New Robust Estimation/Detection Techniques with Applications to Wireless CDMA Communications

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Final Report
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I. Objectives

Current research into the development of receivers for wireless Code-Division Multiple-Access signals had focused on systems where perfect knowledge of communication parameters exists prior to the research funded by the grant in question. It is clear that such a scenario is unlikely as most parameters will be transmitted over the noisy wireless channel or will be estimated. While there had been work evaluating the impact of imperfect knowledge of the necessary parameters on previously derived receivers, there was yet to be research performed on the development of receivers that are insensitive to parameter estimate errors.

The primary aim of the research was to investigate methods of introducing robustness into Direct-Sequence Code-Division Multiple-Access (DS-CDMA) receivers that exploit both some preliminary knowledge of parameter errors as well as the multiple access interference inherent to such multiuser systems. In addition, preliminary research on the novel area of space-time block coding for DS-CDMA systems was also conducted. Transmit diversity (coding across multiple transmit antennae) is another method of increasing robustness and thus improving performance in DS-CDMA wireless channels. To date, there appears to be no work in this area that is specifically focused on designing space-time block codes for DS-CDMA. There has been work devoted to applying previously proposed **narrowband** codes to the **wideband** problem. By exploiting properties of the DS-CDMA signals, we have been able to improve performance.

II. Accomplishments

A. Robust MMSE Receivers

We have developed an improved minimum-mean squared-error (MMSE) based CDMA receiver which relies on parameter estimation techniques and a resulting probability density function for the estimated parameters. We have shown that the previously proposed MMSE (assuming perfect parameter knowledge) is most sensitive to imperfections in timing information, thus our focus has been on delay-error-insensitive receivers. The delay error has been modeled and using a Bayesian framework, the IMMSE receiver developed. This receiver has been compared and contrasted to the conventional MMSE receiver. The probability of error as well as asymptotic performance measures have been computed.

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13. ABSTRACT *(Maximum 200 words)*

Current research into the development of receivers for wireless Code-Division Multiple-Access signals has focused on systems where perfect knowledge of communication parameters exists. It is clear that such a scenario is unlikely as most parameters will be transmitted over the noisy wireless channel or will be estimated. Past work on a new CDMA receiver based on tile minimum-mean squared-error (MMSE) criterion using a Bayesian framework has been furthered. In the prior work it was established that the conventional MMSE receiver is most sensitive to mismatch in time delay information. The improved MMSE (IMMSE) receiver requires statistical information about the time delay estimators employed. In addition an analysis of a promising time-delay estimator based on the MUSIC algorithm was conducted. The new analysis is alternative and further to previous analyses. III particular, bias estimates are provided and better approximations on the variance of the estimators are developed. Recently the research has focused on the creation of timing error insensitive receivers which approximate maximum-likelihood sequence detectors (MLSD). Thus near-optimal receivers can be achieved for mismatched asynchronous channels. Current effort is directed towards a theoretical performance analysis of the modified MLSDs and towards the consideration of fading and multipath channels. Finally, space-time block codes to achieve transmit diversity specifically for DS-CDMA systems are obtained and analyzed.

14. SUBJECT TERMS
Code-Division Multiple-Access, minimum-mean squared-error criterion, MUSIC algorithm, asymptotic analysis, mismatched channels, delay errors, maximum-likelihood sequence detectors, pair-wise error probability analysis, transmit diversity, fading channels

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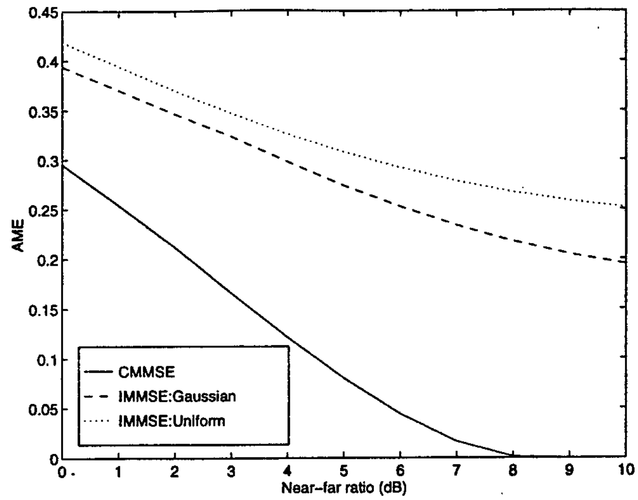

Figure 1: AME of CMMSE and IMMSE receivers with Gaussian and uniform $p(\nu)$ in various near-far situation (N=31; 6 users; $\sigma_\nu = \nu = 0.2T_c$; $SNR_1 = \frac{A_1^2}{\sigma_n^2} = 13dB$). The IMMSE receiver dominates in large near-far situation.

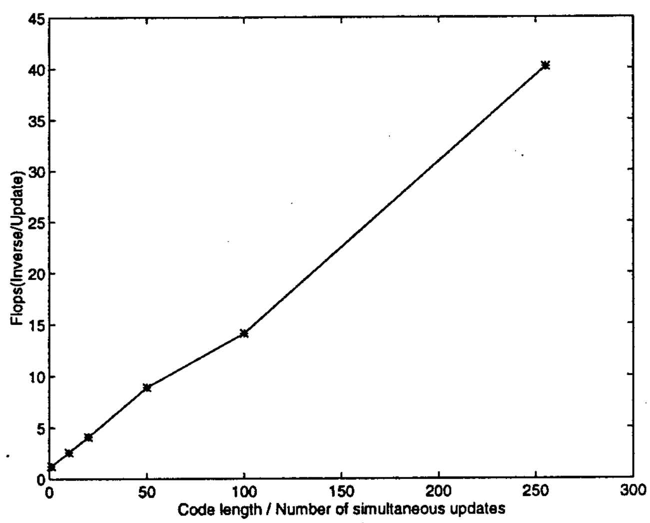

Figure 2: Cost ratio of direct inverse over the proposed mechanism for various ratio of code length over simultaneous updates.

The multi-user performance measure of interest in this study was the asymptotic multi-user efficiency (AME) The AME describes the performance degradation of a receiver due to the multiple-access interference as the additive channel noise diminishes for a fixed set of received powers for all users. Based on the AME definition in, Figure 1 shows the performance of the conventional MMSE (CMMSE) and IMMSE receivers with both Gaussian and uniform distributions for ν. The advantage of the IMMSE receivers for large near-far ratios is well demonstrated.

A method for dynamically updating the IMMSE receiver as channel statistics vary has been proposed. Figure 2 shows the computational cost ratio as a function of the code length N over the number of simultaneous updates. For example, when a system with N=255 has a new user coming in, the proposed update scheme requires 40 times less computation than direct inversion. The proposed mechanism has also been extended to accommodate a delay estimator with varying statistics.

B. Time Delay Estimator Analysis

Motivated by prior work which developed an improved minimum-mean squared-error (MMSE) based CDMA receiver which is less sensitive to errors in timing information than the conventional MMSE receiver, the analysis of timing estimators for CDMA channels has been considered. In particular, the probability density functions of the modified MUSIC delay estimator has been performed. An analysis that is alternative and further to that previously performed has been conducted. In particular, the bias of the modified MUSIC estimator for the CDMA channel has been evaluated (this quantity had not been previously studied) and improved estimates of the variance of the modified MUSIC estimator were also developed.

In general, the exact properties of the time delay estimates with finite sample size or low signal-to-noise ratio (SNR) are intractable. In [6], Parkvall provided an asymptotic performance analysis of the modified MUSIC estimator [5]. In the current work, an alternative perturbation analysis of the second order statistics was pursued which also led to a first order approximation. These new results are provided in our work. It is noted that the analysis in [6] did not consider the estimation of the bias.

Figure 3 shows the analytical and simulated results of the MUSIC estimator with 10 users. The interfering users are 10 dB stronger than the interested user. The signal-to-noise ratio (SNR), defined as symbol energy over the two-sided power spectral density of noise $N_0/2$, is 15 dB. The simulation results are drawn from 4000 Monte-Carlo runs. The symbols of each user are spread by Gold codes of length N=31. The delay set is (25.40, 22.90, 26.95, 9.30, 8.28, 16.21, 19.67, 22.48, 29.05, 11.31) chips. It is clear that the new approximations are quite close to the simulated results and furthermore are closer to the simulation data than the approximations of [6]. Figure 3 (c)-(d) shows the same scheme as in Figure 3 (a)-(b) with the energy of each interfering user 20 dB stronger than the interested user. The new approximations are perceivably better than that of [6] in all situations. It is observed that both the estimator performance and its approximation are fairly insensitive to the energy variation. However, they are sensitive to the change of code sequences and delays. The operation of the IMMSE receiver has been studied using a modified MUSIC delay estimator as a front end. The results have been very promising and further analysis is being undertaken in order to quantify the performance gains of this integrated system over other systems.

C. Robust Sequence Detectors

The initial study on MMSE receivers enabled the consideration of optimal multi-user detector for asynchronous channels. That is, a maximum-likelihood sequence detector (MLSD) have been modified to reduce sensitivity to mismatch in timing errors.

A modified MLSD has been designed and analyzed. Furthermore, a recursive implementation allowing for use in practical systems has also been derived. Performance of the original MSLD and robust MLSD algorithms has been evaluated via simulation and the determination of upper and lower bounds on the bit error rate. This analysis has occurred for both additive white Gaussian

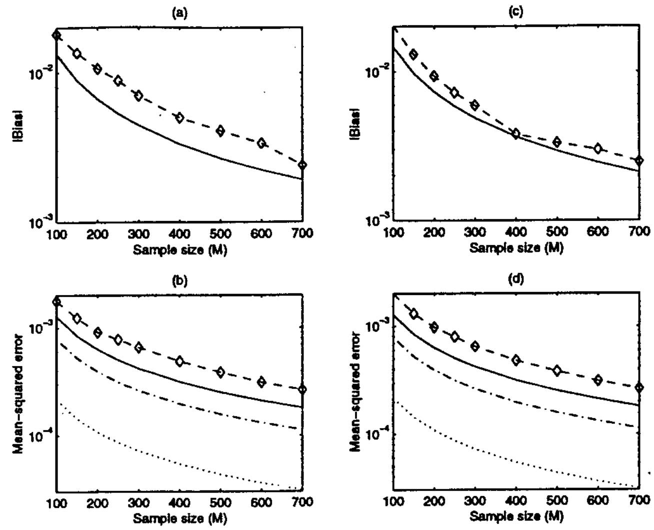

Figure 3: Approximation and simulation of estimator performance with 10 users: The interfering users are 10 dB stronger than the interested user in Figures (a)-(b) and 20dB in Figures (c)-(d). Dashed line with ◇ is simulation; solid line is the new approximation; dashed-dotted line and dotted line are the approximation of mean-squared error and asymptotic Cramèr-Rao bound by Parkvall[], respectively.

noise channels as well as flat fading channels. In the flat fading channel we see a degradation in performance; this was to be expected as there is no form of diversity in the system. However, we anticipate that in multipath (frequency-selective) channels or in systems with multiple sensors, performance would improve significantly for both the original MLSD algorithm as well as the robustified MLSD algorithm.

Efforts towards reducing the impact have been previously considered in [1, 2, 3, 4]. Most, if not all, of the previous works considered single or multi-shot type receivers where edge effects, defined as a partial observation of the data at the edge of the observation window, contribute to the degradation of detection performance. In pursuit of near-optimum performance with complexity linear in the sequence length, we propose a modified maximum-likelihood sequence detector (MLSD) which combats the effects of imperfect synchronization.

