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FEASIBILITY OF THE METHOD OF MOMENT ESTIMATORS

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ABSTRACT.

In this paper we consider the problem of feasibility of the method of moment estimators. We consider three different distributions, namely exponential, uniform and generalized Pareto. In some cases we obtain the explicit probability of the feasible method of moment estimator and other cases we estimate it through simulations. Since the method of moment estimators are always strongly consistent, it is expected that the feasibility probability should also increase to one. But some of the counter intuitive results are obtained. In some cases it is observed that the probability may even converge to zero also.

Key Words and Phrases: *Method of moment estimators, feasibility, consistent estimator, Monte Carlo simulation.*

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1. INTRODUCTION

Method of moments (MM) plays an important role in the estimation of parameters in parametric inference. MM estimator has a long history starting with the work of Fisher (1922). One of the major advantages of using MM estimator is its simplicity in using in practice (Rao (1973)). In many situations it is observed that MM estimators can be obtained by solving simple equations whereas other estimators like maximum likelihood (ML) estimators may be obtained through minimizing or maximizing certain function. Sometimes it may happen that ML estimator may not exist, whereas MM estimator exists. In certain situations, the MM estimator may be used as the initial guess value to obtain the ML estimator in some numerical search procedure. So, it is quite important to study the properties of the MM estimators at least for small sample. In most of the situations MM estimators are consistent and behave reasonably well for large sample sizes, although they may not be as "efficient" as ML estimators (see Fisher 1922). Another major disadvantage of the MM estimators, at least for small sample sizes, is that they may not be feasible. Here, feasibility means that even though there may be some restrictions on the parameter space but the estimators may not satisfy those restrictions. We define formally the feasible estimator as follows:

Definition. Let $X_1, X_2, \ldots, X_n$ be a random sample from the density function $f(x; \theta)$ and $\theta \in \Theta$, where Θ is a subset of R^k. Let be an estimator of θ. If $\hat{\theta}_n \in \Theta$, then we say that $\hat{\theta}_n$ is a feasible estimator of θ. If $\hat{\theta}_n$ is not a feasible estimator of θ, then it is called an infeasible estimator of θ.

Observe that if $\hat{\theta}_n$ converges either in probability or in almost surely and the limit is also an infeasible estimator, then it is inconsistent as well as asymptotically biased. Therefore, it is useful to study the probability of obtaining a infeasible MM estimators in any given situation, at least for small sample sizes. We are not familiar with any literature on this topic except the very recent paper of

Dupuis (1996), although all the text books on statistics mention about the MM estimators. In her paper, Dupuis (1996) considers a very special case of a generalized Pareto distribution and presents some simple programs to predict the probability of obtaining such infeasible estimators for large sample sizes.

The main aim of this paper is to consider first some simple examples and compute the probability of infeasible MM estimators explicitly and then we consider some of the cases when it cannot be computed explicitly. We recommend to use Monte Carlo simulations to compute the probability of obtaining the infeasible MM estimators in those situations. It is known (Hosking and Wallis; 1987) that for the generalized Pareto distribution the MM estimators behave better than the ML estimators for sample sizes up to 500 for certain ranges of parameters. So it is reasonable to use MM estimators for small samples sizes in the case of generalized Pareto distribution. Dupuis (1996) considers the generalized Pareto distribution but she considers only the larger sample (sample size ranges from 500 to 10000) and obtain the probability of infeasible MM estimators. We also consider the generalized Pareto distribution but mainly for small sample sizes and make some comments about Dupuis (1996)'s approach.

The rest of the paper is organized as follows: In Section 2, we consider one parameter and two parameter exponential distributions. In Section 3, we consider uniform distribution, generalized Pareto distribution is considered in Section 4. Some numerical experiments are performed in Section 5. Finally, we draw conclusions in Section 6.

