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# OPTIMIZATION OF GAS-COOLED SUPERCONDUCTING MAGNET LEADS. A METHOD FOR GENERAL MATERIALS

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## Abstract

A method is presented for calculating the length to area ratio of current leads for superconducting magnets that will minimize the heat loss caused by the lead. The method does not assume that the material obeys the Weidemann-Franz Relation; therefore, calculations can be made for ultra high purity metals. Sample calculations for such materials are made and it is shown that it is possible to effect a refrigeration saving over that required fcr W-F materials.

Foreward

A previous analysis by Morgan[1] has led to a method of determining the optimum length to area ratio of a current carrying, gas cooled cryogenic lead for minimum heat loss to the cryogenic system. This analysis was based on the following assumptions:

1. Perfect heat transfer occurs between the cooling gas and the lead.
2. The specific heat of the cooling gas is constant in the temperature range considered.
3. The thermal conductivity and electrical resistivity of the lead material as a function of temperature, is determined by the Weidemann-Franz-Lorenz relation.

The first assumption is a valid approximation providing that the heat transfer surface and flow conditions are such that a small temperature difference exists between the coolant and the lead. These conditions can be generally satisfied. The second assumption is a good approximation particularily for the case of helium whose specific heat at one atm varies from a maximum value of 6 j/gm between $5^{o}K$ and $6^{o}K$ to a constant value of 5.2 j/gm at $40^{o}K$ and higher. The assumption that the material obeys the W-F-L law is, however, only valid for certain classes of materials, i.e., those which have a relatively high residual resistives. Thus, certain alloys, impure and strain hardened metals obey the law fairly well while very pure metals depart widely from this relation. Thus, in Morgan's solution for the W-F-L material, we are restricted to such materials as brass and impure copper and aluminum or pure metals that are highly cold worked. The Morgan solution gives a minimum heat loss that is independent of the actual choice of the material and is the same for all W-F-L materials. The length to area ratio is then determined by supplying either the thermal or electrical properties of the particular choice for the lead material.

Since high purity, strain free metals depart widely from the W-F-L law, Morgan's analytical solution does not provide accurate results for this case. For example Powell, et al[2] have determined that the Lorenz number for 99.999% annealed copper varies from a minimum value of $0.8 \times 10^{-8}$ at $24^{o}K$ to $1.9 \times 10^{-8}$ watt-ohm/$(^{o}K)^{2}$ at higher temperatures. The electrical resistivity of the pure copper, in the range, $5^{o}K$ to $30^{o}K$ drops more rapidly than the thermal resistivity; thus, it can be expected that magnet leads that are more efficient than those which use W-F-L materials can be designed with high purity metals. Thus, the magnet lead problem for steady state conditions has been solved for the general case and is not restricted to W-F-L materials. This solution can be used to obtain the optimum length to area ratio of the lead, for minimum heat influx, provided that measured values of the thermal and electrical resistivities can be supplied for the material under consideration. The solution has been obtained by numerical analysis and examples are compared to the solution for the W-F-L material. It is seen that potential savings of about 20% of refrigeration loss is possible with commercially available high purity metals used for superconducting magnet leads.

## Analysis

The following notation is used:

| | | |
|---|---|---|
| A | - | lead area, $cm^2$ |
| I | - | current per lead, amp. |
| L | - | Lorentz number, typically $2.47 \times 10^{-8}$ watt-ohm/$(^oK)^2$ |
| T | - | Temperature, $^oK$ |
| Q | - | heat flux, watts |
| $T_o$ | - | temperature at cold end of lead, $^oK$ |
| $T_2$ | - | temperature at warm end of lead, $^oK$ |
| C | - | specific heat of coolant gas, j/gm - $^oK$ |
| x | - | distance along lead, cm |
| y | - | thermal impedance along lead |
| m | - | mass of cryogen vaporized at cold end of lead, gm/sec |
| h | - | heat of vaporization of cryogen j/gm |
| l | - | length of lead, cm. x = l is at the cold end |
| ρ | - | electrical resistivity, ohm-cm |
| k | - | thermal conductivity of lead material, watt/cm-$^oK$ |