The performance of the O-MLSD, M-MLSD and RM-MLSD is presented by simulated BER and analytical bounds versus near-far ratio (NFR), SNR, and fixed realization of the delay errors for two active users. The m-sequence of length 15 with two different phases is applied for the chip sequence. The number of sample for each user in each chip interval is 3, *i.e.*, $q_{\rm ch} = 3$. The decision depth of the simulated MLSDs is five times the channel memory. In each figure, the solid lines are simulation results for the O-MLSD and the RM-MLSD. Their corresponding bounds are presented by the dash-dotted lines and the dotted lines, respectively. To quantify the performance loss by the recursive implementation of the modified ML criterion, newly derived bounds for the optimal ML criterion are presented by dashed lines.

Three receivers were studied: the original MLSD algorithm which ignores parameter mismatch (O-MLSD); a modified MLSD algorithm (M-MLSD) which has high complexity, but incorporates the effects of the mismatched channel; and the RM-MSLD (recursive M-MLSD) which incurs further

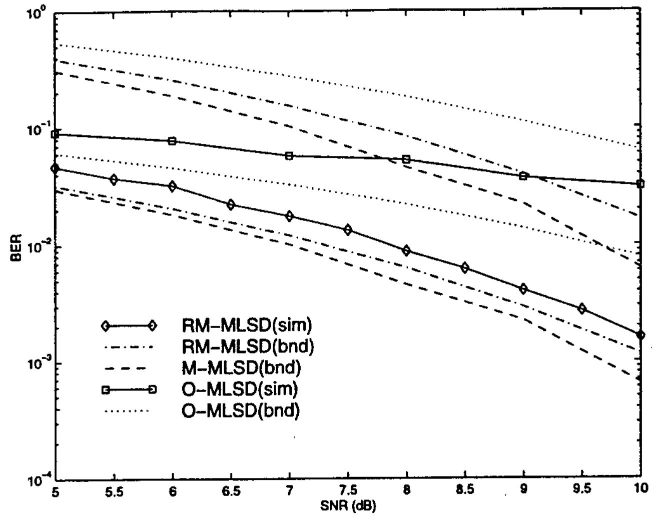

Figure 4: BER and its bounds vs. SNR for the O-MLSD and the (R)M-MLSD. The parameters are: 2 active users; N=15; NFR=20 dB; $\sigma_{\Delta\tau} = 0.6T_c$; $|\Delta\tau| = 0.2T_c$; decision depth=10; and $q_{\text{ch}} = 3$.

approximation to provide a recursive implementation. In Figure 4, the BER is demonstrated as a function of SNR with NFR=20 dB and fixed delay errors of $0.2T_c$. The loss due to recursive implementation is limited, especially in the low SNR regime where the receiver is mostly challenged.

D. *Space-Time Block Coding for DS-CDMA*

Transmit diversity methods have proven effective for combating fading in wireless communication systems [7, 9, 11, 10]. In recent work, the performance criteria and design of space-time block codes have been addressed for single-user systems. There has been some preliminary work to generalize the idea of space-time coding to DS-CDMA systems [9, 8]. These preliminary investigations have typically applied narrowband space–time coding ideas to CDMA systems and do explicitly take advantage of the properties of DS-CDMA.

We have developed a general model for space-time modulation in CDMA systems. From the Chernoff bound of the probability of decoding error, the performance criteria are derived and the design of *optimal* space-time block codes (STBC) is discussed. The presence of spreading codes in the CDMA problem yields interesting differences in code designs and code metrics relative to the single-user narrowband case [7, 10]. Hence the approach for CDMA code design is quite different from the narrowband case.

Optimal codes and equivalent classes are determined. Guidelines for optimal code design are discussed. We have shown that the code design is completely decoupled from user to user, and that full diversity gain is easily achieved and coding gain is a monotonically decreasing function of spreading code correlation $|\rho|$. Optimal codes are presented for some simple cases, it is found that non-unitary codes usually outperform unitary codes in DS-CDMA systems. A systematic way to generate optimal codes is still an open question. Performance analysis by means of numerical integration has also been accomplished. By examining the union bound on the probability of error of the ML decoder (see Figure 5), and by simulation, we can see that, Class 1 will be better than Class 2 when $|\rho|$ is less than about 0.6. For CDMA systems, $|\rho|$ is usually designed to be small, so

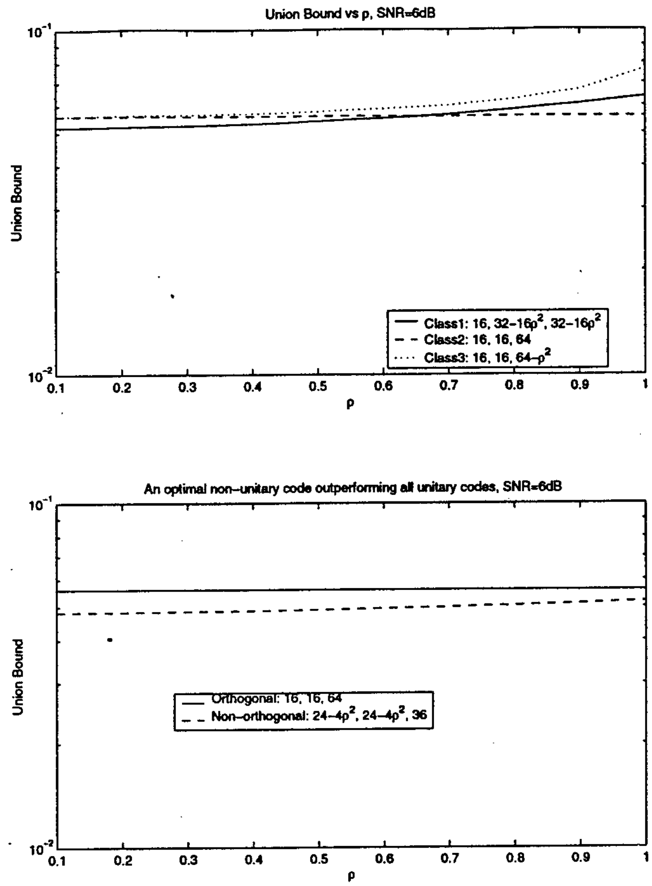

Figure 5: Union bound vs ρ.

we expect that, in general, Class 1 is better for CDMA systems than Class 2.

Figure 6, the numerical integration results are compared with simulation results for BPSK Class 1 codes. The results match very well.

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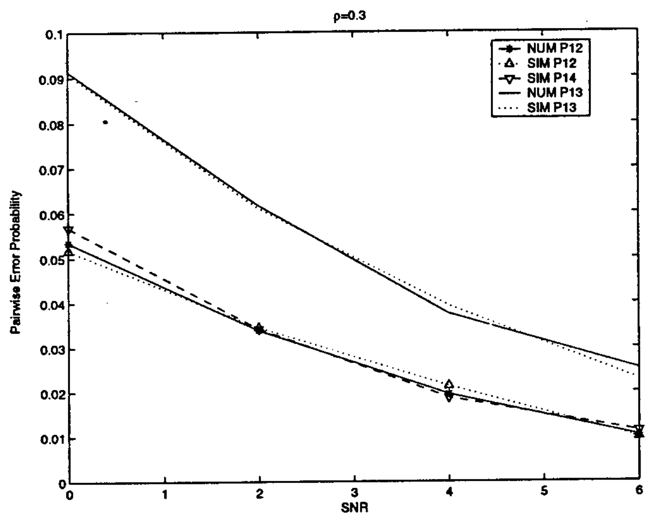

Figure 6: Pairwise probability of error, numerical results vs. simulation resul ts.

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IV. Personnel Supported

1. Urbashi Mitra, PI.
2. Li-Chung Chu, Graduate Student.

V. Technical Publications

A. Journal Publications

J1 L.-C. Chu* and U. Mitra, **Performance Analysis of the Improved MMSE Multi-user Receiver for Mismatched Delay Channels**, *IEEE Transactions on Communications*, vol. 46, no. 10, pp. 1369–1380. October 1998.

J2 L.-C. Chu* and U. Mitra, **Analysis of MUSIC-based Delay Estimators for DS/CDMA Systems**, *IEEE Transactions on Communications*, vol. 47, no. 1, January 1999, pp. 133–138.

J3 L.-C. Chu* and U. Mitra, **Approximate Maximum Likelihood Sequence Detection for DS/CDMA Systems with Tracking Errors**, submitted to the *IEEE Transactions on Communications* May 1999; currently in revision.

B. Reviewed Conference Proceedings

C1 L.-C. Chu* and U. Mitra, **Improved MMSE-based Multi-user Detectors for Mismatched Delay Channels**, *Proceedings of the Thirtieth Annual Conference on Information Sciences and Systems*, Princeton, NJ, March 1996, vol. 1, pp. 326–331.

C2 L.-C. Chu* and U. Mitra, **Performance Analysis of the Improved MMSE Multi-user Receiver for Mismatched Delay Channels**, *Proceedings of the 1997 Conference on Information Sciences and Systems*, Johns Hopkins University, Baltimore MD, March 1997, pp. 474–479.

C3 L.-C. Chu* and U. Mitra, **Further Analysis of MUSIC-based Delay Estimators for DS-CDMA Systems**, *Proceedings of the 1997 Allerton Conference*, Allerton, IL, September 1997, pp. 330 –339.

C4 L.-C. Chu* and U. Mitra, **Improving DS-CDMA Signal Reception in Delay Mismatched Channels**, *Symposium on Interference Rejection and Signal Separation in Wireless Communications*, New Jersey Institute of Technology, Hoboken, NJ, March 1998.

C5 L.-C. Chu* and U. Mitra, **Approximated Maximum Likelihood Detection for DS-CDMA Channels with Tracking Errors**, *Proceedings of the Conference on Information Science and Systems*, Johns Hopkins University, Baltimore MD, March 1999, pp. 638–643. **(invited paper)**.

C6 L.-C. Chu* and U. Mitra, **Trellis-Based Multiuser Detection for DS-CDMA Systems in Mismatched Asynchronous Flat-Fading Channels**, *Proceedings of the IEEE Wireless Communications and Networking Conference*, New Orleans LA, September 1999.

C7 J. Geng*, L.-C. Chu, U. Mitra and M. P. Fitz, **Coding and Decoding Algorithms for CDMA Space-Time Block Codes**, to be presented at *IEEE ISIT 2000*, Sorrento Italy, June 2000.

VI. Interactions/Transitions

A. *Conference Presentations*

Each of the conference articles above involved a public presentation at a conference.

B. *Transitions*

None

VII. Patent Disclosures

None

VIII. Honors

1. Urbashi Mitra: NSF/CISE CAREER Grant, 1996.
2. Urbashi Mitra: Nomination for AFOSR YIP receipt, 1999.
3. Urbashi Mitra: Lumley Research Award, College of Engineering, The Ohio State University, 2000.

CDMA Coding and Decoding Methods for Space–Time Block Codes

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Abstract — **Space-time block code design and decoder design are addressed for Code-Division Multiple-Access (CDMA) systems. Optimal code designs are found by optimizing the Chernoff bound of the probability of decoding error. From this, the diversity gain and the coding gain are determined for the CDMA scenario. The resultant optimal code designs are classified and analyzed. Both optimal and moderate complexity suboptimal decoding algorithms are proposed and evaluated.**

I. Introduction & System Model

Transmit diversity methods have proven effective for combatting fading in wireless communication systems. In this paper, we focus on determining space-time block coding methods [1] for Code-Division Multiple-Access systems. *Due to the assumption of independent fading for different users; the multiuser coding problem decouples to multiple single-user coding problems.* The presence of spreading codes in the CDMA problem yields interesting differences in code designs and code metrics relative to the single-user narrowband case [4].

