

REPORT DOCUMENTATION PAGE

AFRL-SR-BL-TR-01-

0088

Public reporting burden for this collection of information is estimated to average 1 hour per response, including gathering and maintaining the data needed, and completing and reviewing the collection of information. Send collection of information, including suggestions for reducing this burden, to Washington Headquarters Service, Paperwork Project, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Project, Suite 1204, Arlington, VA 22202-4302.

ces,  
this  
rson

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE		3. REPORT PERIOD FROM 01 May 1997 - 31 Dec 1998	
4. TITLE AND SUBTITLE Turbo Decoding of High performance Error-Correcting Codes via Belief Propagation				5. FUNDING NUMBERS F49620-97-1-0313	
6. AUTHOR(S) Robert McEliece					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Electrical Engineering MS 136-93 California Institute of Technology Pasadena, CA 91125				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AFOSR 801 North Randolph Street, Room 732 Arlington, VA 22203-1977				10. SPONSORING/MONITORING AGENCY REPORT NUMBER  F49620-97-1-0313	
11. SUPPLEMENTARY NOTES					
12a. DISTRIBUTION AVAILABILITY STATEMENT Approved for Public Release. Distribution unlimited.				AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFOSR) NOTICE OF TRANSMITTAL DTC: THIS TECHNICAL REPORT HAS BEEN REVIEWED AND IS APPROVED FOR PUBLIC RELEASE LAW AFR 190-12. DISTRIBUTION IS UNLIMITED.	
13. ABSTRACT (Maximum 200 words) We studied AWGN coding theorems for ensembles of coding systems which are built from fixed convolutional codes interconnected with random interleavers. We call these systems turbo-like codes and they include as special cases both the classical turbo codes and the serial concatenation of interleaved convolutional codes. We offered a general conjecture about the behavior of the ensemble (maximum-likelihood decoder) word error probability as the word length approaches infinity. We proved this conjecture for a simple class of rate 1/q serially concatenated codes where the outer code is a q-fold repetition code and the inner code is rate 1 convolutional code with transfer function 1/(1+D). We call these codes "RA" (repeat and accumulate) codes. This was the first rigorous proof of a coding theorem for turbo-like codes.					
14. SUBJECT TERMS				15. NUMBER OF PAGES 1	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT		18. SECURITY CLASSIFICATION OF THIS PAGE		19. SECURITY CLASSIFICATION OF ABSTRACT	
				20. LIMITATION OF ABSTRACT	

20010220 013

NM

**FINAL TECHNICAL REPORT on AFOSR grant no. F49620-97-0313:**

*"Turbo Decoding of High performance Error-Correcting Codes via Belief Propagation"*

**PRINCIPAL INVESTIGATORS:**

Robert J. McEliece  
Department of Electrical Engineering  
MS 136-93  
California Institute of Technology  
Pasadena, CA 91125

Padhraic Smyth  
Department of Information and Computer Science  
University of California  
Irvine, CA

**OBJECTIVES:** No change from those stated in the original proposal.

**STATUS OF EFFORT:** The work went extremely well at both institutions. We had frequent joint meetings (approx. 1 per month) in which each group informed the other about its progress. Technical details follow.

**ACCOMPLISHMENTS/NEW FINDINGS:**

\*\*\*At Caltech:

We studied AWGN coding theorems for ensembles of coding systems which are built from fixed convolutional codes interconnected with random interleavers. We call these systems "turbo-like" codes and they include as special cases both the classical turbo codes and the serial concatenation of interleaved convolutional codes. We offered a general conjecture about the behavior of the ensemble (maximum-likelihood decoder) word error probability as the word length approaches infinity. We proved this conjecture for a simple class of rate  $1/q$  serially concatenated codes where the outer code is a  $q$ -fold repetition code and the inner code is a rate 1 convolutional code with transfer function  $1/(1+D)$ . We call these codes "RA" (repeat and accumulate) codes. This was the first rigorous proof of a coding theorem for turbo-like codes.

These results show that the performance of RA codes with maximum-likelihood decoding is very good. However, the complexity of ML decoding of RA codes, like that of all turbo-like codes, is prohibitively large. But an important feature of turbo-like codes is the availability of a simple iterative, message passing decoding algorithm that approximates ML decoding. We wrote a computer program to implement this "turbo-like" decoding for RA codes with  $q=3$  (rate  $1/3$ ) and  $q=4$  (rate  $1/4$ ), and the results were remarkably good. For example, with an information block length of 16384, the  $q=4$  RA code achieves a decoded word error probability of about  $10^{-5}$  at  $E_b/N_0 = 0.5$  dB with 20 decoding iterations.

This work demonstrated that there is a much less complex way to achieve near Shannon limit performance than was previously suspected.