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STATISTICAL MEASURES OF MARKSMANSHIP

Richard F. Johnson

Military Performance Division

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BACKGROUND

Training in rifle marksmanship is required of all soldiers upon entrance into the U.S. Army. Subsequent demonstration of marksmanship proficiency on an annual basis is also required. This report describes statistical procedures for the measurement of both rifle marksmanship accuracy (the proximity of an array of shots to the center of mass of a target) and marksmanship precision (the dispersion of an array of shots around their own center of impact) often referred to as "shot group tightness." The statistics presented here are intended for use in the field (with live fire) and in the laboratory (with marksmanship simulators).

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EXECUTIVE SUMMARY

Training in rifle marksmanship is required of all soldiers upon entrance into the U.S. Army. Subsequent demonstration of marksmanship proficiency on an annual basis is also required. This report describes objective statistical procedures to measure both rifle marksmanship accuracy, the proximity of an array of shots to the center of mass of a target, and marksmanship precision, the dispersion of an array of shots around their own center of impact, often referred to as "shot group tightness." The statistics presented here are intended for use in the field (with live fire) and in the laboratory (with marksmanship simulators). Although the statistics presented here were developed for use with the Weaponeer Model-70 M16 Marksmanship Trainer, they are generalizable to other marksmanship situations as well. The only requirement is that the target have a clearly described center of mass assigned the coordinates (0,0). Two accuracy measures are presented: (1) constant error, which is defined as the straight line distance between center of the shot group and the target's center of mass; and (2) shooting error, which is defined as the average of the straight line distances between each shot and the center of mass. Eight precision measures are presented. The primary measure of marksmanship precision is the mean radius, which refers to the average of the straight line distances between each shot and the center of the shot group. Secondary measures of marksmanship precision include: the horizontal range, the vertical range, the area of dispersion, the diagonal of dispersion, the standard deviation of the horizontal component, the standard deviation of the vertical component, and the radial standard deviation. The most useful statistics are shooting error, constant error, the shot group coordinates, and variable error (as indicated by the mean radius). These four statistics tell the shooter clearly and concisely how close the shots are to the center of the target (accuracy), and how "tight" the shot group is (precision).

INTRODUCTION

The purpose of this report is to describe statistical procedures to measure (a) *marksmanship accuracy*, the proximity of an array of shots to the center of mass (aiming point) of a target, and (b) *marksmanship precision*, the dispersion of an array of shots (shot group) around their own center of impact, often referred to as "shot group tightness." Although the statistics to be presented here were developed for use with the Weaponeer Model-70 M16 Marksmanship Trainer, they are generalizable to other marksmanship situations as well. The only requirement is that the target have a clearly described center of mass assigned the coordinates (0,0). An early report by Grubbs (1964) was used extensively in the preparation of this report.

The Weaponeer is an M16 rifle marksmanship simulator (manufactured by Spartanics, Ltd., Rolling Meadows, IL) which is used by the U.S. Army for basic rifle marksmanship training of recruits. Marksmanship performance on the Weaponeer is predictive of live fire performance on the rifle range (Schendel, Heller, Finley, & Hawley, 1985), and the U.S. Army Research Institute of Environmental Medicine (USARIEM) has adapted the Weaponeer for evaluating soldier performance in the laboratory (Johnson, 2000). The Weaponeer Model-70 permits evaluation of pop-up, stationary, and moving target marksmanship. Pop-up target marksmanship is automatically scored for accuracy in terms of "hits" and "misses" relative to the center of mass of the target. In the pop-up and moving target configurations the target "pops up," remains in view for a brief interval (1 to 30 seconds), and then drops from view if it is "hit" by the shooter or if the designated interval expires. If the shooter fires after the target falls from view, it is scored as a "late" shot (Spartanics, 1985, 1994). Stationary target marksmanship is assessed when the shooter fires a series of nine self-paced shots at a 25 meter zeroing target; regardless of how close the shots are to the center of mass of the target (accuracy), marksmanship is scored in terms of how many of the shots can fit within a 4 centimeter diameter circle (precision, often referred to as "shot group tightness"). In its standard configuration, the Weaponeer does not provide x,y coordinate data for refined calculations of accuracy and precision. In 1999, Spartanics, Ltd., in conjunction with USARIEM, modified the hardware and software to permit the collection of x,y coordinate data for those Weaponeer marksmanship programs designed for assessment of sentry duty performance (e.g., Johnson & Merullo, 2000). The Weaponeer Model-70, as modified for USARIEM by Spartanics, Ltd., provides x,y output such that each Weaponeer x (and y) unit is equivalent to 0.39 centimeters on a target surface at a simulated distance of 25 meters, and 4.68 centimeters on a target surface at a simulated distance of 300 meters (B. Deutsch, personal communication, May 31, 2000); 300 meters is the simulated distance used in USARIEM sentry duty studies.

MARKSMANSHIP ACCURACY AND MARKSMANSHIP PRECISION

When assessing marksmanship, it is important to distinguish between marksmanship accuracy and marksmanship precision. The difference may be illustrated as follows.

When shooting at a target, various shot patterns may be observed. In one instance, the array of shots may be centered on the center of mass of the target (the aiming point) but the shots are widely dispersed. This situation is illustrated in Figure 1. In this case, the rifleman demonstrates accurate shooting because the average of the shots (the center of the shot group) is close to the aiming point. However, the rifleman is imprecise because his shot group is "not tight;" that is, the shots are widely dispersed. Chapanis (Chapanis, 1951, 1999; Morgan, Cook, Chapanis, & Lund, 1963) has labeled such a cluster of shots as having small constant error (i.e., high accuracy) and large variable error (i.e., low precision).

In another situation, the array of shots may not be centered on the aiming point, or center of mass of the target, but the shots are close to one another (see Figure 2). This is a case of inaccurate shooting (the center of the shot group is far from the aiming point). However, since the shots are consistently in the same place, the shot group is "tight" or precise. This shot group demonstrates low accuracy (large constant error) and high precision (small variable error).

Another possibility includes a shot group that has both large constant error and large variable error indicating both low accuracy and low precision (Figure 3).

A shot group that is both highly accurate and highly precise is the shooter's goal because the shots are centered on the aiming point and they are "tight" (Figure 4). Chapanis (1999) contends that errors in accuracy (as reflected in constant error) are easily corrected, while errors in precision (as reflected in variable error) reflect true instability and are difficult to correct. This is demonstrated during U.S. military basic rifle marksmanship training (U.S. Army Infantry School, 1984) when the instructor's initial goal is to teach soldiers to shoot a tight shot group (precision) and only then to concentrate on hitting the center of the target (accuracy).