2. EXPONENTIAL DISTRIBUTION

First we consider one parameter exponential distribution. Let $X_1, X_2, ..., X_n$ be a random sample of size n from the following distribution

$$f(x) = \begin{cases} e^{-(x-\alpha)} & \text{if } x > \alpha \\ 0 & \text{otherwise} \end{cases}$$

Observe that in this situation the MM estimator of α, say $\hat{\alpha}_n$, is $\bar{X} - 1$. Now there are two kinds of feasibility questions about $\hat{\alpha}_n$. First of all if we have information (prior) that $\alpha \geq 0$ (for example X represents the lifetime) then

$$X_i > \hat{\alpha}_n \geq 0 \Leftrightarrow \bar{X} \geq 1 \text{ and } X_i > \bar{X} - 1 \text{ for all } i = 1,2,...,n \text{ or } X_{(1)} \geq \bar{X} - 1,$$

where $X_{(i)}$ denotes the ith order statistic. Otherwise, $\hat{\alpha}_n$ is not feasible. Therefore,

$$P[\hat{\alpha}_n \text{ is feasible}] = P[\bar{X} \geq 1, \bar{X} \leq X_{(1)} + 1]$$

$$= P[n \leq \sum_{i=1}^{n} X_i \leq n X_{(1)} + n]$$

$$= P[n(1 - X_{(1)}) \leq \sum_{i=1}^{n} (X_{(i)} - X_{(1)}) \leq n]$$

$$= n\int_{\alpha}^{1}\int_{n(1-x)}^{n} g(y)dy\, e^{-n(x-\alpha)}\, dx + n\int_{1}^{\infty}\int_{0}^{n} g(y)dy\, e^{-n(x-\alpha)}\, dx, \quad \text{if } \alpha < 1 \tag{2.1}$$

$$= n\int_{\alpha}^{\infty}\int_{0}^{n} g(y)dy\, e^{-n(x-\alpha)}\, dx, \quad \text{if } \alpha \geq 1 \tag{2.2}$$

$$\text{where } g(y) = \begin{cases} \frac{1}{\Gamma(n-1)} y^{n-2} e^{-y} & y > 0 \\ 0 & \text{otherwise}. \end{cases} \tag{2.3}$$

Using the fact that $\int_0^a g(y)dy = 1 - \sum_{i=0}^{n-2} e^{-a} \frac{a^i}{i!}$, (2.1) can be written as

$$n\int_{\alpha}^{1}\sum_{i=0}^{n-2}\left(e^{-n(1-x)}\frac{n^i(1-x)^i}{i!} - e^{-n}\frac{n^i}{i!}\right) e^{-n(x-\alpha)}\,dx + e^{n(\alpha-1)}\left(1-\sum_{i=0}^{n-2} e^{-n}\frac{n^i}{i!}\right)$$

$$= \sum_{i=0}^{n-1} e^{-n(1-\alpha)}\frac{(n(1-\alpha))^i}{i!} - \sum_{i=0}^{n-2} e^{-n}\frac{n^i}{i!}.$$

Also, (2.2) can be simplified as

$$\int_0^n g(y)\,dy = 1-\sum_{i=0}^{n-2} e^{-n}\frac{n^i}{i!}.$$

In the case we have no information on α,

$$P[\hat{\alpha}_n \text{ is feasible}] = P[\bar{X} < X_{(1)} + 1] = P[\sum_{i=2}^{n}(X_{(i)} - X_{(1)}) < n]$$

$$= \int_0^n g(y)\,dy = 1-\sum_{i=0}^{n-2} e^{-n}\frac{n^i}{i!}.$$

Next we consider the two parameter exponential distribution. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the following density function

$$f(x) = \begin{cases} \frac{1}{\theta}e^{-(x-\alpha)/\theta} & x > \alpha,\ \theta > 0 \\ 0 & \text{otherwise}. \end{cases}$$

It can be easily seen that the MM estimators of α and θ are

$$\hat{\alpha}_n = \bar{X} - S \quad \text{and} \quad \hat{\theta}_n = S,$$

where $nS^2/(n-1)$ is the sample variance. Again, we may or may not have prior information on α being positive. In case $\alpha \geq 0$,

$$P[\hat{\alpha}_n \text{ is feasible}] = P[S < \bar{X} < X_{(1)} + S]. \tag{2.4}$$

If we have no restriction on α, then

$$P[\hat{\alpha}_n \text{ is feasible}] = P[\bar{X} < X_{(1)} + S]. \tag{2.5}$$

Observe that it is not very easy to compute (2.4) and (2.5) explicitly although (2.4) depends only on α/θ and (2.5) is independent of α and θ. We perform some Monte Carlo simulations to estimate (2.4) and (2.5) in Section 5.

