NAVSWC TR 91-205

COMPUTING WITH QUANTUM MECHANICAL OSCILLATORS

BY A. D. PARKS AND J. L. SOLKA
STRATEGIC SYSTEMS DEPARTMENT

MARCH 1991

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FOREWORD

This report describes the results of some preliminary theoretical work in the area of quantum mechanical computing which has been performed by the Strategic Systems (K) Department at the Naval Surface Warfare Center (NAVSWC) as part of a molecular computing focused technology initiative.

This report has been reviewed and approved by Ted Sims, Space Sciences Branch Head, and James L. Sloop, Space and Surface Systems Division Head.

Approved by

R. L. Schmidt

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Strategic Systems Department

CONTENTS

INTRODUCTION

Despite the obvious practical considerations (e.g., stability, controlability), certain quantum mechanical systems seem to naturally lend themselves in a theoretical sense to the task of performing computations. The purpose of this report is to describe one such idealized system-the quantum harmonic computer. As its name might suggest, this theoretical device employs the well known energy characteristics of quantum mechanical oscillators; the associated creation and annihilation operators; and the quantum mechanical axioms of state preparation and observability to perform computations. It is demonstrated that programs can be written for this device in terms of quantum mechanical observables and creation and annihilation operators which will algorithmically manipulate oscillator energy states to perform the desired calculations, the results of which are eigenvalues of a well defined system observable. By definition, these programs are equivalent to Turing machines, so that anything that is Turing computable is also computable with this device.

It is well known that many physical systems (e.g., photon gas, crystals, phonons, diatomic molecules) can be approximated by collections of quantum harmonic oscillators. Although the development herein may suggest such implementations, we emphasize that our intent has been theoretical in nature rather than practical.

QUANTUM MECHANICAL HARMONIC OSCILLATOR

In order to provide a more complete foundation for the discussions which follow, we provide in this section a brief sketch of the one-and m-dimensional quantum mechanical harmonic oscillators. Although we adopt the Dirac notation, our development is standard and can be referenced in more detail in most basic quantum mechanics texts [e.g., References 1, 2].

The ideal classical harmonic oscillator is a particle of mass m constrained to move along an axis and subject to a restoring force proportional to the displacement from the force center. The corresponding quantum mechanical oscillator is a particle of mass m in one dimension with energies generated by the Hamiltonian operator

$$\hat{H}=\frac{1}{2m}(\hat{p}^2+m^2\omega^2\hat{q}^2), \qquad (2.1)$$

where w is the natural angular frequency of oscillation, and $\hat{q}$ and $\hat{p}$ are the position and momentum operators, respectively, related by the commutator

$$[\hat{q},\hat{p}]=i\hbar. \tag{2.2}$$

Here $\hbar$ is Planck's constant divided by 2π.

The energy eigenvalue problem for this system is given by the time-independent Schrodinger equation

$$\hat{H}|E>=E|E>, \tag{2.3}$$

where E is the energy eigenvalue associated with the system state described by the "ket" vector $|E>$.

Using this we may now construct the vector space of the dynamical states of the system. Let

$$\hat{a}=(2\hbar m\omega)^{-1/2}(\hat{p}-im\omega\hat{q}) \tag{2.4}$$

and

$$\hat{a}^{+}=(2\hbar m\omega)^{-1/2}(\hat{p}+im\omega\hat{q}). \tag{2.5}$$

It is readily seen that the operator $\hat{a}$ and $\hat{a}^{+}$ are Hermitean conjugates and that application of (2.2) provides the commutator relation

$$[\hat{a},\hat{a}^{+}]=1. \tag{2.6}$$

Furthermore, it is easily shown that the Hamiltonian operator of (2.1) can be written in terms of $\hat{a}$ and $\hat{a}^{+}$ as

$$\hat{H}=(\hat{N}+1/2)\hbar\omega, \tag{2.7}$$

where

$$\hat{N}=\hat{a}^{+}\hat{a}. \tag{2.8}$$

Thus, the eigenvalue problem of (2.3) is equivalent to constructing the eigenvectors of the operator $\hat{N}$. It should be noted that $\hat{N}$ is Hermitean and a system observable.

