PNE-3012(Vol. 1)

AN ECONOMETRIC STUDY OF SMALL
AND INTERMEDIATE SIZE DIAMETER
DRILLING COSTS FOR THE UNITED STATES

Prepared for
U. S. Atomic Energy Commission
Washington, D. C.

MATHEMATICA

One Palmer Square   Princeton, New Jersey

20000908 174

AN ECONOMETRIC STUDY OF

SMALL AND INTERMEDIATE SIZE DIAMETER

DRILLING COSTS FOR THE UNITED STATES

by

Estela M. Bee Dagum

and

Klaus-Peter Heiss

Prepared for

FOREWORD

A significant part of the cost of conducting a Plowshare project is associated with drilling and preparing the hole needed for the emplacement of the nuclear explosive underground. The cost is determined by the type of rock in which the hole is drilled and the diameter and depth of the hole. A number of the proposed applications for nuclear explosions will require the emplacement of the explosive to great depths. If the explosive also requires a large diameter hole, the costs will be very substantial. Moreover, if casing is required to protect the explosive, the costs will be even higher.

In 1966 the Corps of Engineers published a report on the cost of drilling large diameter holes. However, up to the present time, there have been no readily accessible data on the cost of drilling small and intermediate size holes at different depths and in different rock types. Consequently, it has been difficult to determine, with any accuracy, the savings that would accrue if explosives could be designed to fit in smaller diameter or uncased holes. In order to meet this problem, we engaged Mathematica to conduct a study of actual drilling costs in the United States for both small and intermediate size holes. The resulting study provides significant data required for an evaluation of trade offs that can be made in explosive designs, emplacement hole requirements and other design considerations which will affect the cost of Plowshare projects.

With the expectation that this information will be of general interest, as well as an aid to those involved in the Plowshare program, the AEC is pleased to make this report available.

John S. Kelly, Director
Division of Peaceful
Nuclear Explosives

CONTENT OF VOLUME I

CONTENT OF VOLUME II

SECTION II.   INTERMEDIATE SIZE DIAMETER DRILLING COST FUNCTIONS:

II.1.   Mud Costs: Table 1

II.2.   Cutter Costs: Tables 2 to 6

II.3.   Casing Costs: Table 7

II.4.   Cementing Costs: Table 8

II.5.   Rig Costs: Tables 9 to 13

II.6.   Total Fixed Costs: Table 14

II.7.   Total and Average Costs for Cased Wells: Tables 15 to 19 and the corresponding figures for average costs both as a function of Depth and Diameter.

II.8.   Total and Average Costs for Uncased Wells: Tables 20 to 24 and the corresponding figures for average costs both as a function of depth and diameter.

II.9.   Difference of Total Drilling Costs Between Cased and Uncased Wells: Tables 25 to 29.

II.10.   Marginal Drilling Costs for Cased Wells: Tables 30 to 34.

II.11.   Marginal Drilling Costs for Uncased Wells: Tables 35 to 39.

GENERAL INTRODUCTION

Emplacement costs are a major component of the direct costs in any peaceful application of underground nuclear explosives. As direct costs, we define all costs which will result from each individual shot, independent of the number of shots made in the same general area and/or within the same time period. The other major direct cost of any underground application is the cost of the nuclear device. The level of these direct costs will be of great importance in the long-term outlook of the development of PLOWSHARE techniques to a stage where significant industrial applications will be possible. In particular, these direct costs will determine the areas in which such applications will be economically feasible and the number of explosives demanded (i. e., the general level of activity in the peaceful use of nuclear explosives).

In this study we are mainly concerned with the cost of the emplacement holes of nuclear explosives for peaceful applications. All required diameters for such uses can today be drilled to considerable depth. On the Amchitka Island, a major drilling

program is under way to drill three 90-inch holes to 6000 feet [11].

Holes of up to 140 inches were drilled to smaller depths and, though

the technology in large-diameter hole drilling is progressing to still

larger-diameter capabilities, the available range suffices for any

foreseeable economic application of PLOWSHARE technology.

In general, costs increase with larger hole diameters. As the emplace-

ment hole diameter is strictly related to the required diameter of the nuclear

device, and as the explosive diameter can be reduced in further

research, development, and production programs--and furthermore,

as this diameter may change considerably with the varying restraints

put upon the explosion, depending on its field of application -- it

becomes of great importance to evaluate how drilling costs are

reduced by decreasing the required diameter of the emplacement hole.

The present study tries to estimate this cost reduction and in particular,

it will evaluate intermediate-size diameter drilling costs. (12 to 36

inches) down to standard oil and gas well dimensions.

The theoretical limit in the reduction of emplacement costs

would be reached if all desired field of explosives for peaceful applications

could be put to the desired depth in standard size oil and gas wells,

say 9-7/8 or up to 13-3/4-inch diameters. Information given by the U.S.

Atomic Energy Commission indicates that considerable progress is

possible in this direction:

"Nuclear explosives have not been designed specifically for
underground engineering applications. When conditions
warrant, such special designs could be undertaken. It is
reasonable for industry to assume, for first generation
designs, that yields up to 100 KT could be obtained in a
canister with an outside diameter of 11 inches, suitable
for emplacement in a standard 13-3/8-inch OD casing
designed with at least 12-1/8-inch clear inside diameter,
of when appropriate, open hold of the same minimum size.
Unusual formation pressures and temperatures may present
special problems requiring larger diameters than the above."

A variety of approaches are being, and were,
proposed to determine the drilling costs for wells. The variety of
approaches has in part to do with the aims of the study, in part with
technical reasons, and in part with the form in which the data are
collected.

The aims for establishing drilling cost functions
led to two major subdivisions: drilling costs per day of operation,
and drilling costs per foot drilled. A well operator is more interested

in the costs per day and an overall estimate on the time required to drill a particular well, as he has to plan for an optimal use of his (expensive) equipment and drill crews. Also, most data are available to the operator which makes an estimate of drilling costs per day more readily available. From estimates of required drilling time, trip time, down time, and the costs of the rig, the bits, and crew, the operator does arrive at relatively accurate estimates of the costs per well in a particular region and based on his experience. Bit records are further kept showing individual bit life and time requirements for each well. Also, once the geology of the formation is known, penetration rates per medium can be inferred and, adding up the required time periods, this again will give the operator an overall estimate of expected costs per well.

Thus, an appropriate and, for well operators, a meaningful procedure would be to estimate statistically not the cost per foot at various depths but the time required to reach a certain depth in a particular region. With this information, the operator can then infer costs per well given his company's technical capabilities. As the drilling costs per day are more or less constant given a particular rig, this has the further advantage that well costs are directly proportional to drill time requirements.

This procedure would also allow a much wider and more accurate estimate in all cases where only a <u>limited number of wells were drilled to a particular depth range</u> and no well data are

available for any, or some other, depth ranges. As given, the
time record charts for each well and some estimate of the per day
cost of operating a particular rig, one can now estimate the costs
for any intermediate well depth, even if, for example, only twenty
wells were drilled to 10,000 feet and none in any other class. Thus,
estimates of cost per day and of drill time based on the drill records
of each well will allow much wider cost estimates. From these data,
one can then infer the costs of any well in between and, of course, also
its cost per foot as a result.

This approach has the added advantage that it would,
theoretically, allow also estimates of the costs of wells when their
diameter is increased, based on the time records of deep wells and
disaggregating the costs with regard to depths and diameters used.
From this again, the costs per well and different diameters can be
inferred on the basis of a relatively small sample of total wells in
each class. This is basically also the procedure used in estimating
the costs of intermediate wells in this study.

For an oil and gas company, on the other hand,
drilling costs per foot or total costs of a well to a certain depth
are of more interest and greater meaning. Given an oil and gas
deposit and its depth, the question then is not the drilling time
requirement but the expected cost of tapping the reservoir, which
again is directly a function of its depth. Furthermore, it is also a

question of available data: the Joint Survey collects and publishes

costs per well and footage, and the wealth of the data is, for

standard size holes, such that reasonably accurate estimates of

the cost function per well or per foot can be made, disregarding the

additional information of each well. As mentioned above, a much

more immediate and sufficient improvement could be made with

regard to total well cost data to allow accurate estimates of costs

per well and per foot for standard size diameters.

The study is divided in two volumes. Volume I deals with the

economic interpretation of the estimated drilling cost functions for small

and intermediate size diameter holes. This economic analysis for

small diameter holes is made for total average and marginal costs per

different states and geological regions of the United States. The economic

analysis for intermediate size diameter holes is made considering the

components of total variable and total fixed cost, namely, mud

cutter, casing, cementing, and rig, for the former; and, mobilization,

site preparation surface casing and rig-up and tear-down for the latter.

Two Appendices are also included in volume I, which refer to the

econometric analysis of small and intermediate size diameter holes.

Volume II includes a Statistical Annex of Tables and corresponding

figures that supplement the econometric analysis.

SECTION I

DRILLING COST ANALYSIS FOR SMALL
DIAMATER EMPLACEMENT HOLES

I.1   An Economic Interpretation of the Estimated Average,

Total and Marginal Drilling Cost Functions

The purpose of this section is to provide information pertaining to the cost of drilling small diameter oil and gas wells and dry holes in the United States. By small diameter holes we mean standard size holes drilled by oil and gas companies that range approximately from 4 to 12 inches diameter.

Cost functions per foot drilled were obtained for various states and geological regions. The states comprised are:

        Alabama
        Arkansas
        California (Onshore only)
        Colorado
        Kentucky
        Kansas
        Louisana (Subdivided into South and North)
        Michigan
        Mississippi
        Montana
        New Mexico (Subdivided in West and East)
        North Dakota
        Oklahoma
        Texas (Subdivided in Southwest, East, West,
                North Central, Gulf Coast and Panhandle)
        Utah
        Wyoming
        Appalachian (Includes: New York, Ohio, Pennsylvania,
                and West Virginia)

The geological regions are classified according to the following eras [*] :

---

[*]   This classification is really not representative of the degree of hardness of the soil. Except for the Mesozoic formation, there is not a significant difference between drilling cost per foot in Region I (Carbon-Permian) and Region II (Cenozoic). One of the reasons of this situation is the great variety of kind of rocks involved in the Cenozoic Era, while the Mesozoic seems to be better defined by the presence, in general, of hard rocks.

a) Carbon-Permian (Region I);

b) Cenozoic (Region II); and

c) Mesozoic (Region III)

The final results indicate that for most of the states and for all the regions, the estimated average drilling cost function is a parabolic function of the depth and only seems to behave exponentially for depth greater than 15,000 ft.

Figure I, shows the general pattern followed by drilling costs per foot as a function of depth.

Average
Drilling
Costs
(in $)

Figure I
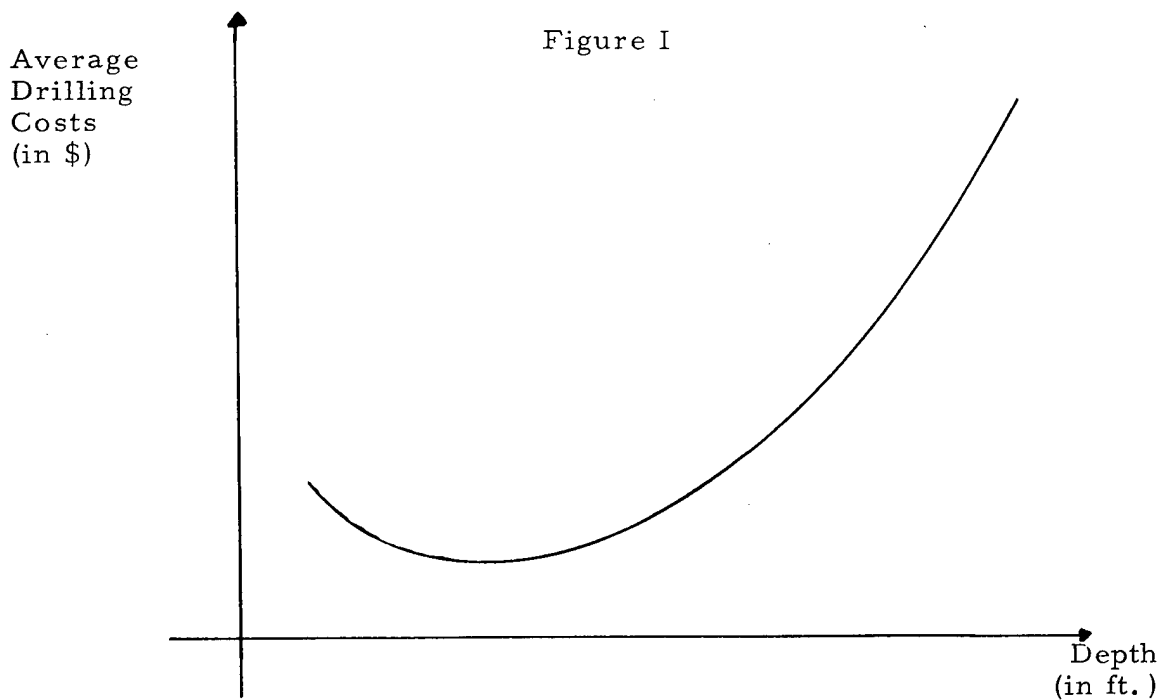
Depth
(in ft.)

As it is clearly indicated, drilling cost per foot will decrease at the outset, then flatten out somewhat and then, rise again.

In our calculations the lower range costs lie in the interval 3,000 - 5,500 ft. of depth for most of the observed cases. This general behavior is due to the presence of average fixed costs as one component of total average costs. The total average cost, i.e., drilling cost per foot, is defined as the sum of the average fixed cost and the average variable cost.

Fixed costs are those whose magnitude does not vary with the level of output, at least within some range. The special features of a fixed cost item are: (1) that it all comes in one big lump once it is decided to enter an operation, and (2) after it is incurred, a further expansion in total production makes no difference in its magnitude. An item per item analysis of fixed costs is given in section II.

Variable costs are given by the variable input services i.e., labor, materials, etc., times their corresponding prices. The variable cost per unit will generally fall at a beginning, then flatten out and then rise again as the equipment or plan capacity is approached and passed.

Therefore, given that fixed cost would remain constant throughout the interval (0 - 5,500 ft.) the larger the footage drilled the smaller the total average cost. Once the minimum interval is passed, cost per foot increases mainly because of the large increments in the average variable cost.

According to our study the two main variables that significantly affect drilling costs per foot are: the depth and the geological medium.

In regard to depth, we can illustrate it with some final results obtained for the states considered. Table I clearly indicates

that drilling cost per foot, in oil wells, increases at a higher

rate than the increment in depth i.e., within the interval

10,000 - 15,000 ft. On the other hand, the general behavior is not so

well defined for 5,000 - 10,000 ft. interval. In effect, the ratio of

increase in drilling costs is smaller here than the one corresponding

to depth greater than 10,000 ft. This situation results from a considerable

difference in the steepness of the estimated parabolic cost function for

each state, within the interval 5,000 - 10,000 ft depth.

Moreover, for Mississippi, East New Mexico and West Texas, the

rate of increase in drilling costs is nearly constant. This is due to the

fact that for these states the minimum average cost depth of their estimated

cost function is reached approximately at 6,000-7,000 ft. (See Table XIII).

This pattern is also observed for gas wells and dry holes (See Table II

and Table III). One important question has still to be dealt with at this

point: given that for oil and gas production wells the diameter of the wells

drilled does change from region to region between every 6 inches[**] and

10 inches, can one observe a significant correlation between drilling costs

of oil wells and these different standard size diameters? Though the data

collected and analyzed here are aggregated in such a form as to make

statistical tests of this hypothesis impossible we did, however, check back

with the casing and bit programs of rotary drilling for each drilling area.

The casing and bit programs for these areas were provided to us by the

National Supply Company and are valid as of June 1968. Conductor strings,

surface strings, intermediate strings and oil strings vary considerably

---

[**]At smaller diameters one may even observe a cost increase again.

according to this information. When compared to drilling costs per area given in the next tables, no significant correlation of drilling costs with these changes in standard size diameters can be observed. This conclusion is further confirmed later on, when analyzing intermediate diameter emplacement costs, where the cost curves at 10 inches to 12 inches coincide roughly with the aggregate cost function estimates of the various geological regions given for small diameter wells. Thus the "take-off" point for cost increases when drilling larger diameter holes occurs when changing from standard size to intermediate size holes i.e., in the region from 10 inches to 12 inches . The progression of drilling costs as a function of diameter is analyzed in detail in the second point of this study. The estimated costs of this section, therefore, can be regarded as the lower limit to emplacement costs. The following cost values were drawn from their corresponding estimated average drilling cost functions shown in Tables of the Annex of Section I.

Example 1

Table I

Drilling Cost Per Foot for Oil Wells in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
Arkansas	6.70	17.45	41.71
Colorado	11.75	40.08	107.75
Kansas	9.60	21.27	51.60
North Louisiana	6.95	17.20	44.20
South Louisiana	13.79	17.11	26.59
Mississippi	9.40	9.77	20.00
Montana	10.65	15.31	28.45
East New Mexico	12.33	13.85	25.39
Oklahoma	9.40	17.28	33.26
East Texas	6.78	19.12	50.38
Gulf of Texas	11.10	16.45	27.90
North Central Texas	8.40	13.30	22.45
Panhandle Texas	9.67	17.54	30.87
Southwest Texas	6.86	17.21	42.86
West Texas	10.94	13.35	24.05

Example 2

Table II

Drilling Cost Per Foot for Gas Wells in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
Appalachian	14.50	29.10	66.50
North Louisiana	7.90	21.20	46.40
South Louisiana	13.15	17.40	32.45
Mississippi	7.05	11.50	25.50
East New Mexico	14	22	35.90
Oklahoma	10.90	16.70	30.90
East Texas	12.80	17.20	36.70
Gulf Coast Texas	10.75	17.10	32.75
North Central Texas	10	18.20	46.20
Panhandle Texas	9.90	18.30	36.90
Southwest Texas	9.70	21.40	42.50
West Texas	10.45	16.80	34.80
Wyoming	17.50	26.50	54.50

Example 3

Table III

Drilling Cost Per Foot for Dry Holes in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
Alabama	5.75	12	27.75
Arkansas	6.65	12.25	30.80
Appalachian	8.75	24	52.75
California	8.85	14.30	27.75
Colorado	6.25	22.20	67.25
South Louisiana	7.05	10.80	26.05
Michigan	6	30.05	74.50
Mississippi	5.60	7.20	16.80
Montana	6.50	13.70	32.40
East New Mexico	9.60	13.60	27.50
West New Mexico	8.90	27.80	70.40
Oklahoma	7.80	15.70	33.20
Gulf Coast Texas	6.80	12.72	37.20
North Central Texas	4	13.90	37.30
Southwest Texas	5.90	15.30	32.80
West Texas	9.80	14.75	34.20
Utah	13.60	22.60	50.10
Wyoming	17.80	26.55	53.70

Considering now the depth as constant, the influence of the geological medium on drilling cost per foot is summarized in Tables IV, V, and VI. The general conclusions are that: (a) for oil wells and gas wells there is not a significant difference between the average drilling costs corresponding to Region I and Region II. On the contrary, drilling costs for Region III are always higher than those of Regions I and II, except only one case, for oil wells of 15,000 ft. depth; and; (b) for dry holes, considerable discrepancies are shown for all three regions which therefore makes it difficult to draw consistent conclusions of the geological medium influence on drilling costs.

We can illustrate this aspect with the following numerical examples for depths of 5,000., 10,000 ft., and 15,000., respectively. The corresponding data were obtained from the estimated average drilling cost functions shown in Tables of the Annex of Section I.

Example 4

Table IV

Drilling Cost Per Foot at 5,000 ft. Depth for all the Regions

(in dollars)

Regions	Oil Wells	Gas Wells	Dry Holes
Region I (Carbon-Permian)	8.95	10.23	6.40
Region II (Cenozoic)	8.71	10.45	4.91
Region III (Mesozoic)	11.27	12.06	7.90

Example 5

Table V

Drilling Cost Per Foot at 10,000 ft. Depth for all Regions

(in dollars)

Regions	Oil Wells	Gas Wells	Dry Holes
Region I (Carbon-Permian)	16.60	16.48	14.90
Region II (Cenozoic)	16.46	17.15	12.26
Region III (Mesozoic)	23.90	19.50	12.63

Example 6

Table VI

Drilling Cost Per Foot at 15,000 ft. Depth for all Regions

(in dollars)

Regions	Oil Wells	Gas Wells	Dry Holes
Region I (Carbon-Permian)	31.25	32.23	32.40
Region II (Cenozoic)	32.71	33.65	29.51
Region III (Mesozoic)	26.27	34.86	33.50

For most of the cases analyzed, given the same depth and geological medium, there is not a substantial difference in drilling costs per foot corresponding to different states. This circumstance supports the choice of these two variables as the most relevant in the determination of drilling costs. The only few exceptions are: dry holes of Colorado, Southwest Texas and East Texas; oil wells for North Central Texas and East Texas, and gas wells for Southwest Texas and North Louisiana.

This point is illustrated with the following numerical examples for oil and gas wells and dry holes. The figures were obtained from their corresponding Tables of the Annex of Section I.

Example 7

Table VII

Drilling Cost Per Foot at 5,000 ft. of Depth for States Belonging
to the Mesozoic Geology (Region III)

(in dollars)

States	Oil Wells	Gas Wells	Dry Holes
Colorado	11.75	-----	4.75
West Texas	10.94	10.80	10.93
East New Mexico	12.33	13.90	10.83
----- indicates unavailable data			

Example 8

Table VIII

Drilling Cost Per Foot at 10,000 ft. of Depth for States With
Carbon-Permian Geology (Region I)

(in dollars)

States	Oil Wells	Gas Wells	Dry Holes
Oklahoma	17.28	16.44	15.70
North Central Texas	13.30	18.22	13.91
Arkansas	17.45	- - - - -	12.11
- - - - - indicates unavailable data			

Example 9

Table IX

Drilling Cost Per Foot at 10,000 ft. of Depth for States Belonging
to the Cenozoic Geology (Region II)

(in dollars)

States	Oil Wells	Gas Wells	Dry Holes
South Louisiana	17.11	17.98	11.02
North Louisiana	17.20	21.23	- - - - -
Panhandle Texas	17.54	18.29	- - - - -
Southwest Texas	17.21	21.42	19.80
East Texas	19.21	- - - - -	17.18
Gulf of Texas	16.45	17.09	13.82
Montana	15.31	- - - - -	13.37
- - - - - indicates unavailable data			

For some states, the estimated average drilling cost functions are linear, i.e., the increase in drilling cost is equal proportionally to the increment of depth, the only independent variable considered.

