Naval Surface Warfare Center
Carderock Division
West Bethesda, MD 20817-5700

NSWCCD-70-TR–2001/060 April 2001

Signatures Directorate

Research and Development Report

Dependence of the Induced Loss Factor on the Coupling Forms and Coupling Strengths: Energy Analysis

by

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20010515 094

NSWCCD-70-TR–2001/060

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
26-Apr-2001

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
-

4. TITLE AND SUBTITLE
Dependence of the Induced Loss Factor on the Coupling Forms and Coupling Strengths: Energy Analysis

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
G. Maidanik

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AND ADDRESS(ES)
Naval Surface Warfare Center
Carderock Division
9500 Macarthur Boulevard
West Bethesda, MD 20817-5700

8. PERFORMING ORGANIZATION REPORT NUMBER
NSWCCD-70-TR-2001/060

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; Distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
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15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			G. Maidanik
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR	0	19b. TELEPHONE NUMBER *(include area code)* 301-227-1292

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Abstract

In two recent papers the induced loss factor is determined via the modification to the loss factor in the linear impedance of a master oscillator caused by its coupling to a set of satellite oscillators. A recapitulation of this linear impedance analysis (LIA) is presented. A loss factor is basically an energetic quantity and, therefore, one may inquire whether the induced loss factor may be estimated via an energy analysis (EA). An answer to this question is sought. It is shown that the two analyses; (LIA) and (EA), yield, in the appropriate frequency range, identical results for the induced loss factor. This frequency range spans the distribution of resonance frequencies of the satellite oscillators. In this frequency range the identity of the results is not only in terms of gross features, but also in details. Finally, the relationship of (EA) to statistical energy analysis (SEA) is explored. The loss factors of the satellite oscillators are directly related to modal overlap parameters. It is found that for the stronger couplings, the validation of (SEA) may require these modal overlap parameters to exceed a certain threshold.

I. Introduction

In two recent papers the influence on the response behavior of a master oscillator due to its coupling to a set of satellite oscillators is derived and examined [1,2]. The complex is sketched in Fig. 1. The examination is conducted in terms of the induced loss factor $\eta_s(y)$, where (y) is the normalized frequency variable; $y=(\omega/\omega_o)$. The normalizing frequency is the resonance frequency (ω_o) of the master oscillator in isolation. The induced loss factor $\eta_s(y)$ accounts for the modification in the loss factor in the linear impedance of the master oscillator caused by its coupling to that set of satellite oscillators. This modification is the focus of the investigation performed in References 1 and 2 and is also the focus of the investigation in this paper. A recapitulation of the linear impedance analysis (LIA) covered in these references may, thus, be in order.

As Fig. 1 indicates, the coupling between the master oscillator and a satellite oscillator allows for stiffness, mass, gyroscopic and a mixture of these forms [3, 4]. With the assistance of Fig. 1, the linear equations of motion for a master oscillator in isolation, for a coupled master oscillator and for a typical satellite oscillator are stated in the forms:

$$Z_o^o(y)\ V_o^o(y) = P_e(y), \tag{1a}$$

$$Z_o(y)\ V_o(y) = P_e(y), \tag{1b}$$

$$V_r(y) = B_r(y)\ V_o(y), \tag{1c}$$

respectively, where

$$Z_o^o(y) = (i\omega M)[1-(y)^{-2}], \tag{2a}$$

$$Z_o(y) = (i\omega M)\Big[1-(y)^{-2}\{[1-S(y)]+i[\eta_o+\eta_s(y)]\}\Big], \tag{2b}$$

$$S(y)-i\eta_s(y) = (y)^2\sum_1^R\Big\{\overline{m}_r\{[1-(z_r)^2(1+i\eta_r)]$$

$$\bullet\,[\overline{m}_{cr}-(z_{cr})^2(1+\eta_{cr})]-(\overline{q}_{cr}/y)^2\}$$

$$\bullet\,[(1+\overline{m}_{cr})-(z_{rr})^2(1+i\eta_{rr})]^{-1}\Big\}, \tag{2c}$$

$$B_r(y) = -[\overline{m}_{cr}+(z_{cr})^2(1+i\eta_{cr})-i(\overline{g}_r/y)]\,[(1+\overline{m}_{cr})-(z_{rr})^2(1+i\eta_{rr})]^{-1}, \tag{2d}$$

$$(\overline{q}_{cr}/y)^2 = 4\overline{m}_{cr}(z_{cr})^2(1+i\eta_{cr}) + (\overline{g}_r/y)^2, \tag{2e}$$

$$(z_{rr})^2(1+i\eta_{rr}) = (z_r)^2(1+i\eta_r)+(z_{cr})^2\ (1+i\eta_{cr}), \tag{2f}$$

$$(z_{rr})^2 = (x_{rr}/y)^2;\quad (z_r)^2 = (x_r/y)^2;\quad (z_{cr})^2 = (x_{cr}/y)^2, \tag{2g}$$

$$(x_r)^2(1+i\eta_r) = (k_r/m_r);\quad (x_{cr})^2(1+i\eta_{cr}) = (k_{cr}/m_r), \tag{2h}$$

$$\overline{m}_r = (m_r/M);\quad \overline{m}_{cr} = (m_{cr}/m_r);\quad \overline{g}_r = [G_r/(\omega_o m_r)]. \tag{2i}$$

In Eq. (1) $Z_o^o(y)$ and $V_o^o(y)$, and $Z_o(y)$ and $V_o(y)$ are the unmodified and the modified linear impedances and responses of the master oscillator, respectively, $V_r(y)$ is the response of the (r)th satellite oscillator and $P_e(y)$ is the external drive; this external drive is applied to the master oscillator only [1, 2]. The other quantities and parameters are defined in Fig. 1 and in Eq. (2). It is convenient, with only a slight loss in generality, to impose a similarity on the stiffnesses (springs) that are placed on either side of the mass (m_r) of the (r)th satellite oscillator; namely

$$(x_r)^2 = \alpha_r (x_r^o)^2; \quad (x_{cr})^2 = \alpha_{cr} (x_r^o)^2, \tag{3}$$

where (x_r^o) defines a designed distribution for the normalized resonance frequencies of the satellite oscillators. In that design the resonance frequencies ascend according to the value of the index (r); i.e., $(x_{rr})^2 \leq (x_{qq})^2$; $q = (r+1)$; $1 \leq r \leq (R-1)$, and the numbers of resonance frequencies on either side of the resonance frequency (ω_o) of the master oscillator in isolation, are equal. As in References 1 and 2, (x_r^o) is assigned the specific form

$$(x_r^o) = [1 + \{(1 - 2\bar{r})\gamma(\bar{R})\}\,]^{(-1/2)}; \;\; \gamma(\bar{R}) = [\gamma / (2(\bar{R})], \tag{4a}$$

where

$$\bar{r} = r(R+1)^{-1}; \;\; \bar{R} = R(R+1)^{-1}; \;\; \gamma = 0.6\,. \tag{4b}$$

Other forms for (x_r^o) are not only permissible, they are readily introduced. Finally, it is recognized that the coupling of the (r)th satellite oscillator to the master oscillator is specified in terms of the stiffness parameter (α_{cr}), the gyroscopic coupling parameter $(\overline{g}_r)$ and the mass coupling parameter $(\overline{m}_{cr})$. The values of these parameters, either individually or in combinations, determine the coupling strengths; there is, by definition, no coupling between a satellite oscillator and another. [cf. Fig. 1.]

The induced loss factor $\eta_s(y)$, investigated via Eqs. (2b) and (2c), reveals the dependence of this quantity on parameters that define the externally driven complex; see Fig. 1. The dependence of this induced loss factor on the coupling forms and coupling strengths is of particular interest in this investigation. The dependence of this quantity on the distribution of the resonance frequencies and on the values of the modal overlap parameters that are assigned to the satellite oscillators is also of interest in this investigation. Computational data of these dependencies are depicted in a number of figures in which $\eta_s(y)$ is presented, as a function of (y). In particular, in these figures, the phenomenon of erosion is defined and demonstrated. In this demonstration the corresponding first order approximations to $\eta_s(y)$ play a role [1, 2]. (The first order approximation is evaluated by replacing the summation in Eq. (2c) by an integration and carrying out the integration subjected to vanishing values of the modal overlap parameters.) In the evaluations presented, certain parametric simplifications are introduced. Largely, these simplifications manifest dropping the dependence of some parameters on the index (r); namely

$$m_r = m; \quad \alpha_r = \alpha; \quad \alpha_{cr} = \alpha_c; \quad \bar{g}_r = \bar{g}; \quad \bar{m}_{cr} = \bar{m}_c; \quad b_r = b_{cr} = b. \tag{5a}$$

Moreover, to ensure that the distribution of the resonance frequencies span a definitive and a constant range, as a function of $(\bar{r})$ $[\bar{r} = r(R+1)^{-1}.]$, the parametric equality

$$(1+\bar{m}_c) = (\alpha + \alpha_c), \tag{5b}$$

is conditioned. Then, under the similarity conditions specified in Eq. (3) and the simplifications and condition introduced in Eqs. (5a) and (5b), respectively, the loss factors assigned to the (r)th satellite oscillator are equal; namely

$$\eta_{rr} = \eta_r = \eta_{cr} = \eta(\bar{r}); \quad \eta(\bar{r}) = (\bar{b}/\pi)[\gamma(\bar{R})(x_r^o)^2]; \quad \bar{b} = [\pi\, b(R+1)^{-1}], \tag{5c}$$

where (x_r^o) and $\gamma(\bar{R})$ are stated in Eq. (4) [1,2]. Although $\eta(\bar{r})$ is not a function of the normalized frequency (y), situations arise in which notationally it is conducive to cast

$$\eta(\bar{r}) \equiv \eta_r(y) = (\bar{b}/\pi)[(y)^2 \gamma(\bar{R})(z_r^o)^2]; \quad z_r^o = (x_r^o / y). \tag{5d}$$

The loss factor $\eta(\bar{r})$ [or $\eta_r(y)$], as a function of $(\bar{r})$, for three values of (b); $b = (0.1), (2.0)$ and (10), is depicted in Fig. 2. With the impositions stated in Eq. (5) the first order approximation (FOA) of the induced loss factor $\eta_s(y)$ assumes the simple form

$$\eta_s(y) = D[C + O\{\eta(y)\}^2]; \quad \eta_I(y) = DC, \tag{6}$$

where

$$D = [\pi/\{2\gamma(\bar{R})\}][M_s/(M\bar{R})[1+\bar{m}_c]^{-1}; \quad M_s = (Rm) \tag{7a}$$

$$C = [(\overline{m}_c + \alpha_c)^2 + (\overline{g}/y)^2], \tag{7b}$$

$$O = [(1 + \overline{m}_c - \alpha_c)\,\alpha_c], \tag{7c}$$

and the term $O\{\eta(y)\}^2$ is suppressed and lumped with the higher order approximations [2]. The validity of Eq. (6) is predicated on the condition that

$$\eta_{r_o}(y) = \eta(\overline{r}_o) = [b(R+1)^{-1}][(y)^2\gamma(\overline{R})] = \eta(y); \quad z^o_{r_o} = 1;$$

$$[1+(\gamma/2)]^{-(1/2)} \le y \le [1-(\gamma/2)]^{-(1/2)}. \tag{8}$$

[cf. Eq. 5d.) It transpires that the first order approximation of $\eta_s(y)$; i.e., $\eta_I(y)$, coincides with the mean-value averaging of the undulations that beset the exact levels of the induced loss factor $\eta_s(y)$ when the modal overlap parameter (b) is small compared with unity [1, 2, 5]. [cf. Fig. 3d of Reference 2.] The first order approximation $\eta_I(y)$ of the induced loss factor is thus independent of (b). Then, the first order approximation $\eta_I(y)$ of $\eta_s(y)$ is free of erosion. (It is speculated that the erosion is manifested in the higher order approximations to $\eta_s(y)$. Since this higher order approximation procedure is geared toward accounting for the higher values of (b), the undulations at the lower values of (b) are not expected to be accounted for in this process of approximations.) In Reference 2 computations of $\eta_s(y)$ are conducted and presented in Figs. 3e and 3f and 4-7. Each figure in this series pertains first to a coupling form and second to a coupling strength. Also, each figure is a superimposition of four curves; three of the four pertain to an exact evaluation of $\eta_s(y)$ for three modal overlap parameters; i.e., $b = (0.1)$, (2.0) and (10). The fourth curve pertains to the first order approximation stated in Eqs. (6) – (8). In this manner the content of information in these figures is high and can be deciphered at a glance. It is assumed that these figures are available to the reader.

A loss factor is basically an energetic quantity and, therefore, one may inquire whether the induced loss factor $\eta_s(y)$, as just determined via the linear impedance analysis (LIA), may be estimated via an energy analysis (EA). In this latter determination the induced loss factor is designated $\eta_s^e(y)$. A question arises: Are $\eta_s(y)$ and $\eta_s^e(y)$, or to that matter the respective first order approximations $\eta_I(y)$ and $\eta_I^e(y)$, respectively, identical quantities? A sense of such an identity, or even a disparity, if any, may help interpret the salient features that they commonly and individually possess. To this end the paper is largely dedicated.

II. Derivation of the Energy Equation of Motion

The linear equation of motion of the complex composed of a master oscillator coupled to a set of satellite oscillators may be largely derived via the Lagrange equations. The Lagrangian describes the difference between the kinetic and potential energies stored in the oscillators and in the couplings [3]. (Notwithstanding that the linear equations are correctly stated in Reference 3, a persistent typographical error in the preceding Langrange's equations needs to be corrected.) Although the kinetic and potential energies are here determined separately, it is the energy and not the Langrangian, as such, that is of immediate interest. The kinetic energy $E_{oK}(y)$ plus the potential energy $E_{oP}(y)$ stored in the master oscillator is given by

$$E_o(y)=E_{oK}(y)+E_{oP}(y); \quad E_{oK}(y)=(1/2)M\,|V_o(y)|^2 ;$$

$$E_{oP}(y)=(y)^{-2}E_{oK}(y); \quad y=(\omega/\omega_o), \tag{9}$$

where $E_o(y)$ is the stored energy in the master oscillator [3]. Similarly, the kinetic energy $E_{rK}(y)$ and the potential energy $E_{rP}(y)$ that is stored in the (r)th satellite oscillator is given by

$$E_r(y)=E_{rK}(y)+E_{rP}(y); \quad E_{rK}(y)=(1/2)\,m_r\,|V_r(y)|^2;$$

$$E_{rP}(y)=(z_r)^2E_{rK}(y); \quad z_r=(x_r/y), \tag{10}$$

where $E_r(y)$ is the stored energy in the (r)th satellite oscillator [3]. In addition, the kinetic energy $E_{crK}(y)$ and the potential energy $E_{crP}(y)$ stored in the coupling, between the master oscillator and the (r)th satellite oscillator, is given by

$$E_{cr}(y) = E_{crK}(y) + E_{crP}(y), \tag{11a}$$

$$E_{crK(y)} = (1/2)m_r \Big[\overline{m}_c \,|\, V_o(y) + V_r(y) \,|^2$$
$$+ \mathrm{Im}\{ (\overline{g}_r / y)[V_o(y) + V_r(y)]\,[V_o(y) - V_r(y)]^* \}\Big], \tag{11b}$$

$$E_{crP} = (1/2)m_r (z_{cr})^2 \,|\, V_o(y) - V_r(y) \,|^2, \tag{11c}$$

where $E_{cr}(y)$ is the stored energy in the coupling between the master oscillator and the (r)th satellite oscillator [3]. Using Eq. (2) one obtains, from Eqs. (9) – (11), the expression for the stored energies

$$\overline{E}_o(y) \cong [1 + (y)^{-2}]; \quad \overline{E}_o(y) = [E_o(y) / E_{oK}(y)], \tag{12a}$$

$$\overline{E}_r(y) = [E_r(y) / E_{oK}(y)] = \overline{m}_r [\{1 + (z_r)^2\} |\, B_r \,|^2], \tag{12b}$$

$$\overline{E}_{cr}(y) = [E_{cr}(y) / E_{oK}(y)]$$
$$= \overline{m}_r [\overline{m}_{cr} \,|\, 1 + B_r \,|^2 + (z_{cr})^2 \,|\, 1 - B_r \,|^2 - i(\overline{g}_r / y)(B_r - B_r^*)], \tag{12c}$$

respectively, where a *single bar* over a stored energy quantity, e.g., $\overline{E}_r(y)$, indicates a normalization by the energy quantity $E_{oK}(y)$. In particular, the so normalized energy $\overline{E}_{or}(y)$, stored in the (r)th satellite oscillator and in the coupling of this satellite oscillator to the master oscillator, is given by

$$\overline{E}_{or}(y) = \overline{E}_r(y) + \overline{E}_{cr}(y)\,; \quad \overline{E}_{os}(y) = \sum_{1}^{R} \overline{E}_{or}(y), \tag{12d}$$

where $\overline{E}_{os}(y)$ is the normalized energy stored in the satellite oscillators and in the couplings of these oscillators to the master oscillator.