BASIC TERMS

THE AIMING POINT OR CENTER OF MASS, CM

The shooter's aiming point is assumed to be the center of mass of the target and is designated CM, where (0,0) are the horizontal (x) and vertical (y) components, or Cartesian coordinates, of the center of the target.

$$CM = (x, y) = (0, 0)$$

THE CENTER OF THE SHOT GROUP, CSG

A shot group is defined as the array of shots in the target area resulting from a series of shots at the target. In the Weaponeer rifle zeroing scenario, the shooter fires 9 shots at the stationary 25-meter zeroing target. The shot group is the array of those 9 shots (Johnson & Kobrick, 1988). In the three-hour USARIEM simulated sentry duty

Figure 1: A shot group with small constant error (high accuracy) and large variable error (low precision).

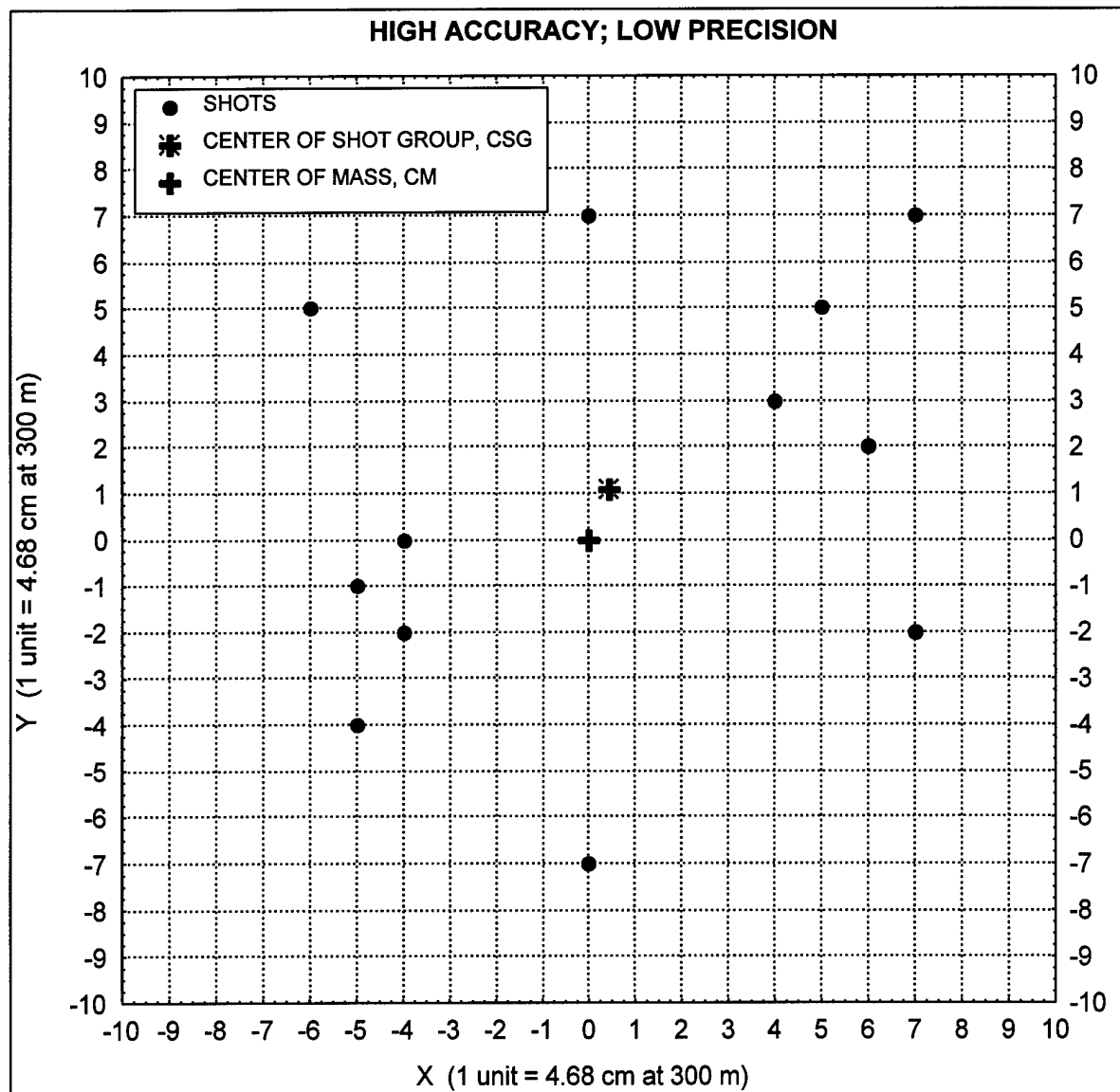


Figure 2. A shot group with large constant error (low accuracy) and small variable error (high precision).