This linear behavior is found for the states indicated in Tables X, XI and XIII for oil wells, gas wells and dry holes respectively.

Example 10

Table X

Drilling Cost Per Foot for Oil Wells in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
Norht Dakota	9.70	16.20	22.70
West New Mexico	15.00	20.00	25.00
Onshore California	19.00	23.40	27.80
Kentucky	17.30	30.80	44.30
Wyoming	13.30	20.30	27.30

Example 11

Table XI

Drilling Cost Per Foot for Gas Wells in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
West New Mexico	11.50	15.50	19.50

Example 12

Table XII

Drilling Cost Per Foot for Dry Holes in Various States

(in dollars)

States	TOTAL DEPTH		
	5,000 ft.	10,000 ft.	15,000 ft.
Kentucky	7.55	10.05	12.55
Panhandle Texas	10.40	17.90	25.40

Based on the estimated drilling cost functions per foot we calculate

total and marginal drilling costs per state and per region.

The general total cost curve is defined as the average

cost function times depth. In other words, it shows total cost as the

sum of both fixed and variable cost at each level of depth. The

form of the total cost function for most of the cases investigated is

depicted in Figure II.

Total
Costs
(in $)

Figure II
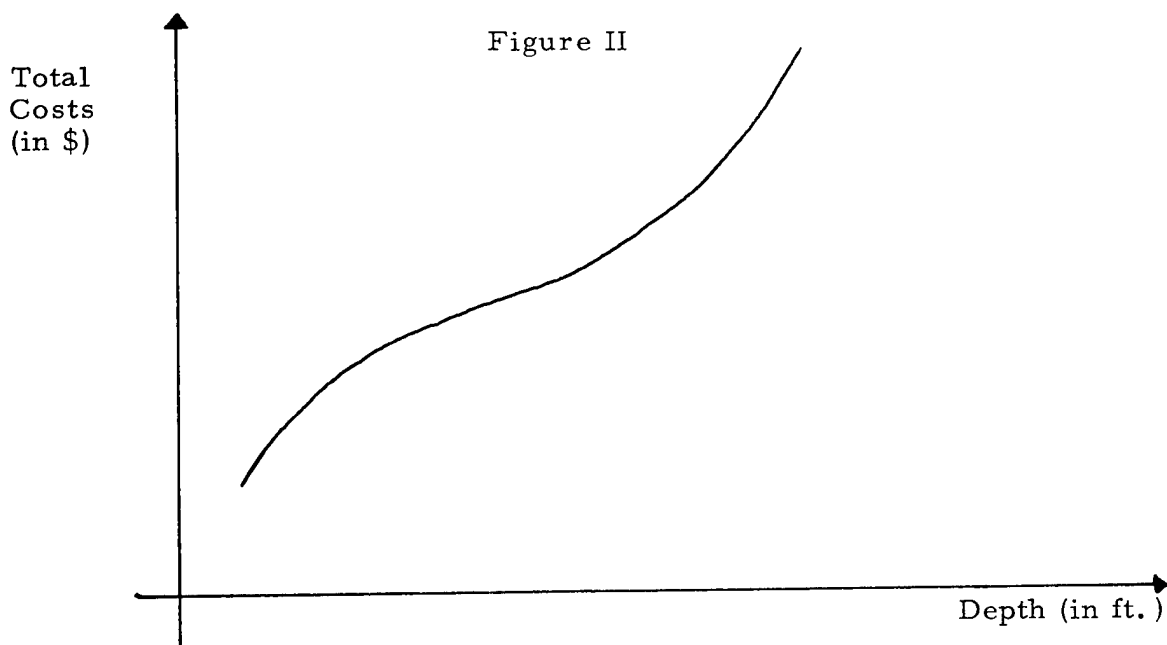
Depth (in ft.)

It is easy to see that total cost does not rise at an even rate. In effect,

it does not cost much more in total to drill 200 feet than 100 feet once the dril-

ling equipment is there. But as depth increases total cost begins to rise

more rapidly because of rapid increases in total variable costs. The main reason

for this trend is that drilling equipment is set up with a capacity and once

this capacity is exceeded, total variable cost becomes very large.

But beyond this one observation, total cost data are not very

illuminating. One reason is that the data does not show either costs

per foot drilled at some selected level of depth or marginal costs, i.e.,

the amount that is added to total cost by each additional foot drilled.

However, once total cost functions are determined it is easy to derive

marginal cost curves which are essential for good decision making.

In effect, by its very nature, marginal information often represents

the answers to hypothetical questions, i.e., information beyond the range

of the firm's actual experience.

In our study, except for the linear cases, marginal cost

curves behave like the average cost curves, i.e., parabolic functions

of depth. The minimum marginal cost always occurs to the left of the

minimum average cost and, except for one state, it lies at some positive

depth which indicates that marginal cost initially decreases. Conse-

quently, this behavior supports the general trend of the total cost given

in Figure II.

We can illustrate this point with some data for various states,

drawn from their corresponding Tables in the Annex of Section I.

Example 13

Table XIII

Minimum, Marginal and Average Costs and their
Depths for Oil Wells in Various States

States	Depth (in ft.)		Cost (in $)	
	Minimum Marginal	Minimum Average	Minimum Marginal	Minimum Average
Arkansas	2,355	3,532	5.00	6.10
Colorado	2,577	3,865	6.60	10.50
North Louisiana	1,985	2,977	5.10	5.90
South Louisiana	3,162	4,743	12.80	13.70
Mississippi	4,926	7,389	4.40	8.30
Montana	2,974	4,461	9.40	10.40
East New Mexico	4,353	6,530	8.60	11.40
Oklahoma	2,211	3,317	8.40	9.00
South West Texas	2,928	4,393	4.70	6.75
West Texas	3,978	5,968	8.50	10.50

I 2.   Summary

We are now able to summarize the major findings of this
part of our study as follows:

A)   Depth and geological medium are the most relevant
variables in the determination of drilling cost functions;

B)   In a vast majority of the analyzed cases, average

and marginal drilling costs are parabolic functions of depth. That is, drilling costs will decrease at the outset, then flatten out somewhat and then rise again. In general, the marginal and average costs reach their minimum between 3,000 and 5,500 ft. of depth. The main reasons supporting this behavior are:

1) The particular care and attention given to the upper part of the well independent of how deep the well will be. The main cost items added in the initial stages of the well, i.e., from 0 to 1,000 ft. are surface, casing and cementing.

2) The equipment available today (rigs, drill pipe strengths, bits, etc.) allow the drilling of deeper wells more efficiently, thus shifting minimum costs per foot and minimum marginal costs to the 3,000 - 5,500 ft. interval.

3) In addition to this, there exists considerable costs mobilization and demobilization for drill rigs and other fixed costs for site preparation, which further accentuate the initial decrease in average costs.

C) In a few cases, average and marginal cost functions are linear. This implies that drilling costs increase proportionally with increasing depth. In other words, of the two main components of average costs, i.e., fixed costs and variable costs, only the latter significantly affect drilling costs.

D) In regard to the geological medium we conclude that there is not a significant difference between drilling costs for Region I (Carbon-Permian) and Region II (Cenozoic). On the contrary, drilling costs for Region III (Mesozoic) are in most of the cases considerably higher than

those of both Regions I and II. One of the reasons for this situation may

be the great variety of kind of rocks involved in the Cenozoic Era; while

the Mesozoic seems to be better defined by the presence, in general,

of hard rocks. Though this classification based on geological ages is

not representative of the hardness of the soil, unfortunately we were

not able to find a better way of aggregating the observed data by types

of medium. (See Figures IIa, IIb and IIc. Additional figures are contained in
Annex of Section I)

E)   We can end this section with some important remarks

in connection to the use of average or marginal costs for the decision-

making.

There is a general understanding in economics that whenever

there is a difference between average and marginal costs, it is the latter

which must be given prior consideration in an optimization problem. The

logic of this statement is not hard to explain. For example, in any

decision making problem, say to continue   drilling a number of feet

more for a given well, the question is not how much money per foot was

already spent in similar wells at that depth but how much has to be spent

for each additional foot drilled.

Unfortunately, marginal data may be difficult and in some

cases, for practical purposes, impossible to come by. It is therefore

sometimes necessary to make decisions with average figures. For this

purpose one must understand the relationship between average and

marginal costs. That is, one must recognize the circumstances under which

the one can be expected to provide a reasonably good approximation

to the other, and   determine when this is not the case, what kind of

adjustments in the average data must be made to bring them closer to

FIGURE II.a.   CARBON-PERMIAN GAS
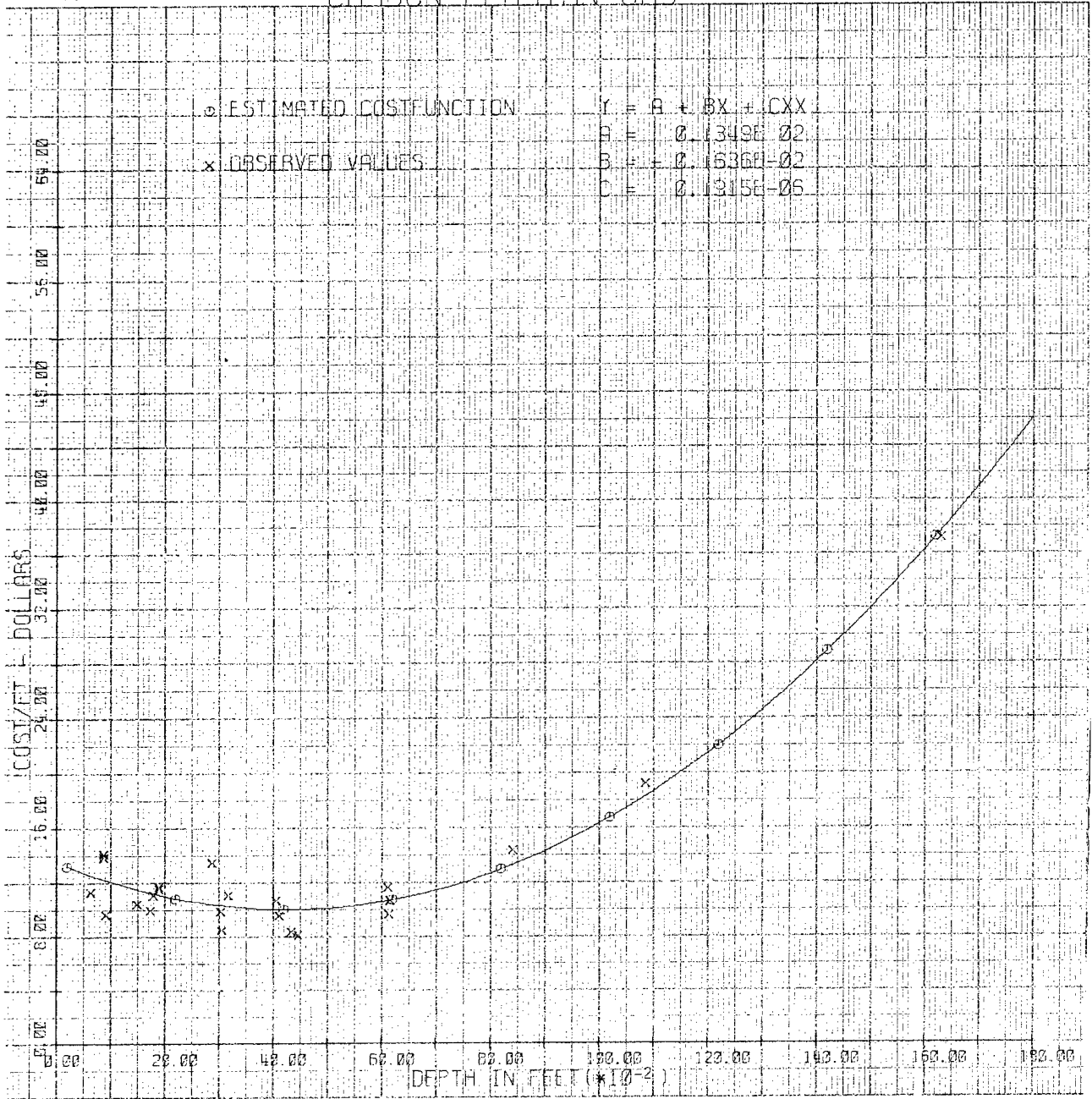
FIGURE II.b.

CENOZOIC GAS

FIGURE II.c.

MESOZOIC GAS

ESTIMATED COSTFUNCTION   Y = A + BX + CXX
                         A = 0.1260E 02
OBSERVED VALUES   B = -0.9058E-03
                         C = 0.1595E-06
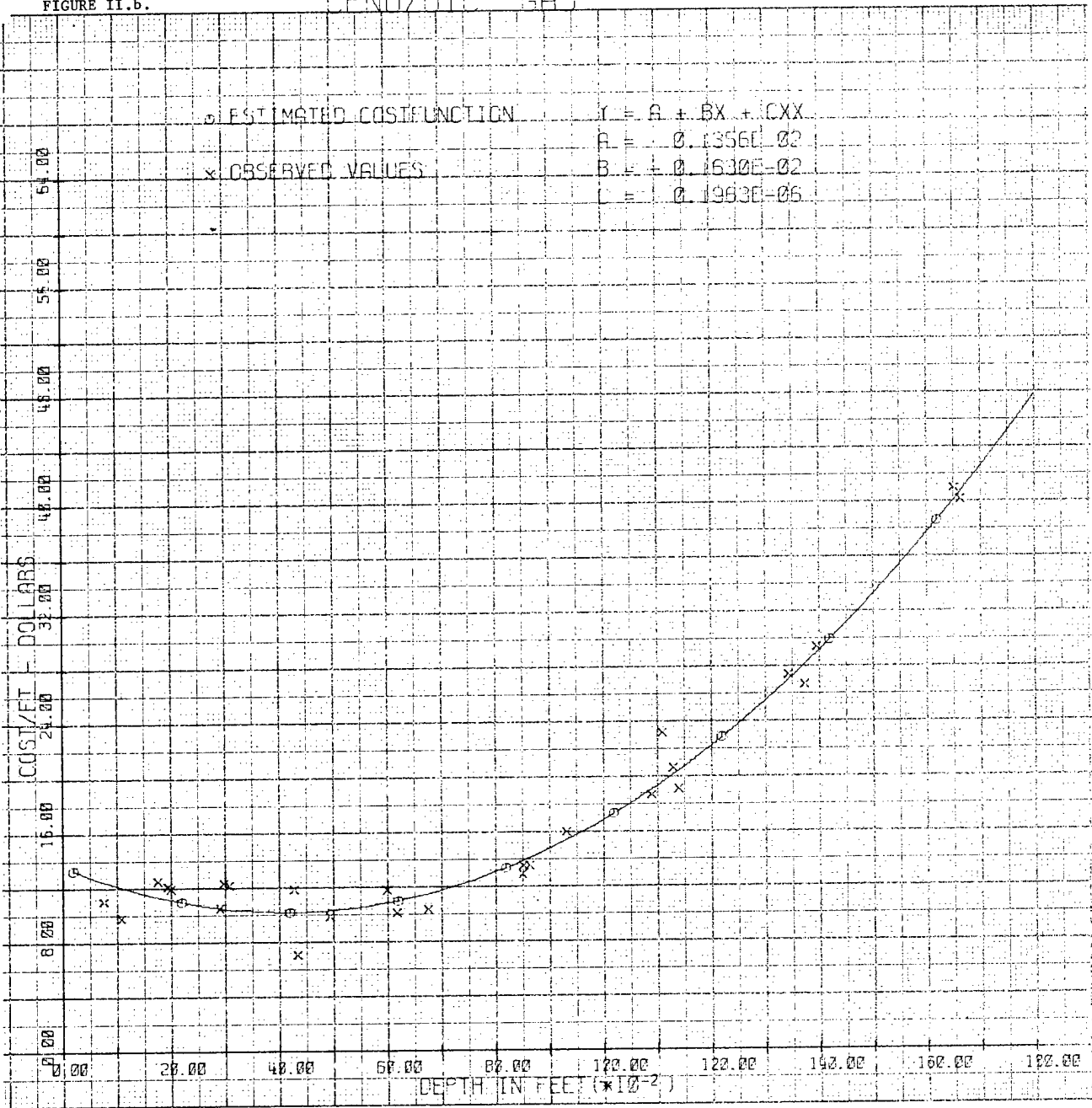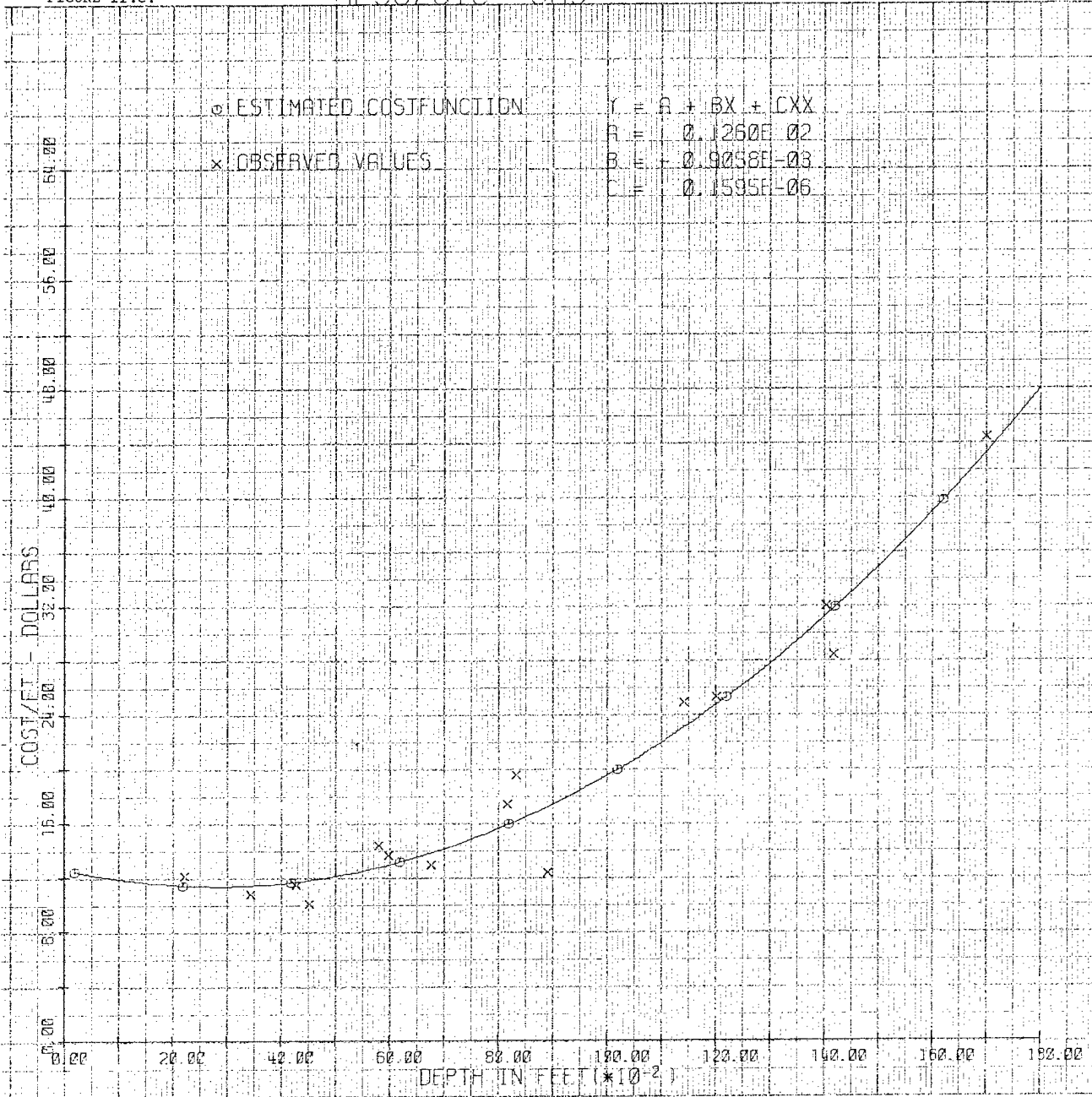
the unknown figures. For example, in cost figures, if there is some reason to believe there are economies of large scale production (falling average costs), marginal costs will be less than average costs. On the other hand, if observation suggests the presence of important diminishing return, the marginal cost will be higher than average cost, so that the average cost figure must be adjusted upward. Experience seems to suggest that such rough adjustments will,in many cases, eliminate the bulk of the error which arises from the use of average costs as a basis for decision-making.

SECTION II

DRILLING COST ANALYSIS FOR INTERMEDIATE
DIAMETER EMPLACEMENT HOLES

II 1.   An Economic Interpretation of Total, Average and Marginal

Drilling Cost Functions

In the previous section, standard size drilling costs for

oil and gas wells and dry holes were analyzed for the United States as

a lower limit to possible emplacement hole costs for the peaceful use

of nuclear devices. Cost studies for large diameter emplacement holes

(larger than 36 inches) already exist, [17, 21] though not in the form of

statistical estimates but at least in the form of costing and estimating

manuals, based on the technological requirements to drill such holes.

There is, however, a lack of studies for intermediate diameter (ap-

proximately 12 - 36 inches) emplacement holes. With the main purpose

to fill that gap, this section provides information pertaining to the cost

of drilling intermediate diameter holes in the United States, particularly,

in fields where data were available. These fields and the corresponding

diameter hole of the wells drilled are:[1]

_____

[1]   About 10 wells of intermediate size were also drilled at the Nevada
Test Site (N. T. S.) in fiscal year 1966 and the first quarter of fiscal
year 1967. However, the cost data of the N. T. S. holes cannot be
considered as representative of industry costs, even if subject to
the same technological requirements. There exists a sizable dif-
ference between industry drilling costs and costs of drilling under
government contract at the N. T. S. The latter are, for some com-
ponents as rig crew costs, nearly twice as large. Consequently,
since the peaceful use of nuclear explosives will be an industrial type
operation, we estimated drilling costs for intermediate diameter em-
placement holes on our industrial type cost basis.

(a) Gomez (Texas) for 12 1/4", 13 3/4" and 17 1/2"

diameter holes.

(b) South Pyote (Texas) for 12 1/4", 13 3/4", and 17 1/2"

diameter holes.