III. Conservation of Energy – the Balance of Power – and the Derivation of the Induced Loss Factor

Remembering that the external drive acts, by definition, only on the master oscillator, the normalized external input power $\bar{\Pi}_e(y)$ that is available to maintain the stored energies in the complex may be derived from the linear equation of motion. This equation is

$$Z_o(\omega)V_o(\omega) = P_e(\omega), \tag{13a}$$

$$Z_o(\omega)=(i\omega M)[1-(y)^{-2}\{[1-S(y)+i[\eta_o+\eta_s(y)]\}] , \tag{13b}$$

where $Z_o(\omega), V_o(\omega)$ and $P_e(\omega)$ are, respectively, the linear impedance, the response and the external drive that induces the response in the master oscillator when coupled. [cf. Eq. (1b) and (2b) and, also Eqs. (5) and (7a) of Reference 2.] This external input power is derived in the form

$$\bar{\Pi}_e(y) = [\Pi_e(y)/\{\omega E_{oK}(y)\}]; \quad \Pi_e(y) = \mathrm{Re}\{P_e(y)\, V_o^*(y)\};$$

$$(y^2/2)\bar{\Pi}_e(y) = [\eta_o+\eta_s(y)] \bullet 1, \tag{14}$$

where, hereafter, a *single bar* over a power quantity; e.g., $\bar{\Pi}_e(y)$, indicates a normalization by the power quantity $\{\omega E_{oK}(y)\}$. [cf. Eq. (12).] The third of Eq. (14) states that a portion $\bar{\Pi}_o(y)$ of $\bar{\Pi}_e(y)$ is dissipated in the master oscillator such that

$$(y^2/2)\, \bar{\Pi}_o(y) = \eta_o^e(y) \bullet 1; \quad \eta_o^e(y) = \eta_o; \quad \bar{\Pi}_o(y) =[\Pi_o(y)/\{\omega E_{oK}(y)\}], \tag{15a}$$

and the portion $\bar{\Pi}_{os}(y)$ of $\bar{\Pi}_e(y)$ is dissipated in the satellite oscillators and in the couplings

$$(y^2/2)\bar{\Pi}_{os}(y) = \eta_s^e(y) \bullet 1, \qquad \bar{\Pi}_{os}(y) = [\Pi_{os}(y)/\{\omega\, E_{oK}(y)\}] \tag{15b}$$

where, as already intimated, the superscript (e) indicates that the loss factors are determined via an energy analysis (EA) and not via a linear impedance analysis (LIA). The conservation of energy (the balance of power) demands that

$$\bar{\Pi}_o(y) + \bar{\Pi}_{os}(y) = \bar{\Pi}_e(y)\,. \tag{16}$$

Employing Eq. (12d), Eq. (15b) may be decomposed in the form

$$\bar{\Pi}_{os}(y) = \sum_1^R \bar{\Pi}_{or}(y); \qquad \bar{\Pi}_{or}(y) = \bar{\Pi}_r(y) + \bar{\Pi}_{cr}(y), \tag{17}$$

where by definition

$$\bar{\Pi}_r(y) = \eta_r\, \bar{E}_r(y)\,; \qquad \bar{\Pi}_{cr}(y) = \eta_{cr}\, \bar{E}_{cr}(y)\,. \tag{18}$$

The loss factors (η_r) and (η_{cr}) are the stiffness control loss factors associated with the springs on the fore and the back sides of the mass (m_r) of the (r)th satellite oscillator. The spring on the back side constitutes the stiffness coupling form. The spring on the fore side renders the satellite oscillator an *oscillator* rather than merely a *mass*. It is assumed that the damping is provided in these springs only [1, 2]. Provisions for other types of damping can be made, but the increase in algebraic complexity can hardly be justified at this stage.

From Eqs. (15b), (17) and (18) one obtains the energetic version of the induced loss factor $\eta_s^e(y)$ in the form

$$\eta_s^e(y) = (y^2/2)\sum_1^R \{\eta_r \, \overline{E}_r(y) + \eta_{cr} \, \overline{E}_{cr}(y)\} \, , \tag{19}$$

where $\overline{E}_r(y)$ and $\overline{E}_{cr}(y)$ are more explicitly expressed in Eqs. (12b) and (12c). The induced loss factor $\eta_s(y)$, derived via (LIA), is explicitly expressed in Eqs. (2b) and (2c) [1, 2]. To what degree then is $\eta_s^e(y)$, stated in Eq. (19), equal to $\eta_s(y)$? To establish analytically the answer to this question may require undue algebraic manipulations which, again, can hardly be justified at this stage. Instead, the equivalence is cursorily tested computationally. Imposing the simplifications stated in Eq. (5), the explicit expression for $\eta_s^e(y)$ may be reduced to the form

$$\eta_s^e(y) = (y^2/2) \sum_1^R \eta(\overline{r}) \, \overline{E}_{or}(y) \equiv (y^2/2) \, \eta^e(y) \overline{E}_{os}(y) \, , \tag{20}$$

where $\overline{E}_{os}(y)$ is stated in Eq. (12d), $\eta^e(y)$ is the loss factor, as a function of (y), of a typical satellite oscillator with its resonance frequency in the vicinity of (y), $\eta(\overline{r})$ is stated in Eq. (5c) and depicted in Fig. 2, and $\overline{E}_{or}(y)$ is the normalized energy stored in the (r)th satellite oscillator and in the coupling of this oscillator to the master oscillator; namely

$$\bar{E}_{or}(y) = \bar{m}_r\,(1+\bar{m}_c)^{-2}\{\,[1-(z_r^o)^2\,]^2 + \,[(z_r^o)^2\,\eta(\bar{r})\,]^2\,\}^{-1}$$

$$\bullet\,\{\,[1+\alpha\,(z_r^o)^2\,]\,[\,\{\bar{m}_c+\alpha_c(z_r^o)^2\}^2+\{\alpha_c\,\eta(\bar{r})(z_r^o)^2-(\bar{g}_r\,/\,y)\}^2\,]$$

$$+\,\bar{m}_c\,[\{1-(1+\bar{m}_c+\alpha_c)\,(z_r^o)^2\}^2 + \{t_+(y)\}^2\,]$$

$$+\,[\alpha_c\,(z_r^o)^2\,]\,[\{(1+2\,\bar{m}_c)-\alpha\,(z_r^o)^2\}^2 + \{t_-(y)\}^2\,]$$

$$-\,(\bar{g}_r\,/\,y)\,[\,\{1-(1+\bar{m}_c+\alpha_c)(z_r^o)^2\}\,\{\alpha\,\eta(\bar{r})(z_r^o)^2 + (\bar{g}_r\,/\,y)\}$$

$$-\,\{(1+2\,m_c)-\alpha\,(z_r^o)^2\}\,\{(1+\bar{m}_c+\alpha_c)\,\eta(\bar{r})-(\bar{g}_r\,/\,y)\}\,]\,\}, \qquad (21a)$$

where the explicit expressions for $t_+(y)$ and $t_-(y)$ are

$$t_+(y) \;=\; (1+\bar{m}_c)\,\eta(\bar{r})(z_r^o)^2 + [\,\alpha_c\,\eta(\bar{r})(z_r^o)^2 - (\bar{g}_r\,/\,y)]\,;$$

$$t_-(y) \;=\; (1+\bar{m}_c)\,\eta(\bar{r})(z_r^o)^2 - [\,\alpha_c\,\eta(\bar{r})(z_r^o)^2 - (\bar{g}_r\,/\,y)]\,. \qquad (21b)$$

Specific parameters involved in Eqs. (20) and (21) are defined in Eqs. (2) – (5). Using Eqs. (20) and (21)and the simplifications and condition stated in Eq. (5), $\eta_s^e(y)$ is computed and the results are presented in Figs. 3-6. These figures are cast in the format of Figs. 3e and 3f and 4-7 of Reference 2. Thus, each figure is the superimposition of four curves: three exact evaluations of $\eta_s^e(y)$ for the three values of the modal overlap parameter (b); b = (0.1), (2.0) and (10) and one for the first order approximation. (The first order approximation for $\eta_s^e(y)$, that is employed, is derived subsequently.) Again, in the manner of this display the content of information in these figures is high and can be deciphered at a glance, notwithstanding that the comparison with Figs. 3e and 3f and 4-7 of Reference 2 is, thereby, facilitated.

In References 1 and 2 the summation for $\eta_s(y)$ is replaced by an integration. The first order approximation of this integral is evaluated and is stated in Eqs. (6) – (8). A similar procedure, replacing the summation by an integration, is now applied to Eq. (20). The first order approximation only is evaluated. The induced loss factor so evaluated yields

$$\eta_s^e(y) = D[C + O^e\{\eta^e(y)\}^2]; \quad \eta_I^e(y) = D\,C, \tag{22}$$

where (D), (C) and the range of validity in (y) are as stated in Eqs. (7) and (8), but

$$O^e = (1/2)\,[(1+\bar{m}_c)(\bar{m}_c + \alpha_c) + 2\,\bar{m}_c\,\alpha_c]. \tag{23}$$

In the frequency range of validity, the loss factors $\eta(y)$ and $\eta^e(y)$ in Eqs. (6) and (22), respectively, are found to be identical. From Eq. (8) one finds

$$\eta(y) = \eta^e(y) = [b(R+1)^{-1}][(y)^2\,\gamma(\bar{R})];$$

$$[1+(\gamma/2)]^{-(1/2)} \le y \le [1-(\gamma/2)]^{-(1/2)}. \tag{24}$$

The evaluations of $\eta^e(y)$ [and $\eta(y)$] for b = (0.1), (2.0) and (10) are presented graphically in Fig. 7. In Fig. 7a, R=27 and in Fig. 7b, R=7. The essential similarity of $\eta^e(y)$ with the corresponding $\eta(\bar{r})$ is clear when Figs. 7 and 2 are appropriately compared. (One recalls that the loss factor, presented graphically in Fig. 2, is the loss factor that is designed and assigned to the (r)the satellite oscillator [2].) Since $\eta(y)$ and $\eta^e(y)$ are identical, the first order approximation (FOA) of $\eta_s(y)$ and of

$\eta_s^e(y)$, stated in Eqs. (6) and (22), respectively, differ in that the coefficients (O) and (O^e) are not identical. Usually, the terms $(DO)\ \{\eta(y)\}^2$ and $(DO^e)\ \{\eta^e(y)\}^2$ are negligible and, therefore, the difference between them is rarely significant. Notwithstanding that these terms are considered to belong with the higher order approximations and, as such, they are removed from the first order approximations [2]. Thus, it is concluded that the first order approximations of $\eta_s(y)$ and $\eta_s^e(y)$ are identical. Moreover, from Eqs. (6) and (22) these first order approximations are independent of the modal overlap parameter (b) [2].

Remarkably, the equivalence between Figs. 3-6 and the corresponding Figs. 3e and 3f and 4-7 of Reference 2 is not only in terms of gross features, but also in details; e.g., the erosions in Figs. 3e and 3f and 4-7 of reference are duplicated, to a tee, in Figs. 3-6, respectively, at least, in the range of the normalized frequency of concern. [cf. Eqs. (8) and (24).]

IV. Renormalization of Stored Energies and Powers – Illegitimate and Legitimate Loss Factors

It has been adopted that quantities; e.g., the stored energy $E_{os}(y)$ and the dissipated power $\Pi_{os}(y)$, when normalized by $E_{oK}(y)$ and $\{\omega E_{oK}(y)\}$, respectively, are designated by a single bar; namely, $\bar{E}_{os}(y)$ and $\bar{\Pi}_{os}(y)$. Situations arise in which a normalization by $E_o(y)$ and $\{\omega E_o(y)\}$, respectively, may be preferred. Such normalizations are to be designated by a *double bar*; namely, $\bar{\bar{E}}_{os}(y)$ and $\bar{\bar{\Pi}}_{os}(y)$, respectively. In particular then

$$\eta_\infty(y) = [\Pi_o(y)/\{\omega E_o(y)\}] = \bar{\bar{\Pi}}_o(y) = 2[1+(y)^2]^{-1}\eta_o, \tag{25a}$$

$$\eta_{os}^e(y) = [\Pi_{os}(y)/\{\omega E_o(y)\}] = \bar{\bar{\Pi}}_{os}(y) = 2[1+(y)^2]^{-1}\eta_s^e(y), \tag{25b}$$

$$\eta_{os}^e(y) = \sum_1^R \eta(\bar{r})\bar{\bar{E}}_{or}(y) = \eta^e(y)\bar{\bar{E}}_{os}(y). \tag{25c}$$

It emerges then that whereas the loss factors (η_o) and (η_s^e) relate to the normalization employing the kinetic energy $E_{oK}(y)$ and the power $\{\omega E_{oK}(y)\}$, the loss factors $\eta_\infty(y)$ and $\eta_{os}^e(y)$ relate to the corresponding quantities normalized by the stored energy $E_o(y)$ and the power $\{\omega E_o(y)\}$. The stored energy $E_o(y)$ is the combined energy, consisting of the kinetic energy $E_{oK}(y)$ and the potential energy $E_{oP}(y)$ stored in the master oscillator; $E_o(y) = E_{oK}(y) + E_{oP}(y)$, as stated in Eq. (9). Equation (25c) is then a renormalized version of Eq. (20). The ratio of the energy $E_{os}(y)$ stored in the satellite oscillators and in the couplings to that of the energy $E_o(y)$ stored in the master oscillator may be dubbed the *global coupling strength* and, as such, is designated $\Xi_o^s(y)$. From Eqs. (25a) and (25b) one obtains

$$\eta_{os}^e(y) = 2\,[1+(y)^2\,]^{-1}\,\eta_s^e(y) = [\eta^e(y)\,\Xi_o^s(y)]\,;$$

$$\Xi_o^s(y) = \overline{\overline{E}}_{os}(y) = [\,E_{os}(y)/E_o(y)\,]. \tag{26a}$$

Were one to construct, from Eqs. (22) and (26a) the hybrid expression

$$[\eta^e(y)\,\Xi_o^s(y)]_h = 2[1+y^2\,]^{-1}\begin{cases}\eta_I^e(y); & b \le 1, & \text{(26b)}\\ \eta_s^e(y); & b \ge 1, & (26c)\end{cases}$$

then, except for erosion at the higher values of the modal overlap parameter; $b \ge 1$, one finds the hybrid quantity $[....]_h$ to be largely undulations free and independent of (b). Referring to Eqs. (26b) and (26c), it follows that in the hybrid milieu, $\eta^e(y)$ is inversely proportional to $\Xi_o^s(y)$ and vice versa. Taking note of Eq. (24), there is no way for a physically acceptable complex to entertain a vanishing $\eta^e(y)$. Of course, Eq. (26a) does not fall under this spell, but neither can one cavalierly ignore the undulations that beset $\eta_s^e(y)$ when (b) is reduced with the intention of rendering $\eta^e(y)$ negligible [2].