With constant specific heat of the coolant and perfect heat transfer between the coolant and lead, the heat balance equation for a gas cooled lead has been given by Morgan and is written here as

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right) - Cm\frac{dT}{dx} - \frac{\rho I^2}{A} = 0 \qquad (1)$$

where x = 0 is at the warm end of the lead. Using the concept of thermal impedance, i.e. $dy = \frac{1}{kA}dx$, the above equation can be written in terms of the heat flux, $Q = \frac{dT}{dy}$. Thus,

$$\frac{dQ}{dy} = - c m Q - \rho k I^2 \tag{2}$$

It is noted here that if the material obeys the W-F-L law, then $\rho k = LT$ and (2) becomes

$$\frac{dQ}{dy} = - c m Q - I^2 LT \tag{3}$$

Since $Q = \frac{dT}{dy}$, this second degree equation can be readily solved analytically, as Morgan has done, and the minimum value of m can be determined for any W-F-L material.

If we wish to solve the general equation (2), a more elaborate procedure is required. We proceed as follows:

Taking the derivative of (2) with respect to y, we obtain

$$\frac{d^2Q}{dy^2} = - c \left[ Q \frac{dm}{dy} + m \frac{dQ}{dy} \right] - I^2 \left[ \rho \frac{dk}{dy} + k \frac{d\rho}{dy} \right] \tag{4}$$

In order to find the minimum value of m that will solve (2), equation (4) is solved with $\frac{dm}{dy} = 0$.

Thus, the problem is described by

$$\frac{d^2Q}{dy^2} = -cm\frac{dQ}{dy} - I^2\left[\rho\frac{dk}{dy} + k\frac{d\rho}{dy}\right]. \qquad (5)$$

The boundary conditions for (5) are:

(a) $Q = mh$ and $T = T_o$ at $x = 1$

(b) $Q = 0$ $T = T_2$ and from (1) $\frac{dQ}{dy} = \rho h I^2$ at $x = 0$.

Since $\frac{dk}{dy} = \frac{dk}{dT} \cdot \frac{dT}{dy}$ and $\frac{d\rho}{dy} = \frac{d\rho}{dy} \cdot \frac{dT}{dy}$

we can write equation (5) in a more convenient form as

$$\frac{d^2Q}{dy^2} = -cm\frac{dQ}{dy} - I^2Q\left[\rho\frac{dk}{dT} + k\frac{d\rho}{dT}\right] \qquad (6)$$

Equation (6) with its boundary conditions must be solved numerically using supplied values of $\rho$, k and their derivatives as a function of T. If this were an initial value problem the solution would be quite easy since one could use the readily available Runge-Kutta numerical integration procedure. However, a method for the numerical solution of ordinary differential equations with values specified on each boundary is available only for certain restricted cases. Although there may be some general method that could be applied, we have solved the problem by converting it to an initial value problem in conjunction with a Newton-Raphson type iteration procedure. The method was as follows:

1. An initial approximate value of the parameter, m is selected. This can be determined by assuming that $\frac{dQ}{dy} = 0$ at $x = 1$ and thus, the trial value of m is

$$m = \sqrt{\frac{\rho k I^2}{c h}}$$

2. Equation (6) is then solved by the Runge-Kutta method for the given initial conditions and the integration proceeds taking increments in y until $T \leq T_o$.

3. At this point, the integration procedure is stopped and the value of Q is compared to the value it must have at the cold end. Thus, the error is $Q(1) - mh$.

4. A correction to m is calculated using a modified Newton-Raphson method based on the variation of the error with m as determined by successive integrations.

5. Step 2 is repeated until the error is less than some pre-assigned value, usually 0.1 watt. Thus, the minimum heat flux length to area ratio and the temperature profile can be determined by the final integration of equation (6).

In addition to the above described iteration procedure, the solution of the problem also requires a method of calculating $\rho$, k, $\frac{d\rho}{dT}$ and $\frac{dk}{dT}$ as a function of T. This is provided as follows:

1. Typically 20 to 25 values of $\rho$ and k at various temperatures are provided.

2. The temperature range is broken up into three sections. In each of these sections a 4th or 5th polynomial is fitted to the data by a least squares method.

