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AN INVESTIGATION OF SCATTER AND DISTRIBUTIONS FOR CRACK GROWTH LIVES UNDER CONSTANT AMPLITUDE STRESS INTENSITY FACTOR CYCLING

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FOREWORD

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This technical memorandum has been reviewed and is approved for publication.

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SECTION I

INTRODUCTION

It has been recognized that scatter in the crack growth phase as well as in the initiation phase is important in assessing the life of aerospace structures using reliability methods. First estimates of the potential effects of this scatter have recently been evaluated (Reference 1).

Selection of potential distributions for the initiation phase evolved over many years both on theoretical bases as a weakest link theory and as an asymptotic distribution (Reference 2) and from fits of actual data (Reference 3 and 4). Two distributions, the Weibull and the lognormal, have been widely used to fit the distribution of times to crack initiation.

For the region of subcritical crack growth, as in the case of initiation, the physical process can provide guidance in selecting potential distributions and investigating their suitability. In initiation, under given cyclic conditions, the first cracking of different structures are considered to be independent events. It still remains to be ascertained whether, under constant amplitude stress intensity factor cycling, incremental crack growth is independent of previous growth. Without independence, distributions would have to include effects of prior crack growth. An investigation of the crack growth process under constant amplitude stress intensity factor cycling as a summation process can limit the class of potential distributions, indicate a relation between crack length and scatter, and help identify experiments for testing independence.

In this report, it will be shown that (1) the summation process of incremental crack growth can be used to restrict the class of distribution functions appropriate for describing cycles for a given increment of crack growth; (2) nonparametric tests for independence of crack growth increments can be developed; and (3) the summation process of crack growth leads to a decreasing standard deviation of cycles as a percentage of the mean number of cycles to grow a crack a given increment (the coefficient of variation) with increasing length of increments of crack growth.

SECTION II

THE CRACK GROWTH PROCESS AND A REQUIREMENT FOR DISTRIBUTIONS

One highly desirable property of a distribution for crack growth scatter is that the chosen distribution represents crack growth over a wide variation of length of crack growth intervals. Only the distribution parameters would be a function of the length of the crack growth interval. If this is not the case, one would have to be fortunate and choose to take data on scatter for the one correct crack growth interval and laboriously evolve the distributions for other crack growth intervals. Simplicity in formulation has been a guiding principle in science.

The crack growth process is an inherently integrative (additive) process. For study, many small increments of growth are summed to form larger increments and finally the crack of interest. Assuming all increments of crack growth, the minimum length studied or longer, are independent of previous growth, the suitability of classes of distributions and nonparametric properties (properties independent of the distribution) can be studied. One convenient formulation of the problem of scatter in crack growth lives is the distribution of the number of constant amplitude stress intensity factor cycles to produce a given increment of crack growth.

2.1 Distributions

Using this formulation of the number of constant amplitude stress intensity factor cycles to produce a given increment of crack growth and the summation process, a restriction on the class of distributions can be derived. If a distribution for the number of cycles is selected for one increment of crack growth, using simplicity of formulation as guidance, the form of the distribution for the sum of two or any number of these increments should be the same. Only the distribution parameters would be a function of the length of the crack growth interval. Indeed the selected interval itself represents this summing process. A convolution integral gives the probability density function for a process which is the sum of two random variables when these two probability density functions are known. In this case, it will be assumed that the processes are identical and independent. If Y is the number of cycles required to grow a crack "2a" units and X is the number of cycles for "a" units of growth, it is indicated in reference 5 that

$$f_Y(y) = \int_{-\infty}^{\infty} f_X(x)\, f_X(y-x)\, dx$$

where $f_Y(y)$ is the probability density function for Y and $f_X(x)$ is the probability density function for X. For simplicity, f_Y should be the same form of distribution as f_X.

Some, but not all distributions which may be considered, satisfy this condition. For instance, it has been shown in reference 5 that the normal distribution satifies this condition. However, it can be demonstrated that the Weibull and lognormal distributions do not satisfy the condition. The Weibull probability density function is

$$f_X(x) = \frac{\alpha}{\beta}(x/\beta)^{\alpha-1} e^{-(x/\beta)^{\alpha}} \qquad \text{for } x \geq 0$$

$$f_X(x) = 0 \qquad \text{for } x < 0.$$

For $\alpha = 1$, this degenerates to the exponential distribution. Assuming x is defined such that $\beta = 1$, the convolution integral for f_Y can be written as

$$f_Y(y) = \int_0^y e^{-x} e^{-(y-x)}\, dx$$

$$= y e^{-y}.$$

Hence Y is Gamma distributed and not Weibull (or exponentially) distributed. Generally the sum of Weibull distributed random variables is not Weibull distributed, and hence, preference should be given to other distributions. Similarly, it can be shown that the lognormal distribution does not satisfy this condition.

The Gamma distribution, itself, has received some interest in terms of describing times to crack initiation. This distribution does satisfy the summing condition (reference 5) and has a particularly convenient property. The density function is

$$f_X(x) = \frac{\lambda(\lambda x)^{r-1} e^{-\lambda x}}{\Gamma(r)} \qquad x > 0$$

$$f_X(x) = 0 \qquad x \leq 0.$$

As indicated in reference 5, the density function for the sum of two Gamma distributed random variables with the same λ is Gamma with the same λ as that of the ones being summed and with the range (r_Y) being the sum of the r's from the summed random variables. Hence for crack growth, λ would be independent of the length of the crack growth interval studied and r would be a linear function of the length of the crack growth interval. The Gamma distribution should be studied as a candidate for expressing cycles to produce given increments of crack growth if any dependency of crack growth on previous growth can be neglected.

