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A NEW SEQUENTIAL GOODNESS-OF-FIT-TEST FOR THE THREE- PARAMETER WEIBULL DISTRIBUTION WITH KNOWN SHAPE PARAMETER VALUE BASED ON SKEWNESS AND Q-STATISTIC G.O.F.TEST STATISTICS

THESIS

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Due to its flexibility, the Weibull distribution has very wide applicability in a lot of disciplines and is very prevalent in reliability theory. Thus, a lot of statistical tests that generally have a substantial degree of computional complexity have been developed to determine if the data at hand can be represented with this distribution. This research presents a new omnibus goodness-of-fit test (G.O.F.) that has less computational complexity than the existing tests for the three-parameter Weibull distribution using a sequential application of two individual tests, sample skewness and Q-Statistic. A Monte Carlo procedure has been employed to generate critical values for the skewness and Q-Statistic (G.O.F.) tests for various Weibull distributions with specified shape parameter values. Additionally, tables or charts of attained significance levels for the new sequential G.O.F. test procedure have been generated. Using the critical values and significance levels, a sequential G.O.F. test procedure can be used to determine if the given sample data agrees with a hypothesized Weibull distribution with known shape. A power study has been conducted against a variety of alternative hypotheses, and the results were compared with those obtained using conventional EDF type Cramer-von Mises, Anderson-Darling and Kolmogorov-Smirnov G.O.F tests, and the sequential procedure by Clough. Since the sequential test demonstrates better or equivalent power, it serves to significantly reduce the computational requirements for powerful G.O.F. testing.

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THESIS

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Above all, I promise to serve my country and people with my best to be able to express my thanks for sending me to this challenge for two years at AFIT.

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List of Terms

- ARE = Asymptotic Relative Efficiency.
- Ave. = Average.
- $\sqrt{b_1}$ = Skewness.
- b_2 = Kurtosis.
- CDF = Cumulative Distribution Function.
- EDF = Empirical Distribution Function.
- G.O.F. = Goodness-of-Fit.
- H_a = The Alternative Hypothesis.
- H_o = The Null Hypothesis.
- MDE = Minimum Distance Estimation.
- MME = Method of Moments Estimation.
- MLE = Maximum Likelihood Estimation.
- MTBF = Mean Time Between Failures.
- MTTF = Mean Time to Failure.
- MTTR = Mean Time to Repair.
- N = The number of samples drawn in Monte Carlo Simulation Procedure.
- n = The Size of Each of the N samples.
- PDF = Probability Distribution Function.
- PIT = Probability Integral Transformation.
- Q-Statistic = Hogg's Q-Statistic.
- RV = Random Variable.

- StDev. = Standard Deviation.

ABSTRACT

A new omnibus goodness-of-fit test for the three-parameter Weibull distribution will be developed using a sequential application of two individual tests, sample skewness, $\sqrt{b_1}$ and Q-Statistic. A Monte Carlo procedure will be employed to generate critical values for the $\sqrt{b_1}$ and Q-Statistic values for various Weibull distributions with specified shape parameter values. Additionally, tables or charts of attained significance levels for the new sequential G.O.F. test procedure will be generated. Using the critical values and significance levels, a sequential G.O.F. test procedure will determine if the given sample data agrees with a hypothesized Weibull distribution with known shape. A power study will be conducted against a variety of alternative hypotheses, and the results will be compared with those obtained using conventional EDF type Cramer-von Mises, Anderson-Darling and Kolmogorov-Smirnov G.O.F tests. If the sequential test demonstrates better or equivalent power, it will serve to significantly reduce the computational requirements for powerful G.O.F. testing.

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I. INTRODUCTION

1.1 Background

An idealization or abstraction of real world, a model is an incomplete representation of the entire population. The model is an analog of reality, made up of factors we assume are representative of the general population [29:1]. The importance of probabilistic and stochastic modeling in the modern world cannot be overvalued. By the help of high-speed computers and highly sophisticated software programs, scientists, engineers and analysts can form and utilize very complicated models. Decision-makers in various fields such as food science, medicine, marketing, management, politics, and weapons development generally make use of these models and associated statistical analyses in their decision-making processes to a great extent.

In order to complete its mission, Turkish Air Force (TUAF) like many other air forces has to depend on a variety of complex weapon systems. Therefore, TUAF is obliged to place more and more emphasis on the system availability, maintainability, and reliability, both in research and development and day to day operations. In order to meet these demands, TUAF needs to analyze its systems or subsystems in detail to improve the efficiency and production level along with the quality. The cost effectiveness of current and new systems is of great importance and interest. With decreasing budgets and increasing costs of new technologies, due to the fact that TUAF is obliged to deal with complicated and analytically hard problems, military analysts increasingly use statistical models or simulation techniques. Considering complex systems, simulation and statistics are hard to tell apart due to the fact that one's input is the other's output [117:1]. Thus, TUAF depends on statistical failure models to describe the failure patterns of various mechanical and electronic components in its weapon systems for use in computer simulations.

Failure distributions are of extreme importance to TUAF since they model the reliability of almost all mechanical and electrical systems in service [31:1]. The mean time to failure and the failure rate of equipment and systems is an important input into the decision making process. Therefore, the selection of a model to represent a population of entities is a critical and difficult task. Indeed, the choice of an incorrect model can make subsequent analysis faulty. Therefore, statistical estimation and failure modeling play an ever-increasing role in the reliability analysis of weapon systems. Of primary concern is the study of life distribution from which predictions of system and component failure can be made. Once failure test data from a particular weapon system is collected, the parameters of the appropriate probability model can be estimated and the system's probable life span determined [142:1].

A statistical failure model is a probability distribution, which attempts to mathematically describe the lifetime of a material, a structure, or a device. In typical reliability and life-span analyses, computations within a simulation are based on the assumption that data collected through experimentation and observation follow a particular theoretical lifetime distribution [53:1]. After being obtained, the data are statistically analyzed and a candidate distribution to model the system is selected. Thus, the problem turns out to be to test how well the sample fits a hypothesized distribution. If we observe a reasonable result, then the analysis can be continued using that specific distribution as the failure model of the system or component. Otherwise, one must search for a better distribution. Therefore, selection of a model that adequately "fits" the data is very important. These input data are the "driving force" for a simulation model [3:355], and improper representation of data will lead to misleading results, which could have a severely detrimental impact on a particular program if key decisions are based on these model results. Accurate model inputs, then are of fundamental importance to any such model-based analyses, be it for TUAF or a commercial establishment.

Thus, a good understanding of reliability science and how to model the data collected via various tests that are done with this science cannot be overvalued. Reliability science deals with longevity of parts, products, and systems. When engineers are faced with questions such as how long a product will last, how the product life can be improved, what warranty can be given for a new product, when is the best time to overhaul a machine, or to replace a production tool, etc., they need reliability methods to find answers. All these questions relate to product life, which can be modeled as a RV. Such questions can be answered by using statistical tools for data analysis and life prediction.

Whereas the variables in ordinary quality control inspection are likely to be normally distributed with its several well-known characteristics, the life variables are not so easily modeled. Several distributions, including the Weibull distribution can describe the time-variable. For one product the failure time may follow different distributions during different periods of the product life. Furthermore, the nature of life data, coming from truncated tests and field experiments, tends to complicate the analysis. Statisticians have developed methods to handle many such complications. Therefore, a good understanding of the statistical distributions and the goodness-of-fit (G.O.F) tests that are used to check the fitness of the sample data to hypothesized distributions is more than necessary.

G.O.F. is concerned with assessing the validity of models involving statistical distributions, an essential and sometimes forgotten aspect of the modeling exercise. G.O.F. tests are used to check the likelihood that the observed data is a sample of the population that is defined by the hypothesized distribution. One can only speculate on how many wrong decisions are made due to the use of an incorrect model. Therefore, we need to come up with better G.O.F. tests that will give us more precise results. In this study, a new sequential G.O.F. test procedure based on the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics is applied to the three-parameter Weibull distribution with known shape parameter value.

1.2 Definitions

This part of the research is intended to give the reader, who could be a rookie in G.O.F. and/or statistics, an overall review of some of the concepts and the definitions that will be covered throughout this thesis effort.

1. A *statistic* is any quantity whose value can be calculated from sample data. Any statistic, being a random variable (RV) has a probability distribution. The probability distribution of a statistic is sometimes referred to as its sampling distribution to emphasize that it describes how the statistic varies in value across all samples that might be selected [41:219].
2. A *distribution* is a single or multi-parameter theoretical, statistical model of data, often used to predict the behavior of a population of entities by studying a sample of it [170:3].
3. The *cumulative distribution function (CDF)* is denoted as *F(x)* and for a continuous RV is defined for every number x by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)dy \tag{1.1}$$

For each x, *F(x)* is the area under the density curve to the left of x [41:145].

4. Given a random sample $X_1, X_2, \ldots \ldots X_n$ drawn from a distribution with CDF F, then *the empirical distribution function (EDF)* is defined as

$$\mathrm{F_n}(x) = (\text{the number of } X_i\text{'s} \leq x) \,/\, n \tag{1.2}$$

For all x values $F_n(x)$ converges for large samples to *F(x)*, the value of the underlying distribution's CDF at x [146:8]. The relationship and comparison between CDF and EDF is graphically represented in Figure 1.1. The y-axis on the figure stands for the cumulative probability. Note that the cumulative probability for both EDF and CDF add up to 1.

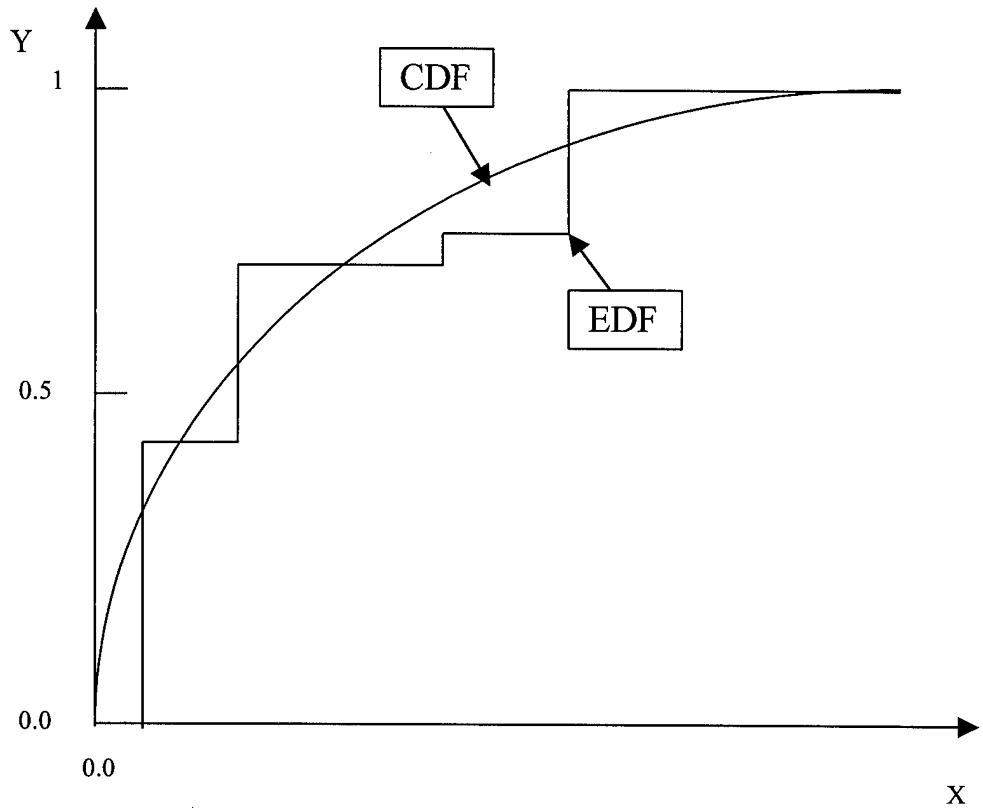


Figure 1.1 Graphical Description and Comparison of EDF and CDF.

5. Let X be a continuous RV. Then a *probability distribution* or *probability density function or PDF* of X is a function f(x) such that for any two numbers a and b with a ≤ b,

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)dx \tag{1.3}$$

That is, the probability that X takes on a value in the interval [a, b] is the area under the graph of the density function.

6. A *statistical hypothesis* or *hypothesis*, is a claim either about the value of a single population characteristic or about the values of several population characteristics.

In any hypothesis-testing problem, there are two contradictory hypotheses under consideration. In testing statistical hypotheses, the problem will be formulated so that one of the claims is initially favored. Therefore, the objective, based on the sample information, is to decide which of the hypotheses is correct. The claim initially favored or believed to be true is called the null hypothesis and is denoted by H_o. The other claim in a hypothesis-testing problem is called the alternative hypothesis and is denoted by H_a. Thus, we might test H_o: $\mu = 0.75$ against the alternative H_a: $\mu \neq 0.75$. Only if sample data strongly suggests that μ is something other than 0.75 should the null hypothesis be rejected. In the absence of such evidence, H_o should not be rejected, since it is quite plausible [41:304-305].

7. A *test procedure* is a rule, based on sample data, for deciding whether to reject H_o. Therefore, its objective is to test a hypothesis concerning the values of one or more population parameters. Therefore, we decide between the H_o and H_a by evaluating a test statistic based on sample data. If the value of our test statistic falls in a certain range called the *rejection region,* we conclude that H_o is false. All statistical tests of hypothesis work in the same way and are composed of the same basic elements:

- Null Hypothesis, H_o
- Alternative Hypothesis, H_a
- Test Statistic
- Rejection Region

The functioning parts of the statistical tests are the test statistic and the rejection region [29:2]. Therefore, the following specifies a test procedure:

1. A *test statistic*, a function of the sample data on which the decision (reject H_o or do not reject H_o) is to be based,
2. A *rejection region*, the set of all test statistic values for which H_o will be rejected [41:306].

The steps involved in hypothesis testing can be summarized as follows [94: 45]:

1. Set up H_o and select an appropriate H_a.
2. Choose an appropriate test statistic.
3. Choose a level of significance.
4. Design the test by specifying the critical region.
5. Select a sample from the population and compute the value of the test statistic.
6. Reject H_o if the observed value of the test statistic falls in the rejection region, otherwise do not reject H_o.

8. *Errors in Hypothesis Testing:* The basis for choosing a particular rejection region lies in understanding the errors that one might be faced with in drawing a conclusion.

There are two types of errors that can be made when reaching a decision about the H_o:

- A type I error consists of rejecting H_o when it is true. The probability of a type I error is denoted by α.
- A type II error involves not rejecting H_o when H_o is false. The probability of a type II error is denoted by β.

Thus, α and β measure the risks associated with making an erroneous decision. As such, they provide a very practical way to measure the goodness of a test [109:429-430]. It is important to note that these possible errors are not consequences of a foolishly chosen rejection region. Either of these two errors might result when any rejection region is used. Suppose an experiment and a sample size are fixed, and a test statistic is chosen. Then decreasing the size of the rejection region to obtain a smaller value of α results in larger value of β for any particular parameter value consistent with H_a.

In the best possible world, test procedures for which neither type of error is possible could be developed. However, this ideal can be achieved only by basing a decision on an examination of the entire population, which is almost always impractical. The difficulty with using a procedure based on sample data is that because of sampling variability, an unrepresentative sample may result. Instead of demanding error-free procedures, we must look for methods where either type is unlikely to occur. That is, a good procedure is one for which the probability of making either type of error is small. The choice of a particular rejection region cutoff value fixes the probabilities of type I and type II errors. Because H_o specifies a unique value of the parameter, there is a single value of α. However, there is a different value of β for each value of the parameter consistent with H_a [41:307-308].

An additional measure of the goodness of a test is called the *power* of the test. It is denoted by $1-\beta$ and will be used extensively in this study. The *power* is the probability of rejecting H_o when in fact, it is false. Therefore, the higher the power of a test, the lower the chance of accepting H_o when in fact, it is false. The power depends on the significance level (α-level) of the test, the components of the calculation of the test statistic, and on the specific alternative hypothesis under consideration [125]. Table 1.1 below summarizes the possible conditional probabilities that we can get during a statistical test.

Table 1.1 The Possible Conditional Probabilities That We Can Have During A Statistical Test.

Action / Given	Reject H_o	Reject H_o
H_o is False	1-β (Power) (Good)	β (Type II Error) (Bad)
H_o is True	α (Type I Error) (Bad)	1-α (Good)

9. An *estimator* is a numerical function of data. There are many ways to specify the form of an estimator for a particular parameter of a given distribution, and many alternative ways to evaluate the quality of an estimator [81:368]. An estimator for a parameter is an algorithm for estimating the value of the parameter from a set of data. An estimate is the value produced by the algorithm for a particular set of data [125].
10. *Goodness-of-fit (G.O.F.) tests* measure how well a statistical distribution fits the sample data. G.O.F. tests are used to check the likelihood that the observed data is a sample of the population that is defined by the hypothesized distribution. Therefore, G.O.F. tests provide helpful guidance for evaluating the suitability of a potential input model [3:375]. An extensive explanation about the goodness of fit tests will be given in Chapter 2.
11. *Skewness* is the degree of asymmetry, or departure from symmetry, of a distribution [144:111]. Data from positively skewed (skewed to the right) distributions have

values that are bunched together below the mean, but have a long tail above the mean. Distributions that are forced to be positive, such as annual income, tend to be skewed to the right. Data from a negatively skewed (skewed to the left) distribution have values that are bunched together above the mean, but have a long tail below the mean. Boxplots may be useful in detecting skewness to the right or to the left; normal probability plots may also be useful in detecting skewness to the right or to the left [125]. For skewed distributions, the mean tends to lie on the same side of the mode as the longer tail [144:111].

Among many measures of skewness, the most widely used measures of a population is given by the third moment about the mean, expressed by $E[(x-\mu)^3]$, where E is the expected value operator. To render this quantity invariant to change in location and scale, it is divided by σ^3 to standardize resulting in the expression (1.4) for skewness measure;

$$\sqrt{\beta_1} = \frac{E(x-\mu)^3}{\sigma^3} \tag{1.4}$$

12. *Kurtosis* is a measure of the heaviness of the tails in a distribution, relative to the normal distribution. A distribution with negative kurtosis (such as the uniform distribution) is light-tailed relative to the normal distribution, while a distribution with positive kurtosis (such as the Cauchy distribution) is heavy-tailed relative to the normal distribution [125].

    Among many measures of kurtosis, the most widely used measures of a population is given by the fourth moment about the mean, expressed by $E[(x-\mu)^4]$, where E is the expected value operator. To render this quantity invariant to change in location and scale, it is divided by σ^4 to standardize resulting in the expression (1.5) for skewness measure;

$$\beta_2 = \frac{E(x-\mu)^4}{\sigma^4} \tag{1.5}$$

13. *Time-to-Failure Distributions:* In survival analysis, data is collected on the time until an event is observed (or censoring occurs). Often this event is associated with a failure (such as death or cessation of function). The probability distribution of such times can be represented by different functions. Three of these are: the survival function, which represents the probability that the event (failure) has not yet occurred; the death density function, which is the instantaneous probability of failure; and the hazard function, which is the instantaneous probability of failure given that it has not yet occurred. The cumulative hazard function is the integral over time of the hazard function, and is estimated as the negative logarithm of the survival function [125].
14. *Survival Function:* In a survival experiment where the event is death, the value of the survival function at time T is the probability that a subject will die at some time greater than T. The survival function always has a value between 0 and 1 inclusive, and is nonincreasing. The function is used to find percentiles for survival time, and to compare the survival experience of two or more groups. The mortality function is simply one minus the survival function. Other names for the survival function are survivorship function and cumulative survival rate. Related functions are the hazard

function and the death density function. Steeper survival curves (faster drop off toward 0) suggest larger values for the hazard or death density functions, and shorter survival times [125].

15. *Shape:* The general form of a distribution, often characterized by its skewness and kurtosis [125].
16. *Scale:* The generalized concept of the variability or dispersion of a distribution. Typical measures of scale are variance, standard deviation, range, and interquartile range. Scale and spread both refer to the same general concept of variability [125].
17. *Reliability:* The measurable capability of an object to perform its intended function in the required timer under specified conditions. Reliability is a complex property that involves availability, operability, reparability, and maintainability. The properties considered in a reliability analysis depend on the goal of the analysis [154: 16].
18. The *MTTF* is the average or mean of the life distribution. In other words, it is the average of life length of all units in the population. The term *MTTF* is normally used for products that have only one life, i.e., not repairable. For products that are repairable, the term mean time between failures (*MTBF)* is used to denote the average time between repairs. The *MTTF* (or the *MTBF*) has a special significance when the life distribution is exponential, because then it is equal to the reciprocal of the parameter of the distribution, the failure rate, which is constant for an exponential distribution. So knowledge of the *MTTF* equates to knowledge of the entire distribution. Evaluation and prediction of all measures relating to reliability can then be made with *MTTF* only [94: 59].

1.3 Scope

The primary objective of this thesis effort, particularly for the three-parameter Weibull distribution, is to acquire the critical values for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics, calculate the attained significance levels that will be used for the sequential G.O.F. test, run and check the power of the proposed new sequential G.O.F. test procedure by using an extensive power study to compare the performance of it with the unique sequential alternative G.O.F. test procedure that was studied by Clough [26]. Clough [26] used $\sqrt{b_1}$ and kurtosis (b_2) as the G.O.F. tests that made up his sequential G.O.F. test procedure. In order to be able to compare the results on a common base, the alternative distributions by Bush's and Wozniak's studies that Clough [26] used in his thesis will be used in this effort. Provided that time permits and the proposed sequential G.O.F. test procedure proves to be powerful against Clough's sequential G.O.F. test procedure, this thesis effort will be extended to include the censored sample analysis.

1.4 Assumptions

Throughout this thesis effort, it is assumed that the shape parameter (β) of the three-parameter Weibull distribution is known while the location (δ) and scale (θ) parameters are to be estimated.

1.5 Research Objective

The objective of this research effort will be to develop, implement, analyze a new sequential G.O.F. procedure for a particular family of the Weibull distribution in which the shape parameter is known, but the location and scale parameters are unknown. The proposed G.O.F. test procedure will utilize the third standardized sample moment known as skewness and Q-Statistic to evaluate the G.O.F. for the Weibull distribution. The G.O.F. test procedure will actually consist of a sequential G.O.F. test, in which one G.O.F. test that will be conducted using $\sqrt{b_1}$ and will be followed by a second G.O.F. test using the Q-Statistic. A sample is considered to fail the sequential G.O.F. test procedure if it fails at least one of the G.O.F. tests. It is highly hoped that the sequential application of these G.O.F. tests will facilitate discrimination against a wider variety of alternate distributions with consistently equal or higher power than existing G.O.F. tests provide. Such a test is known as an omnibus test [81:438]. To assess the effectiveness of the test, an extensive power study will be performed to compare the test's performance to those of the current powerful G.O.F. tests against a wide variety of alternate distributions.

1.6 Support Requirements

This research will require a large amount of AFIT computer and library resources. All of the computer routines will be in MATLAB. For educational purposes and the creation of some of the Figures throughout this thesis effort, Mathcad 7.0 and MATLAB 5.0 will be used.

II. LITERATURE REVIEW

2.1 Weibull Distribution

2.1.1 History

The Weibull Distribution was named after the Swedish scientist Professor Waloddi Weibull [156:293-297], who developed the PDF of the Weibull and published a paper in 1939 suggesting it as a distribution for a variety of applications in data analysis. In his study, he considered the problems of yield strength of Bofors steel, fiber strength of Indian Cotton, length of syrtoideas, fatigue life of St-37 steel, statures of adult males born in the British Isles, and breadth of beans of Phaselous Vulgaris. He mainly concentrated on the distribution of the phenomenon of rupture solids. In a paper published in 1951, Weibull demonstrated the application of the distribution in the investigation of the yield strength and fatigue of steel, the size distribution of fly ash, and the fiber strength of cotton. Although the Weibull distribution was first applied to the fatigue of materials, the distribution is very flexible and has proved very useful in other fields as well. Peto and Lee [122:457-470] used the Weibull distribution in their experiments with continuous carcinogenesis experiments with laboratory animals.

On the other hand, it is important to note that Weibull distribution is also credited to R.A. Fisher and L.H.C. Tippert [17:6] who published a paper in 1928. Furthermore, the distribution was also accepted to be used as early as 1933 by Rosin and Rammler [12:833] in the describing the "laws governing the fineness of powdered coal".

As will be explained in the following pages, the Weibull distribution includes the exponential distribution as a special case and is sometimes thought of as a generalization of the exponential distribution. The exponential model was developed and applied rather extensively in the 1950s and the Weibull began to be seriously considered as a competing model in 1960s, especially in problems in which the time to failure was the response of interest. Today, the Weibull distribution is best known for its application in the field of reliability [17:7].

2.1.2 Statistical Properties and the Application of the Weibull Distribution

The Weibull PDF represents the distribution of *X*, such as the time to failure of a piece of equipment. A RV *X* has a three-parameter Weibull distribution with PDF

$$f(x;\theta,\beta,\delta)=\frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1}e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} \tag{2.1}$$

and CDF

$$F(x;\theta,\beta,\delta)=1-e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}}. \tag{2.2}$$

where $\theta > 0$, $\beta > 0$, $-\infty < \delta < \infty$, and $\delta \le x$.

The three parameters of the Weibull distribution are:

- θ, the scale parameter, which is sometimes called the characteristic life;

- β, the shape parameter, which is sometimes called the Weibull modulus or the exponent;
- δ, the location parameter, which is sometimes called as the minimum life or guaranteed life.

However, often the value of the location parameter is taken to be zero ($\delta = 0$). In component design, this represents a conservative assumption, and yields the more widely used two-parameter Weibull distribution. The PDF and the CDF of the two-parameter Weibull distribution are given respectively below.

$$f(x;\theta,\beta) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{2.3}$$

$$F(x;\theta,\beta) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\beta}} \tag{2.4}$$

where $\theta > 0$, $\beta > 0$, and $x \geq 0$.

The expected value, E(X), and variance, V(X), of the two-parameter Weibull distribution are given by [93:67]:

$$E(X) = \mu = \theta\Gamma\left(1 + \frac{1}{\beta}\right) + \delta \tag{2.5}$$

$$V(X) = \sigma^2 = \theta^2\left(\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right) \tag{2.6}$$

As β increases, μ approaches the characteristic life (θ) and σ^2 approaches to zero [88:293].

Here $\Gamma(Q)$ is the gamma function defined as

$$\Gamma(Q) = \int_0^{\infty} x^{Q^{-1}} e^{-x} dx \tag{2.7}$$

The skewness, $\sqrt{\beta_1}$ and kurtosis, β_2 of a Weibull distribution are given by:

$$\sqrt{\beta_1} = \frac{2\Gamma^3(1+\beta^{-1}) - 3\Gamma(1+\beta^{-1})\Gamma(1+2\beta^{-1}) + \Gamma(1+3\beta^{-1})}{\left[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})\right]^{3/2}} \tag{2.8}$$

$$\beta_2 = \frac{f(a)}{\left[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})\right]^2} \tag{2.9}$$

where $x \in [0, \infty)$ and

$$\begin{aligned} f(a) = &-6\Gamma^4(1+\beta^{-1}) + 12\Gamma^2(1+\beta^{-1})\Gamma(1+2\beta^{-1}) - 3\Gamma^2(1+2\beta^{-1}) \\ &- 4\Gamma(1+\beta^{-1})\Gamma(1+3\beta^{-1}) + \Gamma(1+4\beta^{-1}). \end{aligned} \tag{2.10}$$

If there are n identical equipment components with the same failure distribution $F(x)$, then the hazard function is the percentage of the n components that will fail in the next time interval [17:8]. The hazard function is the instantaneous failure rate [29:6]. The Weibull hazard function can be strictly increasing, strictly decreasing or constant. Since many systems experience a change in the failure rate, the Weibull can adequately model any of the three situations. Therefore, it can be concluded that the ability of the Weibull distribution to model failure situations where non-constant hazard functions apply, makes this distribution one of the most generally useful distributions for analyzing the failure data [93:79]. The hazard function is defined as follows:

$$h(x) = \frac{f(x)}{1 - F(x)} \tag{2.11}$$

or more specifically for the Weibull two parameter case,

$$h(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \tag{2.12}$$

and for the Weibull three parameter case,

$$h`(x) = \frac{\beta(x-\delta)^{\beta-1}}{\theta^{\beta}} \tag{2.13}$$

The shape parameter, β, has a great importance in terms of the Weibull distribution's flexibility in its use in reliability studies. The value of β determines the failure rate direction as follows:

- Constant failure rate implies that the life distribution of units in this region follows exponential law. Thus, when $\beta = 1$, the Weibull PDF reduces to the exponential PDF (with $\lambda = 1 / \theta$). Therefore, with $\beta = 1$, the time to next failure is independent of the time it has already been working. In other words, the amount of time something has already lasted has no effect on how much longer it will last. Here, failures occur not because of inherent defects in the units but because of accidental or chance occurrence of loads in excess of the design strength. This region is known as the period of chance failures [94: 58].

- When $\beta < 1$, the failure rate or the probability of failure decreases with time. If something "breaks in", or there is a wide variation in initial quality, the shape parameter will be less than 1. The burn-in period of systems or components is defined by this initial region with decreasing failure rate. The decrease in failure rate is due to defective units in the population that fail early and are repaired or removed from the population. This region is known as the infant mortality region. A high initial failure rate or long period for infant mortality would indicate inadequate quality control effort for parts and assemblies. The length of this period determines how long a burn-in period is required [177:57].
- When $\beta > 1$, the failure rate or the probability of failure increases with time. If something "wears out", it will have a shape parameter greater than 1. An automobile tire is an example. In short, this describes the wear-out period of systems or components. Failures will show a bell-shaped distribution in time. It is never symmetrical, but appears to be symmetrical for certain values of $\beta > 1$. Such cases are called "pseudo-symmetric" cases by Gumbel [61]. Failures occur due to fatigue, aging and embrittlement in this region known as the wearout

period. Knowledge of when wearout begins helps in planning replacements and overhauls [94: 59].

Figure 2.4 illustrates the descriptions above that are known as the bathtub curve [125:28] [128: 151].

Downtown [44] states, "The bathtub curve is a fascinating example of the impact folklore has in reliability studies. No one can doubt that the bathtub curve represents the subjective views of reliability engineers on the behavior of the systems to failure, both repairable and unrepairable".

The plot in Figure 2.1 illustrates the variety of forms the Weibull PDF could assume by varying the shape parameter (β) while keeping the location parameter (δ) = 0.0 and scale parameter (θ) = 1.0. As can be seen on the plot, β can be varied to obtain a number of vastly different PDF shapes. This flexibility in describing different failure rates makes the Weibull distribution most appealing. This and the fact that the model has simple expressions for the PDF, survivor, and hazard functions partly account for its popularity [97:16]. For $0 < \beta \leq 1$, the Weibull PDF has a reverse *J* shape. For $\beta \leq 1$, the Weibull PDF is highly skewed or asymmetric. As β increases above 1, the skewness decreases until reaching zero at a value between 3.5 and 4.0 [29:7]. When $\beta = 2$, the Weibull distribution reduces to the Rayleigh distribution, which is used to describe the amplitude of sound [43: 44] [61]. In most lifetime estimation problems, the shape parameter lies in the range of $0.5 \leq \beta \leq 3.5$ [72:310]. Lawless [97:16] also states that typical β values vary from application to application, but in many situations, distributions with β in the range 1 to 3 are appropriate. For $\beta = 3.6023494$ the Weibull PDF is "almost normal" [27:27], although Kapur and Lamberson [88:294] warn that a Weibull distribution with this shape parameter value is not a true mathematical approximation to the normal distribution. The Weibull distribution may be regarded as a better model for life testing studies in the sense that it takes into account many situations according to the exponent β. In many applications, β may be known, otherwise it may be estimated using failure data form past inspection and research. For example, from a study of approximately 2000 electron tubes of a certain type, Kao [87] has found that a value of 1.7 may be quiet appropriate. Lieblein and Zelen [99] found that β lies between 1.17 to 1.74 for a certain class of ball bearings. They based their study on 5,000 bearings. As a summary, β, the real driver in determining the form of the PDF curve, and allows the Weibull distribution to assume a broad variety of contours, contributing to its utility to such diverse disciplines as noted earlier [26:2-1].

The scale parameter, θ, affects the dispersion of the RV, x, about its mean and is sometimes called characteristic life in reliability applications [19:48]. The scale parameter divides the area under the PDF curve into the same proportion for all values of β to help locate the Weibull distribution on the x-axis. Now, in order to illustrate this point, consider the two-parameter Weibull CDF that is evaluated at $x = 0$:

$$F(x=0) = 1 - e^{-1} = 0.6321 \tag{2.14}$$

Therefore, we can see the area under the PDF will be divided by θ into 0.6321 and its complement 0.3679. The plot in Figure 2.3 illustrates the effect of varying the scale parameter with $\delta = 0.0$ and $\beta = 1.9$. As can be observed from the plot, variation

Location Parameter (δ) = 0.0 Scale Parameter (θ) = 1.0 Shape Parameter (β) = 0.5, 1.0, 2.0, 3.0, 5.0

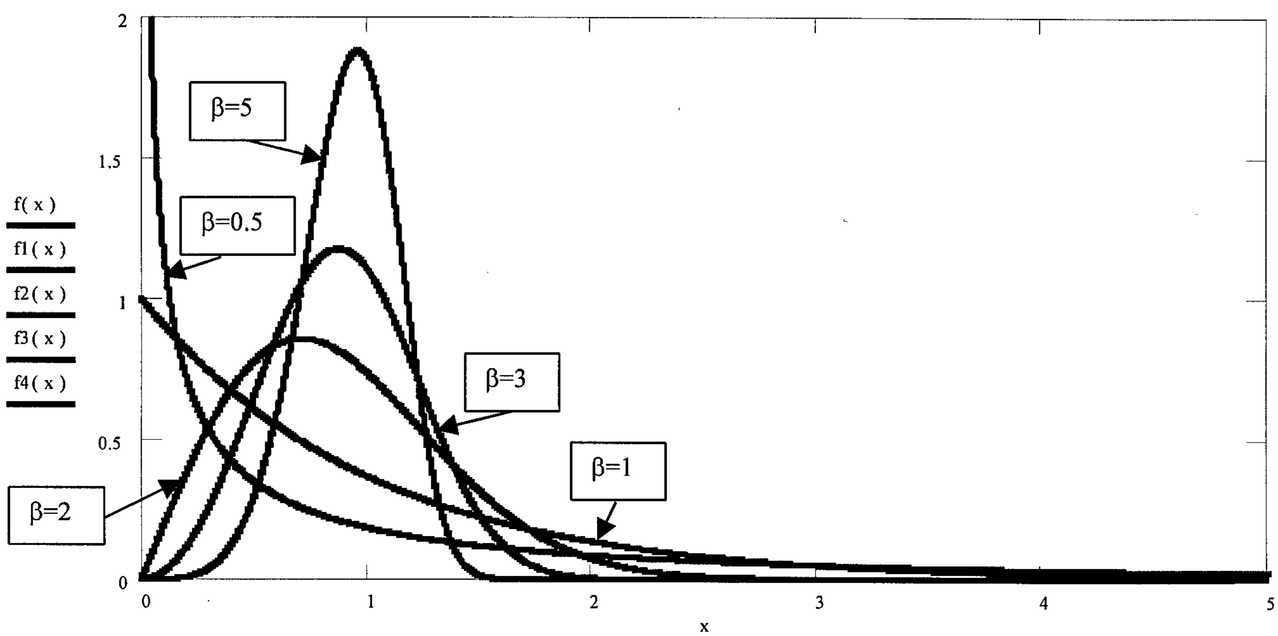


Figure 2.1 Weibull Distributions with Various Shape Parameters

Location Parameter (δ) = -2.0, 0.0, 2.45, 5.25 Scale Parameter (θ) = 1.0 Shape Parameter (β) = 2.0

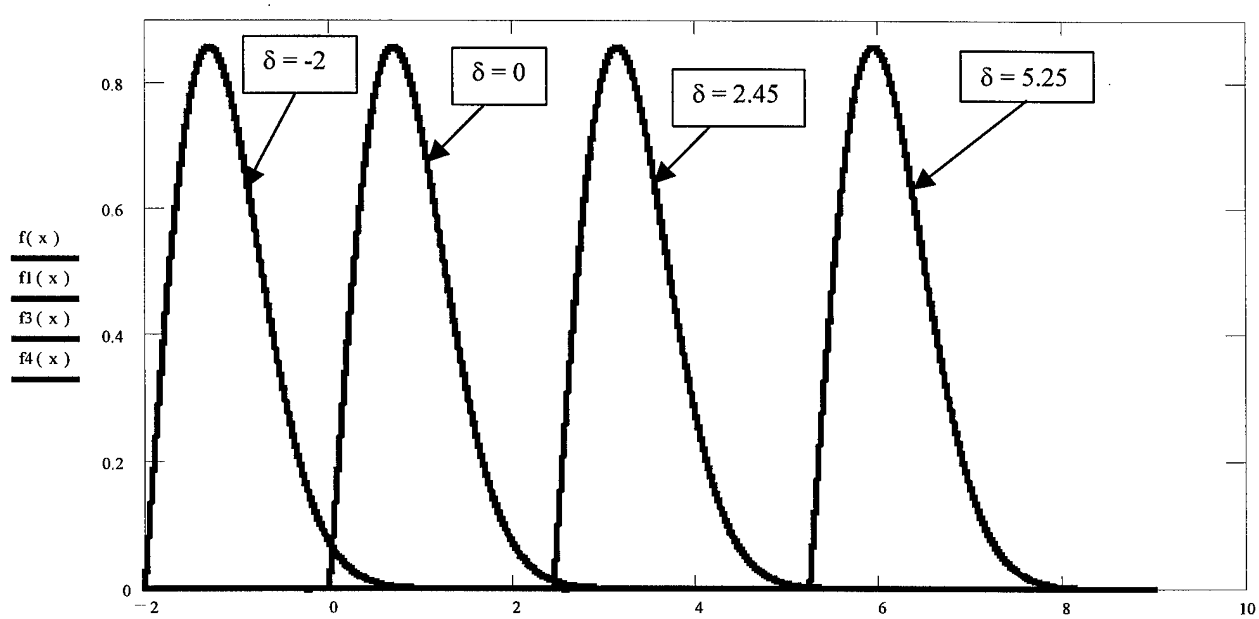


Figure 2.2 Weibull Distributions with Various Location Parameters.

Location Parameter (δ) = 0.0 Scale Parameter (θ) = 0.5, 1.0, 2.2, 3.85 Shape Parameter (β) = 1.9

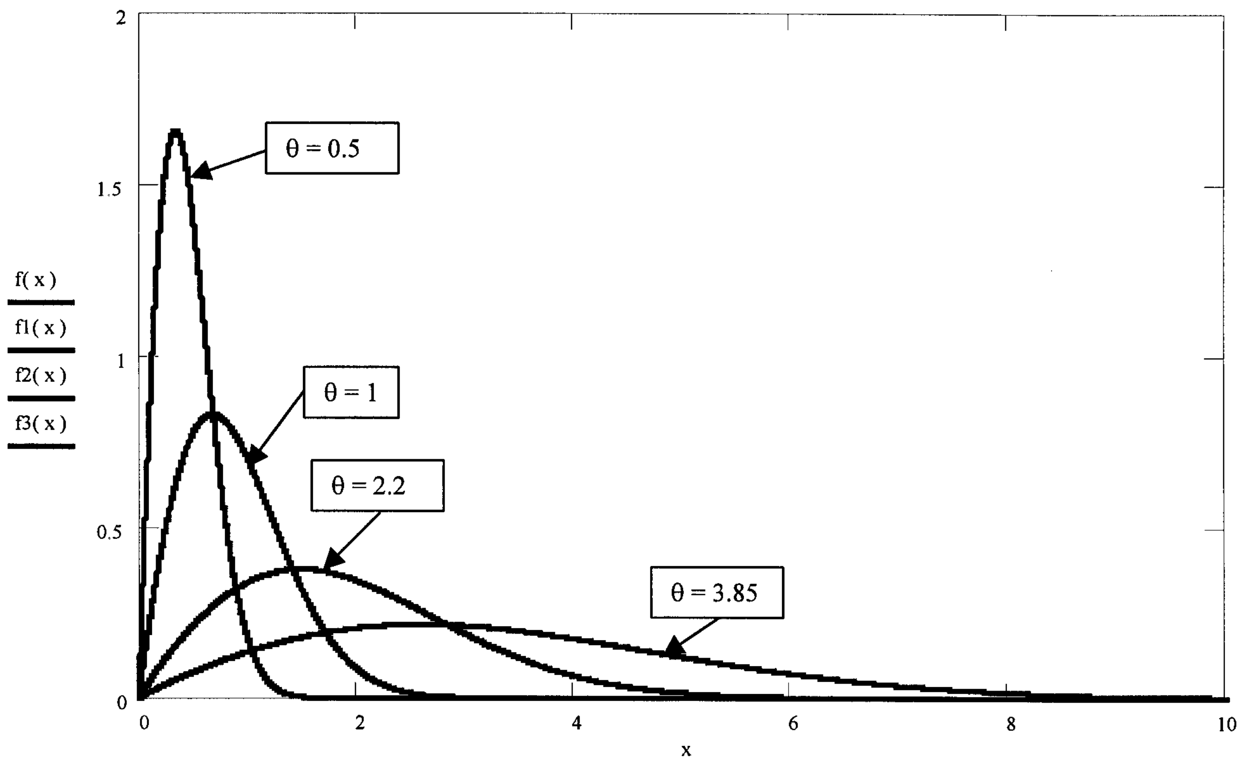


Figure 2.3 Weibull Distributions with Various Scale Parameters

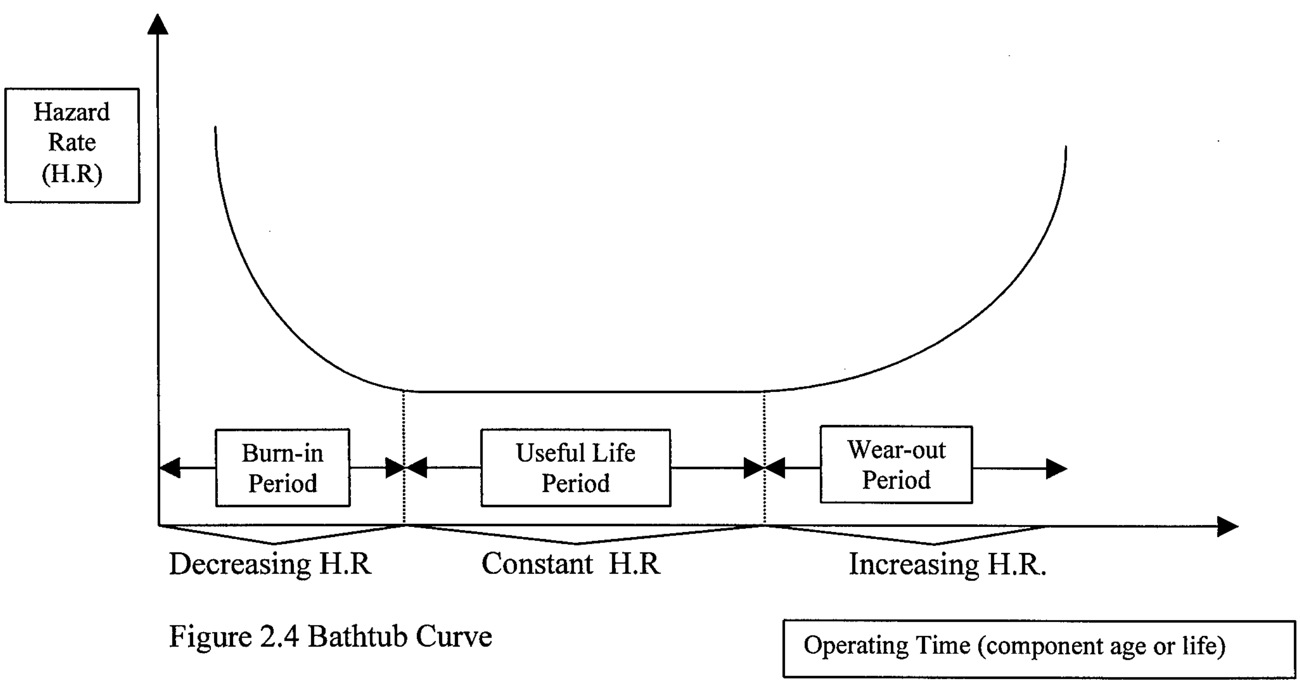


Figure 2.4 Bathtub Curve

in θ causes the Weibull PDF to stretch or compress in the x-axis direction about its central tendency. The higher θ is, the more stretched the Weibull PDF is, and vice versa.

Failures can occur as soon as a component is put into service in most of the life-testing applications. This situation can be modeled with the two-parameter Weibull distribution with $x \geq 0$. On the other hand, there may be some cases where a component can fail and become useless in storage before it is put into service. Or there may be cases where a component may operate with very rare failure or failure-free for some period of time after beginning service. In this case, we need to introduce the location parameter that allows the PDF to be shifted to the left or right along the x-axis depending on when the earliest failure time can occur, and is referred to as the guaranteed life or minimum life [88:292]. In terms of reliability of system components, the location parameter (δ) indicates the value of x at which failures may begin to occur.

- When $\delta = 0$, it describes that failures can occur immediately after a component or device is put into operation. This is the two-parameter Weibull distribution case.
- When $\delta > 0$, it describes a guaranteed period in which a component is failure free. In this case, the PDF is shifted right from the origin.
- When $\delta < 0$, it describes a potential failure in storage or negative failure time.

The effect of the location parameter with $\theta = 1.0$ and $\beta = 2.0$ is illustrated on the plot in Figure 2.2.

The tail of the Weibull distribution may decline more rapidly or less rapidly than that of the exponential distribution. In practice, this means that if there are more large service times than the exponential can account for, a Weibull distribution may provide a better model of these service times [50:132-133].

The Weibull distribution has become important in science and engineering because it represents the distribution of so many things, so well. One explanation for its success as a failure distribution arises as an asymptotic distribution of smallest extremes in sampling from a broad general class of parent distributions [97:16]. Thus, failure of a device at the weakest of a large number of "weak spots" will tend to result in a Weibull distribution of device life [107:1].

The two and three-parameter Weibull distributions are applicable to many phenomenon [71:164]. It has been used to provide a reasonable model for lifetimes of many types of units, such as vacuum tubes, ball bearings, composite materials [50:17], and electric insulation [36] [41]. Furthermore, it can also be used to model the time to complete some task [96:333], and for interarrival and service times [3:132]. It is also widely used in biomedical applications. For example, in studies to model the time to the occurrence of tumors in human populations [159], or in laboratory animals [123] [122].

It should be pointed out that the MTTF and MTBF do not have the same importance when the life distribution is other than the exponential. For example, the Weibull distribution is used to model distributions with nonconstant failure rates where knowledge of MTTF or MTBF alone is not adequate. Then, the parameters of the Weibull distribution must be estimated [94: 59].

Especially in reliability estimation, the Weibull distribution is second in use after the exponential distribution. Unfortunately, in many cases, the exponential distribution is used without a thorough understanding of the fundamentals [88:233]. Therefore, this thesis effort will focus on coming up with a new sequential G.O.F. test procedure for the Weibull distribution because of its usage and flexibility in reliability and other areas.

2.2 Reliability

Reliability of a device is defined as "the probability of a device operating within specified limits for time and operating conditions specified." [126]. The reliability of a system is defined as "the probability that, when operating under stated environmental conditions, the system will perform its intended function adequately for a specified interval of time" [88:1]. Due to the fact that they are complementary events, the sum of the reliability function and the CDF must be always one. The reliability function for the three-parameter Weibull case is given by

$$R(x;\delta,\theta,\beta) = e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} \tag{2.16}$$

which is easily seen to be 1-F(x).

Throughout the history of modern engineering, failures of systems have been observed in every field of engineering. For example, in 1940 the Tacoma Narrows bridge collapsed after just four months of existence because of torsional oscillations induced by a wind velocity of 42 mph. In 1943, the Schenectady, the first welded tanker built at the shipyard of the Kaiser Company, Portland, Oregon, broke into two due to welded structural failure while lying afloat in the calm waters of a fitting-out dock. In recent times, the space shuttle Challenger exploded in midair in January 1986. In 1985, the worst industrial accident in history occurred at the Union Carbide plant in Bhopal, India, in which thousands of people died. In 1986, history's worst nuclear power reactor accident occurred in Chernobyl, USSR, which resulted in the leakage of radioactivity into the atmosphere. The importance of reliability of components and systems is also recognized at every stage of daily life, ranging from consumer products such as TV sets, clothes washers and dryers, lawn mowers, and automobiles to larger systems such as trains and airlines [128: 2-3].

2.3 Goodness-of-Fit Tests

Prior to using a probability model to represent the underlying population, it is important to test the adequacy of the model. One way to do this is by a G.O.F. test [164:113]. The statistical test checks if the hypothesized distribution fits the sample data. G.O.F. tests measure the agreement between the observed sample data and a theoretical statistical distribution by testing the conformity of the observed data's empirical distribution function (EDF) with a posited theoretical distribution function (CDF).

G.O.F tests can be mainly separated into two subgroups as follows:

- Graphical (Visual) G.O.F tests
- G.O.F tests conducted using various G.O.F. test statistics

Shapiro and Brain [139] classified G.O.F. tests into three major groups:

- Regression type G.O.F tests
- Probability transformation tests
- Special features tests.

The most common regression test is probability plotting, in which the ordered sample data is plotted on a graph in which the axes are transformed so that if the data are close to the hypothesized distribution, they lie on a straight line. The deviation of the data from linearity may be visually assessed and the nature of the true distribution may be detected. Even though the judgment of the straight line has been achieved by eye examination so far, more formal techniques to assess the straight line have been put in use. Shapiro and Brain strongly recommended that other G.O.F tests should always be augmented by a probability plot. An informal graphical test described by Law and Kelton [96] and Woeste, Suddarth and Galligan [161] require that the relative frequencies of the sample data and the fitted PDF be plotted on the same graph and visually compared. This graphical test is a variation of the probability-plotting test.

Even though graphical G.O.F tests have wide applicability, it is more formal and common to use G.O.F tests that use various test statistics. G.O.F. tests provide helpful guidance for evaluating the suitability of a potential input model. However, since there is no single correct distribution in a real application, you should not always be a slave to the verdict of such tests. It is especially important to understand the effect of sample size. If very little data are available, then a G.O.F. test is unlikely to reject *any* candidate distribution. On the other hand, with a large amount of data, a G.O.F. test will likely reject *all* candidate distributions. Therefore, failing to reject a candidate distribution should be taken as one piece of evidence in favor of that choice, while rejecting an input model is only one piece of evidence against the choice [3:375].

Frisco [53:2-6] defines the main categories of G.O.F. tests as follows:

- *Chi-Squared Type G.O.F. Tests*

- *EDF Type G.O.F. Tests*
- *Smooth G.O.F. Tests*

In fact, G.O.F tests follow standard hypothesis testing in which:

- H_o = The given sample data are from a particular hypothesized distribution F(x). In this thesis effort F(x) will be the Weibull distribution with known shape.
- H_a (in most of the cases) = H_o is false. Or, depending on the nature of the test, it may actually specify an alternate hypothesized distribution G(x).

Stephens and D'Agostino [29:1] make a clear distinction between a *simple hypothesis,* in which H_o is completely specified (all parameters specified) and a *composite hypothesis,* in which all or some of the parameters are estimated. As an observation, H_a is composite in most cases. A composite alternative hypothesis may still include some specification about the population. For example, H_a may state that the population distribution has positive skewness. When H_a contains such specification, a G.O.F. test should be designed to be sensitive to it.

Stephens and D'Agostino [29:1-2] also emphasize that one aspect of G.O.F tests that distinguishes them from conventional hypothesis tests is the fact that the objective is usually to accept H_o (or fail to reject) as opposed to rejecting it. Therefore, the main focus is on the measure of agreement of the data with the population distribution specified in H_o. Stephens and D'Agostino [29:1-2] list reasons why we want to accept H_o as follows:

- The distribution of sample data may throw light on the process that generated the data.

- Knowledge of the distribution of data allows for application of standard statistical testing and estimation procedures.
- When a distribution can be assumed, extreme tail percentiles, which for example are needed in environmental work, can be computed.

The general procedure for G.O.F tests is as follows [32:2-4]:

- *Make a hypothesis of the theoretical distribution that the data comes from.* This is generally done by a rough examination of the data in the sample.
- *Specify or estimate the parameters of the theoretical distribution from the data unless they are known a priori.* If the parameters are unknown, as is usually the case, then they must be estimated from the data.
- *Compute the test statistic.* Once the parameters of the theoretical distribution have been estimated (if necessary), then the CDF and EDF values can be calculated. The G.O.F test statistic is computed to compare the behavior of the sample data to what one would expect to see if it were actually from the distribution in question. A G.O.F test statistic is a measure of the distance between the CDF and the EDF if it is an EDF type G.O.F. tests like K-S, A-D, and C-vM.
- *Evaluate acceptance or rejection of H_o.* The test statistic is compared to tables constructed specifically to make a decision about the hypothesized distribution. If the test statistic is large, the two distributions are too far apart; hence the distribution being tested is rejected.

The main difference among G.O.F. tests lies in the method of calculation of the test statistic.

G.O.F tests conducted using various test statistics can further be separated in two subgroups:

- *Completely Specified G.O.F. Tests:* The true values of all parameters of the hypothesized distribution are known.
- *Modified G.O.F. Tests:* Some or all of the parameters have to be estimated from the data. Woodruff and Moore make it clear that modified G.O.F. tests must not use tables built on known parameters [164:115], [32:2-4].

There is a significant difference between these two groups in terms of obtaining and using critical values. Using the critical value tables for a completely specified case when parameters are actually estimated, will bias the test toward acceptance, possibly to the extent that it renders the test almost useless [164:115] [26:2-16].

Frisco [53:2-6] states that G.O.F. tests can also be distinguished as *omnibus G.O.F. tests* that are designed to be effective against a broad class of alternate distributions and *directional G. O. F. tests* that are designed to effectively detect specific types of departure from the hypothesized distribution, such as symmetry or skewness.

A substantial number of G.O.F. tests having various unique strengths and weaknesses have been developed in the 1900s. Lawless [97]; Stephens and D'Agostino [146]; and Read and Cressie [130] provide thorough reviews of the G.O.F. techniques and associated test statistics with in-depth bibliographic references. We now take a look at the most commonly used techniques that are used G.O.F. tests: *Chi-squared (χ^2) G.O.F. Test, Kolmogorov-Smirnov (K-S, D) G.O.F. Test, Anderson-Darling (A-D, A^2) G.O.F. Test, and Cramer-Von Mises (C-vM, W^2) G.O.F. Test.*

2.3.1 Chi-Squared Goodness-of-Fit Tests

The chi-squared test, the oldest and most widely used G.O.F. test is covered in almost every statistical book. It was introduced by Karl Pearson, who abandoned the assumption that biological populations were normally distributed and introduced the Pearson system of distributions to provide alternative models in 1900. The need to test the fit of his proposed models resulted in the chi-squared test statistic. This test procedure basically formalizes the intuitive idea of comparing the histogram of the data to the shape of the candidate density or mass function [3:375].

The chi-squared G.O.F. test can be used with grouped or ungrouped data, discrete, continuous, or mixed distributions, and with the parameters estimated or known. It is a less subjective comparison of frequency histograms with fitted distributions than the graphical or visual methods. Kanji [86:12] states the chi-squared G.O.F. test procedure is used to investigate the significance of the differences between observed data arranged in k classes, and the theoretically expected frequencies in the k classes. In this procedure, the range of the sample data, n is divided into a discrete number of intervals k and the number of data points falling in each interval is compared with the expected number predicted by the fitted distribution. The chi-squared test procedure can also be modified for use with censored data or truncated distributions and is well suited for use when parameters are estimated from the sample with a resultant decrease in the degrees of freedom (one for each parameter estimated) via various parameter estimation techniques such as maximum likelihood estimation [149:730].

The test statistic is given by the following equation:

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{2.17}$$

where O_i is the observed frequency in the ith class interval and E_i is the expected frequency in the ith class interval. The expected frequency, E_i is computed as $E_i = np_i$, where p_i is the theoretical, hypothesized probability associated with the ith class interval [3:376].

The null and alternative hypotheses are given as follows [3:376] [60:1-17]:

- H_o: the random variable X conforms to the distributional assumption with the parameter(s) given by the parameter estimate(s).
- H_a: the random variable, X, does not conform to the distributional assumption with the parameter(s) given by the parameter estimate(s).

Normally, we reject H_o if $\chi_o^2 > \chi^2$(k-p-1), where χ^2(k-p-1) refers to the critical value of the chi-squared distribution with *k-p-1* degrees of freedom, *p* being the number of parameters estimated in the specification of H_o. For this to be strictly correct however, the parameters must have been estimated by the minimum chi-squared method that Moore discusses in detail in his article [111:68]. If the other methods are used, then the number of degrees of the chi-squared critical value cannot be stated with certainty, except to say it lies somewhere between *k-1* and *k-p-1*. With *k* large and *p* small (as is often the case), the value of the chi-squared critical value will not change much in this range, so the uncertainty is of little concern.

The chi-squared G.O.F. test is based on the following assumptions and rules [140:95-96] [129:23-24] [86:69]:

- Categorical/nominal data are employed in the analysis. This assumption reflects the fact that the test data should represent frequencies for k mutually exclusive categories and the counts of the number of sample values occurring in each category is recorded.
- The data that are evaluated consist of a random sample of n independent observations and they are identically distributed. This assumption reflects the fact that each observation can only be represented once in the data.
- The hypothesized distribution is specified in advance, so that the number of observations that should appear in each category, assuming the hypothesized distribution is the correct one, can be calculated without reference to the sample values. The expected frequency of each cell is 5 or greater. When this assumption is violated, it is recommended that if $k = 2$, the binomial sign test for single sample test be employed to evaluate the data. When the expected frequency of one or more cells is less than 5 and $k > 2$, the multinomial distribution should be employed to evaluate the data.
- With equiprobable cells, the expected cell frequency should be at least one for $\alpha = 0.05$ and at least two for $\alpha = 0.01$.
- If the cells are not equiprobable, the above cell counts should be doubled.
- If there are only two cells, the test based on the exact binomial distribution should be used in lieu of the chi-squared test.
- The observed and theoretical distributions should contain the same number of elements. Besides, the division into classes must be the same for both distributions.

Law and Kelton [96], Sheskin [140], Greenwood and Nikulin [60] and Banks, Carson and Nelson [3] give details of this test, while Chandra [57] has discussed the effects of correlation between the data points in the sample on the test statistic. These references should be considered for an in-depth understanding of the chi-squared testing procedure.

Besides the assumptions and rules mentioned above, the drawbacks of the chi-squared tests could be listed as below [26:2-17] [150:4]:

- *Relatively low power*: Grouping the sample data into a set of somewhat arbitrary cells discards some useful information causing this test to be less powerful compared to the other tests [146:63] [149:730]. Besides, the distribution of the chi-squared test statistic is known only approximately when the parameters are estimated, and the power of the test is sometimes rather low [3:377]. The chi-squared G.O.F test is the most flexible test dealing with unknown parameters; for each parameter unspecified, a degree of freedom is subtracted. With this flexibility, comes certain drawbacks; as more parameters are estimated, the power of the test is diminished greatly. The lower the power, the greater the possibility that the test will accept a false hypothesized distribution [158:6].
- *Sample size restriction*: In case of limited data, other G.O.F tests have to be used since the chi-squared G.O.F. tests are not recommended for sample sizes less than roughly 20 (or 25 [164:113]) [3:377].
- *No uniqueness*: Due to the fact that the results have been arranged into groups before the test is carried out, the chi-squared G.O.F test results are not

necessarily unique. Since the selection of the groups is somewhat arbitrary with no standard procedure, the results may differ from one analyst to the next [164:113]. A hypothesis may be accepted when the data are grouped one way, but rejected when grouped another way [3:382].

It is important to note that even though the chi-squared test procedure has its weaknesses that can be considered noteworthy, it still has a broad application in G.O.F practices because of its ease of use and flexibility and its economical first cut at trying to figure the sample distribution. Besides, chi-squared tests are more applicable when the sample data is hypothesized to have come from a discrete distribution. In the case of continuous random variables, such as time, tests based on the EDF yield much higher power [146:110].

The books by Stephens and D'Agostino [146], Sheskin [140], Hogg and Craig [76], Read and Cressie [130], Banks and Carson [3], Law and Kelton [96], Kanji [86], and Greenwood and Nikulin [60] should be referenced for chi-squared type G.O.F. test procedures.

2.3.2 Goodness-of-Fit Tests Based on EDF Statistics

Besides chi-squared G.O.F tests, there is a general class of statistics called EDF statistics, which are also used for G.O.F test procedures. Stephens and D'Agostino discuss goodness-of-fit tests based on EDF statistics in a very detailed manner in their book [146]. Goodness-of-fit tests based on EDF statistics that appear to be more powerful than chi-squared type goodness-of-fit tests are based on the comparison of the hypothesized CDF, $F(x)$ to the EDF that is derived from the sample data, $F_n(x)$. They quantify the difference between the two for a measure of agreement [26:2-18]. Two demonstrations of how $F(x)$ and $F_n(x)$ are compared are provided in Figure 1.1 and 2.5. The EDF G.O.F test statistics can be used in cases where all parameters of the hypothesized CDF are known (specified) and where all the parameters of the hypothesized distribution have to be estimated from the sample (unspecified). The EDF function is a step function, which is created by ordering the sample data points and taking a value between 0 and 1 for each member of the ordered sample, to estimate the population CDF [66:2-4]. Suppose a given a random sample of size n is $x_1, x_2, \ldots, x_n$ and let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics; suppose further that the distribution of x is $F(x)$. Then, $F_n(x)$ is defined by

$$F_n(x) = \frac{\text{the number of observations} \leq x}{n}; \qquad -\infty < x < \infty \tag{2.18}$$

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics as defined before. Therefore, as x increases, $F_n(x)$ takes a step up of height $(\frac{1}{n})$ as each point x equals an order statistic. For any x, $F_n(x)$ records the proportion of observations less than or equal to x, while $F(x)$ is the probability of an observation less than or equal to x. We can expect $F_n(x)$ to estimate $F(x)$, and it is in fact a consistent estimator of $F(x)$; as $n \rightarrow \infty$, $|F_n(x) - F(x)|$ decreases to zero with probability one. Then, $F_n(x)$ of n ordered data points $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ is more precisely defined by [146:97-99]

$$F_n(x)=\begin{cases} 0, & x<X_{(1)} \\ \dfrac{i}{n}, & X_{(i)}\le x<X_{(i+1)}, \qquad i=1,....,n-1 \\ 1, & X_{(n)}\le x \end{cases} \tag{2.19}$$

EDF statistics based on the idea of measuring the vertical differences between $F_n(x)$ and $F(x)$ are basically divided into two classes as follows:

- *The Supremum Statistics:* They include the statistics that are based on the largest vertical differences between $F_n(x)$ and $F(x)$ in the two cases where $F_n(x)$ is greater or less than $F(x)$. They are defined in the following sentences and denoted as D^+ and D^-. The most famous and widely used supremum based G.O.F. test statistics are Kolmogorov-Smirnov (K-S) test statistic and Kupier's V statistic.

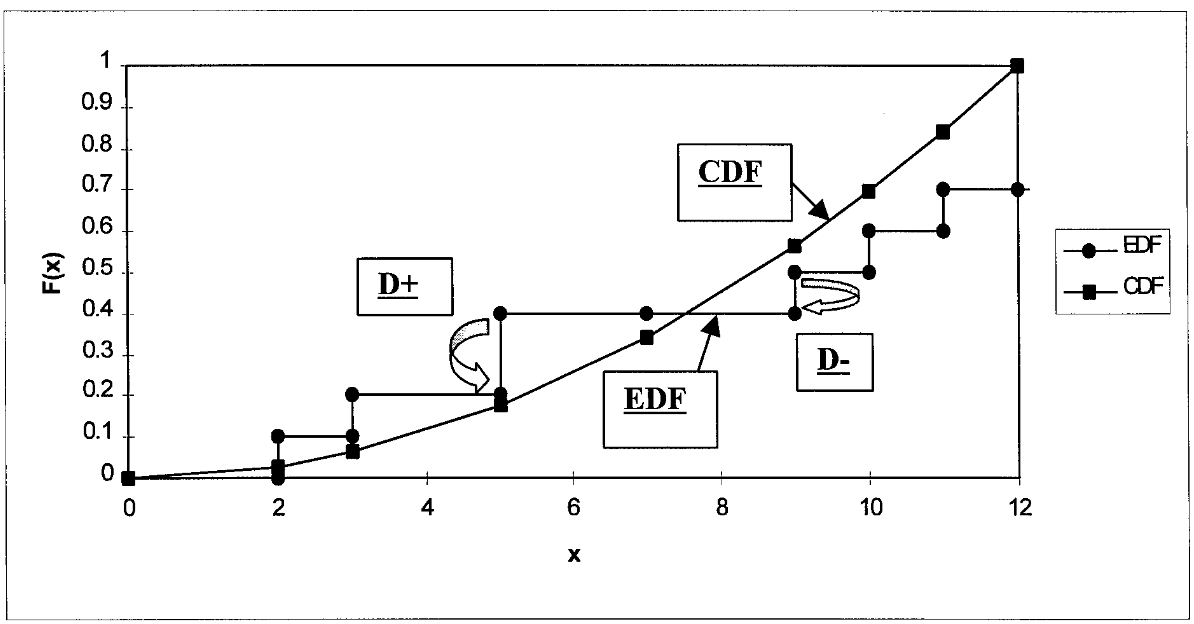


Figure 2.6 Sample EDF vs. Hypothesized (Estimated) CDF

- *The Quadratic Statistics:* They are based on integrating the squared differences between $F_n(x)$ and $F(x)$ where $-\infty \leq x \leq +\infty$. They belong to the Cramer-von Mises family that will be explained in the following sections. The most famous and widely used quadratic based test statistics are the Anderson-Darling (A^2) test statistic and the Cramer-von Mises (W^2) test statistic.

Lawless [97:432] states that when F(x) is completely specified and data are uncensored, the tests are all-distribution-free and percentage points for the various test statistics are generally known. Sections 2.3.2.1 – 2.3.2.3 discuss the G.O.F. tests dealing with completely specified H_o. On the other hand, when the parameters are unspecified and have to be estimated from the sample data, EDF based G.O.F. tests require critical values, which depend upon the hypothesized distribution, the sample size, the parameters estimated, and the method of estimation, as discussed later in this chapter [53:2-7]. Sections 2.3.2.1 – 2.3.2.3 discuss the G.O.F. tests dealing with completely specified H_o whereas section 2.3.2.4 discusses the G.O.F. tests that handle the cases where some or all of the parameters have to be estimated.

2.3.2.1 Kolmogorov-Smirnov (K-S) G.O.F Test Statistic

The Kolmogorov-Smirnov G.O.F test proposed by Kolmogorov in 1933 is used to investigate the significance of the difference between an observed distribution and a specified population distribution [140:108] [86:67]. Being the most famous and mostly used EDF based G.O.F. test statistic, the K-S G.O.F. test statistic formalizes the idea behind examining a q-q plot and is built upon the following two supremum test statistics:

$$D^{+} = \max_x \left\{ \frac{i}{n} - F(x) \right\} = \sup_x \{ \mathrm{F_n}(x) - F(x) \}, and \tag{2.20}$$

$$D^{-} = \max_x \left\{ F(x) - \frac{(i-1)}{n} \right\} = \sup_x \{ \mathrm{F(x) - F_n(x)} \} \quad where \quad i = 1,2,\ldots,n \tag{2.21}$$

D^+ can be defined as the largest vertical difference when $F_n(x)$ is greater than $F(x)$ and D^- can be defined as the largest vertical difference when $F_n(x)$ is smaller than $F(x)$ [146:100]. Examples of D^+ and D^- are illustrated on Figure 2.6. Therefore, the K-S G.O.F. test statistic is the maximum deviation calculated from D^+ and D^- defined in (2.20) and (2.21) as follows:

$$D = \max_x \{ D^{+}, D^{-} \} = \sup_x \left| F_n(x) - F(x) \right| \tag{2.22}$$

Therefore, the maximum difference between $F_n(x)$ and $F(x)$ forms the test statistic and this is compared with the critical values, $D_{crit}(\alpha)$, that are provided in different tables. If $D > D_{crit}(\alpha)$, then the null hypothesis that the sample data came from the assumed population is rejected at the given significance level α [86:67].

Furthermore, being introduced by Kupier in 1960, V statistic is the sum of the maximum deviations both above (D^+) and below (D^-) the CDF.

Advantages of the K-S G.O.F test versus chi-squared G.O.F. can be listed as below:

- *No sample size limitation:* As opposed to the chi-squared testing procedure, there is no sample size restriction in the K-S G.O.F testing procedure, therefore, it can be used with limited data from small samples.
- *Its comparatively high power:* Compared to the chi-squared G.O.F. test, K-S G.O.F. test has a higher power especially with highly skewed null distributions.
- *Less computation:* The K-S G.O.F. test will usually require fewer computations than the chi-squared G.O.F. test.

The K-S G.O.F. test's only disadvantage against the chi-squared G.O.F. test is that in order for the K-S G.O.F test to be employed, all of the parameters must be specified unlike in the chi-squared G.O.F. test where the parameters can be specified or estimated. In the case where the parameters must be estimated, the chi-squared G.O.F test must be preferred to K-S G.O.F test. This limitation has prevented the widespread usage of the EDF statistics. Overall, the K-S G.O.F. test should be used when sample sizes are small and when no parameters have to be estimated instead of the chi-squared G.O.F. test [97:383].

Furthermore, Wood and Alteva [146:176] have discussed asymptotic properties of K-S G.O.F. test statistics D^+, D^-, and D when used with discrete distributions, and have shown how asymptotic percentage points may be simulated.

Being based on the supremum statistics, the K-S G.O.F. test procedure often has less power than G.O.F. test procedures based on the quadratic G.O.F test statistics discussed in the following sections.

2.3.2.2 Cramer-Von Mises (C-vM) G.O.F. Test Statistic

The Cramer-von Mises G.O.F. test statistic is based on the squared integral of the difference between the observed data, $F_n(x)$ and $F(x)$ of the distribution being tested [17:3]. It is a member of the family of quadratic test statistics is defined as

$$Q = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \psi(x) dF(x), \tag{2.23}$$

where $\psi(.)$ is a suitable weighting function for the squared difference $[F_n(x) - F(x)]^2$. Therefore, Q gives us the area discrepancy between $F_n(x)$ and $F(x)$. The unique feature of this G.O.F. test is the incorporation of a weighting function into C-vM G.O.F. test statistics for flexibility. When $\psi(.) = 1$ for all x values, then we have the *C-vM* G.O.F. test statistic denoted by W^2.

Stephens and D'Agostino give the computational form of W^2 using the Probability Integral Transformation (PIT) as:

$$W^2 = \frac{1}{12} + \sum_{i=1}^{n} \left(Z_i - \frac{(2i-1)}{2n} \right)^2, \tag{2.24}$$

where $x_1 < x_2 < \ldots < x_n$ are n ordered observations from a sample and $Z_i = F(x_i)$ for $i = 1, 2, \ldots, n$ [146].

Viviano et al. defines the test procedure as follows [155:6]:

- Let $x_1 \leq x_2 \leq \ldots \leq x_n$ be n observations in the sample,

- Compute W^2,
- If W^2 is too large, then H_o is rejected.

The C-vM G.O.F. test statistic, like the K-S G.O.F. test statistic, is also a function of the vertical distance between the CDF and the EDF. They differ in the fact that the C-vM G.O.F. test goes one step further by considering every difference between the curve formed by the CDF and the curve formed by the EDF. Conover believes that the C-vM G.O.F. test is more appealing than the K-S G.O.F. test because the C-vM G.O.F. test statistic uses more of the sample data [28:306] [66:2-6].

While EDF based G.O.F. test statistics have been recommended on the basis of high efficiency compared to the G.O.F. test statistics designed for grouped data, the preliminary results of Hall (1985) suggest that, for example, Pearson's chi-squared G.O.F. test with overlapping cells can be made more powerful than the C-vM G.O.F. test statistic [130:105] [67].

2.3.2.3 Anderson-Darling (A-D) G.O.F. Test Statistic

When $\psi(.) = [F(x)[1-F(x)]]^{-1}$ for the all x values in equation (2.23), then we have the A-D test statistic. The A-D G.O.F. test statistic is also based on the squared integral of the difference between F (x) and $F_n(x)$. The A-D G.O.F. test statistic is defined to place more emphasis on the tail discrepancy of the distributions, because G.O.F. tests that utilize actual observations without grouping are sensitive to the discrepancies in the tails of the distribution rather than near the median. The A-D G.O.F. test statistic overcomes this problem by accentuating the values of $[F_n(x) - F(x)]$ where the test statistic is desired to have sensitivity. Thus, it is recommended to be used when it is important to detect departures in the tails of distributions in question [146:110] [53:2-9]. The weight function counteracts the decreasing differences between observed data and the hypothesized distribution in the tails by heavily weighting the difference in the tails [158:7]. The A-D G.O.F. test statistic is based on a weighted average of the squared discrepancies between the curve formed by the CDF and the curve formed by the EDF [66:2-8].

$$A^2 = n\int_{-\infty}^{+\infty}[F_n(x) - F(x)]^2 [F(x)[1-F(x)]]^{-1} dF(x) \tag{2.25}$$

Stephens and D'Agostino give the computational form of the A-D G.O.F. test statistic by using the (PIT) as

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[\log(Z_i) + \log(1 - Z_{(n+1-i)})] \tag{2.26}$$

where $x_1 < x_2 < \ldots < x_n$ are n ordered observations from a sample and $Z_i = F(x_i)$ for $i = 1, 2, \ldots, n$ [146].

Viviano et al. defines the test procedure as follows [155:6]:

- Let $x_1 \le x_2 \le \ldots \le x_n$ be n observations in the sample.
- Compute A_n^2
- If A_n^2 is too large, then H_o is rejected.

Lawless [97:432-433] states that the G.O.F. test procedures that are based on K-S, C-vM and A-D test statistics are distribution-free in the sense that the distributions of the statistics under H_o do not depend on F(x) only if H_o is completely specified. This is obvious from (2.20), (2.21) and (2.22), since under H_o the F(x_i)'s are the order statistics in a random sample of size n from the uniform distribution on (0,1). The exact distributions of D^+, D^- and D under H_o are known for all n. The distribution theory for W^2 and A^2 is more difficult; asymptotic distributions are known, but for finite n only partial analytical results are available, supplemented from Monte Carlo studies.

Woodruff, Moore, Dunne, and Cortes [166] made a study on a modified K-S G.O.F. test for the Weibull distributions with unknown location and scale parameters based on the fact that the K-S G.O.F. is one procedure that can be used, provided that there are no unknown parameters in the hypothesized distribution. This research has made a new approach with known shape parameter with unknown location and scale parameters as opposed to the conventional approach with known location parameter and with unknown scale and shape parameters. In this study, the authors note that when the parameters in a continuous distribution are unknown and must be estimated, the standard K-S G.O.F. test tables do not represent the true distribution of the test statistic. Thus, their paper uses Monte Carlo techniques to create tables of critical values for a K-S type test for the Weibull distributions with unknown location and scale parameters, but known shape parameter. They investigated the power of the proposed modified K-S G.O.F. test as well as the relationship between the critical values and the shape parameters. They concluded that the results of this study indicate that the modified K-S G.O.F. test appears to be a reasonable G.O.F. procedure for the Weibull family with unknown scale and location parameters. The same approach was followed for the gamma distributions with unknown location and scale parameters by Woodruff, Viviano, Moore, and Dunne and a conclusion that is parallel to the one for the Weibull distributions was reached [167]. Some of the most referenced research dealing with G.O.F. test procedures for the Weibull distribution with unknown scale and shape parameters, but with known location parameter include papers by Chandra, Singpurwalla and Stephens [21], Littell, McClave and Offen [102], Mann, Scheuer and Fertig [105], and Stephens [148].

Bush, Woodruff, Moore and Dunne presented a paper in which the tables of the critical values are generated for the standard C-vM and A-D G.O.F. test procedures for the Weibull distributions with unknown location and scale parameters and known shape parameters. Subsequently, the powers of the C-vM, A-D, K-S, and chi-squared G.O.F. test procedures for these situations are investigated. They concluded that the C-vM G.O.F. test has the most power when the shape parameter has a value of 1.0 and A-D G.O.F. test has the most power when the shape parameter has a value of 3.5. In addition they presented an equation that shows the relation between the critical value and the inverse shape parameter [18].

As a summary, the test statistic of these tests, which include the K-S, V, C-vM and the A-D tests among others, are measures of the sample deviation from the hypothesized distribution. The Cramer-von Mises test statistic is essentially the sum of the squared deviations from the CDF, while the Anderson-Darling statistic is a weighted sum of deviations, with more weight given to observations in the tails of the distribution [96].

2.3.2.4 Goodness-of-Fit Tests Based on EDF Statistics in the Case of Unknown Parameters (Unspecified Parameters)

Using the PIT, the values of a completely specified cumulative distribution are ordered values from a uniform distribution over the interval from zero to one. G.O.F tests based on EDF statistics test the H_o that a sample has been drawn from a fully specified continuos CDF. Therefore, EDF statistics are a function of ordered uniform RVs. If the CDF is not completely specified, the distribution of the EDF statistics will depend on the sample size n, the null distribution and method of estimation [147:4]. However, in 1948, David and Johnson [37:182] showed that the distribution of any EDF statistic is simplified when the unknown parameters are location and scale parameters. They also showed that the distribution of any test statistic based on the CDF using invariant estimators for the location and the scale will depend on the distribution tested, but not on the specific values of the unspecified scale and location parameters. As a result, the distribution of the A^2, W^2, and K-S G.O.F. test statistic for the Weibull distribution with unknown scale and location parameters will only depend on the sample size n, but will be independent of the true scale and location parameter values [17:5]. The K-S G.O.F. test has been modified by H. W. Lilliefors [100] [101] so that the test can be used with the normal and the exponential distributions when the parameters are unknown and estimated from the sample data. Cortes [31] further extended the K-S G.O.F. test so that it can be used with the gamma and the Weibull distributions when the scale and the location parameters are unknown. In 1969, Green and Hegazy [59] used the K-S, W^2 and A^2 G.O.F. tests to generate rejection tables for the uniform, normal, Laplace, exponential and the Cauchy distributions with unknown parameters. Mann, Scheuer, and Fertig [104] developed a new G.O.F. test for the two-parameter Weibull distribution with unknown parameters in 1973. They called the new statistics the L and S statistic. In 1979, Littlell, McClave, and Offen [102] used K-S, W^2, and A^2 G.O.F. tests to generate the rejection regions for the two-parameter Weibull distribution when the shape and the scale parameters are unspecified. Bush [17] came up with a modified C-vM and A-D test for the Weibull distribution with unknown location and scale parameters.

2.4 Concepts and Types of Censoring (Distinction between Complete and Censored Data)

Even though complete (not censored) data will be used throughout this thesis effort, it is important to make a clear distinction between complete and censored data and understand the concept of censoring. Therefore, knowledge of some of the peculiarities in how failure time data are collected is very important due to the fact that the data used in Weibull goodness-of-fit tests are generally associated with life-testing and reliability studies. In most of the reliability and life-testing experiments, a particular set of components of a system is selected to be tested and the time that takes the component to fail is kept track of. Then, the reliability of a population is basically inferred upon the random sample of failure times for the components. Therefore, as an ideal practice, the components that have a sample size n are chosen, and observed until all of the components fail. As a result of this observation, a sample of n failure times is acquired. The failure data that are collected until all of the components fail is called *complete data* or *a complete sample*. In real life applications, it is impossible to wait until all of the components fail because of the time restrictions and economic unfeasibility. Consequently, almost all of the reliability and life-testing experiments are generally terminated at some point before all of the components fail as we have in the ideal case. The failure data that are collected until some point during the experiment when all of the components have not failed yet, is called *censored data* or *a censored sample.* In this case, failure-time data sample size that is denoted as r is smaller than the total number of components that are being tested, n.

Lifetime data often come with a feature that creates special problems in the analysis of the data. This feature is known as censoring and, broadly speaking occurs when exact lifetimes are known for only a portion of the individuals under study. The remainder of the lifetimes are known to exceed certain values. Therefore, it can be said that lifetime data analysis is often complicated by the introduction of censored samples. Generally, only a portion of under investigation entities have known lifetimes. In some instances, it may be justifiable to disregard the sampling restrictions. If the sampling restrictions are very strict, valid analysis of the data requires the censoring to be considered [27:47].

Formally, an observation is said to be right censored at time L if the exact value of the observation is not known, but only that it is greater than or equal to L. Similarly, an observation is said to be left censored at time L if it is known only that the observation is less than or equal to L. Right-censoring is very common in lifetime data, but left censoring is fairly rare [57:116].

Fortunately, goodness-of-fit test statistics have been adapted for all forms of censoring. The most common and simple censoring schemes involve a planned limit to the magnitude of the variables under study or to the number of observations. These are called Type I and Type II censored data, respectively, sometimes referred to as "time censoring" and "failure censoring" [146:461] [53:2-5].

2.4.1 Type I Censoring (Time Censoring)

If the experiment is terminated at a predetermined period of time and the failed components are replaced during the test period, the resulting data is said to be Type I censored. Lawless [97:34-35] defines Type I censoring as sometimes experiments are run over a fixed time period in such a way that an individual's lifetime will be known exactly only if it is less than some predetermined value. In this case, as the time of the experiment is known before the experiment, the number of components to be tested is not known beforehand, and therefore is a RV. For example, in a life-testing experiment n items may be placed on test, but a decision made to terminate the test after a time L has elapsed. Lifetimes will then be known exactly for those items that fail by time L. Stated more precisely, a Type I censored sample is one that arises when individuals $1,2,\ldots,n$ are subjected to limited periods of observation $L_1,\ldots,L_n$, so that an individual's lifetime T_i is observed only if $T_i \leq L_i$. Type I censoring also frequently arises in medical research where, for example, a decision is made to terminate a study at a date on which not all the individuals' lifetimes will be known. It should be noted that with Type I censoring the number of exact lifetimes observed is random, in contrast to the case of Type II censoring, where it is fixed.

2.4.2 Type II Censoring (Failure Censoring)

As opposed to the Type I censoring, in Type II censoring the experiment is terminated as soon as a predetermined number of observed failures are obtained. Thus, in Type II censoring, the number of observations is predetermined and the length of the experiment is a RV. Lawless [97:33] describes a Type II censored sample as one for which only the r smallest observations in a random sample of n items are observed $(1 \leq r \leq n)$. Experiments involving Type II censoring are often used in life testing. A total of n items is placed on test, but instead of continuing until all n items have failed, the test is terminated at the time of r^{th} item failure. Such tests can save time and money, since it could take a very long time for all items to fail in some instances. It will be seen that statistical treatment of Type II censored data is, at least in principle, straightforward. It should be stressed that with Type II censoring the number of observations r is decided before the data are collected. Formally, the data consist of the r smallest lifetimes $T_{(1)} \leq T_{(2)} \leq \ldots \leq T_{(r)}$ out of a random sample of n lifetimes $T_{(1)},\ldots,T_n$ from the life distribution in question.

2.4.3 Statistical Inference with Censored Data

The presence of censoring creates special problems for statistical inference, some of which are not completely resolved. With Type II censoring, matters are in principle straightforward: properties of likelihoods and procedures associated with them can be readily obtained. The books by Sarhan and Greenberg [137] and David [36] discuss properties of order statistics and give many procedures for Type II censored data. Exact methods of inference for Type II censored data are discussed throughout this book for a variety of parametric and nonparametric models. With Type I censored sampling, complicated distributional problems usually make it impossible to work out exact properties of procedures, and there is consequently a heavy reliance on large-sample methods. As for Type II censored sampling, asymptotic results of the usual type can be shown to hold under essentially the same conditions as for complete samples [5], [97:43]. For a detailed explanation for analysis with censored data and an in-depth analysis of censoring types and their application in G.O.F. testing procedure area, see Lawless [97].

2.5 Parameter Estimation

One of the fundamental tasks of engineering and science is the extraction of information from data. Parameter estimation is needed in the modern world for the solution of the many diverse problems related to the space program, investigation of the atom, and modeling of the economy. Parameter estimation is a discipline that provides tools for the efficient use of data in the estimation of constants appearing in mathematical models and for aiding in modeling of phenomena. The models may be in the form of algebraic, differential, or integral equations and their associated initial and boundary conditions. An estimated parameter may or may not have a direct physical significance. Parameter estimation can also be visualized as a study of inverse problems. In the solution of partial differential equations, one classically seeks a solution in a domain knowing the boundary and initial conditions and any constants.

Fortunately, simultaneous with the development of increased need of parameter estimation, computers have been built that make the parameter estimation practicable for a great array of applications. It should be noted that both digital computational and data acquisition facilities are practical necessities in parameter estimation. Both these facilities have been readily available only since late the 1950s or 1960s, whereas estimation was first extensively discussed by Legendre in 1806 and Gauss in 1809 [41:215]. In Gauss's classic paper, he claimed the usage of *the method of least squares* (still used in parameter estimation) as early as 1795 in connection with the orbit determination of minor planets. For this reason, Gauss is recognized as being the first to use this important tool of parameter estimation [8:2].

Parameter estimation for our own purposes is defined as the process of finding approximations of the true values of the parameters of a probability distribution. These approximations, also called estimates, are found by using rules called estimators. These rules tell "how to calculate an estimate based on the measurements contained in a sample [118:292]. In hypothesis testing, true values of the parameters of a distribution are usually unknown and must be estimated from the observed data by the statisticians who perform the hypothesis testing. Today, parameter estimation is widely used in many applications of G.O.F. test techniques as one of the foundational steps.

Parameter estimation can be classified into two main branches [94: 39]:

1. *Point Estimation:* The value of the parameter is desired. A single value from one sample is proposed as an estimate for the parameter.

2. *Interval Estimation:* An interval is proposed and it is affirmed with a certain degree of confidence that the parameter lies in that interval.

The basic problem of estimation occurs when some random variable, *X*, is modeled by a PDF $f(X;\theta_1,\theta_2,....,\theta_n)$ where the function is of known form but involves certain unknown parameters $(\theta_1,\theta_2,.....,\theta_n)$ [124:391]. It is necessary to somehow estimate the values of these unknown parameters. Needed information is usually obtained from a sample of the population and a rule, or estimator, is used to obtain the estimate [48:3].

It is important to note that the name "parameter estimation" is not universally used. Other terms are nonlinear least squares, nonlinear estimation, nonlinear regression, and identification, although the latter sometimes is given a quite different meaning. Estimation is a statistical term and identification is an electrical engineering term. Parameter estimation is not necessarily nonlinear as implied by some of these terms [8:2].

The Weibull distribution is a particularly important life distribution, and a large body of literature on statistical methods has evolved for it. One reason that so many papers have been written on the Weibull distribution concerns its statistical properties.

There is probably an infinite number of rules or methods that could be used in deriving an estimate from a sample. Some of the more well-known parameter estimation methods are the method of Bayes, least squares, minimum chi-squared, minimum distance, methods of moments, and maximum likelihood estimation [110:273-288]. In this part of this thesis effort, some of the estimation techniques that have been developed for the Weibull distribution will be examined. However, the sequential G.O.F. test procedure proposed here is significant, then, in that it bypasses these parameter estimation methods relying on the sample moments of the observed data.

2.6.1 Maximum Likelihood Estimation

The method of maximum likelihood estimation is today considered to be the most common and accepted method of parameter estimation. It was first introduced by R. A. Fisher, a geneticist and a statistician in the 1920's [41:265]. The method of maximum likelihood estimation, and the principle of maximum likelihood, involves rules for obtaining estimators in models, rather than for constructing models. However, the logic of maximum likelihood estimation contains the seeds of a general modeling strategy that is extremely flexible and quite exciting, as it opens modeling doors that cannot be opened by most other methods [46:1].

Given a set of observations, one often wants to summarize the data by fitting them to a "model" that depends on some adjustable parameters (e.g., coefficients of rate equations in a complex network of chemical reactions). There are, however, important issues that go beyond finding the parameters, since data are generally not exact but subject to measurement errors. Thus, typical data never exactly fit a given model even when the model is correct. One needs to assess whether the model used is correct or not in relation to some objective statistical standard. Finally, we need to know the accuracy with which the parameters are determined by the data at hand. In general, one cannot answer the question, "What is the probability that a particular set of fitted parameters is correct?" The reason is that we do not have the complete set of models from which the parameters are drawn, simply a restricted class of functions. One rather addresses the following question, "Given a set of parameters, what is the probability that the particular data set at hand could have occurred?" In other words, one tries to estimate the probability of obtaining the data given the parameter estimates. Therefore, maximum likelihood estimation is one of the methods for estimating the parameters of the population from which sample data was drawn. It has been proved that the least square procedure is a maximum likelihood estimator of the parameters if the measured errors in the data are independent and normally distributed. This fact accounts for the widespread use of least square fits [26:2-6].

The likelihood function is simply the joint density function evaluated at the values of the given sample [26:2-6].

Parameters are selected so the likelihood function is maximized [109:419]. If x_1, $x_2,\ldots, x_n$ is a set of independent ordered random samples for a distribution with PDF $f(x;\theta)$, where θ is the vector of parameters for the density function, then the likelihood functions is given by

$$L(x_1, x_2, \ldots, x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta) \tag{2.27}$$

The likelihood function for the three-parameter Weibull distribution can be given as

$$L(x_1, x_2, \ldots, x_n|\delta, \beta, \theta) = \prod_{i=1}^{n} f(x_i|\delta, \beta, \theta)$$

$$=\left(\frac{\beta}{\theta^{\beta}}\right)^{n}\prod_{i=1}^{n}(x_i-\delta)^{\beta-1}e^{\left[-\sum_{i=1}^{n}\left(\frac{x_i-\delta}{\theta}\right)^{\beta}\right]} \tag{2.28}$$

Due to the fact that the natural logarithm of L is a monotically increasing function of L, both L and log (L) are maximized by the same parameter values [109:300]. Because the product of the density functions is converted into a summation as the equation (2.29), taking the natural logarithm of the likelihood function simplifies finding the derivative of L [53:2-11].

$$\log(L)=n\log(\beta)-n\beta\log(\theta)+(\beta-1)\sum_{i=1}^{n}\log(x_i-\delta)-\theta^{-\beta}\sum_{i=1}^{n}(x_i-\delta)^{\beta} \tag{2.29}$$

We first find the partial first derivatives of (2.29) with respect to the scale parameter (θ), the shape parameter (β) and the location parameter (δ). Then, the resulting equations are set to zero and solved for the corresponding parameter in order to find the MLEs for the respective parameter.

$$\frac{\partial\log(L)}{\partial\theta}=-\frac{n\beta}{\theta}+\beta\theta^{-(\beta+1)}\sum_{i=1}^{n}(x_i-\partial)^{\beta} \tag{2.30}$$

$$\frac{\partial\log(L)}{\partial\beta}=\frac{n}{\beta}-n\log(\theta)+\sum_{i=1}^{n}\log(x_i-\delta)-\theta^{-\beta}\sum_{i=1}^{n}(x_i-\delta)^{\beta}\log(x_i-\delta) \tag{2.31}$$

$$\frac{\partial\log(L)}{\partial\delta}=(1-\beta)\sum_{i=1}^{n}(x_i-\delta)^{-1}+\beta\theta^{-\beta}\sum_{i=1}^{n}(x_i-\delta)^{\beta-1} \tag{2.33}$$

Law and Kelton [96:370] state that the Maximum-Likelihood estimators (MLEs) have been widely used because they have useful properties that are not shared by other parameter estimation techniques. These statistical properties are summarized below [96:370] [26:2-7] [14:19,20] [2:75-79] [109:402] [41:270] [46:18-20] where L is the likelihood equation and θ is the vector of parameters:

- MLEs are strongly consistent; that is, $\lim_{n\to\infty}\hat{\theta} = \theta$. For large sample sizes, a consistent estimator is as good as an unbiased estimator.
- For most of the common distributions, the MLE is unique; that is, L ($\hat{\theta}$) is strictly greater than L (θ) for any other value of θ. Uniqueness basically means that there are no alternative values that maximize the likelihood function.
- If the expected value of the estimator equals the parameter [E ($\hat{\theta}$) = θ], then it is an unbiased estimator of that parameter. In general, the asymptotic distribution of $\hat{\theta}$ has mean equal to θ as the sample size approaches infinity. Therefore, it can be concluded that even though the MLEs may be biased for small sample sizes, they are usually asymptotically unbiased [E ($\hat{\theta}$) = θ] for larger sample sizes.
- MLEs are asymptotically normally distributed; that is, $\sqrt{n}(\hat{\theta}-\theta)\xrightarrow{D} N(0,\delta(\theta))$, where $\delta(\theta) = -n / E(d^2 L / d\theta^2)$ (the expectation is with respect to X_i, assuming that X_i has the hypothesized distribution) and $\xrightarrow{D}$ denotes convergence in distribution. Furthermore, if $\hat{\theta}$ is any other estimator such that $\sqrt{n}(\hat{\theta}-\theta)\xrightarrow{D} N(0,\sigma^2)$, then $\delta(\theta) \le \sigma^2$.

- MLEs are invariant; that is if $\hat{\theta}$ is an MLE of θ, and that $\phi = h(\theta)$ for some function h, then the MLE of ϕ is $h(\hat{\theta})$. It is important to note that unbiasedness is not equivalent to invariance.
- If a sufficient statistic T exists for the estimator of a parameter θ, the MLE will always be some function of T.

Among many methods for updating values of θ that requires the use of the iterative numerical approaches, the mostly used ones are the variations of what are commonly referred to as quasi-Newton updating methods. Broyden, Fletcher, Goldfarb, and Shanno (BFGS) and Davidon, Fletcher, and Powell (DFP) provide useful methods for stepping toward a solution when θ is far from the maximum [46:44] [39] [153]. Besides, Berndt, Hall, Hall, and Hausman (BHHH) present a very powerful numerical method when θ is close to the maximum [11] [46:44].

Because of the beneficial properties listed above and the fact that calculus-based techniques can usually be used to derive the MLEs (though often numerical methods, such as Newton's method, are necessary), maximum likelihood estimation is the most widely used estimation technique among statisticians [41:270].

Finding a maximum likelihood solution is sometimes more of an art than a science. If a model fails to converge, any number of parts of the iterative process may be suspect. Poor starting values are often to blame [46:45].

The books and articles by Hoaglin, Mosteller and Tukey [73], Eliason [46], Law and Kelton [96], Bain [2], Lawless [97] are some of the good references that can be used for the fundamentals and usage of maximum likelihood estimation methods and algorithms.

2.5.2 Minimum Distance Estimation

Another widely used parameter estimation method, the minimum distance estimation method was introduced by Wolfowitz. He presented a series of papers that developed the Minimum Distance method for obtaining strongly consistent parameter estimates for a distribution [54:20] [162] [163]. According to Wolfowitz, "A great utility of the Minimum Distance method is that, in a wide variety of problems, it will furnish super-consistent estimators even when classical methods, like maximum likelihood method, fail to give consistent estimators" [118:2-7] [14:20].

His technique was to minimize the distance of the discrepancy between the estimated CDF (*F(x)*) and the EDF ($F_n(x)$) defined as [14:20]:

$$\delta(F_n(x), F(x)) = \sup\left|F_n(x) - F(x)\right| \tag{2.34}$$

A good estimate of the parameters for the estimated cumulative density function must be derived from another method, such as MLE, and the better the method's estimate the better result that minimum distance estimations (MDE) can give. A good initial estimate of the parameters may be obtained using the method of maximum likelihood.

The three advantages of MDE in terms of robustness can as be listed as follows [54:21-22] [162:616-617] [119:75]:

The MDE estimates are not very susceptible to outliers. While an erroneous data point greatly increases the distance measure, that value has relatively little effect upon the PDF, which minimizes the G.O.F. test statistic.

The MDE estimates are statistically consistent. Consistency implies that as the number of data points increases, the estimate approaches the true parameter.

Although the MDE was originally used to estimate location, the methodology is easily extended to also estimate shape and scale parameters.

Some of the research efforts that have been done with MDE are given so that the reader can reference them. Matusita [106] proved the consistency of the estimates by MDE in 1959. Sahler [136] proved the conditions for the existence and consistency of MDE estimates. Hobbs, Moore and James [74] used MDE to estimate the parameters for the three-parameter generalized gamma distribution in 1980. Shumaker [142] used MDE to estimate the parameters for the four-parameter generalized gamma distribution in 1982. Charek [22] used MDE in his study about the comparison of the estimation techniques for the three-parameter Pareto distribution as the base estimation technique and found out that the MDE was the best among the alternatives. Gallagher and Moore [54] used MDE to estimate the parameters for the three-parameter Weibull distribution in 1990. Benton-Santo [10] used MDE to estimate the parameters for the mixture of exponential distributions and the mixture of normal distributions in 1986. Mumford [112] used MDE to estimate the parameters for the seven-parameter Mixed Weibull in 1996. Boerrigter [14] used MDE for the nine-parameter mixed generalized gamma distribution in 1997. As these sources indicate, the use of MDE in parameter estimation is getting more widespread everyday.

2.5.3 Method of Moments

Being one of the oldest and simplest general methods of estimating parameters from a sample, the method of moments was proposed by K. Pearson [48:4]. The method of moments is based on the intuitive idea and assumption that sample moments should provide good estimates of the corresponding moments [109:300]. The moments of the distribution of the test results can be used to estimate the unknown parameters of the actual distribution. The number of moments equals the number of unknown parameters and the required number of equations. The method is based on the fact that, as the sample data increases, the values of the sample moments approach the actual values.

Due to its simplicity, when other estimators are bogged down by mathematical manipulative difficulties, the method of moments is especially useful [40:129].

In regular practice, after having equated sample moments with as many as there are parameters to be estimated, one then solves the equations for the unknown parameters. For example, if x is a RV with a PDF $f(x;\theta_1,\theta_2,...,\theta_k)$ and there are i unknown parameters to be estimated, then, i equations have to be solved for the unknown parameters [48:4].

The r^{th} population moment about zero can be defined as

$$\mu_r' = E(x^r) \qquad (2.35)$$

where E denotes the expected value function. If $x_1, x_2, \ldots, x_n$ is a sample of n from the population, the s^{th} sample moment can be defined as [48:4]

$$M_s = \sum \frac{x_i^{\ s}}{n} \tag{2.36}$$

Thus, if there are i parameters to be estimated, the system of equations to be solved is

$$\mu_r = \mu_r^{'} \qquad\qquad r = 1,2,\ldots,i \tag{2.37}$$

where $\mu_r^{'}$ is obviously a function of the parameters of the PDF of x. The i solutions to these equations are the method of moments estimators (MME).

