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ON AN INERTIAL METHOD OF MEASURING VELOCITIES

This is a translation of an article written by Candidate of Technical Sciences, Associate Professor V. N. Drozdovich, of Leningrad Institute of Precision Mechanics and Optics, published in Izvestiya Vysshikh Uchebnykh Zavedeniy, Priborostroyeniye (News of Institutions of Higher Education, Equipment Manufacture), No. 3, 1959, pages 29-39.

Along with the known methods of measuring velocities based on the hydrodynamic effect and on the Doppler effect, there is at present in wide use a method based on the integration of accelerations. The measurement of velocities by means of integrating accelerations constitutes the basis of the so-called inertial system of navigation which serves to determine the complex problem of controlling an object in motion along an assigned trajectory /1/. Notwithstanding a number of undisputed advantages, the use of the method of integrating accelerations under the usual conditions of low velocities and considerable time intervals is associated with the need to account for a number of systematic and fortuitous errors /2/. In this connection it is interesting to determine these errors in the instances when they can accurately be calculated, or to determine their probability characteristics if they are of an accidental nature.

This article is an attempt to investigate systematic errors caused by the Coriolis and centripetal accelerations, for the purpose of finding their values in measuring velocities during long time intervals. The part played by coulomb friction is explained and an estimate is made of static errors caused by the forces of coulomb friction.

Equations of Motion

Let us imagine a gyroscope in a Cardan suspension mounted on a base that is moving with acceleration (Fig. 1). We will assume that the center of gravity of the gyroscope coincides with the fixed point of suspension, and that the gyroscope's spin axis is directed upward and makes small angles with the vertical. Mounted on the housing of the

gyroscope are two precision accelerometers whose sensitive axes are perpendicular to the spin axis and at right angles to each other. The accelerometers are connected to the circuit of moment pick-offs governed by integral connection. Thus, we obtain a gyro vertical with integral correction, which possesses the properties of an astatic system of automatic control. Thanks to astaticism, such a gyro vertical will assume a plumb position with an accuracy to within ballistic deviations.

We will refer the gyroscope to the horizontal system of coordinates $ox_0y_0z_0$, the origin of which will be made to coincide with its center of inertia (Fig. 2). We will place the axes ox_0 and oy_0 in the plane of the horizon in such a manner that they will form, together with axis oz_0 directed vertically upward, the right-hand system of coordinates. Thereafter we will assume that the axis of rotation of the outer gimbal of the Cardan suspension coincides with axis ox_0 . For the time being, we will leave the orientation of the axes ox_0 and oy_0 in the plane of the horizon undetermined.

We will rigidly connect the gyroscope's housing with the system of coordinates $oxyz$, whose axes ox and oy are located in the equatorial plane of the gyroscope and form a right-hand system of coordinates with axis oz directed along the spin axis. Axis oy will be made to coincide with the rotation axis of the gyroscope's housing.

With this selection of the systems of coordinates the gyroscope's housing will be exactly determined by angles α and β equal to angles $\widehat{(y_1y_0)}$ и $\widehat{(x_1x_0)}$, respectively.

We will introduce the following designations:

- $\vec{\omega}_0$ is the vector of angular velocity of rotation of the system of coordinates $ox_0y_0z_0$ relative to a certain inertial system of coordinates;
- $\vec{\omega}_1$ is the vector of angular velocity of rotation of the system of coordinates $oxyz$ relative to the horizontal;
- $\vec{\omega}$ is the vector of the absolute angular velocity of rotation of the system of coordinates $oxyz$;
- φ is the angular velocity of rotation of the gyroscope's rotor relative to the housing;
- J is the polar moment of inertia of the gyroscope's rotor;
- J_1 is the equatorial moment of inertia of the gyroscope's rotor together with the housing;
- M_x, M_y , and M_z are the components of the external torque applied to the gyroscope's housing along the axis of the system of coordinates $oxyz$;

u_x and u_y are the horizontal components of the object's motion velocity relative to the earth's surface;
 ω_1 and ω_2 are the horizontal and vertical components, respectively, of the angular velocity of the earth's rotation;

ψ is the angle between the vector \vec{v} of the object's true velocity and the meridian, reckoned west of the meridian;

γ is the angle between axis ox_0 and vector \vec{v} , reckoned west of axis ox_0 ;

R is the radius of the earth.

