M A T E R I A L S

# Reliable Ceramic Structural Composites Designed with a Threshold Strength

## Technical Report # 5

# Effect of Elastic Properties on the Threshold Strength of Laminar Ceramics

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# Effect of Elastic Modulus on Threshold Strength

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## Abstract

Finite element analysis was carried out to predict the threshold strength of a laminar ceramics loaded parallel to the layers. These materials are composed of alternate layers of two different ceramics in which residual stress is generated. Strength limiting cracks are trapped by the compressive layers and require a minimum (threshold) applied stress to cause them to fail the laminated ceramic. The calculations were utilized to study the influence of the elastic modulus mismatch between the alternate tensile and compressive layers. Good agreement was established between numerical simulations and theoretical results for materials involving layers with the same elastic properties. Results were obtained for a variety of combinations of different ceramics and suggest that threshold strength as high as three times the effective residual stress in the compressive layer is conceivable.

# 1.0 Introduction

Although ceramics have many promising properties such as hardness and high temperature stability, they have the major disadvantage of lacking reliability. The strength of ceramics obeys a statistical description (e.g. Weibull) involving a wide distribution of values, meaning that some components are quite weak and therefore unreliable. The reason for the statistical distribution of strength is the existence of a variety of cracks and crack-like flaws unintentionally introduced during processing or post-processing (such as surface machining) [1, 2]. Unlike ductile materials such as metals, ceramics materials lack significant plastic deformation and hence exhibit low resistance to crack propagation. Thus, the strength of the brittle ceramics correlates directly with the presence of flaws and decreases with increasing size of the flaw.

The reliability of the ceramic could be improved by controlling the size of flaws introduced to the ceramic materials during processing. This can be achieved if a slurry of the designated powder is dispersed and then passed through a filter [1]. Depending on the fineness of the filter only heterogeneties with sizes smaller than a critical value can flow through. Thus, threshold strength (and hence a guaranteed reliability) can be determined by the size of the filter, i.e. by defining the largest flaw that can be present in the material. However, such a material is still subject to damage during machining and the reliability can be degraded accordingly. Recently, Rao et al. [3] have shown that an intrinsic threshold strength can be attained by the introduction of a compressive residual stress in the components of the ceramic. As described below, experiments conducted on two-dimensional layered materials having alternating tensile and compressive segments have shown that threshold strengths as high as 500 MPa can be achieved.

To impose a biaxial compressive residual stress in such a ceramic body, two sets of alternate layers of different materials with different properties were fused together at high temperature. Upon cooling one set of layers has a tensile residual stress while the other has a compressive stress, due to thermal expansion differences. This arrangement can arise also when the layers undergo a differing volume increase due to a crystallographic phase transformation, or undergo a differing increase in their molar volumes due to a chemical reaction.

For the experiments carried out by Rao et al. [3], the layered material was pre-cracked using different indentors and different loads. These indentations were performed in such a way, that the resulting pre-cracks were completely contained in a tensile layer and perpendicular to the plane direction of the layers. The samples were then subjected to 4-point flexural loading tests, such that the top surface of the specimens was subjected to an external tensile load parallel to the layers and perpendicular to the pre-cracks. Independent of the size of the pre-crack it was observed that threshold strength existed and no failure takes place at stresses below this level. This is in contrast to tests on monolithic ceramics where it is observed that the larger the flaw size the lower the applied stress needed to cause the material to fail.

To support and develop the concept of the threshold strength, observed in the experiments, a theory was developed [3]. It was assumed that compressive layers of thickness $t_1$, having a residual stress $\sigma_1$, were sandwiched between the tensile layers of thickness $t_2$ having a residual stress $\sigma_2$ as shown in Fig.1. The biaxial residual stresses arised in the layers are given by:

$$\sigma_1 = \varepsilon . E'_1 . \left(1 + \frac{t_1 . E'_1}{t_2 . E'_2}\right)^{-1} = \sigma_c \text{ and } \sigma_2 = -\sigma_1 . \frac{t_1}{t_2} = \sigma_T \qquad (1)$$

where $\varepsilon = (\alpha_2 - \alpha_1).\Delta T$, $\alpha_i$ is the coefficient of thermal expansion, $T$ is the temperature and $E'_i = \frac{E_i}{(1-\nu_i)}$, $E_i$ is Young's modulus and $\nu_i$ is Poisson's ratio with the subscripts $i = 1$ or 2 designating the relevant layer.

In the theoretical analysis, the crack of length 2a is assumed to span the entire width of the tensile layer and to penetrate some distance into the compressive layer with the feature that smaller cracks would be associated with a higher strength. A tensile load $\sigma_a$ parallel to the layers and perpendicular to the crack is applied. The stress intensity factor is calculated by superposing two stress fields applied to the same crack as shown in Fig.1. The first stress field is a tensile stress of magnitude $(\sigma_a$-$\sigma_c)$ applied to the whole specimen, while the second stress field is a tensile stress of magnitude $(\sigma_c$+$\sigma_T)$ applied only across the tensile layer. The total stress intensity factor for the crack in the tensile layer extending into the compressive layer is determined by summing the stress intensity factors for each of the stress fields mentioned above. In Ref. [3], the stress intensity factors were approximated by results for an elastically homogenous system. This result is given by:

$$K = (\sigma_a - \sigma_c).\sqrt{\pi.a} + (\sigma_c + \sigma_T).\sqrt{\pi.a}\left[\frac{2}{\pi}.\sin^{-1}\left(\frac{t_2}{2.a}\right)\right] \qquad (2)$$

Later on and in the rest of this paper, this equation will be referred as the theoretical model results. The threshold strength, the stress needed for the crack to extend unstably, was assumed in Ref. [3] to occur when it had penetrated through the compressive layer, i.e. when $2.a = t_2 + 2.t_1$ and $K = K_c$. The threshold strength, therefore, was given by:

$$\sigma_{th} = \frac{K_c}{\sqrt{\pi.\frac{t_2}{2}.\left(1+\frac{2.t_1}{t_2}\right)}} + \sigma_c.\left[1-\left(1+\frac{t_1}{t_2}\right)\frac{2}{\pi}.\sin^{-1}\left(\frac{1}{1+\frac{2.t_1}{t_2}}\right)\right] \qquad (3)$$

Alike, we refer later on to this equation as the theoretical model results. McMeeking and Hbaieb[4] have shown that in an elastically homogenous system, the value of the threshold strength for a flaw initially in the tensile layer is lower than its value when the flaw is initially in the compressive layer. As a consequence, a crack in the tensile layer was considered in their work to account for the true threshold strength.

Furthermore, McMeeking and Hbaieb[4] gave the condition that must be met for stable growth to persist until the crack tip penetrates through the compressive layer, thereby validating the estimate of the threshold strength given in equation (3). In other considerations, McMeeking and Hbaieb[4] provided results that optimize the threshold strength for an elastically homogenous system. They found that the threshold strength is maximized by selection of a good combination of materials to give the highest values for $K_c$, $E'$ (where $E' = E'_1 = E'_2$ due to homogeneity) and $\varepsilon$. Further maximization of the threshold strength is possible by choosing the thicknesses of the tensile and compressive layers. McMeeking and Hbaieb [4] demonstrated that optimization is achieved by choosing the layers as thin as possible. However, they assumed that there is a technological limit to how thin the layers can be and therefore considered the case where one or other or both of the sets of layers are made at the technological limit of thickness.