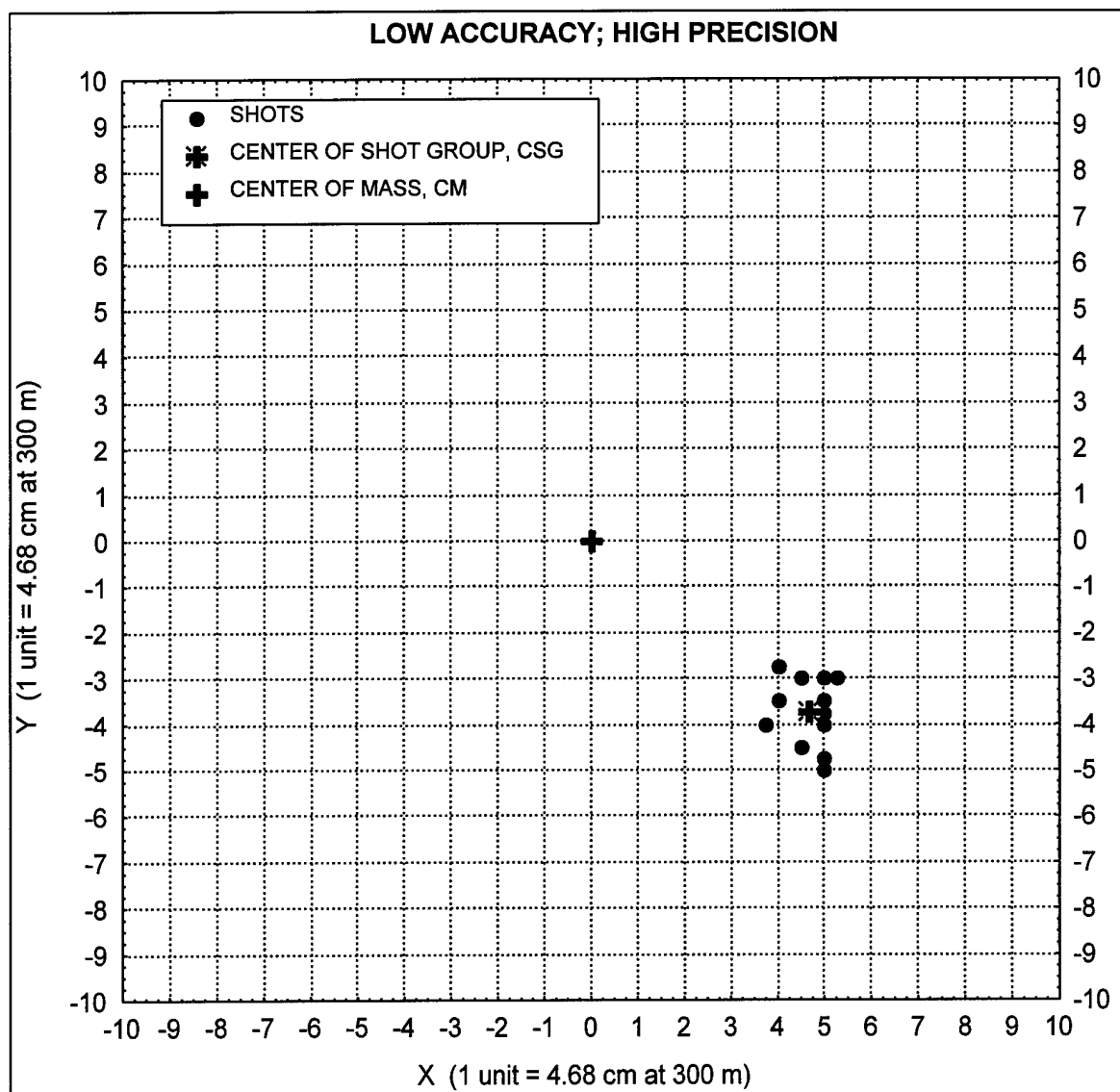


Figure 3. A shot group with large constant error (low accuracy) and large variable error (low precision).

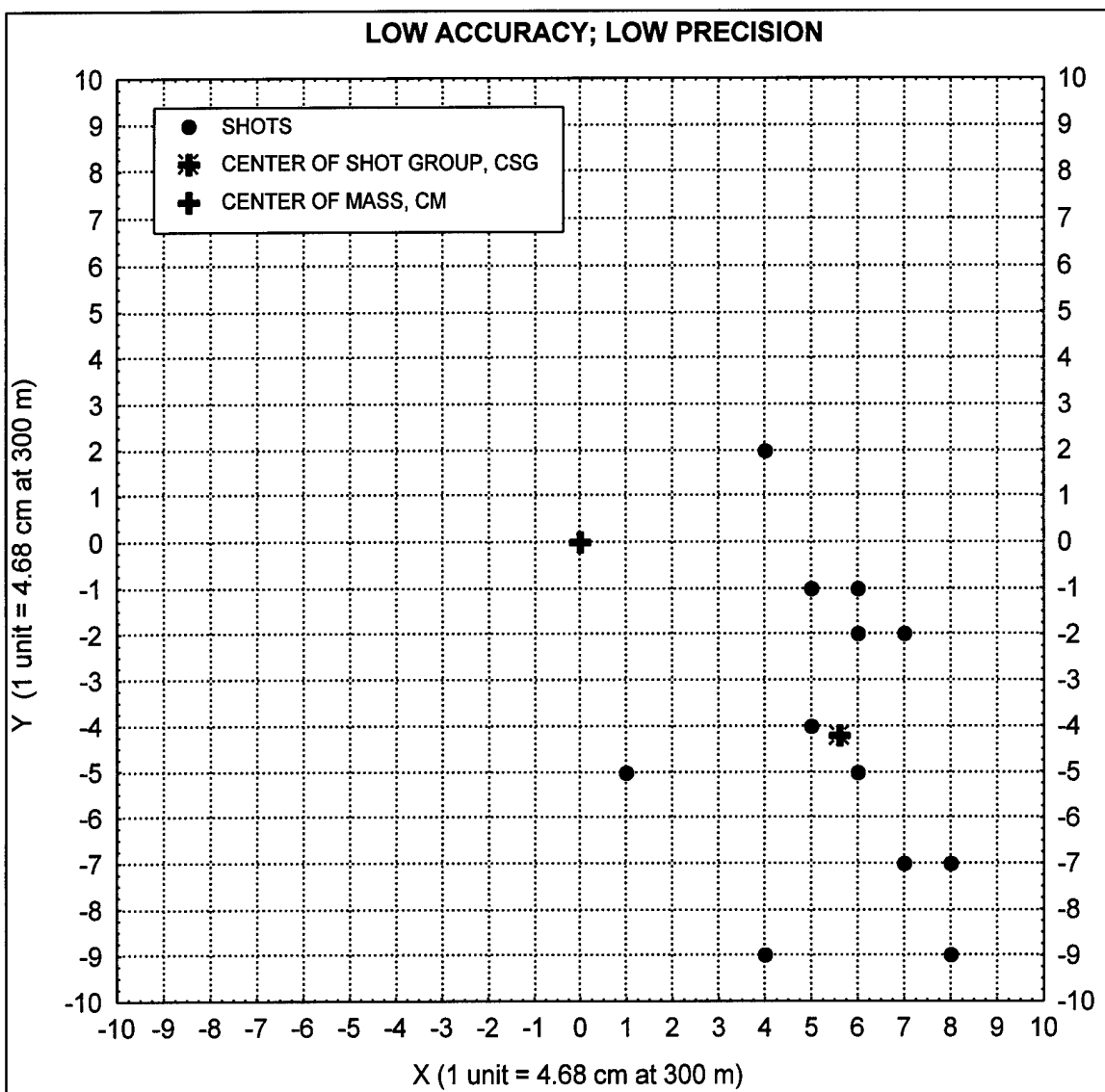
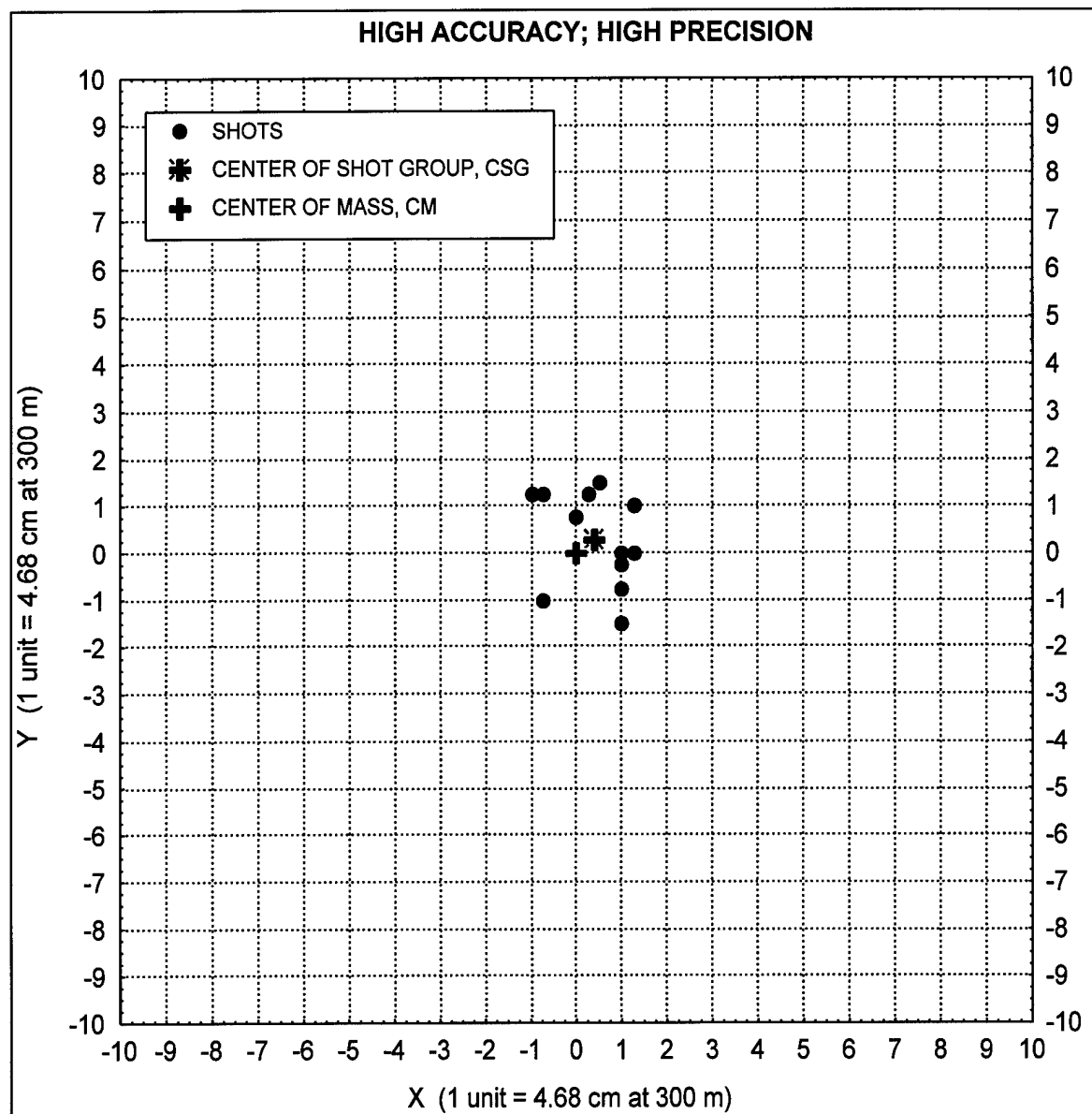