Being cognizant of the second of Eq. (26a), the global coupling strength $\Xi_o^s(y)$ may feature in another renormalization in which the entire energy $E_e(y)$ stored in the complex is employed. This stored energy is given by

$$E_e(y) = E_o(y) + E_{os}(y) = E_o(y)\,[1+\Xi_o^s(y)], \tag{27}$$

and the corresponding renormalization for the powers is given by $\{\omega\, E_e(y)\}$. Such renormalizations are to be designated by a *tilde* over the quantity; e.g.

$$\tilde{E}_{os}(y) = [E_{os}(y)/E_e(y)]; \quad \tilde{\Pi}_{os}(y) = [\Pi_{os}(y)/\{\omega\, E_e(y)\}]. \tag{28a}$$

From Eqs. (25), (27), and (28a) one may derive

$$\tilde{E}_{os}(y) = [E_{os}(y)/E_e(y)] = \bar{\bar{E}}_{os}(y)[1+\Xi_o^s(y)]^{-1}, \tag{28b}$$

$$\tilde{\Pi}_{os}(y) = [\Pi_{os}(y)/\{\omega\, E_e(y)\}] = \bar{\bar{\Pi}}_{os}(y)[1+\Xi_o^s(y)]^{-1}, \tag{28c}$$

$$\bar{\bar{E}}_e(y) = [1+\Xi_o^s(y)]. \tag{28d}$$

The two renormalizations just introduced, in Eqs. (25) and (28), lead to the definition of two distinct loss factors. The first is defined in terms of the energy $E_o(y)$ stored in the master oscillator only; namely

$$\eta_t^e(y) = [\Pi_e(y)/\{\omega\, E_o(y)\}] = [\eta_\infty(y) + \eta^e(y)\ \Xi_o^s(y)], \tag{29a}$$

where use is made of Eqs. (14) and (16) . The second is defined in terms of the energy $E_e(y)$ stored in the entire complex; namely

$$\eta_e^e(y) = \Pi_e(y)/\{\omega\, E_e(y)\} = \eta_t^e(y)[1+\Xi_o^s(y)]^{-1}, \tag{30a}$$

where use is made of Eqs. (28) and (29a). These two loss factors were previously defined and discussed [6, 7]. In these discussions it was claimed that $\eta_t^e(y)$ is not a legitimate loss factor. This designation was primarily predicated on assigning to the definition of this loss factor the entire external input power $\Pi_e(y)$, but accounting for the stored energy $E_o(y)$ that is maintained in the master oscillator only, thereby, ignoring the portion of the stored energy that is maintained in the satellite oscillators and in the couplings. (No wonder the question of "where did the energy go?" arose from accepting $\eta_t^e(y)$ as a legitimate loss factor [8-13].) In this connection one recalls that

$$\eta_t^e(y) = [\eta_\infty(y) + \eta_{os}^e(y)]; \qquad \eta_{os}^e(y) = [\eta^e(y)\Xi_o^s(y)]. \tag{29b}$$

Again, since $\eta_{os}^e(y)$ is found to be largely independent of $\eta^e(y)$, one concludes, from Eq. (29b), that $\eta_t^e(y)$ is also independent of $\eta^e(y)$. Equation (29) makes clear that to change $\eta_t^e(y)$ either the coupling parameters, the mass ratio (M_s / M) or both, need changing. In this assessment, η_o is assumed to be fixed, and $\eta_\infty(y) = 2\eta_o(1+y^2)^{-1}$. [cf. Eq. (25a).] On the other hand, the second loss factor, dubbed the *effective* loss factor and designated $\eta_e^e(y)$, does take into account the whole energy stored in the complex [6, 7]. It may thus be designated a legitimate loss factor. From Eqs. (27) and (30a) one finds

$$\eta_e^e(y) = \eta_t^e(y)\,\{\eta^e(y)\,[\eta^e(y) + \eta_{os}^e(y)]^{-1}\} \leq \eta_t^e(y). \tag{30b}$$

In particular, if the induced loss factor $\eta_{os}^e(y)$ exceeds the indigenous loss factor $\eta_\infty(y)$ of the master oscillator, Eq. (30b) may be further reduced to

$$\eta_e^e(y) \cong [\eta^e(y)\eta_{os}^e(y)]\,[\eta^e(y)+\eta_{os}^e(y)]^{-1} < \begin{cases} \eta_{os}^e \\ \eta^e(y) \end{cases} ;\quad \eta_\infty(y) < \eta_{os}^o(y). \tag{30c}$$

Thus, the effective loss factor $\eta_e^e(y)$, under this condition, is a parallel combination of the induced loss factor $\eta_{os}^e(y)$ and the loss factor $\eta^e(y)$ assigned to a typical satellite oscillator. It follows that $\eta_e^e(y)$ is less than either one of these loss factors. Consider a reasonable complex for which $\eta_{os}^e(y) > \eta^e(y)$. Under this additional condition, Eq. (30c) may be further reduced to

$$\eta_e^e(y) \cong \eta^e(y);\quad \eta_{os}^e(y) > \eta^e(y). \tag{30d}$$

Were a noise control effort intended to achieve an effective loss factor $\eta_e^e(y)$ that exceeds the indigenous loss factor $\eta_\infty(y)$ of the basic master oscillator, a number of design criteria need be implemented. In this exercise the noise control is to be derived from the attachment of a number of satellite oscillators to this master oscillator [12, 15]. [cf. Fig. 1.] If the two conditions stated in Eqs. (30c) and (30d) are to be satisfied, it follows, from Eqs. (22) and (27), that the three design criteria

$$\eta_\infty(y) < \eta^e(y)\,\Xi_o^s(y) = \eta_{os}^e(y)\,, \tag{31a}$$

$$\Xi_o^s(y) > 1, \tag{31b}$$

$$\eta_{\infty}(y) < \eta^{e}(y), \tag{31c}$$

must be simultaneously satisfied [7]. The first of these criteria, then, demands that the induced loss factor $\eta_{os}^{e}(y)$ exceeds the loss factor $\eta_{\infty}(y)$ of the isolated master oscillator. To satisfy this criterion, it is observed that both $\eta^{e}(y)$ and $\Xi_{o}^{s}(y)$ be designed to possess high values, as dictated in Eq. (31a). How high? As stated in Eq. (31b), the energy $E_{os}(y)$ stored in the satellite oscillators and in the couplings need be in excess of the energy $E_{o}(y)$ stored in the master oscillator. This is achieved by increasing the coupling parameters and the modal density (number of satellite oscillators per unit frequency) of the satellite oscillators [7]. Simultaneously, a typical loss factor $\eta^{e}(y)$ of the satellite oscillator need be maintained in excess of $\eta_{\infty}(y)$. This condition ensures that the dampings in the satellite oscillators can handle the excess in the stored energy that they are designed to harbor [7]. This handling needs to exceed that which is indigenous to the master oscillator, and hence the criterion stated in Eq. (31c). Details of these criteria may be cast in terms of the parameters that describe the master oscillator, the satellite oscillators and the couplings between them. These details may be drawn from the preceding equations. To help with the interpretation of these details, a number of computations are performed. In these computations the loss factors $\eta_{t}^{e}(y)$ and $\eta_{e}^{e}(y)$, as functions of (y), are evaluated and contrasted. Again, only a few representative cases are displayed. The displays are given in Figs. 8-11. The presentation covers cases that parametrically conform to those governing Figs. 3-6, respectively. The two loss factors; $\eta_t^e(y)$ and $\eta_e^e(y)$, are contrasted on the same figure and each is compounded by a superimposition of curves that pertain, in turn, to the modal overlap parameters $b = (0.1)$, (2.0) and (10)). [cf. Fig. 7.] In addition to the evaluations of $\eta_t^e(y)$ and $\eta_e^e(y)$, the corresponding levels of $[(R)^{-1}\,\Xi_o^s(y)]$ are also evaluated, as a function of (y), and are displayed in Figs. 12-15. Again, three curves that pertain to the modal overlap parameters $b = (0.1)$, (2.0) and (10) are superimposed in these figures. The undulations in the levels when (b) is less than unity and the suppression of the undulations when (b) is in excess of unity is clearly apparent in these figures too. The first order approximation of $[(R)^{-1}\,\Xi_o^s(y)]$ are also superimposed on these figures. This superimposition exposes, once again, the phenomena that the mean-value averaging of the undulations coincide with the first order approximation (FOA) and that erosion commences and increases as (b) approaches and increases beyond unity [1, 2, 5, 12].

V. Relationship to the Statistical Energy Analysis (SEA)

It may be conducive to include in this thesis a possible relationship between the energy analysis (EA) dealt with in the preceding three sections and the statistical energy analysis (SEA). The latter analysis was initiated in the early 1960s at BBN and subsequently has become a major tool in noise control engineering [3, 16]. A rudimentary (SEA) is applied to a complex comprising of a master oscillator coupled to a set of individual satellite oscillators; the satellite oscillators are neither coupled to each other nor externally driven. [cf. Fig. 1.] In terms of (SEA) the equations that govern the energy distribution in the complex are

$$[\eta_\infty(y)+\sum_1^R \eta_{ro}(y)]\,E_o(y)-\sum_1^R \eta_{or}(y)\,E_{or}(y)=[\Pi_e(y)/\omega], \tag{32a}$$

$$[\eta_r(y)+\eta_{or}(y)]E_{or}(y)-\eta_{ro}(y)\,E_o(y)=0, \tag{32b}$$

where $E_o(y)$, $E_{or}(y)$, $\eta_\infty(y)$, $\eta_r(y)$ and $\Pi_e(y)$ are previously defined in Eq. (9), (12d), (25a), (5d) and (14), respectively [3, 16]. In Eq. (32) $\eta_{or}(y)$ and $\eta_{ro}(y)$ are the *coupling loss factors* from the (r)th satellite oscillator to the master oscillator and vice versa, respectively. In a conservative coupling $\eta_{or}(y)=\eta_{ro}(y)$ [3]. After a straightforward algebraic manipulation of Eq. (32) one obtains

$$\eta_t^e(y)=[\eta_\infty(y)+\eta_{os}^e(y)]; \qquad \eta_{os}^e(y)=\sum_1^R \eta_r(y)\,\zeta_o^r(y);$$

$$\zeta_o^r(y)=[E_{or}(y)/E_o(y)]=\eta_{ro}(y)[\eta_r(y)+\eta_{or}(y)]^{-1}, \tag{33}$$

where Eq. (29) is consulted and $\zeta_o^r(y)$ is the *modal coupling strength* of the (r)th satellite oscillator to the master oscillator [3, 16]. From Eqs. (29) and (33) one further obtains that

$$\eta_{os}^e(y) = \eta^e(y)\Xi_o^s(y); \quad \Xi_o^s(y)=\sum_1^R \zeta_o^r(y). \tag{34a}$$

In this role the stored energy ratio $\Xi_o^s(y)$ may be designated, as already mentioned, the *global coupling strength*. Again, with a slight of *statistical* hand, Eq. (34a) is approximated

$$\Xi_o^s(y) = \zeta_o^s(y)\,[N_s(y)U\{R-N_s(y)\} + RU\{N_s(y)-R\}], \tag{34b}$$

where $N_s(y)$ is the number of satellite oscillators that contribute viable modal coupling strengths. A typical modal coupling strength is designated $\zeta_o^s(y)$; the viable modal coupling strengths are assumed to be statistically indistinguishable. The use of viability is in reference to investigations revealing that the coupling loss factor, typically $\eta_{or}(y)$ or $\eta_{ro}(y)$, decreases as the frequency disparity increases between the resonance frequency of a satellite oscillator and the resonance frequency of the master oscillator to which it is coupled [3]. As the disparity between the resonance frequencies increases beyond a specific threshold, the modal coupling strength decreases sharply [3, 4]. The threshold is set by either a critical value of the frequency disparity when typically $\eta_r(y) > \eta_{or}(y)$ or a critical value of the frequency of transition (y_t) when typically $\eta_r(y) < \eta_{or}(y)$, both inequalities are validated only in a frequency range for which (y) is at and in the vicinity of unity. In the latter case the frequency of transition is defined by $\eta_r(y_t) \simeq \eta_{or}(y_t)$. Consulting Reference 1 and imposing the simplification rendered in Eq. (5a), one may derive

$$N_s(y) \simeq (\Delta\omega\ b)[\omega_r\,\eta^e(y)]^{-1} \simeq (R+1)(\Delta y/y)[(1+\overline{m}_c)^{1/2}(y^2)\gamma(\overline{R})]^{-1}. \qquad (35a)$$

Using Eqs. (4) and (24) one finds that

$$\gamma(\overline{R}) = [\gamma/(2\overline{R})]; \quad (\Delta y/y) \simeq (\gamma/2); \quad \gamma < 1, \qquad (36a)$$

and, therefore, from Eqs (35a) one obtains

$$N_s(y) \simeq R\,[(1+\overline{m}_c)^{1/2}(y)^2]^{-1} \simeq R, \qquad (35b)$$

indicating that all the satellite oscillators which resonance frequency distribution is defined in Eqs. (2) and (4), may be designated viable. Equations (34b) and (35b) yield

$$\zeta_o^s(y) \simeq (R)^{-1}\,\Xi_o^s(y); \quad \Xi_o^s(y) \simeq (R)\,\zeta_o^s(y), \qquad (37)$$

Thus, the typical modal coupling strength $\zeta_o^s(y)$ is one $(1/R)$th the global coupling strength, both quantities are appropriately averaged [3]. From Eqs. (20), (25), (26) and (37) one obtains

$$\zeta_o^s(y) = (y)^2[(1+y^2)R]^{-1}\sum_1^R \overline{E}_{or}(y), \qquad (38a)$$

where $\overline{E}_{or}(y)$ is explicitly stated in Eq. (21). Under the condition that allows the summation in Eq. (19) to be replaced by integration, one derives for the first order approximation of a typical modal coupling strength

$$\zeta_o^s(y) \Rightarrow \zeta_I^s(y) = (4\pi/b)\,(M_s/M)[\gamma^2\,y^2\,(1+y^2)\,(1+\bar{m}_c)]^{-1}(C);$$

$$C = [(\bar{m}_c + \alpha_c)^2 + (\bar{g}/y)^2];$$

$$[1+(\gamma/2)]^{-(1/2)} \leq y \leq [1-(\gamma/2)]^{-(1/2)}, \tag{38b}$$

where use is made of Eqs. (6), (7), (22) and (23). A number of sets of parametric values that define previous figures are identified and applied to Eq. (38a). The results of this application are presented, for three values of (b); namely, $b = (0.1)$, (2.0) and (10), in Figs. 12-15. Also superimposed in these figures are the levels of the first order approximation of $\zeta_o^s(y)$, as stated in Eq. (38b). Moreover, if there are undulations in levels in curves based on Eq. (38a), these undulations are suppressed in the corresponding curves based on Eq. (38b). Again, neglecting the phenomenon of erosion, this suppression is commensurate with the appropriate mean-value averaging, notwithstanding that the phenomenon of erosion is found in these figures too. An immediate violation, however, is revealed in Figs. 12-15; in particular, in figures pertaining to strong couplings and a modal overlap parameters (b) that are small. Since the maximum value of a modal coupling strength $\zeta_o^r(y)$ in a rudimentary (SEA); e.g., in Eq. (33), cannot, by definition, exceed unity, (SEA) is invalidated for a complex for which a typical modal coupling strength $\zeta_o^s(y)$, as stated in Eq. (38), exceeds unity. For example, the levels pertaining to $b = (0.1)$ in Figs. 12a, 13a and 15a, which display $\zeta_o^s(y)$, as a function of (y), stand in obvious violation of a core tenet of (SEA). The contradictions that arise in Fig. 12a and others, in which levels of $\zeta_o^s(y)$ that are in excess of unity are found, do not, however, negate the analysis under which they are derived, they merely lie outside the range of validity for the use of (SEA). It, thus, emerges that (SEA) has a modal overlap parameter threshold; situations arise in which (SEA) is not valid for certain degrees of coupling strengths unless the modal overlap parameters assigned to the satellite oscillators exceed that threshold.