They found that for high toughness materials, the optimal threshold strength is associated with layers of equal thickness (but at the technological limit of thickness). In contrast, for low toughness materials the optimal threshold strength occur when the tensile layers in the system are thicker than the compressive layers by a ratio lying in the range 1 to 2.8. Furthermore, McMeeking and Hbaieb [4] provided estimates of the optimal threshold strength, which they found to be at least -0.3 $E'\varepsilon$ and to be significantly higher than this for high toughness materials.

This paper is an extension of the work done by McMeeking and Hbaieb [4] to develop the theoretical basis for the experimental observation made by Rao et al. [1]. However, the effort described here is based on a finite element modeling rather than analytical calculations. Consequently the heterogeneous case where the tensile layer has a different elastic modulus from the compressive layer can be analyzed properly. Predictions are given for the threshold strength as it depends on the compressive layer toughness and the ratio of the elastic modulus of the tensile layer to the elastic modulus of the compressive layer.

## 2.0 Overview

To model a crack in the layered material composed of alternate tensile layers fused together with compressive layers a finite element analysis is carried out. It is assumed that the crack has already tunnelled down the tensile layer so that the configuration analyzed is a through crack as depicted in Fig.1. For simplicity only a quarter of the specimen is modeled which accounted for the whole body by way of symmetry. Several tensile and compressive layers (the number of layers varied for each case and was usually between 6 and 11) are present in the model to represent adequately the specimen used in the experiments. The length of the model is more than three times larger than the width; the latter is in turn much larger than the crack, so that the finite element calculations effectively simulate an infinite body fracture analysis. Displacement boundary conditions are imposed on the symmetry line of the model, while no constraint is applied to the external lateral and the top surfaces. The cracksurface is free of traction. An external tensile load was applied on the top surface. The path independent J integral [6] is calculated for several cracks with differing lengths and hence the relationship of the stress intensity factor versus the crack length is investigated.

When layers of different elastic modulii are considered, the mesh near the interface between the layers is greatly refined to make sure that the J integral is path independent and accurate. Only crack tips displaced from the layer interface are investigated to avoid non-square root singular crack tip fields arising for crack tips exactly at the material interface. However, since the mesh is very fine the accuracy of the stress intensity factor for crack tips near the interface is good.

To have results that are generally valid, several parameters are varied. The ratio of elastic modulus in the tensile layer, $E_2$, to the elastic modulus in the compressive layer, $E_1$, was varied from *1/10* to *10*. However, in view of the results for $E_1 = E_2$ obtained by McMeeking and Hbaieb [4], only thickness ratios $\frac{t_2}{t_1}$ equal to 1, 3/2, 2, 5/2 are accounted. The expectation is that optimal threshold strengths will occur within this thickness ratio range. Finally, by assuming that the toughness of the ceramics materials composing the layers would fall in the range *1* to *10* $MPa.\sqrt{m}$ the threshold strength dependence on toughness, thickness ratio and elastic modulus ratio is investigated.

## 3.0 Model description:

The computer simulation is carried out using the finite element code ABAQUS [5] to perform linear elastic calculations. Isotropy is assumed for all materials so that the only mechanical properties needed are the elastic modulus and the Poisson's ratio. All layers are given the value of *0.32* for the Poisson's ratio.

A different coefficient of thermal expansion is given to the alternating layers. To the tensile layers a value of $9 * 10^{-6}$ $1/K$ is assigned, whereas the compressive layers are given the value of $6.025 * 10^{-6}$ $1/K$. The same values are used throughout the analysis. For calculating the residual stresses, a temperature 1200 °C lower than the stress-free state is used and ABAQUS employed to calculate the thermal stresses in the layers. As a consequence, the layers with thickness $t_1$ have a biaxial residual compressive stress of magnitude $\sigma_c$ and the layers of thickness $t_2$ have a biaxial tensile stress $\sigma_T$, relieved only by the presence of the crack, as depicted in Fig.1. As a result, a value of $\varepsilon$ equal to *0.357%* is generated. The externally applied stress in the material is caused by an external applied tensile load on the top surface of the model. It is simulated by a stress of

*400 MPa* applied uniformly on the top surface. Linearity of the solutions allows scaling and superposition of the residual stress loading and the applied loading in arbitrary ways. The model used is two dimensional and plane strain. Eight noded plane strain quadrilateral elements are used in the mesh such as the one shown in Fig.2. The elements near the crack tip are very small, as shown in Fig.3, especially in the compressive layer containing the tip. Such an arrangement permits the accurate calculations of J by the domain integral method [7] as well as accurate solution of the near tip stresses.

## 4.0 Simulation Results

For crack length with a tip in the material with modulus $E_i$, the stress intensity factor is calculated from the value of J through $K = \sqrt{J.E_i/(1-\nu^2)}$ and then plotted versus the crack length. The calculations are first conducted for layers having the same elastic properties. This is intended to verify the finite element results in comparison with the exact theoretical model results for an infinite body [4]. The finite element results for the elastically homogeneous case along with the theoretical model results are plotted in Fig.4 for a thickness ratio $\frac{t_2}{t_1} = 1$. To obtain accurate trends for results having crack tips near the interface between the compressive and tensile layers, many calculations were carried out for tips located in the vicinity of the interface.

Results are shown in Fig.4 for both the stress intensity factor due to the residual stress and the stress intensity factor due the externally applied load. The good agreement between the finite element results and theoretical model results shown in Fig.4 implies reliability of the finite element solution of the model for the crack problems.

Results for the stress intensity factor for cracks in the heterogeneous material are plotted in Fig.5. In this case situations with crack tips very close to the layer interface are avoided because of the non square root singularity arising when the crack tip is exactly at the interface and because of path dependence of J when the domain for its calculation encompasses the neighboring layers. A comparison between the finite element results and the theoretical model results (assuming both tensile and compressive layers have the same elastic properties, $E_2$ and $\nu_2$) is also giving.

As depicted by Fig.6 the total stress intensity factor is split into two terms, the thermal term driving by the residual stress and the applied stress term. Obviously the "applied stress" intensity factor is always positive as the crack propagates through the material. However, the compressive residual stress -in one corresponding set of layers- serves to close the crack by applying a negative "thermal" stress intensity factor. For the material to resist failure due to crack propagation the combination of both "residual thermal" stress intensity factor $K_{th}$ and "applied stress" intensity factor $K_{app.}$ must not exceed the fracture toughness $K_c$ of the compressive layer material. Using our calculations along with applying this condition enable us to determine the maximum critical applied strength to propagate a crack, i.e. threshold strength, as follows. By assuming a known Stress intensity factor for the compressive layer material and with the help of Fig.6 we can determine the difference between the fracture toughness $K_c$ and the "thermal residual" stress intensity factor $K_{th}$. Let this value be $\Delta K_{max}(=K_c\text{-}K_{th}.)$ This calculated value $\Delta K_{max}$ is exactly how high the "applied stress" intensity factor is allowed to attain. Let the value of current "applied stress" intensity factor, depicted in Fig.6, be $K_{app.}$. Thus, the ratio $\Delta K_{max}$ to $K_{app}$ is also the ratio of the critical strength $\sigma_{crit.}$ to the value of the applied stress $\sigma_a$ used in our calculations (Note that the applied stress intensity factor has a linear relationship with stress.) By doing this for each point in Fig.6, we can determine the threshold strength as the highest critical stress value calculated. Expressed in a mathematical form and according to the above description, the threshold strength can be giving by:

$$\sigma_{th} = \left[ \frac{(K_c - K_{th}).\sigma_a}{K_{app.}} \right]_{max.} \tag{4}$$

By varying values of fracture toughness of the compressive layer material, a plot of the threshold strength $\sigma_{th}$ versus fracture toughness could be constructed. To express graphically the effect of the elastic modulus mismatch -between tensile and compressive layer- on threshold strength, multiple curves are plotted for a variety of layer modulus ratios. Since the thickness ratio is the next parameter that is believed to influence the magnitude of the threshold strength, a couple of other plots are as well constructed in the are as well constructed in the same manner but for different thickness ratios. A total of eight plots for $\frac{t_2}{t_1} = 1$, 1.5, 2, 2.5 are presented in Fig.7-14.