For example, with a two-parameter Weibull distribution with the PDF formula in equation (2.3), the first and second initial moments are given by

$$m_1 = a\Gamma\left(1+\frac{1}{b}\right) \qquad\qquad m_2 = a^2\Gamma\left(1+\frac{2}{b}\right) \tag{2.38}$$

where $a = \beta$ and $b = \theta$. Therefore, the following two equations ca n be used for the Weibull distributions with parameters $\hat{a}$ and $\hat{b}$ [40: 290]:

$$\hat{a}\Gamma\left(1+\frac{1}{\hat{b}}\right) = \frac{1}{n}\sum_{1\le i\le n} x_i, \qquad\qquad \hat{a}^2\Gamma\left(1+\frac{2}{\hat{b}}\right) = \frac{1}{n}\sum_{1\le i\le n} x^2{}_i \tag{2.39}$$

where $a = \beta$ and $b = \theta$.

Because the method of moments estimators do not possess desirable asymptotic properties like the maximum likelihood estimators, it has relatively low popularity, even though the method of moments has the advantage of being simple to use.

2.5.4 Method of Quantiles

Parameter estimates are obtained in the method of quantiles with the same equations used in the method of moments. Theoretical distribution quantiles are found for the empirical quantiles. The number of empirical quantiles equals the number of equations and also equals the number of parameters to be estimated. To estimate parameters $\hat{a}$ and $\hat{b}$ of a Weibull distribution, two equations are used

$$1-e^{-\left(\frac{t_1}{\hat{a}}\right)^{\hat{b}}}=F_1 \qquad 1-e^{-\left(\frac{t_2}{\hat{a}}\right)^{\hat{b}}}=F_2 \tag{2.40}$$

where $a = \beta$ and $b = \theta$, t_1 and t_2 are the quantiles of the empirical distribution, and F_1 and F_2 are the values of the empirical distribution function corresponding to the quantiles t_1 and t_2. The method of quantiles obviously can be used with any sample, but the estimates may have high variances. The estimates depend on the levels of the chosen quantiles [154:292]. For extensive explanation about the graphical and analytical determination of the Weibull distribution parameters, see Knezevic [93: 166-198].

2.5.5 Some Other Parameter Estimation Methods

Cheng and Amin [24] proposed an alternative method for three-parameters, called maximum product of spacing (MPS) estimation method that can be used in cases when MLE fails. This method solves three equations in three unknowns using a numerical approach.

Dubey [45] suggested another method for the three-parameter estimation problem.. In practice, this method first estimates the location parameter γ by

$$\tilde{\gamma} = \frac{X_{(1)}X_{(n)} - X_{(k)}^2}{X_{(1)} + X_{(n)} - 2X_{(k)}} \tag{2.41}$$

where k is the smallest integer in 2, 3,..., n -1 such that $X_{(k)} > X_{(1)}$. It is shown by Dubey that $\tilde{\gamma} < X_{(1)}$ if and only if $X_{(k)} < \left[\frac{X_{(1)} + X_{(n)}}{2}\right]$. Zanakis [171] concluded that $\tilde{\gamma}$ was accurate for the Weibull distribution. Given $\tilde{\gamma}$ as the location parameter, two parameter MLEs can be found for the shape and scale parameter after subtracting out $\tilde{\gamma}$ for all the observations.

Furthermore, McCool [107:256] discusses some other alternative estimators based on order statistics. He notes that the MLEs are regular (in the sense of having the usual asymptotic distribution) only for $\beta > 2$. If it is known that $0 < \beta < 1$, then $\min(X_1, \ldots X_n)$ is a super efficient estimator for the location parameter.

Usually the location parameter, δ is assumed to be zero. On the other hand, it is important to note that a value of location less than zero, $0 > \delta$ could indicate a failure in storage. Hirose [72:310] discusses the location parameter in his paper as follows: In failure analysis (especially in electrical engineering), it is well known that failures follow the Weibull CDF and there seems to exist a certain point, greater than zero, in the Weibull CDF under which a breakdown will not occur, or at least will be very rare. Since very low probabilities are expected in electrically powered equipment, electrical engineers consider it very crucial to estimate that point.

Hirose [72:330] proposed an algorithm of MLE comprised of three parts below.

- Determining appropriate initial values for Newton-Raphson method.
- Finding the approximate values by using the line search algorithm.
- Solving the three simultaneous likelihood equations by Newton-Raphson method.

He concluded that the larger the shape parameter value, β, the more often the parameters fail to converge in MLE.

For other estimation methods, reference Engelhardt and Bain [47], Smith and Naylor [143], Cohen and Whitten [27: 31-32], Wozniak [168], and Winkler [160: 350].

2.6 The Concept and Applications of Sequential G.O.F. Testing

The sequential G.O.F. tests utilize present G.O.F. tests in sequence with the goal of coming up with an omnibus test procedure that is a more powerful and dependable G.O.F. test procedure against a broader range of alternative hypotheses, and as such are not considered to be new G.O.F tests. Sequential G.O.F. tests are based on the idea that two different G.O.F. tests are employed independently in any order. The sequential

G.O.F. testing procedure that utilizes two independent G.O.F. tests in sequence has its own significance level and power. Denote the first G.O.F. test procedure's significance level as α_1 and the following G.O.F. test procedure's significance level as α_2. By Bonferroni's inequality, the significance level of the sequential test is bounded as follows:

$$\alpha \leq \alpha_1 + \alpha_2 . \tag{2.42}$$

Therefore, Bonferroni's inequality can be interpreted as an upper limit for the overall significance level of the sequential test and with sequential G.O.F. test significance level no larger than the sum of the individual G.O.F. tests' significance levels. Notice Bonferroni's inequality does not give us an exact value for the overall significance level of the sequential test procedure. Therefore, the question of concern is how to obtain an exact measure for the overall significance level of the sequential test procedure. From the previous work done in sequential G.O.F. testing, we can see that there are three approaches to solve this problem:

- *Approximation of the overall significance level of the sequential test procedure:* For example, for Pearson's R-test for normality, an overall significance level was approximated (see Equations 2.43 and 2.44), but contingent upon starting with identical significance levels with each test and making an independence assumption that did not hold even though skewness and kurtosis are not correlated. Determining an exact value for the actual attained significance levels of this test and other sequential tests are elusive. Therefore, this alternative should not be preferred over other alternatives.

- *Using Monte-Carlo simulation techniques to develop tables of empirically observed attained overall significance levels of the sequential test procedure:* Onen, Gunes and Clough utilized Monte-Carlo simulation techniques to develop tables of empirically observed attained overall significance levels of the sequential test procedure. The readers are able to determine several levels in the given tables that would lie close to a desired value and then choose which combination of α-levels for the individual tests would be best for a particular situation. If user has some additional insights into his data that might lead him to presume it would be skewed, for example, he could pick a greater significance level, meaning a larger rejection region, for the skewness test to be more powerful against skewed alternatives in the sequential test [26:2-28] [117:5-7] [64:74].
- *Using contour plots of the overall significance level of the sequential test procedure based on the significance levels obtained via Monte Carlo simulation:* Stephens and D'Agostino use this technique for normality tests based on the skewness and kurtosis with each axis representing the significance level of each test's significance level [146:282, 302-305]. Clough [26] did a marvelous job employing this idea in his thesis effort. Throughout his thesis effort, he created various contour plots with the individual test significance levels are the axes that can be used to find the overall significance level for the sequential test procedure. He states that the generation of the contour plots to depict attained significance level via Monte Carlo simulation provide a simple and more user-friendly means of

determining the appropriate levels for the component tests, and represent a substantial improvement over earlier tabular formats. He also states that these contour plots yield tactical insights into the proper selection of significance levels for the two tests such that potentially higher power can be achieved.

The research, the applications, and publications about sequential G.O.F. test procedures are very rare and waiting to explored and employed. Some of the studies that have been done by using sequential G.O.F. test procedures will be discussed in the following discussion.

Being the only author who has ever been written a book on sequential hypothesis testing and G.O.F. procedures, Ghosh [57] mentions various sequential testing procedures in his book. He refers to Abraham Wald as the pioneer in sequential analysis. A very well known and simple example of sequential G.O.F. test procedure is the R-test for normality that was introduced by Pearson, D'Agostino and Bowman in 1977. This sequential G.O.F. test procedure utilizes the separate skewness and kurtosis tests in sequence. If either of these tests is rejected, then the normality assumption is rejected. By the help of equation (2.42) Pearson, D'Agostino and Bowman showed that if $\alpha_1 = \alpha_2 = \alpha^*$, then a good approximation to the overall level of significance is:

$$\alpha \approx 4\left[\alpha^* - \left(\alpha^*\right)^2\right] \tag{2.43}$$

where one can determine α^* as

$$\alpha^* = \frac{1}{2}\left[1-(1-\alpha)^{\frac{1}{2}}\right] \tag{2.44}$$

Equation (2.43) holds exactly if skewness and kurtosis are independent. Skewness and kurtosis are uncorrelated but not independent and use of (2.43) to determine the overall level of significance produces a conservative test. Tables of corrected values for the skewness and kurtosis employed in R-test for normality are given in the paper by Pearson [120] for $n = 20$, 50 and $\alpha = 0.05$ and 0.10.

Now let us take a look at more applications of sequential G.O.F. test procedures. Woodruff and Moore [164] came up with a new sequential goodness-of-fit test technique for symmetric distributions based on a study that Schuster (JASA 1973) proposed reflecting data points about the sample mean as a way to improve the power for fully specified symmetric distributions against symmetric alternatives. The improved power against symmetric alternatives also holds when the tests are modified to accommodate distributions with unknown parameters. However, the reflection technique decreases the power against non-symmetric alternatives. A sequential approach is proposed for tests against alternatives with unknown symmetry properties. The standard Anderson- Darling test is applied, and if the null hypothesis is not rejected, the reflected test is applied. Results for the normal and uniform distributions show that this approach results in increased power against symmetric alternatives while showing only a slight decrease in power against non-symmetric alternative.

Onen [117] used various combinations of EDF statistics to be able to employ a series of sequential G.O.F test procedures for the Cauchy distribution. He concluded that the power of the sequential G.O.F. test procedure was less than that of its most powerful component test at the same significance level, but almost always greater than the worst of the two at identical α levels for symmetric distributions. Thus, it increased the power of the weaker G.O.F test procedure. On the other hand, he also stated that the sequential G.O.F test procedures did not improve the power of G.O.F test procedures for the non-symmetric distributions. His conclusion agrees with the conclusion that Moore and Woodruff reached in their study about sequential G.O.F test procedures for the symmetric distributions. Gunes [64] used EDF based G.O.F. tests and Watson's W test to be able to employ six sequential modified G.O.F. tests for the inverse Gaussian distribution. As a noteworthy alternative to employing only one G.O.F. test procedure, he concluded that a sequential G.O.F. test procedure that combines two G.O.F. tests in sequence gives very powerful results at opposite levels of symmetry. Most recently, Clough [26] employed a sequential G.O.F. test procedure for the three-parameter Weibull distribution with known shape based on skewness and kurtosis. In his conclusion, which parallels Onen's, he found in most cases of the two-sided G.O.F test procedures, the sequential test power was less than of its most powerful component test at the same significance level, but almost always greater than the worse of the two. This result might seem to force us to favor choosing a single test that is the most powerful of the two, versus the sequential G.O.F test procedure. Clough [26] also states that since knowing which G.O.F test procedure is more powerful might be problematic, the sequential G.O.F test procedure ensures higher average power against a spectrum of alternatives. He also states that in a couple of cases where the skewness and kurtosis G.O.F test procedures have almost identical power, the sequential G.O.F test procedure's power exceeded both separate tests at a given significance level. Thus, the cooperative nature of the tests overcame the natural lag in power due to the differences in significance levels between separate and sequential employment. Furthermore, he states that sequential G.O.F test procedures substantially increase the power in the case of one-sided G.O.F test procedures.

As a result of investigating these studies, the main concern in sequential G.O.F. test procedures that arises about the power of the procedure has been cleared. Sachs [135:126] states that due to the fact that the probability of Type II error (β) decreases as the probability of Type I error (α) increases, the power (1-β) increases. Onen [117:5-7], Gunes [64:73] and Clough [26:2-29] came to the same conclusion that the power of a sequential G.O.F. test procedure is between the powers of the G.O.F. test procedures that make up the sequential test procedures at the identical significance levels. Clough [26] answers the question that one may have in mind that is, why use a sequential test if it tends to be less powerful at a given α level than the best test that it combines? He states that the answer is in the fact that the sequential G.O.F. test procedure may be more powerful on average against a wider range of alternatives than either of the two separate G.O.F. test procedures. For example, one of the component G.O.F. tests that makes up the sequential G.O.F. test procedure may be powerful against symmetric alternatives but weak against the skewed alternatives while the other G.O.F. test may be effective against the skewed alternatives. The combination of these two G.O.F tests in the sequential procedure may give a very useful omnibus G.O.F test that gives better results against a broader spectrum of alternatives.

Some other examples of the sequential G.O.F. test procedures and the studies in this area include Corradini [30], Neuhaus and Kremer [115], Irle and Wertz [82], Xiong [169], Basu [6], McWiliams and Pollak [108], Iuculano and Catelani [83], Harter, Moore and Wiegand [68], Govindarajulu [58], DeGroot [38], Jackson [84], Aroian and Robinson [1], Hoel, Weiss and Simon [75], Lai [95], and Germogenov and Ronzhin [55], Kanji [86:95, 97, 99, 144].

In this thesis, a new sequential goodness-of-fit test for a family of three-parameter Weibull distributions with known shape based on skewness and Q-Statistic will be developed. We expect to see an increase in power over Clough due to replacing kurtosis with the Q-Statistic. A more complete discussion of Q-Statistic G.O.F. test statistic is contained in section 2.8.

2.7 Adaptive Methods

Adaptive methods use the sample data to make a decision on what form of estimator to use. This is accomplished by using the sample data to make a preliminary classification of the underlying model [127:771]. Hogg's opinion is that [77:1185] [48: 11]:

> In this age of the excellent computing devices, the statistician should take a broader view and not select a narrow model prior to observing the sample Items. Let the analysis of the sample play a role in the classification of the underlying distribution; this, in turn can dictate the most desirable statistics to use for inference.

Most of the success of adaptive estimation depends on a good preliminary classification of the underlying distribution [80:597].

2.8 Discriminants: Skewness, Kurtosis and Q-Statistics

The preliminary classification is determined by the value of a measure of non-normality obtained from the sample. Such statistics are called discriminants, and they are the key elements of adaptive estimation. Typically for this application, discriminants are used to determine whether the underlying distribution has a short tail, a medium tail, or a long tail. Several useful discrimants are skewness, kurtosis, and the Q-Statistic.

Stephens and D'Agostino [146:279-318, 375-391] provide a comprehensive explanation about the skewness and kurtosis. These two well-known characteristics of the statistical distributions are often employed to describe the shape of the PDF of the statistical distribution in question. Shape statistics such as skewness and kurtosis measure how the shape of the underlying population differs from the shape of a normal distribution with the same mean and variance. Both sample skewness and sample kurtosis statistics make use of all the data values, and, like the mean and standard deviation, are sensitive to outliers [125].

Skewness is the degree of asymmetry, or departure from symmetry, of a distribution [144:111]. A symmetric distribution has zero skewness. Data from a positively skewed (skewed to the right) distribution are bunched together below the mean, a long tail above the mean. Distributions that are forced to be positive, such as annual income, tend to be skewed to the right. Data from a negatively skewed (skewed to the left) distribution are bunched together above the mean, but have a long tail below the mean. Outliers in a sample from a symmetric distribution can produce a non-zero sample skewness statistic. Boxplots, histograms and normal probability plots may be useful in detecting skewness to the right or to the left and the presence of the outliers in terms of symmetry [125]. For skewed distributions, the mean tends to lie on the same side of the mode as the longer tail [144:111].

Kurtosis is the degree of peakedness of a distribution, usually taken relative to the normal distribution [144:113]. Therefore, the sample kurtosis measures heavy-tailedness or light-tailedness relative to the normal distribution. A light-tailed distribution like the Beta distribution has fewer values in the tails than the normal distribution, and will have negative kurtosis. Heavy-tailed distributions, like the Cauchy distribution, have more values in the tails than the normal distribution, and will have positive kurtosis. Like skewness, outliers in a sample from a distribution with normal tails can produce a non-zero sample kurtosis statistic. Boxplots, histograms and normal probability plots of the sample can provide information as to whether this might be the case. A sample from a distribution with long tails (positive kurtosis) may also have a sizable non-zero skewness statistic, even if the underlying distribution is symmetric. When the distribution from which the sample is selected is light-tailed, the largest and smallest observations are usually not as extreme as would be expected from a normal random sample [41:187].

Among many measures of skewness, the most widely used measures of a population is given by the third and the fourth moment about the mean, expressed by $E[(x\text{-}\mu)^3]$ and $E[(x\text{-}\mu)^4]$ where E is the expected value operator. To render this quantity invariant to change in location and scale, it is divided by σ^3 and σ^4 respectively to standardize resulting in;

$$\sqrt{\beta_1} = \frac{E(X-\mu)^3}{\sigma^3} \tag{2.45}$$

for Skewness, and

$$\beta_2 = \frac{E(X-\mu)^4}{\sigma^4} \tag{2.46}$$

for Kurtosis.

For a perfectly symmetric distribution, such as the normal distribution, $\sqrt{\beta_1}$ is zero. When $\sqrt{\beta_1} > 0$, the distribution is said to be positively skewed, or skewed to the right. When $\sqrt{\beta_1} < 0$, the distribution is said to be negatively skewed, or skewed to the left.

For the normal distribution $\beta_2 = 3$ where the distribution appears to be not very peaked or flat-topped and it is called *mesokurtic*. Thus, in order for the kurtosis to be a comparative measure to the normal curve, it is generally corrected by subtracting 3 from it. This correction makes the normal distribution a reference distribution in terms of the sample kurtosis. Since $\beta_2 = 3$ for the normal distribution, the corrected measure is 0 for normally distributed populations. By the help of this correction, kurtosis can be used as a measure of departure from normal kurtosis. When corrected $\beta_2 > 3$, the distribution appears to have a relatively high peak and heavy tail and it is called *leptokurtic*. When corrected $\beta_2 < 3$, the distribution appears to have relatively flat-top and light tail and it is called *platykurtic* [144:112]. Clough [26:2-13] notes that the platykurtic examples exhibit more peakedness than the normal distribution but has no tails to speak of, demonstrating that kurtosis is heavily influenced by the tail thickness.

Stephens and D'Agostino [146:279-280] derive and define the sample skewness and kurtosis in their book. Let's let $x_1, x_2 \ldots x_n$ be samples from a random sample of size n from a given distribution. The population's distinguishing characteristics may be evaluated by the use of the sample moments. The sample mean is the first moment about the origin is given by

$$m_1 = \frac{1}{n}\sum_{j=1}^{n} x_j \qquad (2.47)$$

The sample central moments or moments about the mean are defined by

$$m_i = \frac{1}{n}\sum_{j=1}^{n}\left(x_j - m_1\right)^i, \qquad i = 2,3,4,\ldots.. \qquad (2.48)$$

By the use of the formulas defined above, Stephens and D'Agostino define the sample skewness, $\sqrt{\beta_1}$, and kurtosis, β_2, as follows [146:279]:

$$\sqrt{\beta_1} = \frac{m_3}{m_2^{3/2}} \qquad\qquad \beta_2 = \frac{m_4}{m_2^2} \qquad (2.50)$$

Stephens and D'Agostino [146:279] place an important emphasis on the fact that $\sqrt{\beta_1}$ and β_2 are invariant under origin and scale changes. They also emphasize that a considerable amount of research and study has been done on expressing the distributions of $\sqrt{\beta_1}$ and β_2. Skewness and Kurtosis are well-defined and used to a great extent as G.O.F. test statistics for testing the normality. It is noted that in random samples from normal distributions, there may be wide variations from the theoretical values for $\sqrt{\beta_1}$ and β_2, especially for small sample sizes ($n < 25$). The sample may arise from some nonnormal distribution, such as a uniform, negative exponential, or Weibull, etc. Symmetric distributions, or those with non-zero densities extending over negative and positive values, are likely to produce samples with small skewness, whereas distributions corresponding to positive valued random variables (such as the negative exponential) are likely to produce samples with large skewness. In sampling from fairly symmetric distributions, one might expect the kurtosis to reflect nonnormality. Therefore, a combination of the skewness and kurtosis test statistics might provide a more comprehensive test than either taken by itself. They also note that skewness and kurtosis are two of the test statistics whose distributions in normal sampling are still known exactly [146:375-376].

Skewness and kurtosis are correlated, and although for normal sampling the correlation is zero, they are still dependent variables. In other words, there will be situations in which $\sqrt{b_1}$ will dominate the test decision about normality, b_2 playing a minor role, and vice versa. For example, monthly rainfall amounts in certain climates are well fitted by a negative exponential distribution so that one might expect the skewness to play a major role in testing for non-normality [146:283]. Since they are dependent on each other, the overall attained significance level cannot be calculated only by adding the two significance level of the individual tests ($\alpha_1 + \alpha_2 \neq \alpha_{overall}$). An extensive explanation will be given in the attained significance level derivation section in Chapter 3.

The Q-statistic was originally introduced by Hogg and was analyzed and improved by several other researchers [78:424]. Throughout this research, Hogg's Q-Statistic will be stated as the Q-Statistic. The Q-Statistic is based on discriminating between the distributions on the basis of the order statistics. Hogg [164] states that the Q-Statistic is a more powerful discriminant of the tail length than kurtosis, and the Q-Statistic's convergence properties are better than those of kurtosis, since it is a linear function of the order statistics.

Having used the Q-Statistic as a measure of dispersion to discriminate between the uniform, normal, and double exponential distributions, Hogg and his student Davenport [34:12] came to the following conclusions. A low value of Q-Statistic is associated with the uniform distribution. An intermediate value of Q-Statistic suggests the use of the MLE for the normal distribution. A high value of Q-Statistic indicates that the sample came from the double exponential distribution. Therefore, we can come to the conclusion that an increasing value of Q-Statistic indicates an increasing tail length.

Hogg [78:424] defines the Q-Statistic as follows:

$$Q = \frac{U(\alpha) - L(\alpha)}{U(\beta) - L(\beta)} \tag{2.51}$$

where $U(\beta)$ is the average of the largest $n\beta$ order statistics of the sample size n, and $L(\beta)$ is the average of the smallest $n\beta$ order statistics [78:424]. $U(\alpha)$ and $L(\alpha)$ have similar definitions. $U(\alpha)$ and $L(\alpha)$ are the means of the upper and lower $100\alpha\%$ tails of the underlying distribution [79:913]. It is important to note that the α and β used in this formula are not related to the Type I and Type II errors. Thus, α and β used in Q-Statistic will be denoted as Qα and Qβ respectively in this research. Let's assume that Qα = 0.05 and Qβ = 0.5 that are advised by Hogg and Davenport [79:913] for the best results and the sample size is 100. Then, $U(\beta)$ is the average of the smallest 50 sample points whereas $L(\beta)$ is the average of the largest 50 sample points and $U(\alpha)$ is the average of the largest 5 [1- (100*0.05)] sample points whereas $L(\alpha)$ is the average of the smallest 5 (100*0.05) sample points. After these values are found, the Q-Statistic value can be found by substituting these values in equation (2.51). It is also important to note that the fractional order statistics are used when $n\alpha$ and $n\beta$ order statistics do not yield integers [34:13].

Hogg [78: 424] concluded that the Q-Statistic is the best when deciding between the normal and the double exponential distributions. As a matter of fact, the Q-Statistic is good when the double exponential distribution is competing with the normal or logistic distribution but is very poor when the uniform distribution is involved. They also state that if the analyst knows that light-tailed distributions are definitely not under consideration and the analyst wants to discriminate between distributions around the normal/logistic range and those with heavier tails, then the Q-Statistic would be an appropriate statistic.

Curry [34:13] used the Q-Statistic as one of his discriminators where he assumed that the sample could have come from the uniform, normal, or double exponential distributions in his thesis on adaptive robust estimation of location and scale parameters. He used the values of Qα = .04, Qβ = .5 in his analysis and stated that Rugg [134] used these values as well for n = 4(4)16.

The following example shows how he calculated the Q-Statistic for $n = 4$. Given a sample of size four, where $X_{(1)}, X_{(2)}, X_{(3)}, X_{(4)}$ are the sample values ranked in ascending order, $U(\beta)$ is the average of the largest two order statistics which is $\left(\frac{X_{(4)} + X_{(3)}}{2}\right)$. $U(\alpha)$ is the average of the largest .16 order statistics, or $U(\alpha) = 0.16\left(\frac{X_{(4)}}{0.16}\right)$, while $L(\alpha) = 0.16\left(\frac{X_{(1)}}{0.16}\right)$ and $L(\beta) = \frac{\left(X_{(1)} + X_{(2)}\right)}{2}$.

Curry [34:21] defines the Q-Statistic for the fractional order statistics case as follows

$$Q = \frac{x_{(n)} - x_{(1)}}{2\sum_{i=1}^{\frac{n}{2}} \frac{\left[x_{(n-i+1)} - x_i\right]}{n}} \tag{2.52}$$

where n stands for the sample size selected. For $n = 4$.

$$\begin{aligned} Q &= \frac{x_{(4)} - x_{(1)}}{\frac{2\left[\left(x_{(4)} - x_{(1)}\right) + \left(x_{(3)} - x_{(2)}\right)\right]}{4}} \\ &= \frac{2\left(x_{(4)} - x_{(1)}\right)}{x_{(4)} - x_{(1)} + x_{(3)} - x_{(2)}} \end{aligned} \tag{2.53}$$

From the definitions of $L(\alpha)$, $U(\beta)$, $U(\alpha)$, and $L(\beta)$, he shows that the equations (2.53) and (2.54) are equal to each other.

$$Q = \frac{\dfrac{\left(0.16x_{(4)} + 0.16x_{(1)}\right)}{0.16}}{\dfrac{x_{(4)} + x_{(3)}}{2} - \dfrac{x_{(1)} + x_{(2)}}{2}}$$

$$= \frac{2\left(x_{(4)} - x_{(1)}\right)}{x_{(4)} - x_{(1)} + x_{(3)} - x_{(2)}} \tag{2.54}$$

Rugg [134: 58-75] empirically determined two values (upper/lower) of the Q-Statistic during his thesis study. He attempted to maximize the effectiveness of the Q-Statistic as follows [134: 58-75] [34: 14]:

(1) 5000 draws of sample size *n* were made from one of the three distributions.

(2) The Q-Statistic was computed for each sample.

(3) Values for Q(lower) and Q(upper) were picked so that when the underlying distribution was uniform, the relative efficiency of the estimator was at least 50 percent.

(4) When the true underlying distribution was normal or double exponential, steps (1) and (2) were repeated and a relative efficiency of 80 percent was used to select the values of Q-Statistic [134:36].

Fitting a polynomial to Rugg's resulting values using least squares yielded the following formulas [34:14]:

$$Q(Lower) = -0.00183035n^2 + 0.0928214n + 1.024 \tag{2.55}$$

$$Q(Upper) = 0.0012946 - 0.0071785n + 2.332 \tag{2.56}$$

where n is the sample size.

Hogg [80:599] also developed a formula for computing the Q-Statistic values. They used Hogg's Q-Statistic in a two-step adaptive procedure that picked the population having the largest location parameter. In that paper he developed the following formulas

$$Q(Lower) = 2.08 - \frac{2}{n} \tag{2.57}$$

$$Q(Upper) = 2.96 - \frac{5.5}{n} \tag{2.58}$$

where n is the sample size being considered [134:771].

In his thesis, Curry [34] demonstrates that equations (2.55) and (2.56) are competitive with Rugg's [134] values given in equations (2.57) and (2.58). Curry lists the relative efficiencies associated with equations (2.55) and (2.56) under the variable Q section of Tables I, III, and V in his study.

Other researchers have made use of the Q-Statistic. D'Agostino and Lee [93:393-396] showed that Q-Statistic is a better discriminator than sample kurtosis. Wegmann and Carrol [44:1-20] obtained excellent results using Q-Statistic as a discriminator for Hampel M estimators.

Ethridge [48] studied adaptive estimation of life distributions based on a family of logarithmic power distributions by using six various discriminants, including the Q-Statistic in his thesis. He concluded that sample kurtosis should be used as a discriminant if the underlying transformed population is suspected to be mesokurtic or leptokurtic; and Q-Statistic should be used if a platykurtic transformed population is suspected. If nothing is known about the underlying population, the Q-Statistic appears to be the best overall discriminant.

2.9 Monte Carlo Simulation Methods

A computer simulation is often used to predict the behavior of equipment whose operation processes are difficult or impossible to represent with analytical relationships that are known to be valid. A reliability simulation model ordinarily consists of a governing sequence of discrete events, such as failures, repairs, and switchings. Development of a formal model requires creation of an algorithm that define mathematical relationships that mirror the real world and that transforms a set of initial data into a sequence of discrete events. These events may have the characteristics that are computed. Statistical inferences are made about the equipment or process after the runs are completed [154: 456].

Monte Carlo simulation can also be used for numerically solving differential equations. It is used extensively in finance for such tasks as pricing derivatives or estimating the value at risk of a portfolio. The technique tends to be computer intensive, with many problems taking minutes or hours to solve on a high-speed computer. For this reason, Monte Carlo simulation is avoided when closed form or other simple solutions exist for a problem. Monte Carlo simulation, however, has the advantage that it is a brute force technique that will solve many problems for which no other solutions exist.

Bratley et al states [15:114]:

> Monte Carlo simulation is now widely used to solve certain problems in statistics that are not tractable. For example, it has been applied to estimate the critical values for the Kolmogorov-Smirnov G.O.F. test for normality.

Much of the work with G.O.F. tests use Monte Carlo simulation to create data that mimics many different populations (e.g. normal, lognormal, exponential, double exponential, uniform, Weibull). It is one thing to have data that are representative of the distribution (e.g., ten samples from an exponential distribution with $\lambda = 10$), but it is quite another to have that sample is randomly generated again and again and again [131] [66:2-14].

In a more elaborate definition, a Monte Carlo simulation procedure generates a random sample from a particular distribution and uses the sample to evaluate some measures of interest, as if the sample were experimental data from an actual problem. It repeats this process for N total samples or trials and combines the measures calculated from them to draw a conclusion or approximate a quantity [26:3-2]. Note that in this thesis study, N will denote the number of samples drawn or the number of replications, while n specifies the size of each the N samples or replications. Sheskin [140:143] states that as a result of the Monte Carlo simulations, we have "pseudorandom" samples that provide the scientists or the researchers who perform the study with a mechanism for studying natural processes or evaluating problems that otherwise would be impossible or more problematic to evaluate.

Statisticians and mathematicians often utilize Monte Carlo simulation methods to approximate the sampling distribution of test statistics. In G.O.F. test procedures, Monte Carlo methods have been extensively used to estimate the significance levels of test statistics. In order to achieve this goal, the sampling distribution of a test statistic under a given H_o is approximated by Monte Carlo simulation of random samples from the distribution. With this estimate, the significance level or critical values may be derived

[116:50, 63-65]. Monte Carlo simulation has been widely used to study the properties of robust estimators and to test their performance (26:19). By using Monte Carlo simulation, parameter estimates may be calculated and then compared to the true parameters of the underlying distribution. The basic steps in a Monte Carlo simulation for testing parameter estimation techniques are as follow [14:125]:

1. Generate sample variates from the selected underlying distribution.
2. Determine the parameter estimates for each sample, using each estimation technique.
3. Compare the performance of the estimators.

Gwinn [66:2-4] states that the main weakness in Monte Carlo simulation is that the answers it produces are to some degree uncertain since they are based on the seeds and random variate generation techniques of the software program in use. Shooman [141: 259] states that in a perfect model with perfect random numbers, the error will decrease proportional to the square root of the number of scenarios used.

As a result, we can conclude that there is usually no concern if the uncertainty is negligible for practical purposes. One way of reducing the uncertainty is to base the Monte Carlo work on a larger number of observations. This is done because the law of the large numbers states that, as the sample size increases, the difference between the sample mean and the population mean becomes smaller [131:176] [66:2-15].

Advancements in computer technology have positively affected Monte Carlo simulation methods, since the speed and the number of the trials of the Monte Carlo simulation method directly depends on speed and computing capability. The more simulation runs we have, the better approximations to the theoretical critical values for the test statistic we reach. Therefore, the benefit of having large simulation runs cannot be overvalued in Monte Carlo simulation methods. Thus, modern computational power permits the generation of tens or hundreds of thousands of samples of a given sample size from a distribution. For these samples, one can calculate an equivalent number of test statistics so that by the sheer number, their sampling distribution closely resembles the population distribution. It is in essence a brute-force approach to determining the sampling distribution [26:3-2,3]. Due to the nature of the method, the simulated critical values that are created by Monte Carlo simulation method must be used with care and sound judgment, especially when high precision is needed.

The advancements in Monte Carlo simulation methods and the increase in the replication sizes can be observed from the G.O.F. test procedures related with the Weibull distribution in Table 2.1. Note that the sample sizes in the Monte Carlo simulation methods have increased as the years have passed, as would be expected. Clough et al also emphasized this point via examples in his study [26].

Much research is currently being performed on "efficient" techniques for generating scenarios. These "quasi-random" techniques attempt to minimize the number of observations that must be used in order to achieve a desired level of precision.

Clough [26:3-3] notes that Noreen [116:49] has affirmed the utility of this approach by the following statement:

> In general, a valid Monte Carlo significance level can be computed for any test statistic that is a function of data drawn from any specified population. The population does not have to have a familiar, well-behaved distribution studied by statisticians; population can be entirely arbitrary.

Table 2.1 The Increase in Sample Sizes in Parallel to Technology Improvement.

Research Date	Author and the Reference	Number of the Runs
1981	CORTES [31]	2,000
1981	REAM [131]	5,000
1981	BUSH [17]	5,000
1982	VIVIANO [155]	5,000
1986	WHITSEL [158]	5,000
1991	CROWN [32]	5,000
1993	YUCEL [170]	10,000
1993	GWINN [170]	10,000
1994	ONEN [117]	50,000
1997	CLOUGH [26]	100,000
1997	FRISCO [53]	100,000

All work done in this area has not been done to satisfy one particular sample from the Weibull distribution. But instead the researchers were trying to establish general principles that applied, if not across all probability distribution families, then at least across one distribution family – regardless of the sample size. In order to be able to reach this end, Monte Carlo simulation was the key [66:2-15].

In this thesis study, for each sample size in question, 100,000 simulation trials will be run in order to generate the critical values for each of the $\sqrt{b_1}$ and the Q-Statistic test statistics for a Weibull distribution with known shape parameter. Depending on the computer time, the sample size for the critical values may be increased to have better precision.

2. 10 Plotting Positions

Various plotting positions that mainly differ in the analyses with small sample sizes have been developed so far to be able to be used in specific applications. Now, let's take a look at several plotting positions formulas that could be used in this thesis effort in the literature and the historical development for them.

Given a series of observations ordered from smallest to largest (or vice versa), each event may be assigned a plotting position, which is its cumulative probability. Recurrence intervals, which are reciprocals of the cumulative probabilities (or of their complements) may also be used to define the plotting positions. A selection of plotting positions in the range of $\frac{(i-1)}{n}$ to $\frac{i}{n}$ is generally used, because given a sample size, *n,* the EDF of *n* can be interpreted as a step function that basically jumps from $\frac{(i-1)}{n}$ to $\frac{i}{n}$ at the *i*th order statistic of the sample data. It is important to note that if the plotting position $\frac{i}{n}$ is used, the largest value cannot be plotted, while if $\frac{(i-1)}{n}$ is used, the smallest value cannot be plotted, since the probability values of 1 and O are off the scale. Hazen [70] therefore proposed the compromise position $\frac{(2i-1)}{2n} = \frac{\left(i-\frac{1}{2}\right)}{n}$, the value midway through the jump from $\frac{(i-1)}{n}$ to $\frac{i}{n}$.

Gumbel [62] showed that the most probable, *modal plotting position* is $\frac{(i-1)}{(n-1)}$, and the *mean plotting position* (expected value of the popu1ation CDF at the i*th* order statistic of a sample of size n) is $\frac{i}{n+1}$. Weibull [157] also advocated use of the mean plotting position.

Beard [7] proposed use of the *median position*, $1-(0.5)^{\frac{1}{n}}$. Gumbel [63] considered the relative merits of the modal plotting position, the mean plotting position, and the Hazen [70] compromise position, which was also advocated by Pearson [121]. Gumbel [63] chose the mean plotting position. Kimball [89] used the plotting Hazen compromise position without any comment.

Kimball [89] recommended the mean plotting position for general use, but noted that if F[E(x_i)], the population CDF at the expected value of the i_{th} order statistic of the sample, can be estimated independently of the unknown parameters, such points might prove more desirable for graphical fitting near the extremes.

Johnson [85] tabulated the median plotting positions to 4 decimal places, which he called "median ranks", for $n \leq 20$, and gave an approximate formula for $n > 20$. Benard & Bos-Levenbach [9] showed that the median rank is closely approximated by the plotting position $\frac{(i-0.3)}{(n+0.4)}$. About the same time, this plotting position was also advocated in Russian publications by Lebedev [98] and Chegodayev [23]. Kapur and Lamberson [88:300] recommend the use of the median ranks, or the median plotting position.

Chernoff & Lieberman [25] showed that estimates of σ based on the plotting position $\frac{i}{n+1}$ are much less efficient than those based on the plotting position $\frac{\left(i-\frac{1}{2}\right)}{n}$.

Blom [13] applied what he called "the α, β - correction" to the plotting position $\frac{i}{n+1}$, obtaining $\frac{(i-\alpha)}{(n-\alpha-\beta+1)}$. If, for symmetry, one takes $\alpha=\beta$, this becomes $\frac{(i-\alpha)}{(n-2\alpha+1)}$. Many of the previously proposed plotting positions are special cases of this formula; $\alpha =$ O gives the mean plotting position, $\alpha = 1$ gives the modal position, $\alpha = 0.5$ gives Hazen's compromise position, and $\alpha = 0.3$ gives a good approximation to the median plotting position. Blom [13] showed that his formula can also be used to approximate the plotting position F[E(x_i)] proposed by Kimball [89] . For the normal distribution, the required value of α varies from 0.33 for $n = 2$ to a limiting value of $\frac{\pi}{8} \approx .39$ for $n \to \infty$. Blom [13] suggested the compromise value $\alpha = \frac{3}{8}$ for, giving the plotting position $\frac{\left(i-\frac{3}{8}\right)}{\left(n+\frac{1}{4}\right)}$. He obtained similar results for several other distributions, e.g. Cauchy, $1 \le \alpha \le 1.23$ and the extreme-value, $0.25 \le \alpha \le 0.5$.

Harter [69] states that much more has been written on the question of plotting positions during the past 50 years, but no consensus has been reached yet. It is clear from this body of literature that the optimum plotting position depends on the use that is to be made of the results and may also depend on the underlying distribution. He also states that hydrologists concerned with the return period of floods or other hydrologic events have usually favored the Weibull plotting position, while statisticians concerned with estimating the standard deviation or other scale parameter have noted the bias of estimates based on that plotting position and have preferred various other positions, depending on the underlying distribution. Harter [68] presents some of the results of the studies on the plotting positions in the next few paragraphs.

Kimball [89] investigated the choice of plotting positions on normal and extreme-value probability paper, with accent on the latter. He considered several plotting positions, including (I) Weibull; (IIA) Blom compromise; (II) Haze compromise; (III) F[E(W_i)]; (IV) F(median of W_i); and (V) F(mode of W_i), where W_i is the reduced variate [$W_i = \frac{(x_i - \mu)}{\beta}$], where x_i is the i_{th} order statistic of the sample and μ and β are location and scale parameters, respectively]. He enumerated three purposes of plotting: (1) a test of fit to the hypothetical distribution; (2) estimation of the slope of the fitted straight line; and (3) extrapolation at the extremes. He concluded, with qualifications, that method (IIA) is the best for all three purposes in the normal case, while in the extreme-value case, either method (III) or method (IIA) is the best for purposes (1) and (2), but for purpose (3) judgment will have to be exercised that cannot be guided precisely by formula.

Filliben [49] developed a systematic method for determining Blom's correction factor α and applied it to a set of 34 prototype distributions (all continuous and symmetric), consisting of the rectangular (uniform), Gaussian (normal), logistic, double exponential, and Cauchy distributions, augmented by the λ distribution, defined by $R_p = \frac{\left[p^{\lambda} - (1-p)^{\lambda}\right]}{\lambda}$ where $R_p = F^{-1}(p)$ is the percent point function for $\lambda = .9(-.1).1($-

.2)-.1(-.1)-1.0(-.2)-3.0. He found $\alpha = .5$ for double exponential, $\frac{\pi^2}{8}$ for Cauchy, $\frac{\pi^2}{8}$ for Gaussian, and $\frac{(\lambda - 1)(\lambda - 2)}{4}$ for the λ distribution [valid for rectangular ($\lambda = 1$) and logistic ($\lambda = 0$)]. He noted that, for all 34 distributions studied, the correction factor increases monotonically with tail length [49].

Gerson [56] noted the fact that when n sample values are ordered to form the set $\{y_i\}$ it is necessary also to decide on the percentiles $\{p_i\}$ to which they correspond in order to evaluate the quantiles $\{q_i\}$ of the relevant distribution. Probably the earliest value for the percentiles was $p_i = \frac{\left(i - \frac{1}{2}\right)}{n}$. This is the middle of the interval $\frac{(i-1)}{n}$ to $\frac{i}{n}$, and is still very widely used [56]. All other suggested values are very similar to this in the middle of the range, and differ significantly only for the most extreme points. Kimball [89] considered the problem particularly in relation to the normal distribution and the extreme-value distributions of type I. Chernoff and Lieberman [25] also looked at this problem as it affects normal plotting. They were especially concerned with the use of the plot as a good estimator of σ. They assumed that a line fitted by eye to a set of points will be the least squares line, and then looked for plotting positions that produce either minimum variance unbiased (MVU) estimates or minimum mean square deviation (MSE) of a biased estimate. Kimball's suggestion of $\frac{i}{n+1}$ produces a significant bias in $\hat{\sigma}$, whereas a proposal due to Blom of using his compromise method meets this objection and would appear to have a low mean square deviation of the estimate. Furthermore, it is thought to have the property that it produces points that lie approximately on a line which deviates little from a straight line if the $\{y_i\}$ are in fact normally distributed. If this is indeed the case, it is clearly a good value to use in testing for normality. Gerson [156] uses the expression $p_i = \frac{(3i-1)}{(3n+1)}$, which is almost identical in value to Blom's suggestion even for high, and low values of i.

Sutcliffe [151] commented at some length on graphical estimation and plotting positions for particular distributions. They recommended the following plotting position:

For the exponential distribution and the extreme value Type I (Gumbel) distribution, $F_i = \frac{(i-.44)}{(n+.12)}$ or $F[E(Y_{(i)})]$; for the normal and lognormal distributions, Blom compromise method; for the two-parameter gamma distribution and the Pearson Type III distribution (a special case of the gamma distribution), $F_i = \frac{\left(i-\frac{2}{5}\right)}{\left(n+\frac{1}{5}\right)}$; this is a compromise between the values for the normal distribution (gamma with shape parameter $= \infty$) and the exponential population (gamma with shape parameter = 1). For the EVI (Gumbel) distribution, they tabulated exact values of $E(y_{(i)})$ for n = 1(1)35 and approximate values $y_i = -In(-lnF_i)$, where $F_i = F_i = \frac{(i-.44)}{(n+.12)}$ for $n = 36(1)50$.

Cunnane [33] reviewed the unbiased plotting positions. He noted that the existing attitude that the criterion for choice of plotting position is arbitrary is rebuked and it is shown that a worthwhile criterion can be based on desired statistical properties of the plot, rather than on comparison of plotting positions with estimates of probability for individual sample values. These properties are that any quantile estimate made from the plot should be unbiased and should have the smallest mean square error (MSE) among all such estimates. This leads to specification of plotting position initially in terms of reduced variate rather than probability value. The unbiased plotting position is $E(Y_{(i)})$, the mean of the i_{th} order statistic in samples from the reduced variate population, which differs from the distribution to another. A good approximation for each distribution is available in the probability domain. These take the general form $\frac{(i-\alpha)}{(n-2\alpha+1)}$ with $\alpha = \frac{3}{8}$ in the normal case and $\alpha = 0.44$ in the extreme-value type-1 (EV1) and exponential cases. The Weibull formula, $\alpha = 0$, is correct for the uniform distribution alone and is shown to be biased for other distributions. Hazen's formula, $\alpha = \frac{1}{2}$, shows up much better in terms of bias than many would expect. If a single simple distribution free formula were required then $\alpha = \frac{2}{5}$ would be the best compromise. The probability position postulates that have supported the Weibull formula for many years are examined and some are seen to be unreasonable.

King [92] noted that the determination of a probability plotting position was given simply as $\frac{i}{n+1}$ where i is the rank order number of an item of data in the numerically ordered sample data. As sample sizes increase above $n = 20$, the differences among the positions determined by any method of estimation decrease to the point where they are practically unimportant. Harter [69] notes that in order to evaluate Cunnane's recommendations, they have conducted a series of Monte Carlo experiments using the Weibull formula recommended by Gumbel, and the Hazen formula recommended by Johnson. He notes that the test results indicate that there is no practical difference in the estimates of the mean obtained by any of the three methods, but there is a highly significant difference among the estimates of the sample standard deviation. The Weibull formula consistently overestimates the standard deviation. Although, for some purposes, this is primarily a conservative error, it also the Weibull formula consistently overestimates the standard deviation. Although, for some purposes, this is primarily a conservative error, it also reduces the sensitivity for making test comparisons. On the other hand, the Hazen formula consistently underestimates the standard deviation, which consistently results in unconservative errors. The Cunnane formulas give estimates which average 1-2% high for the standard deviation. These estimates are, therefore, slightly conservative and their bias is practically irrelevant. The Cunnane estimates are also consistently closer to the true population parameters than either of the other formula. King [92] also noted that the value of α recommended for Type II extreme value and for the Weibull distribution is 0.5. The use of this value leads to the Hazen formula as the formula of preference for plotting Weibull distribution data, giving $\chi_i = \frac{(i-0.5)}{n}$.

Nelson [114] defined the "midpoint" plotting position as $F_i = 100\frac{\left(i-\frac{1}{2}\right)}{n}$, i = 1,...,n [Hazen's compromise position expressed as a percentage]. He noted that different plotting positions have been zealously advanced. In general, the i_{th} plotting position is a 'typical' population percentage near to which the i_{th} ordered observation falls. The mean plotting position is popular and is $F_i' = 100\ \frac{i}{n+1}$, $i = 1, 2, ..., n$. King [92] tabulates Fi', the expected percentage of the sample below the *ith* ordered observation. Johnson [148] advocates and tabulates median plotting positions, well approximated by

$F_i" \approx 100\frac{(i-0.3)}{(n+0.4)}$. In practice, plotting positions differ little compared with the randomness of the data.

Bush [17:2-14] used the average of the mean plotting position and the modal plotting position as a part of his thesis study. The calculation formula can be given as follows:

$$\frac{\left[\left(\frac{i}{N+1}\right)+\left(\frac{i-1}{N-1}\right)\right]}{2} \tag{2.59}$$

It seems to Harter [69] that much of the disagreement and confusion as to the choice of plotting positions is due to the fact that the CDF at the expected value of the *ith* order statistic is not equal to the expected value of the CDF at the *ith* order statistic, i.e.

$$F[E(x_i)] \neq E[F(x_i)] = \frac{i}{(n+1)} \tag{2.60}$$

except in the case of the uniform distribution (0,1). In the case of distribution-free G.O.F. tests based on the sample CDF, the PIT converts the hypothetical distribution into a uniform distribution, and the proper plotting position is $\frac{i}{N+1}$.

The plotting position $F[E(x_i)]$ gives unbiased estimates of the values of a measured variable corresponding to particular values of the cumulative probability. In this context, Barnett has shown (in the case of a normal distribution) that $F[E(x_i)]$ is a convenient and nearly optimum plotting position, yielding estimates of the regression parameters which are the alternative linear unbiased estimates of Gupta [65] [4]. This method is also applicable to other distributions. If the expected values of the order statistics have not been tabulated for the distribution and sample size of interest, $F[E(x_i)]$ can be approximated by Blom's formula.

The plotting position $E[F(x_i)]$ gives unbiased estimates of the cumulative probabilities corresponding to particular values of the measured variable. Most hydrologists favor this plotting position because it fulfills Gumbel's second postulate, giving a return period of $(n + 1)$ for the largest observation. It should be noted, however, that an unbiased estimate of the cumulative probability does not transform into an unbiased estimate of the return period $T(x_i) = \dfrac{1}{[1 - F(x_i)]}$, since $E[T(x_i) \neq \dfrac{1}{\{(1 - E[F(x_i)]\}}$

One possible way out of this difficulty lies in the fact that $F[median(x_i)] = median[F(x_i)]$.

The median plotting position yields median unbiased estimates of x_i for a specified $F(x_i)$ and of $F(x_i)$ for a specified x_i. It also yields median unbiased estimates of x_i for a specified $T(x_i)$ and of $T(x_i)$ for a specified x_i, since

$$median[T(x_i)] = \frac{1}{\{1 - median[F(x_i)]\}} = \frac{1}{\{1 - F[median(x_i)]\}}$$

and hence

$$median(x_i) = F^{-1}\left\{1 - \frac{1}{median[T(x_i)]}\right\}$$

Johnson [85] has tabulated the median plotting positions for samples of size $n = 1(1)20$. Values for $n > 20$ can be approximated by Blom's formula with $\alpha = .3$.

Table 2.2 Comparison of Some of the Plotting Positions for $i = 4$.

The Plotting Position Method	$N = 5$	$N = 10$	$N = 25$	$N = 50$	$N = 100$	$N = 250$
EDF $F_n(X_{(1)})$	0.200	0.100	0.040	0.020	0.010	0.004
EDF $F_n(X_{(3)})$	0.600	0.300	0.120	0.060	0.030	0.012
Modal	0.750	0.333	0.125	0.061	0.030	0.012
Mean	0.667	0.364	0.154	0.078	0.040	0.016
Average of Modal and Mean	0.708	0.348	0.139	0.070	0.035	0.014
Median Ranks	0.685	0.356	0.146	0.073	0.037	0.015

In summary, Harter [69] states that the optimum choice of plotting positions depends upon the purpose of the investigation and may also depend upon the distribution of the variable under consideration. One may wish to avoid the difficulties associated with unbiased estimates by obtaining median unbiased estimates instead.

2.6 The Conclusion of the Literature Review

This chapter has taken a look at the statistical properties of the Weibull distribution, the importance and the flexibility of the Weibull distribution in reliability and modeling applications, some of the G.O.F. test procedures and the parameter estimation for the Weibull distribution, and the concept and importance of sequential G.O.F. test procedures using skewness, kurtosis and Q-Statistic. Since it is very complicated to estimate the parameters of two and especially, three-parameter Weibull distributions, most of the goodness-of-fit techniques discussed involve some expensive and computationally demanding numerical methods. However, as Stephens and D'Agostino [146] make it very clear in their book, moment-based G.O.F. tests that are well documented for the normal distribution such as skewness and kurtosis are free from these numerical methods and they seem to have pretty good power compared to other G.O.F. test procedures. Clough [26] concludes in his thesis study that the sequential G.O.F. test procedure using the skewness and kurtosis in sequence provides very good power against a range of alternatives without the typical computational outlays for the hypothesized three-parameter Weibull distributions with known shape. Hogg [79] states that the Q-Statistic is a better G.O.F. test statistic than kurtosis in terms of power. Therefore, it is hoped that the new sequential G.O.F. test procedure that will be conducted using skewness and Q-Statistic as a sequential omnibus G.O.F. test procedure will provide very good power against a range of alternatives. It is also hoped that the power of this new sequential G.O.F. test procedure will be better than that of Clough's.

III. METHODOLOGY

3.1 Introduction

As discussed in detail in Chapter 2, the distributions of the EDF test statistics have been studied in the field of the G.O.F. testing for years. Unfortunately, the researchers haven't been able to come up with any user-friendly and computationally effective G.O.F test procedures that utilize these test statistics due to the fact that any closed form of the distribution functions of the EDF test statistics are rather complicated to handle. Thus, there arises a need for a user-friendly and computationally effective analysis tool that that can be used in G.O.F. testing for the Weibull distribution with known shape parameter value. This thesis is oriented to come up with a new sequential G.O.F. test procedure for the three-parameter Weibull distribution with known shape parameter value based on $\sqrt{b_1}$ and Q-Statistic that has the potential of providing this useful analysis tool that will result in higher power against a range of alternatives without the typical computational complexity.

As in most formal G.O.F. tests, determining the sampling distribution of the G.O.F. test statistic to be used in the G.O.F. test procedure for a specified H_o is the first thing to accomplish in the foundation of a new G.O.F. test procedure. After this step is accomplished, the critical values of the new G.O.F. test at specified significance levels can be derived and tabled. However, it is a known fact that coming up with a new sequential G.O.F. test procedure that involves using two G.O.F. test statistics in sequence, as in our study, complicates this procedure to some extent. In addition to the fact that the critical values for the individual G.O.F. tests can be derived and tabled, there will be numerous combinations of the significance levels for the component G.O.F test statistics that will yield the same overall significance level in the sequential G.O.F. test approach. Therefore, determining the attained significance levels for a variety of combinations of levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests that are employed in sequence will take up a considerable amount of the new sequential G.O.F. test development.

After the new sequential G.O.F. test is developed, it will have to be tested against the existing G.O.F. tests to see if this new sequential G.O.F. test improved the discriminatory power against a broad range of alternate distributions with the null hypotheses of Weibull distributions. The power, $(1-\beta)$, the probability of correctly rejecting a false H_o, stands out as the main discriminant in comparison between the new sequential G.O.F test and the existing G.O.F. tests. Ideally, it is expected that the new sequential G.O.F. test will result in very good power discriminating against a broad range of alternate distributions when compared to the existing G.O.F. test. Clough's [26] alternate distributions in his power study that are based on the alternate distributions from Bush's [17] and Wozniak's [168] researches will be utilized in order to see if the new sequential G.O.F. test that utilizes the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests improved power compared to Clough's [26] sequential G.O.F. test that uses the $\sqrt{b_1}$ and the b_2 G.O.F. tests. Based on the discussion in Chapter 2, since Q-Statistic is considered to be a better discriminant than the sample kurtosis, replacing b_2 with the Q-Statistic in the sequential G.O.F test is expected to result in better overall power. On the other hand, the fact that most of the G.O.F tests' power fluctuates significantly based on the alternate distributions studied, and one particular G.O.F test rarely demonstrates superiority over all other G.O.F. tests in all of the possible cases, should be kept in mind [146:2].

The objective of this thesis effort comes down to formally developing a new omnibus sequential G.O.F. test employing the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in sequence for a set of hypothesized three-parameter Weibull distributions with known shape parameter values and evaluate its power against a variety of alternate distributions compared to the power results of the existing EDF based K-S, C-vM and A-D G.O.F. tests. This procedure will have the benefit of eliminating the need for location and scale parameter estimation prior to initiating the sequential G.O.F. test procedure that the EDF tests require.

3.2 Critical Value Determination

3.2.1 Plotting Positions

In parallel to Crown's [32], Bush's [17] and Clough's [26] work, the mechanics of the procedure to determine the critical values in this thesis involves plotting a piecewise linear approximation to the CDF of the G.O.F. test statistic and employing linear interpolation at a given significance level. The order statistics of the sample of the G.O.F. test statistic values generated by Monte Carlo simulation are plotted on the abscissa versus a particular plotting position on the ordinate that represents its cumulative probability in order to construct this procedure. Clough [26-3,4] states that these plotting position values are similar to those of the EDF at a given order statistic and are bounded on [0,1]. Hence, one can extrapolate the critical value for, say the 95% level, by linearly interpolating between the order statistics whose plotting positions bound the value 0.95.

One of the methods of calculating the critical values would be to simply pull off the desired percentiles from the ranked sample data values. For example, given a sample size of 100,000 as in this thesis, the 95,000th order statistic from the ranked sample data values would be pulled off if the 95th percentile were desired. This seemingly easy method of calculating the critical values assumes that the range of the test statistic is fixed between the 1st and the 100,000th order statistics values. In other words, it is bound by the first and the last order statistic, which is obviously a false representation. Bush [17], Crown [32], and Clough [26] have emphasized this fact. They all chose to use median ranks or median plotting position method that was explained in Chapter 2. Therefore, the median ranks or median plotting position technique defined in Equation (3.1) will be used in this thesis for determining the critical values for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests instead of the percentile method. Harter [69:1624] notes that as the sample size increases above $n > 20$, the differences in the plotting positions become negligible, but recommends the median plotting position.

$$\frac{(i-0.3)}{(n+0.4)} \tag{3.1}$$

For example, in order to be able to find the 85th percentile, the median rank value that falls just below 0.85 and its associated ith order statistic should be identified. As a result of this, the median plotting position corresponding to the $(i+1)^{st}$ order statistic will be greater than or equal to 0.85. Therefore, by the help of the linear interpolation between the ith and $(i+1)^{st}$ ordered sample values, the critical value at the $\alpha = 0.85$ level can be found. Therefore, given that y_i is the median rank position for X_i in the sample and y_{i+1} is that for X_{i+1}, then the slope of the line joining the two points is given by [26:3-6]

$$m = \frac{y_{i+1} - y_i}{X_{i+1} - X_i}$$

and the intercept can be formulated by

$$b = y_i - mX_i$$

Thus, in order to find the critical value x for the $100(1\text{-}\alpha)\%$ level, we use the equation $y = mx + b$ and let $y = (1\text{-}\alpha)$ and solve for x. Thus,

$$\text{critical value } = \frac{(1-\alpha)-b}{m}$$

3.2.2 The Monte Carlo Procedure and its Implementation in MATLAB

In order to determine the critical values for the individual $\sqrt{b_1}$ and the Q-Statistic G.O.F. tests, the following Monte Carlo method steps are employed. This 8-step procedure, which can also be seen as a flowchart in Figure 3.1, is modified from the one used in Clough's research to fulfill our research goals [26:3-7].

1. Generate a sample size n from the Weibull (β, θ, δ) distribution where $\delta = 0$ and $\theta = 1$, for each value of the Weibull shape parameter $\beta = 0.5$ (0.5) 4, and for sample sizes $n = 5$ (5) 50. The values for the location and scale parameters are chosen for convenience since the $\sqrt{b_1}$ and Q-Statistic test statistic are location and scale invariant [146:279]. Therefore, the critical values generated from this particular distribution will apply for all values of δ and θ.
2. Calculate the $\sqrt{b_1}$ and Q-Statistic test statistic values for the given sample.
3. In order to generate $N = 100{,}000$ for each of the $\sqrt{b_1}$ and Q-Statistic test statistics, repeat steps (1) and (2) 100,000 times.
4. Order the $\sqrt{b_1}$ and Q-Statistic test statistics values in individual vectors.
5. By using the median rank formula that is defined in equation (3.1), calculate the median rank plotting position for each ordered value.
6. Use linear interpolation to find the corresponding critical values for both component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at lower tail significance levels $\alpha = 0.005(0.005)0.10$ and $0.10(0.01)0.20$ and at upper tail significance levels $\alpha = 0.80(0.01)0.90$ and $0.90(0.005)0.995$. Critical values for both tails of sampling distributions will be needed since each test will be a two-sided test. The results of the Monte Carlo simulation will be reported as a table of the critical values at the levels of the sample size and the Weibull distribution shape parameter values. Although, this study seems to deal with the significance levels in too much detail, the importance of this detail will be realized in the following pages.
7. Repeat the process for $n = 5(5)50$.

8. Repeat the process for $\beta = 0.5(0.5)4$.

The Monte Carlo procedure illustrated in Figure 3.1 was implemented in the MATLAB programming language. Since this study can be considered to be a follow-on study for Clough's [26] thesis study, the implementation of this routine was done with the modifications on his critical value derivation coding in MATLAB that provides concise and powerful coding tools for computationally intensive Monte Carlo routines such as this routine.

The *weibrnd* function in the MATLAB Statistics Toolbox was used in order to generate the Weibull random variates. The *weibrnd* function basically uses the inverse transformation to generate its so-called pseudo-random variates. Banks and Carson [3:327] define the following equation as an inverse transformation for the two-parameter Weibull distribution ($\delta = 0$) as follows:

$$X = \theta[-\ln(1-R)]^{\frac{1}{\beta}}$$

where R is a uniform random variable on (0,1), β is the shape parameter value, and θ is the scale parameter. MATLAB uses exactly the same formula; therefore no transformation was needed in the algorithm. In order to represent each sample size of n as an $1 \times n$ vector and handle the samples and the corresponding test statistics in this form, MATLAB's powerful matrix handling were utilized throughout the algorithm for generating the critical values for the test statistics, $\sqrt{b_1}$ and Q-Statistic. The computer coding for generating the critical values in MATLAB can be seen in Appendix J-2 and the resulting critical values and the standard deviations for $\sqrt{b_1}$ and Q-Statistic with the specified shape parameter values and sample sizes can be viewed in Appendix B.

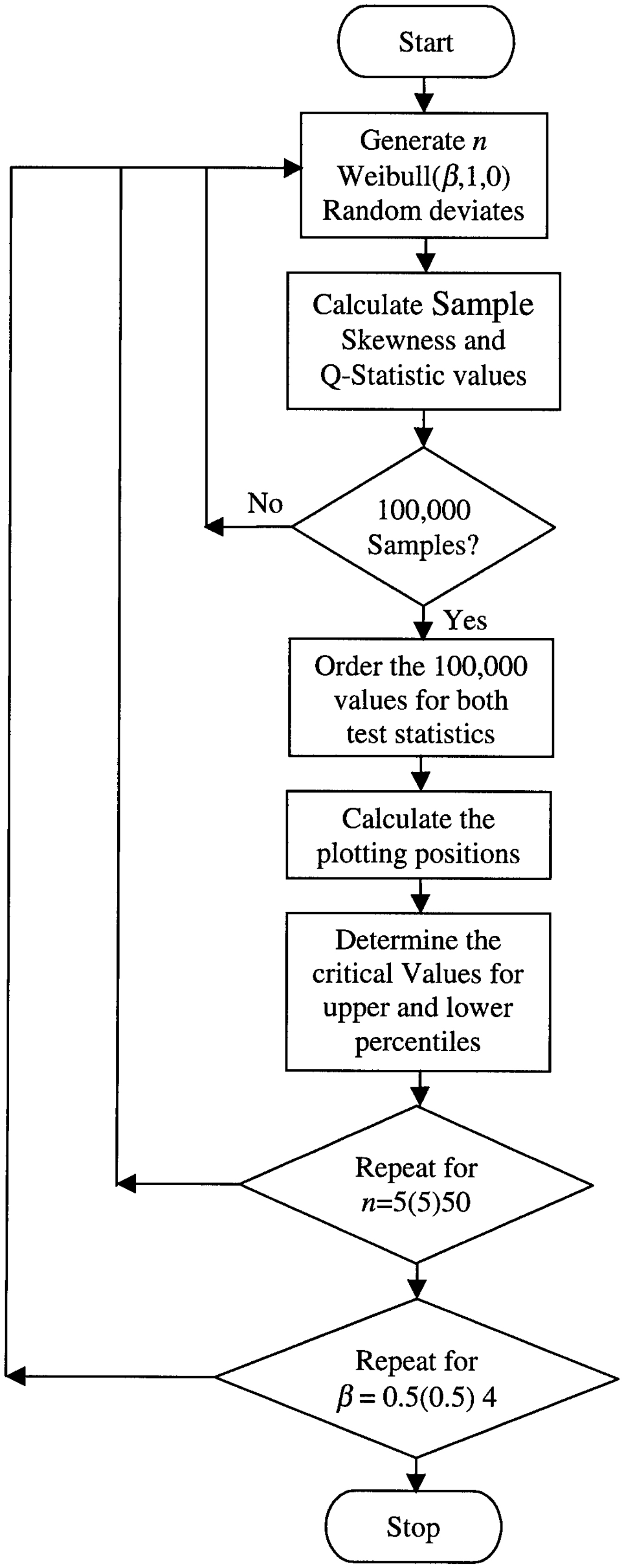


Figure 3.1 Flowchart for the Critical Value Derivation.

3.3 Formal Statements of Component $\sqrt{b_1}$ and Q-Statistic and the Sequential G.O.F. Tests

The individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests that make up the sequential G.O.F. test and the new sequential G.O.F. test can be formally presented, since the critical values for these tests have been generated by use of the Monte Carlo simulation.

3.3.1 $\sqrt{b_1}$ G.O.F. Test

Given a random sample $X_1, X_2, \ldots, X_n$, and hypothesized Weibull distribution with a shape parameter value β, the sample $\sqrt{b_1}$ G.O.F. test can be formally stated as follows:

Hypotheses:

$H_o = X \sim$ Weibull (β). The random sample of n X-values follows a Weibull distribution with known shape parameter β, versus

$H_a = X \nsim$ Weibull (β). The random sample of n X-values does not follow a Weibull distribution with known shape parameter β.

Skewness Test Statistic:

$$\sqrt{b_1} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^3}{\left[\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2\right]^{\frac{3}{2}}} \tag{3.2}$$

Rejection Region:

Due to the fact that a two-tailed skewness G.O.F. test is employed, for a significance level α, the rejection region is defined by

$$\sqrt{b_1} > \sqrt{b_1}_{\left(1-\frac{\alpha_1}{2}\right)} \qquad or \qquad \sqrt{b_1} < \sqrt{b_1}_{\left(\frac{\alpha_1}{2}\right)}$$

upper tail *lower tail*

where the values of the lower and upper tail $\sqrt{b_1}$ and Q-Statistic G.O.F. tests critical values and the standard deviations can be located in the tables in Appendix B for each sample size n and the Weibull distribution shape parameter β. It is important to place an emphasis on the fact that the two-sided nature of the G.O.F. test procedure requires the critical values in increments of $\frac{\alpha}{2}$; the need for increments of 0.005 in the α-levels of the previous simulation.

3.3.2 Q-Statistic G.O.F. Test

Given a random sample $X_1, X_2, \ldots, X_n$ and hypothesized Weibull distribution with a shape parameter value β, the sample Q-Statistic G.O.F. test can be formally stated as follows:

Hypotheses:

$H_o = X \sim$ Weibull (β). The random sample of n X-values follows a Weibull distribution with known shape parameter β, versus

H_a = X $\nsim$ Weibull (β). The random sample of n X-values does not follow a Weibull distribution with known shape parameter β.

Q-Statistic Test Statistic:

$$Q = \frac{U(\alpha) - L(\alpha)}{U(\beta) - L(\beta)} \tag{3.3}$$

where $U(\beta)$ is the average of the largest $n\beta$ order statistics of the sample size n, and $L(\beta)$ is the average of the smallest $n\beta$ order statistics. $U(\alpha)$ and $L(\alpha)$ are the average of the largest and smallest $n\alpha$ order statistics [78:424]. See Chapter 2, for further explanation of this test statistic.

Rejection Region:

Due to the fact that a two-tailed Q-Statistic G.O.F. test is employed, for a significance level α, the rejection region is defined by

$$\underset{upper \quad tail}{Q > Q_{\left(1-\frac{\alpha_2}{2}\right)}} \qquad or \qquad \underset{lower \quad tail}{Q < Q_{\left(\frac{\alpha_2}{2}\right)}}$$

where the values of the lower and upper tail critical values and the standard deviations can be located in the tables that are provided in Appendix B for each sample size n and the Weibull distribution shape parameter β. As in $\sqrt{b_1}$ G.O.F. test, it is important to place an emphasis on the fact that the two-sided nature of the test procedure requires the critical values in increments of $\frac{\alpha}{2}$; the need for increments of 0.005 in the α-levels of the previous simulation.

3.3.3 Proposed Sequential G.O.F. Test

The proposed sequential G.O.F. test studied in this thesis consists of employing the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in sequence in any order. If the sample data passes both $\sqrt{b_1}$ and Q statistic G.O.F tests at their individual significance levels, the sample data passes the sequential G.O.F. test.

Given a random sample $X_1, X_2, \ldots, X_n$, and hypothesized Weibull distribution with a shape parameter of β, the formal proposed sequential G.O.F. test can be stated as follows:

<u>*Hypotheses:*</u>

$H_o = X \sim$ Weibull (β). The random sample of n X-values follows a Weibull distribution with known shape parameter β, versus

$H_a = X \not\sim \text{Weibull}(\beta)$. The random sample of n X-values does not follow a Weibull distribution with known shape parameter β.

The Test Statistic:

The $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics are defined in equations (3.2) and (3.3).

Rejection Region:

For an overall significance level α, the rejection region can be presented by

$$\underbrace{\sqrt{b_1} > \sqrt{b_1}_{\left(1-\frac{\alpha_1}{2}\right)}}_{upper\ tail} \qquad or \qquad \underbrace{\sqrt{b_1} < \sqrt{b_1}_{\left(\frac{\alpha_1}{2}\right)}}_{lower\ tail}$$

OR

$$\underbrace{Q > Q_{\left(1-\frac{\alpha_2}{2}\right)}}_{upper\ tail} \qquad or \qquad \underbrace{Q < Q_{\left(\frac{\alpha_2}{2}\right)}}_{lower\ tail}$$

where α_1 and α_2 are the selected significance levels of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. The selection of the α_1 and α_2 is up to the analyst among several combinations producing the desired overall α. Here arises a legitimate question about the selection of α_1 and α_2. The relationship between them and the selection's effect on the α will be explained in the next chapter.

3.4 Attained Significance Levels

3.4.1 Background

In order to proceed with the sequential G.O.F. test procedure, we need to identify the overall significance levels of the sequential G.O.F. test procedure, given α_1 and α_2 levels. As mentioned before, Bonferroni's inequality, $\alpha \leq \alpha_1 + \alpha_2$, can be interpreted as a lower bound for the overall significance level of the sequential G.O.F. test. On the other hand, Bonferroni's inequality does not give us an exact value of the overall significance level. Besides, various combinations of α_1 and α_2 may be utilized to obtain very similar values for α, letting the analyst opt for particular α_1 and α_2 levels based on the alternate distributions considered. Considering characteristics of the sample it may be possible to choose α_1 and α_2 to maximize the power. For example, being mainly concerned with greater power in distinguishing among the skewed alternatives, the analyst could choose to use a higher α_1 and find the appropriate α_2 to achieve the desired overall significance level. Due to the fact that increasing Type I error α increases the power, this choice would result in a higher power for the $\sqrt{b_1}$ G.O.F. test because of the larger rejection region for the $\sqrt{b_1}$ G.O.F. test and the same overall significance level with a higher overall power for the sequential G.O.F. test. On the other hand, if accuracy in the tails was a key concern, the analyst could choose to use a higher α_2 in the Q-Statistic G.O.F. test find the appropriate α_1 for the $\sqrt{b_1}$ G.O.F. test to achieve the desired overall significance level.

In light of the discussion above, the empirical determination of these attained significance levels for the sequential G.O.F. test will be determined via Monte Carlo simulation. Onen [117], Gunes [64] and Clough [26] have successfully used this approach. With modifications to fit the goal of this research, the method that was devised by Clough [26] that is a variation on Onen's[117] and Gunes's [64] procedures will be used in this thesis. This approach is initially similar to the one used in the determination of critical values. On the other hand, this procedure will actually conduct the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests using the critical values found in the previous simulations. As done before, a large number of Weibull samples are going to be generated, and the $\sqrt{b_1}$ and Q-Statistic test statistics for each calculated. Each sample then, will be subjected to the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in sequence at all possible combinations of α_1 and α_2 levels. It is important to keep in mind that the given sample fails the sequential G.O.F. test if it fails one or both of the component G.O.F. tests. The analyst can evaluate the attained significance levels for the sequential G.O.F. test by counting the number of samples that fail at each combination of individual significance levels. Due to the fact that α is simply the probability of rejecting a true H_o, and the samples generated are indeed Weibull samples, α can be found by calculating the percentage of all samples that are rejected at a given combination of α_1 and α_2 levels. The attained significance levels will have to be established for each sample size and shape parameter value combination, since we are concerned with cases of known shape parameter value and various sample sizes.

Clough [26] mentions the historical development of the attained significance level coding he used in his study. The computer algorithm that was developed and used by Onen [117] and then used by Gunes [64] is made up of counting the number of the samples that passed both component G.O.F. tests and storing the results in an array structure indexed by the individual significance levels. For example, the element A_{ij} held the count of the samples that pass both tests at level $\alpha_1 = \frac{i}{100}$ for the first test and $\alpha_2 = \frac{j}{100}$ for the second test. Onen [117] subjected each of the 50,000 samples to all levels of each test from $\alpha = 0.01(0.01)0.20$, instead of generating new samples for each increment in the α values. Therefore, each sample was tested at 20 significance levels for each of the two component G.O.F. tests and therefore, a total number of 40 times. Clough [26: 3-12,13] notes that the flowchart in Onen's text seems to indicate that Onen actually tested each sample at every possible combination of significance levels for the two component G.O.F. tests, meaning 400 tests on each given sample. However, with a closer look at Onen's code, we can see that he actually conducted 40 tests on each sample, as noted above, as opposed to 400 tests that is defined in Onen's flowchart. After all 50,000 samples were tested, the array structure contained the number of samples that passed the sequential G.O.F. test at each of the 400 combinations of significance levels for the two component G.O.F. tests. As a summary, Onen [117] found the proportion of samples that incorrectly failed each component G.O.F. test by dividing each element in the array by 50,000 and then taking the complement of these values.

A modified version of the method that was implemented by Clough [26] will be used in this thesis. In this implementation, the counter array A tracks the number of the samples failing the sequential G.O.F. test (failing one or both of the component $\sqrt{b_1}$ and/or Q-Statistic G.O.F. test(s)) and store the results in an array structure indexed by the individual significance levels. This approach is more direct and simplifies the coding logic eliminating the pitfall that Onen's algorithm had. Besides, the number of tests each sample is subjected to was reduced to a great extent by applying some simple results from hypothesis testing with this approach.

If a given sample fails any G.O.F. test at a significance level α, say $\alpha = 0.05$, then it will, of course, fail the G.O.F. test at any higher significance level, such as $\alpha = 0.07$, because increasing α causes the rejection region to get enlarged for the G.O.F. test. On the other hand, the given sample would be guaranteed to pass any hypothesis test at any significance level $\alpha < 0.05$ for this particular example. This logic is the basis of this study here. If a G.O.F. test presented here is conducted starting with the lowest significance level and working toward higher significance levels (smaller to larger α-levels), then as soon as a sample fails at a particular significance level, it follows that it also fails at every subsequent level. As a result, that G.O.F. test does not actually have to be conducted any further. Also, once a failure is determined for one G.O.F. test, the analyst may conclude that the given sample will fail for all remaining larger α levels for the given G.O.F. test and any α levels for the second G.O.F. test. Based on the discussion above, the implementation of this logic is simplified and not as CPU intensive as that of Onen's algorithm, and therefore it is computationally effective.

There are four possible outcomes of the application of the sequential G.O.F. test at all combinations of significance levels for the two component G.O.F. tests. Tables 3.1 to 3.4 illustrate the possible results of the two G.O.F. tests over a range of α levels between 0.01 and 0.10 help us see these four potential outcomes visually. These Pass – Fail tables actually reflect the results for a single sample in the counter array *A* mentioned above. An "F" indicates a failure of the sequential G.O.F. test and an "A" indicates passing the sequential G.O.F. test at the given combination of the significance levels α_1 and α_2. The four possible outcomes are explained and illustrated as follows [26: 3-14]:

1. The sample could fail both component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at some α_1 and α_2 levels. For example, let's consider a given sample that fails G.O.F. Test # 1 at $\alpha_1 = 0.05$ and fails G.O.F. Test # 2 at $\alpha_2 = 0.08$. The sample fails the sequential G.O.F. test for all combinations where $\alpha_1 \geq 0.05$ or $\alpha_2 \geq 0.08$. See Table 3.1.

2.) The sample could fail G.O.F. Test #1 at some significance level, say $\alpha_1 = 0.05$ and pass G.O.F. Test # 2 at all levels. In this case, the sample fails the sequential G.O.F. test at the significance levels for $\alpha_1 \geq 0.05$ across all significance levels of α_2. See Table 3.2.

3.) The sample could pass G.O.F. Test #1 at all significance levels and fail G.O.F. Test # 2 at some significance level, say $\alpha_2 = 0.08$. In this case, the sample fails the sequential G.O.F. test for all combinations where $\alpha_2 \geq 0.08$ across all significance levels of α_1. See Table 3.3.

Table 3.1 Pass – Fail Table: Fail Both G.O.F. tests: G.O.F. Test # 1 at α_1 = 0.05 and G.O.F. Test # 2 at α_1 = 0.08.

G.O.F TEST # 1	G.O.F. TEST # 2 α_2 – LEVEL									
α_1 – LEVEL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	A	A	A	A	A	A	A	F	F	F
0.02	A	A	A	A	A	A	A	F	F	F
0.03	A	A	A	A	A	A	A	F	F	F
0.04	A	A	A	A	A	A	A	F	F	F
0.05	F	F	F	F	F	F	F	F	F	F
0.06	F	F	F	F	F	F	F	F	F	F
0.07	F	F	F	F	F	F	F	F	F	F
0.08	F	F	F	F	F	F	F	F	F	F
0.09	F	F	F	F	F	F	F	F	F	F
0.10	F	F	F	F	F	F	F	F	F	F

Table 3.2 Pass – Fail Table: Fail G.O.F. Test # 1 Only at α_1 = 0.05.

G.O.F TEST # 1	G.O.F. TEST # 2 α_2 – LEVEL									
α_1 – LEVEL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	A	A	A	A	A	A	A	A	A	A
0.02	A	A	A	A	A	A	A	A	A	A
0.03	A	A	A	A	A	A	A	A	A	A
0.04	A	A	A	A	A	A	A	A	A	A
0.05	F	F	F	F	F	F	F	F	F	F
0.06	F	F	F	F	F	F	F	F	F	F
0.07	F	F	F	F	F	F	F	F	F	F
0.08	F	F	F	F	F	F	F	F	F	F
0.09	F	F	F	F	F	F	F	F	F	F
0.10	F	F	F	F	F	F	F	F	F	F

4.) The sample could pass both component G.O.F. tests at all α-levels. Thus, no elements in the counter array A would be incremented. See Table 3.4.

Table 3.3 Pass – Fail Table: Fail G.O.F. Test # 2 Only at α_2 = 0.08.

G.O.F TEST # 1	G.O.F. TEST # 2 α_2-LEVEL									
α_1- LEVEL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	A	A	A	A	A	A	A	F	F	F
0.02	A	A	A	A	A	A	A	F	F	F
0.03	A	A	A	A	A	A	A	F	F	F
0.04	A	A	A	A	A	A	A	F	F	F
0.05	A	A	A	A	A	A	A	F	F	F
0.06	A	A	A	A	A	A	A	F	F	F
0.07	A	A	A	A	A	A	A	F	F	F
0.08	A	A	A	A	A	A	A	F	F	F
0.09	A	A	A	A	A	A	A	F	F	F
0.10	A	A	A	A	A	A	A	F	F	F

Table 3.4 Pass – Fail Table: Pass Both G.O.F. Tests at all α_1 and α_2 values.

G.O.F TEST # 1	G.O.F. TEST # 2 α_2-LEVEL									
α_1- LEVEL	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.01	A	A	A	A	A	A	A	A	A	A
0.02	A	A	A	A	A	A	A	A	A	A
0.03	A	A	A	A	A	A	A	A	A	A
0.04	A	A	A	A	A	A	A	A	A	A
0.05	A	A	A	A	A	A	A	A	A	A
0.06	A	A	A	A	A	A	A	A	A	A
0.07	A	A	A	A	A	A	A	A	A	A
0.08	A	A	A	A	A	A	A	A	A	A
0.09	A	A	A	A	A	A	A	A	A	A
0.10	A	A	A	A	A	A	A	A	A	A

3.4.2 Algorithmic Implementation

In the light of the discussion above, pass/fail for each combination of the $\sqrt{b_1}$ and the Q-Statistic G.O.F. tests can be determined without actually running them all. As mentioned before, Clough's algorithmic implementation with the necessary modifications is used to fit this research's purposes [26:3-15-17]. Clough [26:3-15-17] states that this procedure can be visualized as testing "down" column 1 with every G.O.F. Test # 1 and "across" row 1 with G.O.F. Test # 2 without considering the other G.O.F. test's significance level. The analyst not having to conduct each sequential G.O.F. test can fill the squares for failures for the other combinations as noted above. As a result, only 20 tests would be needed, vice 100, for α-levels examined between 0.01 and 0.10 in the examples presented above.

On the other hand, Onen's method basically counted passes instead of the failures and conducted each G.O.F. test at all 20 α levels regardless of whether a failure point was reached or not. The fact that the flag variable that he used to identify the α-level where the G.O.F. test initially failed was not reset between the sample sizes indicates that Onen has a flaw in his coding and creates a very big problem if the given sample fails at the lowest significance level, $\alpha = 0.01$, where the flag variable is not assigned and keeps its value from the previous sample tested. Therefore, the new sample is mistakenly counted as passing at the same significance levels as the previous one that results in the counter array being incremented for numerous combinations of significance levels, which the sample actually failed to pass. The outcome is an underestimation of the attained significance levels and of the power for the G.O.F. test when Onen used similar code for his power study. It is important to note that this problem only arises if the new sample fails at the lowest significance level, $\alpha = 0.01$, not in the cases it passes. The algorithm that was used in this thesis has rectified this potential flaw in Onen's code. Besides, the algorithm used in this thesis is different than that of Onen's in the sense that both $\sqrt{b_1}$

and the Q-Statistic G.O.F. tests in this new sequential G.O.F. test are two-sided, thereby requiring the aforementioned fine detail in the levels of the critical values [26:3-17]. The results of the attained significance level calculations are presented in Appendix C and discussed in detail in the next chapter.

The method to derive the attained significance levels that is illustrated by the flowchart in Figure 3.2 can be summarized as follows [26:3-16-17]. As before, due to the fact that $\sqrt{b_1}$ and the Q-Statistic G.O.F. tests can be conducted in any order, they are generally referred as G.O.F. Test # 1 and G.O.F. Test # 2.

1. As before, begin by selecting a shape parameter β and sample size n for the Weibull samples according to the specific null hypothesis being investigated. Generate a sample size n from the Weibull (β, θ, δ) distribution where $\delta = 0$ and $\theta = 1$, for each value of the Weibull shape parameter $\beta = 0.5\ (0.5)\ 4$, and for sample sizes $n = 5\ (5)\ 50$. The values for the location and scale parameters are chosen for convenience since the component $\sqrt{b_1}$ and Q-Statistic test statistics are location and scale invariant [146:279]. Therefore, the attained significance values generated from this particular distribution will apply for all values of δ and θ.
2. Calculate the sample $\sqrt{b_1}$ and Q-Statistic test statistic values for the given sample.
3. Initialize counters to track the current levels of the two component $\sqrt{b_1}$ and the Q-Statistic G.O.F. tests; i_{curr} for Test # 1 and j_{curr} for Test # 2. The individual significance levels are $\alpha_1 = \frac{i_{curr}}{100}$ and $\alpha_2 = \frac{j_{curr}}{100}$. Start with both i_{curr} and $j_{curr} = 1$.

Another pair of indices will indicate the level of the first failure for each G.O.F. test; i_{stop} for Test # 1 and j_{stop} for Test # 2.

4. Conduct the first G.O.F. test (Test # 1) on the sample for $\alpha_1 = \frac{i_{curr}}{100}$. If the sample fails the G.O.F. test at this significance level, record the current level in i_{stop}. Then proceed to Step (6).

5. If the sample passes at the current level, increment i_{curr} by 1. If the range of desired levels has been tested ($i_{curr} > 20$), then leave $i_{stop} = 21$, indicating no failures, and proceed to Step (6). Otherwise, return to Step (4) with the new value for i_{curr}.

6. Conduct the second G.O.F. test (Test # 2) on the sample for $\alpha_2 = \frac{j_{curr}}{100}$. If the sample fails the G.O.F. test at this significance level, record the current level in j_{stop}. Then proceed to Step (8).

7. If the sample passes at the current level, increment j_{curr} by 1. If the range of desired levels has been tested ($j_{curr} > 20$), then leave $j_{stop} = 21$, indicating no failures, and proceed to Step (8). Otherwise, return to Step (4) with the new value for j_{curr}.

8. Now that the failure points have been determined, increment the appropriate counters in the array A. Specifically, increment A_{ij} for all (i,j) such that $i \geq i_{stop}$ and $j \geq j_{stop}$, avoiding duplication in the intersection of the two sets.

9. Repeat Steps (1) through (8) for 100,000 samples.

10. When finished, the array element A_{ij} will hold the counts for the number of failures (the rejection of the true null hypothesis) for the corresponding combinations of the significance levels $\alpha_1 = \frac{i}{100}$ and $\alpha_2 = \frac{j}{100}$. To find the attained significance level for

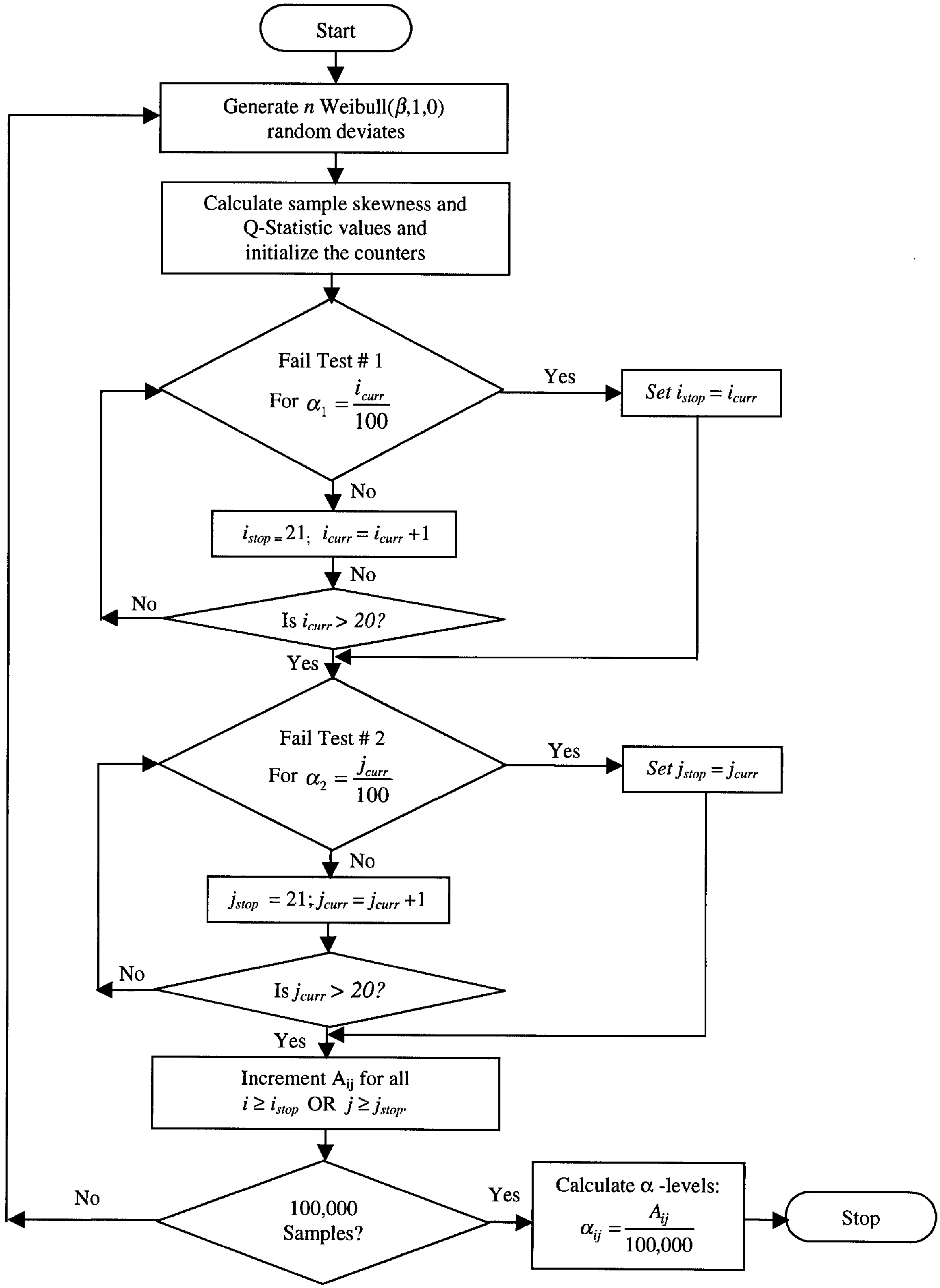


Figure 3.2 Flowchart for the Attained Significance Levels Derivation.

a given combination, α_{ij}, simply divide A_{ij} by the total number of samples, 100,000.

3.5 Power Study

In order to measure the effectiveness and utility of the new sequential G.O.F. test compared to that of other existing G.O.F. tests, there exists a necessity for conducting an extensive power study against a broad set of alternate distributions, because the power of a G.O.F. test indicates how useful and helpful it is compared to other existing G.O.F. tests. The power depends on the significance level of the G.O.F. test, the components of the calculation of the G.O.F. test statistic, and on the specific alternative hypothesis under consideration [125]. Recall that the power of the new sequential G.O.F. test developed in this thesis concerns the hypotheses

$H_o = X \sim$ Weibull (β). The random sample of n X-values follows a Weibull distribution with known shape parameter β, versus

$H_a = X \nsim$ Weibull (β). The random sample of n X-values does not follow Weibull distribution with known shape parameter β.

For a good omnibus G.O.F. test, we seek power values as close to 1.0 for any alternate distribution that is not the Weibull distribution specified in the null hypothesis. After determining the attained significance levels of the new sequential G.O.F. test, the power studies could be conducted by evaluating its power against particular alternate distributions to be able to determine if the new sequential G.O.F. test actually performs equally or better than the existing G.O.F. procedures. This research effort makes use of the widely used EDF based K-S, Cv-M, and A-D G.O.F. test power results by Bush [17] and Wozniak [168], and the unique sequential G.O.F. test example for the hypothesized Weibull distribution with known shape parameter value that uses $\sqrt{b_1}$ and b_2 in sequence by Clough [26] for this comparative power study. Based on the discussion in Chapter 2, the A-D G.O.F. test and Cv-M G.O.F. tests appear to be very powerful. On the other hand, even though the K-S G.O.F. test appears to be less powerful than these two G.O.F. tests, it is very-well documented and very widely used in the field of G.O.F. testing. Furthermore, the power results of this research will be compared to those of Clough's to see if any improvement in the power was achieved by replacing the b_2 G.O.F. test with Q-Statistic G.O.F. test. It is expected that the new sequential G.O.F. test will demonstrate roughly equivalent or better power than those of the aforementioned existing G.O.F. tests across a wide range of alternate distributions that will be discussed later so that there will be a noteworthy contribution to the field of G.O.F. testing. Its best contribution could be in the sense that this new sequential G.O.F. test procedure does not require any complex parameter estimation routines. Therefore, if the new G.O.F. test is proven to have the same or better power than the existing G.O.F. test, a computationally efficient means of conducting G.O.F. testing for the three-parameter Weibull distribution with the known shape parameter value will be introduced for use.

The alternate distributions and the sample sizes covered in this research match with those of these earlier studies in order to have a common ground to compare the power study results. This research also goes beyond these alternate distributions to be able to present some additional power results against some other selected alternate distributions for making a future reference for follow-on studies. These will be discussed in detail later in this chapter.

It is important to note that due to the fact that Wozniak [168] conducted her research for the two-parameter Weibull distribution using an extreme-value G.O.F. test, it is obvious that her power study results are not directly comparable to the those of the ones found in this new sequential G.O.F. test study. On the other hand, due to the fact that the alternate distributions that she used in her research often appear in the statistics literature and are considered to be benchmark alternate distributions in G.O.F. test studies for the Weibull distribution, the power study results that she got in her research do provide a general context for evaluating the power study results that are obtained via this new sequential G.O.F. test. Following a systematic approach, this thesis evaluated the two component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests individually in addition to the new sequential G.O.F. test. This part of the thesis resulted in very useful and helpful information in the sense that examining the results together show us the existence of clues indicating the better of the two G.O.F. tests against particular alternate distributions and insights into the complementary role they play as partners in the omnibus sequential G.O.F. test. This thesis also took the directional versions of the separate $\sqrt{b_1}$ and Q-Statistic G.O.F. tests into consideration by conducting them in a one-sided manner. Presuming that there is enough a priori information to justify a one-sided G.O.F. test, it ought to improve the power of the corresponding G.O.F. test, because of the well-known fact that a one-sided G.O.F. test enlarges the rejection region in the tail where a potential discrepancy is expected. If this is the case, there exists an opportunity of using either one or both directional G.O.F. tests in a sequential fashion that could be a very good and effective means of improving its power against the alternate distributions studied here. For example, if the analyst believes that the given sample data cannot be represented with

Weibull distribution with $\beta = 1$, that it will be more highly skewed (higher $\sqrt{\beta_1}$ value), he or she could choose to utilize the directional upper tail $\sqrt{b_1}$ G.O.F. test in the sequential G.O.F. test and anticipate higher discriminatory ability. Extensive directional power studies were conducted to quantify the degree of improvement from using one-sided $\sqrt{b_1}$ and Q-Statistic G.O.F. tests individually and in the sequential G.O.F. test. Stephens and D'Agostino [146] discuss the improved power of such directional variants of the $\sqrt{b_1}$ and b_2 G.O.F. tests [146:403-404].

3.5.1 Monte Carlo Procedure

The Monte Carlo algorithm used in determining the attained significance levels was slightly modified to accomplish the implementation of the power study. The Monte Carlo procedure used to determine the attained significance levels estimated the proportion of times the new sequential G.O.F. test procedure incorrectly rejected a null hypothesis by generating Weibull samples and comparing the component G.O.F. test statistics values to the corresponding critical values for that hypothesized Weibull distribution with the specified shape parameter value. However, the purpose of the power study is to measure the proportion of times the new sequential G.O.F. test correctly rejects samples from distributions other than that in the null hypothesis. Therefore, the data generated in the power study Monte Carlo simulation will come from specific alternate distributions instead of generating Weibull samples with a shape parameter value specified in that null hypothesis. The sample $\sqrt{b_1}$ and Q-Statistic values are compared to the critical values for the null Weibull hypothesis as in the attained significance level calculations. Several combinations $\sqrt{b_1}$ and Q-Statistic G.O.F. tests can be evaluated at various combinations of the individual significance levels for a desired significance level of the overall G.O.F. test. It is crystal clear that by changing the generator for the samples being tested, the algorithm used in the attained significance level study can be used in the power study conducted in this thesis.

Hence, in order to determine the power of the new sequential G.O.F. test procedure in discriminating between a given sample of size n from a hypothesized Weibull distribution with shape parameter value β and that of some specified alternate distribution, repeat the steps for the attained significance level algorithm with the exception of step (1), which is replaced by [26:3-20,21]:

1. Generate n random deviates from the alternate distribution.

An array of powers for each of the possible combinations of each of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at levels $\alpha = 0.01(0.01)0.20$ will be the resulting product of the procedure described above.

Two other modifications to the algorithm were done to cut down on the run time of the simulation from in excess of 15 hours to 0.5-3 hours. First, the only sample sizes studied in this power study will be the ones that were studied by Bush [17] or Wozniak [168] with the addition of $n = 50$ that Clough [26] studied in his research for the particular alternate distributions considered instead of calculating power results for all sample sizes $n = 5(5)50$ as before. Second, the power study does not necessitate 100,000 samples to be generated, due to the fact that the accuracy in the power estimates is not as crucial as for the critical values and the attained significance levels. Consequently, only 40,000 samples were generated in the power study in order to be able to make the power comparison with Clough [26] as accurate as possible, because he also used $N = 40{,}000$ in his power study.

It is crucial that reducing the sample size of the Monte Carlo simulation runs be justified. Due to the fact that the objective of the power study is to generate points of comparison with other published G.O.F. tests, the accuracy in the estimates is not as critical. Correct estimates in just first two decimal places are considered sufficient; therefore, as long as they fall within ∓ 0.005 of the true values, they are considered valid. Power is fundamentally a proportion, and estimation theory informs us that the standard error of an estimate of proportion, $\hat{p}$ is: [109:326] [26:3-21]

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{N}} \tag{3.4}$$

A rough 95% confidence interval for p can be formed by $\hat{p} \mp 2\sigma_{\hat{p}}$, according to the empirical rule [109:9]. Thus, the objective is then to ensure N is large enough so that $2\sigma_{\hat{p}} \leq 0.005$. When $\hat{p} = \frac{1}{2}$, $\sigma_{\hat{p}}$ is maximized. By substituting in the expression for $\sigma_{\hat{p}}$ and solving for N, one finds that $N \geq 40{,}0000$ will keep the maximum half-width of the confidence interval within the criteria specified. Consequently, this rationale led to the use of 40,000 samples vice 100,000 in the power study.

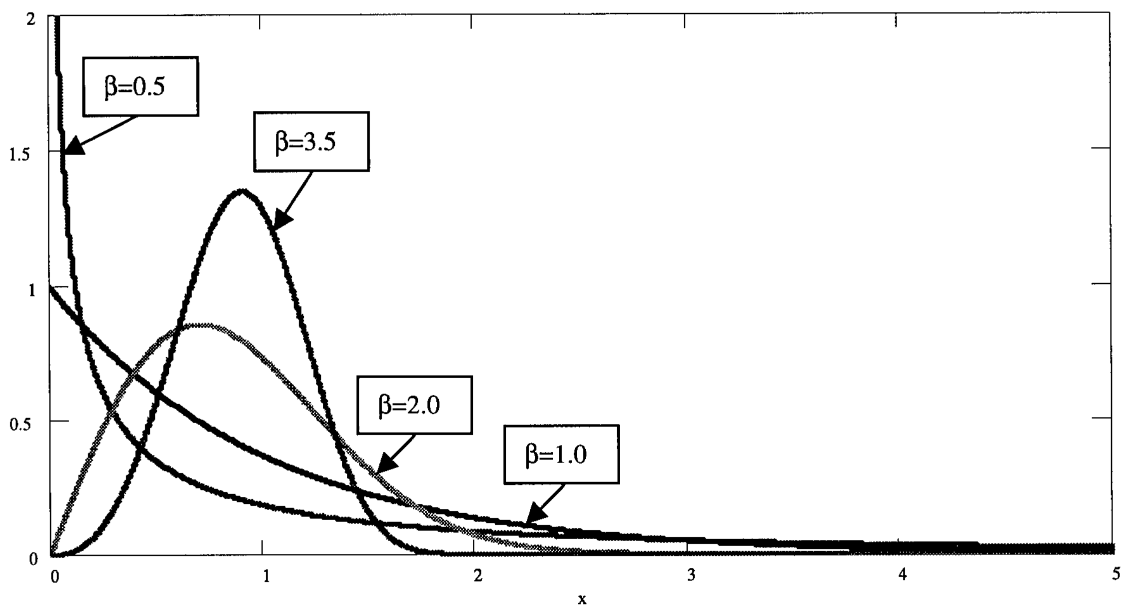
3.5.2 Alternate Distributions

A considerable number of alternate distributions were evaluated in the power study conducted in this research. In order to be able to compare the power study results of the new sequential G.O.F. test to the EDF studies of Bush [17] and Wozniak [168], those alternate distributions were used here. Since Clough [26] did not use any other alternate distributions in his power study than Bush's and Wozniak's, no other alternate distributions were needed to allow comparison with Clough's sequential G.O.F. test. On the other hand, there are some alternate distributions studied in this power study that were not used by Bush [17] or Wozniak [168] in their power studies to prepare a reference for the future power comparisons.