3. UNIFORM DISTRIBUTION

Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from $U(0,\theta)$. Then observe that the MM estimator of θ is $\hat{\theta}_n = 2\bar{X}$. Therefore,

$$P[\ \hat{\theta}_n \text{ is feasible }] = P[\ X_i \le 2\bar{X} \text{ for all } i = 1,2,\ldots,n\ ]$$

$$= P[\ nX_{(n)} \le 2\sum_{i=1}^{n} X_{(i)}\ ] = 1 - P[\ \sum_{i=1}^{n-1} X_{(i)} \le \frac{n-2}{2} X_{(n)}\ ]$$

$$= 1 - \frac{n}{\theta^n \Gamma(n-1)} \int_0^{\theta} \int_0^{\frac{n-2}{2}x} \sum_{i=0}^{[\frac{n-2}{2}]} (-1)^i \binom{n-1}{i} (z-ix)_+^{n-2}\, dz\, dx$$

$$= 1 - \frac{n}{\theta^n \Gamma(n-1)} \int_0^{\theta} \sum_{i=0}^{[\frac{n-2}{2}]} (-1)^i \binom{n-1}{i} \int_{ix}^{\frac{n-2}{2}x} (z-ix)_+^{n-2}\, dz\, dx$$

$$= 1 - \frac{n}{\theta^n\ \Gamma(n)} \int_0^{\theta} \sum_{i=0}^{[\frac{n-2}{2}]} (-1)^i \binom{n-1}{i} (\frac{n-2}{2} - i)^{n-1} x^{n-1}\, dx$$

$$= 1 - \sum_{i=0}^{[\frac{n-2}{2}]} (-1)^i \frac{1}{\Gamma(i+1)\ \Gamma(n-i)} (\frac{n-2}{2} - i)^{n-1}, \qquad (3.1)$$

where $(z - ix)_+$ denotes $\max\{0, (z - ix)\}$. Observe that (3.1) does not depend on θ and it is a decreasing function of n. Simple numerical computations on MAPLE show that (3.1) decreases to ½ as n tends to ∞.

Now suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n from $U(\theta\text{-}a, \theta\text{+}a)$, where a is known and θ is unknown. The corresponding density function is

$$f(x) = \begin{cases} (2a)^{-1} & \text{if } \theta - a \le x \le \theta + a, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

Without loss of generality let us assume that a = ½. The MM estimator of θ is $\hat{\theta}_n = \bar{X}$. It is interesting to note that ML estimator is not unique in this case, for example any value between ($X_{(n)}$ -½) to ($X_{(1)}$+ ½) maximizes the likelihood function although MM estimator is unique.

$$P[\ \hat{\theta}_n \text{ is feasible }] = P[\ \bar{X} - \frac{1}{2} \le X_{(1)} \le X_{(n)} \le \bar{X} + \frac{1}{2}\ ]$$

$$= P[\ (n-1)X_{(n)} - X_{(1)} - \frac{n}{2} \le \sum_{i=2}^{n-1} X_{(i)} \le (n-1)X_{(1)} - X_{(n)} + \frac{n}{2}\ ] \tag{3.3}$$

Observe that the conditional distribution of $\sum_{i=2}^{n-1} X_{(i)}$ given $X_{(1)}$, $X_{(n)}$ is the same as the distribution of the sum of random sample of size (n-2) from $U(X_{(1)}, X_{(n)})$. Therefore, (3.3) can be evaluated explicitly but the actual expression become very messy. However, it is clear that it does not depend on θ. We have performed simulations to estimate (3.3) in section 5.

