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REVIEW OF DEVELOPMENTS IN FRACTURE AND FATIGUE OF CERAMIC MATRIX COMPOSITES

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13. ABSTRACT *(Maximum 200 words)*

A review of recent developments and state of the art in damage, fatigue and particularly, high-temperature fatigue of ceramic matrix composites is presented. Both laminated, as well as woven configurations are considered, while whisker and particle-reinforced materials are not covered in this review. Based on a detailed discussion of the mechanisms of failure, the problems that have to be addressed for a successful implementation of ceramic matrix composites in design are outlined.

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TABLE OF CONTENTS

LIST OF FIGURES

FOREWORD

This report was prepared by Dr. Victor Birman of the University of Missouri-Rolla and Dr. Larry Byrd of the Structures Division, Air Vehicles Directorate, Wright-Patterson AFB,OH. The work was performed under Contract Number F49620-93-C-0063. This report covers work accomplished during the period June-August 1997.

Introduction

Ceramic matrix composites (CMCs) have found numerous applications in a variety of areas where structural elements are subjected to high temperature combined with mechanical loading. In particular, aerospace structures often experience such loads. The applications of CMCs include:

- Internal chamber walls and nozzles for rocket motors, nozzles of cryogenic engines for satellite launchers, engine after-burner components and exhaust nozzles for turbojets (Lamicq et al. 1995);
- Intake ramps for hypersonic propulsion systems (Kochendorfer and Krenkel, 1995);
- Low-pressure turbine nozzles (Ortiz et al. 1997);
- Exhaust cones and nozzles, flame holders and flaps (Spiret et al., 1995).

Historically, carbon-carbon composites were the first CMCs, but a higher oxidation resistance eventually brought SiC-fiber/SiC-matrix composites to the attention of industry (Lamicq et al. 1995). Examples of the materials of the matrices are lithium aluminosilicate (LAS) and calcium aluminosilicate (CAS). Fiber coatings may be based on carbon, boron nitride (BN) or other materials capable to reduce oxidation and provide a weak interface.

The opportunities offered by ceramic matrix composites (CMCs) were outlined in a number of papers. Prewo (1988) gave toughness, environmental stability, low density and low cost as important advantages. The principal challenges for developers of CMCs according to this paper are matrix cracking and the associated loss of stiffness. Coating the fibers to provide an environmentally stable fiber-matrix interface was suggested as a possible solution. The development of woven CMCs was listed as an important potential tool for improving structural integrity. Schwartz (1997) listed such advantages as high specific strength and stiffness, high strength at elevated temperatures, toughness and improved fatigue and creep properties.

One of the unique features of CMCs is their ability to withstand initial damage without immediate catastrophic consequences. This is due to a deflection of the matrix cracks that occurs when they reach the fiber-matrix interfaces where they turn and begin to propagate along the interface. A related factor is a redistribution of the stresses in the areas of a high stress concentration associated with a nonlinear stress-strain behavior (Genin and Hutchinson, 1995). However, although the interface facilitates the propagation of cracks, it is also subject to oxidation at high temperatures. A stronger interface that results from oxidation causes embrittlement of the material. This dictates a choice of the fiber and matrix materials as well as an introduction of special interface layers capable of minimizing oxidation as well as producing a weak interface.

The high toughness of CMCs is usually explained by the frictional energy dissipation during fracture or fatigue-related damage propagation. As explained elsewhere (Allen et al., 1993), a low interfacial stiffness is necessary to increase the fiber pull-out lengths. This results in a larger sliding surface area, a higher energy dissipation and a higher toughness. An increase of the interfacial frictional resistance that may be due to oxidation reduces the toughness and overall strength of CMCs.

In this review, we present the state-of-the-art in research and understanding of the mechanisms involved in the mechanical and thermomechanical performance of CMCs. The problems that have to be addressed for a successful implementation of CMCs in design are discussed with a particular emphasize on such critical areas as constitutive relationships and fatigue and fracture.

Brief Outline of Approach to Fatigue of Polymeric Composites

The present section is included to show the concepts and approach to fatigue of polymeric composite materials. While the mechanics of failure and fatigue in PMCs and CMCs are not necessarily identical, a lot can be learned from the existing knowledge and information related to the former materials. The section concentrates on several general reviews on failure and fatigue of PMCs that represent the state-of-the-art in this area.

Reifsnider (1990) presents a general discussion on the modes of fatigue in composites. In particular, the following categories of damage that can be fatigue-related are suggested:
a. Constituent damage: micro-cracking, chemical degradation, plastic deformation, crazing;
b. Boundary separation: debonding, delamination, interphase cracking;
c. Inhomogeneous deformation: yielding of boundary material, discontinuous deformation gradients, discontinuous rotation gradients.
Note that typical damage modes of CMCs that will be discussed in this review can be assigned to the items in this list. For example, oxidation of the interface between the fiber and the matrix represents a chemical degradation, while wear of the fibers due to particles in the wake of the propagating interface crack is a consequence of the interphase cracking.

Development of fatigue damage in a composite laminate usually begins with matrix cracking occurring at an angle to the load direction. Usually primary cracks do not cause a significant reduction of the overall stiffness of the laminate. However, the stress concentration in front of the crack tip may result in the fiber fracture in adjacent layers that has been observed at about one-third of the life of a laminate. Propagating cracks result in debonding along the fiber-matrix interface and sometimes fiber breaking. Delamination between layers can be observed as the damage accumulates. Delamination prevents load redistribution among individual layers as some of them begin to fail. Final fracture occurs as a result of the strength reduction of the laminate.

The concept of a "characteristic damage state" (CDS) was introduced by Reifsnider in 1977. According to this concept, a development of primary cracks in off-axis layers during a fatigue test follows a certain law. As the number of cycles increased, the number of cracks in a layer "saturates" and a stable pattern is formed. This pattern experiences only relatively minor changes and the crack spacing remains almost constant with subsequent cycling. According to Reifsnider (1990), the characteristic damage state is a property of the particular laminate. Note that a similar phenomenon has been observed in CMCs, as shown below.

Another review of fatigue of laminated composite materials was published by Stinchcomb and Bakis (1990). They showed an interesting phenomenon of an increase of the strength around local discontinuities as a result of cycling. In the experiments, a graphite epoxy specimen with a 9.5 mm diameter center hole was subjected to a preliminary cyclic loading at a maximum stress of 80% of the monotonic tensile strength and the stress ratio $R = 0.1$. An increase in the tensile strength was attributed to matrix cracks and delaminations around the hole that result in a reduction of the stiffness. Accordingly, the stress concentration around the hole decreases.

Three phases of the damage process were specified. The first phase lasts 10-15% of the fatigue life and is characterized by matrix cracking in layers with the fibers oriented perpendicular to the tensile stress direction. Matrix crack saturation occurs at the end of this phase. Fiber fracture was also observed during this phase in graphite epoxy and boron epoxy, although it does not seem to be a damage mode in CMCs.

During the second phase (70-80% of the fatigue life), the matrix cracks turn and delamination begins. Damage development during this phase is relatively slow. Fiber breakage also takes place during this phase.

An accelerated damage propagation resumes in the third phase that occurs during the last 10-15% of the fatigue life. Now, damage localization and delamination growth are dominant modes. Laminate stiffness degradation finally results in overall failure accompanied by buckling or microbuckling of the fibers and shear crippling. It was emphasized that while fiber fractures occur during all phases, the majority are associated with the stress concentrations due to matrix cracks in adjacent layers.

Thermal effects on fatigue of polymeric composites are related to a degradation of the stiffness of the matrix (Ryder and Walker, 1979). However, although a high temperature reduces the fatigue life of PMCs, it can also increase its residual tensile strength (Porter, 1991) due to a softening of the matrix and a corresponding decrease of the stress concentration in front of the crack tip.

A review of fatigue mechanisms in metal matrix composites was published by Johnson, 1990. The principal difference, as compared to fatigue of PMCs, is related to a relatively large contribution of the matrix to the behavior of the material. The response of a metal matrix composite under cyclic loading depends on a relative failure cyclic strain of the fibers and the matrix. If the matrix fails first, failure does not occur immediately even though the stiffness is severely reduced. However, if the fibers fail prior to the matrix, the failure follows as soon as a sufficiently large number of fibers are broken.

Sendeckyj (1990) outlined methods for a prediction of fatigue life of PMCs. The remaining part of this section is based on his paper where the methods of analysis were divided into a number of categories that will be traced throughout the following discussion.

Empirical fatigue methods are usually based on uniaxial S-N curves. These methods can be extended to cover multiaxial cases as was done by Sims and Brogdon (1977), Rotem and Hashin (1976) and Hahn (1979).

Residual strength degradation fatigue theories have also been reviewed. Sendeckyj (1990) presented backgrounds for theories that are based on the assumptions that the static strength can be represented by a two-parameter Weibull model and the residual strength is related to the applied load by a deterministic equation. The failure is assumed to occur when the residual strength decreases to the maximum applied cyclic stress. According to the author, the weaknesses of the fatigue theories based on the residual strength degradation are that they require an extensive experimental program for each individual laminate. The other problem is related to observations that showed a slow change of the residual strength until the final phase of the fatigue life. An extrapolation of residual strength degradation fatigue theories to the cases of two-stage and spectrum fatigue loading was proposed by Sendeckyj (1990).

The last weakness of the residual strength approach listed in the previous paragraph can be addressed by fatigue theories based on stiffness degradation (Chou et al., 1982, Wang et al., 1984, 1985). The stiffness changes continuously during cycling and therefore, it may be better suited for an estimate of damage. This class of theories is based on the assumption that fatigue life of the laminate is determined by the critical layers oriented along the load direction. Furthermore, the fatigue life of the critical element is assumed to be defined by a residual strength degradation. These theories account for an increase of the load on the critical element due to a redistribution caused by stiffness changes of the layers. Buckling of the critical element is another possible failure mode during compression.

The theory developed by Chou et al. 1982; Wang et al. 1984 and 1985 is based on a fracture mechanics approach. The model of damage includes randomly distributed matrix cracks parallel to the fibers and utilizes linear fracture mechanics. The history of damage accumulation prescribed by this theory was in an agreement with experimental data. On the negative side, the application of the theory requires a complicated numerical solution and it does not address the issue of fatigue life.

Cumulative damage theories attempt to predict the fatigue life in the case of a spectrum fatigue loading. Sendeckyj (1990) indicated that Miner's rule produced disappointing results in predicting the fatigue life of resin-matrix composites.

Two methods reviewed by Sendeckyj were the theory of Hashin and Rotem (1978) and the approach of Broutman and Sahu (1972). The former theory is based on an assumed form of the damage curve, while in the latter method a decrease of the instantaneous strength was taken as a linear function of the fractional life at a current stress level.

Observations of Damage in Ceramic Matrix Composites

The subjects considered in this section refer to constitutive laws for CMCs and to their fracture observed under static loading. Both issues are important for the subsequent analysis of fatigue. In particular, similar mechanisms are involved in static fracture and fatigue failure of CMCs.

A detailed discussion of the mechanical properties of CMCs was published by Evans and Marshall (1988). They emphasized the importance of the fiber/matrix interface in the performance of CMCs. The mechanics of cracks at the interface involves both the opening and shearing fracture modes and two different materials (bimaterial interface). Accordingly, debond resistance can be characterized by the critical strain energy release rate and the phase angle of loading that represents the inverse of tangent of the ratio between shearing and opening stress intensity factors. Both these parameters are affected by residual stresses. The external tensile stress (Mode I fracture) corresponding to matrix cracking has already been derived as a function of the interfacial shear stress used to characterize the interface sliding resistance (Budiansky et al., 1986). Evans and Marshall (1988) noticed that when the interface is subject to residual compression, the interface shear stress becomes a function of the applied stress and the corresponding solution for the matrix cracking stress is more complex. The same paper emphasized a necessity to minimize a mismatch in thermal expansion between the fibers and matrix to avoid high residual stresses.

The paper of Thouless and Evans (1988) presents an analysis of the response of the fibers within the bridging zone. In particular, a stress-displacement relationship was derived for weakly bonded fibers with a low interfacial shear stress. Thouless (1989) considered the toughness of a cracked CMC accounting for a contribution of the interfacial friction.

The fact that CMCs are processed at a very high temperature implies the presence of thermally-induced residual stresses. These stresses can reach very large values and result in a damage of the material even before it is subject to an external load. Bischoff et al. (1989) observed debonding of the fibers from the matrix in SiC fiber lithium aluminosilicate (LAS) matrix composite after its processing. Post-processing cracking was also reported by Nishiyama et al. (1989) who also showed that the coating on the fibers can affect the process.

In spite of the fact that post-processing residual stresses should be reduced as a result of an increasing temperature, it is usually difficult to definitely predict an effect of temperature on the strength and the stiffness of CMCs. For example, the results presented by Warren (1990) illustrate that the bending strength of C/SiC and SiC/SiC composites increases with temperature to 1300^0C. However, this strength abruptly dropped in SiC/SiC specimens at higher temperatures, probably due to oxidation (the results for higher temperatures were not presented for the C/SiC material). The monograph of Schwartz (1997) contains examples of the influence of temperature on tensile strength (based on the paper of Riccitiello et al., 1992, and other references listed in the book). While the strength of C/SiC specimens was not significantly affected even at 1538^0C, the strength of composites with Nicalon and Nextel fibers decreased. The stiffness of these materials was not significantly affected by temperature, with the exception of Nicalon fiber SiC matrix composites where it decreased (It was suggested that this behavior results from an imperfect interphase bonding.). Results were also presented for the flexural strength of SiC fiber composites with CAS or LAS matrices. Both material showed a moderate reduction in strength as temperature increased. However, while the strength of the SiC/LAS material abruptly dropped at temperatures above about 1100^0C, the strength of SiC/CAS specimens slightly increased. Although some of the results discussed here were obtained for woven composites, the general trend, when it comes to the effect of temperature on strength and stiffness, or rather the absence of such trend, is obvious. Note that such inconsistent results reflect several factors involved in high-temperature behavior of CMCs, i.e. residual stresses, post-processing cracking and oxidation at high temperatures. The response of the material can be altered, if one of the factors is absent. For example, the same SiC/LAS-III material showed an increase of the tensile strength with temperature when it was tested in argon, while in the air the strength decreased (Prewo et al., 1989). In this case the difference in the response was probably related to oxidation that was prevented in argon.

A review of various mechanisms of fracture in CMCs was suggested by Osmani et al. (1990) who considered the fracture energy for continuous and short-fiber composites. Predictably, the fracture energy was related to the properties of the fibers and matrix and to the processing conditions. The test temperature had a pronounced effect on the fracture toughness.