Hereafter we will assume that motion takes place along a large circle, and we will connect axes ox_0 and oy_0 with the object in such a manner that axis ox_0 extends along the longitudinal axis of the object. We will assume that γ remains constant during the entire period of motion.

Taking into consideration that

$$\vec{\omega} = \vec{\omega}_e + \vec{\omega}_r$$

and projecting this vector equality on the axes of the system of coordinates $oxyz$, we find

$$\omega_x = \dot{\alpha} + \omega_{ex} \cos \beta - \omega_{ez} \sin \beta,$$

$$\omega_y = \dot{\beta} + \omega_{ey} \cos \alpha + \omega_{ex} \sin \beta \sin \alpha + \omega_{ez} \cos \beta \sin \alpha, \quad (1)$$

$$\omega_z = \dot{\gamma} \sin \alpha + \omega_{ex} \sin \beta \cos \alpha + \omega_{ey} \sin \alpha + \omega_{ez} \cos \alpha \cos \beta,$$

where

$$\omega_{ex} = \omega_1 \cos \psi - \frac{u_y}{R},$$

$$\omega_{ey} = \omega_1 \sin \psi + \frac{u_x}{R}, \quad (2)$$

$$\omega_{ez} = \omega_2.$$

Hereafter we will consider α , β , $\dot{\alpha}$ and $\dot{\beta}$ as infinitely small values of the same order. Limiting our analysis to small oscillations of the gyroscope's axis in the vicinity of the vertical and neglecting terms of higher orders of smallness, we obtain, for the projections of absolute angular velocity of the gyroscope's housing (2), linearized expressions

$$\begin{aligned}
\omega_x &= \dot{\alpha} + \omega_{ex} - \omega_{ez} \beta; \\
\omega_y &= \dot{\beta} + \omega_{ey} + \omega_{ez} \alpha; \\
\omega_z &= \omega_{ez} + \omega_{ex} \beta + \omega_{ey} \alpha.
\end{aligned}
\tag{2'}$$

Applying the Resal theorem, we obtain the following equations in projection along the axes of the system of coordinates oxyz of the gyroscope's motion:

$$\begin{aligned}
J_e \frac{d\omega_x}{dt} + J\omega_y(\omega_z + \dot{\varphi}) - J_e \omega_z \omega_y &= M_x; \\
J_e \frac{d\omega_y}{dt} - J\omega_x(\omega_z + \dot{\varphi}) + J_e \omega_z \omega_x &= M_y; \\
J \frac{d(\omega_z + \dot{\varphi})}{dt} &= 0.
\end{aligned}
\tag{3}$$

Considering that $\omega_z = \omega_z \ll \dot{\varphi}$ and assuming that $M_z = 0$, we get

$$J(\omega_z + \dot{\varphi}) = J\dot{\varphi} = H = \text{const.} \tag{4}$$

We further assume that $J = J_e$, and neglecting the nutational terms in Eqs. (3), we obtain the following equation of motion on the basis of (4):

$$H\omega_y = M_x; \quad -H\omega_x = M_y. \tag{5}$$

Torques M_x and M_y represent here values which are proportional to the integrals of accelerations measured by the accelerometers. The coefficients should be selected in such a way as to make ballistic deviations minimal. The latter requirement is equal to the fulfillment of the Schuler condition.

Projecting all accelerations on axes ox and oy, including the acceleration of the force of gravity which we will consider to be directed along the earth's radius, we obtain the following expressions for M_x and M_y :

$$\begin{aligned}
M_x &= k_1 \int_0^t (W_x - g\beta) dt + M_{1x}; \\
M_y &= k_2 \int_0^t (W_y + g\alpha) dt + M_{1y}.
\end{aligned}
\tag{6}$$

Here g is the acceleration of the force of gravity neglecting the centripetal acceleration;

\underline{W}_x and \underline{W}_y are the components of the acceleration vector

in absolute motion along axis ox and oy ;

\underline{M}_{lx} and \underline{M}_{ly} are certain supplementary torques applied to

the gyroscope which should compensate for the horizontal component of the earth's rotation; these torques should be produced by a calculating-determinating device.

