $\operatorname{div} \vec{F}(w) = \operatorname{div} \vec{F}(x,t,w)$ is a convective term, $G(w) = G(x,t,w)$ is a reactive term and $h = h(x,t)$ is an external control or disturbance.

We are interested in very general boundary feedback controls acting though the boundary Γ of Ω. In particular we consider two types of (nonlinear) feedback controls from our recent work [24]:

Feedback Control Type I: $\quad u(x,t) = -k(x, y(x,t)) \qquad (70)$

where $k \in C^{1,m}(\Gamma \times \mathbb{R})$

Feedback Control Type II: $\quad u(x,t) = -\int_\Gamma \mathcal{K}(x,\gamma,y(\gamma,t))\,d\Gamma_\gamma \qquad (71)$

where $\mathcal{K} \in C^{1,1,m}(\Gamma \times \Gamma \times \mathbb{R})$

Employing the **Type I** control law, the input (68) and the output (69), we obtain a Closed Loop System consisting of (66), (67) subject to the (nonlinear) boundary condition

$$\left[\mathcal{B}(w)(x,t) + k(x, w(x,t))\right]\Big|_{x\in\Gamma} = 0. \tag{72}$$

For the **Type II** control law, the input (68) and the output (69), we obtain a Closed Loop System consisting of (66), (67) subject to the (nonlinear) boundary condition

$$\left[\mathcal{B}(w)(x,t) + \int_\gamma \mathcal{K}(x,\gamma,w(\gamma,t))\, d\Gamma_\gamma\right]\Big|_{x\in\Gamma} = 0, \tag{73}$$

In either case, we define the zero dynamics, obtained by constraining the output (69) to zero, to be the system consisting of (66), (67) and Dirichlet boundary conditions

$$w(x,t) = 0. \tag{74}$$

In [24] we have been able to establish global in time existence and regularity of solutions for the above class of initial boundary value problems.

In our work on existence of steady state responses for convective reaction diffusion processes, in a joint work [1] (with Professor John Burns at VT& State University), we have also made a startling discovery concerning the existence of so-called numerical stationary solutions for Burgers' equation with Neumann boundary conditions. In particular, we illustrate with this example that, because of finite precision arithmetic, a convergent numerical algorithm can produce false (purely numerical) solutions. The main purpose of this work is to give an in-depth examination of this model problem and to give warning in the use of numerical based proofs of uniqueness for hydrodynamic problems.

Burgers' equation on the interval $(0,1)$ subject to Neumann Boundary Conditions is given by the dynamical system

$$\begin{aligned} &w_t - \epsilon w_{xx} + w w_x = 0, \\ &x \in (0,1), \quad t > 0,\ \epsilon > 0, \\ &w_x(0,t) = w_x(1,t) = 0, \\ &w(x,0) = \phi(x). \end{aligned} \tag{75}$$

We are interested in the corresponding steady state problem

$$\begin{aligned} &-\epsilon v_{xx}(x) + v(x) v_x(x) = 0, \\ &v_x(0) = v_x(1) = 0. \end{aligned} \tag{76}$$

Solutions of (76) are called *stationary* (or equilibrium) solutions of the unsteady problem (75). One approach to the development of numerical methods for solving (76) is to solve the time dependent problem (75) and assume that $w(\cdot, t) \to v(\cdot)$ as $t \to +\infty$. In order to construct fast and accurate "time marching" schemes based on this idea, a number of points must be considered. In particular, one should address the following issues.

(a) If possible, the questions of existence and uniqueness of stationary solutions to the boundary value problem (76) needs to be answered. These are still open questions for many fluid and gas dynamic problems.

(b) One needs to know that for reasonable initial data $\phi(\cdot)$, the time varying solution $w(\cdot, t)$ exists for all $t > 0$, $\lim_{t \to +\infty} w(\cdot, t) = v(\cdot)$ exists, and $v(\cdot)$ is a stationary solution.

(c) The rate at which $w(\cdot, t) \to v(\cdot)$ is important because it can influence the efficiency of the scheme.

