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SELECTION OF A COMPENSATING NETWORK TRANSFER FUNCTION OF A STABILIZATION SYSTEM

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The stabilization of the position of a body in inertial space represent a problem in automatic control. In this problem, the object of regulation is the stabilized body, which is under the influence of certain external disturbing forces.

Usually the role of the sensitive elements, which determine the deviation of the body from a given position, is performed by a gyroscope. The correcting pulses produced by the gyroscope are converted and amplified and then applied to several motors, which produce torques that counteract the disturbing forces.

Thus, the stabilization system under consideration is a system that follows the disturbing torques. The more accurately the correcting network reproduces the torques opposing the disturbing forces, the better stabilization of the body.

The quality of stabilization depends substantially on the choice of correcting networks. In position stabilization systems, a criterion of the quality is usually the gain, an increase in which leads to a deterioration in the transient and to instability.

To eliminate this contradiction differentiating links, are connected in parallel to the correcting network and this changes the dynamic properties of the closed loop somewhat for the better.

Experience shows that the inclusion of differentiating networks in the correcting network does not always improve the transient.

In this connection, interest attaches to the conditions which must be satisfied before such a complication of the stabilization system can give the desirable effect. The problem of improving the quality of automatic control system by inclusion of additional feedbacks, which include certain links of the corrective network, were considered in many papers [1, 2, 3]. It was established in these investigations that by suitable choice of local feedbacks it is possible to obtain the desired transient for any gain.

In the present article we report on an investigation of the question of the choice of the number of differentiating links necessary in order to make it feasible to obtain any presecribed transient.

It is proved that by choosing ideal differentiating and intergrating links it is possible to realize a synthesis of the desired correcting network with a non-uniformity coefficient equal to zero.

Transfer Function of the Closed-Loop Stabilization System

In the investigation of the question of the choice of the number of differentiating links needed for the synthesis of the desired system, we start from the following simplifying assumptions:

1) The system of stabilization does not have any time-lag links.

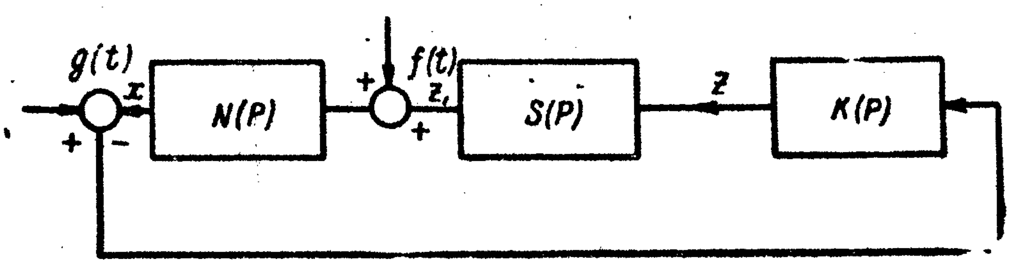
2) The corrective network consists of only ideal differentiating and integrating links.

3) The control signal equals zero.

4) The disturbing and correcting signals have a common imput.

These assumptions can be later on eliminated by generalizing the results.

Let us consider a system of stabilization relative to one coordinate where the transfer function of the object N(p) (Fig. 1). The disturbing and controlling signals are denoted respectively by f(t) and g(t), while the correcting signal is denoted by z.



According to the assumptions we can put

$$S(p) \equiv 1, \quad g(t) \equiv 0, \quad z_1 = z.$$

The closed-loop transfer function is represented in the form

$$\Phi(p) = \frac{N(p)}{1 + N(p)\,k(p)}. \tag{1}$$

Here $k(p) = \frac{\bar{z}}{\bar{x}}$ — is the transfer function of the correcting network, and $\Phi(p) = \frac{\bar{x}}{\bar{f}}$ — is the transfer function of the closed-loop system; $\bar{x}, \bar{z}$ and $\bar{f}$ are the Laplace transforms of the corresponding quantities $x(t)$, $z(t)$ and $f(t)$.