## 5.0 Discussion

The simulation results showed a good agreement with the theoretical model results when the tensile and compressive layers have same elastic properties. As mentioned above, the theory ceases to be exact for the heterogeneous case where the real experimental conditions are generally represented. The model showed to be a reliable alternate for the theoretical framework. It has gained our confidence because it simulated the homogeneous material fairly well and gave results that are satisfactory close to the exact theoretical ones.

As shown in the figures depicting threshold strength versus toughness, the simulation results showed that the threshold strength increases with increasing toughness. This is expected since a tougher material is more apt to resist against crack propagation and failure. This is true for all calculations.

The ambiguity of the results at the interface (see Fig.5) is related to the difference in material properties of the tensile and compressive layers. Consider two different elements belonging to two different layers at the interface of the compressive/tensile layer. These elements although having different material properties are strained equally since both layers are fused together (i.e. due to the continuity of the materials.) The two different layers are subject to the same external load. However, due to their different stiffness the two elements are subject to two different stresses. When the compressive layer is more compliant than the tensile layer the element in the compressive layer is subject to a lower stress. Thus, the driving force for crack propagation is made lower for materials with compressive layers more compliant than tensile layers.

The thickness ratio $\frac{t_2}{t_1}$ is varied within the range of 1 to 2.5. This range is, on one hand, chosen based on results of previous work. McMeeking and Hbaieb [4] claimed that it is within the range of 1 and 2.8 that the optimal threshold strength is to be expected.

On the other hand, since no major variety in the calculation results is detected, there was no need to further the range of $\frac{t_2}{t_1}$.

In our previous work [4] an attempt was made to optimize the threshold strength when both tensile and compressive layers have same elastic properties. We supposed that $\sigma_{th}$ is optimized when maximizing both toughness and residual stress and minimizing layer thicknesses. Moreover, the thickness ratio $\frac{t_2}{t_1}$ is to be chosen within the range of 1 to 2.8. Nonetheless, this work disregarded any effect of an elastic mismatch on the threshold strength. This current work has, however, proven that such an assumption is not acceptable when relative large elastic modulus ratio is at hand. For a range of $E_1/E_2$ between 1 and 10, threshold strength can be multiplied by a factor of 3. Thus, the elastic modulus ratio is an essential parameter that must be included in searching for the maximum $\sigma_{th}$ for a thorough estimation to be valid. Since many parameters are now necessary to account for the best value of $\sigma_{th}$, a construction of a map in the fashion of Ashby to identify the most promising combination of these parameters is increasingly motivated. Such an effort would be a topic of future research task.

## 6.0 Conclusion

As implied in the previous section, the elastic modulus mismatch is a relevant parameter to include in optimizing the threshold strength. Although we don't have results for a wide range of thickness ratios, we suppose that the thickness ratio variation hardly influences the value of the threshold strength. The best way to maximize the threshold strength including selecting materials that give the highest $K_c$, $E'_1$ and $\varepsilon$ and making both tensile and compressive layers as thin as possible, is to choose a tensile layer much stiffer than a compressive layer.

## 7.0 References

[1] F. F. Lange, *J. Am. Ceram. Soc.* **72** (no. 1), 3 (1989).

[2] D. J. Green, *Introduction to mechanical properties of ceramics* ( Cambridge Univ. Press, Cambridge, UK, 1998).

[3] Rao M., A. J. Sanchez-Herencia, G. E. Beltz, R. M. McMeeking, F. F. Lange, *Science*, **286**, (no. 5437), 102-5 (1999).

[4] R. M. McMeeking, Hbaieb K., *Zeitshrift fuer Metalkunde,* **90** (no.12), 1031 (1999).

[5] ABAQUS Users and Examples Manual. Version 5.8. Hibbitt, Karlsson and Sorensen Inc. Pawtucket, R. I.

[6] Rice J. R., "A path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks." *J. App. Mech.*, v. 34, **68**, pp. 379-386.

[7] Moran, B. and Shih, C. F., "A General Treatment of Crack Tip Contour Integral." *International Journal of Fracture*, Vol.35, 1987, pp. 295-310.

## 8.0 Figure capture

Fig.1: A laminar ceramic involving a through crack in the tensile layer partially penetrating in the compressive layer, is loaded parallel to the layers. Linearity allows superposition of two known fracture mechanics solutions to account for the total stress intensity factor.

Fig.2: Mesh of two-dimensional plane-strain model. The mesh is refined at the region around the crack.

Fig.3: Refined mesh showing very small elements in the compressive layers. A portion of the tensile layer is also shown. The elements in this region are also small, however a bit bigger than the compressive layer elements.

Fig.4: Comparison of simulation results with theoretical model results for a homogeneous material. Both tensile and compressive layers have same thickness.

Fig.5: Simulation results for elastic modulus in the tensile layer 1.7 times higher than the elastic modulus in the compressive layer. The theoretical model results for homogeneous material is also plotted for comparison.

Fig.6: Example of "applied" and "residual thermal" intensity factor results and their combination. From this plot the threshold strength can be estimated.

Fig.7-14: Threshold strength vs. compressive layer toughness and ratio of the elastic modulus of the tensile layer to the elastic modulus of the compressive layer. Threshold strength can be made three times higher when a tensile layer is chosen to be 10 times stiffer than the compressive layer.