(d) If one introduces a numerical approximation (with spatial mesh size Δx) and constructs the numerical solution $w^{\Delta x}(\cdot, t)$ with the property that as $\Delta x \to 0$ (i.e. mesh refinement) $w^{\Delta x}(\cdot, t) \to w(\cdot, t)$, then $\lim_{t \to +\infty} w^{\Delta x}(\cdot, t) = v^{\Delta x}(\cdot)$ needs to exist.

(e) The limit $v^{\Delta x}(\cdot)$ is assumed (or proven) to be a good approximation to $v(\cdot)$. This issue is more complicated than one might guess and it can fail in surprisingly simple problems.

Items (a) - (e) above do not address all of the of important issues. For example, as we have shown in our paper [1], even if items (a)-(d) are satisfied and one can prove (this means using infinite precision arithmetic) that $v^{\Delta x}(\cdot) \to v(\cdot)$, then problem sensitivity and finite precision arithmetic can produce numerical solutions $v^{\Delta x}(\cdot)$ that do not approximate any stationary solution! Thus, it is possible for a perfectly sound theoretical algorithm to produce "false" numerical solutions to the steady state problem. We demonstrate this point by a complete analysis of Burgers' equation with Neumann boundary conditions.

3 References

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4 Participating Professionals

1. Principal Investigators
   - Christopher I. Byrnes
   - Alberto Isidori
2. Senior Personnel
   - D. Gilliam, A. Lindquist, V. Shubov
3. Postdocs
   - S. Pandian
4. Graduate Students
   - J. Ramsey, Ph.D. Washington University, Fall 2000

5 Scientific Publications

a. *Peer Reviewed Journal:*

A. Isidori, "A remark on the problem of semiglobal nonlinear output regulation," IEEE Transaction on Automatic Control, AC-42, 1734-1738, 1997.

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H.T. Banks, D.S. Gilliam and G. Pinter, "Theoretical and numerical results for parameter estimation in nonlinear elastomers," 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997), 3733-3738, 1997.

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c. *Books:*

A. Isidori, "Nonlinear control systems, II," Communications and Control Engineering Series. Springer-Verlag London, Ltd., London, 1999. xii+293 pp. ISBN: 1-85233-188-7.

(Co-editor David S. Gilliam) "Dynamical Systems, Control, Coding, Computer Vision: New trends, Interfaces, and Interplay," to appear in *Progress in Systems and Control*, Birkhäuser, 1999, (501 pages), Eds. G. Picci and D.S. Gilliam.

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6 Scientific Interactions/Transitions

During the month of November 1997, Dr. Christopher I. Byrnes visited the Flight Controls Lab at Wright Patterson Air Force Base to learn about new AF initiatives in flight control from Dr. Siva Banda. Dr. Byrnes has also provided technical advice to Dr. James Cloutier and other Air Force personnel at Ft. Eglin Air Force Base.

The Boeing/Washington University Graduate Education and Research Partnership, a $3.1 Million program sponsored by the Boeing/McDonnell Douglas Foundation, provides a novel, formal opportunity to interact scientifically with research engineers at the Boeing corporation and to transition technology developed at Washington University to the defense industry. This program was inaugurated in December 1998, under the co-direction of Dr. Allen Atkins (Vice President (VP) of Production Application Technologies, Boeing) and Dr. Byrnes, with a joint meeting of Washington University (WU) and Boeing researchers and the announcement of twelve joint research projects in the areas of Aerospace, Automatic Control, Computational Science, Materials and Image Processing. In May 1999, Dr. Atkins and Dr. Byrnes co-hosted the First Research Day for the Partnership. There were twelve presentations by research teams led by a WU Professor of Engineering and a Boeing research engineer who codirect a doctoral thesis in the School of Engineering and applied Science. In this program, Dr. A. Isidori chaired and Dr. Kevin Wise served on A. Serrani's dissertation committee. The dissertation aimed to transition research on robust output regulation to methodologies for robust asymptotic tracking and disturbance rejection for aircraft.for flutter control. In the same program, Dr. C. Byrnes chaired and Dr. Kevin Wise served on J.