Theoretical analyses of CMCs subjected to a tensile load reviewed in the corresponding section below are often based on the concept of steady-state cracking, i.e., long cracks oriented perpendicular to the fibers (the load is applied along the fibers). The stress-strain curve is assumed to deviate from the linear section when these cracks appear. As the load is increased, the cracks bridging the fibers eventually reach saturation (another linear section of the stress-strain curve is observed at post-saturation loads, although the modulus is obviously smaller than the initial value). Experimental evidence of such steady-state cracking was observed by Marshall and Evans (1985). However, Kim and Pagano (1991) found in their experiments that matrix cracking occurs at a much smaller strain than the strain corresponding to the proportionality limit. The values of the strain corresponding to the initiation of the matrix cracking were in the range of failure strains for monolithic matrix materials. It was observed that in the materials where the coefficient of thermal expansion of the matrix is smaller than that of the fibers, the microcracking stress is higher. This was explained by axial compressive residual stresses in the matrix in such materials. Matrix crack initiation stresses predicted using the Aveston-Cooper-Kelly (Aveston et al., 1971) theory were higher than the experimental values, although Kim and Pagano (1991) indicated that the values of the matrix fracture energy and the interfacial shear stress used in their computations could be inaccurate. Note that in the subsequent paper of Pagano and Brown (1993), a different model of cracking was introduced. This model, called the full-cell cracking mode, is based on an annular matrix crack propagating from the interface. According to the authors, when the crack reaches adjacent fibers it causes debonding due to the very high radial tension.

Subsequent experiments of Dutton et al. (1996) addressed the effect of the relationship between the coefficients of thermal expansion of the fibers and glass matrix on cracking. Although two different types of fiber coatings were used, the authors assumed that the coating does not affect the stresses in the undamaged matrix, based on the results of Berriche and Dutton (1995). It was found that if the coefficient of thermal expansion of the matrix exceeds that of the fiber, an increase of the fiber volume fraction resulted in a higher matrix cracking stress. Predictably, a higher tensile axial residual stress resulted in a decrease of the matrix cracking stresses. In the situation where the coefficient of thermal expansion of the fiber exceeds its counterpart for the matrix, the cracking stresses were probably affected by a partial post-processing debonding. The authors listed such debonding as a reason for smaller experimental values of the matrix cracking stress, as compared to the stress predicted by the semiempirical model.

An experimental study of matrix cracking in CMCs at room temperatures was performed by Barsoum et al. (1992). One of the motivations of this research was to validate existing theoretical models (see discussion on the paper by Wang et al., 1992 in the section Analysis of Failure and Fatigue in Ceramic Matrix Composites). According to the observations of the authors, cracks in the matrix first appear near the edge of the specimen or between adjacent fibers having the largest spacing. These isolated cracks are usually arrested at the fiber/matrix interfaces. However, at higher loads, the cracks propagate along the interface, the length of the debonded region usually being on the order of the fiber spacing. If the load continues to increase, the cracks begin to propagate into the matrix and cause debonding of the next fiber/matrix interface. Eventually, the local cracks link and form a long crack extending across the width of the specimen, although the authors indicated that the mechanism of this process is not clear yet. Note that the mechanism of cracking observed in this paper correlates with the findings of Pagano and his colleagues (see, for example, Kim and Pagano, 1991) who found localized fiber-bridging cracks at the level of stresses much below than the stresses necessary for formation of long cracks according to the Aveston-Cooper-Kelly model).

Kagawa and Goto (1995) considered static cracking of unidirectional CMCs with SiC (Nicalon) fibers and borosilicate glass matrix. They observed that matrix cracking occurred perpendicular to the fiber (and load) direction at the stress 90-100 MPa that was close to the proportionality limit. The cracking started with formation of short cracks in the matrix-rich regions. As the load was increased, these local cracks linked and formed long cracks. The authors indicated that the initial formation of short cracks is not usually covered by available theoretical solutions, although they probably could consider the approach of Pagano and Brown (1993) to interpret their results. The mechanism of cracking described in this paper is in agreement with the observations of Barsoum et al. (1992).

Experimental study of Nicalon fiber calcium aluminosilicate (CAS) matrix by Sorensen and Talreja (1993a) illustrated the following development of damage (uniaxial tension). The linear elastic response without apparent damage was retained until the longitudinal strain was 0.13%. This was followed with a multiple matrix cracking with saturation reached at a strain of 0.5%. Frictional sliding dominated damage development as longitudinal strains increased from 0.5% to 0.7%. At higher strains, fiber and composite failure were observed. The authors suggested that the effects of interfacial debonding and sliding are negligible until the matrix cracks reach the saturation.

The response of various classes of CMCs to mechanical loading has been reviewed by Chou and Karandikar (1995). First matrix cracks in unidirectional CMCs were observed in the matrix-rich regions at longitudinal strains of an order of 0.12% (for Nicalon-fiber/calcium alumino silicate matrix). The cracks grow and new cracks appear, as the load increases. The stress-strain relationship becomes nonlinear at longitudinal strains exceeding 0.2%. An increase of the load beyond the level corresponding to the saturation of matrix cracks resulted in a linear section of the stress-strain curve, although the modulus of elasticity corresponding to this section is significantly smaller than that for the intact material.

The principal difference between the response of unidirectional and cross-ply CMCs is in the magnitude of strains corresponding to the initiation of matrix cracks. In cross-ply configurations, these strains may be almost an order of magnitude higher than their counterparts for unidirectional materials (Chou and Karandikar, 1995). Cracks first appear in the transverse layers. As the strain increases, cracks begin to extend into the longitudinal layers and the interfaces begin to fail.

Temperature effects on the matrix cracking were considered by Xu et al. (1992) for Nicalon-fiber SiC matrix composites. The fracture toughness of the matrix decreased significantly when test temperatures increased from room temperature to 800^0C. The regions of a high fiber concentration were particularly susceptible to cracking. A theoretical model gave values for matrix fracture toughness in good agreement with finite element and experimental results.

Davies et al. (1993) presented an experimental study of the onset of damage and the ultimate failure of CMC laminates at room temperature. It was shown that the elastic modulus and the thermal expansion coefficient can be evaluated by the rule of mixtures. The onset of cracking in unidirectional materials was close to that predicted by the Aveston-Cooper-Kelly model (Aveston et al. 1971). For $0/\theta/0$ laminates, two cases were distinguished. At small values of θ, the onset of damage was found at the stress level close to that for unidirectional composites, while in the case of a larger angle, the onset of damage could be predicted by the Tsai-Hill strength criterion (of course, the Aveston-Cooper-Kelly theory developed for unidirectional composites is not applicable in this case). The ultimate strength of the laminate was found in a good agreement with that predicted by the Tsai-Hill criterion for small values of the angle. At larger values of the angle, the strength appeared almost constant at approximately two thirds of the strength of a unidirectional composite.

The damage mechanism in CMCs was also discussed in detail by Sorensen and Holmes (1996). During the first cycle, matrix cracks can appear, if the maximum stress exceeds the matrix strength. The nonlinear constitutive response, associated hysteresis and permanent strains observed in experiments are due to cracks in the matrix and debonding. A reduction of the modulus is observed during cracking, but it usually stabilizes when the matrix crack density reaches saturation (note a similarity of this saturation to CDS, i.e. characteristic damage state, introduced by Reifsnider, 1977, for polymeric composites). The material fails when the residual composite strength declines to the level of the maximum applied stress as predicted by the residual strength degradation theories (Sendeckyj, 1990). Following Sutcu (1989), Curtin (1991) and Rouby and Reynaud (1993), it was suggested the composite strength could be predicted by models representing the fiber strength by the two-parameter Weibull distribution.

It was indicated that some of the shear-lag models used to predict a cyclic stress-strain response of unidirectional CMCs are based on a number of assumptions, i.e., a frictional interface, negligible fiber roughness and Poisson's effect, etc. The interfacial shear stress is often assumed constant within a current cycle. Note however, that some of these assumptions are relaxed in recent papers reviewed in the section on the analysis of CMCs. The paper outlines the formulae for the average composite modulus, permanent strain for $R = 0$ and the frictional energy dissipation per unit time for different combinations of full and partial interfacial slips during cycling.

The deterioration of the fiber-matrix interface was identified as the primary damage mode at room temperatures. The mechanism of damage emphasized is abrasive wear of roughness along the interface resulting in a decrease of the interfacial shear stress. A decrease in the composite residual strength and a longer pull-out fiber length are the consequences of a smaller interfacial shear stress.

Lynch and Evans (1996) studied the effect of off-axis tensile loading on the response of CMCs. The cross-ply laminate considered had Nicalon fibers and borosilicate doped cordierite matrix. The load was applied at the angles 0°, $30^\circ/60^\circ$, and 45° with respect to the fiber direction. In all cases, the inelastic behavior was due to matrix cracking. The dominant matrix cracks were perpendicular to the tensile stress when the load was applied at 0° or 45°. For the angle of $30^\circ/60^\circ$, the cracks were tilted relative to the applied load due to the loss of symmetry of the load with respect to the fibers. IWhen the angle between the load and the fibers was equal to 45°, the cracks were oriented along the fibers and appeared just before failure. While in the $30^o/60^o$ configuration, cracks were also oriented along the fibers, they were detected load levels below the failure limit and experienced a stable growth when the load was increased. Based on these observations, the authors concluded that debonding occurs when deformation exceeds the level corresponding to the matrix cracking.

Several papers have recently been published where measurements of hysteresis in CMCs were suggested as means to predict their mechanical properties (Domergue et al., 1995, 1996; Vagaggini et al., 1995). Vagaggini et al. (1995) presents a theoretical treatment of the problem (see section "Analysis of Failure and Fatigue of Ceramic Matrix Composites"), two other papers are experimental and will be discussed here. Domergue et al. (1995) considered unidirectional SiC/CAS and SiC/SiC composites subjected to tensile cycling. They showed that measurements obtained from hysteresis loops generated in a single tension test can be used to predict stiffness loss, residual misfit stress, interfacial shear stress and debond energy. A subsequent paper (Domergue et al. 1996) applied similar principles to the analysis of cross-ply CMCs. The failure mechanism is more complicated as compared to unidirectional materials in this case. It was suggested the stress-strain response can be predicted based on four parameters associated with the change of the elastic compliance due to cracking, interfacial frictional slip, debonding and relief of the residual stresses using hysteresis measurements.

Creep of fibers, matrix and interfaces is a phenomenon that has to be considered when analyzing high-temperature fatigue. Jakus and Nair (1990) considered high-temperature cracking attributed to creep in SiC whisker reinforced alumina composites. The cracks initiated at a critical creep strain that was affected by temperature. Tensile creep in cross-ply Nicalon-fiber/calcium aluminosilicate matrix CMCs subjected to tensile loading was studied by Wu and Holmes (1993). They showed that at 1200^oC the creep rate increases with stress. Surprisingly, the creep rate was approximately the same for unidirectional composites with fibers oriented in the load direction and cross-ply laminates. The results of these studies emphasize a significant effect that creep may play in failure of CMCs subjected to large peak stresses.

A comprehensive overview of creep in CMCs was presented by Chermant and Holmes (1995). As indicated in this review, CMCs have a very good creep resistance at temperatures up to 1473^oK. At higher temperatures, thermodynamically stable materials for fibers and their coatings becomes important. Also, stress redistribution in the process of creep has to be addressed, although the authors emphasized difficulties involved in the development of an analytical model necessary to study this phenomenon.

Tressler and DiCarlo (1995) considered creep of several types of ceramic fibers. They indicated that high-temperature applications imply a creep strain limit on the order of 1% and therefore, the creep research should concentrate on the primary stage of the phenomenon. The analytical modeling of creep in CMCs is addressed in the section on the analysis of failure and fatigue.

In conclusion of this section, typical stress-strain curves for unidirectional CMCs are shown in Figs. 1 and 2. The former figure models the behavior of the material with SiC Nicalon fibers with polyvinylacetate coating and aluminosilicate matrix that was observed by Zawada et al. (1991). The material whose behavior is depicted in Fig. 2 has Nicalon fibers and CAS matrix (Davies et al., 1993; Chou and Karandikar, 1995). The principal difference between two types of the stress strain curves is related to a presence of one (Fig. 1) or two (Fig. 2) linear sections.

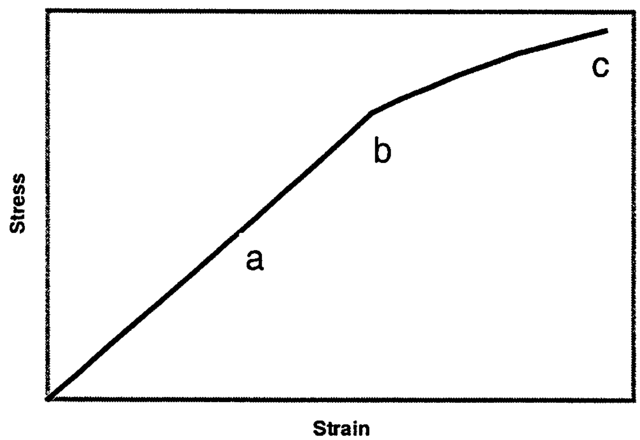

Figure 1. Stress-strain curve of a unidirectional CMC with a single linear section; a = matrix cracking stress, b = proportionality limit, c = failure.

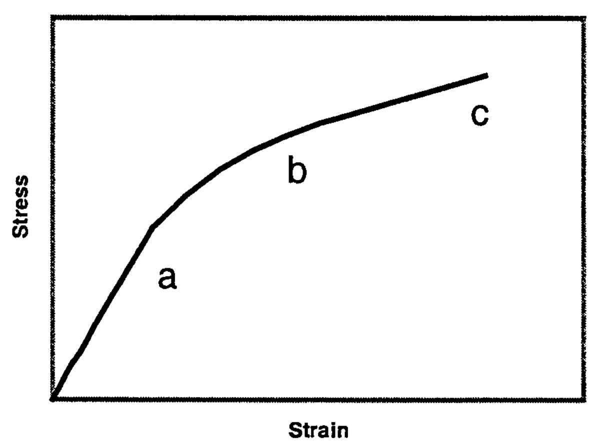

Figure 2. Stress-strain curve for a unidirectional CMC with two linear sections; a = matrix cracking stress, a-b = progressive cracking region, b = matrix crack saturation strain, b-c = linear region with a lower modulus of elasticity, c = failure.

Mechanical, Thermomechanical and Thermal Fatigue in Ceramic Matrix Composites

The studies reviewed in this section consider fatigue of CMCs under mechanical loads at room temperature (mechanical fatigue), under mechanical loads at elevated temperatures (thermomechanical fatigue) and thermal fatigue that occurs as a result of temperature cycling. It should be emphasized that because CMCs are produced at high temperatures, high residual thermal stresses can exist at the room temperature. These stresses can result in the matrix cracking prior to the application of loads. Although an elevated temperature can reduce these thermal stresses, it accelerates oxidation and in general, fatigue resistance of CMCs decreases with temperature.