Figure 4. A shot group with small constant error (high accuracy) and small variable error (high precision).



scenario, sentries are typically permitted 12 shots per half hour at the 300-meter pop-up target (Johnson & McMenemy, 1989). For this scenario, the shot group consists of an array of 12 shots. The center of the shot group is designated CSG, the average point about which the individual shots cluster. CSG is determined by *first* averaging the x components and then *separately* averaging the y components of the bullet holes that comprise the shot group. The average x and average y are then used to designate the grid coordinates for the CSG. The CSG is indicated in Figures 1-4. For illustration, if $CSG = (-3, -1)$, then CSG is 3 units to the left and 1 unit below the CM. If the marksman consistently hits the CM, then $CSG = CM = (0,0)$. In the following equation, x_i and y_i represent the values of the horizontal and vertical components of each bullet hole; and n represents the total number of bullet holes in the shot group.

$$CSG = \left(\frac{\sum x_i}{n}, \frac{\sum y_i}{n} \right) = (\bar{x}, \bar{y})$$

MARKSMANSHIP ACCURACY: STATISTICAL MEASURES OF PROXIMITY TO THE CENTER OF THE TARGET

Marksmanship accuracy is reflected in statistics that measure proximity to the aiming point, or center of mass of the target. Two accuracy measures are presented.

SHOT GROUP DISTANCE FROM CENTER OF MASS (CONSTANT ERROR), DCM_{SG}

The shot group distance from center of mass, DCM_{SG} , is the straight line distance between CSG and CM, and is the constant error component of marksmanship accuracy. The smaller the DCM_{SG} the more accurate the marksman. Following the Pythagorean Theorem for calculating the hypotenuse of a right triangle, DCM_{SG} is calculated:

$$DCM_{SG} = \sqrt{(\bar{x})^2 + (\bar{y})^2}$$

where the mean of x and the mean of y are the horizontal and vertical sides of a right triangle, and DCM_{SG} is the hypotenuse. Figures 5-8 provide the values of DCM_{SG} for the shot groups illustrated in Figures 1-4.

SHOT DISTANCE FROM CENTER OF MASS (SHOOTING ERROR), DCM_s

Situations may arise where constant error, or DCM_{SG} , is small, but variable error is so large such that each shot is a "miss" (see Figure 1). Shooting error, DCM_s , refers to the average of the separate straight line distances between each shot and the CM. Shooting error is a very useful measure of marksmanship accuracy because, compared to DCM_{SG} , it communicates whether *each shot* actually comes close to the center of the

target. This concept of marksmanship accuracy is readily and intuitively understood by the shooter. DCM_S is calculated as follows:

$$DCM_S = \frac{\sum \sqrt{(x_i)^2 + (y_i)^2}}{n}$$

Figures 5-8 provide the values of DCM_S for the shot groups illustrated in Figures 1-4.

MARKSMANSHIP PRECISION: STATISTICAL MEASURES OF DISPERSION: "SHOT GROUP TIGHTNESS"

The magnitude of dispersion of a group of shots, or variable error, is reflected in statistics of precision. "Shot group tightness" is the popular term for marksmanship precision. The basic difference between marksmanship precision and marksmanship accuracy is as follows. Statistics of marksmanship accuracy, such as constant error (DCM_{SG}) and shooting error (DCM_S), express performance in terms of distance from the CM. Statistics of marksmanship precision, on the other hand, express performance in terms of distance from the CSG. Eight measures of marksmanship precision are presented. Mean radius, MR, about the CSG is the primary measure of marksmanship precision, or shot group tightness. MR is an efficient statistic because (a) it incorporates into one number both the horizontal and vertical components of marksmanship, (b) it is unlikely to be affected by extreme scores, and (c) like DCM_{SG} and DCM_S , it is a statistic which expresses marksmanship performance in terms of distance from a clearly defined reference point on the target. The remaining seven measures of marksmanship precision largely focus on the separate horizontal and vertical components of the shot group. These secondary measures of marksmanship precision are useful when coaching a shooter who is particularly difficult to train.