Ramsey's dissertation committee. The dissertation aims to transition research on output regulation near a stable periodic motion of unknown frequency to methodologies for flutter control. The highlights of this research day were the subject of a briefing from 14:30 to 17:30 on December 20, 1999 to high level personnel in Boeing and the Boeing/McDonnell Douglas Foundation. Dr. C. Byrnes participated in a briefing to Boeing personnel on these research projects and possible transitions. The Boeing personnel included:

Dr. Allen Atkins	Vice President (VP) of Production Application Technologies
Dr. Kevin Wise	Manager, Integrated Flight Control
Jerry Halley	Assoc. Tech. Fellow, Advanced Manufac. Res. and Devel.
Toni Bailey	Pres., Boeing-McDonnell Found., VP, Comm. and Educ. Relations
Ron Shelley	Dir., Supplier Management and Procurement, Prod. Operations
Dan Grossman	Director of Strategic Planning
Dr. Diane Chong	Department Manager, Engineering
Dr. Matt Thomas	Manager, University Liaison
John Van Gels	VP and General Manager of Production Operations/Gen. Serv.
Flake Campbell	Director of Advanced Manufacturing Research and Development
Charles Saff	Staff Manager, Advanced Materials Structures Technology
Jimmy Williams	Senior Manager, Advanced Manufac. Research and Devel.

On February 12, 2001, Dr. Atkins and Dr. Byrnes co-hosted the Second Research Day for the Partnership. There were thirteen presentations by research teams on new research projects in the same five areas. In this program, Dr. C. Byrnes and Dr. A. Isidori co-chair and Dr. Kevin Wise serves on F. Celani's dissertation committee. The dissertation aims to transition research on the stability of inner-loop/outer-loop feedback systems and of dynamic inversion schemes to methodologies for for analyzing stability of flight control systems and for modeling and correcting small deviations from trim conditions such as occur in "nose slippage" at certain high angle-of-attack maneuvers.

From 1997-1999, Dr. Byrnes served as a scientific advisor to Drs. Alan Cain, Yutaka Ikeda and Kevin Wise at Boeing Aerospace on an AFOSR funded initiative in flow control. Among the intended applications are the development of a robust nonlinear control strategy for the control of airflow to increase lift on tail flaps on aircraft such as the C17 and the development of a nonlinear control strategy to shorten the exhaust plume on fighter aircraft in order to render the aircraft more stealthy.

During the same period, Dr. Byrnes served as a consultant to Dr. Rowena Eberhardt in support of an AFOSR funded program at Lockheed-Martin. on the stability of nonlinear adaptive controllers.

Drs. Gilliam and Shubov are involved in a collaborative effort with researchers in the

AFOSR Center for Optimal Design and Control at Virginia Tech & State University on the use of "numerical based" methods for hydrodynamic models. As part of this collaboration, Dr. Gilliam collaborated visited the AFOSR Center for Optimal Design and Control at Virginia Tech & State University in November of 2000 and collaborated with researchers in the center.

Through collaboration with AFOSR sponsored researcher Dr. Tryphon Georgiou at University of Minnesota, Drs. C. I. Byrnes and A.G. Lindquist have developed a new methodology for designing rational positive real functions satisfying the Nevanlinna-Pick constraints, a problem with multiple applications to circuit design, robust control design and signal processing. A transition of this technology has commenced with a patent application for a high-resolution spectral estimator, discussed in the next section. On March 1, 2000 a written disclosure of this technology was made to Dr. Richard Albanese of Armstrong Laboratory.

In addition to collaborative research with engineering research and development personnel at Boeing, St. Louis, MO, and scientific interaction with AFOSR personnel at Bolling AFB, Ft. Eglin AFB and Wright Patterson AFB, we have presented many invited lectures and colloquia nationally and internationally:

December 1997:

- "Global L_2-gain State Feedback Design for a Class of Nonlinear Systems," 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997), Lecture presented by Professor A.Isidori
- "Harmonic Forcing for a Class of Nonlinear Systems," 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997), Lecture presented by Professor A.Isidori
- "Computational Methods for Feedback Control of Distributed Parameter Systems," Special Session at 36th IEEE Conf. Decision and Control (San Diego, CA, December 1997), Organizer and Chair: Professor David Gilliam.

February 1998:

- "Semiglobal Stabilization by output feedback of non-minimum phase uncertain nonlinear systems" Invited lecture presented by Dr. Alberto Isidori at the Seminar "Nonlinear Control", Centre E. Borel, Universite Pierre et Marie Curie, Paris, 1998.