Substituting (6) into Eqs. (5) and dividing the left-hand and right-hand parts by H , we obtain, on the basis of (2') and (2),

$$\dot{\beta} + \omega_1 \sin \phi + \frac{v_x}{R} + \omega_2 \alpha = \frac{k_1}{H} \int_0^t (W_x - g\beta) dt + \frac{M_{lx}}{H}; \quad (7)$$

$$\dot{\alpha} + \omega_1 \cos \phi - \frac{v_y}{R} - \omega_2 \beta = -\frac{k_2}{H} \int_0^t (W_y + g\alpha) dt - \frac{M_{ly}}{H}.$$

Hence it is clear that the supplementary torques should be as follows:

$$M_{lx} = H\omega_1 \sin \phi; \quad M_{ly} = -H\omega_1 \cos \phi. \quad (8)$$

In order to determine the coefficients of proportionality \underline{k}_1 and \underline{k}_2 , we will proceed as follows: for the time

being we will assume that the base is moving along a large circle on a certain imaginary sphere of radius equal to the earth's radius, the motion not taking part in the rotational motion of the earth (2). Then the accelerations \underline{W}_x and \underline{W}_y will be in the form

$$\underline{W}_x = \dot{v}_x; \quad \underline{W}_y = \dot{v}_y. \quad (9)$$

In this case the equations of motion (7) will assume a very simple form:

$$\begin{aligned} \dot{\beta} + \frac{v_x}{R} &= \frac{k_1}{H} \int_0^t (\dot{v}_x - g\beta) dt, \\ \dot{\alpha} - \frac{v_y}{R} &= -\frac{k_2}{H} \int_0^t (\dot{v}_y + g\alpha) dt. \end{aligned} \quad (10)$$

Differentiating these equations with respect to time and selecting the undetermined coefficients \underline{k}_1 and \underline{k}_2 in such a way that the equations will not contain accelerations $\underline{\ddot{x}}$ and $\underline{\ddot{y}}$, we find

$$k_1 = k_2 = \frac{H}{R}. \quad (11)$$

Equations of motion (10) with coefficients \underline{k}_1 and \underline{k}_2 from (11) are written as follows after differentiation:

$$\ddot{\beta} + \frac{g}{R} \beta = 0; \quad \ddot{\alpha} + \frac{g}{R} \alpha = 0. \quad (12)$$

Hence we see that under condition (11) and circumstances discussed above, the gyro vertical will be free from ballistic deviations but will perform harmonic oscillations about the true vertical within a period equal to the Schuler period of 84.4 min.

We will now return to the case of motion along a large circle, and assuming that conditions (8) and (11) are fulfilled, we will write Eqs. (7) in the following form:

$$\begin{aligned} \ddot{\beta} + \frac{v_x}{R} + \omega_2 \alpha &= \frac{1}{R} \int_0^t (W_x - g\beta) dt; \\ \ddot{\alpha} - \frac{v_y}{R} - \omega_2 \beta &= -\frac{1}{R} \int_0^t (W_y + g\alpha) dt. \end{aligned} \quad (13)$$

Accelerations \underline{W} and \underline{W} will be somewhat more complex in these equations than $\underline{\ddot{x}}$ and $\underline{\ddot{y}}$ in Eqs. (10), since they will contain in addition to the relative acceleration, also the centripetal and Coriolis accelerations, produced by the rotation of the earth.

Accelerations During Motion Along a Large Circle on the Sphere of the Earth

Assuming that the vector of transfer velocity \underline{v} lies in the plane of the horizon at all times, we will neglect the vertical component of the relative acceleration.

According to the Coriolis theorem, absolute acceleration can be found from the following formula:

$$\underline{W}_{abs} = \underline{W}_{transf} + \underline{W}_{rel} + \underline{W}_{cor} \quad (14)$$

The acceleration of transfer motion is a function of the rotation of the earth. The vector of this acceleration lies in the plane of the meridian and its direction is perpendicular to the axis of the earth's rotation, its value being dependent on the latitude of the location φ . For its components along the axes of the horizontal system of coordinates $\begin{smallmatrix} \text{ox} & \text{y} & \text{z} \\ 0 & 0 & 0 \end{smallmatrix}$, oriented in the manner indicated

above, we will have the following expressions:

$$\begin{aligned} W' \text{ trans } x &= \frac{1}{2} R \omega_z^2 \sin 2\varphi \cos(\psi + \gamma); \\ W' \text{ trans } y &= \frac{1}{2} R \omega_z^2 \sin 2\varphi \sin(\psi + \gamma); \\ W' \text{ trans } z &= -\frac{1}{2} R \omega_z^2 \cos^2 \varphi. \end{aligned} \quad (15)$$