The up-link between K users and one base station is considered. For each user, k_c information bits are mapped to one of 2^{k_c} Space–Time Block Codes (STBC), D. The codeword D is a matrix of dimension $L_t \times M$; where M is the number of transmit antennae and L_t is the duration, in symbol intervals, of the code. It is assumed that the base station has N receive antennae. We further assume that: the fading processes associated with each transmit antenna are independent; the channel is constant over the duration of the block code (quasi-static) and is known perfectly; and that the transmission is synchronous. For the system under consideration, a different spreading code is employed for each transmit antenna and it is assumed that the receiver has full knowledge of these spreading codes.

II. Performance Criteria & Code Designs

It can be shown that optimizing the upper bound on probability of decoding error yields two criteria for space-time block code design. Performance is determined by a key matrix $\Phi = \Delta D^H \Delta D \odot R$ [1], where ΔD is a codeword difference matrix and R is the spreading code cross-correlation matrix. The resultant design "metrics" are:

diversity gain $\Delta_H = N r_{min}$, where r_{min} is the minimum rank of Φ.

coding gain $\Delta_p = \left(\prod_{i1}^{r_{min}} \lambda_i\right)^N$ is the smallest product of all the non-zero eigenvalues of Φ.

These "metrics" are analogous to those obtained in [1] for the narrowband case, but due to the presence of the cross-correlation matrix R, some new features appear in the resulting optimal codes. The goal of code design is to find 2^{k_c} distinct STBCs such that r_{min} is maximized and given this rank, that Δ_p is also maximized. Codes satisfying these conditions are deemed *optimal*. The following two propositions can be proved regarding diversity gain and coding gain:

PROPOSITION 1 *If ΔD has no zero columns and if R is positive definite, full diversity gain is always achieved.*

PROPOSITION 2 *If $R_{ij} = \rho$ for $i \neq j$, the coding gain is a monotonically decreasing function of ρ.*

Thus for the CDMA case, we focus on maximizing Δ_p. An interesting observation is that *non-unitary code usually outperformes unitary code.* Consider the optimal code sets for BPSK modulation. We discuss the case of $k_c = 2$, $M = 2$ and $L_t = 2$. The resulting optimal codes can be partitioned into three equivalence classes. Each element of the equivalence class can be transformed into another element via simple isometries. Each class is geometrically uniform. Class 1 and 2 are optimal for all $|\rho| \leq 1$ while Class 3 is optimal for $|\rho| \leq \sqrt{3}/2$. Class 2 is essentially Alamouti's orthogonal code set [3] while Class 1 is non-unitary [4]. Interestingly, for QPSK modulation, we can find an optimal non-unitary code set which outperforms all unitary codes for all $|\rho| \leq 1$. The optimal codes are tabulated below.

symbol	Class	D1		D2		D3		D4	
BPSK	1	1	1	1	-1	-1	1	-1	-1
		1	1	-1	-1	1	-1	-1	1
BPSK	2	1	1	1	-1	-1	1	-1	-1
		1	-1	-1	-1	1	1	-1	1
BPSK	3	1	1	1	-1	-1	1	-1	-1
		1	1	-1	1	1	-1	-1	-1
QPSK		1	1	1	-1	-1	i	-i	-i
		1	i	-1	i	-i	1	i	-i

III. Decoding Algorithms

Three types of decoders are considered: the optimal maxmimum-likelihood (ML) decoder, a joint multiuser minimum mean-squared error decoder and a combined interference cancellation/ML decoder. These algorithms perform as predicted with the ML decoder offering the best performance at the expense of computational complexity. The two suboptimal algorithms offer solid performance with reduced complexity.

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[1] Here $\odot$ represents Schur product.

Analysis of MUSIC-Based Delay Estimators for Direct-Sequence Code-Division Multiple-Access Systems

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***Abstract*— Most receiver designs for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems exploit timing information to simultaneously detect the desired signal and suppress the interference. In this paper, a previously proposed MUSIC delay estimation algorithm is considered which requires no initial information and no exhaustive search of the delays. Based on a Taylor series expansion, approximations of the first and second moments of the delay estimation error are derived for this MUSIC algorithm. The analysis is alternative and further to that previously performed.**

***Index Terms*— Channel estimation, code-division multiple-access, multiaccess communication.**

I. Introduction

A significant number of the previously proposed receiver designs for multiuser receivers in direct-sequence code-division multiple-access (DS-CDMA) environments assumed that any necessary communication parameters were perfectly known (see, e.g., [1] and [2]). This parameter knowledge was exploited to successfully suppress multiple access interference (MAI) as well as detect the signal of the user of interest. Of the various relevant parameter estimation problems in a multiuser context, acquisition of the signals is one of the most challenging. Thus, there has been considerable effort devoted to the development of delay estimators for DS-CDMA systems. The conventional sliding correlator suffers in a near–far scenario when the signal power of the interfering users overwhelms that of the desired user [3]. Other parameter estimation algorithms either assume initial information [4] or require long observation periods [5]. Our analysis is focused on the modified MUSIC delay estimator proposed in [3], which requires no side information about the delays. This estimator is of particular interest due to its good performance with a moderate number of observations. In this paper, the study of the modified MUSIC delay estimator is motivated by the need for estimator statistics to drive the construction of the delay-error insensitive minimum-mean squared-error (MMSE) based receivers proposed in [6]. To properly construct the receivers in [6], both bias and variance information about the delay estimators are necessary.

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In general, the exact properties of the estimates with finite sample size or low signal-to-noise ratio (SNR) are intractable. In [7], Parkvall provided an asymptotic performance analysis of the modified MUSIC estimator. In this paper, we pursue an alternative perturbation analysis of the second-order statistics, which also leads to a first-order approximation. The analysis in [7] does not provide an approximation for the bias of the estimator, whereas the approach considered herein lends itself to the approximation of the bias.

This paper is organized as follows. In Section II, the system model and assumptions are provided. In addition, the modified MUSIC algorithm is reviewed. In Section III, the new performance analysis of the modified MUSIC algorithm is provided. The bias and mean-squared error estimates are compared to previous estimates and simulation results in Section IV. Final conclusions are found in Section V.

II. System Model and Modified MUSIC Algorithm

The system model and the MUSIC algorithm summarized in this section are adopted from [3]. In a K-user asynchronous DS-CDMA system, each user employs a spreading waveform $b_k(t)$ with unit energy to form the baseband signal. These spreading waveforms are formed by modulating rectangular chip waveforms with binary pseudonoise sequences [8]. The total received signal is the sum of the signals from all active users plus white Gaussian noise. The baseband signal can be represented as

$$r(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} \sqrt{P_k} \cos\theta_k d_k(m) b_k(t - mT - \tau_k) + n(t) \tag{1}$$

where $\sqrt{P_k}$ is the amplitude, θ_k is the phase, τ_k is the time delay, and the data $d_k(m) \in \{+1, -1\}$, for BPSK modulation. Without loss of generality, it is assumed $\tau_k \geq 0$. We assume that the data $d_k(m)$ are identically distributed and independent for all k (users) and m (time). Furthermore, the data and the noise processes are also independent. The received signal is matched filtered with the chip pulse shape without any timing alignment with the delay τ_k. The matched filter output is sampled at the chip-rate T_c. The samples corresponding to the mth frame of the received signal from the kth user are given by

$$\mathbf{r}_k(m) = [\mathbf{a}_{2k-1}\ \mathbf{a}_{2k}]\begin{bmatrix}\beta_k & 0\\ 0 & \beta_k\end{bmatrix}\begin{bmatrix}z_{2k-1}(m)\\ z_{2k}(m)\end{bmatrix} \tag{2}$$

where the received amplitude is $\beta_k = \sqrt{P_k}\cos\theta_k$. The data frames are of length N, which correspond to the number of chips in the spreading waveform. The vectors $\mathbf{a}_{2k-1}$ and $\mathbf{a}_{2k}$ represent the shifted signal sequences,

$$\mathbf{a}_{2k-1} = \left[\frac{\delta_k}{T_c}D_{+1}^{(p_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right)D_{+1}^{(p_k)}\right]\mathbf{c}_k \tag{3}$$

$$\mathbf{a}_{2k} = \left[\frac{\delta_k}{T_c}D_{-1}^{(p_k+1)} + \left(1 - \frac{\delta_k}{T_c}\right)D_{-1}^{(p_k)}\right]\mathbf{c}_k \tag{4}$$

where p_k and δ_k denote the integer and fractional part of the delay, respectively, i.e., $\tau_k = p_kT_c + \delta_k$, where p_k is a nonnegative integer and $\delta_k \in [0, T_c)$. The length N vector $\mathbf{c}_k$ is the spreading vector of user k. The permutation matrix $\mathbf{D}_s^{(p)}$ is defined in block form as

$$\mathbf{D}_s^{(p)} = \begin{bmatrix}\mathbf{0} & \mathbf{I}_{N-p}\\ s\mathbf{I}_p & \mathbf{0}\end{bmatrix} \tag{5}$$

where $\mathbf{I}_N$ is the identity matrix of dimension N. The vectors $\mathbf{a}_{2k-1}$ and $\mathbf{a}_{2k}$ are modulated by different combinations of two adjacent binary symbols, z_{2k-1} and z_{2k}, respectively,

$$\begin{aligned} z_{2k-1}(m) &= \frac{1}{2}(d_k(m) + d_k(m-1))\\ z_{2k}(m) &= \frac{1}{2}(d_k(m) - d_k(m-1)). \end{aligned} \tag{6}$$

The autocorrelation matrix of the received signal $\mathbf{r}(m)$, where $\mathbf{r}(m) = \sum_k \mathbf{r}_k(m) + \mathbf{n}$, is given by $\mathbf{R} = E\{\mathbf{r}(m)\mathbf{r}^H(m)\}$. The superscript "$H$" denotes the conjugate transpose. For $N > 2K$, the matrix $\mathbf{R}$ can be decomposed into nontrivial signal and noise subspaces. We assume that the signal vectors $\mathbf{a}_i$, $i = 1, \cdots, 2K$, are linearly independent, so the signal subspace has dimension $2K$. In practice, the autocorrelation matrix must be estimated. Given M frames of observation data, an unbiased estimate of the correlation matrix and its eigenvalue decomposition (EVD) are given by

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{r}(m)\mathbf{r}^H(m) = \hat{\mathbf{E}}_s\hat{\mathbf{\Lambda}}_s\hat{\mathbf{E}}_s^H + \hat{\mathbf{E}}_n\hat{\mathbf{\Lambda}}_n\hat{\mathbf{E}}_n^H \tag{7}$$

where the columns of $\hat{\mathbf{E}}_s$ and $\hat{\mathbf{E}}_n$ are the estimated eigenvectors of the signal and noise space, respectively; $\hat{\mathbf{\Lambda}}_s$ and $\hat{\mathbf{\Lambda}}_n$ are diagonal matrices of the corresponding eigenvalues on the diagonal. In a K-user system with $N > 2K$, $\hat{\mathbf{E}}_s$ is $N \times 2K$ and $\hat{\mathbf{E}}_n$ is $N \times (N - 2K)$. Their column vectors are denoted by $\hat{\mathbf{E}}_s = [\hat{\mathbf{e}}_1, \cdots, \hat{\mathbf{e}}_{2K}]$ and $\hat{\mathbf{E}}_n = [\hat{\mathbf{e}}_{2K+1}, \cdots, \hat{\mathbf{e}}_N]$.