March 1998:

- "Output regulation for nonlinear distributed parameter systems," Department of Systems Science and Math, Washington University, St. Louis, Lecture presented by Professor D.S. Gilliam.

- "Why subspace identification algorithms sometimes don't work and what we can do instead" presented by Professor Anders Lindquist at Washington University, St. Louis.

April 1998:

- "What is Optimization and Systems Theory, and what are we doing at KTH?", presented by Professor Anders Lindquist at Physics Colloquium, KTH, Stockholm Sweden.

May 1998:

- "Output regulation for nonlinear distributed parameter systems," SIAM Conference on Systems Control, Jacksonville, Florida. Lecture presented by Professor D.S. Gilliam.
- "Covariance extension and speech processing", presented by Professor Anders Lindquist at Third Russian-Swedish Control Conference, Stockholm.
- AFOSR Contractors Workshop, Pasadena, CA. Attended by Professor C.I. Byrnes.
- "The geometry of positive real functions with applications to systems and signals," Plenary lecture presented at SIAM Conference on Systems Control, Jacksonville, Florida, by Professor C.I. Byrnes.

June 1998:

- "Conditions for solvability of the output regulator problem for distributed parameter systems," Invited Lecture presented at the IEEE Mediterranean Conference by Professor C.I. Byrnes.

July 1998:

- "The separation principle in nonlinear stabilization" Plenary lecture presented by Dr. Alberto Isidori at the Conference: "Mathematical Theory of Networks and Systems", Padova, Italy.
- "On the geometry of Nevanlinna-Pick interpolation." One hour invited lecture presented at International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy by Professor C.I. Byrnes.
- "Output regulation for nonlinear distributed parameter systems," International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy. Lecture presented by Professor D.S. Gilliam.

- "A generalized entropy criterion for rational Nevanlinna-Pick interpolation with applications to systems and control," International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy. Lecture presented by Professor C.I. Byrnes.
- "Experimental evidence showing that stochastic subspace identification methods may fail," International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy. Lecture presented by Professor A. Lindquist.
- D.S. Gilliam, Organizer of of special session entitled "Applications of Numerical, Functional and Stochastic Methods to the Analysis of Data" for International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy.
- "Exponential nonlinear observer design for bifurcating systems," Lecture presented at International Conference on the Mathematical Theory of Networks and Systems, Padova, Italy by Professor C.I. Byrnes.
- "Persistence of stability of periodic motion for nonlinear systems," Invited lecture presented at the Systems Theory Days Conference held in Montalcina, Italy by Professor C.I. Byrnes.

August 1998:

- "Output regulation for nonlinear distributed parameter systems," 6th Conference on Computation and Control in Bozeman, Montana. Lecture presented by Professor D.S. Gilliam.

October 1998:

- "Global analysis of linear systems," Plenary lecture, Dr. C. Byrnes at *Perspectives in Control*, (In honor of R.W. Brockett's 60th Birthday) BU/Harvard, Cambridge, MA, Oct. 98.
- "A convex optimization approach to Nevanlinna-Pick interpolation problems in robust stabilization," Plenary lecture, Dr. A. Lindquist at the *International Conference Dynamical Systems: Stability, Control, Optimization*, Minsk, Belarus, Oct. 98.
- "Stabilization of nonlinear systems using output feedback," Invited lecture, Dr. A. Isidori,*MIT*, Cambridge, MA, Oct. 98.

November 1998:

- "Advances in nonlinear control theory," Invited lecture, Dr. A. Isidori at the *Case Western Reserve University*, Cleveland, OH.

- "Conditions for solvability of the regulator problem for linear distributed parameter systems," Invited lecture, Dr. D.S. Gilliam at *Texas System Days.*
- "The geometry of Nevanlinna-Pick interpolation, with applications to control and signal processing," Dr. C. Byrnes at *The Royal Institute of Technology*, Stockholm, Sweden, Invited lecture on the occasion of receiving an Honorary Doctorate.