For the sake of brevity, we omit its detailed development and simply state the well known result that the spectrum of eigenvalues of $\hat{N}$ is the set of non-negative integers {0, 1, 2, ..., n, ...} from which we may form the orthonormal state vectors $|0>,|1>,|2>,...,|n>,...$, where

$$\hat{N}|n>=n|n>. \tag{2.9}$$

For the same reason as above, we also state without proof the following well-known properties of the operators $\hat{a}^{+}$ and $\hat{a}$:

$$\hat{a}^{+}|n>=(n+1)^{1/2}|n+1> \tag{2.10}$$

$$\hat{a}|n\rangle = (n)^{1/2}|n-1\rangle \tag{2.11}$$

$$\hat{a}|0\rangle = 0 \tag{2.12}$$

and

$$|n\rangle = (n!)^{-1/2}(\hat{a}^{+})^{n}|0\rangle, \tag{2.13}$$

where $|0\rangle$ is called the ground state of the system.

Upon application of (2.7) and (2.9) to (2.3), it is easily verified that

$$\hat{\mathrm{H}}|n\rangle = (n+1/2)\hbar\omega\,|n\rangle, \tag{2.14}$$

i.e., the energy states of the oscillator are quantized into discreet multiples of $\hbar w$ with ground state energy $1/2\hbar w$.

Consider now the m-dimensional system of m distinguishable, noninteracting oscillators with system Hamiltonian

$$\hat{\mathrm{H}} = \sum_{i=1}^{m} \hat{\mathrm{H}}_i\,, \tag{2.15}$$

where each $\hat{\mathrm{H}}_i$ in the summation has the form of (2.7) with all associated operators having the appropriate i subscript. If $|n_i\rangle$ denotes the eigenvectors of $\hat{\mathrm{H}}_i$, then the tensor product of eigenvectors

$$|n_1 n_2 \ldots n_m\rangle \equiv |n_1\rangle |n_2\rangle \ldots |n_m\rangle \tag{2.16}$$

forms a complete orthonormal set S_m of eigenvectors of $\hat{\mathrm{H}}$ in (2.15) so that

$$\hat{\mathrm{H}}|n_1 n_2 \ldots n_m\rangle = \sum_{i=1}^{m} (n_i + 1/2)\hbar\omega\,|n_1 n_2 \ldots n_m\rangle \tag{2.17}$$

and

$$\hat{\mathrm{N}}|n_1 n_2 \ldots n_m\rangle = \sum_{i=1}^{m} \hat{\mathrm{N}}_i |n_1 n_2 \ldots n_m\rangle = \sum_{i=1}^{m} n_i |n_1 n_2 \ldots n_m\rangle. \tag{2.18}$$

Analogous to (2.6), it can be shown that the operators $\hat{a}_i$ and $\hat{a}_i^{+}$ obey the following commutation rules:

$$\left.\begin{array}{l}[\hat{a}_i,\hat{a}_j] \;=0\\ [\hat{a}_i^+,\hat{a}_j^+]=0\\ [\hat{a}_i,\hat{a}_j^+]=\delta_{ij}\end{array}\right\} (i,j\ =\ 0,1,...,m) \tag{2.19}$$

It is convenient at this point to introduce several new operators which will prove useful in the following sections. Define the new operator $\hat{\beta}_i$ as

$$\hat{\beta}_i|n_i>\equiv\begin{cases}(n_i)^{-1/2}|n_i>, & n_i\neq 0\\ 0 & , n_i=0\end{cases} \tag{2.20}$$

and

$$\hat{\alpha}_i^+\equiv\hat{\beta}_i\hat{a}_i^+ \tag{2.21}$$

$$\hat{\alpha}_i\equiv\hat{a}_i\hat{\beta}_i \tag{2.22}$$

so that

$$\hat{\alpha}_i^+|n_i>=|n_i+1> \tag{2.23}$$

and

$$\hat{\alpha}_i|n_i>=\begin{cases}|n_i-1>, & n_i\neq\ 0\\ 0 & , n_i=\ 0\end{cases} \tag{2.24}$$