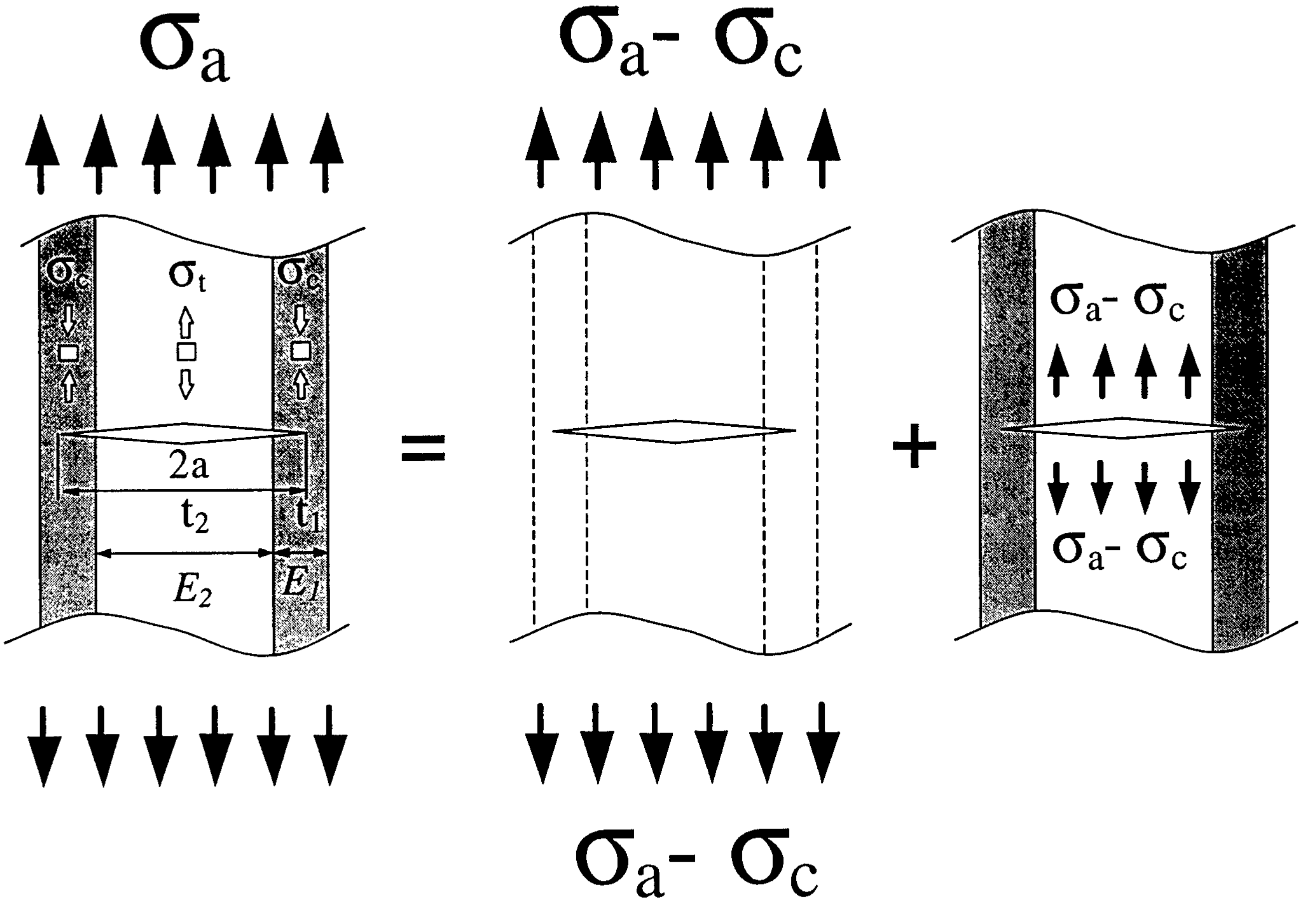



**Fig.1:**

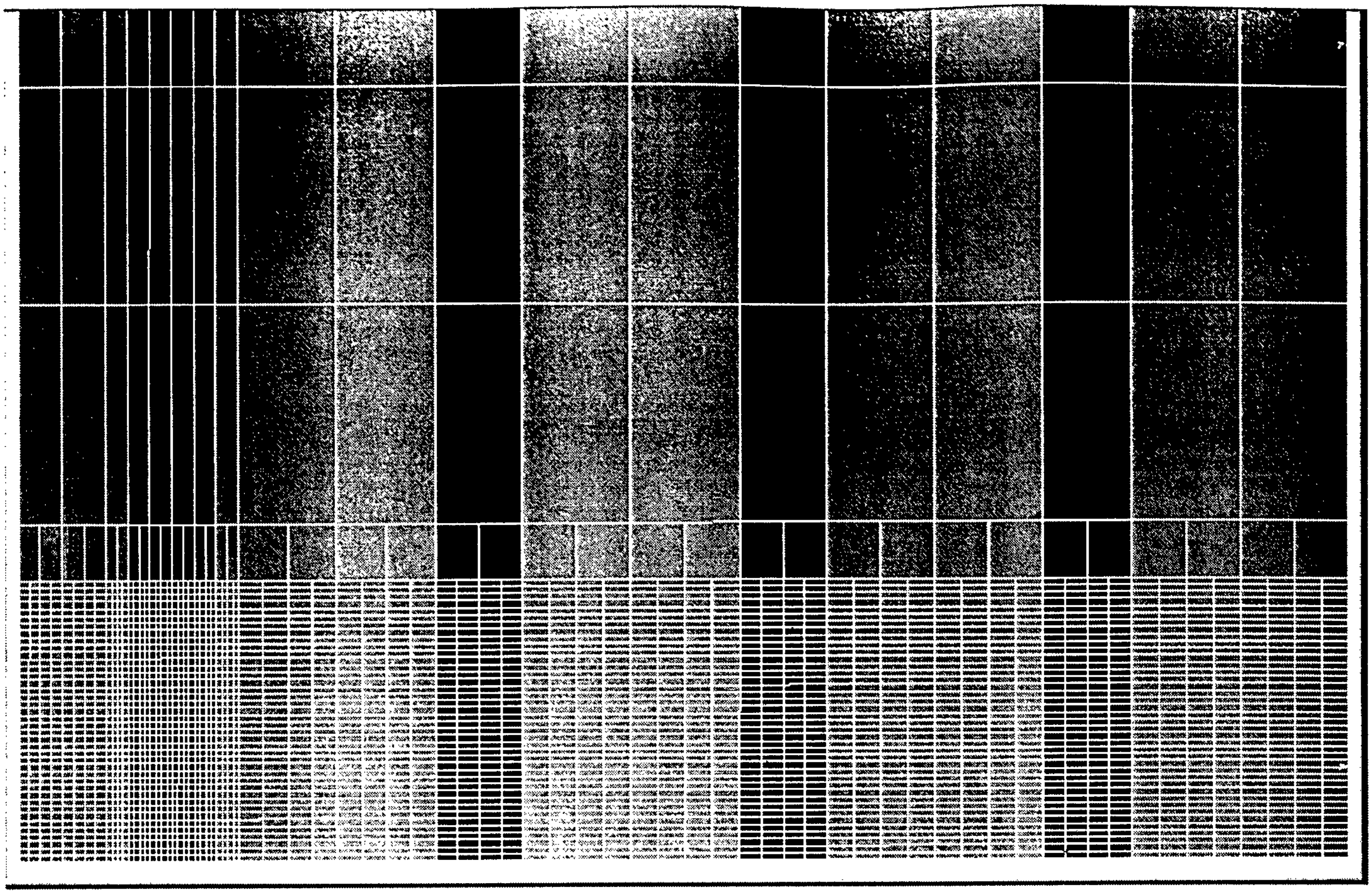


**Fig.2:**