December 1998:

- "Output regulation for nonlinear systems: an overview," Invited lecture, Dr. A. Isidori at the *37th IEEE Conference on Decision and Control*, Tampa, FL, Dec. 1998.
- "Conditions for solvability of the output regulator problem for siso distributed parameter systems," Lecture, Dr. D.S. Gilliam at *37th IEEE on Decision and Control*, Tampa, FL, Dec. 1998.
- "Zero dynamics for relative degree one siso distributed parameter systems," Lecture, Dr. D.S. Gilliam at *37th IEEE on Decision and Control*, Tampa, FL, Dec.1998.

February 1999:

- "Analytic interpolation with degree constraints, with applications to systems and control," Dr. A. Lindquist at the *Texas Tech University*, Lubbock, TX, Feb. 99.

March 1999:

- "A duality between filtering and interpolation," Invited lecture, Dr. C. Byrnes at *Kyoto University*, Japan, Mar. 99.
- "A duality between filtering and interpolation," Invited lecture, Dr. C. Byrnes at *Tokyo University*, Japan, Mar. 99.
- "Stabilization by output feedback revisited," Dr. A. Isidori, *Oberwolfach Math. Inst.*, Mar. 99.

April 1999:

- "The geometry of Nevanlinna-Pick interpolation, with applications to control and signal processing," Plenary lecture, Dr. C. Byrnes at *Advances in Mathematical Systems Theory*, (in honor of D. Hinrichsen's 60th Birthday), Borkum, Germany, Apr. 99.
- "Analytic interpolation with degree constraints, with applications to systems and control," Dr. A. Lindquist at *Moscow State University*, Apr. 99.

- "A Duality Between Filtering and Interpolation," Invited lecture, Dr. C. Byrnes at *Universiteit Groningen*, Netherlands, Apr. 99.

May 1999:

- "Output regulation for periodic systems," J. Ramsey and Dr. C. Byrnes at the *Boeing/WU Research Day*, St. Louis, May 99.
- "Robust Output Regulation," A. Serrani and Dr. C. Byrnes at the *Boeing/WU Research Day*, St. Louis, May 99.
- "The geometry of Nevanlinna-Pick interpolation, with applications to control and signal processing," Plenary lecture, Dr. C. Byrnes at the *Michigan Interdisciplinary Mathematics Meeting*, University of Michigan, Ann Arbor, May 99.
- "On the control of vortices in tornado flows," Invited lecture, Dr. V.I. Shubov at the *SIAM Annual Meeting*, Atlanta, May 99.

June 1999:

- "A duality between filtering and Interpolation," Invited lecture, Dr. C. Byrnes at the *University of Rome*, Italy, Jun. 99.
- "Analytic interpolation with degree constraints, with applications to systems and control," Dr. A. Lindquist at the *Int. Conf. on Rat. Approx.*, Antwerpen, Belgum, Jun. 99.
- "A comment on numerical based 'proofs' for hydrodynamic flows," Invited lecture, Dr. D.S. Gilliam at *NSF Workshop on Control of Fluids* at UCSD, Jun. 99.
- "Output regulation of nonlinear systems," and "Stabilization of nonlinear systems using output feedback," Invited lectures, Dr. A. Isidori at *COSY*, Arrabida, Portugal, Jun. 99.

August 1999:

- "Nonlinear control systems," Invited lecture, Dr. C. Byrnes at *The 1999 AFOSR Annual Contractors Meeting on Dynamics and Control*, Wright-Patterson Air Force Base, Aug. 99.
- "On the stabilization of uncertain linear systems by output feedback," Invited Lecture by Dr. A. Isidori at *The Åström Symposium on Control* in Lund, Sweden, Aug. 99.

September 1999:

- "Toward a nonequilibrium theory for nonlinear control," Plenary Invited Lecture given by Dr. C. Byrnes at *The Mathematics of Systems and Control: From Intelligent Control to Behavioral Systems* (In honor of J.C. Willem's 60th Birthday), Groningen, The Netherlands, Sept. 99.
- "Analytic interpolation with degree constraints, with applications to systems and control," Plenary lecture, Dr. A. Lindquist at the *Sixth St. Petersburg Symposium on Adaptive Systems Theory*, St. Petersburg, Russia, Sept. 99.
- "On the problem of residual generation for fault detection in nonlinear systems and some related facts," Dr. A. Isidori, *5th European Control Conference.*

October 1999:

- "Advances in high-resolution spectral estimation," Invited Lecture given by Dr. A. Lindquist *Advances in Systems Theory*, Cambridge, Massachusetts, (In honor of S.K. Mitter's 65th Birthday)).