The acceleration of the relative motion will be centripetal and rotary. The vector of this acceleration will lie in the plane of a large circle and its components along the axis of the system of coordinates $\begin{smallmatrix} \text{ox} & \text{y} & \text{z} \\ 0 & 0 & 0 \end{smallmatrix}$ will

be written as follows:

$$\begin{aligned} W' \text{ rel } x &= \dot{v}_x; \\ W' \text{ rel } y &= \dot{v}_y; \\ W' \text{ rel } z &= -\frac{1}{R} (v_x^2 + v_y^2). \end{aligned} \quad (16)$$

Finally, the Coriolis acceleration will be a function of the rotation of the earth and the relative motion of the object along the large circle. The vector of this acceleration can be found according to the formula

$$W' \text{ cor} = 2\vec{\omega}_z \times \vec{v}. \quad (17)$$

Projecting the vector equality (17) along the axes of the horizontal system of coordinates, we find the components of the Coriolis acceleration:

$$\begin{aligned} W' \text{ cor } x &= -2\omega_z v_y; \\ W' \text{ cor } y &= 2\omega_z v_x; \\ W' \text{ cor } z &= 2\omega_z \sqrt{v_x^2 + v_y^2} \cdot \sin \varphi. \end{aligned} \quad (18)$$

On the basis of expressions (15), (16) and (18) we find the components of the vector of absolute acceleration along the axis of the horizontal system of coordinates, which have the form

$$\begin{aligned} W_x &= R \omega_1 \omega_2 \cos(\psi + \gamma) - 2 \omega_2 v_y + \dot{v}_x; \\ W_y &= R \omega_1 \omega_2 \sin(\psi + \gamma) + 2 \omega_2 v_x + \dot{v}_y; \\ W_z &= -R \omega_2^2 - \frac{v^2}{R} + 2 \omega_1 v \sin \psi. \end{aligned} \quad (19)$$

The designations here are

$$\omega_1 = \omega_2 \cos \varphi; \quad \omega_2 = \omega_0 \sin \varphi; \quad v = \sqrt{v_x^2 + v_y^2}.$$

Hereafter, as before, we will neglect the nonspherical shape of the earth. It should however, be borne in mind that the calculation of the deviation of the true shape of the earth's surface from a sphere is very important in a quantitative evaluation of errors.

Projecting the components of absolute acceleration along the axes oxyz connected with the gyroscope's housing, and separating the terms that characterize the centripetal and Coriolis accelerations, we obtain, in first approximation, the following expressions for accelerations acting on the accelerometers:

$$\begin{aligned} W_x &= W_x^* + v_x; \\ W_y &= W_y^* + v_y; \end{aligned} \quad (20)$$

where

$$\begin{aligned} W_x^* &= R \omega_1 \omega_2 \cos(\psi + \gamma) - 2 \omega_2 v_y; \\ W_y^* &= R \omega_1 \omega_2 \sin(\psi + \gamma) + 2 \omega_2 v_x. \end{aligned} \quad (21)$$

Determination of Errors Produced by Centripetal and Coriolis Accelerations

Analyzing the simplest case of motion along a large circle of a nonrotating sphere, we saw that in fulfilling conditions (11) the gyro vertical is free from deviations. This does not mean, however, that we will get at the

output of each accelerometer an accurate value of the velocity component. This could happen only in the case of zero initial conditions for Eqs. (12) of the gyro vertical's motion. But if the initial conditions are different than zero, the gyro vertical will produce harmonic oscillations, which will result in an error of velocity measurement that also changes according to the harmonic function.

Let us assume, for example, that $\beta|_{t=0} = \beta_0$; $\dot{\beta}|_{t=0} = 0$; then we will get the following solution from the first Eq. (12):

$$\beta = \beta_0 \cos \sqrt{\frac{g}{R}} t. \quad (22)$$

The magnitude of the oscillation error of velocity measurement equals

$$\Delta v_x \equiv \int_0^t g \beta dt = \beta_0 \sqrt{gR} \sin \sqrt{\frac{g}{R}} t. \quad (23)$$

The error of measurement of the velocity \underline{v} is determined in exactly the same manner.

It is interesting to note that the amplitude of this oscillation error is proportional to the magnitude of the first cosmic velocity of a zero-latitude satellite which, as is known, equals \sqrt{gR} . Regardless of this, the error amplitude can, in principle, be made as small as desired, by means of decreasing the initial angle of deviation β_0 . The systematic nature of this error makes possible its elimination, if only it does not become distorted in the process of motion under the effects of accidental disturbances.