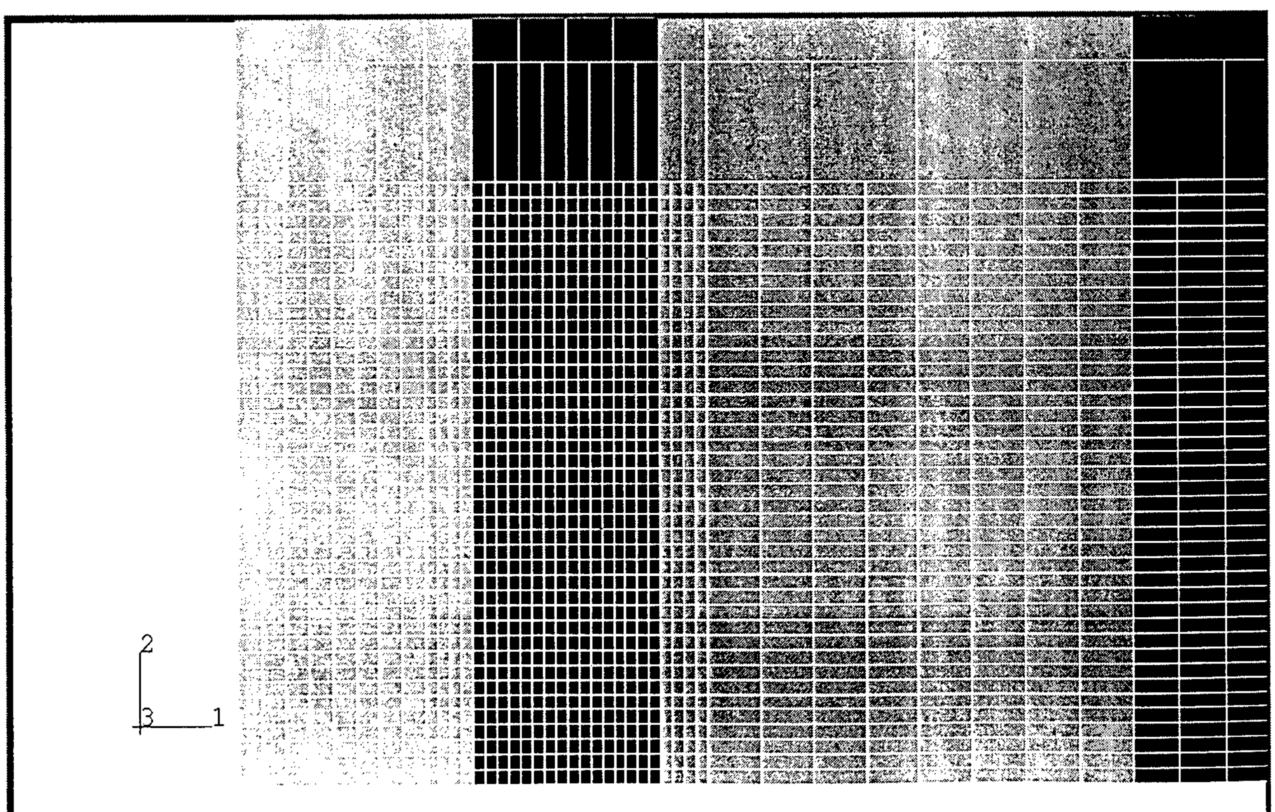



**Fig.3:**

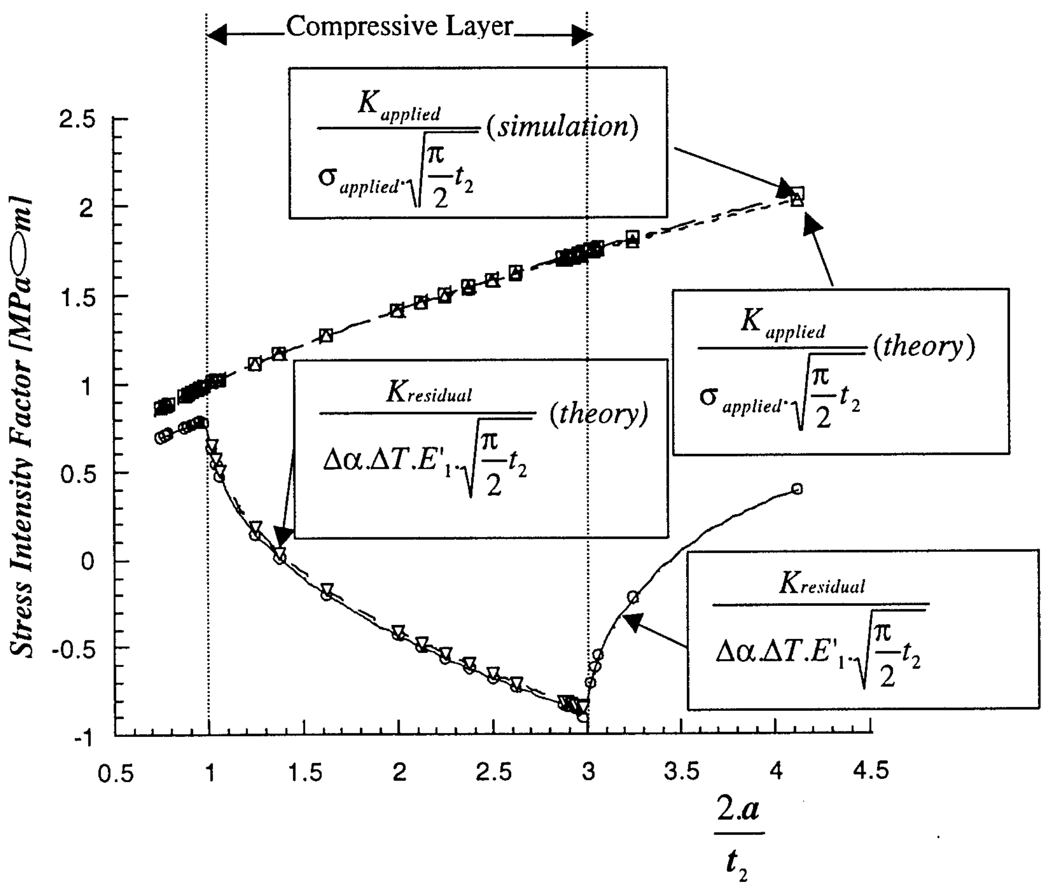
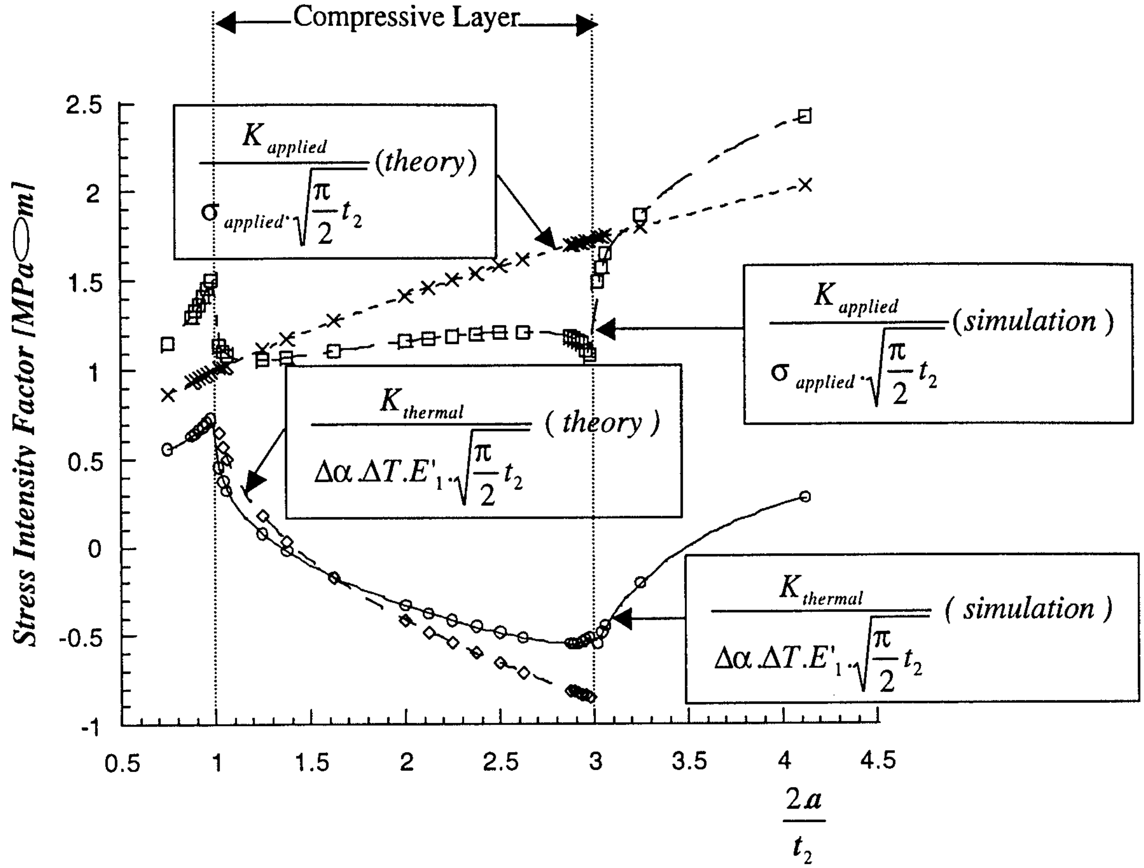