It is easily verified that $\hat{\alpha}_i$ and $\hat{\alpha}_j^+$ obey the following commutation relation for $1\leq i,\ j\leq m$:

$$[\hat{\alpha}_i,\hat{\alpha}_j^+]=\begin{cases}1, & i=j,\ n_i\ =\ 0\\ 0, & otherwise\end{cases} \tag{2.25}$$

We also define the identity operator $\hat{\mathrm{I}}_i$ as

$$\hat{\mathrm{I}}_i|n_i>=|n_i> \tag{2.26}$$

so that for any operator $\hat{\gamma}_i$,

$$\hat{\gamma}_i\hat{\mathrm{I}}_i=\hat{\mathrm{I}}_i\hat{\gamma}_i=\hat{\gamma}_i\ . \tag{2.27}$$

QUANTUM HARMONIC COMPUTER: DEFINITIONS

In this section we develop a set of definitions that will provide for a precise description of the quantum harmonic computer. Let S_m be as defined in the previous section and $Q_m \subset S_m$, where

$$Q_m = \{|n_1 n_2 ... n_m> | n_i \in \{0,1\}, 3 \le i \le m; 3 \le n_2 \le m\}. \tag{3.1}$$

Each eigenvector in Q_m is called an instantaneous description (ID). The quantum numbers n_1 and n_2 in each ID are called the state and pointer, respectively. Also define the following sets of operators:

$$\mathcal{L}_m = \{\hat{\alpha}_i, \hat{\alpha}_i^+ | 1 \le i \le m\}, \tag{3.2}$$

$$\eta_m = \{\hat{N}_1, \hat{N}_2, ... \hat{N}_m, \hat{T}\} \tag{3.3}$$

where

$$\hat{T} = \sum_{i=3}^{m} \hat{N}_i, \tag{3.4}$$

and

$$R_m = \mathcal{L}_m \cup \eta_m. \tag{3.5}$$

We note that $\hat{T}$ is Hermitean and is a system observable.

An R_m-quadruple is a 4-tuple of one of the following three types:

$$n_1 n_{n_2} A_{n_2}^k A_1^l, \tag{3.6}$$

$$n_1 n_{n_2} \hat{\alpha}_2^+ A_1^l, \tag{3.7}$$

$$n_1 n_{n_2} \hat{\alpha}_2 A_1^l, \tag{3.8}$$

where n_{n_2} is the quantum number describing the eigenstate of the n_2^{th} oscillator; k and l are non-negative integers; $A_i \in \mathcal{L}_m$; and

$$A_i^k = \begin{cases} \underbrace{A_i A_i \ldots A_i}_{k\ times}, & k \neq 0 \\ \hat{I}_i, & k=0. \end{cases} \tag{3.9}$$

Each R_m-quadruple may be interpreted as being the instruction: "if the state is n_1 and the eigenstate of the n_2^{th} oscillator is n_{n_2} for some ID, then apply the following operators to the ID."

A program P_m is a finite nonempty set of R_m-quadruples, no two of which have the same state quantum number. Let $p=n_1 n_{n_2} \hat{\tau}$ represent any R_m-quadruple, where $\hat{\tau}$ is any of the operator pairs in (3.6) - (3.8), and X, Y ϵ Q_m. A program P_m induces a basic P_m transition $X \xrightarrow{\hat{\tau}} Y$ from X to Y if there is a pϵP_m such that $\hat{N}_1 X = n_1 X$; $\hat{N}_{n_2} X = n_{n_2} X$; and $Y = \hat{\tau} X$. A finite sequence $X_1 \xrightarrow{\hat{\tau}} Y \xrightarrow{\hat{\tau}'} \ldots \xrightarrow{\hat{\tau}''} Z$ of such transitions is called a P_m-computation if there is no p''' ϵP_m or $W \epsilon Q_m$ with $Z \xrightarrow{\hat{\tau}'''} W$. In this case the eigenvector Z is called the resultant. The eigenvector X is called the initiator. It will always be assumed here that for every initiator, the pointer quantum number $n_2 = 3$ and the state quantum number $n_1 = 1$.