(c) Lockridge (Texas) for 12 1/4", 13 3/4", and 17 1/2"

diameter holes.

The total fixed cost comprises: mobilization and

demobilization cost, site preparation cost, rig-up and tear-down

cost and surface casing cost. Though, as it was already pointed out,

in most of the literature intermediate diameters are defined as those

ranging from 12 to 36 inches, our calculations are made for hole

diameters ranging from 10 to 45 inches. The main reasons supporting our

decision are: (a) the possibility of a straightforward comparison

between cased and uncased wells of 30 inches inside diameter (for

a 30 inch diameter cased well, the diameter of the drilled hole must

be approximately 45 inches) and, (b) the possibility of using the figures

obtained for all the variable costs to supplement and compare to

the information provided in Section I for small holes, whenever the

diameter of the well drilled lies in the interval of 10-12 inches.

II 1.2 Total Variable Costs

Variable costs are regarded in this study mainly as a function

of depth, diameter and hardness of the soil drilled and the corresponding

penetration rate. There exist some other important factors which influence

penetration rates and drilling costs, which are of a less tangible nature:

the hydrology of the drilling region, the possibility of excessive loss of

circulation fluid, hole deviation restraints, the weight, speed, torque used

in drilling, to mention some of these factors. To overcome the difficulties

in estimating the influence of these factors on drilling costs we develop

later on a method to estimate directly drilling time per well and depth in a

particular region. The economic significance of the main components is

analyzed as follows:

II 1.2a Mud Costs

In our calculations we assume that drilling mud is used as a

circulating medium for cleaning the hole bottom and the bit and for the bit

and returning the cuttings to the surface. The volume of the material

drilled at the bottom of the hole has to be removed continuously and lifted

to the surface. Various systems are available at present to achieve this.

According to the mud cost function (see Appendix II), they increase pro-

portionally with the depth drilled and more than proportionally with the

diameter of the hole. This is due to the fact that mud costs are mainly a

function of the volume of the hole. However, the quantity of mud needed

is about twice the volume of the well because of surface storage losses

and losses of drilling fluid along the hole.

We can best illustrate this point with the following mud costs for

different depths and diameters. The corresponding values were obtained

from Table 1 of the Annex of Section II where the price of mud is fixed

at about $4 per barrel for moderately treated mud with long term drilling:

Table I

Mud Costs per Well for Different Depths and Diameters

(in dollars)

Depth (in feet)	Diameter (in inches)			
	10	20	30	45
1,000	777.10	3,108.45	6,994.00	15,736.50
5,000	3,885.55	15,542.20	34,969.95	78,682.40
10,000	7,771.10	31,084.40	69,939.95	157,364.90

II 1.2b Cutter Costs

Cutters used on bits for drilling intermediate diameter holes are a major component of total costs, particularly when the formation is hard and abrasive or the bit is not properly cleaned by the circulating fluid. Cutter costs are a function of the volume of the hole (i.e., diameter and depth) and the hardness of the geological medium. Consequently, they increase more than proportionally with increasing diameter and proportionally (linearly) with depth. Moreover, according to the values given by Dellinger [17], cutter costs seem also to have an increasing rate of growth with respect to the hardness of the soil.

We can illustrate the cutter cost function behavior with some numerical examples, the corresponding data of which were obtained from Tables 2 to 6 of the Annex of Section II.

Table II

Cutter Costs per Well for Different Depths;
Diameters and Geological Media

(in dollars)

Geological Medium	Depth (in feet)	Diameter (in inches)			
		10	20	30	45
Soft	1,000	272.70	1,090.80	2,454.40	5,522.35
	5,000	1,363.55	5,454.15	12,271.85	27,611.65
	10,000	2,727.10	10,908.30	24,543.70	55,223.25
Medium Soft	1,000	409.05	1,636.25	3,681.55	8,283.50
	5,000	2,045.30	8,181.20	18,407.75	41,417.95
	10,000	4,090.60	16,362.45	36,815.50	82,834.90
Medium Hard	1,000	545.40	2,181.05	4,908.75	11,044.65
	5,000	2,727.07	10,908.30	24,543.70	55,223.25
	10,000	5,454.15	21,816.60	49,087.35	110,446.50
Hard	1,000	818.10	3,272.50	7,363.10	16,567.00
	5,000	4,090.60	16,362.45	36,815.50	82,834.90
	10,000	8,181.20	32,724.90	73,631.00	165,669.80
Very Hard	1,000	1,090.85	4,363.30	9,817.50	22,089.30
	5,000	5,454.30	21,816.60	49,087.35	110,446.50
	10,000	10,908.30	43,633.20	98,174.70	220,893.05

II 1.2c Casing Costs

Some of the intermediate size diameter holes require a steel liner
or casing whose main function is to protect the hole, equipment and personel
as well as to resist internal pressure where incompetent ground or water
must be sealed off.

Casings to total depth are today also required in order to protect
the nuclear explosive against the outside environment and to ensure its
predicted, controlled performance. Total depth casing requirements are
a very expensive cost item in the emplacement of nuclear explosives. But,

even if these casing requirements are overcome, there would still remain
the intermediate casing requirements which are dictated by the geological
and hydrological conditions of the drill area. In the following we analyse
casing costs to total well depth and other cost items implied by casing
requirements, and compare these costs to open hole costs. The intermediate
casing requirements would then give rise to emplacement costs somewhere
between these two limiting cases.

In our calculations, casing costs are a linear function of the depth
drilled and of the diameter of the open hole. Therefore, the rate of in-
crease is constant with respect to each independent variable. Casing
costs represent a major part of total variable costs. Later on in this
section we will show the substantial difference existing between the total
costs of cased and uncased wells. The situation results from a considerable
saving in cementing costs, casing costs and costs of drilling the hole which
are all strictly connected with casing requirements. In particular, the
diameter of the nuclear explosive will determine the minimum size of the
required hole to emplace the explosive, whether the hole needs or does not
need casing. However, if the hole has to be cased, then the inside diameter
of the casing has to fulfill the same minimum diameter requirements. Since
the space between the steel liners (casing) and the hole in which they are
emplaced is about 1/3 of the hole diameter, this implies that for the same
required dimensions of the emplacement hole, e.g. 30 inches, in our case
(open holes) we just drill a hole fulfilling these requirements, while in the

second case (cased holes) the inside diameter of the casing has to

be 30 inches, the required diameter of the hole, therefore, about

45 inches and to this casing and cementing costs have to be added to give

the full cost increase for casing requirements.

We can now illustrate the behavior of the casing cost

function with some numerical examples obtained from Table 7

of the Annex of Section II.

Table III

Casing Costs per Well for Different Depths and Diameters[*]

(in dollars)

Depth (in feet)	Diameter (in inches)			
	10	20	30	45
1,000	2,083.30	12,916.70	23,750.00	40,000.00
5,000	10,416.70	64,583.30	118,750.00	200,000.00
10,000	20,833.30	129,166.70	237,500.00	400,000.00

II 1.2d Cementing Costs

Steel liners are sealed into the hole with a cement grout

placed in the annulus between the liner and the boring wall. In our cal-

culations we assume that the annular space plus 30% for the boring wall

washouts determine the total quantity of cement required. The price of

[*] The diameter considered is that of the open hole. The corre-
sponding inside diameter (ID) of the casing is approximately equal
to 2/3 of the diameter of the hole. For example, if the diameter
of the hole is 45 inches, the ID casing is about 30 inches.

the cement is estimated at about $2.00 per cubic foot. Cementing costs,
like mud and cutter costs discussed above, are a second degree and poly-
nomial function of the diameter of the hole. This aspect is clearly shown
by the cementing costs indicated in Table IV. The corresponding values
were obtained from Table 8 of the Annex of Section II.

Table IV

Cementing Costs per Well for Different
Depths and Diameters

(in dollars)

Depth (in feet)	Diameter (in inches)			
	10	20	30	45
1,000	787.80	3,151.30	7,090.40	15,953.40
5,000	3,939.10	15,756.40	35,452.00	79,766.95
10,000	7,878.20	31,512.90	70,903.95	159,533.90

II 1.2e Rig Costs

Rig costs are given by the product of total drilling time
and the rig cost rate. In our study, the estimation of the drilling time
was made in two different ways, namely:

(a) As a Ratio Between Total Depth of the Well and

Penetration Rate

The penetration rate was first calculated for medium

soft rock by extrapolating from 30 inch diameter to

10 inch diameter the corresponding curve function

given by Dellinger [17]. For the other geological

media, namely; soft, medium hard, hard and very

hard rocks we multiplied the estimated penetration

rate function for medium soft soil by a constant

which varies according to the hardness of the rock.

This way of determining drilling time allows a

straightforward estimation of total drilling costs

whenever the geological medium is known.

(b) By a Multi-linear Regression of the Drilling Time

Function

We also performed a multi-linear regression

analysis.of drilling time as a function of the depth,

for those fields where available data exist, namely;

Gomez, S. Pyote and Lockridge in Texas. The

advantage of this procedure is the possibility of

"predicting" the drilling time required in the

investigated fields, given the desired depth of the

well. According to our estimations, for a given

diameter, drilling time functions behave parabol-

ically with depth. In other words, the drilling time

increases proportionally more than the increment of

depth . For all the cases investigated but one, the

goodness of fit of our regression equation, i.e., a

second degree polynomial, measured by the

coefficient of multiple determination (R^2) was very

high. In effect, we obtained an $R^2 \geq 0.90$ which

indicates that the least squares regression of

drilling time on depth and square of the depth,
accounts for at least 90% of the variance in drilling
time. (For a detailed econometric analysis see
Appendix II). Though, we did not perform drilling
cost calculations for these particular fields, namely;
Gomez, Lockridge and S. Pyote, the procedure is the
one followed for the determination of general inter-
mediate-diameter size drilling costs with the only
exception on the estimation of the drilling time
requirement where now this second degree poly-
nomial function is substituted in.

In regard to the rig hour rates (basic costs per unit of time
for the drill rig and support equipment package) we estimated that there
is a linear function of the rig horsepower which is in itself a function
of the volume of the hole.

We illustrate now with some numerical examples the
total rig costs as a function of drilling time and rig hour rate. The
corresponding values were obtained from Tables 9 to 13 of the Annex of
Section II.

Table V

Rig Costs per Well for Different Depths,
Diameters and Geological Media*

(in dollars)

Geological Medium	Depth (in feet)	Diameter (in inches)			
		10	20	30	45
Soft	1,000	2,250.00	3,500.00	4,750.00	8,593.20
	5,000	13,406.25	25,214.60	40,137.50	68,361.70
	10,000	48,375.00	83,970.80	125,795.85	193,229.20
Medium Soft	1,000	3,000.00	4,666.70	6,333.30	11,457.60
	5,000	17,875.00	33,619.45	53,516.70	91,149.00
	10,000	64,500.00	111,961.10	167,727.80	257,638.90
Medium Hard	1,000	3,913.05	6,086.95	8,260.90	14,944.70
	5,000	23,315.20	43,851.45	69,804.35	118,889.95
	10,000	84,130.40	146,036.20	218,775.40	336,050.70
Hard	1,000	6,923.08	10,769.20	14,615.40	26,440.55
	5,000	41,250.00	77,583.30	123,500.00	210,343.75
	10,000	148,846.15	258,371.80	387,064.10	594,551.30
Very Hard	1,000	9,000.00	14,000.00	19,000.00	34,372.70
	5,000	53,625.00	100,858.30	160,550.00	273,446.90
	10,000	193,500.00	335,883.30	503,183.35	772,916.70

Now we are able to determine the amount of total variable drilling costs, for different depths, diameters and hardness of the soil, by summing up the costs for: mud, cutter, cementing, casing and rig.

Total variable costs are mainly a function of the volume of the hole and the hardness of the soil and, therefore, increase more proportionally than in relation to each independent variable. Table IV indicates total variable costs for some depths, diameters and geological media.

Table VI

Total Variable Drilling Costs for Different
Depths, Diameters and Geological Media

(in dollars)

Geological Medium	Depth (in feet)	Diameter (in inches)			
		10	20	30	45
Soft	1,000	6,170.90	23,767.25	45,038.80	85,805.45
	5,000	33,011.15	126,550.65	241,581.30	454,422.70
	10,000	87,584.70	286,643.10	528,683.45	965,351.25
Medium Soft	1,000	7,057.25	25,479.40	47,849.25	91,431.00
	5,000	38,161.65	137,682.55	261,096.40	491,016.30
	10,000	105,073.20	320,087.55	582,887.20	1,057,372.60
Medium Hard	1,000	8,106.65	27,445.05	51,004.05	97,679.25
	5,000	44,283.62	150,641.65	283,520.00	532,562.55
	10,000	126,067.15	359,616.80	646,206.65	1,163,396.00
Hard	1,000	11,389.38	33,218.15	59,812.90	114,697.45
	5,000	63,581.95	189,827.65	349,487.45	651,628.00
	10,000	193,509.95	482,860.70	839,039.00	1,477,119.90
Very Hard	1,000	13,739.05	37,539.75	66,651.90	128,151.90
	5,000	77,320.65	218,556.80	398,809.30	742,342.75
	10,000	240,890.90	571,280.50	979,701.95	1,710,708.55

II 1.3   Total Fixed Costs

The special features of fixed costs are: (a) that they all
come in one big lump once a rig size suitable for the required hole diameter
and depth is selected and (b) that after they are incurred a further expansion
in the total output makes no difference to their magnitude.

Thus, the definition of fixed costs is very much a function of
technology and also the scale and time barrior assumed when looking at such
cost components. Nevertheless, at any particular time and for any particular
enterprise, some cost components will be given as or lump sum which
is not dependent at all, (or only to an insufficient degree) on the scale and
rate of operation. For drilling costs these fixed cost components can be

identified as: mobilization and demobilization, rig-up and tear-down, site preparation and surface casing. To estimate the total fixed costs we consider all its components, except surface casing, as a function of the rig horsepower requirement which in itself depends on the depth and diameter of the hole. The final results indicate that total fixed costs are rather independent of the hardness of the soil and that they increase very slowly with regard to depth and diameter. Thus, for example, fixed costs less than double for a hole diameter four and one half times larger. This point is clearly indicated in the table below. The corresponding fixed cost values were derived from Table 14 of the Annex of Section II.

Table VII

Total Fixed Costs [*] For Different Depths and Diameters

(in dollars)

Depth (in feet)	Diameter (in inches)			
	10	20	30	45
1,000	10,002.70	10,486.10	11,075.10	16,768.00
5,000	13,827.10	20,282.05	27,400.00	39,320.05
10,000	30,471.70	40,106.40	51,067.15	68,084.70

II 1.4   Total Drilling Costs

Total Costs are obtained for cased and uncased holes as the sum of total variable costs and total fixed costs. Total variable costs for uncased holes do not include costs of: cementing and casing.

---

[*]   Includes costs for mobilization and demobilization, site preparation, surface casing and rig-up and tear-down.

In both cases, the total cost function is a third degree polynomial in two variables, depth and diameter. The general form is the same observed for small diameter holes as it is depicted in Figure I. It is easy to see that total costs do not rise at a constant rate. In the first part of the curve, before the point of inflection is reached, the rate of growth is decreasing because of the presence of fixed cost. Thereafter, the total cost function starts to rise much more rapidly because of the increase in variable costs. The main reason for this trend is that drilling equipment is set up with a given capacity and once this capacity is exceeded, total variable costs become very large in proportion to the fixed cost component.

Total
Drilling
Cost
(in $)

Figure I

Depth (in ft.)
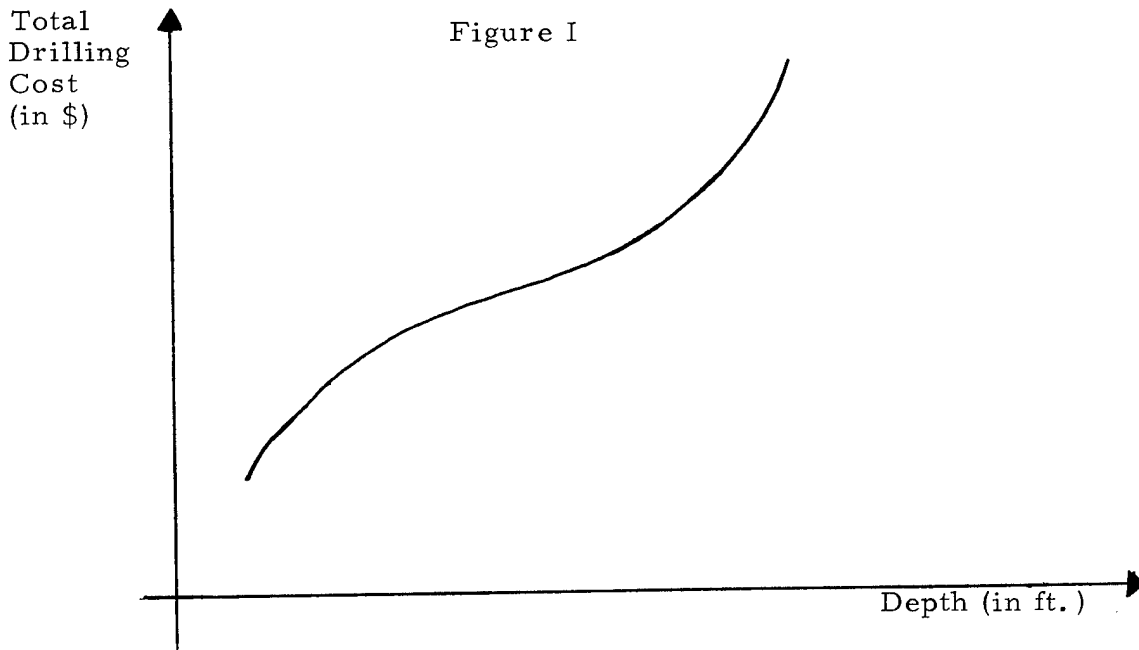
The final results indicate that there is a substantial difference in total costs for uncased and cased holes. We have to stress that it is not only the amount of both cementing and casing

costs that will be saved but also costs for drilling the open hole. In effect, if we want to compare total costs for both 30 inch diameter cased and uncased holes, the diameter of the open hole for the cased one must be approximately equal to 45 inches. This is due to the fact that the inside diameter (ID) of casing is about 2/3 of the diameter of the hole drilled. Moreover, some of the fixed costs will also diminish,mainly mobilization and demobilization costs, site preparation, and rig-up and tear-down costs because of the reduction in the required rig capacity.

Now we illustrate with some numerical examples total costs for cased and uncased holes and the difference between both, for various geological media. The corresponding values were drawn from Tables 15 to 19 of the Annex of Section II.

Table VIII

Total Drilling Costs for Cased and Uncased
20 inch Diameter Wells as a Function of Depth,
and Geological Medium

(in dollars)

Geological Medium	Depth (in feet)	Cased Well ϕ=30 in. *	Uncased Well ϕ=20 in. *	Difference
Soft	1,000	55,964.70	18,093.45	37,871.25
	5,000	267,932.80	65,890.70	202,042.10
	10,000	576,812.50	164,245.60	412,566.85
Medium Soft	1,000	58,845.50	19,848.35	43,327.60
	5,000	287,935.80	77,300.95	247,786.95
	10,000	632,371.35	198,526.15	529,558.40
Medium Hard	1,000	62,079.10	21,863.20	40,215.90
	5,000	310,919.95	90,584.00	220,335.95
	10,000	697,273.75	239,043.65	458,230.10
Hard	1,000	71,108.20	27,780.60	38,997.15
	5,000	378,536.65	130,749.70	210,634.85
	10,000	894,926.98	365,368.60	433,845.20
Very Hard	1,000	78,118.15	32,210.25	45,907.90
	5,000	429,091.50	160,197.05	268,894.45
	10,000	1,039,106.50	455,998.95	583,107.55

* ϕ = Diameter of the open hole

Table IX

Total Drilling Costs for Cased and Uncased
30 inch Diameter Wells as a Function of Depth
and Geological Medium

(in dollars)

Geological Medium	Depth (in feet)	Cased Well* ϕ=45 in.	Uncased Well* ϕ=30 in.	Difference
Soft	1,000	102,276.50	25,124.30	77,152.20
	5,000	491,789.30	113,730.85	378,058.45
	10,000	1,028,484.90	268,408.50	760,076.40
Medium Soft	1,000	108,042.70	28,005.10	91,623.05
	5,000	529,297.15	103,152.50	469,590.05
	10,000	1,122,806.80	323,967.40	966,524.75
Medium Hard	1,000	114,447.15	31,238.70	83,208.45
	5,000	571,882.65	156,717.99	415,164.65
	10,000	1,231,480.90	388,869.80	842,611.10
Hard	1,000	131,890.85	40,267.80	80,037.60
	5,000	693,924.70	224,334.70	395,563.35
	10,000	1,553,047.80	586,523.05	798,839.40
Very Hard	1,000	145,681.70	47,277.75	98,403.90
	5,000	786,907.35	274,889.55	512,017.80
	10,000	1,792,476.20	730,702.50	1,061,773.70

* ϕ = Diameter of the open hole

II 1.5 Average Drilling Costs

The average drilling costs or costs per foot drilled behave
parabolically with increasing depth.   This general trend was already
observed for small-diameter drilling costs (see Section I) and it is
here again indicated by Figure II.