THE PRIMARY MEASURE OF MARKSMANSHIP PRECISION: THE MEAN RADIUS

The mean radius, MR, refers to the average of the straight line distances between the CSG and each shot (Grubbs, 1964). The mean radius, MR, is calculated as follows:

$$MR = \frac{\sum \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{n}$$

Figures 5-8 provide the values of MR for the shot groups illustrated in Figures 1-4; the circle described by the MR about the CSG is also illustrated in each of the figures.

Figure 5. A shot group with small constant error (high accuracy) and large variable error (low precision): $DCM_{SG}=1.35$; $MR=6.38$; $DCM_S=6.45$; $R_H=13$; $R_V=14$; $AD=182$; $DD=19.10$; $S_H=5.12$; $S_V=4.48$; $S_R=6.82$.

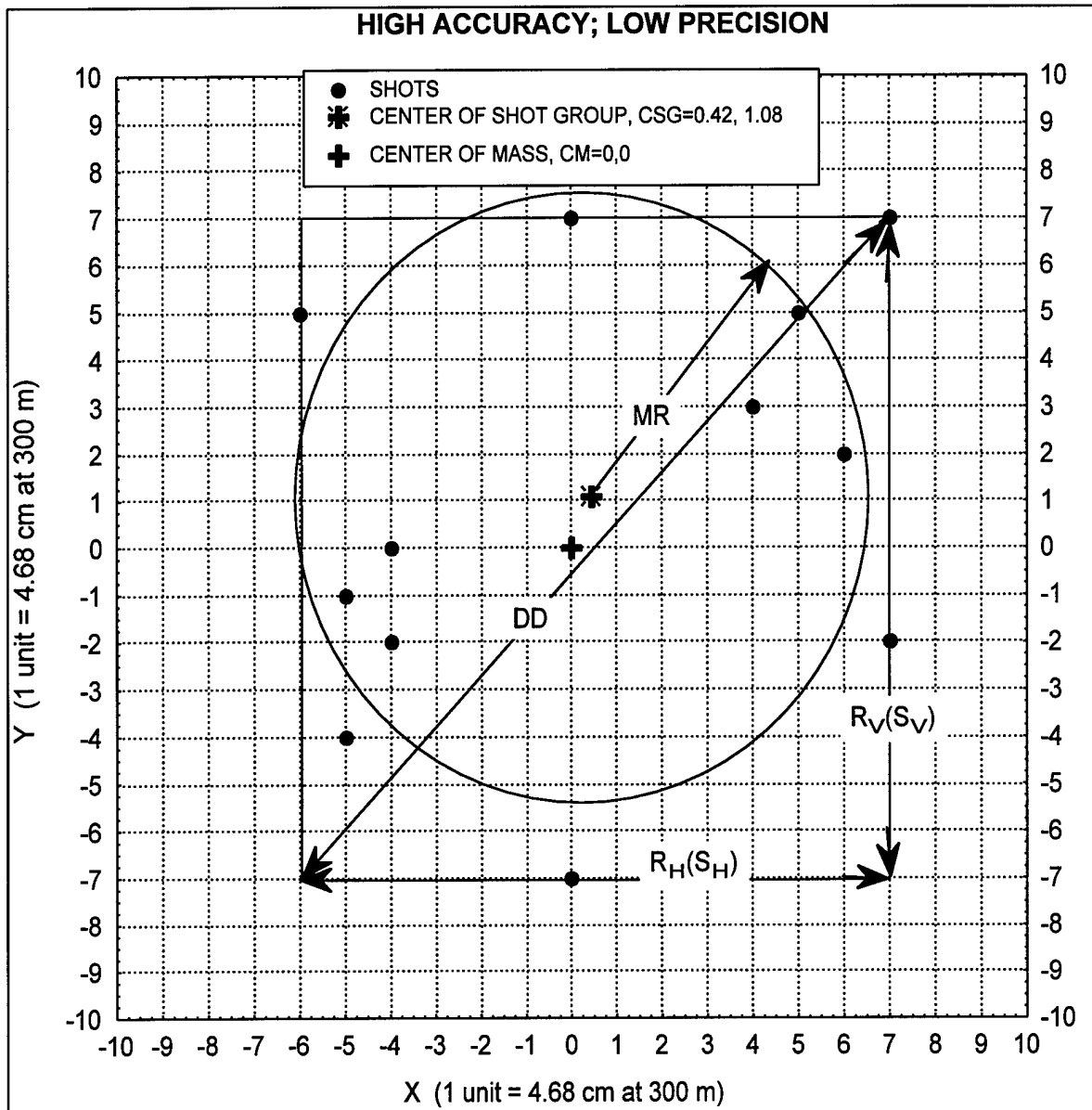


Figure 6. A shot group with large constant error (low accuracy) and small variable error (high precision): $DCM_{SG}=5.60$; $MR=0.81$; $DCM_S=6.00$; $R_H=1.50$; $R_V=2.25$; $AD=3.38$; $DD=2.70$; $S_H=0.50$; $S_V=0.74$; $S_R=0.90$.