We may now define the m-dimensional quantum harmonic computer Γ_m as the 4-tuple

$$\Gamma_m \equiv (Q_m, R_m, P_m, X) \tag{3.10}$$

Informally, Γ_m can be thought of as an m-dimensional quantum mechanical harmonic oscillator which serves as a memory/storage medium for any ID in Q_m. This oscillator interfaces with a "device" which imposes upon it energy eigenstate transitions via the application of a sequence of operators from R_m as algorithmically dictated by P_m.

NUMERICAL COMPUTATION USING Γ_m: SIMPLE EXAMPLES

It is obvious that there is a one-to-one correspondence between P_m programs and Turing machines [3, 4]. Hence, as long as we view m as being potentially infinite (i.e., adding extra oscillators as needed), anything Turing computable is also P_m computable. In this section we provide examples of P_m program which compute several non-negative integer arithmetic functions and illustrate the associated P_m computations using simple initiators.

In order to perform these computations, we make use of the following symbolic representation for a non-negative integer J:

$$J \leftrightarrow J' \equiv \underbrace{11...1,}_{J+1\ times} \tag{4.1}$$

(thus $5' \equiv 111111$). Also, we may assume without violating any quantum mechanical principles that initiators may be prepared with the required state and pointer quantum numbers, as well as any combination of quantum numbers $n_i \in \{0,1\}$ for $3 \leq i \leq m$. Initiators may therefore be prepared which can contain within the energy states of oscillators 3 through m symbolic representations for non-negative integers of the form (4.1). We adopt the convention that if J and K are two non-negative integers which are to be symbolically represented within an initiator with J appearing first, then the initiator will be prepared with $n_i = 1$ for $3 \leq i \leq J+3$, $J+5 \leq i \leq J+K+5$, and $n_i = 0$ elsewhere for $i > 2$ (thus $m \geq J+K+5$). An initiator prepared in this manner is said to be properly prepared.

Each of the sample P_m programs described below are designed to produce a resultant from a properly prepared initiator via the sequential application of operators in R_m. The computed solution is the eigenvalue of $\hat{T}$ when applied to the resultant.

Example 1. The set A is a P_m program which computes $J+K$ for J,K non-negative integers, where

$$A=\left\{11\hat{\alpha}_{n_2}(\hat{\alpha}_1)^{\circ},\ 10\hat{\alpha}_2^{+}\hat{\alpha}_1^{+},\ 21\hat{\alpha}_2^{+}(\hat{\alpha}_1)^{\circ},\ 20\hat{\alpha}_2^{+}\hat{\alpha}_1^{+},\ 31\hat{\alpha}_{n_2}(\hat{\alpha}_1)^{\circ}\right\}.$$

For the sake of clarity we have used $(\hat{\alpha}_1)^{\circ}$ in A, but note that by (3.9) and (2.27) it need not be made explicit. Effectively, this P_m program produces a resultant in which oscillators 3 and $J+5$ have transitioned to their ground states and (neglecting the state and pointer) $J+K$ oscillators remain in their first excited state. Thus the computed solution $J+K$ is the eigenvalue of $\hat{T}$ when applied to the resultant.

In order to illustrate this let $J=2$ and $K=1$ and choose $m=J+K+5=8$. The associated P_8 computation is:

$$|13111011\rangle \xrightarrow{\hat{\alpha}_3(\hat{\alpha}_1)^{\circ}} |13011011\rangle \xrightarrow{\hat{\alpha}_2^{+}\hat{\alpha}_1^{+}} |24011011\rangle \xrightarrow{\hat{\alpha}_2^{+}(\hat{\alpha}_1)^{\circ}}$$

$$|25011011\rangle \xrightarrow{\hat{\alpha}_2^{+}(\hat{\alpha}_1)^{\circ}} |26011011\rangle \xrightarrow{\hat{\alpha}_2^{+}\hat{\alpha}_1^{+}} |37011011\rangle \xrightarrow{\hat{\alpha}_7(\hat{\alpha}_1)^{\circ}}$$

$$|37011001\rangle,$$

so that

$$\hat{T}|37011001\rangle = 3|37011001\rangle.$$

As an aside, we note that in general the composite operator $\hat{\alpha}_{J+5}\hat{\alpha}_3(\hat{\alpha}_2^+)^{J+2}(\hat{\alpha}_1^+)^2$ will always produce the desired resultant for J+K for any properly prepared initiator.

