## AFOSR F49620-96-1-0287 Stable Feature Classification in the Wavelet Domain: Final Invention Report

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2 March 2000

## 1 Introduction

In the past year the AFOSR has directly supported the research of Professors Wickerhauser and Weiss, as well as the work of temporary visitors, graduate students and postdoctoral researchers. In this third year of the 3-year research plan, there have been no patents granted or applied for.

## 2 Patents and Disclosures

No patentable inventions were produced by the research which was directly supported by this grant. No patent applications were filed as a direct result of the research.

## 3 Availability of Software

Abstracts, preprints and software produced under this contract are available by anonymous file transfer from the following URL:

http://math.wustl.edu/~victor/index.html

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#### STABLE FEATURE CLASSIFICATION IN THE WAVELET DOMAIN AFOSR grant F49620-96-1-0287 Final Report for 1 August 1998 to 31 May 1999

Principal Investigators: Guido L. Weiss, M. Victor Wickerhauser

Department of Mathematics Campus Box 1146 Washington University St. Louis, MO 63130

#### OVERVIEW

This research project studied the mathematics of adapted wavelet transforms, which underlie some highly successful algorithms for feature detection and classification. Feature analysis in the wavelet domain is made difficult by the instability of wavelet coefficients under shifts, rotations, and other geometric transformations. Successful algorithms can be built of tools from classical harmonic analysis such as decreasing rearrangements, local averages, and maximal operators. The research surveyed five feature detection methods, evaluating them empirically on reasonably large data sets to find their weakest points.

In the course of this study, new methods have been found for characterizing and constructing wavelets. Several sets of newly discovered equations characterizing orthonormal wavelet bases can be applied, along with more classical constructions, to create new families of wavelets and similar functions. These new constructions not only provide connectivity, but also allow new dilation structures and translations by lattices other than those already studied.

#### STATUS OF EFFORT

Further progress has been made in characterizing and constructing new classes of wavelets. Progress has also been made in devising and understanding new algorithms based on adapted wavelet analysis.

Weiss' research results belong to three areas:

(1) The characterization of wavelets, scaling functions and filters. Weiss and his collaborators have obtained complete characterizations of all wavelets, all scalar functions and all low pass filters in one dimension. That two simple equations characterize all such wavelets was reported previously, and this is a result obtained with Hernandez and X. Wang. The characterization of all scaling functions is presented in Chapter 7 of the reference [HW]. The characterization of all low pass filters has just been completed [PSW]. This last characterization extends the partial one obtained by A. Cohen and has several applications.

(2) The construction of wavelets. The characterizations just described offer several different methods for the construction of wavelets, scaling functions and filters. In a collaboration with L. Brandolini, G. Garrigos, and Z. Rzeszotnik, Weiss has obtained several classes of wavelets that have not appeared in the literature. In particular, they have discovered several classes on non-MRA wavelets as well as certain low pass filters having interesting features explaining the behaviour of MRA wavelets. In particular they show how very general functions of absolute value not exceeding one on certain intervals completely determine a wavelet. This work has just been written and will be submitted for publication.

#### SUMMARY OF ACCOMPLISHMENTS

Research supported by this grant in its final year has resulted in one encyclopedia chapter, three software applications which have transitioned to commercial use, six articles in scholarly journals, seven academic conference proceedings, seven preprints of work in progress, and dozens of lectures, colloquia and tutorials.

Ten referred publications appeared in 1999 or were accepted for publication.

PERSONNEL SUPPORTED

In 1998-1999, funds from AFOSR grant F49620-96-1-0287 were spent on two computers plus salaries for the following personnel:

Guido L. Weiss, PI M. Victor Wickerhauser, PI, 2 months Maceiej Paluszynski, post-doc, 5 months Hrvoje Sikic, post-doc, 2 months Zioma Rzeszotnik, graduate student, 1 month Wojciech Czaja, graduate student, 1 month

#### TECHNICAL PUBLICATIONS

Books and Book Chapters

[Wi1] M. V. Wickerhauser. Wavelet Analysis. In Henry F.~Schaeffer III, ed., Encyclopedia of Computational Chemistry. John Wiley & Sons, Limited, Chichester, England. September 1998.