In the following, the user of interest is labeled as user 1 and the subscript $k = 1$ is dropped. According to the MUSIC algorithm (e.g., see [9]), the delay estimate is determined such that the projection of the signal vectors onto the noise space is minimized. However, the minimization of the corresponding cost function $J(\tau)$ is problematic due to the nature of the rectangular chip waveforms employed. To be specific, the first derivative of $J(\tau)$ with respect to τ, denoted by $\dot{J}(\tau)$, is not continuous at the chip boundary. A practical approach is to separate the estimation problem into N subestimation problems [3]. In each of the problems, the delay estimate is constrained to be within one chip interval, where $J(\tau)$ is analytic, so the MUSIC algorithm may be applied directly. As asynchronous transmission is considered, the received signal from the first user is a linear combination of two vectors, $\mathbf{a}_1$ and $\mathbf{a}_2$. Hence, the generalized cost function for the pth subestimator is a weighted squared norm of the 2-D manifold defined by Span($\mathbf{a}_1$, $\mathbf{a}_2$) onto the noise space $\hat{\mathbf{E}}_n$ [3]

$$\begin{aligned} J_p(\tau_p) &= \mathrm{tr}\left\{\mathbf{U}_p(\tau_p)\mathbf{A}_p^H(\tau_p)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^H\mathbf{A}_p(\tau_p)\right\}\Big|_{\tau_p\in I_p}\\ &= \mathrm{tr}\{\mathbf{U}_p(\tau_p)\mathbf{A}_p^H(\tau_p)(\mathbf{I} - \hat{\mathbf{E}}_s\hat{\mathbf{E}}_s^H)\mathbf{A}_p(\tau_p)\}\Big|_{\tau_p\in I_p} \end{aligned} \tag{8}$$

where the weighting matrix $\mathbf{U}_p$ is a 2×2 real matrix, and $\mathbf{A}_p = [\mathbf{a}_1\ \mathbf{a}_2]$ is $N \times 2$ with column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ defined in (3) and (4). The set I_p defines the interval $[(p-1)T_c, pT_c]$, and $\mathrm{tr}\{\cdot\}$ denotes the trace operator. The resulting overall estimate is given by

$$\hat{\tau} = \arg\min_{\hat{\tau}_1\cdots\hat{\tau}_N} J_p(\hat{\tau}_p)$$

where

$$\hat{\tau}_p = \arg\min_{\tau_p\in I_p} J_p(\tau_p). \tag{9}$$

That is, the estimate is selected from one of the N subestimates. For $\tau = p_0T_c + \delta$, $\hat{\tau}_{p_0}$ is a consistent estimate of τ [9], thus, $\hat{\tau}$ is also a consistent estimate of τ provided that the minimum of J is unique. However, for a finite number of samples, $\hat{\tau}_p$ is not assured to be the consistent estimate $\hat{\tau}_{p_0}$. We shall refer to the inconsistent estimates $\hat{\tau}_p$ ($p \neq p_0$) as "outliers." That is, these outliers correspond to the incorrect chip interval altogether. Since the statistical properties of the outliers are intractable, the analytical performance provided in Section III excludes the outliers as done in [3] and [7].

III. Performance Analysis of the MUSIC Estimator

In the remainder of this paper, we only consider the properties of the consistent estimate $\hat{\tau}_p$, where $p = p_0$, and drop the subscript p. In this section, we present a performance analysis of the modified MUSIC estimator. Our objective is to develop estimates for the first and second moments of the estimation error. This analysis is alternative and further to that considered in [7]. The approach is similar to [10], which follows a perturbation technique based on a Taylor series expansion.

The Taylor series expansion of $\dot{J}$ about the point $(\tau, \mathbf{E}_s)$ is given by

$$\dot{J}\left(\hat{\tau}, \hat{\mathbf{E}}_s\right) \approx \dot{J}(\tau, \mathbf{E}_s) + \ddot{J}(\tau, \mathbf{E}_s)\Delta\tau + 2\mathrm{Re}\sum_{i=1}^{2K}\mathbf{g}_i^T\Delta\mathbf{e}_i \tag{10}$$

where $\Delta\tau = \hat{\tau} - \tau$, and $\Delta\mathbf{e}_i = \hat{\mathbf{e}}_i - \mathbf{e}_i$, $i = 1, \cdots, 2K$. The vector $\mathbf{g}_i$ is the gradient of $\dot{J}(\tau, \mathbf{E}_s)$ along $\mathbf{e}_i$

$$\begin{aligned} \mathbf{g}_i &\triangleq \nabla_{\mathbf{e}_i}\dot{J}(\tau, \mathbf{E}_s)\\ &= -\left\{\mathbf{a}^*\dot{\mathbf{U}}\mathbf{A}^T + \dot{\mathbf{A}}^*\mathbf{U}\mathbf{A}^T + \mathbf{a}^*\mathbf{U}\dot{\mathbf{A}}^T\right\}\mathbf{e}_i^*. \end{aligned} \tag{11}$$

The details of the derivation and appropriate definitions are provided in the Appendix. The superscripts $*$ and T denote the conjugate and the transpose, respectively. Given the estimated signal space $\hat{\mathbf{E}}_s$, the estimate of τ by the MUSIC algorithm satisfies

$$\hat{\tau} = \arg \min_{\tau \in [(p-1)T_c, pT_c]} J(\tau, \hat{\mathbf{E}}_s). \tag{12}$$

In the following, we assume that due to the definition of $\hat{\tau}$ in (12), it is necessarily a stationary point of $J(\tau, \hat{\mathbf{E}}_s)$. Thus $\dot{J}(\hat{\tau}, \hat{\mathbf{E}}_s) = 0$. Furthermore, $\dot{J}(\tau, \mathbf{E}_s) = 0$ where τ is the true delay value and $\mathbf{E}_s$ is the true signal subspace. Thus, by setting (10) to zero, we obtain

$$\Delta\tau \approx \frac{-2\mathrm{Re}\sum_{i=1}^{2K} \mathbf{g}_i^T \Delta \mathbf{e}_i}{\ddot{J}(\tau, \mathbf{E}_s)}. \tag{13}$$

The second moment of $\Delta\tau$ can then be obtained by standard manipulation of (13)

$$E[\Delta\tau^2] \approx \frac{2}{(\ddot{J}(\tau, \mathbf{E}_s))^2} \mathrm{Re} \sum_{i=1}^{2K} \sum_{j=1}^{2K} \cdot \left\{ \mathbf{g}_i^T E\left[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^T\right] \mathbf{g}_j + \mathbf{g}_i^T E\left[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^H\right] \mathbf{g}_j^* \right\} \tag{14}$$

where $E[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^T]$ and $E[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^H]$ can be approximated by two functions of the eigenvalues (λ_i) and eigenvectors $(\mathbf{e}_i)$ of the true autocorrelation matrix $\mathbf{R}$ [11]

$$E\left[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^H\right] \approx \delta_{ij} \frac{\lambda_i}{M} \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{\lambda_k}{(\lambda_i - \lambda_k)^2} \mathbf{e}_k \mathbf{e}_k^H, \tag{15}$$

$$E\left[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^T\right] \approx (1 - \delta_{ij}) \frac{-\lambda_i \lambda_j}{M(\lambda_i - \lambda_j)^2} \mathbf{e}_j \mathbf{e}_i^T. \tag{16}$$

The Kronecker delta function is denoted by δ_{ij}.

Bias of the *modified* MUSIC delay estimator of [3] has not been previously addressed in the literature; though bias analysis has been considered for the original MUSIC estimator for direction of arrival [10], as it is an important performance characteristic for limited sample size or low SNR. In addition, such information is necessary for the construction of the delay error insensitive estimators of [6]. To fully utilize the statistical properties of the eigenvectors up to the second order, we expand $\dot{J}$ by a second-order Taylor series expansion about the point $(\tau, \mathbf{e}_s)$

$$\begin{aligned} \dot{J}\left(\hat{\tau}, \hat{\mathbf{E}}_s\right) \approx{}& \dot{J}(\tau, \mathbf{E}_s) + \ddot{J}(\tau, \mathbf{E}_s)\Delta\tau + 2\mathrm{Re} \sum_{i=1}^{2K} \mathbf{g}_i^T \Delta \mathbf{e}_i \\ &+ \tfrac{1}{2} \dddot{J}(\tau, \mathbf{E}_s)\Delta\tau^2 + 2\mathrm{Re} \sum_{i=1}^{2K} \left[\mathbf{q}_i^T \Delta \mathbf{e}_i \Delta\tau\right] \\ &+ \mathrm{Re}\,\mathrm{tr}\left[\sum_{i,j=1}^{2K} \mathbf{H}_{i,j} \Delta \mathbf{e}_j \Delta \mathbf{e}_i^H + \breve{\mathbf{H}}_{i,j} \Delta \mathbf{e}_j \Delta \mathbf{e}_i^T\right] \end{aligned} \tag{17}$$

where the vector $\mathbf{q}_i$, calculated in the Appendix, is the gradient of $\ddot{J}$ along $\mathbf{e}_i$

$$\begin{aligned} \mathbf{q}_i &\triangleq \nabla_{\mathbf{e}_i} \ddot{J}(\tau, \mathbf{E}_s) \\ &= -\mathbf{A}^* \ddot{\mathbf{U}} \mathbf{A}^T + 2\left(\dot{\mathbf{A}}^* \dot{\mathbf{U}} \mathbf{A}^T + \mathbf{A}^* \dot{\mathbf{U}} \dot{\mathbf{A}}^T + \dot{\mathbf{A}}^* \mathbf{U} \dot{\mathbf{a}}^T\right) \mathbf{e}_i^*. \end{aligned} \tag{18}$$

The Hessian matrices are derived in the Appendix as well,

$$\begin{aligned} \mathbf{H}_{i,j}(k, l) &= \frac{\partial^2 \dot{J}(\tau, \mathbf{E}_s)}{\partial \mathbf{e}_i^*(k) \partial \mathbf{e}_j(l)} \\ &= \begin{cases} -\left[(\mathbf{A}^* \dot{\mathbf{U}} \mathbf{A}^T + \dot{\mathbf{A}}^* \mathbf{U} \dot{\mathbf{A}}^T + \mathbf{A}^* \mathbf{U} \dot{\mathbf{A}}^T)\right]_{(l,k)}, & i = j \\ \mathbf{0}, & i \neq j \end{cases} \end{aligned} \tag{19}$$

$$\begin{aligned} \breve{\mathbf{H}}_{i,j}(k, l) &= \frac{\partial^2 \dot{J}(\tau, \mathbf{E}_s)}{\partial \mathbf{e}_i(k) \partial \mathbf{e}_j(l)} \\ &= \mathbf{0}. \end{aligned} \tag{20}$$

By setting (17) equal to zero, we obtain the approximation of $E[\Delta\tau]$

$$\begin{aligned} E[\Delta\tau] \approx{}& \frac{-1}{\ddot{J}(\tau, \mathbf{E}_s)} \sum_{i=1}^{2K} \mathrm{Re}\{2\mathbf{g}_i^T E[\Delta \mathbf{e}_i] + 2\mathbf{q}_i^T E[\Delta \mathbf{e}_i \Delta\tau] \\ &+ \mathrm{tr}\{\mathbf{H}_{i,i} E[\Delta \mathbf{e}_i \Delta \mathbf{e}_i^H]\}\} + \tfrac{1}{2} \dddot{J}(\tau, \mathbf{E}_s) E[\Delta\tau^2] \end{aligned} \tag{21}$$

where the bias of the eigenvectors can be approximated by [11]

$$E[\Delta \mathbf{e}_i] \approx \frac{-\lambda_i}{2M} \sum_{\substack{k=1 \\ k \neq i}}^{N} \frac{\lambda_k}{(\lambda_i - \lambda_k)^2} \mathbf{e}_i. \tag{22}$$

The approximation of $E[\Delta \mathbf{e}_i \Delta\tau]$ is given by premultiplying (13) with $\Delta \mathbf{e}_i$ before taking the expectation [10]

$$E[\Delta \mathbf{e}_i \Delta\tau] \approx \frac{-1}{\ddot{J}(\tau, \mathbf{E}_s)} \sum_{j=1}^{2K} \cdot \left\{E[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^H] \mathbf{g}_j^* + E[\Delta \mathbf{e}_i \Delta \mathbf{e}_j^T] \mathbf{g}_j\right\}. \tag{23}$$

Finally, the bias is obtained by substituting (14), (15), (22), and (23) into (21).