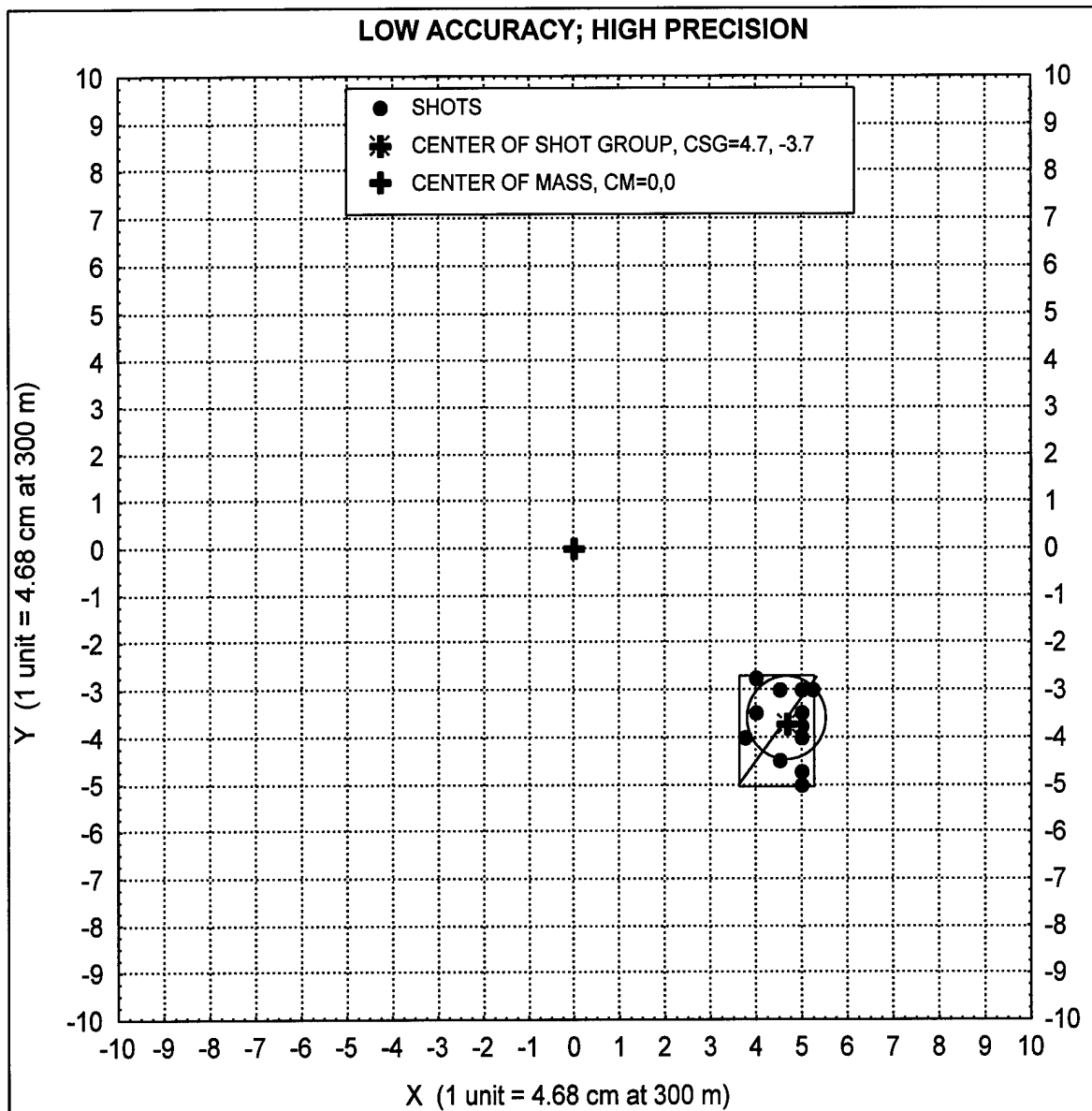


Figure 7. A shot group with large constant error (low accuracy) and large variable error (low precision): $DCM_{SG}=6.95$; $MR=3.43$; $DCM_S=7.58$; $R_H=7.00$; $R_V=11.00$; $AD=77.00$; $DD=13.04$; $S_H=1.98$; $S_V=3.45$; $S_R=3.98$.