Figure II
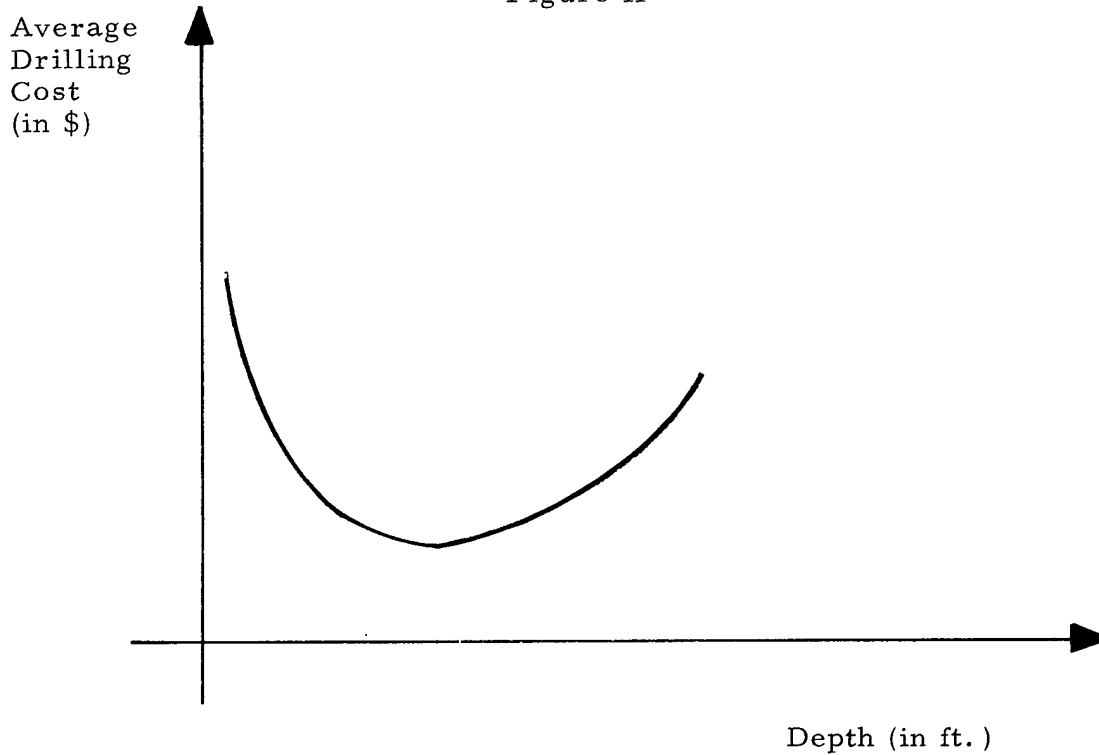
Drilling costs per foot decrease at first, then flatten out somewhat and increase again. According to our study, the main variables that significantly affect the average drilling costs for a given medium are: depth and diameter. They are: (a) quadratic function of the diameter and therefore increase more than proportionally with increasing diameter; and (b) a linear function of depth.

We performed calculations for average cased and uncased holes and here again we observed a large difference between the costs for both cases. We can illustrate now with numerical examples average costs which were derived from Tables 15 to 19 of the Annex of Section II.

Table X

Cost per Foot of Drilling for Cased and Uncased
30 inch Diameter Wells as a Function of Depth
and Geological Medium

(in dollars)

Geological Medium	Depth (in feet)	Cased Well ϕ=45 in.*	Uncased Well ϕ=30 in.*	Difference
Soft	1,000	102.30	25.10	77.20
	5,000	98.35	22.75	75.60
	10,000	101.40	26.85	74.55
Medium Soft	1,000	108.05	28.00	80.05
	5,000	105.85	26.75	79.10
	10,000	112.30	32.40	79.90
Medium Hard	1,000	114.45	31.25	83.20
	5,000	114.40	31.35	83.05
	10,000	123.15	38.90	84.25
Hard	1,000	131.90	40.30	91.60
	5,000	138.80	44.90	93.90
	10,000	155.30	58.65	96.65
Very Hard	1,000	145.70	47.30	98.40
	5,000	157.40	55.00	102.40
	10,000	179.25	73.10	106.15

* ϕ = Diameter of the open hole

II 1.6 Marginal Drilling Costs

Since our total drilling cost function is a third degree
polynomial in two variables, namely, diameter and depth, we only
were able to obtain "partial" marginal drilling costs defined as the
derivative of the total drilling cost function with respect to only one of the
independent variables in each instance. We calculate the marginal drilling
costs for changes in diameter, i. e., how much is the amount added
to total costs by each additional inch of diameter. We have already
pointed out in the previous section for small-diameter emplacement
holes, the importance of marginal costs for good decision making.

In effect, by its very nature, marginal cost information often represents the answers to hypothetical questions; i.e., information beyond the range of the industry's actual experience. The final results indicate that marginal drilling costs for intermediate-diameter holes are a second degree polynomial in two variables, diameter and depth.

In the table below, we indicate some numerical examples of marginal drilling costs for cased and uncased wells as a function of depth, diameter and hardness of the soil. The corresponding values were obtained from Tables 30 to 39 of the Annex of Section II.

Table XI

Marginal Drilling Costs for Cased and Uncased
30 inch Diameter Wells as a Function of Depth
and Geological Medium

Geological Medium	Depth (in feet)	Cased Well $\phi=45$ in.*	Uncased Well $\phi=30$ in.*
Soft	1,000	3,498.60	823.60
	5,000	16,465.75	5,562.30
	10,000	31,605.00	11,972.80
Medium Soft	1,000	3,733.70	951.60
	5,000	17,805.05	6,556.95
	10,000	34,094.50	14,371.40
Medium Hard	1,000	3,992.90	1,088.85
	5,000	19,300.30	7,675.20
	10,000	36,854.90	17,106.15
Hard	1,000	4,686.10	1,430.75
	5,000	23,423.30	10,808.90
	10,000	44,342.00	25,016.25
Very Hard	1,000	5,241.50	1,719.55
	5,000	26,653.80	13,225.50
	10,000	50,279.30	31,002.80

* ϕ = Diameter of the open hole

II 1.7 Summary

Now we are able to summarize the major findings of
our study on drilling costs for intermediate-diameter emplacement
holes:

(a) Diameter, depth and hardness of the soil are the
most relevant variables in the determination of the
drilling cost functions;

(b) Drilling costs, in general, increase more than
proportionally with the increment in diameter and
the increment in the hardness of the soil, while they
increase equal proportionally (linearly) with depth:

(c) A substantial difference in total, average and
marginal costs for cased and uncased wells is
observed. For example, the marginal drilling cost
for a fully cased well is in average about three times
larger than the corresponding marginal drilling cost
for the same well uncased. The main reason for this
situation is the substantial saving for uncased holes
that results from the elimination of costs for cementing
and casing as well as a sizable reduction in the costs
of drilling a comparatively smaller diameter hole. Conse-
quently, we can conclude that casing costs are a

major part in the total drilling cost of a well.

(d) The general shape of total costs, average costs

and marginal costs, for drilling intermediate-

diameter emplacement holes is equal to the

corresponding ones observed for small-diameter

holes in Section I. That is, total drilling costs

are a third degree polynomial function in two

variables, depth and diameter, and average and

marginal drilling cost functions are a second degree

polynomial, also in two variables, namely, diameter

and depth. As a consequence of the observed

similarities in the trend of the drilling cost functions

corresponding to Sections I and II, we performed

our calculations for diameters starting at 10 inches

instead of 12 inches.[2] The main purpose of this

decision is to use the figures obtained on the variable

cost components; i. e., mud, cementing, cutter,

casing and rig, to supplement the information provided

in Section I for small holes, whenever the diameter of

the well drilled lies in the interval 10 - 12 inches.

On the other hand, in this study we fixed 45 inches as an

upper limit of this variable, i. e., diameter for open holes. We based

this decision on the possibility of a straightforward comparison of

drilling costs for both 30 inch diameter cased and uncased wells, given

---

[2] Intermediate-diameter emplacement holes are actually those
ranging approximately from 12 to 36 inches.

that the inside diameter (ID) of casing is approximately about 2/3
of the diameter of the hole drilled. Therefore, to have a 30 inch
diameter cased well, the hole drilled must be about 45 inches in
diameter.

SECTION III

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

III 1.   Conclusions

The analysis of standard size diameter wells of the first

section were found to be a representative cross section of the long

term, theoretical lower limit for the emplacement hole costs of nuclear

devices for industrial use.   Thus, the variance in the individual

costs given for each of the states in which significant drilling activity

took place, and the aggregation of drilling costs to three major

geological groupings, do reflect the expected variance in emplacement

hole costs in the long run if this theoretical limit can be reached.   For

each individual state and the aggregated costs, we therefore gave a

complete analysis of total well costs, costs per foot (average costs),

and marginal costs as well as their minima and points of inflection.

To facilitate the analysis and the use of the results, we also drew for

each individual state and geological region the estimated cost-per-foot

curves and the empirical data points.   For all cases, these costs are

significantly lower than present emplacement hole costs in PLOWSHARE

experiments, even when discounting the fact that these are single

experimental events and, at present at least, to some extent

governmental operations.

When analyzing intermediate size hole costs, one has to

bear in mind that the U. S. Atomic Energy Commission is faced with

two possible ways in which to reduce the presently high costs of

emplacement: first, by reducing the diameter of the nuclear explosives at given yields, and second, by advancing research and development to allow the controlled detonation of nuclear explosives in an adverse environment and thereby eliminate the need for complete casing.

With regard to the first case, the most immediately significant results are given by the marginal costs with regard to diameter. These marginal costs are the partial derivative of total costs in the direction of diameter changes, and are shown in Appendix II for each depth class (from 1,000 to 10,000 feet), for all given different geological conditions, and for diameters ranging from 10 to 30 (or 45) inches. These marginal costs reflect the cost savings if, at any particular point shown, the diameter of the well is reduced by 1 inch. In order to find the optimum diameter reduction program for nuclear explosives, we can directly apply these costs and--in combination with the increased costs of research, development, and production of such explosives--an optimal economic solution can be found by linear and/or dynamic programming techniques. Reducing the diameter of nuclear devices to fit them in holes smaller than 12 inches should bring no significant additional cost savings with regard to emplacement hole costs. The optimum solution can therefore lie anywhere in the intermediate diameter interval, and may be different for various yields of the explosives.

As further cost savings from the minimum intermediate sizes to smaller diameters are insignificant, the absolute minimum

will be, at best, around this intermediate size limit ($\approx$ 12 inches).
Assuming that the diameter of nuclear explosives can be reduced for
all yields considered in completely-contained underground PLOWSHARE
applications, Table XII, the extreme case for savings if it is possible to
reduce, shows the required diameter from 30 to 12 inches. (See also
Figures III, IV). At various depths, these savings vary considerably in
both cased and uncased holes. For example, in moderately hard rock, the
reduction in cost ranges from \$80,000 (at 1,000) to nearly \$900,000
(at 10,000 feet) for cased holes, and from \$15,000 (at 1,000 feet) to
about \$250,000 (at 10,000 feet) for open holes. Other figures are easily
read from Table XII. As these cost savings accrue for each single
emplacement hole, the overall reduction in cost for any one of the potential
fields of use for peaceful nuclear explosives is large indeed and will be,
ultimately, very decisive in determining the success and scale of such
industrial applications.

With regard to the second case (i.e., a success in the open hole
vs. cased hole program), the size of the savings is to a large extent
dependent on the success of the diameter reduction program. Table XIII
shows the savings in costs due to the dropping of the casing requirement for
10, 20 and 30 inch diameter wells, and the following depths: 1,000 ft.,
5,000 ft. and 10,000 ft. (See also Figures V, VI) In Annex of Section II
all possible combinations of diameters and depths and their corresponding
cost savings are shown in Tables 25 to 29. If a complete success
in the diameter reduction program is scored, say, to 12 inches for all

FIGURE III. INTERMEDIATE SIZE OPEN HOLES
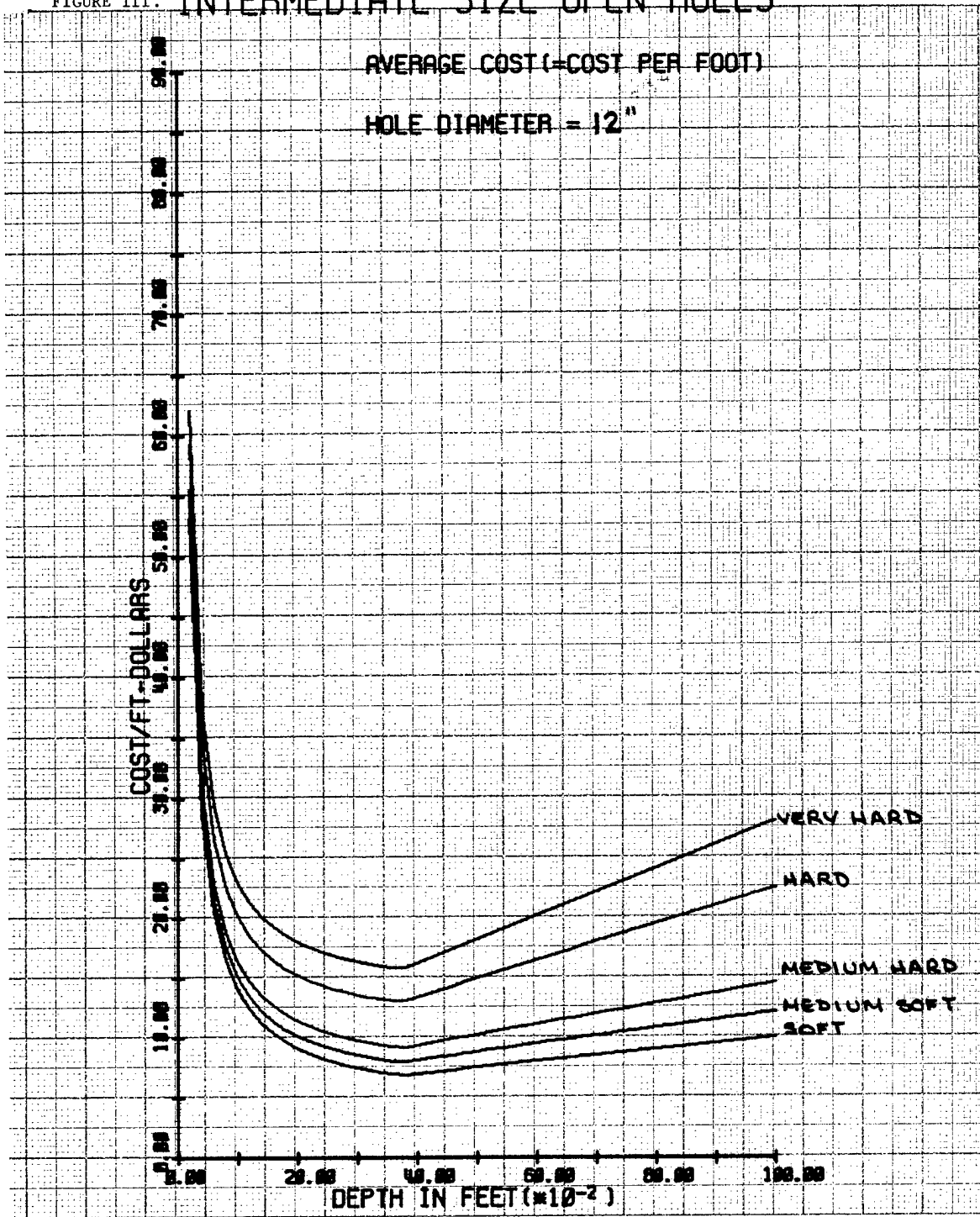
AVERAGE COST (=COST PER FOOT)

HOLE DIAMETER = 12"

FIGURE IV.

INTERMEDIATE SIZE OPEN HOLES

AVERAGE COST(=COST PER FOOT)

HOLE DIAMETER = 30"

VERY HARD

HARD

MEDIUM HARD

MEDIUM SOFT

SOFT

COST/FT-DOLLARS

DEPTH IN FEET($\times 10^{-2}$)
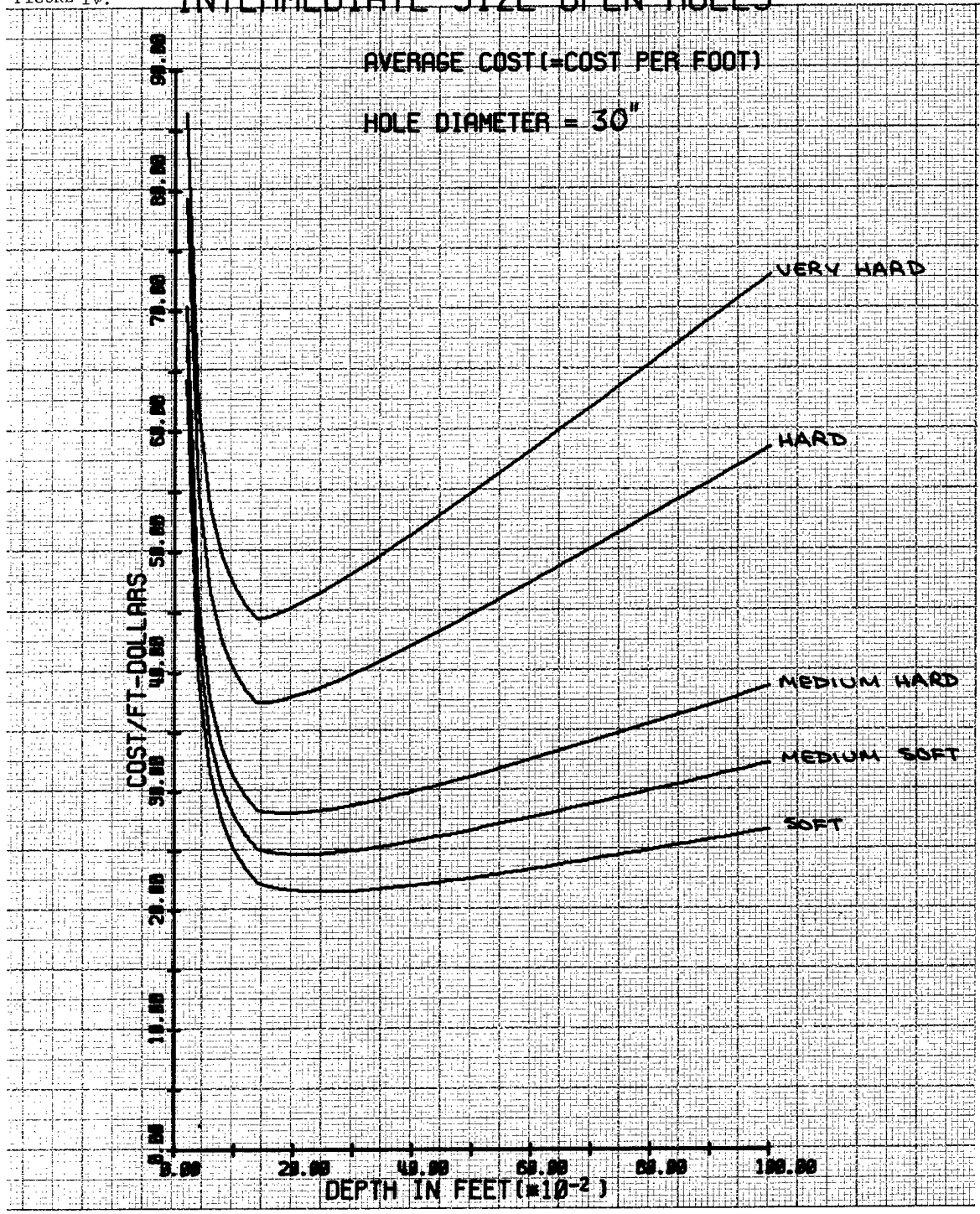
FIGURE V. INTERMEDIATE SIZE CASED HOLES

AVERAGE COST (=COST PER FOOT)

HOLE DIAMETER = 18 INCHES
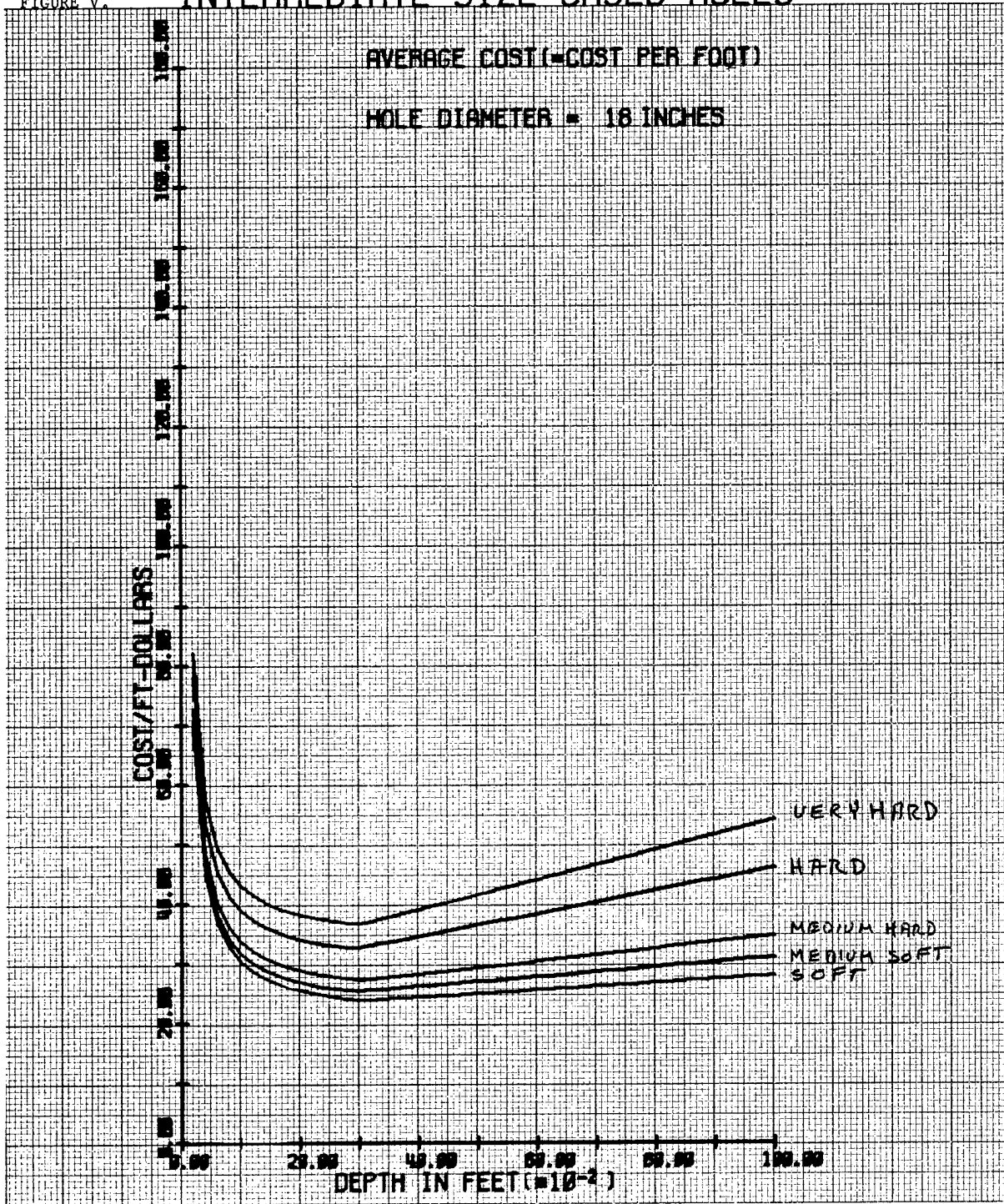
FIGURE VI.