Example 2. The set B is a P_m program which computes J-K for J$\geq$K where J and K are non-negative integers:

$$B=\{11\hat{\alpha}_{n_2}(\hat{\alpha}_1)^\circ,\ 10\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 21\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ,\ 20\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 31\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ,\ 30\hat{\alpha}_2\hat{\alpha}_1^+,\ 41\hat{\alpha}_{n_2}(\hat{\alpha}_1)^\circ,\ 40\hat{\alpha}_2\hat{\alpha}_1^+,\ 51\hat{\alpha}_2\hat{\alpha}_1^+,$$

$$61\hat{\alpha}_2(\hat{\alpha}_1)^\circ,\ 60\hat{\alpha}_2\hat{\alpha}_1^+,\ 71\hat{\alpha}_2\hat{\alpha}_1^+,\ 70\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2,\ 81\hat{\alpha}_2(\hat{\alpha}_1)^\circ,\ 80\hat{\alpha}_2^+(\hat{\alpha}_1)^7,\ 90\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ,\ 91\hat{\alpha}_2(\hat{\alpha}_1)^\circ\}\ .$$

Although not necessary, we have again made $(\hat{\alpha}_1)^\circ$ explicit. Here B produces a resultant in which oscillators 3 through K+3 and J+5 through J+K+5 have transitioned to their ground states and J-K oscillators remain in their first excited state. Thus the eigenvalue of $\hat{T}$ when applied to the resultant is J-K. B also needs a ground state oscillator after the representation of K in the initiator. Thus we choose m=J+K+6.

As an illustration, let J=2 and K=1, as before, so that m=9. P_9 produces the following computation:

$$|131110110\rangle \xrightarrow{\hat{\alpha}_3(\hat{\alpha}_1)^\circ} |130110110\rangle \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |240110110\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ}$$

$$|250110110\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ} |260110110\rangle \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |370110110\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ}$$

$$|380110110\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ} |390110110\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |480110110\rangle \xrightarrow{\hat{\alpha}_8(\hat{\alpha}_1)^\circ}$$

$$|480110100\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |570110100\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |660110100\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+}$$

$$|750110100\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |840110100\rangle \xrightarrow{\hat{\alpha}_2(\hat{\alpha}_1)^\circ} |830110100\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^7}$$

$$|140110100\rangle \xrightarrow{\hat{\alpha}_4(\hat{\alpha}_1)^\circ} |140010100\rangle \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |250010100\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ}$$

$$|260010100\rangle \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |370010100\rangle \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1)^\circ} |380010100\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+}$$

$$|470010100\rangle \xrightarrow{\hat{\alpha}_7(\hat{\alpha}_1)^\circ} |470010000\rangle \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |560010000\rangle,$$

so that

$$\hat{T}|560010000\rangle = 1|560010000\rangle.$$

Again we note that the composite operator

$$\hat{\alpha}_{J+K+5} \cdots \hat{\alpha}_{J+5}\hat{\alpha}_{K+3} \cdots \hat{\alpha}_3(\hat{\alpha}_2^+)^{J+1}(\hat{\alpha}_1^+)^4$$

will always produce the desired resultant for *J-K*, $J \geq K$, for any properly prepared initiator.

Γ_4 LOGIC GATES

Because of the obvious equivalence between Turing machines and P_m programs, we can construct P_m-programs which produce logic function computations. In this section we prove a series of theorems which state that specific 4-dimensional quantum harmonic computers can serve as basic logic gates. Let us assume that the energy states of the third and fourth oscillators represent truth values for propositions r and s, respectively, where $n_i=0$ means "false" and $n_i=1$ means "true" for $i \in \{3, 4\}$. The truth value obtained from a Γ_4 logic gate computation is the eigenvalue of $\hat{T}$ when applied to the associated resultant.