Let us now return to our case of motion along the surface of the earth and attempt to determine the effect of centripetal and Coriolis accelerations. Substituting the expressions for \underline{W} and \underline{W} from (20) into Eqs. (13), we obtain

$$\ddot{\beta} + \omega_z \dot{\alpha} = \frac{1}{R} \int_0^t (W_x' - g \beta) dt; \quad (24)$$

$$\ddot{\alpha} - \omega_z \dot{\beta} = -\frac{1}{R} \int_0^t (W_y' + g \alpha) dt.$$

Differentiating Eqs. (24), we obtain the equations of motion of the gyro vertical,

$$\begin{aligned}\ddot{\beta} + \omega_2 \dot{\alpha} + \dot{\omega}_2 \alpha + \frac{g}{R} \beta &= \frac{1}{R} W_x^*, \\ \ddot{\alpha} - \omega_2 \dot{\beta} - \dot{\omega}_2 \beta + \frac{g}{R} \alpha &= -\frac{1}{R} W_y^*.\end{aligned}\quad (25)$$

We assume that we know the steady motion of the gyro vertical and its corresponding quotient solution:

$$\alpha_r = \alpha_r(t); \quad \beta_r = \beta_r(t). \quad (26)$$

Then, stating $\xi = \alpha - \alpha_r$, $\eta = \beta - \beta_r$, we can write the equations in variations which reflect the small oscillations of the gyro vertical about the steady motion (26). The equations will be held in the following form of variations:

$$\begin{aligned}\ddot{\xi} + \omega_2 \dot{\xi} + \dot{\omega}_2 \xi + \frac{g}{R} \eta &= 0; \\ \ddot{\eta} - \omega_2 \dot{\xi} - \dot{\omega}_2 \xi + \frac{g}{R} \xi &= 0.\end{aligned}\quad (27)$$

The equations obtained (27) are analogous to Eqs. (12) which determine the oscillation error. Unlike the latter, Eqs. (27) are dependent. The terms containing the multiple ω_2 are very small if the relative velocity v is small, and they can be discarded. Without considering in detail the analysis of Eqs. (27), we only wish to note that the oscillations of error will take place within two periods, 24 hours and 84.4 min and that they are of a pulsating nature.

In order to calculate the errors produced by centripetal and Coriolis accelerations, we have formulas

$$\begin{aligned}\Delta v_x &= \int_0^t (W_x^* - g \beta_r) dt; \\ \Delta v_y &= \int_0^t (W_y^* + g \alpha_r) dt.\end{aligned}\quad (28)$$

where

$$\begin{aligned}W_x^* &= R \omega_1 \omega_2 \cos(\psi + \gamma) - 2 \omega_2 v_y; \\ W_y^* &= R \omega_1 \omega_2 \sin(\psi + \gamma) + 2 \omega_2 v_x.\end{aligned}$$

In order to calculate the errors according to formulas (28) it is first necessary to find the quotient

solution (26) of Eqs. (25). For this purpose, it is first of all necessary to concern ourselves with the laws governing the motion of the object. Formulas (28) can be presented in the following form on the basis of Eqs. (25):

$$\begin{aligned}\Delta v_x &= R(\omega_2 \alpha_r + \dot{\beta}_r); \\ \Delta v_y &= R(\omega_2 \dot{\beta}_r - \alpha_r).\end{aligned}\quad (29)$$

At constantly small magnitudes of velocities \underline{v} and \underline{v}_x , the functions $\underline{W}^*(\underline{t})$ and $\underline{W}^*(\underline{t})$ will be slowly changing functions of time; accordingly, $\underline{\alpha}$ and $\underline{\beta}_r$ will also be slowly changing functions of time. Under these conditions, we can approximately state

$$\Delta v_x \approx R \omega_2 \alpha_r; \quad \Delta v_y \approx R \omega_2 \dot{\beta}_r;$$

where

$$\alpha_r \approx -\frac{W_y^*}{g}; \quad \dot{\beta}_r \approx \frac{W_x^*}{g}.$$