IV. NUMERICAL RESULTS

Figs. 1 and 2 show the analytical and simulated results of the MUSIC estimator with five and ten users. The SNR, defined as the symbol energy over the two-sided power spectral density of noise $N_0/2$, is 15 dB. The simulation results are drawn from 4000 Monte Carlo runs. The symbols of each user are spread by Gold codes of length $N = 31$. The first two sequences, $\mathbf{m}_1$ and $\mathbf{m}_2$, of the Gold code family are the primitive m-sequences with octal representation of their generator polynomials being 45, 75 and their loading is 06, 35, respectively. The third to the twenty-ninth sequences are generated by $\mathbf{m}_i = \mathbf{m}_1 + D^{i-3}\mathbf{m}_2$, $i = 3, \cdots, 33$, where $+$ denotes bitwise exclusive-or operation and D^j denotes a phase shift of the m-sequence by j units [8]. The weighting matrix $\mathbf{U}$ defined in (A.4) was employed. Fig. 1(a) and (b) shows the bias, $-E[\Delta\tau]$ (with reverse sign to be shown in log scale), and the mean-squared error (MSE), $E[\Delta\tau^2]$, with Gold code sequences $\mathbf{m}_{20}$ to $\mathbf{m}_{24}$ and delays of (10.67, 2.13, 26.46, 27.00, 15.69, 16.21) chips. Each interfering user is 10 dB stronger than the desired user. The scheme in Fig. 1(c) and (d) is the same except that a different set of spreading codes, $\mathbf{m}_{11}$ to $\mathbf{m}_{15}$, and delays of (25.40, 22.90, 26.95, 9.30,

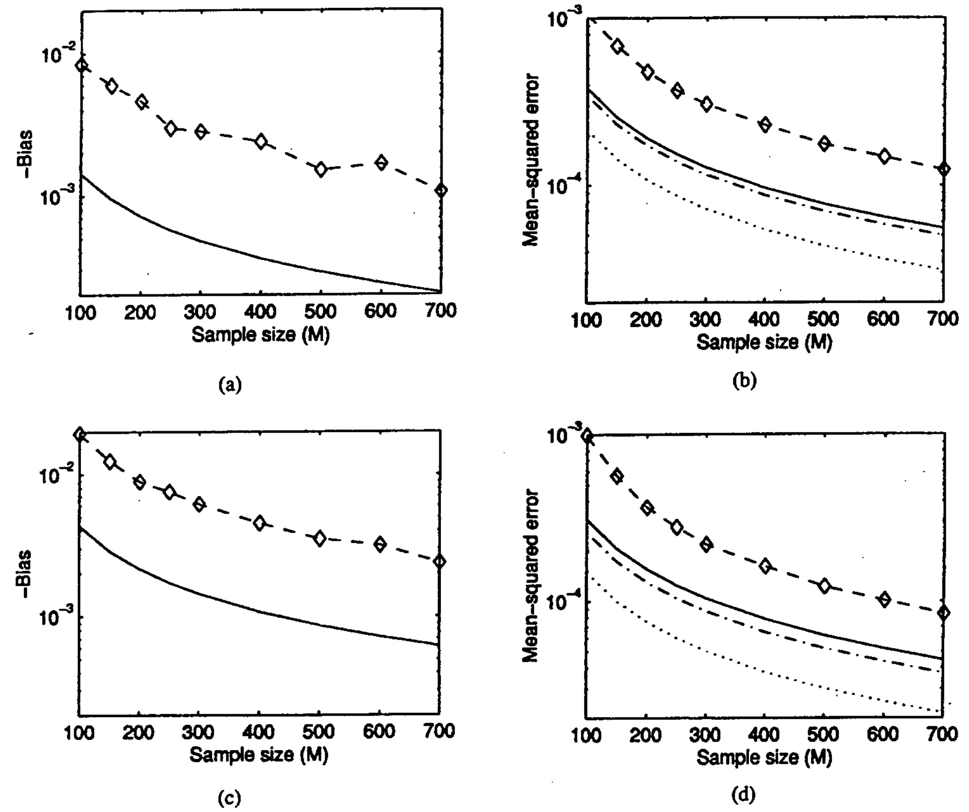

Fig. 1. Approximation and simulation of estimator performance with five users: (a) and (b) are for the first set of code sequences and delays, and (c) and (d) are for the second set of code sequences and delays. Dashed line with ◇ is simulation; solid line is approximation by (14) or (21); dashed-dotted line and dotted line are approximation of mean-squared error and asymptotic Cramèr–Rao bound by Parkvall [7], respectively.

8.28) chips are used. The accuracy of the approximation, which depends on the convergence rate of the Taylor series expansion to the true $J(\tau, \mathbf{E}_s)$, is dependent on the particular realization of spreading codes and delays. This convergence has not been fully studied at this time.

Fig. 2(a) and (b) shows the same scheme as in Fig. 1(a) and (b) with five additional interfering users, each with energy 10 dB stronger than the interested user. They are spread by the sequences $\mathbf{m}_{25}$ to $\mathbf{m}_{29}$ with delays of (16.21, 19.67, 22.48, 29.05, 11.31) chips. Discrepancy between the approximations by (14), (21), and the simulated results appears to be far less than an order of magnitude. Fig. 2(c) and (d) shows the same scheme as in Fig. 2(a) and (b) with the energy of each interfering user 20 dB stronger than the interested user. The approximation by (14), while incidentally achieving better accuracy in the ten-user channel than in the five-user channel, is perceivably better than that of [7] in all four situations. It is observed that both the estimator performance and its approximation are fairly insensitive to the energy variation. However, they are sensitive to the change of code sequences and delays.

V. Conclusions

In this paper, we have provided further analysis of the statistics of the modified MUSIC estimator [3]; in particular, approximations of the first and second moments of the estimation error are developed. The study was motivated by the need for delay estimator statistics to drive the construction of delay-error insensitive MMSE-based multiuser receivers [6]. The results have been compared to a previously derived approximation [7] and simulation data. In addition to the bias approximation, the new approach appears to provide improved accuracy of the approximation of the mean-squared error.

Appendix

A. Derivation of the Gradients $\mathbf{g}_i$, $\mathbf{q}_i$, and the Matrices $\mathbf{H}$, $\breve{\mathbf{H}}$

For simplicity, we drop the subscript p throughout the derivation. While the derivation is valid for all users, the indexes used in the derivation are for user 1 to simplify expressions. The matrix $\mathbf{A}$ is defined as $[\mathbf{a}_1 \ \mathbf{a}_2]$ with $\mathbf{a}_1$ and $\mathbf{a}_2$ given by (3) and (4) when $k = 1$. The first and second partial derivatives of $\mathbf{A}$ with respect to the fractional delay δ are

$$\dot{\mathbf{A}} \triangleq \frac{\partial \mathbf{A}}{\partial \delta} = \frac{1}{T_c}\left[\left(\mathbf{D}_{+1}^{(p_1+1)} - \mathbf{D}_{+1}^{(p_1)}\right)C_1 \quad \left(\mathbf{D}_{-1}^{(p_1+1)} - \mathbf{D}_{-1}^{(p_1)}\right)\right]\mathbf{c}_1 \tag{A1}$$

$$\ddot{\mathbf{A}} \triangleq \frac{\partial \dot{\mathbf{A}}}{\partial \delta} = 0. \tag{A2}$$

With the subscripts of p_1 and $\mathbf{c}_1$ dropped, the squared norm of the code sequences as a function of δ is given by

$$\mathbf{a}_i^H \mathbf{a}_i = \mathbf{c}^H \left[\frac{\delta^2}{T_c^2}(2\mathbf{I} - \overline{\mathbf{D}}_i) + \frac{\delta}{T_c}(\overline{\mathbf{D}}_i - 2\mathbf{I}) + \mathbf{I}\right]\mathbf{c} \tag{A3}$$

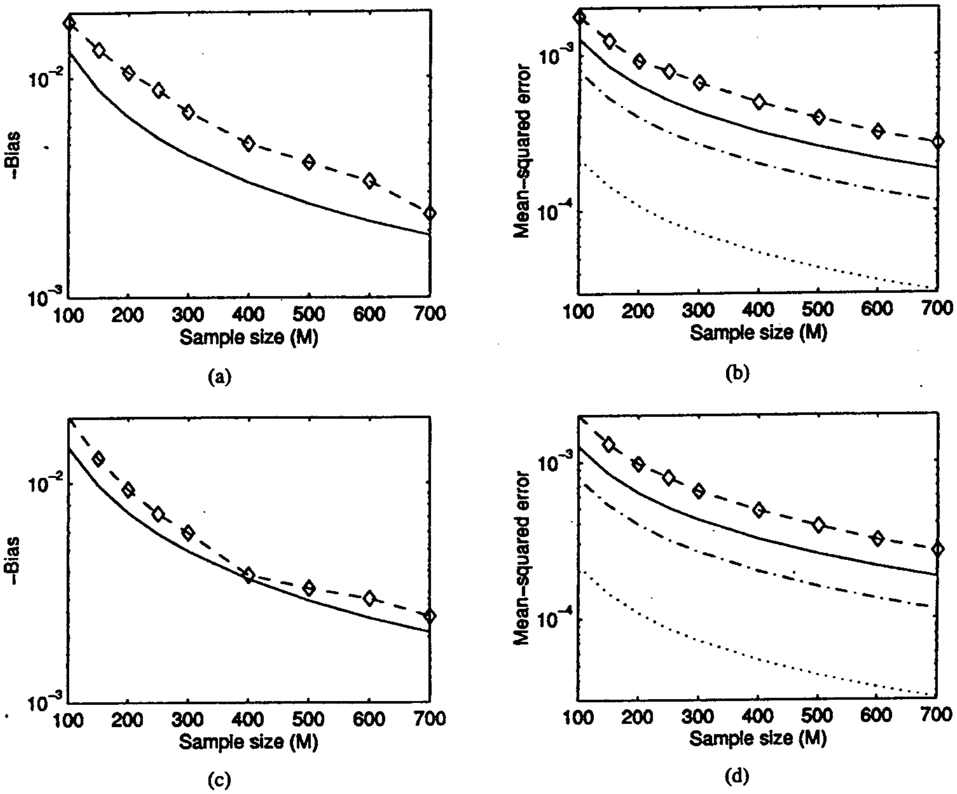

Fig. 2. Approximation and simulation of estimator performance with ten users: the interfering users are 10 dB stronger than the interested user in (a) and (b) and 20 dB in (c) and (d). Dashed line with ◇ is simulation; solid line is approximation by (14) or (21); dashed-dotted line and dotted line are approximation of mean-squared error and asymptotic Cramèr–Rao bound by Parkvall [7], respectively.