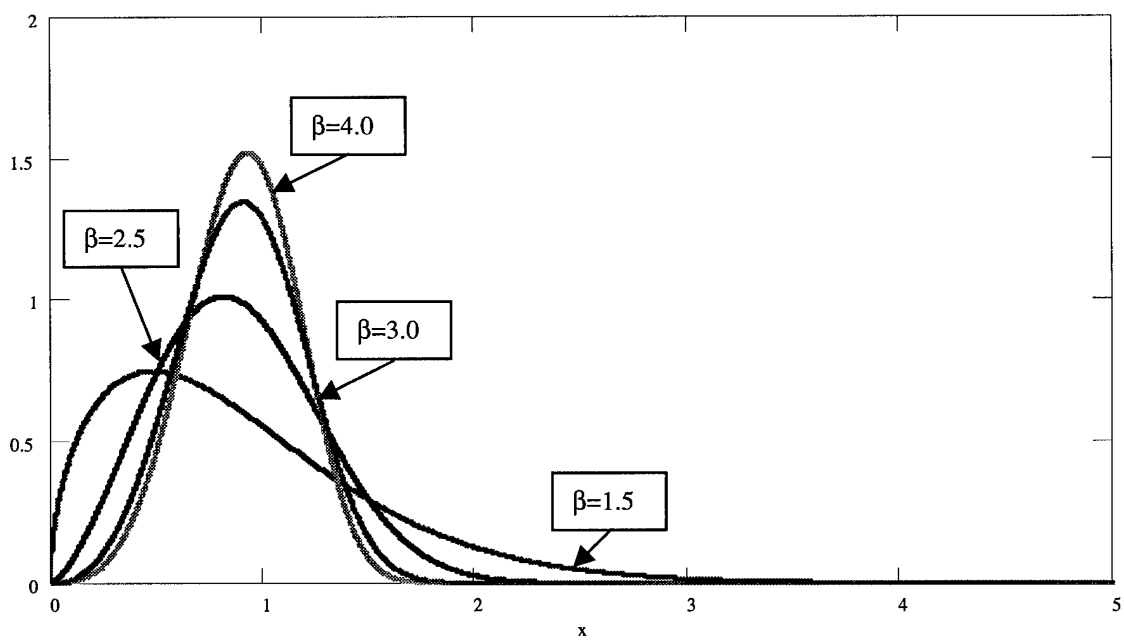
In order to serve as a means to verify the simulation code and validate the critical values, the power study included the studies for H_o = Weibull(β = 0.5(0.5)4.0) against H_a = Weibull(β = 0.5(0.5)4.0). Figure 3.3 (a) and (b) demonstrate the PDFs of the Weibull distribution with the shape parameters considered here. The power results in these cases should be nearly identical (within some small random error, ε) to the attained significance levels calculated in the previous simulation for a given shape and sample size. If so, the critical values, the attained significance levels, and the simulation coding for all three facets of this thesis can be considered verified and validated.

3.5.2.1 The Bush Alternate Distributions

In order to conduct his power study, Bush compared the hypothesized Weibull distributions with shape $\beta = 1$ and 3.5 with sample sizes n = 5, 15, 25 against the alternate



(a) PDF of Weibull $\beta = 0.5, 1.0, 2.0, 3.5$



(b) PDF of Weibull $\beta = 1.5, 2.5, 3.0, 4.0$

Figure 3.3 PDFs of the Weibull Distributions studied for the verification and validation.

distributions in Table 3.5.

Table 3.5 Bush's Alternate Distributions for the Power Study.

Weibull(β=1)	Beta(2,2)
Weibull(β=2)	Beta(2,3)
Weibull(β=3.5)	Uniform(1,2)
Gamma(2,1,0)	Normal(10,1)

The same alternate distributions were used for the power study in this research except for a few minor variations that do not affect the power study results. Due to the fact that the normal and uniform distributions are location and scale-parameter distributions, and the new sequential G.O.F. test is scale and location invariant, the parameters for these distributions are of no importance in terms of their values. Therefore, in order to make this research parallel Clough's work as close as possible and for convenience, the Uniform(0,2) and Normal(0,1) alternate distributions were used instead of the Uniform(1,2) and Normal(10,1) that Bush originally used. The gamma distribution with shape parameter value equal to one is equivalent to the Weibull distribution with shape parameter value equivalent to one, since both distributions reduce to the exponential distribution. Thus, for this particular shape parameter value, the Gamma(1,1,0) distribution was not studied separately in the power study, because the Weibull(1,1,0) distribution is studied here. Besides, the Beta(1,1) distribution that was studied by Bush was not included in this power study, because it was assessed to be an unnecessary alternate distribution via examination of the uniform PDF that can be seen in Figure 3.4 and Clough [26] did not consider this alternative in his power study. Again, in order to be able to compare the power study result of the new sequential G.O.F. test to those of Clough's work, the sample size 50 was studied to quantify the power at large sample sizes, even though this sample size was not studied by Bush.

The PDFs for the non-Weibull alternate distributions that Bush used in his power study are presented below. See Figure 3.3 for the PDFs of the Weibull alternate distributions. The appropriate routines in the MATLAB Statistics Toolbox were used to generate the random variates from each of the alternate distributions that Bush used in his power study. Table 3.6 summarizes the specific built-in MATLAB functions for random number generation and the generation method they employ to generate random variates from the alternate distributions studied here.

Table 3.6 MATLAB Random Variate Generation Methods

Distribution	MATLAB Function	Random Variate Generation Method [26:3-23]
BETA	betarnd	Ratio of gamma deviates
GAMMA	gamrnd	(Depending on the shape parameter, α) Accept-Reject or Inverse Transformation
NORMAL	normrnd	Based on the work of George Marsaglia
UNIFORM	unifrnd	Lagged Fibonacci and shift register generator
WEIBULL	weibrnd	Inverse Transformation

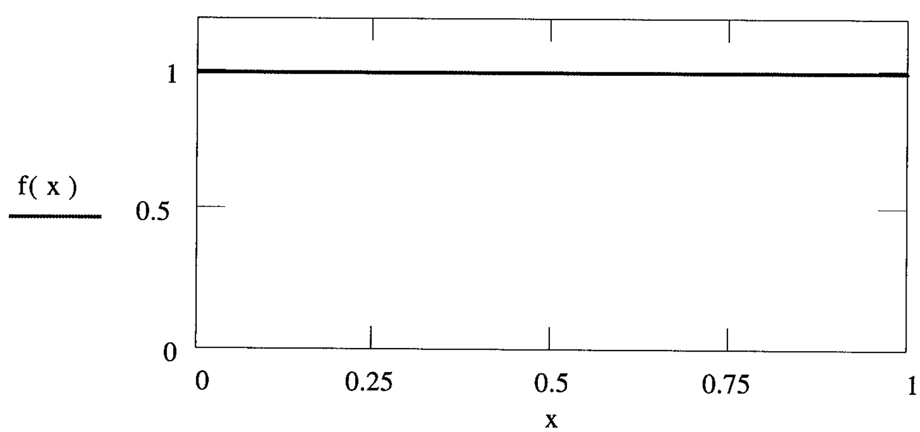


Figure 3.4 Beta (1,1) PDF

1. Beta Distributions:

$$f(x,\alpha,\beta)=\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]x^{\alpha-1}(1-x)^{\beta-1}, \qquad 0<x<1; \qquad \beta>0$$

a.) When $\alpha=2$ and $\beta=2$, Beta distribution has the PDF presented in Figure 3.5.

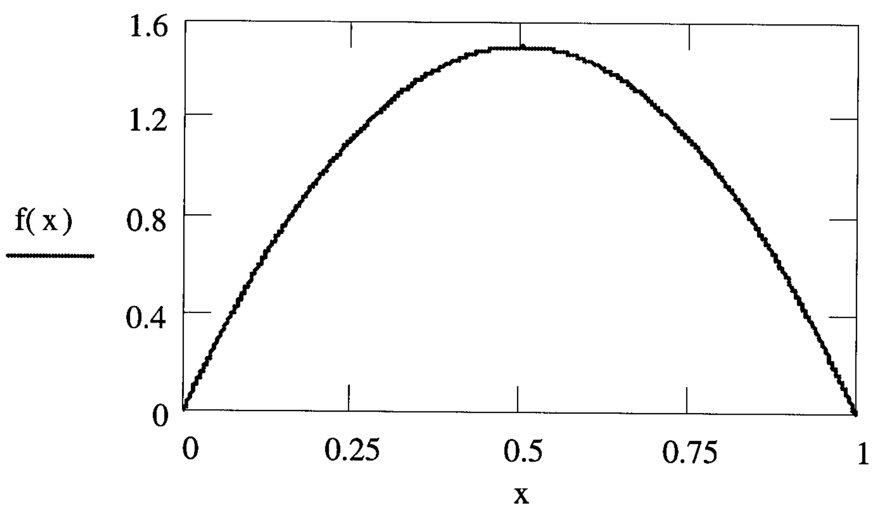


Figure 3.5 Beta (2,2) PDF

b.) When $\alpha=2$ and $\beta=3$, Beta distribution has the PDF presented in Figure 3.6.

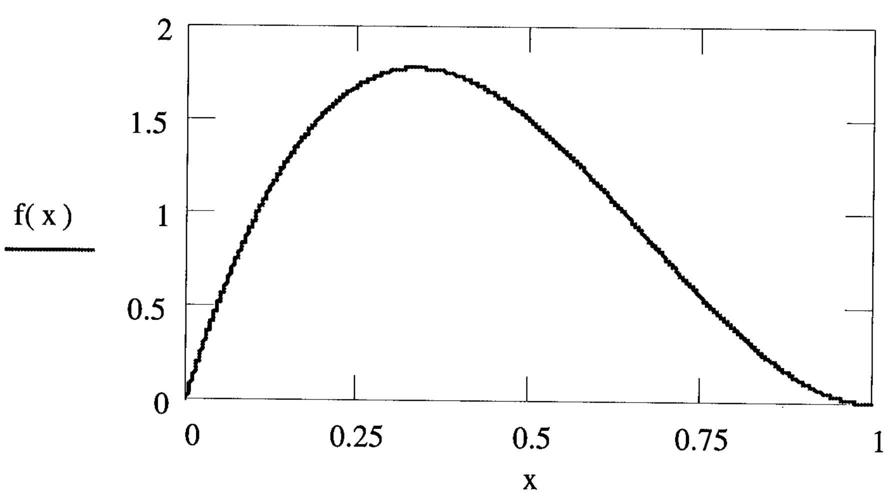


Figure 3.6 Beta (2,3) PDF

2. *Gamma Distribution:*

$$f(x;\delta,\beta,\theta)=\frac{1}{\theta\Gamma(\beta)}\left(\frac{x-\delta}{\theta}\right)^{\beta-1}e^{-\left(\frac{x-\delta}{\theta}\right)};\quad x>\delta;\quad \theta>0;\quad \beta>0$$

When $\delta = 2$ and $\beta = 1$, Gamma distribution has the PDF presented in Figure 3.7.

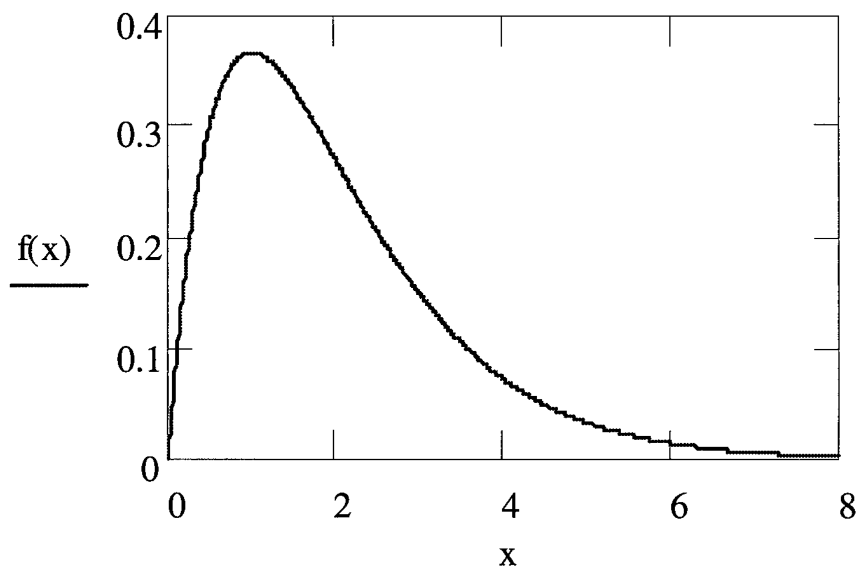


Figure 3.7 Gamma(2,1,0) PDF

3. *Normal Distribution:*

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\left[\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)\right]},\quad -\infty<x<\infty;\quad -\infty<\mu<\infty;\quad \sigma>0$$

When $\mu = 0$ and $\sigma = 1$, Normal distribution has the PDF presented in Figure 3.8

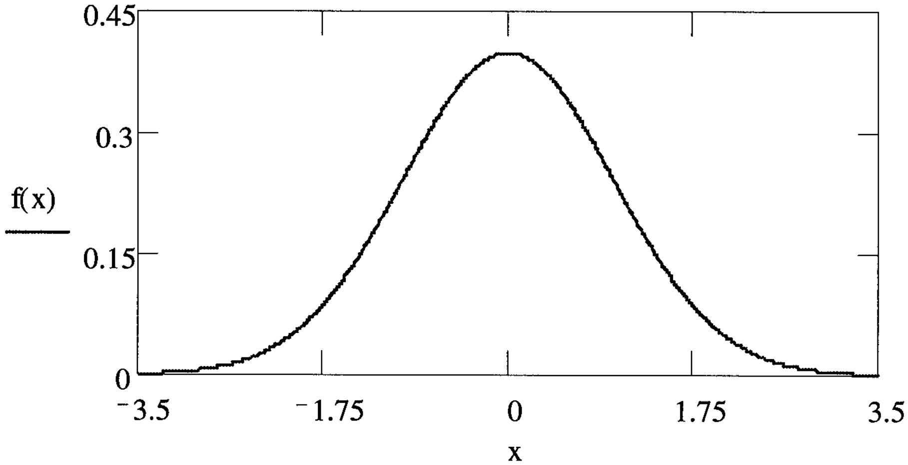


Figure 3.8 Normal(0,1) PDF.

4. *Uniform Distribution:*

$$f(x;\delta_1,\delta_2)=\frac{1}{\delta_2-\delta_1},\qquad \delta_1 \le x \le \delta_2$$

When $\delta_1 = 0$ and $\delta_2 = 2$, Normal distribution has the PDF presented in Figure 3.9.

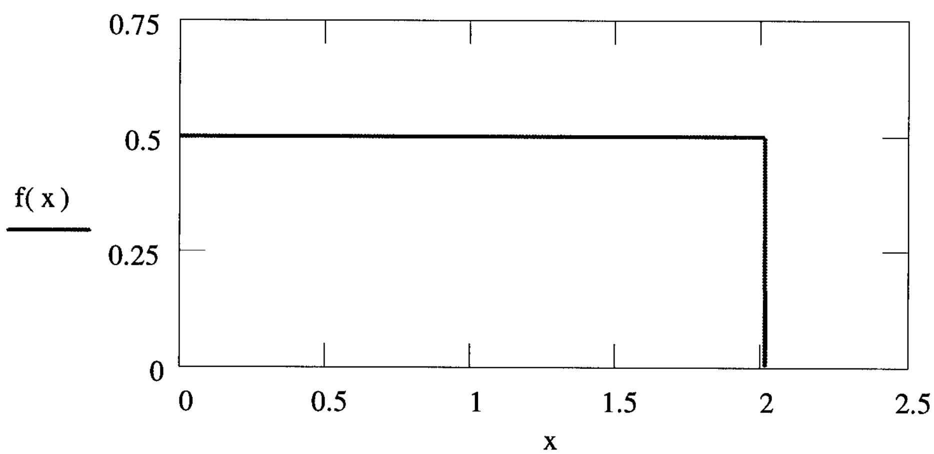


Figure 3.9 Uniform(0,2) PDF.

For a thorough explanation of the various techniques used in random variate generation, reference Law and Kelton [96:462-493], Banks and Carson [3:321-347], Bratley [16], Ripley [132], Schmeiser [138], Devyore [42], Dagpunar [35], and Fishman [50].

3.5.2.2 The Wozniak Alternatives

The second part of the power study was conducted against the alternate distributions studied by Wozniak that demanded more research and thinking for any type of comparison between the new sequential G.O.F. test and the EDF based K-S, C-vM, and A-D G.O.F. tests. Table 3.7 below shows the alternate distributions that are commonly used in power studies to evaluate G.O.F. tests for the Weibull distribution and that she studied for sample sizes, n = 20 and 30 in her study [168:152].

Table 3.7 Wozniak's Alternatives for the Power Study.

Chi-squared with 1 degree of freedom ($\chi^2(1)$)	Lognormal(0,1)
Chi-squared with 1 degree of freedom ($\chi^2(4)$)	Transformed Logistic(0,1)
Transformed Double Exponential	Transformed Cauchy(0,1)

Wozniak's research only deals with the hypothesized two-parameter Weibull distribution (no location parameter) addressing the G.O.F. problem by testing for the extreme-value distribution on log-transformed data from the alternate distributions noted above. Therefore, her study is fundamentally different than the sequential G.O.F. test studied in this thesis. This fundamental difference creates two major problems. The first problem is to identify the Weibull shape to utilize as the null hypotheses. The second problem is to generate samples from some of the alternate distributions she used in her research.

If β and θ are Weibull shape and scale parameters, then $\delta = \frac{1}{\beta}$ and $\xi = \log(\theta)$ are the scale and location parameters for the extreme-value distribution respectively

[168:153]. Thus, it is important to make a note of the fact that the log-transformation that converts the two-parameter Weibull data to extreme-value data converts the Weibull β and θ parameters to scale and location parameters respectively in order to recognize the first problem defined above. Since the EDF based K-S, C-vM, and A-D G.O.F. tests require estimation of the location and the scale parameters, Wozniak had to use a method that was based on estimating the corresponding Weibull β and θ parameters. Consequently, it can be inferred that a direct comparison with her results is not possible because the sequential G.O.F. test proposed in this thesis is based on known β values. Clough [26:3-25] addressed that some comparison is possible using the known β values in the range studied that are close to those that Wozniak's procedure would have essentially estimated. In order to determine these values, by following Clough's [26:3-25] methodology, 100 samples of sizes n = 20, 30 from each of Wozniak's alternate distributions were generated. Subsequently, the MLE for the Weibull β parameter for each of these samples was calculated. The appropriate null hypotheses to utilize in comparing powers with Wozniak's published results could be chosen by further evaluation of the means of these MLEs. The 100-sample means for each alternative and the values of β selected for comparison can be seen in Table 3.8 [26:3-26]. For example, since the MLEs for Weibull β on $\chi^2(4)$ for n =20 and 30 are 1.6263 and 1.5684 respectively, the closest Weibull β =1.5 was used as the H_o for the power studies with the $\chi^2(4)$ alternate distribution.

Therefore, it is clear that the comparisons that will be accomplished in this research will need to be interpreted based on the findings pointed out above. As can be seen in Table 3.8, even though most of the estimated β values are very close to the discrete β values considered in this thesis, there are some instances where the estimated β values fall between two of the discrete values. The power assessment is subjective in the cases where the estimated β values fall between two of the discrete values as opposed to be taken somewhat at face value in the cases where the estimated β values are very close to the discrete β values considered here. Clough [26:3-26] legitimately states that even if both G.O.F. tests that are being compared are identically powerful, the fact that Wozniak's results came from estimated parameters and the sequential G.O.F. test results from fixed β values, would lead the analyst to anticipate higher power for the sequential G.O.F. test where the discrepancies between the H_o and H_a were exaggerated. Even though the power comparison here has the problems mentioned above, the power comparison results considering Wozniak's alternate distributions will yield good insights as for the performance of the new sequential G.O.F. test against common alternate distributions generally used for Weibull G.O.F. tests.

The second problem of generating samples from the distributions Wozniak used in her research only exists for the transformed distributions. Therefore, samples from the two chi-squared distributions, $\chi^2(1)$ and $\chi^2(4)$, and the Lognormal(0,1) distributions, can be generated by utilizing MATLAB's statistics toolbox in which *chi2rnd* and *lognrnd* built-in routines exist. The chi-squared distribution with k degrees of freedom is a special case of the gamma distribution with shape $\frac{k}{2}$ and scale 2 [96:332]. Therefore, the *chi2rnd* function makes use of the *gamrnd* routine. The *lognrnd* function is based on a simple exponential transformation of the *normrnd* generator output [152].

Table 3.8 Shape Estimates for Wozniak's Alternate Distributions.

Alternate Distribution	n	Mean Shape Estimate	Standard Deviation	Shape Parameter(s) Used for H_o in the power studies here.
$\chi^2(1)$	20	0.6658	0.1263	$\beta = 0.5, 1.0$
	30	0.6494	0.0942	
$\chi^2(4)$	20	1.6263	0.3146	$\beta = 1.5$
	30	1.5684	0.2448	
Lognormal(0,1)	20	1.1426	0.2199	$\beta = 1.0$
	30	1.0819	0.1872	
X Cauchy(0,1)	20	0.3467	0.5224	$\beta = 0.5, 1.0$
	30	0.3783	0.5473	
X Double Exponential	20	0.8031	0.2455	$\beta = 0.5, 1.0$
	30	0.7616	0.2079	
X Logistic(0,1)	20	0.6038	0.1699	$\beta = 0.5, 1.0$
	30	0.5812	0.1173	

The Cauchy, the double exponential and the logistic alternate distributions are rather complicated to deal with. In order to make the random deviates nonnegative like Weibull random deviates are, Wozniak converted the random deviates to the positive real line by using an exponential transformation, $X_i = \exp(Y_i)$ given that Y_i was a random deviate from one of the distributions she studied. As a result of this transformation, the transformed random deviate X_i was tested as Weibull data, not the original random deviate Y_i. Subsequently, X_is are log-transformed to Y_is and tested as extreme-value random deviates in Wozniak's method. Therefore, X_is from the exponentially transformed Cauchy, double-exponential, and logistic random deviates will have to be generated and tested via the new sequential G.O.F. test procedure that tests hypothesized Weibull data as if they come from a Weibull population.

In this part of the thesis, the random number generators that were coded by Clough [26:H-14] for Cauchy, double-exponential, and logistic alternate distributions that utilize the inverse transformation method were used. Applying the exponential function to these generators derived the new generators for the transformed distributions.

The PDFs of the transformed distributions were derived in order to observe the PDFs of the transformed distributions visually. It is important to note that an examination of the shapes of the transformed distributions comply with the prior determination of the appropriate Weibull shapes used in this part of the power study. The PDFs of the original and the transformed distributions, and the random variate generators for the transformed distributions used in this part of the power study are in Appendix K. It can be noted that the PDFs of the transformed distributions are mostly alike the Weibull distributions with $\beta = 0.5$ and 1 that goes with the previous conclusion.

3.5.2.3 Additional Power Study

In additional to Clough's [26] alternate distributions that include alternate distributions from Bush [17] and Wozniak [168], the null and the alternate distributions that were studied for better power assessment of the new sequential G.O.F. test and preparing a reference database for the future studies are presented in Table 3.9.

3.5.2.4 Theoretical Skewness and Q-Statistic Values of the Alternate Distributions

Due to the fact that the new sequential G.O.F. test is based on sample $\sqrt{b_1}$ and sample Q-Statistic, it appears to be crucial to investigate the theoretical skewness, $\sqrt{\beta_1}$ (third central moment about the mean) and Q-Statistic values of the alternate

Table 3.9 The Distribution used in Additional Power Study.

H_o	H_a	n	H_o	H_a	n
$\beta = 0.5$	Weibull $\beta = 0.5$	5,15,25,50	$\beta = 2.5$	Weibull $\beta = 0.5$	5,15,25,50
	Weibull $\beta = 1.0$	5,15,25,50		Weibull $\beta = 1.0$	5,15,25,50
	Weibull $\beta = 1.5$	5,15,25,50		Weibull $\beta = 1.5$	5,15,25,50
	Weibull $\beta = 2.0$	5,15,25,50		Weibull $\beta = 2.0$	5,15,25,50
	Weibull $\beta = 2.5$	5,15,25,50		Weibull $\beta = 2.5$	5,15,25,50
	Weibull $\beta = 3.0$	5,15,25,50		Weibull $\beta = 3.0$	5,15,25,50
	Weibull $\beta = 3.5$	5,15,25,50		Weibull $\beta = 3.5$	5,15,25,50
	Weibull $\beta = 4.0$	5,15,25,50		Weibull $\beta = 4.0$	5,15,25,50
$\beta = 1.0$	Weibull $\beta = 0.5$	5,15,25,50	$\beta = 3.0$	Weibull $\beta = 0.5$	5,15,25,50
	Weibull $\beta = 1.5$	5,15,25,50		Weibull $\beta = 1.0$	5,15,25,50
	Weibull $\beta = 2.5$	5,15,25,50		Weibull $\beta = 1.5$	5,15,25,50
	Weibull $\beta = 3.0$	5,15,25,50		Weibull $\beta = 2.0$	5,15,25,50
	Weibull $\beta = 4.0$	5,15,25,50		Weibull $\beta = 2.5$	5,15,25,50
	$\chi^2(4)$	20,30,50		Weibull $\beta = 3.0$	5,15,25,50
$\beta = 1.5$	Weibull $\beta = 0.5$	5,15,25,50		Weibull $\beta = 3.5$	5,15,25,50
	Weibull $\beta = 1.0$	5,15,25,50		Weibull $\beta = 4.0$	5,15,25,50
	Weibull $\beta = 1.5$	5,15,25,50	$\beta = 3.5$	Weibull $\beta = 0.5$	5,15,25,50
	Weibull $\beta = 2.0$	5,15,25,50		Weibull $\beta = 1.5$	5,15,25,50
	Weibull $\beta = 2.5$	5,15,25,50		Weibull $\beta = 2.5$	5,15,25,50
	Weibull $\beta = 3.0$	5,15,25,50		Weibull $\beta = 3.0$	5,15,25,50
	Weibull $\beta = 3.5$	5,15,25,50		Weibull $\beta = 4.0$	5,15,25,50
	Weibull $\beta = 4.0$	5,15,25,50	$\beta = 4.0$	Weibull $\beta = 0.5$	5,15,25,50
$\beta = 2.0$	Weibull $\beta = 0.5$	5,15,25,50		Weibull $\beta = 1.0$	5,15,25,50
	Weibull $\beta = 1.0$	5,15,25,50		Weibull $\beta = 1.5$	5,15,25,50
	Weibull $\beta = 1.5$	5,15,25,50		Weibull $\beta = 2.0$	5,15,25,50
	Weibull $\beta = 2.0$	5,15,25,50		Weibull $\beta = 2.5$	5,15,25,50
	Weibull $\beta = 2.5$	5,15,25,50		Weibull $\beta = 3.0$	5,15,25,50
	Weibull $\beta = 3.0$	5,15,25,50		Weibull $\beta = 3.5$	5,15,25,50
	Weibull $\beta = 3.5$	5,15,25,50		Weibull $\beta = 4.0$	5,15,25,50
	Weibull $\beta = 4.0$	5,15,25,50			

distributions used in this thesis. The $\sqrt{\beta_1}$ and theoretical Q-Statistic values of the alternate distributions are reported in Table 3.10. Since the $\sqrt{\beta_1}$ and theoretical Q-Statistic values fail to exist for the three Wozniak alternate distributions, transformed

Table 3.10 $\sqrt{\beta_1}$ and Theoretical Q-Statistic Values of the Alternate Distributions.

Distribution	$\sqrt{\beta_1}$ Values	Theoretical Q-Statistic Values
Weibull(β=0.5)	6.6189	4.5424
Weibull(β=1.0)	1.9990	2.8622
Weibull(β=1.5)	1.0722	2.5537
Weibull(β=2.0)	0.6314	2.4782
Weibull(β=2.5)	0.3592	2.4752
Weibull(β=3.0)	0.1684	2.4759
Weibull(β=3.5)	0.0248	2.4894
Weibull(β=4.0)	0.0874	2.5057
Beta(2,2)	0.0000	2.1898
Beta(2,3)	0.2863	2.2733
Gamma(2,1)	1.4144	2.7042
Normal(0,1)	0.0000	2.5839
Uniform(0,2)	0.0000	1.8993
$\chi^2(1)$	2.8278	3.2565
$\chi^2(4)$	1.4143	2.7031
Lognormal(0,1)	6.1841	82.2360
X Logistic	DNE	DNE
X Double Exponential	DNE	DNE
X Cauchy	DNE	DNE

Logistic (X Logistic), transformed Double Exponential (X Double Exponential) and Cauchy (X Cauchy), they do not appear in Table 3.10. On the other hand, even though the $\sqrt{\beta_1}$ and theoretical Q-Statistic values for these distributions fail to exist, the sample $\sqrt{b_1}$ and Q-Statistic that are used as the G.O.F. test statistics in this thesis usually exist for them. There were not any instances in which the sample $\sqrt{b_1}$ and Q-Statistic values fail to exist for these alternate distributions, even though they exhibit a great deal of variability. Therefore, $\sqrt{b_1}$ and Q-Statistic G.O.F. tests can still be conducted with these alternate distributions. Clough [26:3-30] notes that for the transformed Cauchy distributions, the sample moments failed to exist somewhat frequently, and that frequency increased as the sample size increased and this problem originates from the nature of the random variate generator that is capable of producing values exceeding machine precision, which MATLAB reports as infinity and because of this condition, the sample skewness and kurtosis cannot be calculated. This condition was also observed for the sample $\sqrt{b_1}$ and sample Q-Statistic values for the transformed Cauchy distribution in this thesis.

Table 3.11 below summarizes the mean percentage of transformed Cauchy samples obtained via simulation that had either sample $\sqrt{b_1}$ or sample Q-Statistic values fail to exist. By following Clough's [26:3-30] logic, the power study algorithm was modified not to include the samples that did not have sample $\sqrt{b_1}$ and sample Q-Statistic values in the sample size of 40,000 selected for the power study effort to make the power study results with X Cauchy alternate distribution valid.

Table 3.11 Cauchy Samples With Non-Existent Skewness or Q-Statistic Values.

X Cauchy Sample Size	Mean Number Observed	Percentage (%)
20	1482.5	3.705
30	1915.3	5.8
50	3141.2	9.38

3.5.3 Power Study Implementation

After identifying the structure of the power study and the types of the alternate distributions, the new sequential G.O.F. test could be tested against both Bush's [17] and Wozniak's [168] alternate distributions to see if the new sequential G.O.F. test improved the power compared to that of the aforementioned EDF based G.O.F. test. As with Bush cases, the sample size 50 was added in order to be able to compare the power study results of the new sequential G.O.F. test to those of Wozniak's and Clough's power results. In order to determine which individual G.O.F. tests are most powerful against particular distributions, the next task was to analyze the two-sided $\sqrt{b_1}$ and Q-Statistic G.O.F. tests individually after deriving the power study results of the new sequential G.O.F. test. The information that could be derived from this part of the analysis would particularly help the analyst determine the individual significance levels for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests when they are conducted in sequence. If the analyst is trying to decide the appropriate combination of significance levels to pick out and one of the G.O.F. tests demonstrates significantly better power against specific alternatives, choosing a higher significance level for the more powerful of the two G.O.F. tests will result in better power for the overall analysis.

The last but not the least part of the power study was based on using directional one-sided versions of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests individually and sequentially to quantify and observe any change, especially an increase in power. Due to the fact that a one-sided G.O.F. test presupposes that the analyst has a priori information in order to use either the upper or the lower tail, a one-tailed G.O.F. test version should result in better power than a two-sided G.O.F. test. There are several means to determine these *a priori* information observations on the system from which the data are collected to yield some theoretical knowledge about the data and the use of computer software packages to come up with the histograms of the data in question. As mentioned before, Weibull shape parameter, $\beta = 1$, is used for evaluating failure times under the assumption of a constant failure rate. Now, let's assume that an analyst has this constant failure rate assumption to start his analysis with and during his analysis suspects that the data are collected from a system with increasing failure rate, in which a Weibull model with $\beta > 1$ is appropriate to proceed with his analysis. In this case, since larger Weibull shapes lead to lower skewness and Q-Statistic values, the analyst uses lower-tailed versions of one or both of the G.O.F. tests to specifically address this potential. Furthermore, the analyst, keeping in mind that the power of the sequential G.O.F. test would not exceed that of its strongest component at the same significance level, could only use the more powerful of the two G.O.F. tests individually, if *a priori* knowledge about the data were available before the analysis. Of course, we cannot always have *a priori* information about the sample data, therefore, the example above could not occur for each analysis problem. On the other hand, observing the power increase from use of the two-sided skewness and Q-Statistic G.O.F. tests to the use of one-sided skewness and Q-Statistic G.O.F. tests was worth the investigation. The analyst should always remember that the sequential G.O.F. test procedures hypothetically provide much more consistent power results across a wider range of alternate distributions than the individual G.O.F. tests might. Examining the $\sqrt{\beta_1}$ and theoretical Q-Statistic values of the alternative distributions as noted in Table 3.10, and comparing those to the particular Weibull null hypotheses being tested was the measure of choice of the appropriate tail, either upper or lower, to test for the alternate distributions. For example, for testing H_o = Weibull distribution shape parameter $\beta = 1.5$ with $\sqrt{b_1} = 1.072$ and Q-Statistic = 2.5537 against H_a = Beta (2,2) with $\sqrt{\beta_1} = 0.000$ and theoretical Q-Statistic = 2.1898, the analyst could use a lower-tail $\sqrt{b_1}$ G.O.F. test or a lower-tail Q-Statistic G.O.F. test. Likewise, for testing H_o = Weibull distribution shape parameter $\beta = 3.5$ with $\sqrt{b_1} = 0.025$ and theoretical Q-Statistics = 2.4894 against H_o =

Gamma (2,1) with $\sqrt{\beta_1} = 1.414$ and Q-Statistics = 2.7042, the analyst could use upper-tail tests. The power studies for the individual one-sided G.O.F. tests conducted in this thesis effort are listed in Tables 3.12 and 3.13. Upper tail G.O.F. tests were used in the verification runs. The use of the upper tail tests against the transformed Logistic distribution in both cases was based upon observation of the considerably large values of the sample skewness and Q-Statistic values in the earlier Monte Carlo simulations. The impact of incorporating directional versions of the component G.O.F. test procedures into the sequential G.O.F. test procedure was also examined. Note, this was conducted only for H_o = Weibull(β = 1 and 3.5) against H_a = Beta(2,2). Finally, Tables 3.14, 3.15 and 3.16 summarize all of the power studies conducted in this thesis.

3.6 Conclusion

The critical values for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for the particular Weibull distribution shape parameters, $\beta = 0.5(0.5)4$ and sample sizes, $n = 5(5)50$ were derived for the development and the implementation of the proposed sequential G.O.F. test procedure. The tool of choice was a set of large Monte Carlo simulations using a technique that was developed and implemented by Clough [26] who corrected the aforementioned conspicuous mistake in Onen's [117] method. Therefore, an extensive set of Monte Carlo simulations were coded and run in MATLAB5 to generate the upper and lower tail critical value tables for of both the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in a detail sufficient for the two-sided component G.O.F. tests. Of course, due to the fact that a sequential G.O.F. test procedure introduces the need for determining the attained siginificance levels resulting from combining two separate tests at their own levels of significance, deriving the critical values was not enough to implement the new sequential test. Once again, the tool of choice was a set of large Monte Carlo simulations using a technique that was developed and implemented by Clough [26]. Therefore, an extensive set of Monte Carlo simulations were coded and run in MATLAB5 to generate the attained significance tables for the aforementioned Weibull shape values and sample sizes. After the critical values and the attained significance levels had been empirically estimated in this fashion, the new sequential G.O.F. test could eventually be conducted against samples from various alternate distributions to determine how much utility it has in terms of power. Therefore, numerous power studies were conducted against a set of alternate distributions that were used by Clough [26] that were based on Bush's [17] and Wozniak's [168] alternate distributions, and some other selected alternate distributions to prepare a database for future studies. The individual and various one-sided variants of all $\sqrt{b_1}$ and Q-Statistic G.O.F. tests were conducted to document their performances and compare to those of the existing EDF based K-S, C-vM and A-D G.O.F. test results. The results of these power studies provide the reader with the means to implement the new sequential G.O.F. test procedure and assess its effectiveness in the field of G.O.F. testing.

Table 3.12 One-sided $\sqrt{b_1}$ G.O.F. Tests conducted

H_o	H_a	Tail Tested	H_o	H_a	Tail Tested
Weib(β=0.5)	$\chi^2(1)$	Lower	Weib(β=1.5)	Weib(β=1)	Upper
	X Logistic	Upper		Weib(β=1.5)	Upper
	Weib(β=0.5)	Upper		Weib(β=2)	Lower
	Weib(β=1)	Lower		Weib(β=3.5)	Lower
	Weib(β=1.5)	Lower	Weib(β=2)	Weib(β=0.5)	Upper
	Weib(β=2)	Lower		Weib(β=1)	Upper
	Weib(β=3.5)	Lower		Weib(β=1.5)	Upper
Weib(β=1)	Beta(2,2)	Lower		Weib(β=2)	Upper
	Beta(2,3)	Lower		Weib(β=3.5)	Lower
	Gamma(2,1)	Lower	Weib(β=3.5)	Beta(2,2)	Lower
	Norm(0,1)	Lower		Beta(2,3)	Upper
	Unif(0,2)	Lower		Gamma(2,1)	Upper
	Weib(β=0.5)	Upper		Normal(0,1)	Lower
	Weib(β=1)	Upper		Uniform(0,2)	Lower
	Weib(β=1.5)	Lower		Weib(β=0.5)	Upper
	Weib(β=2)	Lower		Weib(β=1)	Upper
	Weib(β=3.5)	Lower		Weib(β=1.5)	Upper
	$\chi^2(1)$	Upper		Weib(β=2)	Upper
	$\chi^2(4)$	Lower		Weib(β=3.5)	Upper
	Lognorm(0,1)	Upper		$\chi^2(1)$	Upper
	X Logistic	Upper		$\chi^2(4)$	Upper
Weib(β=1.5)	$\chi^2(4)$	Upper		X Logistic	Upper
	Weib(B=0.5)	Upper			

Table 3.13 One-sided Q-Statistic G.O.F. tests conducted.

H_o	H_a	Tail Tested	H_o	H_a	Tail Tested
Weib(β=0.5)	$\chi^2(1)$	Lower	Weib(β=1.5)	Weib(β=1)	Upper
	X Logistic	Upper		Weib(β=1.5)	Upper
	Weib(β=0.5)	Upper		Weib(β=2)	Lower
	Weib(β=1)	Lower		Weib(β=3.5)	Lower
	Weib(β=1.5)	Lower	Weib(β=2)	Weib(β=0.5)	Upper
	Weib(β=2)	Lower		Weib(β=1)	Upper
	Weib(β=3.5)	Lower		Weib(β=1.5)	Upper
Weib(β=1)	Beta(2,2)	Lower		Weib(β=2)	Upper
	Beta(2,3)	Lower		Weib(β=3.5)	Lower
	Gamma(2,1)	Lower	Weib(β=3.5)	Beta(2,2)	Lower
	Norm(0,1)	Lower		Beta(2,3)	Upper
	Unif(0,2)	Lower		Gamma(2,1)	Upper
	Weib(β=0.5)	Upper		Norm(0,1)	Upper
	Weib(β=1)	Upper		Unif(0,2)	Lower
	Weib(β=1.5)	Lower		Weib(β=0.5)	Upper
	Weib(β=2)	Lower		Weib(β=1)	Upper
	Weib(β=3.5)	Lower		Weib(β=1.5)	Upper
	$\chi^2(1)$	Upper		Weib(β=2)	Upper
	$\chi^2(4)$	Lower		Weib(β=3.5)	Upper
	Lognorm(0,1)	Upper		$\chi^2(1)$	Upper
	X Logistic	Upper		$\chi^2(4)$	Upper
Weib(β=1.5)	$\chi^2(4)$	Upper		X Logistic	Upper
	Weib(β=0.5)	Upper			

Table 3.14 Summary for Power Studies for H_o = Weibull with $\beta = 0.5$ and 1.0.

			G.O.F. Tests Evaluated				
H_o	H_a	Sample Sizes	Sequential	Two-Sided Skewness	Two-Sided Q-Statistic	One-Sided Skewness	One-Sided Q-Statistic
$\beta = 0.5$	$\chi^2(1)$	20,30,50	X	X	X	X	X
	X Cauchy	20,30,50	X	X	X		
	X Double Exp.	20,30,50	X	X	X		
	X Logistic	20,30,50	X	X	X	X	X
	Weib $\beta = 0.5$ *	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 4.0$	5,15,25,50	X	X			
$\beta = 1.0$	Beta (2,2) #	5,15,25,50	X	X	X	X	X
	Beta (2,3)	5,15,25,50	X	X	X	X	X
	Gamma (2,1,0)	5,15,25,50	X	X	X	X	X
	Normal (0,1)	5,15,25,50	X	X	X	X	X
	Uniform (0,2)	5,15,25,50	X	X	X	X	X
	Weib $\beta = 0.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.0$ *	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 4.0$	5,15,25,50	X	X			
	Lognormal(0,1)	20,30,50	X	X	X	X	X
	$\chi^2(1)$	20,30,50	X	X	X	X	X
	$\chi^2(4)$	20,30,50	X	X	X	X	X
	X Cauchy	20,30,50	X	X	X		
	X Double Exp.	20,30,50	X	X	X		
	X Logistic	20,30,50	X	X	X	X	X

* Verification Runs.

One-sided variants of the sequential G.O.F. test were also studied for this distribution.

Table 3.15 Summary for Power Studies for H_o = Weibull with $\beta = 1.5$, 2.0, 2.5 and 3.0.

H_o	H_a	Sample Sizes	G.O.F. Tests Evaluated				
			Sequential	Two-Sided Skewness	Two-Sided Q-Statistic	One-Sided Skewness	One-Sided Q-Statistic
$\beta = 1.5$	Weib $\beta = 0.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.5$ *	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 4.0$	5,15,25,50	X	X			
	$\chi^2(4)$	20,30,50	X	X	X	X	X
$\beta = 2.0$	Weib $\beta = 0.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.0$ *	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 4.0$	5,15,25,50	X	X			
$\beta = 2.5$	Weib $\beta = 0.5$	5,15,25,50	X	X			
	Weib $\beta = 1.0$	5,15,25,50	X	X			
	Weib $\beta = 1.5$	5,15,25,50	X	X			
	Weib $\beta = 2.0$	5,15,25,50	X	X			
	Weib $\beta = 2.5$ *	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X			
	Weib $\beta = 4.0$	5,15,25,50	X	X			
$\beta = 3.0$	Weib $\beta = 0.5$	5,15,25,50	X	X			
	Weib $\beta = 1.0$	5,15,25,50	X	X			
	Weib $\beta = 1.5$	5,15,25,50	X	X			
	Weib $\beta = 2.0$	5,15,25,50	X	X			
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$ *	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X			
	Weib $\beta = 4.0$	5,15,25,50	X	X			

* Verification Runs

Table 3.16 Summary for Power Studies for H_o = Weibull with $\beta = 3.5$ and 4.0.

			G.O.F. Tests Evaluated				
H_o	H_a	Sample Sizes	Sequential	Two-Sided Skewness	Two-Sided Q-Statistic	One-Sided Skewness	One-Sided Q-Statistic
$\beta = 3.5$	Beta (2,2) #	5,15,25,50	X	X	X	X	X
	Beta (2,3)	5,15,25,50	X	X	X	X	X
	Gamma (2,1,0)	5,15,25,50	X	X	X	X	X
	Normal (0,1)	5,15,25,50	X	X	X	X	X
	Uniform (0,2)	5,15,25,50	X	X	X	X	X
	Weib $\beta = 0.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.0$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 1.5$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2$	5,15,25,50	X	X	X	X	X
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$ *	5,15,25,50	X	X	X	X	X
	Weib $\beta = 4.0$	5,15,25,50	X	X			
	Lognormal(0,1)	20,30,50	X	X	X		
	$\chi^2(1)$	20,30,50	X	X	X	X	X
	$\chi^2(4)$	20,30,50	X	X	X	X	X
	X Cauchy	20,30,50	X	X	X		
	X Double Exp.	20,30,50	X	X	X		
	X Logistic	20,30,50	X	X	X	X	X
$\beta = 4.0$	Weib $\beta = 0.5$	5,15,25,50	X	X			
	Weib $\beta = 1.0$	5,15,25,50	X	X			
	Weib $\beta = 1.5$	5,15,25,50	X	X			
	Weib $\beta = 2.0$	5,15,25,50	X	X			
	Weib $\beta = 2.5$	5,15,25,50	X	X			
	Weib $\beta = 3.0$	5,15,25,50	X	X			
	Weib $\beta = 3.5$	5,15,25,50	X	X			
	Weib $\beta = 4.0$ *	5,15,25,50	X	X			

* Verification Runs.

One-sided variants of the sequential G.O.F. test were also studied for this distribution.

IV. RESULTS AND ANALYSIS

4.1 Introduction

Monte Carlo simulations discussed in the preceding chapter were employed to implement the new sequential G.O.F. test and to measure the effectiveness and utility of it in the field of G.O.F. testing.

By carefully assessing the derived critical values, attained significance levels, and power-study results for the particular alternate distributions presented in the preceding chapter, the analyst can make prudent decisions on how, when, and whether to utilize this new sequential G.O.F. test. Thus, this chapter is intended for summarizing the analysis results and providing the analyst with some useful insights into the usage of the new sequential procedure. Specifically, the results of each objective set forth in Chapter 1 are presented along with select tables and figures, which highlight these results. Complete sets of tables and figures can be found in the appendices.

4.2 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic

In order to explore the joint behavior of the sample $\sqrt{b_1}$ and sample Q-Statistic values, they were derived and plotted for each Weibull($\beta = 0.5(0.5)4$) distribution at sample sizes $n = 5(5)50$ that can be viewed in Appendix A. Ninety and ninety-five percent contour plots on the ($\sqrt{b_1}$,b_2) plane for the joint distribution of sample $\sqrt{b_1}$ and sample b_2 for normal distribution was presented by D'Agostino and Stephens [146:282].

Clough [26:4-1-7] presents the joint distributions of sample $\sqrt{b_1}$ and sample b_2 for Weibull(β = 0.5, 1, 2, and 4) at sample sizes n = 5, 15, 25 and 50 by following a less sophisticated approach. He first generated 25,000 samples by using the same seed that he used for critical value determination via Monte Carlo simulation to plot on the ($\sqrt{b_1}$, b_2) plane and compared them to the theoretical $\sqrt{\beta_1}$ and β_2 values that he reported in his thesis. No such work exists for the Weibull distribution for the joint distributions of the sample $\sqrt{b_1}$ and sample Q-Statistic in the statistics literature. Thus, in order to visualize the general form of the joint distribution and relationship between the component test statistics, the first 30,000 samples from the Monte Carlo simulation to generate the critical values were plotted on the ($\sqrt{b_1}$, Q-Statistic) plane for the joint distribution in which the generated samples are plotted with an '*' and compared to the theoretical values for the true skewness and Q-Statistic values that are plotted with '+' where scaling permits. These plots do give us insights into identifying trends in the joint distributions and behavior of the sample $\sqrt{b_1}$ and sample Q-Statistic values.

The plots in Appendix A exhibit a very unique nature of the joint distributions for n = 5, regardless of shape parameter values. Even though the scatterplot appears to have very fixed boundaries that seem to be the same for all shape parameter values, the density of the joint distribution is mostly concentrated on the positively skewed portion for small shape parameter values and becomes more uniform as shape parameter value increases. At this point, a need for investigating the nature of the joint distributions for some of the alternate distribution arises to see if the new sequential G.O.F. test procedure potentially has considerable discriminatory power at small sample sizes against the alternate

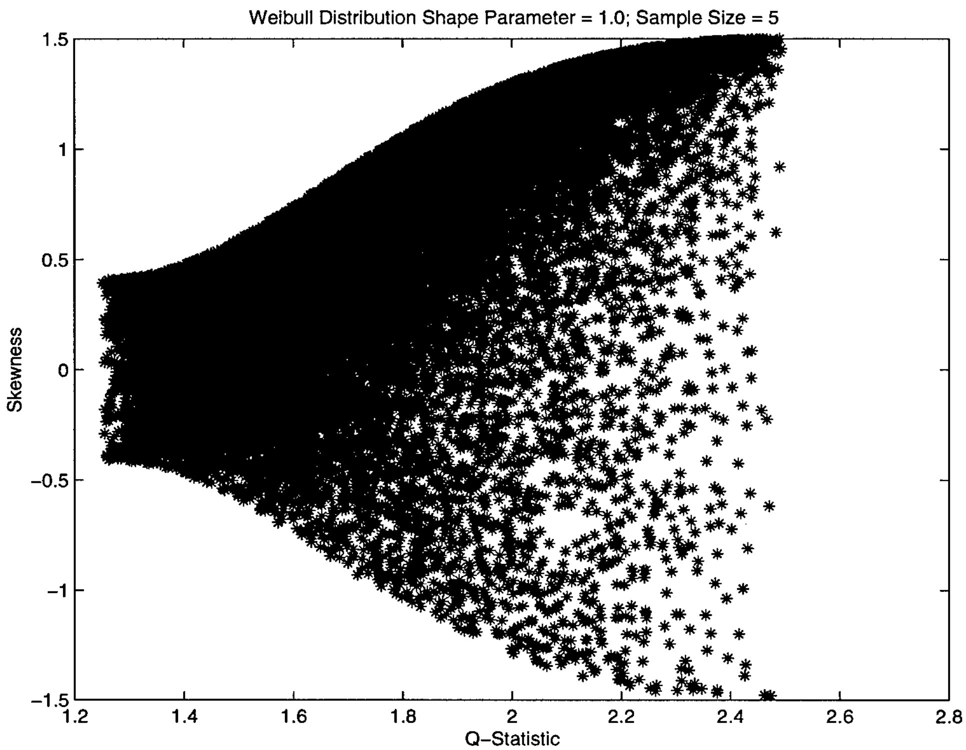


Figure 4.1 Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 5$.

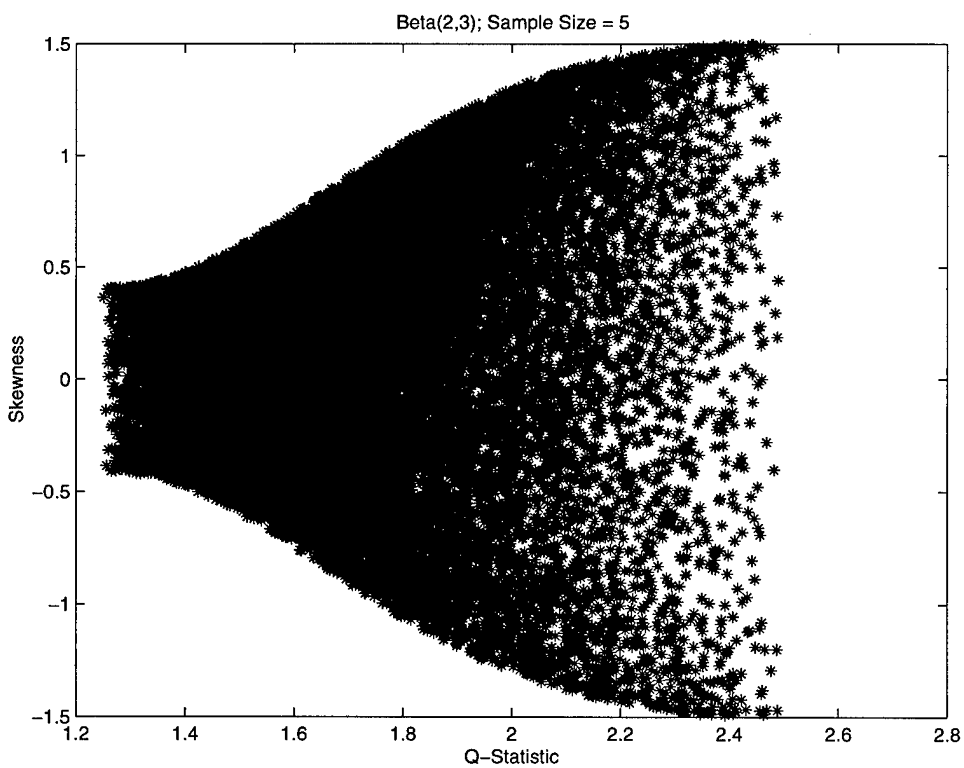


Figure 4.2 Distribution of $\sqrt{b_1}$ and Q-Statistic for Beta(2,3) ; $n = 5$.

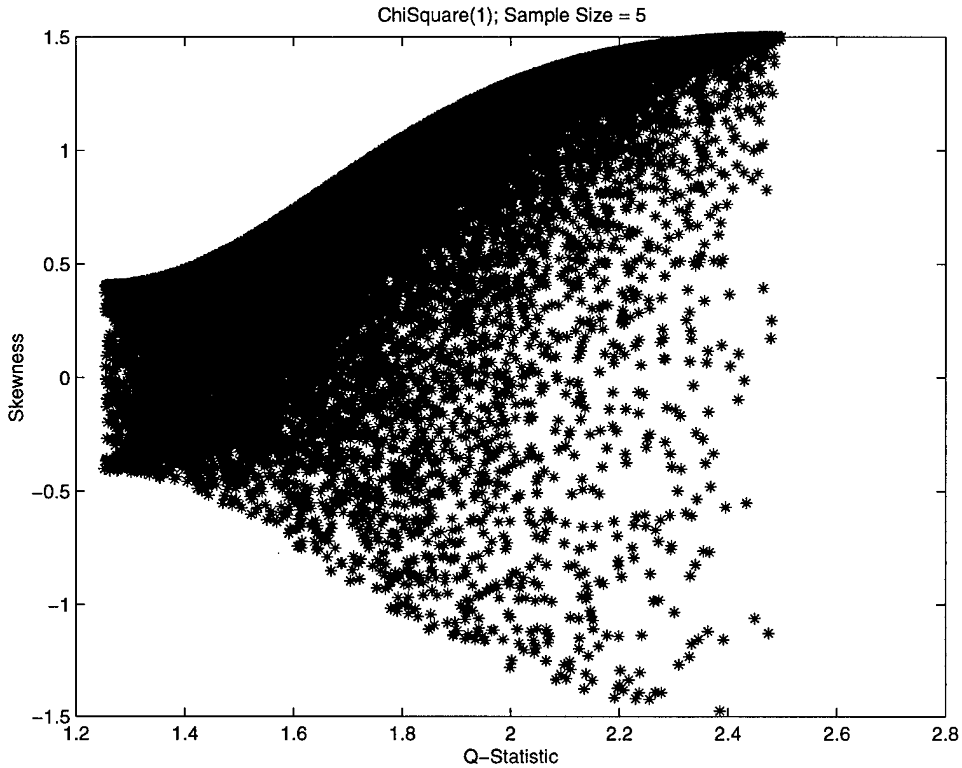


Figure 4.3 Distribution of $\sqrt{b_1}$ and Q-Statistic for $\chi^2(1)$; $n = 5$.

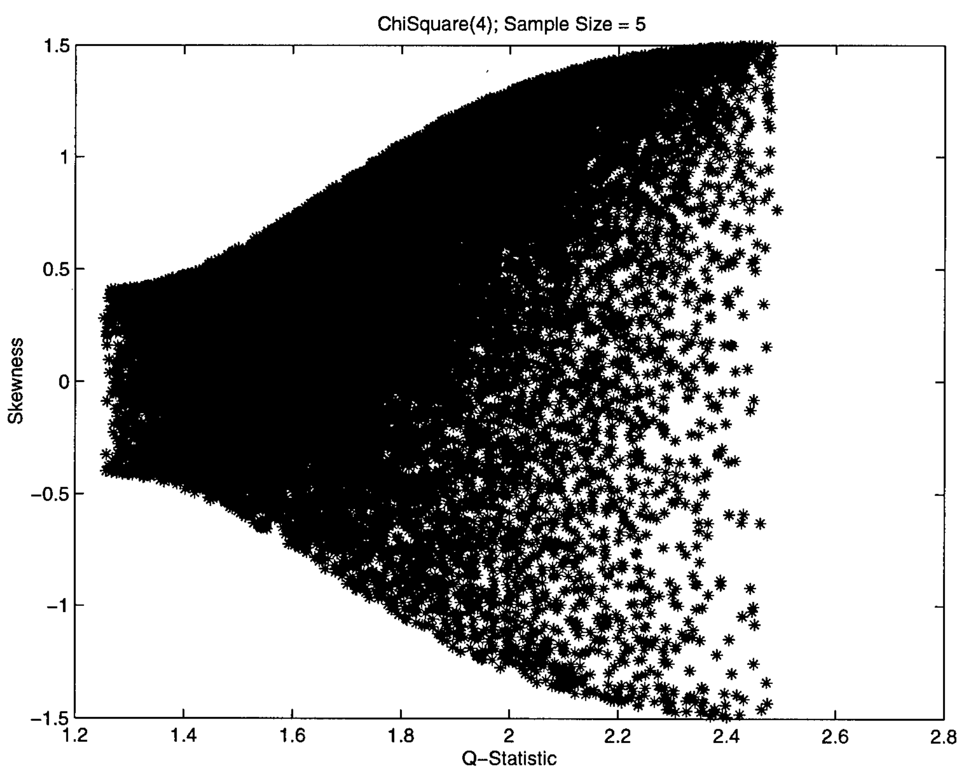


Figure 4.4 Distribution of $\sqrt{b_1}$ and Q-Statistic for $\chi^2(4)$; $n = 5$.

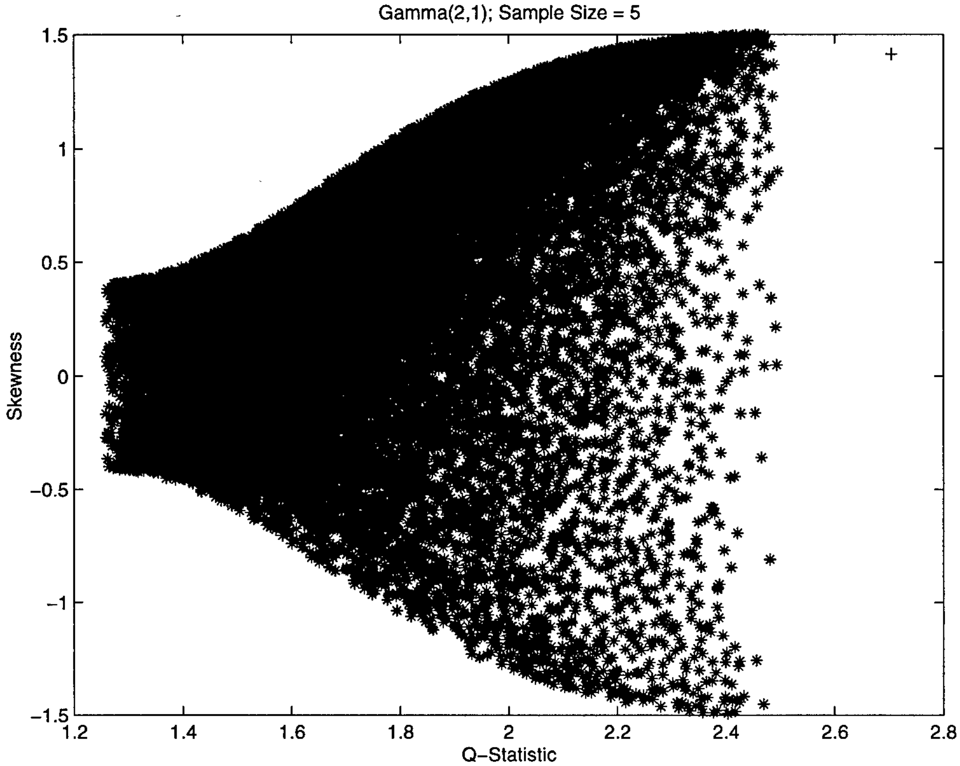


Figure 4.5 Distribution of $\sqrt{b_1}$ and Q-Statistic for Gamma(2,1) ; $n = 5$.

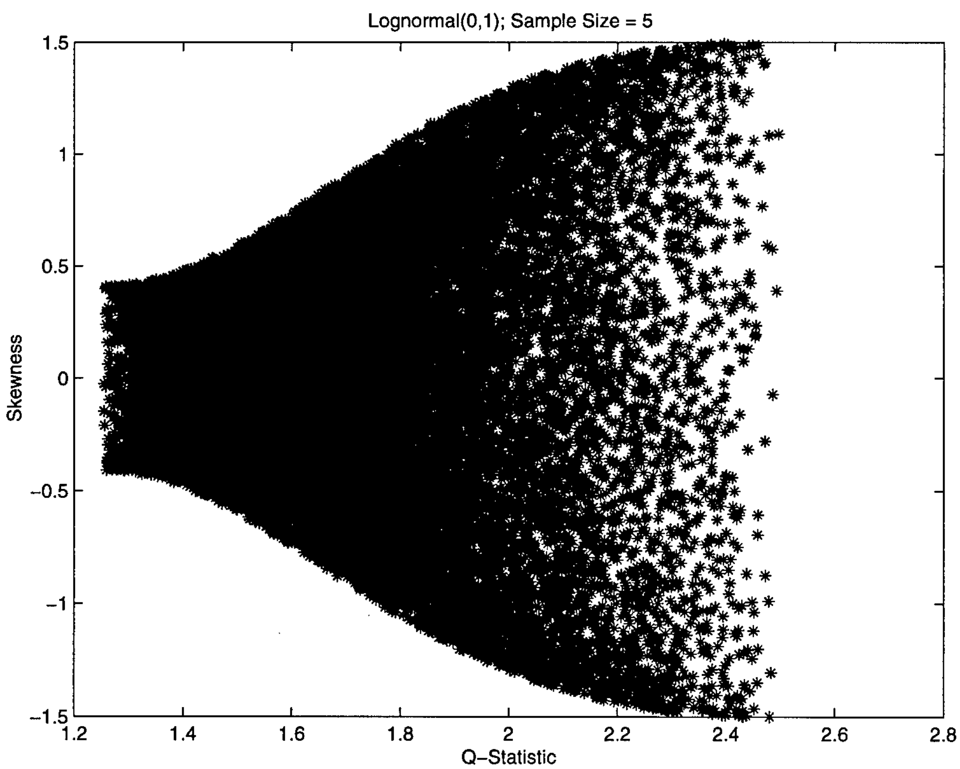


Figure 4.6 Distribution of $\sqrt{b_1}$ and Q-Statistic for Lognormal(0,1) ; $n = 5$.

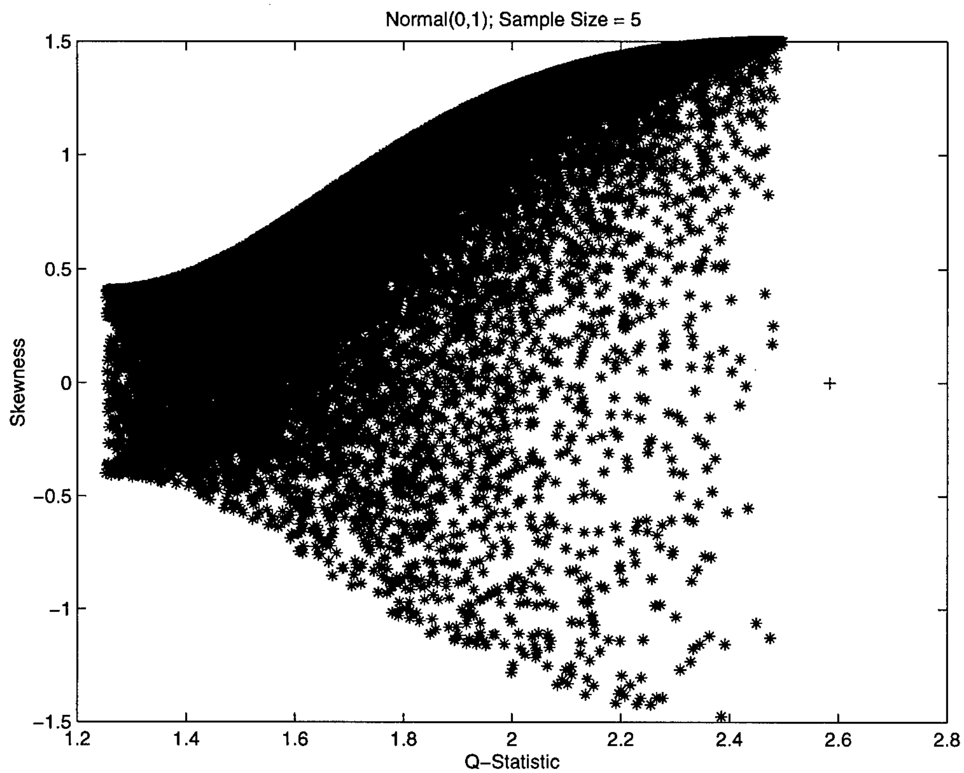


Figure 4.7 Distribution of $\sqrt{b_1}$ and Q-Statistic for Normal(0,1) ; $n = 5$.

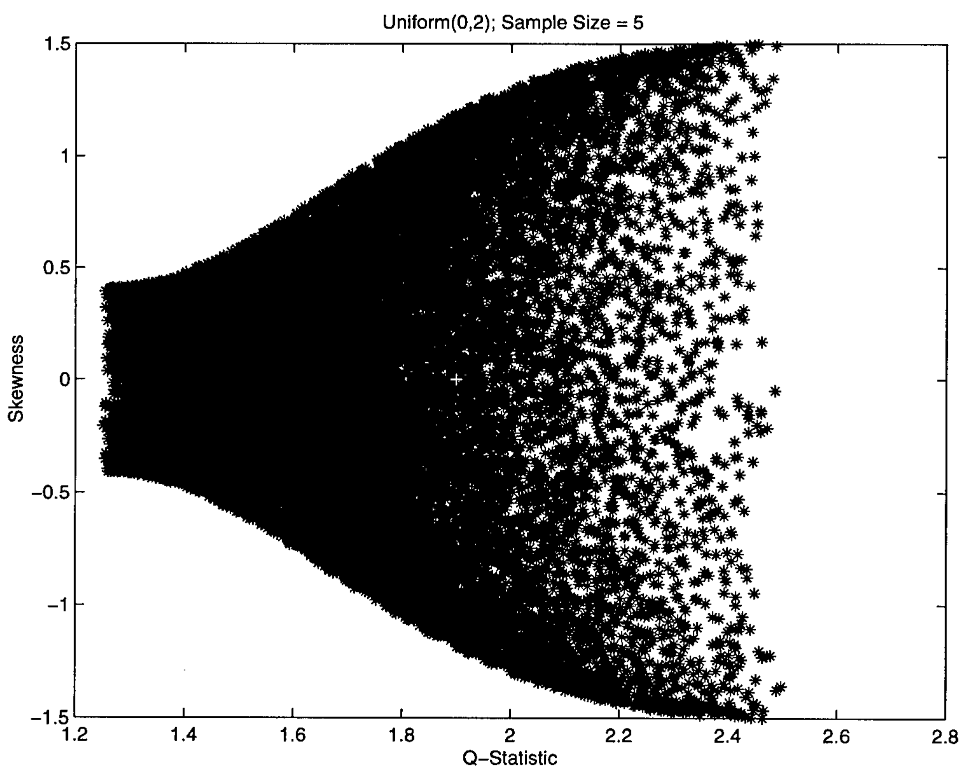


Figure 4.8 Distribution of $\sqrt{b_1}$ and Q-Statistic for Uniform(0,2) ; $n = 5$.

distributions used in this thesis. If they appear to be notably different than each other, it means that the new sequential G.O.F. test has good discriminatory power against a spectrum of alternate distributions. In order to investigate this question, 30,000 samples of size 5 were generated from each of the Weibull($\beta = 1$), Beta(2,3), $\chi^2(1)$, $\chi^2(4)$, Gamma(2,1), Lognormal(0,1), Normal(0,1) and Uniform(0,2) distributions and plotted on the ($\sqrt{b_1}$, Q-Statistic) as can be seen in Figures 4.1-4.8. Examining these plots, one finds identical boundaries, but differing patterns in the densities. The power study will show if these differences in the joint distribution densities illustrate the fact that the new sequential G.O.F. test has notably good power at small sample sizes as indicated in these scatter plots.

The fact that the sample $\sqrt{b_1}$ and sample Q-Statistic values are inclined to be highly correlated for smaller shape parameter values and less so for larger shape parameter values can clearly be observed from these scatterplots in Appendix A. The apparent trend that as sample $\sqrt{b_1}$ values increase in magnitude so do the sample Q-Statistic values, becomes less observable for shape parameter values greater than 2.5 at all sample sizes considered here. The extent of correlation for smaller shape parameter values do indicate that due to the fact that $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics are not measuring independent traits of the given sample data, one of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests may prove to be dominant in terms of discriminatory power for these lower shape parameter values. The test statistic whose distribution has the least variability will probably demonstrate this dominance since it tolerates less deviation from its central tendency. However, as the shape parameter values increase, this effect should be diminished. The power study will quantify this observation.

It was also observed that the degree of correlation of $\sqrt{b_1}$ and Q-Statistic vary in relation to the sample size and the shape parameter value. For $\beta = 0.5$, the degree of the correlation increases from $n = 5$ to $n = 20$ and keeps decreasing slightly from $n = 25$ to $n = 50$. For $\beta = 1.0$, the degree of correlation increases from $n = 5$ to $n = 25$ and keeps decreasing from $n = 30$ to $n = 50$. For $\beta = 1.5$, the degree of the correlation increases notably from $n = 5$ to $n = 20$, decreases for $n = 25$, increases from $n = 30$ to $n = 35$, decreases for $n = 40$ and increases from $n = 45$ to $n = 50$. For $\beta = 2.0$, the degree of the correlation increases from $n = 5$ to $n = 20$ and stays the same from $n = 25$ to $n = 50$. For $\beta = 2.5$, the degree of the correlation increases from $n = 5$ to $n = 25$, decreases for $n = 30$, increases for $n = 35$ and starts slightly decreasing from $n = 40$ to $n = 50$. For $\beta = 3.0$, the degree of the correlation increases from $n = 5$ to $n = 30$, decreases slightly for $n = 35$, notably increases for $n = 40$ and starts decreasing from $n = 45$ to $n = 50$. For $\beta = 3.5$, the degree of the correlation increases from $n = 5$ to $n = 30$, notably decreases for $n = 35$, increases from $n = 40$ to $n = 45$ and slightly decreases for $n = 50$. For $\beta = 4.0$, the degree of the correlation increases from $n = 5$ to $n = 25$, slightly decreases from $n = 30$ to $n = 50$. These findings mean that the power of the new sequential G.O.F. test procedure may vary for different sample sizes with the same hypothesized Weibull shape parameter value as expected. Again, the power study section in this thesis will help quantify this observation.

Another notable feature of the joint distributions derived here is the existence of changing variability of sample $\sqrt{b_1}$ and sample Q-Statistic values in relation to the sample size as the shape parameter value increases. Both $\sqrt{b_1}$ and Q-Statistic values seem to cover a wider range of values as the sample size increases for Weibull distribution shape parameter values less than 2. The trend stops at Weibull distribution shape parameter 2, and reverses when the shape parameter is greater than 2, after which the variability in both $\sqrt{b_1}$ and Q-Statistic values decrease as the sample size increases. This observation can be accounted for based on the fact that when shape parameter is 0.5 or 1, the Weibull distribution behaves exponential or exponential-like, indicating a high degree of variance. Thus, it is an expected result for the sample $\sqrt{b_1}$ and sample Q-Statistic to reflect this variability in the joint distribution. Moreover, since the magnitudes of the true skewness and Q-Statistic values are fairly large at these shape parameter values, the sample $\sqrt{b_1}$ and sample Q-Statistic values are supposed to increase to approach the theoretical values. Leading to less variability in $\sqrt{b_1}$ and tail length, the Weibull PDFs become more and more mound-shaped when the shape parameter value is 2 or greater. It was also observed that the joint distributions tend to be concentrated on the theoretical skewness and Q-Statistic values as the Weibull shape parameter values increase. The findings here indicate that since both $\sqrt{b_1}$ and Q-Statistic tend to have a comparatively high variability, the new sequential G.O.F. test procedure may have low discriminatory power for these G.O.F. test statistics for tests with known shape parameter values less than 2. Consequently, it remains to be seen in the power study whether these observations relate to any power performance determination.

Some useful insights into how Q-Statistic test statistic performs compared to b_2 test statistic along with the $\sqrt{b_1}$ test statistic can be obtained when the joint distributions here are compared to Clough's joint distributions. The reader should refer to Clough's thesis to see his joint distributions [26:4-2]. Clough [26] only reported the joint distributions of $\sqrt{b_1}$ and b_2 for Weibull ($\beta = 0.5, 1.0, 2.0$ and 4.0) and $n = 5, 15, 25$ and 50. Thus, the comparison here is limited to those shape parameter values and sample sizes.

The unique nature of the joint distributions observed here for $n = 5$ can also be observed in Clough's joint distributions. However, while the joint distributions here have fixed extending outward (fishtail) boundaries, the ones in Clough's have fixed straight boundaries. The difference in these boundaries could not be interpreted. On the other hand, the sample $\sqrt{b_1}$ and b_2 follow the same concentration patterns as the ones for $\sqrt{b_1}$ and Q-Statistic. This observation suggests that Q-Statistic may perform at least as well as b_2 does. Furthermore, the correlation pattern between $\sqrt{b_1}$ and b_2 seems to be the same between $\sqrt{b_1}$ and Q-Statistic. This observation suggests that one of the component tests in both studies may dominate the other in terms of power. The most noticeable difference between the joint distributions here and Clough's is the different degrees of variability. While sample $\sqrt{b_1}$ values cover the same area, sample b_2 values cover two or three times more area than Q-Statistic values do. This observation suggests that the Q-Statistic may be a better discriminator than the b_2 since sample Q-Statistic values exhibit less variability than the sample b_2 values do. As a result, we can conclude that the differences in the joint distributions suggest that the Q-Statistic will probably perform equally well or better. Of course, the power study will quantify these observations.

4.3 Critical Values

The critical value tables that were generated via Monte Carlo simulations with $N = 100{,}000$ provide the upper and lower tail percentage points for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at sample sizes $n = 5(5)50$ for each Weibull shape parameter value $\beta = 0.5(0.5)4$. In these tables, the critical values are listed for all significance levels $\alpha = 0.005(0.005)0.10$ and $0.10(0.01)0.20$ that reflect the level of granularity required by the two-sided nature of each component G.O.F test. Appendix B provides the complete critical value tables for $\sqrt{b_1}$ and Q-Statistic G.O.F. test procedures for each hypothesized Weibull distribution shape parameter, sample size, and significance level. The critical values derived for $\sqrt{b_1}$ G.O.F. here are exactly the same as the ones in Clough's [26] research verifying and validating the critical value derivation procedure.

The simulations were run under MATLAB 5 on a UNIX-based Sun SPARC-20 workstation on different machines simultaneously. Clough [26:3-9] noted it took roughly 12 hours to run for each value of shape parameter for $\sqrt{b_1}$ and b_2 test statistics. The simulation runs for this research took roughly from 10 to 20 hours depending on the shape parameter value. It is worth noting that for H_o = Weibull ($\beta = 1$), the simulation took about 20 hours, that is roughly 5 to 10 hours longer than the simulations for the other shape parameter values. For each given shape with a replication number of $N = 100{,}000$ samples of sizes $n = 5(5)50$, each simulation run generated 27,500,000 Weibull deviates.

4.3.1 Use of the Critical Value Tables for $\sqrt{b_1}$ and Q-Statistic G.O.F. Test Procedures

This section describes the use of the tabled critical values for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests that make up the proposed sequential G.O.F. test. As mentioned before, if either or both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests reject the given sample data against the hypothesized Weibull distribution with the specified shape parameter value, the sequential G.O.F. test procedure rejects the sample data. These G.O.F. tests are summarized in the following procedure that illustrates how the individual two-tailed $\sqrt{b_1}$ and Q-Statistic G.O.F. test procedures would work separately in the proposed sequential G.O.F. test:

- Based on the previous theoretical knowledge or analysis of the system under consideration, specify the shape parameter for the hypothesized Weibull distribution.
- Compute the sample skewness via MATLAB built-in *skewness* function or via equation (3.2) and Q-Statistic via the routine in MATLAB that is presented in Appendix J-1 or via equation (3.3).
- Keeping in mind that the level of significance is the probability of incorrectly rejecting the null hypothesis, specify the desired significance level, α.
- Find the lower and upper critical values from the appropriate lower and upper tail critical value table based on the G.O.F. test employed, the shape parameter β of the hypothesized Weibull distribution, the sample size n, and the desired significance level of the G.O.F. test procedure, α.

- Define the rejection region with these values and check to see if the computed test statistic is in the rejection region. If the computed test statistic is smaller than the lower tail or larger than the upper tail critical value, reject H_o; otherwise accept H_o.

In order to demonstrate how to use the lower and upper tail critical value tables derived in this thesis, the complete upper and lower critical values tables for $\sqrt{b_1}$ G.O.F. test for the hypothesized Weibull shape $\beta = 1.5$ are presented in Table 4.1 and 4.2. Now, in the light of the steps discussed above, let's take a look at an example that demonstrates the use of the critical value tables. Let's assume that an analyst is performing a two-tailed test of H_o = Weibull($\beta = 1.5$) and has a sample data of size $n = 40$ with sample skewness value of $\sqrt{b_1} = 1.644$. In order to conduct the $\sqrt{b_1}$ G.O.F. test at $\alpha = 0.04$ desired significance level, the analyst would take a look at the $\frac{\alpha}{2} = 0.02$ column of the Table 4.1 to be able to identify $\sqrt{b_1}_{\,(0.02)} = 0.192$ and in the $\left(1-\frac{\alpha}{2}\right) = 0.98$ column of the Table 4.2 to be able to identify $\sqrt{b_1}_{\,(0.98)} = 1.970$. The rejection region for the $\sqrt{b_1}$ G.O.F. test for this particular example is defined by these two values. The conclusion by the analyst would be to fail to reject H_o, because $0.192 < \sqrt{b_1} = 1.644 < 1.970$. It is important to note that if a different significance level were used, say $\alpha = 0.12$, H_o would be rejected due to the fact that the sample data fall out of the rejection region by failing the upper tail portion of the $\sqrt{b_1}$ G.O.F. test, since $\sqrt{b_1}_{\,(0.94)} = 1.638$. The Q-Statistic G.O.F. test would have to be conducted similarly to make up the new sequential G.O.F. test procedure. Similar examples will be presented at the presentation of the attained significance levels

Table 4.1 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 1.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.282	-1.163	-1.070	-0.998	-0.936	-0.879	-0.827	-0.785	-0.744	-0.702
10	-0.801	-0.653	-0.570	-0.511	-0.458	-0.418	-0.381	-0.348	-0.318	-0.292
15	-0.486	-0.376	-0.309	-0.261	-0.223	-0.189	-0.158	-0.130	-0.106	-0.084
20	-0.310	-0.209	-0.146	-0.103	-0.066	-0.036	-0.008	0.016	0.038	0.056
25	-0.182	-0.095	-0.043	-0.002	0.031	0.059	0.085	0.106	0.125	0.142
30	-0.082	-0.005	0.044	0.078	0.107	0.133	0.153	0.172	0.193	0.210
35	-0.003	0.071	0.111	0.147	0.176	0.200	0.222	0.241	0.259	0.275
40	0.043	0.114	0.159	0.192	0.218	0.241	0.262	0.279	0.295	0.312
45	0.099	0.165	0.208	0.239	0.265	0.286	0.305	0.322	0.338	0.352
50	0.138	0.204	0.243	0.273	0.300	0.320	0.340	0.356	0.372	0.384

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.667	-0.633	-0.603	-0.575	-0.546	-0.525	-0.503	-0.483	-0.463	-0.446
10	-0.271	-0.248	-0.227	-0.208	-0.190	-0.175	-0.155	-0.139	-0.124	-0.109
15	-0.064	-0.045	-0.027	-0.010	0.005	0.020	0.034	0.047	0.060	0.073
20	0.074	0.090	0.106	0.120	0.134	0.147	0.160	0.172	0.184	0.196
25	0.159	0.174	0.189	0.202	0.216	0.228	0.240	0.251	0.262	0.274
30	0.226	0.241	0.255	0.268	0.280	0.292	0.303	0.314	0.324	0.333
35	0.289	0.303	0.315	0.327	0.338	0.349	0.359	0.369	0.379	0.389
40	0.325	0.337	0.349	0.360	0.371	0.382	0.392	0.402	0.412	0.421
45	0.365	0.378	0.390	0.401	0.411	0.422	0.431	0.440	0.449	0.457
50	0.396	0.409	0.420	0.431	0.442	0.452	0.460	0.469	0.479	0.487

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.414	-0.389	-0.363	-0.338	-0.314	-0.292	-0.269	-0.233	-0.223	-0.199
10	-0.081	-0.057	-0.031	-0.006	0.016	0.038	0.058	0.077	0.096	0.115
15	0.098	0.122	0.143	0.164	0.184	0.203	0.221	0.238	0.254	0.272
20	0.217	0.237	0.256	0.275	0.292	0.309	0.326	0.341	0.357	0.372
25	0.295	0.314	0.332	0.349	0.366	0.381	0.397	0.411	0.425	0.439
30	0.353	0.371	0.388	0.404	0.420	0.436	0.450	0.464	0.478	0.491
35	0.406	0.424	0.439	0.454	0.469	0.482	0.495	0.509	0.522	0.535
40	0.437	0.454	0.469	0.484	0.499	0.513	0.526	0.539	0.552	0.564
45	0.473	0.489	0.503	0.517	0.531	0.544	0.557	0.570	0.582	0.594
50	0.503	0.518	0.532	0.546	0.560	0.572	0.584	0.596	0.608	0.619

Table 4.2 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 1.5$

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.458	1.434	1.412	1.392	1.373	1.355	1.338	1.320	1.301	1.285
10	2.152	2.025	1.940	1.874	1.816	1.765	1.718	1.678	1.641	1.607
15	2.357	2.184	2.064	1.976	1.909	1.846	1.795	1.751	1.710	1.672
20	2.445	2.229	2.103	2.016	1.943	1.886	1.835	1.790	1.747	1.710
25	2.422	2.227	2.094	2.009	1.935	1.880	1.832	1.785	1.748	1.709
30	2.448	2.238	2.101	2.007	1.936	1.887	1.828	1.784	1.746	1.711
35	2.416	2.205	2.079	1.989	1.917	1.859	1.808	1.764	1.726	1.696
40	2.384	2.181	2.058	1.970	1.905	1.852	1.803	1.760	1.722	1.691
45	2.363	2.160	2.035	1.947	1.880	1.825	1.781	1.739	1.706	1.674
50	2.338	2.131	2.007	1.924	1.861	1.809	1.760	1.721	1.687	1.656

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.268	1.253	1.238	1.221	1.206	1.193	1.177	1.163	1.149	1.136
10	1.575	1.542	1.512	1.487	1.462	1.438	1.415	1.393	1.372	1.354
15	1.640	1.608	1.578	1.552	1.527	1.504	1.481	1.460	1.439	1.420
20	1.676	1.647	1.616	1.591	1.566	1.542	1.521	1.498	1.479	1.461
25	1.675	1.646	1.617	1.590	1.566	1.544	1.522	1.501	1.480	1.461
30	1.679	1.648	1.622	1.597	1.574	1.550	1.531	1.511	1.492	1.474
35	1.666	1.638	1.612	1.587	1.564	1.543	1.522	1.503	1.486	1.468
40	1.663	1.638	1.611	1.586	1.564	1.543	1.524	1.505	1.487	1.470
45	1.645	1.618	1.597	1.575	1.555	1.535	1.517	1.498	1.482	1.466
50	1.629	1.603	1.580	1.558	1.538	1.520	1.502	1.486	1.470	1.454

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	1.108	1.080	1.054	1.026	0.999	0.973	0.948	0.923	0.899	0.873
10	1.315	1.279	1.246	1.216	1.186	1.159	1.133	1.108	1.084	1.060
15	1.382	1.348	1.317	1.287	1.259	1.233	1.207	1.185	1.161	1.139
20	1.424	1.391	1.359	1.329	1.301	1.274	1.251	1.228	1.207	1.186
25	1.426	1.394	1.365	1.337	1.311	1.287	1.265	1.243	1.223	1.203
30	1.441	1.411	1.381	1.353	1.328	1.304	1.282	1.260	1.241	1.222
35	1.435	1.408	1.379	1.354	1.330	1.308	1.286	1.266	1.246	1.228
40	1.438	1.408	1.382	1.357	1.333	1.311	1.290	1.271	1.252	1.235
45	1.436	1.407	1.381	1.356	1.333	1.313	1.292	1.274	1.256	1.239
50	1.425	1.399	1.375	1.353	1.332	1.312	1.293	1.276	1.259	1.243

results for the proposed sequential G.O.F. test.

One possible disagreement on the critical values derived here could be that the procedure could be affected by the choice of the plotting positions and the choice of seeds. Although this may be partially true, the effect of different plotting position methods has been reduced greatly by having a replication size of $N = 100{,}000$. Harter [83: 1624] notes that the differences in plotting positions become negligible as replication size increases beyond $N > 20$. To further examine this question, the median plotting positions used here was replaced with MATLAB's built-in percentile function and mean plotting position one at a time and the critical values via these two modified codes for Weibull($\beta = 0.5$) were derived. When these resultant critical values were compared to those of the original run, they were found to be exactly the same to the third decimal point. Therefore the resultant critical values from these runs were not reproduced. On the other hand, different seeds were found to not change the derived critical values to the third decimal point that will be discussed later in the following section.

4.3.2 Estimates of Variability in the Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. Tests

It is also essential to examine the degree of variability present in the critical values derived in this thesis to let the analyst have strong confidence in their use, because the critical values here were derived via a set of large Monte Carlo simulations in which the precision is defined by the number of replications performed. In order to quantify this variability four additional such runs with 100,000 trials each were made at all hypothesized shape parameter values, $\beta = 0.5(0.5)4$ to provide at least 5 samples of each critical value from which a rough estimate of variance could be calculated. Clough [26:4-9] calculated the standard deviation for only the shape values $\beta = 0.5$ and $\beta = 1$, since they seemed to demonstrate the greatest variability in the sample $\sqrt{b_1}$ and b_2 in the joint distributions he created. He states that the estimates of the standard deviations at $\beta = 0.5$ and $\beta = 1$ would presumably serve as an upper bound on the estimates for the larger shapes, which seemed to exhibit less variability in the sample moments. The author, not objecting to and being parallel to Clough's [26] idea, thinks that variability in critical values can be better quantified for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. test critical values with 5 simulation runs for all shape values. Thus, the estimates of the variability for upper and lower tail $\sqrt{b_1}$ and Q-Statistic critical values calculated based on the original and four additional runs are given in Appendix B along with the upper and lower critical values for the original run. For illustrative purposes, the upper and lower tail variance tables for the $\sqrt{b_1}$ and the Q-Statistics G.O.F. tests for Weibull($\beta = 1.0$) are given in Tables 4.3 through 4.6.

As a summary, an examination of the lower and upper tail standard deviations shows us the fact that the critical values for the smaller hypothesized shape parameter values exhibit more variability as expected. It is observed that the standard deviations generally increase with the sample size, consistent with the scatter plots for the shape parameter values presented before and greater variability can be observed in the Q-Statistic critical values than $\sqrt{b_1}$, especially at smaller shape parameter values. When the variability of the critical values observed here is compared to that of the $\sqrt{b_1}$ and b_2 G.O.F. test critical values in Clough's [26] thesis, we find almost identical results for

Table 4.3 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.003	0.005	0.007	0.000	0.004	0.006	0.006	0.002	0.000	0.004
10	0.002	0.001	0.002	0.000	0.002	0.004	0.003	0.002	0.000	0.001
15	0.006	0.003	0.004	0.007	0.006	0.011	0.004	0.004	0.007	0.003
20	0.005	0.005	0.004	0.006	0.006	0.008	0.005	0.006	0.006	0.004
25	0.004	0.003	0.003	0.003	0.004	0.003	0.003	0.005	0.003	0.003
30	0.005	0.003	0.005	0.003	0.003	0.005	0.005	0.003	0.003	0.003
35	0.006	0.006	0.006	0.007	0.004	0.009	0.006	0.004	0.007	0.006
40	0.002	0.003	0.003	0.005	0.002	0.007	0.002	0.002	0.005	0.002
45	0.005	0.005	0.006	0.009	0.006	0.008	0.006	0.006	0.009	0.005
50	0.007	0.006	0.005	0.004	0.006	0.006	0.007	0.006	0.004	0.007

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.002
10	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
15	0.003	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.004	0.004
20	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.004	0.000	0.001
25	0.002	0.003	0.003	0.003	0.002	0.002	0.001	0.004	0.003	0.003
30	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.003	0.001	0.002
35	0.002	0.003	0.003	0.003	0.002	0.003	0.002	0.003	0.003	0.002
40	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
45	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.006
50	0.005	0.006	0.005	0.005	0.002	0.003	0.003	0.006	0.005	0.004

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.003	0.004	0.003	0.003	0.004	0.003	0.004	0.003	0.003	0.003
10	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002
15	0.001	0.001	0.000	0.001	0.000	0.001	0.001	0.000	0.001	0.000
20	0.001	0.000	0.002	0.001	0.002	0.002	0.001	0.002	0.000	0.002
25	0.002	0.003	0.001	0.001	0.002	0.001	0.001	0.002	0.003	0.001
30	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.002	0.002
35	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.003	0.002
40	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.002	0.001
45	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.002	0.003	0.002
50	0.004	0.005	0.005	0.005	0.003	0.005	0.004	0.004	0.006	0.005

Table 4.4 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.017	0.004	0.002	0.002	0.014	0.010	0.003	0.003	0.003
10	0.003	0.009	0.002	0.010	0.009	0.011	0.010	0.002	0.001	0.005
15	0.003	0.003	0.001	0.000	0.002	0.002	0.003	0.002	0.002	0.002
20	0.006	0.009	0.004	0.004	0.008	0.012	0.005	0.005	0.004	0.005
25	0.005	0.019	0.008	0.009	0.004	0.012	0.014	0.004	0.003	0.005
30	0.007	0.009	0.001	0.006	0.006	0.006	0.011	0.003	0.002	0.005
35	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.002	0.002
40	0.011	0.006	0.010	0.008	0.007	0.013	0.009	0.012	0.010	0.014
45	0.009	0.006	0.008	0.011	0.009	0.009	0.009	0.008	0.007	0.007
50	0.005	0.006	0.004	0.007	0.006	0.008	0.008	0.006	0.006	0.004

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.004	0.003	0.001	0.003	0.002	0.004	0.002	0.003	0.002	0.001
10	0.005	0.003	0.004	0.004	0.006	0.005	0.005	0.002	0.007	0.004
15	0.008	0.009	0.010	0.008	0.007	0.007	0.007	0.009	0.006	0.010
20	0.006	0.006	0.007	0.005	0.003	0.004	0.003	0.006	0.004	0.007
25	0.007	0.004	0.005	0.007	0.006	0.006	0.006	0.004	0.005	0.005
30	0.004	0.003	0.004	0.004	0.003	0.005	0.004	0.003	0.002	0.004
35	0.003	0.001	0.002	0.001	0.003	0.001	0.004	0.001	0.001	0.001
40	0.006	0.006	0.006	0.007	0.007	0.006	0.005	0.008	0.006	0.006
45	0.002	0.002	0.002	0.003	0.004	0.004	0.003	0.003	0.003	0.002
50	0.009	0.008	0.008	0.009	0.006	0.008	0.006	0.010	0.006	0.008

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002
10	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.001
15	0.005	0.003	0.004	0.003	0.002	0.003	0.002	0.005	0.003	0.002
20	0.003	0.002	0.003	0.004	0.004	0.003	0.004	0.003	0.003	0.004
25	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.006	0.005	0.005
30	0.003	0.002	0.003	0.001	0.001	0.001	0.002	0.003	0.002	0.002
35	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.004	0.004	0.002
40	0.005	0.005	0.004	0.006	0.006	0.005	0.007	0.006	0.007	0.006
45	0.004	0.004	0.002	0.004	0.003	0.004	0.004	0.004	0.003	0.005
50	0.003	0.006	0.003	0.005	0.005	0.005	0.005	0.004	0.006	0.006

Table 4.5 Q-Statistic Lower Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.002	0.001	0.001	0.001	0.000	0.001	0.002	0.004	0.000	0.001
10	0.003	0.002	0.003	0.000	0.002	0.002	0.001	0.002	0.000	0.005
15	0.003	0.005	0.004	0.003	0.004	0.004	0.005	0.006	0.002	0.007
20	0.000	0.005	0.006	0.005	0.003	0.000	0.004	0.008	0.000	0.005
25	0.006	0.004	0.003	0.007	0.002	0.002	0.005	0.005	0.003	0.008
30	0.004	0.008	0.004	0.004	0.003	0.005	0.004	0.004	0.007	0.005
35	0.010	0.014	0.007	0.009	0.007	0.011	0.008	0.007	0.004	0.007
40	0.009	0.010	0.009	0.010	0.009	0.007	0.007	0.010	0.008	0.008
45	0.013	0.011	0.011	0.014	0.008	0.007	0.008	0.006	0.009	0.014
50	0.024	0.018	0.013	0.014	0.012	0.012	0.009	0.015	0.028	0.022

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.001	0.001	0.000	0.001	0.002	0.000	0.000	0.001	0.001	0.000
10	0.003	0.002	0.001	0.002	0.001	0.000	0.000	0.002	0.000	0.001
15	0.003	0.005	0.002	0.002	0.003	0.003	0.004	0.004	0.003	0.002
20	0.000	0.004	0.002	0.003	0.001	0.003	0.002	0.003	0.004	0.003
25	0.004	0.003	0.004	0.004	0.002	0.003	0.002	0.006	0.003	0.004
30	0.003	0.005	0.005	0.003	0.003	0.005	0.003	0.004	0.005	0.005
35	0.009	0.006	0.006	0.005	0.005	0.010	0.009	0.002	0.006	0.010
40	0.014	0.007	0.008	0.006	0.008	0.009	0.005	0.008	0.005	0.008
45	0.008	0.009	0.007	0.009	0.006	0.009	0.010	0.006	0.005	0.006
50	0.012	0.013	0.011	0.008	0.009	0.011	0.006	0.010	0.008	0.007

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.001	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.000	0.002
10	0.002	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.000
15	0.002	0.001	0.004	0.002	0.003	0.002	0.001	0.007	0.001	0.000
20	0.003	0.002	0.003	0.003	0.004	0.003	0.003	0.004	0.004	0.002
25	0.000	0.004	0.004	0.003	0.003	0.004	0.005	0.003	0.003	0.004
30	0.002	0.003	0.005	0.002	0.002	0.003	0.002	0.005	0.002	0.005
35	0.005	0.003	0.004	0.004	0.004	0.006	0.005	0.004	0.005	0.008
40	0.003	0.002	0.009	0.006	0.005	0.007	0.008	0.008	0.007	0.005
45	0.008	0.007	0.008	0.007	0.006	0.009	0.007	0.006	0.005	0.007
50	0.009	0.011	0.009	0.008	0.009	0.008	0.009	0.010	0.007	0.004

Table 4.6 Q-Statistic Upper Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.001	0.002	0.000	0.003	0.001	0.003	0.000	0.001	0.000
10	0.000	0.003	0.004	0.002	0.001	0.000	0.001	0.000	0.000	0.003
15	0.000	0.000	0.003	0.005	0.002	0.000	0.004	0.004	0.004	0.005
20	0.005	0.003	0.003	0.007	0.003	0.003	0.000	0.005	0.003	0.003
25	0.003	0.005	0.004	0.009	0.001	0.002	0.003	0.003	0.005	0.002
30	0.004	0.002	0.005	0.004	0.004	0.004	0.005	0.007	0.007	0.004
35	0.005	0.006	0.007	0.006	0.005	0.006	0.005	0.004	0.007	0.000
40	0.010	0.005	0.005	0.007	0.006	0.007	0.006	0.010	0.013	0.008
45	0.009	0.004	0.006	0.008	0.007	0.006	0.010	0.006	0.008	0.009
50	0.009	0.010	0.011	0.010	0.012	0.008	0.008	0.008	0.010	0.013

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001
10	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001
15	0.004	0.005	0.001	0.003	0.003	0.005	0.002	0.001	0.002	0.000
20	0.005	0.006	0.003	0.006	0.003	0.007	0.002	0.006	0.004	0.005
25	0.004	0.006	0.004	0.002	0.003	0.004	0.003	0.003	0.003	0.003
30	0.003	0.005	0.006	0.006	0.007	0.005	0.006	0.007	0.006	0.008
35	0.005	0.006	0.003	0.006	0.003	0.007	0.002	0.006	0.004	0.005
40	0.004	0.010	0.008	0.008	0.007	0.011	0.008	0.009	0.008	0.008
45	0.008	0.013	0.007	0.009	0.005	0.011	0.005	0.009	0.007	0.007
50	0.009	0.011	0.009	0.011	0.008	0.013	0.009	0.010	0.011	0.010

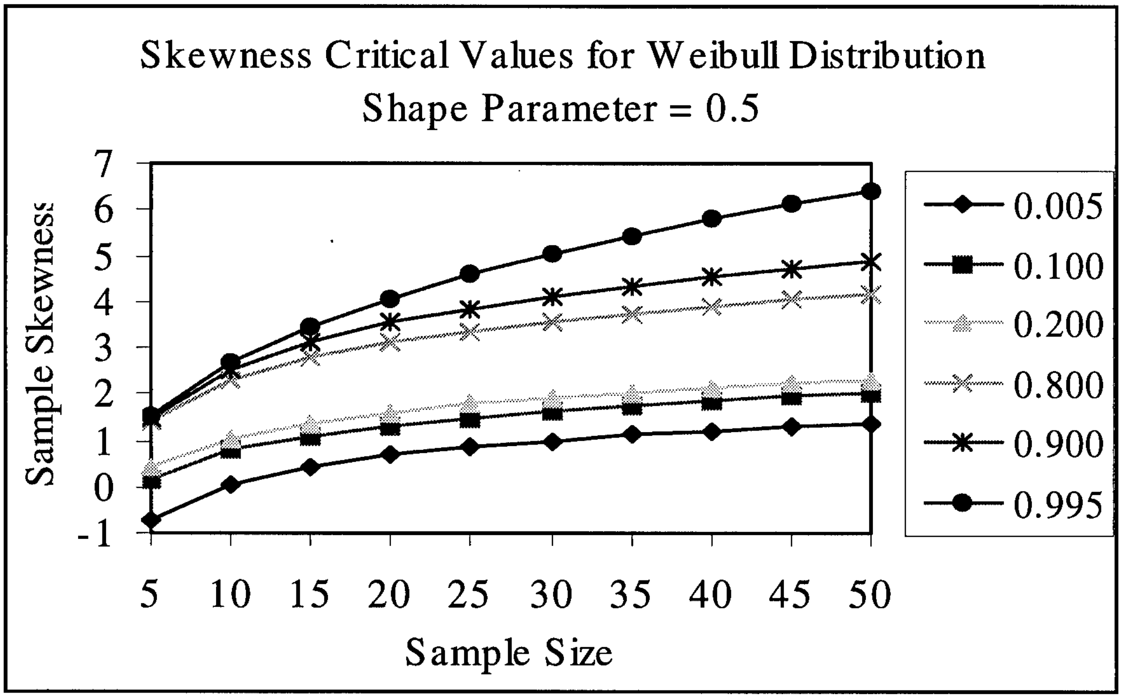
Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003
10	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
15	0.003	0.003	0.003	0.001	0.001	0.002	0.001	0.002	0.001	0.002
20	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.001
25	0.000	0.001	0.001	0.001	0.000	0.001	0.001	0.000	0.000	0.000
30	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.003	0.002	0.002
35	0.002	0.000	0.000	0.002	0.002	0.001	0.001	0.002	0.002	0.002
40	0.005	0.003	0.004	0.003	0.001	0.001	0.001	0.001	0.001	0.001
45	0.004	0.005	0.008	0.006	0.006	0.003	0.004	0.005	0.004	0.004
50	0.008	0.006	0.010	0.006	0.005	0.005	0.006	0.007	0.006	0.005

$\sqrt{b_1}$ G.O.F. test critical values while the Q-Statistic G.O.F. test critical values appeared to exhibit less variability than the b_2 critical values did. This observation suggests that Q-Statistic G.O.F. test statistic may be a better discriminant than b_2. Besides, due to the fact that most of the standard deviations presented in the tables in Appendix B are only significant in the third decimal place, they are quite acceptable.