A final remark briefly challenges a noise control estimate. The estimate is guided by all three analyses (LIA), (EA) and (SEA). The challenge is an attempt to dispel an assumption that often leads to overestimation of the benefits that may be accrued were the noise control guide implemented. Focusing on Eqs. (1), (2), (9), (25) and (28) one may define the ratio $\varepsilon_o(y)$, of the kinetic energy $E_{oK}(y)$ stored in the *coupled* to the kinetic energy $E_{oK}^o(y)$ stored in the *uncoupled* master oscillator, in the form

$$\varepsilon_o(y) = [E_{oK}(y) / E_{oK}^o(y)] = [|V_o(y)|^2 / |V_o^o(y)|^2]$$

$$\varepsilon_o(y) = P^o(y)[\eta_\infty(y) / \eta_t^e(y)]; \quad P^o(y) = [\Pi_e(y) / \Pi_e^o(y)], \qquad (39)$$

where $P^o(y)$ is the ratio of the external input powers into the *coupled* and the *uncoupled* master oscillator. The loss factor $\eta_t^e(y)$ is stated in Eq. (29); namely

$$\eta_t^e(y) = [\eta_\infty(y) + \eta_{os}^e(y)]; \quad \eta_{os}^e(y) = \eta^e(y)\ \Xi_o^s(y). \qquad (40)$$

The blessings of the noise control are assessed in terms of the smallness of $\varepsilon_o(y)$ as compared with unity; a beneficial noise control is one for which $\varepsilon_o(y) << 1$. Often one tends, in this assessment, to argue that the external input power ratio $P^o(y)$ is largely equal to unity [7]. Using Eq. (40) and this argument, Eq. (39) reduces to

$$\varepsilon_o(y) \Rightarrow \varepsilon_o^o(y) = [\eta_\infty(y)/\eta_t^e(y)] = \eta_\infty(y)\,\{\eta_\infty(y)+\eta_{os}^e(y)\}^{-1};\;\; P^o(y)\equiv 1. \qquad (41a)$$

Employing the first order approximation of $\eta_{os}^e(y)$ in Eqs. (40) and (41a), one further obtains

$$\varepsilon_o^o(y) \Rightarrow \varepsilon_I^o(y) = \eta_o\,\{\eta_o + \eta_I^e(y)\}^{-1};\;\; P_I^o(y) \equiv 1. \qquad (41b)$$

From Eq. (41) it becomes clear that if $\eta_\infty(y)$ and (η_o) are much less than $\eta_{os}^e(y)$ and $\eta_I^e(y)$, respectively, the noise control that can be projected is impressive. For example, as Figs. 3-6 and 8-11 testify, if $\eta_o = (10^{-3})$ $[\eta_\infty(y) = 2(1+y^2)^{-1} \bullet (10^{-3})$.] then for strong and even moderate couplings, the projected noise control is two to three orders of magnitude. In passing, except for a minor erosion consideration, these noise control benefits are independent of the modal overlap parameter (b), notwithstanding that the undulations at low values of (b); $b << 1$, need to be recognized and handled with care. An obvious question then arises: Are these projections of noise control real? Is the ratio $P^o(y)$ or its first order approximation $P_I^o(y)$, equal to unity as assumed? After all, the external input power injected into a dynamic system is critically dependent on the loss factor that the system presents to the external drive. As Eq. (29) shows, the coupling of a set of satellite oscillators to the master oscillator enhances the loss factor that the master oscillator presents to the external drive. It is, thus, expected that the external input power ratio $P^o(y)$ harbors levels that exceed unity, thereby, mitigating the noise control achievement predicted in Eq. (41). This kind of mitigation conforms with Le Chatelier's Principle [18.] If the mass of the satellite oscillators remain within reasonable limits, the rosy noise control projections are mitigated in the manner

$$P^o(y) = [1+\Xi_o^s(y)]; \quad \varepsilon_o(y) = [1+\Xi_o^s(y)]\varepsilon_o^o(y) = [\eta_\infty(y)/\eta_e^e(y)];$$

$$\Xi_o^s(y) = R\zeta_o^s(y), \tag{42a}$$

and the equivalent first order approximation

$$P_I^o(y) = [1+\Xi_I^s(y)]; \quad \varepsilon_o(y) = [1+\Xi_I^s(y)]\,\varepsilon_I^o(y);$$

$$\Xi_I^s(y) = R\zeta_I^s(y), \tag{42b}$$

where use is made of Eqs. (6), (7), (22), (30) and (37)-(39) [7, 17]. It is recalled that $\Xi_o^s(y)$ is the ratio of the energy stored in the satellite oscillator and in the couplings to the energy stored in the master oscillator. Design conditions may be found to render this ratio large compared with unity [7]. Except for minor erosions and undulations that may need to be recognized and handled for low values of (b), Figs. 12-15 exemplify that for strong and even moderate coupling strengths one may find levels of $\Xi_o^s(y)$ and $\Xi_I^s(y)$ that exceed unity by an order or two of magnitude [7]. Such large levels may mollify much of what Eq. (41) promises. Indeed, Eqs. (42a) and (42b) suggest that if $\eta^e(y)$, which is the loss factor assigned to a typical satellite oscillator, is less than the loss factor $\eta_\infty(y)$ of the master oscillator in isolation, even a noise control reversal may result were the coupling instituted. The unusual, but possible, conditions for such a reversal are

$$\varepsilon_o(y) \simeq [\eta_\infty(y)/\eta^e(y)] > 1, \quad \Xi_o^s(y) >> 1; \quad \eta_{os}^e(y) = \eta^e(y)\Xi_o^s(y) >> \eta_\infty(y), \quad (43a)$$

$$\varepsilon_I(y) \simeq [\eta_\infty(y)/\eta^e(y)] > 1, \quad \Xi_I^s(y) >> 1; \quad \eta^e(y)\Xi_I^s(y) >> \eta_\infty(y), \quad (43b)$$

respectively [7, 17]. [cf. Eqs. (30) and (31).] Of course, such noise control reversal may, when it occurs, be highly surprising to those who factually accept the validity of Eq. (41) .

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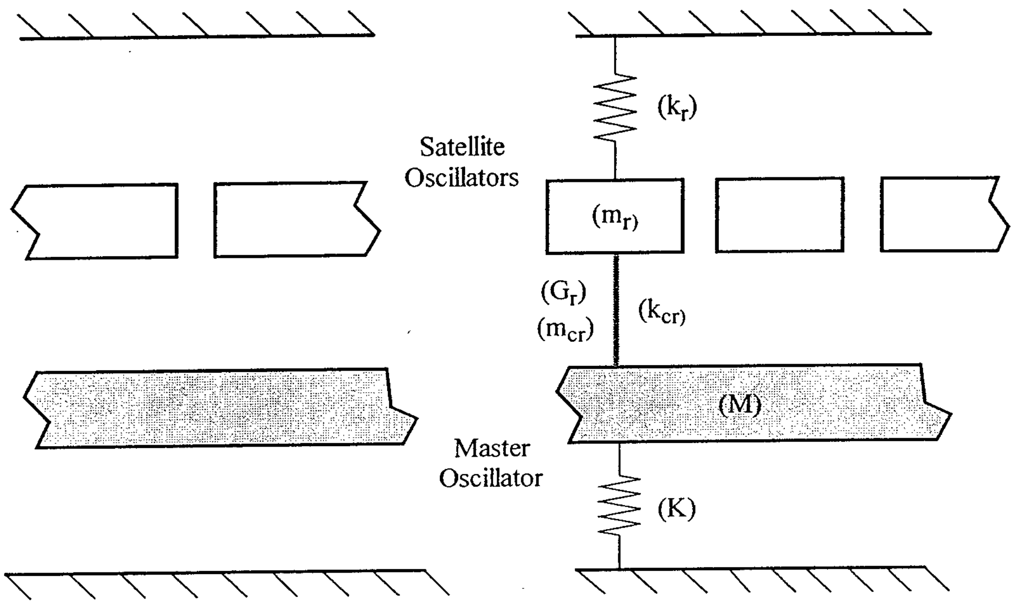
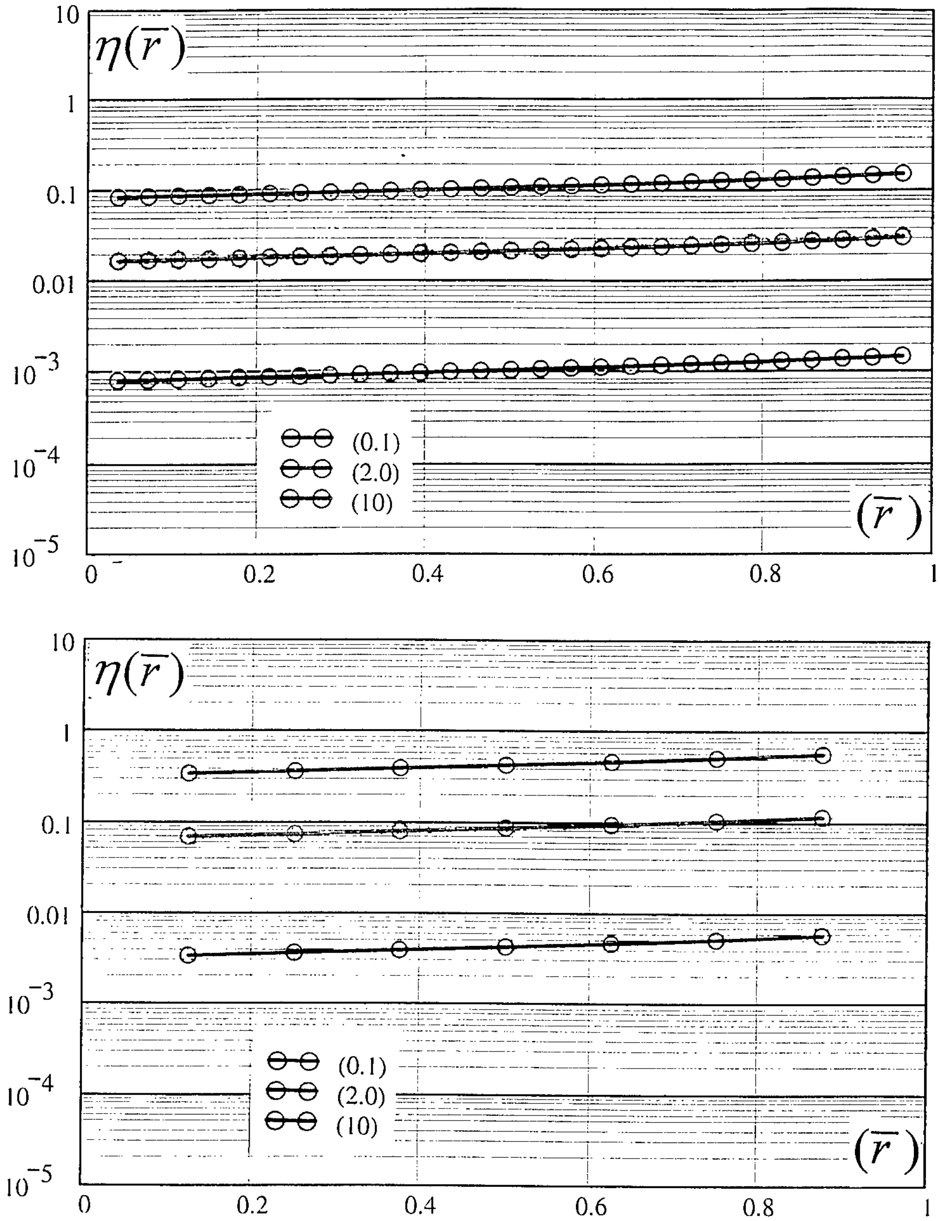


Fig. 1. A set of satellite oscillators coupled to a master oscillator.

a.



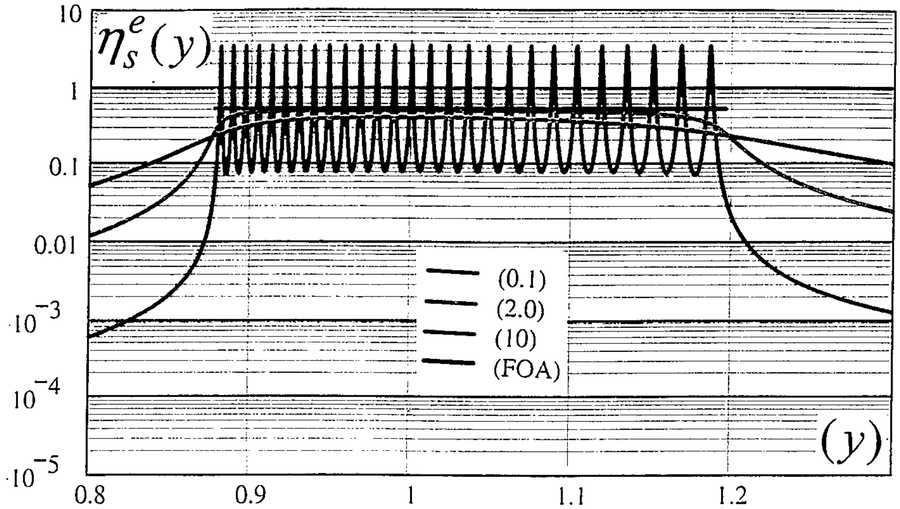
b.

Fig. 2. Loss factors $\eta(\bar{r})$ assigned to the satellite oscillators as a function of the normalized index $(\bar{r})$ and with the mass coupling parameter $(\bar{m}_c)$ equal to zero.

a. R = 27.

b. R = 7.

a.



b.

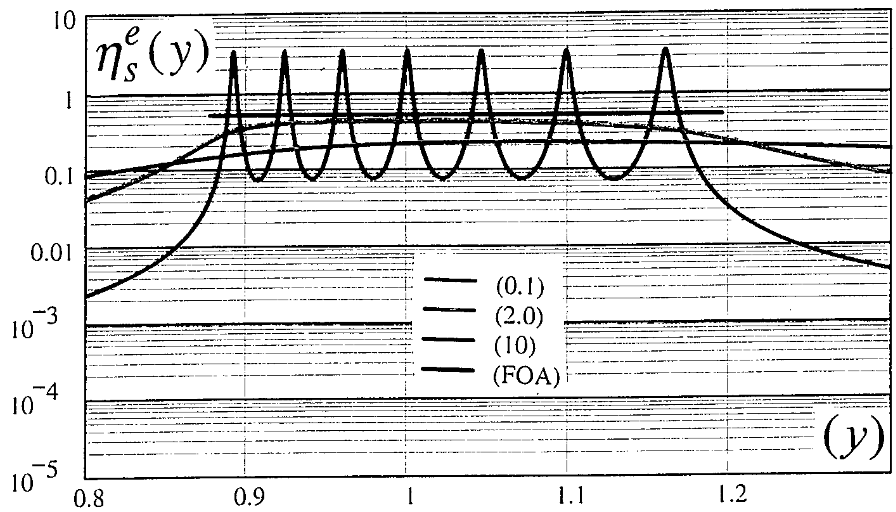


Fig. 3. Induced loss factor $\eta_s^e(y)$, as a function of (y), with stiffness control couplings. [$(M_s/N) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).]

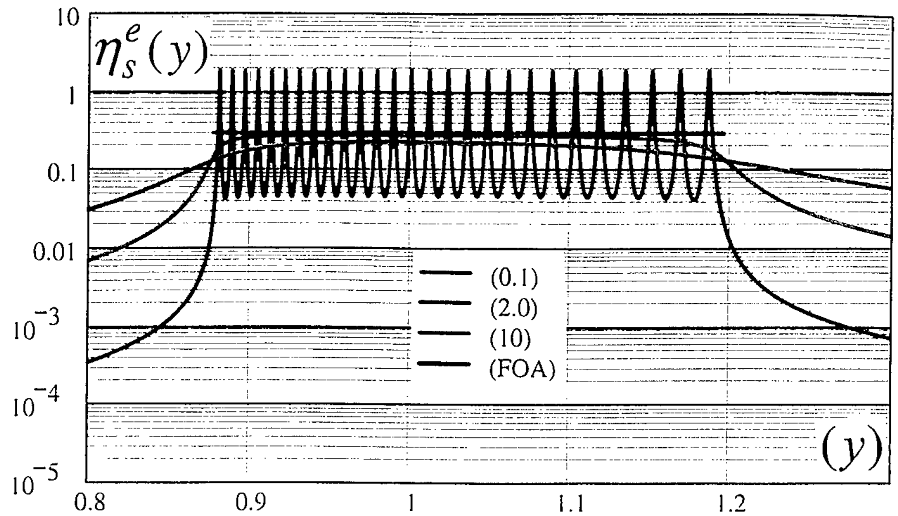
a. (R = 27) Sprung-masses: $\alpha_c = 1.0[\alpha = 0.0.]$, $\bar{g} = \bar{m}_c = 0$.

[Very strong coupling.]

b. (R = 7) Sprung-masses: $\alpha_c = 1.0[\alpha = 0.0.]$, $\bar{g} = \bar{m}_c = 0$.

[Very strong coupling.]

c.