for $i = 1$ and 2, where $\mathbf{I}$ is the identity matrix

$$\overline{\mathbf{D}}_1 = \left(\mathbf{D}_{+1}^{(p)}\right)^H \mathbf{D}_{+1}^{(p+1)} + \left(\mathbf{D}_{+1}^{(p+1)}\right)^H \mathbf{D}_{+1}^{(p)}$$

and

$$\overline{\mathbf{D}}_2 = \left(\mathbf{D}_{-1}^{(p)}\right)^H \mathbf{D}_{-1}^{(p+1)} + \left(\mathbf{D}_{-1}^{(p+1)}\right)^H \mathbf{D}_{-1}^{(p)}.$$

In the following discussion, we adopt the weighting matrix in [3]:

$$\mathbf{U} = \begin{bmatrix} \dfrac{1}{\mathbf{a}_1^H \mathbf{a}_1} & 0 \\ 0 & \dfrac{1}{\mathbf{a}_2^H \mathbf{a}_2} \end{bmatrix} \tag{A4}$$

with the subscript p of $\mathbf{U}_p$ dropped. By substituting (A3) into (A4), followed by taking derivatives with respect to δ, we obtain the following quantities:

$$\dot{\mathbf{U}}(i, i) = \frac{\partial}{\partial \delta} \frac{1}{\mathbf{a}_i^H \mathbf{a}_i} = \frac{-\dfrac{\partial}{\delta \partial}(\mathbf{a}_i^H \mathbf{a}_i)}{(\mathbf{a}_i^H \mathbf{a}_i)^2} = \frac{-u_i}{(\mathbf{a}_i^H \mathbf{a}_i)^2} \tag{A5}$$

$$\ddot{\mathbf{U}}(i, i) = \frac{\partial}{\partial \delta} \dot{\mathbf{U}}(i, i) = \frac{2u_i^2 - (\mathbf{a}_i^H \mathbf{a}_i) u_i'}{(\mathbf{a}_i^H \mathbf{a}_i)^3} \tag{A6}$$

$$\dddot{\mathbf{U}}(i, i) = \frac{\partial}{\partial \delta} \ddot{\mathbf{U}}(i, i) = \frac{6u_i[(\mathbf{a}_i^H \mathbf{a}_i) u_1' - u_i^2]}{(\mathbf{a}_i^H \mathbf{a}_i)^4} \tag{A7}$$

for $i = 1$ and 2, where

$$u_i \triangleq \left(\frac{2\delta}{T_c^2} - \frac{1}{T_c}\right) \mathbf{c}^H (2\mathbf{I} - \overline{\mathbf{D}}_i) \mathbf{c} \tag{A8}$$

$$u_i' \triangleq \frac{2}{T_c^2} \mathbf{c}^H (2\mathbf{I} - \overline{\mathbf{D}}_i)\, \mathbf{c}. \tag{A9}$$

The (2, 1) and (1, 2) elements of $\mathbf{U}$ and thus their derivatives are zero.

Next, we will find the derivatives of the cost function $J_p(\tau_p, \mathbf{E}_s)$ with respect to δ. With subscript p dropped, the cost function J is given by $J(\tau, \mathbf{e}_s) = \mathrm{tr}\{\mathbf{U}\mathbf{A}^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{A}\} = \mathrm{tr}\{\mathbf{U}\mathbf{A}^H (\mathbf{I} - \mathbf{E}_s \mathbf{E}_s^H) \mathbf{A}\}$. Employing the derivatives of $\mathbf{A}$ and $\mathbf{U}$ given by (A1), (A2), (A5), and (A6), finding the derivatives of J is straightforward

$$\begin{aligned} \dot{J}(\tau, \mathbf{e}_s) &\triangleq \frac{\partial J(\tau, \mathbf{E}_s)}{\partial \delta} \\ &= \mathrm{tr}\left\{\dot{\mathbf{U}}\mathbf{A}^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{A} + \mathbf{U}(\dot{\mathbf{A}}^H \mathbf{e}_n \mathbf{E}_n^H \mathbf{A} + \mathbf{A}^H \mathbf{E}_n \mathbf{E}_n^H \dot{\mathbf{A}})\right\} \\ &= 0 \end{aligned} \tag{A10}$$

$$\begin{aligned} \ddot{J}(\tau, \mathbf{E}_s) &\triangleq \frac{\partial \dot{J}(\tau, \mathbf{E}_s)}{\partial \delta} \\ &= \mathrm{tr}\Big\{\ddot{\mathbf{U}}\mathbf{A}^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{A} + 2\dot{\mathbf{U}}\Big(\dot{\mathbf{A}}^H \mathbf{E}_n \mathbf{e}_n^H \mathbf{A} + \mathbf{A}^H \mathbf{E}_n \mathbf{E}_n^H \dot{\mathbf{A}}\Big) \\ &\quad + 2\mathbf{U}\dot{\mathbf{A}}^H \mathbf{E}_n \mathbf{e}_n^H \dot{\mathbf{A}}\Big\} \\ &= \mathrm{tr}\Big\{2\mathbf{U}\dot{\mathbf{A}}^H \mathbf{E}_n \mathbf{E}_n^H \dot{\mathbf{A}}\Big\} \end{aligned} \tag{A11}$$

$$
\begin{aligned}
&\dddot{J}(\tau, \mathbf{E}_s) \\
&\triangleq \frac{\partial \ddot{J}(\tau, \mathbf{E}_s)}{\partial \delta} \\
&= \mathrm{tr}\Big\{\dddot{\mathbf{U}}\mathbf{A}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{A} + 3\ddot{\mathbf{U}}\Big(\dot{\mathbf{A}}^H\mathbf{e}_n\mathbf{E}_n^H\mathbf{A} + \mathbf{A}^H\mathbf{E}_n\mathbf{E}_n^H\dot{\mathbf{a}}\Big) \\
&\qquad + 6\dot{\mathbf{U}}\dot{\mathbf{A}}^H\mathbf{e}_n\mathbf{E}_n^H\dot{\mathbf{A}}\Big\} \\
&= \mathrm{tr}\Big\{6\dot{\mathbf{U}}\dot{\mathbf{A}}^H\mathbf{E}_n\mathbf{E}_n^H\dot{\mathbf{A}}\Big\}. \qquad \text{(A12)}
\end{aligned}
$$

The gradient of a complex scalar function $f(\mathbf{v})$ is defined as the directional derivative along the direction pointed by a vector, say $\mathbf{v}_0$. Thus, the gradient $\nabla_{\mathbf{v}_0} f(\mathbf{v})$ is a vector whose ith element is $\partial f/\partial \mathbf{v}_0(i)$. As part of the system of complex vector operations, the convention related to the derivatives of a complex number z, $dz/dz = 1$, $dz/dz^H = dz^H/dz = 0$ [12], is applied. By taking the gradient of $\dot{J}(\tau, \mathbf{E}_s)$ and $\ddot{J}(\tau, \mathbf{E}_s)$ along the signal-space eigenvector $\mathbf{e}_i$, we obtain the expressions for $\mathbf{g}_i$ and $\mathbf{q}_i$

$$
\begin{aligned}
\mathbf{g}_i &\triangleq \nabla_{\mathbf{e}_i} \dot{J}(\tau, \mathbf{E}_s) \\
&= \frac{\partial}{\partial \mathbf{e}_i}\mathrm{tr}\Big\{\dot{\mathbf{U}}\mathbf{A}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{A} + \mathbf{U}\dot{\mathbf{A}}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{A} \\
&\qquad + \mathbf{U}\mathbf{A}^H\mathbf{E}_n\mathbf{E}_n^H\dot{\mathbf{A}}\Big\} \\
&= -\frac{\partial}{\partial \mathbf{E}_i}\mathrm{tr}\Big\{\dot{\mathbf{U}}\mathbf{A}^H\mathbf{E}_s\mathbf{E}_s^H\mathbf{A} + \mathbf{U}\dot{\mathbf{A}}^H\mathbf{E}_s\mathbf{E}_s^H\mathbf{A} \\
&\qquad + \mathbf{U}\mathbf{A}^H\mathbf{E}_s\mathbf{E}_s^H\dot{\mathbf{A}}\Big\} \\
&= -\Big\{\mathbf{A}^*\mathbf{U}\dot{\mathbf{A}}^T + \mathbf{A}^*\mathbf{U}\dot{\mathbf{A}}^T\Big\}\mathbf{e}_i^* \\
&\triangleq \mathbf{\Psi}\mathbf{e}_i^* \qquad \text{(A13)} \\
\mathbf{q}_i &\triangleq \nabla_{\mathbf{e}_i} \ddot{J}(\tau, \mathbf{E}_s) \\
&= -\frac{\partial}{\partial \mathbf{e}_i}\mathrm{tr}\Big\{\ddot{\mathbf{U}}\mathbf{A}^H\mathbf{E}_s\mathbf{E}_s^H\mathbf{A} \\
&\qquad + 2\dot{\mathbf{U}}\Big(\dot{\mathbf{A}}^H\mathbf{E}_s\mathbf{E}_s^H\mathbf{A} + \dot{\mathbf{A}}^H\mathbf{E}_s\mathbf{E}_s^H\dot{\mathbf{A}}\Big) \\
&\qquad + 2\mathbf{U}\dot{\mathbf{A}}^H\mathbf{E}_s\mathbf{E}_s^H\dot{\mathbf{A}}\Big\} \\
&= -\Big\{\mathbf{A}^*\ddot{\mathbf{U}}\mathbf{A}^T + 2\Big(\dot{\mathbf{A}}^*\dot{\mathbf{U}}\mathbf{A}^T + \mathbf{A}^*\dot{\mathbf{U}}\dot{\mathbf{A}}^T \\
&\qquad + \dot{\mathbf{A}}^*\mathbf{U}\dot{\mathbf{A}}^T\Big)\Big\}\mathbf{e}_i^*. \qquad \text{(A14)}
\end{aligned}
$$

Next, we consider the derivatives of the Hessian matrices in (17). The (k, l)th element of $\mathbf{H}_{i,j}$ is defined by $\mathbf{H}_{i,j}(k, l) \triangleq \partial/\partial \mathbf{e}_i^*(k)(\partial \dot{J}/\mathbf{e}_j(l))$ [10], where the inner partial derivative is equal to the lth element of (A13) for $\mathbf{e}_j^*$. Since $\partial \dot{J}/\partial \mathbf{e}_j(l)$ is only a function of $\mathbf{e}_j^*$, $\partial/\partial \mathbf{e}_i^*(k)(\partial \dot{J}/\partial \mathbf{e}_j(l))$ is $\mathbf{\Psi}(l, k)$ for $i = j$ and zero otherwise. For the same reason, $\breve{\mathbf{H}}_{i,j}(k, l) \triangleq \partial/\partial \mathbf{e}_i(k)(\partial/\partial \dot{J}\mathbf{e}_j(l)) = 0$, $\forall k, l$ because the expression inside the parenthesis is a function of $\mathbf{e}_j^*$ only. Therefore, $\mathbf{H}$ and $\breve{\mathbf{H}}$ are given by (19) and (20).