INTERMEDIATE SIZE CASED HOLES

AVERAGE COST (=COST PER FOOT)

HOLE DIAMETER = 44 INCHES

VERY HARD

HARD

MEDIUM HARD

MEDIUM SOFT

SOFT

COST/FT-DOLLARS

DEPTH IN FEET (=10⁻²)
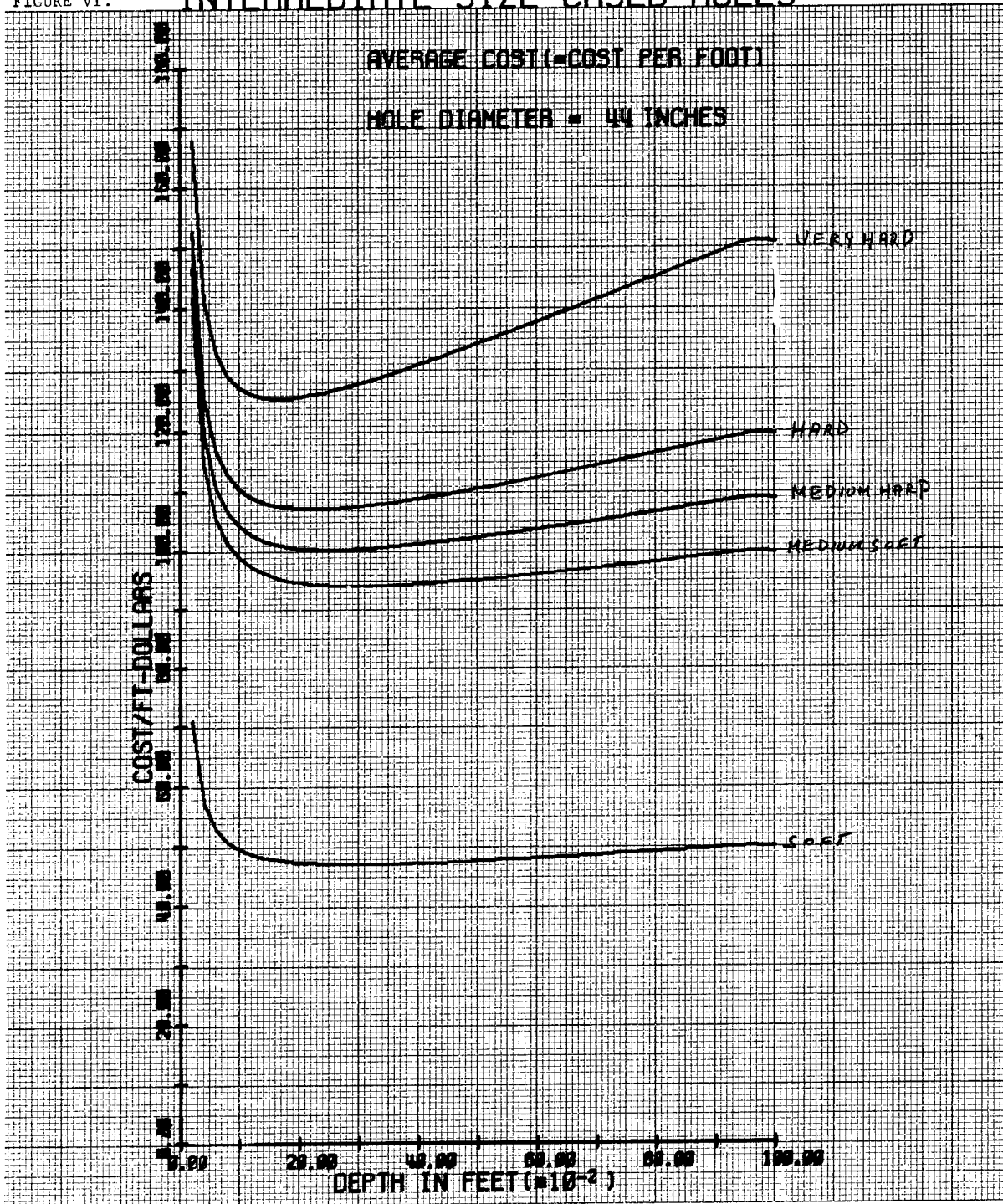
PLOWSHARE yields, these savings will range from $18,000 (for a 1,000-foot hole) to about $200,000 (for a 10,000-foot hole) for each well.

Table XII

Maximum Cost Savings for Diameter Reduction

From 30 Inches to 12 Inches

Cased and Open Holes--Five Different Media

CASED

FEET	SOFT	MEDIUM SOFT	MEDIUM HARD	HARD	VERY HARD
1,000	72023.71	76226.65	80826.47	92907.96	102718.14
	145044.49	154318.07	164574.02	192219.36	214242.58
	220474.93	235661.31	252598.61	299187.70	335756.05
	293202.35	314334.92	337994.25	403661.08	454867.07
	367255.88	394776.66	425696.13	512214.79	579282.49
	442635.52	476986.54	515704.27	624848.78	709002.26
	519341.21	560964.54	608018.65	741563.07	844026.41
	597373.01	646710.63	702639.29	862357.62	984354.91
	676730.92	734224.84	799566.23	987232.50	1129987.80
10,000	748650.43	812356.62	884743.87	1092555.60	1250685.40

OPEN

FEET	SOFT	MEDIUM SOFT	MEDIUM HARD	HARD	VERY HARD
1,000	11077.69	12903.04	14895.52	20094.01	24336.07
	25027.18	29051.16	33490.53	45385.60	54903.39
	42384.31	49353.46	57148.55	78736.34	95597.21
	60337.80	70703.96	82404.86	115498.84	140954.07
	75924.21	89618.64	105141.64	149465.83	183325.02
	92394.68	109712.05	129415.90	186153.02	229232.20
	109749.20	130984.22	155227.66	225560.40	278675.65
	127987.80	153435.13	182576.93	267687.98	331655.35
	147110.45	177064.81	211463.70	312535.74	338171.30
10,000	167117.17	201873.22	241887.97	360103.69	448223.47

Table XIII

Differences of Total Drilling Costs Between Cased and Uncased Wells

For Different Diameter Depths and Geological Media

(in Dollars)

Geological Medium	Depth (in ft.)	Diameter		
		10-inch	20-inch	30-inch
SOFT	1,000	11,418.35	37,871.30	77,152.20
	5,000	61,439.30	202,042.10	378,058.45
	10,000	127,119.90	412,566.85	760,076.35
MEDIUM SOFT	1,000	13,447.70	43,327.60	91,623.05
	5,000	76,673.60	247,786.95	469,590.05
	10,000	170,339.45	529,558.35	966,524.75
MEDIUM HARD	1,000	12,241.25	40,215.90	83,208.45
	5,000	67,364.40	220,335.95	415,164.65
	10,000	143,507.90	458,230.10	842,611.10
HARD	1,000	11,806.60	38,997.15	80,037.60
	5,000	64,197.05	210,634.85	395,563.35
	10,000	134,681.85	433,845.20	798,839.40
VERY HARD	1,000	14,388.45	45,907.90	98,403.95
	5,000	83,638.55	268,894.45	512,017.80
	10,000	189,936.35	583,107.60	1,061,773.70

This represents the very minimum of cost savings due to no-casing requirements. If, however, diameter requirements can be reduced as presently indicated by the U.S. Atomic Energy Commission, (e.g., to an outside diameter cannister of 11 inches for a 100KT explosive; see also introductory statement) then these cost savings are considerably increased, particularly for higher yield nuclear devices. For example, for a required 18-inch diameter hole, the reduction in cost will range from about $35,000 for a 1,000-foot hole to about $400,000 for a 10,000-foot hole; with a required 30-inch hole, these costs savings increase to $80,000 and $850,000 respectively.

Table XIV gives a summary of these general comparisons.

Table XIV

Cost Reduction per Emplacement Hole: Diameter vs. Casing

(in U. S. Dollars, 1965 Prices)

Hole Depth	MAXIMUM DIAMETER COST SAVING		SAVINGS OF OPEN VS. CASED HOLES Diameter		
	Cased	Uncased	12-inch	18-inch	30-inch
1,000 ft.	$ 80,000	$ 15,000	$ 18,000	$ 35,000	$ 80,000
↓	↓	↓	↓	↓	↓
10,000 ft.	$900,000	$250,000	$200,000	$400,000	$850,000

III 2. Recommendations

Based on the analysis given in this report and on the

previous work done by MATHEMATICA on the peaceful use of

completely-contained underground nuclear explosives, we recommend

the following:

First: The costs in research and development, and the

cost increases for devices at different fields, shall be determined as

far as possible for both a device diameter reduction program and a

"no-casing" program. On the basis of this and the cost reductions

analyzed and outlined in this study, a rational decision can be made as

to where the maximum research and development effort should go.

Second: In order to determine the optimum research and

development effort for both the "no-casing" program and the diameter

reduction program, we recommend that this be done by an operations

research effort using linear or dynamic programming techniques, as

both programs are, in their effects, highly interdependent. A reduction

in diameter requirements affects potential savings in casing costs, and

a success in the "no-casing" program affects significantly the further

savings in diameter reduction efforts. No a priori decision as to

which program should be preferred can be given without such an

analysis, due to their high interdependence, which would be based on

the results of this study and those obtained from the first recommendation.

The following points illustrate the difficulty of this decision further:

(a) The larger cost savings in the "no-casing" program would accrue fully with a successful research and development effort in all cases where geological conditions permit open holes, while diameter cost reductions listed above reflect only maximum possible savings for intermediate size holes.

(b) The savings in casing costs would also be applicable in some cases in the experimental testing program undertaken by the U.S. Atomic Energy Commission for larger diameter requirements.

(c) Furthermore, at larger depth, the casing of the hole to total depth may become impossible due to technical restraints imposed on casing technology. This would then exclude at least part of the potential PLOWSHARE applications.

(d) On the other hand, the technical feasibility of a no-casing effort may be quite uncertain (if not impossible in some cases) while even a partially successful reduction in diameters would yield significant results in the form of costs saved.

(e) In many cases geological and hydrological conditions in the drilling area are such that intermediate casing would be required to some depth of the hole for technical reasons when drilling such holes. This holds even if nuclear explosives as such would not require protective casing. This reduces, of course, the potential economies of a "no-casing" program effort.

This proposed analysis is in particular called for if either program can be realized only to the exclusion of the other due to financial restrictions in the research and development effort.

Third: Both the reduction of diameters and the easing of

casing requirements will have a very important effect on the scale

and economic feasibility of PLOWSHARE applications.   We recommend,

therefore, that these programs should be realized as soon as possible

and, if finances allow to do so, this should be a simultaneous effort.

APPENDIX OF SECTION I

ECONOMETRIC ANALYSIS OF THE ESTIMATED

DRILLING COST FUNCTIONS

1.   Specification of the Model

A large number of oil wells, gas wells, and dry wells

are drilled each year in the United States. The depth of oil and gas

wells drilled depends, of course, on where the oil and gas field is

expected to be, based on geological information derived from earlier

wells in the same region and/or on a variety of more or less sophis-

ticated geological tests. Both the average depth of wells drilled and

the depth of particular oil and gas fields tapped by drilling have con-

sistently increased over time. Today in some regions of Texas, in

particular the Devonian Basin in the western part of Texas, wells

down to 20,000 feet are common, and the deepest wells drilled today

exceed 25,000 feet. In 1967 alone, 402 deep wells (i.e., more than

15,000 feet) were drilled in the U.S., and the total number of deep

wells drilled in the U.S. is about 3,500 [36]. Thus, based on the data

of wells drilled in the U.S., it would be possible to estimate with suf-

ficient accuracy the cost of drilling wells to various depths and, if

the information is collected in an appropriate way, also for different

geographical regions. Up to 1965, this information was, however,

collected on a state-by-state basis, and for some states the well data

were further broken down into some additional geographical, but

not geological, areas. This nationwide survey is made jointly by the
American Petroleum Institute, the Independent Petroleum Association
of America, and Mid-Continent Oil and Gas Association, and data were
collected and published for the years 1953, 1955-56, and 1959 through
1965. The information collected is published in aggregated form for nine
depth ranges (0-1,250, 1,251-2,500, 2,501-3,750, 3,751-5,000,
5,001-7,500, 7,501-10,000, 10,000-12,500, 12,501-15,000, 15,001 and
over) and about 30 different geographical areas (states and some subdivisions).
These joint survey figures do contain a considerable wealth of statistical
information. In addition to the number of wells drilled for each depth
range, it also gives the total costs of these wells and their total footage.
These figures are furthermore given separately for oil, gas, and dry wells.

Costs per foot drilled in each state were plotted against the
average depth per well in each class interval and then, regression cost
curves were fitted for each case.

Cost functions were calculated whenever the sample size
and the number of wells drilled were considered representative. The
same procedure was carried out for regions classified according to the
following geological eras:

a)   Carbon-Permian (Region I)

b)   Cenozoic (Region II)

c)   Mesozoic (Region III)

For each state we worked with small samples whose sizes range from 5 to 9 data.   It is important, however, to point out that each one of these data is an "average" of the drilling costs per foot observed for each class interval.   This average of observations resulted from the number of wells drilled that ranged from 5 to 888 and which therefore, in part, contribute to the smooth behavior of the empirical observations. For all these reasons, though we worked with small samples, we considered them as highly representative.   For the region analysis the size samples vary form 15 to 41 data.

The final results indicate that the average cost drilling function is parabolic and becomes exponential only for depth approximately larger than 15, 000 ft.

The model is a uniequational cross-section model of the following mathematical form for nearly all the cases investigated:

(1) $Y = A + B X_1 + C X_2 + u$

Where

   Y = Drilling cost per foot of depth;

   X_1 = Depth;

   X_2 = Square of the depth, and

   u = Stochastic variable

By cross-section model we mean a model supposed to be valid for each of several different individual firms or consumers, or geographical regions or the like, and intended to be used in connection with data describing economic features of those individuals firms, consumers or regions, etc., [13].

The exogenous variables are those unexplained by the model, in our case, X_1 and X_2; i.e., depth and the square of the depth.

The endogenous variable is Y, i.e., drilling cost per foot; and it is the one explained by the model.

The stochastic variable u represents: (1) all the other missing exogenous variables that do exist but are not relevant in the determination of Y, i.e., those due to errors of omission; and, (2) all errors of observation that arise because the data are never exactly correct. The bulk of conventional economic theory, whether expressed in diagrammatic or algebraic form, postulates exact functional relationship between variables. The most elementary acquaintance with economic data, however, indicates that points do not lie exactly on straight lines or other smooth functions. Therefore, we face the need of introducing a stochastic term into economic relationships. There are three possible, though not mutually exclusive, ways of rationalizing the insertion of the stochastic term "u" in (1). First we may say that drilling costs of each and every well could be fully explained if we knew all the factors at work and had all the necessary data. However, many of the factors could not be quantifiable, and even if they were, it is not usually possible in practice to obtain data on them all. Even if one can do that, the number of factors is still almost certain to

exceed the feasible number of observations, so that no statistical means exists for estimating their actual influence. Moreover, many variables may have very slight effects so that we choose to represent Y as an explicit function of just a small number of what are the more relevant X's and let the net effect of the excluded variables be represented by "u", i.e., the stochastic variable.

A, B and C are the structural unknown parameters. We estimated these parameters statistically on the basis of our sample observations on X and Y applying the least-squares method. According to the Gauss-Markov Theorem least-squares estimators, in uniequational models with exogenous variables, are best linear unbiased estimators (BLUE). That is, of the class of linear unbiased estimators, the least-squares estimators have the smallest variance.

We also tested the null hypothesis about these parameters, that is to say we determined if any of our estimated parameters was obtained from a population where its true value is zero. For example, A = 0 means that Y is proportional to X_1 and X_2 ; B = 0 means that there is no relation between drilling cost, Y, and the depth X_1 ; C = 0 means that there is no relation between drilling costs Y and the square of the depth X_2. Using the "Student's t" distribution we performed tests for each of the parameters, A, B and C separately and also we made joint tests for all of them with the F distribution.

The null hypothesis was rejected at a significant level of 5%, which implies that specification (1) explains with a probability of 95% the cost function investigated.

To measure the goodness of fit of our equation, i.e., if the second degree polynomial or parabolic curve is an adequate representation of the data, we used the coefficient of multiple determination R^2. The coefficient of multide determination is equal to the proportion of Y variance accounted for by the simultaneous influence of X_1 and X_2. In nearly all the cases we obtained an $R^2 \geq 0.90$ which indicates that the least-squares regression of Y or X_1 and X_2 accounts for at least 90% of the variance in Y.

We also obtained enough information to compute "confidence intervals" for each cost function. This means that if we take repeated samples where the X'_i s as well as the Y_i may change from sample to sample, approximately $100(1-\epsilon)$ percent of our confidence interval will contain the observed sample values, where ϵ is the significance level, generally equal to 5%. A confidence interval of Y_1, i.e., drilling cost per foot, with a 95% of probability and symmetrical around the sample estimate $\hat{Y}$ is given by the following expression:

(2) $$\hat{Y} - t_{0.025}\, \hat{\sigma}_u \;\leq\; Y \;\leq\; \hat{Y} + t_{0.025}\, \hat{\sigma}_u$$

Where

$$t_{0.025} = \text{t value at 2.5\% for n-3 degrees of freedom.}^{*}$$

_____

[*] The number of degrees of freedom is equal to the sample size minus the number of the estimated parameters.

$$\hat{\sigma}_u = \frac{\Sigma u^2}{n-3}$$ standard error of the residual equal to the square root of the ratio between the sum of squares of the residual and the number of degrees of freedom.

For a few cases the plotted empirical data of drilling costs per foot indicated a linear trend. Consequently we fitted them with a simple uniequational linear model. That is;

(3) $Y = A + BX + v$

Where

     Y = Drilling costs per foot

     X = Depth

     v = Stochastic variable.

We have now only one endogenous variable Y and one exogenous variable X , i.e., depth. The insertion of the stochastic variable v has the meaning pointed out above.

For the linear equation cases, we performed the same analysis of statistical inference as applied to the parabolic equation (1).

2.   Workability of the Model

In this section we illustrate, with some numerical examples, the workability of our average drilling cost function model.

Example 1

Using Table 15 we find the following estimated average drilling cost function[*] for gas wells in North Louisiana;

$$(4) \quad \hat{Y} = 7.4 - 0.0011 \, X_1 + 0.25 \, (10^{-6}) \, X_2$$

Where

$\hat{Y}$ = Estimated drilling cost per foot;

$$
\left.
\begin{aligned}
\hat{A} &= 7.4 \\
\hat{B} &= 0.11 \, (10^{-2}) \\
\hat{C} &= 0.25 \, (10^{-6})
\end{aligned}
\right\}
\quad
\begin{aligned}
&\text{Estimates of the structural} \\
&\text{parameters.}
\end{aligned}
$$

---

[*] The estimated drilling cost function or regression curve of Y on X_1 and X_2 is the expected value of Y for given values of X_1 and X_2.

The expected value in this case is the one that has the greatest probability to occur. We denote the estimated variable and parameters with a $\wedge$ on them. Starting from equation (1) ;

$$(1) \quad Y = A + B \, X_1 + C \, X_2 + u$$

taking expected value;

$$(2) \quad \hat{Y} = E \, (Y \mid X_1, \, X_2) = \hat{A} + \hat{B} \, X_1 + \hat{C} \, X_2$$

$E \, (u) = 0$ by the specification of the model.

$\widehat{A}$, $\widehat{B}$ and $\widehat{C}$ are the regression coefficients and they measure the impact of each exogenous or independent variable on the endogenous or dependent variable. In our case, how much is the influence of the depth and the square of the depth in the drilling cost per foot.

Using equation (2) we construct a confidence interval with a 95% of probability for Y.

That is

$$(5) \qquad \widehat{Y} - 1.50 \leq Y \leq \widehat{Y} + 1.50$$

which implies that of 100 observations of drilling cost per foot at a given depth, in North Louisiana, 95 of them will lie in that interval. For example, for a depth of 5,000 ft., the confidence interval is

$$(6) \qquad 8.15 - 1.50 \leq Y \leq 8.15 + 1.50$$

and therefore, the observed drilling cost per foot will lie, in 95% of the total cases, between $6.65 and $9.65.

The confidence interval is shown graphically in Figure III.