A number of early experimental studies of fatigue of CMCs was referred to in the monograph of Chawla (1993). A comprehensive review of fatigue of ceramics can be found in the book of Suresh (1991). The latest research on fatigue of CMCs has been outlined by Sorensen and Holmes (1996).

An experimental study of the tensile fatigue of Nicalon fiber lithium aluminosilicate (LAS) glass-ceramic matrix composites was published by Prewo et al. (1986). Two different matrices were considered in this paper with a linear and nonlinear constitutive stress-strain response, the latter being attributed to the matrix cracking. The former specimen did not illustrate a reduction in the residual tensile strength and stiffness after cycling, while the latter specimen showed a reduction of the stiffness after cycling to stresses exceeding the proportionality limit. This was explained by matrix microcracking at high stresses (Chawla, 1993).

Zawada et al. (1991) presents experimental results of Nicalon fiber aluminosilicate matrix composites subjected to tensile and fatigue loadings. The fibers were coated with a layer of protective polyvinilacetate. A careful manufacturing procedure described in the paper resulted in a material free of porosity and matrix-rich regions where the cracks could initiate at low stresses. The proportionality limit of a uniaxial material subjected to tension was much higher than the matrix microcracking stress. Two proportionality limits were detected in cross-ply laminates: the first limit corresponded to the matrix cracking of the 90° plies, while the second limit reflected the damage in the longitudinal plies. The changes of the material modulus of elasticity detected in fatigue tests were attributed to the onset and accumulation of damage. However, modulus recovery was observed close to the end of fatigue life of specimens loaded by intermediate stresses. The authors suggested several possible reasons for this phenomenon, an increase in the interfacial sliding friction seems to be most logical.

The fatigue limits for both unidirectional and cross-ply composites were close to the highest proportionality limit, i.e., they exceeded the matrix microcracking stress. This implies that fatigue failure occurs when the laminate is subject to the stresses that are sufficiently high to cause failure of all plies (including longitudinal plies in cross-ply configurations). The fact that the fatigue strain limits were very close for unidirectional and cross-ply specimens caused the authors to suggest that the strain in longitudinal plies may be the controlling fatigue failure mechanism.

Fatigue of cross-ply CMCs was also investigated by Wang et al. (1991) who observed that the mode of damage included matrix cracking and spalling. Tensile cyclic creep was also detected, particularly in the vicinity of notches.

Karandikar and Chou (1992) considered the modulus reduction rate defined as $(-1/E_0)(dE/dN)$ at a given value of E/E_0 where E_0 is the initial modulus, E is the current modulus and N is the number of cycles. A correlation between crack density and stiffness reduction was observed for unidirectionally reinforced Nicalon fiber calcium aluminosilicate (CAS) composites.

Fatigue of unidirectional and cross-ply Nicalon fiber (CAS) matrix composites at room temperature was considered by Habib et al. (1993). The saturation matrix crack spacing was identical both in tension and in flexural cycling. This spacing was close to 0.2 mm, i.e., about 13 fiber diameters, while the spacing in carbon/Pyrex composites found in the previous work of the authors was about 0.3 mm. The damage nature did not differ in a monotonical and fatigue loading which resulted in the suggestion to use a single model for predicting failure in both monotonic and cycling loading.

Suresh (1991) discussed the issue of fatigue in brittle solids and pointed out that it can be traced to microcracking, interfacial sliding and creep. In particular, in composites (CMCs) the frictional sliding along the interface between the fibers and the matrix can be responsible for fatigue damage. In addition, it was indicated that debris particles formed by the repeated contact of the crack faces can promote fatigue damage. An additional mechanism of fatigue damage was related to thermal residual stresses along the interfaces. Finally, glassy phases that can originate during processing or due to oxidation become dangerous during thermomechanical or thermal fatigue, because they result in a stronger fiber-matrix interface. As is shown in the present review, all mechanisms identified by Suresh (1991) are responsible for fatigue damage of CMCs.

An important factor that affects both the matrix cracking as well as the interfacial sliding during cycling is the frequency of loading. Holmes et al. (1994) illustrated that a higher frequency results in a serious decrease of the fatigue life. On the other hand, Gomina (1995) who considered fatigue of cross-ply Nicalon fiber lithium magnesium aluminosilicate matrix composites at low frequencies (1Hz and 10Hz) did not record a noticeable effect of the rate of loading on the modulus of elasticity for an identical number of cycles.

Evans et al. (1995) emphasized that fatigue occurs as a consequence of matrix cracks that appear if the stresses exceed the matrix cracking strength. The interface between the fiber and the matrix is very important since it is involved in the debonding process responsible for fatigue damage. Three possible mechanisms of fatigue were identified with the changes in the interface sliding resistance due to cycling, abrasion-induced degradation of fiber strength due to cyclic sliding along the interface, and crack growth within the matrix. The effect of temperature was associated with embrittlement, that is the result of oxidation of fiber coatings or fibers themselves. The paper presented a detailed analysis of the process of interface degradation of ceramic fibers. This is the dominant fatigue model if the matrix has low toughness. In this case crack extension occurs if the energy release rate at the tip becomes equal to the matrix fracture energy. It was suggested to consider this criterion in conjunction with the interfacial shear stress variations during cycling. On the other hand, if the matrix is tough, a Paris law type relations govern the crack propagation. The authors proposed generating the S-N curve, determining the permanent strain and a reduction of the modulus using measurements of the hysteresis loop.

Mechanical fatigue of carbon/carbon CMCs was studied by Tallaron et al. (1995). They established the presence of a fatigue limit for the material and compared the hysteresis loops at the maximum stresses below and above this limit. While the loops associated with the stresses below the fatigue limit were not affected by the number of cycles, a large change in the loop area was observed as material loaded above the fatigue limit began to fail.

The mechanism of mechanical fatigue in CMCs was also outlined by Sorensen et al. (1995). If the maximum applied stress is below the fatigue limit, cycling does not result in damage. However, if the stress is sufficiently high, cracks appear during the first cycle. Existing cracks develop and new cracks are added during subsequent cycles until the cracking saturates. If the applied stress is high, saturation can even occur during the first cycle. Once crack saturation is achieved, interfacial sliding becomes the dominant mode of damage. The interfacial frictional shear stress increases with the rate of loading. At high temperature, oxidation occurs along the fiber-matrix interface resulting in a higher interfacial bonding. This stiffening of the interface further decreases damage tolerance of the material. The wear of the fibers due to progressive damage of the interface becomes important. Failure occurs when the strength of the composite layer decreases to the maximum stress during the cycle.

A series of papers by Pagano (1997a,b) considered failure in unidirectional axially loaded "ideal" CMCs with a controlled fiber spacing. The material considered in the former paper consisted of SiC fibers, two coatings (carbon coating inside and TiB_2 coating outside) and a glass-ceramic matrix. In the latter paper, the fibers were uncoated. The author observed the presence of matrix microcracks that were perpendicular to the fibers at the stress level below the proportionality limit. While the spacing of the matrix cracks varied at their initiation (statistical phenomenon), at failure it was approximately 100μm (note that this spacing is of the same order as that observed by Habib et al., 1993). Debonding along the carbon-SiC interface was triggered by radial tensile stresses and propagated as a result of interfacial shear. Fiber breaks were observed in the regions where a continuous interface was preserved during loading so that the stress transfer was not imperiled.

A comprehensive review of fatigue of CMCs with continuous fibers has been published by Sorensen and Holmes (1996). They emphasized that the performance of CMCs can be imperiled at high temperature as a result of oxidation of the fiber-matrix interface. This oxidation results in strong bonding that is detrimental to the fatigue resistance of the material. A weak interfacial layer necessary for an improved damage tolerance can be achieved by using special fiber coatings. Time-dependent oxidation, diffusion and creep were identified as complicating mechanisms that are present during high-temperature fatigue.

Wear damage on the fiber surface accumulates with cycling. A reduction in the stress level at which 63.2% of the fibers fail (this number was employed in the model for the composite strength based on the two parameter Weibull distribution) and a change of the Weibull modulus can serve as the indicators of the fiber wear. Alternatively, it is possible to detect the fiber wear from a decrease of the overall composite strength that is related to the two previously mentioned parameters. The wear rate decreases with a decrease in the stress ratio R. A large number of cycles is usually required to cause wear damage of typical ceramic fibers. Therefore, two mechanisms that are distinguished in the review of Sorenson and Holmes are low-cycle fatigue associated with a decrease in interfacial shear stress and a high-cycle fatigue due to wear of the fibers.

Summarizing fatigue methodology outlined in the review, the authors suggested 8 parameters necessary to describe and predict the composite response. These are: moduli of the matrix and the fibers, volume fraction and radius of the fibers, axial residual stress in the fibers, a regular matrix crack spacing, interfacial shear stress, and the characteristic strength introduced in the composite strength model based on the Weibull distribution. All these parameters are easily determined or can be evaluated experimentally using tensile and fatigue tests.

Sorensen and Holmes (1996) presented a discussion on high-temperature fatigue. In the case of a high-temperature isothermal mechanical fatigue, creep of the fibers and matrix is an additional damage mechanism discussed by Wu and Holmes (1993). Accelerated oxidation of the interface is another possible damage mode. The issue of thermal cycling at a constant level of external stresses that was first considered by Cox (1990) was also discussed. In this case, interfacial sliding associated with a mismatch between the coefficients of thermal expansion of the fibers and the matrix and a variation of the interfacial shear stress with temperature become the dominant factors affecting fatigue damage. The concepts of in-phase and out-of-phase thermomechanical cycling were introduced in the paper. The former is characterized by opposing thermal axial strains in the fibers and the matrix, while the latter corresponds to the same sign of these strains.

High temperature mechanical fatigue was investigated by Holmes (1991). In this paper, the results of fatigue tests conducted at 1000-1200°C on unidirectional CMCs were reported. When the specimens were cyclically loaded with maximum stresses above the proportionality limit, the elastic modulus decreased as the fatigue damage progressed. However, at a low stress level, below the fatigue limit, the elastic modulus remained constant for up to 2,000,000 cycles, although creep was present. Fatigue that occurred at sufficiently high stresses was characterized by decreasing stiffness, increasing stress-strain hysteresis and strain ratchetting (cyclic creep). A decrease of the tensile stress ratio R was shown to result in a shorter fatigue life.

Prewo, et al. (1989) considered high temperature mechanical fatigue of a cross-ply SiC fiber high temperature LAS glass-ceramic matrix composite. They observed a temperature-related reduction of the material ultimate strength in air, although this strength was not affected by high temperatures, up to 1300°C, in argon. Note that such differences between otherwise identical tests can be explained by oxidation that occurs in the specimen tested in air. The failure stress for the specimens tested in air was significantly lower than their tensile strength.

The effect of temperature on fatigue of Nicalon fiber CAS matrix CMCs was also considered by Allen et al. (1993). A difference in the mechanism of damage at room and elevated temperatures was emphasized in this paper. While at room temperature, the damage was characterized by multiple cracking and subsequent fiber debonding. Localized cracking associated with a high interfacial sliding resistance was observed at 1000°C. Considering that the experiments were carried out in air, it is probable that an increase in the interface frictional resistance was due to oxidation.

Reynaud et al. (1995) considered mechanical fatigue of cross-ply and cross-weave woven CMCs at room and elevated temperatures. It was observed that the area of the hysteresis loop has a maximum at a certain value of the interfacial shear stress. The dominant damage mechanism at both the room and elevated temperatures was related to wear at the fiber-matrix interface. However, the wear was modified at a high temperature due to a release of the radial thermal residual stresses.

An experimental study of high temperature mechanical fatigue of silicon carbide (SiC) fiber barium magnesium aluminosilicate matrix (BMAS) composites was reported by Vanswijgenhoven et al. (1996). This material had a carbon-rich reaction layer at the interface after the production. At temperatures below 500°C (embrittlement temperature), the material experienced damage development and fatigue, damage without fatigue or no damage at all, dependent on the magnitude of external stresses. Above the embrittlement temperature, the regime at which damage occurrence was not accompanied by fatigue failure gradually disappeared and the fatigue resistance abruptly decreased.

Contrary to the previous papers dealing with mechanical fatigue at a constant high temperature, the issue of thermal fatigue involves cycling of temperature at a constant mechanical load (in a particular case, this load can be absent). In the paper of Zawada and Wetherhold (1991), the experiments were conducted on Nicalon fiber aluminosilicate glass matrix composites subjected to temperature variations from 250°C to 700°C and from 250°C to 800°C. The reason for the low limit of 250°C selected in experiments was related to a necessity to avoid longer cooling periods due to nonlinear cooling rates that occur below 250°C. The tensile modulus was not affected by thermal cycling. However, the specimens heated to 700°C exhibited a drastic reduction in the strength, while the specimens tested to 800°C did not have such a large strength deterioration. The authors explained this contradictory result by a large reduction of the matrix viscosity at higher temperatures and the subsequent flow at the stress concentration and flaw locations. This matrix flow results in a lesser oxygen penetration within the specimen and the corresponding reduction of the embrittlement rate. Accordingly, it was suggested that the strength of the material could be improved by applying a higher temperature.

Both thermal and theromomechanical fatigue of 16-layer unidirectional Nicalon fiber CAS composites were investigated by Butkus and Holmes (1993). The thermomechanical fatigue experiments were conducted at 1100°C and involved tensile loading with the stress ratio $R = 0.1$. Thermal fatigue experiments were carried out between 500°C and 1100°C. The authors concluded that matrix creep was the principal damage mechanism while stresses in the fibers were below the level necessary to trigger creep.

Erturk, et al. (1995) studied the effects of thermal cycling at a constant applied tensile stress. Thermal fatigue of SiC continuous fiber Si_3N_4 matrix composites was considered under impinged jet fuel flame, a constant applied tensile stress and thermal cycling in the temperature range 500-1300°C. It was shown that failure may appear in the region exposed to the highest temperature (SCS-9 SiC fiber Si_3N_4 composites failed within the flame impinged zone). On the other hand, failure may also occur in the adjacent region characterized by high thermal stresses (SCS-6 fiber Si_3N_4 composites failed outside the impinged region, in the area characterized by high-temperature gradients). The dominant fatigue failure mechanism of SCS-9 SiC fiber Si3N4 composites was associated with partial degradation of the columnar structure of the fibers. High temperature resulted in a chemical reaction between the outer carbon layer of the fiber and the matrix. The thin layer produced by chemical reaction consisted of mainly amorphous SiO_2 and some $MgY_4Si_{13}O_{13}$ phase.