Thus, it can be concluded, the analyst could have a great amount of confidence in the critical values tabled from this research. As noted before, according to the Shooman's rule, $\frac{1}{\sqrt{N}}$, we have to have a replication size of over 1,000,000 to be able to reduce the significance of the error to the fourth decimal place [141:259]. This big a replication size was not the choice in this research, because given the relatively low variance that can be observed from the tables mentioned and illustrated above, this additional cost in the computer times was deemed unacceptable.

4.3.3 Observations on the Critical Values

Some useful information can also be gathered by carefully examining the tabled critical values and observing the notable trends in their behavior according to Weibull shape and sample sizes and comparing them to the observations from the aforementioned joint distributions. One means to visualize these trends is to plot the lower and upper critical values at the selected significance levels against all the sample sizes for a given Weibull distribution shape that can be viewed in Appendix L. H_o: Weibull ($\beta = 0.5$ and 3.5) were presented in Figure 4.9 and 4.10 to exemplify these plots. Another means is to plot the lower and upper critical values at the same selected significance levels against all



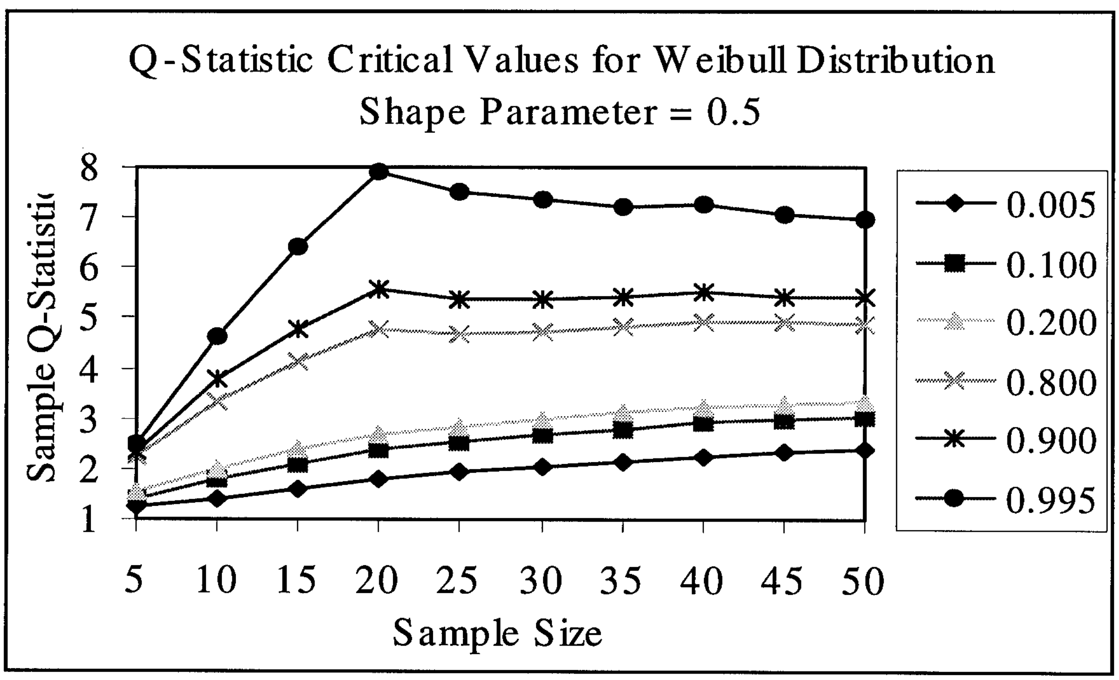
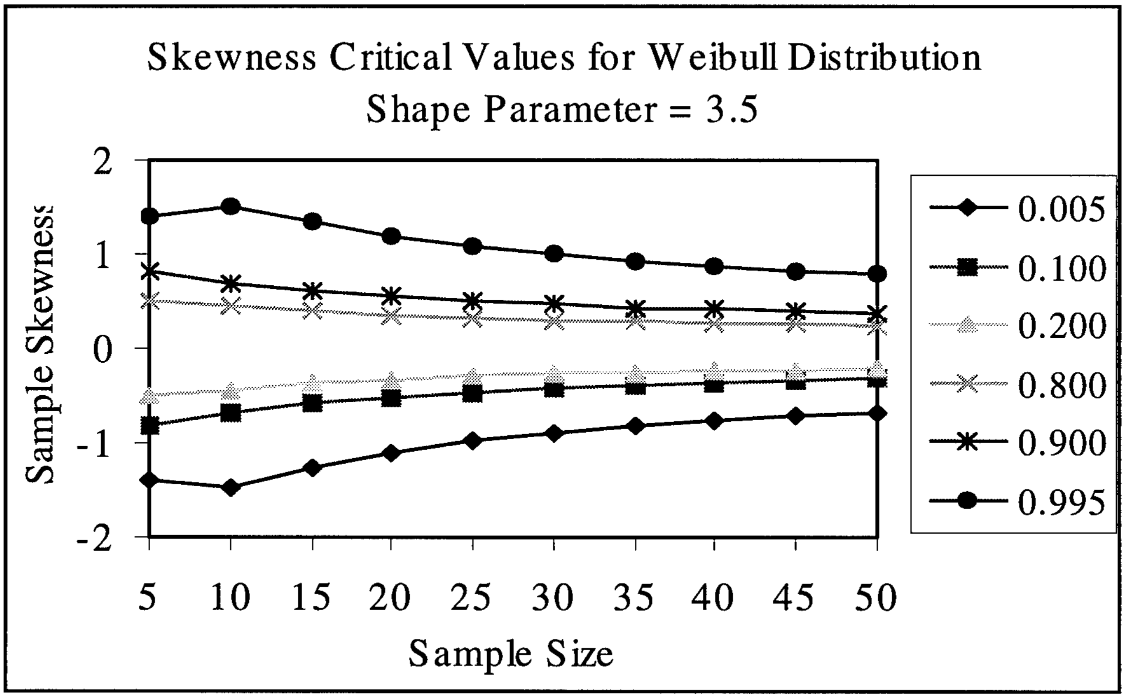


Figure 4.9 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_o : Weibull($\beta = 0.5$); $n = 5(5)50$.



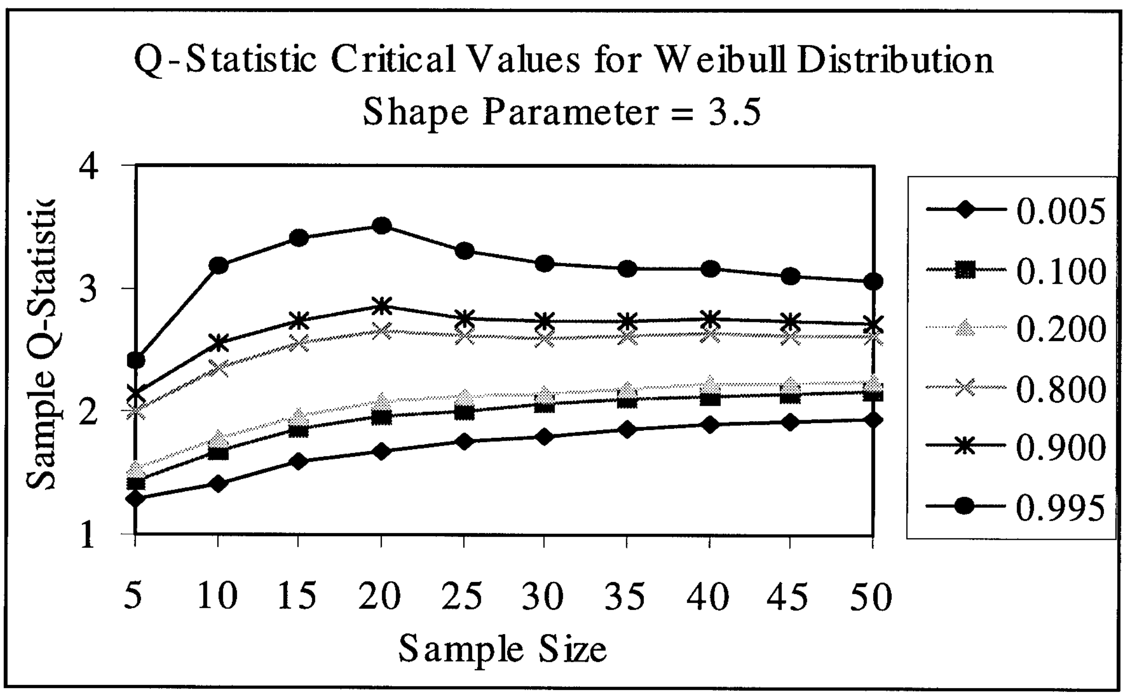
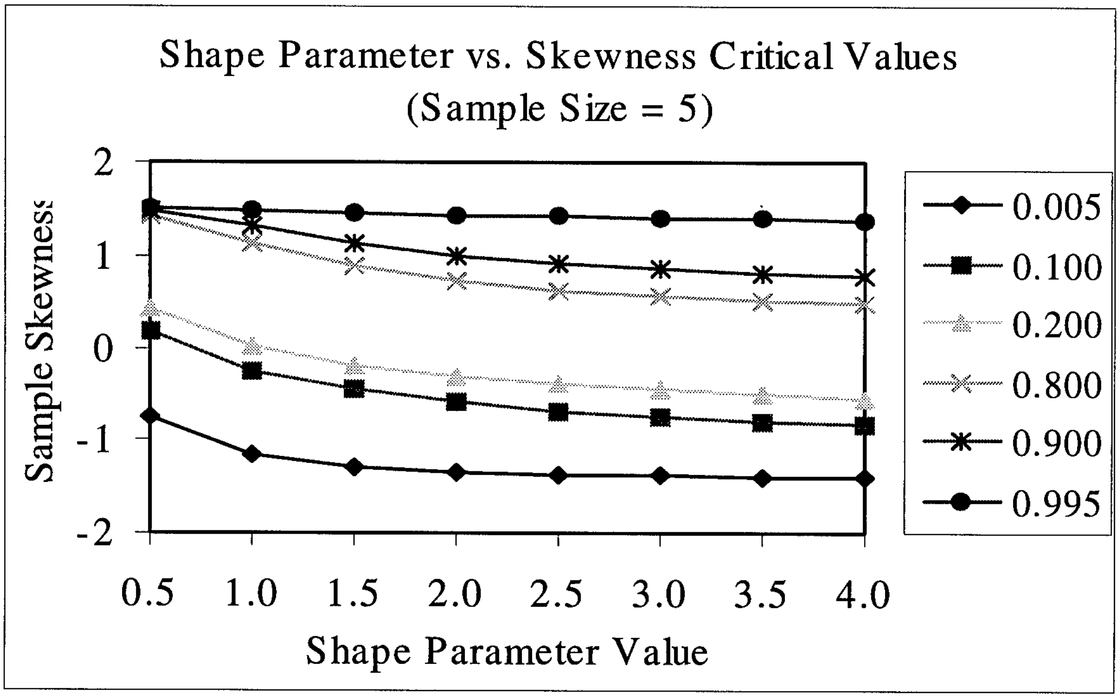


Figure 4.10 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_o : Weibull($\beta = 3.5$); $n = 5(5)50$.



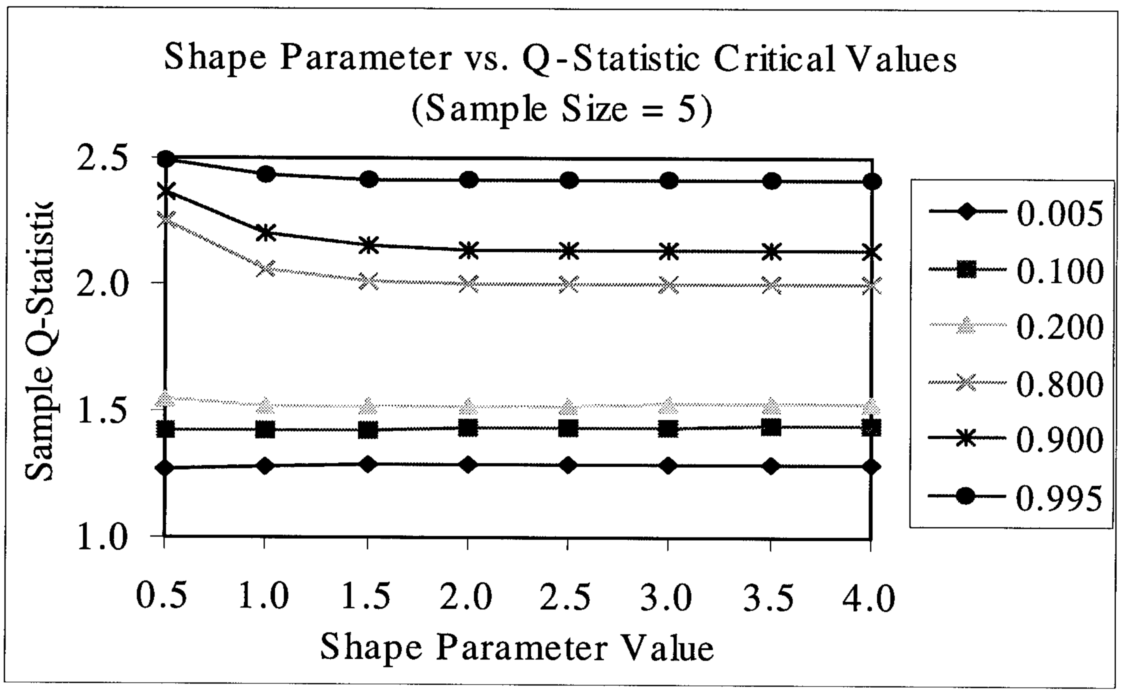
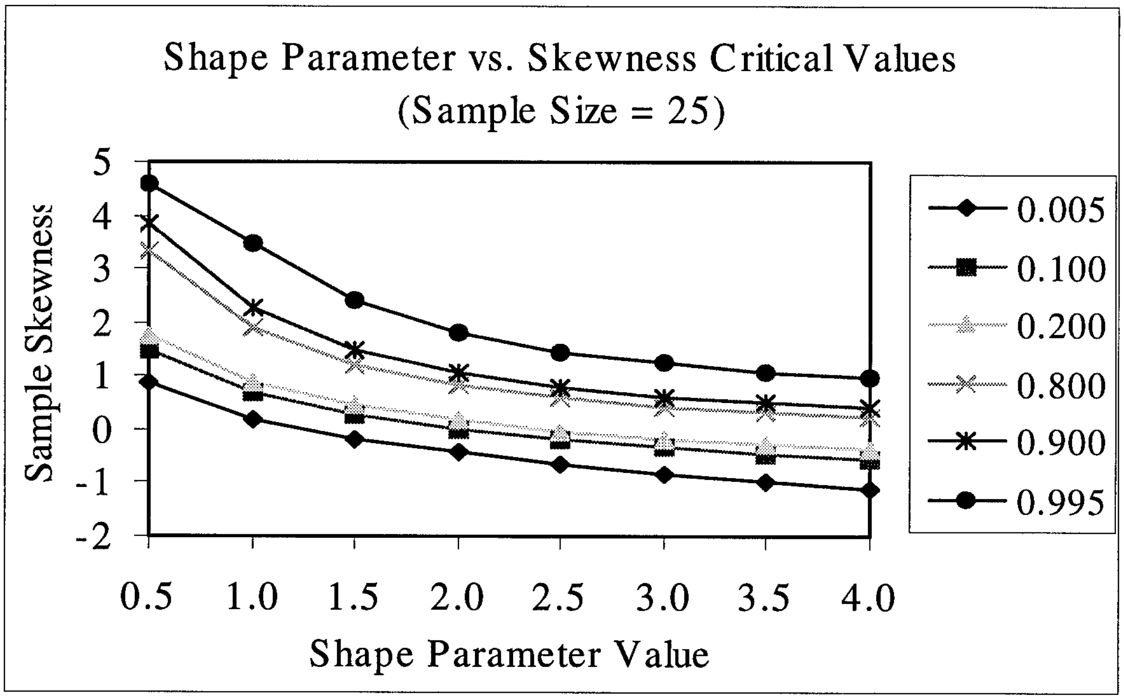


Figure 4.11 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull($\beta = 0.5(0.5)4.0$); $n = 5$.



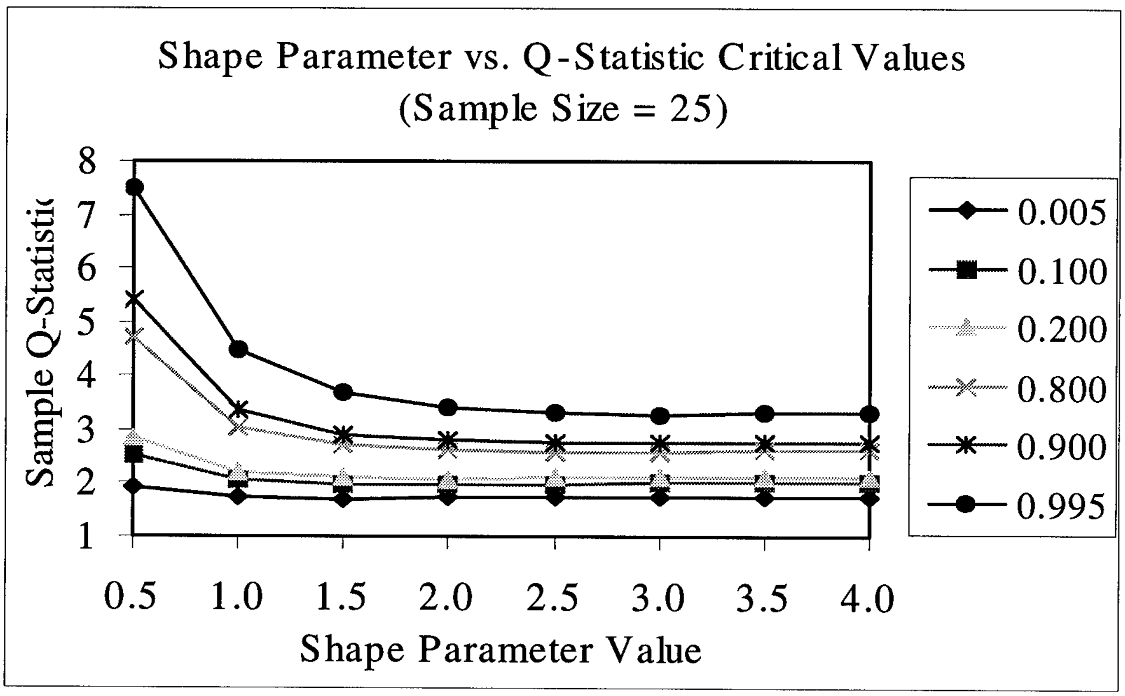


Figure 4.12 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_0 : Weibull($\beta = 0.5(0.5)4.0$); $n = 25$.

the Weibull distribution shape parameter values for a given sample size that can be viewed in Appendix M. $n = 5$ and 25 were presented in Figure 4.11 and 4.12 to exemplify these plots. These plots for $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics should be examined by keeping the true theoretical values for $\sqrt{\beta_1}$ and Q-Statistic G.O.F. test statistics in mind that can be seen in Table 3.9. In each plot, the critical values at 0.005, 0.100, and 0.200 α-levels were selected to be plotted in the lower tail critical values and the critical values at 0.800, 0.900 and 0.995 α-levels were selected to be plotted in the upper critical values. In order to find which significance level the desired trend line corresponds with, one should refer to the legend located on the right side of these plots.

The behavior of the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics can be observed via these plots. As the sample size increases, both $\sqrt{b_1}$ and Q-Statistic lower tail critical values are monotonically increasing for all shape parameter values except the $\sqrt{b_1}$ lower tail critical values for $\beta = 3.5$ and 4.0 decrease from $n = 5$ to $n = 10$, then increase for the rest of the sample sizes. On the other hand, the behavior of the upper tail critical values depends on the shape parameter values. For $\beta = 0.5$ and 1.0, while the $\sqrt{b_1}$ upper tail critical values monotically increase, the Q-Statistic upper tail critical values increase up to $n = 20$ and then decrease at $\alpha = 0.995$ and stay nearly the same at $\alpha = 0.900$ and 0.800 for the rest of the sample sizes. For $\beta = 1.5$, while the Q-Statistic upper tail critical values increase up to $n = 20$ and then decrease for the rest of the sample sizes for the critical values at $\alpha = 0.995$ and stay nearly the same for $\alpha = 0.900$ and 0.800 , the $\sqrt{b_1}$ upper tail critical values at the $\alpha = 0.995$ increase up to $n = 20$ and then decrease very slightly. The

$\sqrt{b_1}$ upper tail critical values at $\alpha = 0.900$ and 0.800 increase up to n = 10 and stay nearly the same for the remaining sample sizes. For $\beta = 2.0$, 2.5, and 3.0, while the Q-Statistic upper tail critical values increase up to n = 20 and then decrease for the rest of the sample sizes for the critical values at $\alpha = 0.995$ and stay nearly the same for $\alpha = 0.900$ and 0.800, the $\sqrt{b_1}$ upper tail critical values at the $\alpha = 0.995$ increase up to n = 10 and decrease very slightly. The $\sqrt{b_1}$ upper tail critical values at the $\alpha = 0.900$ and 0.800 stay nearly the same for all of the sample sizes after the initial increase up to n =10. For $\beta = 3.5$ and 4.0, while the Q-Statistic upper tail critical values behave the same as they did for $\beta = 2.0$, 2.5, and 3.0, the $\sqrt{b_1}$ upper tail critical values at $\alpha = 0.995$ increase up to n = 10 and then decrease very slightly for the rest of the sample sizes. The $\sqrt{b_1}$ upper tail critical values at $\alpha = 0.900$ and 0.800 decrease slightly for all of the sample sizes.

As can be seen in Table 3.9, the true skewness and Q-Statistic values do decrease as the Weibull distribution shape parameter value goes up. Therefore, the lower and upper tail critical values are expected to converge about these true theoretical values as the sample size increases. The plots above do demonstrate this fact. By examining the trend lines in the plots above, it is observed that the sample $\sqrt{b_1}$ lower and upper critical values converge faster than those do for the Q-Statistic. Besides, the comparatively high variability present in the scatter plots for the Q-Statistic is also observed from these plots. The upper tail critical values at $\alpha = 0.995$ demonstrate spread from those at $\alpha = 0.900$ and 0.800, and the slope of convergence toward the true skewness and Q-Statistic values with the increasing sample size is very slight in all cases. Even though the slope of convergence toward the true skewness and Q-Statistic values with the increasing sample size is not as slow as the upper tail critical values at $\alpha = 0.995$, the lower tail critical values at $\alpha = 0.005$ demonstrate comparatively higher spread from those at $\alpha = 0.100$ and 0.200. These observations give some useful insights about the characteristic of the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics considered in this research. The Q-Statistic G.O.F. test will probably show greater discriminatory power when the alternate distributions used in the study have lower Q-Statistic values. The lower variability in the $\sqrt{b_1}$ G.O.F. test statistic compared to the Q-Statistic G.O.F. test statistic will probably make the $\sqrt{b_1}$ G.O.F. test have a greater discriminatory power than that of Q-Statistic G.O.F. test in most cases. Besides, the fact that all the Q-Statistic upper tail critical values at sample size $n = 5$ are almost identical for each shape value considered here and the noticeable jump in the upper tail Q-Statistic critical values from $n = 5$ to 10, show us that the distribution of the Q-Statistic G.O.F. test statistic is tightly constrained within particular limits. Suggesting good power for the small sample sizes, this observation agrees with the unusual shape of the fixed boundaries in the scatter plots for the sample size 5 on the ($\sqrt{b_1}$,Q-Statistic) plane. The power study will validate these observations empirically.

4.4 Attained Significance Levels

4.4.1 Attained Significance Level Tables vs Attained Significance Level Contour Plots

The next course of action was to come up with the attained significance levels for the sequential G.O.F. test after deriving and analyzing the critical values for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Due to the nature of the sequential G.O.F. testing, the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests can be utilized at any number of significance levels, each of which yields a different overall significance level for the new sequential G.O.F. test procedure studied here, as mentioned in Chapter 3 in detail. Therefore, it is crystal clear that the proposed sequential G.O.F. test procedure could not be advanced any further without any sufficient assessment of the attained significance levels for the sequential G.O.F. test. Because the analyst needs to choose the appropriate significance levels of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics so that they result in the desired overall significance level for the sequential G.O.F. test procedure.

Monte Carlo simulation was the tool of choice to approximate the desired overall significance levels. The Monte Carlo simulation method was employed for all combinations of significance levels for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests from $\alpha =$ 0.01(0.01)0.20, Weibull($\beta = 0.5(0.5)4.0$), and $n = 5(5)50$. The Monte Carlo simulation results for the attained significance levels are presented in tabular format in Appendix C. For illustrative purposes, the attained significance simulation results for the Weibull($\beta =$ 1.0) and $n = 30$ are presented in Table 4.7.

As can be realized easily by taking a look at the attained significance level tables, the use of these tables is very inefficient in practice. The analyst has to find an attained significance level that is nearly equal to the desired significance level for the sequential G.O.F. test, and then pick out the Q-Statistic G.O.F. test significance level from the column heading and the $\sqrt{b_1}$ G.O.F. test significance from the row heading. Understanding the difficulty level of this procedure, a more user-friendly and efficient method of presenting the results of Monte Carlo simulation for the attained significance levels was developed in the form of contour plots that depict the data in the attained significance level tables in parallel to Clough's [26] effort. Both the tables and the corresponding contour plots for the combinations of all Weibull($\beta = 0.5(0.5)4.0$) and n = 5(5)50 were produced and presented in the Appendix C and D, respectively. Only presenting the contour plots that corresponds to the attained significance levels for each shape/sample size would be a viable option due to its ease of use. On the other hand, it is preferred to present both attained significance tables and the corresponding contour plots to take care of different preferences of analysts and to prepare a means for a cross-reference. Again, the contour plot for the attained significance levels for Weibull($\beta = 1.0$) and $n = 30$ are presented in Figure 4.13 for illustrative purposes and for giving the reader a feel for which one would be more convenient for him/her to use in his/her studies. Keeping in mind that the only real challenge is deciding upon which combination of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. test significance levels to utilize, the visual approach via the contour plots corresponding to the attained significance levels should simplify the utilization of the new sequential G.O.F. test procedure to a great extent. The analyst only needs to identify the contour line corresponding to the desired attained significance level and then pick out the appropriate combination of significance levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics by referring to the corresponding axes.

4.4.2 Estimates of the Variability of the Attained Significance Levels

The same method that was used for estimating the variability in the power study results was used in order to approximate the variance estimates for the attained significance levels. Due to the fact that the attained significance levels are proportions, the expression (4.1) can be used for finding the estimate of the standard error.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{N}} \tag{4.1}$$

where $\hat{p}$ is the estimate of the true attained significance level value and p is the true attained significance level. The N in the denominator in this formula stands for the number of trials of each sample size or replication size that were run to generate the attained significance level results. Due to the fact that it is p that is being estimated, the formula cannot be evaluated directly by the analyst. On the other hand, because the formula is maximized using $p = 0.5$, it is possible to come up with an upper bound. As can be observed from Appendices C and D, since the largest significance level observed rarely exceeds 0.30, the estimate of the variation will be an extremely conservative one in practice. Consequently, $\sigma_{\hat{p}} = 0.00158$ can be found by substituting $N = 100{,}000$ and $p = 0.5$ in equation (4.1). As a result, the analyst can be confident in at least the first two decimal places of these attained significance level results derived via Monte Carlo simulation method.

4.4.3 Example of Use of the Attained Significance Level Tables/Plots

At this point of this research, an illustration of how to utilize the attained significance tables and/or contour plots may be beneficial to the reader. As mentioned before, the first course of action in order to employ the sequential G.O.F. test is to decide upon what significance level is desired for the overall sequential G.O.F. test. Given that desired overall attained significance level, the analyst makes use of the attained significance level tables and/or contour plots to be able to come up with the significance levels of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests.

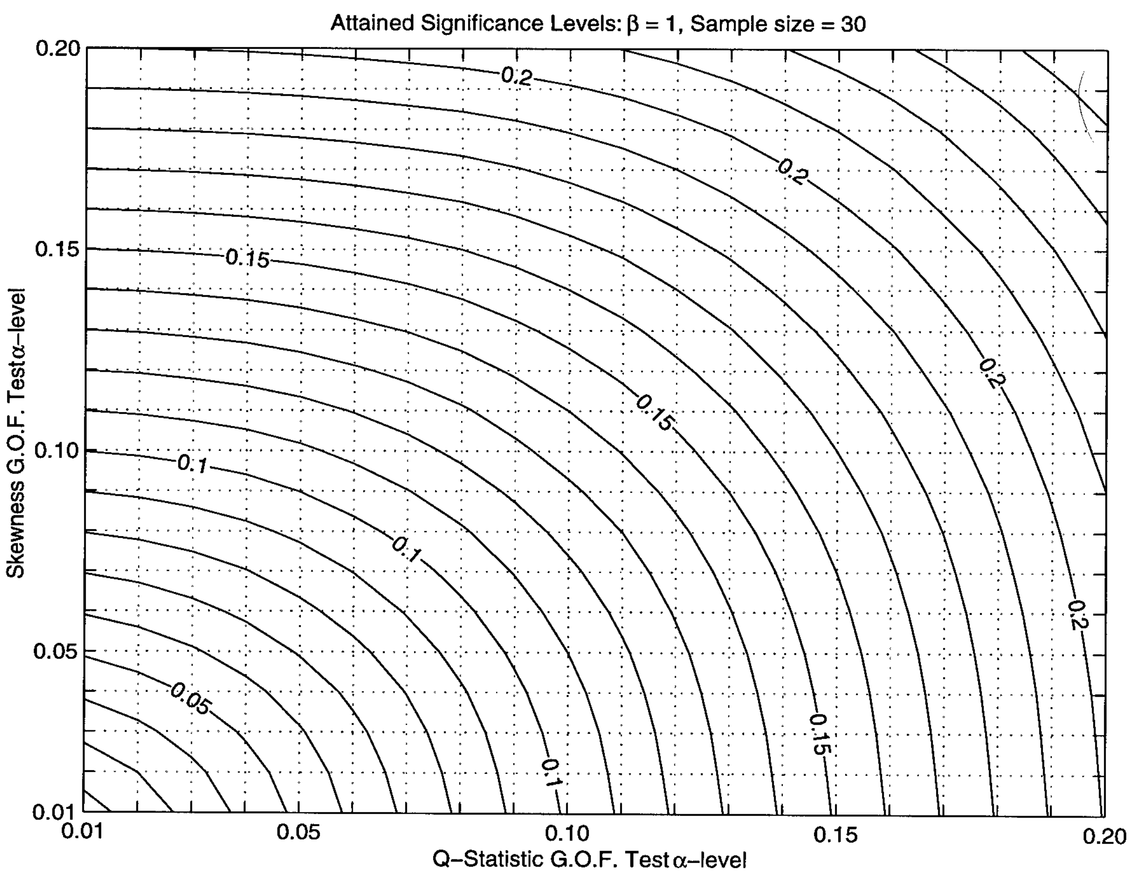


Figure 4.13 The Attained Significance Level Contour Plot for $\beta = 1.0$ and $n = 30$.

Table 4.7 (a) Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 25.

		Q-Statistic G.O.F. Test Significance Level (alpha)																			
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
Skewness G.O.F. Test Significance Level	0.01	0.015	0.024	0.033	0.043	0.053	0.063	0.072	0.082	0.092	0.102	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.181	0.191	0.201
	0.02	0.024	0.030	0.038	0.046	0.056	0.065	0.075	0.085	0.094	0.104	0.114	0.123	0.133	0.143	0.153	0.162	0.172	0.182	0.192	0.202
	0.03	0.033	0.038	0.044	0.051	0.060	0.068	0.078	0.087	0.096	0.106	0.116	0.125	0.135	0.145	0.154	0.164	0.174	0.183	0.193	0.203
	0.04	0.042	0.046	0.051	0.057	0.065	0.072	0.081	0.090	0.099	0.108	0.118	0.127	0.137	0.146	0.156	0.166	0.175	0.185	0.194	0.204
	0.05	0.052	0.055	0.059	0.064	0.070	0.077	0.085	0.093	0.102	0.111	0.120	0.129	0.139	0.148	0.158	0.167	0.177	0.186	0.196	0.206
	0.06	0.062	0.064	0.068	0.072	0.077	0.083	0.090	0.098	0.106	0.114	0.123	0.132	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207
	0.07	0.071	0.073	0.076	0.080	0.084	0.090	0.096	0.102	0.110	0.118	0.126	0.134	0.143	0.152	0.162	0.171	0.180	0.189	0.199	0.208
	0.08	0.081	0.083	0.085	0.088	0.092	0.097	0.102	0.108	0.115	0.122	0.130	0.138	0.146	0.155	0.164	0.173	0.182	0.191	0.200	0.210
	0.09	0.091	0.092	0.095	0.097	0.101	0.104	0.109	0.114	0.120	0.127	0.134	0.142	0.150	0.158	0.167	0.175	0.184	0.193	0.202	0.212
	0.10	0.101	0.102	0.104	0.106	0.109	0.112	0.116	0.121	0.126	0.132	0.139	0.146	0.154	0.161	0.170	0.178	0.187	0.195	0.204	0.213
	0.11	0.111	0.112	0.113	0.115	0.118	0.121	0.124	0.128	0.133	0.138	0.144	0.151	0.158	0.165	0.173	0.181	0.189	0.198	0.206	0.215
	0.12	0.120	0.121	0.123	0.125	0.127	0.129	0.133	0.136	0.140	0.145	0.150	0.157	0.163	0.170	0.177	0.185	0.192	0.201	0.209	0.218
	0.13	0.130	0.131	0.132	0.134	0.136	0.138	0.141	0.144	0.148	0.152	0.157	0.162	0.168	0.175	0.181	0.189	0.196	0.204	0.212	0.220
	0.14	0.140	0.141	0.142	0.143	0.145	0.147	0.150	0.152	0.156	0.159	0.164	0.169	0.174	0.180	0.186	0.193	0.200	0.207	0.215	0.223
	0.15	0.150	0.151	0.152	0.153	0.154	0.156	0.159	0.161	0.164	0.167	0.171	0.176	0.180	0.186	0.192	0.198	0.204	0.211	0.219	0.227
	0.16	0.160	0.161	0.161	0.163	0.164	0.166	0.168	0.170	0.172	0.175	0.179	0.183	0.187	0.192	0.197	0.203	0.209	0.216	0.223	0.230
	0.17	0.170	0.171	0.171	0.172	0.173	0.175	0.177	0.179	0.181	0.184	0.187	0.190	0.194	0.199	0.204	0.209	0.215	0.221	0.227	0.234
	0.18	0.180	0.180	0.181	0.182	0.183	0.184	0.186	0.188	0.190	0.192	0.195	0.198	0.202	0.206	0.210	0.215	0.220	0.226	0.232	0.239
	0.19	0.190	0.190	0.191	0.192	0.193	0.194	0.195	0.197	0.199	0.201	0.204	0.207	0.210	0.213	0.217	0.222	0.227	0.232	0.237	0.243
	0.20	0.200	0.200	0.201	0.201	0.202	0.203	0.205	0.206	0.208	0.210	0.212	0.215	0.218	0.221	0.224	0.228	0.233	0.237	0.242	0.248

Table 4.7 (b) Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 30.

		Q-Statistic G.O.F. Test Significance Level																			
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
Skewness G.O.F. Test Significance Level	0.01	0.016	0.024	0.033	0.042	0.052	0.062	0.072	0.082	0.091	0.101	0.111	0.121	0.131	0.141	0.151	0.161	0.171	0.181	0.191	0.201
	0.02	0.024	0.030	0.038	0.046	0.055	0.064	0.074	0.083	0.093	0.103	0.112	0.122	0.132	0.142	0.152	0.162	0.171	0.181	0.191	0.201
	0.03	0.033	0.038	0.044	0.051	0.059	0.068	0.076	0.086	0.095	0.104	0.114	0.124	0.133	0.143	0.153	0.163	0.172	0.182	0.192	0.202
	0.04	0.042	0.046	0.051	0.057	0.064	0.072	0.080	0.089	0.098	0.107	0.116	0.125	0.135	0.144	0.154	0.164	0.173	0.183	0.193	0.203
	0.05	0.051	0.055	0.059	0.064	0.071	0.078	0.085	0.093	0.102	0.110	0.119	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.204
	0.06	0.061	0.064	0.067	0.072	0.078	0.084	0.091	0.098	0.106	0.114	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.186	0.195	0.205
	0.07	0.071	0.073	0.076	0.080	0.085	0.090	0.096	0.103	0.111	0.118	0.126	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.197	0.206
	0.08	0.080	0.082	0.085	0.088	0.092	0.097	0.103	0.109	0.116	0.123	0.130	0.138	0.146	0.154	0.163	0.171	0.180	0.189	0.199	0.208
	0.09	0.090	0.092	0.094	0.096	0.100	0.105	0.110	0.115	0.122	0.128	0.135	0.142	0.150	0.158	0.166	0.175	0.183	0.192	0.201	0.210
	0.10	0.100	0.101	0.103	0.105	0.108	0.112	0.117	0.122	0.128	0.134	0.140	0.147	0.155	0.162	0.170	0.178	0.186	0.195	0.203	0.212
	0.11	0.110	0.111	0.113	0.114	0.117	0.120	0.124	0.129	0.134	0.140	0.146	0.152	0.159	0.166	0.174	0.181	0.189	0.197	0.206	0.215
	0.12	0.120	0.121	0.122	0.124	0.126	0.129	0.132	0.136	0.141	0.146	0.152	0.158	0.164	0.171	0.178	0.186	0.193	0.201	0.209	0.218
	0.13	0.130	0.131	0.132	0.133	0.135	0.138	0.140	0.144	0.148	0.153	0.158	0.164	0.170	0.176	0.183	0.190	0.197	0.204	0.212	0.220
	0.14	0.140	0.141	0.141	0.143	0.144	0.146	0.149	0.152	0.156	0.160	0.165	0.170	0.175	0.182	0.188	0.194	0.201	0.208	0.216	0.224
	0.15	0.150	0.150	0.151	0.152	0.154	0.155	0.157	0.160	0.163	0.167	0.171	0.176	0.181	0.187	0.193	0.199	0.206	0.213	0.220	0.227
	0.16	0.160	0.160	0.161	0.162	0.163	0.165	0.166	0.168	0.171	0.175	0.179	0.183	0.188	0.193	0.199	0.204	0.211	0.217	0.224	0.231
	0.17	0.170	0.170	0.171	0.171	0.172	0.174	0.175	0.177	0.179	0.183	0.186	0.190	0.194	0.199	0.204	0.210	0.216	0.222	0.228	0.235
	0.18	0.180	0.180	0.181	0.181	0.182	0.183	0.184	0.186	0.188	0.191	0.194	0.197	0.201	0.206	0.210	0.215	0.221	0.227	0.233	0.239
	0.19	0.190	0.190	0.191	0.191	0.192	0.193	0.194	0.195	0.197	0.199	0.202	0.205	0.208	0.212	0.217	0.221	0.227	0.232	0.238	0.244
	0.20	0.200	0.200	0.200	0.201	0.202	0.202	0.203	0.205	0.206	0.208	0.210	0.213	0.216	0.220	0.224	0.228	0.233	0.238	0.243	0.249

Now, let's take a look at an example that demonstrates how to use this approach. Let's assume that an analyst wants to conduct the sequential G.O.F. test at a desired overall significance level $\alpha = 0.10$ with H_o: Weibull($\beta = 1.0$) for $n = 30$ and his prior analysis shows that the sample data come from a skewed distribution. He could realize that there are many possibilities by taking a look at the Table 4.7 and contour plot in Figure 4.13. Some of the combinations of the individual G.O.F. test significance level that result in nearly 0.10 are presented in Table 4.8. Note that if significance levels of the component tests are chosen using the contour plots, they should be verified in the corresponding tables to acquire better precision.

Table 4.8 Some of the Potential Attained Significance Level Combinations for the Sequential G.O.F. Test Procedure.

Significance Levels (α)		
$\sqrt{b_1}$ G.O.F. Test	Q-Statistic G.O.F. Test	Sequential G.O.F. Test
0.10	0.01	0.100
0.09	0.05	0.100
0.01	0.10	0.101
0.05	0.09	0.102
0.07	0.07	0.096

Since the analyst knows that the data come from a skewed distribution via his prior analysis, he will have better discriminatory power if he chooses to utilize the first combination in Table 4.8, because he wants to have the largest rejection region for the skewness G.O.F. test. Therefore, the sequential G.O.F. test should be conducted at overall desired significance level, $\alpha = 0.10$ by conducting the $\sqrt{b_1}$ G.O.F. test at the significance level $\alpha_1 = 0.10$ and the Q-Statistics G.O.F. test at the significance level $\alpha_2 = 0.01$. Next, the corresponding critical values can be obtained from the critical value tables in Appendix B. For the $\sqrt{b_1}$ G.O.F. test at $\alpha_1 = 0.10$, the lower critical value can be found in the $\frac{\alpha_1}{2} = 0.5$ column and is 0.628; the upper tail critical value is located in the $\left(1-\frac{\alpha_1}{2}\right) = 0.95$ column and is 2.658. For the Q-Statistic G.O.F. test at $\alpha_2 = 0.01$, the lower critical value can be found in the $\frac{\alpha_1}{2} = 0.005$ column, and is 1.784; the upper tail critical value is located in the $\left(1-\frac{\alpha_1}{2}\right) = 0.995$ column and is 4.303.

Let's now assume that the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistic values for the given sample data are 1.444 and 3.333, respectively. Neither test statistic falls in the rejection regions that are defined by the upper and lower tail critical values for each G.O.F. test statistic since $0.628 < 1.444 < 2.658$ and $1.784 < 3.333 < 4.303$. As a result, the analyst can conclude that there is no evidence to reject H_o at $\alpha = 0.10$.

If there is no *a priori* information available about the potential alternate distributions before the application of the G.O.F. test, picking the largest equivalent α-levels for both of the component individual G.O.F. tests will be the best and the safest choice. One should realize that this choice could, of course, result in somewhat lower overall power for the sequential G.O.F test. Continuing with the previous example, the combination that can be seen in the fourth row of Table 4.8 in which $\alpha_1 = 0.07$ and $\alpha_2 = 0.07$ that yields $\alpha = 0.096$ should be selected to employ the sequential G.O.F. test. In our example, if the analyst came to the conclusion with his beforehand analysis that the sample data came from a symmetric or similarly skewed alternate distribution for which

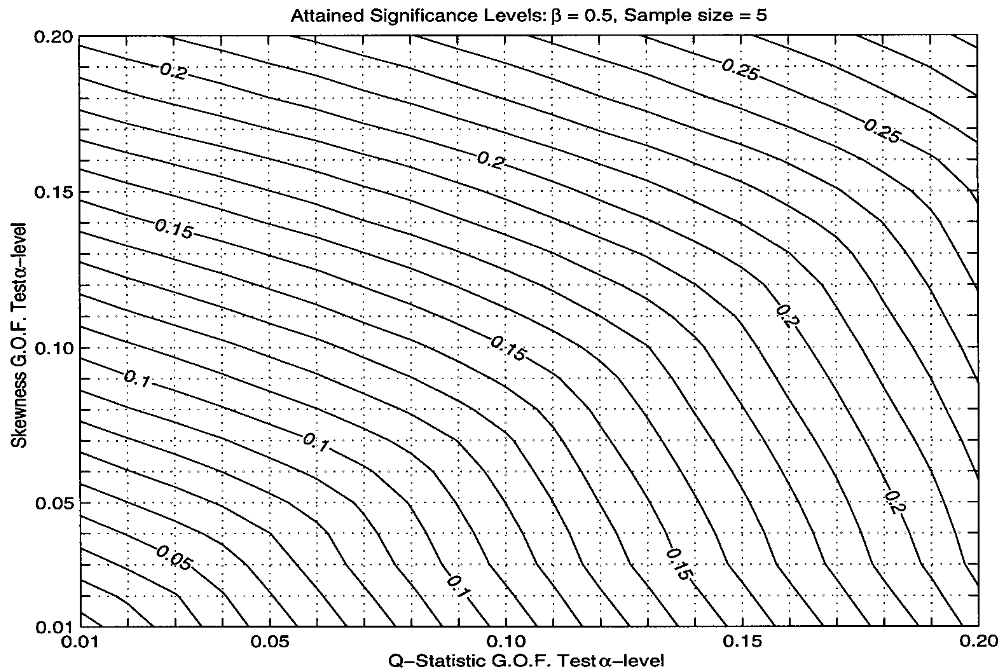
Q-Statistic G.O.F. test would probably have a greater discriminatory power, he would look for a combination in which the α_2-level is the highest possible. He would probably choose the third alternative in Table 4.8 in which $\alpha_1 = 0.01$ and $\alpha_2 = 0.10$ that yields $\alpha = 0.101$. This choice would yield a larger Type-I error for the Q-Statistic G.O.F. test, and, therefore, higher discriminatory power in rejecting the symmetric alternate distributions with the differing Q-Statistic values. As mentioned before, the analyst should not limit himself/herself to these combinations and compare the power study results for different combinations of the individual component G.O.F. test significance levels that yield the desired overall significance level.

One has to confess that use of the contour plots is somewhat simpler and time-efficient than use of the attained significance level tables, because one does not have to get drowned in the sea of numbers that the tables present. In order to use the contour plot in Figure 4.13, the analyst only needs to find the 0.10 contour line and move along it until he/she finds a suitable combination of α_1 and α_2 levels for the individual G.O.F. tests. In our example, all α_1 and α_2 combinations for $\alpha = 0.10$ that are reported in Table 4.8 can also be picked out in Figure 4.13.

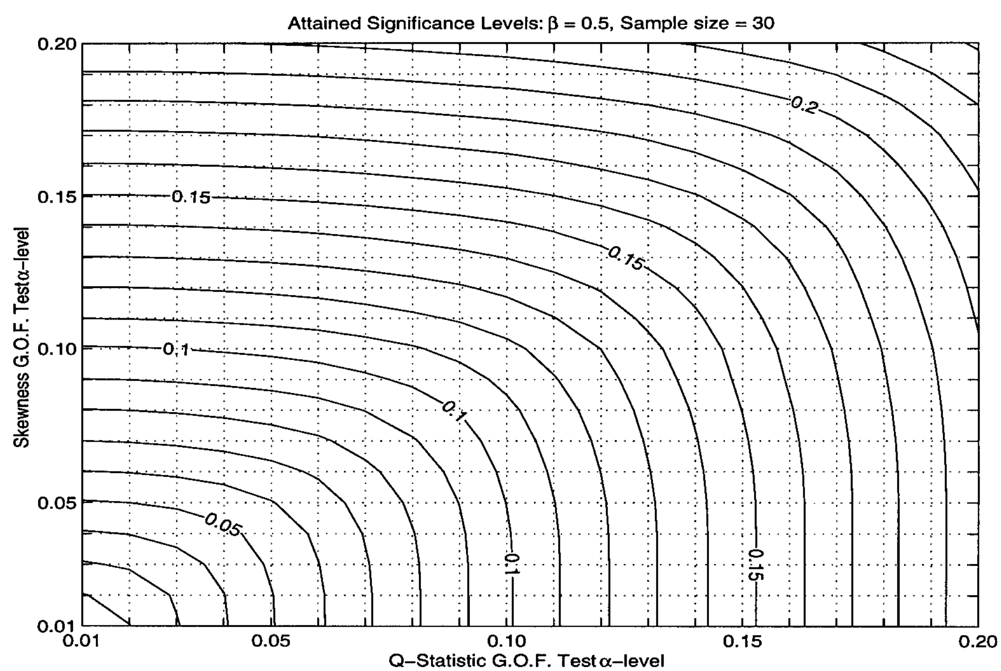
The power study against a variety of alternate distributions in the following pages will result in some useful results and insights into how each component G.O.F. test performs against particular alternate distributions that will yield useful information that could help the analyst in making these attained significance level choices.

4.4.4 Observations on the Attained Significance Levels

A careful examination of the contour plots that depict the attained significance levels do yield some useful information and observations that can chip in using the sequential G.O.F. test procedure efficiently and effectively. In parallel to the findings in the critical values, there is a very noticeable difference between the behaviors for Weibull distribution shape parameter values less than 2 and those greater. As can be observed in Appendix D, the contour level lines do demonstrate an increasing curvature as the sample size increases to the point they nearly align with the matching significance levels of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for $\beta = 0.5$, 1.0 and 1.5. However, for $\beta = 0.5$, the curvature of the contour level lines increases as the sample size increases until the sample size $n = 30$ where they have the most curvature and the curvature starts disappearing slightly for the rest of the sample sizes. Figure 4.14 illustrates the contour plots for $\beta = 0.5$ and (a) $n = 5$ and (b) $n = 30$. As we can see in this figure, α_1 and $\alpha_2 =$ 0.10 contour level line lines up almost with the x-y grid lines for $\alpha = 0.10$ for both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for $n = 30$. When these contour plots are compared, our difference between them can be seen very clearly. This difference basically tells us that both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests can be conducted at relatively high significance levels close to that of the overall sequential G.O.F. test attained significance level. When $n = 30$ with H_o: Weibull($\beta = 0.5$), the analyst can employ both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at the significance level of $\alpha_1 = 0.08$ and $\alpha_2 = 0.09$ rather than some more biased combination of the individual G.O.F. test significance levels like $\alpha_1 = 0.05$ and



(a) $n = 5$



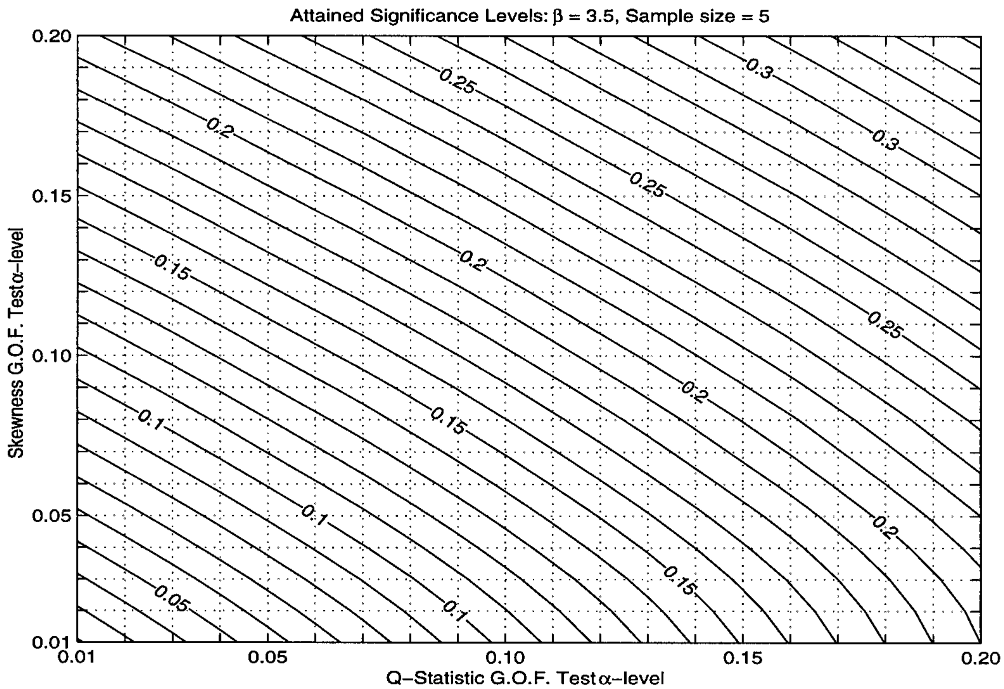
(b) $n = 30$

Figure 4.14 Contour plots for H_o: Weibull($\beta = 0.5$) at (a) $n = 5$ and (b) $n = 30$

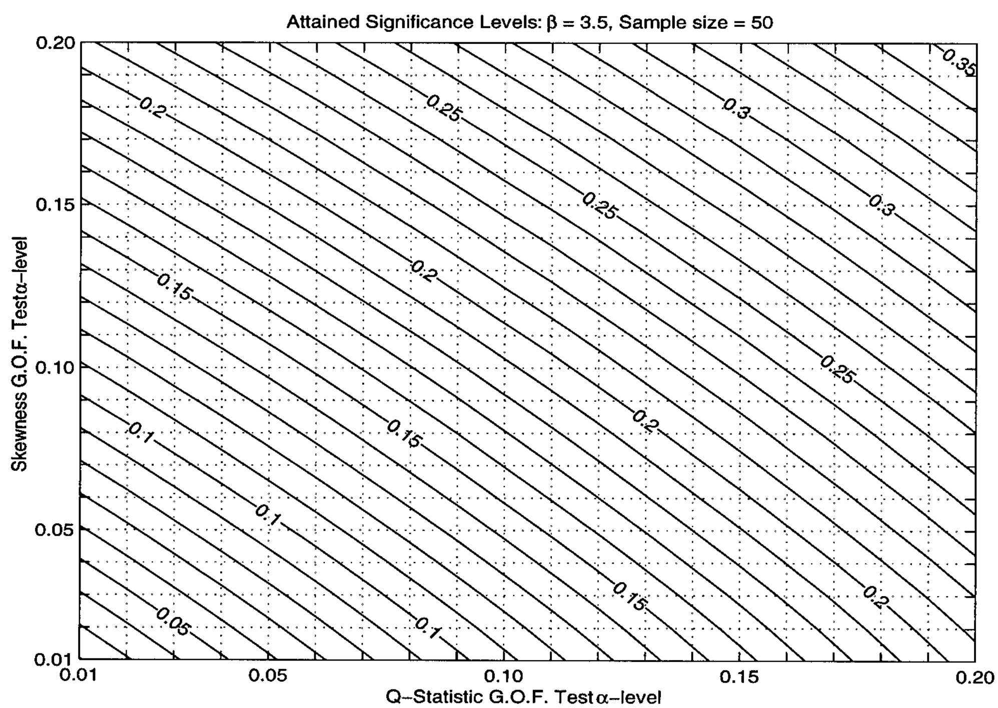
$\alpha_2 = 0.08$ when $n = 5$ with H_o: Weibull($\beta = 0.5$). In order for the analyst to preserve power, especially in cases where there is no evidence to suggest which of the two component G.O.F tests may be more powerful, he/she would like to choose nearly equal α levels for the individual G.O.F. tests.

Due to the effect of Bonferroni's inequality, the sequential G.O.F. test procedure always has comparatively lower power against a particular alternative than the most powerful component G.O.F. test it utilizes. Therefore, the sequential G.O.F. test does not always obtain the power of the best of its component G.O.F. tests when compared at the same or very close significance levels. On the other hand, as will be seen in the power study results in the next chapter, the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests complement each other and yield better power when employed in sequential manner than when employed by itself. With $\beta \leq 1.5$ and large sample sizes, the analyst can employ $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at individual significance levels close to that of the sequential G.O.F. test accounting for the lag between power of the sequential G.O.F. test and that of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. This indication means that the sequential G.O.F. test will be beneficial for small shape parameter values and large sample sizes.

As illustrated in Figure 4.15 (a)/(b) for $\beta = 3.5$ for (a) $n = 5$ and (b) $n = 50$, there does not exist any trend toward increasing curvature of the contour levels as the sample size increases for $\beta \geq 2.0$. On the other hand, the contour lines become slightly more linear or stay at the same curvature as the sample size increases for these greater shape parameter values. Therefore, while using these hypothesized Weibull distribution

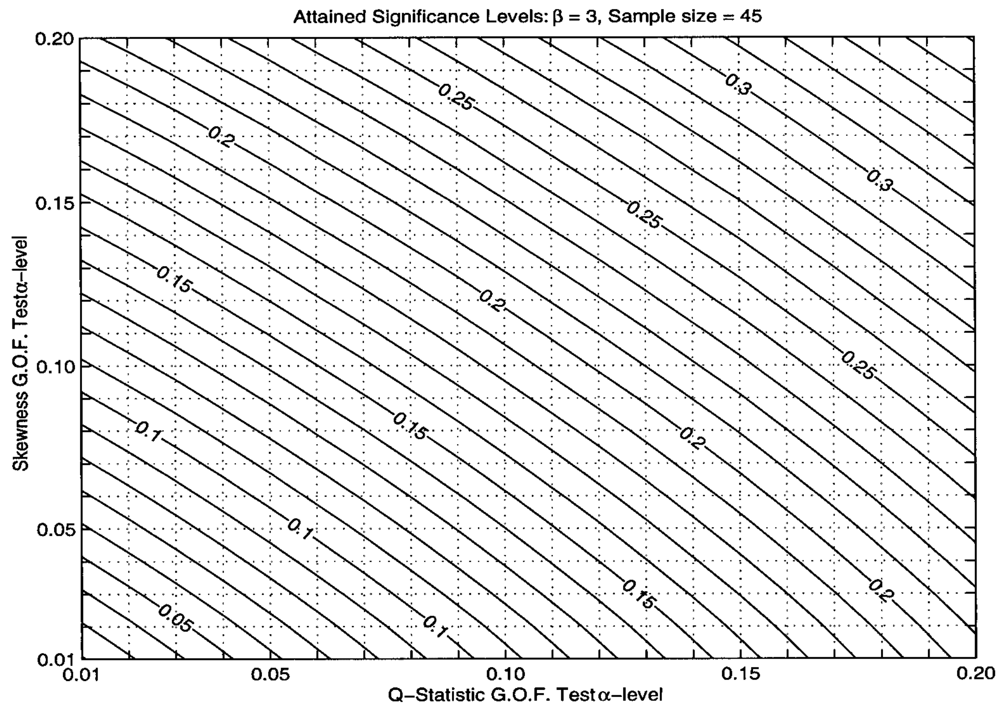


(a) $n = 5$

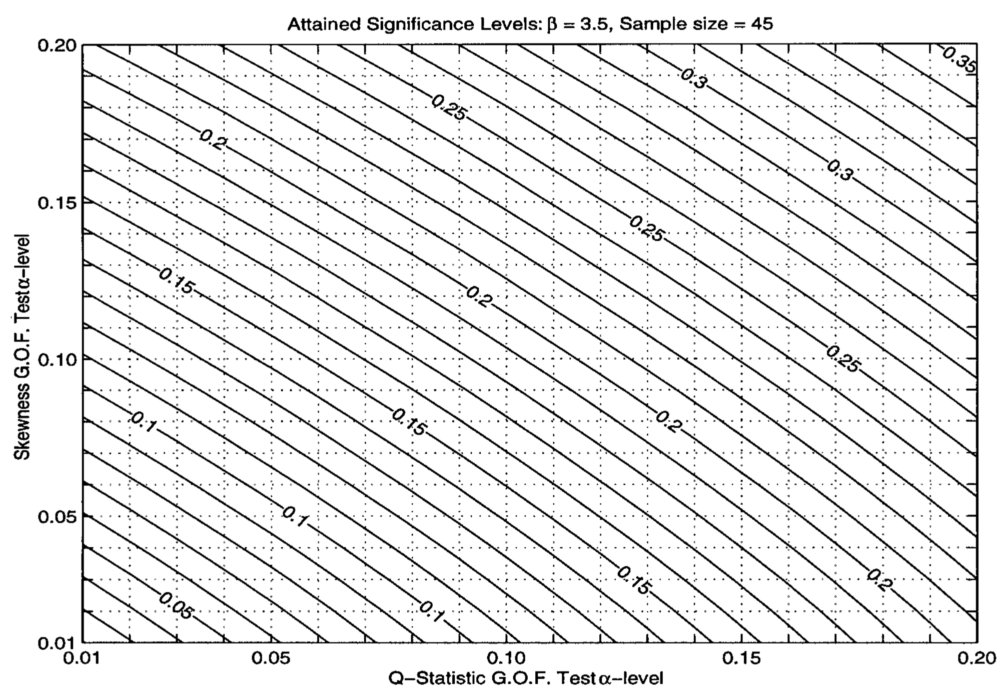


(b) $n = 50$

Figure 4.15 Contour Plots for H_o: Weibull($\beta = 3.5$) at (a) $n = 5$ and (b) $n = 50$



(a) H_o: Weibull($\beta = 3.0$)



(b) H_o: Weibull($\beta = 3.5$)

Figure 4.16 Contour Plots for $n = 45$ with (a) H_o: Weibull($\beta = 3.0$) (b) H_o: Weibull($\beta = 3.5$)

shape parameter values, the analyst should choose a combination of α levels in which the more powerful component G.O.F. test's significance level is the highest possible. For example, continuing with Figure 4.15, let's assume that the analyst has n = 50 and wants to have $\alpha = 0.15$ for H_o: Weibull($\beta = 3.5$). Being considerably lower than $\alpha = 0.15$, one possible combination could be $\alpha_1 = 0.08$ and $\alpha_2 = 0.08$. If there is no a priori information that might indicate which of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests has better discriminatory power, the analyst is better off using equal or nearly equal significance levels and expect lower power from the sequential G.O.F. test procedure. However, if the analyst is employing the sequential G.O.F. test against the skewed alternate distributions, he should choose combinations where the $\sqrt{b_1}$ G.O.F. test has the highest individual significance level such as $\alpha_1 = 0.14$ and $\alpha_2 = 0.01$ whereas $\alpha_1 = 0.05$ and $\alpha_2 = 0.13$ combination if he is employing the sequential G.O.F. test against symmetric or similarly skewed alternate distributions to opt for the Q-Statistic with a higher significance level.

Furthermore, there exists an increasing density of the contour lines along the diagonal of equal significance levels ($\alpha_1 = \alpha_2$) in the contour plots for $\beta \geq 2.0$. Figure 4.16 (a)/(b) for (a) $\beta = 3.0$ and (b) β = 3.5 for n = 45 illustrates this point. Examining these plots, one can note that there are more contour lines in the upper right corner of these plots implying that the attained significance level is higher for higher Weibull distributions shape parameter values at a given combination of equivalent contour levels (e.g. $\alpha_1 = \alpha_2 = 0.25$) even though the particular contour levels are positioned in nearly identical locations. As a result, based on the previous and current observations for $\beta \leq$ 1.5 for such equal or nearly equal significance levels for the component G.O.F. tests, the analyst should expect more power with H_o: Weibull($\beta \leq 1.5$) due to the fact that the lag between the significance levels of the sequential G.O.F. test and its component G.O.F. tests is considerably smaller for $\beta \leq 1.5$.

As a summary, some useful insights into selecting the significance levels of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests can be obtained based on examination of the form of the contour plots to have a overall better power for the sequential G.O.F. test. With $\beta \leq 1.5$ and n large, the analyst should select equal or nearly equal significance levels for both component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests and not lose any overall power for the sequential G.O.F. test in cases when no a priori information is available on the alternate distributions. If a priori information is available, a higher significance level for the component G.O.F. test with more discriminatory power should be used. However, the analyst should not use equal or nearly equal significance levels for the contour plots with more linear contour levels with $\beta \geq 2$. In this case, the analyst should opt for a higher significance level for the G.O.F. test that may be identified via prior analysis as the most powerful against the alternate distributions studied. The next section that deals with the power study results will help identify which of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests has better discriminatory power against which particular alternate distributions.

4.5 Power Study Results

The power study results obtained here for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests and the proposed sequential G.O.F. test procedure do help the analyst pick out the appropriate significance levels for the component G.O.F. tests in order to have a greater overall power for the sequential G.O.F. test procedure. The first part of the power study conducted here compared the discriminatory power performance of the new sequential G.O.F. test to those of Bush's EDF based A^2, W^2, and K-S G.O.F. tests against a set of particular alternate distributions. Her power study results were not directly comparable to that of the new sequential G.O.F. test due to the fact that Wozniak's EDF based A^2, W^2, and K-S G.O.F. tests were designed for the two-parameter Weibull distribution and were based on the extreme-value distribution. The second part of the power study considered the alternate distributions used by Wozniak [168] because they stand out as the benchmark alternate distributions in power studies with the hypothesized null Weibull distributions. After all the power results were tabled and discussed in the previous parts of the power study, the third part of the study included comparing the power results of Clough's research. This section provides useful insights into the change in the power of the sequential G.O.F. test procedure when the kurtosis test statistic was replaced with Q-Statistic test statistic. The fourth part of this power study included some of the null and alternate distributions that were not studied by Bush [17], Wozniak [168], or Clough [26] to be able to prepare a database for power comparisons in the future. Subsequently, the individual skewness and Q-Statistic G.O.F. tests were evaluated against a set of the aforementioned alternate distributions to determine which of the component G.O.F. test was more powerful against certain alternate distributions. Eventually, one-sided versions of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests and the sequential G.O.F. tests against a subset of the aforementioned alternate distributions were studied in order to explore what kind of impact these changes had on the power.

4.5.1 Power versus Bush's Alternate Distributions

Bush used H_o = Weibull (β = 1 and 3.5) in his research and presented his power study results in tabular form for the significance level $\alpha = 0.05$ and for the three EDF based A^2, W^2 and K-S G.O.F. test statistics for the sample sizes n = 5, 15, 25 against the alternate distributions discussed in the previous chapter.

The new sequential G.O.F. test was evaluated for the same sample sizes with the addition of n =50 in order to prepare a common ground for the comparison of the results of this power study to those of Clough's for each H_o and H_a combination Bush evaluated. The results of the power study with these H_o and H_a combinations are tabled and can be seen in Table 4.9 – 4.25 for all combinations of the significance levels for the two component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Since Bush [17] studied $\alpha = 0.05$ in his research, three different combinations of the significance levels for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests to acquire an attained significance level as close as possible to α = 0.05 are used to be able to compare the power study results of this research to those of Bush's. It is important to note that in order to be able to facilitate a direct comparison of the power study results of Bush's with those of the new sequential G.O.F. test, the same set of α-level combinations were repeated for each alternate distribution. It is also important to make a note of the fact that even though the three reported power results for the new sequential G.O.F. test are aligned with the A^2, W^2 and K-S G.O.F. test results, this does not mean that the comparison of the power study results is only restricted to those power study results lined up horizontally with them when reading these tables. Consequently, the new sequential G.O.F. test power for each combination of individual significance levels needs to be compared to all three of the EDF based A^2, W^2 and K-S G.O.F. test power study results by Bush [17]. Two of the tables below show the results from the verification runs for Weibull($\beta = 1$ and 3.5) where H_o is the same as the H_a in which the power should be equal to the attained significance level within some random error. Indeed, the reader can see from Table 4.14 and 4.25 that the verification runs demonstrated good agreement between the power and the attained significance levels as expected, indicating valid code and accurate critical values.

For H_o = Weibull($\beta = 1$), the new sequential G.O.F. test procedure performed extremely well. As can be seen in Table 4.9 and 4.12, the sequential G.O.F. test performed much better than all of the EDF based A^2, W^2, and K-S G.O.F. tests for every sample size against H_a = Beta(2,2) and Uniform(0,2). As can be seen in tables 4.10, the sequential G.O.F. test outperformed EDF based A^2, W^2, and K-S G.O.F. tests in terms of power for small sample sizes, against H_a = Beta(2,3), even though the power performance went down for larger sample sizes. On the other hand, the new sequential G.O.F. test surprisingly did not perform well against H_a = Normal(0,1) and Gamma(2,1) relative to the EDF based A^2, W^2, and K-S G.O.F. tests as can be seen in Tables 4.11 and 4.13. Even though the new sequential G.O.F. test outperformed the W^2 and K-S G.O.F. tests against these alternate distributions for $n = 5$, it failed to have the same performance for all other sample sizes. Lagging the EDF based A^2, W^2, and K-S G.O.F. tests by approximately 20% for both of these alternate distributions, the new sequential G.O.F. test still performed acceptably against the normal alternative, but really could not keep up with the EDF based A^2, W^2, and K-S G.O.F. tests for the gamma alternative. Even though the new sequential G.O.F. test performed worse than the EDF based A^2, W^2, and K-S G.O.F. tests

Table 4.9 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Beta(2,2).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.031	0.04	0.02	0.048	0.107	0.104	0.04	0.01	0.047
	W^2	0.082	0.05	0.01	0.054	0.123	0.124	0.05	0.01	0.057
	K-S	0.069	0.02	0.04	0.048	0.071	0.079	0.02	0.04	0.053
15	A^2	0.508	0.04	0.02	0.048	0.597	0.591	0.04	0.02	0.048
	W^2	0.578	0.05	0.01	0.054	0.633	0.629	0.05	0.01	0.053
	K-S	0.454	0.02	0.04	0.049	0.500	0.493	0.02	0.04	0.048
25	A^2	0.867	0.04	0.04	0.051	0.904	0.904	0.04	0.03	0.051
	W^2	0.899	0.05	0.01	0.052	0.920	0.923	0.05	0.01	0.052
	K-S	0.771	0.03	0.04	0.047	0.881	0.878	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	1.000	1.000	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	1.000	1.000	0.05	0.01	0.051

Table 4.10 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Beta(2,3).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.014	0.04	0.02	0.048	0.082	0.082	0.04	0.01	0.047
	W^2	0.066	0.05	0.01	0.054	0.094	0.094	0.05	0.01	0.057
	K-S	0.057	0.02	0.04	0.048	0.055	0.064	0.02	0.04	0.053
15	A^2	0.359	0.04	0.02	0.048	0.371	0.372	0.04	0.02	0.048
	W^2	0.428	0.05	0.01	0.054	0.406	0.408	0.05	0.01	0.053
	K-S	0.340	0.02	0.04	0.049	0.291	0.286	0.02	0.04	0.048
25	A^2	0.715	0.04	0.04	0.051	0.675	0.668	0.04	0.03	0.051
	W^2	0.773	0.05	0.01	0.052	0.711	0.706	0.05	0.01	0.052
	K-S	0.628	0.03	0.04	0.047	0.629	0.624	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.976	0.975	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.982	0.981	0.05	0.01	0.051

Table 4.11 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Normal(0,1).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.202	0.04	0.02	0.048	0.116	0.116	0.04	0.01	0.047
	W^2	0.099	0.05	0.01	0.054	0.135	0.138	0.05	0.01	0.057
	K-S	0.079	0.02	0.04	0.048	0.073	0.083	0.02	0.04	0.053
15	A^2	0.685	0.04	0.02	0.048	0.569	0.568	0.04	0.02	0.048
	W^2	0.699	0.05	0.01	0.054	0.602	0.602	0.05	0.01	0.053
	K-S	0.606	0.02	0.04	0.049	0.482	0.477	0.02	0.04	0.048
25	A^2	0.938	0.04	0.04	0.051	0.816	0.816	0.04	0.03	0.051
	W^2	0.950	0.05	0.01	0.052	0.839	0.839	0.05	0.01	0.052
	K-S	0.904	0.03	0.04	0.047	0.790	0.789	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.984	0.984	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.986	0.986	0.05	0.01	0.051

Table 4.12 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Uniform(0,2).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.050	0.04	0.02	0.048	0.115	0.110	0.04	0.01	0.047
	W^2	0.084	0.05	0.01	0.054	0.127	0.129	0.05	0.01	0.057
	K-S	0.072	0.02	0.04	0.048	0.088	0.094	0.02	0.04	0.053
15	A^2	0.384	0.04	0.02	0.048	0.622	0.618	0.04	0.02	0.048
	W^2	0.446	0.05	0.01	0.054	0.644	0.644	0.05	0.01	0.053
	K-S	0.328	0.02	0.04	0.049	0.564	0.563	0.02	0.04	0.048
25	A^2	0.703	0.04	0.04	0.051	0.924	0.917	0.04	0.03	0.051
	W^2	0.746	0.05	0.01	0.052	0.926	0.926	0.05	0.01	0.052
	K-S	0.575	0.03	0.04	0.047	0.909	0.904	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	1.000	1.000	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	1.000	1.000	0.05	0.01	0.051

Table 4.13 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Gamma(2,1).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.114	0.04	0.02	0.048	0.050	0.048	0.04	0.01	0.047
	W^2	0.043	0.05	0.01	0.054	0.057	0.058	0.05	0.01	0.057
	K-S	0.040	0.02	0.04	0.048	0.042	0.050	0.02	0.04	0.053
15	A^2	0.122	0.04	0.02	0.048	0.079	0.078	0.04	0.02	0.048
	W^2	0.116	0.05	0.01	0.054	0.090	0.090	0.05	0.01	0.053
	K-S	0.099	0.02	0.04	0.049	0.064	0.060	0.02	0.04	0.048
25	A^2	0.231	0.04	0.04	0.051	0.109	0.105	0.04	0.03	0.051
	W^2	0.245	0.05	0.01	0.052	0.120	0.120	0.05	0.01	0.052
	K-S	0.196	0.03	0.04	0.047	0.094	0.090	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.157	0.155	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.175	0.175	0.05	0.01	0.051

Table 4.14 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Weibull($\beta = 1$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.049	0.04	0.02	0.048	0.051	0.048	0.04	0.01	0.047
	W^2	0.045	0.05	0.01	0.054	0.056	0.058	0.05	0.01	0.057
	K-S	0.045	0.02	0.04	0.048	0.051	0.054	0.02	0.04	0.053
15	A^2	0.043	0.04	0.02	0.048	0.047	0.046	0.04	0.02	0.048
	W^2	0.053	0.05	0.01	0.054	0.052	0.055	0.05	0.01	0.053
	K-S	0.049	0.02	0.04	0.049	0.047	0.049	0.02	0.04	0.048
25	A^2	0.047	0.04	0.04	0.051	0.050	0.050	0.04	0.03	0.051
	W^2	0.056	0.05	0.01	0.052	0.051	0.051	0.05	0.01	0.052
	K-S	0.057	0.03	0.04	0.047	0.046	0.050	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.049	0.053	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.050	0.052	0.05	0.01	0.051

Table 4.15 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Weibull($\beta = 2$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.049	0.04	0.02	0.048	0.069	0.067	0.04	0.01	0.047
	W^2	0.055	0.05	0.01	0.054	0.079	0.081	0.05	0.01	0.057
	K-S	0.051	0.02	0.04	0.048	0.048	0.057	0.02	0.04	0.053
15	A^2	0.321	0.04	0.02	0.048	0.243	0.241	0.04	0.02	0.048
	W^2	0.346	0.05	0.01	0.054	0.270	0.269	0.05	0.01	0.053
	K-S	0.277	0.02	0.04	0.049	0.184	0.179	0.02	0.04	0.048
25	A^2	0.655	0.04	0.04	0.051	0.411	0.407	0.04	0.03	0.051
	W^2	0.700	0.05	0.01	0.052	0.447	0.447	0.05	0.01	0.052
	K-S	0.575	0.03	0.04	0.047	0.369	0.365	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.714	0.714	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.745	0.745	0.05	0.01	0.051

Table 4.16 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1$); H_a: Weibull($\beta = 3.5$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.052	0.04	0.02	0.048	0.113	0.112	0.04	0.01	0.047
	W^2	0.097	0.05	0.01	0.054	0.133	0.135	0.05	0.01	0.057
	K-S	0.079	0.02	0.04	0.048	0.073	0.082	0.02	0.04	0.053
15	A^2	0.624	0.04	0.02	0.048	0.570	0.569	0.04	0.02	0.048
	W^2	0.667	0.05	0.01	0.054	0.605	0.604	0.05	0.01	0.053
	K-S	0.568	0.02	0.04	0.049	0.481	0.476	0.02	0.04	0.048
25	A^2	0.930	0.04	0.04	0.051	0.842	0.841	0.04	0.03	0.051
	W^2	0.947	0.05	0.01	0.052	0.864	0.864	0.05	0.01	0.052
	K-S	0.885	0.03	0.04	0.047	0.814	0.813	0.03	0.04	0.051
50	N/A	N/A	0.04	0.04	0.051	0.992	0.992	0.04	0.03	0.053
	N/A	N/A	0.05	0.01	0.052	0.994	0.994	0.05	0.01	0.051

by more than approximately 20% against H_a = Weibull(β = 2), it performed reasonably well against H_a = Weibull(β = 3.5) as evidenced by Tables 4.14 and 4.15. As expected from any G.O.F. test procedure, the discriminatory power increased as the sample size increased. For almost all of the alternate distributions against H_o = Weibull(β = 1), better power was acquired when the $\sqrt{b_1}$ G.O.F. test was employed at a higher significance level than the Q-Statistic G.O.F. test emphasizing the importance of the appropriate choice of the significance levels for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. The findings here confirm the findings in the scatter plots for smaller Weibull distribution shape parameter values in the sense that $\sqrt{b_1}$ G.O.F. test might result in better discriminatory power. As can be observed from the scatter plots, the correlation between $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics yielded the fact that one of the two component G.O.F. tests would yield better discriminatory power. This observation and the smaller variability in $\sqrt{b_1}$ G.O.F. test statistic point toward this test statistic being the superior one. This finding will be verified later in this section with the examinations of the component G.O.F. tests individually.

A summary of the power study results against the Bush's alternate distributions against H_o = Weibull(β = 1) are presented in Table 4.17 in which the discriminatory power of the new sequential G.O.F. test is compared to those of the EDF based A^2, W^2, and K-S G.O.F. tests. "Light chart" visual format [26] was used in this table. A clear circle (O) was used to indicate the cases in which the new sequential G.O.F. test performed better in terms of the discriminatory power than the EDF based A^2, W^2, and K-S G.O.F. tests did in at least two of the three combinations of the individual significance levels of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests as presented in the previous tables. A circle with a dot in its center (⊙) was used to indicate the cases in which the sequential G.O.F. test outperformed EDF based A^2, W^2, and K-S G.O.F. tests only in one combination or it was outperformed by the EDF based A^2, W^2, and K-S G.O.F. tests by no more than 5% in terms of power. As solid circle (●) was used to indicate the cases in which the sequential G.O.F. test was outperformed by the EDF based A^2, W^2, and K-S G.O.F. tests by more than 5% in all three combinations. In summary, the clearer the circles are, the better the new sequential G.O.F. test performed against the alternate distributions considered here compared to the EDF based A^2, W^2, and K-S G.O.F. tests by Bush [17]. Presenting a means for an easier power comparison of the new sequential G.O.F. test to each of the EDF based A^2, W^2, and K-S G.O.F. tests by Bush [17], this light chart visual summary format [26] makes the comparison results more digestible by the alternate distribution and the sample size. Furthermore, power comparison histograms for each H_o and sample size considered here were prepared to provide the reader with additional analysis means that can be seen in Figures 4.17- 4.22. In these figures, the highest sequential G.O.F. test power results were compared to those of the Clough's highest power results by his sequential procedure and the EDF based A^2, W^2, and K-S G.O.F. tests by Bush [17].

In addition to the findings above, Table 4.17 and Figures 4.17-4.19 do present insights about the power comparison accomplished here. As indicated by this table and figures, the new sequential G.O.F. test performed extremely well against almost all of the alternate distribution for $n = 5$. Considering the existing G.O.F. tests generally suffer for

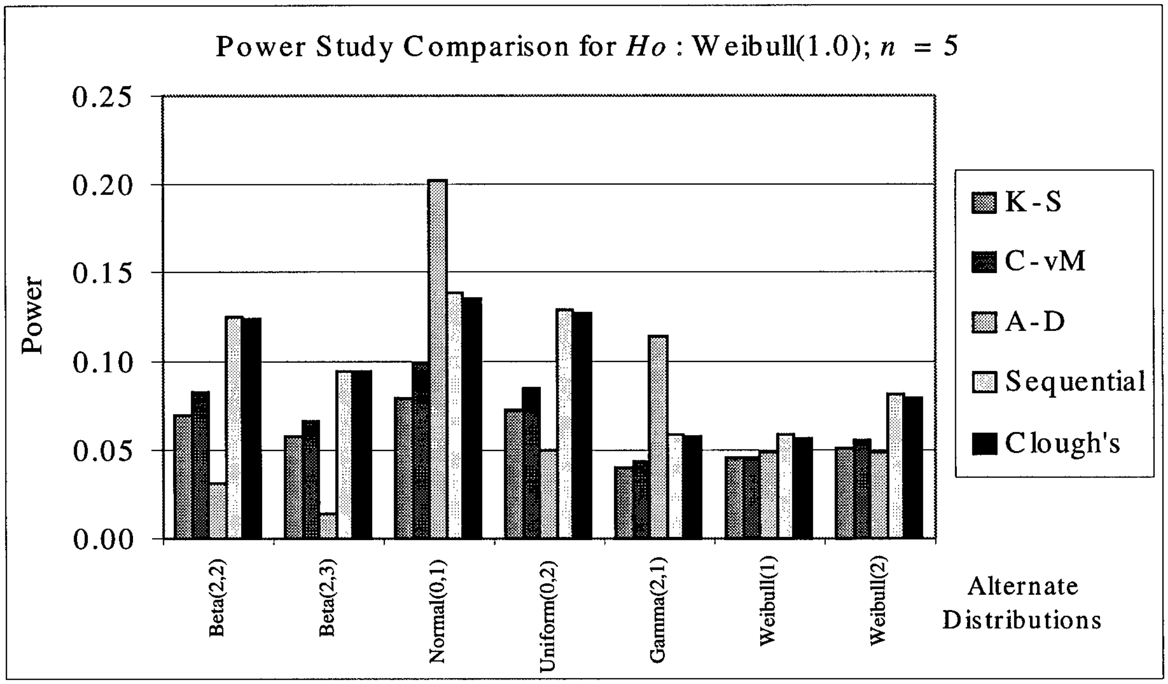


Figure 4.17 Power Comparison for H_o = Weibull(β = 1) at n = 5 with those of Bush's and Clough's.

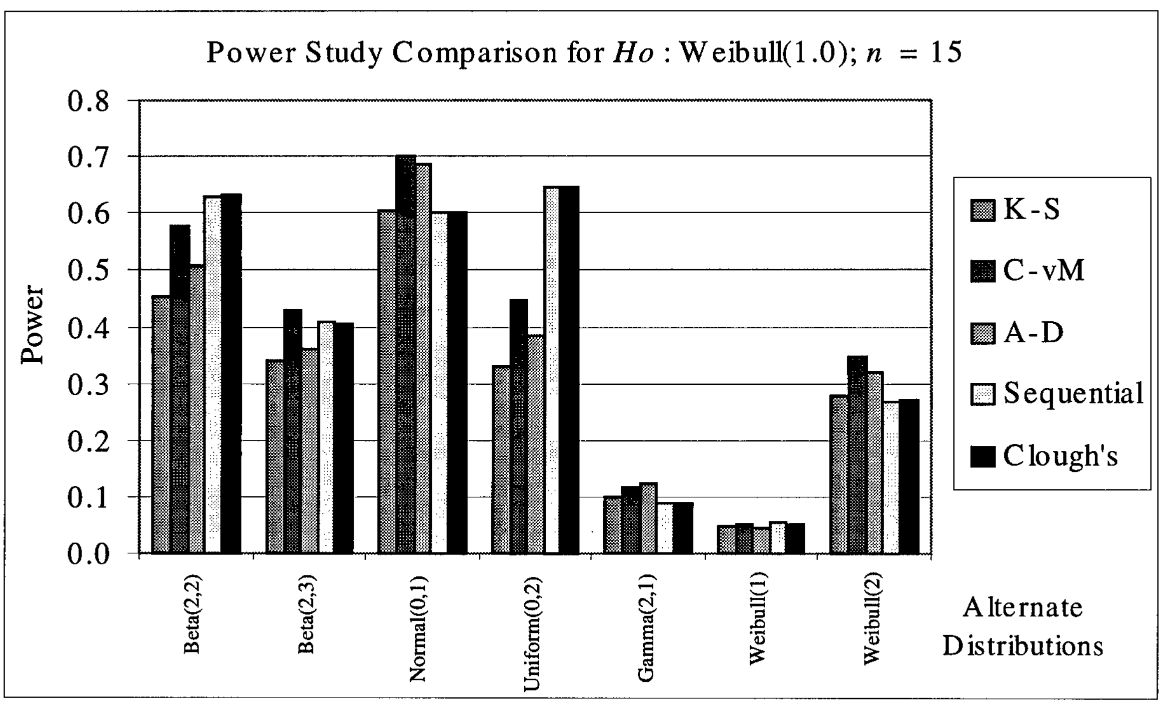


Figure 4.18 Power Comparison for H_o = Weibull(β = 1) at n = 15 with those of Bush's and Clough's.

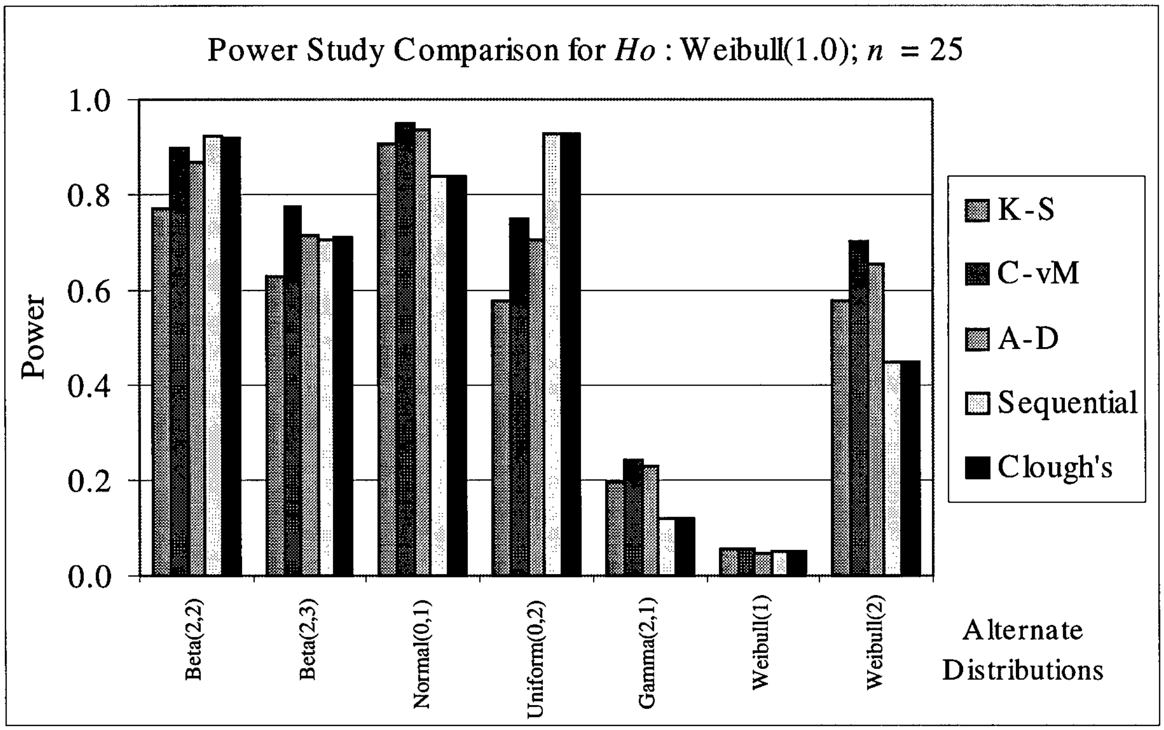


Figure 4.19 Power Comparison for H_o = Weibull(β = 1) at n = 25 with those of Bush's and Clough's.

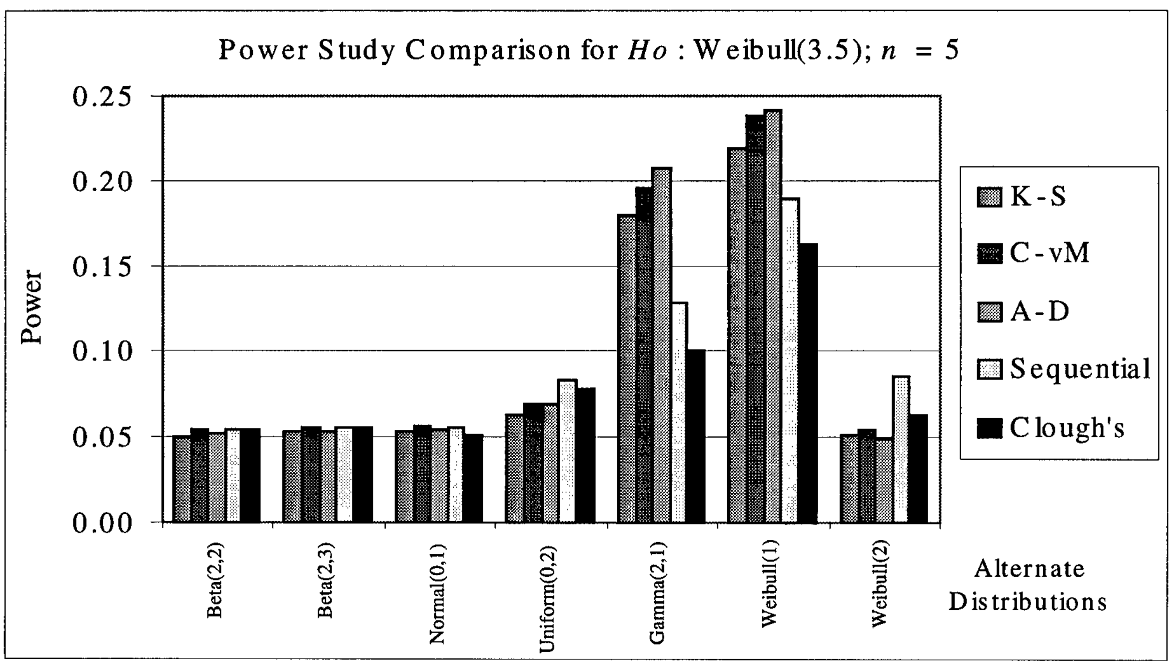


Figure 4.20 Power Comparison for H_o = Weibull(β = 3.5) at n = 5 with those of Bush's and Clough's.

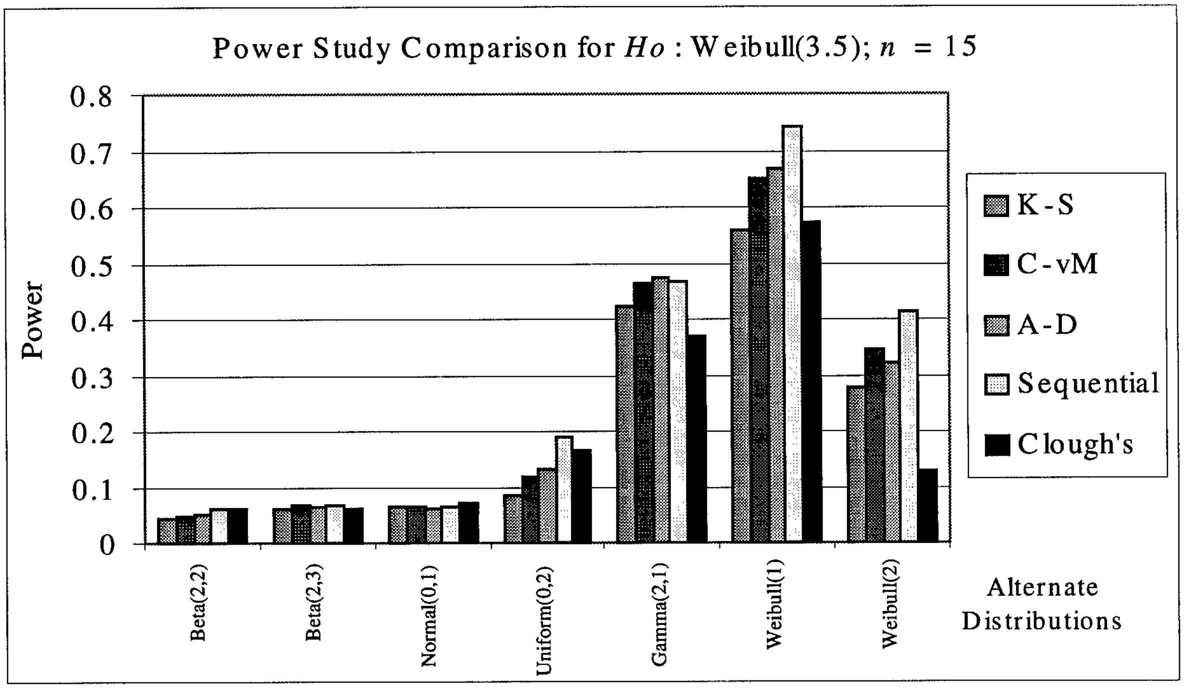


Figure 4.21 Power Comparison for H_o = Weibull(β = 3.5) at n = 15 with those of Bush's and Clough's.

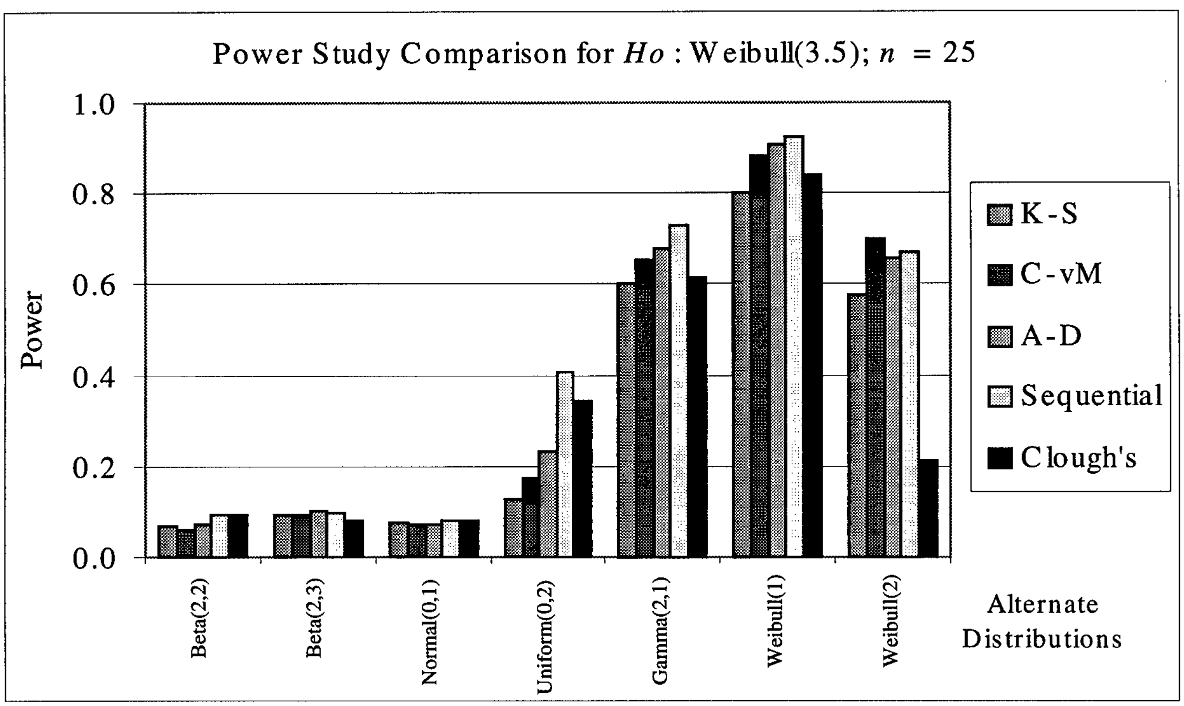


Figure 4.22 Power Comparison for H_o = Weibull(β = 3.5) at n = 25 with those of Bush's and Clough's.

the small sample sizes, this achievement is quite extraordinary. However, in parallel to the findings in the scatter plots that the variability in $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics increased as the sample size increased, leading to reduced power for the hypothesized Weibull distributions with small shape parameter values, the power of the new sequential G.O.F. test fell below those of the EDF based A^2, W^2, and K-S G.O.F. tests by a larger margin as the sample size increased. It is very important to note a (●) does not necessarily mean that the use of the sequential G.O.F. test always result in low power against a broad range of the alternate distributions, but simply indicates the cases in which the new sequential G.O.F. test power lagged those of the EDF based A^2, W^2, and K-S G.O.F. tests for that particular alternate distribution and the sample size considered. For example, as evidenced by Table 4.15, the new sequential G.O.F. test obviously had considerably good power against the H_a = Weibull(β = 3.5), but lagged the power of the EDF based A^2, W^2, and K-S G.O.F. tests. In summary, the sequential G.O.F. test performed admirably for H_o = Weibull(β = 1) against Bush's alternate distributions when compared to the famous EDF based A^2, W^2, and K-S G.O.F. tests by Bush [17] in terms of power and utility.

Due to the fact that H_o = Weibull(β = 3.5) has a PDF shape that is very close to that of the normal distribution and almost all of the alternate distributions considered by Bush are mound-shaped, the new sequential G.O.F. test faced a greater challenge here than it did for H_o = Weibull(β = 1). Consequently, the reader can note that both sequential G.O.F. test and the EDF based A^2, W^2, and K-S G.O.F. tests have lower power for this particular Weibull parameter shape value, but again the proposed sequential G.O.F. test demonstrates some certain superiority against other famous conventional

Table 4.17 Summary of the Power Study Results for H_o = Weibull(β = 1).

Sample Size	EDF Tests	Alternate Distributions						
		Beta(2,2)	Beta(2,3)	Gamma(2,1)	Normal(0,1)	Weibull(2)	Weibull(3.5)	Uniform(0,2)
5	A^2	○	○	●	●	○	○	○
	W^2	○	○	○	○	○	○	○
	K-S	○	○	○	○	○	○	○
15	A^2	○	○	●	●	●	⊙	○
	W^2	○	⊙	●	●	●	●	○
	K-S	○	○	⊙	⊙	⊙	○	○
25	A^2	○	⊙	●	●	●	●	○
	W^2	○	●	●	●	●	●	○
	K-S	○	○	●	●	●	⊙	○

○ = The sequential G.O.F test outperformed the EDF based G.O.F. test in at least two of the three combinations.

⊙ = The sequential G.O.F. test outperformed the EDF based G.O.F. test only in one of the three combinations or its power is lower than that of the EDF based G.O.F. test by no more than 5%.

● = The sequential G.O.F. test was outperformed by the EDF based G.O.F. test in every combination by more than 5%.