4. GENERALIZED PARETO DISTRIBUTION

In this section we consider some feasibility problem concerning Generalized Pareto (GP) distribution. The GP distribution was introduced by Pickands (1975) and it has applications in a number of fields including reliability studies and the analysis of environmental extreme events. It is a two parameter distribution and it contains uniform, exponential and Pareto distribution as special cases.

Let the random variable X follows a GP distribution, then X has the density function

$$f(x) = \begin{cases} \frac{1}{\alpha}(1 - \frac{kx}{\alpha})^{\frac{1}{k} - 1} & \text{if } k \neq 0 \\ \frac{1}{\alpha}\exp(-x/\alpha) & \text{if } k = 0, \end{cases} \qquad (4.1)$$

the range of x is $0 \le x < \infty$ for $k \le 0$ and $0 \le x \le \alpha/k$ for $k > 0$. The parameters of the distribution are α, the scale parameter, and k, the shape parameter. The special cases $k = 0$ and $k = 1$ yield, the exponential distribution and uniform distribution on $[0, \alpha]$, respectively. For $k < 0$ the ordinary Pareto distribution is obtained.

It is important to observe that for $k < 0$, the r^{th} moment of X exists if $k > -1/r$ (see Hosking and Wallis; 1987) and it can be easily seen that if $0 > k > -1/2$, then

$$E(X) = \alpha/(1+k) \text{ and } Var(X) = \alpha^2 / ((1+k)^2 (1+2k)) \qquad (4.2)$$

On the other hand if $k \ge 0$ then all the moments of X exist, which can be easily seen as follows for $k > 0$

$$E(X^m) = \frac{1}{\alpha} \int_0^{\alpha/k} x^m (1 - \frac{kx}{\alpha})^{\frac{1}{k} - 1} dx$$

$$= \frac{\alpha^m}{k^{m+1}} \int_0^1 y^m (1-y)^{\frac{1}{k} - 1} dy$$

$$= \frac{\alpha^m}{k^{m+1}} B(m+1, \frac{1}{k}) = \frac{m!\ \alpha^m}{(km+1)(k(m-1)+1)...(k+1)} \qquad (4.3)$$

and for exponential distribution ($k = 0$) all the moments exist. However, Hosking and Wallis (1987, p. 340) commented that for $k \ge 1/2$, X has infinite variance, which is not correct.

Therefore for $-1/2 < k < 0$, the MM estimators of α and k exist and they are as follows: (Hosking and Wallis; 1987)

$$\hat{\alpha}_n = \frac{\bar{X}}{2}\left(\frac{\bar{X}^2}{S^2} + 1\right) \quad \text{and} \quad \hat{k}_n = \frac{1}{2}\left(\frac{\bar{X}^2}{S^2} - 1\right) \tag{4.4}$$

where $\bar{X}$ and $nS^2/(n-1)$ are sample mean and variance respectively.

For $k > 0$, the MM estimators exist and they are the same as in defined (4.4). It is important to note that for the GP distribution the ML estimators do not exist for $k > 1$ (Grimshaw (1993)). This is because for $k > 1$ the likelihood function converges to ∞ as α/k tends to $X_{(n)}$ from the upper side. However, the MM estimators do exist in this case.

Now we would like to consider the feasibility of the MM estimators in two different situations, namely for $k > 0$ and for $-\frac{1}{2} < k < 0$. This is because for $k = 0$, the MM estimator of α is always feasible.