Figure III
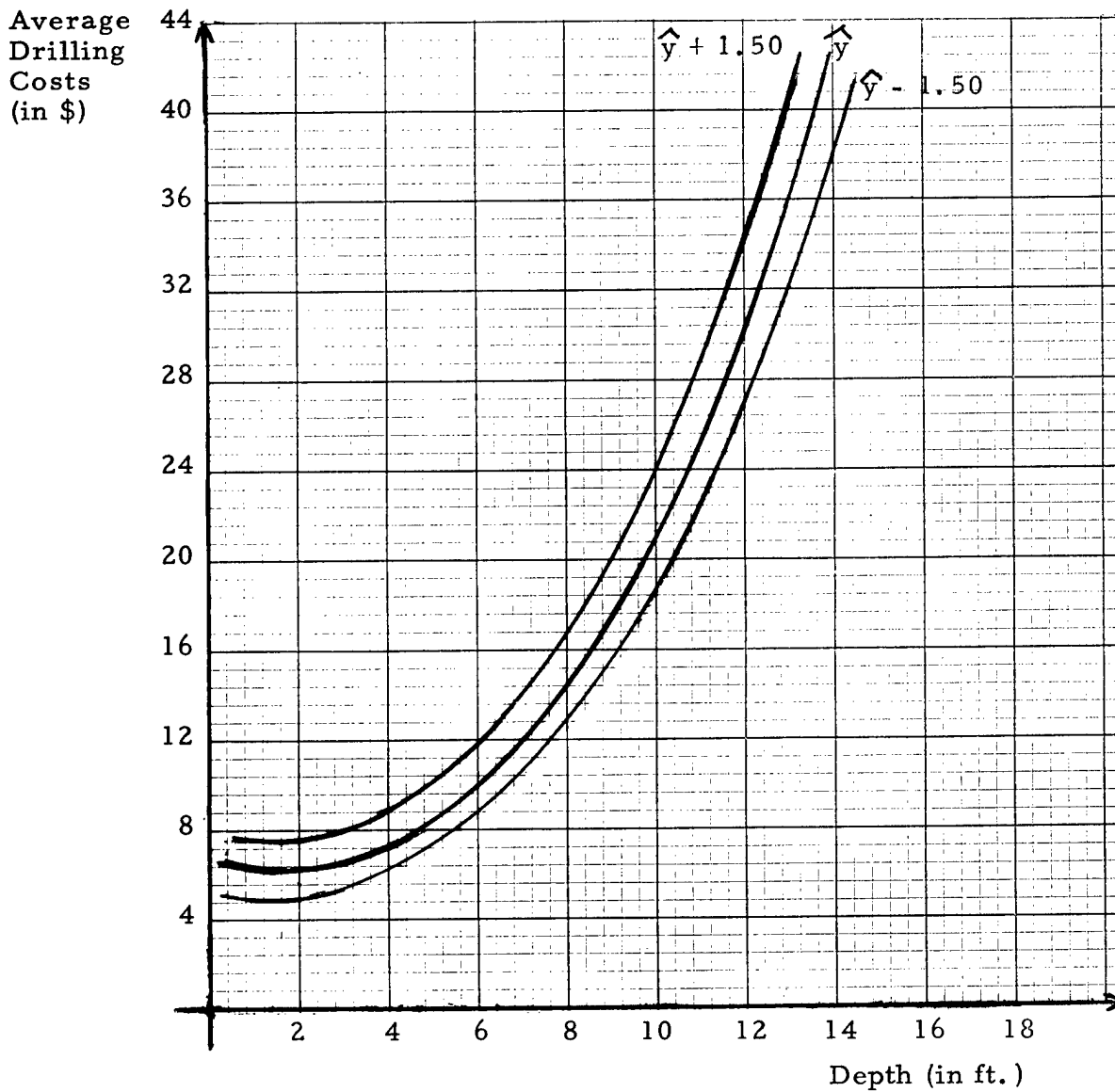
The coefficient of multiple determination for (4) is $R^2 = 99.5\%$ which confirms the goodness of fit of our regression function.

Example 2

According to Table 55 the average estimated drilling cost function for oil in Region I (Carbon-Permian) is:
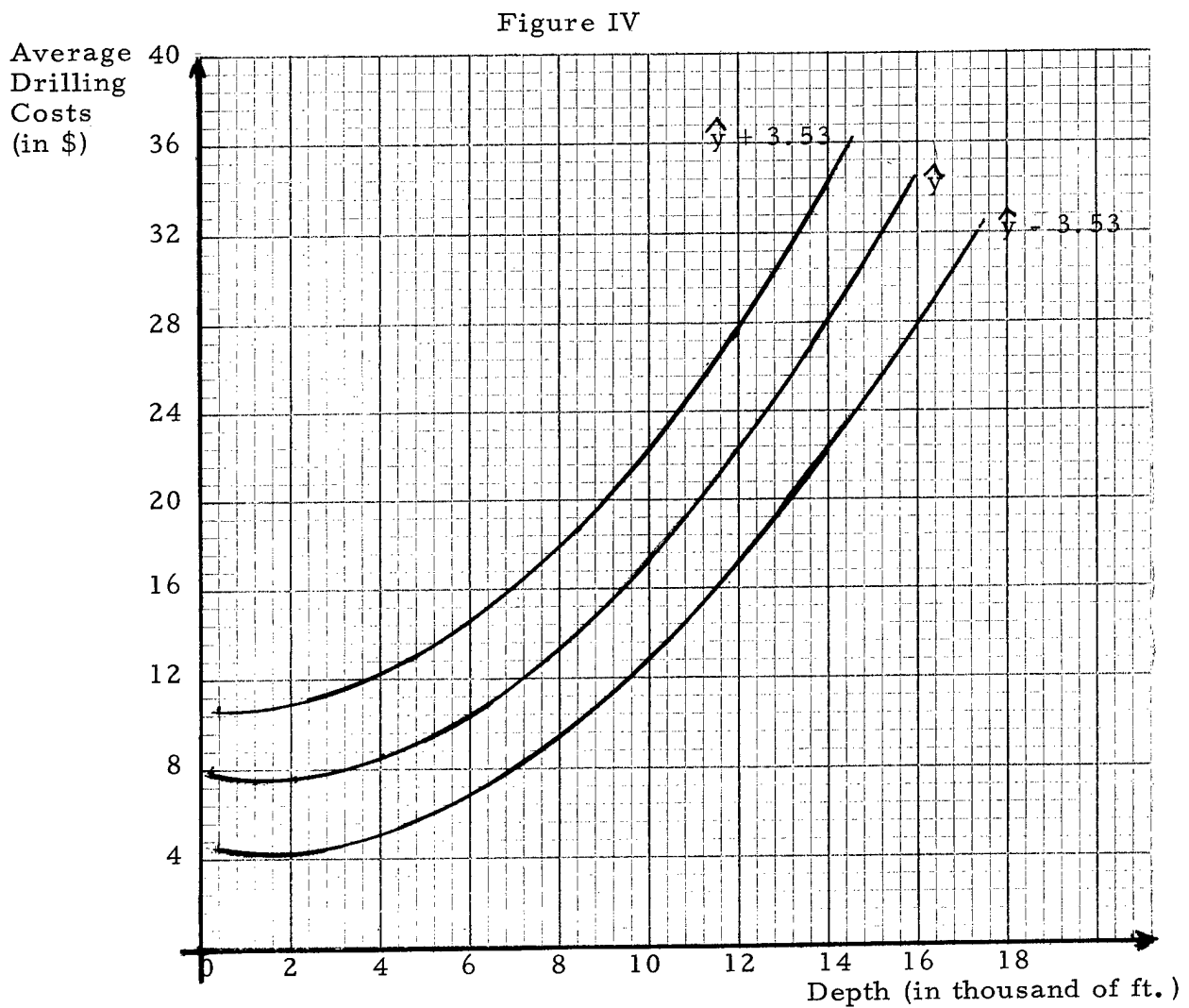
(7) $\widehat{Y} = 8.3 - 0.57 (10^{-3}) X_1 + 0.14 (10^{-6}) X_2$

The confidence interval with a 95% of probability is:

(8) $\widehat{Y} - 3.53 \leq Y \leq \widehat{Y} + 3.53.$

Which implies that at a given depth, for example 10,000 ft., the observed average drilling cost will be between $13.27 and $20.33 in 95% of the total observations.

Figure IV indicates the confidence interval for the average drilling cost of oil wells in Region I.

Figure IV

The coefficient of multiple determination for this case is $R^2 = 84.36\%$.

Example 3

Only in few cases we did find linear average drilling cost functions. For example, for oil wells in Onshore California we obtained according to Table 6 the following regression function:

$$(9) \qquad \hat{Y} = 14.58 + 0.88 (10^{-3}) X_1$$

The confidence interval with a 95% of probability is

$$(10) \qquad \hat{Y} - 2.60 \leq Y \leq \hat{Y} + 2.60$$

Then, given a depth of 6,000 ft., the observed drilling cost per foot will lie between $17.26 and $22.46 in 95% of the observations.

Figure V shows the confidence interval for the drilling cost per foot of oil wells in Onshore California.

The coefficient of determination for the regression function (9) is $R^2 = 95.27\%$.

Figure V
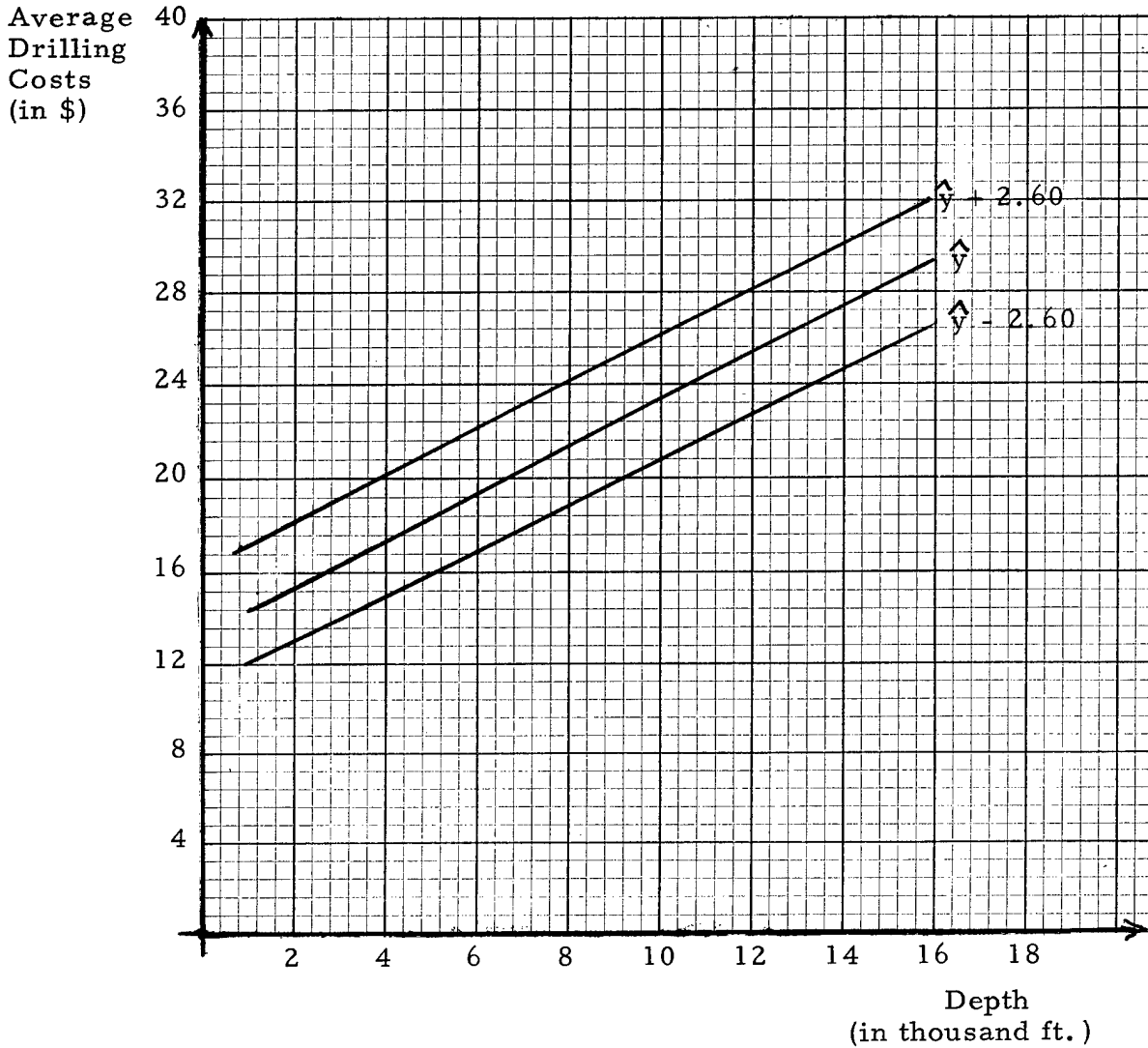
Average Drilling Cost Curves and Their Relation to Marginal and Total Drilling Costs

3.

As to the shape of the cost function related to footage, there seems to be a general agreement in the literature that drilling costs increase more than proportionately with increasing depth.

Given this nonlinear relationship, Franklin R. Fisher [22] inferred an exponential increase in the total costs per well based on the assumption that marginal costs per well increase linearly as a function of total costs per well. Fisher's hypotheses were developed on the

assumption that marginal costs are propotional to total costs and allowing for non-zero marginal costs as the depth decreases to zero. Fisher thus writes

(11) $dY/dX = H + \alpha Y$

where α and $H > 0$, X is the depth of the well, and Y is the total cost of the well, H the limit of marginal costs when the depth of the well approaches zero, and α gives the rate at which marginal costs increase with increasing depth. Substituting $K = H/\alpha$ and integrating Franklin Fisher derives his basic relation between total costs and depth as:

(12) $Y = K (e^{\alpha X} - 1)$

To derive equation (12), he rewrites equation (11) substituting $K = H/\alpha$ as follows:

(11 a.) $\dfrac{d(Y + K)}{dX} = \alpha (Y + K),$

(11 b.) $\dfrac{d (Y + K)}{(Y + K)} = \alpha \, dX,$ and

(11 c.) $d \log (Y + K) = \alpha \, dX.$

Integrating equation (11 c.), it becomes:

(11 d.) $\log (Y + K) = \alpha X + \log C,$ and

(11 e.) $Y + K = C \, e^{\alpha X}$

When X is zero, costs Y are zero, which implies that:

$$K = C$$

and therefore,

$$Y = K (e^{\alpha X} - 1)$$

The statistical problems in determining equation (12) based on the data of the Joint Survey are quite considerable as equation (12) involves a basically non-linear function which cannot be reduced to a linear relationship by some transformation, e.g., by taking logarithms.

Fisher estimated the cost function (12) by iterative techniques and in general he obtained a good fit to his observed data (1959).

The assumption of an exponential increase in drilling costs, however, has an important shortcoming in relation to our observed data (1965), which induced us to take a different approach. From the form of equation (12) and given that $\alpha_1 H > 0$ and therefore $K > 0$, both Y (total costs) and dY/dX (marginal costs) are regularly increasing functions. This implies that average costs, or costs per foot, when related to depth were also a regularly increasing function. The basic assumption in (11), furthermore, is that marginal costs increase in an exponential way, i.e., proportionally to Y. The minimum in these cost functions regularly occurs at 0 depth. When we tested the 1965 data of the Joint Survey, we found, however, that in the vast majority of cases, average costs behaved parabolically and reached their minimum between 3,000 and 5,500 feet of depth.

The parabolic regression on the observed average drilling costs gave excellent statistical fits.

Based on those results, we obtained the following general conclusions on the behavior of total and marginal estimated drilling cost functions:

First, the estimated total drilling cost function, defined as the average cost times its depth is:

(13) $$\hat{Y}_T = \hat{A} X + \hat{B} X^2 + \hat{C} X^3 \quad , X \geq 1$$

i.e., a third degree polynomial cost function having a point of inflection where marginal costs reach their minimum.

.Where:

$\hat{Y}_T$ = Estimated total drilling cost or total cost per well

X = Depth

Second, the estimated marginal drilling costs are given by the first order derivative of the total cost function. That is:

(14) $$\hat{Y}_M = \frac{d\hat{Y}_T}{dX} = \hat{A} + 2 \hat{B} X + 3 \hat{C} X$$

a parabolic function or second degree polynomial with respect to depth. In both total and marginal cost functions, the values of the regression coefficients $\hat{A}$, $\hat{B}$ and $\hat{C}$ are equal to those already estimated for the average drilling cost functions. Therefore, they can be used to calculate marginal and total costs in a very straightforward way.

Third, the minimum of the marginal cost function can be determined by the first and second order conditions.[*] In our case, for the marginal cost curve, the critical point or first order condition is given by:

$$(15) \qquad \frac{d\hat{Y}_M}{dX} = 2\hat{B} + 6\hat{C}X = 0$$

i.e., at

$$(15.a) \qquad X = \frac{\hat{B}}{3\hat{C}}$$

In all cases, except one single state, this minimum occurs at some positive depth, since all the $\hat{B}'$s are negative. This general result indicates that marginal costs initially decrease, then flatten out somewhat and then, rise again. This behavior supports the pattern of our total cost functions already depicted in Figure II, in the first part of this study.

As the second derivative of the marginal costs at the critical point, we have

$$(16) \qquad \frac{d^2\hat{Y}_M}{dX^2} = 6\hat{C} > 0$$

and therefore positive in all cases. We have reached at this point indeed a minimum of the marginal cost function.

---

[*] The first order condition is the usual requirement that the first derivative be zero and the second order condition requires that the second derivative be negative in the case of a maximum and positive for a minimum.

Fourth, the minimum now for the <u>average cost function</u> can be determined in two ways, both yielding identical results:

(a) By taking the derivative of the average cost function and setting it equal to zero, we have:

(17) $$\frac{dY}{dX} = B + 2CX = 0$$

which gives as our critical point:

(17.a) $$X = \frac{\hat{B}}{2\hat{C}}$$

again positive for all but one regression and, in particular, positive for all the estimated cost functions per regions. This also is true for the marginal cost curves. Checking on second order conditions, we have again reached a minimum cost point as:

(18) $$\frac{d^2\hat{Y}}{dX^2} = 2\hat{C} > 0$$

in all our cases.

(b) A second way of finding the minimum of average costs is given by the point where average costs equal marginal costs, i.e.:

(19) $$\hat{A} + \hat{B}X + \hat{C}X^2 = \hat{A} + 2\hat{B}X + 3\hat{C}X^2$$

which gives, if solved for X, again

(19.a) $$X = -\frac{\hat{B}}{2\hat{C}}$$

Furthermore, the minimum of the average cost function $(X = -\dfrac{\hat{B}}{2\,\hat{C}}$ occurs at a larger depth than the minimum of the marginal cost curve $(X = -\dfrac{\hat{B}}{3\,\hat{C}}$ as:

(20) $\qquad -\dfrac{\hat{B}}{2\,\hat{C}} > -\dfrac{\hat{B}}{3\,\hat{C}}$

with $\hat{B}$ real and negative, and $\hat{C}$ real and positive. This indicates that the minimum of the marginal cost curves is to the left of (and below) the minimum of the average cost curve. The point of inflection of the total cost curves occurs in each case at the same depth where marginal costs reach their minimum, i.e., X_1.

Figure III shows the general case of total cost functions with X_1 as the minimum depth of marginal costs and with X_2 as the point where average costs are minimum. Figure IV shows the general pattern follows by our marginal and average estimated drilling cost functions.

Total drilling costs and marginal drilling costs for each state and region analyzed are given in Tables of the Annex I.

Figure III

Figure IV

APPENDIX OF SECTION II
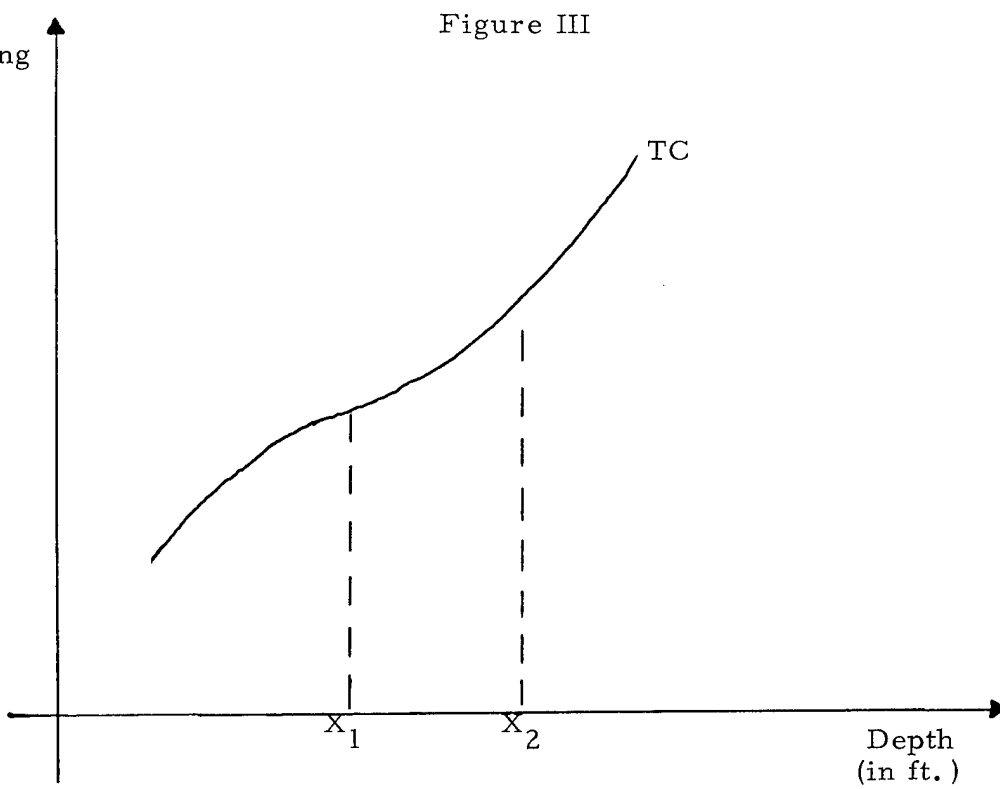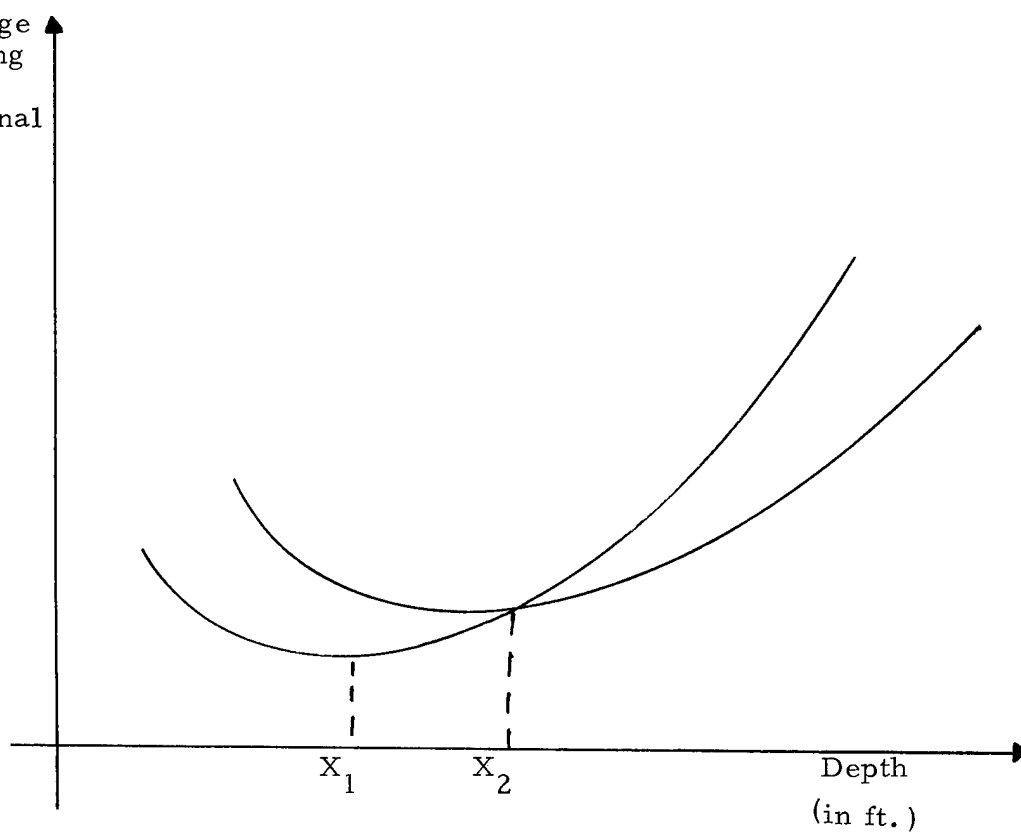
ECONOMETRIC ANALYSIS OF TOTAL, AVERAGE AND MARGINAL DRILLING COST FUNCTIONS

1. Specifications of Both Variable and Fixed Drilling Cost Functions

There exists, a remarkable lack of studies for intermediate size hole costs say, 12-1/4 inches to 36 inches . In this section, estimates on the progression of these costs are made, as a function of the diameter of wells, the depth drilled (shown to and including 10, 000 feet), and the geological medium.