G.O.F. tests. The power comparisons here will be proceeded by following the logic in the light chart presentation. The sequential G.O.F. test performed better than all of the EDF based A^2, W^2, and K-S G.O.F. tests for every sample size against H_a = Beta(2,2) and Uniform(0,2) and as it did for H_o = Weibull(β = 1) against these alternate distributions. Besides, the new sequential G.O.F. surpassed the powers of the EDF based A^2, W^2, and K-S G.O.F. tests against H_a = Weibull(2) except for the W^2 G.O.F. test at n = 25. The sequential G.O.F. test performed better than all of the EDF based A^2, W^2, and K-S G.O.F. tests for every sample size against H_a = Normal(0,1) except for the W^2 G.O.F. test for n = 5 and 15. Although the proposed sequential G.O.F. test suffered at n = 5, it then outperformed the EDF based A^2, W^2, and K-S G.O.F. tests at $n = 15$ and 25. Again, while the new sequential G.O.F. test suffered at $n = 5$ against all EDF based A^2, W^2, and K-S G.O.F. tests and A^2 and W^2 G.O.F. tests at $n = 15$ and performed superior against K-S G.O.F. test at this sample size, it outperformed the EDF based A^2, W^2, and K-S G.O.F. tests at $n = 25$ against H_a = Gamma(2,1). Although the proposed sequential G.O.F. test suffered against the W^2 G.O.F. test and outperformed the other EDF based G.O.F. tests at $n = 5$ and 15, it suffered against the A^2 G.O.F. test and outperformed the other EDF based G.O.F. tests at $n = 25$ against H_a = Beta(2,2). It is noteworthy that in all cases in which the proposed sequential G.O.F. test failed to outperform the EDF based A^2, W^2, and K-S G.O.F. tests, the sequential G.O.F. test power was usually within roughly 7.5% from those of the EDF based A^2, W^2, and K-S G.O.F. tests. This fact is very important in the sense that the proposed sequential G.O.F. test does not have the computational complexity that the existing G.O.F. tests have.

As with H_o = Weibull($\beta = 1$), the power of the sequential G.O.F. test increased as the sample size increases with H_o = Weibull($\beta = 3.5$) against all of the alternate distributions and it performed consistently better across all sample sizes with H_o = Weibull($\beta = 3.5$), whereas with H_o = Weibull($\beta = 1$), it seemed to lose its lead as the sample size increased. Moreover, the new sequential G.O.F. test performed best against some alternate distributions where the Q-Statistic G.O.F. test was conducted at higher significance levels except for the gamma and Weibull alternate distributions. As opposed to the findings for H_o = Weibull($\beta = 1$), this observation indicates that Q-Statistic dominates the $\sqrt{b_1}$ G.O.F. test against specific alternate distributions. These observations should be taken into account when the individual significance levels for the component

$\sqrt{b_1}$ and Q-Statistic G.O.F. tests are chosen to conduct new sequential G.O.F. test at the desired overall significance level to have a better overall discriminatory power for the sequential G.O.F. test. These observations also show us the strengths and weaknesses of the component $\sqrt{b_1}$ and Q-Statistics G.O.F. tests that enhance the appeal of the proposed sequential G.O.F. test.

As with H_o = Weibull(β = 1), the comparison of the power study results of this study and those of Bush are summarized in Table 4.26 in the format of a light chart and power comparison histograms that can be seen in Figures 4.20-4.22 for H_o = Weibull(β = 3.5). These figures and the table clearly show that the new sequential G.O.F. test performed better for H_o = Weibull(β = 3.5) relative to the EDF based A^2, W^2, and K-S G.O.F. tests than it did for H_o = Weibull(β = 1). The previous observations from the scatter plots that the test statistic variability diminished as the sample size increased directly translates directly to better power for H_o = Weibull(β = 3.5). The fact that the new sequential G.O.F. test outperformed the EDF based A^2, W^2, and K-S G.O.F. tests at a very high frequency is very noteworthy and certainly marks an important result.

The results of this part of the power study against Bush's alternate distributions with H_o = Weibull(β = 1 and 3.5) are very satisfactory. For almost all cases, the new sequential G.O.F. test performed equally or better than the famous and much more computationally expensive EDF based A^2, W^2, and K-S G.O.F. tests did. Selecting higher significance levels for the $\sqrt{b_1}$ G.O.F. test resulted in higher overall power for H_o = Weibull(β = 1) against all alternate distributions, whereas selecting higher significance levels for the Q-Statistic G.O.F. test seemed to improve the overall power for

Table 4.18 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Beta(2,2).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.052	0.04	0.02	0.049	0.047	0.045	0.03	0.02	0.047
	W^2	0.054	0.03	0.04	0.050	0.052	0.051	0.02	0.03	0.046
	K-S	0.050	0.02	0.05	0.049	0.054	0.054	0.01	0.04	0.047
15	A^2	0.050	0.04	0.02	0.052	0.046	0.042	0.03	0.02	0.047
	W^2	0.048	0.03	0.03	0.050	0.054	0.052	0.02	0.03	0.046
	K-S	0.044	0.02	0.04	0.050	0.062	0.062	0.01	0.04	0.047
25	A^2	0.073	0.04	0.01	0.048	0.039	0.057	0.03	0.02	0.047
	W^2	0.061	0.03	0.03	0.053	0.077	0.076	0.02	0.03	0.046
	K-S	0.067	0.02	0.04	0.052	0.092	0.094	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.184	0.181	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.218	0.215	0.01	0.04	0.047

Table 4.19 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Beta(2,3).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.053	0.04	0.02	0.049	0.052	0.049	0.03	0.02	0.047
	W^2	0.056	0.03	0.04	0.050	0.055	0.052	0.02	0.03	0.046
	K-S	0.053	0.02	0.05	0.049	0.056	0.056	0.01	0.04	0.047
15	A^2	0.065	0.04	0.02	0.052	0.056	0.056	0.03	0.02	0.047
	W^2	0.069	0.03	0.03	0.050	0.059	0.063	0.02	0.03	0.046
	K-S	0.061	0.02	0.04	0.050	0.062	0.067	0.01	0.04	0.047
25	A^2	0.103	0.04	0.01	0.048	0.060	0.076	0.03	0.02	0.047
	W^2	0.094	0.03	0.03	0.053	0.077	0.094	0.02	0.03	0.046
	K-S	0.093	0.02	0.04	0.052	0.081	0.098	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.132	0.138	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.134	0.143	0.01	0.04	0.047

Table 4.20 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Normal(0,1).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.054	0.04	0.02	0.049	0.050	0.056	0.03	0.02	0.047
	W^2	0.057	0.03	0.04	0.050	0.051	0.054	0.02	0.03	0.046
	K-S	0.053	0.02	0.05	0.049	0.050	0.050	0.01	0.04	0.047
15	A^2	0.061	0.04	0.02	0.052	0.072	0.066	0.03	0.02	0.047
	W^2	0.066	0.03	0.03	0.050	0.067	0.063	0.02	0.03	0.046
	K-S	0.063	0.02	0.04	0.050	0.065	0.061	0.01	0.04	0.047
25	A^2	0.073	0.04	0.01	0.048	0.077	0.080	0.03	0.02	0.047
	W^2	0.071	0.03	0.03	0.053	0.082	0.076	0.02	0.03	0.046
	K-S	0.076	0.02	0.04	0.052	0.077	0.072	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.106	0.102	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.107	0.101	0.01	0.04	0.047

Table 4.21 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Uniform(0,2).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.069	0.04	0.02	0.049	0.066	0.068	0.03	0.02	0.047
	W^2	0.069	0.03	0.04	0.050	0.075	0.076	0.02	0.03	0.046
	K-S	0.063	0.02	0.05	0.049	0.078	0.083	0.01	0.04	0.047
15	A^2	0.131	0.04	0.02	0.052	0.117	0.129	0.03	0.02	0.047
	W^2	0.117	0.03	0.03	0.050	0.144	0.168	0.02	0.03	0.046
	K-S	0.086	0.02	0.04	0.050	0.167	0.191	0.01	0.04	0.047
25	A^2	0.232	0.04	0.01	0.048	0.179	0.300	0.03	0.02	0.047
	W^2	0.172	0.03	0.03	0.053	0.303	0.360	0.02	0.03	0.046
	K-S	0.129	0.02	0.04	0.052	0.345	0.406	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.725	0.780	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.764	0.812	0.01	0.04	0.047

Table 4.22 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Gamma(2,1).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.207	0.04	0.02	0.049	0.100	0.128	0.03	0.02	0.047
	W^2	0.195	0.03	0.04	0.050	0.094	0.111	0.02	0.03	0.046
	K-S	0.180	0.02	0.05	0.049	0.090	0.094	0.01	0.04	0.047
15	A^2	0.476	0.04	0.02	0.052	0.371	0.469	0.03	0.02	0.047
	W^2	0.465	0.03	0.03	0.050	0.337	0.433	0.02	0.03	0.046
	K-S	0.423	0.02	0.04	0.050	0.298	0.346	0.01	0.04	0.047
25	A^2	0.679	0.04	0.01	0.048	0.613	0.728	0.03	0.02	0.047
	W^2	0.652	0.03	0.03	0.053	0.578	0.682	0.02	0.03	0.046
	K-S	0.602	0.02	0.04	0.052	0.526	0.592	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.899	0.957	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.869	0.922	0.01	0.04	0.047

Table 4.23 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 1$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.241	0.04	0.02	0.049	0.162	0.189	0.03	0.02	0.047
	W^2	0.238	0.03	0.04	0.050	0.156	0.169	0.02	0.03	0.046
	K-S	0.219	0.02	0.05	0.049	0.143	0.134	0.01	0.04	0.047
15	A^2	0.667	0.04	0.02	0.052	0.572	0.743	0.03	0.02	0.047
	W^2	0.651	0.03	0.03	0.050	0.532	0.714	0.02	0.03	0.046
	K-S	0.558	0.02	0.04	0.050	0.479	0.668	0.01	0.04	0.047
25	A^2	0.907	0.04	0.01	0.048	0.837	0.925	0.03	0.02	0.047
	W^2	0.881	0.03	0.03	0.053	0.810	0.912	0.02	0.03	0.046
	K-S	0.801	0.02	0.04	0.052	0.767	0.839	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.990	0.998	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.984	0.986	0.01	0.04	0.047

Table 4.24 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 2$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.049	0.04	0.02	0.049	0.063	0.085	0.03	0.02	0.047
	W^2	0.055	0.03	0.04	0.050	0.062	0.078	0.02	0.03	0.046
	K-S	0.051	0.02	0.05	0.049	0.061	0.066	0.01	0.04	0.047
15	A^2	0.321	0.04	0.02	0.052	0.130	0.412	0.03	0.02	0.047
	W^2	0.346	0.03	0.03	0.050	0.116	0.375	0.02	0.03	0.046
	K-S	0.277	0.02	0.04	0.050	0.105	0.283	0.01	0.04	0.047
25	A^2	0.655	0.04	0.01	0.048	0.210	0.668	0.03	0.02	0.047
	W^2	0.700	0.03	0.03	0.053	0.196	0.660	0.02	0.03	0.046
	K-S	0.575	0.02	0.04	0.052	0.169	0.581	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.386	0.927	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.335	0.913	0.01	0.04	0.047

Table 4.25 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 3.5$).

Sample Size	EDF Tests	Bush's Power	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
			$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
5	A^2	0.052	0.04	0.02	0.049	0.051	0.047	0.03	0.02	0.047
	W^2	0.053	0.03	0.04	0.050	0.051	0.045	0.02	0.03	0.046
	K-S	0.052	0.02	0.05	0.049	0.051	0.046	0.01	0.04	0.047
15	A^2	0.056	0.04	0.02	0.052	0.051	0.046	0.03	0.02	0.047
	W^2	0.058	0.03	0.03	0.050	0.050	0.045	0.02	0.03	0.046
	K-S	0.057	0.02	0.04	0.050	0.049	0.045	0.01	0.04	0.047
25	A^2	0.046	0.04	0.01	0.048	0.047	0.049	0.03	0.02	0.047
	W^2	0.045	0.03	0.03	0.053	0.053	0.047	0.02	0.03	0.046
	K-S	0.051	0.02	0.04	0.052	0.052	0.047	0.01	0.04	0.047
50	N/A	N/A	0.03	0.03	0.055	0.052	0.046	0.02	0.03	0.046
	N/A	N/A	0.02	0.04	0.054	0.054	0.045	0.01	0.04	0.047

Table 4.26 Summary of the Power Study Results for H_o = Weibull(β = 3.5).

Sample Size	EDF Tests	Alternate Distributions						
		Beta(2,2)	Beta(2,3)	Gamma(2,1)	Normal(0,1)	Weibull(1)	Weibull(2)	Uniform(0,2)
5	A^2	○	⊙	●	○	●	○	○
	W^2	○	⊙	●	⊙	●	○	○
	K-S	○	⊙	●	○	●	○	○
15	A^2	○	⊙	⊙	○	○	○	○
	W^2	○	⊙	⊙	⊙	○	○	○
	K-S	○	○	○	○	○	○	○
25	A^2	○	⊙	○	○	○	○	○
	W^2	○	○	○	○	○	○	○
	K-S	○	○	○	○	○	○	○

○ = The sequential G.O.F test outperformed the EDF based G.O.F. test in at least two of the three combinations.

⊙ = The sequential G.O.F. test outperformed the EDF based G.O.F. test only in one of the three combinations or its power is lower than that of the EDF based G.O.F. test by no more than 5%.

● = The sequential G.O.F. test was outperformed by the EDF based G.O.F. test in every combination by more than 5%.

H_o = Weibull(β = 3.5) against symmetric alternate distributions. Based on the discussion in Chapter 2, this finding is an expected one because Q-Statistic G.O.F. test statistic was devised as a discriminant against symmetric or similarly skewed alternate distributions. The following part of the power study will help determine the particular strengths and weaknesses of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests better. As a result, the proposed sequential G.O.F. test here stands out as a very successful candidate to be the analysis tool that is user-friendly and computationally effective in the field of G.O.F. testing. The power plots for this part of the power study are provided in Appendix F.

Power comparison with the Clough's results against Bush's alternate distributions will be discussed in section 4.5.3.

4.5.2 Power Against Wozniak's Alternate Distributions

Even though one can note that the comparison between the new sequential G.O.F. test power results and those obtained by Wozniak [168] is not as clear as with Bush's power study results, the power study results against Wozniak's alternate distributions can be considered valuable for the future comparative studies due to the fact that these alternate distributions constitute a set of benchmark alternate distributions that have been used to evaluate the G.O.F. of the Weibull distribution. As noted before, Wozniak's power study results are not directly comparable to those in this power study, because Wozniak [168] used an extreme-value G.O.F. test procedure to evaluate the two-parameter Weibull distribution. As opposed to the procedure used here that accounts for a location parameter, in order to address the sample data with a nonzero location parameter, Wozniak's procedure requires parameter estimation of this parameter. Then, the estimated location parameter has to be subtracted from the entire given sample data points that results in the loss of the first order statistic when utilizing the MLE of the location parameter. If the analyst has sample data with a small sample size, this loss would be very costly. On the other hand, even when the location parameter is less than unity, the extreme-value G.O.F. tests are inclined to reject the Weibull sample data mistakenly, if the location parameter is assumed to be zero when in fact it is not. Even though it is dissimilar to Wozniak's extreme-value G.O.F. test procedure for the two-

Table 4.27 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 0.5$); H_a: $\chi^2(1)$.

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.0762	0.05	0.01	0.051	0.116	0.126	0.05	0.01	0.051
	W^2	0.0647	0.05	0.03	0.052	0.117	0.125	0.04	0.04	0.050
	K-S	0.0672	0.01	0.05	0.051	0.094	0.102	0.01	0.05	0.051
30	A^2	0.0932	0.05	0.01	0.051	0.153	0.162	0.05	0.02	0.050
	W^2	0.0803	0.05	0.04	0.053	0.154	0.162	0.04	0.04	0.047
	K-S	0.0745	0.01	0.05	0.051	0.122	0.134	0.02	0.05	0.050
50	N/A	N / A	0.05	0.01	0.051	0.233	0.245	0.05	0.01	0.052
	N/A	N / A	0.05	0.04	0.052	0.234	0.242	0.04	0.03	0.049

Table 4.28 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.0$); H_a: $\chi^2(1)$.

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.0762	0.05	0.01	0.054	0.089	0.097	0.05	0.01	0.053
	W^2	0.0647	0.04	0.03	0.050	0.079	0.092	0.03	0.04	0.051
	K-S	0.0672	0.01	0.05	0.054	0.087	0.103	0.01	0.05	0.054
30	A^2	0.0932	0.05	0.01	0.050	0.098	0.102	0.05	0.01	0.053
	W^2	0.0803	0.04	0.04	0.050	0.085	0.113	0.04	0.03	0.051
	K-S	0.0745	0.01	0.05	0.051	0.088	0.123	0.01	0.05	0.052
50	N/A	N / A	0.05	0.01	0.052	0.111	0.124	0.05	0.01	0.051
	N/A	N / A	0.04	0.04	0.051	0.096	0.156	0.03	0.04	0.053

parameter Weibull distribution, the power study results from the extreme-value G.O.F. test based on normalized spacing by Lockhart, Reilly and Stephens [168:419] that demonstrated very high power against the Weibull distributions with various location parameters shows this deficiency. Thus, the location-and scale-free nature of the proposed sequential G.O.F. test is advantageous for these situations.

Since Wozniak's procedure involves a log conversion to the extreme-value data, which eliminates the shape parameter, this power study utilizes known shape parameter values as close as possible to those that sample data from the alternate distributions exhibited by

Table 4.29 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.5$); H_a: $\chi^2(4)$.

Sample Size	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	0.05	0.01	0.055	0.079	0.074	0.04	0.02	0.050
	0.04	0.02	0.050	0.069	0.065	0.03	0.03	0.048
	0.02	0.04	0.050	0.067	0.065	0.02	0.04	0.050
30	0.05	0.01	0.054	0.084	0.081	0.04	0.02	0.050
	0.04	0.02	0.048	0.075	0.074	0.03	0.03	0.049
	0.02	0.04	0.049	0.071	0.076	0.02	0.04	0.050
50	0.05	0.01	0.053	0.104	0.102	0.04	0.02	0.050
	0.03	0.04	0.051	0.088	0.095	0.02	0.04	0.051

Table 4.30 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.0$); H_a: Lognormal(0,1).

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.2156	0.05	0.01	0.054	0.156	0.165	0.05	0.01	0.053
	W^2	0.2059	0.04	0.03	0.050	0.140	0.182	0.03	0.04	0.051
	K-S	0.1661	0.01	0.05	0.054	0.149	0.201	0.01	0.05	0.054
30	A^2	0.3335	0.05	0.01	0.050	0.197	0.224	0.05	0.01	0.051
	W^2	0.3031	0.04	0.04	0.050	0.180	0.242	0.03	0.04	0.051
	K-S	0.2199	0.01	0.05	0.051	0.179	0.269	0.01	0.05	0.052
50	N/A	N / A	0.05	0.01	0.052	0.270	0.346	0.04	0.03	0.053
	N/A	N / A	0.04	0.04	0.051	0.248	0.363	0.03	0.04	0.053

Table 4.31 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 0.5$); H_a: X Cauchy(0,1).

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.8752	0.05	0.01	0.051	0.667	0.676	0.05	0.01	0.051
	W^2	0.8764	0.05	0.03	0.052	0.667	0.674	0.04	0.04	0.050
	K-S	0.8460	0.01	0.05	0.051	0.663	0.669	0.02	0.05	0.051
30	A^2	0.9676	0.05	0.01	0.051	0.751	0.749	0.05	0.02	0.050
	W^2	0.9666	0.05	0.04	0.053	0.751	0.748	0.04	0.04	0.047
	K-S	0.9486	0.01	0.05	0.051	0.748	0.745	0.02	0.05	0.050
50	N/A	N / A	0.05	0.01	0.051	0.823	0.821	0.04	0.02	0.050
	N/A	N / A	0.05	0.04	0.052	0.823	0.819	0.04	0.03	0.052

Table 4.32 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.0$); H_a: X Cauchy(0,1).

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.8752	0.05	0.01	0.054	0.830	0.838	0.05	0.01	0.053
	W^2	0.8764	0.04	0.03	0.050	0.819	0.834	0.03	0.04	0.051
	K-S	0.8460	0.01	0.05	0.054	0.807	0.830	0.01	0.05	0.054
30	A^2	0.9676	0.05	0.01	0.050	0.913	0.935	0.05	0.01	0.051
	W^2	0.9666	0.04	0.04	0.050	0.906	0.932	0.03	0.04	0.051
	K-S	0.9486	0.01	0.05	0.051	0.887	0.933	0.01	0.05	0.052
50	N/A	N / A	0.05	0.01	0.052	0.929	0.992	0.05	0.01	0.051
	N/A	N / A	0.04	0.04	0.051	0.928	0.981	0.03	0.04	0.049

using the Weibull MLE estimate for the shape parameter values as presented in Table 3.8. If Wozniak has operated from a known scale parameter in her extreme-value G.O.F. test, the power study results here would be directly comparable.

The use of the nearest shape parameter within a half-unit interval in our power study

Table 4.33 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 0.5$); H_a: X Double Exp.

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.4964	0.05	0.01	0.051	0.201	0.213	0.05	0.01	0.051
	W^2	0.4702	0.05	0.03	0.052	0.201	0.209	0.04	0.04	0.050
	K-S	0.4002	0.01	0.05	0.051	0.186	0.194	0.02	0.05	0.051
30	A^2	0.6535	0.05	0.01	0.051	0.229	0.239	0.05	0.02	0.050
	W^2	0.6547	0.05	0.04	0.053	0.229	0.237	0.04	0.04	0.047
	K-S	0.5614	0.01	0.05	0.051	0.219	0.227	0.02	0.05	0.050
50	N/A	N / A	0.05	0.01	0.051	0.276	0.289	0.04	0.02	0.050
	N/A	N / A	0.05	0.04	0.052	0.276	0.290	0.04	0.03	0.052

Table 4.34 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.0$); H_a: X Double Exp..

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.4964	0.05	0.01	0.054	0.473	0.654	0.05	0.01	0.053
	W^2	0.4702	0.04	0.03	0.050	0.449	0.644	0.04	0.03	0.051
	K-S	0.4002	0.01	0.05	0.054	0.442	0.547	0.01	0.05	0.054
30	A^2	0.6535	0.05	0.01	0.050	0.613	0.752	0.05	0.01	0.051
	W^2	0.6547	0.04	0.04	0.050	0.585	0.745	0.04	0.03	0.051
	K-S	0.5614	0.01	0.05	0.051	0.547	0.675	0.01	0.05	0.052
50	N/A	N / A	0.05	0.01	0.052	0.792	0.867	0.05	0.01	0.051
	N/A	N / A	0.04	0.04	0.051	0.770	0.829	0.01	0.05	0.052

exaggerates the difference between H_o and the given sample data, and could result in possible bias in favor of the new sequential G.O.F. test. As a result, the use of the Wozniak's power results in the resultant tables ought to be taken as a general reference to a related but not equivalent G.O.F. test. Due to the fact that the power study results here quantify the new sequential G.O.F. test's performance in terms of discriminatory power against the

Table 4.35 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 0.5$); H_a: X Logistic(0,1).

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.3109	0.05	0.01	0.051	0.185	0.192	0.05	0.01	0.051
	W^2	0.3002	0.05	0.03	0.052	0.186	0.191	0.04	0.04	0.050
	K-S	0.2375	0.01	0.05	0.051	0.181	0.184	0.02	0.05	0.051
30	A^2	0.4602	0.05	0.01	0.051	0.221	0.228	0.05	0.02	0.050
	W^2	0.4392	0.05	0.04	0.053	0.221	0.227	0.04	0.04	0.047
	K-S	0.3382	0.01	0.05	0.051	0.217	0.220	0.02	0.05	0.050
50	N/A	N / A	0.05	0.01	0.051	0.272	0.275	0.04	0.03	0.049
	N/A	N / A	0.05	0.04	0.052	0.273	0.269	0.01	0.05	0.052

Table 4.36 Comparison of the Sequential G.O.F. Test Power Results to those of Wozniak and Clough with the significance levels used for H_o: Weibull($\beta = 1.0$); H_a: X Logistic(0,1).

Sample Size	EDF Tests	Wozniak's Power	Clough's Significance Levels $\sqrt{b_1}$	b_2	Attained	Clough's Power	Current Power	Current Significance Levels $\sqrt{b_1}$	Q-Statistic	Attained
20	A^2	0.3109	0.05	0.01	0.054	0.498	0.699	0.05	0.010	0.053
	W^2	0.3002	0.04	0.03	0.050	0.475	0.677	0.04	0.030	0.051
	K-S	0.2375	0.01	0.05	0.054	0.463	0.539	0.01	0.050	0.054
30	A^2	0.4602	0.05	0.01	0.050	0.637	0.827	0.05	0.010	0.051
	W^2	0.4392	0.04	0.04	0.050	0.610	0.812	0.04	0.030	0.051
	K-S	0.3382	0.01	0.05	0.051	0.563	0.716	0.01	0.050	0.052
50	N/A	N / A	0.05	0.01	0.052	0.814	0.932	0.05	0.010	0.051
	N/A	N / A	0.04	0.04	0.051	0.791	0.882	0.01	0.050	0.052

benchmark alternate distributions, the power study results for the new sequential G.O.F. test are no less valuable.

Keeping in mind the fact that Wozniak [168] did not study $n = 50$ but Clough [26] studied this sample size, the Monte Carlo simulation runs for the power study generated results for all combinations of individual significance levels of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for $\alpha = 0.01(0.01)0.20$ and n = 20, 30 and 50 for each alternate distribution. The power study results for the proposed and Clough's [26] sequential G.O.F. test and those of Wozniak's [168] EDF based A^2, W^2, and K-S extreme-value G.O.F. tests are presented in Tables 4.27-4.36.

The $\chi^2(1)$ alternate distribution was evaluated first. Due to the fact that the shape parameter estimate reported in Table 3.8 was roughly 0.65 for n = 20 and 30 and nearby $\chi^2(2)$ becomes Weibull(β = 1), H_o: Weibull(β = 0.5 and 1.0) were used here. As can be seen from the power study results in Tables 4.27 and 4.28, the sequential G.O.F. test outperformed the EDF based A^2, W^2, and K-S G.O.F. extreme-value tests across the board for all of the sample sizes for H_o: Weibull(β = 0.5 and 1.0). With H_o: Weibull(β = 0.5), the $\sqrt{b_1}$ G.O.F. emerged as the dominant component G.O.F. test, whereas the Q-Statistic G.O.F. emerged as the dominant component G.O.F. test with H_o: Weibull(β = 1.0). While the power for all G.O.F. tests was low, the new sequential G.O.F. test demonstrated higher power for H_o: Weibull(β = 1.0) than it did for H_o: Weibull(β = 0.5) against this alternate distribution. This is an expected result due to the fact that $\chi^2(1)$ alternate distribution is closer to Weibull(β = 0.5) than Weibull(β = 1.0).

For the $\chi^2(4)$ alternate distribution, the shape parameter estimates from Table 3.8 were roughly 1.6 resulting in the use of H_o: Weibull(β = 1.5) for the sequential G.O.F. test. Due to the fact that Wozniak [168] did not consider this alternate distribution in her power study, and this alternate distribution was studied by Clough [26] whose findings will be discussed in the following section, there exist no power study results by Wozniak

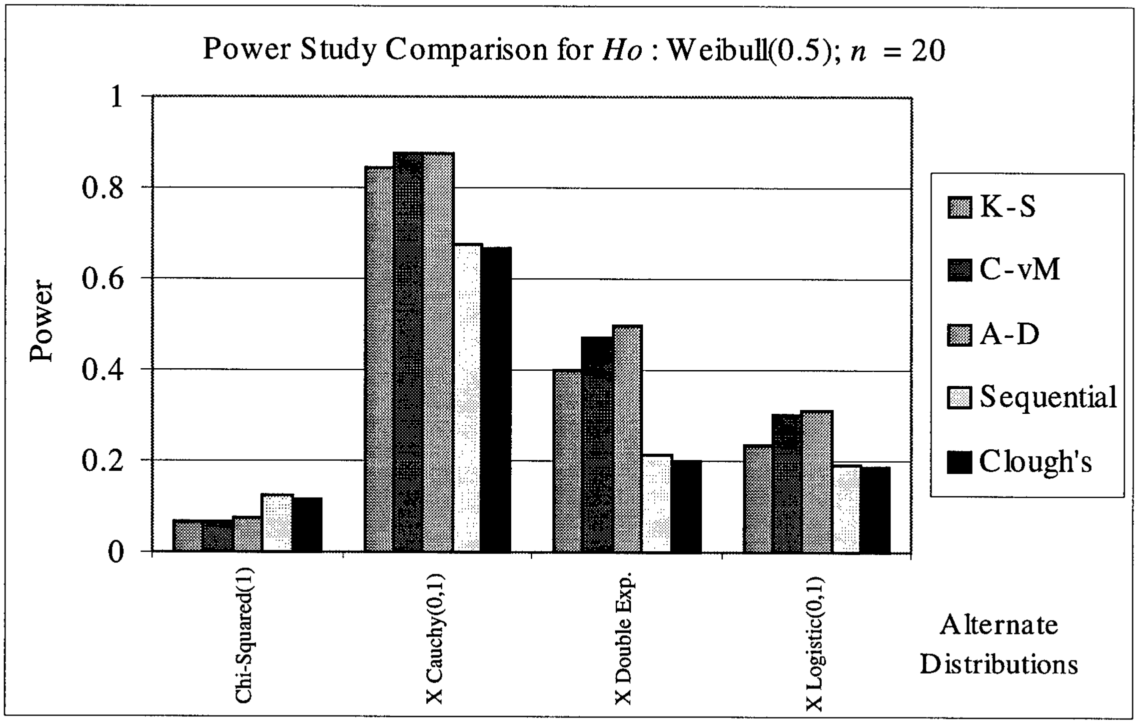


Figure 4.23 Power Comparison for H_o = Weibull($\beta = 0.5$) at $n = 20$ with those of Wozniak's and Clough's.

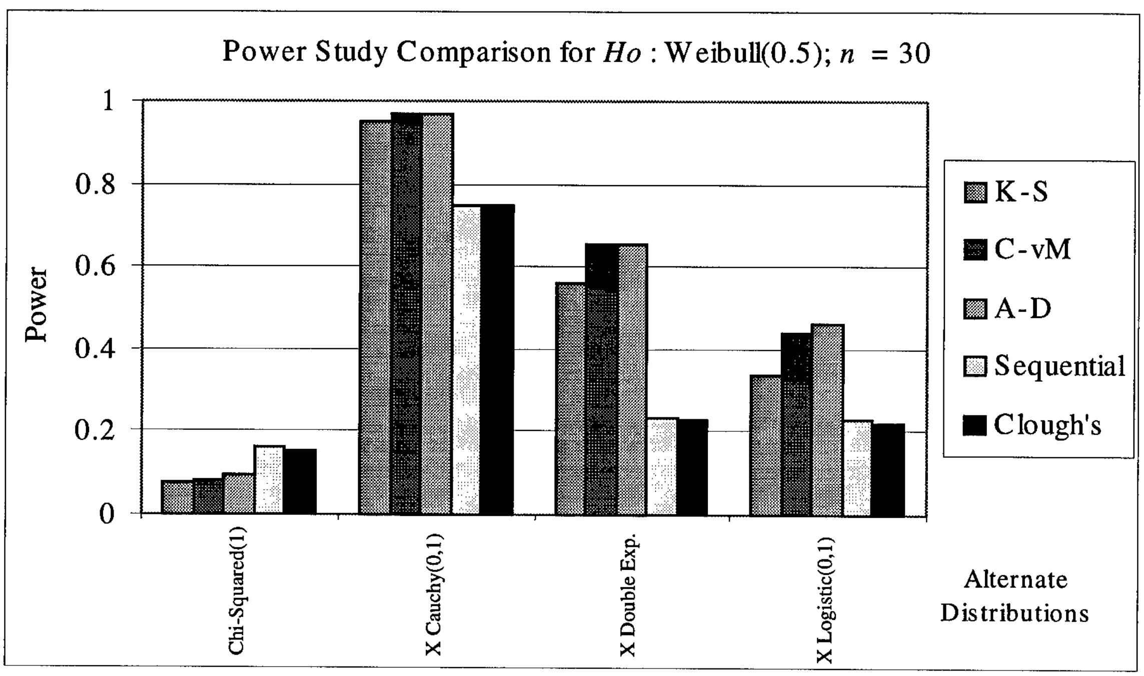


Figure 4.24 Power Comparison for H_o = Weibull($\beta = 0.5$) at $n = 30$ with those of Wozniak's and Clough's.

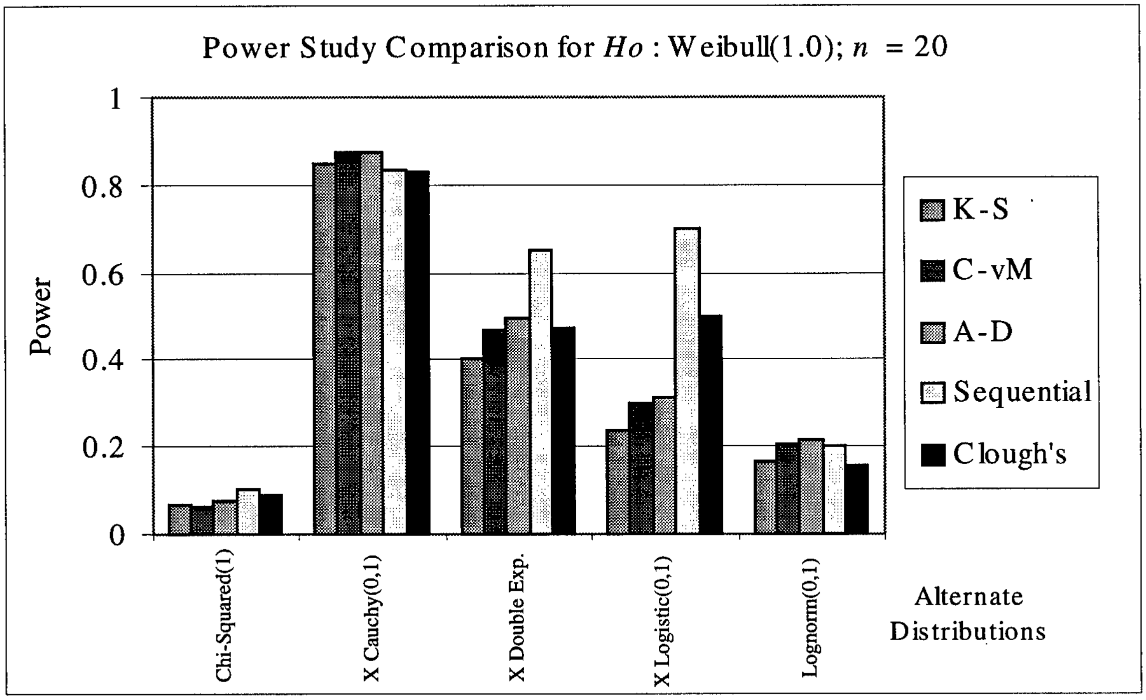


Figure 4.25 Power Comparison for H_o = Weibull(β = 1.0) at n = 20 with those of Wozniak's and Clough's.

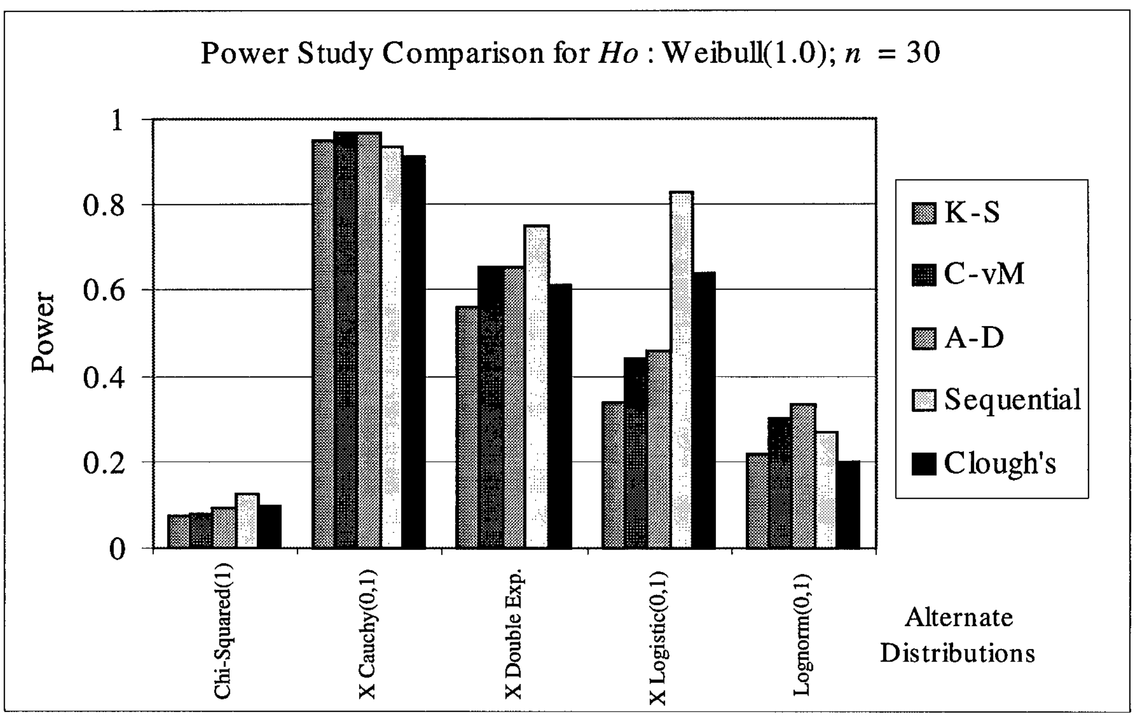


Figure 4.26 Power Comparison for H_o = Weibull(β = 1.0) at n = 30 with those of Wozniak's and Clough's.

Table 4.37 Summary of the Power Study Results for H_o = Weibull(β = 0.5).

Sample Size	EDF Tests	Alternate Distributions $\chi^2(1)$	X Cauchy(0,1)	X Double Exp.	X Logistic(0,1)
20	A^2	○	●	●	●
	W^2	○	●	●	●
	K-S	○	●	●	●
30	A^2	○	●	●	●
	W^2	○	●	●	●
	K-S	○	●	●	●

Table 4.38 Summary of the Power Study Results for H_o = Weibull(β = 1.0).

Sample Size	EDF Tests	Alternate Distributions $\chi^2(1)$	Lognorm(0,1)	X Cauchy(0,1)	X Double Exp.	X Logistic(0,1)
20	A^2	○	●	⊙	○	○
	W^2	○	●	⊙	○	○
	K-S	○	○	⊙	○	○
30	A^2	○	●	⊙	○	○
	W^2	○	●	⊙	○	○
	K-S	○	○	⊙	○	○

to compare with. On the other hand, Wozniak [168:159] commented that the EDF based A^2, W^2, and K-S extreme-value G.O.F. tests yielded less power against the $\chi^2(4)$ alternate distribution than against the $\chi^2(1)$ alternate distribution, with no power greater than 10% even with a sample size of 50. As can be seen in Table 4.29, the same could almost be said for the new sequential G.O.F. test that demonstrated less power against the $\chi^2(4)$ alternate distribution than against the $\chi^2(1)$ alternate distribution, but it achieved better than 10% for the n = 50 case.

For the lognormal(0,1) alternate distribution, the shape parameter estimates from Table 3.8 were roughly 1.1 resulting in the use of H_o: Weibull(β = 1.0) for the sequential

G.O.F. test. As can be seen in Table 4.30, the new sequential G.O.F. test had fairly low power against this alternate distribution and was clearly outperformed by the EDF based A^2, W^2, and K-S extreme-value G.O.F. tests across the board. Using a high significance level for the Q-Statistic G.O.F. test did yield a better power for the sequential G.O.F. test than a high significance level for the $\sqrt{b_1}$ G.O.F. test.

As can be seen in Table 3.8, the X Cauchy alternate distribution samples produced shape estimates near 0.4, the X double exponential near 0.8 and the X logistic near 0.6 resulting in the use of H_o: Weibull($\beta = 0.5$ and 1.0). Even though the best power against the transformed (X) alternate distribution was found against the X Cauchy alternate distribution, the sequential G.O.F. test was still clearly outperformed by the EDF based A^2, W^2, and K-S extreme-value G.O.F. tests across the board. The use of different combinations of individual significance levels did not seem to cause any significant fluctuations in power for this alternate distribution. The sequential G.O.F. test yielded resoundingly better power than the EDF based A^2, W^2, and K-S extreme-value G.O.F. tests for H_o: Weibull($\beta = 1.0$), but considerably less for H_o: Weibull($\beta = 0.5$) against the X double exponential and X logistic alternate distributions. For X double exponential and X logistic alternate distributions, using a high significance level for the $\sqrt{b_1}$ G.O.F. test did yield a better power for the sequential G.O.F. test than a high significance level for the Q-Statistic G.O.F. test. The reader should always remember that Wozniak's power study results from the EDF based A^2, W^2, and K-S extreme-value G.O.F. tests are not directly comparable to those of this sequential G.O.F. test here. However, the power study results provide ballpark figures for the performance of commonly used EDF based G.O.F. tests. As with Bush's case, power comparison histograms were produced to facilitate easy power comparison for this procedures' power results against Wozniak's as can be seen in Figures 4.23-4.26 for H_o = Weibull($\beta = 0.5$ and 1.0) and $n = 20$ and 30. Furthermore, the powers of the two studies were compared based on the light chart visual format as can be seen in Tables 4.37 and 4.38. These tables illustrate the fact that the sequential test performed poorly with H_o = Weibull($\beta = 0.5$) and fairly well with H_o = Weibull($\beta = 1.0$) against Wozniak's alternate distributions. Poor performance with H_o = Weibull($\beta = 0.5$) originates from the fact that the functional forms of the transformed alternate distributions were almost the same as that of the null distribution. Power comparison with the Clough's results against Wozniak's alternate distributions will be discussed in the next section.

4.5.3 Comparison of the Current Research's Powers with Those of Clough's

Clough's power study results against Bush's and Wozniak's alternate distributions can be compared to those of new sequential G.O.F. test proposed here in the light of the following discussion. Since Clough [26] did not study any other alternate distribution other than those of Bush's and Wozniak's, the sequential G.O.F. test was not conducted against any other alternate distributions for the comparison purposes except for case where H_o = Weibull($\beta = 1.5$) and $H_a = \chi^2(4)$. Clough [26] studied the same sample sizes as the ones in Bush's and Wozniak's researches with the addition of $n = 50$ for the attained significance level of $\alpha = 0.05$; therefore $n = 5$, 15, 25 and 50 were studied throughout this research.

$$\sigma_{\hat{p}_{mc}} = \sqrt{\frac{\hat{p}_m(1-\hat{p}_m)+\hat{p}_c(1-\hat{p}_c)}{N}} \tag{4.2}$$

Due to the fact that the power is fundamentally a proportion, expression 4.2 can be used to find the estimate of the standard deviation of the least significant difference $|\hat{p}_m - \hat{p}_c|$, where the power result of this research is denoted as $\hat{p}_m$ and the power result of Clough's as $\hat{p}_c$. It is important to note that $N = 40{,}000$ was used in both power studies. Here, $\hat{p}_m$ and $\hat{p}_c$ are the estimates of the respective powers where p_m and p_c are the true powers. Therefore, due to the fact that it is p_m and p_c that are being estimated, the power results from both studies can only be compared directly based on the difference of the two power study results. Because $\sigma_{\hat{p}_{mc}}$ is maximized using $\hat{p}_m = \hat{p}_c = 0.5$, it is possible to come up with an upper bound (least significant difference) for the comparison here. Therefore, $\sigma_{\hat{p}_{mc}} = 0.0035$ can be found by substituting $N = 40{,}000$ and $\hat{p}_m$ and $\hat{p}_c = 0.5$ in equation (4.2). A rough 95% confidence interval about $|p_m - p_c|$ can be formed by $|\hat{p}_m - \hat{p}_c| \pm 2\hat{\sigma}_{\hat{p}_{mc}}$ according to the empirical rule [109:9]. Consequently, if the least significance difference, $|\hat{p}_m - \hat{p}_c|$ is greater than 0.07, the power results of these researches are assessed to be statistically different than each other with 95% confidence level based on the discussion above.

As can be seen in the previous tables where the power study results of Bush and Wozniak are compared to those of Clough and our current research, the power comparison here will not be exact because the exact same significance levels for the individual component G.O.F. tests of the two sequential G.O.F. tests could not be

Table 4.39 Comparison of the Sequential G.O.F. Test Power Results to those of Bush and Clough with the significance levels used for H_o: Weibull($\beta = 1.5$); H_a: $\chi^2(4)$.

Sample Size	Clough's Significance Levels			Clough's Power	Current Power	Current Significance Levels		
	$\sqrt{b_1}$	b_2	Attained			$\sqrt{b_1}$	Q-Statistic	Attained
20	0.05	0.01	0.055	0.079	0.074	0.04	0.02	0.050
	0.04	0.02	0.050	0.069	0.065	0.03	0.03	0.048
	0.02	0.04	0.050	0.067	0.065	0.02	0.04	0.050
30	0.05	0.01	0.054	0.084	0.081	0.04	0.02	0.050
	0.04	0.02	0.048	0.075	0.074	0.03	0.03	0.049
	0.02	0.04	0.049	0.071	0.076	0.02	0.04	0.050
50	0.05	0.01	0.053	0.104	0.102	0.04	0.02	0.050
	0.03	0.04	0.051	0.088	0.095	0.02	0.04	0.051

used. It is important to note that the power results from both studies at the same or similar individual significance levels for the component G.O.F. test are compared. Table 4.39 presents the power study results conducted for H_o: Weibull($\beta = 1.5$) and H_a: $\chi^2(4)$ that was not originally studied by Wozniak. Clough studied this H_o and H_a combination, because the MLE shape estimates discussed in the previous section required so. Since $|\hat{p}_m - \hat{p}_c|$ for all of the combinations in Table 4.39 is within 0.007, we can conclude that the two sequential procedure power results are not statistically different. Nevertheless, the comparison here will yield useful insights into the difference in the power results by replacing the b_2 G.O.F. test statistic used by Clough [26] with the Q-Statistic used in our sequential procedure.

The first part of the power comparison was conducted considering the performances of these studies against Bush's alternate distributions. Here α_1 is used to denote the individual significance of the $\sqrt{b_1}$ G.O.F. test whereas α_2 is used to denote the individual significance level of Q-Statistic G.O.F. test. The reader can refer to the aforementioned corresponding power comparison tables to see the individual and the attained significance levels for Clough's power results with reference to the ones in this research on the same line. The new sequential procedure performed equally well or better against Clough' sequential procedure against Bush's alternate distributions. More specifically, while our sequential procedure performed equally against H_a: Uniform(0,2) and Weibull(β = 1.0 and 3.5) at all sample sizes, it outperformed Clough's sequential procedure against H_a : Beta(2,2), Beta(2,3), Normal(0,1) and Gamma(2,1) at α_1= 0.02 and α_2 = 0.04 at n = 5 and performed equally for the rest of the sample sizes for H_o : Weibull(β = 1.0). More promising observations were made for H_o : Weibull(β = 3.5). The current sequential procedure performed equally against H_a: Beta(2,2) and outperformed Clough's sequential procedure at α_1= 0.03 and α_2 = 0.02 at n = 25. While our current sequential procedure performed equally against H_a: Beta(2,3) at n = 5 and 15, it outperformed Clough's procedure at n = 25 and 50. Both sequential procedures performed equally against H_a: Normal(0,1). While the new sequential procedure performed equally at n = 5, it outperformed Clough's sequential procedure for the rest of the sample sizes against H_a: Uniform(0,2). The new sequential procedure outperformed Clough's procedure for all sample sizes except for the case it performed equally where α_1 = 0.01 and α_2 = 0.04 at n = 5 against H_a: Gamma(2,1) and Weibull(β = 2.0). The new sequential procedure outperformed Clough's procedure for all sample sizes except for the case it performed worse where α_1 = 0.01 and α_2 = 0.04 at n = 5 against H_a : Gamma(2,1). As a result, it can be assessed that some power improvement over Clough's procedure has been accomplished via the use of the proposed sequential G.O.F. test studied in this research against Bush's alternate distributions. The comparison results can be summarized in Table 4.40 and 4.41. In these tables, $Q > S$ stands for the cases where our sequential procedure performed better than Clough's for the combination with a higher significance level for the Q-Statistic or b_2 than the $\sqrt{b_1}$ G.O.F. test and performed equally well for the other combinations. On the other hand, $S>Q$ stands for the cases where our sequential procedure performed better than Clough's for the combination with a higher significance level for $\sqrt{b_1}$ G.O.F. test than the Q-Statistic or b_2 and performed equally well for the other combinations. (--) stands for the cases where both sequential procedures performed equally well. *ALL* stands for the cases where our sequential procedure performed better than Clough's for all combinations. Note that our sequential procedure did not perform worse than Clough's sequential procedure for any combination. These tables illustrate the fact that our sequential procedure especially improved the power for H_o: Weibull($\beta = 3.5$).

The final part of the power comparison with those of Clough's was accomplished against Wozniak's alternate distributions. Again, the reader may refer to the corresponding tables and figures to visually see the power study results obtained for both of these efforts. Our sequential procedure outperformed Clough's procedure with H_o : Weibull($\beta = 0.5$ and 1.0) and H_a:$\chi^2(1)$ and performed equally with H_o : Weibull($\beta = 1.5$) and H_a:$\chi^2(4)$. The proposed sequential procedure outperformed Clough's procedure with H_o: Weibull($\beta = 1.0$) and H_a: Lognorm(0,1) for every sample size. While Clough's sequential procedure performed equally with H_o: Weibull($\beta = 0.5$) and H_a: X Cauchy(0,1) and X Logistic(0,1), it was outperformed by our procedure with H_o: Weibull($\beta = 1.0$) and H_a: X Cauchy(0,1) and X Logistic(0,1). In addition, our sequential procedure outperformed Clough's procedure with H_o: Weibull($\beta = 0.5$ and 1.0) and H_a: X Double

Exponential for every sample size. Again, the power comparison results are summarized in Tables 4.42 and 4.43.

Table 4.40 Power comparison with those of Clough's against Bush's alternate distributions for H_o: Weibull($\beta = 1.0$).

Sample Size	Alternate Distributions						
	Beta(2,2)	Beta(2,3)	Gamma(2,1)	Normal(0,1)	Weibull(1)	Weibull(2)	Uniform(0,2)
5	Q > S	Q > S	Q > S	Q > S	Q > S	Q > S	--
15	--	--	--	--	--	--	--
25	--	--	--	--	--	--	--
50	--	--	--	--	--	--	--

Table 4.41 Power comparison with those of Clough's against Bush's alternate distributions for H_o: Weibull($\beta = 3.5$).

Sample Size	Alternate Distributions						
	Beta(2,2)	Beta(2,3)	Gamma(2,1)	Normal(0,1)	Weibull(1)	Weibull(2)	Uniform(0,2)
5	--	--	ALL	--	ALL	ALL	ALL
15	--	--	ALL	--	ALL	ALL	ALL
25	S > Q	ALL	ALL	--	ALL	ALL	ALL
50	--	Q > S	ALL	--	Q > S	ALL	ALL

Table 4.42 Power comparison with those of Clough's against Wozniak's alternate distributions for H_o: Weibull($\beta = 0.5$).

Sample Size	Alternate Distributions			
	$\chi^2(1)$	X Cauchy(0,1)	X Double Exp.	X Logistic(0,1)
20	ALL	--	ALL	--
30	ALL	--	ALL	--
50	ALL	--	ALL	--

Table 4.43 Power comparison with those of Clough's against Wozniak's alternate distributions for H_o: Weibull($\beta = 1.0$).

Sample	Alternate Distributions				
Size	$\chi^2(1)$	Lognorm(0,1)	X Cauchy(0,1)	X Double Exp.	X Logistic(0,1)
20	ALL	ALL	ALL	ALL	ALL
30	ALL	ALL	ALL	ALL	ALL
50	ALL	ALL	ALL	ALL	ALL

These observations indicate that our new sequential G.O.F. test improved the discriminatory power over Clough's sequential procedure. The Q-Statistic G.O.F. test statistic did prove itself as being an equal or better discriminator than the b_2 G.O.F. test statistic. Consequently, based on the previous observations the sequential procedure is assessed to be a very useful and user-friendly tool for use in analyses concerned with hypothesized Weibull distributions with known shape parameter value. The next section will provide some more power results derived by the application of our new sequential procedure.

4.5.4 Additional Power Study Results

In addition to the previous power studies conducted against Bush's, Wozniak's and Clough's alternate distributions, we performed numerous power studies with the null and alternate hypotheses listed in Table 3.9. This effort is expected to provide the future researcher with power study results that he can utilize to compare to his results to see how the G.O.F. test in question did in terms of power. Appendix E presents power study results for these H_o and H_a combination results at $\alpha = 0.05$ and sample sizes $n = 5, 15, 25$

Table 4.44 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 0.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.020	0.051
	0.049	0.030	0.030	0.048
	0.050	0.020	0.040	0.049
15	0.050	0.050	0.010	0.050
	0.049	0.040	0.040	0.050
	0.050	0.010	0.050	0.051
25	0.049	0.050	0.010	0.050
	0.047	0.040	0.040	0.049
	0.050	0.010	0.050	0.050
50	0.052	0.050	0.010	0.052
	0.049	0.040	0.030	0.049
	0.051	0.010	0.050	0.052

and 50 as in Bush's case. The verification run results where H_o: Weibull(0.5) and H_a: Weibull(0.5) are presented in Table 4.44 as a sample result table. Two-sided and one-sided versions of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests were studied for some of these H_o and H_a combinations that can be seen in Appendices G and H. Power plots for all of the H_o and H_a combinations studied for the additional power study can be seen in Appendix F as well as those against Bush's alternate distributions.

4.5.5 Individual $\sqrt{b_1}$ and Q-Statistic G.O.F. Test Results

In order to help quantify which component G.O.F. test is most powerful against specific alternate distributions and to result in useful insights that would guide the analyst for selecting the appropriate significance level combinations of the component G.O.F. tests when used sequentially, another power study was conducted using the $\sqrt{b_1}$ and Q-

Statistic G.O.F. test individually for selected combination of H_o and H_a reported in Table 3.14, 3.15 and 3.16. The results of each of these power studies are tabled in Appendix G. Furthermore, the power study results for the new sequential G.O.F. test and individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests were plotted together by significance level for each sample size examined for Bush's alternate distributions in order to effectively compare the individual component G.O.F. tests results with those of the new sequential G.O.F. test. Even though some of Wozniak's alternate distributions were selected to be studied for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests, their power study results were not chosen to be plotted with the sequential G.O.F. test power study results due to the fact that her power study results are not directly comparable to the those of this study. Furthermore, the sequential and the individual G.O.F. test power study results were plotted together for some of the H_o and H_a combinations that are examined in the additional power study section. The resulting plots that are grouped according to the null hypothesis evaluated gave useful insights into the behavior and usefulness of the individual and sequential G.O.F. tests are presented in Appendix F.

For H_o: Weibull($\beta = 0.5$, 1.0, 1.5 and 2.0), the resultant power plots were very similar to those presented in Figure 4.27 for H_a: Beta(2,3). It is crystal clear that the $\sqrt{b_1}$ G.O.F. test dominates the Q-Statistic G.O.F. test in terms of power for every alternate distribution considered for these null hypotheses except for the interesting case where H_o: Weibull($\beta = 1.0$) and H_a: Weibull($\beta = 0.5$). Although the $\sqrt{b_1}$ G.O.F. test dominates the Q-Statistic G.O.F. test at $n = 5$, the Q-Statistic G.O.F. test dominates the $\sqrt{b_1}$ G.O.F. test for the rest of the sample sizes. This case is illustrated in Figure 4.28. The jagged lines in these plots indicating the sequential G.O.F. test power demonstrate the fact that the power of the sequential G.O.F. test fluctuates considerably depending on what combinations of significance levels the analyst selects for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for a given attained significance level. Due to the fact that the $\sqrt{b_1}$ G.O.F. test is generally more powerful, selecting a higher significance level for it and correspondingly lower significance level for the Q-Statistic G.O.F. test will result in higher power for the sequential G.O.F. test, whereas selecting a higher significance level for the weaker Q-Statistic G.O.F. test yields lower power at the same overall significance level. Of course, the situation depicted in Figure 4.28 is an exception and the combination of individual significance levels where the Q-Statistic G.O.F. test is employed at the highest individual α-level and $\sqrt{b_1}$ G.O.F. is employed at the individual α-level that comes with this selection should be selected with this H_o and H_a combination when $n > 5$.

The analyst should note that even though the $\sqrt{b_1}$ G.O.F. test is generally the most powerful against these set of alternate distributions, the difference between the two individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests was dramatically less for the particular alternate distributions that are very similar in shape to the hypothesized Weibull shape. Figure 4.29 illustrates the power plots where H_o: Weibull($\beta = 2.0$) and H_a: Weibull($\beta = 1.5$). The PDF of the H_o and H_a are similar in shape and both individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests perform equally well, especially at small sample sizes. It can be concluded that the Q-Statistic G.O.F. test can be valuable as well for the alternate distributions with similar skewness characteristics. This explains the case illustrated in Figure 4.28.

The reader should note that the sequential G.O.F. test power is bound by the power of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for all these H_o and H_a combinations considered here. This condition exists because of the fact that at a fixed attained significance level, the corresponding significance levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests are usually less or equal to the overall attained significance level, yielding less power than the most powerful individual G.O.F. test employed individually at that same significance level but better power than the least powerful individual G.O.F. test. In other words, if the significance levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests are denoted as α_1 and α_2 respectively that yield an overall attained significance level α, one notes from the contour plots that normally both $\alpha_1 \leq \alpha$ and $\alpha_2 \leq \alpha$. As a result, the power of the both of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests is less than they would be if they had been employed at the level α. The complementary nature of two individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests working together usually prevents the power of the sequential G.O.F. test from going below that of the weaker component G.O.F. test. There were only few cases (see the power plots where H_o: Weibull($\beta = 1.0$) against H_a: Weibull($\beta = 0.5$), H_o: Weibull($\beta =$ 1.5) against H_a: Weibull($\beta = 0.5$ and 1.0) and H_o: Weibull($\beta = 2.0$) against H_a: Weibull(β = 1.5)) where the power of the sequential G.O.F. test appeared to be less than both of the individual component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. On the other hand, these cases are assessed to a result of the variability in the third place of the power estimates rather than any real decrease in power.

There are a few cases where the sequential G.O.F. test power is actually higher than those of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests because of the fact that there is a particularly effective complementary relationship between the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Only one such case where H_o: Weibull(β = 3.5) and H_a: Beta(2,3) alternate distribution for all sample sizes was this observed as presented in Figure 4.30. Both of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests demonstrated nearly the same power against this particular alternate distribution, and they clearly complemented each other to the point that they reject more frequently when employed in sequence against a given sample than each component G.O.F. test does individually.

The results are somewhat more mixed for H_o: Weibull(β = 3.5) than those for H_o: Weibull(β = 1.0). Against H_a: Beta(2,2), Beta(2,3) and Uniform(0,2), the Q-Statistic G.O.F. test yielded higher power than that of the $\sqrt{b_1}$ G.O.F. test unlike the findings for H_o: Weibull(β = 1.0). Both of the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests demonstrated nearly equivalent power against H_a: Normal(0,1). The Q-Statistic G.O.F. test apparently turns out to be a better discriminator against similarly-shaped alternate distributions for this mound-shape null Weibull distribution. This observation is not surprising due to the fact that H_a: Beta(2,2), Beta(2,3), Uniform(0,2) and Normal(0,1) are all more or less symmetric yielding very similar skewness values. Besides, it also supports the previous finding that the reduced correlation between the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics for larger hypothesized Weibull shape parameter values may lead to less dominance of one over the other. As can be seen in Appendix F, the $\sqrt{b_1}$ G.O.F. test performed better than the Q-Statistic G.O.F. test against the rest of the alternate distributions that are more skewed than the aforementioned similarly skewed alternate distributions. These observations help us realize the usefulness and feasibility of the new sequential G.O.F. test studied in this thesis. The component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests each have their own specific strengths and weaknesses, and when confronted with a broad field of alternate distributions, they are enhanced by being conducted together in sequence. While the $\sqrt{b_1}$ G.O.F. test is generally powerful against the skewed alternate distributions, the Q-Statistic G.O.F. test is useful in distinguishing among nearly symmetric or very similarly skewed alternate distributions. Although these are not particularly shocking observations, the fact that the initial expectations were verified by the empirical results is very satisfying.

4.5.6 Directional $\sqrt{b_1}$ and Q-Statistic G.O.F. Test Power

The power study results from the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests that are two-sided statistical tests in their original form assist in determining which of the component G.O.F. tests are most powerful for specific H_o and H_a combinations. The discriminatory power of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests would be expected to go up using one-sided versions of these component G.O.F. tests. Thus, in order to quantify the potential increase in the discriminatory power with the one-sided versions of these G.O.F. tests, various additional power studies were carried out. However, the analyst has to have some specific knowledge about the skewness and Q-Statistic values of the alternate distributions studied in order to be able to utilize the one-sided versions of these component G.O.F. tests. Although this a priori information about the alternate distributions will not always be at hand, quantifying the potential increase in discriminatory power was worthy of investigation.

This research considered the one-sided $\sqrt{b_1}$ and Q-Statistic G.O.F. test for the H_o and H_a combinations that can be seen in Table 3.12 and 3.13. The resultant powers are tabled in Appendix H, with some relevant observations discussed here. To facilitate a means for the reader to be able to see the increase in power with the one-sided versions of the individual G.O.F. tests over their two-sided versions, these comparisons are presented in Appendix I.

As expected, the one-sided versions of the component G.O.F. tests improved power, in some cases, considerably so. Table 4.45 shows us the power increase observed in percentage with one-sided over two-sided individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. As can be seen in Table 4.46, the increase in power with the one-sided version of the individual Q-Statistic G.O.F. test was higher that of the $\sqrt{b_1}$ G.O.F. test with H_o: Weibull(β = 0.5, 1.0 and 1.5), whereas the increase in power with the one-sided version

Table 4.45 Power Increase Observed with One-Sided over Two-Sided Individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests.

H_o	Power Increase Observed with One-Sided over Two-Sided $\sqrt{b_1}$ G.O.F. Test (%)	Power Increase Observed with One-Sided over Two-Sided Q-Statistic G.O.F. Test (%)
Weibull($\beta = 0.5$)	0.1% ~ 100%	0.5% ~ 186%
Weibull($\beta = 1.0$)	0.1% ~ 113%	0.0% ~ 200%
Weibull($\beta = 1.5$)	0.6% ~ 94%	0.8% ~ 100%
Weibull($\beta = 2.0$)	0.1% ~ 68%	0.0% ~ 36%
Weibull($\beta = 3.0$)	0.0% ~ 130%	0.0% ~ 83%

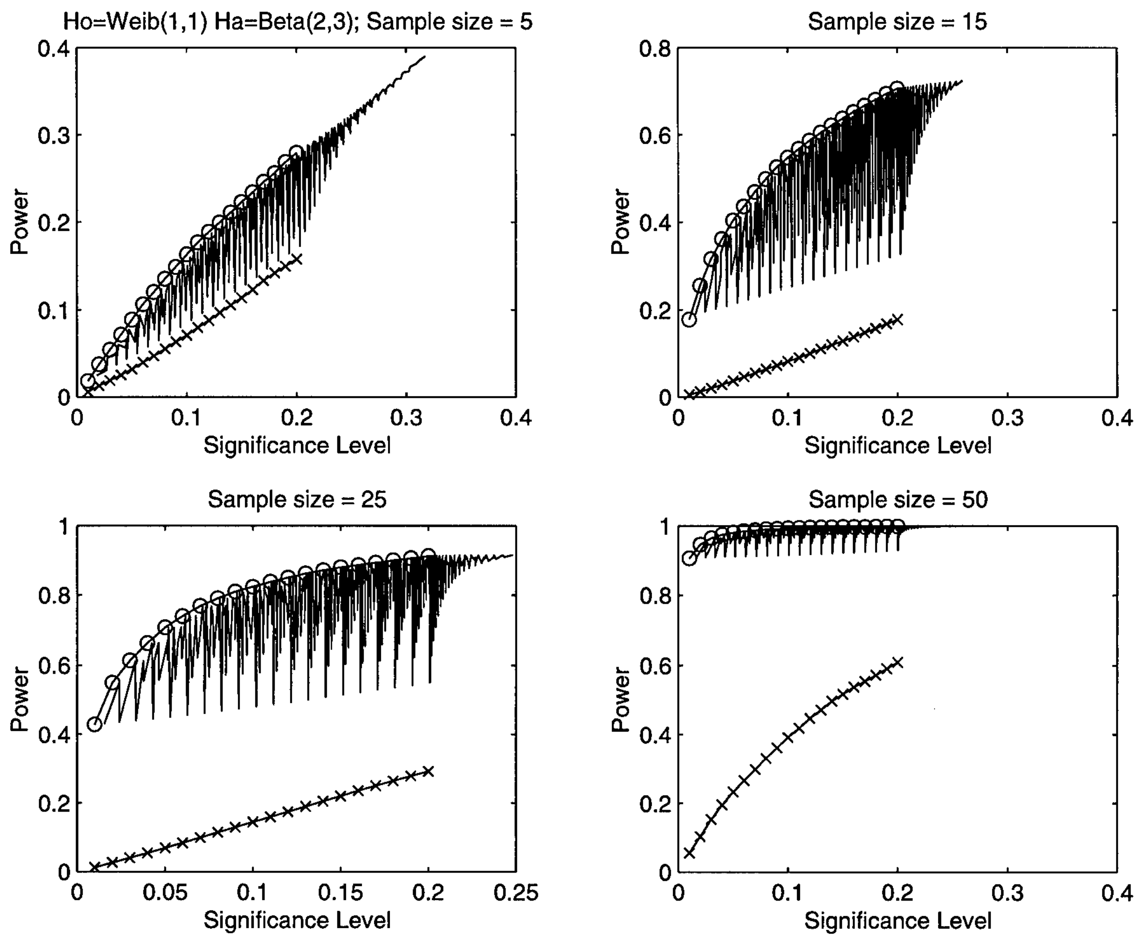


Figure 4.27 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Beta(2,3).

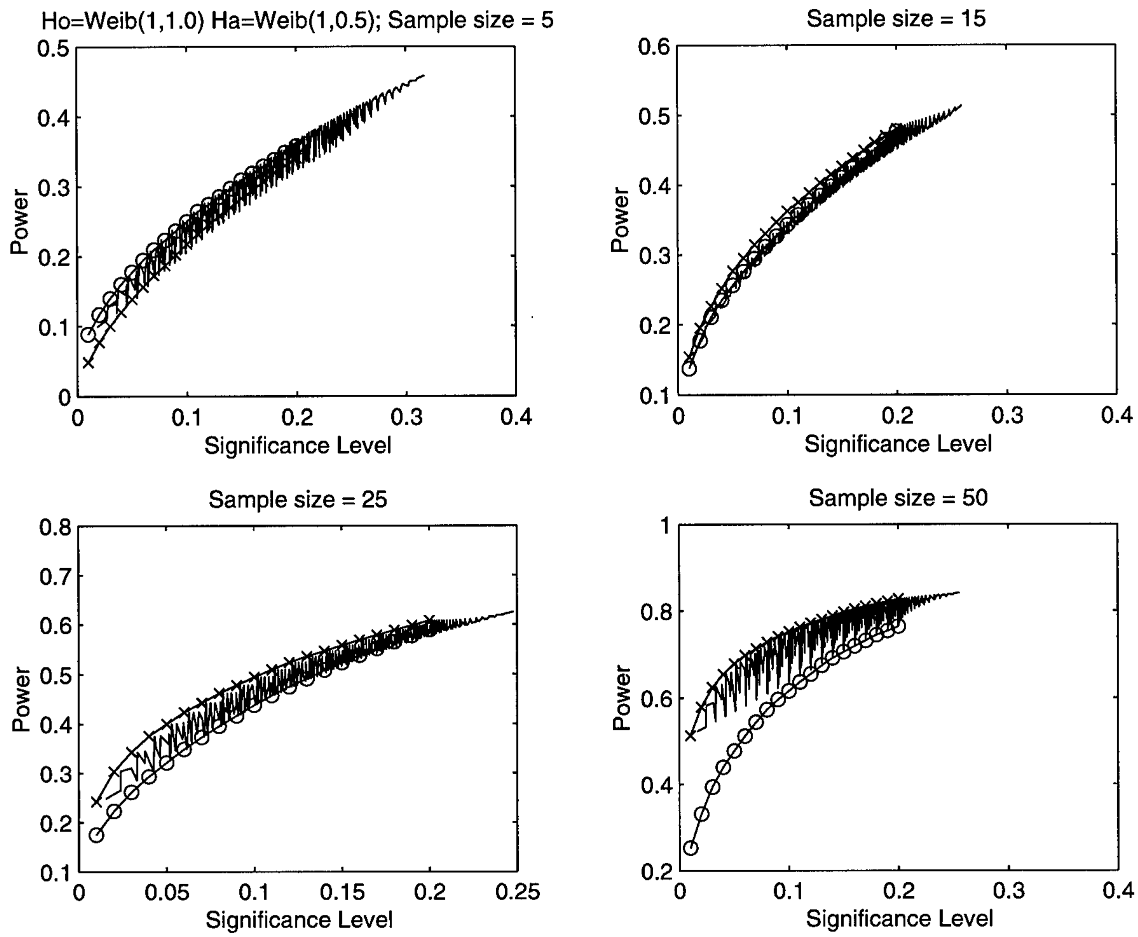


Figure 4.28 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 0.5$).

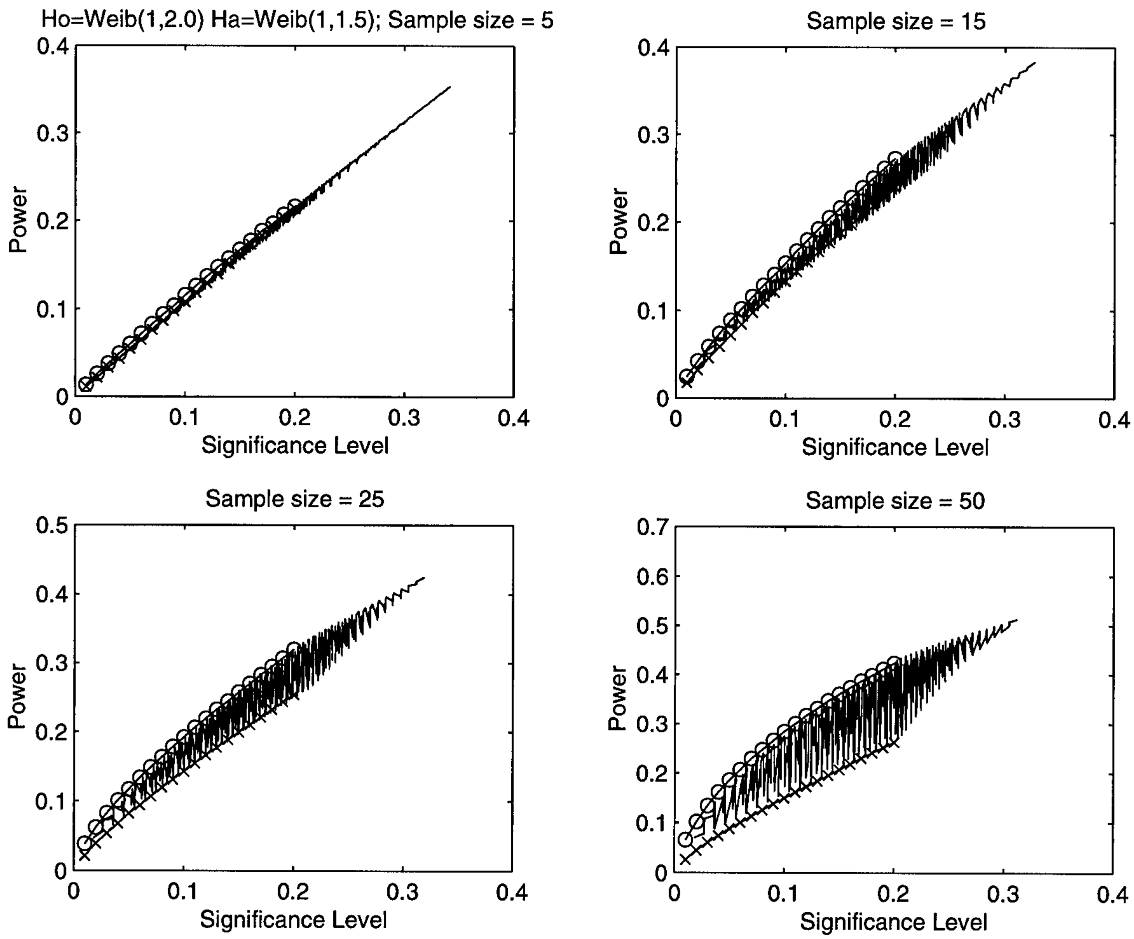


Figure 4.29 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 1.5$).

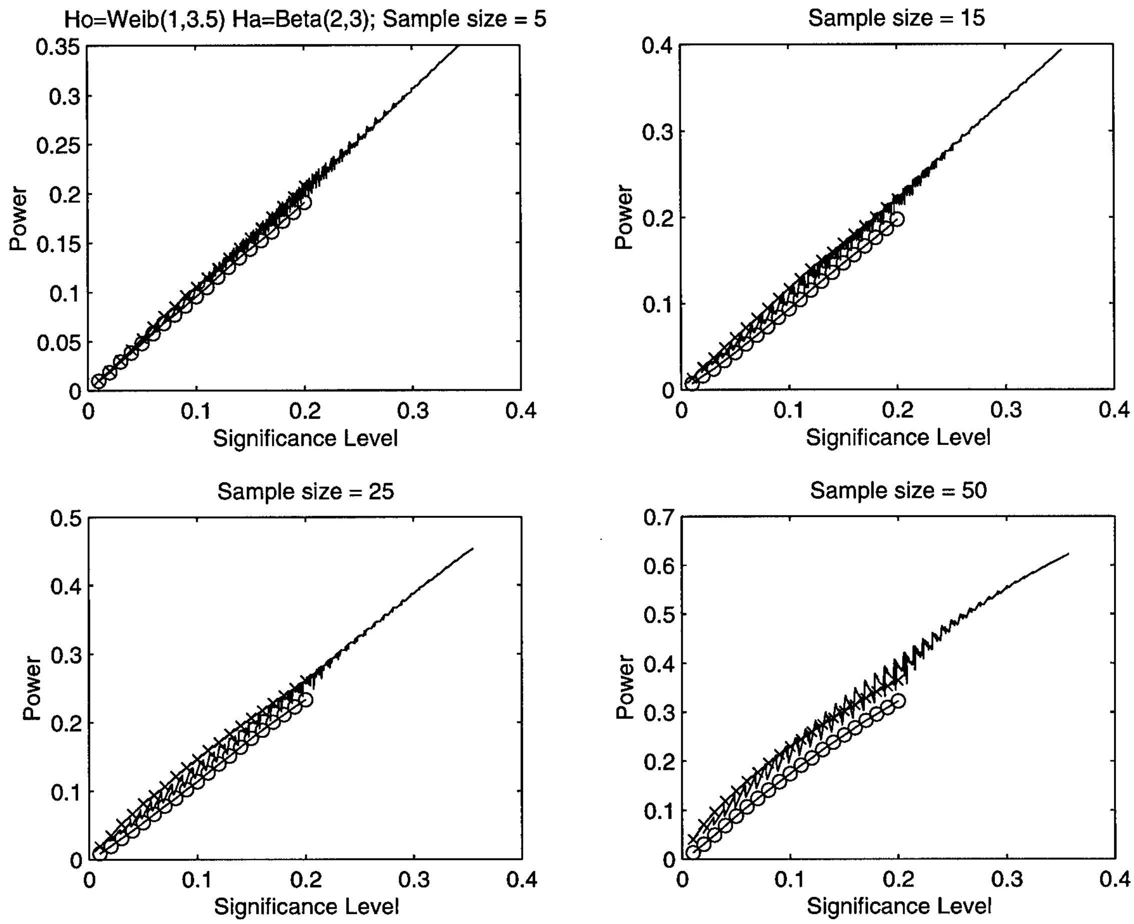


Figure 4.30 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Beta(2,3).

of the individual $\sqrt{b_1}$ G.O.F. test was higher that of the Q-Statistic G.O.F. test with H_o: Weibull(β = 2.0 and 3.5). The power increase was most noticeable against the alternate distributions for which two-sided individual G.O.F. test should have performed well, but did not do so. Table 4.46 shows the one- and two-sided power results for H_o: Weibull(β = 0.5; H_a: Weibull(β = 3.5) where the power increase with the usage of one-sided Q-Statistic can be observed. True Q-Statistic value of these distributions are notably different (4.5424 versus 2.4894, respectively), and one would normally expect the two-sided Q-Statistic G.O.F. to yield good power. It did have acceptable power, however the lower tail one-sided version improved this power by up to 186%, which can be considered a very significant increase in power. Most of the improvements were not as much as the one in the example, but were nonetheless significant. An example of the one-sided $\sqrt{b_1}$ G.O.F. test performance over its two-sided version for H_o: Weibull(β = 0.5; H_a: Weibull(β = 3.5) are presented in Table 4.47. In this case, the

Table 4.46 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.002	0.007	0.045	0.474
	One-Tailed	0.004	0.020	0.099	0.666
0.05	Two-Tailed	0.014	0.073	0.254	0.877
	One-Tailed	0.032	0.168	0.460	0.962
0.10	Two-Tailed	0.035	0.171	0.466	0.964
	One-Tailed	0.085	0.361	0.722	0.995
0.15	Two-Tailed	0.064	0.271	0.616	0.987
	One-Tailed	0.151	0.530	0.860	0.999
0.20	Two-Tailed	0.098	0.367	0.723	0.995
	One-Tailed	0.227	0.665	0.928	1.000

Table 4.47 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(O.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.115	0.820	0.982	1.000
	One-Tailed	0.196	0.875	0.989	1.000
0.05	Two-Tailed	0.307	0.937	0.996	1.000
	One-Tailed	0.426	0.966	0.999	1.000
0.10	Two-Tailed	0.426	0.968	0.999	1.000
	One-Tailed	0.600	0.985	1.000	1.000
0.15	Two-Tailed	0.526	0.980	0.999	1.000
	One-Tailed	0.703	0.992	1.000	1.000
0.20	Two-Tailed	0.602	0.986	1.000	1.000
	One-Tailed	0.754	0.996	1.000	1.000

power was improved up to 71%. Even though the power increase is not as high as the one with usage of the one-tailed Q-Statistic G.O.F. for these null and alternate distributions, this degree of increase can be considered significant. Conclusive remarks cannot be made due to the fact that not all null and alternate hypotheses were evaluated. Nevertheless, there were considerable increases in power for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests when the appropriate one-sided variants were utilized.

4.5.7 Directional Variants of the Sequential G.O.F. Test

The impact of incorporating one-sided $\sqrt{b_1}$ and Q-Statistic G.O.F. tests into the sequential G.O.F. test to measure the increase in power was also examined. Obviously, such cases would be very rare due to the fact that such modifications would mandate a considerable amount of a priori information on the alternate distributions considered.

This approach was complicated by the fact that each modification to the sequential G.O.F. test requires a reevaluation of the attained significance levels. As a result, given these facts, only two such power studies on directional variants of the sequential G.O.F. tests were considered in this research where H_o = Weibull(β = 1 and 3.5) and H_a = Beta(2,2). Three different modifications were assessed for each study here: one using an lower-tail $\sqrt{b_1}$ G.O.F. test, another with only the Q-Statistic G.O.F. test in a one-tailed format, and a final one with both component G.O.F. tests conducted in the lower-tail manner. As mentioned before, the new attained significance levels had to be calculated first. Contour plots or tables of these results were not reproduced in this thesis. Table 4.48 and 4.49 compare the power results of these modified sequential G.O.F. tests to those of the

Table 4.48 Power for Sequential G.O.F. Test Variants: H_o: Weibull(β = 1.0); H_a : Beta(2,2).

Sequential G.O.F. Test Variants				
$\sqrt{b_1}$ G.O.F. Test	2-Sided	Lower Tail	2-Sided	Lower Tail
Q-Statistic G.O.F. Test	2-Sided	2-Sided	Lower Tail	Lower Tail
$n = 5$	0.096	0.131	0.114	0.163
$n = 15$	0.406	0.482	0.429	0.501
$n = 25$	0.754	0.883	0.801	0.901
$n = 50$	1.000	1.000	1.000	1.000

Table 4.49 Power for Sequential G.O.F. Test Variants: H_o: Weibull(β = 3.5); H_a : Beta(2,2).

Sequential G.O.F. Test Variants				
$\sqrt{b_1}$ G.O.F. Test	2-Sided	Lower Tail	2-Sided	Lower Tail
Q-Statistic G.O.F. Test	2-Sided	2-Sided	Lower Tail	Lower Tail
$n = 5$	0.024	0.056	0.092	0.121
$n = 15$	0.189	0.202	0.278	0.304
$n = 25$	0.235	0.262	0.341	0.369
$n = 50$	0.401	0.464	0.543	0.672

original version for a particular case of $\alpha = 0.05$ with nearly same combinations of individual significance levels for the individual G.O.F. tests. Recall that $\sqrt{b_1}$ G.O.F. test yielded a higher power than that of the Q-Statistic with H_o = Weibull(β = 1), whereas individual Q-Statistic G.O.F. test yielded a higher power than that of the $\sqrt{b_1}$ G.O.F. test with H_o = Weibull(β = 3.5) against H_a = Beta(2,2). As we can see from Table 4.99 and 100, the one-sided $\sqrt{b_1}$ G.O.F. test contributes more power than the Q-Statistic G.O.F. test does for H_o = Weibull(β = 1), whereas the one-sided Q-Statistic G.O.F. test contributes more power than the $\sqrt{b_1}$ G.O.F. test for H_o = Weibull(β = 3.5). However, when using both one-sided versions of these component G.O.F. tests, the resulting power is significantly better than the original two-sided versions for both cases. Therefore, if the analyst has enough a priori information to justify utilizing directional versions of the $\sqrt{b_1}$ and/or Q-Statistic G.O.F. tests in the sequential format, he can have better resulting power. However, these cases would be very rare. Thus, further examination was assessed to be beyond the scope of this research.

4.6 Verification and Validation

Various verification and validation techniques were utilized throughout this thesis to determine if the Monte Carlo simulation models utilized in this thesis function properly and are an accurate representation of the statistical theory.

4.6.1 Verification

Verification is determining that a simulation computer code performs according to the intended logic [17: 299]. In other words, it answers the question, "was the model built right?" The computer programs were written and debugged for each model developed in this research to assure an affirmative answer to that question. Each computer program started as a moderately detailed model and debugged in modules to accomplish further objectives. The execution of the previous level was tested rigorously before another level of complexity was added to the computer program. This included replicating the computer output with hand calculations when feasible.

4.6.2 Validation

Validation is concerned with determining whether the conceptual computer model is as accurate representation of the system under investigation [17: 299]. In other words, it answers the question, " was the right model built?" In this thesis, a valid model accurately represents what we would find from statistical theory. Fortunately, some natural "cross-checks" can be accomplished with hypothesis tests and tests of fit. For instance, the critical values for a given G.O.F. test statistic should increase as the significance level of the test decreases, regardless of the sample size. This is indeed what one finds for all of the lower and upper tail critical values of the component $\sqrt{b_1}$ and Q-Statistic G.O.F. test. For example, suppose we are testing the null hypotheses that a random sample of size 40 follows a Weibull distribution with $\beta = 3.5$ at $\alpha = 0.10$. If in our power study algorithm we draw numerous samples of size 40 from a Weibull distribution as our alternate distribution, we would expect 10 percent of the samples to result in rejection of the null hypothesis. This fact is observed in power studies where the null and the alternate hypothesis are the same in corresponding power plots and resultant power study tables.

Since Clough [26] used the $\sqrt{b_1}$ as one the component G.O.F. test in his sequential G.O.F. test, he derived the upper and lower tail critical values for the $\sqrt{b_1}$ test with the replication size of 100, 000. The critical values derived in this study and in Clough's study are the same verifying and validating the critical value derivation algorithms.

4.7 Conclusion

The new sequential G.O.F. test that is based on conducting the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in sequence for the Weibull distribution with known shape parameter could be employed and evaluated via extensive Monte Carlo simulations as mentioned before. By first examining the joint distributions of the $\sqrt{b_1}$ and Q-Statistic values from the Monte Carlo samples, some preliminary observations were made on their correlation and variability. Then, the critical values for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics, with a demonstrated two decimal accuracy, were tabled, providing the means to employ the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Closer examination of these values confirmed some of the early indications seen in the ($\sqrt{b_1}$, Q-Statistic) scatter plots, particularly those relating to changes in variability with increasing shape and sample size. Since the critical values for the component $\sqrt{b_1}$ and Q-Statistic G.O.F. tests alone were not sufficient to be able to utilize the new sequential G.O.F. test, the attained significance levels were also computed. Once the attained significance levels were derived, they were tabled and presented in the form of contour plots that simplify the table look-up approach and yield useful insights regarding the importance of choosing appropriate significance levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests to be able to optimize the overall power. A number of power studies were conducted to yield some idea of the optimum choice of the significance levels for the individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. They also indicated that at a substantially lower computational expense, the new sequential G.O.F. test performed admirably when compared to popular EDF based G.O.F. tests, A^2, W^2 and K-S, in many cases exceeding their power. Moreover, based on the comparison of the power study results from the current and Clough's sequential procedures, replacing b_2 with the Q-Statistic G.O.F. test statistic has improved the power over Clough's sequential procedure. The individual two-sided and one-sided $\sqrt{b_1}$ and Q-Statistic G.O.F. test versions of the new sequential G.O.F. test exhibited very useful information about the utilization of the new sequential G.O.F. test. The individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests demonstrated various complementary strengths and weaknesses, with the $\sqrt{b_1}$ G.O.F. test exhibiting more power than that for Q-Statistic G.O.F. test against the majority of alternate distributions considered in this study. The Q-Statistic G.O.F. test performed very well in distinguishing among the symmetric or similarly skewed distributions. Of course, these findings are very useful in picking out the appropriate selection of significance levels for the individual G.O.F. tests when they are used sequentially. In order to address those cases when sufficient information exists on the alternatives to warrant the use of the upper or lower tailed versions of the G.O.F. tests, one-sided versions of the individual G.O.F. tests and the new sequential G.O.F test were examined and they exhibited substantial enhancements in the power.

V. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

G.O.F. is a field of statistics that warrants attention, especially as more and more organizations in TUAF, and in industry, are making decisions based on the outputs of computer simulations. Even in the hands of the best analyst, such outputs can be misleading if inputs are not modeled properly. No matter what the application, it is important for input models, typically theoretical probability distributions, to be consistent with the data collected through experimentation and observation.

Even though there are numerous G.O.F. tests for the Weibull distribution due to its wide applicability in the field of reliability as a "failure model", most of these G.O.F. tests do not consider the three-parameter form of the Weibull distribution and can only be conducted with a considerably high degree of computational load making them appear unappealing. The new sequential G.O.F. test procedure presented in this thesis provides a computationally simple means of testing for the three-parameter Weibull with a known shape parameter and exhibits good power when compared to the famous and more complex EDF type G.O.F. tests widely used today.

The sequential G.O.F. test procedure has some advantages in that it gets rid of the need for parameter estimation, it is invariant to location and scale parameters, and it requires no transformation of the data. In reliability theory, the location parameter of the Weibull distribution indicates the value of the random variable X for which failures may begin to occur. If a particular component or system truly has a period of operation that is failure free or has the possibility of failing prior to operation, it is important for the failure distribution to be flexible enough to model this. Most of the G.O.F. tests are based on the extreme-value distribution and assume a location parameter of zero. In cases where a non-zero location parameter is assumed, the location parameter must be estimated that results in the sacrifice of the original sample data. Besides, the raw data must also be log-transformed prior to the employment of the G.O.F. test procedure. Thus, in these cases, fixing the location parameter may limit the suitability of such G.O.F. tests. In many circumstances, G.O.F. tests for the Weibull distributions with known shape parameter (unknown scale and location parameters) may be more appropriate, since there are various applications of the Weibull distribution in which the shape parameter is known.

Our new sequential G.O.F. test for the Weibull distribution with known shape parameter is based on applying $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in sequence eliminates these computationally unnecessary steps. Besides, since the G.O.F. test statistics are based on the sample skewness and sample Q-Statistic values, the need for parameter estimation techniques is eliminated. Most of the statistical software packages provide the sample skewness in their summary statistics. Besides, the routine in MATLAB5 that can be seen in Appendix J.1 can be used to determine the Q-Statistics values of the sample data in consideration. The G.O.F. tests that use $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics can be found in the literature, especially for the normal distribution. On the other hand, there exist no documented G.O.F. test for the Weibull distribution that use these G.O.F. test statistics except for Clough [26] who used $\sqrt{b_1}$ G.O.F. test procedure as a part of his research. Sequential G.O.F. test procedures offer the potential of better performance against a wide range of alternative distributions by combining the G.O.F. test statistics that complement each other in terms of power characteristics.

In the light of the advantages mentioned above, a new sequential G.O.F. test for the three-parameter Weibull distribution based on $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistics was developed, implemented and tested against data from 13 alternate distributions. More specifically, the new sequential G.O.F. test procedure was created for Weibull distributions with shape parameter values $\beta = 0.5(0.5)4$ and the sample sizes n = 5(5)50. Monte Carlo simulations for $\alpha = 0.005(0.005)0.10(0.01)0.20$ and 0.80(0.01)0.90(0.005)0.995 were run in order to be able to find the critical values for both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests that make up our new sequential G.O.F. test procedure. Power studies using a true H_o resulting in power identical to the significance level of the G.O.F. test and assessment of the critical values' variability confirmed the validity of the critical values found.

Likewise, Monte Carlo simulations were run in order to find the attained significance levels for all potential combinations of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. The attained significance levels are presented in tabular and graphical form as can be seen in Appendices C and D. The attained significance level contour plots provide the analyst with easier means to identify the appropriate choice of significance levels for the component tests that yields the desired significance levels for the sequential procedure so that higher discriminatory power can be reached with the new sequential procedure.

The selection of individual significance levels for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests has direct influence on the performance of the sequential G.O.F. test to be able to discriminate against a variety of distributions in terms of power. Keeping in mind that the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests have particular strengths and weaknesses, the overall power of the sequential G.O.F. test can be increased by conducting it with a choice of higher significance level for the more powerful of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Power studies conducted in this research examined the H_o = Weibull (β = 1 and 3.5) against the alternative distributions studied by Bush's research and the H_o = Weibull (β = 0.5, 1.0 and 1.5) against most commonly used alternatives in power studies documented by Wozniak. In the end, the results from this thesis were compared to Clough's work who also studied a sequential G.O.F. test procedure that is made up of $\sqrt{b_1}$ and b_2 G.O.F. tests. Furthermore, some additional distributions were studied to present some comparison for future studies in this field.

The new sequential G.O.F. test consistently outperformed the EDF based A^2, W^2, and K-S G.O.F. tests for $n = 5$ against all Bush alternate distributions with H_o = Weibull(β = 1). The new sequential procedure also performed admirably well against H_a = Beta(2,2), Beta(2,3)and Uniform(0,2) for the remaining sample sizes. On the other hand, the new sequential G.O.F. test surprisingly did not perform well against H_a = Normal(0,1) and Gamma(2,1) for n = 15 and 25. Even though the new sequential procedure performed reasonably well against H_a = Weibull(β = 3.5), it performed poorly against H_a = Weibull(β = 2).

Better power performance was achieved against Bush alternate distributions with H_o = Weibull(β = 3.5). Even though the sequential G.O.F. test performed poorly against H_a = Gamma(2,1) and Weibull(β = 1) for n = 5, its power matched or exceeded those of the EDF based A^2, W^2, and K-S G.O.F. tests against the remaining alternate distributions for $n = 5$ and against all of Bush alternate distributions for $n = 15$ and 25. It is noteworthy that in all cases in which the proposed sequential G.O.F. test failed to outperform the EDF based A^2, W^2, and K-S G.O.F. tests, the sequential G.O.F. test power was usually within roughly 7.5% from those of the EDF based A^2, W^2, and K-S G.O.F. tests. This fact is very important in the sense that the proposed sequential G.O.F. test does not have the computational complexity that the existing G.O.F. tests have. Consequently, it can be concluded that the new sequential procedure performing well compared to the EDF based A^2, W^2, and K-S G.O.F. tests against Bush alternate distributions was a noteworthy accomplishment.

Even though the power study results against Wozniak's alternate distributions are not directly comparable to the new sequential procedure, the trends matched her EDF results fairly closely. The new sequential procedure exhibited very low power against $H_a = \chi^2(1)$ and χ^2 (4) and low power against $H_a =$ Lognormal(0,1). While moderate to good power was obtained against H_a = Transformed (X) Logistic(0,1) and X Double Exponential, the best power was achieved against H_a = X Cauchy(0,1). Again, the new sequential procedure performed fairly well compared to the famous EDF based A^2, W^2, and K-S G.O.F. tests.

The third part of the power study included the comparison of the power study results by Clough. The new sequential G.O.F. test clearly performed equally as well or better than those of Clough's against these alternate distributions by the comparison criterion that is based on the least significance difference explained in the previous chapter. The improvement in power was most noticeable for the cases where

H_o = Weibull(3.5) was used against Bush alternate distributions and where H_o = Weibull(1.0) was used against Wozniak alternate distributions. There was no case where the new sequential procedure performed worse than Clough's. Consequently, this research shows that replacing b_2 with the Q-Statistic in the sequential G.O.F. test results in equal or better overall power.

In addition to the power studies conducted against Bush, Wozniak and Clough alternatives, there were numerous power studies with additional H_o and H_a combinations that are listed in Table 3.9 to prepare a reference for the future studies.

Some useful insights were obtained when the power study results of the new sequential G.O.F. test were compared to those of the individual G.O.F. tests. Even though the new sequential G.O.F. test power was less than that of its most powerful component G.O. F. test in almost all of the cases, it was greater than the worse of the two component G.O.F. tests. This may lead the reader to favor utilizing the most powerful component test individually versus utilizing the sequential G.O.F. test. However, the use of the new sequential G.O.F. test guarantees a higher average power against a wide range of H_as, because it is impossible to always have *a priori* information about H_a. As a matter of fact, there were a few cases where $\sqrt{b_1}$ and Q-Statistic G.O.F. tests had nearly identical power, and the new sequential G.O.F. test outperformed both individual $\sqrt{b_1}$ and Q-Statistic G.O.F. tests at a given significance level.

Consequently, the overall power was increased to a great extent by incorporating one-sided variants of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests in the new sequential G.O.F. test. These modifications were accomplished to account for the cases in which the analyst has *a priori* information about H_a so that he or she can identify the proper upper or lower tail G.O.F. test(s). However, it is rare to have *a priori* information about H_a in real life applications.

5.2 Recommendations for Use

Based on the results of the significance level determination and the power study, the following recommendations on the employment of the new sequential G.O.F. test can be made:

- Since the $\sqrt{b_1}$ G.O.F. test has the best discriminatory power between $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_o = Weibull ($\beta \leq 1$), a higher significance level for the $\sqrt{b_1}$ G.O.F. test should be chosen to ensure higher overall power for the sequential G.O.F. test.
- When testing for H_o = Weibull ($\beta \geq 1.5$), the first course of action should be to acquire some additional information that can be derived from computer-based graphical analysis of the sample data using probability plots or histograms or from theoretical knowledge on the source or the process that generated the data. This information can be used to yield some additional insights into the possible alternate distributions that the analyst wishes to discriminate against, allowing the analyst to select a combination of individual significance levels favoring the most powerful component tests. The power plots that are presented in Appendix F can be used as a basis for determining the more powerful G.O.F. test statistic, if the alternative distributions are known specifically. If no priori information is at hand about the possible alternate

distributions, there exist two optional courses of action. The first optional course of action is choosing an attained significance level combination that is made up of nearly equal individual significance levels for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. Even though this optional course of action gives $\sqrt{b_1}$ and Q-Statistic G.O.F. tests equal weight in the new sequential G.O.F. test, selecting smaller but equivalent individual significance levels will yield less power for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests. It has been observed that the nature of the attained significance levels leads the analyst to select near equality at much lower individual significance level of the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests or a very biased combination heavily favoring one G.O.F. test over the other G.O.F. test. The second optional course of action is to conduct the new sequential G.O.F. test twice at the opposite extremes of the possible attained significance level combinations. Conducting the new sequential G.O.F. test twice in which the first trial is conducted with a higher significance level for $\sqrt{b_1}$ G.O.F. test and the second trial is conducted with a higher significance level for Q-Statistic G.O.F. test will create a chance for both $\sqrt{b_1}$ and Q-Statistic G.O.F. tests to reject the sample data in consideration.

5.3 Recommendations for Future Research

Some of the follow-on study opportunities for this research are as follows:

- This new sequential G.O.F. test could be modified to test sample data with small and large sample sizes that have been censored to different degrees.
- Since the sequential G.O.F. test studied in this research only considered small sample sizes ($n \leq 50$), the next logical step is to investigate it for larger sample sizes. It would be especially useful to determine and present the asymptotic critical values in at least tabular form for the $\sqrt{b_1}$ and Q-Statistic G.O.F. tests so our sequential test can be conducted at sample sizes over 50. Of course, such an effort would require a new scheme to generate the expected values of the $\sqrt{b_1}$ and Q-Statistic G.O.F. test statistic values of the large sample Weibull random variables for various known shapes. In addition, a set of functions to allow the analyst to approximate the critical values as a function of sample size and Weibull distribution shape parameter could be derived.
- More work can be done for determining the utility of the sequential procedure for the small sample sizes by studying more null and alternate distribution combinations and comparing the power results to the existing ones.
- Although the Q-Statistic does not exhibit as much variability as the b_2 G.O.F. test statistic, it still demonstrates more variability than the $\sqrt{b_1}$ test statistic, especially for smaller Weibull shape values. An option could be to replace the Q-Statistic with Royston's t_4 test statistic that was developed as a substitute for b_2 test statistic [143]. The $\sqrt{b_1}$ test statistic could also be replaced with Hogg's Q_1 test statistic [80]. These replacements will possibly improve the power of the sequential G.O.F. test introduced here.

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Appendix A. Joint Distribution of $\sqrt{b_1}$ and Q-Statistic

A.1 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$).