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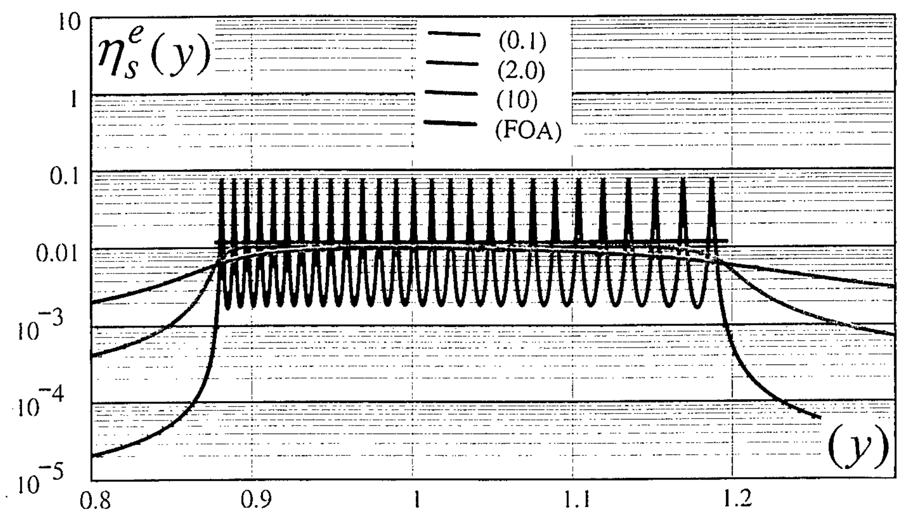


Fig. 3. Induced loss factor $\eta_s^e(y)$, as a function of (y), with stiffness control couplings. [$(M_s/N) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).]

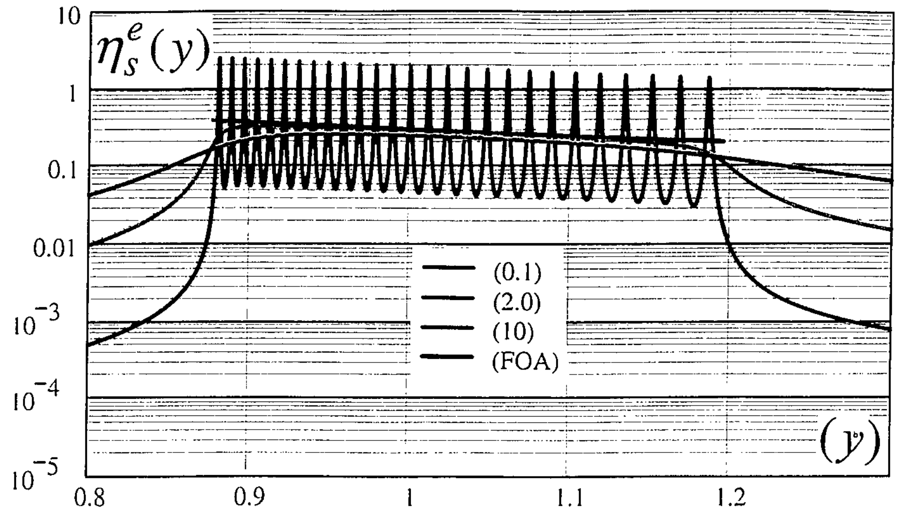
c. (R = 27) Satellite oscillators: $\alpha_c = 0.75$ [$\alpha = 0.25.$], $\overline{g} = \overline{m}_c = 0.$

[Strong coupling.]

d. (R = 27) Satellite oscillators: $\alpha_c = 0.15$[$\alpha = 0.85.$], $\overline{g} = \overline{m}_c = 0.$

[Moderate coupling.]

a.



b.

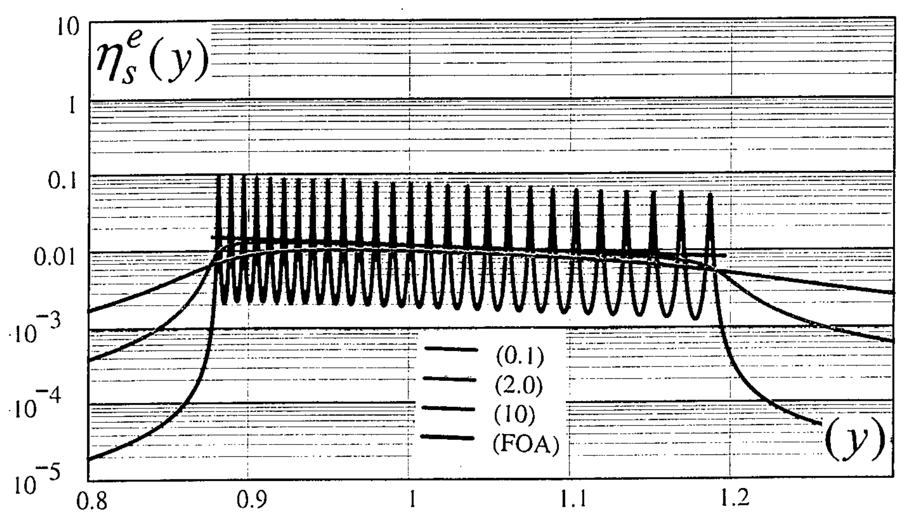
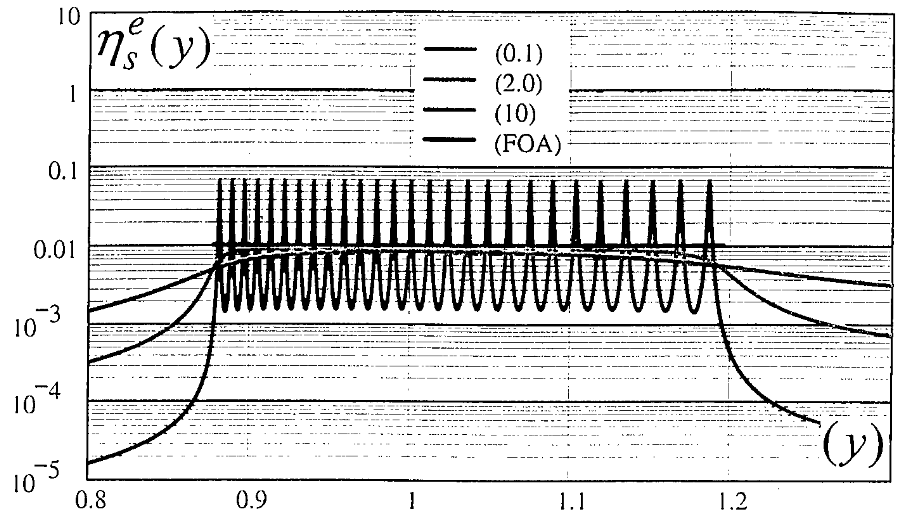


Fig. 4. Induced loss factor $\eta_s^e(y)$, as a function of (y), with gyroscopic control couplings. [R = 27 and $(Ms/M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).]

a. $\alpha_c = \overline{m}_c = 0$ [α=1.0.], $\overline{g} = 0.75$. [Strong coupling.]

b. $\alpha_c = \overline{m}_c = 0$ [α=1.0.], $\overline{g} = 0.15$. [Moderate coupling.]

a.



b.

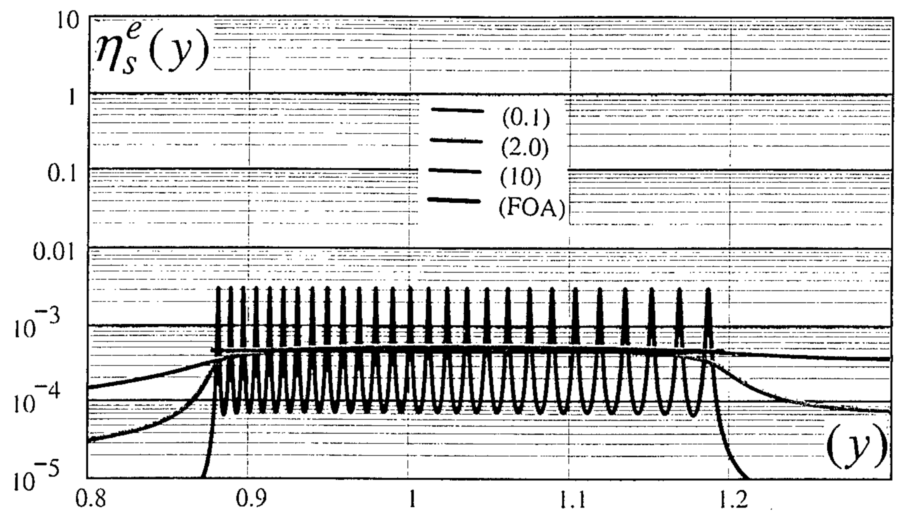
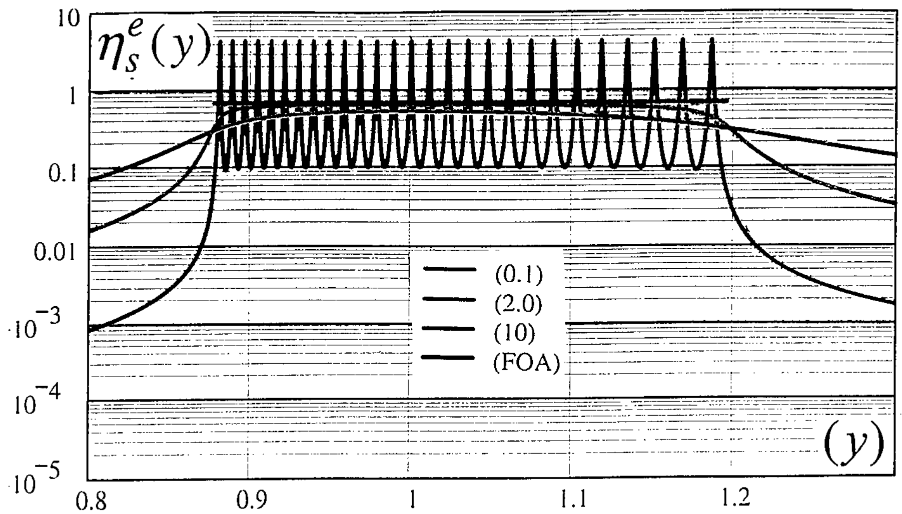


Fig. 5. Induced loss factor as a function of (y), with mass control couplings.

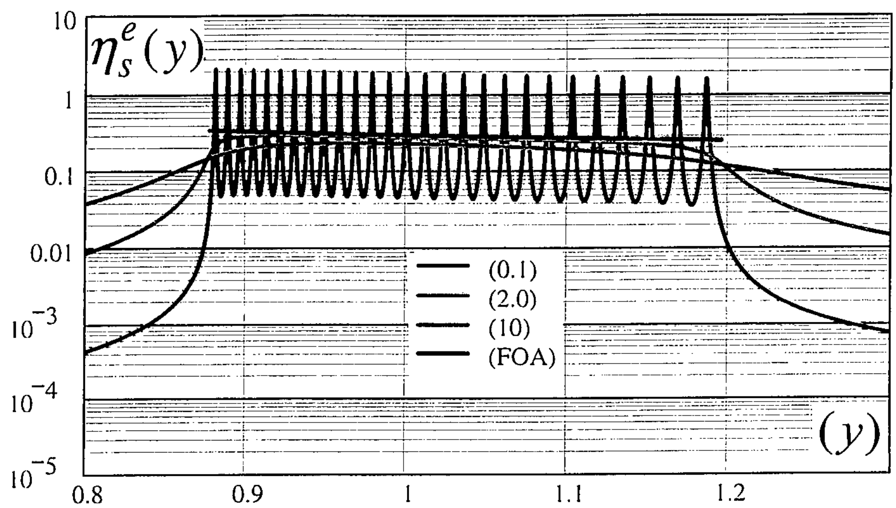
[R = 27 and $(M_s/M)=0.1$; modal overlap parameter (b) and first order approximation (FOA).]

a. $\alpha_c = \bar{g} = 0$ [α=1.5.], $\bar{m}_c = 0.15$. [Moderate coupling.]

b. $\alpha_c = \bar{g} = 0$ [α=1.03.], $\bar{m}_c = 0.03$. [Weak coupling.]



a.



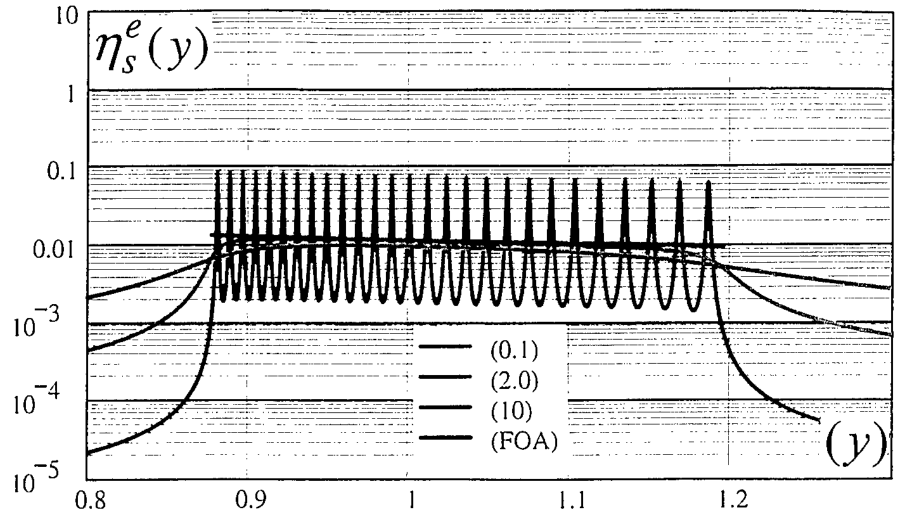
b.

Fig. 6. Induced loss factor $\eta_s^e(y)$, as a function of (y), with mix control coupling forms. [R = 27 and $(M_s / M) = 0.1$; modal overlap parameter (b) and first order approximations (FOA).]

a. $\alpha_c = \overline{m}_c = 0.75$ [α=1.0.], $\overline{g} = 0$. [Very strong coupling.]

b. $\alpha_c =$ 0.53 [α=.47.], $\overline{g} = 0.54$, $\overline{m}_c = 0$. [Strong coupling.]

c.



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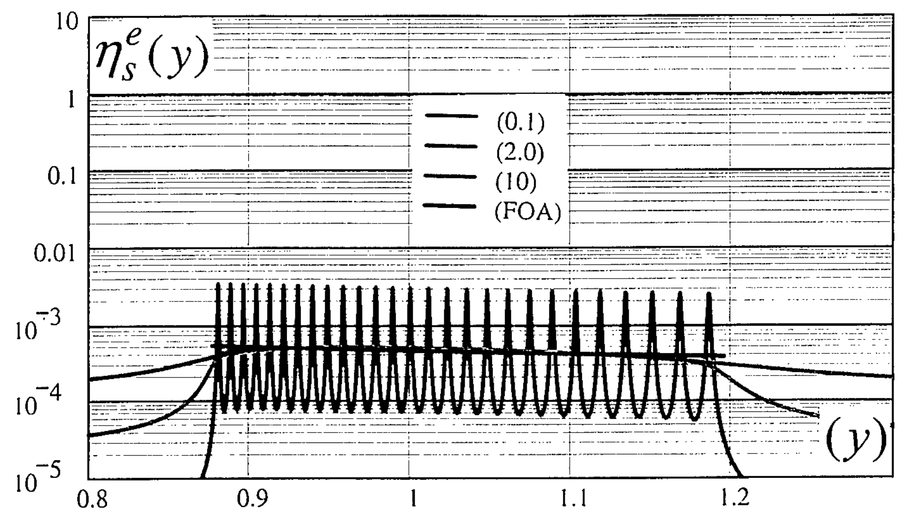
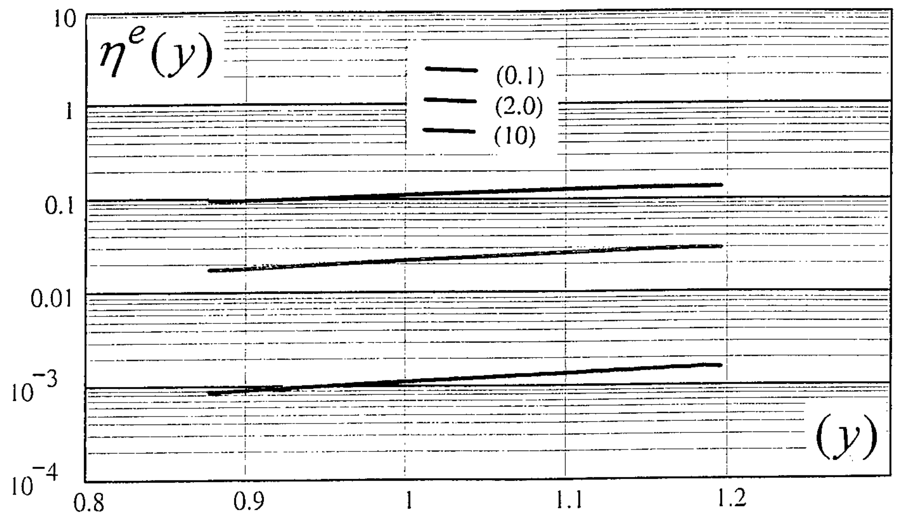


Fig. 6. Induced loss factor $\eta_s^e(y)$, as a function of (y), with mix control coupling forms. [R = 27 and $(M_s / M) = 0.1$; modal overlap parameter (b) and first order approximations (FOA).]

c. $\alpha_c =$ 0.1 [α=.9.], $\overline{g} = 0.11$, $\overline{m}_c = 0$. [Moderate coupling.]

d. $\alpha_c =$ 0.02 [α=.98.], $\overline{g} = 0.22$, $\overline{m}_c = 0$. [Weak coupling.]

a.



b.