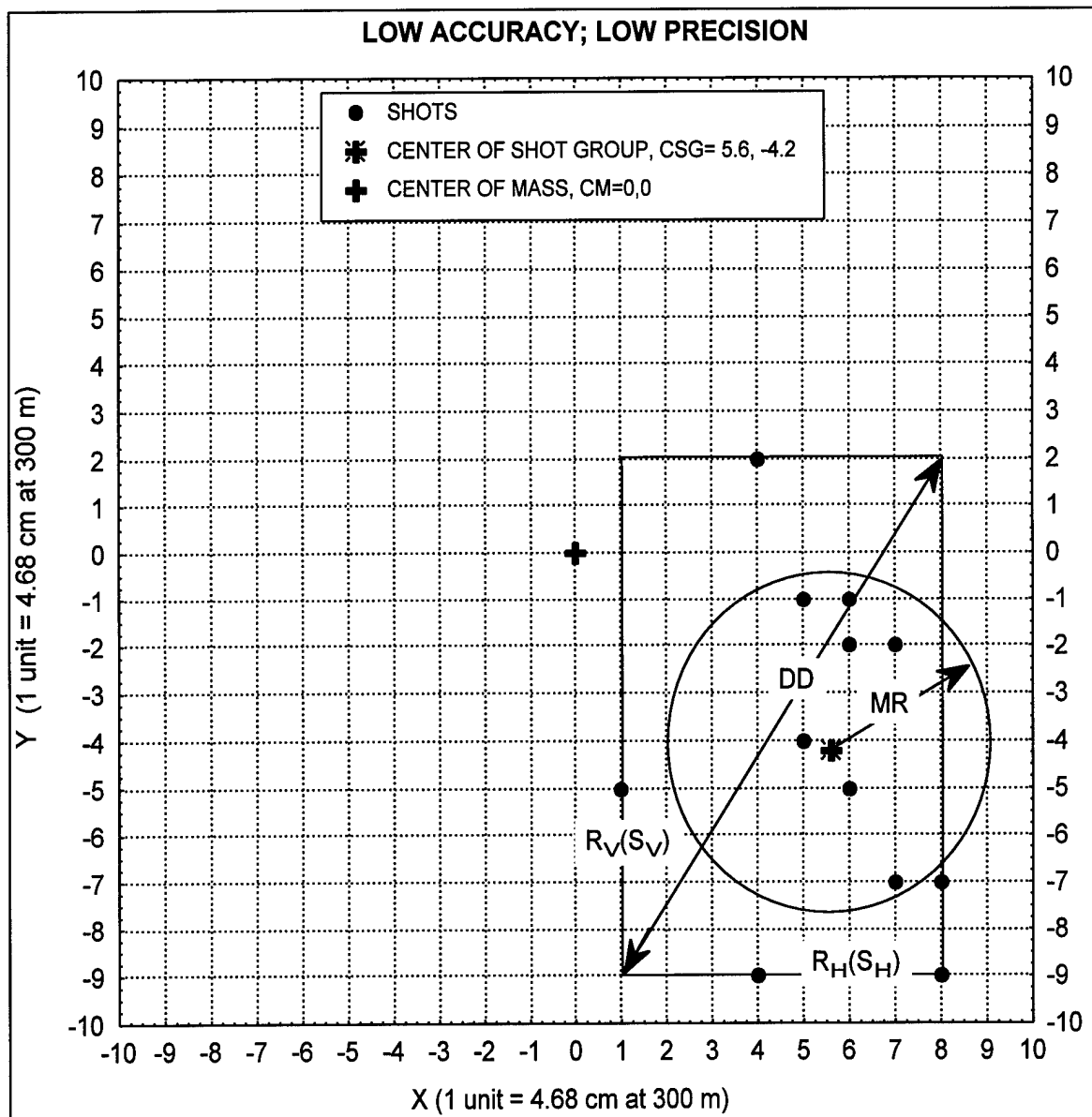
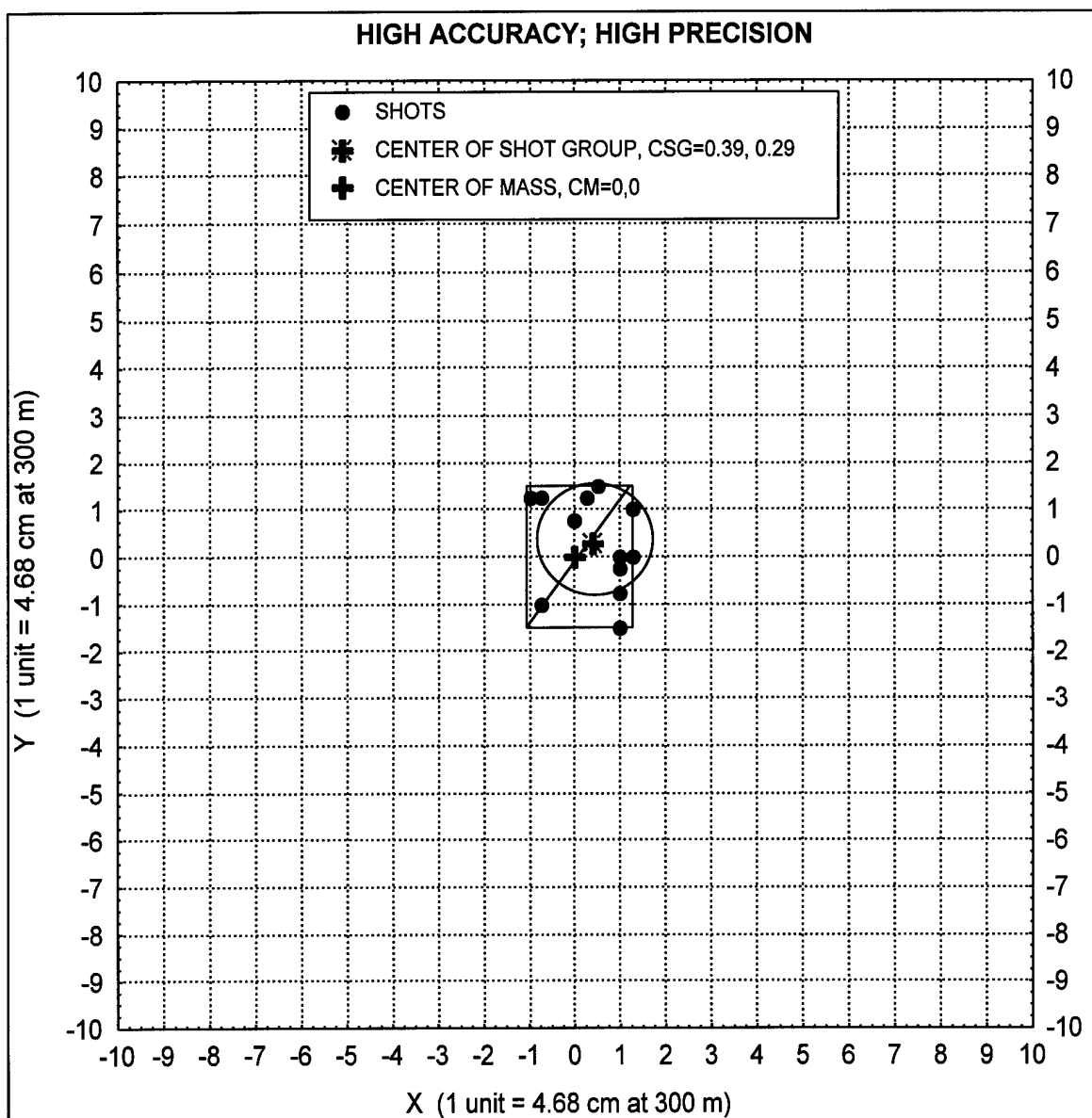


Figure 8. A shot group with small constant error (high accuracy) and small variable error (high precision): $DCM_{SG}=0.49$; $MR=1.19$; $DCM_S=1.32$; $R_H=2.25$; $R_V=3.00$; $AD=6.75$; $DD=3.75$; $S_H=0.84$; $S_V=1.01$; $S_R=1.31$.



SECONDARY MEASURES OF MARKSMANSHIP PRECISION

MR tells the shooter important information about overall shot group tightness. However, if the shooter is particularly difficult to train, it may be useful to discuss with the shooter secondary measures of precision, including the horizontal and vertical components of shot group tightness. Figures 5-8 provide the values of these secondary measures for the shot groups illustrated in Figures 1-4.

Horizontal Range, R_H

For a shot group, the horizontal range = R_H = (the maximum x component observed) - (the minimum x component observed). This statistic is sometimes referred to as horizontal shot group tightness (Tharion, Hoyt, Marlowe, and Cymerman, 1992). Although R_H is simple to calculate and its meaning is clear and easy to communicate, R_H is readily influenced by extreme scores.

$$R_H = (x_{\max}) - (x_{\min})$$

Vertical Range, R_V

For a shot group, the vertical range = R_V = (the maximum y component observed) - (the minimum y component observed). This statistic has been referred to as vertical shot group tightness (Tharion et al., 1992). Like R_H , the R_V is simple to calculate and its meaning is clear and easy to communicate, but it is readily influenced by extreme scores.

$$R_V = (y_{\max}) - (y_{\min})$$

Area of Dispersion, AD

For a shot group, the area of dispersion = $AD = (R_H)(R_V)$. AD describes the smallest rectangle which contains all the shots (impact points) of a shot group and has its sides parallel to the x and y axes (for examples, see Figures 5-8). AD is not efficient because it is readily influenced by extreme scores, and two or more rectangles of equal area may be very different in appearance.