Compressive Layer
$\dfrac{K_{applied}}{\sigma_{applied}\cdot\sqrt{\frac{\pi}{2}t_2}}$ (simulation)
$\dfrac{K_{applied}}{\sigma_{applied}\cdot\sqrt{\frac{\pi}{2}t_2}}$ (theory)
$\dfrac{K_{residual}}{\Delta\alpha.\Delta T.E'_1.\sqrt{\frac{\pi}{2}t_2}}$ (theory)
$\dfrac{K_{residual}}{\Delta\alpha.\Delta T.E'_1.\sqrt{\frac{\pi}{2}t_2}}$
Stress Intensity Factor [MPa√m]
$\dfrac{2.a}{t_2}$

**Fig.4:**



Compressive Layer
Stress Intensity Factor [MPa√m]
2.5
2
1.5
1
0.5
0
-0.5
-1
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\frac{K_{applied}}{\sigma_{applied} \cdot \sqrt{\frac{\pi}{2} t_2}}$ (theory)
$\frac{K_{applied}}{\sigma_{applied} \cdot \sqrt{\frac{\pi}{2} t_2}}$ (simulation)
$\frac{K_{thermal}}{\Delta\alpha . \Delta T . E'_1 \cdot \sqrt{\frac{\pi}{2} t_2}}$ ( theory )
$\frac{K_{thermal}}{\Delta\alpha . \Delta T . E'_1 \cdot \sqrt{\frac{\pi}{2} t_2}}$ ( simulation )
$\frac{2a}{t_2}$


**Fig.5:**

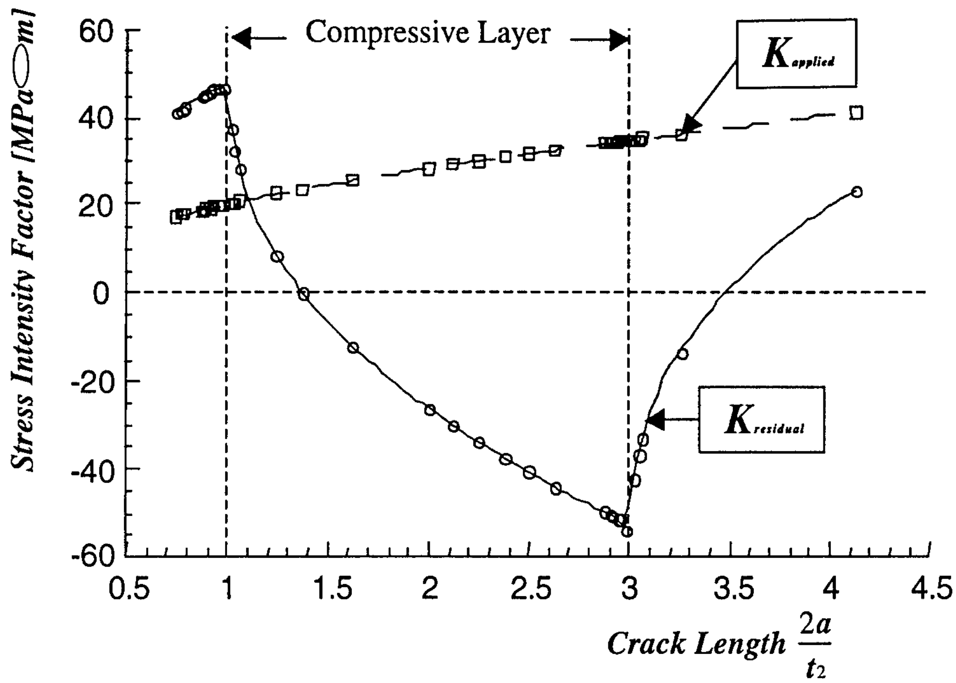

Compressive Layer
$K_{applied}$
$K_{residual}$
Stress Intensity Factor [MPa√m]
Crack Length $\frac{2a}{t_2}$
60
40
20
0
-20
-40
-60
0.5
1
1.5
2
2.5
3
3.5
4
4.5

**Fig.6:**

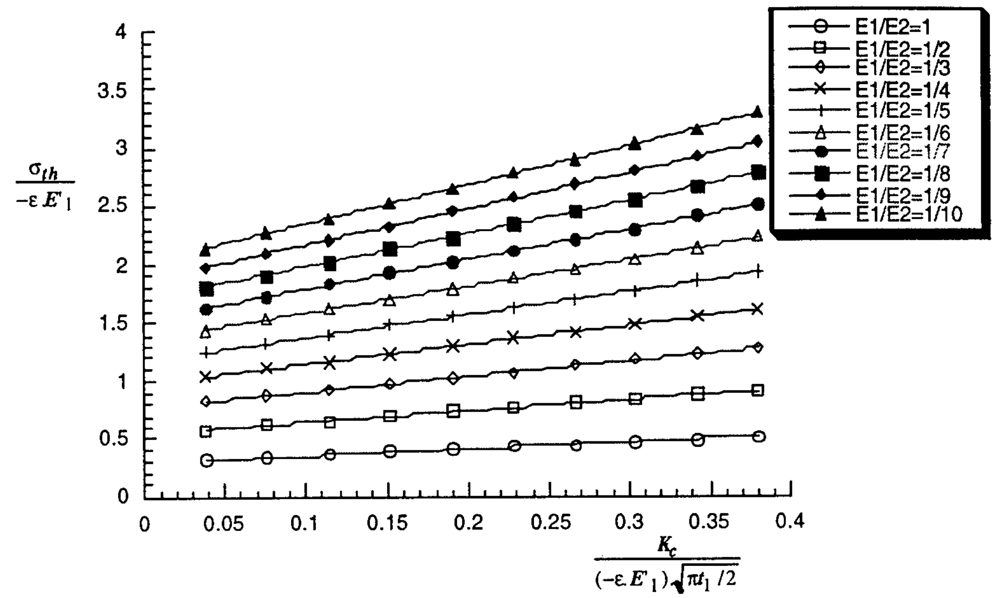



**Fig.7**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1/10$ to1. These calculations are made for a thickness ratio $t_2/t_1 = 1$.