The study encountered the following difficulties:

(a) There exists hardly any significant drilling experience in intermediate size holes down to 10, 000 feet for any particular geological region to allow econometric estimates of the cost function for all depth classes based on total costs of the wells and their depth alone, as was the case for standard size holes where there exists a wealth of data (though sometimes poorly collected and aggregated).

(b) The wells actually drilled in the intermediate range were in most cases combinations of different size diameter wells, as depth changes which, when combined, gave total well costs as an indistinguishable and, for our purposes, meaningless total number. Furthermore, as the costs of these wells could not be broken down proportionately to depth (due to the nonlinearities involved), we had to devise a novel method to come to estimates for intermediate size wells.

(c) Except for areas in Western Texas and a few wells in Louisiana, no representative data for drilling intermediate size wells could be found. Table I summarizes the data analyzed for all the fields where significant drilling in intermediate size diameter wells took place and the depths to which these diameters extend. About ten wells of intermediate size were also drilled at NTS in fiscal year 1966 and the first quarter of fiscal year 1967. However, the cost data of NTS cannot be regarded as representative of industry costs, even if subject to the same technological requirements (e.g., hole deviation, geology, etc.). We detected a sizable difference between industry drilling costs and costs of drilling under government contract (as is the case at NTS). The latter costs are, for some components, nearly twice as large (e.g., rig crew costs). As the peaceful use of nuclear explosives will be an industrial type operation, we had to estimate intermediate size emplacement costs on an industrial type cost basis.

(d) Marked differences exist already for standard size oil well, gas well, and dry well costs when the geological medium differs, mainly due to the hardness of the rock encountered at various depths, but also due to the hydrology of the area and the chemical and physical properties of the medium. The importance of the geological medium is considerably increased when the diameter of the emplacement hole is enlarged. For large diameter holes, therefore, at present, standard size exploratory holes are drilled around the emplacement site for the very purpose of determining the exact geological, chemical, physical, and hydrological profile of the well area. Based on these

Table I

Summary of Data for Intermediate - Size Diameter Wells

Fields	Diameter (in inches)						
	18 1/2	17 1/2	15	14 3/4	13 3/4	12 1/4	
Gomez (Texas)	---	9	---	---	5	9	Number Wells
	---	0-4225ft	---	---	0-9500ft	4,000-10,000 ft	Depth Range
Aggregated Lifette and Crowley (Louisiana)	2	1	1	1	---	2	Number of Wells
	0-11700ft	0-7833ft	3050-11245ft	2640-14523ft	---	7833-12924ft	Depth Range
Aggregated S. Pyote and Lockridge (Texas)	---	4	---	---	4	4	Number of Wells
	---	0-4900	---	---	1100-11100ft	2400-11200ft	Depth Range

- - -   Indicates un-available data

data, the costs for each large diameter hole are estimated separately.
For intermediate size wells, and in large-scale industrial applications,
such procedures would be too expensive and also not called for, when
some a priori knowledge of the geology of the region is given. Thus,
in the case of intermediate size holes, we had first to arrive at an
estimate of penetration rates in various geological media and, based
on these, we arrived then at cost estimates for these wells for each
of the media. This generalized procedure is further complemented
by a second econometric approach to estimating drilling time of wells
for each particular well separately if other wells were drilled nearby.
To our knowledge, this is another first attempt and, again, gave
surprisingly satisfactory results.

Given the above shortcomings, we decided to estimate the
drilling cost function for intermediate well diameters by estimating
cost functions for the separate technical cost components as a function
of depth, diameter, and hardness of geological medium within the
ranges of interest for our study. As the wealth of technical data on
various cost components was much larger than on total costs, we
were enabled, for the major part, to arrive at econometric estimates
of both individual costs and technical ratios of interest (e. g., pene-
tration rates).

Where empirical data were lacking, we reconstructed the
individual cost functions on the basis of technical requirements based
on various manuals. The individual cost functions were then aggregated
to yield total costs, total cost per foot (average cost), and marginal
costs for cased holes and for open (uncased) holes for the desired
intervals.

The cost components, their cost functions, and their aggregation are shown below in more detail, but the following theoretical remarks can be made on their functional form and aggregation.

We found that the cost functions for the technical components of the drilling process fall into one of the following four cases: fixed costs, linear cost functions (L), second degree polynomial cost functions (Q), or third degree polynomial cost functions (T). The general mathematical form of the cost functions for the component costs are:

For rig costs based on estimates of penetration rates:

$$Y_{RIG} (\phi, D, H) = L_1 (\phi, D, H) o L_2(\phi, D)$$

where: L_1 = function for drilling time

L_2 = function for rig day rate

ϕ = diameter

D = depth

H = hardness

and o symbolizes the convolution of the functions. Thus, we have as the general rig cost function based on penetration rates:

$$Y_{RIG} (\phi, D, H) = Q_1(\phi, D, H),$$

a second degree polynomial function in both depth and diameter.

Where econometric drilling time estimates are possible (see sections on rig costs), the rig costs are given by:

$$Y_{RIG}(\phi, D, H) = Q_1(D, \phi, H) o L_2(D, \phi),$$

which, when combined, results in a third degree polynomial function:

$$Y_{RIG}(\phi, D, H) = T_1(D, \phi, H).$$

For casing costs, we have:

$$Y_{CAS}(\phi, D, H) = L_3(\phi, D)$$

and, similarly, for the other cost components:

Mud costs: $Y_{MU}(\phi, D, H) = T_2(D, \phi)$

Cementing costs: $Y_C(\phi, D, H) = T_3(\phi, D)$

Cutter costs: $Y_{CU}(\phi, D, H) = T_4(\phi, D)$

Mobilization-demobilization costs: $Y_{MOB}(\phi, D, H) = L_4(\phi, D)$*

Site preparation costs: $Y_{SP}(\phi, D, H) = L_5(\phi, D)$*

Rig up and teardown costs: $Y_{RT}(\phi, D, H) = L_6(\phi, D)$*

Surface casing costs: $Y_{SC}(\phi, D, H) = T_5(\phi, D, H)$

As the individual cost components are additive, the total costs for either cased holes or open holes turn out to be a third degree polynomial function:

$$Y_{TOTAL}(\phi, D, H) = T_6(\phi, D)$$
$$= \alpha_0 + \alpha_1 D + \alpha_2 \phi + \alpha_3 D \phi + \alpha_4 D^2 + \alpha_5 D^2 \phi + \alpha_6 D \phi^2 + \alpha_7 D^3$$

i. e., the general form of total costs, average costs, and marginal costs is identical to the one found in the econometric analysis of standard size diameter wells given in the previous section, resulting in parabolically shaped average and marginal cost curves.   The theoretical analysis on total costs, average costs, and marginal costs given in the

---

* but nearly fixed cost.

first section holds also for intermediate size diameter holes. The

determination of the minimum of average and marginal costs is

identical , with the exception that now we have to take partial

derivatives to determine the marginal costs with regard to depth

and diameter increases; that is:

(1) $\dfrac{\partial\, Y(\phi,\ D,\ H)}{\partial\, \phi}$ = marginal costs of diameter changes when depth and geological medium are constant

and   (2) $\dfrac{\partial\, Y(\phi,\ D,\ H)}{\partial\, D}$ = marginal costs of depth changes when diameter and geological medium are constant

The second expressions is comparable to the marginal costs of standard

size diameter wells.

Furthermore, this allows us to make a direct comparison

between the results obtained for standard size and for intermediate size

wells, and important direct, general conclusions can be drawn from

the results.

The accuracy of the estimates for intermediate size wells

is underlined by the surprisingly accurate correspondence between the

intermediate size cost estimates of up to 12 inches and those estimated

from the sample data of U. S. drilling costs for standard size diameters:

the average costs at 10, 11, and 12 inches are practically identical to

the aggregated empirical cost curves for standard size diameters.

Again, the minimum of average costs per foot is reached in the general

range between 3,000 and 6,000 feet. This exact correspondence is

important for the following reasons:

(a) It shows that for standard size diameter holes, the progression of drilling costs (between 6 and 12 inches) is negligible; i. e. , reducing the diameter of nuclear devices to fit them in holes smaller than 12 inches should bring no additional cost savings with regard to emplacement hole costs(see also Section I. 1.)

(b) It underlines the accuracy of the estimates obtained by the second method used in this section and shows, for practical purposes, the equivalence of the two methods.

(c) It is a further proof that drilling cost functions at depths of interest do increase, with average and marginal costs both behaving parabolically (as in the classical case of cost functions) and not exponentially (the other major hypothesis advanced in other studies).[22]

We now proceed to analyze each of the components of total variable cost.

Mud Cost Function

The volume of the material drilled at the bottom of the hole has to be continuously removed and lifted to the surface. Various systems are available at present to achieve this. The circulated medium can be fluid (drilling mud) or air, or a combination of both, such as foam. In our calculations, we assume that the circulating medium is mud. Mud costs are determined by the price of the materials generally expressed per barrel and the volume of the well. The latter is due to the fact that when mud is used, the hole is usually maintained full of mud. Moreover, it is considered that for long-term drilling the amount of mud needed will be about twice the volume of the hole because of fluid losses and surface storage.

The general mud cost function used in calculations indicated in Table 1 of Annex of Section II is:

$$Y_{MU} = (\pi (\phi^2/4) 2D \ P)/5.6148$$

Where

Y_{MU} = cost of mud per well

ϕ = diameter of the hole in feet. This variable ranges from 10 ft. to 45 ft. in our calculations.

D = depth in feet ranging from 1,000 ft. to 10,000 ft.

P = price of mud equal to $4 per barrel in our calculations.

5.6148 = number of cubic feet contained in a barrel used to correct used cost per barrel into cost per cubic feet.

Cutter Cost Function

One of the most important parts of the equipment for rotary drilling operations, is the drill bit. The most preferred bit for big hole rotary drilling is the "rolling cutter bit," which is also available for conventional sizes, from 3 3/4 inches to 26 inches [21]. The cutter design is related to various formation characteristics: hardness, resiliency, abrasiveness, etc., and in its selection all these aspects must be taken into account. Cutter costs are mainly a function of the volume of material removed and the hardness of the soil. They increase more than proportionally with increasing diameter and hardness.

Our general cutter cost function used for the calculations indicated in Tables 2 to 6 of Annex of Section II is:

$$Y_{CU} = \pi \,(\phi^2/4)\; D \; C_H$$

Where

Y_{CU} = cutter costs per well

ϕ = diameter of the hole drilled ranging from 10 inches to 45 inches.

D = depth ranging from 1,000 ft. to 10,000 ft.

C_H = cost per foot of linear cut as a function of the geological medium. Based on Dellinger [16] estimations, we use the following cutter costs: $0.50, $0.75, $1, $1.50 and $2 for soft rock, medium soft rock, medium hard rock, hard rock and very hard rock, respectively.

Casing Cost Function

Steel casing is the only satisfactory casing material, at this time, based upon cost, performance properties and design experience.

For work at the Nevada Test Site of the U. S. Atomic Energy Commission an intensive program is undergoing to develop plastic casing tubes, which would have great advantages over steel casing in two respects: reduced weight and larger collapse strength. However, at present, steel casing is yet the most economic and readily available casing material. Two basic design approaches exist:

(a)   Unstiffened shell casing.

(b)   Stiffened shell casing.

Considering safety factors the needed wall thickness of the casing is

nearly 3 inches at the bottom of an 800 ft. and 45 inch casing string.

In this limit, the casing would weight about 1650 lbs. per linear foot

and cost some $250 per foot. Stiffened shell casing would weight

about one half as much and the costs would be appreciably less.

Regardless of the type of casing design, the weight of our 8,000 ft.,

45 inch casing string will greatly exceed the lifting capacity of any

rig or casing jack presently available on the market, though for

particular jobs, like the large hole program on Amchitka showed that

such capabilities can be ordered, if necessary. There are, thus,

some of the existing limits in casing capacity.

For intermediate size wells we only considered casing

diameters from about 8 inches to 30 inches with corresponding required

well diameter of 10 inches to 45 inches (i. e., casing diameter $\approx 2/3$

of well diameter). Once casing diameter and the depth of the well are

determined, then the costs of the casing are a function of the weight

per linear foot of casing, the costs of the steel used, per lb., the depth

of the hole, and the location of the hole. For our purposes we choose

casing costs for Odessa, Texas. As for the steel quality used, the

weight per foot and resulting cost per foot, the next Figure shows

the possible spread of casing costs from 8 inches to 30 inches for 1,000,

and 10,000 feet. The heavy line reflects the approximation of overall

casing costs used in our calculations. As the diameter of the casing

increases, the wall thickness of the casing and the quality of the steel

Cost of Casing given 1,000 ft. depth
(in thousand dollars)

Figure 1

Casing Costs

ID Casing (in inches)

Costs of Casing, given 10,000 ft. depth
(in thousand dollars)
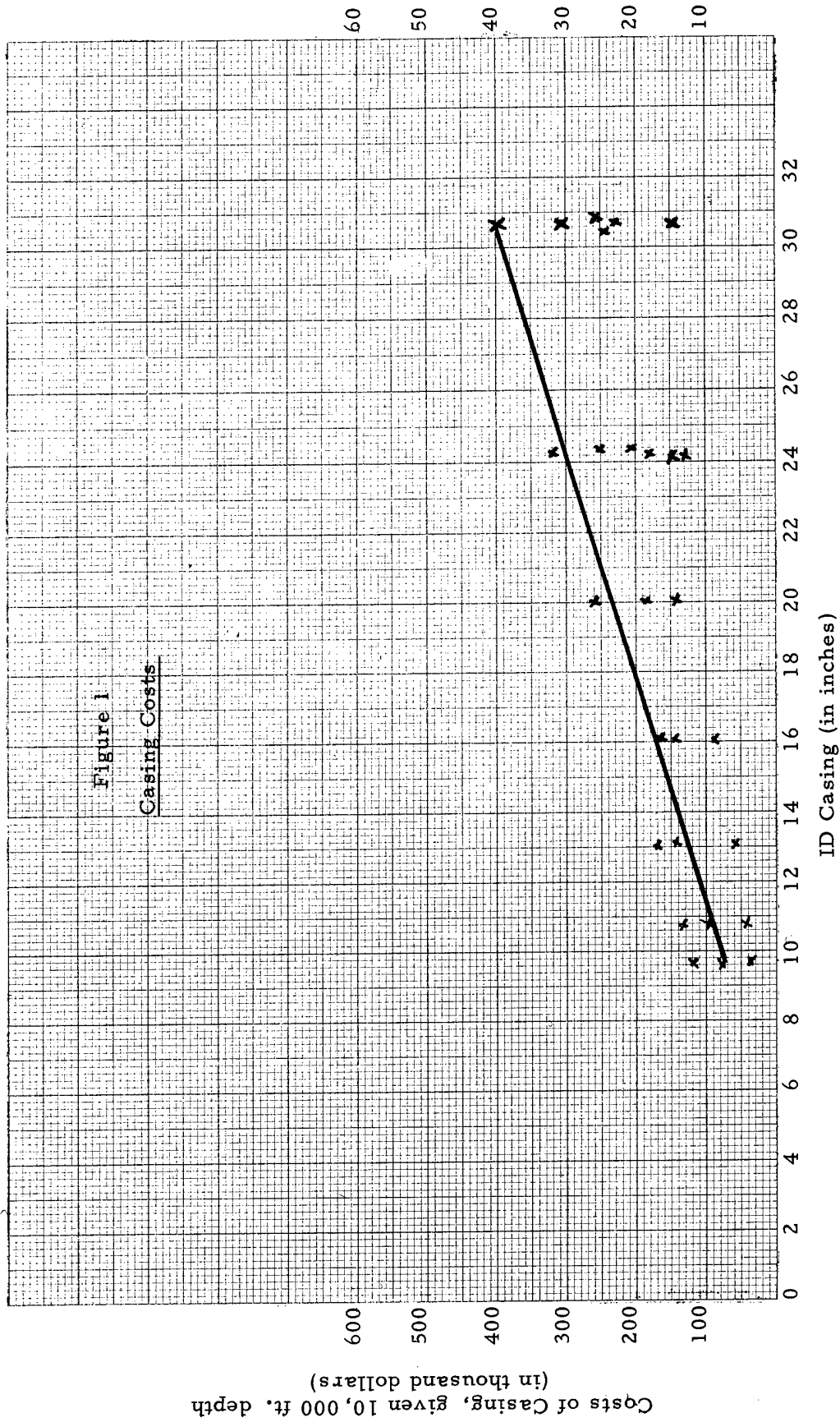
x = Cost of casing calculated on the basis of the official prices for various steel qualities given
by Continental-Emsco Co., Division of the Youngtown Sheet and Tube Co., for Odessa, Texas.

used have to meet very stringent technical requirements, particularly
with regard to minimum collapse resistance. Intermediate size holes
were never fully cased down to 10,000 feet and thus the cost figures
derived from Figure 1 reflect rather lower limits to actual casing costs
for longer diameter holes, as the costs of the casing holes and related
costs of linear emplacement may increase considerably in order to
meet minimum collapse resistance requirements. Based on the above
assumptions and on the interpretation of the data shown in Figure 1,
we arrived at the following general casing cost function, for inter-
mediate size wells used for the calculation indicated in Table 7 in
Annex of Section II. The casing cost function is given in two equiva-
lent forms, first, as a function of the hole diameter and second, as
a function of the casing diameter:

$$Y_{CAS} = (7,500 + 1,625 ((2\phi_1/3) - 10)) \ D/1,000$$

$$Y_{CAS} = (7,500 + 1,625 (\phi_2 - 10)) \ D/1,000$$

Where

Y_{CAS} = casing costs on a function of depth and diameter.

ϕ_1 = the diameter of the hole drilled.

ϕ_2 = the diameter of the casing and

D = depth in feet.

Not reflected in these costs are the technical limits of casing length
due to the collapse strength of the casing, though we believe that for
intermediate size wells these technical problems can be solved at
some additional costs. Thus, the above casing costs reflect rather
a lower limit to possible casing costs at extreme limits (i. e., 10,000
ft. and 30 inch casing).

Casing costs to total depth and given in the above equations,
have to be distinguished from intermediate casing requirements due to
adverse drilling conditions. At a given diameter and at a given
intermediate length of the casing the above two equations <u>can also</u>
<u>be used to determine intermediate size casing costs.</u> However,
for each well this length of intermediate casing may be different and,
in order to determine the costs of such a hole, the total depth has
now to be broken down into the cased hole length and the open hole
length. Than the appropriate values can be derived from the tables
and figures of the Annex of Section II and added up to give an estimate
of hole costs with intermediate casing. The distinction between
casing requirements due to nuclear explosive protection and casing
requirement due to drilling conditions is very important.

Cementing Cost Function

Cementing costs are mainly a function of the quantity
of material required to fill the annular space between the casing and
the boring wall. In our calculations we assume that the total quantity
of cement is equal to the annular volume plus 30 percent for the
boring wall washouts. The annular volume is determined by the
difference between the volume of the open hole and the volume of the
cased hole, where the inside diameter of the casing is approximately
equal to 2/3 of the hole diameter. Therefore, our general cementing
cost function becomes:

$$Y_C = \frac{13}{18} \pi \ P \ D \ (\phi^2/4)$$

Where

Y_C =   cementing cost per well

ϕ =   diameter of the hole drilled ranging from 10 to 45 inches in our calculations while the corresponding inside diameter (ID) of the casing ranges from 8 to 30 inches.

D   =   depth ranging from 1,000 to 10,000 ft. in our calculations.

P   =   price of the cement material equal to $2.00 per cubic foot in our calculations.

The corresponding calculations are shown in Table 8 in Annex of Section II.

Rig Cost Function

Rig costs are mainly a function of the drilling time and the rig day rate.

(A)   Drilling Time Estimation

In our study, the estimation of the drilling time was made in two different ways:

1.   As a Ratio Between Total Depth of the Hole and Penetration Rate

To calculate the expected penetration rate for different geological medium we proceeded as follows:

First we extrapolated from 30 inch to 10 inch diameters, the expected penetration rate curve for medium soft rock given by Dellinger[17] and approximated this function by varying the constant on the left hand side between 5 and 15 and the following hyperbolic function gave the best fit:

$$PR \ (\phi + 8) = C$$

Where

PR $\quad$ = $\quad$ expected penetration rate

ϕ $\quad$ = $\quad$ diameter of the hole

C $\quad$ = $\quad$ constant equal to 300 in our calculations

Second, for the other geological media we multiplied the penetration rate function obtained for medium soft, by a constant that varies according to the hardness of the soil. They are: $\frac{3}{4}$; $\frac{3}{2.3}$; $\frac{3}{1.3}$ and 3 for soft rock, medium hard rock, hard rock and very hard rock respectively. These ratios were derived from various manuals on large diameter drilling costs [49,17]. This way of determining drilling time allows a straightforward estimation of drilling costs whenever the geological medium is known.

(b) By a Multi-linear Regression of the Drilling
      Time Function

Only for a few fields we were able to get sufficient
data on drilling time per well to perform regression
analyses like those made for small holes in the previous
section. Drilling time in hours for different hole depths
were obtained for the fields of: Gomez, Lockridge and
S. Pyote in Texas. The drilling time was plotted
against depth and then regression curves were fitted.
The final results indicate that the drilling time is a
least function of the depth.