3. The integration procedure then uses the polynomial approximations for $\rho$, k, $\frac{d\rho}{dT}$ and $\frac{dk}{dT}$ to calculate the values at any temperature in the range considered.

This solution has been programmed for the CDC-6600. It has been checked with the solution for the W-F-L materials, and agrees almost exactly. The method has been found to converge quite well; for helium temperature pure copper leads, 10 iterations were required for convergence and 6 iterations were required for convergence with hydrogen cooled leads.

Numerical Examples

For the numerical examples, 6000 amp., helium cooled leads have been chosen for various materials for which published data was available. These materials are listed in Table I.

TABLE I

| Material No. | Description |
|---|---|
| ✓ 1 | 99.999% Copper Annealed |
| 2 | 99.999% Copper, 26% Cold-drawn |
| ✓3 | OFHC Copper, annealed |
| ✓ 4 | 1100-0 Aluminum |
| 5 | W-F-L material with k value similar to 2 |
| 6 | 99.995% Aluminum single crystal |

Thermal conductivity and electrical resistivity data for materials 1 and 2 were taken from Powell et al.[2] Data for 3 was taken from two sources; the resistivity from Clark et al[3] which had measurements to $4^{o}K$. Thermal conductivity for the OFHC copper (condition unknown) was taken from the NBS circular.[4] However, the curve did not extend below $23^{o}K$ so that extrapolated values down to $4^{o}K$ were estimated from the shape of other curves. Furthermore, since corresponding electrical and thermal measurements were not made on the same sample, the calculation for this material may not be reliable. The data for 1100-0 aluminum was taken from curves in Hall's paper.[5] Also, this same reference gave curves for the aluminum single crystal. Table II summarizes the thermal and electrical data for the above materials.

The points shown were computed from the polynominal approximations. Calculated results for a 6000 amp., helium cooled lead for these materials are given in Table II. The temperature at the top of the lead is 300°K with no heat input, the temperature of the bottom was 4.5°K. Values used for the specific heat of helium gas and heat of vaporization of the liquid were 5.194 j/gm - °K and 18.69 j/gm.

TABLE III

6000 amp. Helium Cooled Leads for Various Materials

| MATERIAL | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Optimum Length: Area Ratio, $cm^{-1}$ | 260.2 | 63.7 | 29.3 | 14.4 | 63.0 | 46.0 |
| Heat Input at Lead Bottom, Watts | 4.7 | 5.1 | 4.6 | 5.6 | 5.8 | 5.4 |
| Mass of Helium Vaporized, gm/sec. | .252 | .275 | .247 | .300 | .311 | .287 |

The optimization calculations show that one could expect to achieve a helium refrigeration saving of about 20% over that required for leads made from a W-F-L material by using a material such as high purity annealed copper. In this case actual samples of the material would have to be checked at low temperature in order to verify the resistance ratio. The resistance ratio for material 1 is 1500. The cold-drawn high purity copper 2, having a resistance ratio of 138 represented a possible refrigeration saving of 11%. Although OFHC copper 3, was calculated to have a heat loss of only 4.7 w, which is almost as low as material 1, these results are questionable until a re-calculation can be made with reliable electrical resistivity and thermal conductivity data. The commercially pure aluminum 4, gave the same result as for a W-F-L material as did the data used for the aluminum single crystal, 6.