Theorem 1. (Q_4, R_4, V, X) is an OR gate, where

$$V=\left\{11\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2,\ 20(\hat{\alpha}_2)^\circ\hat{\alpha}_1^+,\ 21\hat{\alpha}_{n_2}\hat{\alpha}_1^+\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2411> \xrightarrow{\hat{\alpha}_4\hat{\alpha}_1^+} |3410>; \hat{T}|3410> = 1|3410>.$$

$$|1310> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2410> \xrightarrow{(\hat{\alpha}_2)^\circ\hat{\alpha}_1^+} |3410>; \hat{T}|3410> = 1|3410>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3401> \qquad ; \hat{T}|3401> = 1|3401>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3400> \qquad ; \hat{T}|3400> = 0|3400>.$$

■

Theorem 2. (Q_4, R_4, Λ , X) is an AND gate, where

$$\Lambda = \left\{11\hat{\alpha}_2^+\hat{\alpha}_1^+, 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2, 21\hat{\alpha}_{n_2}(\hat{\alpha}_1^+)^2, 20\hat{\alpha}_2(\hat{\alpha}_1^+)^3, 31\hat{\alpha}_{n_2}\hat{\alpha}_1^+, 30(\hat{\alpha}_{n_2})^\circ\hat{\alpha}_1^+, 51\hat{\alpha}_{n_2}\hat{\alpha}_1\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2411> \xrightarrow{\hat{\alpha}_4(\hat{\alpha}_1^+)^2} |4410>; \hat{T}|4410> = 1|4410>.$$

$$|1310> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2410> \xrightarrow{\hat{\alpha}_2(\hat{\alpha}_1^+)^3} |5310> \xrightarrow{\hat{\alpha}_3\hat{\alpha}_1} |4300>; \hat{T}|4300> = 0|4300>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3401> \xrightarrow{\hat{\alpha}_4\hat{\alpha}_1^+} |4400>; \hat{T}|4400> = 0|4400>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3400> \xrightarrow{(\hat{\alpha}_4)^\circ\hat{\alpha}_1^+} |4400>; \hat{T}|4400> = 0|4400>.$$

■

Theorem 3. $(Q_4, R_4, \not{V}, X)$ is an XOR gate, where

$$\not{V}=\left\{11\hat{\alpha}_2^+\hat{\alpha}_1^+, 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2, 21\hat{\alpha}_{n_2}(\hat{\alpha}_1^+)^2, 20(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^3, 31(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^2, 30(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^2, 40\hat{\alpha}_2(\hat{\alpha}_1^+)^2, 61\hat{\alpha}_{n_2}\hat{\alpha}_1\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2411> \xrightarrow{\hat{\alpha}_4(\hat{\alpha}_1^+)^2} |4410> \xrightarrow{\hat{\alpha}_2(\hat{\alpha}_1^+)^2} |6310> \xrightarrow{\hat{\alpha}_3\hat{\alpha}_1} |5300>; \hat{T}|5300> = 0|5300>.$$

$$|1310> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2410> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^3} |5410>; \hat{T}|5410> = 1|5410>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3401> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^2} |5401>; \hat{T}|5401> = 1|5401>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^2} |3400> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^2} |5400>; \hat{T}|5400> = 0|5400>.$$

■

Theorem 4. $(Q_4, R_4, \sim V, X)$ is a NOR gate, where

$$\sim V=\left\{11\hat{\alpha}_{n_2}\hat{\alpha}_1^+,\ 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^3,\ 20\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 31\hat{\alpha}_{n_2}(\hat{\alpha}_1^+)^2,\ 30(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^2,\ 41\hat{\alpha}_{n_2}\hat{\alpha}_1^+,\ 40\hat{\alpha}_{n_2}^+\hat{\alpha}_1^+\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_3\hat{\alpha}_1^+} |2301> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |3401> \xrightarrow{\hat{\alpha}_4(\hat{\alpha}_1^+)^2} |5400>; \hat{T}|5400> = 0|5400>.$$

$$|1310> \xrightarrow{\hat{\alpha}_3\hat{\alpha}_1^+} |2300> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |3400> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^2} |5400>; \hat{T}|5400> = 0|5400>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^3} |4401> \xrightarrow{\hat{\alpha}_4\hat{\alpha}_1^+} |5400> \qquad ; \hat{T}|5400> = 0|5400>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^3} |4400> \xrightarrow{\hat{\alpha}_4^+\hat{\alpha}_1^+} |5401> \qquad ;\hat{T}|5401>=1|5401>.$$