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Trellis-Based Multiuser Detection for DS-CDMA Systems in Mismatched Asynchronous Flat-Fading Channels †

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Abstract- **The problem of multi-user detection in Direct-Sequence Code-Division Multiple-Access systems with channel state mismatch is addressed. An approximate maximum-likelihood criterion is derived based on knowledge of the statistical properties of the residual channel estimation errors. The time of arrival is assumed to be a constant during the time frame of interest. Errors in both timing information and fading coefficient information are assumed. A Viterbi-like recursive demodulation scheme is derived and analyzed. Performance of the new algorithm is studied via simulation and the development of performance bounds.**

I. INTRODUCTION

Multi-user detection for uncertain environments has evolved in two directions. The first approach is based on joint data detection and parameter estimation, which is computationally challenging especially when *nonlinear* parameters, such as time delays are to be estimated [1]. When bandwidth efficient chip pulses are employed, the sampled and filtered observation signals are non-linear functions of the relative multi-user time delays. Thus we term such parameters as *nonlinear* parameters. Another approach is to perform blind or decision-directed parameter estimation followed by detection algorithms which are optimized under the assumption of perfect parameter estimates. In practice, especially in dynamic channels which are typically experienced in mobile communication systems, perfect estimation can hardly be achieved. Mismatch in parameter estimates might greatly impact detector performance [2, 3]. In this paper, we pursue modification of the second approach to reduce detector sensitivity to parameter mismatch.

Due to the nonlinear nature of timing in detector formulation, isolation of the timing estimation is desired to reduce the computational effort. Without any presumptions based on the tracking mechanism, we model the residual errors in the timing estimates as independent Gaussian random variables. The flat fading process is modeled as a second-order autoregressive (AR) process. Efforts towards reducing the impact of imperfect parameter estimation have been previously considered in

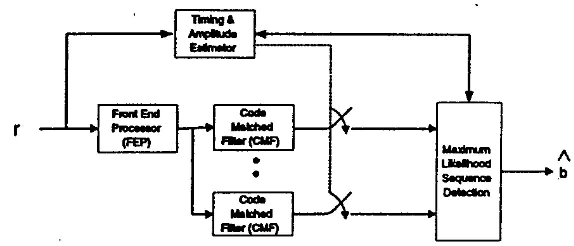

Figure 1: Receiver diagram

[3, 4, 5, 6]. Most of the previous works for asynchronous channels considered single or multi-shot type receivers where edge effects, defined as a partial observation of the data at the edge of the observation window, contribute to the degradation of detection performance. For additive white Gaussian noise (AWGN) channels, we proposed a trellis-based sequence detector based on an approximate maximum-likelihood (AML) metric, which incorporates statistics of residual timing errors [7]. In this paper, we consider extension of these ideas to flat-fading channels.

This paper is organized as follows. The system model is discussed in Section II. In Section III, the receiver structure is presented. Bounds on performance are developed in Section IV and numerical results are provided in Section V. Final conclusions are drawn in Section VI.

II. SYSTEM MODEL

Processing of the continuous-time baseband signal is illustrated in Figure 1. We shall assume coherent reception of the signal, which is transmitted through channels without resolvable multipath [1]. The received signal is given by

$$r(t) = \sum_{k=1}^{K} \sum_{m=-M}^{M} a_k(m) b_k(m) s_k(t - mT - \tau_k) + n(t), \quad (1)$$

where $a_k(m)$ is the amplitude, $b_k(m)$ is the data with differential BPSK modulation, τ_k is the delay, and $n(t)$ is an

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[1]While multipath channel models are more appropriate for wideband signaling such as DS-CDMA, we shall study flat fading channels as a necessary precursor to the study of multipath channels.

AWGN random process. Both $n(t)$ and $a_k(m)$ are complex Gaussian with values varying in time. The spreading waveform $s_k(t)$ is given by

$$s_k(t) = \sum_{j=0}^{N-1} \beta_k(j)\psi(t - jT_c), \qquad (2)$$

where a chip-pulse waveform, $\psi(t)$, is modulated by the binary spreading sequence $\beta_k(j)$ The square-root raised cosine pulse shape is employed for $\psi(t)$. The front-end processor (FEP) at the receiving end is a linear filter with impulse response $\psi^*(-t)$. For the convenience of analysis, we will approximate the fading process by a second-order AR process [8],

$$a_k(t) = \rho_{k1}a_k(t-1) + \rho_{k2}a_k(t-2) + w(t), \qquad (3)$$

where $a_k(t)$ is the kth user's fading coefficient at time t, ρ's are the parameters in the AR process, and $w(t)$ is the Gaussian process. The coefficients in (3) are given by

$$\rho_i = R_a(0)J_0(2\pi(i-1)vT/cf_c), \qquad (4)$$

where $R_a(0)$ is the average power of the process, $J_0(\bullet)$ is the zero-order Bessel function, v is the vehicle speed, T is the symbol period, c is the speed of light, and f_c is the center frequency. We assume that the fading is slow enough so that $a_k(m)$ remains a constant within one symbol period.

The vectors of delay and amplitude are written as the sum of their estimates and residual errors, *i.e.*,

$$\begin{aligned} \tau &= \hat{\tau} + \delta_\tau \\ \mathbf{a} &= \hat{\mathbf{a}} + \delta_a, \end{aligned} \qquad (5)$$

where δ_τ and δ_a are zero-mean Gaussian vectors with independent elements, *i.e.*, $\delta_\tau \sim \mathcal{N}(\mathbf{0}, \Sigma_\tau)$, $\delta_a \sim \mathcal{N}(\mathbf{0}, \Sigma_a)$ with diagonal covariance matrices Σ_τ and Σ_a. This scheme corresponds to adopting the maximum average uncertainty principle subject to fixed variance for the residual errors [9].

For bandwidth limited signals, Nyquist sampling provides sufficient statistics for data detection regardless of the time of arrival. By the same token, Nyquist sampling takes no advantage of timing information when it is available. If the timing is perfectly known, symbol-rate sampling is sufficient for data detection. In this paper, we assume knowledge of the timing estimates prior to the detection procedure. It is further assumed that these timing estimates contain residual errors. As the residual errors are small, it is beneficial to slightly modify the scheme of symbol-rate sampling. The proposed sampling scheme is shown in Figure 2. The output of each code matched filter (CMF) is sampled non-uniformly at a rate of q_{sym} times per symbol, or equivalently, q_{sym} times per chip at the FEP output for each user [7]. The q_{sym} samples for each symbol is T_c/q_{ov} apart, where q_{ov} is denoted the over-sampling factor with $q_{ov} \geq q_{sym}$. The sampled signal for a channel with perfect timing and amplitude information may be expressed as

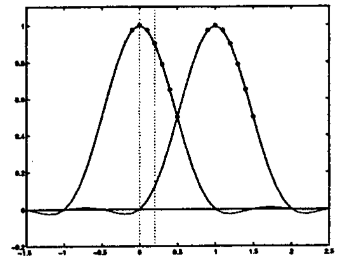

Figure 2: Sampling instants at the output of FEP is marked by the asterisk. ($q_{\text{ov}} = 10$; $q_{\text{ch}} = 7$).

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n}, \qquad (6)$$

where $\mathbf{y}$ is the vector of CMF outputs; $\mathbf{R}$ is the cross-correlation matrix, $\mathbf{A}$ is the diagonal amplitude matrix, $\mathbf{n}$ is noise vector with zero mean and covariance matrix Σ_n, denoted by $\mathcal{N}(\mathbf{0}, \Sigma_n)$; and $\mathbf{b}$ is the data vector (*e.g.*[10]). The received signal is arranged in the order,

$$\mathbf{y} = [\mathbf{y}_1^T(-M)\ldots\mathbf{y}_K^T(-M)\ldots\mathbf{y}_1^T(M)\ldots\mathbf{y}_K^T(M)]^T. \qquad (7)$$

III. RECEIVER STRUCTURE

For imperfect timing and amplitude information, a linear approximation of the signal portion of $\mathbf{y}$ in (6) is obtained via a Taylor expansion,

$$\begin{aligned} \mathbf{y} &\approx \hat{\mathbf{R}}\mathbf{B}\hat{\mathbf{a}} + (\hat{\mathbf{R}}\mathbf{B})\delta_a + (\dot{\mathbf{R}}\mathbf{B}\mathbf{A})\delta_\tau + \mathbf{n} \\ &\triangleq \hat{\mathbf{y}}_0 + \mathbf{Q}_a(\mathbf{b})\delta_a + \mathbf{Q}_\tau(\mathbf{b})\delta_\tau + \mathbf{n}, \end{aligned} \qquad (8)$$

where $\mathbf{B} = \text{diag}(\mathbf{b})$. The matrix $\hat{\mathbf{R}}$ is obtained by substituting the true signal space after sampling with the signal space corresponding to estimated timing; similarly, substituting the true signal space with its derivative with respect to timing would give $\dot{\mathbf{R}}$. That is,

$$\begin{aligned} \hat{\mathbf{R}}(\tau) &= \mathbf{R}(\tau)|_{\tau=\hat{\tau}} \\ \dot{\mathbf{R}}(\tau) &= \left.\frac{\partial \mathbf{R}(\tau)}{\partial \tau}\right|_{\tau=\hat{\tau}}. \end{aligned} \qquad (9)$$

In this paper, $\hat{\mathbf{R}}$ and $\dot{\mathbf{R}}$ are rectangular matrices since $q_{\text{sym}} > 1$ is used.

By applying $p(\delta_a) \sim \mathcal{N}(\mathbf{0}, \Sigma_a)$, $p(\delta_\tau) \sim \mathcal{N}(\mathbf{0}, \Sigma_\tau)$ and (8), we may write the approximate likelihood function as

$$p_{\mathbf{b}}(\mathbf{y}) \sim \mathcal{N}(\hat{\mathbf{y}}_0, \Sigma_b), \qquad (10)$$

where $\hat{\mathbf{y}}_0 = \hat{\mathbf{R}}\hat{\mathbf{A}}\mathbf{b}$ and

$$\mathbf{\Sigma}_b = \mathbf{\Sigma}_n + \mathbf{Q}_a(\mathbf{b})\mathbf{\Sigma}_a\mathbf{Q}_a^H(\mathbf{b}) + \mathbf{Q}_\tau(\mathbf{b})\mathbf{\Sigma}_\tau\mathbf{Q}_\tau^H(\mathbf{b}). \quad (11)$$

Thus the ML estimate of $\mathbf{b}$ is given by

$$\begin{aligned}\hat{\mathbf{b}} &\triangleq \arg_{\mathbf{b}} \max \Omega_d(\mathbf{b}) \\ &= \arg_{\mathbf{b}} \min \left\{(\mathbf{y}-\hat{\mathbf{y}}_0)^H \mathbf{\Sigma}_b^{-1}(\mathbf{y}-\hat{\mathbf{y}}_0) + \log|\mathbf{\Sigma}_b|\right\}.\end{aligned} \quad (12)$$

The cost function for the original ML formulation with perfect knowledge will be denoted by $\Omega_c(\mathbf{b})$. As we are considering asynchronous transmission, the complexity of the ML detection in (12) is exponential in the number of total symbols transmitted. It is noted that the Viterbi algorithm (VA) can not be applied directly to this problem since $\mathbf{\Sigma}_b$ in (12) is sequence dependent. We may transform the received signal $\mathbf{y}$ into a new random vector $\mathbf{z}$ according to an innovations-based transformation (IBT) [11],

$$\mathbf{z} = \mathbf{\Gamma}(\mathbf{y} - \hat{\mathbf{y}}_0), \quad (13)$$

where $\hat{\mathbf{y}}_0$ is the estimated mean of $\mathbf{y}$ and $\mathbf{\Gamma}$ is a causal, whitening transformation matrix. The *pdf* of $\mathbf{z}$ is given by

$$p_{\mathbf{b}}(\mathbf{z}) = \pi^{-(2M+1)q_{sym}} \exp\{-\mathbf{z}^H\mathbf{z}\}. \quad (14)$$

Since the elements in $\mathbf{z}$ are independent and, furthermore, they are not explicitly sequence dependent, a conventional VA can be applied in terms of $\mathbf{z}$ with slight modification. It is noted in [10] that $\mathbf{z}$ is causal function of $\mathbf{n}$ and $\mathbf{y}$, but not of $\mathbf{b}$. However, we may still apply the VA with future K symbols, as well as the past K symbols, represented in the states at each node. The trellis for a 2-user system is shown in Figure 3. The size of the trellis is equal to that for a channel with inter-symbol interference memory of $2K$ symbols.