The result for the high-purity annealed copper has verified that one can expect to achieve a lower loss design with ultra high purity materials since at the low temperature end the resistivity decreases at a faster rate than the thermal conductivity increases. Indeed, it would be interesting to speculate on the possibility of designing low loss leads of high purity single crystals of copper. Gniewek and Clark[6] have measured the resistance ratio of such samples at 20,000. One might even effect an additional enhancement by using crystals which are grown to be oriented along certain principal axes.

annealed 99.999 Cu   Annealed OFHC Cu   Material   1100-0 Al

| | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temp, °K | $\rho \times 10^6$ | k | $\rho \times 10^6$ | k | $\rho \times 10^6$ | k | $\rho \times 10^6$ | k | $\rho \times 10^6$ | k | $\rho \times 10^6$ | k |
| 4. | .00110 | 66.4 | .0123 | 6.35 | .0160 | 2.00 | .2000 | .4539 | .0119 | 7.28 | .0245 | 3.44 |
| 8. | .00115 | 120.5 | .0126 | 12.52 | .0162 | 3.35 | .2019 | .9190 | .0115 | 15.10 | .0251 | 6.76 |
| 12. | .00127 | 133.1 | .0130 | 18.20 | .0164 | 5.16 | .2043 | 1.385 | .0120 | 21.75 | .0259 | 10.00 |
| 16. | .00150 | 116.9 | .0134 | 22.42 | .0167 | 7.06 | .2073 | 1.83 | .0132 | 26.24 | .0271 | 12.83 |
| 20. | .00200 | 87.0 | .0141 | 24.49 | .0170 | 9.00 | .2110 | 2.22 | .0154 | 28.11 | .0290 | 14.92 |
| 24. | .00282 | 60.8 | .0159 | 24.03 | .0190 | 11.24 | .2165 | 2.56 | .0190 | 27.42 | .0305 | 15.96 |
| 32. | .00645 | 33.0 | .0233 | 18.90 | .0224 | 12.53 | .2314 | 3.02 | .0322 | 21.50 | .0422 | 14.28 |
| 40. | .0160 | 18.5 | .0386 | 14.18 | .0275 | 12.32 | .2515 | 3.22 | .0547 | 15.86 | .0651 | 11.96 |
| 60. | .0800 | 8.40 | .1130 | 6.99 | .1210 | 6.50 | .3300 | 2.99 | .1712 | 7.60 | .1400 | 6.38 |
| 80. | .2036 | 4.71 | .2218 | 5.25 | .2234 | 6.34 | .4521 | 2.43 | .2810 | 6.18 | .2103 | 3.41 |
| 100. | .3425 | 4.60 | .3509 | 4.49 | .3487 | 4.44 | .6433 | 2.24 | .4132 | 5.25 | .3747 | 2.89 |
| 140. | .6309 | 4.21 | .6312 | 4.22 | .6282 | 4.21 | 1.152 | 2.12 | .6546 | 4.64 | .8751 | 2.60 |
| 180. | .9033 | 4.21 | .9030 | 4.21 | .9056 | 4.20 | 1.699 | 2.10 | .9138 | 4.27 | 1.421 | 2.60 |
| 240. | 1.273 | 4.12 | 1.273 | 4.11 | 1.284 | 4.08 | 2.407 | 2.06 | 1.284 | 4.06 | 2.063 | 2.52 |
| 300. | 1.700 | 4.00 | 1.700 | 4.00 | 1.700 | 4.00 | 3.000 | 2.02 | 1.627 | 4.00 | 2.495 | 2.40 |

$\rho$ - ohm-cm

K - Watt/cm K

TABLE II

References


1. G. H. Morgan, "Optimization of Current Leads with a Cryogenic System", Report AADD-171, November 24, 1970.

2. R. L. Powell et al., "Low Temperature Transport Properties of Copper and Its Dilute Alloys: Pure Copper, Annealed and Cold-Drawn"

NBS 127 3. Clark et al, "Low Temperature Electrical Resistivity of Some Engineering Alloys", Advances in Cryogenic Engineering, Vol. 15, Plenum Press (1970).

4. "Thermal Conductivity of Metals and Alloys at Low Temperatures", NBS Circular 556, Sept. 1, 1954.

5. W. J. Hall et al, "Thermal Conductivities of Common Commercial Aluminum Alloys", Advances in Cryogenic Engineering, Vol. 3, Pg. 408 (1960).

6. J. J. Gniewek and A. F. Clark, "Preparation of Copper Crystals with Low Electrical Resistivity", J. App. Phys., Vol. 36, Pg. 3358 (1965).