December 1999:

- "Output regulation of nonlinear systems" 8 hour Workshop presented by C.I. Byrnes, A. Isidori, Jie Huang, A. Serrani, and L. Marconi, *38rd IEEE Conf. Decision and Control (CDC)*, Phoenix.
- "Recent advances in output regulation for distributed parameter systems," Dr. D.S. Gilliam, *Recent Advances in Systems and Control Theory*, Washington University, St. Louis, (*Invited Lecture*).
- "Adaptive stabilization of nonlinear non-minimum phase systems," Dr. A. Isidori, *Recent Advances in Systems and Control Theory*, Wash. Univ., St. Louis, (*Invited Lecture*).
- "Analytic interpolation with degree constraints, with applications to systems and control," Dr. A. Lindquist, *Recent Advances in Systems and Control Theory*, Washington University, St. Louis, (*Invited Lecture*).
- "A necessary condition for local asymptotic stability of equilibria of nonlinear systems with parameters," Lecture presented by C.I. Byrnes at the Proc. of the 38th IEEE Conference on Decision and Control, Phoenix, Dec. 1999.
- "Output regulation for linear systems with anti-stable eigenvalues in the presence of input saturation," R. De Santis, A. Isidori, *38rd IEEE CDC.*
- "Analytic interpolation with degree constraint: A constructive theory with applications to control and signal processing," A. Lindquist, *38rd IEEE CDC.*

- "A convex optimization approach to the covariance extension problem with degree constraint," Dr. A. Lindquist, *38rd IEEE CDC.*
- "Global output regulation for a class of nonlinear systems," A. Serrani, A. Isidori, *38rd IEEE CDC.*
- "Example of output regulation for a system with unbounded inputs and outputs," Dr. D.S. Gilliam, *38rd IEEE CDC*, Phoenix, AZ, (Invited Lecture).
- "Semiglobal Stabilization of a boundary controlled viscous Burgers' equation," Dr. D.S. Gilliam, *38rd IEEE CDC*, Phoenix, AZ.

March 2000:

- "Why Nevanlinna-Pick interpolation theory is important in applications to systems and control and how it can be modified to be more useful," Dr. A. Lindquist *Helsinki University of Technology*, Helsinki, finland, (*Colloquium lecture*).

May 2000:

- "Fine dust limit for coupled systems of Navier-Stokes and Euler equations." Dr. V.I. Shubov, *3rd International Conference on Nonlinear Problems in Aviation and Aerospace*, Daytona Beach, FL, (*Invited lecture*).
- "Stability of airflow containing dust and applications to tornado dynamics," Dr. V.I. Shubov, *International Conference on Differential Equations and Dynamical Systems*, Kennesaw State University, Atlanta, GA, (*Invited lecture*).
- "Analytic interpolation with degree constraint with applications to systems and control and signal processing," The 2000 Zaborszky Lectures (three lectures) Dr. A. Lindquist Washington University, St. Louis (*Invited Lectures*).

June 2000:

- "Toward a nonequilibrium theory for nonlinear control," Dr. C. Byrnes, *Nonlinear Control in the Year 2000*, Paris, France, (*Plenary Lecture*).
- "Toward a nonequilibrium theory for nonlinear control," Dr. C. Byrnes, *14th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2000)*, Perpignan, France, (*Plenary Lecture*).
- "Some New Methods and Concepts in High-resolution Spectral Estimation," Dr. A. Lindquist *Third Asian Control Conference (ASCC)*, Shanghai, China (*Plenary lecture*).