■

Theorem 5. (Q_4, R_4, $\sim\Lambda$, X) is a NAND gate, where

$$\sim\Lambda=\left\{11\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^4, 21\hat{\alpha}_{n_2}\hat{\alpha}_1^+, 20(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^4, 30\hat{\alpha}_2\hat{\alpha}_1^+, 41\hat{\alpha}_{n_2}(\hat{\alpha}_1^+)^2, 51(\hat{\alpha}_{n_2})^\circ\hat{\alpha}_1^+, 50\hat{\alpha}_{n_2}^+\hat{\alpha}_1^+\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2411> \xrightarrow{\hat{\alpha}_4\hat{\alpha}_1^+} |3410> \xrightarrow{\hat{\alpha}_2\hat{\alpha}_1^+} |4310> \xrightarrow{\hat{\alpha}_3(\hat{\alpha}_1^+)^2} |6300>;\hat{T}|6300>=0|6300>.$$

$$|1310> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |2410> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^4} |6410>;\hat{T}|6410>=1|6410>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^4} |5401> \xrightarrow{(\hat{\alpha}_4)^\circ\hat{\alpha}_1^+} |6401>;\hat{T}|6401>=1|6401>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+(\hat{\alpha}_1^+)^4} |5400> \xrightarrow{\hat{\alpha}_4^+\hat{\alpha}_1^+} |6401>;\hat{T}|6401>=1|6401>.$$

■

Theorem 6. (Q_4, R_4, $\sim\!\mathbb{V}$, X) is a NXOR gate, where

$$\sim\!\mathbb{V} = \left\{11\hat{\alpha}_{n_2}\hat{\alpha}_1^+, 10\hat{\alpha}_2^+(\hat{\alpha}_1^+)^3, 20\hat{\alpha}_2^+\hat{\alpha}_1^+,\ 30(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^2,\ 31(\hat{\alpha}_{n_2})^\circ(\hat{\alpha}_1^+)^2,\ 41\hat{\alpha}_{n_2}\hat{\alpha}_1^+,\ 40\hat{\alpha}_{n_2}^+\hat{\alpha}_1^+\right\}.$$

Proof:

$$|1311> \xrightarrow{\hat{\alpha}_3\hat{\alpha}_1^+} |2301> \xrightarrow{\hat{\alpha}_2^+\hat{\alpha}_1^+} |3401> \xrightarrow{(\hat{\alpha}_4)^\circ(\hat{\alpha}_1^+)^2} |5401>;\hat{T}|5401>=1|5401>.$$

$$|1310> \xrightarrow{\hat{\alpha}_3 \hat{\alpha}_1^+} |2300> \xrightarrow{\hat{\alpha}_2^+ \hat{\alpha}_1^+} |3400> \xrightarrow{(\hat{\alpha}_4)^{\circ} (\hat{\alpha}_1^+)^2} |5400>; \hat{T}|5400> = 0\,|5400>.$$

$$|1301> \xrightarrow{\hat{\alpha}_2^+ (\hat{\alpha}_1^+)^3} |4401> \xrightarrow{\hat{\alpha}_4 \hat{\alpha}_1^+} |5400>; \hat{T}|5400> = 0\,|5400>.$$

$$|1300> \xrightarrow{\hat{\alpha}_2^+ (\hat{\alpha}_1^+)^3} |4400> \xrightarrow{\hat{\alpha}_4^+ \hat{\alpha}_1^+} |5401>; \hat{T}|5401> = 1\,|5401>.$$

$\blacksquare$

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This report describes an idealized quantum mechanical computer which performs computations via the algorithmic manipulation of the energy states of quantum harmonic oscillators. These algorithms are constructed in terms of observables and creation and annihilation operators and are Turing machine equivalents. Computational results are eigenvalues of a well-defined Hermitean operator.

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