The transfer function of a sufficiently broad class of stabilized objects can be represented in the form

$$N(p) = \frac{1}{b_0 p^r + b_1 p^{r-1} + \ldots + b_r}. \tag{2}$$

The transfer function of the correcting network, made up of ideal differentiating and integrating links, is written in the following form

$$k(p) = d_0 p^n + d_1 p^{n-1} + \ldots + d_n + \frac{e_1}{p} + \frac{e_2}{p^2} + \ldots + \frac{e_m}{p^m}. \tag{3}$$

Inserting expressions (2) and (3) into (1), we obtain the transfer function of the closed loop in the following form

$$\Phi(p) = \frac{p^m}{p^m(b_0p^r + b_1p^{r-1} + \ldots + b_r) + p^m(d_0p^n + d_1p^{n-1} + \ldots + d_n) + e_1p^{m-1} + \ldots e_m}. \tag{4}$$

On examining expression (4) we note that the degree a of the polynomial in the denominator equals the largest of the numbers $m+r$ or $m+n$, i. e.,

$$\alpha = \sup \cdot \{m + r; \quad m + n\}. \tag{5}$$

Thus, the denominator of the transfer function of the closed system can be represented in the form

$$a_0 (p - p_1)(p - p_2) \ldots (p - p_\alpha). \tag{6}$$

Let us assume that $n > r$, and then $\alpha = m + n$, and according to the well-known theorem on the roots of an algebraic equation, we can write

$$\begin{aligned}
d_0 &= a_0, \\
d_1 &= -\sum_{j_1} p_{j_1} \\
&\ldots\ldots \\
d_{n-r} + b_0 &= (-1)^{(n-r)} \sum p_{j(1)} \ldots p_{j(n-r)}, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
d_n + b_r &= (-1)^n \sum p_{j(1)} \ldots p_{j(n)}, \\
e_1 &= (-1)^{n+1} \sum p_{j(1)} \ldots p_{j(n+1)}, \\
&\ldots\ldots\ldots\ldots\ldots \\
e_m &= (-1)^{n+m} \sum p_{j(1)} \ldots p_{j(n+m)},
\end{aligned} \tag{7}$$

where the summation extends over all the indices that are not equal to each other.

From expressions (7) it follows that for $n > r$ we can arbitrarily specify all the zeros of polynomial (6) and find the corresponding coefficients $d_1 \ldots d_n$, $e_1 \ldots e_m$ with the aid of formulas (7).

Let us note now that when $n < r$ this cannot be done, since the system of linear algebraic equations (7) may not be compatible in this case.

On the basis of this we can state the following propositions:

1) To obtain any desirable transient by choosing a corresponding distribution of the roots of the characteristic equation it is necessary to introduce into the correcting network differentiating links up to order r incluseve, where r is the order of the transfer function of the object.

2) The number of integrating links can be arbitrary and is determined by the order of the required astatism.

Transfer Function of Correcting Network for Critical Transients

By way of illustration of the application of the established proposition let us examine an example of a single-axis gyroscopic gimbal.

Let it be required to realize a correcting network for a gyroscopic gimbal, satisfying the following requirements:

a) The system should be astatic;
b) The transient should be critical.

In the absence of a controlling signal, the equations of motion of the gyroscopic gimbal are written in the form

$$A\ddot{\alpha} + h_a\dot{\alpha} + H\dot{\beta} = 0,$$

and

$$B\ddot{\beta} + h_b\dot{\beta} - H\dot{\alpha} = M - M^*. \tag{8}$$

Here α and β are respectively the angles of rotation of the gyroscope housing and of the gimbal, H is the kinetic moment of the gyroscope, and A and B are respectively the moments of inertia of the gyroscope and of the gimbal, h_a and h_b are the coefficients of viscous friction respectively for the gyroscope and for the gimbal, M is the correcting tongue, and M^* the disturbing tongue.