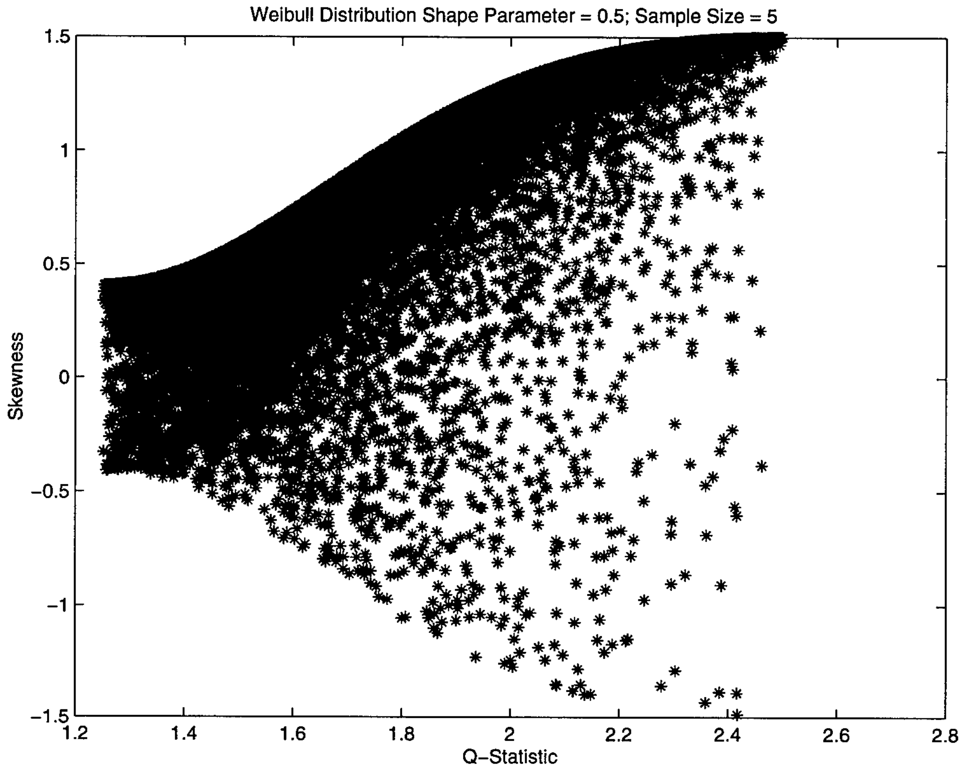


Figure A.1 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 5$.

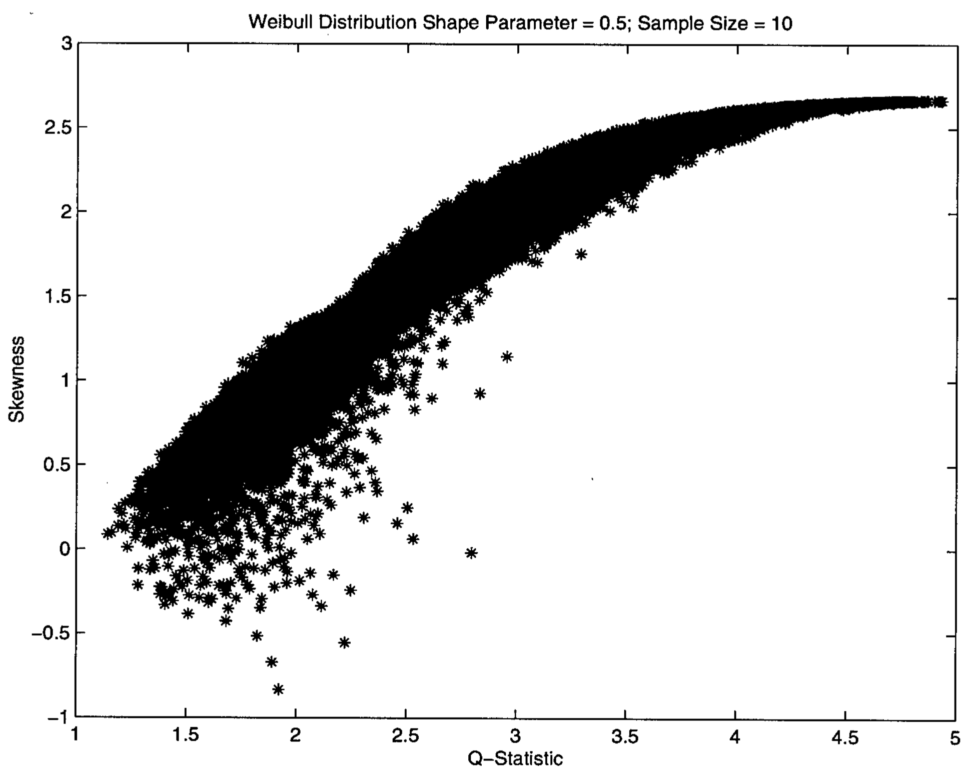


Figure A.2 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 10$.

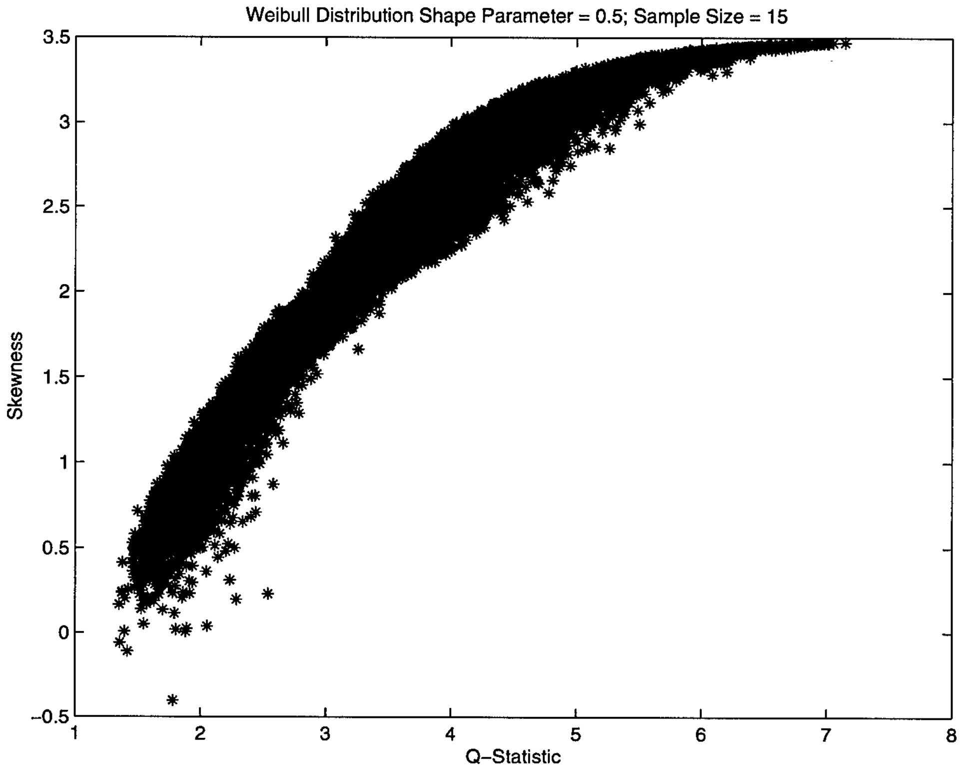


Figure A.3 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 15$.

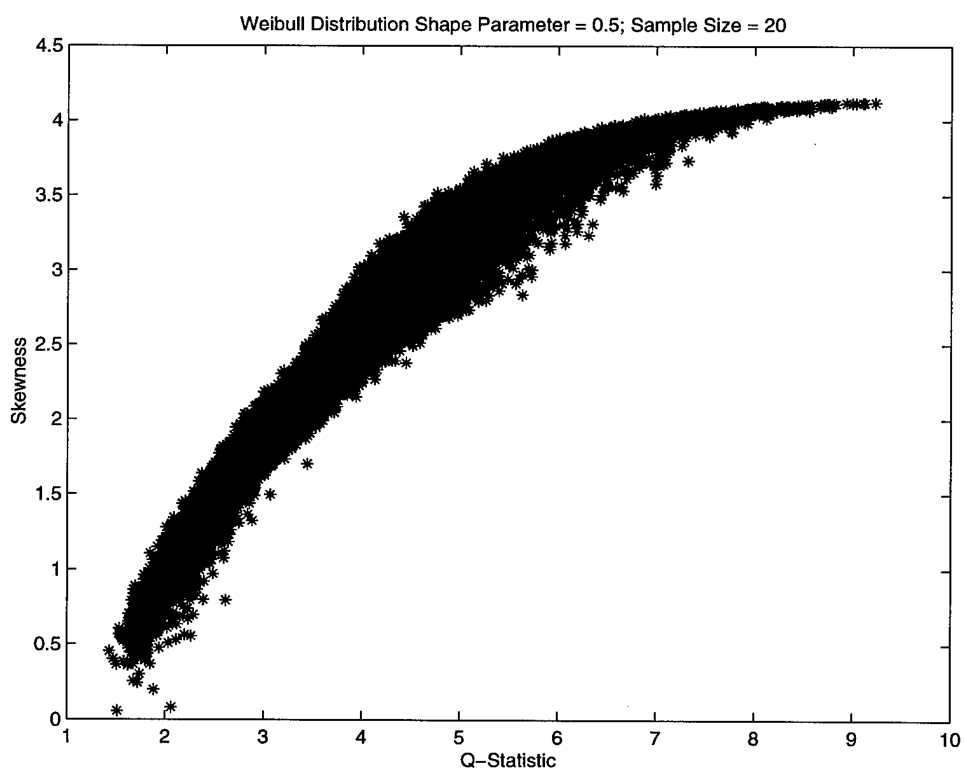


Figure A.4 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 20$.

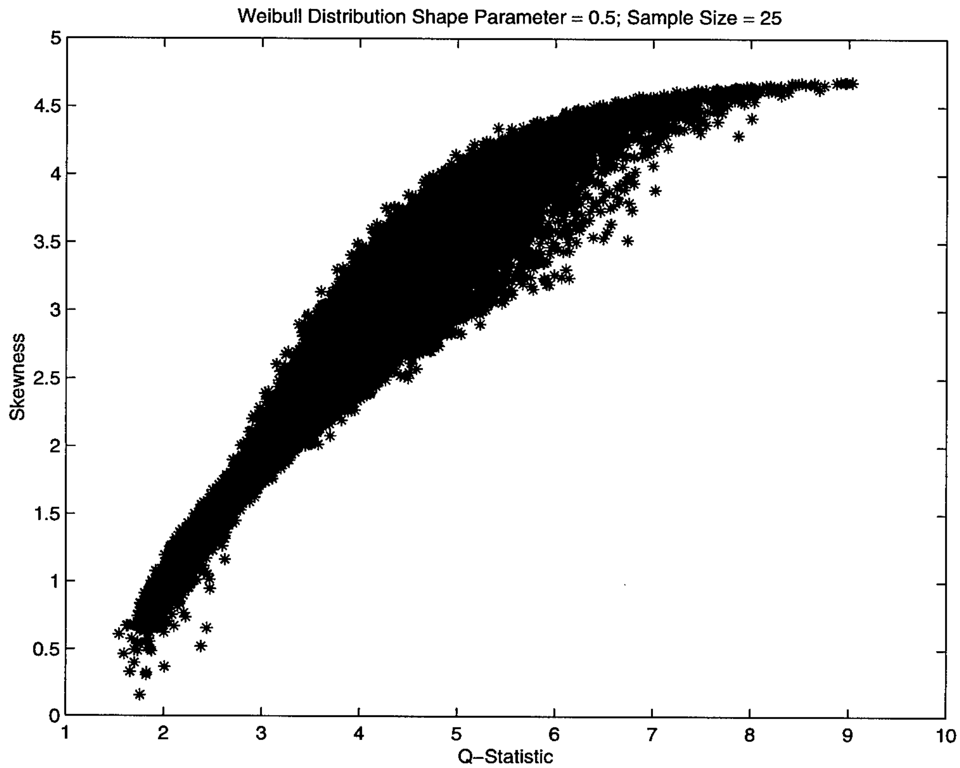


Figure A.5 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 25$.

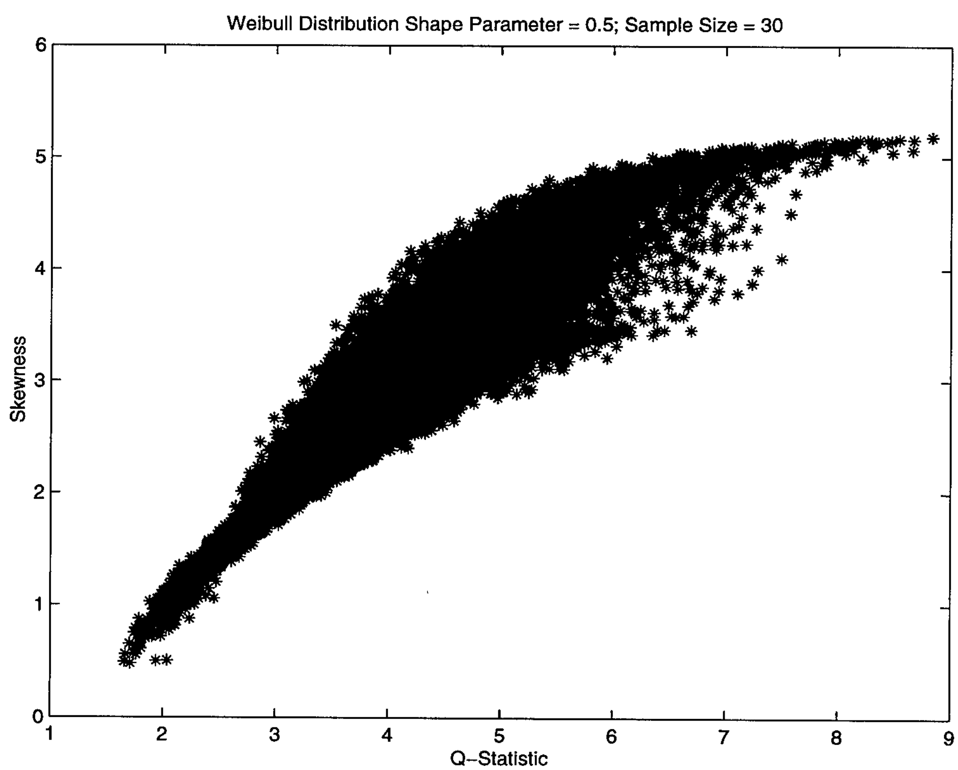


Figure A.6 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 30$.

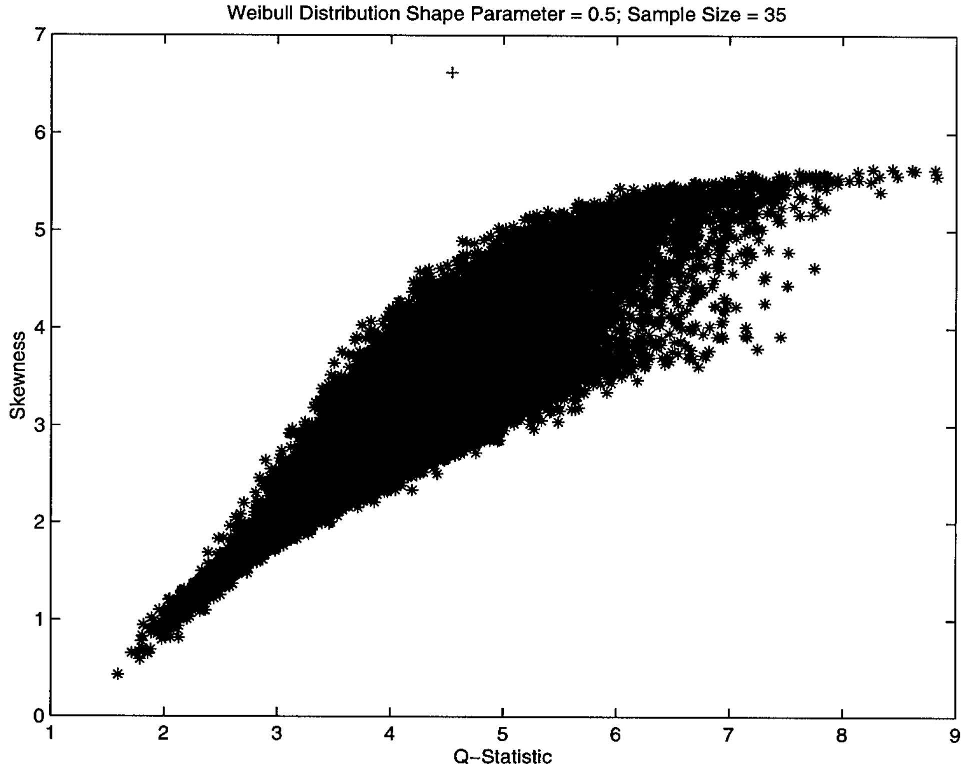


Figure A.7 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 35$.

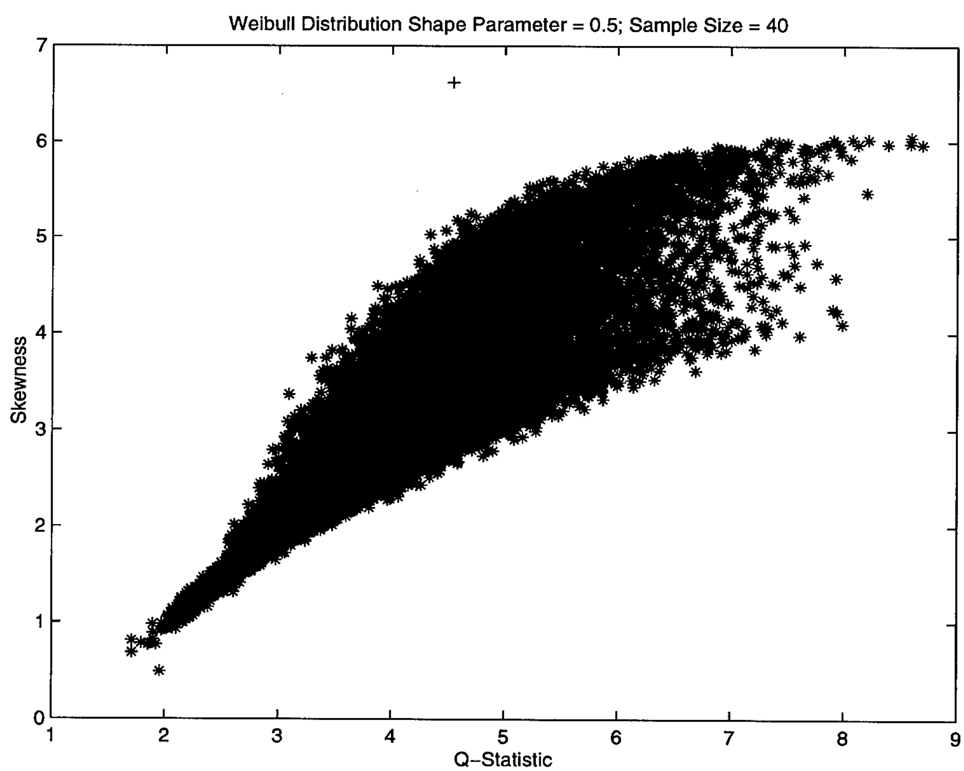


Figure A.8 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 40$.

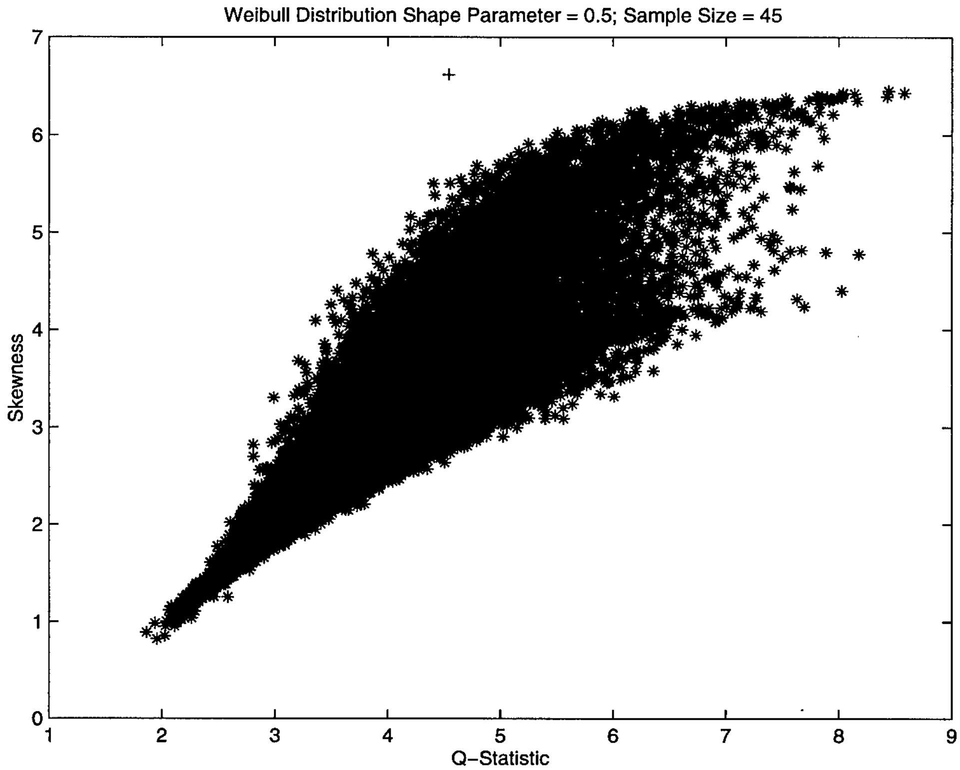


Figure A.9 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 45$.

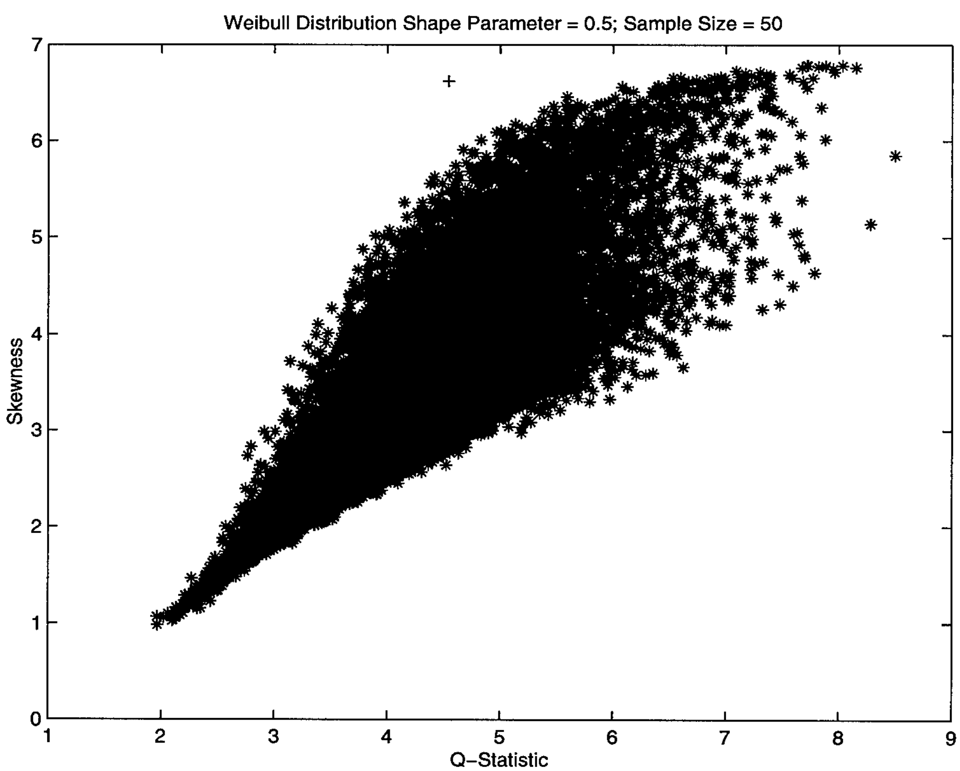


Figure A.10 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 0.5$) ; $n = 50$.

A.2 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$).

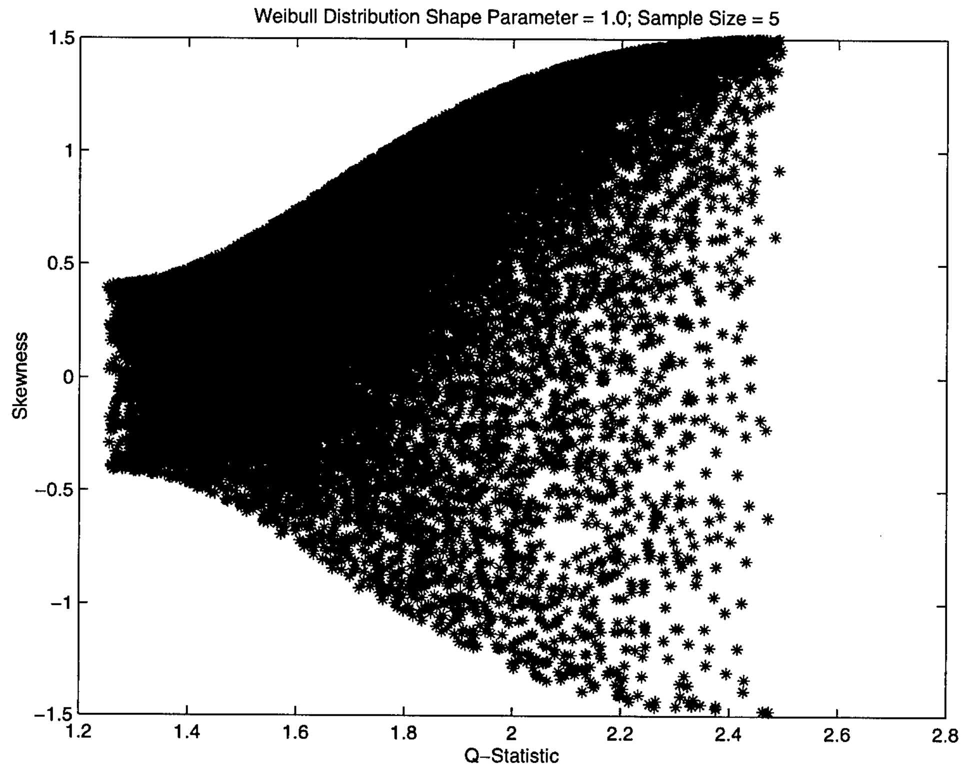


Figure A.11 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 5$.

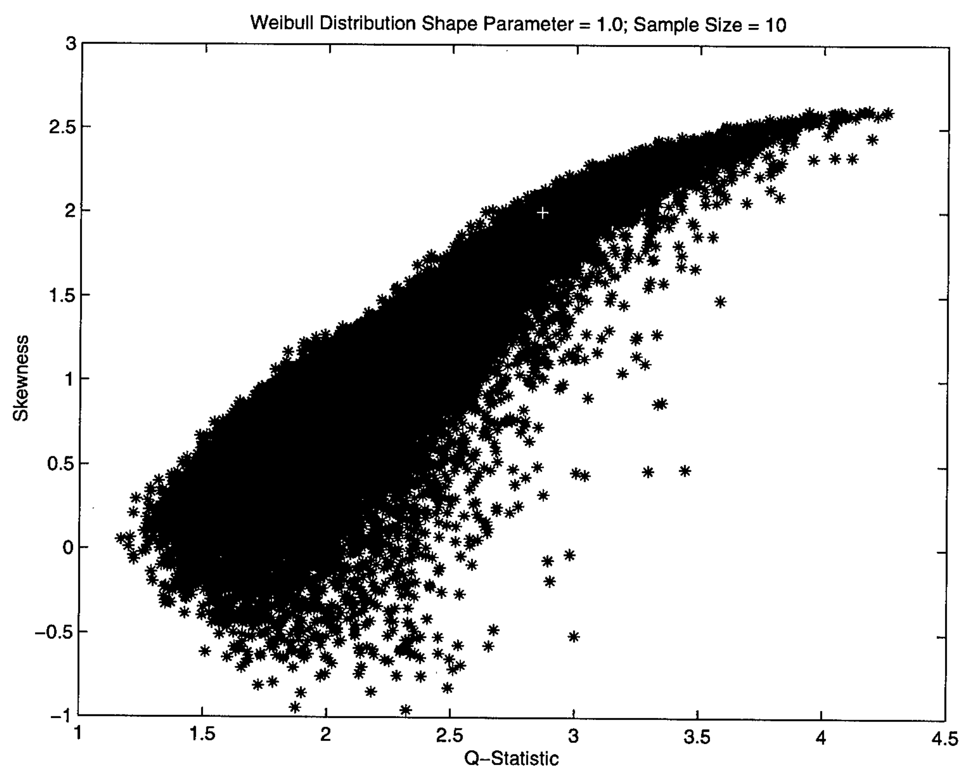


Figure A.12 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 10$.

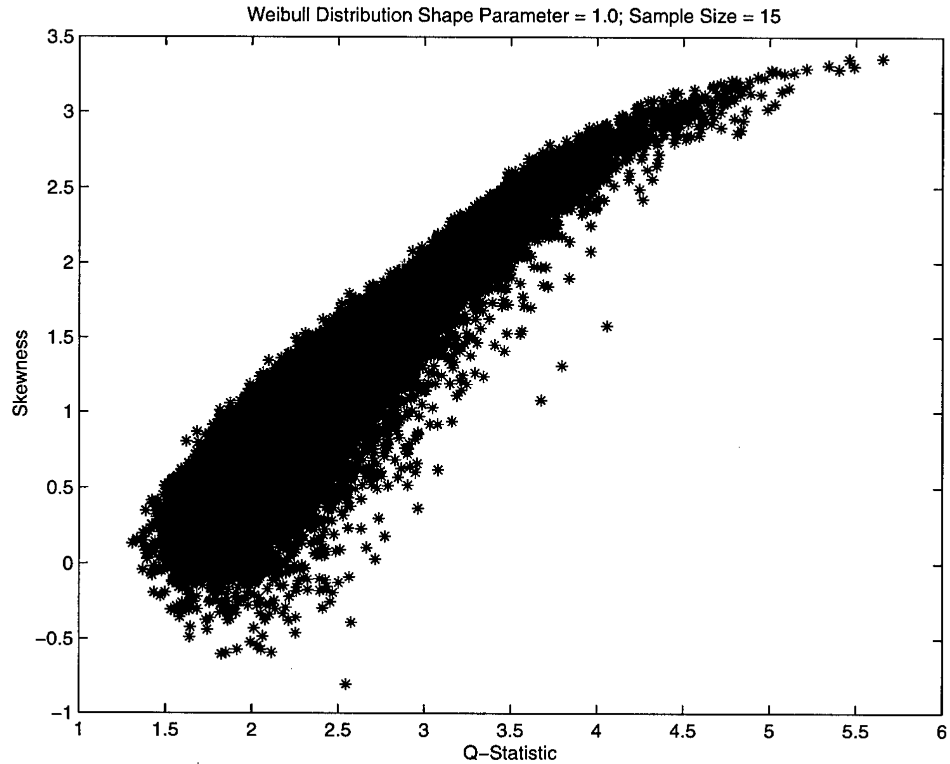


Figure A.13 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 15$.

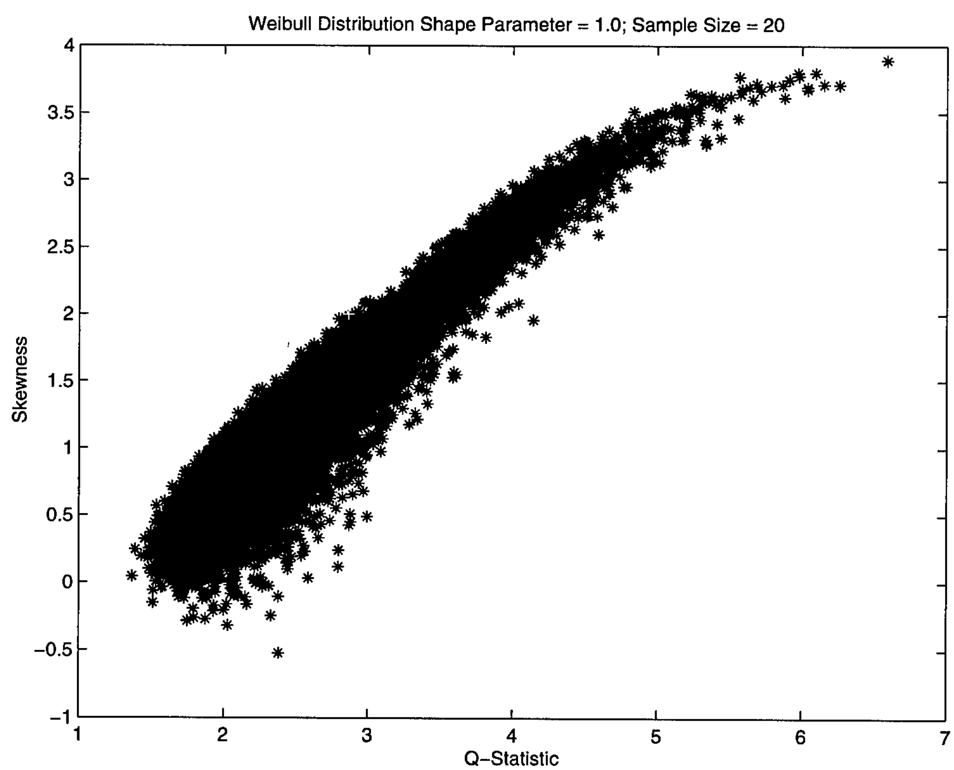


Figure A.14 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 20$.

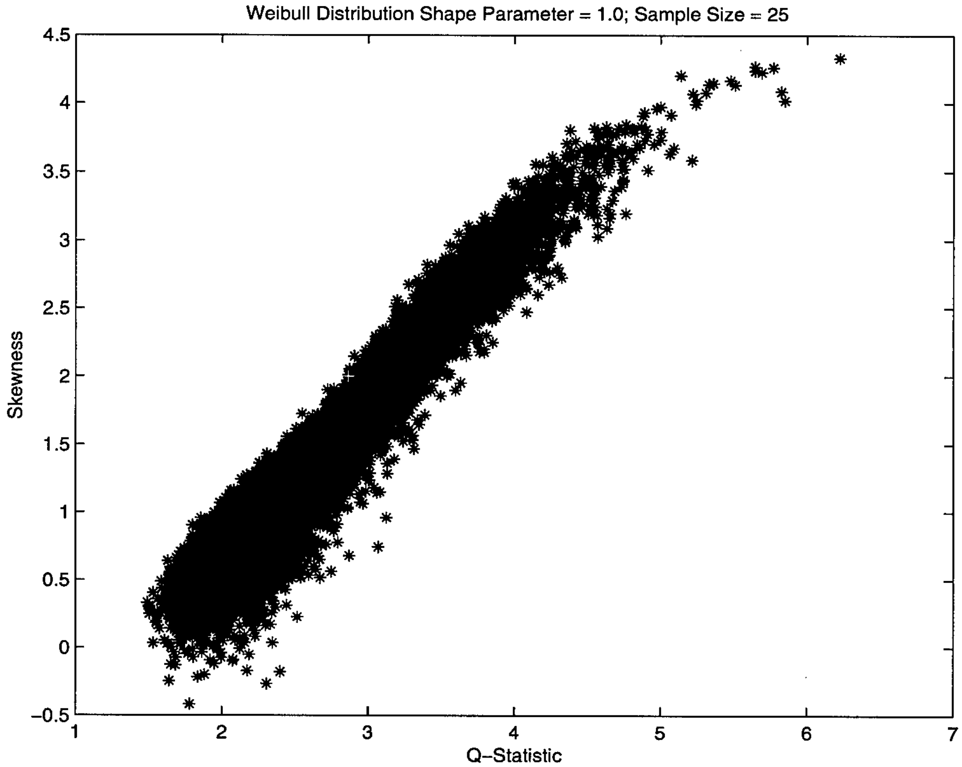


Figure A.15 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 25$.

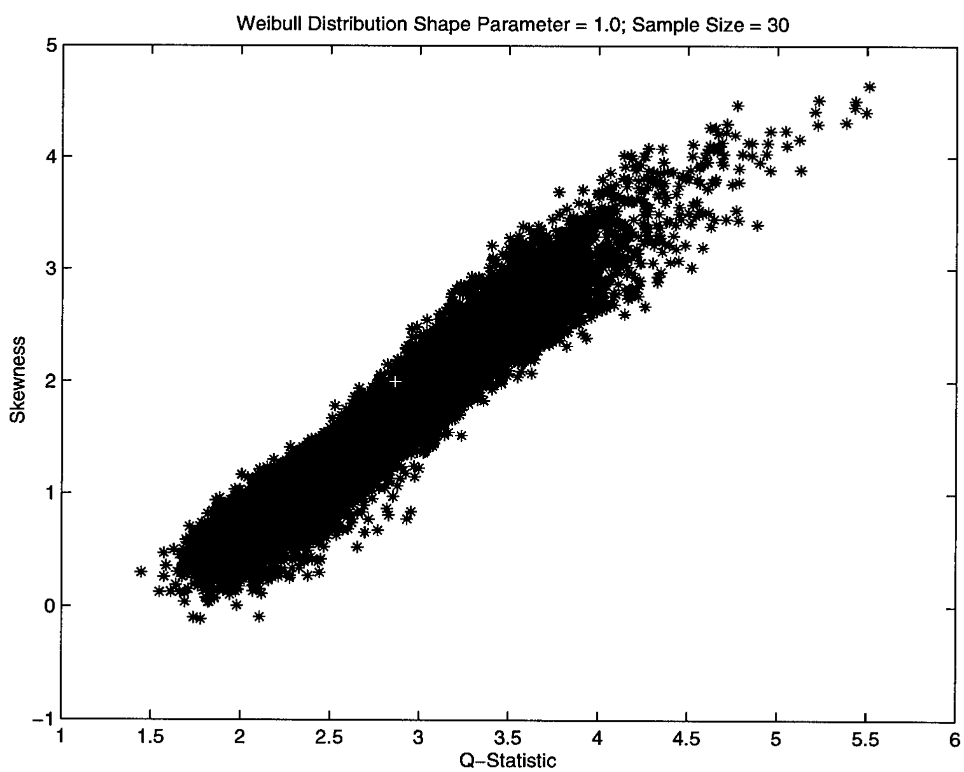


Figure A.16 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 30$.

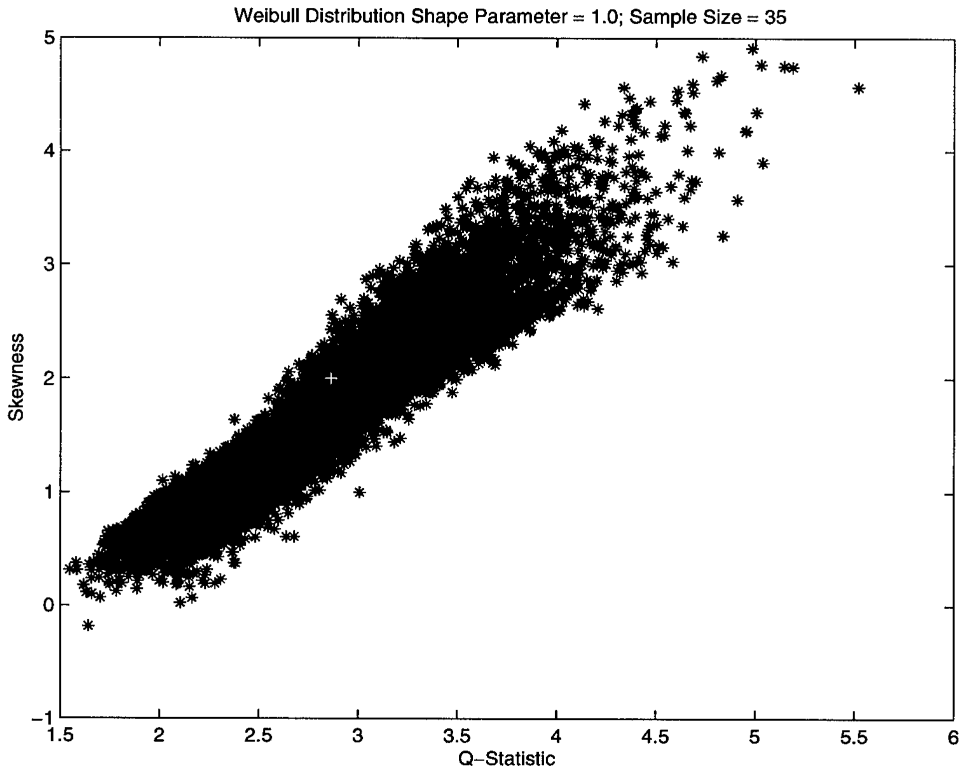


Figure A.17 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 35$.

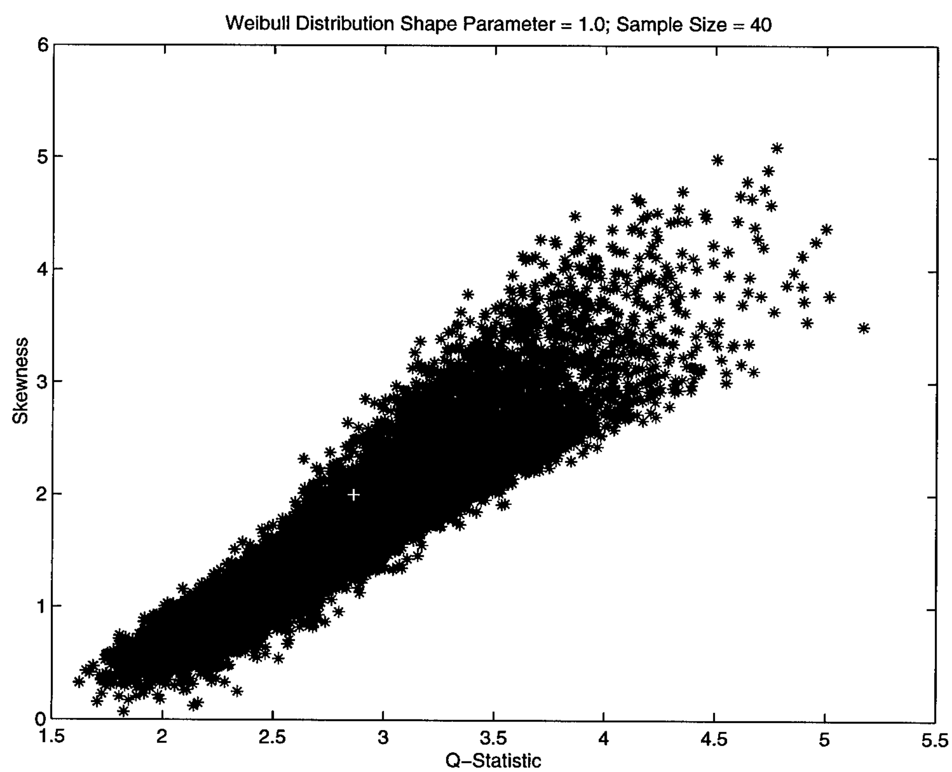


Figure A.18 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 40$.

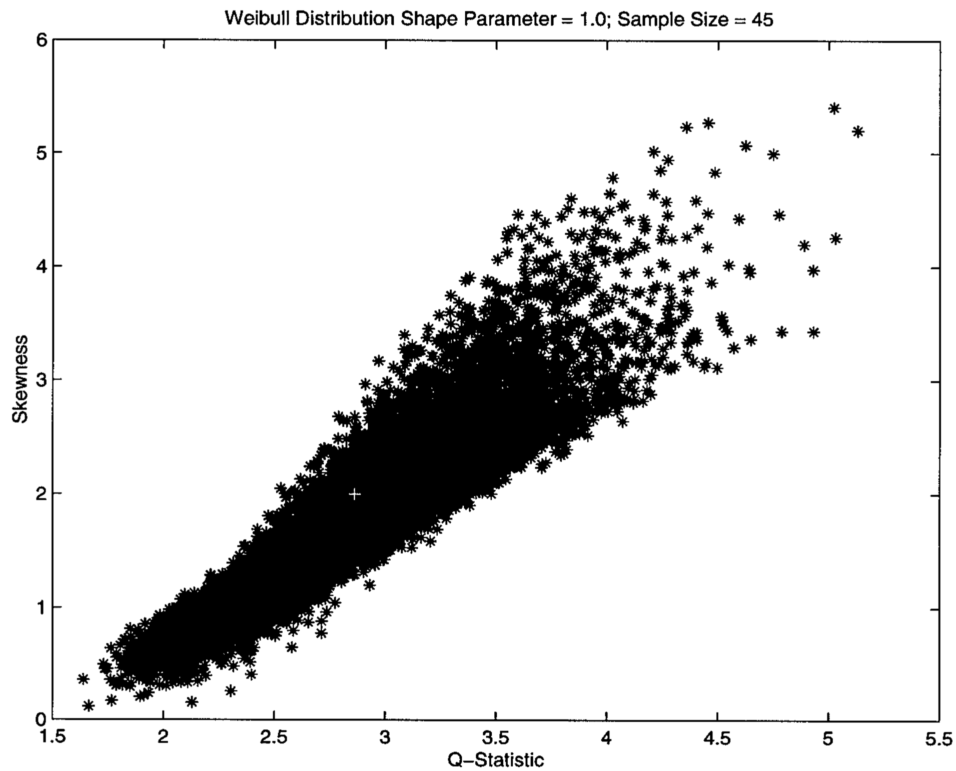


Figure A.19 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 45$.

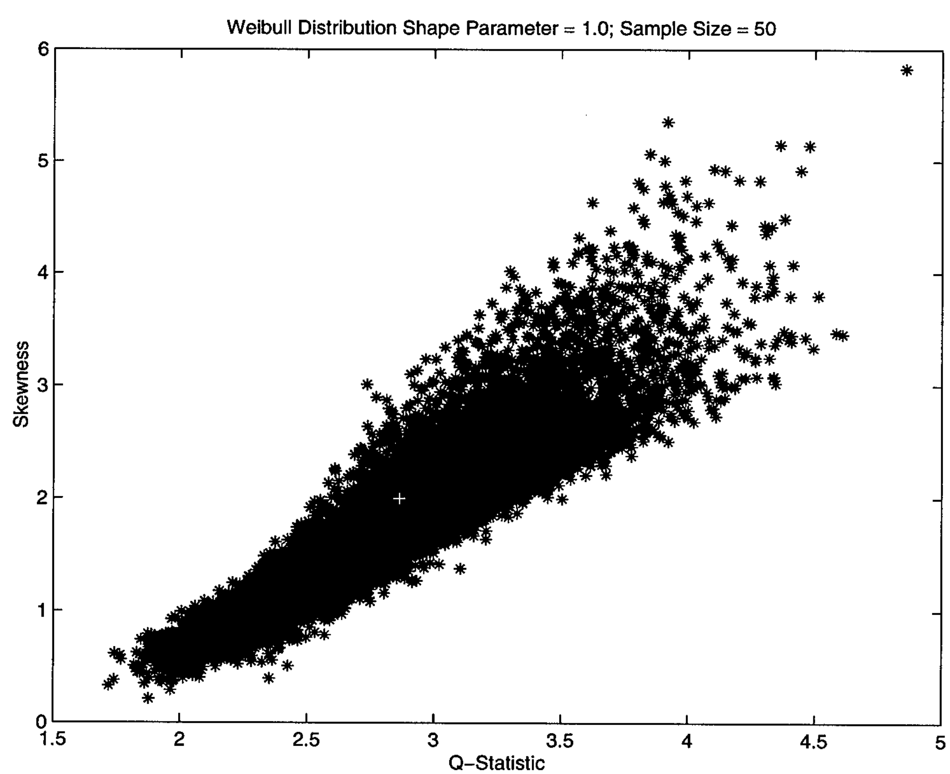


Figure A.20 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.0$) ; $n = 50$.

A.3 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$).

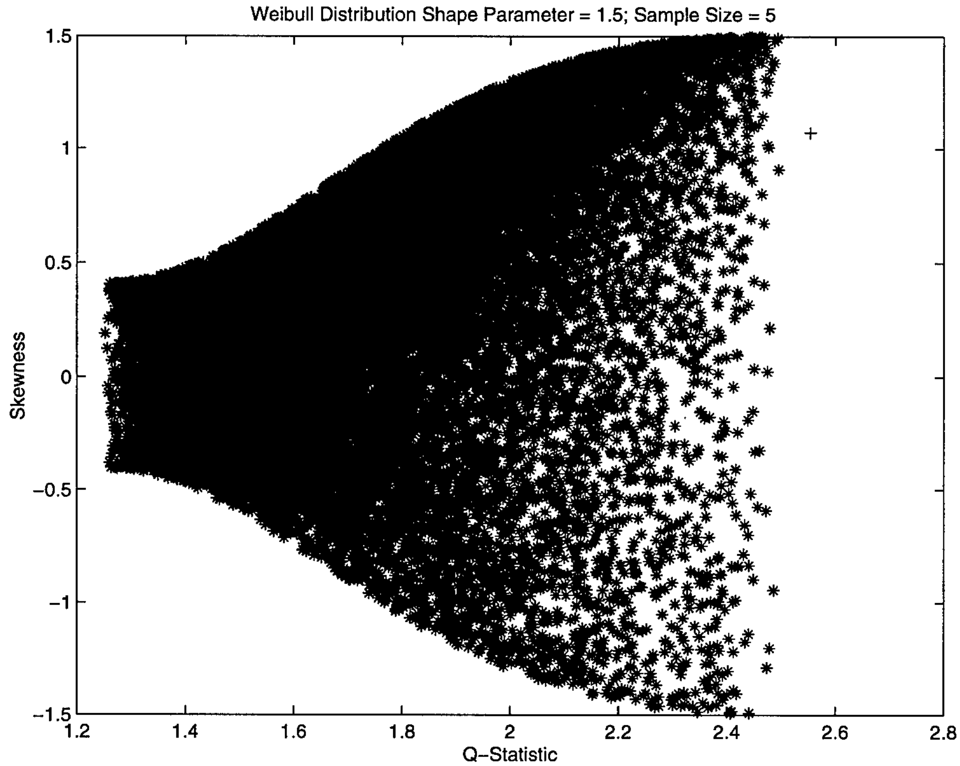


Figure A.21 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 5$.

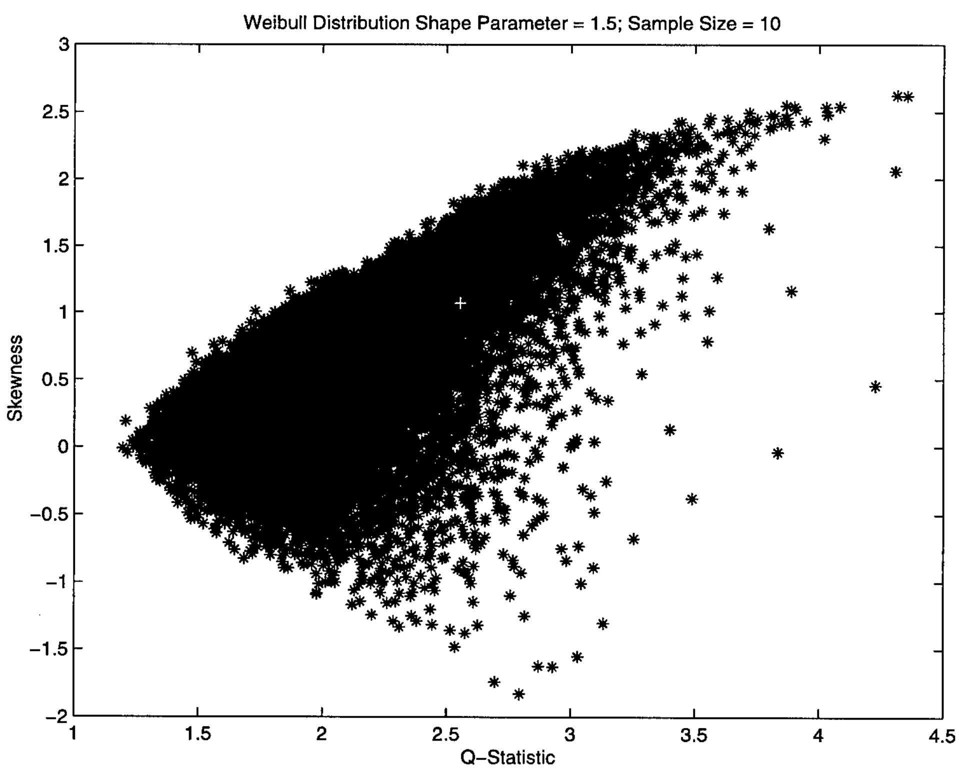


Figure A.22 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 10$.

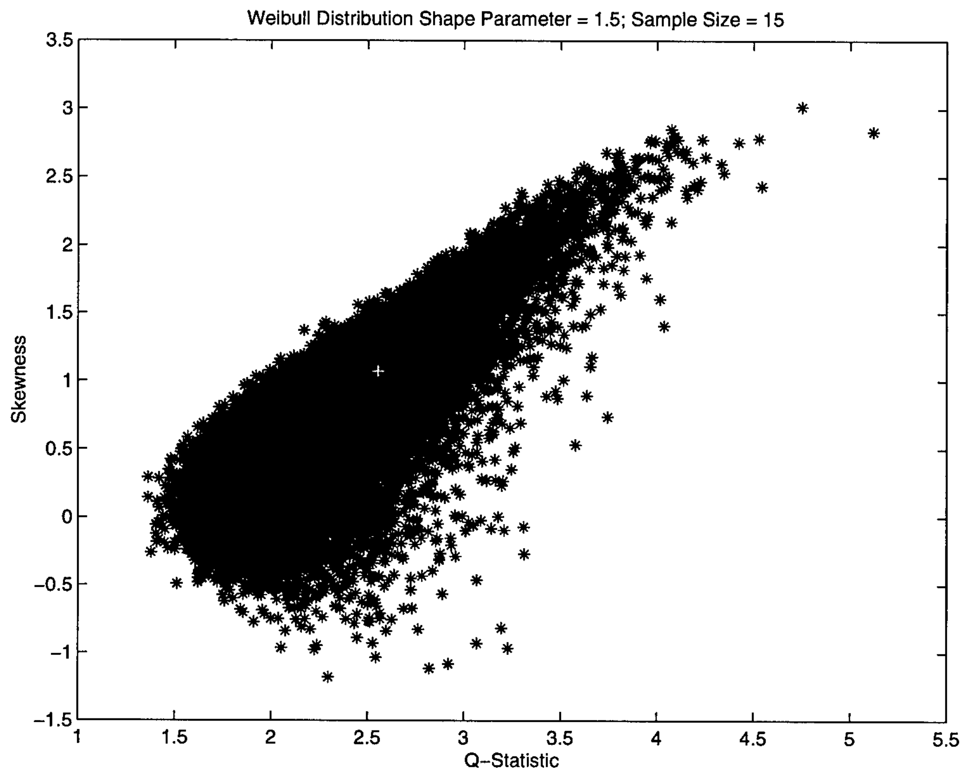


Figure A.23 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 15$.

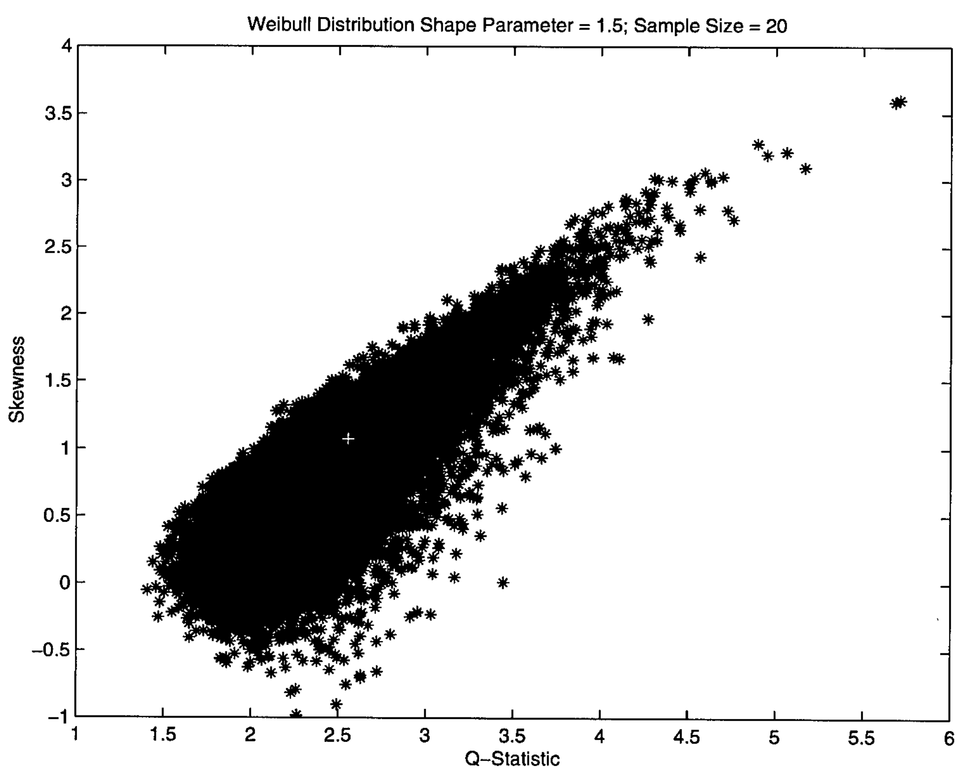


Figure A.24 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 20$.

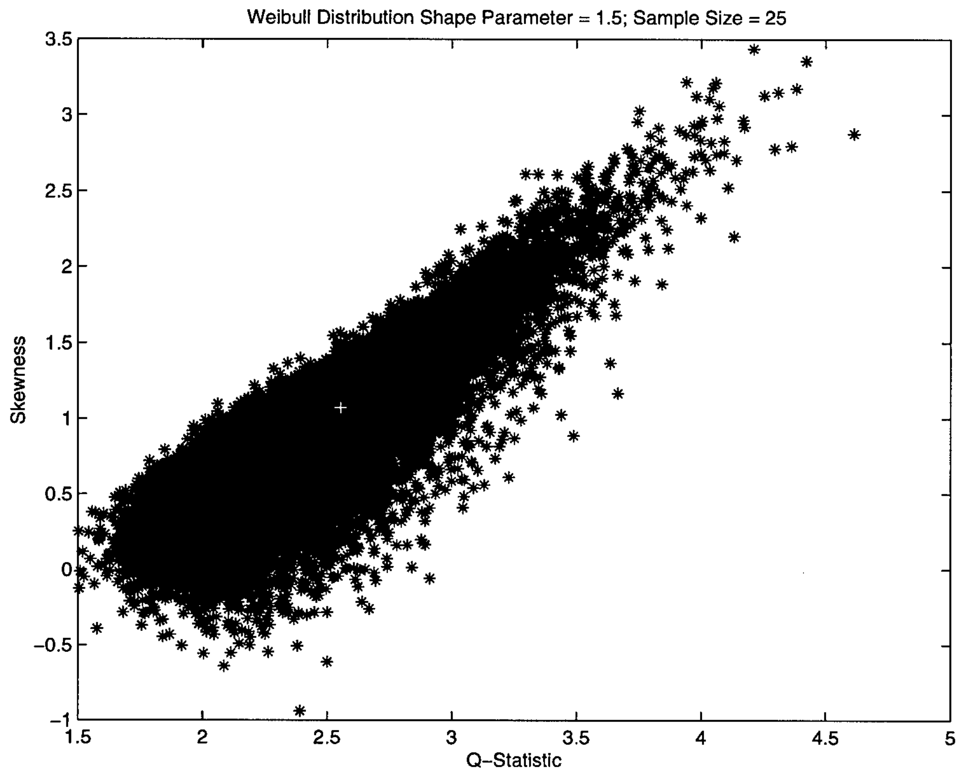


Figure A.25 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 25$.

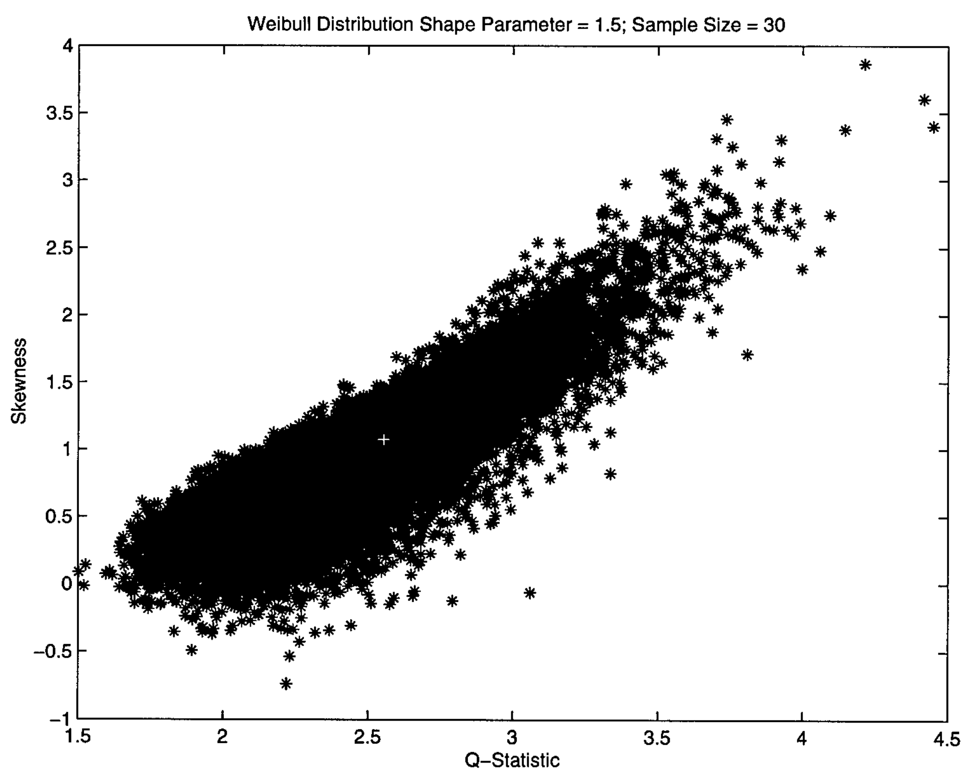


Figure A.26 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 30$.

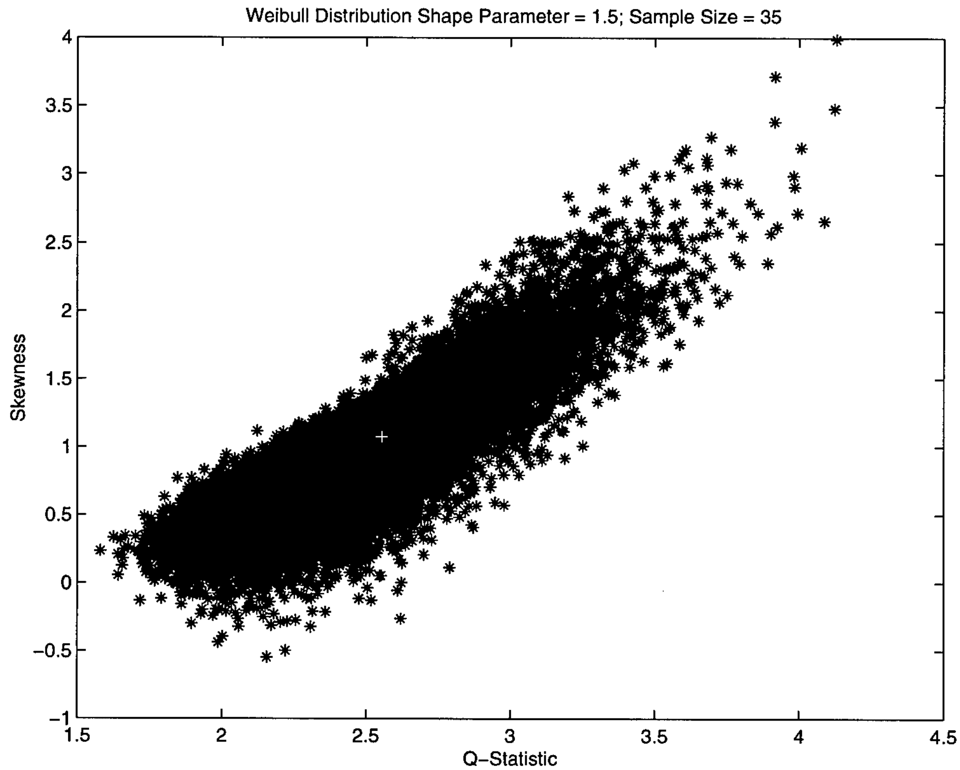


Figure A.27 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 35$.

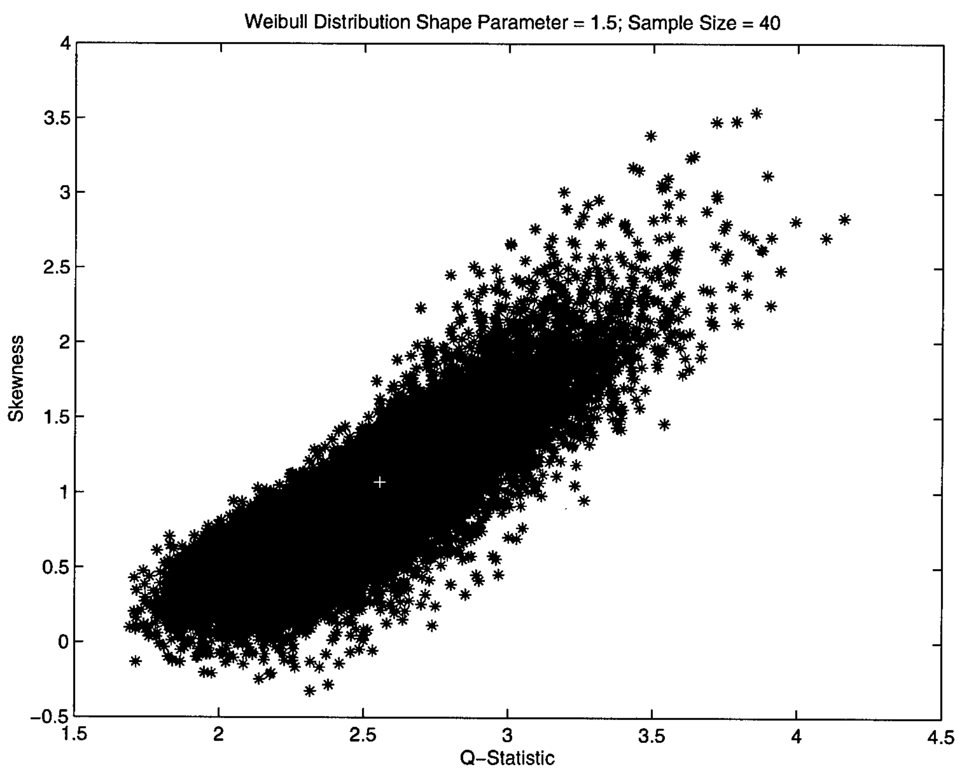


Figure A.28 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 40$.

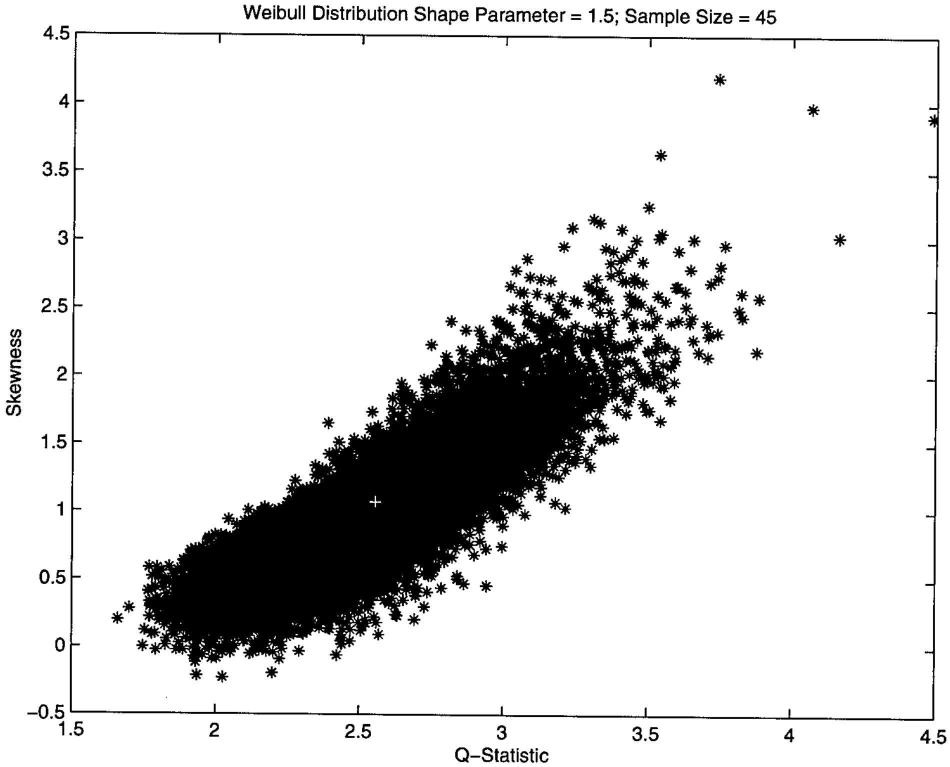


Figure A.29 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 45$.

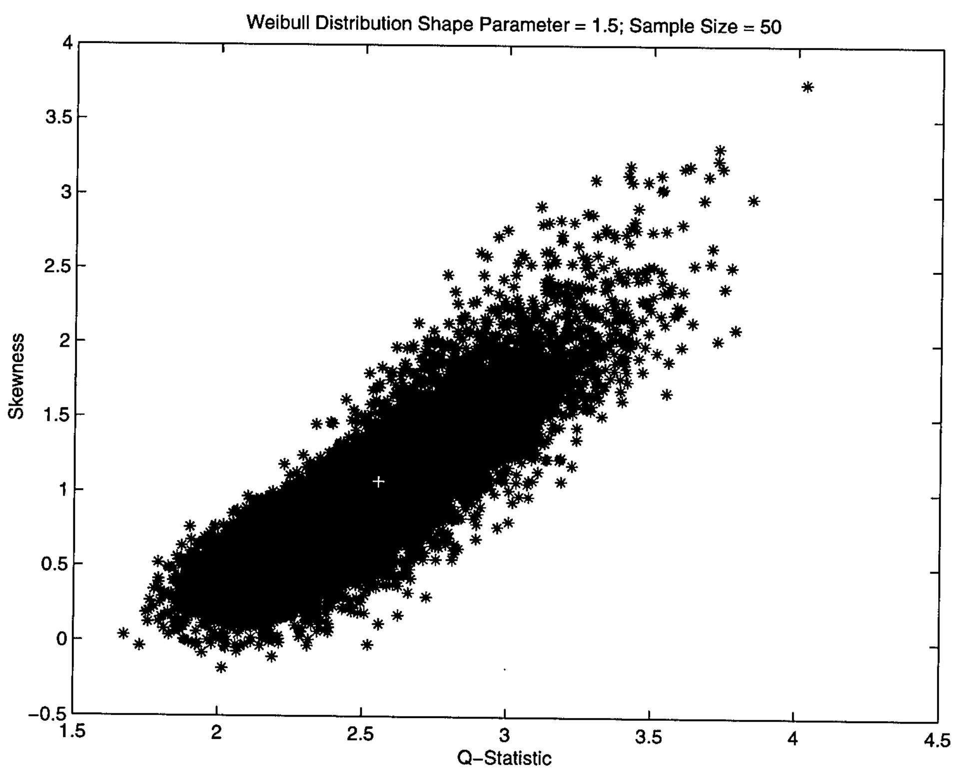


Figure A.30 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 1.5$) ; $n = 50$.

A.4 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$).

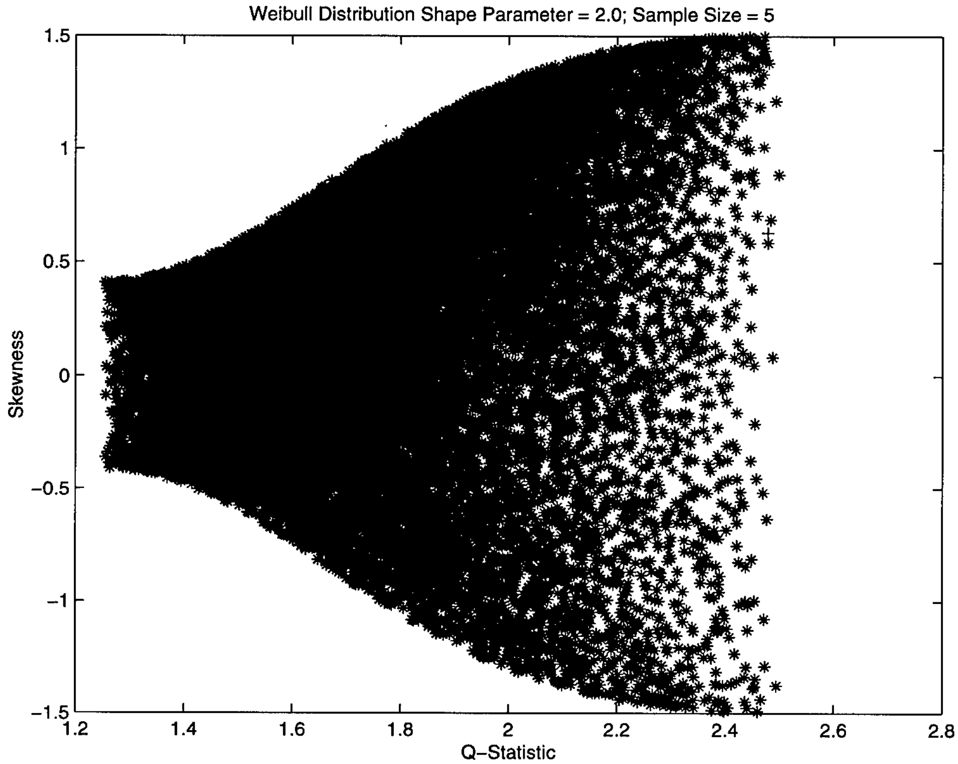


Figure A.31 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 5$.

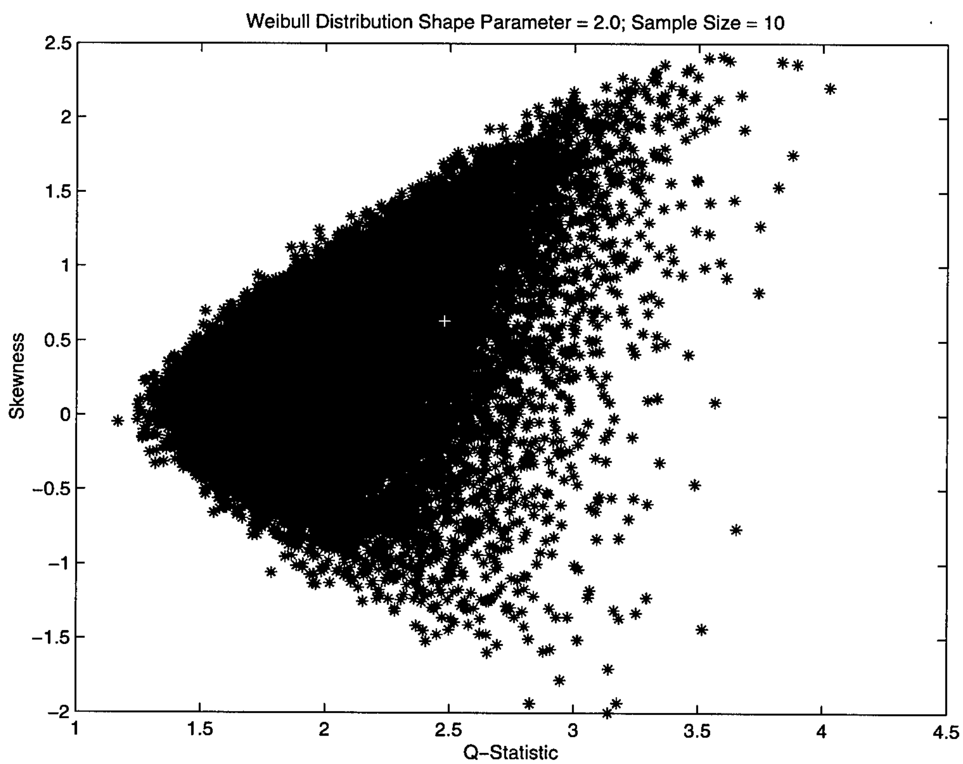


Figure A.32 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 10$.

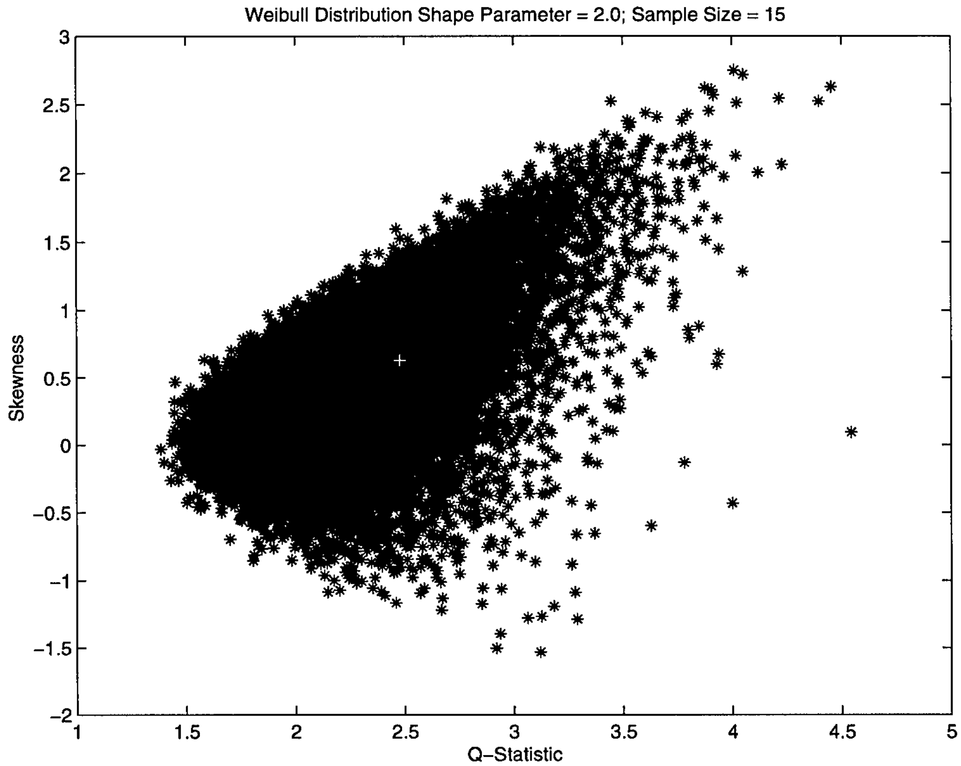


Figure A.33 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 15$.

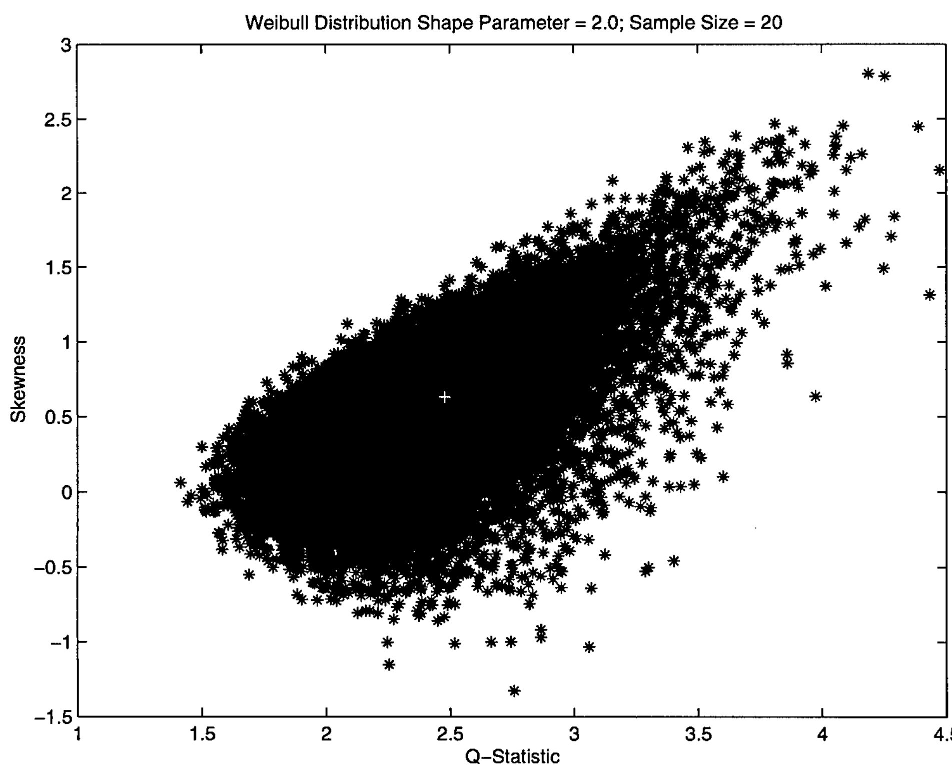


Figure A.34 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 20$.

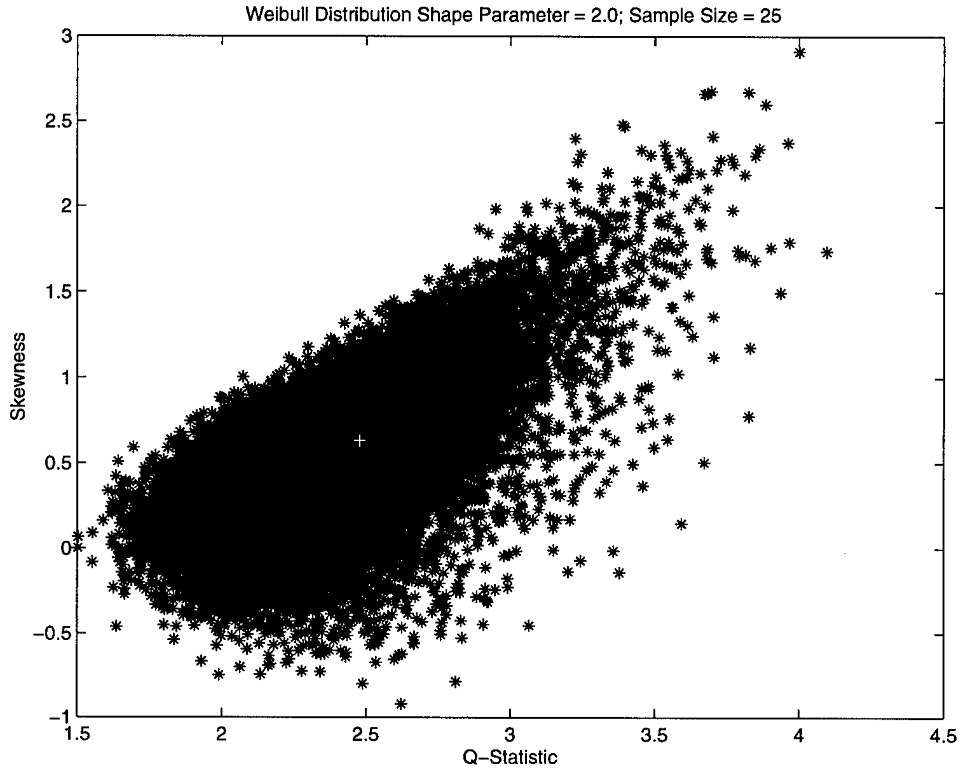


Figure A.35 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 25$.

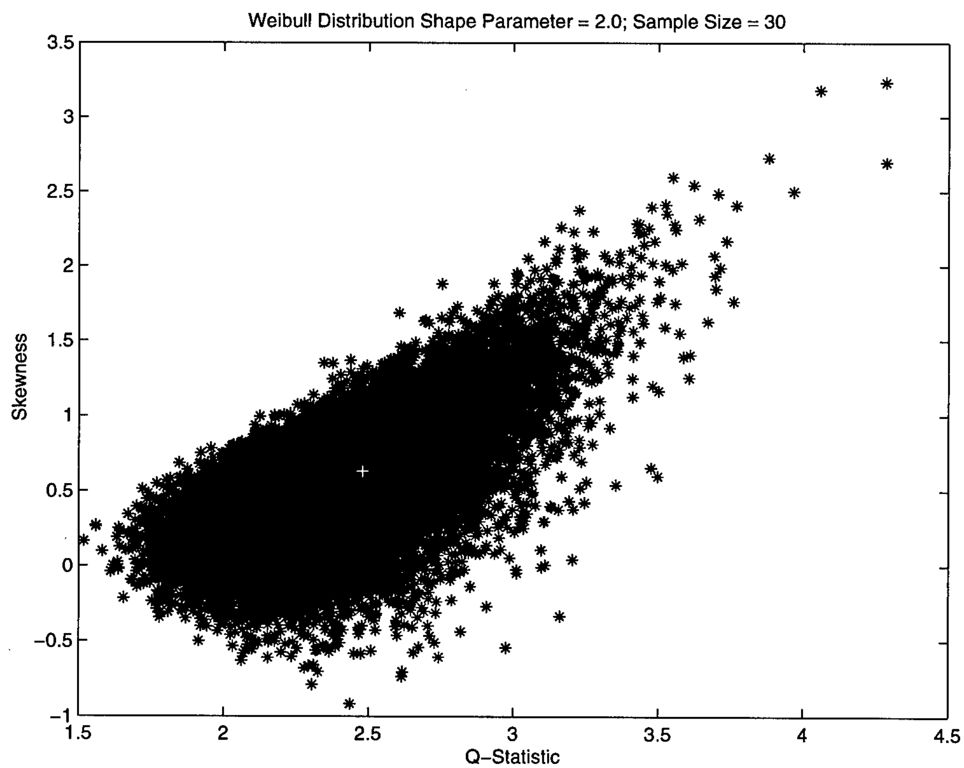


Figure A.36 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 30$.

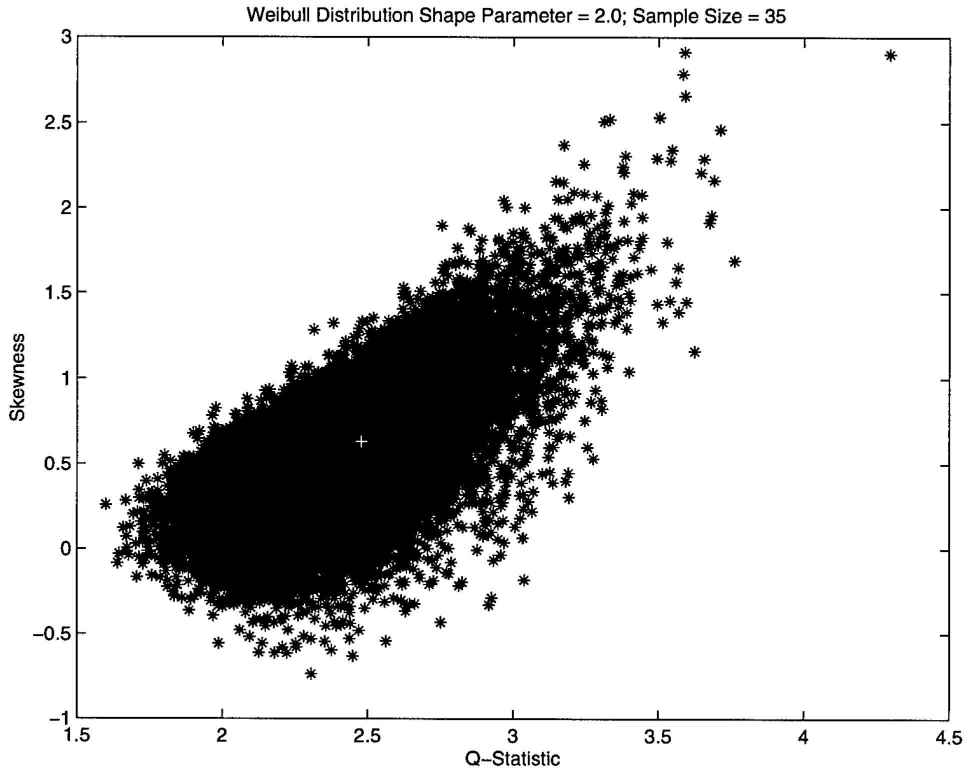


Figure A.37 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 35$.

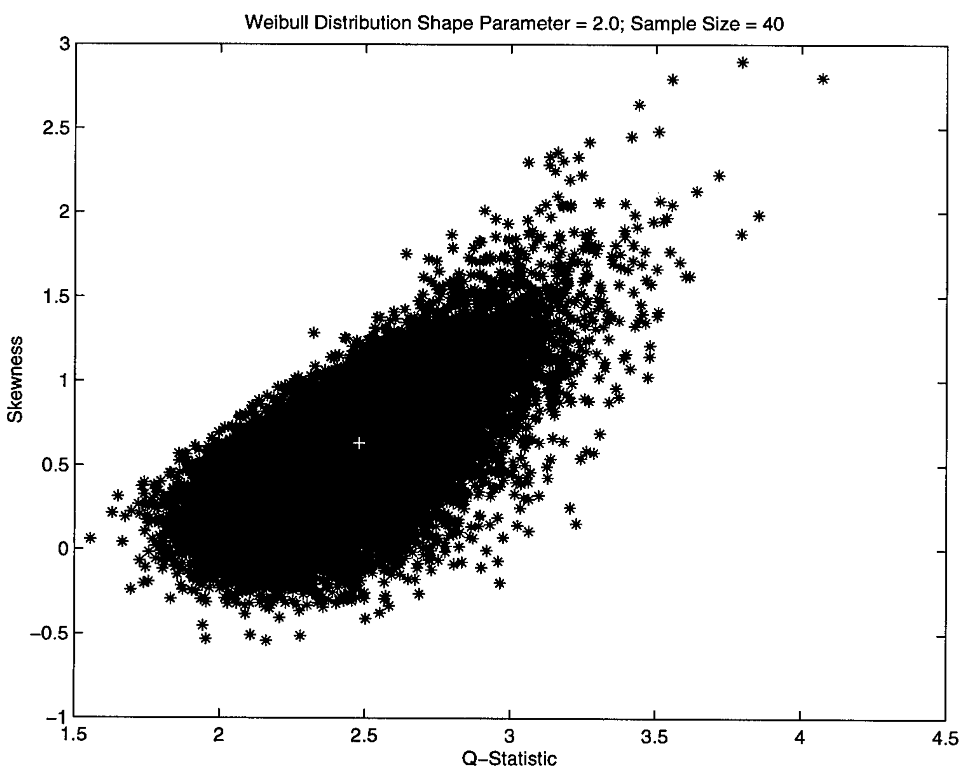


Figure A.38 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 40$.

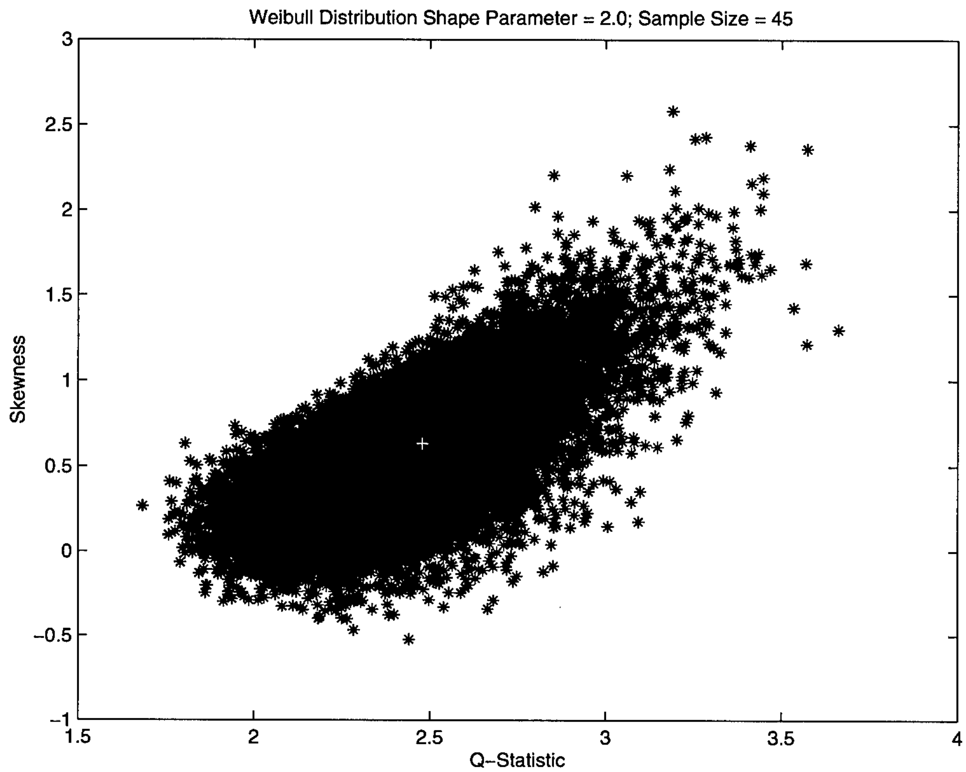


Figure A.39 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 45$.

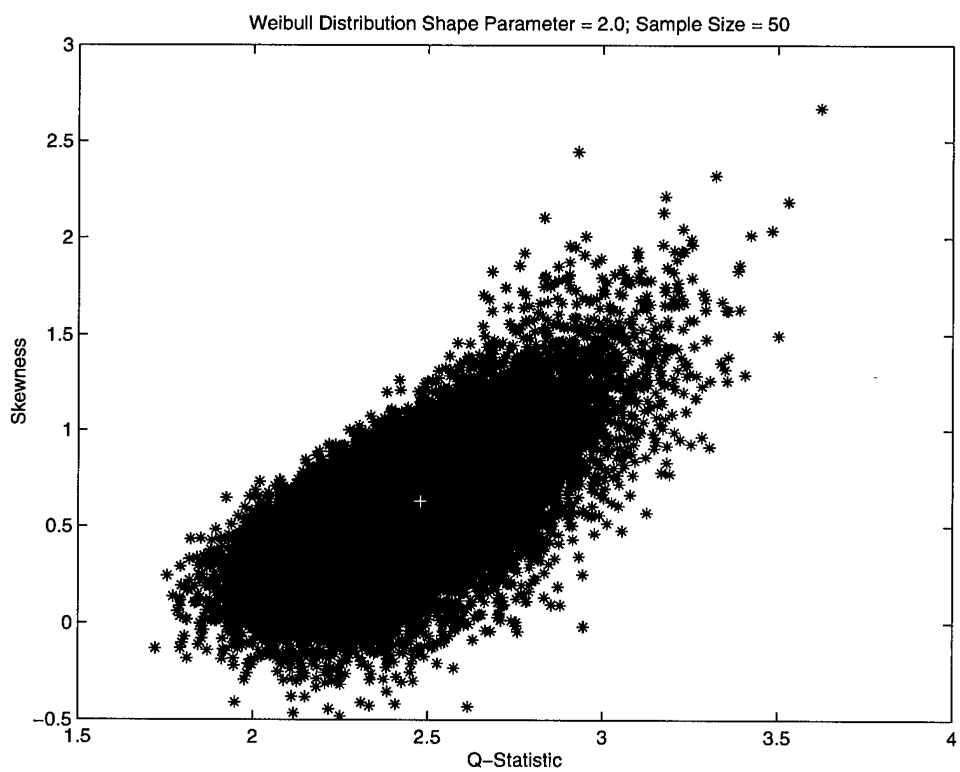


Figure A.40 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.0$) ; $n = 50$.

A.5 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$).

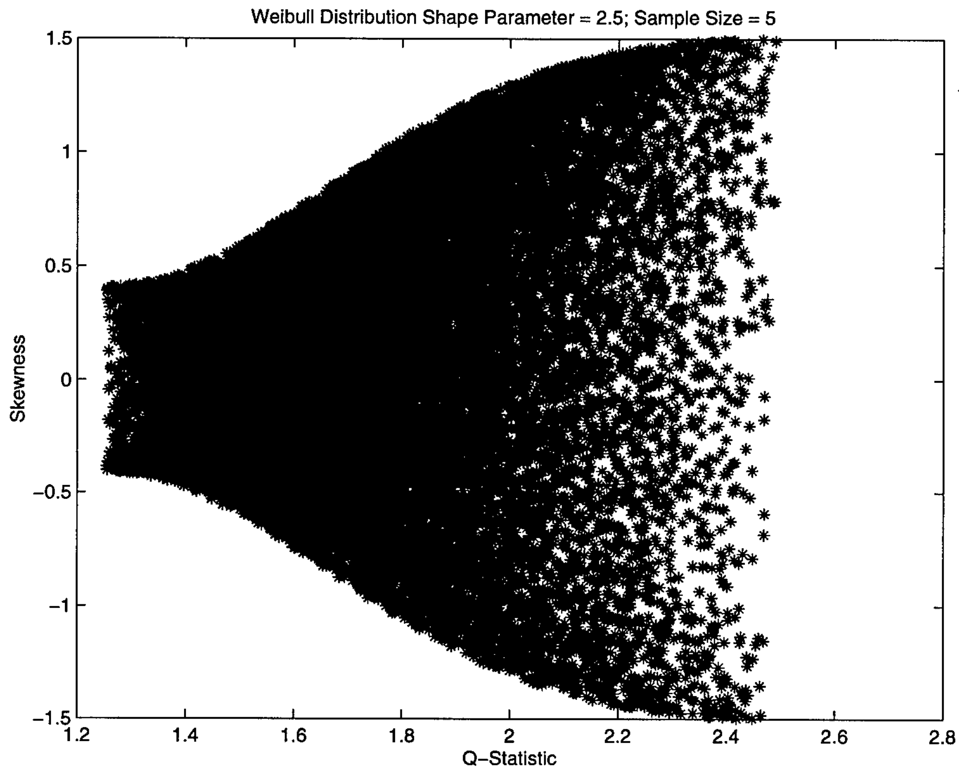


Figure A.41 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 5$.

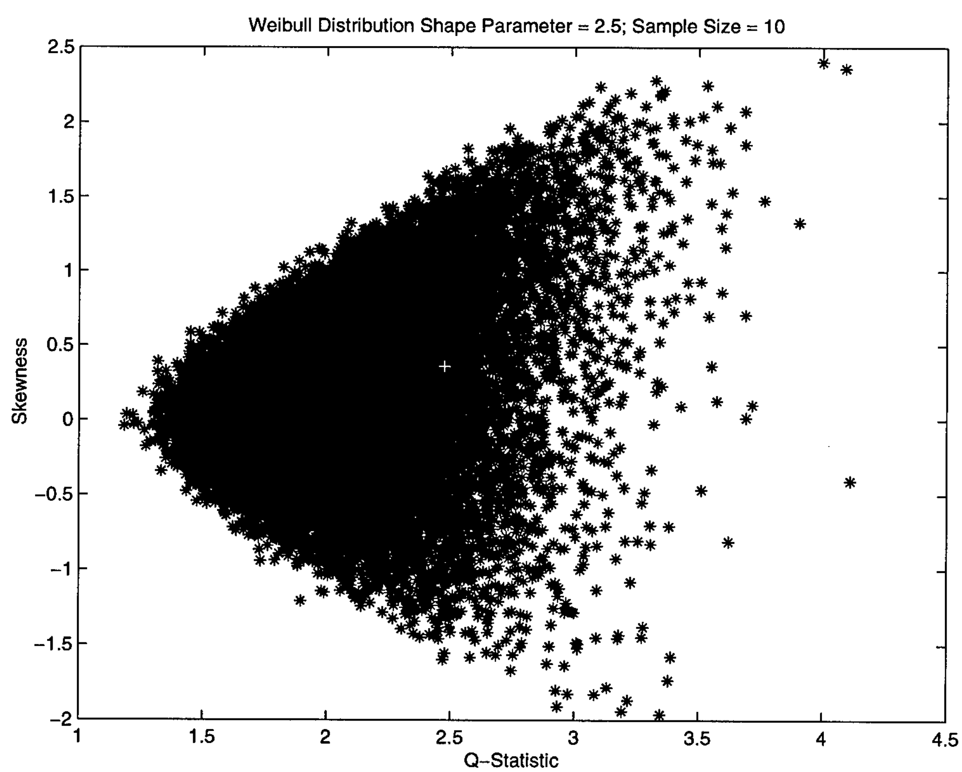


Figure A.42 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 10$.