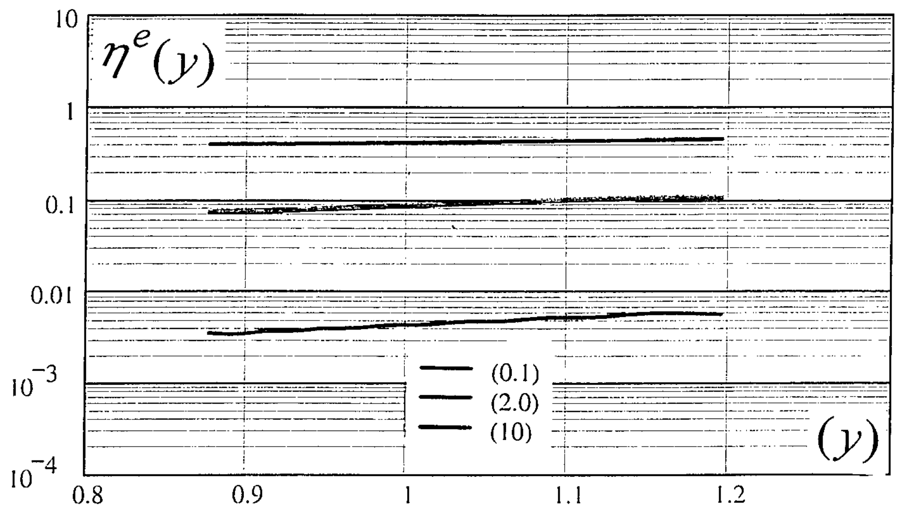
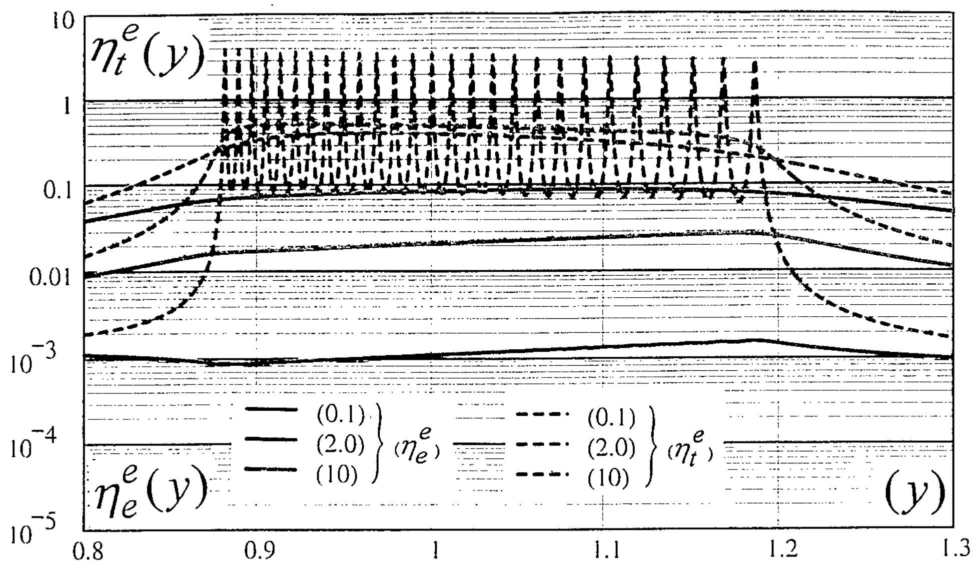


Fig. 7. First order approximation of the loss factor $\eta^e(y)$ that is assigned to individual satellite oscillators, as a function of (y), in the appropriate frequency range. [Modal overlap parameter (b).] [cf. Figs. 2a and 2b.]

a. R = 27.

b. R = 7.

a.



b.

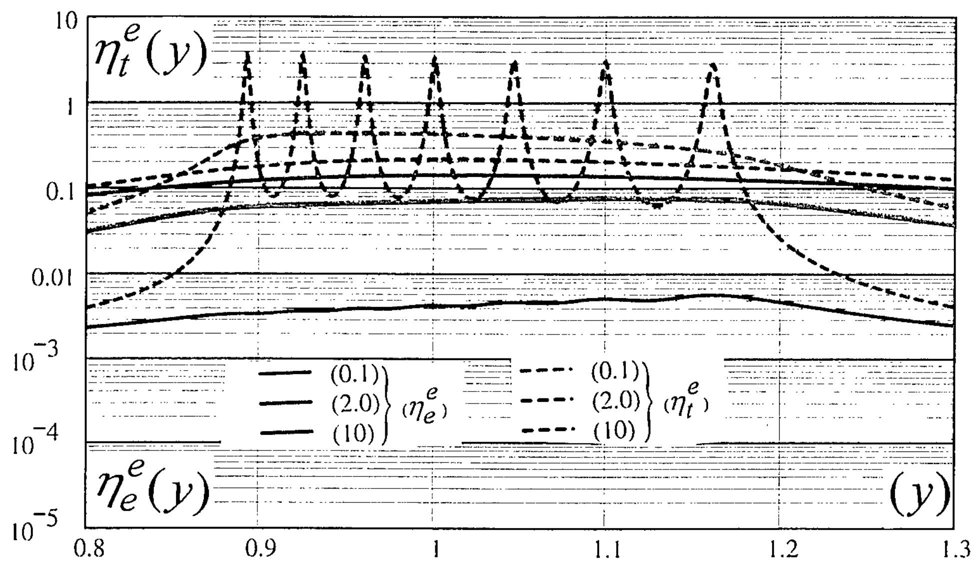
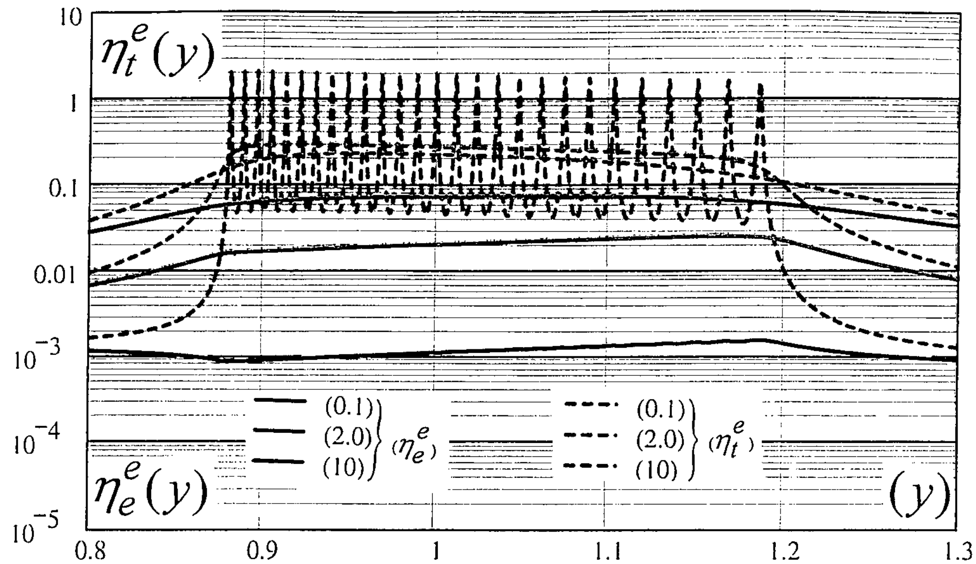


Fig. 8. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y), with stiffness control couplings. [$(M_s/M) = 0.1$ and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 3.]

a. [R = 27] Sprung-masses; $\alpha_c = 1.0$ [$\alpha = 0.0.$], $\bar{g} = \bar{m}_c = 0$. [cf. Fig. 3a.].

b. [R = 7] Sprung-masses; $\alpha_c = 1.0$ [$\alpha = 0.0.$], $\bar{g} = \bar{m}_c = 0$. [cf. Fig. 3b.].

c.



d.

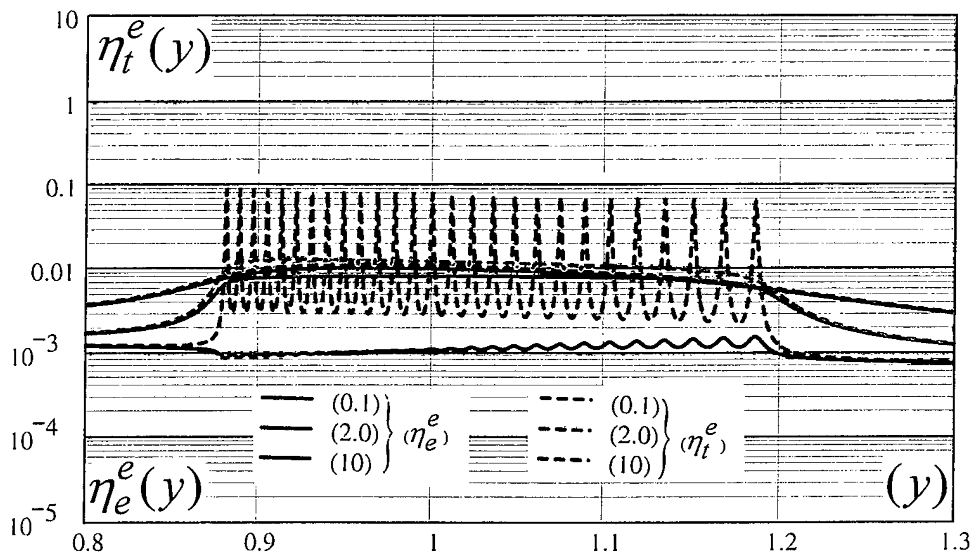
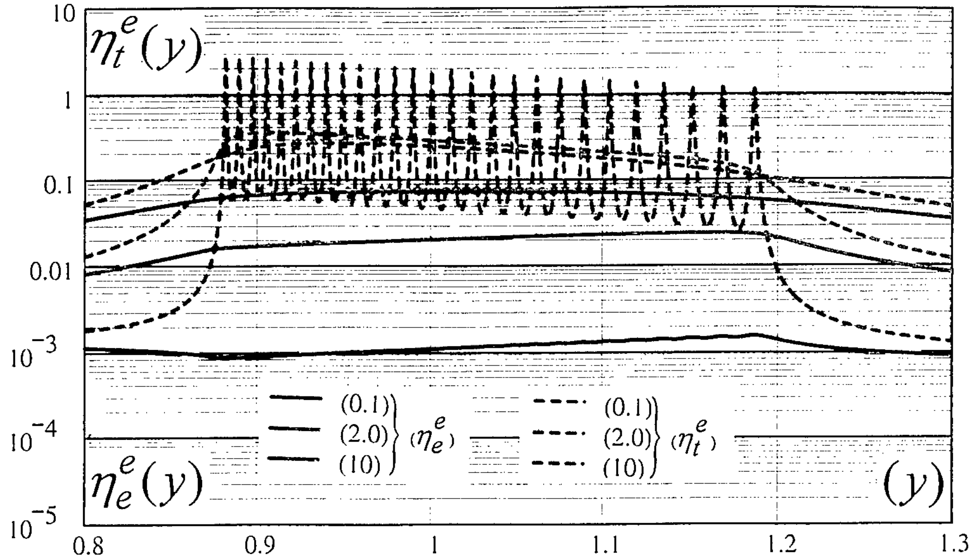


Fig. 8. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y), with stiffness control couplings. $[(M_s/M) = 0.1$ and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 3.]

c. Coupling as specified in Fig. 3c. [R = 27.]

d. Coupling as specified in Fig. 3d. [R = 27.]

a.



b.

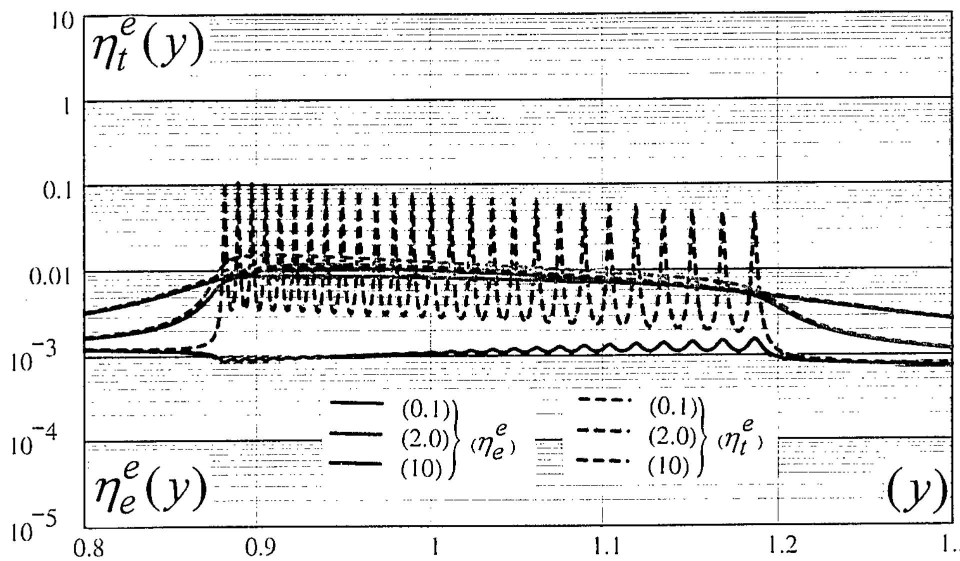


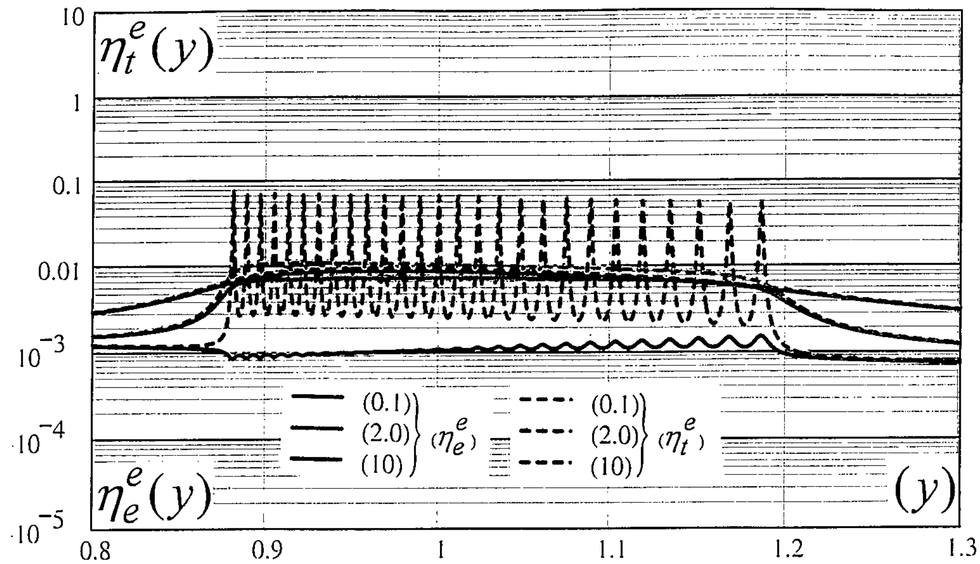
Fig. 9. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y) with gyroscopic control couplings. [R=27, $(M_s / M) = 1$ and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 4.] (*)

a. Coupling as specified in Fig. 4a.

b. Coupling as specified in Fig. 4b.

(*) When regions of curves clearly overlap, the color of the one with the higher modal overlap parameter (b) wins.

a.



b.