Experiments on thermal cycling fatigue of SiC (Nicalon) fiber ceramic matrix cross-ply composites have been reported by Kharrat et al. (1996). The material was processed at 1300°C and thermally cycled in air or argon. High residual stresses were recorded after processing. These stresses were attributed to ply anisotropy as well as to a mismatch between the coefficients of thermal expansion of the fibers and the matrix. Thermal cycling was carried out between 500 and 1150°C. Microcrack saturation in the matrix was achieved after 300 cycles and explained by a CTE mismatch between the phases constituting the matrix. While the effect of this cracking on the overall composite properties was small, it promoted the chemical degradation of the interfaces and fibers. An enhanced fiber oxidation accompanied by strength loss was observed in air and vacuum. The overall strength loss in argon was 18%, while the corresponding number for air and vacuum environments was close to 70%.

A detailed discussion of cyclic fatigue of CMCs at room and elevated temperatures was suggested by Reynaud (1996). This discussion was based on the experiments with 2-D woven and laminated composites. At room temperature, the average modulus decreased with cycling, while the area of the stress-strain loop and residual strain increased. However, the area of the loop did not significantly change after 10,000 cycles. Discussion on fatigue at elevated temperatures was limited to the case where temperature is not high enough to cause chemical reactions (oxidation). In this case, the principal effect of temperature is confined to a reduction of the thermal residual stresses and interfacial wear is the dominant fatigue mechanism. According to Reynaud (1996), in this situation, the total interfacial shear stress is composed of short-term and long-term contributions. The former is associated with the fiber roughness and morphology of the interface, while the latter is due to residual thermal stresses and such factors as fiber waviness, Poisson's effects, etc. The interfacial shear stresses increase with temperature if the radial thermal expansion coefficient of the matrix is lower than that of the fibers.

The illustrations presented below reflect effects of cycling on the modulus of elasticity of unidirectional CMCs (Fig. 3) as well as the changes of the matrix crack density and the interfacial shear stress (Fig. 4). Note that the behavior shown in Fig. 3 (Zawada et al., 1991) is typical if the maximum applied stress exceeds the matrix cracking stress. A noticeable decrease of the modulus of elasticity in the first phase of the fatigue life corresponds to an accumulation of the cracks in the matrix. Once the matrix crack saturation is achieved, the changes of the modulus of elasticity are relatively small. However, the modulus declines abruptly at the end of the fatigue life. Fig. 4 illustrates a stabilization of both the interfacial shear stress as well as the matrix crack density that occur at the end of the first phase of the fatigue life (a similar figure appears in the paper of Sorensen and Holmes, 1996).

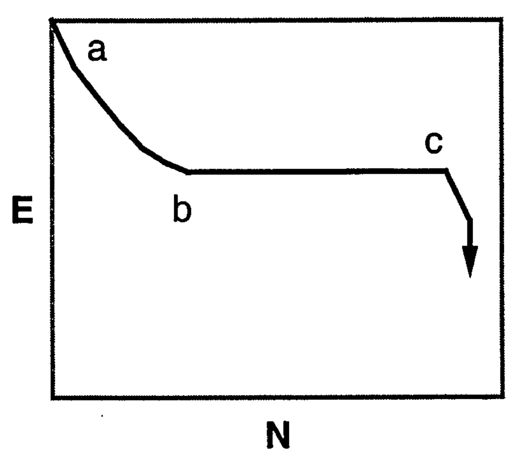

Figure 3. Effect of cycling on the modulus of elasticity of a unidirectional CMC; a-b = progressive matrix cracking, b = matrix crack saturation strain, b-c = zone of relatively small or negligible changes of the modulus of elasticity, c = failure.

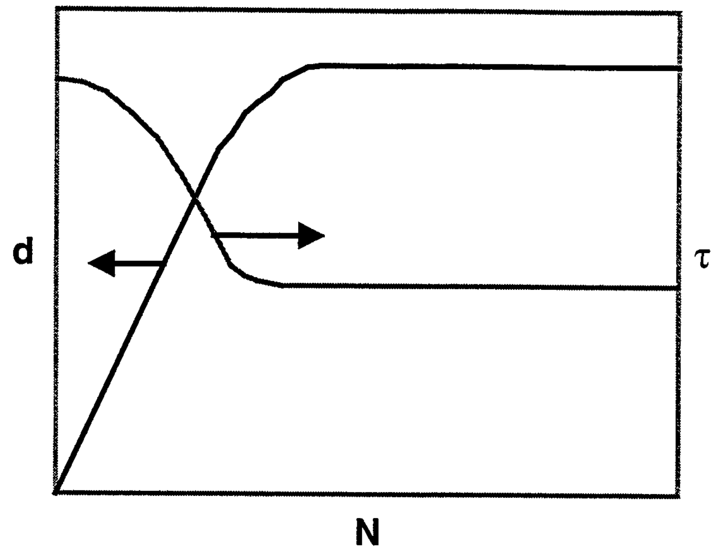

Figure 4. Effect of cycling on the interfacial shear stress (τ) and the matrix crack density (d). The matrix crack density is an inverse of the crack spacing.

Effect of Oxidation at High Temperatures

As follows from the previous discussion on thermomechanical and thermal fatigue, oxidation presents a major problem at high temperatures. In particular, SiC fiber-reinforced SiC matrix composites are often considered primary candidates for elevated temperature applications due to their attractive properties. If a graphitic fiber coating is used, the problems related to the interfacial debonding and fiber pullout can be reduced or prevented and both toughness and damage tolerance of the material improved. However, a degradation of properties is observed at an elevated temperature (400°C and above) as a result of oxidation of graphite and formation of a SiO_2 bond sublayer between the fiber and the matrix. This results in a higher interfacial strength and consequently, lowering of the toughness. In this section, the results from several papers that specifically refer to the effect of oxidation are reviewed. Additional references to this phenomenon are present elsewhere in the text.

Wetherhold and Zawada (1991) and Zawada and Wetherhold (1991) studied the behavior of ceramic grade Nicalon fiber in aluminosilicate glass matrix under isothermal and thermal cycling. Between 650-700°C all samples showed rapid oxidation and loss of strength. Oxidation behavior overshadowed any thermal cycling effects for these test conditions. The embrittlement was attributed to oxygen infiltration from the surface which destroyed the weak carbon-rich interface in the composite. At 800°C, less embrittlement was observed and the fiber-toughening effect was retained due to an enhanced matrix flow reducing the oxygen pemetration.

Fiber reinforced ceramic composites considered for gas turbine engine applications were investigated by Vaidyanathan et al. (1995). The material had Nextel™

312 fibers with a BN rich surface layer and a Blackglas^{TM} matrix. After the material was subjected to 600°C for 20-1000 hours, oxidation and the corresponding degradation of tensile strength and modulus were observed. Short-term exposure to the oxidation environment was relatively harmless. After oxidation for 96 hours the tensile strength of the composite was not affected. The flexural strength remained unaffected by oxidation for 111 hours, although the modulus of elasticity decreased in both tensile and flexural experiments after a relatively short exposure to oxidation. The effects of oxidation became apparent with time. For example, the strength was reduced by 50% relative to as-prepared specimens after 500 hours of exposure to oxidation. Based on these results, the authors concluded that oxidation beyond 200 hours may be embrittling the composite.

The effect of oxidation on creep of the Nicalon fiber reinforced SiC composite with 0.3 micron graphite fiber coating was studied by Lin et al. (1995). The lifetime of the specimens was affected by the load when applied stresses exceeded the proportionality limit (100 MPa). An increase of the test temperature was detrimental to the lifetime and the authors related this phenomenon to oxidation of the graphite interfacial layer in the intermediate range of temperatures (400-700°C). At higher temperatures, in addition to oxidation of the graphite coating, a silicate interfacial layer was formed due to oxidation of the SiC matrix and the Nicalon fiber.

An example of a development of an oxidation resistance CMC is presented in the paper by Ishikawa et al. (1995). The paper represents an outline of technology for a manufacture of 3-D woven composites whose ambient tensile strength reached 440 MPa.

In a recent paper, Kahraman (1997) studied tensile response of cross-ply CMCs under high temperature. The material consisted of Nicalon fibers and a glass-ceramic matrix (CAS-II). Embrittlement was observed and attributed to the oxidation of the carbon-rich interphase in the longitudinal layers that occurs in the regions where the matrix cracks approach the fibers. As a result of oxidation and the consequent higher interfacial bond strength, the cracks begin to grow through the fibers. These observations are in an agreement with the previous paper of Kahraman and Mandell (1996) where embrittlement of a unidirectional Nicalon/CAS-II composite was observed at high temperatures.

A review of existing efforts directed toward a protection of carbon fibers in CMCs from oxidation at temperatures above 400°C was published by Westwood et al. (1996). The authors concluded that an efficient fiber coating should consist of several ceramic layers that they called "functionally active layers." While silicon based layers (SiC or Si_3N_4) can provide erosion protection; these layers should be in turn protected from oxidation. Such protection can be offered by a layer with boron and/or silicon components capable of sealing microcracks by forming glassy phases when exposed to oxygen.

An analytical model for the crack growth in the presence of oxidation was suggested by Evans et al. (1996). Two classes of crack growth were considered. Reaction controlled cracking occurs at a relatively low stress and because propagation is slow, oxygen concentration within the crack is uniform. The fibers fail as their strength is reduced to the value of the stress calculated accounting for the stress concentration. At higher stresses, the oxygen consumption near the crack mouth increases with temperature and fracture is related to diffusion of oxygen.

Sun et al. (1997) have recently published an analysis of the effect of temperature on the properties of material composed of Nicalon fibers with a dual boron nitride and SiC coating and barium magnesium aluminosilicate matrix. Creep tests conducted at 600 and 950^{o} C revealed a noticeable degradation of the strength after 500 hours in the specimen subjected to the lower temperature. However, the strength of the specimen tested at the higher temperature was not affected. This phenomenon was due to a relatively slow oxidation within the cracks along the BN-fiber interface at 600^{o}C that left the cracks unsealed, so that further damage and oxidation could not be prevented. At the higher temperature, the interface was quickly protected by the oxidation products of SiC fibers that filled the cracks. These findings raise an interesting question regarding a possible artificial preventive oxidation through a periodically applied high temperature.

Interfaces

As follows from the previous paragraphs, the interface between the fiber and the matrix is the region where damage accumulates after the saturation of the matrix cracks. In addition to the interfacial sliding, oxidation can result in a reduction of the fatigue life and the ultimate loading capacity. Papers considered here discuss interfaces and their performance under thermal and mechanical loads. Analytical work on the evaluation of the interfacial shear stresses is included in the papers referenced in the section on the analysis of failure and fatigue of ceramic matrix composites.

The effectiveness of BN (boron nitride) fiber coatings was considered by Singh (1988) for SiC-fiber composites with mullite and zircon matrices. The interfacial shear stress was reduced compared to uncoated specimens.

Two different SiC/SiC composites were compared by Frety and Bousage (1990). The difference being in the carbon coating of the fibers both materials were subjected to elevated temperature. While the material with the carbon coating did not show a loss of strength after a continuous exposure to 1400^{o}C in air, a significant decrease of strength was observed in its counterpart without the coating.

Fischbach and Lemoine (1990) found that a thin coating applied to Nicalon fibers resulted in a strength decrease of the fiber by about 40%. However, the authors emphasized potential advantages of a weak interface created by the coating and the resulting enhancement of the composite toughness.

Morscher et al. (1997) studied BN interphases that were previously found superior to C interphases for $SiC_F/BN/SiC_M$ composites under stress-oxidation conditions. They concluded that higher processing temperatures on the order of 1400°C, can improve durability of the interphase and increase life of CMCs.

The influence of various nitrogen glass matrices on fracture of CMCs was compared by Zhang and Thompson (1996). The fibers were Nicalon or Hi-Nicalon. The interface was affected by the thermal expansion coefficients of the constituents. The authors observed that yttrium sialon matrices have a higher thermal expansion coefficients than the fibers used in the specimens and the resulting compression of the interface resulted in a high frictional stress. Accordingly, the failure of such composites was brittle. This problem did not exist for material with Nicalon fibers and a lithium sialon matrix. The fact that Hi-Nicalon fibers were susceptible to a reaction with the matrix illustrates a potential difficulty involved in processing of CMCs.

Eldridge et al. (1997) focussed on the cyclic push-in behavior of C and BN (boron nitride) coated fibers. The cyclic fiber push-in tests conducted on the material with a carbon-rich interface showed a progressive decrease of interfacial frictional sliding stresses and increased fiber sliding distances during cyclic loading at room temperature. The environment was also considered and it was found that a fiber-bridged matrix crack exhibits a rapid growth with cyclic loading in air but would stabilize or even show decreasing crack-opening displacements in nitrogen. In particular, experiments with BN-coated fiber CMCs showed that the interface is susceptible to chemical reactions and oxidation, if it is exposed to air.

Jaskowiak et al. (1997) considered composites with sapphire fibers (volume fraction 30%, random distribution), and zirconia interface. The objective was to evaluate the mechanical and interfacial properties of sapphire reinforced alumina at elevated temperatures in an oxidizing environment. Test temperatures varied from 800 to 1200°C. The loss of composite strength at elevated temperatures was directly correlated to the loss of strength of sapphire fibers. Although precise measurements of interfacial shear stress were not conducted, zirconia was believed to remain effective as an interfacial coating which may produce crack deflection around the fibers within the range of temperatures considered in the paper.

Work toward experimental evaluations of the interfacial shear stress from mechanical tests has also been conducted. Mentioned here are the papers by Hsueh (1993) and Hsueh et al. (1995) where an approach to a prediction of the interfacial shear stress based on the fiber push-out tests was considered. The former paper presented the solution for smooth fibers, while the second paper incorporated the roughness effect into the analysis. Zhou et al. (1995) applied an energy method to predict interface interfacial shear stress and debonding during the fiber push-out tests. The solutions presented in the paper can be applied to the task of predicting interfacial shear stresses from push-out experiments.

A study by Goettler and Faber (1990) presented the results of tests on CMCs formed from SiC fibers and soda-boronosilicate glass matrices. In particular, the effect of a carbon coating was considered. While the coating reduced oxidation, an increase of the carbon content in the coating yielded a stronger bond, i.e., it was shown that the optimum properties of the coating should be sought. An increase in the thermal residual strain mismatch resulted in a higher interfacial shear strength and stress. However, the interfacial shear stress dropped at a certain level of the residual strain due to matrix cracking. The interfacial shear strength was shown to be an almost linear function of the loading rate. This observation was in agreement with the previous work of Phillips (1974) who showed that a higher loading rate results in a decreasing work of fracture and a shorter pull-out fiber lengths.