Journal Publications

[BPW] E.Berkson, M.Paluszynski, and G.Weiss. Decompositions des Multiplicateurs de Fourier en Ondelettes. C. R. Acad. Sci. Paris, 1999.

[DLLPSW] X. Dai, D. Larson. R. Liang, M. Papadakis, H. Sikic and G. Weiss. Basic Properties of Wavelets. To appear in J. Fourier Anal. Appl.(1999).

[FGWW] M. Frazier, G. Garrigos, K. Wang and G. Weiss. A Characterization of Functions that Generate Wavelet and Related Expansions. J. Fourier Anal. Appl. Vol. 3 (1998), pp. 883-906

[GHHLWW] G. Garrigos, E. Hernandez, Y-S Han, J. Lakey, D. Weiland and G. Weiss. The Calderon Reproducing Formula and Frames With Applications to Singular Integrals, pp. 1-75. To appear in Memoirs of the AMS (1999)

[HYW] E. Hernandez, D. Yang and G. Weiss. The Phi Transform and Wavelet Characterizations of Herz-Type Spaces. Colectanea Matematica Vol.5 (1998) pp. 47--73

[PSW] M. Papadakis, H. Sikic, and G. Weiss. The Characterization of Low-pass Filters, Scaling Sets and Basic Properties of Wavelets and Scaling Functions, To appear in J. Fourier Anal. Appl. (1999)

#### Reviewed Conference Proceedings

[HWW] E.Hernandez, X.Wang, and G.Weiss. Characterization of Wavelets, Scaling Functions and Wavelets Associated with Multiresolution Analyses. Israel Math. Conf. Proc., Workshop on Function Spaces, Interpolation Spaces, and Related Topics. Technion, Haifa, Israel, June, 1995.

[PW] V. Perrier and M. V. Wickerhauser. Multiplication of Short Wavelet Series Using Connection Coefficients. In K-S. Lau, ed., Advances in Wavelets: Proceedings of the Workshop on Wavelets and Their Applications Inst. Math. Sci., Chinese University of Hong Kong. Springer-Verlag. (1998)

[We] G. Weiss. The Characterization of Wavelets and Related Functions. To appear in the proceedings Functional Analysis V, Dubrovnik. (1999)

[Wi2] M. V. Wickerhauser. A Primer on Wavelet Theory and Its Applications. In Bonami et al., Proceedings of IWC-Tangier 98, International Wavelets Conference 'Wavelets and Multiscale Methods', pages 53--66. Tangier, Morocco, 13--17 April 1998. INRIA, Rocquencourt, France.

[Wi3] M. V. Wickerhauser. Designing a Custom Wavelet Packet Image Compression Scheme, with Applications to Fingerprints and Seismic Data. In M. Holschneider and G. Saracco, eds. Perspectives in Mathematical Physics: Conference in honor of Alex Grossmann, Marseille-Luminy, France, July 1997. CFML, CRC Press. (1998)

[WWB] E. Wesfreid, M. V. Wickerhauser, and R. Bouguerra. Well-Adapted Non-Dyadic Local Spectrum for Some Acoustic Signals. In Bonami et al., eds. Proceedings of IWC-Tangier 98, International Wavelets Conference 'Wavelets and Multiscale Methods', pages 223--225. Tangier, Morocco, 13--17 April 1998. INRIA, Rocquencourt, France.

[WW] E. Wesfreid and M. V. Wickerhauser. Vocal command signal segmentation and phoneme classification. In A. A. Ochoa., ed., Proc. II Artificial Intelligence Symp. at CIMAF 99. Institute of Cybernetics, Mathematics and Physics (ICIMAF), Habana, Cuba, 1999.

#### Preprints

[BGRW] L.Brandolini, G.Garrigos, Z.Rzeszotnik, and G.Weiss. A detailed description of all wavelets having band within [-4 pi,4 pi]. Preprint, Washington University, St. Louis, 1998. To appear.

[C] W.Czaja. Characterization of Gabor Systems. Preprint, Washington University, St. Louis, 1998.