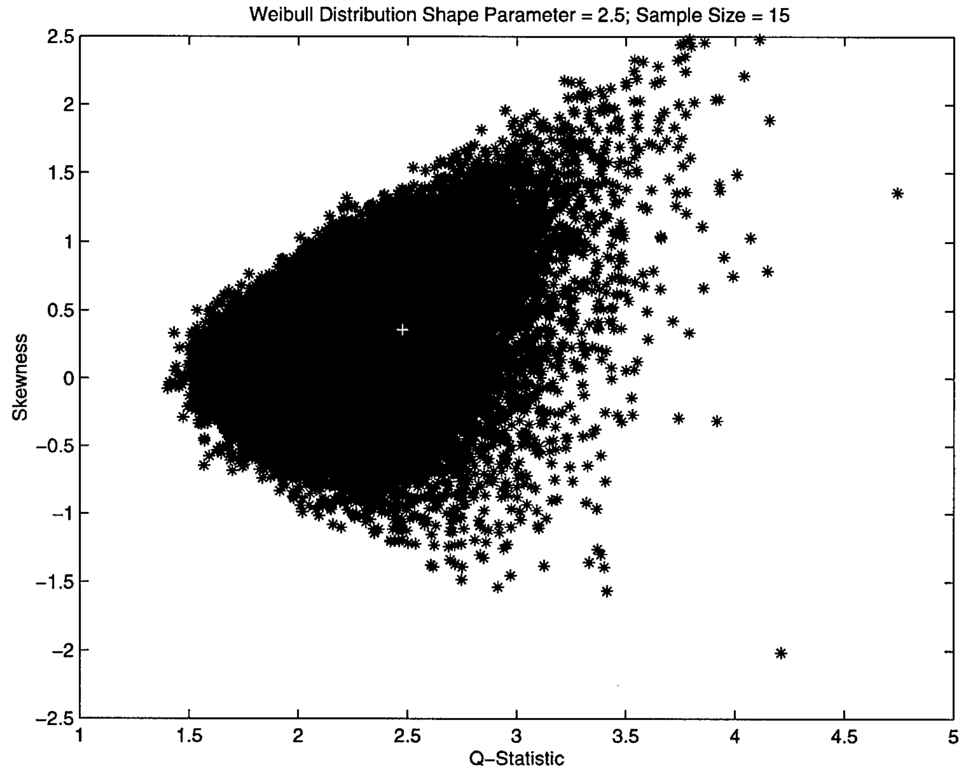


Figure A.43 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 15$.

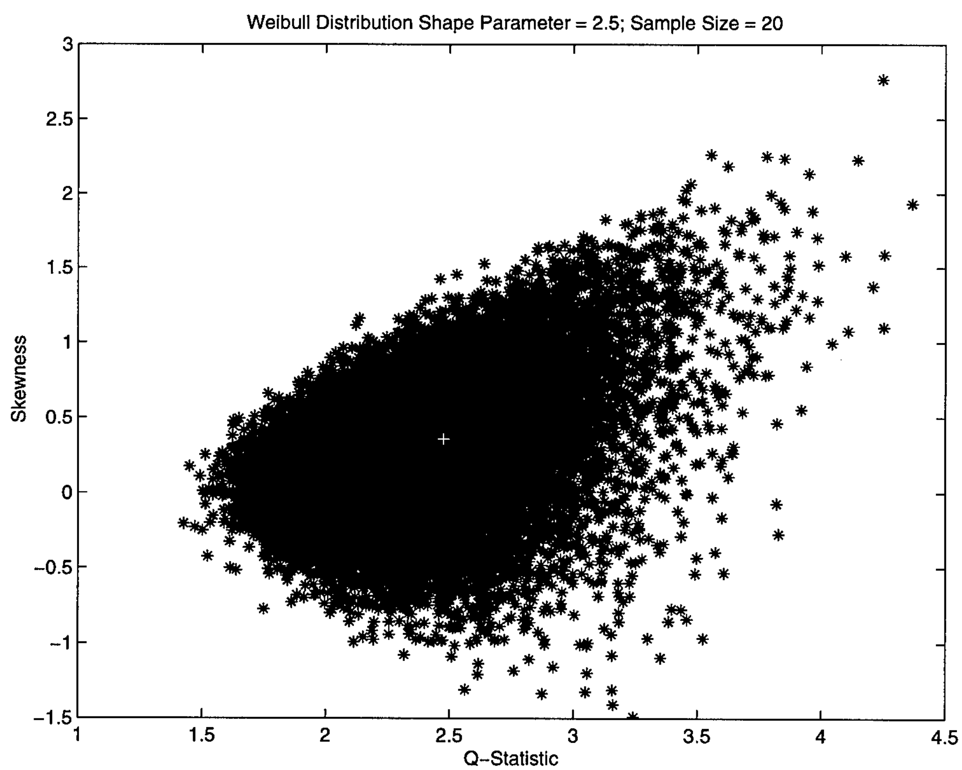


Figure A.44 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 20$.

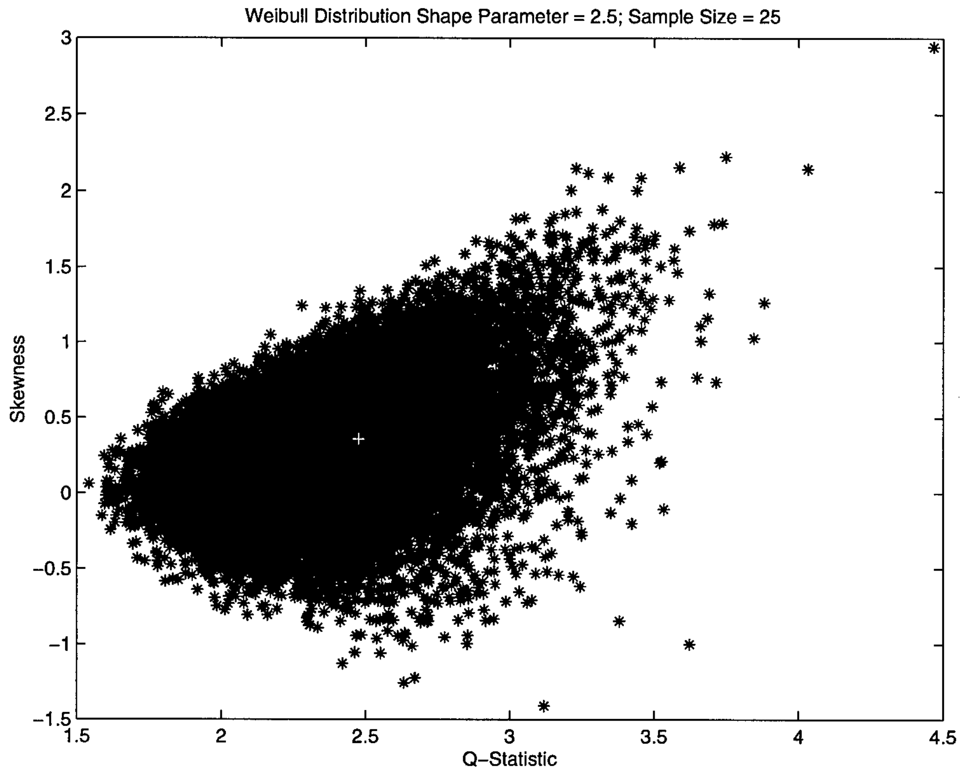


Figure A.45 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 25$.

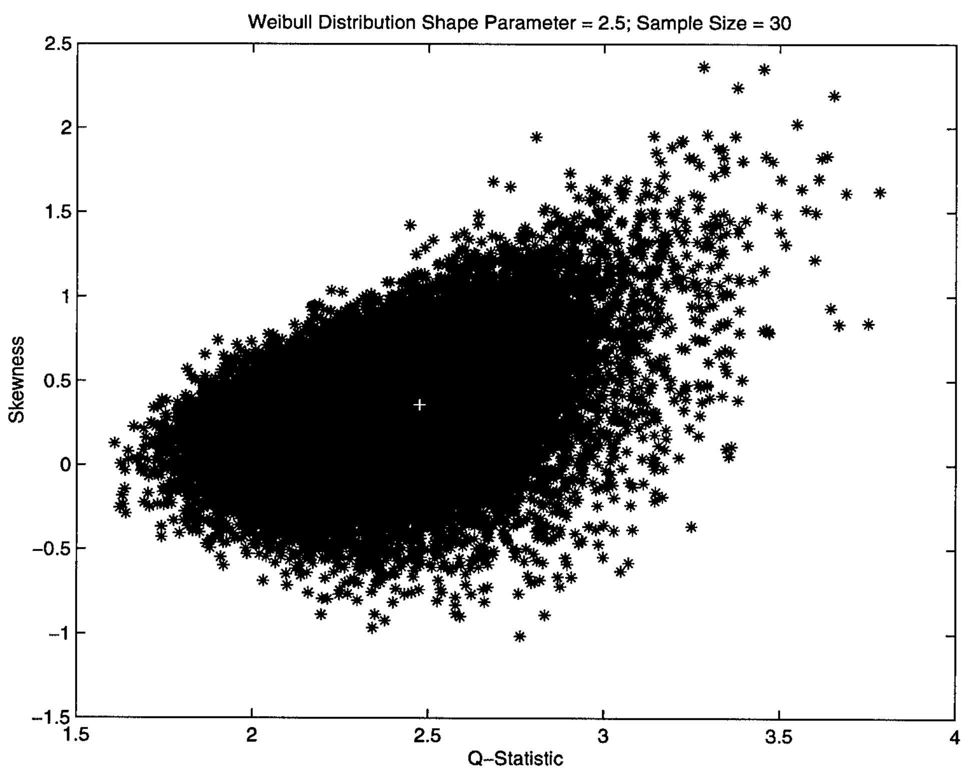


Figure A.46 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 30$.

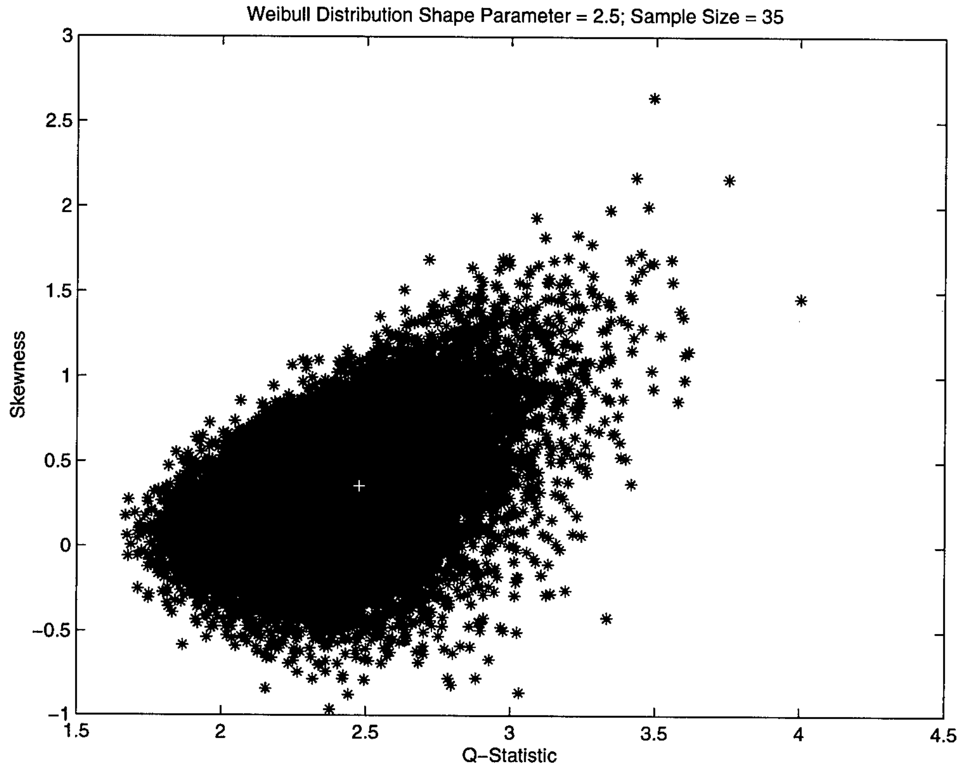


Figure A.47 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 35$.

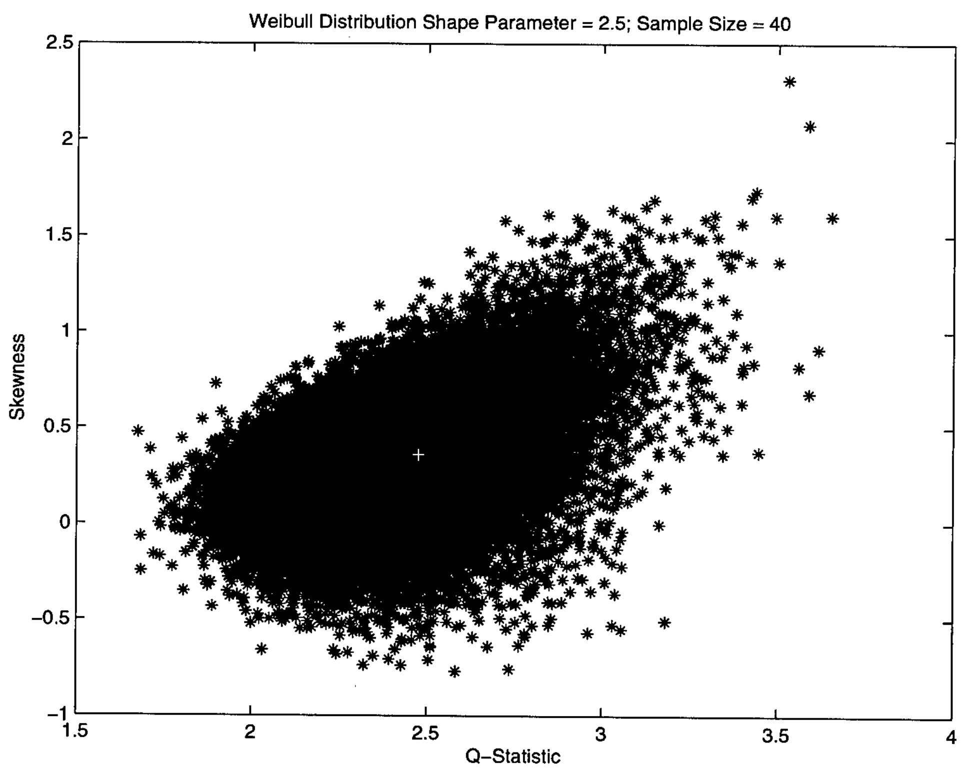


Figure A.48 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 40$.

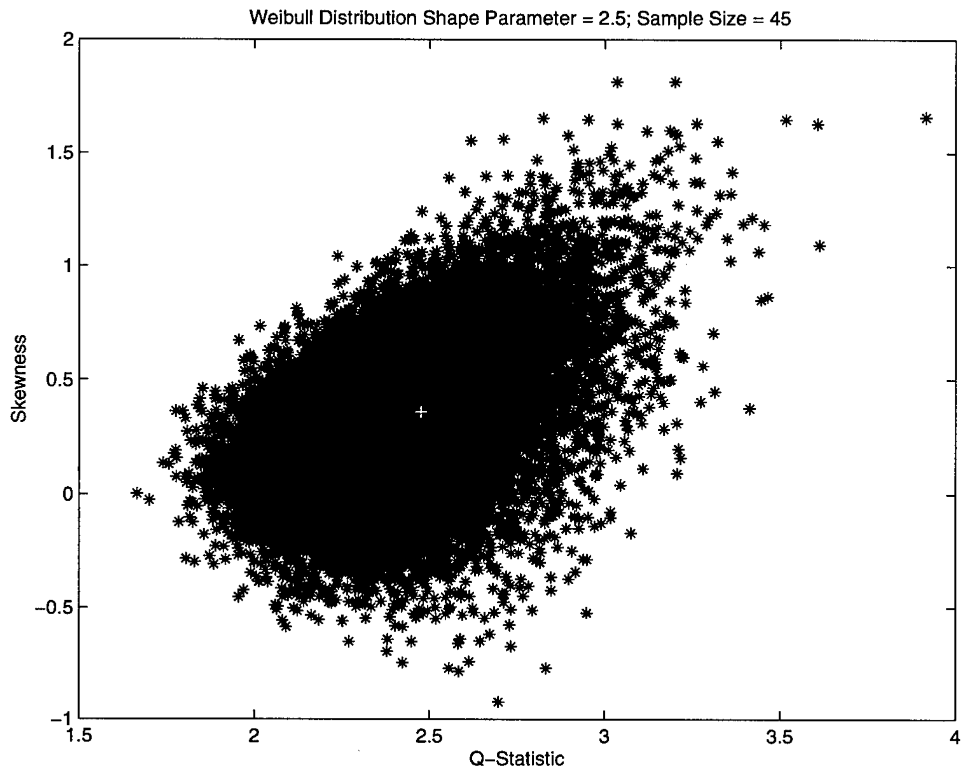


Figure A.49 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 45$.

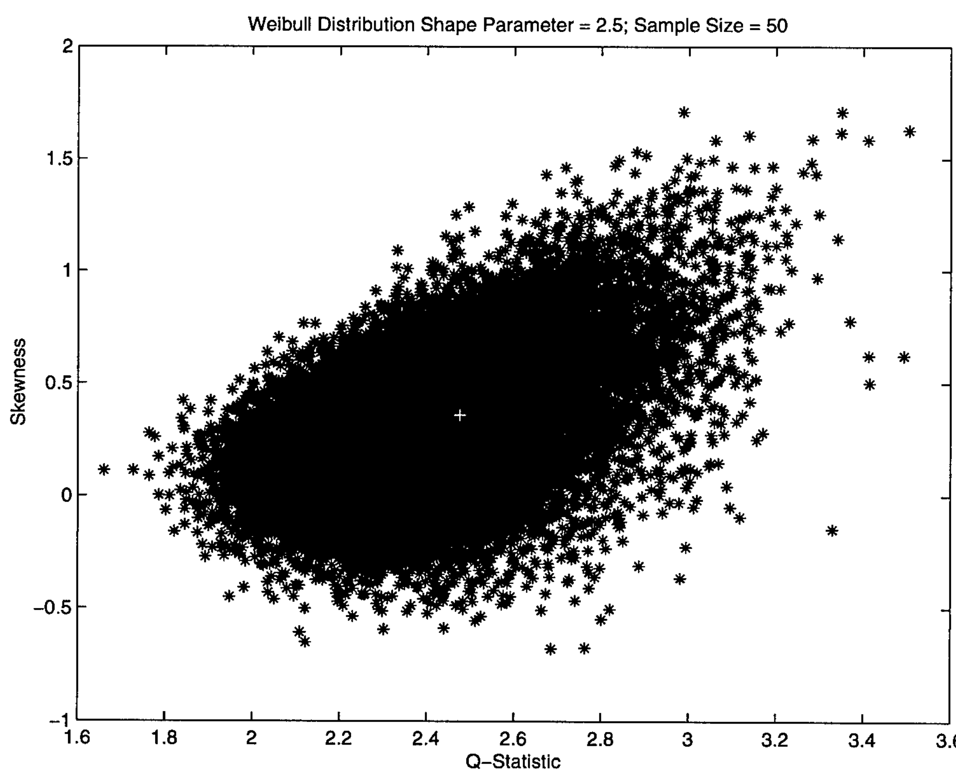


Figure A.50 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 2.5$) ; $n = 50$.

A.6 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$).

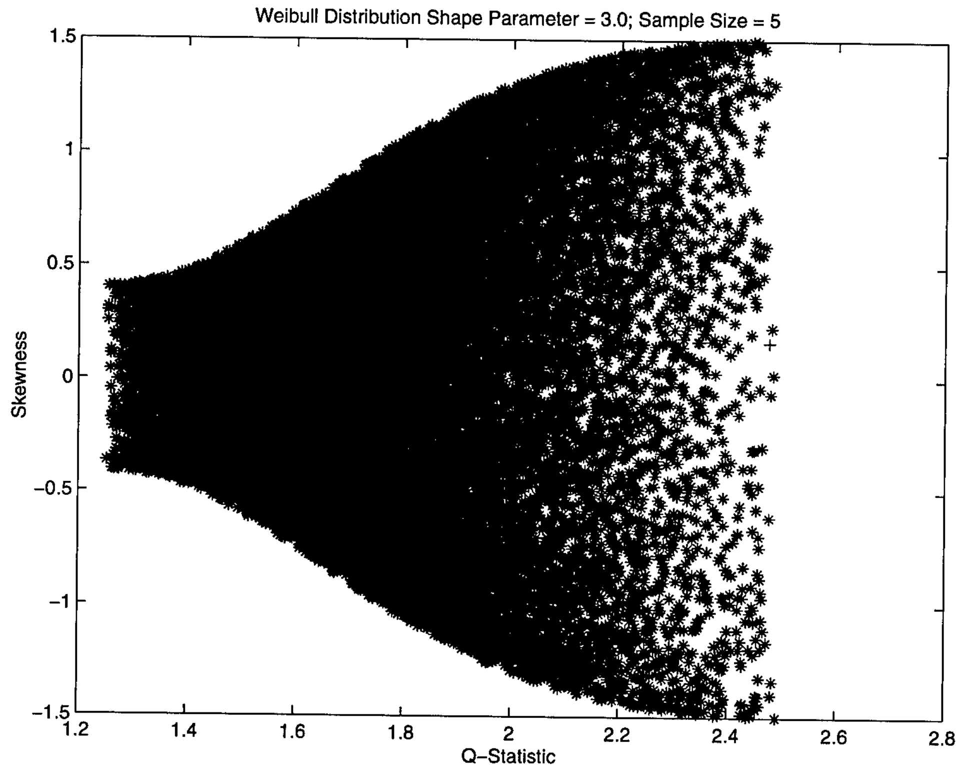


Figure A.51 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 5$.

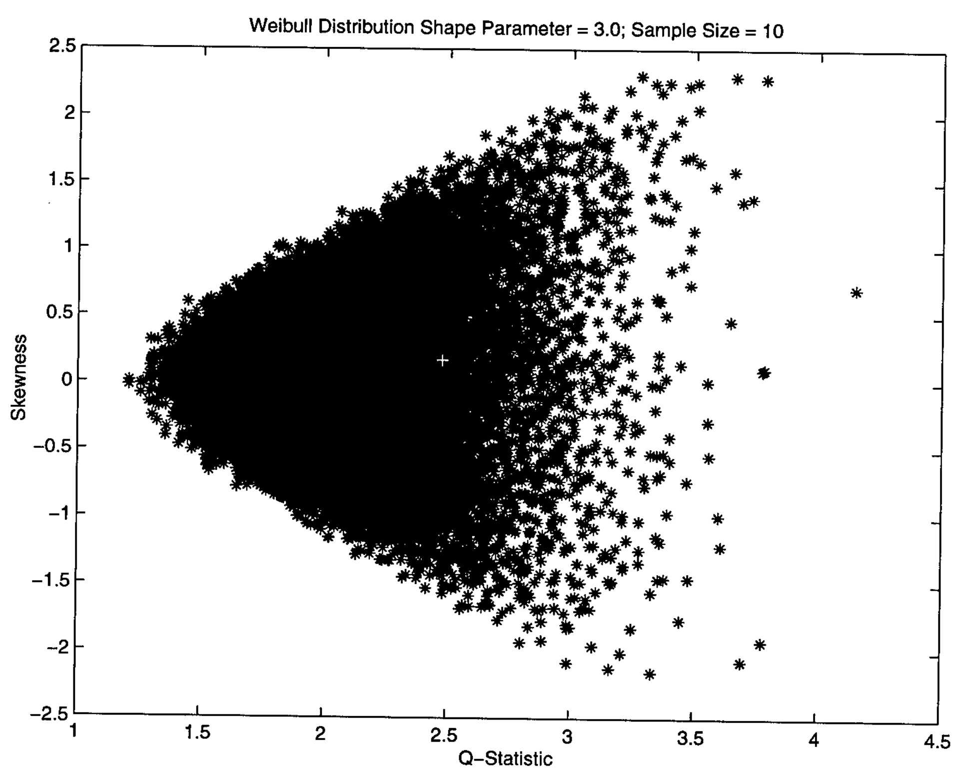


Figure A.52 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 10$.

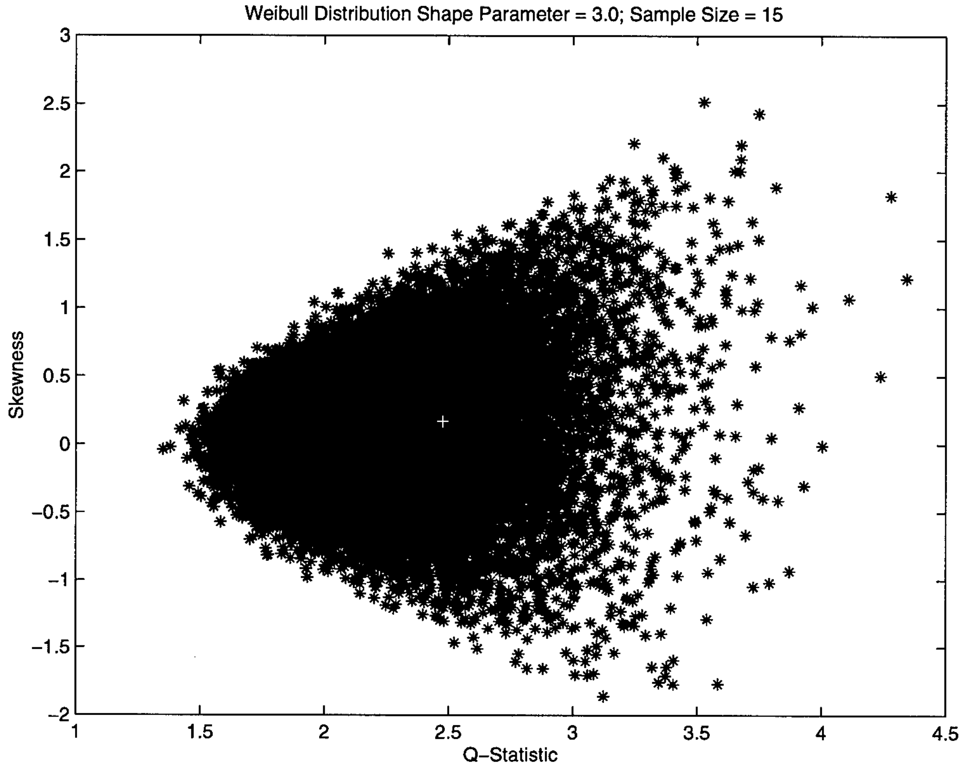


Figure A.53 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 15$.

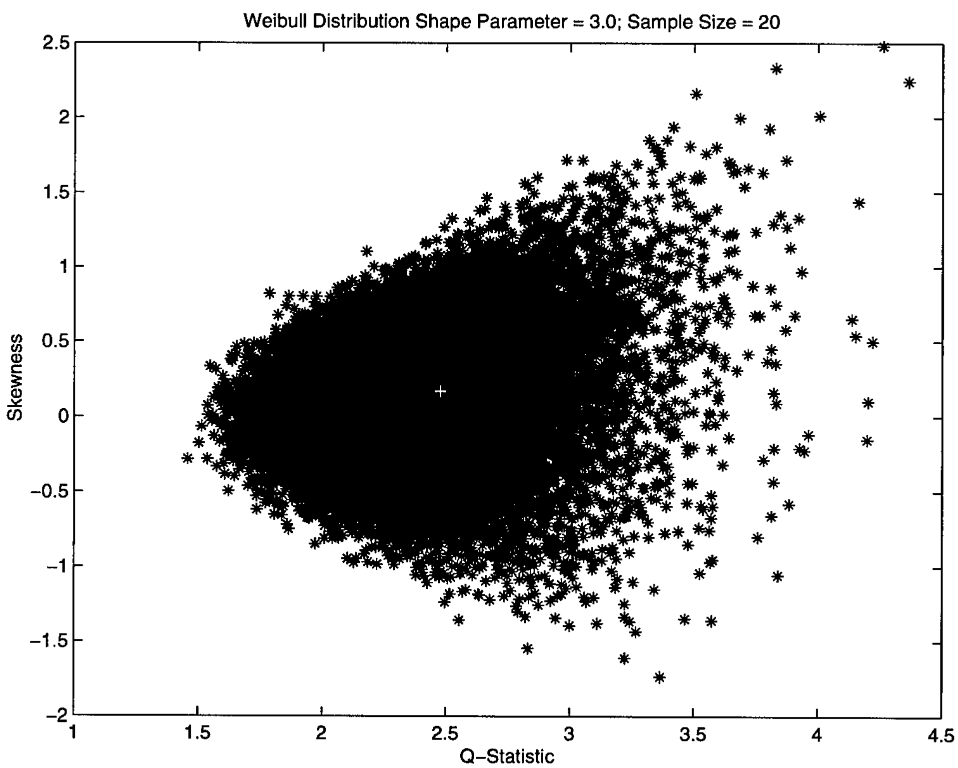


Figure A.54 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 20$.

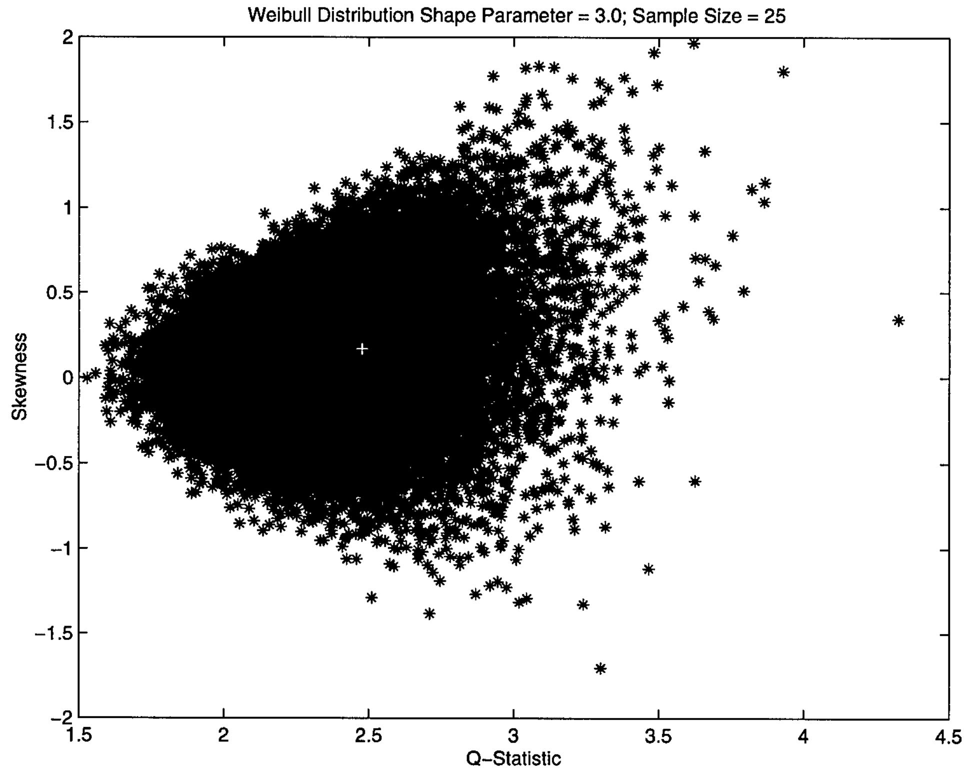


Figure A.55 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 25$.

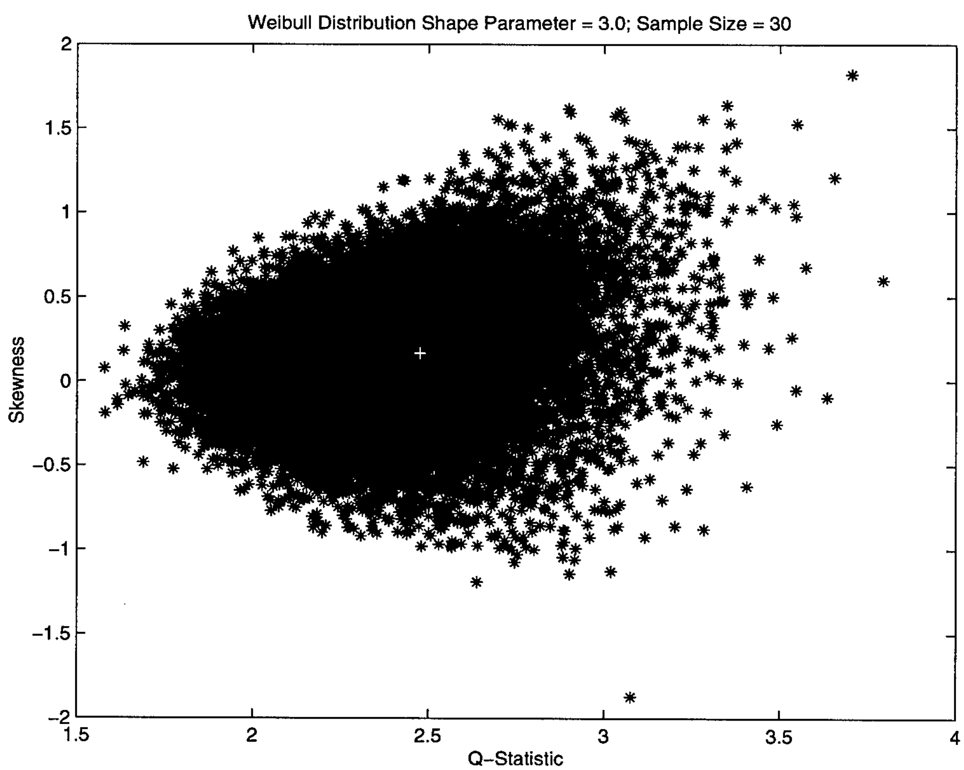


Figure A.56 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 30$.

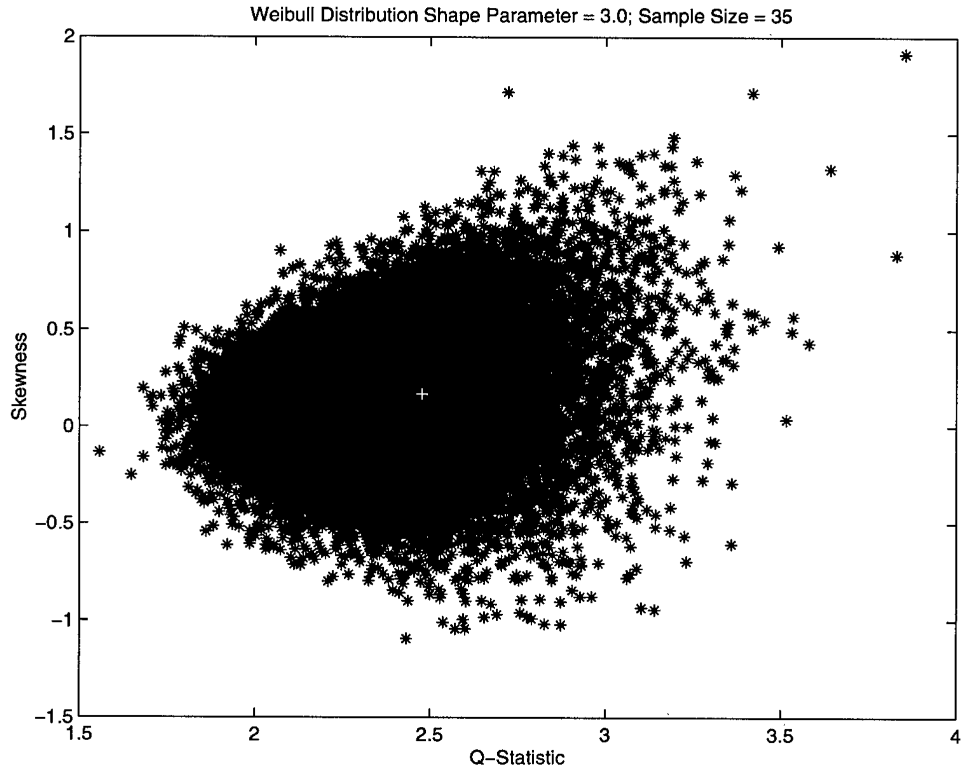


Figure A.57 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 35$.

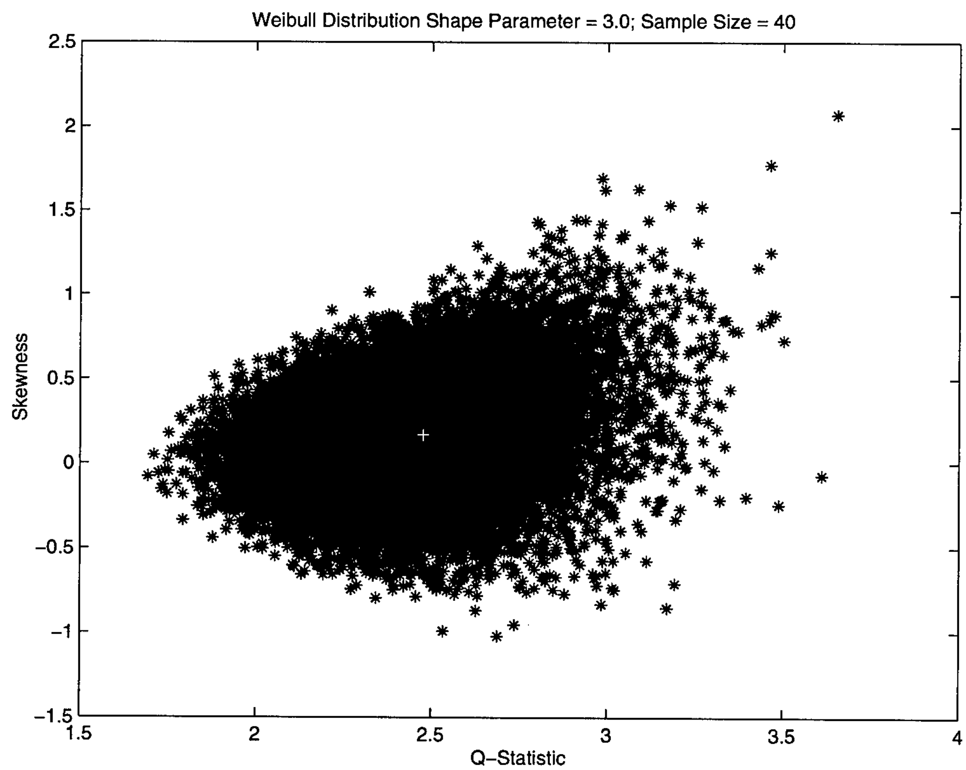


Figure A.58 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 40$.

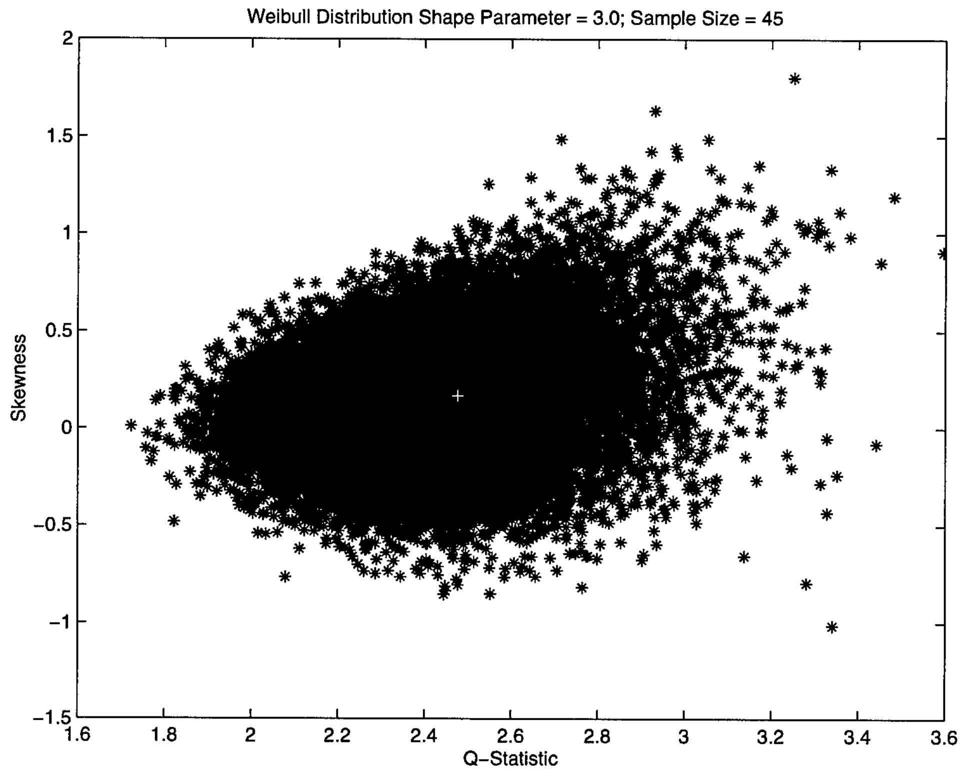


Figure A.59 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 45$.

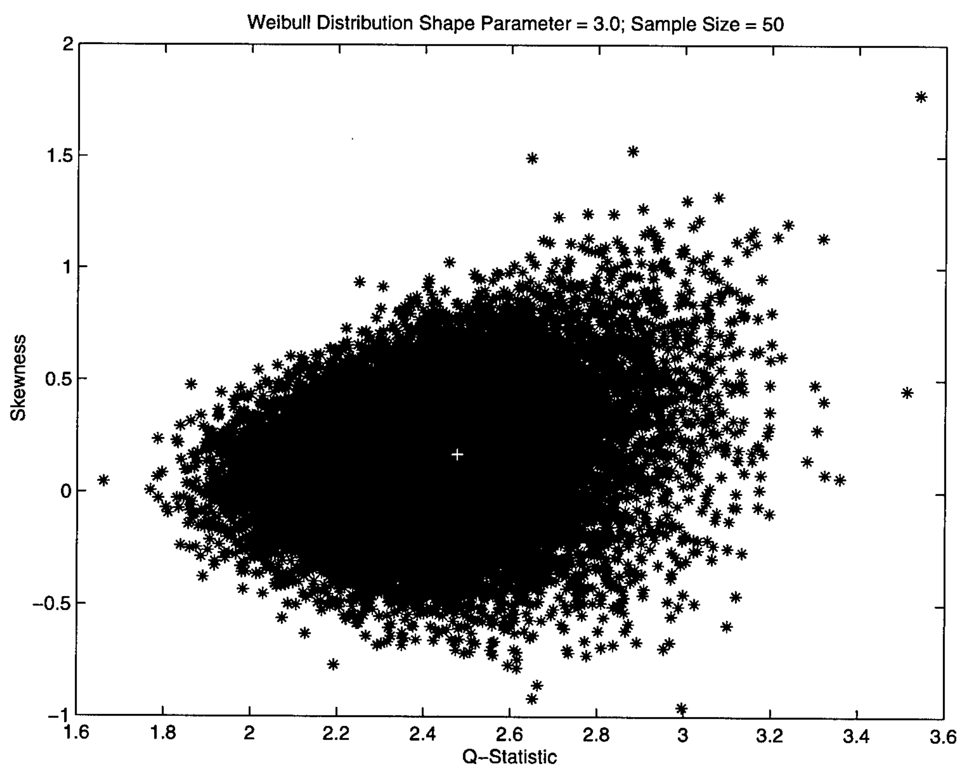


Figure A.60 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.0$) ; $n = 50$.

A.7 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$).

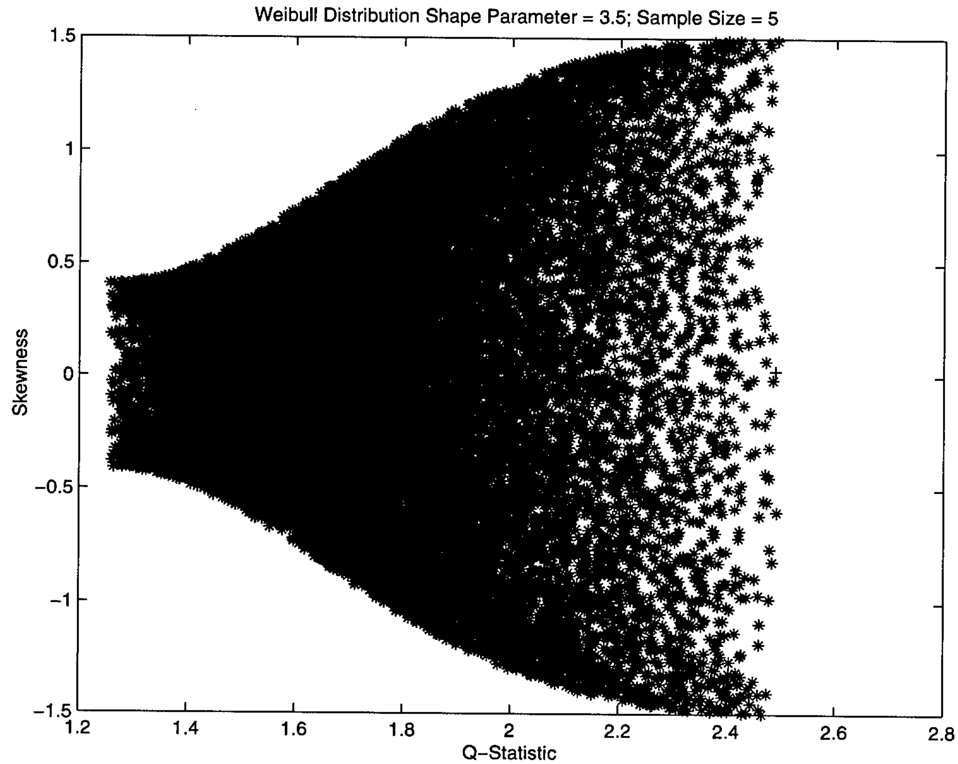


Figure A.61 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 5$.

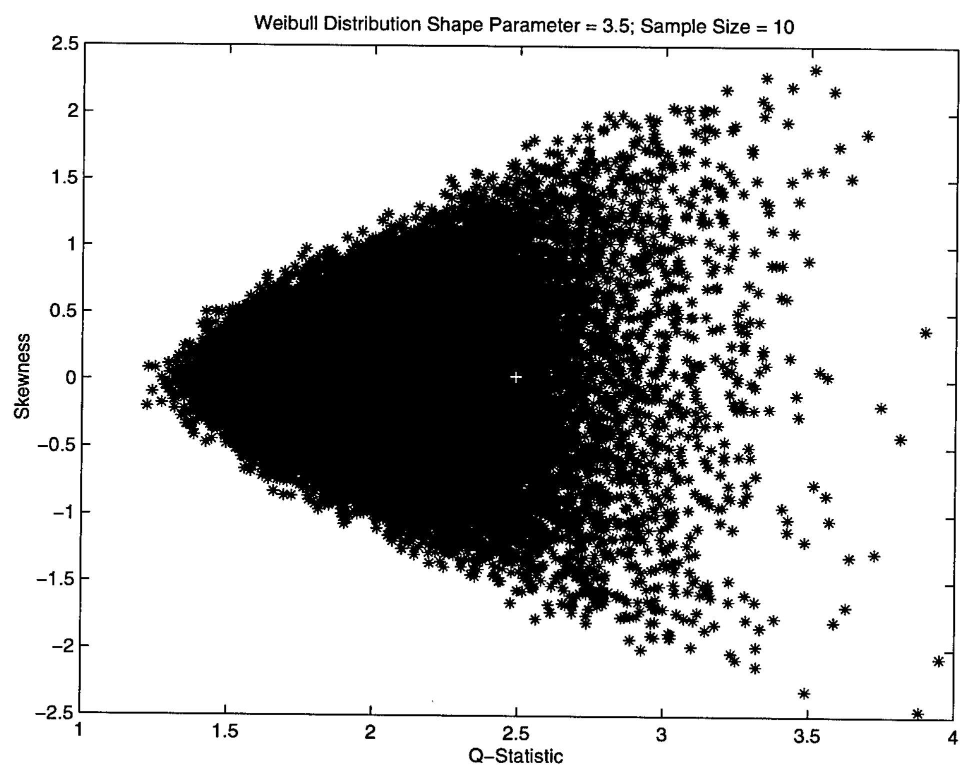


Figure A.62 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 10$.

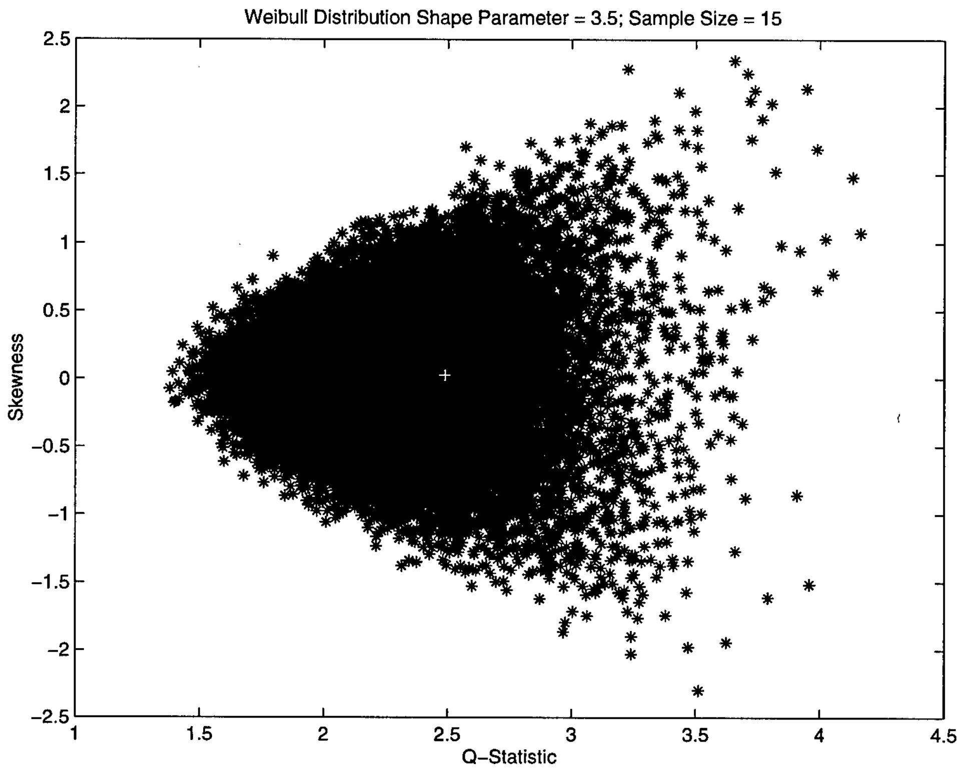


Figure A.63 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 15$.

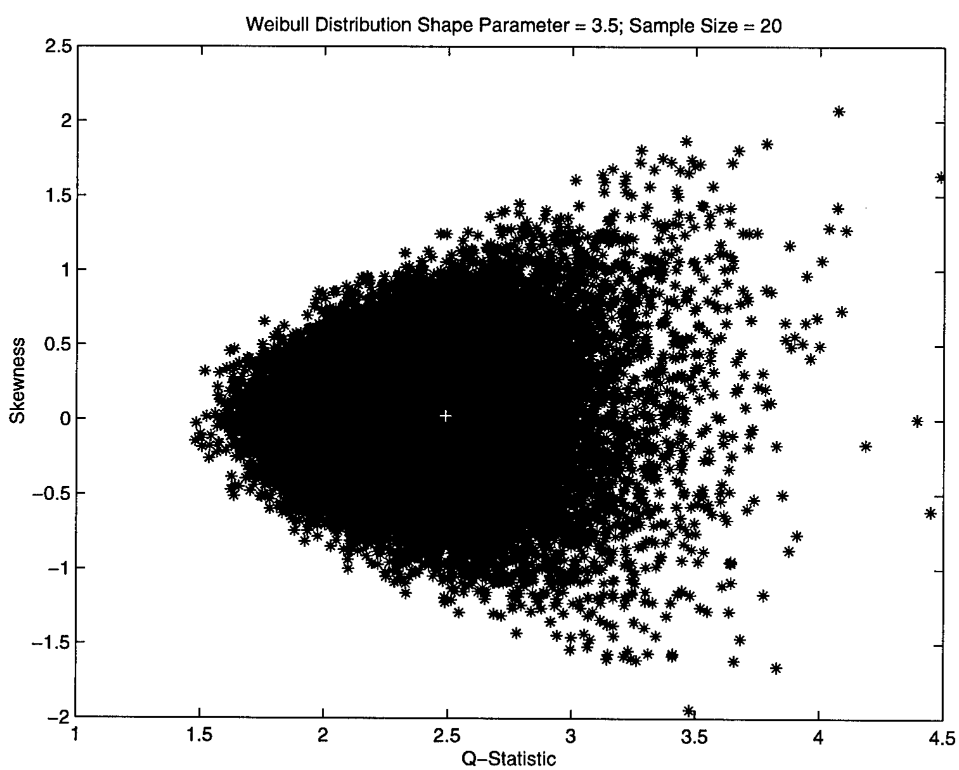


Figure A.64 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 20$.

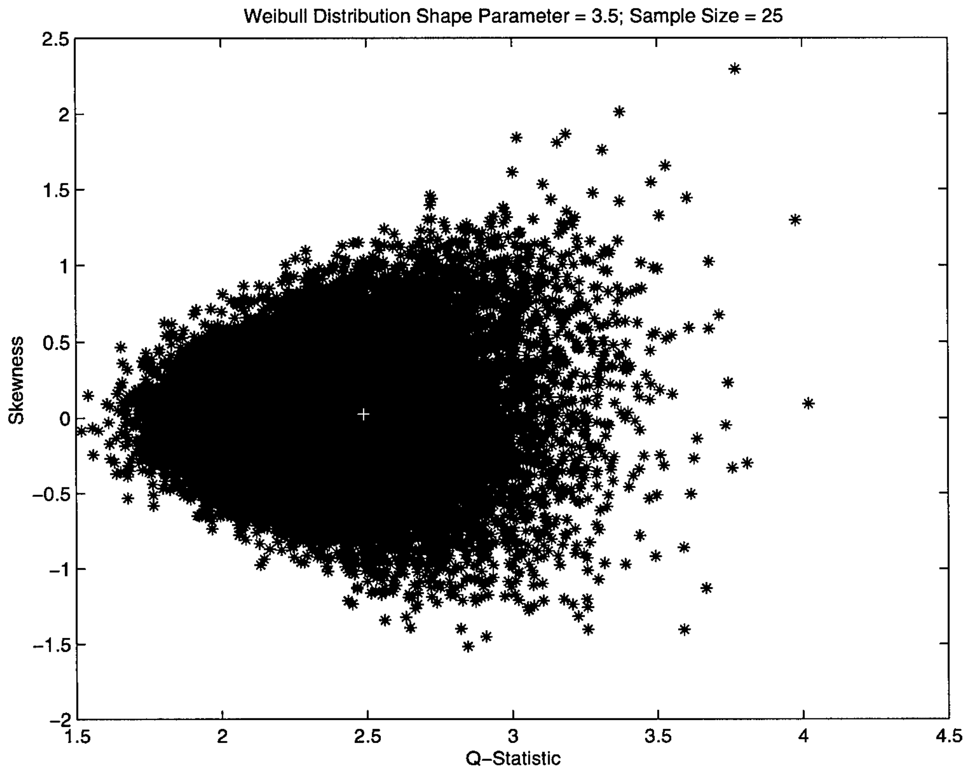


Figure A.65 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 25$.

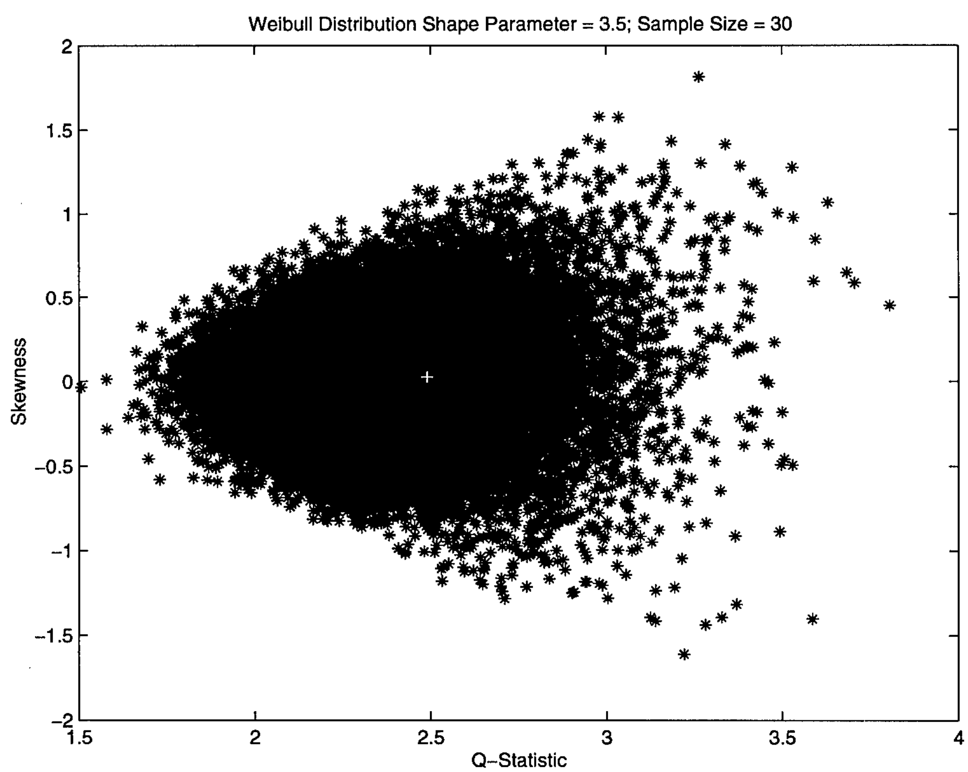


Figure A.66 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 30$.

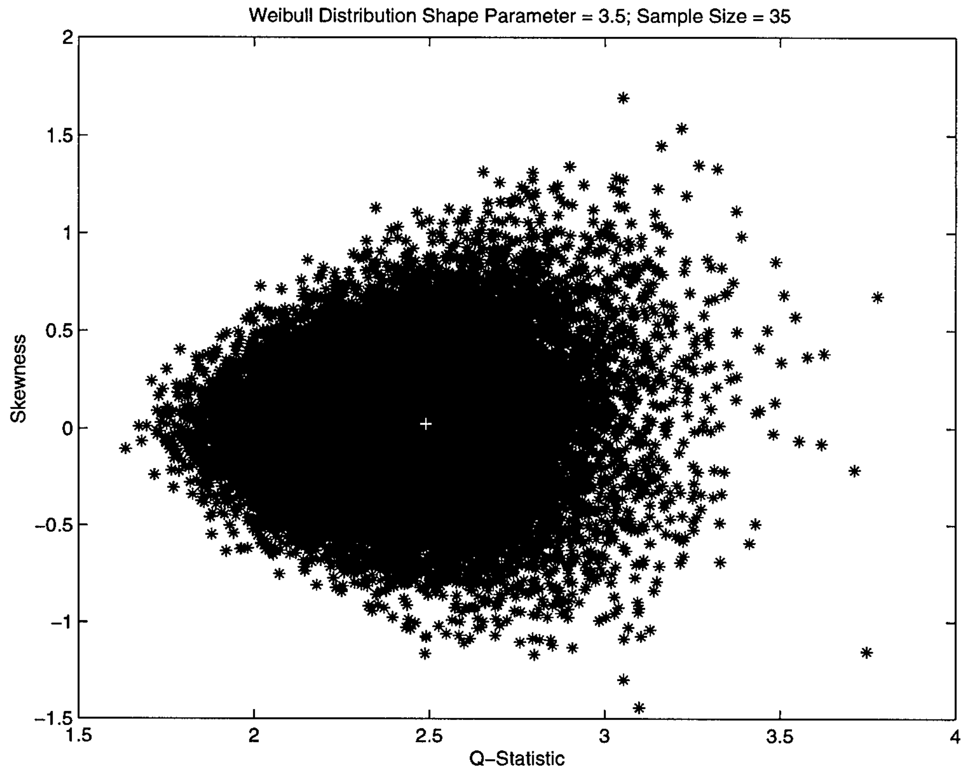


Figure A.67 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 35$.

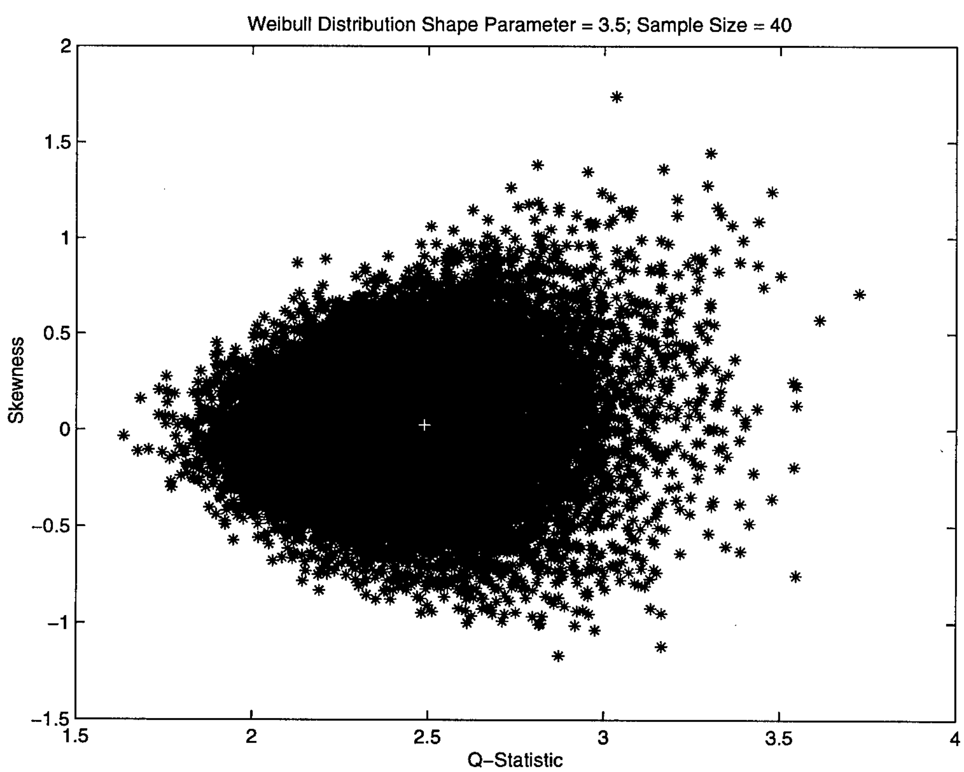


Figure A.68 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 40$.

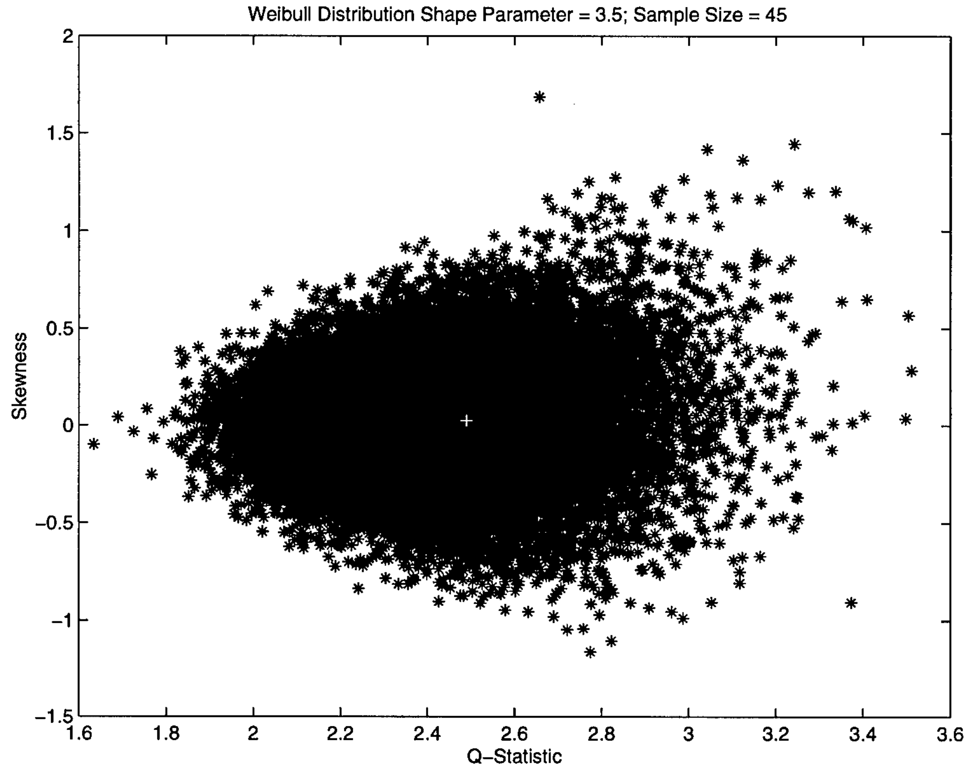


Figure A.69 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 45$.

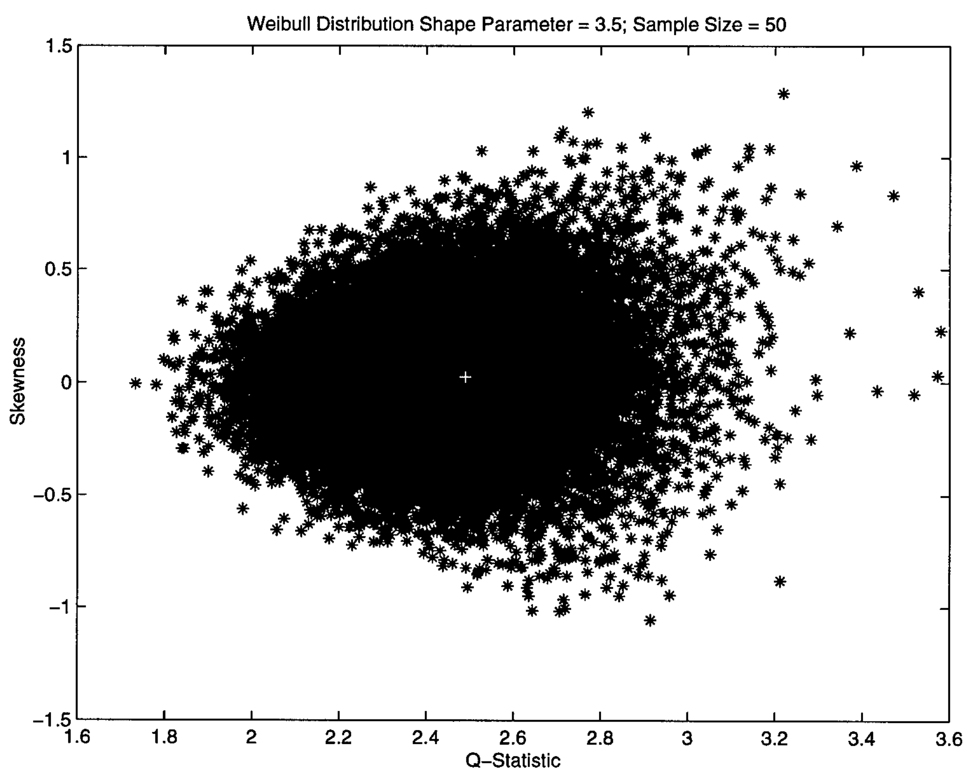


Figure A.70 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 3.5$) ; $n = 50$.

A.8 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$).

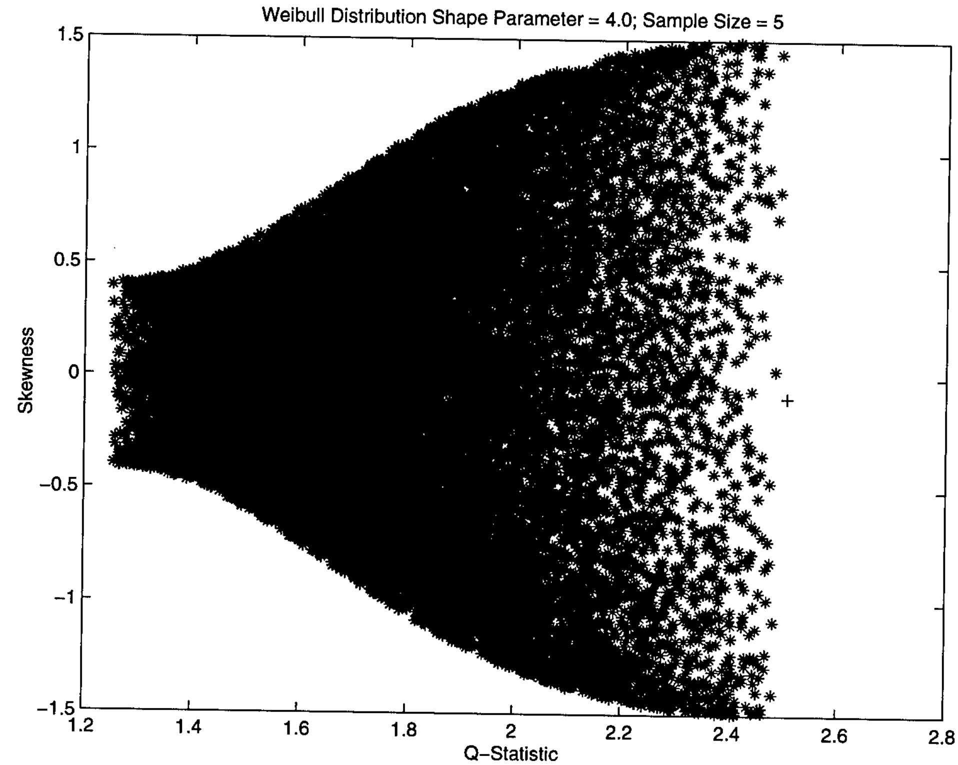


Figure A.71 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 5$.

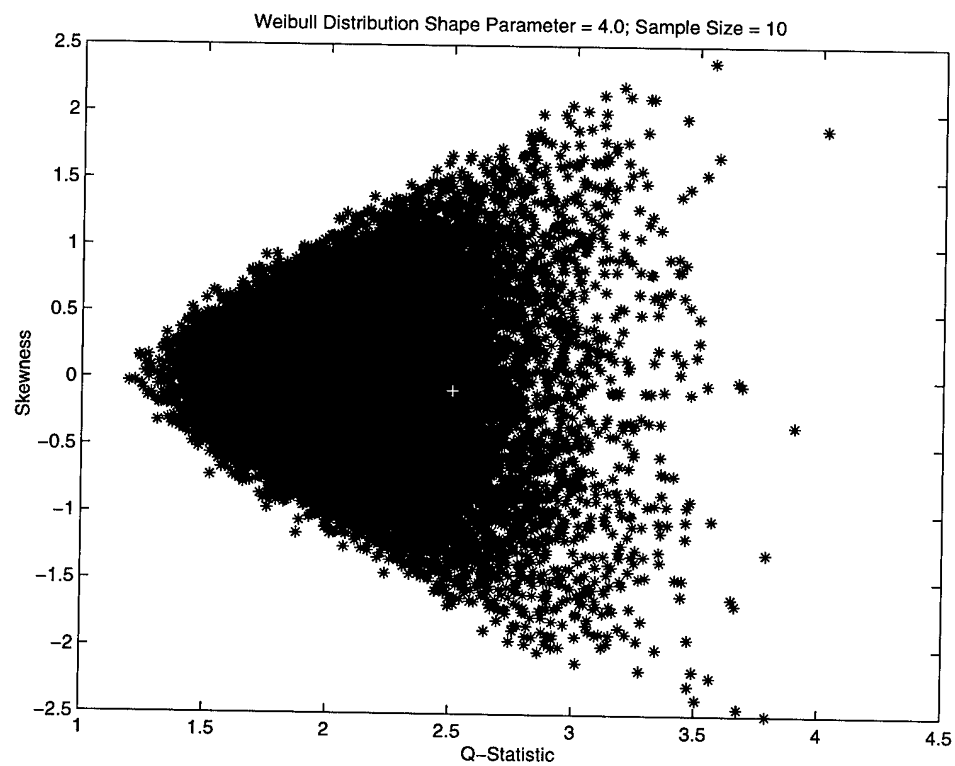


Figure A.72 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 10$.

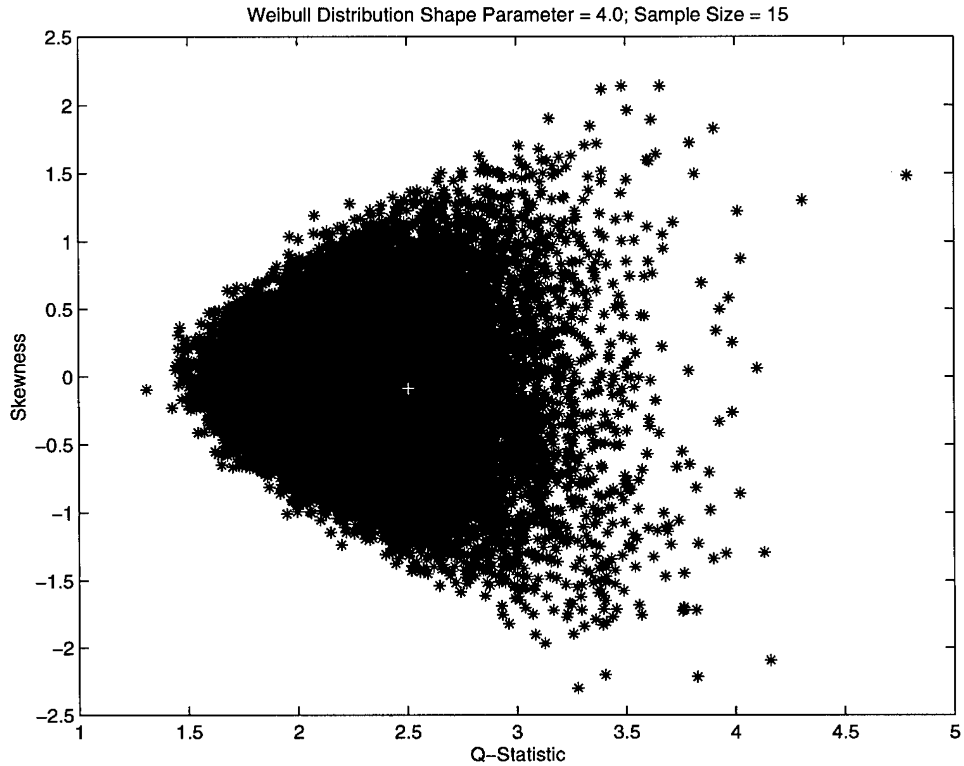


Figure A.73 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 15$.

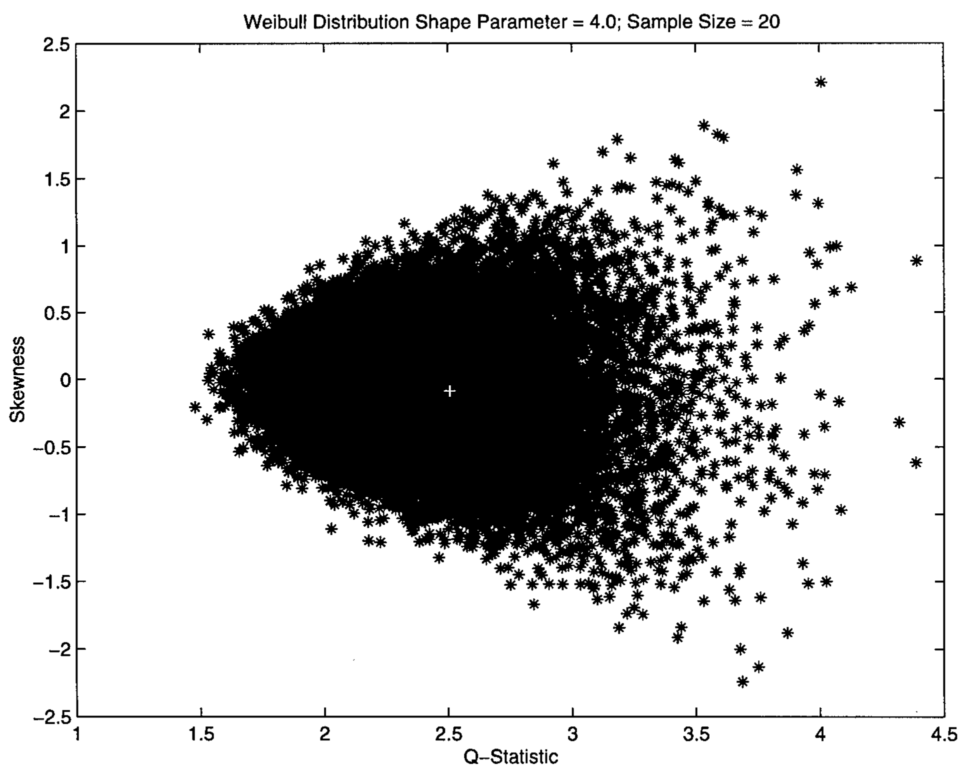


Figure A.74 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 20$.

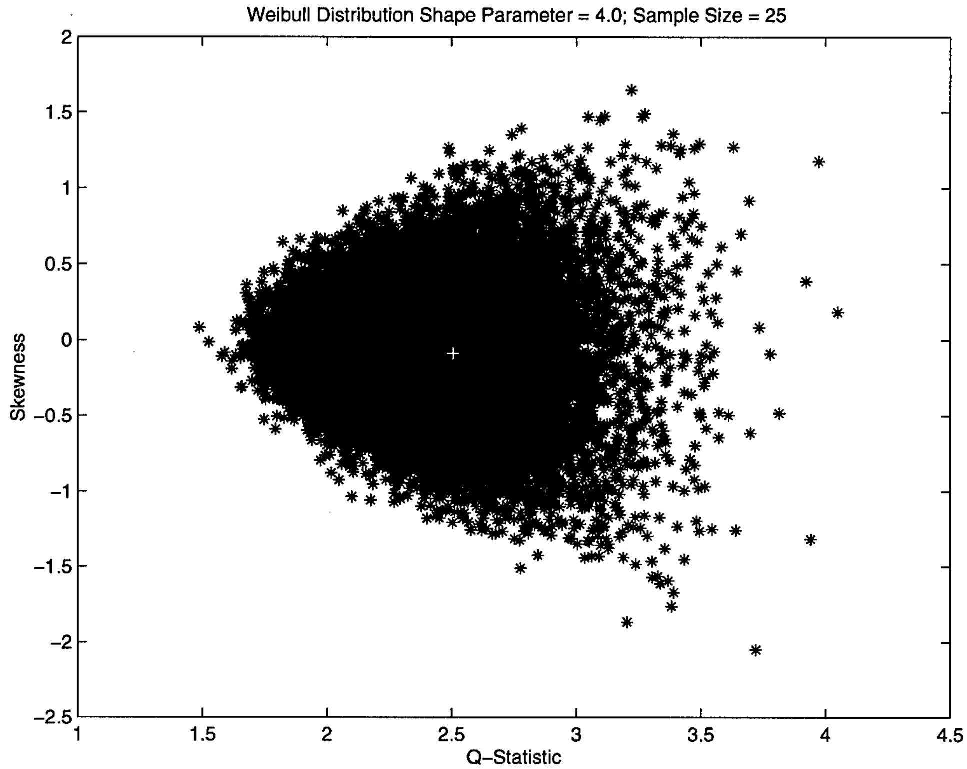


Figure A.75 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 25$.

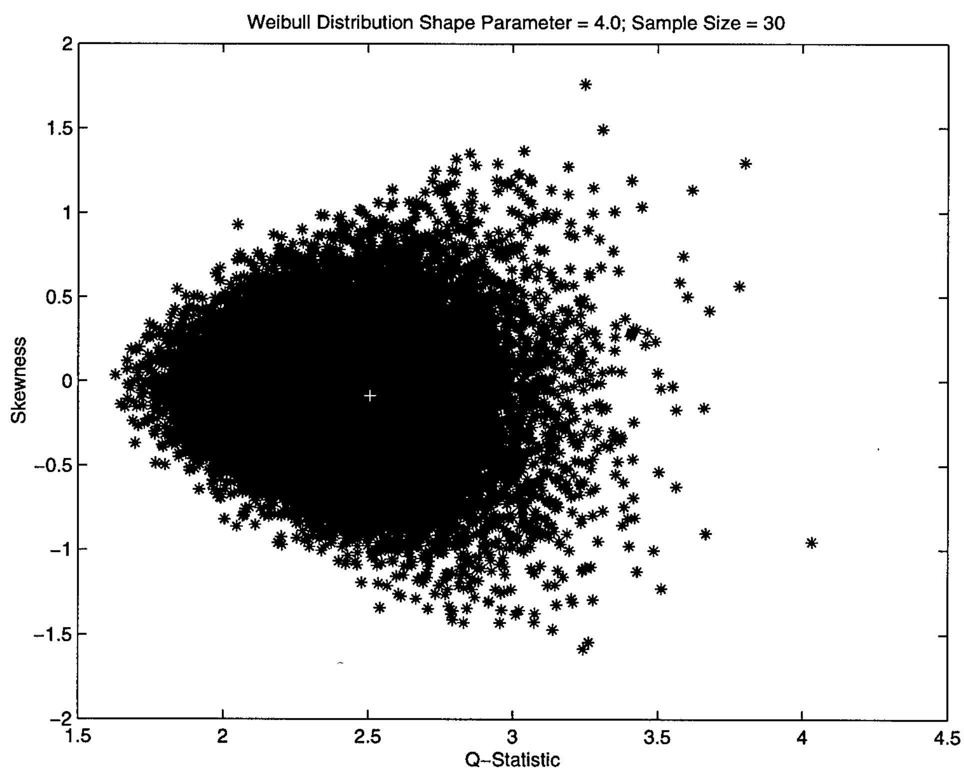


Figure A.76 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 30$.

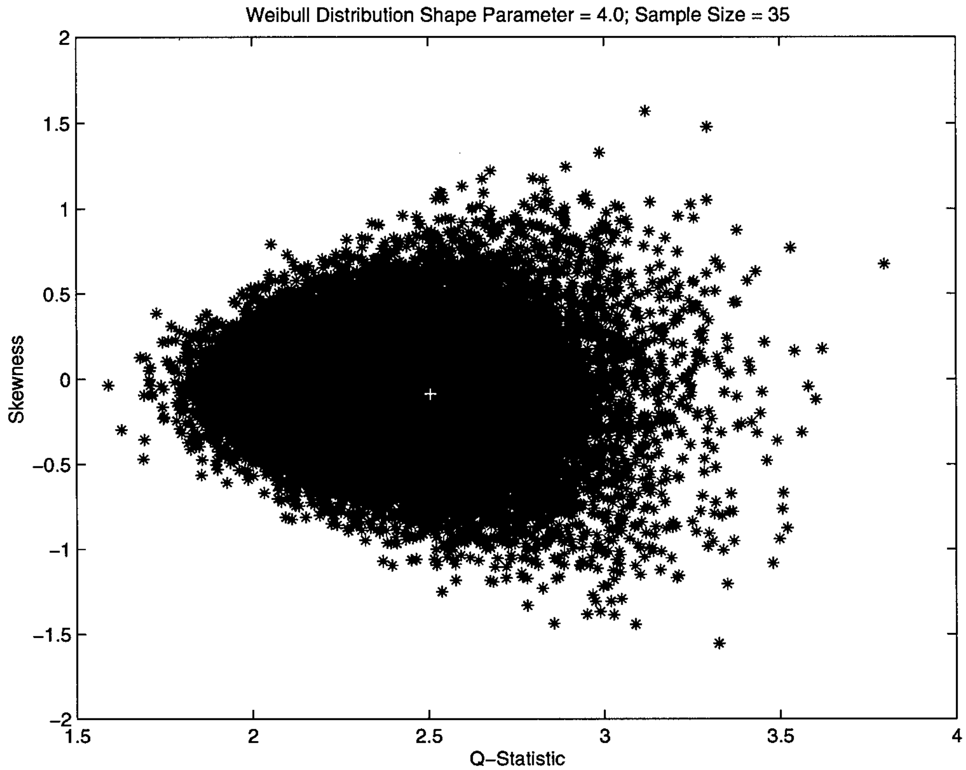


Figure A.77 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 35$.

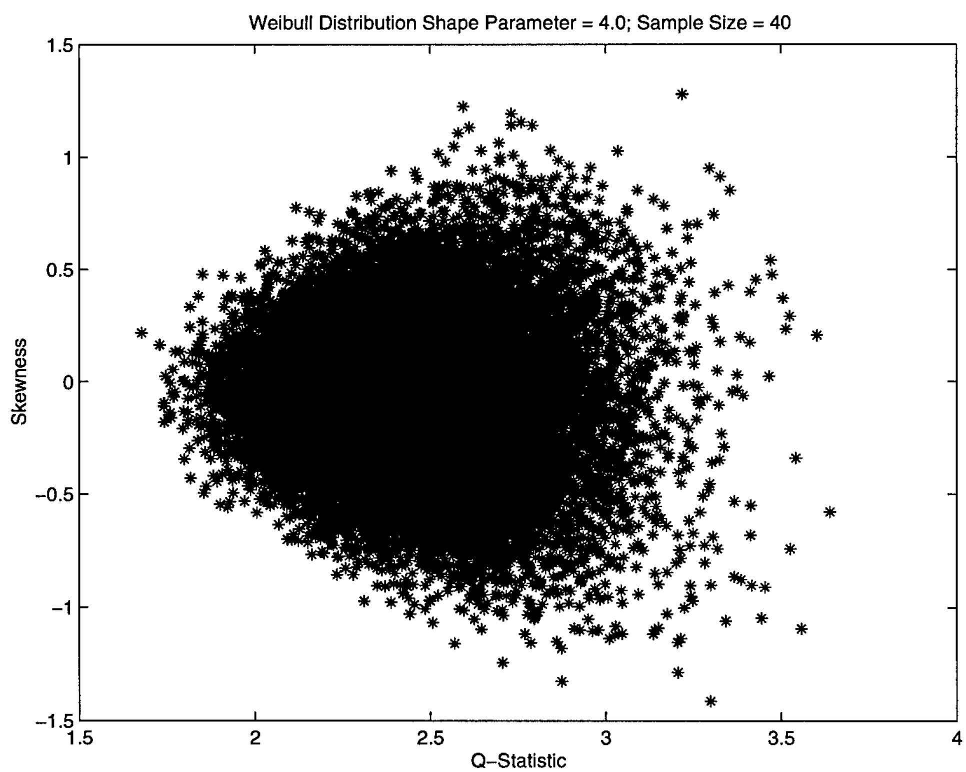


Figure A.78 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 40$.

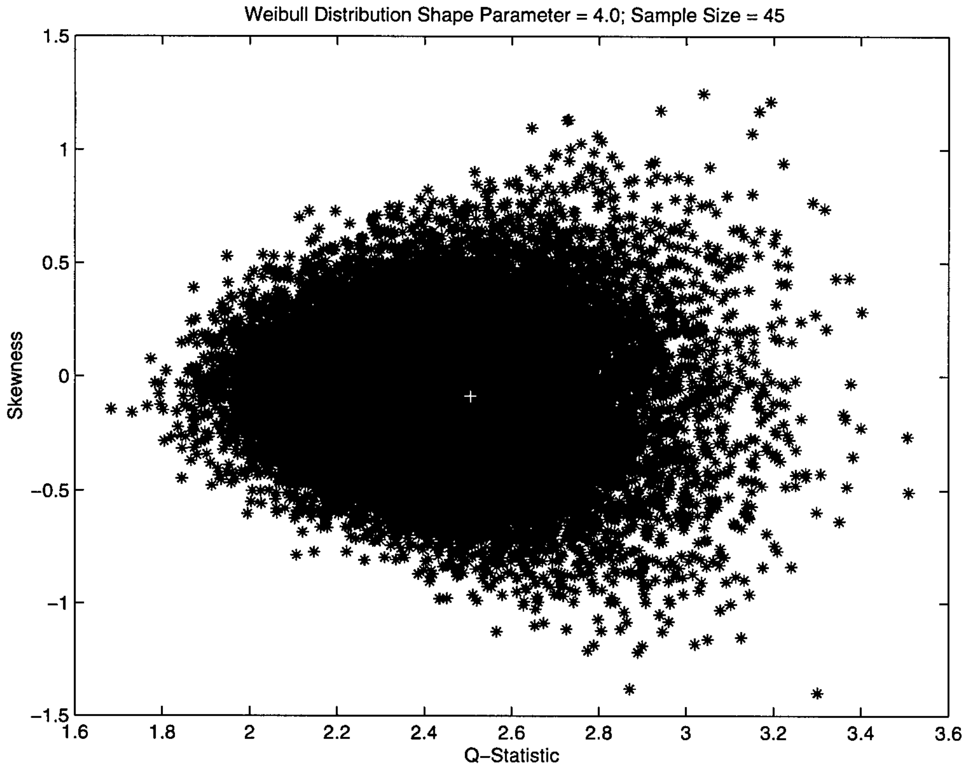


Figure A.79 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 45$.

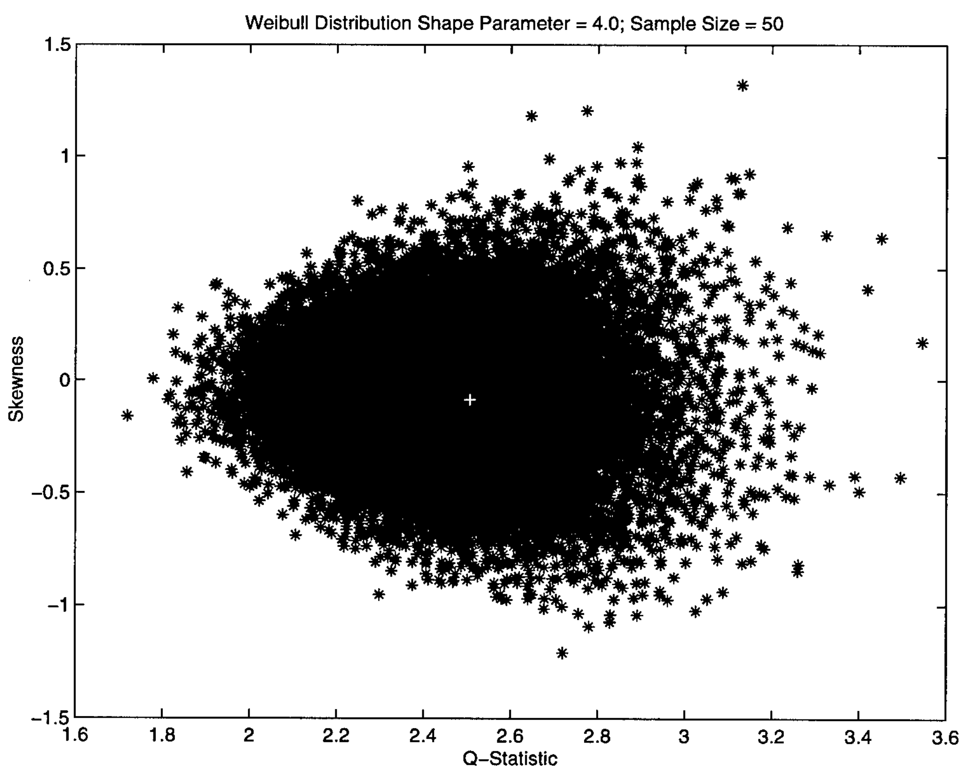


Figure A.80 Joint Distribution of $\sqrt{b_1}$ and Q-Statistic for Weibull($\beta = 4.0$) ; $n = 50$.

Appendix B. Upper and Lower Tail Critical Values For $\sqrt{b_1}$ and Q-Statistic G.O.F. Tests.