Kumar and Singh (1994, 1996) presented the results of their experimental (first paper) and finite element (second paper) analyses of the influence of the fiber coating parameters on interfacial cracking of Nicalon SiC matrix composites. The work of fracture and the ultimate strength increased with coating thickness in both experimental and numerical studies. Conclusions on the effects of the modulus and thickness of coating on the hoop and tangential stresses were also presented for the cracks at the fiber-coating and matrix-coating interfaces.

In conclusion of this section, two reviews of design, theoretical developments and characterization of the interfaces should be mentioned (Lewis and Murthy, 1991; Evans et al., 1991). In particular, Evans et al. (1991) emphasized a controversial aspect of the interfaces in the problems of fracture and creep. A weak interface is necessary to improve the fracture resistance of the material over that of the matrix. But, a strong interface is needed to enhance the creep behavior. Two independent parameters, i.e., the debond energy and the sliding shearing stress were specified as the characteristics of the interfacial effects.

Frictional Heating in Ceramic Matrix Composites

One of the principal failure modes of CMCs is related to a deterioration of the interface between the fibers and the matrix. This results in sliding of the fibers relative to the matrix due to a local debonding. Fiber slip is a function of a number of parameters, such as the maximum nominal stress, frequency of loading, etc. The interface is usually analyzed using a shear-lag model and an interfacial shear stress is used to characterize the friction between the fiber and matrix. A stress acting between two surfaces that are moving with respect to each other produces work and increases the local temperature.

Holmes and Shuler (1990) and Holmes and Cho (1992) observed internal heat generation during fatigue of carbon-fiber/SiC matrix composites. They conducted experiments at variable frequencies and a constant stress amplitude, and at a constant frequency and a variable stress amplitude. The effect of cycling on the material temperature was considered. Testing in the range of frequencies between 1Hz and 85Hz illustrated that the temperature of the material increases almost proportionally to the rate of loading. A very significant increase of temperature was also found in response to an increase of the maximum stress. Finally, although temperature increased rapidly in the initial phase of cycling, it remained stable and slightly decreased with continuous cycling. The latter result can be attributed to initial cracking of the matrix to saturation (temperature increases accordingly) and a subsequent wear of the interface as damage accumulates (this results in a decrease of friction and temperature).

Cho et al. (1991) developed a solution relating the interfacial shear stress to the rise of the temperature of a specimen. As indicated in this paper, the model can be used to continuously monitor the interfacial shear stress as a function of temperature. Two different cases were considered dependent on a relation between the active interfacial slip length and the matrix crack spacing. These cases include a partial slip (slip length is less than one-half the matrix crack spacing), and a partial/full slip (slip length exceeds one-half the matrix crack spacing).

Analysis of Failure and Fatigue of Ceramic Matrix Composites

This section outlines the analytical studies on the mechanics of failure and fatigue of CMCs. The research considered here refers only to laminated CMCs, while the theories of woven CMCs are reviewed in the separate section.

The strength of fibers in CMCs is usually evaluated based on the Weibull statistics (Sutcu, 1989). The statistical approach was suggested because of a variable pull-out length of fibers observed in experiments. This implies a dominant effect of flaws on the fiber failure and accordingly, a statistical nature of the problem. The results generated using the model were in a reasonable agreement with experimental data for a Nicalon fiber LAS ceramic matrix composite. It should be noted that attempts has been made to develop statistical models for residual strengths and fatigue life of laminates (Diao et al., 1995). However, such models have not been applied to CMCs.

A computational model for the stress analysis of high temperature laminated CMCs was presented by Mital et al. (1990) and Murthy et al. (1995). The computational code (CEMCAN) was developed based on this model. The code utilizes a multilevel substructuring technique with four levels: from laminate to ply, from ply to subply and from subply to fiber. The fiber-matrix interphase can be included into the analysis. The effect of temperature on the properties of the constituent materials is also accounted for via a power law similar to that employed in the micromechanical model of Chamis for polymeric composites. The results obtained using this code were in an excellent agreement with available experimental data. Damage modes were not incorporated into this model.

The analytical model for a stress-strain response of composites with a brittle matrix was developed by Aveston, Cooper and Kelly (Aveston et al., 1971, ACK model) and Aveston and Kelly (1973). They considered the fiber-matrix debonding as well as the matrix crack spacing. The sliding resistance of the interface was characterized by an interfacial shear stress assumed independent of the microstructure. The shear-lag model employed in this study was based on the assumption of a constant frictional interfacial stress in the debonded region. Within the intact region, this stress was proportional to a difference of fiber and matrix axial displacements (in the fiber direction). The matrix crack considered in this model was long and bridged by the fibers capable of sliding along the matrix. While in the former paper elastic displacements of the fibers and the matrix were assumed independent (unbonded fibers), an attempt was made to qualitatively address this problem in the latter paper.

Budiansky et al. (1986) also considered conditions for matrix cracking in unidirectional CMCs subjected to tensile loading. Two types of problems were studied: unbonded fibers that are restrained due a mismatch of thermal strains but can slip along the matrix and weakly bonded fibers. The former case represents a generalization of the solution of Aveston et al. (1971). The authors concluded that in the presence of friction, there exists an optimal strain mismatch that can maximize the matrix cracking strength. The solution was extended by Sutcu and Hillig (1990) who included the effect of the fiber-matrix adhesive debonding into the energy formulation. The adhesion was shown to reduce the matrix crack spacing. The existence of a constant critical fiber-matrix shear strength was illustrated. A residual axial stress that results in a spontaneous debonding was also determined.

Note that shear-lag models used in these and other analyses include a solid cylinder (fiber) and the outer cylindrical shell (matrix) To account for an interaction between the fibers that becomes important at high fiber volume fractions, a medium with the overall composite properties should be introduced around the cylindrical shell.

Another approach to the matrix cracking called a "local flaw model" was proposed by Wang et el. (1989) and Wang (1987). According to this approach, the flaw represents a crack in the matrix between two fibers and two symmetric interface cracks. This model is symmetric with respect to the axis drawn through the matrix at the "midspan" between the fibers. The flaw propagates when the strain energy release rate at the tip of the matrix flaw is equal to the critical value for the matrix which is defined as a double crack surface energy.

Tsai and Mura (1993) analyzed the effect of the thermal residual strains on the matrix cracking and fiber failure as well as the interfacial debonding. The analysis was based on the shear-lag composite cylinders model and an approach utilizing the Eshelby inclusion method. A comparison of fracture maps for a representative material corresponding to the differences in the processing and room temperature equal to 980°C and 490°C, demonstrated thermal residual stresses had a noticeable effect on the performance of the material.

A comparison between several matrix cracking models and experimental results was presented in the paper of Wang et al. (1992). The matrix cracking stress found from the experiments was shown to be within rather broad bounds formed by the application of the Aveston-Cooper-Kelly model using the values of the interfacial shear stress chosen to provide the best correspondence between experimental and theoretical data and the stress obtained using the model based on the paper of Budiansky et al. (1986). The local flaw model of Wang provided a qualitative agreement with experimental results.

Solti et al. (1995), modified a shear-lag model to predict a development of damage in a unidirectional CMC. The Kuo-Chou shear-lag model utilized includes a fiber in the shell formed by the matrix. The axial boundaries are formed by two matrix cracks that are perpendicular to the interface and the interfacial debonding starts from the cross-sections of these cracks. The concept of critical matrix strain energy is utilized to identify the energy level corresponding to the progressive matrix failure. The external stress corresponding to this energy level is determined from experimental stress-strain curves and corresponds to a deviation of these curves from a linear behavior. The analysis results in a closed-form solution for the debond length, the crack spacing, and the fiber stresses, as long as the proportionality limit of the stress-strain response is known.

Peters et al. (1995) presents an analysis of the interfacial shear stresses in CMCs based on the previously introduced model of McCartney (1989). This is an approximate model that satisfies some of the constitutive and strain-displacement relations in the integral sense. Nevertheless, the results generated using this model were found in a good agreement with finite element predictions. However, the comparison with experimental results revealed a significant difference in terms of the interfacial shear stress and the slip length. The authors attributed this difference to an effect of the fiber surface roughness that was not accounted for in the model.

Besides the work listed above, various approaches to mechanics of the frictional interfaces include Gao et al. (1988) who used the Coulomb friction to model the sliding resistance. Sigl and Evans (1989) incorporated thermal residual stresses into their analysis, while Hutchinson and Jensen (1990) described the process of debonding along the interface as a mode II fracture and applied the fracture mechanics techniques. Wijeyewickrema and Keer (1993) applied an axisymmetric theory of elasticity approach based on the assumption of a constant interfacial shear stress in the slip region. Tandon and Pagano (1996) used a concentric cylinders model to investigate the assembly with a frictionally constrained interface. They showed that the radial stress remains compressive along the continuous zone, i.e., outside the slip and open regions. The fiber axial stresses at the crack tip were large but finite. The authors emphasized that, contrary to their results, an assumption of a constant interfacial shear stress would produce a singular axial fiber stress at the crack tip.

Cox (1990) developed an analytical shear lag model for interfacial sliding in a uniaxial composite material subjected to thermal cycling. The interface was characterized by a temperature dependent interfacial shear stress. It was shown that the residual axial displacement increases with cycling prior to approaching a stable value.

The paper of McCartney (1993) represents an interest, although it does not specifically deal with CMCs. In this paper, a methodology for determining thermoelastic constants of a cross-ply composite with cracks in the 90° layer is proposed. Thermal residual stresses can be included into the analysis. Progressive cracking can also be considered using the theory presented in the paper.

Sorensen et al. (1993) considered micromechanics of a unidirectional CMC under mechanical and thermal cycling. A finite element analysis was performed using a concentric cylinder model. The variations of the interfacial sliding shear stress over the sliding distance were limited to about 15% indicating that this stress may be assumed constant. This is an important observation, because this conclusion has been employed in such theories as that of Aveston-Cooper-Kelly (Aveston et al., 1971) and Budiansky-Hutchinson-Evans (Budiansky et al., 1986). The interfacial stress was reduced, as the axial external stress increased, as a result of the Poisson's contraction of the fiber. Therefore, the authors concluded that the effect of the external stresses on the interfacial stress cannot be neglected. A comparative effect of several parameters on the magnitude of the interfacial shear stress was considered. The effect of a 50% reduction of the thermal expansion mismatch was larger than a 50% reduction of the fiber Poisson ratio or a similar reduction of the interfacial friction coefficient. It was shown that contact between the fiber and matrix can be lost during mechanical loading as a result of a diameter contraction.

The paper of Pagano and Brown (1993) modelled a single annular (penny-shaped) matrix crack surrounding the fiber. This approach was based on the experimental evidence that illustrated the existence of matrix cracks that are arrested or deflected when they encounter the fiber-matrix interface. This mode of cracking is different from the mode based on long bridging cracks employed in the Aveston-Cooper-Kelly model (Aveston et al., 1971) and the subsequent research of Budiansky et al. (1986) who assumed that the cracks bridge the fibers. The analysis is based on the Reissner variational equation and the solution is obtained for the class of materials where the coefficient of thermal expansion of the matrix is higher than the corresponding coefficient of the fiber, as is the case for Nicalon fibers and glass-ceramic matrices. The model was expanded to account for interfacial friction by Tandon and Pagano (1996). The forthcoming papers of Pagano (1997a,b) present a further experimental verification of the axisymmetric damage model developed in the previous paper (Pagano and Brown, 1993).

A semi-empirical model suggested in the paper of Dutton et al. (1996) predicts the onset of the penny-shaped cracking mode observed by Kim and Pagano (1991) and analyzed by Pagano and Brown (1993). The model is based on the experimental evidence that related the total matrix stress, including the residual stress to the fracture toughness of the monolithic matrix material. The results predicted by the model were found in a good agreement with experimental data presented in the paper.

The stress-strain response of CMC materials with interfacial cracks was analyzed by Pryce and Smith (1993). The model utilized a step-wise shear lag approach to generate relationships between increments of stresses and strains. The stress-strain curve was subdivided into a number of stress increments and the matrix crack density within each increment was assumed constant. The resulting constitutive stress-strain predictions depend on the interfacial shear stress and the residual thermal stress in the fibers that were selected by determining the best fit with the experimental data. The model was also employed to analyze hysteresis. It was shown that model predictions are little affected by the initial thermal stresses, but they are very sensitive to the interfacial shear. This implies that a development of reliable methods to determine this shear is an important task.

A theory for the prediction of the ultimate tensile strength of unidirectional CMCs was proposed by Curtin (1991, 1993). This theory is based on the Weibull model for the fiber strength and accounts for fiber-matrix sliding. The broken fibers are assumed to equally distribute the excess load between the remaining intact fibers (global load sharing assumption). As a result, a formula for the ultimate strength is obtained that was found in a reasonably good agreement with experimental data. In a subsequent paper (Curtin, 1994), an approach is proposed that can be used to derive the failure strength of composites after the processing as a function of the fracture mirror strength distribution.

Genin and Hutchinson (1995) suggested a constitutive model for cross-ply and quasi-isotropic CMC laminates in the state of plane stress with matrix cracks. The values of parameters employed in this model can be obtained from uniaxial tests. The model that is valid for the case of proportional loading can be applied to evaluate the problems related to notches and holes in laminates.

Rouby and Reynaud (1993) proposed a micromechanical model for unidirectional CMCs subjected to loading in the fiber direction. The model incorporated a decrease of the interfacial shear stress as a result of interfacial wear caused by sliding of fibers along the matrix. The model is based on an analysis of the material with regularly spaced matrix cracks bridged by the fibers. The probability of failure of individual fibers increases with cycling due to a deterioration of the interfacial shear stress. The instability and global failure are related to a critical fraction of broken fibers.

Vagaggini et al. (1995) suggested an approach to predict the constituent properties of CMCs from hysteresis and matrix crack measurements taken during a single tension test where the maximum stress is kept in the range between the matrix cracking and crack saturation stresses. Based on these data, the authors evaluated the properties of the material.

Sorensen and Talreja (1993b) employed a three-dimensional finite element solution to analyze residual thermal stresses in a unidirectional CMC. They analyzed three possible types of cracks, i.e., circumferential debonds of fibers from the matrix, radial cracks into the matrix and cracks with the plane normal to the fiber axis and extending into the matrix (as in the paper of Pagano and Brown (1993)). For SiC/CAS (calcium aluminosilicate) composites, the last two types of cracks are likely to appear because compressive radial stress at the interface would prevent debonding. However, in SiC/LAS (lithium aluminosilicate) composites, the radial stress is tensile and circumferential debond cracks can be expected. It was concluded that the cracks extending from the interface in a radial direction will appear in the case where the coefficient of thermal expansion of the matrix is larger than that of the fibers. If the relation between CTEs is reversed, debond cracks are more likely.