Eliminating β from (8) we reduce the equation of the gyroscopic gimbal to the form

$$\dddot{\alpha} + b_1\ddot{\alpha} + b_2\dot{\alpha} = z^* - z,$$
$$b_1 = \frac{h_a}{A} + \frac{h_b}{B}, \quad b_2 = \frac{h_a h_b}{AB} + \frac{H^2}{AB}, \tag{9}$$

where

$$z^* = \frac{H}{AB}M^*, \quad z = \frac{H}{AB}M.$$

The equation of the correcting network will have the following form

$$z = e_1\int_0^t \alpha dt + d_0\dddot{\alpha} + d_1\ddot{\alpha} + d_2\dot{\alpha} + d_3\alpha. \tag{10}$$

Our problem consists of determining the relation between the coefficients e_1, d_0, d_1, d_2 and d_3.

The transfer function (4) of the closed system will be of fourth order and is written in this case in the form

$$\Phi(p)=\frac{p}{a_0(p-p_1)(p-p_2)(p-p_3)(p-p_4)}. \qquad (11)$$

For a critical transient we have

$$p_1=p_2=p_3=p_4=-\rho \ (\rho>0),$$

which corresponds to the case when all four roots are real and lie in the left half plane of the complex variable p.

To determine the coefficients e_1, d_0, d_1, d_2 and d_3 we write down equations (7), For our case, when $r=n=3$, we have

$$\begin{aligned} d_0+b_0&=a_0,\\ d_1+b_1&=4\rho,\\ d_2+b_2&=6\rho^2,\\ d_3&=4\rho^3,\\ e_1&=\rho^4,\\ b_0&=1. \end{aligned} \qquad (12)$$

Determining the unknowns from the system of equations (12) and inserting them in expression (10), we obtain the following equation for the correcting network

$$z=\rho^4\int_0^t \alpha dt+(a_0-1)\dddot{\alpha}+(4\rho-b_1)\ddot{\alpha}+(6\rho^2-b_2)\dot{\alpha}+4\rho^3\alpha. \qquad (13)$$

If we put $a_0=1$, then E. (13) becomes simple. From the expression obtained it follows that to realize a critical transient with any damping ρ it is necessary to introduce the first and second derivatives into the law of regulation. Considering that b_2 is large, one can choose ρ in such a way that the coefficient of the first derivative vanishes. Here the damping will be sufficiently strong, and the law of regulation becomes somewhat simpler and assumes the form

$$z=\rho^4\int_0^t \alpha dt+(4\rho-b_1)\ddot{\alpha}+4\rho^3\alpha, \qquad (14)$$

where $\rho=\sqrt{\frac{b_2}{6}}$.

If we wish to obtain a critical trasient mode with small damping without introducing a second derivative into the law of regulation, it is necessary to set the coefficient of $\ddot{\alpha}$ equal to zero, i. e., to put $4p-b_1=0$. The relation (13) becomes

$$z=\rho^4\int_0^t \alpha dt+(6\rho^2-b_2)\dot{\alpha}+4\rho^3\alpha,$$

where

$$\rho=\frac{b_1}{4}. \tag{15}$$

Conclusions

1. To produce stabilization of a prescribed nature in a closed-loop transient it is necessary to introduce into the correcting network differentiating and integrating elements, the number of which must be chosen in accordance with the established condition ($n>r$).

2. When all the assumptions are satisfied, it is possible, on the basis of the premises established in this article, to effect a choice of correcting links by choosing the roots of the characteristic equation of the closed system.

3. The choice of the best parameters must be based on some convenient criterion.

4. The results given in the article require further generalization to the case of real differentiating and integrating links, and also for the case of time delay.

Bibliography

1. Meyerov, M. V. Concerning control systems that are stable under a gain as large as desired. Avtomatika i telemekhanika [Automation and Telemechanics], No 4, Academy of Sciences USSR, 1947.
2. Solodovnikov, V. V. Izvestiya AN SSSR, OTN [News of the Academy of Sciences, USSR, Department of Technical Sciences], No 4, 1947.
3. Meyerov, M. V. Certain problems in the choice of a structural scheme for an automatic control system. Avtomatika i telemekhanika, No 4, Academy of Sciences USSR, 1952

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