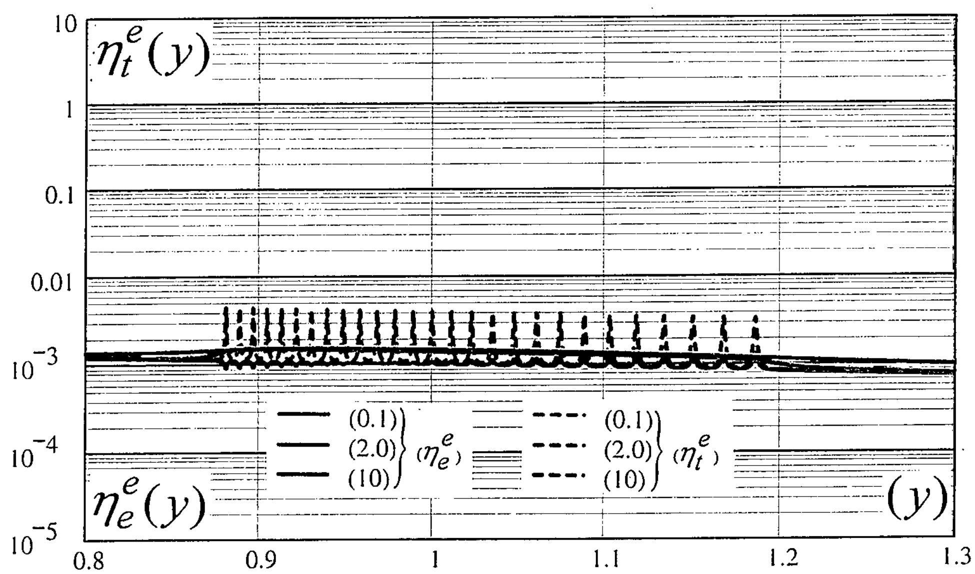
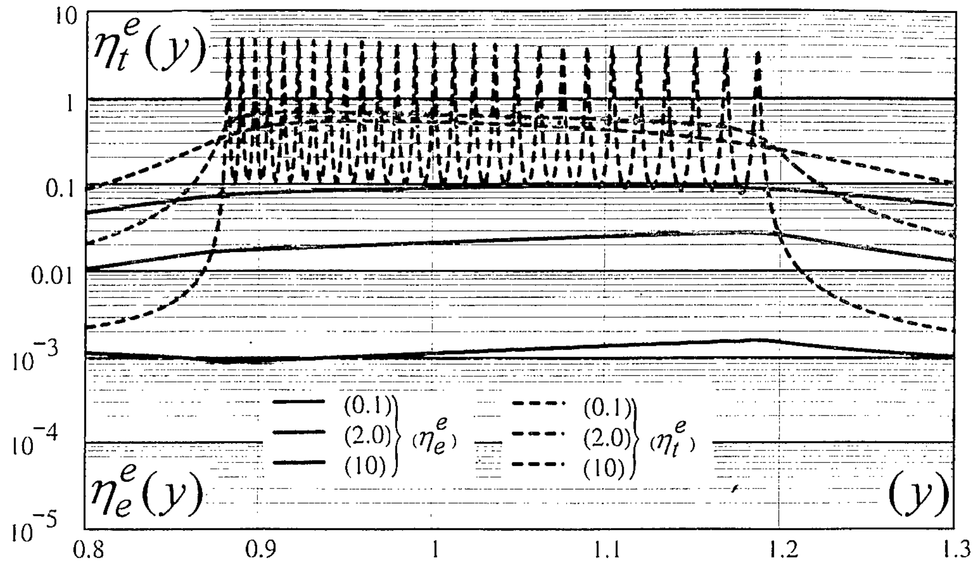


Fig. 10. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y), with mass control couplings. [R=27, $(M_s/M) = 0.1$ and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 5.] (*)

a. Coupling as specified in Fig. 5a.

b. Coupling as specified in Fig. 5b.

a.



b.

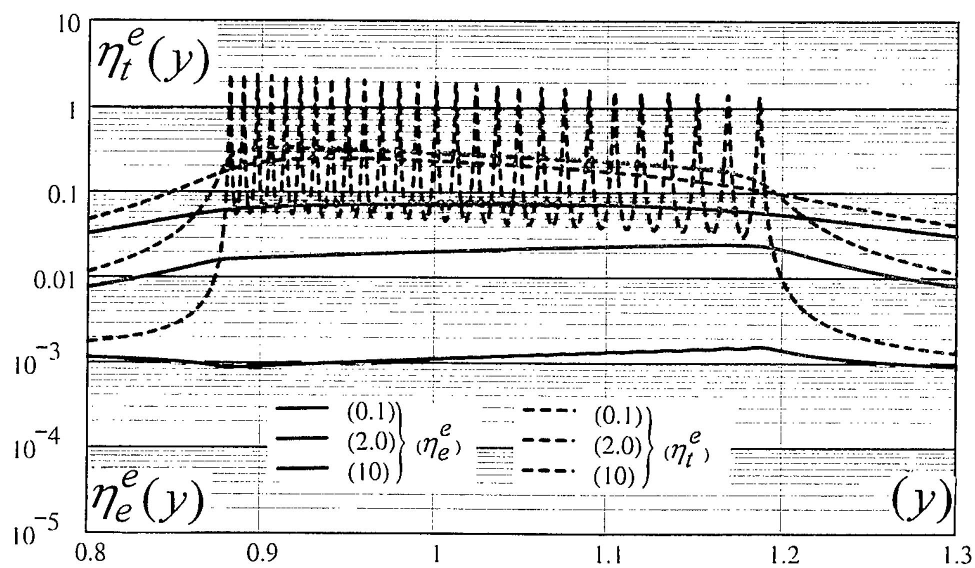
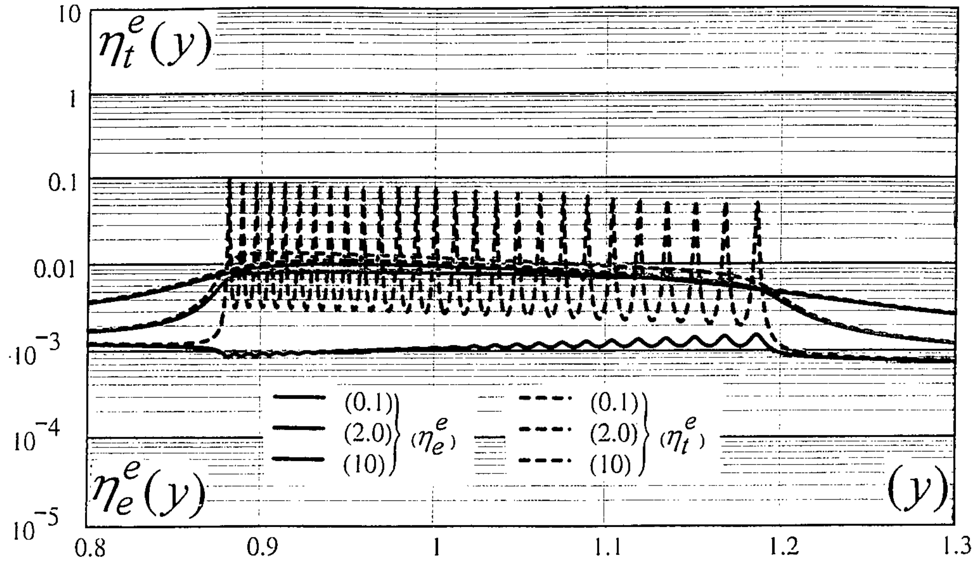


Fig. 11. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y), with mix control coupling forms. [R=27, (M_s/M) =0.1 and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 6.] (*)

a. Coupling as specified in Fig. 6a.

b. Coupling as specified in Fig. 6b.

c.



d.

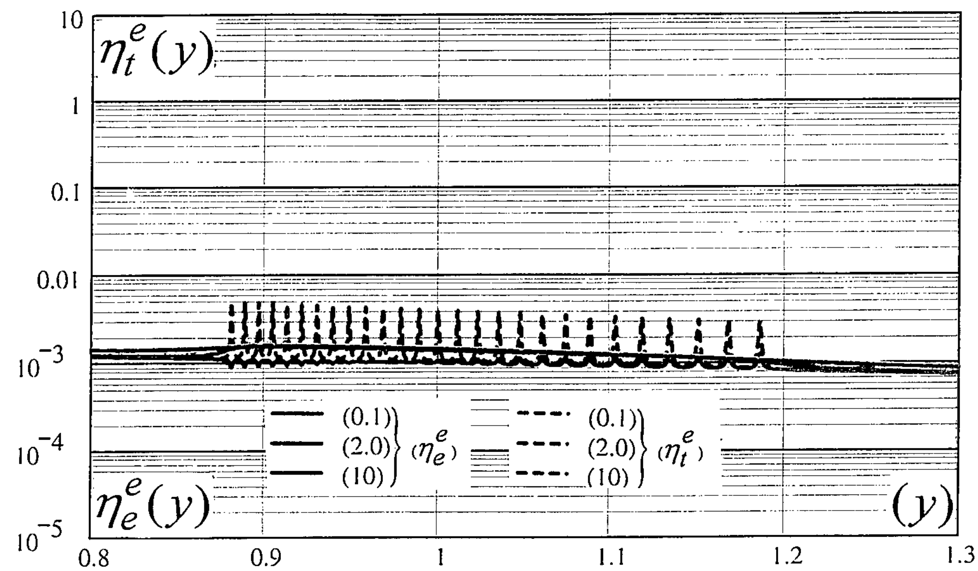
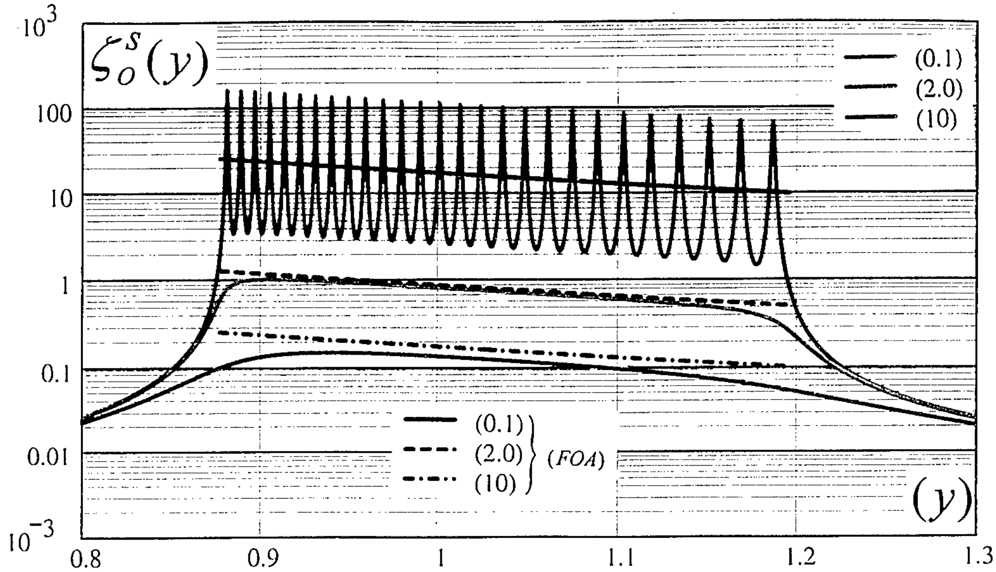


Fig. 11. Loss factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of (y), with mix control coupling forms. [R=27, $(M_s/M)=0.1$ and $\eta_o = (10^{-3})$; modal overlap parameter (b).] [cf. Fig. 6.] (*)

c. Coupling as specified in Fig. 6c.

d. Coupling as specified in Fig. 6d.

a.



b.

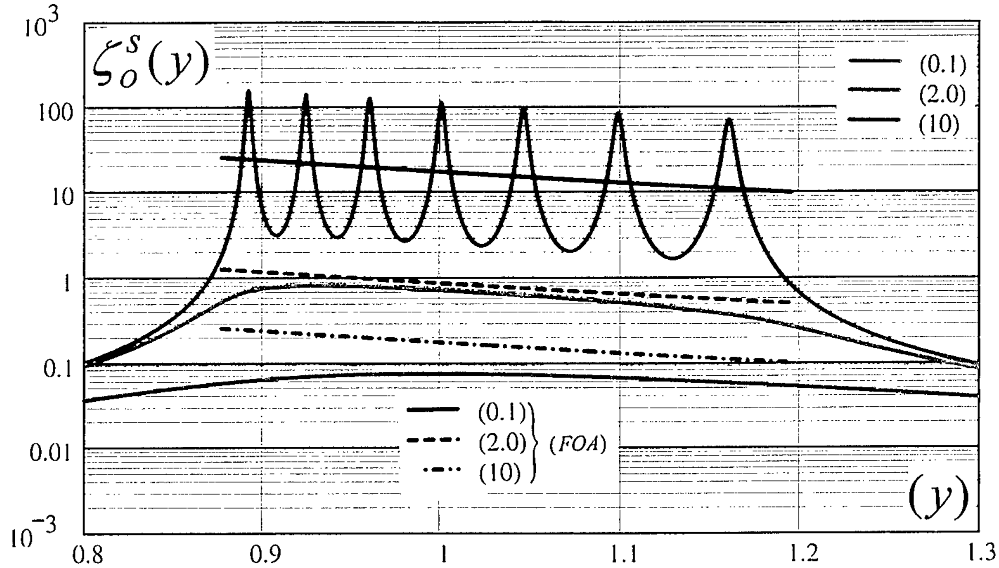
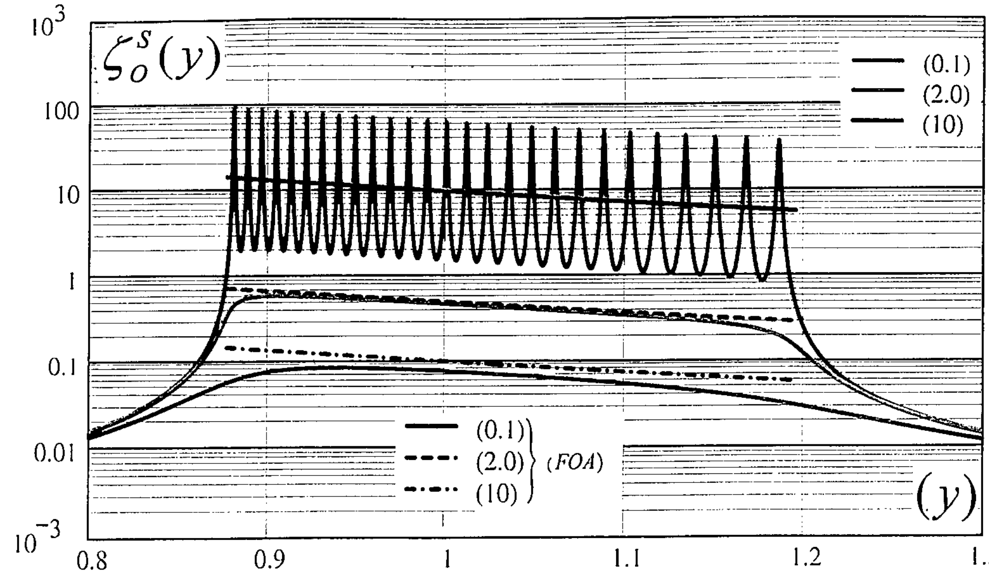


Fig. 12. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for stiffness control couplings. [$(M_s/M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] (*)

a. Coupling and number of satellite oscillators as specified in Fig. 3a.

b. Coupling and number of satellite oscillators as specified in Fig. 3b.

c.



d.