- "An H_∞-suboptimal fault detection filter for bilinear systems," C. De Persis, A. Isidori, *Nonlinear Control in the Year 2000*, (*Invited lecture*).
- "Advances in nonlinear output regulation," A. Serrani, A. Isidori, C.I.Byrnes, L. Marconi at *Nonlinear Control in the Year 2000*, (*Invited lecture*).
- "Robust global stabilization of a class of uncertain feedforward nonlinear systems," A. Marconi, A. Isidori, *Nonlinear Control in the Year 2000.*
- "The design of filters for fault detection in nonlinear systems," Dr. A. Isidori, *IFAC Symposia "Safeprocess 2000".*
- "Generalizations of Hadamard's theorem and Dirichlet's principle for finite dimensions," Dr. C. Byrnes, *(MTNS 2000)*, Perpignan, France.
- "Robust observer design for nonlinear systems that change with the disturbance," Dr. C. Byrnes *(MTNS 2000)*, Perpignan, France.
- "Covariances, cepstral coefficients and pole-zero models for signal processing," Dr. A. Lindquist, *(MTNS 2000)*, Perpignan, France.
- "Recent results on Interpolation in the class of positive real functions: A geometric approach," Dr. A. Lindquist *(MTNS 2000)*, Perpignan, France.

August 2000:

- "Output regulation for systems with delays," Dr. D.S. Gilliam, *8th Conf. on Computation and Control* at Montana State University.
- "Fine dust limit for coupled systems of Navier-Stokes and Euler equations." Dr. V.I. Shubov, *8th Conf. on Computation and Control* Montana State University.
- "Toward a nonequilibrium theory for nonlinear control systems," 2000 AFOSR Contractors Meeting: Dynamics and control, Cal Tech University, Lecture presented by Dr. C.I. Byrnes.

September 2000:

- "Cepstral geometry and global analysis of ARMA parameterizations," Lecture presented by Professor Anders Lindquist at the Workshop for the European Research Network of Systems Identification, vadstena, Sweden.

November 2000:

- "Geometric Theory of Output Regulation for Linear Distributed Parameter Systems," Dr. D.S. Gilliam, Future Directions in Distributed Parameter Systems in honor of H.T. Banks 60th birthday, North Carolina State University, November 2000 (Invited Address).

- "Synthetic speech and modern mathematics: What is the connection?," Colloquium lecture presented by Professor Anders Lindquist at the Royal Institute of Technology, Stockholm, Sweden.

7 New Discoveries, Inventions or Patent Disclosures

On May 8, 1998, Drs. Christopher I. Byrnes and Anders G. Lindquist submitted, through Washington University, a patent application, US Serial No. 08/854,150, "A new method and apparatus for speech analysis and synthesis." On February 11, 1999, the patent application US Serial No. 08/854,150 was allowed by the U.S. Patent Office. On May 13, 1999 a positive International Preliminary Examination Report was issued for the extension (PCT/US98/09576) of this patent to Australia, Canada, Europe, and Japan, on the basis of the previous disclosure, US Serial No. 08/854,150. On September 2, 1999, US Patent No 5,940,791, "Method and apparatus for speech analysis and synthesis using lattice ladder notch filters," was awarded to Washington University by the U.S. Patent Office as . On November 9, 1999 an application to the national stage was made for the extension (PCT/US98/09576) of this patent to Australia, Canada, Europe, and Japan.

On August 5, 1998, Drs. Christopher I. Byrnes and Anders G. Lindquist submitted, through Washington University, the patent disclosure, US Serial No. No. 09/117,721: "A new method and apparatus for speech analysis and synthesis including speaker recognition," to the U.S. Patent Office. On May 9, 2000, they made a response to the favorable review of patent application US Serial No. 09/117,721.

On October 8, 1998, the patent application US Serial No. 09/176,984, "Method and apparatus for a tunable high-resolution spectral estimator," was filed by Drs. C.I. Byrnes, T. Georgiou and A.G. Lindquist through the University of Minnesota and Washington University to the U.S. Patent Office. On October 22, 1999 an application was made for the international extension of the patent application US Serial No. 09/176,984.

8 Additional Information, Awards and Honors

In November 1998, Dr. Byrnes received an Honorary Doctor of Technology from the Swedish Royal Institute of Technology, along with the Nobel Laureate, Dr. Claude Cohen-Tanoudji. In 1998, Dr. Byrnes was also elected a Fellow of the Academy of Sciences of St. Louis.

The paper "A convex optimization approach to the rational covariance extension problem", by C. I. Byrnes, S. V. Gusev and A. Lindquist, published in SIAM J. Control and Opimization, 37 (1999), 211–229, has been chosen by the editors of SIAM Review to be the second "SIGEST" paper from SICON.