Sorensen and Talreja (1995) presented a study of the energy absorption, i.e. toughness, in unidirectional CMCs including a number of mechanisms, such as multiple matrix cracking, debonding and interfacial frictional sliding. Although the contribution of the strain energy in the fibers was dominant, the components associated with the frictional sliding and matrix cracking were also significant. Conversely, the energy due to debonding was negligible. The theoretical predictions were found in a qualitative agreement with experiments.

Matrix cracking in unidirectional and cross ply CMCs was analyzed by Kuo and Chou (1995). The initiation of cracks was considered based on the concentric cylinders micromechanical model and the energy balance approach. The critical stresses for crack initiation were predicted analytically and compared to experimental data for SiC/CAS cross-ply composites. Predictably, the presence of transverse plies (with the fibers oriented perpendicular to the load direction) resulted in a decrease of the crack initiation stress.

The effective properties of cross-ply CMCs were measured by Beyerle et al. (1992). The material considered in this paper had Nicalon fibers and a CAS matrix. The mechanism of damage involved cracking in the transverse (90^o) layers. However, these cracks were arrested along the layer interfaces. As the load increased, additional cracks were generated in the transverse layers without an apparent damage to the longitudinal layers. Existing transverse cracks started to propagate in the longitudinal layers, as the load continued to increase. The fibers started to fail only when the cracks extended over two or more transverse layers. Tunneling cracks in transverse layers propagating in the plane perpendicular to the load direction along the fibers were also observed at stress levels below those corresponding to the penetration of the cracks into the longitudinal layers.

Experimental data generated in this research was compared to the theoretical predictions (see the paper for details). While the onset of cracking was accurately predicted by the models, the theoretical values of the modulus of elasticity differed significantly from the test data. On the other hand, the ultimate strength was accurately calculated using the global load-sharing model with an appropriately reduced fiber volume fraction in the direction of the load.

An analytical and numerical (FEA) study of tunnel matrix cracking in the transverse layers of a cross-ply brittle matrix laminate was published by Xia et al. (1993). This paper was concerned with the onset and evolution of tunnel cracks and their effect on the constitutive response of the laminate.

Robertson et al. (1996) analyzed failure in CMCs using the damage criteria developed in the previous study (Solti et al. 1995). The criteria of damage included matrix cracking determined from the critical matrix strain energy, fiber failure based on Weibull statistics, and fiber pullout as a function of cycling. The overall failure was identified with the situation where the work required for the fiber pullout decreases below the available energy.

Finally, creep of CMCs has also been modeled in a number of studies. Pachalis et al. (1990) used a shear-lag model to analyze the steady-state creep of short-fiber CMCs. This model refers to the case of aligned fibers that can exhibit creep as well as the matrix. A power creep law selected for the study was justified by previous experiments. The equations of equilibrium were solved by the Runge-Kutta procedure.

Sutherland et al. (1995) suggested a simple creep model based on a power law for the creep rate in the matrix and a time-exponential law for the strain in the fibers. The analytical results generated using this model were in a good agreement with experiments.

El-Azab and Ghoniem (1995), considered creep of continuous CMCs suggested for the use in fusion nuclear power reactors using a shear-lag model and experimentally determined relationships for the irradiation creep components for the fiber and matrix. Thermal creep of fibers was also included while creep of the matrix was neglected. Numerical simulation illustrated that thermally-induced stresses relaxed within several days as a result of irradiation creep.

Fracture and Fatigue of Woven Ceramic Matrix Composites

While laminated CMCs offer a combination of attractive features, woven composites have potential advantages due to a higher strength in the thickness direction, as well as excellent impact and damage resistance. This is because woven composites do not have layer interfaces where delamination propagates in laminated composites (Bogdanovich and Pastore, 1996b). The cost of manufacturing these composites may also be reduced by an automation of the process (Smith and Swanson, 1996). In-plane stiffness and strength of these composites may be lower compared to their laminated counterparts. Therefore, it is important to develop a reliable experimental and analytical basis for a judgment on the areas where woven composites should be selected as primary candidates for structural applications. The present section outlines experimental data collected on woven composites, particularly on CMCs. A survey of theoretical developments in the field of woven composites (not limited to CMCs) is found in the next section.

1. Fracture of woven CMCs

Experimental studies of woven SiC-fiber/SiC-matrix composites with Nicalon fiber coating were reported by Brehm et al. (1991). They tested carbon-coated and uncoated specimens from room temperature to 1200°C. The comparison of these specimens illustrated that coating may provide a protection against environmental damage to woven CMCs at high temperatures.

The paper of R'Mili et al. (1990) addressed the issue of fracture of 2-D woven carbon-carbon CMCs. In this paper, the methods of generating so-called R-curves that reflect the energy required for crack growth have been illustrated. According to this paper nonlinear effects associated with the post-processing matrix cracking should not be disregarded. On the other hand, Bouquet et al. (1990) studied the toughness of various 2-D and 3-D CMCs and concluded that it can be adequately estimated using linear elastic fracture mechanics.

Fujii and Lin (1990) considered mechanical fatigue of a plain-woven glass fabric composite specimens under a combined tension and torsion loading. They analyzed S-N curves and concluded that their slope is affected by the presence of torsional loading, although this effect was relatively small. Probably, this limited effect reflects different fracture modes in tension and shear. Shearing stresses had a relatively higher effect on the decay of the shear modulus than the corresponding effect of tensile stresses on the modulus of elasticity. Initial cracks oriented perpendicular to the direction of the maximum principal stress were observed in resin-rich regions. The orientation of the cracks in the case of a combined tension/torsion fatigue was difficult to predict.

The effect of the interfacial shear stress on the mode of failure of cross-ply woven CMCs was considered by Turner et al. (1995). It was found that tensile failure of composites with a high interfacial shear stress involves matrix cracking in transverse layers. At higher loads, these cracks begin to penetrate longitudinal layers and subsequently, the fibers begin to fail. In the case of a low interfacial shear stress, matrix cracking was not observed. The failure occurred in the fibers oriented in the load direction. In-plane shear of the materials with a low interfacial shear stress resulted in a mode of failure characterized by cracking of the fiber coating. In matrix-rich regions, cracking of the matrix was observed in all specimens.

The effect of the matrix microcracking on the properties of a ceramic matrix material was also discussed by Baste and El Bouazzaoui (1996). They used ultrasonic technique to measure the stiffnesses of a 2-D carbon-fiber SiC-matrix woven composite. When a tensile load was applied in one of the fiber directions, cracks appeared in the planes perpendicular to this direction. This resulted in a specially orthotropic material. However, when the tensile load was applied at 45° to the fiber directions, the system of microcracks was not parallel to any of the fibers. As a result, the material exhibited an anisotropic elastic degradation. Note that although the study discussed here dealt with a ceramic matrix cloth, similar changes of the lamina properties that will be present in laminated composites whenever one of the plies is generally orthotropic may significantly affect the response of the material.

Camus et al. (1997) and El Bouazzaoui et al. (1997) present a further investigation of 2-D C/SiC woven composites subjected to tensile and compressive loads at room temperature. In particular, it was illustrated that while the tensile stress-strain response is nonlinear, the compressive response remains linear to failure. Microcracks observed in the post-processed composite were responsible for a nonlinear tensile response. These microcracks were found both in the matrix between fiber bundles as well as in the fiber bundles and they were attributed to a mismatch of the coefficients of thermal expansion of the fibers and the matrix. The result of this mismatch were tensile residual stresses in the matrix and subsequent cracking. One family of cracks observed in these experiments included those in the matrix-rich areas between the fiber bundles. The second family consisted of regularly spaced cracks parallel to the fibers within the fiber bundles. These observations emphasize an importance of predicting post-processing cracks in woven CMCs. A nonlinear tensile stress-strain response was related to the phases of damage development that included an elastic (though slightly nonlinear) response, multiple transverse matrix cracking, fiber/matrix, bundle/matrix and interbundle debonding, progressive fracture of fibers and the subsequent failure.

Another experimental study of woven CMCs was published by Dalmaz et al. (1996). The material considered in this study consisted of carbon fibers coated in a thin Pyrocarbon layer. The authors reported the presence of microcracks resulting from the post-processing cooling and a porosity of 7%. Transverse bundles and fibers exhibited some rotation in the peripheral areas under loading. The failure occurred at approximately the same maximum stress and strain for both monotonic and cycling tests.

Guillaumat and Lamon (1996) presented a description of experimentally observed damage accumulation in 2-D woven SiC/SiC composites. Microcracks were first detected at macropores (large pores between the plies and within the plies at yarn intersections) at the strain level of 0.025%. All macropores contained cracks, as the strain increased to 0.06% and these cracks propagated into the longitudinal yarns at the strain equal to 0.085%. The saturation of the cracks in the longitudinal yarns corresponded to the strain of 0.12%. As the strain continued to increase, the cracks started to appear in the transverse yarns as well as in the interply matrix regions. At the strain approaching 0.2%, a new family of cracks appeared in the longitudinal yarns and the material failed.

A relatively recent addition to investigations of damage in CMCs is research on their response to external shearing stresses. Keith and Kedward (1997) presented an experimental and analytical study of a woven CMC under a quasi-static shear load. The principal damage mechanism was matrix cracking, although both interlaminar cracks as well as ply delaminations were also observed. It was concluded that debonding and fiber sliding are the principal factors that affect shear strength and fatigue characteristics of the material. Notably, the failure shear strains were remarkably high, exceeding 3%.

An interesting suggestion was made by Smith and Swanson (1996) based on their experimental study of 2-D woven carbon-fiber/epoxy composites subject to biaxial tension or compression. The authors of this paper suggested that failure can be predicted based on methods applicable to continuous laminated composites using degraded strength properties accounting for the woven structure. While this work did not address CMCs, it may be interesting to consider whether the conclusions of the paper could be extrapolated to this class of materials. In particular, the question is whether it is possible to analyze the interface degradation that is the principal mode of failure (Mizuno et al., 1996) using the corresponding theories for continuous CMCs and degraded material properties of a woven composite.

Lamouroux et al. (1995) considered an interaction between creep and damage in 2-D woven alumina fiber SiC matrix composites. They observed a progressive debonding during creep and attributed it to a creep rate mismatch between the fibers and the matrix. Therefore, the authors suggested to tailor the interphase of the composite to optimize its creep resistance. Another suggestion made in the paper was to select the coefficients of thermal expansion of the fibers and the matrix to achieve the same goal. The effect of temperature on the creep rate has also been considered (Schwartz, 1997). In particular, for 2-D SiC/SiC composites, this rate increased by 33-50% as a result of an increase of temperature from 982°C to 1204°C.

2. Fatigue of woven CMCs

Fatigue of 2-D woven SiC/SiC composites (Nicalon fibers) was considered by Rouby and Reynaud (1993). The accumulation of fatigue damage was clearly observed on the hysteresis curves whose area increased with cycling, while the main slope (modulus of elasticity) decreased. The authors found that the fatigue limit was much higher than the tensile proportionality limit of the material.

The influence of the rate of loading on fatigue of cross-ply woven composites at room temperature was considered by Shuler et al. (1993). A higher loading frequency was shown to result in a reduction of the fatigue limit of the material. Frictional heating increased with the loading frequency and with an increase of the maximum stresses. In addition, a rapid decrease of the matrix spacing was observed in the beginning of fatigue life for the range of loading frequencies from 1 to 10Hz.

Experimental investigation of fatigue of graphite fiber SiC matrix 2-D woven composites at the room temperature and at 830°C carried out by Morris et al. (1993) illustrated an accelerated high-temperature wear due to SiC debris within the cracks. The authors emphasized a dilemma faced by the designer due to this phenomenon. While a weak interface and low interfacial friction are desirable at room temperatures, they can increase the fatigue damage at elevated temperatures.

Rodrigues et al. (1995) considered fatigue of 2-D CMCs at room temperature and at 1200°C. They detected strain accumulation as a result of cycling that increased with temperature. On the other hand, the authors did not observe an effect of temperature on the fatigue life.

Lang et al. (1997) published an investigation of SiC/SiC 8-harness woven composite for the use in hot sections of high-speed civil transport. Fatigue tests carried out at room temperature and a stress ratio equal to $R = 0.1$ showed over 1,000,000 cycles endurance under 50% ultimate stress. However, at 75% ultimate stress, the specimen survived only two cycles. Although some material degradation was detected at 60% ultimate stress, the specimen survived over 1,000,000 cycles at this stress.

Mitrovic and Carman (1996) considered a correlation between the degradation of the coefficient of thermal expansion and the residual compressive strength in the process of thermomechanical fatigue of woven composites. Although the paper does not refer to CMCs, the conclusion on a possible correlation between these two properties may provide a useful tool for experimental assessment of the residual strength.

A very detailed analysis of both room and high-temperature mechanical fatigue of woven CMCs has recently been published by Mizuno et al. (1996). They performed tensile fatigue tests (stress ratio $R = 0.1$) of woven SiC-fiber SiC-matrix composites. The tests at room temperature were conducted in air, while at 1000°C, the specimens were in argon. The interface bonding between the fibers and the matrix was reduced by a carbon coating. The authors observed an increase in the ultimate tensile strength and strain in the specimens tested at 1000°C, as compared to their counterparts at room temperature. This was probably due to a compensation of the post-processing residual stresses at an elevated temperature, while oxidation was prevented. However, the effect of temperature on the fatigue limit was significant: while this limit was equal to 160 MPa at room temperature (80% of the static strength), it was reduced to just 75 MPa at 1000°C (30% of the static strength). There was little difference in low-cycle fatigue life at room and elevated temperatures, but high-temperature fatigue life decreased rapidly at the stresses below 180 MPa when the number of cycles exceeded 10,000. The fact that the fatigue limit at room temperature is much higher than the stress corresponding to the matrix cracking implies that the cracks formed during the first cycle were arrested in the subsequent phase of the composite life.

The dominant failure mode corresponded to cracks in 90° bundles at high stresses and cracks in 0° bundles at crossover points at low stresses. The authors suggested the presence of a critical stress for the transition from one dominant damage mode to the other. They explained the mechanism of damage at high stresses as a result of matrix cracking saturation in 90° bundles during the first cycle and the consequent interface debonding. At low stresses, matrix cracking develops during cycling and stress concentration at the pores may result in deflections of the cracks. Creep of 0° fibers and a reduction of the sliding resistance were also identified as possible explanations for their cracking. Contrary to carbon-fiber composites, fiber degradation is not one of the failure mechanisms in their ceramic-fiber counterparts. However, the interface degradation that was identified as another dominant failure mechanism for carbon-fiber composites is also the dominant failure mechanism for ceramic-fiber composites.