[CR]W.Czaja and Z.Rzeszotnik. Spectral Asymptotics of Pseudodifferential Operators. Preprint, Washington University, St. Louis, 1998.

[PSWX] M.Paluszynski, H.Sikic, G.Weiss, and S.Xiao. Generalized Low-Pass Filters. Preprint, Washington University, St. Louis, 1998. To appear.

[LCKWG] J-P. Leduc, J. Corbett, M. Kong, M. V. Wickerhauser, and B-K. Ghosh. Accelerated Spatio-Temporal Wavelet Transforms: An Iterative Trajectory Estimation. Preprint, Washington University, 1997.

[KLGCW] M. Kong, J-P. Leduc, B-K. Ghosh, J. Corbett, and M. V. Wickerhauser. Wavelet Based Analysis of Rotational Motion in Digital Image Sequences. Preprint, Washington University, 1997.

[SW] H. Sikic and M. V. Wickerhauser. Information Cost Functions. To appear in Appl. Comp. Harmonic Anal., 2000.

### INTERACTIONS AND TRANSITIONS

Conference Presentations

Weiss:

- + Zygmund Lecturer at the University of Chicago.
- + Colloquium speaker at University of Canterbury, New Zealand; University of Auckland, New Zealand; University of Milan; University of Genoa; Politecnico di Torino; University of Rome; University of Orleans, France.
- + Invited conference lectures at the Fourth Australasian Mathematical Conference, Auckland, New Zealand; Regional Conference of the AMS, Philadelphia; Harmonic Analysis Conference (CNR sponsored) in Milano; Wavelet and Computer-Aided Design Conference in Montecatini, Italy; Show-Me Conference at the the University of Missouri, St. Louis; Harmonic Analysis and Function Spaces Conference, Ponza, Italy.

Wickerhauser:

- + Principal speaker at Mathematics Awareness Week, Illinois State University.
- + Colloquium speaker at Illinois State University and the University of Maryland.
- + Seminar speaker at the Technical University of Berlin and Aarhus University, Denmark; National Taiwan Normal University, Taipei;
- Invited conference lecturer at University of Wisconsin at Milwaukee (AMS Meeting 927); International Wavelet Conference, Tangier, Morocco; NSF-CBMS Wavelet Conference, University of Central Florida, Orlando; and the Danish Symposium on Partial Differential Equations, Odense University; Functional Analysis V, Inter-University Center, Dubrovnik; Lockheed Martin Corporation, Orlando, Florida; AMS Sectional Meeting 941, University of Illinois at Urbana-Champaign; Functional Analysis VI, Inter-University Center, Dubrovnik; Zhongshan University, Guangzhou, P.R. China.
- + Tutorial presentation at WIS Technologies, Inc., San Jose, California; National Taiwan Normal University, Taipei;

Transitions

(1) The implementation of the WSQ fingerprint image compression algorithm standard based on Adapted Wavelet Analysis Library 3.0, which is maintained and updated by Wickerhauser, was licensed to Pegasus Imaging Corporation in June, 1998. Contact information:

> Mr Jack Berlin, President Pegasus Imaging Corporation 4010 Boy Scout Blvd., Suite 400 Tampa, FL 33607 USA email: jberlin@jpg.com tel: +1(813)875-7575 ext. 303

(2) A data compression algorithm developed and implemented by Wickerhauser for both raw and reduced seismic data was licensed by a new company, GeoEnergy Inc., started in January, 1998 by a former researcher for Amoco Corporation. Contact information:

> Dr Anthony A. Vassiliou GeoEnergy, Inc. 2530 E. 71st Street, Suite E Tulsa, Oklahoma 74136 USA email: anthony@intcon.net tel: +1(918)494-4641

(3) Image compression algorithms developed for still photographs was improved using wavelet packet methods developed by Wickerhauser and is being distributed by a new company, WIS Technologies, Inc. Contact information:

Dr Jack He Ouyang WIS Technologies, Inc. 2033 Concourse Drive San Jose, CA 95131 email: activeo@yahoo.com tel: +1(408)435-5580