Case I: $k > 0$.

$$P[\ \hat{k}_n \text{ and } \hat{\alpha}_n \text{ are feasible }]$$

$$= P[\ \hat{k}_n > 0 \text{ and } X_i \le \hat{\alpha}_n / \hat{k}_n \text{ for all } i = 1, 2, \ldots, n\ ]$$

$$= P[\ \hat{k}_n > 0 \text{ and } X_{(n)} \le \hat{\alpha}_n / \hat{k}_n\ ]$$

$$= P\left[\ \frac{\bar{X}^2}{S^2} > 1 \text{ and } \frac{\bar{X}\left(\frac{\bar{X}^2}{S^2} + 1\right)}{\left(\frac{\bar{X}^2}{S^2} - 1\right)} \ge X_{(n)}\ \right]$$

$$= P\left[\ \frac{X_{(n)} - \bar{X}}{X_{(n)} + \bar{X}} \le (C.V.)^2 < 1\ \right], \tag{4.5}$$

where C.V. denotes the coefficient of variation.

Case II: $-\frac{1}{2} \le k < 0$.

In this case $\hat{\alpha}_n$ is always feasible.

$$P[\ \hat{k}_n \text{ is feasible }] = P\left[\ -\frac{1}{2} < \hat{k}_n < 0\ \right]$$

$$= P[\bar{X}^2 < S^2] = P[(C.V.)^2 > 1]. \tag{4.6}$$

In both the cases (4.5) and (4.6) it is not easy to evaluate the probabilities explicitly. We propose to use Monte Carlo simulation to estimate (4.5) and (4.6). However, observe that (4.6) converges to 1 as n tends to ∞ due to the strong law of large numbers.

Dupuis (1996) considers the case when $0 < k \le ½$ but unfortunately she does not consider the feasibility of $\hat{k}_n$. Instead, she considers the feasibility of $\hat{\alpha}_n$ and $\hat{k}_n$ given that $\hat{k}_n > 0$, which may not be correct. Observe that it is important to know the range of k because the likelihood function, the range of the data vector and also the feasibility questions are quite different in different ranges. For example when $k = 0$, there is no question of feasibility of MM estimators. In fact in our simulation it is observed that P[$\hat{k}_n$ is infeasible] may not be negligible for small positive k. Another point regarding the work of Dupuis (1996) we would like to mention that she made the following statement

If $X_1, X_2, \ldots, X_n$ is a random sample of size n from GP distribution, then

$$P[\text{All } X_i < \hat{\alpha}_n/\hat{k}_n \text{ for all } i = 1, 2, \ldots, n] \tag{4.7}$$

$$= (P[X < \hat{\alpha}_n/\hat{k}_n])^n \tag{4.8}$$

Here $\hat{\alpha}_n$ and $\hat{k}_n$ are same as defined in (4.4), and X is GP with parameters α and k. We feel that (4.7) does not imply (4.8) due to the fact that the events in (4.7) are not independent. Unfortunately her most of the analysis and approximations are based on (4.8).

5. NUMERICAL EXPERIMENTS

In this section, we present some Monte Carlo simulation results to estimate (2.4), (2.5), (3.3), (4.5) and (4.6) where the explicit expressions are not possible. All these simulations are performed on Sun Workstation using the random deviate generator RAN2 of Press et al (1986). In all the cases we use RAN2 as the uniform random deviate generator and using the proper transformation we obtain the different distributions. All the results reported here are based on 10,000 replications. We use the sample sizes 10, 15, 50, 100, 500, and 1,000 in all the cases.

In the case of two parameters exponential distribution, to estimate the probability of feasible MM estimator, when there is a prior information on α (i.e. (2.4)) and when there is no prior information on α (i.e. (2.5)), we consider α = .25, .50, 1.0, 2.0 and θ = 1.0 (without loss of generality). The results are reported in Table 1. It is observed that when we have restrictions on α, namely $\alpha > 0$, then for fixed α as n increases the probability of feasible MM estimator of α increases and it seems it is increasing to ½ as in the case of one parameter exponential family. For fixed θ, the probability of feasible estimator of α increases as α increases to 1 and after that (for $\alpha > 1$) it remains constant. We compute the result for $\alpha = 3$ and $\alpha = 4$ also and it is exactly same as $\alpha = 2$ and we do not report the results separately. As sample size increases the probability becomes independent of α. When we do not have restrictions on α, then the feasible probability is independent of α and θ and it is the same probability as we obtain in the previous case for $\alpha > 1$ (last row of Table 1).