Assuming, for example,

$$v_y = 10 \text{ m/sec}; \quad v_x = 0; \quad \varphi = 60^\circ; \quad R = 6.4 \cdot 10^6 \text{ m};$$

$$\omega_2 = 7.3 \cdot 10^{-5} \text{ 1/sec}; \quad \dot{\varphi} + \dot{\gamma} = 0,$$

we will obtain

$$W_x^* = 6.4 \cdot 10^6 \cdot 7.3^2 \cdot 0.87 \cdot 0.5 \cdot 10^{-10} - 2 \cdot 7.3 \cdot 987 \cdot 10^{-5} \cdot 10 =$$

$$= 0.0136 \text{ m/Sec}^2 \quad W_y^* = 0;$$

$$\alpha_r \approx 0; \quad \dot{\beta}_r \approx 0.0014;$$

$$v_y \approx 6.4 \cdot 10^6 \cdot 7.3 \cdot 10^{-5} \cdot 987 \cdot 1.4 \cdot 10^{-3} \approx 0.67 \text{ m/Sec}. \quad \Delta v_x \approx 0.$$

This example shows that errors produced by centripetal and Coriolis accelerations must be taken into account.

The Effect of Coulomb Friction

It is a known fact that friction in the bearings of suspension of the sensing element plays an important part.

in most precision gyroscopic devices. The accuracy and sensitivity of the device depends in large measure on coulomb friction, and this makes it necessary to develop especially sensitive bearings.

In this connection, it is interesting to estimate the effect of the forces of coulomb friction on the accuracy of velocity measurement in an inertial system. For this purpose, we will consider the simplest case of motion of the base, when Coriolis and centripetal accelerations are absent. Turning to the equations of motion (10) applicable to this case under conditions (11), and taking into account the friction torque in the bearings of Cardan suspension, we obtain the following equations of motion:

$$\ddot{\beta} + \frac{g}{R} \int_0^t \dot{\beta} dt = -\varphi_1 \operatorname{sign} \dot{\beta}; \quad (30)$$

$$\ddot{\alpha} + \frac{g}{R} \int_0^t \dot{\alpha} dt = \varphi_2 \operatorname{sign} \dot{\alpha}.$$

Here φ_1 and φ_2 are the torque forces of coulomb friction referred to the gyroscope's kinetic moment, i.e.,

$$\varphi_1 = \frac{M_{\tau x}}{H}; \quad \varphi_2 = \frac{M_{\tau y}}{H}. \quad (31)$$

It is not difficult to see that coulomb friction does not change the position of dynamic equilibrium of the gyro vertical. Matters are different with integrators, at the outputs of which we will have, in addition to useful components of the measured velocity, also supplementary components which compensate for friction torque. These supplementary components will appear as static errors, produced by friction torques $\underline{M}_{\tau x}$ and $\underline{M}_{\tau y}$. The limit values of static errors of measuring the velocity components will be as follows:

$$\begin{aligned} |\Delta v_x| &\equiv g \left| \int_0^\infty \dot{\beta} dt \right| = \frac{M_{\tau x} R}{H}; \\ |\Delta v_y| &\equiv g \left| \int_0^\infty \dot{\alpha} dt \right| = \frac{M_{\tau y} R}{H}. \end{aligned} \quad (32)$$

Let us assume, for example, that $|\Delta v_x| = 0.01 \text{ m/sec}$; the corresponding drift produced by coulomb friction will

be equal to

$$\frac{M_{\text{ex}}}{H} \cdot \frac{|\Delta \omega|}{R} = \frac{0.01}{6.4 \cdot 10^6} \approx 1.6 \cdot 10^{-9} \text{ 1/sec.}$$

Hence it is clear that the requirement applicable to the drift of the gyroscope in an inertial system should be higher than an analogous requirement applicable to the bearings of modern precision gyrocompasses.

Conclusions

The introduction of integral correction satisfying the Schuler condition is insufficient to obtain an accurate measurement of velocity by the method of inertia during motion along a large circle. In order to compensate for systematic errors produced by Coriolis and centripetal accelerations, it is necessary to introduce into the system additional devices which take into account the parameters of the trajectory and the position of the object along same.

Integral correction ensures absence of errors of velocity measurement, errors which increase without limit with the passage of time.

Bibliography

1. G. A. Slomyanskiy and Yu. N. Pryadilov, Poplavkovyye giroskopy i ikh primeneniye (Floating Gyroscopes and their Application), Oborongiz (State Press of the Defense Industry) 1958.
2. A. Yu. Ishlinskiy, ob uravneniyakh zadachi opredeleniya mestopolozheniya dvizhushchegosya obyektu posredstvom giroskopov i izmeriteley uskoreniya (Concerning the Equation of the Problem of Determining the Location of a Moving Object by Means of Gyroscopes and Velocimeters), PMM, Vol. XXI, issue 6, 1957.

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