IV. PERFORMANCE ANALYSIS

In the following, the detector based on the original VA is denoted by conventional maximum-likelihood sequence detector (C-MLSD); and the modified version presented in Section III is by modified MLSD (M-MLSD). Pairwise error probabilities (PEP's) are obtained by considering the situation when the detector erroneously chooses $\hat{\mathbf{b}}$ instead of the transmitted data $\mathbf{b}$. For a fixed realization of delay errors, we will present the PEP only for the C-MLSD in Rayleigh fading channels. Comparable analysis for the C-MLSD in AWGN channels can be found in [2]. The difficulty in calculating the PEP for the M-MLSD stems from the following: the PEP is given by the average probability of the following event over the *pdf* of $\hat{\mathbf{a}}$ and $\boldsymbol{\delta}_a$,

$$\Omega_d(\hat{\mathbf{b}}) - \Omega_d(\mathbf{b}) > 0, \quad (15)$$

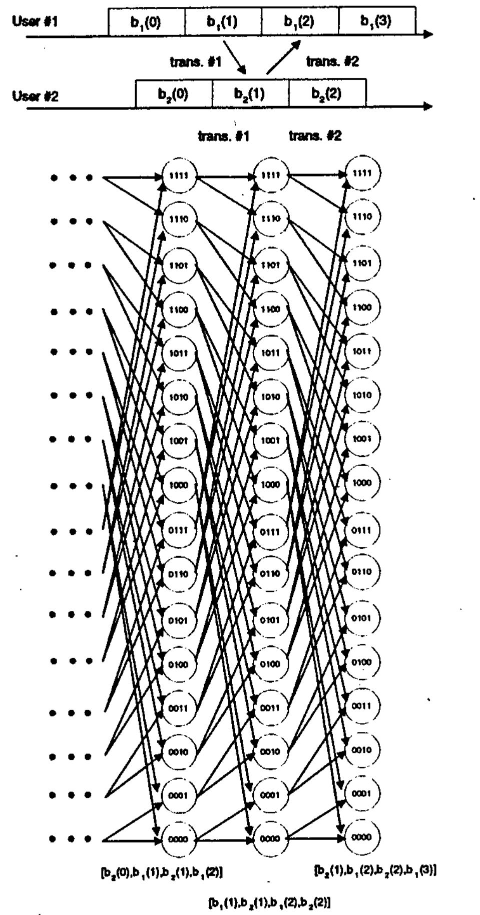

Figure 3: Top: transmitted data sequence by two users; Bottom: corresponding trellis diagram of the MLSD.

where $\Omega_d(\mathbf{b})$ is given in (12). Since $\mathbf{\Sigma}_b$ is a quadratic form in $\hat{\mathbf{a}}$, where $\hat{\mathbf{a}}$ is a Gaussian vector, the overall expression in (15) is a complicated function of $\hat{\mathbf{a}}$, which can not be analyzed by conventional second-order techniques.

By substituting $\mathbf{y} = \mathbf{R}\mathbf{B}(\hat{\mathbf{a}} + \boldsymbol{\delta}_a) + \mathbf{n}$ and $\mathbf{y}_0 = \hat{\mathbf{R}}\mathbf{B}\hat{\mathbf{a}}$, we may write the pairwise error event as

$$\begin{aligned}0 < \zeta_{cr} &\triangleq \Omega_c(\hat{\mathbf{b}}) - \Omega_c(\mathbf{b}) \\ &= \mathbf{x}^H\mathbf{P}_b\mathbf{x} - \mathbf{x}^H\mathbf{P}_{\hat{b}}\mathbf{x} \triangleq \mathbf{x}^H\mathbf{P}_{cr}\mathbf{x}\end{aligned} \quad (16)$$

where

$$\begin{aligned}\mathbf{x} &= \left[\hat{\mathbf{a}}^T \ \boldsymbol{\delta}_a^T \ \mathbf{n}^T\right]^T \\ \mathbf{P}_b &= \left[(\mathbf{R}-\hat{\mathbf{R}})\mathbf{B} \ \ \mathbf{R}\mathbf{B} \ \ \mathbf{I}\right]^H \mathbf{\Sigma}_n^{-1}\left[(\mathbf{R}-\hat{\mathbf{R}})\mathbf{B} \ \ \mathbf{R}\mathbf{B} \ \ \mathbf{I}\right] \\ \mathbf{P}_{\hat{b}} &= \left[\mathbf{R}\mathbf{B}-\hat{\mathbf{R}}\hat{\mathbf{B}} \ \ \mathbf{R}\mathbf{B} \ \ \mathbf{I}\right]^H \mathbf{\Sigma}_n^{-1}\left[\mathbf{R}\mathbf{B}-\hat{\mathbf{R}}\hat{\mathbf{B}} \ \ \mathbf{R}\mathbf{B} \ \ \mathbf{I}\right].\end{aligned} \quad (17)$$

All the quantities in (17) are real, except $\mathbf{x}$ which is a complex Gaussian vector. The PEP is given by

$$P[\zeta_{cr} > 0] = \sum_{\substack{i \\ \lambda_i > 0}} \prod_{\substack{j \\ j \neq i}} \left(1 - \frac{\lambda_j}{\lambda_i}\right)^{-1}, \tag{18}$$

where λ_i's are the eigenvalues of $\mathbf{\Sigma}_x \mathbf{P}_{cr}$, with $\mathbf{\Sigma}_x$ being the covariance matrix of $\mathbf{x}$ [6]. It is assumed in (18) that none of the eigenvalues are identical.

A practical example for $\mathbf{\Sigma}_x$ can be obtained by considering approximation of the flat Rayleigh fading process in (3). We assume an ideal scenario where $\hat{a}_k(m) = \rho_{k1} a_k(m-1) + \rho_{k2} a_k(m-2)$ and $\delta_{a,k}(m) = e(t)$; that is, the estimator is capable of tracking the predictable part of the fading coefficients. Thus, the corresponding PEP is a lower bound of that for a practical system which employs a causal channel tracking process. According to the AR model, $\delta_{a,k}(m)$ is a white Gaussian process with covariance matrix $\text{diag}([\sigma_1^2, \ldots, \sigma_K^2, \sigma_1^2, \cdots, \sigma_K^2, \ldots])$, where $\sigma_k^2 = E\{|\delta_{a,k}(m)|^2\}$. We assume that the fading process is independent amongst the users. Thus, we focus on one of the users and drop the index k. The correlation function of $\hat{a}(m) = a(m) - \delta_a(m)$ is given by

$$\begin{aligned} R_{\hat{a}}(n) &= E\{\hat{a}(n)\hat{a}^*(n-m)\} \\ &= E\{[\rho_1 a(m-1) + \rho_2 a(m-2)] \\ &\quad \cdot[\rho_1 a(m-1) + \rho_2 a(m-2)]^*\} \\ &= (|\rho_1|^2 + |\rho_2|^2) R_a(m) + \rho_1 \rho_2^* R_a(m+1) \\ &\quad + \rho_1^* \rho_2 R_a(m-1). \end{aligned} \tag{19}$$

Given that $\mathbf{\Sigma}_n$ and $\mathbf{\Sigma}_{\delta_a}$ are known and $\mathbf{\Sigma}_{\hat{a}}$ can be obtained from (19), the covariance matrix of $\mathbf{x}$ is given by

$$\mathbf{\Sigma}_x = \begin{bmatrix} \mathbf{\Sigma}_{\hat{a}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_{\delta_a} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_n \end{bmatrix}. \tag{20}$$

To determine a lower bound of performance, consider a scheme where the vector $\mathbf{b}$ is transmitted. Assume a genie tells the receiver that either $\mathbf{b}$ or $\hat{\mathbf{b}}_M$ is transmitted, where $\hat{\mathbf{b}}_M = \arg_{\hat{\mathbf{b}}} \max P\{\Omega_d(\hat{\mathbf{b}}) - \Omega_d(\mathbf{b}) > 0|\mathbf{b}\}$. That is, $\hat{\mathbf{b}}_M$ is the most probable symbol to be mistakenly detected when $\mathbf{b}$ is transmitted. The BER is bounded by the performance of this genie-aided receiver,

$$\text{BER}_1 > \frac{1}{2^{(2M+1)K}} \sum_{\mathbf{b}} \max_{\substack{\hat{\mathbf{b}} \\ \hat{b}_1(0)\hat{b}_1(1) \neq b_1(0)b_1(1)}} P_{pw},$$

where

$$P_{pw} \triangleq P[\Omega_d(\hat{\mathbf{b}}) - \Omega_d(\mathbf{b}) > 0|\mathbf{b}]. \tag{21}$$

The upper bound employed is simply the union bound,

$$\text{BER}_1 < \frac{1}{2^{(2M+1)K}} \sum_{\mathbf{b}} \sum_{\substack{\hat{\mathbf{b}} \\ \hat{b}_1(0)\hat{b}_1(1) \neq b_1(0)b_1(1)}} P_{pw}.$$

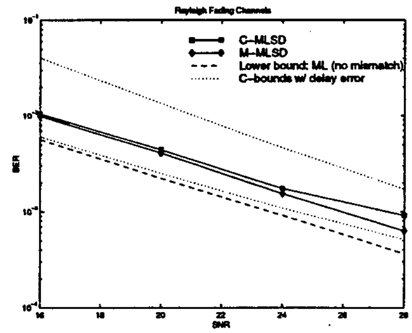

Figure 4: BER and analytical bounds vs. SNR for C-MLSD and M-MLSD in flat Rayleigh fading channels.

These bounds are accurate for a sequence detector only when data of infinite horizon is considered. To reduce the computational effort, a finite horizon of five data bits ($M = 2$) per user is considered with the interested bit, $b_1(0)$, in the middle. Thus, the bounds presented here are expected to be slightly higher than their true values, which correspond to the infinite horizon case.

V. NUMERICAL RESULTS

In this section, preliminary results are obtained under the assumption that the only source of prediction error in the fading coefficients is the innovations process $\mathbf{w}(t)$ and the ambient noise. Thus the channel estimator perfectly tracks the predictable portion of the AR process in (3). Therefore, the results presented here provide a lower bound of the bit-error rate (BER). Due to complexity of the algorithm, a two-user system with Gold codes of length 31 is considered. The decision depth of the simulated MLSD's is 10 times the channel memory. The SNR is defined as the peak signal energy at the CMF output over the variance of the noise. Due to timing uncertainty, the "effective" SNR is lower than that with regular SNR definition.

Figure 4 shows the performance in flat Rayleigh fading channels approximated by a second-order AR process with the following system parameters: center frequency $f_c = 900$ MHz, vehicle speed $v = 100$ km/hr, symbol rate $T^{-1} = 9.6K\text{sec}^{-1}$, and NFR=4 dB. We acknowledge the fact that the accuracy of timing estimation should improve with increased SNR, and thus $\sigma_{\tau_k}^2$ should change with SNR. However, without studying any specific timing estimators in this paper, we consider fixing the value of $\sigma_{\tau_k}^2$ as an artificial but acceptable choice. The system is tested with fixed tracking error of $0.1T_c$, which is equal to the standard deviation of tracking errors used to construct the receiver. To reduce the error of the linear approximation for large SNR, we found the parameters $q_{sym} = 7$ and $q_{ov} = 10$ to be sufficiently large for