In case of uniform distribution to estimate the probability of feasible MM estimator (i.e. (3.3)) we consider θ = ½ without loss of generality because (3.3) is independent of θ. The results are reported in Table 2. From the table it is clear that as sample size increases the probability of feasible MM estimator is gradually decreasing and it seems the probability is converging to zero. This may be due to the fact that the convergence of $X_{(1)}$ ($X_{(n)}$) to θ - ½ (θ + ½) is much faster than the convergence of $\bar{X}$ to θ.

In case of GP distribution to estimate the probability of feasible MM estimators for -½ < k

< 0 (i.e. (4.6)) we consider $k = -.4, -.3, -.2, -.1$ and for $k > 0$ (i.e. (4.5)) we take $k = .1, .2, .3, .4, .5, .75, 1.0, 1.25, 1.50$, and $\alpha = 1$ (without loss of generality) in both the cases. The results for $-½ < k < 0$ and for $k > 0$ are reported in Table 3 and Table 4 respectively.

In Table 3, when $-½ < k < 0$, it is observed that as sample size increases the probability is increasing to one. It is also observed that as k approaches zero, the probability decreases and it is quite prominent at least for small sample sizes. The results are quite different for $k > 0$ in Table 4. Distinct features are observed for $0 < k < ½$ and for $k \geq ½$. For $0 < k < ½$ it is observed that as n increases the probability is increasing and it seems it is increasing to one. On the other hand if $k \geq ½$ the probability is quite erratic and no such conclusions can be made.

6. CONCLUSIONS

In this paper we consider the problem of feasibility of MM estimators when there are some restrictions on the parametric space and/or the range of the data depending on parameters. We feel it is an important problem although it did not get enough attention in the literature. Since the MM estimators are always consistent (by strong law of large numbers), it is expected that they will be feasible also. But we observe some of the counter intuitive results in some situations. Simulation results indicate that the feasibility might even converge to zero also. Since the MM estimators do not take into consideration the restrictions on the parametric space and/or the range depending on the parameters, this work clearly suggests that practitioner must check out the feasibility of the MM estimators in such situations before using them.

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Table 1

Two parameters exponential (Restrictions on α)

$\theta = 1$

α\n	10	15	50	100	500	1000
.25	.3450	.3456	.4003	.4358	.4739	.4758
.50	.3914	.4022	.4262	.4455	.4739	.4758
1.0	.4049	.4118	.4275	.4457	.4739	.4758
2.0	.4060	.4122	.4275	.4457	.4739	.4758

Table 2

Uniform [θ - ½, θ + ½]

n	10	15	50	100	500	1000
Prob	.6998	.6093	.3701	.2665	.1203	.0876

Table 3

Generalized Pareto: $\alpha = 1, k < 0$

k \ n	10	15	50	100	500	1000
-.4	.5621	.6894	.9506	.9954	1.00	1.00
-.3	.5024	.6015	.8901	.9756	1.00	1.00
-.2	.4143	.5120	.7920	.9130	.9996	1.00
-.1	.3328	.4043	.6079	.7264	.9667	.9967

Table 4

Generalized Pareto k > 0, $\alpha = 1$

k \ n	10	15	50	100	500	1000
.10	.7169	.7351	.8317	.8986	.9932	.9998
.20	.7469	.7901	.8974	.9461	.9808	.9865
.30	.7524	.7903	.8744	.8909	.9182	.9333
.40	.7375	.7722	.8088	.8143	.8261	.8275
.50	.7179	.7336	.7354	.7314	.7221	.7275
.75	.6390	.6443	.6145	.5924	.5669	.5539
1.0	.5777	.5600	.5378	.5275	.5134	.5118
1.25	.5187	.5255	.4997	.5016	.4964	.4974
1.50	.5038	.4865	.4886	.4786	.4923	.4961