The mathematical form of the model, for all
the cases analyzed is:

$$Y_{DT} = A + B X_1 + C X_2 + u$$

Where

Y_{DT} = drilling time in hours

X_1 = depth in feet

X_2 = square of the depth

u = stochastic variable

A, B and C are the structural unknown parameters, which
we estimated on the basis of our sample observations on
X and Y_{DT} applying the least-square method. This method
of estimation gives the best linear unbiased estimators, in
uniequational models with exogenous variables. That is,

of the class of linear unbiased estimators, the least-square estimators have the smallest variance.

Using the "student's t" distribution we performed the null hypothesis test for each of the parameters A, B and C separately and we performed also joint tests for all ot them with the F distribution. The null hypothesis was rejected at a significant level of 5%, which implies that the specification or mathematical form of our model explains

with a probability of 95% the drilling time function investigated. The goodness of the fit was measured by the coefficient of multiple determination which is equal to the proportion of this Y variance accounted for the simultaneous influence of X_1 and X_2. Except for one case, the coefficient of multiple determination was greater than 90% which indicates the outstanding goodness of fit of our equation i.e., a second degree polynomial. In all the cases we used large samples that range from 67 to 231 observed data per regressions.

The estimated drilling time functions for each of the fields where available data existed are:

(a)   Estimated Drilling Time Function for Gomez Field
       (Texas) for 12 1/4 inch Diameter Wells (in hours)

$$Y_{DT} = 0.126 (10^{-5}) X^2 + 0.06 X - 31.00$$

Where

Y_{DT} = drilling time in hours

X = depth in feet

(b) Estimated Drilling Time Function for Gomez Field (Texas) for 13 3/4 inch Diameter Wells (in hours)

$$Y_{DT} = 10.45 + 0.88 (10^{-2}) X + 0.60 (10^{-5}) X^2$$

Where

Y_{DT} = drilling time in hours

X = depth in feet

(c) Estimated Drilling Time Function for Gomez Field (Texas), for 17 1/2 inch Diameter Wells (in hours)

$$Y_{DT} = 0.107 (10^{-2}) X^2 + 0.056 X - 13$$

Where

Y_{DT} = drilling time in hours

X = depth in feet

(d) Estimated Drilling Time Function for Aggregated Lockridge and S. Pyote (Texas) for 12 1/4 inch Diameter Wells (in hours)

$$Y_{DT} = 42 + 0.02 X + 0.40 (10^{-3}) X^2$$

Where

Y_{DT} = drilling time in hours,

X = depth in feet

(e) Estimated Drilling Time Function for Aggregated
Lockridge and S. Pyote (Texas) for 13 3/4 inch
Diameter Wells (in hours)

$$Y_{DT} = 62 + 0.24 (10^{-2}) X + 0.81 (10^{-5}) X^2$$

Where

Y_{DT} = drilling time in hours

X = depth in feet

(B) Rig Day Rate

The rig day rate which is the basic cost per time

unit for the drill rig and support equipment package,

is calculated as a linear function of the rig horsepower

requirement. There are essentially three basic

rig rates: (1) the operating rate while the rig is at

work; (2) the standby rate while the rig is not working

but while the drilling crew is being held at the drill

site in anticipation of working and (3) the standby secured

rate, which is applicable when the rig is being held at

a location without a crew or in transit from one location

to another [21]. In our calculations we only use the

operating rig rate and expressed per hour instead of

per day. To calculate the latter we extrapolate the

values given by Fisher [21], for a horsepower require-

Table II

Rig Horsepower Versus Depth and Various Diameters

Depth (in feet)	Hole and Casing Diameter			
	12" Hole 8" Cased	24" Hole 16" Cased	28" Hole 18" Cased	45" Hole 30" Cased
1,000		500 hp	500 hp	
2,000	500 hp			1,000 hp
2,500		1,000 hp	1,000 hp	
3,000				
3,500				1,375 hp
4,000			1,375 hp	
5,000	1,000 hp	1,375 hp		
6,000				
6,500				
7,000				
8,000	1,325 hp	2,500 hp	2,500 hp	2,500 hp
9,000				
10,000				

ment greater than 1,500 as is indicated in Figure 2.

Table II shows some of the particular assumptions on rig

horsepower requirements down to 10,000 feet and at various

diameters on which the interpolations of Figure 2 are based.

The approximation of horsepower requirements by this

function proved to be very useful, though by necessity it

does not reflect the very discrete changes in presently

available horsepowers. The general mathematical form

for our rig day rate is:

Rig day rate   =   1,200 + 1.15 HPR

Where

HPR   =   26 (ϕ - 10) + D/5 - 800 is the rig horse-
power requirement defined for the interval
500 - 2,500. ϕ is the diameter of the hole
drilled in feet and D is the depth in feet.

Finally, our rig cost function per well expressed as the

product of drilling time and rental hour rate becomes:

$$Y_{RIG} = \frac{D H_i (\phi + 8)}{7,200} (1,200 + 1.15 HPR)$$

Where

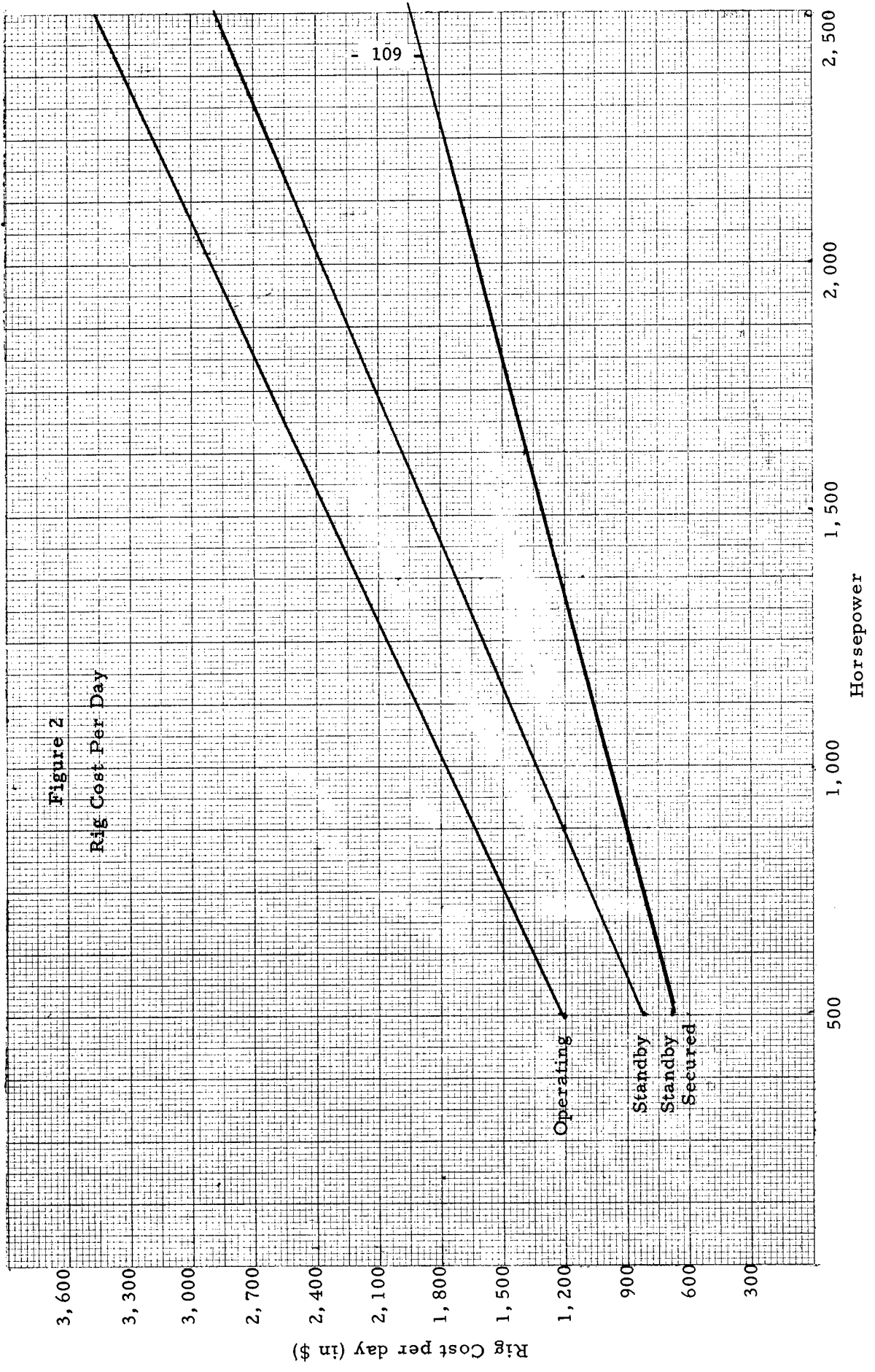Y_{RIG} =   rig cost per well

D   =   depth ranging from 1,000 ft. to 10,000 ft.

ϕ =   diameter of the drilled hole ranging from
10 inches to 45 inches.

Figure 2

Rig Cost Per Day

H_i = factor of the hardness of the soil equal to $\frac{3}{4}$; 1; $\frac{3}{2.3}$; $\frac{3}{1.3}$ and 3 for the following geological media, respectively; soft, medium soft, medium hard, hard and very hard.

HPR = rig horsepower as defined above.

Total Fixed Cost Function

Total fixed costs are obtained by the sum of the following components: mobilization and demoblization, rig-up and tear-down, site preparation and surface casing. The three first components are estimated as a linear function of the rig horsepower requirement which itself depends on the depth and diameter of the hole. They are expressed in the following mathematical form:

(a) Mobilization and Demobilization Cost Function

$$Y_{MOB} = 4,000 + 4.0 \ (HPR - 500)$$

Where

Y_{MOB} = mobilization and demobilization costs per well, in dollars.

HPR = rig horsepower requirement defined for the interval 500-2,500 with the following expression; HPR = 26 (ϕ-10) + D/5 - 800.

In particular applications of Plowshare, the repetitive emplacement of nuclear devices may allow a continuous drilling operation, in which case, mobilization and demobilization costs could be substantially reduced. As in such operations, the allocation of these costs to

individual wells would be reduced with an increase in
the number of wells drilled, it is difficult to make
accurate estimates of such costs per well. For our
purposes we include a fixed amount of $4,000 per well
and increasing linearly with the rig horsepower require-
ment as the latter increases with the volume of the hole.

(b)   Site Preparation Cost Function

Site preparation costs include, for intermediate
wells, the leveling of terrain, the preparation of mud
pits and the foundations of the rig. We assume that these
costs are also a linear function of the rig horsepower and
we include a fixed amount of $2,000 as independent term.
The final estimated site preparation cost function is:

$$Y_{SP} = 2,000 + 6.5 \; (HPR-500)$$

(c)   Rig-up and Tear-down Cost Function

The procedure of erecting and tearing down a drill
rig requires varying amounts of support equipment,
depending on the type and size of a drill rig. In our
calculations we assume that rig-up and tear-down
costs are a linear function of the rig horsepower require-
ment. The final cost function is ;

$$Y_{RT} = 3,800 + 4.1 \; (HPR-500)$$

(d)   Surface Casing Cost Function

Prior to moving in the big hole drill rig is the opening of a surface hole and its casing which is always done independently of whether the drilling operation will later require intermediate or full casing. The surface casing allows a better handling of the drilling equipment close to the surface. The diameter of the surface cased hole is considerably larger than the ultimate diameter of the well (about 1 to 2 feet larger for intermediate wells). The depth of surface casing can be set from 50 ft. to 100 ft. or, even more, depending on the particular conditions. In our calculations we assume that surface casing is a linear function of the total variable costs. That is:

$$Y_{S.C} = 0.025 \text{ (Total Variable Costs)}$$

Where

Total variable costs = mud costs, cutter costs, cementing costs, casing costs and rig costs.

2.   Total, Average and Marginal Cost Functions

The total drilling cost function is determined by the sum of total variable costs and total fixed costs. Their general mathematical form individually and as aggregates we discussed at the beginning of this Appendix. Given that, many of its components are mainly a function of the volume of the hole, and the resulting horse-

power requirements. As was already largely discussed above the total cost function is a third degree polynomial in two variables, diameter and depth. We determine total drilling costs in our calculations for cased and uncased wells as follows:

$$Y_{TC} = Y_{MUD} + Y_{CAS} + Y_C + Y_{CU} + Y_{RIG} + Y_{FC}$$

Where

Y_{TC}	=	total drilling costs for cased wells in dollars
Y_{MUD}	=	mud costs per well
Y_{CAS}	=	casing costs per well
Y_C	=	cementing costs per well
Y_{CU}	=	cutter costs per well
Y_{RIG}	=	rig costs per well
Y_{FC}	=	"fixed costs" per well

For uncased holes, total drilling cost functions do not include cementing and casing costs, that is:

$$Y_{TUC} = Y_{MUD} + Y_{CU} + Y_{RIG} + Y_{FC}$$

Where

Y_{TUC}	=	total drilling costs for uncased wells in dollars
Y_{CU}	=	cutter costs per well
Y_{RIG}	=	rig costs per well
Y_{FC}	=	fixed costs per well

It is extremely important, however, to bear in mind that casing and cementing costs are not the only cost increases for comparable emplacement holes as the diameter of the holes drilled differs by 50% of the open hole diameter. This difference is of particular relevance when reading the enclosed Tables for cased and open holes. The average drilling cost function or cost per foot drilled is obtained by dividing total drilling costs by the corresponding depth. The general form of the function is a second degree polynomial. For cased wells we have:

$$Y_{AV.C} = Y_{TC}/D$$

Where

$Y_{AV.C}$ = average drilling costs for a cased well in dollars

Y_{TC} = total drilling cost for a cased well

D = depth

and for uncased wells;

$$Y_{AV.UC} = Y_{TU}/D$$

Where

$Y_{AV.UC}$ = average drilling costs for uncased wells in dollars

Y_{TU} = total drilling costs for uncased wells

D = depth

Since our total cost function is a third degree polynomial in two variables, we only are able to obtain marginal costs defined as the partial derivatives of the total drilling cost function in the direction of only one of the independent variables. We calculate the marginal drilling costs for changes in diameter, i.e., how much is the amount added to total costs by each additional inch of diameter. Given the complicate form of the final function for total drilling costs, we approximate marginal costs by the ratio between the increment of the total drilling cost function and the increment of the variable (diameter), that is:

$$Y_{MC/\phi} = \frac{\Delta Y_{TC}}{\Delta \phi}$$

Similarly, the marginal costs of depth changes can be calculated for any of the depth ranges to 10,000 feet.

BIBLIOGRAPHY

[1]   Allen, James H., "The Development of Large Diameter Rotary Drilling Machines and Equipment for the Mining and Construction Industries," Ninth Annual Drilling Symposium, Bulletin of the Mineral Industries Experiment Station, Pennsylvania State University, October 1957.

[2]   Allen, James H., "The Special Equipment and Problems Associated With Large Diameter Rotary Drilling," American Society of Mechanical Engineers, Paper No. 61-PET-34.

[3]   American Association of Oil Well Drilling Contractors, "Rotary Drilling Bid Proposal and Contract Form, Dallas, Texas, March 1967.

[4]   American Petroleum Institute, Independent Petroleum Association of America, Mid-Continent Oil and Gas Association, "Joint Association Survey of Industry Drilling Costs," 1965.

[5]   Aran-Nuri, A.I., "Russia Improves Drilling Technology," World Oil, July 1962, pp. 112-115.

[6]   Baumol, W., Economic Theory and Operational Analysis, 2nd Edition, Prentice Hall, N.J., 1965.

[7]   Bawcom, Z. W., "Rotary Drilling of Large Diameter Vertical Holes," Symposium on Salt, Cleveland, Ohio, Northern Ohio Geological Society, May 1962.

[8]   Bentson, H. G., "Experience--Important in Development Of Drilling Tools for Large Diameter Holes," Drilling Magazine's 1960 Exposition-In-Print.

[9]   Bowman, G. A., "Large Diameter Drilling Methods, Equipment and Problems," 24th Annual Meeting of American Association of Oil Well Drilling Contractors, San Antonio, October 1964.

[10]   Bowman, G. A., "Big Hole Drilling Methods, Equipment and Problems," Drilling Magazine, April 1965, pp. 60-62, p. 65, pp. 97-99, pp. 101-106.

[11]   Burke, Robert G., "Big Holes Point to Deep Wells," The Oil and Gas Journal, January 1968, pp. 65-70.

[12]   Campbell,. . .., Private Communication on Cost Reduction Per 1-Inch Hole Diameter Reduction, Dallas, Texas, February 1968.

[13]   Christ, C. F., Econometric Models and Methods, John Wiley, New York, 1966.

[14] Clinedinst, W. O., "The Effect of Internal Pressure and Bending on the Tensile Strength of API Round Thread Casing," presented at the Symposium on Mechanical Properties of Pipe, API Division of Production, 1967 Midyear Standardization Conference, San Francisco, California, June 1967.

[15] Combs, George D., "New Testing Tool Improves Mud Control, Penetration Rate" in The Oil and Gas Journal, October 2, 1967.

[16] Continental-EMSCO Company, "Oil Country Tubular Goods, Comprehensive Price List #8," January 1967.

[17] Dellinger, Thomas B., "Economic Factors Relating to Drilling Large-Diameter Holes," prepared for the Second Symposium on Salt, Northern Ohio Geological Society, May 3-5, 1965, Ferrix & Scisson, Inc., Tulsa, Oklahoma, June 1965.

[18] Eftring, N., and Hult, J., "Transverse Strength of Steel Tubes," presented at the API Division of Production, 1967 Midyear Standardization Conference, San Francisco, California, June 1967.

[19] Fenix & Scisson, Inc., Private Communication on Cost Per Foot Data for All Nevada Test Site-CPFF Holes Drilled in Fiscal Year 1966 and First Quarter of Fiscal Year 1967, Las Vegas, January 1967.

[20] Fenix & Scisson, Inc., "Generalized Time and Cost Curves for Large-Diameter Based Holes," prepared for U. S. Atomic Energy Commission, Las Vegas, Nevada, January 1968.

[21] Fisher, P. R., "Large Diameter Drilling for Emplacing Nuclear Explosives," U. S. Army Corps of Engineers, Fort Worth, Texas, 1964.

[22] Fisher, Franklin M., "Supply and Costs in the U. S. Petroleum Industry--Two Econometric Studies," Resources for the Future, Inc., Washington, D. C., 1968.

[23] Grovenburg, W. W., et al, "Proposed Construction Methods-- Large Diameter Holes to 6,000 and 8,000 Feet," Ferrix & Scisson, Inc., prepared for the U. S. Atomic Energy Commission, Las Vegas, Nevada, October 1966.

[24] Gstalder, S., "Measurement of Some Mechanical Properties of Rocks and Their Relationship to Rock Drillability," Journal of Petroleum Technology, August 1966.

[25] Henderson, J. M. and Quandt, R. E., Microeconomic Theory, McGraw Hill, New York, 1958.

[26]   Hughes Tool Company, "Scientific and Technical Applications
       Forecast--Excavation," prepared for Office of the Chief of Research
       and Development, Department of the Army.

[27]   Hughes Tool Company, "Price List," Houston, Texas, February 1967.

[28]   Hughes Tool Company, Bid Records for Intermediate Hole
       Diameter Wells Drilled in Texas and Louisiana.

[29]   Johnson, W. H., "Project Plowshare--Opportunity for the
       Drilling Industry," Drilling Magazine, March 1964, pp. 62-65.

[30]   Johnston, J., Statistical Cost Analysis, McGraw Hill,
       New York, 1960.

[31]   Johnston, J., Econometric Methods, McGraw Hill, New York,
       1963.

[32]   McCaslin, John C., "Deep-Hole History made in Rockies" in
       The Oil and Gas Journal, November 20, 1967.

[33]   McCaslin, John C., "Journal's Survey of Active Fields" in
       The Oil and Gas Journal, October 9, 1967.

[34]   Langston, J. W., "A Method of Utilizing Existing Information to
       Optimize Drilling Procedure" in Journal of Petroleum Technology,
       May 1966.

[35]   Lorenzoni, A. B. and Desnoyers, G. R., "A New Look at Cost
       Control for Constructed Construction" in The Oil and Gas Journal,
       October 2, 1967.

[36]   Personal Communication by John Scot, Associate Editor
       of Petroleum Engineer, on 1967 Deep Well Operators.

[37]   Platt, D. H., "Shaft Sinking by Rotary Drilling," Bureau of
       Mines Circular I. C. 7336, September 1945.

[38]   Presley, C. K., "Big Hole Drilling Techniques of the Atomic
       Energy Commission," API Paper 851-37-E, presented at
       Amarillo, March 1963.

[39]   Presley, C. K., "Recent Trends in Big Hole Drilling," AIME
       paper, presented at Moab, May 1964.

[40]   Rogers, Leslie C., "27 Tips from the Rig" in The Oil and Gas
       Journal, October 9, 1967.

[41]   _____, "Better Muds can Cut Costs", The Oil and Gas Journal,
       November 1966.

[42]   _____, "New Bits can Drill More Hole," <u>The Oil and Gas Journal</u>,
November 1967.

[43]   _____, "New Down-Hole Tools," <u>The Oil and Gas Journal</u>,
November 1967.

[44]   _____, "Improve Drilling," <u>The Oil and Gas Journal</u>,
November 1967.

[45]   _____, "Rotary Rig Due for Face-Lifting," <u>The Oil and Gas
Journal</u>, November 1967.

[46]   Wardlaw, H. W. R., "Simplified Analysis Aids in Optimizing
Drilling Factors for Minimum Costs," in <u>Journal of Petroleum
Technology</u>, May 1961.

[47]   Wilks, Samuel S., Mathematical Statistics, John Wiley,
New York, 1962.

[48]   Wilson, G. M., "Standard Rig Used to Drill 46-Inch Holes,"
World Oil, February 1, 1960, pp. 85-88.

[49]   Wirtz, H. A., "Circulation System for Large Diameter Holes,"
Hughes Tool Company Report.

[50]   The National Supply Company, "Casing and Bit Programs for
Various Oil Producing Areas Drilled with Rotary Tools,"
Revised June 1956.

[51]   Samuelson, W. Z., Hair, J. R., Fisher, P. R., "Construction
Techniques and Costs for Underground Emplacement of Nuclear
Explosives," U. S. Army Corps of Engineers--Nuclear Cratering
Group, Fort Worth, Texas, and Livermore, California, April 1966.

[52]   Stigler, George J., Theory of Price, Macmillan, New York, 1954.