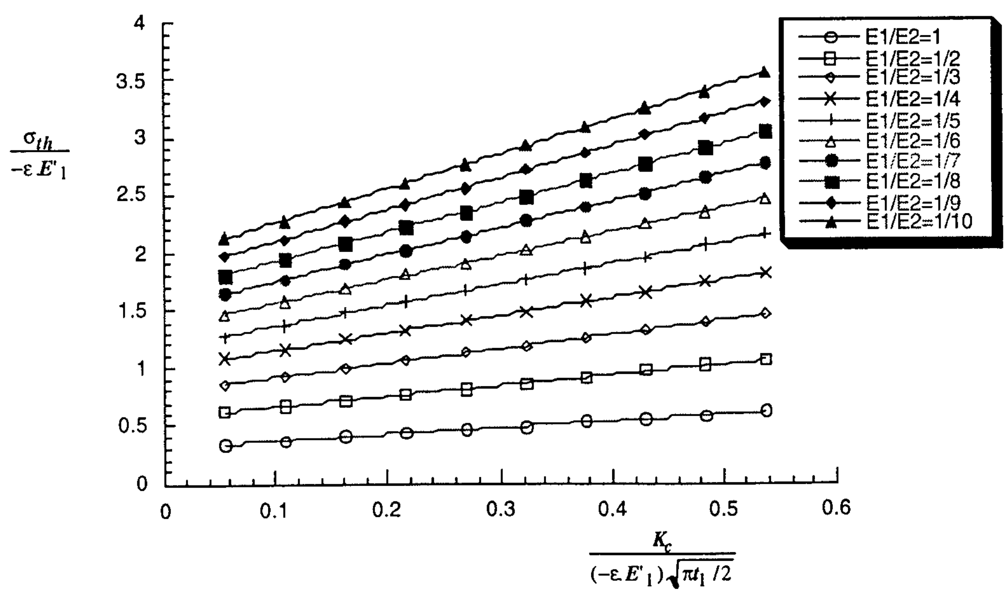



**Fig.8**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1/10$ to1. These calculations are made for a thickness ratio $t_2/t_1 = 1.5$.

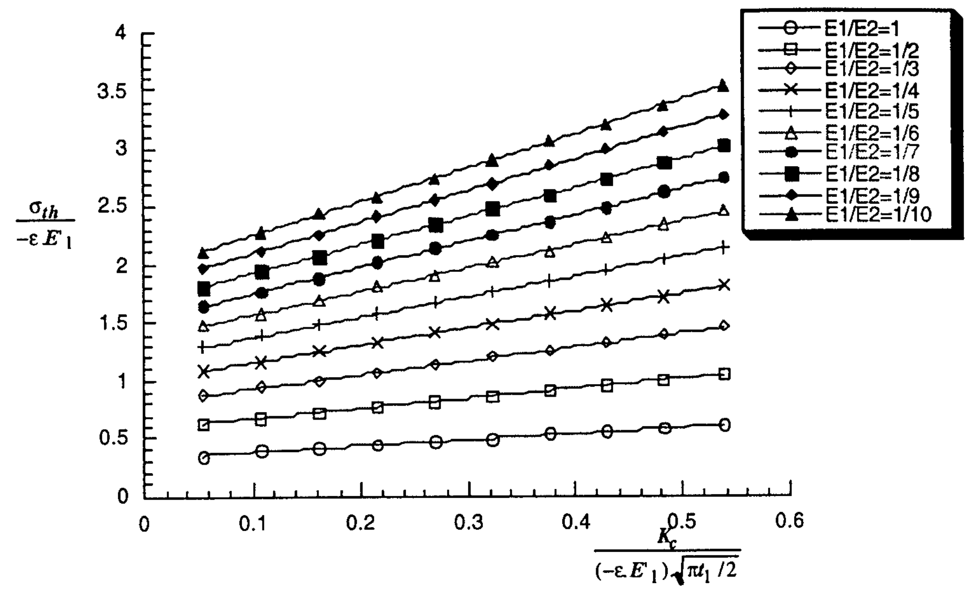



**Fig.9**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1/10$ to $1$. These calculations are made for a thickness ratio $t_2/t_1 = 2$.

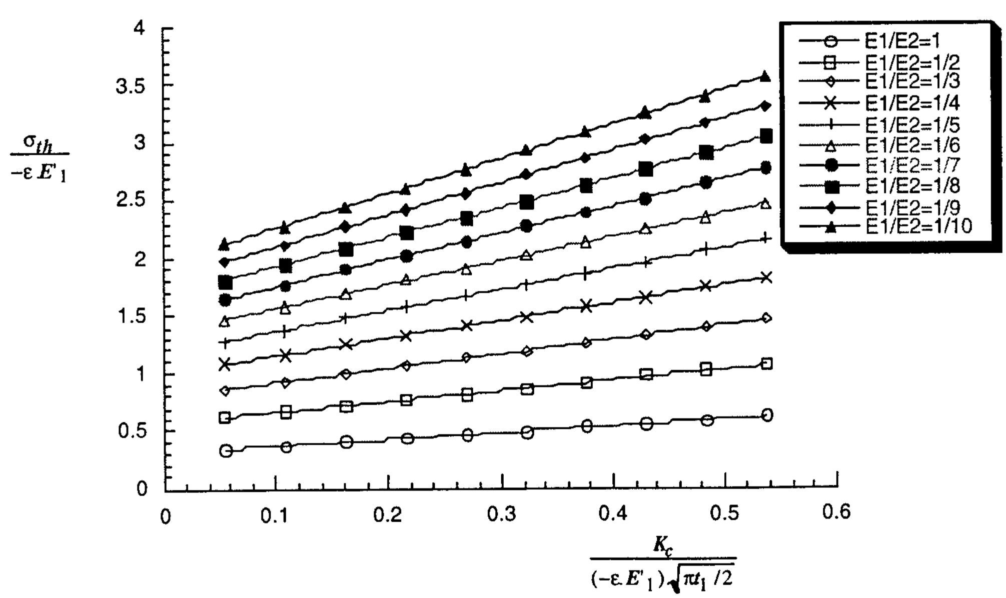



**Fig.10**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1/10$ to $1$. These calculations are made for a thickness ratio $t_2/t_1 = 2.5$.

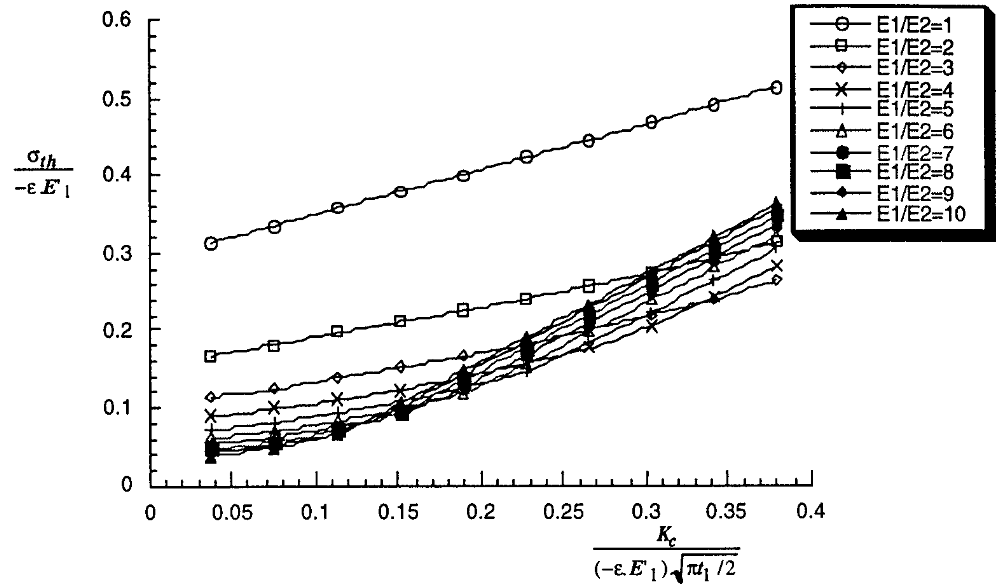



**Fig.11**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1$ to10. These calculations are made for a thickness ratio $t_2/t_1 = 1$.