2.2 Nonparametric Properties

If it is assumed that the crack growth increments are independent, certain properties of a summing process can be obtained which are independent of the distributions and indeed independent of whether the simplicity guideline is obeyed. As indicated in reference 5, the characteristic function for a random variable composed of the sum of N independent random variables is

$$\phi_Y(u) = \prod_{i=1}^{N} \phi_{X_i}(u).$$

All moments of the random variable Y can be calculated from this function. The moments are given by

$$E[Y^k] = \frac{1}{i^k} \frac{d^k}{du^k} \phi_Y(0)$$

where E implies expected value. Recognizing from the definition

$$\phi_X(u) = \int_{-\infty}^{\infty} e^{ixu} f(x)\,dx$$

that $\phi_X(0)$ is one, the moments of Y can be calculated in terms of the moments of the X's without knowing these distributions.

The first moment (k = 1) is the mean. For the sum of two independent random variables

$$\begin{aligned} E[Y] &= -i\frac{d}{du}\left(\phi_{X_1}(0)\,\phi_{X_2}(0)\right) \\ &= -i\left[\phi_{X_2}(0)\frac{d}{du}\phi_{X_1}(0) + \phi_{X_1}(0)\frac{d}{du}\phi_{X_2}(0)\right] \\ &= E[X_1] + E[X_2]\,. \end{aligned} \tag{1}$$

This can be extended to the sum of N independent random variables. The second moment is related to the variance. The variance is the $E(Y^2) - E^2(Y)$. This can be calculated for the sum of two random variables as

$$\begin{aligned} E[Y^2] &= -\frac{d^2}{du^2}\left(\phi_{X_1}(0)\,\phi_{X_2}(0)\right) \\ &= \frac{-d}{du}\left[\phi_{X_2}(u)\frac{d}{du}\phi_{X_1}(u) + \phi_{X_1}\frac{d}{du}\phi_{X_2}(u)\right]\Big|_{u=0} \\ &= -\left[2\frac{d}{du}\phi_{X_2}(0)\frac{d}{du}\phi_{X_1}(0) + \phi_{X_2}(0)\frac{d^2}{du^2}\phi_{X_1}(0) + \phi_{X_1}(0)\frac{d^2}{du^2}\phi_{X_2}(0)\right] \\ &= 2E[X_2]\,E[X_1] + E[X_1^2] + E[X_2^2]\,. \end{aligned}$$

Hence the variance

$$E[Y^2]-E^2[Y]=E[X_1^2]-E^2[X_1]+E[X_2^2]-E^2[X_2] \quad (2)$$

is the sum of the variances of the two random variables X_1 and X_2. This can be extended to the sum of N independent random variables. Similarly all higher moments can be calculated for Y from knowing the moments of the X's without knowing the specific distributions.

The hypothesis of independence can be tested by checking moments when combining intervals. If crack growth increments are independent, the moments of distributions for 2, 3, . . . intervals should be related to the moments for a single interval as derived from the above characteristic function.

2.3 The Crack Growth Process

From Equations 1 and 2, assuming independence, two interesting properties of the crack growth process can be determined. First, under constant amplitude stress intensity factor cycling, the mean number of cycles required to grow a crack N increments is N times the mean number of cycles required to grow a crack one increment. That is,

$$E[Y]=N\,E[X].$$

Second, the ratio of the standard deviation of the number of cycles to the mean of the number, the coefficient of variation, decreases as the square root of the number of increments. That is,

$$\frac{\sqrt{E[Y^2] - E^2[Y]}}{E[Y]} = \frac{1}{\sqrt{N}} \frac{\sqrt{E[X^2] - E^2[X]}}{E[X]} .$$

Note that the length of a crack growth increment is proportional to N.

If increments of crack growth are dependent, the coefficient of variation may (although not necessarily) vary with the length of the crack growth increment at a faster or slower rate. For example, if an increment of fast crack growth is closely associated with a prior or subsequent increment of slow crack growth, the coefficient of variation will decrease at a rate faster than one over the square root of the length of the crack growth increment.

SECTION III

CONCLUSIONS

The summation nature of the crack growth process can provide strong guidance for selecting distributions to represent the number of cycles required to produce given increments of crack growth.

Neither the Weibull nor the lognormal distribution is appropriate to represent a summation process.

The Gamma distribution is appropriate to represent a summation process of independent events and has additional convenient properties. It should be tested on actual crack growth data.

Nonparametric tests (independent of the actual distribution) can be devised to test for the independence of crack growth increments.

The standard deviation of cycles to produce a given increment of crack growth divided by the mean number of cycles, the coefficient of variation, should decrease as larger increments are selected. If increments are independent, this decrease should be proportional to one over the square root of the length of the increment. If an increment of fast crack growth is associated with a prior or subsequent increment of slow crack growth, the coefficient of variation will decrease faster than one over the square root of the length of the increment.

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