Modeling Woven Composites

Although existing models of woven composites usually do not specifically refer to CMCs, these models can provide an estimate of the constitutive properties for all classes of woven composites. Analytical work on woven composites is mostly limited to their constitutive modeling. The analysis of fracture and fatigue of woven composites have received relatively little attention.

Chou and Ishikawa (1989) outlined the result of their previous work in a comprehensive paper presenting the stiffnesses, and the coefficients of thermal expansion for 2-D woven composites. Three analytical models were considered, i.e., a mosaic model, a fiber crimp model and a bridging model. The models are based on the classical laminated theory which seems appropriate for the global analysis of typical thin woven (textile) laminates. Note, however, that the local analysis at the crossover points should account for a three-dimensional state of stresses.

The mosaic model replaces the actual woven laminate with a collection of patches of piece-wise asymmetric laminates. Accordingly, this model neglects the fiber continuity and crimp of the actual fabric. The crimp model incorporates the fiber continuity and crimps into the analysis. This model was developed for one-dimensional problems, but it can be expanded to two-dimensional approximations as well. Predictably, the fiber waviness results in a reduction of the in-plane stiffness, although the coupling stiffness remains unaffected.

The bridging model was developed for composites where interlaced regions are separated from each other. Therefore, it is suitable for eight-harness woven composites. The model treats a typical cell as an assembly of five regions where the four regions with straight threads surround the centrally located interlaced region. The results generated using this model were in a good agreement with experimental results generated for an eight-harness satin-reinforced glass/polymide material. Note that the analysis of the knee behavior associated with the failure of fibers may not be relevant for CMCs where one of the principal forms of failure is a degradation of the fiber-matrix interface.

Another review of analytical models for 2-D and 3-D woven composites was published by Byun and Chou (1989). In addition to the review of the mosaic, crimp and bridging models, they discussed two approaches to the analysis of 3-D braided composites, i.e., the energy and fiber inclination methods. It was emphasized that 3-D braided metal matrix composites (CMCs were not considered) have excellent impact tolerance and fracture toughness.

The response of woven composites to temperature represents an important extension of the theories described in the previous two reviews if they are to be applied to CMCs that often operate in high-temperature environments. This can be found in the earlier paper of Ishikawa and Chou (1983a) who derived constitutive equations in the presence of thermal loads for three different models, i.e., the mosaic model, the one-dimensional fiber undulation model and the two-dimensional bridging model.

Another feature that may become necessary to model in the analysis is a nonlinearity of the stress-strain constitutive curves in woven composites that is due to cracking. This problem was considered by Ishikawa and Chou (1983b) whose stress-strain relations were in an excellent agreement with experiments. The nonlinear contributions considered in this paper included the shear deformation of the threads, the extensional deformation of the pure matrix region that was shown relatively less important, and transverse cracking of the warp regions.

Dumont et al. (1987) considered damage in 3-D woven carbon-carbon composites consisting of periodically arranged fiber yarns and a carbon matrix filling the voids. The model was developed based on the assumptions that the fiber-yarns are elastic and brittle, the matrix is elastoplastic in the Prandtl-Reuss sense and can sustain damage. The interfaces are elastic and are also capable of sustaining damage. After identification of model parameters, a homogenization was used to describe mean behavior. The results generated using the model were found in a good agreement with experimental data.

The approach developed in the previous paper (Dumont et al., 1987) was modified by Aubard et al. (1994) who considered nonlinear behavior of 2-D SiC-SiC composites. In this paper, the independent damage variables account for the anisotropy of the material, loading history and damage coupling.

Naik and Shembekar (1992) extended one dimensional models of Chou and Ishikawa to a two-dimensional lamina. They utilized their model to analyze woven laminates consisting of a number of laminae (Shembekar and Naik, 1992). In later papers, Naik et al. (1994) and Naik (1995) presented a code TEXCAD for the analysis of woven composites. The code utilizes a repeated unit cell, discretizing the yarns within the cell into the slices and using a volume averaging technique based on an iso-strain state within the cell.

An analytical model of microcracking damage in SiC/SiC woven composites was proposed by Gasser et al. (1996) using the loss of the stiffness observed in uniaxial tests as an indicator of a damage. However, both uniaxial and biaxial tests were necessary to evaluate all parameters necessary to use this model.

Analysis of the strength of woven composites subjected to uniaxial or biaxial tension was presented by Pan (1996). Assumptions included a Weibull form of the distribution of yarn strengths, material and geometric characteristics of the composite were insensitive to the fabric extension, and stress concentrations were negligible. The interaction between yarns was found to be predominantly of a frictional nature, while the adhesive effects were negligible. The stress-strain relationship that was obtained takes into account the strength variation between the yarns.

Examples of numerical simulation using the finite element method can be found in the papers of Lamon and Thommereset (1995) and Guillaumat and Lamon (1995, 1996). The former paper addressed the problem of matrix cracking in 2-D SiC/SiC composites. Cracks were accounted for by splitting the corresponding nodes. The results were in a qualitative agreement with experimental data. Earlier solutions using the finite element analysis or standard codes can be found in the papers of Zhang and Harding (1990), Tan et al. (1991) and Baldwin et al. (1992).

Note that in woven and braided CMCs, a high degree of porosity between yarns (interyarn porosity) and between individual fibers affects the response. While the compressive stress-strain behavior is linear to failure for all material systems, tensile behavior is affected by the orientation of cracks relative to the direction of the load which is determined by the architecture (Chou and Karandikar, 1995). This important fact should be reflected in the analytical models.

Finite element and analytical models were employed by Ishikawa et al. (1995) to evaluate the stiffnesses of woven CMCs. The analytical approach based on the parallel-serial approximation (details in the paper) yielded results in a very close agreement with the finite element prediction.

A continuous effort to accurately characterize complicated woven geometries is reflected in a number of recently published papers. This effort is motivated by discrepancies between theoretical predictions of the properties of woven composites and the measured values (Cox and Dadkhah, 1995). Typically, the authors begin with a characterization of geometry. In particular, improvements over existing solutions may include accounting for the actual strand geometry, gaps between strands, etc. Once the geometry has been described, the analysis is presented based on proper micromechanical and macromechanical models. Examples of recent work include the series of papers by Ganesh and Naik (1996a,b) and Naik and Ganesh (1996) the papers of Vandeurzen et al. (1996a,b) and the paper of Glaessgen et al. (1996).

Bogdanovich and Pastore (1996a,b) presented reviews of the constitutive theories for textile (woven) composites. They also suggested a methodology based on a meso-volume approach (see for details, the papers of Bogdanovich and Pastore, 1992 and Pastore et al., 1993) and suitable for the incorporation into a finite element model. This approach is based on a hierarchical treatment of a woven architecture so that the structure can be presented as a combination of meso-volumes. Each meso-volume has its own averaged properties, i.e., its matrix of stiffnesses is identified through the position vector. Once the discretization mesh is specified, the solution in terms of displacements is sought in triple series. The system of linear algebraic equations from which the amplitude of the terms in the series for displacements can be determined is obtained from the variational equation.

A simple meso-model was recently suggested by Aubard (1995) for 2-D SiC/SiC composites. The model replaced a meso-volume representing a woven fabric with a laminate composed according to experimentally measured proportions of matrix, fibers and pores of the layer consisting of the matrix and pores and four unidirectional composite layers. Experimental justification and verification of the model is presented in the paper.

In the recent paper, Jiang et al. (1997) modeled the nonlinear stress-strain relationships of plain weave composites (a unidirectional nonlinear stress-strain problem was first considered by Ishikawa and Chou, 1983b). Their approach is based on meso-volumes, i.e., a representative volume cell is specified first and its properties are averaged through the thickness using the assumptions of iso-stresses. The subsequent averaging is performed at the subcell level yielding average subcell strains and stresses. The material nonlinearity induced by in-plane shear due to microcracking is introduced through the Hahn-Tsai equations (Hahn and Tsai, 1973). According to this approach, in-plane shear strains are represented as cubic functions of the corresponding shear stresses. The incremental solution is obtained by prescribing an increment of the global average strain. The corresponding strain increments in each subcell are subsequently determined as well as the total average subcell strains. The total stresses for each subcell are found from the nonlinear constitutive equations using Newton's method.

Conclusions

The survey presented above illustrates that although mechanics of CMCs has been intensively studied, an additional research thrust is needed to achieve a reliable modeling of the processes that take place under thermal and mechanical loading. Some of the areas where improved design procedures or additional research are needed are outlined in this section. These areas are related to design and development of CMCs that are required to withstand cyclic thermomechanical loading. The issues of manufacturing and such important problems as thermal shock of CMCs (on the latter subject, see Singh, R.N. and Wang, H., "Thermal Shock Behavior of Fiber-Reinforced Ceramic Matrix Composites," Composites Engineering, Vol. 5, pp. 1287-1297, 1995) are not considered.

Although the processing of CMCs is outside the scope of this survey, post-processing matrix cracking is an important consideration in both laminated and woven configurations. It may be necessary to account for a difference between the processing and room temperature properties of the constituent materials, because such differences could affect thermal residual stresses. An important task is to develop fiber coatings that would reduce thermal residual stresses to an acceptable level and prevent post-processing matrix cracking. Post-processing diagnostics of CMCs is another important issue. In particular, matrix cracking may be diagnosed using such techniques as acoustic emission or thermoelasticity.

A design of interfaces is a particularly important issue for CMCs, because of oxidation at high temperatures and the mechanism of damage propagation that involves debonding. CMCs are produced to operate at elevated temperatures and therefore, oxidation has to be reduced or prevented. This problem may be addressed by producing coatings consisting of several layers (functionally graded coatings). In such configurations, the analysis of cracking becomes particularly complicated and one of the possible damage modes may be debonding between two adjacent layers of coating.

A reliable model for variations of the interfacial resistance with temperature is needed. For example, as was indicated by Sorensen and Holmes (1996), two processes involved are the sliding of the fiber due to a thermal mismatch and changes of the interfacial shear stress with temperature.

The progress of damage in angle-ply or cross-ply CMCs as well as in woven configurations has to be analyzed in detail. For example, it is known that in cross-ply laminates under tensile loading the cracks in transverse layers propagate into longitudinal layers, once the load reached a certain level. This raises a question about designing laminates with features that would deter the cracking in longitudinal layers. In particular, functionally graded composites with a nonuniform distribution of fibers within longitudinal layers may be worth consideration.

Considering fatigue of CMCs, including both thermal and thermomechanical fatigue, it is necessary to account for a number of factors. For example, the effect of the rate of loading on the fatigue life is an important issue that should be addressed both analytically and experimentally. A related issue is creep in CMCs that should be studied based on the creep characteristics of the fibers and the matrix materials and an appropriate micromechanics to produce overall composite creep characteristics.

Spectrum loading corresponding to several different regimes should be considered for future practical applications. Such analysis should incorporate an accurate theory of cumulative damage. Reliable procedures for monitoring the damage and predicting the fatigue life are needed for practical applications. While these procedures should be nondestructive, they also should be sufficiently simple and inexpensive to be feasible in the field conditions.

Design of woven CMCs presents interesting possibilities to delay fatigue damage through functional grading of the yarns. This may be achieved by a nonuniform distribution of fibers providing for a higher damage resistance at the crossover points where damage usually originates. One possibility is to combine continuous fibers and whiskers within each yarn using a higher whisker concentration at the crossover points to enhance the damage resistance.

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Appendix: Mathematical foundations of a nondestructive thermal monitoring of matrix cracks in CMCs

The purpose of the present solution is to determine a relationship between the matrix crack spacing in a unidirectional CMC and its surface temperature during a nondestructive dynamic test. The amplitudes of cyclic stresses applied during the test are assumed below the matrix cracking limit of the material so that cycling does not change the matrix crack spacing. An elevated temperature is due to frictional heating that is triggered by a relative movement of bridged fibers with respect to the matrix. The solution assumes the mode of cracking employed in the theories of Aveston-Cooper-Kelly (Aveston et al., 1971) and Budiansky-Hutchnison-Evans (Budiansky et al., 1986), i.e. long, regularly spaced cracks. The formulation of the problem is shown in Fig. A1 that identifies the crack spacing (s), the length of the sliding distance (x_0), and a distribution of stresses in the fibers and the matrix.

The solution involves the following steps. First, the modulus of elasticity of the material is determined as a function of the interfacial shear stresses, matrix crack spacing and the residual thermal stresses in the fiber using a modified approach of Pryce and Smith (1993). Residual thermal stresses are evaluated accounting for the effect of temperature on the properties of the constituent materials. Then the experimental findings of Karandnikar and Chou (1992) are used to justify a simple relationship between the modulus of elasticity and the matrix spacing. Combining the two solutions referred to above, the modulus of elasticity can be eliminated and a single equation relating the matrix crack spacing to the interfacial shear stress obtained. Subsequently, the balance between the rate of heat flow and the rate of dissipation of the frictional work is employed, as suggested by Cho et al. (1991), to obtain a relationship between the surface temperature and the interfacial shear stress. This procedure enables us to evaluate both the shear stress and the matrix crack spacing as functions of the surface temperature.

A distribution of stresses in a specimen subjected to an external stress σ_c is shown in Fig. A1. Note that this distribution corresponds to a partial slip, i.e., $2x_0 < s$ that is considered in the present analysis. The stresses in the fibers and the in the matrix can be evaluated based on their values at the points A and B:

$$\begin{aligned}
\sigma_{fA} &= \frac{\sigma_c}{V_f} \\
\sigma_{fB} &= \frac{\sigma_c E_f}{E_c} + \sigma_f^T \\
\sigma_{mA} &= 0 \\
\sigma_{mB} &= \frac{\sigma_c E_m}{E_c} + \sigma_m^T
\end{aligned} \tag{1}$$

where the subscripts "f" and "m" refer to the fibers and matrix, respectively, σ_c is the stress applied to the composite, V_f is the volume fraction of the fibers, E_f, E_m and E_c are the moduli of elasticity of the fibers, matrix and undamaged composite, respectively, and σ_f^T and σ_m^T are the residual thermal post-processing stresses outside the slippage region. Note that during cycling the composite stress σ_c varies continuously. Therefore, the stresses given by eqn. (1) represent instantaneous values, although dynamic (viscous) effects are not included in the present analysis.