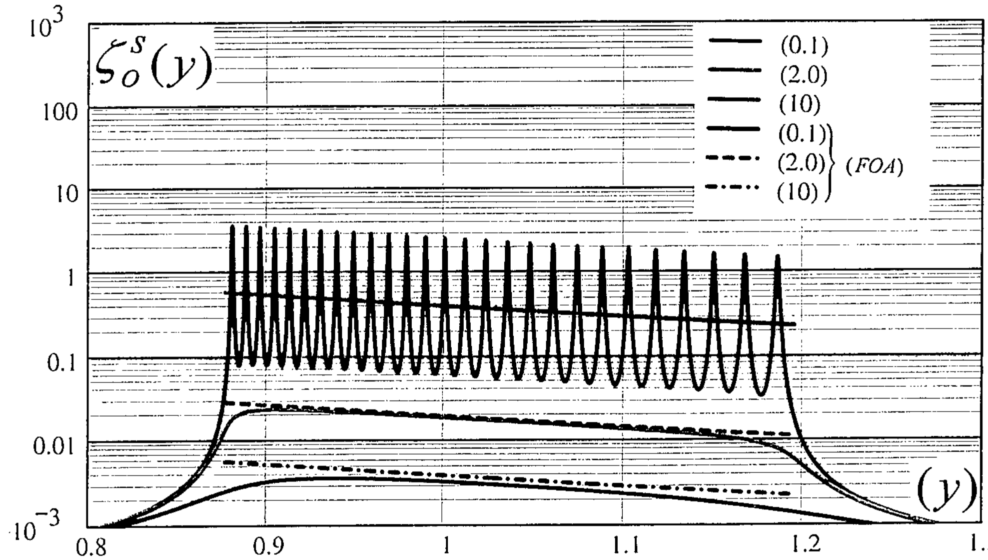
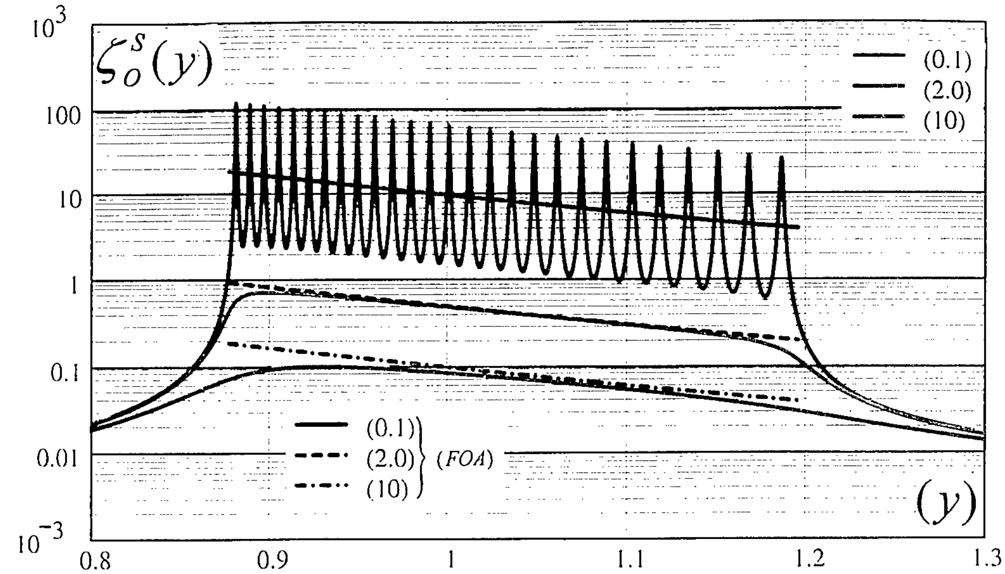


Fig. 12. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for stiffness control couplings. [$(M_s / M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] (*)

c. Coupling and number of satellite oscillators as specified in Fig. 3c.

d. Coupling and number of satellite oscillators as specified in Fig. 3d.

a.



b.

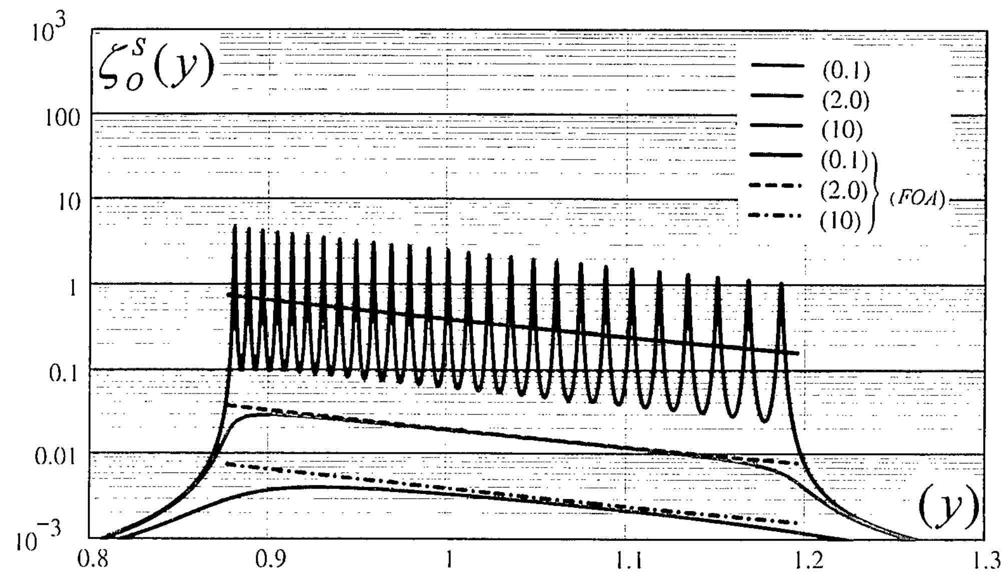
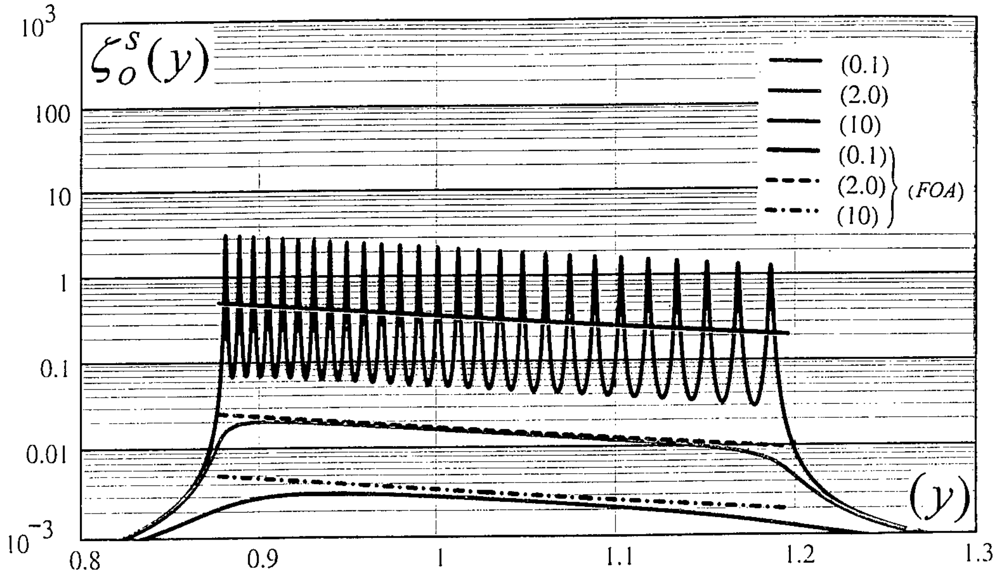


Fig. 13. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for gyroscopic control couplings. [R=27, $(M_s/M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] [cf. Fig. 4.] (*)

a. Coupling as specified in Fig. 4a.

b. Coupling as specified in Fig. 4b.

a.



b.

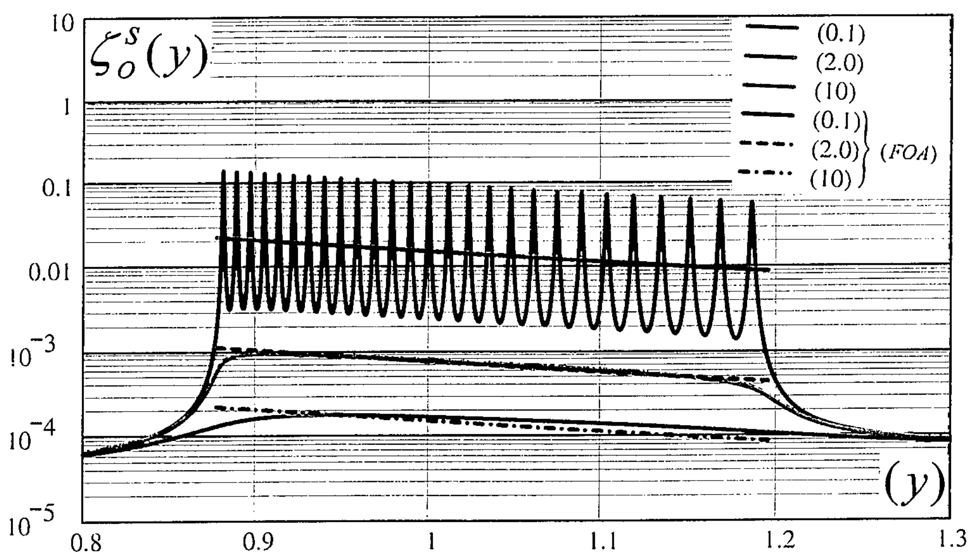
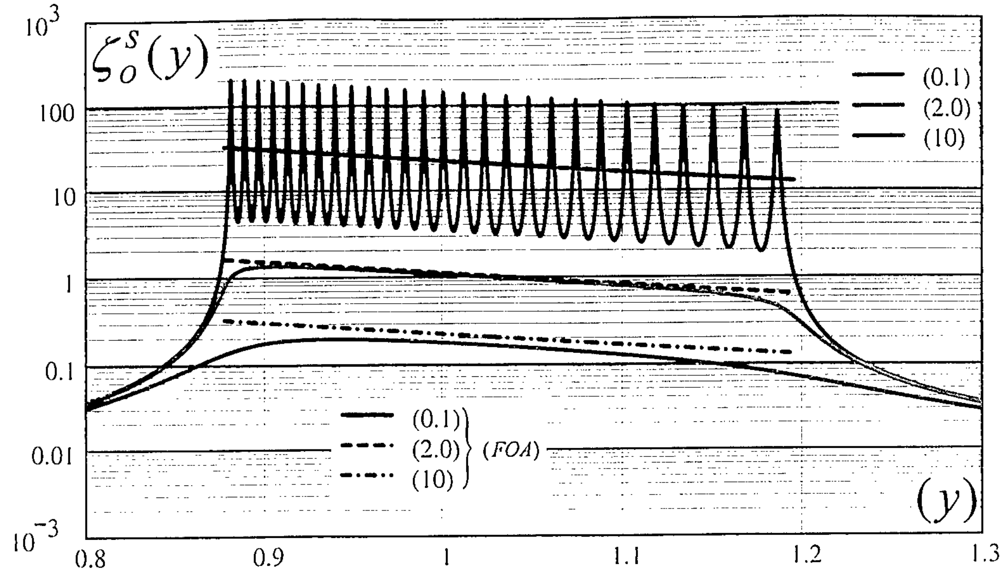


Fig. 14. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for mass control couplings. [R=27, $(M_s / M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] [cf. Fig. 5.] (*)

a. Coupling as specified in Fig. 5a.

b. Coupling as specified in Fig. 5b.

a.



b.

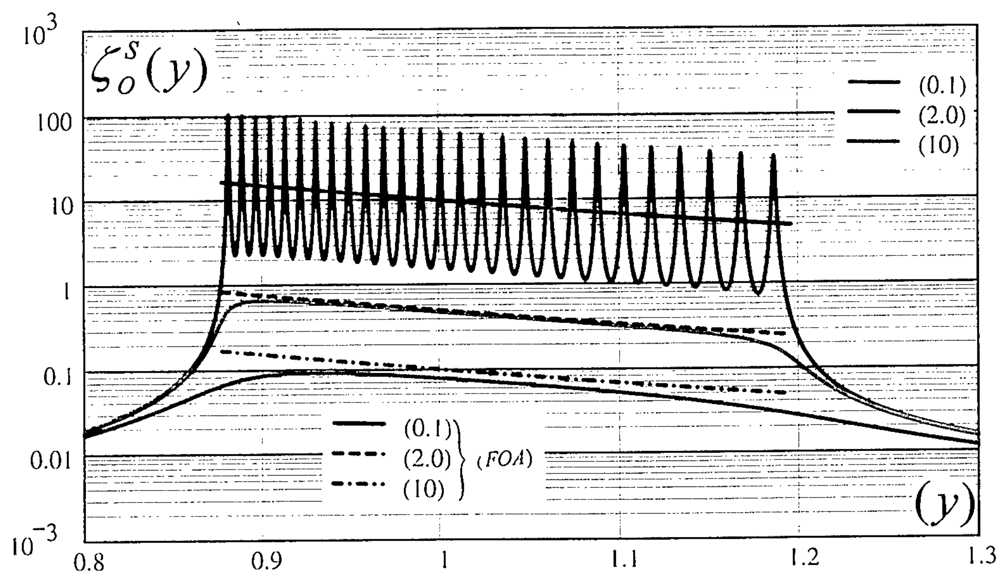
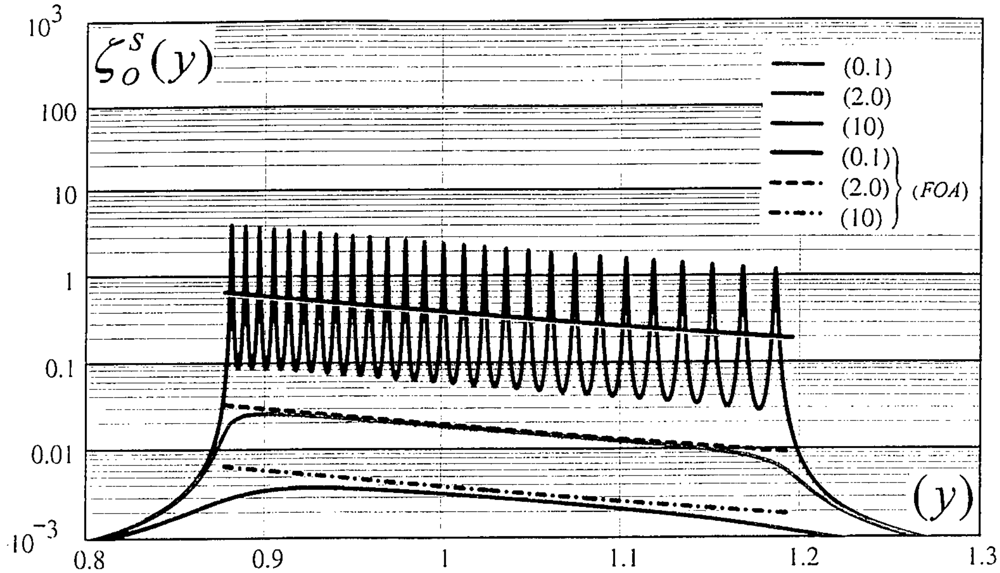


Fig. 15. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for mix control coupling forms. [R=27, $(M_s / M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] [cf. Fig. 6.] (*)

a. Coupling as specified in Fig. 6a.

b. Coupling as specified in Fig. 6b.

c.



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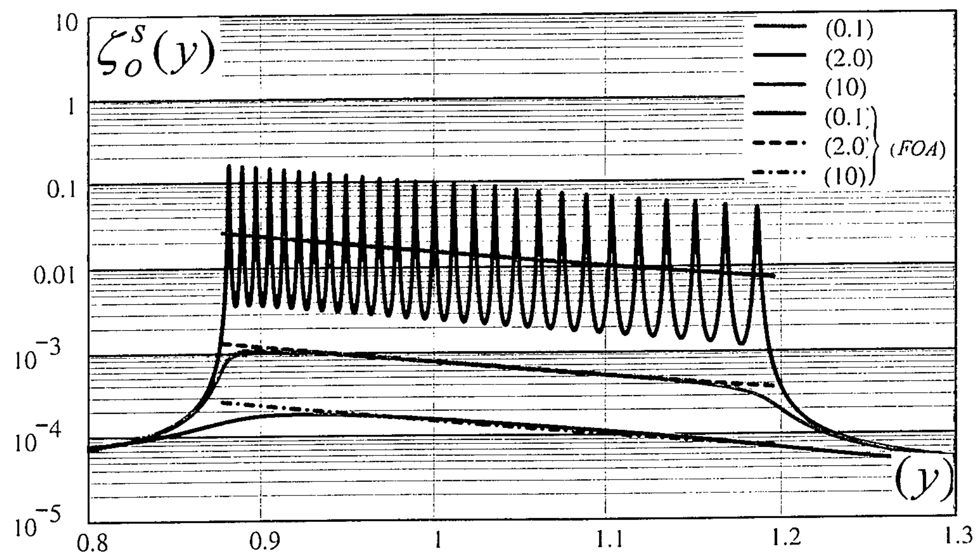

Fig. 15. Modal coupling strength $\zeta_o^s(y)$, as a function of (y), for mix control coupling forms. [R=27, $(M_s/M) = 0.1$; modal overlap parameter (b) and first order approximation (FOA).] [cf. Fig. 6.] (*)

c. Coupling as specified in Fig. 6c.

d. Coupling as specified in Fig. 6d.

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