$$AD = (R_H)(R_V)$$

Diagonal of Dispersion, DD

For a shot group, the diagonal of dispersion = DD = the square root of the sum of the square of the horizontal range (R_H) and the square of the vertical range (R_V). This measure is the diagonal of the smallest rectangle which contains all the shots of a shot group and has its sides parallel to the x and y axes (see Figures 5-8 for examples). DD

incorporates both horizontal and vertical dispersion into one statistic. DD is not as efficient as MR (Grubbs, 1964). DD is expressed as follows:

$$DD = \sqrt{(R_H)^2 + (R_V)^2}$$

Standard Deviation, Horizontal Component, S_H

The standard deviation of the horizontal component of the shot group = S_H. This statistic is not easily influenced by extreme scores. S_H is calculated by

$$S_H = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

Standard Deviation, Vertical Component, S_V

The standard deviation of the vertical component of the shot group = S_V. Again, this statistic is not easily influenced by extreme scores. S_V is calculated by

$$S_V = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n - 1}}$$

Radial Standard Deviation, S_R

The radial standard deviation of the shot group = S_R = the square root of the sum of the square of S_H and the square of S_V. This measure reflects both the horizontal and vertical components of dispersion in one number (as does the mean radius, MR, the area of dispersion, AD, and the diagonal of dispersion, DD) and is not easily influenced by extreme scores. Although it is a very efficient measure of variable error, it is not easily communicated to the rifleman. S_R is calculated as follows:

$$S_R = \sqrt{(S_H)^2 + (S_V)^2}$$

DISCUSSION

DETERMINING FUNCTIONAL ACCURACY OF DCM_{SG}

The Weaponer Model-70, as modified for USARIEM, provides x,y coordinate output such that each x (and y) unit is equivalent to 0.39 centimeter on a target 25

meters from the shooter and 4.68 centimeters on a target 300 meters from the shooter. A 48 centimeter wide silhouette at 300 meters is approximately equivalent to the shoulder breadth of the 90th percentile U.S. Army male soldier (U.S. Army Natick R&D Laboratories, 1980). Assuming a shot group with a random distribution of shots about the CM, a DCM_{SG} greater than 5 Weaponeer units may be considered off-target because the circle described by the DCM_{SG} extends beyond the shoulder breadth of 90 per cent of U.S. Army male soldiers. For the purpose of this report, *a shot group may be considered functionally accurate if the constant error, DCM_{SG} , is ≤ 5 Weaponeer units ($5 \times 0.39 \text{ cm} = 1.95 \text{ cm}$ at 25 m; $5 \times 4.68 \text{ cm} = 23.4 \text{ cm}$ at 300 m).*

DETERMINING FUNCTIONAL PRECISION OF MR

According to the U.S. Army Unit Rifle Marksmanship Training Guide FC23-11 (U.S. Army Infantry School, 1984), a shot group is considered acceptably "tight" if most shots fit within a 4 centimeter diameter circle at 25 meters. This converts to a 48 centimeter diameter circle at 300 meters, which is approximately equivalent to the shoulder breadth of the 90th percentile male U.S. soldier (U.S. Army Natick R&D Laboratories, 1980). For the purposes of this report, *a shot group may be considered precise, or "tight," if the mean radius, MR, is ≤ 5 Weaponeer units ($5 \times 0.39 \text{ cm} = 1.95 \text{ cm}$ at 25 m; $5 \times 4.68 \text{ cm} = 23.4 \text{ cm}$ at 300 m).*

ADVANTAGES OF THESE STATISTICAL MEASURES OF MARKSMANSHIP

Although the above statistics have been developed for use with the Weaponeer Model-70 Rifle Marksmanship Simulator, they are also applicable to a variety of other marksmanship situations. For example, the same formulas may be used to assess marksmanship when the target is a circular bull target. The only requirement is that the target have a clearly described center of mass and that the center of mass coordinates are defined as (0,0). If x,y coordinate data are unavailable, but a paper printout of the shots is available, then the measures described in this report may be generated and calculated with the aid of a computer, digitizing tablet, and digitizing software. A previously published technical report by Marlowe, Tharion, Harman, & Rauch (1989) describes such a procedure.

CONCLUSIONS

This report describes statistical procedures to measure (a) *marksmanship accuracy*, the proximity of an array of shots to the center of mass (aiming point) of a target, and (b) *marksmanship precision*, the dispersion of an array of shots (shot group) around their own center of impact, often referred to as "shot group tightness." Although the statistics to be presented here were developed for use with the Weaponeer Model-70 M16 Marksmanship Trainer, they are generalizable to other situations as well. The only requirement is that the target have a clearly described center of mass and that the researcher assign the center of mass the coordinates (0,0).

RECOMMENDATIONS

When discussing marksmanship performance, the most useful statistics are shooting error (DCM_S), constant error (DCM_{SG}), the location of the center of the shot group (CSG), and variable error as indexed by the mean radius (MR). DCM_S , DCM_{SG} , and CSG tell the shooter how close the shots are to the center of the target (accuracy), while MR tells the shooter how "tight" the shot group is (precision).

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APPENDIX

COMMENTS ON TERMINOLOGY

The terms used to describe marksmanship accuracy and precision in this paper have their counterparts in the writings of other authors, principally Grubbs (1964) and Tharion (e.g., Tharion et al., 1992). In order to clarify the terminology here and elsewhere, the following comments on selected terms are offered.

The Aiming Point or Center of Mass, CM

In stationary and pop-up target configurations, authors agree that the shooter's aiming point is the center of mass of the target, where (0,0) are the horizontal (x) and vertical (y) components, or Cartesian coordinates, of the center of the target. In non-traditional situations, for example, where the target is moving (e.g., Schendel & Johnston, 1983), the shooter's aim may lead the target so that the aim is off center.

The Center of the Shot Group, CSG

Authors agree that center of the shot group is the average point about which the individual shots cluster. Grubbs (1964) refers to this location as the *center of impact*. When Tharion (e.g., Tharion et al., 1992) describes the shot group's x and y components, he uses the terms *horizontal and vertical deviation* from center of mass.

Shot Group Distance from Center of Mass (Constant Error), DCM_{SG}

The shot group distance from center of mass, DCM_{SG} , is the straight line distance between CSG and CM. Until 1992, Tharion's term "distance from center of mass, DCM" was calculated this way; since 1992, Tharion's DCM has been calculated the same as shooting error, DCM_S (W.J. Tharion, personal communication, February 5, 2001).

Shot Distance from Center of Mass (Shooting Error), DCM_S

Shooting error, DCM_S , refers to the average of the separate straight line distances between each shot and the CM. Since 1992, Tharion's term "distance from center of mass, DCM" has been calculated this way; prior to 1992, Tharion's DCM was calculated the same as constant error, DCM_{SG} (W.J. Tharion, personal communication, February 5, 2001).

Horizontal Range, R_H

This statistic is sometimes referred to as *horizontal shot group tightness* (Tharion et al., 1992) or *extreme horizontal dispersion* (Grubbs, 1964).

Vertical Range, R_v

This statistic is sometimes referred to as *vertical shot group tightness* (Tharion, et al., 1992) or *extreme vertical dispersion* (Grubbs, 1964).

Diagonal of Dispersion, DD

Grubbs (1964) refers to this statistic as the *diagonal*, D .

Radial Standard Deviation, S_R

Grubbs (1964) refers to this statistic as the *RSD*.