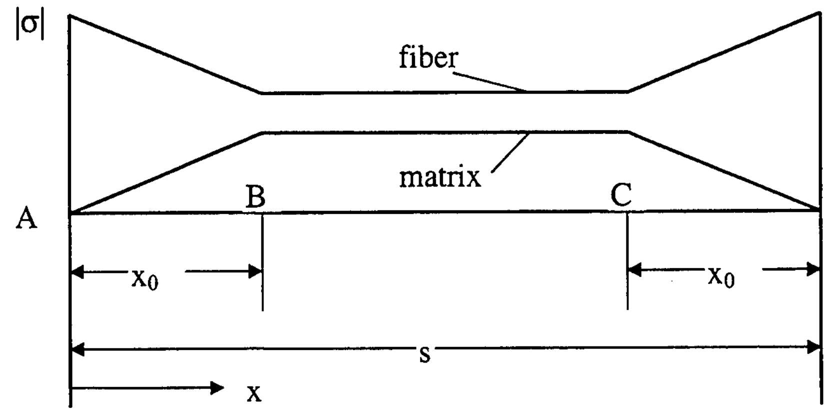

Fig. A1. Distribution of stresses in the fibers and in the matrix during cycling (not to scale). The stresses are shown along the fiber length between two parallel bridging cracks.

It can be immediately observed that the equilibrium of forces in the cross sections outside the slippage region is satisfied. The stresses within the slippage region are also balanced at each cross section. Within the slippage region, the stresses in the fiber and the matrix are linear functions of the distance from the plane of the crack, i.e., x. The stress in the fiber is (Budiansky et al., 1986):

$$\sigma_f = \frac{\sigma_c}{V_f} - \frac{2\tau x}{r} \qquad (2)$$

where r is the fiber radius and τ is the interfacial shear stress that is assumed constant, as in the theories of Aveston-Cooper-Kelly and Budiansky-Hutchinson-Evans. Note that the finite element solution of Sorensen et al. (1993) that showed that the maximum variation of the interfacial shear stress is 15% supports the theoretical solutions based on the assumption is that this stress is constant.

The stress in the matrix within the slippage region is

$$\sigma_m = \left(\sigma_c \frac{E_m}{E_c} + \sigma_m^T \right) \frac{x}{x_0} \qquad (3)$$

where x_0 is the half-length of the slippage region that can be determined according to Pryce and Smith (1993) as

$$x_0 = \frac{r}{2\tau}\left(\sigma_c \frac{V_m E_m}{V_f E_c} - \sigma_f^T \right) \tag{4}$$

V_m being the matrix volume fraction. The residual thermal stresses in the fiber and in the matrix can be found from the force equilibrium that should be preserved at an arbitrary cross section, i.e.,

$$E_f V_f (\varepsilon - \alpha_f \Delta T) + E_m V_m (\varepsilon - \alpha_m \Delta T) = 0 \tag{5}$$

where α_f and α_m are the coefficients of thermal expansion of the fibers and the matrix, respectively, and ΔT is a difference between the processing and operating temperatures. The strain, ε, can be immediately evaluated from eqn. (5). However, considering the fact that the processing temperature of CMCs is usually above 1200^0C, it is necessary to account for an effect of the temperature on the properties of the materials of the fibers and the matrix. In this case, the strain will be found from

$$\varepsilon = \int_{T_p}^{T_0} \frac{E_f(T) V_f \alpha_f(T) + E_m(T) V_m \alpha_m(T)}{E_f(T) V_f + E_m(T) V_m} dT \tag{6}$$

where the integration is carried out from the processing (T_p) to the operating (T_o) temperature. An example of analytical expressions for the moduli of elasticity and the coefficients of thermal expansion of CMCs as functions of temperature can be found in the report of NASA TM 106789.

The residual stresses in the region that is not affected by the slip are found as

$$\begin{aligned} \sigma_f^T &= E_f(\varepsilon - \alpha_f \Delta T) = E_f \int_{T_p}^{T_o} \frac{E_m V_m (\alpha_m - \alpha_f)}{E_f V_f + E_m V_m} dT \\ \sigma_m^T &= E_m(\varepsilon - \alpha_m \Delta T) = E_f \int_{T_p}^{T_o} \frac{E_f V_f (\alpha_f - \alpha_m)}{E_f V_f + E_m V_m} dT \end{aligned} \tag{7}$$

It is now necessary to evaluate an instantaneous modulus of elasticity of the material. This procedure follows the approach adapted by Pryce and Smith (1993), although the problem considered in the present solution is different which dictates modifications outlined below.

The instantaneous mean strain in the fiber is found by averaging the strains over the spacing length, i.e.,

$$\bar{\varepsilon}_f = \frac{2x_0}{s}\varepsilon_{AB} + \frac{s-2x_0}{s}\varepsilon_{BC} \tag{8}$$

where the mean strain within the slippage region ε_{AB} and the strain outside the slippage region ε_{BC} can be expressed in terms of the fiber stresses. The substitution of these strains into eqn. (8) yields

$$\bar{\varepsilon}_f = \frac{x_0}{s}\left[\frac{\sigma_c(2V_f E_f + V_m E_m)}{V_f E_f E_c} + \frac{\sigma_f^T}{E_f}\right] + \frac{s-2x_0}{s}\left(\frac{\sigma_c}{E_c} + \frac{\sigma_f^T}{E_f}\right) \tag{9}$$

Note that the strain given by eqn. (9) includes the post-processing residual and the additional cycling components. The increase of the strain due to cyclic loading is represented by the latter component. The mean residual strain is found from an analog of eqn. (8) where the mean strain in the region AB is given by $\sigma_f^T/2E_f$, while the strain within the region BC is σ_f^T/E_f (see Fig. A2). Accordingly,

$$\bar{\varepsilon}_f^T = \frac{s-x_0}{s}\frac{\sigma_f^T}{E_f} \tag{10}$$

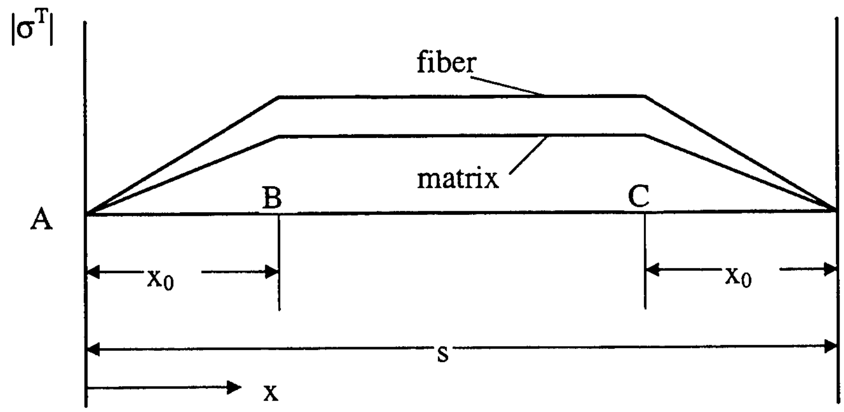

Fig. A2. Thermal residual stresses in a unidirectional material with regularly spaced matrix cracks (not to scale).

The additional strain associated with cyclic loading can now be found as a difference of the strains given by eqns. (9) and (10), i.e.,

$$\bar{\varepsilon} = \left[1 + \frac{r}{2s\tau} \frac{\left(\sigma_c \frac{V_m E_m}{V_f E_c} - \sigma_f^T \right) V_m E_m}{V_f E_f} \right] \frac{\sigma_c}{E_c} \tag{11}$$

The instantaneous modulus of elasticity is determined as

$$\mathrm{E} = \frac{\mathrm{d}\sigma_c}{\mathrm{d}\bar{\varepsilon}} \tag{12}$$

yielding

$$E = \left[1 + \frac{r}{2s\tau} \frac{E_m V_m}{E_f V_f} \left(2\sigma_c \frac{E_m V_m}{E_c V_f} - \sigma_f^T \right) \right]^{-1} E_c \tag{13}$$

Note that eqn. (13) presents the elastic modulus of a cracked material that is not affected by cycling. Accordingly, it is possible to compare this result to experimental findings of Karandikar and Chou (1992) who showed that the change of the modulus of a unidirectional Nicalon fiber CAS matrix composite is a linear function of the matrix crack density (1/s):

$$\Delta E = E_c - E = k_1 + k_2 (1/s) \tag{14}$$

where k_1 and k_2 are constants. For Nicalon fiber CAS matrix, $k_1 = -6.7350$ and $k_2 = 6.2754$ (the modulus of elasticity is measured in MPa and the crack density in number/mm). Note that the present approach to the solution remains valid as long as an arbitrary analytical relationship $\Delta E = f(s)$ is available.

A combination of eqns. (13) and (14) yields the relationship between the interfacial shear stress and the matrix crack spacing:

$$\tau = \left[\frac{E_c - k_1 - k_2 / s}{k_1 + k_2 / s} \right] \left[\frac{r}{2s} \frac{E_m V_m}{E_f V_f} \left(2\sigma_c \frac{E_m V_m}{E_c V_f} - \sigma_f^T \right) \right] \tag{15}$$

Now it is necessary to find a relationship between the interfacial shear stress and the surface temperature of the specimen. This problem can be addressed by considering the equilibrium between the rate of the steady state heat loss from a unit volume of the specimen and the rate of work performed by the interfacial friction within this volume (Cho et al., 1991). The former quantity was presented in the paper of Cho et al. (1991) for the general case where the flow loss occurs through conduction in the fiber direction and free convection and radiation from the surface. In the case of a uniform matrix crack distribution and small-amplitude vibrations excited in the course of a nondestructive test prior to any additional fatigue loading, the temperature may be assumed independent of the axial coordinate oriented along the fibers. Therefore, no conduction takes place in the fiber direction. At the sea level the radiation is typically small compared to free convection (although the relative contribution of radiation increases with altitude). Therefore, radiation from the surface may be neglected, although this is not necessary for the solution. Retaining the radiation contribution, the rate of the heat loss from the element with the surface area A_s (including both surfaces of the specimen) and the volume $V = A_s t/2$ where t is the thickness becomes

$$\dot{q} = \left[h(T_s - T_a) + \varepsilon\beta_0(T_s^4 - T_a^4)\right]\frac{2}{t} \tag{16}$$

In eqn. (16), T_s and T_a are the surface and ambient air temperatures, respectively, h is the heat transfer coefficient, ε is the emissivity and $\beta_0 = 5.67*10^{-8}$ W/m^{2}K^4 is the Stefan-Boltzman constant. As indicated above, the second term in the square brackets in eqn. (16) may be negligible. Note that the heat transfer coefficient refers to the mean properties of the film adjacent to the surface of the specimen, i.e., its evaluation requires the knowledge of $T = (T_s + T_a)/2$.

The heat transfer coefficients from the surfaces of a representative element can be determined as

$$h_i = \frac{(Nu)(k)}{L} \tag{17}$$

where the subscript identifies the surface, Nu is the Nusselt number, L is the ratio of the surface area of the element to its perimeter and k is the thermal conductivity of the film. The Nusselt number can be found as a function of the Rayleigh number (see, for example, Incropera, F.P. and DeWitt, D.P., "Fundamentals of Heat and Mass Transfer," 4th edition, Wiley, New York, 1996). In particular, if the surfaces losing heat are horizontal, the following formulae apply:

Upper surface: $Nu = 0.27Ra^{1/4}$ for $10^5 < Ra < 10^{10}$

Lower surface: $Nu = 0.54\,Ra^{1/4}$ for $10^4 < Ra < 10^7$

$0.15Ra^{1/3}$ for $10^7 < Ra < 10^{11}$ (18)

The Rayleigh number can be calculated by

$$Ra = \frac{g\beta(T_s - T_a)L^3}{\nu\alpha} \tag{19}$$

where g is the gravity acceleration, β is the volumetric thermal expansion coefficient, ν is the kinematic viscosity and α is the thermal diffusivity of the film at its average temperature. The total heat transfer coefficient is a sum of the coefficients of the opposite surfaces.

The instantaneous work produced by the interfacial friction on the slippage length of one fiber is obtained as (Cho et al., 1991)

$$W = 2\int_0^{x_0} (\pi d_f \tau)(u_f - u_m)dx \tag{20}$$

where u_f and u_m are dynamic components of the axial displacements of the fiber and the matrix due to cyclic loading that are functions of the x-coordinate. Note that the upper limit of integration given by eqn. (4) is affected by the magnitude of the interfacial shear stress. The difference between dynamic components of the axial fiber and matrix displacements can be found as

$$u_f - u_m = \frac{1}{2}[\delta\varepsilon_f(x) - \delta\varepsilon_m(x)](x_0 - x) \tag{21}$$

where $\delta\varepsilon_f$ and $\delta\varepsilon_m$ are the dynamic strains at the cross section x. This equation reflects the fact that the fiber and the matrix experience identical axial displacements at $x = x_0$, and the change of the length of the element $(x_0 - x)$ can be found as the mean strain within this element multiplied by its length.

The dynamic strain in the fiber is determined as a difference between the total and residual strains. The former can be found from eqn. (2) but it is more convenient to use the following expression that immediately follows from Fig. A1:

$$\varepsilon_f(x) = \left[\frac{\sigma_c}{V_f} - \left(\frac{\sigma_c}{V_f} - \frac{\sigma_c E_f}{E_c} - \sigma_f^T\right)\frac{x}{x_0}\right]E_f^{-1} \tag{22}$$

The latter strain is of course (see Fig. A2), $(\sigma_f^T/E_f)(x/x_0)$.

Now the dynamic components of the strains in the fibers and in the matrix can be found as

$$\begin{aligned} \delta\varepsilon_f(x) &= \left[\frac{1}{V_f} - \left(\frac{1}{V_f} - \frac{E_f}{E_c}\right)\frac{x}{x_0}\right]\frac{\sigma_c}{E_f} \\ \delta\varepsilon_m(x) &= \frac{x}{x_0}\frac{\sigma_c}{E_c} \end{aligned} \tag{23}$$

Substituting dynamic strains given by eqns. (23) into eqn. (21) and subsequently, integrating eqn. (20) one obtains

$$W = \frac{\pi d_f \tau \sigma_c}{3V_f E_f} x_0^2 \tag{24}$$

Note that the composite stress in eqn. (24) represents the maximum stress during the cycle, while the minimum stress is assumed equal to zero. This is probably the most practical case for a nondestructive dynamic testing where a unidirectional load is periodically applied to the component (like in the case of acoustic pressure). In the case of a stress ratio different from zero, the composite stress should be replaced with the stress range.

The rate of the frictional energy dissipation per unit volume can be found as recommended by Cho et al. (1991), i.e.,

$$\dot{w} = 2fW / (\pi r^2 s / V_f) \tag{25}$$

where f is the frequency of loading and the factor "2" in the numerator accounts for the fact that equal amounts of energy are generated during the loading and unloading phases of each cycle.

The solution can be obtained by prescribing the surface temperature. Then a relationship between the matrix crack spacing and the interfacial shear stress can be obtained from the requirement that the rate the heat loss given by eqn. (16) must be equal to the rate of the frictional energy dissipation according to eqn. (25). This relationship should be considered together with eqn. (15) to specify both the interfacial shear stress as well as the matrix crack spacing.