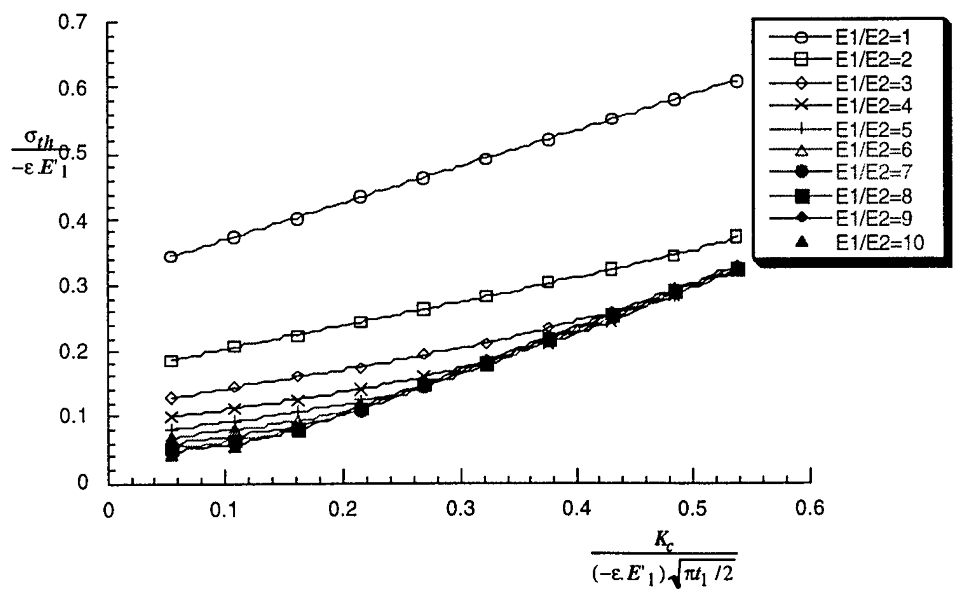



**Fig.12**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1$ to10. These calculations are made for a thickness ratio $t_2/t_1 = 1.5$.

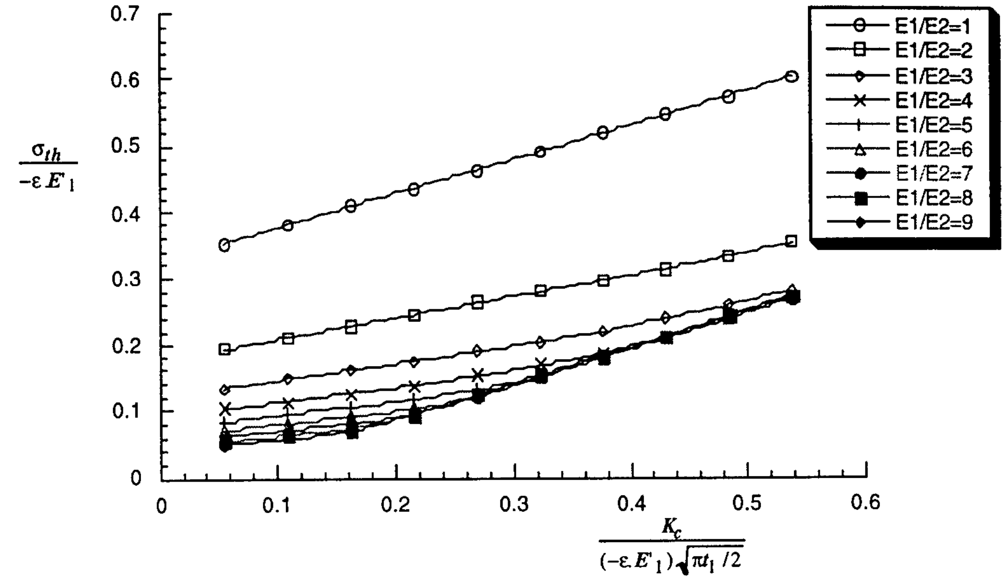



**Fig.13**: Threshold strength versus fracture toughness of the compressive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1$ to10. These calculations are made for a thickness ratio $t_2/t_1 = 2$.

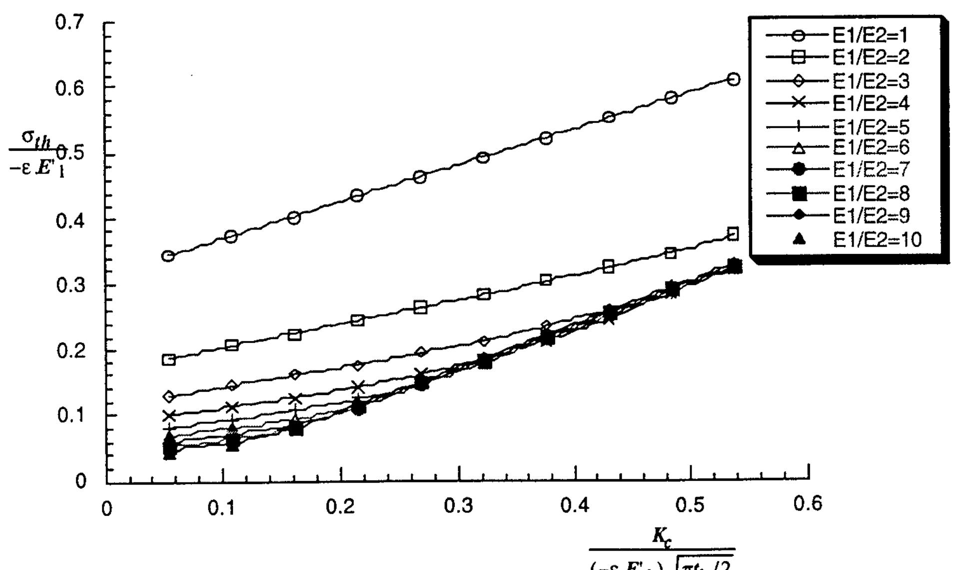



**Fig.14**: Threshold strength versus fracture toughness [illegible]ive layer material. The multiple curves depicted in this figure are for different elastic modulus ratio lying in the range $E_1/E_2 = 1$ to10. These calculations are made for a thickness ratio $t_2/t_1 = 2.5$.

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The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an offician Department of the Army position, policy or decision, unless so designeated by other documentation.

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13. ABSTRACT (Maximum 200 words)

Finite element analysis was carried out to predict the threshold strength of a laminar ceramics loaded parallel to the layers. These materials are composed of alternate layers of two different ceramics in which residual stress is generated. Strength limiting cracks are trapped by the compressive layers and require a minimum (threshold) applied stress to cause them to fail the laminated ceramic. The calculations were utilized to study the influence of the elastic modulus mismatch between the alternate tensile and compressive layers. Good agreement was established between numerical simulations and theoretical results for materials involving layers with the same elastic properties. Results were obtained for a variety of combinations of different ceramics and suggest that threshold strength as high as three times the effective residual stress in the compressive layer is conceivable.


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