B.1 Sample Skewness $\left(\sqrt{b_1}\right)$ Percentage Points

Table B.1 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 0.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-0.748	-0.516	-0.404	-0.359	-0.314	-0.272	-0.230	-0.190	-0.153	-0.118
10	0.068	0.201	0.286	0.344	0.395	0.431	0.466	0.499	0.530	0.558
15	0.422	0.543	0.620	0.675	0.722	0.766	0.805	0.837	0.865	0.891
20	0.674	0.784	0.856	0.913	0.960	0.999	1.031	1.062	1.089	1.115
25	0.852	0.958	1.025	1.078	1.124	1.164	1.199	1.230	1.260	1.288
30	0.979	1.086	1.158	1.217	1.264	1.303	1.340	1.371	1.400	1.427
35	1.112	1.220	1.293	1.343	1.391	1.430	1.465	1.493	1.522	1.548
40	1.200	1.312	1.382	1.434	1.479	1.518	1.555	1.585	1.613	1.641
45	1.300	1.406	1.474	1.527	1.573	1.613	1.650	1.683	1.711	1.737
50	1.374	1.480	1.548	1.603	1.647	1.688	1.722	1.752	1.782	1.807

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.083	-0.049	-0.017	0.015	0.043	0.068	0.094	0.120	0.144	0.165
10	0.585	0.612	0.634	0.656	0.677	0.699	0.717	0.736	0.753	0.796
15	0.916	0.939	0.963	0.984	1.003	1.204	1.042	1.060	1.077	1.094
20	1.140	1.164	1.185	1.209	1.230	1.249	1.267	1.285	1.304	1.321
25	1.313	1.337	1.361	1.382	1.403	1.422	1.440	1.458	1.476	1.492
30	1.453	1.477	1.498	1.518	1.538	1.557	1.575	1.594	1.612	1.630
35	1.572	1.596	1.620	1.642	1.664	1.683	1.703	1.722	1.739	1.759
40	1.666	1.692	1.715	1.735	1.755	1.776	1.794	1.813	1.831	1.849
45	1.761	1.784	1.805	1.828	1.850	1.871	1.890	1.908	1.926	1.943
50	1.832	1.856	1.878	1.900	1.921	1.941	1.960	1.979	1.997	2.014

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.208	0.244	0.276	0.305	0.331	0.355	0.374	0.390	0.403	0.412
10	0.801	0.831	0.858	0.884	0.908	0.935	0.959	0.983	1.007	1.031
15	1.127	1.156	1.185	1.213	1.239	1.265	1.291	1.315	1.339	1.363
20	1.353	1.383	1.413	1.442	1.469	1.496	1.520	1.546	1.571	1.596
25	1.526	1.558	1.588	1.616	1.644	1.671	1.696	1.721	1.746	1.772
30	1.662	1.695	1.728	1.757	1.785	1.814	1.842	1.867	1.893	1.917
35	1.793	1.823	1.852	1.881	1.911	1.938	1.966	1.993	2.017	2.041
40	1.882	1.914	1.948	1.977	2.006	2.034	2.061	2.087	2.115	2.141
45	1.977	2.008	2.040	2.068	2.096	2.123	2.151	2.177	2.203	2.228
50	2.048	2.081	2.111	2.140	2.170	2.199	2.226	2.252	2.281	2.308

Table B.2 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 0.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.012	0.013	0.010	0.009	0.008	0.010	0.010	0.008	0.007	0.007
10	0.011	0.009	0.007	0.009	0.009	0.006	0.005	0.005	0.004	0.006
15	0.010	0.009	0.011	0.005	0.006	0.004	0.001	0.002	0.002	0.003
20	0.005	0.003	0.004	0.002	0.004	0.004	0.004	0.004	0.004	0.005
25	0.004	0.002	0.008	0.003	0.004	0.003	0.003	0.003	0.004	0.002
30	0.006	0.005	0.005	0.004	0.003	0.004	0.002	0.002	0.002	0.002
35	0.010	0.003	0.005	0.005	0.007	0.008	0.008	0.007	0.008	0.006
40	0.008	0.007	0.004	0.005	0.005	0.005	0.006	0.004	0.004	0.005
45	0.013	0.010	0.014	0.010	0.009	0.010	0.009	0.008	0.006	0.009
50	0.014	0.013	0.011	0.009	0.011	0.011	0.012	0.012	0.012	0.011

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.006	0.003	0.003	0.004	0.005	0.007	0.005	0.004	0.005	0.004
10	0.004	0.004	0.005	0.004	0.003	0.003	0.004	0.002	0.002	0.003
15	0.005	0.006	0.005	0.005	0.004	0.004	0.005	0.003	0.003	0.003
20	0.002	0.002	0.001	0.002	0.001	0.001	0.003	0.001	0.001	0.001
25	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002
30	0.002	0.000	0.002	0.002	0.003	0.003	0.002	0.003	0.004	0.003
35	0.008	0.006	0.007	0.008	0.008	0.007	0.006	0.009	0.007	0.008
40	0.002	0.003	0.003	0.003	0.003	0.003	0.001	0.001	0.001	0.003
45	0.002	0.002	0.003	0.002	0.002	0.001	0.003	0.000	0.001	0.002
50	0.004	0.005	0.005	0.004	0.002	0.003	0.006	0.002	0.002	0.002

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.004	0.004	0.003	0.002	0.002	0.002	0.002	0.001	0.003
10	0.005	0.001	0.005	0.001	0.003	0.003	0.003	0.004	0.004	0.004
15	0.004	0.005	0.005	0.004	0.002	0.003	0.006	0.002	0.002	0.002
20	0.000	0.004	0.001	0.003	0.001	0.001	0.002	0.002	0.001	0.003
25	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.001
30	0.003	0.000	0.003	0.001	0.001	0.003	0.002	0.004	0.004	0.000
35	0.004	0.003	0.003	0.002	0.003	0.004	0.004	0.004	0.002	0.002
40	0.003	0.001	0.002	0.001	0.003	0.004	0.002	0.003	0.004	0.003
45	0.002	0.007	0.008	0.009	0.004	0.004	0.006	0.005	0.003	0.006
50	0.008	0.009	0.009	0.010	0.007	0.010	0.010	0.011	0.009	0.009

Table B.3 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 0.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.500	1.499	1.499	1.499	1.498	1.497	1.497	1.496	1.495	1.494
10	2.658	2.650	2.643	2.636	2.629	2.622	2.616	2.608	2.600	2.592
15	3.441	3.418	3.398	3.378	3.360	3.341	3.324	3.306	3.288	3.271
20	4.062	4.020	3.983	3.951	3.920	3.890	3.861	3.833	3.805	3.779
25	4.571	4.508	4.457	4.414	4.369	4.327	4.287	4.250	4.212	4.174
30	5.027	4.946	4.874	4.812	4.752	4.697	4.645	4.598	4.551	4.510
35	5.418	5.306	5.221	5.142	5.073	5.010	4.944	4.883	4.828	4.774
40	5.780	5.658	5.554	5.459	5.378	5.298	5.228	5.160	5.097	5.034
45	6.102	5.942	5.817	5.711	5.620	5.539	5.455	5.379	5.305	5.244
50	6.397	6.217	6.073	5.952	5.847	5.745	5.655	5.578	5.494	5.423

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.493	1.492	1.491	1.490	1.489	1.487	1.486	1.485	1.483	1.481
10	2.584	2.575	2.567	2.559	2.550	2.541	2.532	2.523	2.514	2.505
15	3.254	3.235	3.216	3.198	3.181	3.164	3.149	3.131	3.114	3.096
20	3.751	3.724	3.698	3.672	3.648	3.623	3.598	3.573	3.550	3.524
25	4.141	4.107	4.072	4.036	4.004	3.973	3.942	3.909	3.879	3.849
30	4.463	4.421	4.379	4.341	4.298	4.262	4.225	4.189	4.152	4.118
35	4.723	4.676	4.624	4.579	4.534	4.492	4.449	4.407	4.367	4.329
40	4.978	4.924	4.870	4.817	4.769	4.722	4.675	4.631	4.587	4.543
45	5.181	5.119	5.063	5.008	4.957	4.903	4.853	4.808	4.761	4.714
50	5.353	5.286	5.217	5.058	5.101	5.046	4.991	4.937	4.887	4.838

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	1.478	1.475	1.471	1.467	1.462	1.457	1.452	1.446	1.440	1.434
10	2.487	2.467	2.446	2.427	2.045	2.384	2.362	2.341	2.319	2.297
15	3.061	3.028	2.993	2.956	2.922	2.887	2.855	2.822	2.789	2.756
20	3.479	3.429	3.383	3.340	3.297	3.257	3.215	3.173	3.134	3.093
25	3.792	3.735	3.679	3.629	3.579	3.530	3.481	3.432	3.386	3.343
30	4.049	3.985	3.922	3.865	3.807	3.751	3.698	3.647	3.597	3.551
35	4.255	4.185	4.115	4.048	3.986	3.926	3.867	3.815	3.764	3.718
40	4.457	4.381	4.305	4.239	4.172	4.110	4.048	3.988	3.934	3.882
45	4.629	4.548	4.466	4.393	4.323	4.257	4.194	4.136	4.078	4.025
50	4.748	4.662	4.580	4.505	4.436	4.372	4.309	4.249	4.193	4.138

Table B.4 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 0.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.002	0.001
10	0.003	0.003	0.001	0.003	0.003	0.002	0.004	0.005	0.002	0.003
15	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.000	0.001
20	0.003	0.002	0.005	0.004	0.005	0.003	0.003	0.003	0.004	0.005
25	0.002	0.004	0.003	0.002	0.002	0.003	0.001	0.004	0.002	0.002
30	0.006	0.006	0.010	0.005	0.003	0.005	0.004	0.002	0.006	0.003
35	0.003	0.003	0.002	0.003	0.004	0.003	0.002	0.001	0.003	0.003
40	0.012	0.012	0.009	0.014	0.008	0.009	0.009	0.004	0.007	0.008
45	0.005	0.008	0.004	0.008	0.002	0.002	0.005	0.002	0.003	0.002
50	0.011	0.011	0.006	0.012	0.010	0.009	0.007	0.003	0.009	0.010

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.002	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.001
10	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
15	0.003	0.009	0.003	0.006	0.004	0.006	0.007	0.007	0.004	0.002
20	0.002	0.008	0.014	0.012	0.006	0.012	0.009	0.009	0.013	0.011
25	0.001	0.004	0.004	0.004	0.002	0.004	0.002	0.003	0.004	0.003
30	0.004	0.006	0.004	0.006	0.006	0.005	0.004	0.004	0.003	0.007
35	0.014	0.015	0.013	0.015	0.014	0.013	0.013	0.014	0.014	0.016
40	0.006	0.006	0.006	0.004	0.006	0.006	0.006	0.008	0.008	0.006
45	0.014	0.015	0.013	0.015	0.011	0.010	0.013	0.012	0.014	0.013
50	0.016	0.014	0.009	0.014	0.013	0.012	0.011	0.011	0.012	0.011

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.001	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.000
10	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001
15	0.008	0.010	0.007	0.008	0.008	0.011	0.008	0.009	0.008	0.008
20	0.003	0.006	0.003	0.002	0.004	0.004	0.003	0.003	0.003	0.006
25	0.000	0.005	0.003	0.003	0.001	0.005	0.002	0.001	0.002	0.004
30	0.008	0.005	0.007	0.006	0.006	0.005	0.006	0.007	0.006	0.005
35	0.005	0.006	0.003	0.006	0.003	0.007	0.004	0.006	0.002	0.007
40	0.006	0.006	0.002	0.006	0.003	0.004	0.004	0.006	0.002	0.005
45	0.007	0.013	0.005	0.009	0.007	0.011	0.007	0.009	0.005	0.009
50	0.010	0.011	0.008	0.011	0.009	0.013	0.011	0.010	0.009	0.010

Table B.5 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 1.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.161	-0.988	-0.874	-0.780	-0.704	-0.639	-0.588	-0.541	-0.499	-0.464
10	-0.459	-0.333	-0.250	-0.188	-0.140	-0.096	-0.061	-0.029	-0.003	0.022
15	-0.138	-0.034	0.034	0.082	0.123	0.156	0.188	0.218	0.244	0.266
20	0.051	0.156	0.216	0.268	0.306	0.337	0.365	0.389	0.411	0.431
25	0.191	0.286	0.337	0.380	0.420	0.451	0.479	0.503	0.524	0.543
30	0.301	0.384	0.436	0.477	0.511	0.540	0.564	0.588	0.610	0.628
35	0.398	0.473	0.520	0.561	0.594	0.623	0.648	0.670	0.689	0.707
40	0.452	0.528	0.578	0.616	0.647	0.673	0.698	0.718	0.738	0.756
45	0.511	0.591	0.638	0.674	0.705	0.731	0.755	0.776	0.794	0.811
50	0.568	0.637	0.684	0.719	0.750	0.777	0.796	0.816	0.833	0.850

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.435	-0.411	-0.392	-0.374	-0.355	-0.337	-0.318	-0.302	-0.284	-0.268
10	0.046	0.069	0.088	0.108	0.126	0.143	0.161	0.178	0.193	0.208
15	0.288	0.307	0.326	0.344	0.360	0.378	0.393	0.409	0.424	0.437
20	0.450	0.470	0.487	0.502	0.519	0.535	0.549	0.563	0.576	0.589
25	0.562	0.579	0.596	0.610	0.625	0.638	0.651	0.664	0.676	0.689
30	0.647	0.663	0.679	0.693	0.708	0.721	0.733	0.746	0.758	0.770
35	0.724	0.739	0.755	0.769	0.783	0.795	0.809	0.820	0.832	0.843
40	0.772	0.788	0.803	0.817	0.831	0.843	0.856	0.867	0.877	0.888
45	0.827	0.842	0.856	0.869	0.883	0.895	0.907	0.917	0.928	0.939
50	0.866	0.881	0.895	0.908	0.921	0.933	0.944	0.955	0.966	0.977

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.232	-0.200	-0.169	-0.139	-0.108	-0.080	-0.053	-0.025	0.003	0.027
10	0.238	0.265	0.291	0.315	0.338	0.360	0.381	0.402	0.422	0.441
15	0.463	0.488	0.512	0.536	0.557	0.577	0.596	0.615	0.633	0.651
20	0.612	0.635	0.657	0.678	0.699	0.718	0.736	0.756	0.773	0.790
25	0.713	0.735	0.755	0.776	0.796	0.814	0.832	0.850	0.867	0.884
30	0.792	0.813	0.835	0.854	0.873	0.891	0.910	0.927	0.943	0.959
35	0.866	0.886	0.906	0.924	0.942	0.960	0.976	0.992	1.008	1.023
40	0.910	0.930	0.950	0.969	0.986	1.003	1.019	1.035	1.050	1.066
45	0.959	0.977	0.996	1.014	1.031	1.048	1.064	1.081	1.095	1.110
50	0.997	1.016	1.035	1.053	1.070	1.087	1.103	1.119	1.134	1.148

Table B.6 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.003	0.005	0.007	0.000	0.004	0.006	0.006	0.002	0.000	0.004
10	0.002	0.001	0.002	0.000	0.002	0.004	0.003	0.002	0.000	0.001
15	0.006	0.003	0.004	0.007	0.006	0.011	0.004	0.004	0.007	0.003
20	0.005	0.005	0.004	0.006	0.006	0.008	0.005	0.006	0.006	0.004
25	0.004	0.003	0.003	0.003	0.004	0.003	0.003	0.005	0.003	0.003
30	0.005	0.003	0.005	0.003	0.003	0.005	0.005	0.003	0.003	0.003
35	0.006	0.006	0.006	0.007	0.004	0.009	0.006	0.004	0.007	0.006
40	0.002	0.003	0.003	0.005	0.002	0.007	0.002	0.002	0.005	0.002
45	0.005	0.005	0.006	0.009	0.006	0.008	0.006	0.006	0.009	0.005
50	0.007	0.006	0.005	0.004	0.006	0.006	0.007	0.006	0.004	0.007

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.002
10	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
15	0.003	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.004	0.004
20	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.004	0.000	0.001
25	0.002	0.003	0.003	0.003	0.002	0.002	0.001	0.004	0.003	0.003
30	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.003	0.001	0.002
35	0.002	0.003	0.003	0.003	0.002	0.003	0.002	0.003	0.003	0.002
40	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
45	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.006
50	0.005	0.006	0.005	0.005	0.002	0.003	0.003	0.006	0.005	0.004

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.003	0.004	0.003	0.003	0.004	0.003	0.004	0.003	0.003	0.003
10	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002
15	0.001	0.001	0.000	0.001	0.000	0.001	0.001	0.000	0.001	0.000
20	0.001	0.000	0.002	0.001	0.002	0.002	0.001	0.002	0.000	0.002
25	0.002	0.003	0.001	0.001	0.002	0.001	0.001	0.002	0.003	0.001
30	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.002	0.002
35	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.003	0.002
40	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.002	0.001
45	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.002	0.003	0.002
50	0.004	0.005	0.005	0.005	0.003	0.005	0.004	0.004	0.006	0.005

Table B.7 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.486	1.477	1.468	1.459	1.450	1.441	1.432	1.424	1.415	1.406
10	2.464	2.389	2.335	2.291	2.249	2.214	2.176	2.143	2.112	2.080
15	2.971	2.834	2.731	2.652	2.586	2.529	2.475	2.427	2.383	2.340
20	3.286	3.097	2.966	2.868	2.785	2.713	2.654	2.600	2.548	2.520
25	3.458	3.241	3.097	2.993	2.905	2.826	2.759	2.698	2.645	2.588
30	3.646	3.379	3.214	3.088	2.985	2.901	2.826	2.762	2.708	2.658
35	3.714	3.425	3.260	3.135	3.030	2.943	2.869	2.801	2.744	2.689
40	3.794	3.510	3.336	3.195	3.082	2.999	2.924	2.857	2.795	2.741
45	3.846	3.526	3.334	3.203	3.095	3.008	2.929	2.865	2.805	2.758
50	3.869	3.558	3.357	3.217	3.114	3.021	2.939	2.869	2.810	2.759

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.396	1.388	1.379	1.371	1.362	1.353	1.344	1.336	1.326	1.317
10	2.052	2.023	1.998	1.971	1.948	1.924	1.901	1.879	1.857	1.835
15	2.301	2.263	2.228	2.196	2.165	2.137	2.110	2.083	2.058	2.035
20	2.458	2.418	2.382	2.348	2.314	2.286	2.255	2.227	2.199	2.174
25	2.541	2.500	2.461	2.423	2.388	2.358	2.328	2.300	2.274	2.250
30	2.612	2.568	2.530	2.493	2.458	2.425	2.394	2.367	2.340	2.312
35	2.643	2.599	2.564	2.527	2.490	2.456	2.429	2.399	2.371	2.344
40	2.690	2.646	2.604	2.566	2.528	2.495	2.464	2.436	2.408	2.380
45	2.711	2.666	2.625	2.590	2.552	2.523	2.492	2.462	2.434	2.407
50	2.712	2.668	2.626	2.586	2.551	2.520	2.490	2.460	2.435	2.411

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	1.299	1.280	1.262	1.241	1.222	1.202	1.183	1.162	1.142	1.123
10	1.795	1.754	1.717	1.680	1.646	1.613	1.584	1.553	1.524	1.495
15	1.991	1.949	1.909	1.872	1.837	1.804	1.772	1.739	1.712	1.686
20	2.125	2.080	2.041	2.001	1.965	1.930	1.900	1.869	1.841	1.813
25	2.201	2.155	2.113	2.075	2.039	2.004	1.971	1.940	1.910	1.882
30	2.263	2.218	2.177	2.137	2.100	2.065	2.032	2.001	1.971	1.942
35	2.296	2.250	2.207	2.168	2.134	2.099	2.067	2.035	2.006	1.980
40	2.333	2.288	2.244	2.205	2.168	2.133	2.101	2.072	2.043	2.014
45	2.358	2.313	2.270	2.231	2.196	2.163	2.131	2.101	2.073	2.045
50	2.365	2.323	2.283	2.246	2.210	2.176	2.146	2.117	2.088	2.061

Table B.8 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.017	0.004	0.002	0.002	0.014	0.010	0.003	0.003	0.003
10	0.003	0.009	0.002	0.010	0.009	0.011	0.010	0.002	0.001	0.005
15	0.003	0.003	0.001	0.000	0.002	0.002	0.003	0.002	0.002	0.002
20	0.006	0.009	0.004	0.004	0.008	0.012	0.005	0.005	0.004	0.005
25	0.005	0.019	0.008	0.009	0.004	0.012	0.014	0.004	0.003	0.005
30	0.007	0.009	0.001	0.006	0.006	0.006	0.011	0.003	0.002	0.005
35	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.002	0.002
40	0.011	0.006	0.010	0.008	0.007	0.013	0.009	0.012	0.010	0.014
45	0.009	0.006	0.008	0.011	0.009	0.009	0.009	0.008	0.007	0.007
50	0.005	0.006	0.004	0.007	0.006	0.008	0.008	0.006	0.006	0.004

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.004	0.003	0.001	0.003	0.002	0.004	0.002	0.003	0.002	0.001
10	0.005	0.003	0.004	0.004	0.006	0.005	0.005	0.002	0.007	0.004
15	0.008	0.009	0.010	0.008	0.007	0.007	0.007	0.009	0.006	0.010
20	0.006	0.006	0.007	0.005	0.003	0.004	0.003	0.006	0.004	0.007
25	0.007	0.004	0.005	0.007	0.006	0.006	0.006	0.004	0.005	0.005
30	0.004	0.003	0.004	0.004	0.003	0.005	0.004	0.003	0.002	0.004
35	0.003	0.001	0.002	0.001	0.003	0.001	0.004	0.001	0.001	0.001
40	0.006	0.006	0.006	0.007	0.007	0.006	0.005	0.008	0.006	0.006
45	0.002	0.002	0.002	0.003	0.004	0.004	0.003	0.003	0.003	0.002
50	0.009	0.008	0.008	0.009	0.006	0.008	0.006	0.010	0.006	0.008

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002
10	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.001
15	0.005	0.003	0.004	0.003	0.002	0.003	0.002	0.005	0.003	0.002
20	0.003	0.002	0.003	0.004	0.004	0.003	0.004	0.003	0.003	0.004
25	0.005	0.005	0.005	0.05	0.004	0.005	0.005	0.006	0.005	0.005
30	0.003	0.002	0.003	0.001	0.001	0.001	0.002	0.003	0.002	0.002
35	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.004	0.004	0.002
40	0.005	0.005	0.004	0.006	0.006	0.005	0.007	0.006	0.007	0.006
45	0.004	0.004	0.002	0.004	0.003	0.004	0.004	0.004	0.003	0.005
50	0.003	0.006	0.003	0.005	0.005	0.005	0.005	0.004	0.006	0.006

Table B.9 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 1.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.282	-1.163	-1.070	-0.998	-0.936	-0.879	-0.827	-0.785	-0.744	-0.702
10	-0.801	-0.653	-0.570	-0.511	-0.458	-0.418	-0.381	-0.348	-0.318	-0.292
15	-0.486	-0.376	-0.309	-0.261	-0.223	-0.189	-0.158	-0.130	-0.106	-0.084
20	-0.310	-0.209	-0.146	-0.103	-0.066	-0.036	-0.008	0.016	0.038	0.056
25	-0.182	-0.095	-0.043	-0.002	0.031	0.059	0.085	0.106	0.125	0.142
30	-0.082	-0.005	0.044	0.078	0.107	0.133	0.153	0.172	0.193	0.210
35	-0.003	0.071	0.111	0.147	0.176	0.200	0.222	0.241	0.259	0.275
40	0.043	0.114	0.159	0.192	0.218	0.241	0.262	0.279	0.295	0.312
45	0.099	0.165	0.208	0.239	0.265	0.286	0.305	0.322	0.338	0.352
50	0.138	0.204	0.243	0.273	0.300	0.320	0.340	0.356	0.372	0.384

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.667	-0.633	-0.603	-0.575	-0.546	-0.525	-0.503	-0.483	-0.463	-0.446
10	-0.271	-0.248	-0.227	-0.208	-0.190	-0.175	-0.155	-0.139	-0.124	-0.109
15	-0.064	-0.045	-0.027	-0.010	0.005	0.020	0.034	0.047	0.060	0.073
20	0.074	0.090	0.106	0.120	0.134	0.147	0.160	0.172	0.184	0.196
25	0.159	0.174	0.189	0.202	0.216	0.228	0.240	0.251	0.262	0.274
30	0.226	0.241	0.255	0.268	0.280	0.292	0.303	0.314	0.324	0.333
35	0.289	0.303	0.315	0.327	0.338	0.349	0.359	0.369	0.379	0.389
40	0.325	0.337	0.349	0.360	0.371	0.382	0.392	0.402	0.412	0.421
45	0.365	0.378	0.390	0.401	0.411	0.422	0.431	0.440	0.449	0.457
50	0.396	0.409	0.420	0.431	0.442	0.452	0.460	0.469	0.479	0.487

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.414	-0.389	-0.363	-0.338	-0.314	-0.292	-0.269	-0.233	-0.223	-0.199
10	-0.081	-0.057	-0.031	-0.006	0.016	0.038	0.058	0.077	0.096	0.115
15	0.098	0.122	0.143	0.164	0.184	0.203	0.221	0.238	0.254	0.272
20	0.217	0.237	0.256	0.275	0.292	0.309	0.326	0.341	0.357	0.372
25	0.295	0.314	0.332	0.349	0.366	0.381	0.397	0.411	0.425	0.439
30	0.353	0.371	0.388	0.404	0.420	0.436	0.450	0.464	0.478	0.491
35	0.406	0.424	0.439	0.454	0.469	0.482	0.495	0.509	0.522	0.535
40	0.437	0.454	0.469	0.484	0.499	0.513	0.526	0.539	0.552	0.564
45	0.473	0.489	0.503	0.517	0.531	0.544	0.557	0.570	0.582	0.594
50	0.503	0.518	0.532	0.546	0.560	0.572	0.584	0.596	0.608	0.619

Table B.10 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 1.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.000	0.005	0.002	0.003	0.004	0.002	0.000	0.003	0.004	0.002
10	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
15	0.005	0.010	0.005	0.009	0.003	0.008	0.011	0.004	0.003	0.009
20	0.006	0.009	0.005	0.004	0.004	0.004	0.005	0.005	0.004	0.008
25	0.003	0.003	0.002	0.000	0.002	0.001	0.003	0.002	0.002	0.002
30	0.007	0.009	0.005	0.006	0.012	0.001	0.011	0.003	0.002	0.006
35	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
40	0.005	0.006	0.004	0.007	0.007	0.004	0.008	0.009	0.006	0.008
45	0.009	0.004	0.007	0.011	0.007	0.010	0.009	0.006	0.008	0.009
50	0.008	0.006	0.011	0.008	0.009	0.008	0.009	0.009	0.010	0.005

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.003	0.003	0.004	0.002	0.001	0.002	0.001	0.002	0.002	0.002
10	0.001	0.002	0.003	0.006	0.002	0.003	0.003	0.002	0.003	0.003
15	0.003	0.004	0.004	0.004	0.004	0.005	0.004	0.004	0.005	0.004
20	0.000	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.000	0.004
25	0.004	0.003	0.004	0.004	0.004	0.004	0.004	0.002	0.004	0.004
30	0.002	0.002	0.001	0.001	0.003	0.002	0.002	0.002	0.001	0.003
35	0.003	0.001	0.003	0.004	0.002	0.000	0.003	0.003	0.003	0.004
40	0.001	0.000	0.004	0.001	0.001	0.000	0.001	0.001	0.000	0.001
45	0.005	0.004	0.006	0.005	0.006	0.005	0.005	0.006	0.005	0.008
50	0.006	0.008	0.005	0.007	0.002	0.008	0.006	0.004	0.005	0.006

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.003	0.003	0.003	0.004	0.003	0.004	0.003	0.003	0.003
10	0.001	0.002	0.002	0.002	0.003	0.004	0.002	0.002	0.004	0.002
15	0.002	0.003	0.000	0.004	0.005	0.003	0.002	0.002	0.001	0.001
20	0.003	0.001	0.002	0.001	0.000	0.002	0.001	0.002	0.003	0.003
25	0.001	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.002
30	0.000	0.002	0.000	0.005	0.002	0.001	0.001	0.000	0.000	0.000
35	0.002	0.003	0.001	0.001	0.003	0.004	0.003	0.002	0.001	0.002
40	0.001	0.002	0.002	0.004	0.002	0.002	0.001	0.001	0.001	0.001
45	0.004	0.004	0.006	0.005	0.004	0.005	0.004	0.004	0.005	0.006
50	0.003	0.003	0.004	0.003	0.002	0.003	0.001	0.005	0.004	0.002

Table B.11 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 1.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.458	1.434	1.412	1.392	1.373	1.355	1.338	1.320	1.301	1.285
10	2.152	2.025	1.940	1.874	1.816	1.765	1.718	1.678	1.641	1.607
15	2.357	2.184	2.064	1.976	1.909	1.846	1.795	1.751	1.710	1.672
20	2.445	2.229	2.103	2.016	1.943	1.886	1.835	1.790	1.747	1.710
25	2.422	2.227	2.094	2.009	1.935	1.880	1.832	1.785	1.748	1.709
30	2.448	2.238	2.101	2.007	1.936	1.887	1.828	1.784	1.746	1.711
35	2.416	2.205	2.079	1.989	1.917	1.859	1.808	1.764	1.726	1.696
40	2.384	2.181	2.058	1.970	1.905	1.852	1.803	1.760	1.722	1.691
45	2.363	2.160	2.035	1.947	1.880	1.825	1.781	1.739	1.706	1.674
50	2.338	2.131	2.007	1.924	1.861	1.809	1.760	1.721	1.687	1.656

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.268	1.253	1.238	1.221	1.206	1.193	1.177	1.163	1.149	1.136
10	1.575	1.542	1.512	1.487	1.462	1.438	1.415	1.393	1.372	1.354
15	1.640	1.608	1.578	1.552	1.527	1.504	1.481	1.460	1.439	1.420
20	1.676	1.647	1.616	1.591	1.566	1.542	1.521	1.498	1.479	1.461
25	1.675	1.646	1.617	1.590	1.566	1.544	1.522	1.501	1.480	1.461
30	1.679	1.648	1.622	1.597	1.574	1.550	1.531	1.511	1.492	1.474
35	1.666	1.638	1.612	1.587	1.564	1.543	1.522	1.503	1.486	1.468
40	1.663	1.638	1.611	1.586	1.564	1.543	1.524	1.505	1.487	1.470
45	1.645	1.618	1.597	1.575	1.555	1.535	1.517	1.498	1.482	1.466
50	1.629	1.603	1.580	1.558	1.538	1.520	1.502	1.486	1.470	1.454

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	1.108	1.080	1.054	1.026	0.999	0.973	0.948	0.923	0.899	0.873
10	1.315	1.279	1.246	1.216	1.186	1.159	1.133	1.108	1.084	1.060
15	1.382	1.348	1.317	1.287	1.259	1.233	1.207	1.185	1.161	1.139
20	1.424	1.391	1.359	1.329	1.301	1.274	1.251	1.228	1.207	1.186
25	1.426	1.394	1.365	1.337	1.311	1.287	1.265	1.243	1.223	1.203
30	1.441	1.411	1.381	1.353	1.328	1.304	1.282	1.260	1.241	1.222
35	1.435	1.408	1.379	1.354	1.330	1.308	1.286	1.266	1.246	1.228
40	1.438	1.408	1.382	1.357	1.333	1.311	1.290	1.271	1.252	1.235
45	1.436	1.407	1.381	1.356	1.333	1.313	1.292	1.274	1.256	1.239
50	1.425	1.399	1.375	1.353	1.332	1.312	1.293	1.276	1.259	1.243

Table B.12 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 1.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.001	0.002	0.004	0.002	0.006	0.004	0.003	0.001	0.002	0.003
10	0.002	0.000	0.001	0.004	0.003	0.002	0.003	0.004	0.002	0.003
15	0.000	0.003	0.003	0.000	0.001	0.000	0.002	0.000	0.001	0.002
20	0.004	0.002	0.005	0.004	0.001	0.004	0.004	0.003	0.003	0.005
25	0.003	0.003	0.002	0.003	0.003	0.003	0.003	0.002	0.005	0.003
30	0.001	0.002	0.002	0.002	0.000	0.001	0.002	0.001	0.001	0.005
35	0.002	0.004	0.003	0.003	0.004	0.002	0.003	0.004	0.002	0.003
40	0.007	0.003	0.008	0.009	0.008	0.011	0.007	0.003	0.006	0.008
45	0.005	0.005	0.010	0.004	0.006	0.004	0.005	0.002	0.005	0.004
50	0.010	0.009	0.008	0.007	0.011	0.008	0.009	0.005	0.011	0.010

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.002	0.000	0.001	0.000	0.002	0.001	0.001	0.002	0.002	0.003
10	0.002	0.003	0.002	0.002	0.003	0.001	0.003	0.002	0.003	0.004
15	0.001	0.000	0.002	0.002	0.001	0.001	0.002	0.000	0.000	0.001
20	0.003	0.002	0.002	0.002	0.004	0.001	0.002	0.004	0.002	0.004
25	0.004	0.002	0.002	0.004	0.002	0.002	0.001	0.002	0.001	0.000
30	0.003	0.003	0.003	0.000	0.003	0.002	0.002	0.003	0.003	0.002
35	0.005	0.005	0.002	0.002	0.001	0.003	0.003	0.002	0.002	0.000
40	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.002
45	0.005	0.004	0.005	0.003	0.004	0.005	0.005	0.004	0.003	0.003
50	0.006	0.007	0.004	0.006	0.004	0.008	0.005	0.006	0.005	0.005

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.002
10	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.002	0.002	0.001
15	0.005	0.003	0.004	0.003	0.002	0.003	0.002	0.005	0.003	0.002
20	0.003	0.002	0.003	0.004	0.004	0.003	0.004	0.003	0.003	0.004
25	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.006	0.005	0.005
30	0.003	0.002	0.003	0.001	0.001	0.001	0.002	0.003	0.002	0.002
35	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.004	0.004	0.002
40	0.005	0.005	0.004	0.006	0.006	0.005	0.007	0.006	0.007	0.006
45	0.004	0.004	0.002	0.004	0.003	0.004	0.004	0.004	0.003	0.005
50	0.003	0.006	0.003	0.005	0.005	0.005	0.005	0.004	0.006	0.006

Table B.13 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 2.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.334	-1.242	-0.169	-0.108	-0.153	-1.006	-0.960	-0.920	-0.882	-0.844
10	-1.041	-0.890	-0.796	-0.732	-0.680	-0.639	-0.602	-0.567	-0.538	-0.510
15	-0.739	-0.624	-0.555	-0.502	-0.463	-0.427	-0.396	-0.371	-0.348	-0.326
20	-0.568	-0.465	-0.403	-0.358	-0.322	-0.292	-0.267	-0.243	-0.221	-0.201
25	-0.447	-0.357	-0.307	-0.266	-0.234	-0.206	-0.180	-0.160	-0.141	-0.123
30	-0.355	-0.275	-0.227	-0.191	-0.164	-0.138	-0.115	-0.096	-0.078	-0.063
35	-0.278	-0.205	-0.161	-0.129	-0.098	-0.076	-0.057	-0.038	-0.023	-0.008
40	-0.232	-0.163	-0.121	-0.089	-0.065	-0.044	-0.024	-0.007	0.008	0.023
45	-0.182	-0.117	-0.076	-0.045	-0.021	-0.001	0.016	0.032	0.047	0.060
50	-0.142	-0.083	-0.043	-0.014	0.011	0.030	0.046	0.063	0.077	0.090

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.811	-0.780	-0.751	-0.724	-0.695	-0.671	-0.646	-0.625	-0.602	-0.583
10	-0.486	-0.463	-0.443	-0.423	-0.403	-0.385	-0.369	-0.353	-0.337	-0.321
15	-0.305	-0.285	-0.267	-0.252	-0.235	-0.220	-0.206	-0.192	-0.179	-0.167
20	-0.183	-0.166	-0.151	-0.137	-0.122	-0.110	-0.098	-0.086	-0.074	-0.063
25	-0.106	-0.091	-0.076	-0.063	-0.049	-0.037	-0.026	-0.015	-0.005	0.005
30	-0.048	-0.034	-0.021	-0.009	0.002	0.014	0.025	0.035	0.044	0.055
35	0.006	0.019	0.031	0.042	0.052	0.063	0.073	0.083	0.091	0.100
40	0.037	0.048	0.060	0.071	0.082	0.092	0.102	0.111	0.119	0.128
45	0.072	0.084	0.094	0.105	0.115	0.125	0.134	0.143	0.151	0.159
50	0.103	0.113	0.123	0.133	0.143	0.151	0.160	0.169	0.176	0.183

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.546	-0.511	-0.481	-0.453	-0.427	-0.404	-0.384	-0.364	-0.344	-0.325
10	-0.293	-0.266	-0.242	-0.218	-0.195	-0.173	-0.153	-0.132	-0.113	-0.096
15	-0.143	-0.120	-0.099	-0.080	-0.060	-0.041	-0.023	-0.006	0.010	0.026
20	-0.043	-0.023	-0.004	0.014	0.031	0.047	0.062	0.078	0.092	0.106
25	0.024	0.041	0.058	0.074	0.090	0.104	0.119	0.133	0.147	0.159
30	0.073	0.089	0.106	0.121	0.136	0.150	0.163	0.176	0.189	0.201
35	0.117	0.132	0.147	0.161	0.174	0.186	0.199	0.211	0.223	0.235
40	0.144	0.159	0.173	0.187	0.199	0.212	0.224	0.235	0.246	0.257
45	0.174	0.188	0.202	0.215	0.227	0.239	0.250	0.260	0.270	0.281
50	0.198	0.212	0.224	0.237	0.248	0.259	0.270	0.281	0.292	0.301

Table B.14 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 2.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.000	0.002	0.000	0.001	0.002	0.000	0.001	0.000	0.000
10	0.001	0.002	0.001	0.002	0.000	0.001	0.002	0.002	0.002	0.002
15	0.000	0.003	0.004	0.003	0.002	0.000	0.001	0.003	0.003	0.002
20	0.003	0.003	0.001	0.002	0.005	0.001	0.002	0.002	0.003	0.001
25	0.002	0.003	0.002	0.002	0.003	0.002	0.004	0.004	0.001	0.003
30	0.004	0.002	0.002	0.005	0.003	0.001	0.001	0.002	0.002	0.003
35	0.006	0.003	0.000	0.002	0.002	0.003	0.003	0.001	0.004	0.002
40	0.002	0.002	0.003	0.003	0.004	0.002	0.002	0.003	0.002	0.001
45	0.005	0.004	0.003	0.003	0.004	0.006	0.004	0.004	0.003	0.008
50	0.008	0.006	0.007	0.009	0.006	0.011	0.008	0.009	0.008	0.012

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.002	0.002	0.000	0.003	0.001	0.000	0.001	0.001	0.000	0.003
10	0.000	0.003	0.001	0.002	0.000	0.002	0.000	0.002	0.002	0.001
15	0.001	0.001	0.002	0.004	0.001	0.001	0.002	0.000	0.001	0.002
20	0.003	0.002	0.003	0.003	0.004	0.003	0.004	0.003	0.003	0.002
25	0.004	0.006	0.005	0.004	0.003	0.004	0.002	0.003	0.004	0.005
30	0.002	0.002	0.004	0.003	0.004	0.003	0.004	0.003	0.002	0.002
35	0.003	0.003	0.002	0.005	0.003	0.002	0.003	0.005	0.003	0.003
40	0.005	0.005	0.004	0.003	0.004	0.002	0.004	0.003	0.004	0.000
45	0.004	0.004	0.003	0.003	0.003	0.004	0.004	0.005	0.003	0.003
50	0.007	0.008	0.006	0.007	0.005	0.006	0.006	0.006	0.009	0.006

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.000	0.003	0.002	0.002	0.000	0.001	0.000	0.000	0.001
10	0.000	0.000	0.002	0.001	0.000	0.001	0.002	0.001	0.002	0.002
15	0.002	0.001	0.000	0.001	0.001	0.002	0.001	0.002	0.001	0.000
20	0.005	0.003	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.003
25	0.002	0.003	0.002	0.002	0.002	0.001	0.002	0.001	0.004	0.000
30	0.003	0.001	0.003	0.003	0.004	0.001	0.003	0.004	0.002	0.002
35	0.001	0.002	0.004	0.002	0.002	0.002	0.003	0.002	0.002	0.003
40	0.002	0.004	0.003	0.001	0.004	0.001	0.005	0.003	0.003	0.002
45	0.004	0.002	0.004	0.002	0.002	0.003	0.002	0.002	0.004	0.001
50	0.006	0.007	0.006	0.005	0.006	0.009	0.006	0.007	0.006	0.005

Table B.15 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 2.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.434	1.398	1.365	1.335	1.307	1.282	1.258	1.236	1.214	1.190
10	1.896	1.748	1.651	1.574	1.509	1.457	1.412	1.371	1.335	1.302
15	1.921	1.742	1.633	1.554	1.492	1.442	1.397	1.357	1.321	1.289
20	1.881	1.702	1.600	1.520	1.464	1.415	1.373	1.335	1.303	1.271
25	1.810	1.644	1.541	1.470	1.417	1.368	1.330	1.295	1.263	1.234
30	1.766	1.614	1.518	1.441	1.387	1.340	1.304	1.269	1.239	1.213
35	1.731	1.570	1.475	1.405	1.349	1.309	1.271	1.238	1.210	1.188
40	1.668	1.522	1.427	1.364	1.317	1.279	1.245	1.215	1.190	1.166
45	1.631	1.482	1.398	1.339	1.292	1.254	1.221	1.192	1.168	1.147
50	1.592	1.452	1.368	1.310	1.266	1.230	1.197	1.171	1.147	1.124

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.169	1.148	1.128	1.110	1.091	1.073	1.054	1.037	1.019	1.003
10	1.271	1.243	1.218	1.195	1.173	1.151	1.130	1.108	1.089	1.071
15	1.259	1.232	1.205	1.181	1.159	1.138	1.118	1.099	1.082	1.064
20	1.244	1.219	1.195	1.171	1.148	1.129	1.111	1.093	1.076	1.061
25	1.208	1.184	1.163	1.142	1.122	1.104	1.086	1.070	1.053	1.038
30	1.190	1.167	1.146	1.128	1.110	1.092	1.076	1.060	1.045	1.031
35	1.163	1.143	1.124	1.105	1.087	1.071	1.056	1.042	1.028	1.014
40	1.145	1.125	1.106	1.088	1.073	1.059	1.044	1.030	1.016	1.003
45	1.126	1.107	1.090	1.073	1.057	1.041	1.028	1.014	1.002	0.990
50	1.104	1.085	1.068	1.052	1.038	1.024	1.011	0.999	0.987	0.976

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.971	0.939	0.907	0.879	0.851	0.821	0.793	0.767	0.740	0.715
10	1.035	1.002	0.971	0.944	0.915	0.891	0.866	0.844	0.821	0.799
15	1.033	1.004	0.975	0.950	0.927	0.902	0.880	0.860	0.839	0.820
20	1.031	1.002	0.977	0.953	0.930	0.909	0.889	0.869	0.851	0.834
25	1.010	0.984	0.959	0.937	0.916	0.895	0.877	0.860	0.842	0.826
30	1.003	0.978	0.956	0.936	0.916	0.898	0.880	0.864	0.846	0.830
35	0.988	0.966	0.945	0.924	0.906	0.887	0.870	0.854	0.840	0.825
40	0.980	0.958	0.938	0.919	0.901	0.883	0.868	0.852	0.837	0.822
45	0.967	0.946	0.926	0.908	0.891	0.875	0.861	0.846	0.832	0.819
50	0.954	0.935	0.917	0.900	0.885	0.870	0.855	0.841	0.827	0.815

Table B.16 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 2.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.001	0.002	0.004	0.000	0.000	0.000	0.004	0.002	0.003	0.004
10	0.000	0.000	0.002	0.003	0.003	0.001	0.000	0.001	0.001	0.003
15	0.002	0.001	0.001	0.001	0.001	0.003	0.002	0.002	0.002	0.001
20	0.003	0.005	0.004	0.005	0.004	0.003	0.004	0.003	0.004	0.005
25	0.004	0.003	0.002	0.002	0.001	0.002	0.002	0.002	0.002	0.001
30	0.001	0.005	0.002	0.002	0.007	0.003	0.004	0.001	0.003	0.004
35	0.005	0.006	0.005	0.004	0.003	0.004	0.001	0.003	0.003	0.006
40	0.004	0.007	0.004	0.005	0.005	0.004	0.006	0.004	0.008	0.004
45	0.008	0.005	0.009	0.006	0.006	0.005	0.008	0.005	0.007	0.006
50	0.012	0.010	0.008	0.007	0.007	0.006	0.009	0.005	0.006	0.005

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.001	0.000	0.001	0.002	0.001	0.001	0.002	0.000	0.001
10	0.000	0.001	0.002	0.002	0.001	0.002	0.002	0.001	0.001	0.003
15	0.002	0.003	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.000
20	0.000	0.003	0.003	0.001	0.001	0.001	0.002	0.002	0.003	0.001
25	0.004	0.005	0.002	0.004	0.005	0.004	0.005	0.005	0.004	0.002
30	0.002	0.003	0.003	0.002	0.002	0.003	0.005	0.002	0.003	0.004
35	0.004	0.004	0.003	0.003	0.004	0.003	0.004	0.004	0.003	0.001
40	0.002	0.003	0.002	0.004	0.000	0.004	0.003	0.004	0.004	0.004
45	0.006	0.005	0.003	0.004	0.005	0.006	0.007	0.006	0.005	0.004
50	0.005	0.006	0.005	0.006	0.008	0.005	0.006	0.006	0.008	0.004

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.001	0.002	0.000	0.002	0.001	0.000	0.001	0.002	0.001	0.003
10	0.002	0.000	0.001	0.002	0.002	0.001	0.002	0.001	0.002	0.001
15	0.001	0.002	0.002	0.002	0.001	0.001	0.002	0.000	0.001	0.001
20	0.000	0.003	0.002	0.004	0.003	0.002	0.003	0.002	0.002	0.002
25	0.000	0.002	0.003	0.003	0.000	0.002	0.001	0.001	0.001	0.004
30	0.003	0.002	0.003	0.001	0.002	0.001	0.003	0.003	0.000	0.003
35	0.001	0.003	0.004	0.002	0.000	0.003	0.003	0.004	0.002	0.004
40	0.003	0.002	0.002	0.003	0.003	0.002	0.004	0.004	0.000	0.002
45	0.003	0.004	0.004	0.003	0.004	0.005	0.005	0.003	0.003	0.004
50	0.004	0.006	0.004	0.005	0.006	0.007	0.004	0.007	0.004	0.008

Table B.17 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 2.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.363	-1.285	-1.223	-1.171	-1.122	-1.080	-1.039	-1.002	-0.967	-0.934
10	-1.223	-1.066	-0.968	-0.901	-0.850	-0.805	-0.766	-0.729	-0.700	-0.673
15	-0.941	-0.819	-0.749	-0.693	-0.649	-0.612	-0.579	-0.550	-0.527	-0.505
20	-0.773	-0.670	-0.603	-0.559	-0.520	-0.487	-0.460	-0.436	-0.414	-0.394
25	-0.659	-0.566	-0.506	-0.465	-0.431	-0.402	-0.379	-0.357	-0.338	-0.321
30	-0.567	-0.481	-0.430	-0.395	-0.364	-0.340	-0.318	-0.299	-0.281	-0.266
35	-0.490	-0.415	-0.370	-0.334	-0.309	-0.286	-0.264	-0.245	-0.229	-0.214
40	-0.441	-0.374	-0.331	-0.299	-0.275	-0.251	-0.232	-0.216	-0.200	-0.186
45	-0.395	-0.328	-0.286	-0.257	-0.232	-0.212	-0.193	-0.178	-0.164	-0.152
50	-0.356	-0.294	-0.255	-0.225	-0.203	-0.182	-0.166	-0.151	-0.136	-0.123

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.903	-0.872	-0.846	-0.818	-0.793	-0.768	-0.746	-0.724	-0.703	-0.682
10	-0.646	-0.621	-0.599	-0.580	-0.560	-0.541	-0.523	-0.505	-0.490	-0.475
15	-0.485	-0.466	-0.446	-0.428	-0.412	-0.397	-0.382	-0.368	-0.355	-0.342
20	-0.374	-0.358	-0.342	-0.327	-0.313	-0.300	-0.286	-0.274	-0.263	-0.251
25	-0.304	-0.288	-0.274	-0.260	-0.246	-0.234	-0.221	-0.210	-0.200	-0.191
30	-0.250	-0.236	-0.223	-0.211	-0.198	-0.186	-0.175	-0.165	-0.155	-0.146
35	-0.200	-0.188	-0.176	-0.165	-0.154	-0.144	-0.134	-0.125	-0.116	-0.107
40	-0.172	-0.160	-0.148	-0.137	-0.127	-0.117	-0.108	-0.098	-0.090	-0.082
45	-0.140	-0.127	-0.116	-0.106	-0.095	-0.086	-0.077	-0.068	-0.060	-0.053
50	-0.111	-0.100	-0.090	-0.080	-0.072	-0.063	-0.054	-0.046	-0.039	-0.031

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.644	-0.609	-0.575	-0.544	-0.517	-0.489	-0.463	-0.440	-0.419	-0.401
10	-0.446	-0.418	-0.391	-0.368	-0.345	-0.325	-0.304	-0.284	-0.264	-0.245
15	-0.319	-0.295	-0.274	-0.253	-0.234	-0.216	-0.198	-0.180	-0.164	-0.148
20	-0.231	-0.211	-0.193	-0.175	-0.157	-0.141	-0.125	-0.110	-0.094	-0.081
25	-0.171	-0.153	-0.137	-0.122	-0.107	-0.092	-0.078	-0.064	-0.051	-0.038
30	-0.129	-0.112	-0.096	-0.081	-0.066	-0.053	-0.040	-0.027	-0.015	-0.003
35	-0.091	-0.075	-0.060	-0.046	-0.034	-0.021	-0.009	0.002	0.014	0.025
40	-0.066	-0.052	-0.037	-0.024	-0.011	0.001	0.013	0.024	0.035	0.045
45	-0.038	-0.025	-0.011	0.001	0.013	0.025	0.035	0.046	0.056	0.066
50	-0.017	-0.004	0.009	0.020	0.032	0.042	0.053	0.063	0.073	0.083

Table B.18 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 2.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.002	0.001	0.000	0.002	0.002	0.001	0.000	0.001	0.000	0.001
10	0.003	0.000	0.000	0.001	0.000	0.002	0.001	0.003	0.001	0.006
15	0.003	0.001	0.002	0.004	0.002	0.000	0.000	0.003	0.003	0.004
20	0.001	0.002	0.001	0.004	0.001	0.001	0.003	0.003	0.002	0.002
25	0.001	0.000	0.000	0.001	0.002	0.000	0.001	0.004	0.000	0.001
30	0.002	0.002	0.001	0.003	0.003	0.002	0.002	0.001	0.001	0.001
35	0.004	0.004	0.003	0.003	0.003	0.005	0.003	0.002	0.003	0.004
40	0.004	0.003	0.004	0.004	0.005	0.004	0.003	0.004	0.004	0.004
45	0.005	0.006	0.006	0.008	0.006	0.005	0.004	0.005	0.004	0.005
50	0.006	0.008	0.005	0.006	0.005	0.008	0.006	0.005	0.008	0.010

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.002	0.000	0.001	0.000	0.002	0.001	0.001	0.002	0.002	0.003
10	0.002	0.003	0.002	0.002	0.003	0.001	0.003	0.002	0.003	0.004
15	0.001	0.000	0.002	0.002	0.001	0.001	0.002	0.000	0.000	0.001
20	0.003	0.002	0.002	0.002	0.004	0.001	0.002	0.004	0.002	0.004
25	0.004	0.002	0.002	0.004	0.002	0.002	0.001	0.002	0.001	0.000
30	0.003	0.003	0.003	0.000	0.003	0.002	0.002	0.003	0.003	0.002
35	0.005	0.005	0.002	0.002	0.001	0.003	0.003	0.002	0.002	0.000
40	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.002
45	0.005	0.004	0.005	0.003	0.004	0.005	0.005	0.004	0.003	0.003
50	0.006	0.007	0.004	0.006	0.004	0.008	0.005	0.006	0.005	0.005

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.000	0.000	0.000	0.001	0.000	0.003	0.002	0.002	0.000	0.001
10	0.002	0.002	0.002	0.002	0.002	0.000	0.002	0.002	0.003	0.001
15	0.001	0.002	0.001	0.000	0.003	0.002	0.002	0.001	0.002	0.003
20	0.003	0.003	0.001	0.002	0.002	0.001	0.003	0.000	0.001	0.003
25	0.002	0.000	0.002	0.003	0.001	0.003	0.003	0.002	0.001	0.001
30	0.003	0.004	0.003	0.003	0.003	0.002	0.003	0.003	0.003	0.003
35	0.005	0.002	0.001	0.003	0.001	0.003	0.004	0.005	0.001	0.002
40	0.002	0.000	0.002	0.001	0.004	0.004	0.002	0.004	0.002	0.002
45	0.005	0.002	0.007	0.005	0.004	0.003	0.005	0.003	0.005	0.003
50	0.004	0.006	0.006	0.006	0.007	0.006	0.004	0.005	0.005	0.005

Table B.19 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 2.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.415	1.369	1.330	1.292	1.261	1.228	1.200	1.173	1.147	1.122
10	1.716	1.558	1.454	1.376	1.316	1.265	1.220	1.182	1.144	1.116
15	1.645	1.469	1.374	1.300	1.245	1.193	1.147	1.112	1.079	1.050
20	1.539	1.387	1.294	1.227	1.176	1.133	1.096	1.061	1.031	1.005
25	1.448	1.300	1.218	1.158	1.109	1.070	1.034	1.002	0.974	0.951
30	1.389	1.257	1.174	1.115	1.067	1.032	0.998	0.969	0.943	0.920
35	1.337	1.204	1.129	1.070	1.027	0.991	0.961	0.934	0.910	0.889
40	1.267	1.149	1.078	1.024	0.986	0.954	0.927	0.902	0.880	0.861
45	1.218	1.113	1.042	0.996	0.961	0.931	0.905	0.882	0.859	0.840
50	1.189	1.080	1.015	0.969	0.930	0.900	0.874	0.853	0.833	0.815

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.098	1.075	1.052	1.031	1.011	0.989	0.969	0.949	0.930	0.912
10	1.087	1.058	1.032	1.010	0.988	0.967	0.947	0.928	0.910	0.891
15	1.024	0.998	0.975	0.954	0.934	0.915	0.897	0.879	0.863	0.847
20	0.983	0.958	0.935	0.916	0.898	0.879	0.863	0.847	0.832	0.817
25	0.930	0.909	0.890	0.873	0.855	0.839	0.824	0.808	0.793	0.779
30	0.900	0.881	0.863	0.846	0.830	0.815	0.801	0.788	0.775	0.763
35	0.869	0.852	0.835	0.819	0.805	0.790	0.776	0.764	0.751	0.739
40	0.842	0.826	0.810	0.796	0.782	0.769	0.757	0.745	0.734	0.724
45	0.822	0.805	0.790	0.776	0.764	0.751	0.740	0.727	0.716	0.706
50	0.799	0.785	0.770	0.757	0.745	0.733	0.721	0.710	0.701	0.692

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.877	0.843	0.811	0.781	0.751	0.722	0.693	0.666	0.638	0.613
10	0.857	0.826	0.799	0.772	0.747	0.723	0.698	0.676	0.655	0.635
15	0.817	0.790	0.763	0.737	0.716	0.695	0.675	0.655	0.637	0.620
20	0.791	0.766	0.742	0.720	0.699	0.680	0.662	0.644	0.627	0.611
25	0.754	0.731	0.711	0.690	0.672	0.655	0.638	0.623	0.607	0.592
30	0.740	0.719	0.699	0.681	0.663	0.646	0.630	0.615	0.600	0.586
35	0.718	0.698	0.680	0.664	0.648	0.631	0.616	0.602	0.588	0.575
40	0.704	0.686	0.668	0.651	0.635	0.620	0.606	0.593	0.580	0.567
45	0.686	0.669	0.653	0.637	0.622	0.608	0.595	0.582	0.571	0.560
50	0.674	0.657	0.642	0.627	0.613	0.600	0.587	0.574	0.564	0.552

Table B.20 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 2.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.000	0.002	0.001	0.003	0.001	0.002	0.002	0.002	0.000
10	0.002	0.001	0.001	0.002	0.001	0.002	0.001	0.000	0.001	0.001
15	0.001	0.002	0.002	0.001	0.000	0.001	0.002	0.003	0.003	0.003
20	0.003	0.003	0.003	0.003	0.002	0.002	0.003	0.001	0.003	0.002
25	0.002	0.003	0.005	0.003	0.004	0.004	0.004	0.004	0.004	0.002
30	0.003	0.004	0.003	0.002	0.004	0.002	0.005	0.003	0.003	0.003
35	0.004	0.002	0.001	0.003	0.003	0.004	0.004	0.002	0.003	0.002
40	0.007	0.006	0.005	0.004	0.006	0.005	0.007	0.003	0.005	0.004
45	0.006	0.005	0.005	0.005	0.003	0.004	0.006	0.003	0.004	0.005
50	0.009	0.004	0.007	0.006	0.005	0.005	0.005	0.007	0.003	0.003

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.001
10	0.000	0.001	0.002	0.002	0.002	0.002	0.000	0.002	0.000	0.002
15	0.002	0.000	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003
20	0.003	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.004	0.002
25	0.002	0.000	0.001	0.003	0.002	0.001	0.004	0.001	0.003	0.001
30	0.003	0.002	0.002	0.002	0.001	0.004	0.002	0.002	0.004	0.004
35	0.002	0.003	0.004	0.003	0.002	0.002	0.002	0.004	0.002	0.002
40	0.002	0.001	0.001	0.001	0.000	0.001	0.000	0.001	0.002	0.001
45	0.005	0.006	0.004	0.004	0.004	0.003	0.004	0.003	0.005	0.003
50	0.006	0.004	0.005	0.004	0.005	0.005	0.004	0.005	0.006	0.005

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.001	0.001	0.003	0.003	0.002	0.000	0.000	0.002	0.002	0.002
10	0.003	0.002	0.004	0.002	0.000	0.002	0.002	0.002	0.001	0.002
15	0.000	0.002	0.001	0.002	0.002	0.000	0.001	0.001	0.001	0.002
20	0.003	0.003	0.003	0.001	0.000	0.002	0.001	0.002	0.003	0.001
25	0.004	0.002	0.003	0.002	0.001	0.001	0.002	0.002	0.001	0.002
30	0.003	0.002	0.003	0.002	0.004	0.003	0.003	0.002	0.002	0.002
35	0.001	0.002	0.002	0.001	0.003	0.001	0.003	0.002	0.002	0.003
40	0.000	0.001	0.002	0.004	0.002	0.002	0.001	0.001	0.001	0.002
45	0.005	0.005	0.004	0.004	0.003	0.003	0.003	0.004	0.004	0.004
50	0.004	0.002	0.003	0.003	0.002	0.004	0.001	0.005	0.005	0.005

Table B.21 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 3.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.382	-1.312	-1.256	-1.213	-1.167	-1.128	-1.090	-1.057	-1.023	-0.993
10	-1.368	-1.201	-1.106	-1.035	-0.980	-0.930	-0.891	-0.856	-0.824	-0.792
15	-1.108	-0.984	-0.903	-0.842	-0.797	-0.761	-0.727	-0.698	-0.670	-0.647
20	-0.952	-0.840	-0.769	-0.721	-0.679	-0.646	-0.615	-0.588	-0.565	-0.544
25	-0.835	-0.736	-0.671	-0.626	-0.592	-0.562	-0.537	-0.514	-0.494	-0.476
30	-0.739	-0.654	-0.599	-0.559	-0.527	-0.502	-0.479	-0.459	-0.441	-0.423
35	-0.670	-0.587	-0.539	-0.503	-0.473	-0.449	-0.427	-0.408	-0.393	-0.378
40	-0.615	-0.541	-0.498	-0.465	-0.439	-0.418	-0.398	-0.380	-0.363	-0.348
45	-0.568	-0.499	-0.455	-0.423	-0.400	-0.379	-0.360	-0.343	-0.328	-0.315
50	-0.527	-0.464	-0.424	-0.395	-0.372	-0.352	-0.332	-0.316	-0.302	-0.288

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-0.964	-0.937	-0.910	-0.885	-0.860	-0.837	-0.815	-0.794	-0.773	-0.753
10	-0.767	-0.743	-0.722	-0.700	-0.679	-0.661	-0.642	-0.625	-0.609	-0.592
15	-0.625	-0.606	-0.586	-0.567	-0.551	-0.535	-0.520	-0.505	-0.492	-0.478
20	-0.525	-0.507	-0.491	-0.475	-0.461	-0.449	-0.436	-0.422	-0.410	-0.399
25	-0.459	-0.442	-0.428	-0.415	-0.401	-0.389	-0.377	-0.365	-0.354	-0.343
30	-0.407	-0.394	-0.381	-0.368	-0.357	-0.344	-0.333	-0.322	-0.311	-0.301
35	-0.363	-0.351	-0.338	-0.326	-0.315	-0.304	-0.295	-0.285	-0.276	-0.266
40	-0.334	-0.321	-0.309	-0.298	-0.288	-0.279	-0.269	-0.260	-0.251	-0.243
45	-0.303	-0.291	-0.279	-0.269	-0.259	-0.249	-0.240	-0.232	-0.223	-0.216
50	-0.276	-0.264	-0.254	-0.245	-0.236	-0.227	-0.219	-0.211	-0.203	-0.195

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.713	-0.678	-0.645	-0.615	-0.585	-0.556	-0.529	-0.503	-0.481	-0.458
10	-0.561	-0.533	-0.507	-0.482	-0.458	-0.435	-0.414	-0.393	-0.375	-0.356
15	-0.453	-0.429	-0.406	-0.386	-0.366	-0.346	-0.328	-0.310	-0.294	-0.279
20	-0.376	-0.355	-0.336	-0.318	-0.301	-0.284	-0.267	-0.251	-0.236	-0.222
25	-0.324	-0.305	-0.287	-0.270	-0.255	-0.241	-0.226	-0.212	-0.199	-0.185
30	-0.283	-0.266	-0.250	-0.235	-0.221	-0.207	-0.193	-0.180	-0.168	-0.155
35	-0.250	-0.234	-0.219	-0.205	-0.192	-0.179	-0.166	-0.154	-0.143	-0.132
40	-0.227	-0.212	-0.197	-0.183	-0.171	-0.159	-0.146	-0.135	-0.125	-0.114
45	-0.201	-0.187	-0.173	-0.161	-0.149	-0.138	-0.127	-0.116	-0.106	-0.097
50	-0.181	-0.168	-0.155	-0.143	-0.132	-0.121	-0.111	-0.101	-0.090	-0.081

Table B.22 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 3.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.000	0.000	0.001
10	0.001	0.002	0.002	0.003	0.003	0.000	0.001	0.003	0.001	0.002
15	0.002	0.003	0.003	0.004	0.002	0.000	0.002	0.000	0.002	0.001
20	0.002	0.000	0.001	0.003	0.001	0.002	0.003	0.000	0.001	0.002
25	0.001	0.001	0.000	0.001	0.004	0.000	0.002	0.001	0.000	0.000
30	0.003	0.005	0.001	0.001	0.003	0.002	0.000	0.001	0.000	0.000
35	0.003	0.002	0.003	0.023	0.002	0.003	0.003	0.002	0.003	0.003
40	0.004	0.003	0.004	0.004	0.004	0.004	0.005	0.003	0.002	0.003
45	0.005	0.005	0.004	0.008	0.005	0.005	0.004	0.004	0.002	0.006
50	0.004	0.007	0.008	0.006	0.003	0.002	0.005	0.006	0.004	0.007

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.002	0.002	0.003	0.002	0.003	0.002	0.003	0.002	0.002	0.002
10	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
15	0.001	0.003	0.003	0.001	0.002	0.002	0.001	0.002	0.001	0.001
20	0.002	0.002	0.003	0.002	0.001	0.002	0.002	0.002	0.002	0.002
25	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.000	0.000	0.000
30	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.002	0.002
35	0.001	0.000	0.000	0.002	0.002	0.001	0.001	0.002	0.002	0.002
40	0.001	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
45	0.004	0.005	0.006	0.005	0.003	0.004	0.005	0.004	0.005	0.005
50	0.004	0.004	0.003	0.003	0.004	0.003	0.003	0.003	0.003	0.003

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.001	0.003	0.003	0.002	0.000	0.000	0.002	0.002	0.001
10	0.002	0.001	0.002	0.004	0.002	0.002	0.001	0.001	0.001	0.000
15	0.002	0.002	0.001	0.002	0.002	0.000	0.001	0.001	0.001	0.000
20	0.001	0.003	0.003	0.001	0.000	0.002	0.001	0.002	0.003	0.003
25	0.002	0.002	0.003	0.002	0.004	0.003	0.003	0.002	0.002	0.003
30	0.002	0.002	0.003	0.002	0.001	0.001	0.002	0.002	0.001	0.004
35	0.002	0.002	0.004	0.002	0.000	0.002	0.002	0.002	0.001	0.003
40	0.003	0.002	0.002	0.001	0.003	0.001	0.003	0.002	0.002	0.001
45	0.004	0.005	0.004	0.004	0.003	0.003	0.003	0.004	0.004	0.005
50	0.005	0.002	0.003	0.003	0.002	0.004	0.001	0.005	0.005	0.004

Table B.23 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 3.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.401	1.350	1.303	1.261	1.225	1.191	1.158	1.128	1.100	1.073
10	1.591	1.428	1.322	1.250	1.188	1.138	1.095	1.057	1.022	0.993
15	1.468	1.293	1.205	1.137	1.076	1.031	0.988	0.954	0.923	0.895
20	1.330	1.183	1.101	1.038	0.993	0.952	0.916	0.885	0.858	0.832
25	1.223	1.093	1.016	0.957	0.913	0.876	0.846	0.817	0.793	0.770
30	1.151	1.036	0.960	0.910	0.869	0.832	0.803	0.776	0.753	0.731
35	1.092	0.977	0.910	0.860	0.824	0.790	0.761	0.738	0.717	0.698
40	1.024	0.920	0.857	0.816	0.782	0.751	0.724	0.702	0.681	0.663
45	0.973	0.878	0.821	0.783	0.751	0.726	0.702	0.681	0.660	0.643
50	0.949	0.850	0.794	0.753	0.721	0.693	0.669	0.649	0.632	0.616

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.046	1.021	0.997	0.975	0.951	0.931	0.910	0.888	0.868	0.849
10	0.963	0.937	0.911	0.888	0.864	0.843	0.824	0.806	0.789	0.772
15	0.868	0.847	0.823	0.803	0.785	0.767	0.749	0.732	0.716	0.701
20	0.809	0.787	0.768	0.749	0.731	0.716	0.699	0.684	0.671	0.657
25	0.748	0.728	0.710	0.694	0.679	0.663	0.649	0.636	0.622	0.610
30	0.712	0.695	0.678	0.663	0.649	0.635	0.622	0.610	0.598	0.588
35	0.680	0.662	0.645	0.631	0.617	0.605	0.592	0.581	0.570	0.560
40	0.648	0.632	0.618	0.606	0.594	0.582	0.571	0.560	0.550	0.540
45	0.627	0.611	0.598	0.584	0.573	0.561	0.550	0.539	0.530	0.520
50	0.601	0.588	0.576	0.564	0.553	0.542	0.532	0.522	0.513	0.504

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.812	0.777	0.744	0.713	0.682	0.653	0.623	0.597	0.573	0.547
10	0.740	0.709	0.681	0.656	0.631	0.608	0.585	0.564	0.543	0.524
15	0.673	0.646	0.622	0.599	0.577	0.556	0.537	0.518	0.500	0.482
20	0.632	0.608	0.586	0.565	0.544	0.526	0.509	0.492	0.476	0.460
25	0.587	0.565	0.545	0.527	0.509	0.494	0.478	0.463	0.447	0.433
30	0.565	0.545	0.527	0.509	0.493	0.478	0.463	0.449	0.435	0.421
35	0.541	0.523	0.505	0.488	0.473	0.458	0.443	0.430	0.418	0.405
40	0.521	0.503	0.487	0.472	0.457	0.443	0.429	0.417	0.405	0.393
45	0.502	0.487	0.471	0.456	0.442	0.429	0.417	0.405	0.394	0.383
50	0.487	0.471	0.457	0.444	0.431	0.419	0.407	0.396	0.385	0.375

Table B.24 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: β = 3.0.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.001	0.001	0.002	0.001	0.001	0.002	0.002	0.000	0.000
10	0.001	0.000	0.002	0.001	0.003	0.002	0.000	0.000	0.001	0.001
15	0.003	0.000	0.004	0.002	0.002	0.002	0.004	0.002	0.001	0.002
20	0.004	0.002	0.002	0.000	0.003	0.001	0.001	0.001	0.002	0.001
25	0.001	0.000	0.001	0.001	0.004	0.000	0.001	0.002	0.000	0.000
30	0.002	0.002	0.001	0.003	0.001	0.001	0.000	0.001	0.001	0.001
35	0.001	0.002	0.004	0.004	0.003	0.000	0.003	0.002	0.003	0.002
40	0.004	0.006	0.002	0.008	0.0054	0.005	0.005	0.006	0.003	0.005
45	0.005	0.005	0.004	0.005	0.004	0.004	0.002	0.004	0.004	0.004
50	0.006	0.009	0.008	0.006	0.006	0.009	0.006	0.005	0.007	0.005

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.000	0.002	0.000	0.001	0.001	0.001	0.001	0.001	0.000
10	0.000	0.000	0.002	0.002	0.002	0.000	0.004	0.001	0.002	0.001
15	0.002	0.003	0.004	0.002	0.003	0.002	0.002	0.002	0.001	0.002
20	0.003	0.002	0.000	0.002	0.001	0.003	0.001	0.002	0.004	0.003
25	0.004	0.000	0.001	0.003	0.002	0.003	0.002	0.000	0.003	0.002
30	0.003	0.002	0.002	0.002	0.004	0.003	0.003	0.003	0.002	0.001
35	0.002	0.003	0.001	0.000	0.002	0.003	0.002	0.000	0.001	0.001
40	0.003	0.004	0.003	0.003	0.004	0.005	0.004	0.006	0.005	0.003
45	0.002	0.003	0.001	0.001	0.002	0.002	0.003	0.002	0.001	0.003
50	0.005	0.003	0.006	0.005	0.006	0.005	0.007	0.004	0.005	0.005

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.000
10	0.002	0.002	0.003	0.002	0.002	0.002	0.004	0.000	0.000	0.002
15	0.000	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.001
20	0.001	0.000	0.002	0.002	0.002	0.001	0.002	0.001	0.002	0.002
25	0.002	0.002	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.002
30	0.004	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.003	0.002
35	0.003	0.002	0.003	0.002	0.002	0.001	0.000	0.002	0.002	0.002
40	0.002	0.001	0.003	0.001	0.001	0.002	0.003	0.001	0.001	0.001
45	0.003	0.005	0.004	0.004	0.004	0.003	0.004	0.004	0.005	0.005
50	0.002	0.004	0.003	0.005	0.003	0.003	0.004	0.003	0.003	0.003

Table B.25 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 3.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.394	-1.332	-1.280	-1.239	-1.198	-1.161	-1.128	-1.095	-1.063	-1.035
10	-1.481	-1.318	-1.214	-1.140	-1.081	-1.034	-0.993	-0.955	-0.920	-0.891
15	-1.255	-1.108	-1.029	-0.970	-0.921	-0.882	-0.848	-0.818	-0.789	-0.764
20	-1.106	-0.984	-0.909	-0.850	-0.810	-0.774	-0.744	-0.715	-0.690	-0.668
25	-0.986	-0.879	-0.810	-0.762	-0.727	-0.696	-0.667	-0.643	-0.622	-0.603
30	-0.887	-0.799	-0.738	-0.698	-0.665	-0.635	-0.612	-0.591	-0.572	-0.555
35	-0.818	-0.735	-0.683	-0.644	-0.611	-0.586	-0.566	-0.546	-0.528	-0.511
40	-0.763	-0.685	-0.639	-0.605	-0.577	-0.553	-0.534	-0.515	-0.497	-0.480
45	-0.721	-0.642	-0.597	-0.564	-0.539	-0.517	-0.498	-0.480	-0.463	-0.449
50	-0.674	-0.609	-0.567	-0.535	-0.511	-0.489	-0.470	-0.452	-0.437	-0.423

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-1.008	-0.982	-0.957	-0.932	-0.909	-0.886	-0.866	-0.844	-0.825	-0.804
10	-0.864	-0.840	-0.817	-0.795	-0.774	-0.754	-0.734	-0.716	-0.699	-0.682
15	-0.739	-0.718	-0.698	-0.679	-0.661	-0.644	-0.629	-0.614	-0.599	-0.585
20	-0.649	-0.631	-0.613	-0.597	-0.582	-0.566	-0.553	-0.541	-0.528	-0.516
25	-0.586	-0.569	-0.554	-0.539	-0.526	-0.512	-0.500	-0.488	-0.476	-0.466
30	-0.539	-0.523	-0.508	-0.496	-0.483	-0.471	-0.460	-0.448	-0.439	-0.427
35	-0.496	-0.482	-0.468	-0.457	-0.445	-0.434	-0.424	-0.413	-0.404	-0.395
40	-0.466	-0.454	-0.441	-0.430	-0.419	-0.409	-0.399	-0.389	-0.381	-0.372
45	-0.435	-0.423	-0.412	-0.402	-0.392	-0.382	-0.372	-0.362	-0.353	-0.346
50	-0.410	-0.399	-0.389	-0.378	-0.368	-0.359	-0.351	-0.342	-0.334	-0.326

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.766	-0.730	-0.697	-0.667	-0.635	-0.608	-0.579	-0.554	-0.529	-0.506
10	-0.652	-0.623	-0.595	-0.570	-0.546	-0.522	-0.500	-0.480	-0.459	-0.440
15	-0.559	-0.534	-0.511	-0.489	-0.469	-0.450	-0.432	-0.413	-0.396	-0.379
20	-0.492	-0.470	-0.450	-0.430	-0.412	-0.395	-0.379	-0.363	-0.347	-0.332
25	-0.446	-0.426	-0.407	-0.390	-0.374	-0.358	-0.344	-0.330	-0.316	-0.302
30	-0.408	-0.390	-0.373	-0.357	-0.343	-0.328	-0.314	-0.301	-0.288	-0.276
35	-0.377	-0.361	-0.345	-0.331	-0.317	-0.303	-0.291	-0.279	-0.267	-0.255
40	-0.355	-0.339	-0.325	-0.311	-0.298	-0.285	-0.273	-0.261	-0.250	-0.239
45	-0.331	-0.316	-0.302	-0.290	-0.278	-0.266	-0.255	-0.244	-0.234	-0.224
50	-0.312	-0.298	-0.285	-0.273	-0.261	-0.251	-0.240	-0.228	-0.219	-0.209

Table B.26 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 3.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001
10	0.000	0.003	0.001	0.000	0.002	0.001	0.001	0.003	0.002	0.001
15	0.002	0.000	0.002	0.000	0.000	0.001	0.001	0.000	0.002	0.001
20	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
25	0.001	0.002	0.000	0.000	0.001	0.002	0.002	0.000	0.000	0.002
30	0.002	0.003	0.002	0.003	0.002	0.003	0.003	0.002	0.004	0.003
35	0.004	0.002	0.003	0.002	0.001	0.002	0.002	0.003	0.003	0.004
40	0.003	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.003
45	0.005	0.004	0.003	0.006	0.004	0.004	0.004	0.004	0.004	0.004
50	0.006	0.010	0.005	0.005	0.005	0.005	0.005	0.005	0.006	0.005

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.001	0.002	0.000	0.002	0.002	0.003	0.000	0.002	0.001	0.003
10	0.003	0.001	0.002	0.002	0.000	0.004	0.002	0.002	0.002	0.002
15	0.000	0.001	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.004
20	0.003	0.003	0.002	0.001	0.000	0.003	0.001	0.002	0.003	0.001
25	0.003	0.002	0.003	0.002	0.004	0.003	0.003	0.002	0.002	0.002
30	0.004	0.001	0.001	0.002	0.001	0.003	0.002	0.002	0.002	0.002
35	0.001	0.002	0.001	0.003	0.003	0.002	0.003	0.002	0.002	0.001
40	0.000	0.001	0.000	0.002	0.002	0.001	0.001	0.001	0.002	0.002
45	0.004	0.005	0.004	0.005	0.002	0.003	0.001	0.005	0.002	0.003
50	0.005	0.004	0.003	0.004	0.003	0.004	0.003	0.004	0.005	0.004

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.000	0.002	0.002	0.003	0.002	0.003	0.000	0.002	0.002
10	0.003	0.003	0.003	0.003	0.002	0.001	0.002	0.002	0.000	0.000
15	0.001	0.000	0.001	0.001	0.003	0.002	0.001	0.000	0.001	0.001
20	0.002	0.002	0.003	0.002	0.001	0.004	0.002	0.002	0.002	0.000
25	0.001	0.000	0.001	0.002	0.000	0.001	0.002	0.000	0.000	0.000
30	0.003	0.002	0.002	0.003	0.00	0.002	0.002	0.003	0.002	0.002
35	0.002	0.002	0.000	0.002	0.002	0.002	0.001	0.000	0.002	0.002
40	0.001	0.002	0.002	0.003	0.001	0.001	0.002	0.001	0.001	0.001
45	0.004	0.006	0.005	0.005	0.005	0.004	0.004	0.005	0.004	0.004
50	0.005	0.004	0.003	0.003	0.004	0.003	0.003	0.002	0.003	0.002

Table B.27 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 3.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.391	1.335	1.281	1.237	1.197	1.160	1.126	1.094	1.064	1.035
10	1.496	1.337	1.231	1.158	1.100	1.051	1.007	0.970	0.936	0.907
15	1.340	1.177	1.088	1.022	0.964	0.918	0.879	0.845	0.816	0.790
20	1.185	1.048	0.970	0.912	0.868	0.829	0.795	0.764	0.736	0.711
25	1.069	0.949	0.876	0.825	0.781	0.746	0.717	0.691	0.665	0.644
30	0.998	0.888	0.820	0.772	0.734	0.700	0.671	0.644	0.621	0.601
35	0.931	0.828	0.764	0.719	0.683	0.653	0.625	0.603	0.582	0.563
40	0.862	0.766	0.712	0.670	0.640	0.612	0.588	0.566	0.547	0.529
45	0.810	0.724	0.674	0.637	0.608	0.583	0.561	0.540	0.522	0.506
50	0.790	0.700	0.647	0.608	0.576	0.550	0.529	0.510	0.494	0.479

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	1.009	0.981	0.956	0.933	0.909	0.886	0.866	0.844	0.823	0.804
10	0.878	0.851	0.827	0.801	0.780	0.759	0.740	0.722	0.705	0.688
15	0.766	0.740	0.719	0.700	0.681	0.663	0.646	0.631	0.614	0.598
20	0.690	0.670	0.651	0.633	0.617	0.602	0.587	0.573	0.558	0.545
25	0.623	0.604	0.587	0.571	0.555	0.541	0.528	0.515	0.503	0.492
30	0.582	0.565	0.550	0.536	0.522	0.509	0.497	0.486	0.475	0.464
35	0.546	0.530	0.516	0.503	0.489	0.477	0.465	0.454	0.444	0.433
40	0.513	0.498	0.486	0.475	0.463	0.451	0.440	0.430	0.420	0.411
45	0.492	0.476	0.463	0.450	0.438	0.428	0.417	0.407	0.398	0.389
50	0.465	0.452	0.439	0.428	0.418	0.407	0.398	0.389	0.380	0.372

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.765	0.730	0.697	0.663	0.633	0.604	0.575	0.551	0.526	0.502
10	0.656	0.627	0.600	0.574	0.550	0.527	0.505	0.484	0.465	0.444
15	0.571	0.547	0.524	0.501	0.478	0.459	0.439	0.420	0.402	0.386
20	0.520	0.497	0.476	0.455	0.436	0.417	0.400	0.383	0.367	0.352
25	0.470	0.448	0.430	0.411	0.395	0.379	0.363	0.348	0.332	0.319
30	0.444	0.425	0.406	0.388	0.372	0.358	0.343	0.328	0.314	0.301
35	0.415	0.397	0.380	0.364	0.348	0.335	0.321	0.308	0.296	0.284
40	0.393	0.376	0.361	0.345	0.331	0.317	0.304	0.291	0.279	0.268
45	0.374	0.357	0.342	0.328	0.314	0.301	0.289	0.277	0.266	0.256
50	0.355	0.340	0.327	0.314	0.301	0.289	0.278	0.267	0.256	0.246

Table B.28 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 3.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.001	0.001	0.001	0.002	0.001	0.000	0.001	0.001	0.002
10	0.001	0.001	0.002	0.002	0.001	0.002	0.002	0.003	0.000	0.001
15	0.003	0.003	0.002	0.001	0.001	0.001	0.003	0.001	0.000	0.002
20	0.004	0.003	0.003	0.004	0.000	0.004	0.002	0.004	0.002	0.004
25	0.004	0.005	0.005	0.004	0.005	0.004	0.002	0.002	0.004	0.005
30	0.004	0.003	0.003	0.003	0.003	0.003	0.002	0.000	0.002	0.003
35	0.003	0.004	0.004	0.003	0.004	0.003	0.003	0.001	0.004	0.004
40	0.003	0.003	0.005	0.002	0.002	0.003	0.003	0.004	0.002	0.002
45	0.005	0.005	0.007	0.004	0.005	0.006	0.003	0.004	0.006	0.006
50	0.008	0.006	0.006	0.006	0.008	0.005	0.005	0.004	0.005	0.006

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.001
10	0.001	0.002	0.001	0.000	0.002	0.000	0.000	0.002	0.002	0.000
15	0.002	0.000	0.000	0.000	0.001	0.002	0.002	0.001	0.001	0.000
20	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.002	0.002
25	0.000	0.003	0.002	0.000	0.001	0.001	0.002	0.001	0.001	0.003
30	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.002
35	0.004	0.002	0.001	0.002	0.002	0.003	0.003	0.004	0.002	0.003
40	0.002	0.003	0.002	0.003	0.003	0.002	0.004	0.003	0.003	0.002
45	0.004	0.008	0.005	0.004	0.005	0.005	0.006	0.005	0.005	0.005
50	0.005	0.004	0.004	0.006	0.004	0.003	0.004	0.004	0.004	0.004

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.002	0.001	0.002	0.001	0.001	0.001	0.000	0.001	0.001	0.001
10	0.004	0.002	0.003	0.002	0.000	0.002	0.002	0.000	0.002	0.002
15	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000
20	0.000	0.002	0.003	0.002	0.002	0.001	0.003	0.002	0.002	0.002
25	0.000	0.002	0.000	0.000	0.000	0.001	0.002	0.001	0.000	0.002
30	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.000
35	0.002	0.002	0.002	0.003	0.003	0.002	0.004	0.002	0.002	0.001
40	0.003	0.001	0.003	0.001	0.001	0.002	0.002	0.001	0.001	0.001
45	0.004	0.003	0.003	0.005	0.003	0.003	0.002	0.003	0.003	0.004
50	0.004	0.005	0.004	0.004	0.005	0.003	0.003	0.004	0.004	0.005

Table B.29 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Critical Values: $\beta = 4.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	-1.402	-1.347	-1.298	-1.258	-1.221	-1.186	-1.154	-1.123	-1.093	-1.065
10	-1.571	-1.408	-1.298	-1.226	-1.165	-1.118	-1.075	-1.033	-1.002	-0.969
15	-1.376	-1.221	-1.138	-1.078	-1.025	-0.982	-0.947	-0.914	-0.885	-0.858
20	-1.236	-1.105	-1.023	-0.967	-0.922	-0.884	-0.851	-0.823	-0.796	-0.773
25	-1.122	-1.005	-0.932	-0.880	-0.839	-0.806	-0.779	-0.753	-0.732	-0.711
30	-1.018	-0.924	-0.864	-0.818	-0.784	-0.751	-0.725	-0.702	-0.683	-0.665
35	-0.955	-0.864	-0.806	-0.765	-0.732	-0.706	-0.681	-0.659	-0.640	-0.623
40	-0.894	-0.812	-0.760	-0.722	-0.693	-0.670	-0.647	-0.628	-0.610	-0.594
45	-0.850	-0.768	-0.722	-0.687	-0.659	-0.634	-0.613	-0.595	-0.578	-0.562
50	-0.802	-0.734	-0.691	-0.655	-0.629	-0.606	-0.585	-0.567	-0.551	-0.536

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	-1.040	-1.016	-0.992	-0.968	-0.945	-0.924	-0.904	-0.883	-0.862	-0.843
10	-0.942	-0.917	-0.893	-0.872	-0.850	-0.830	-0.809	-0.789	-0.770	-0.754
15	-0.834	-0.812	-0.790	-0.770	-0.752	-0.734	-0.718	-0.702	-0.687	-0.673
20	-0.752	-0.732	-0.714	-0.696	-0.681	-0.666	-0.651	-0.637	-0.624	-0.611
25	-0.692	-0.675	-0.659	-0.643	-0.629	-0.616	-0.603	-0.591	-0.578	-0.567
30	-0.646	-0.630	-0.614	-0.601	-0.589	-0.576	-0.564	-0.553	-0.541	-0.531
35	-0.607	-0.592	-0.578	-0.566	-0.554	-0.542	-0.531	-0.520	-0.510	-0.501
40	-0.579	-0.564	-0.551	-0.539	-0.528	-0.517	-0.507	-0.497	-0.487	-0.477
45	-0.548	-0.535	-0.523	-0.512	-0.501	-0.490	-0.480	-0.471	-0.463	-0.453
50	-0.524	-0.512	-0.500	-0.489	-0.478	-0.469	-0.460	-0.451	-0.443	-0.435

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	-0.806	-0.771	-0.737	-0.706	-0.676	-0.647	-0.619	-0.594	-0.569	-0.545
10	-0.723	-0.695	-0.667	-0.640	-0.615	-0.590	-0.567	-0.547	-0.527	-0.507
15	-0.645	-0.620	-0.595	-0.573	-0.553	-0.533	-0.515	-0.496	-0.478	-0.460
20	-0.586	-0.564	-0.542	-0.523	-0.505	-0.487	-0.469	-0.453	-0.437	-0.422
25	-0.545	-0.525	-0.506	-0.488	-0.471	-0.455	-0.440	-0.425	-0.411	-0.396
30	-0.511	-0.493	-0.475	-0.458	-0.442	-0.428	-0.413	-0.399	-0.386	-0.373
35	-0.483	-0.465	-0.449	-0.434	-0.418	-0.405	-0.392	-0.380	-0.368	-0.356
40	-0.460	-0.444	-0.430	-0.415	-0.401	-0.388	-0.376	-0.363	-0.352	-0.341
45	-0.438	-0.423	-0.409	-0.395	-0.383	-0.371	-0.359	-0.348	-0.338	-0.327
50	-0.420	-0.405	-0.392	-0.379	-0.367	-0.356	-0.345	-0.334	-0.324	-0.314

Table B.30 Skewness $\left(\sqrt{b_1}\right)$ Lower Tail Standard Deviations: $\beta = 4.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.002	0.002	0.002	0.002	0.003	0.003	0.000	0.000	0.001	0.001
10	0.002	0.002	0.001	0.002	0.004	0.000	0.002	0.001	0.002	0.000
15	0.002	0.001	0.001	0.002	0.004	0.002	0.002	0.001	0.001	0.000
20	0.000	0.003	0.002	0.001	0.001	0.003	0.002	0.001	0.003	0.003
25	0.004	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.002	0.003
30	0.002	0.002	0.001	0.002	0.004	0.000	0.002	0.001	0.002	0.000
35	0.003	0.002	0.002	0.003	0.001	0.002	0.001	0.003	0.002	0.001
40	0.001	0.001	0.002	0.002	0.002	0.003	0.001	0.002	0.002	0.004
45	0.003	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.005	0.005
50	0.002	0.005	0.005	0.005	0.003	0.003	0.004	0.001	0.002	0.004

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.002	0.003	0.003	0.002	0.00	0.003	0.002	0.002	0.002	0.002
10	0.001	0.002	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.002
15	0.001	0.001	0.001	0.000	0.003	0.000	0.001	0.002	0.001	0.001
20	0.003	0.002	0.002	0.002	0.001	0.002	0.000	0.004	0.002	0.002
25	0.003	0.003	0.003	0.003	0.002	0.002	0.000	0.001	0.000	0.002
30	0.002	0.002	0.002	0.000	0.003	0.000	0.002	0.002	0.002	0.003
35	0.000	0.002	0.002	0.002	0.002	0.000	0.002	0.002	0.002	0.001
40	0.002	0.003	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.002
45	0.005	0.005	0.004	0.006	0.005	0.005	0.004	0.004	0.004	0.004
50	0.003	0.003	0.005	0.004	0.004	0.002	0.002	0.003	0.003	0.003

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.000	0.001	0.001	0.000	0.000	0.000	0.001	0.001	0.000	0.000
10	0.002	0.000	0.004	0.002	0.002	0.002	0.003	0.000	0.002	0.002
15	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.000
20	0.002	0.002	0.002	0.002	0.002	0.001	0.002	0.001	0.000	0.001
25	0.000	0.000	0.000	0.000	0.002	0.001	0.000	0.001	0.002	0.002
30	0.002	0.003	0.002	0.003	0.002	0.002	0.002	0.002	0.001	0.004
35	0.002	0.002	0.000	0.002	0.002	0.001	0.003	0.002	0.002	0.003
40	0.001	0.001	0.003	0.001	0.001	0.002	0.003	0.001	0.001	0.002
45	0.004	0.005	0.004	0.004	0.005	0.003	0.004	0.004	0.005	0.003
50	0.003	0.003	0.004	0.005	0.003	0.003	0.003	0.003	0.004	0.002

Table B.31 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Critical Values: $\beta = 4.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	1.382	1.323	1.266	1.218	1.176	1.138	1.101	1.068	1.037	1.008
10	1.430	1.267	1.165	1.092	1.033	0.986	0.943	0.909	0.874	0.843
15	1.243	1.098	1.001	0.934	0.883	0.841	0.799	0.766	0.738	0.713
20	1.083	0.953	0.879	0.823	0.779	0.742	0.708	0.676	0.648	0.626
25	0.973	0.849	0.780	0.726	0.689	0.655	0.624	0.597	0.574	0.553
30	0.891	0.784	0.720	0.673	0.636	0.603	0.573	0.549	0.528	0.507
35	0.820	0.721	0.663	0.619	0.584	0.554	0.527	0.505	0.485	0.466
40	0.752	0.663	0.609	0.567	0.537	0.511	0.487	0.467	0.449	0.431
45	0.698	0.619	0.569	0.534	0.505	0.481	0.458	0.439	0.423	0.405
50	0.678	0.592	0.541	0.504	0.474	0.449	0.429	0.410	0.394	0.378

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.978	0.952	0.925	0.900	0.877	0.853	0.831	0.808	0.789	0.770
10	0.814	0.787	0.763	0.740	0.719	0.699	0.681	0.662	0.644	0.626
15	0.688	0.665	0.644	0.624	0.606	0.589	0.573	0.554	0.539	0.524
20	0.605	0.585	0.566	0.548	0.533	0.517	0.503	0.489	0.476	0.463
25	0.533	0.515	0.498	0.483	0.468	0.453	0.440	0.427	0.416	0.404
30	0.490	0.472	0.457	0.443	0.430	0.417	0.406	0.394	0.383	0.372
35	0.450	0.435	0.421	0.408	0.394	0.382	0.371	0.361	0.350	0.340
40	0.416	0.403	0.389	0.378	0.366	0.354	0.344	0.334	0.325	0.316
45	0.391	0.377	0.365	0.352	0.340	0.330	0.321	0.312	0.302	0.293
50	0.364	0.351	0.339	0.328	0.318	0.309	0.300	0.291	0.282	0.274

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.730	0.695	0.661	0.628	0.596	0.567	0.540	0.515	0.492	0.469
10	0.595	0.567	0.538	0.514	0.490	0.467	0.445	0.426	0.405	0.386
15	0.497	0.473	0.450	0.427	0.406	0.386	0.367	0.348	0.331	0.314
20	0.438	0.416	0.394	0.374	0.354	0.337	0.320	0.303	0.287	0.271
25	0.382	0.363	0.343	0.326	0.310	0.293	0.277	0.263	0.248	0.235
30	0.352	0.334	0.317	0.301	0.284	0.268	0.253	0.239	0.226	0.213
35	0.322	0.305	0.288	0.272	0.257	0.243	0.229	0.216	0.204	0.193
40	0.298	0.282	0.266	0.251	0.236	0.222	0.210	0.198	0.186	0.174
45	0.276	0.260	0.246	0.232	0.218	0.205	0.194	0.182	0.171	0.160
50	0.258	0.243	0.229	0.216	0.204	0.192	0.181	0.170	0.159	0.149

Table B.32 Skewness $\left(\sqrt{b_1}\right)$ Upper Tail Standard Deviations: $\beta = 4.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.001	0.000
10	0.000	0.000	0.002	0.001	0.001	0.000	0.001	0.002	0.002	0.002
15	0.002	0.001	0.000	0.000	0.002	0.001	0.002	0.000	0.000	0.000
20	0.003	0.000	0.001	0.001	0.000	0.001	0.002	0.000	0.002	0.001
25	0.004	0.000	0.000	0.000	0.000	0.002	0.003	0.001	0.000	0.002
30	0.003	0.003	0.002	0.002	0.002	0.002	0.003	0.002	0.000	0.001
35	0.002	0.002	0.002	0.001	0.003	0.002	0.003	0.001	0.003	0.002
40	0.002	0.002	0.002	0.003	0.000	0.002	0.003	0.001	0.004	0.002
45	0.004	0.004	0.003	0.004	0.003	0.004	0.004	0.004	0.003	0.004
50	0.004	0.003	0.004	0.004	0.004	0.006	0.007	0.006	0.005	0.006

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.000
10	0.000	0.001	0.003	0.001	0.002	0.001	0.002	0.000	0.000	0.001
15	0.003	0.002	0.000	0.001	0.003	0.002	0.003	0.000	0.004	0.002
20	0.002	0.001	0.000	0.001	0.002	0.001	0.002	0.002	0.002	0.000
25	0.000	0.000	0.000	0.001	0.000	0.000	0.002	0.001	0.001	0.000
30	0.002	0.002	0.002	0.001	0.002	0.002	0.003	0.001	0.001	0.002
35	0.002	0.002	0.000	0.001	0.000	0.002	0.003	0.002	0.002	0.002
40	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001
45	0.004	0.004	0.004	0.003	0.003	0.004	0.002	0.002	0.003	0.002
50	0.002	0.004	0.003	0.002	0.003	0.003	0.002	0.003	0.004	0.003

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000
10	0.002	0.002	0.002	0.002	0.002	0.000	0.003	0.002	0.000	0.002
15	0.000	0.001	0.002	0.001	0.001	0.001	0.002	0.000	0.002	0.000
20	0.001	0.000	0.001	0.002	0.002	0.001	0.001	0.001	0.002	0.002
25	0.002	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000	0.000
30	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001
35	0.003	0.002	0.000	0.002	0.000	0.001	0.003	0.001	0.000	0.002
40	0.000	0.000	0.003	0.001	0.001	0.001	0.001	0.002	0.002	0.001
45	0.003	0.003	0.003	0.004	0.002	0.004	0.004	0.002	0.003	0.003
50	0.000	0.004	0.002	0.001	0.003	0.003	0.003	0.003	0.002	0.003

B.2 Sample Q-Statistic Percentage Points.

Table B.33 Q-Statistic Lower Tail Critical Values: $\beta = 0.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.272	1.285	1.295	1.305	1.313	1.321	1.329	1.337	1.345	1.352
10	1.392	1.449	1.490	1.521	1.549	1.573	1.594	1.613	1.631	1.648
15	1.605	1.677	1.729	1.772	1.807	1.839	1.865	1.889	1.913	1.935
20	1.793	1.873	1.931	1.975	2.016	2.053	2.087	2.117	2.144	2.171
25	1.922	2.012	2.070	2.118	2.162	2.200	2.231	2.263	2.290	2.318
30	2.030	2.127	2.192	2.246	2.292	2.329	2.361	2.394	2.421	2.452
35	2.146	2.254	2.326	2.379	2.423	2.460	2.496	2.529	2.558	2.586
40	2.244	2.349	2.423	2.476	2.521	2.561	2.598	2.629	2.660	2.690
45	2.335	2.440	2.509	2.564	2.610	2.649	2.687	2.720	2.750	2.778
50	2.402	2.508	2.579	2.634	2.678	2.720	2.754	2.785	2.816	2.843

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.359	1.367	1.374	1.381	1.387	1.394	1.400	1.407	1.413	1.420
10	1.665	1.681	1.696	1.710	1.723	1.736	1.749	1.761	1.772	1.783
15	1.955	1.975	1.995	2.013	2.032	2.048	2.065	2.083	2.098	2.113
20	2.194	2.216	2.237	2.260	2.279	2.300	2.319	2.338	2.357	2.376
25	2.342	2.368	2.391	2.414	2.434	2.453	2.474	2.492	2.511	2.529
30	2.479	2.503	2.527	2.548	2.568	2.589	2.609	2.629	2.648	2.666
35	2.614	2.639	2.663	2.686	2.708	2.729	2.750	2.770	2.789	2.807
40	2.718	2.745	2.768	2.790	2.813	2.834	2.854	2.874	2.893	2.912
45	2.805	2.830	2.852	2.874	2.896	2.917	2.935	2.955	2.973	2.991
50	2.868	2.893	2.919	2.938	2.959	2.981	3.001	3.019	3.037	3.055

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.434	1.446	1.459	1.471	1.485	1.497	1.510	1.522	1.534	1.546
10	1.807	1.830	1.852	1.875	1.895	1.916	1.936	1.956	1.975	1.995
15	2.143	2.172	2.200	2.226	2.251	2.276	2.301	2.326	2.350	2.375
20	2.412	2.447	2.479	2.510	2.543	2.574	2.602	2.631	2.659	2.687
25	2.564	2.599	2.632	2.664	2.695	2.724	2.753	2.779	2.808	2.835
30	2.700	2.733	2.767	2.800	2.830	2.861	2.889	2.917	2.945	2.972
35	2.841	2.877	2.909	2.940	2.968	2.998	3.027	3.056	3.083	3.111
40	2.948	2.983	3.015	3.047	3.077	3.108	3.138	3.167	3.196	3.224
45	3.026	3.058	3.089	3.119	3.149	3.177	3.204	3.232	3.259	3.286
50	3.089	3.122	3.154	3.183	3.212	3.242	3.269	3.297	3.323	3.348

Table B.34 Q-Statistic Lower Tail Standard Deviations: $\beta = 0.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.000	0.002	0.000	0.003	0.002	0.000	0.003	0.001	0.000
10	0.002	0.003	0.001	0.002	0.005	0.001	0.002	0.001	0.000	0.000
15	0.005	0.006	0.007	0.004	0.006	0.006	0.007	0.005	0.005	0.010
20	0.004	0.006	0.006	0.004	0.001	0.003	0.005	0.004	0.006	0.004
25	0.008	0.005	0.005	0.006	0.003	0.004	0.006	0.007	0.004	0.006
30	0.003	0.004	0.005	0.003	0.004	0.006	0.005	0.005	0.008	0.006
35	0.009	0.011	0.009	0.010	0.008	0.013	0.009	0.011	0.008	0.013
40	0.012	0.013	0.013	0.008	0.012	0.016	0.011	0.009	0.009	0.012
45	0.014	0.017	0.016	0.012	0.009	0.008	0.007	0.012	0.012	0.016
50	0.015	0.012	0.018	0.009	0.009	0.007	0.010	0.013	0.016	0.021

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.000	0.002	0.000	0.001	0.003	0.003	0.001	0.002	0.002	0.002
10	0.003	0.004	0.001	0.002	0.002	0.001	0.002	0.001	0.000	0.001
15	0.004	0.006	0.003	0.003	0.004	0.004	0.006	0.005	0.004	0.008
20	0.003	0.005	0.002	0.004	0.001	0.003	0.005	0.002	0.005	0.004
25	0.006	0.002	0.005	0.005	0.002	0.002	0.005	0.007	0.004	0.005
30	0.003	0.004	0.004	0.003	0.004	0.005	0.005	0.005	0.006	0.006
35	0.010	0.008	0.007	0.006	0.007	0.010	0.011	0.001	0.007	0.009
40	0.009	0.009	0.009	0.006	0.009	0.013	0.006	0.009	0.008	0.007
45	0.012	0.013	0.012	0.011	0.008	0.012	0.008	0.005	0.007	0.007
50	0.014	0.014	0.010	0.010	0.010	0.015	0.011	0.014	0.010	0.014

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.003	0.000	0.002	0.002	0.003	0.000	0.000	0.000	0.002	0.006
10	0.002	0.002	0.001	0.001	0.004	0.000	0.001	0.002	0.004	0.001
15	0.004	0.006	0.004	0.003	0.005	0.008	0.003	0.006	0.005	0.006
20	0.004	0.005	0.007	0.004	0.001	0.006	0.004	0.005	0.003	0.002
25	0.006	0.006	0.005	0.005	0.002	0.005	0.006	0.008	0.004	0.006
30	0.003	0.004	0.004	0.004	0.004	0.012	0.004	0.004	0.007	0.005
35	0.007	0.010	0.011	0.009	0.007	0.009	0.008	0.007	0.006	0.009
40	0.011	0.012	0.008	0.012	0.008	0.008	0.009	0.009	0.008	0.008
45	0.008	0.019	0.008	0.008	0.011	0.009	0.008	0.006	0.006	0.014
50	0.011	0.013	0.006	0.013	0.010	0.006	0.016	0.015	0.009	0.013

Table B.35 Q-Statistic Upper Tail Critical Values: $\beta = 0.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.486	2.477	2.470	2.462	2.455	2.448	2.442	2.436	2.430	2.424
10	4.633	4.528	4.450	4.382	4.324	4.275	4.225	4.179	4.136	4.094
15	6.406	6.156	5.978	5.831	5.713	5.612	5.526	5.446	5.372	5.297
20	7.902	7.498	7.232	7.021	6.856	6.720	6.596	6.473	6.374	6.275
25	7.508	7.138	6.894	6.708	6.550	6.424	6.301	6.196	6.105	6.011
30	7.361	6.976	6.731	6.549	6.398	6.280	6.172	6.075	5.989	5.909
35	7.216	6.874	6.654	6.500	6.367	6.240	6.147	6.061	5.981	5.908
40	7.250	6.910	6.706	6.552	6.423	6.316	6.225	6.141	6.066	6.001
45	7.079	6.780	6.577	6.413	6.294	6.194	6.102	6.026	5.956	5.896
50	6.966	6.664	6.478	6.329	6.215	6.115	6.028	5.950	5.881	5.822

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.418	2.411	2.405	2.399	2.393	2.387	2.381	2.375	2.370	2.364
10	4.057	4.022	3.988	3.956	3.924	3.893	3.862	3.834	3.807	3.780
15	5.223	5.162	5.106	5.051	4.999	4.946	4.898	4.852	4.808	4.766
20	6.186	6.094	6.015	5.947	5.878	5.815	5.750	5.687	5.627	5.571
25	5.922	5.848	5.772	5.705	5.646	5.592	5.541	5.491	5.440	5.393
30	5.840	5.777	5.715	5.657	5.604	5.553	5.505	5.456	5.410	5.368
35	5.841	5.782	5.726	5.674	5.621	5.576	5.526	5.481	5.442	5.401
40	5.935	5.875	5.820	5.767	5.713	5.669	5.624	5.579	5.536	5.496
45	5.835	5.779	5.729	5.683	5.635	5.589	5.548	5.509	5.471	5.432
50	5.765	5.713	5.661	5.620	5.577	5.537	5.499	5.463	5.430	5.398

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.352	2.341	2.329	2.318	2.306	2.294	2.283	2.272	2.261	2.249
10	3.728	3.677	3.627	3.580	3.537	3.495	3.453	3.416	3.378	3.341
15	4.686	4.607	4.534	4.463	4.401	4.337	4.280	4.226	4.171	4.118
20	5.463	5.364	5.274	5.192	5.112	5.036	4.966	4.896	4.828	4.765
25	5.300	5.213	5.138	5.066	4.999	4.933	4.871	4.813	4.752	4.698
30	5.287	5.211	5.144	5.075	5.014	4.955	4.897	4.841	4.791	4.742
35	5.326	5.257	5.189	5.129	5.071	5.015	4.962	4.910	4.862	4.815
40	5.420	5.352	5.285	5.224	5.166	5.111	5.062	5.013	4.965	4.918
45	5.364	5.300	5.238	5.183	5.127	5.077	5.030	4.984	4.942	4.900
50	5.333	5.274	5.215	5.159	5.112	5.063	5.020	4.974	4.933	4.892

Table B.36 Q-Statistic Upper Tail Standard Deviations: $\beta = 0.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.002	0.000	0.000	0.002	0.000	0.000	0.000	0.001	0.002
10	0.003	0.004	0.001	0.001	0.004	0.000	0.002	0.002	0.000	0.001
15	0.004	0.006	0.005	0.004	0.004	0.005	0.005	0.006	0.004	0.005
20	0.006	0.007	0.006	0.003	0.002	0.002	0.005	0.005	0.005	0.006
25	0.006	0.006	0.007	0.005	0.005	0.006	0.004	0.008	0.003	0.004
30	0.004	0.005	0.005	0.004	0.004	0.008	0.005	0.006	0.008	0.005
35	0.010	0.009	0.008	0.011	0.009	0.004	0.012	0.009	0.007	0.005
40	0.013	0.008	0.009	0.009	0.020	0.034	0.008	0.011	0.015	0.008
45	0.009	0.016	0.015	0.014	0.012	0.011	0.009	0.008	0.009	0.019
50	0.016	0.010	0.022	0.013	0.016	0.009	0.009	0.009	0.018	0.007

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.004	0.001	0.000	0.001	0.002	0.000	0.001	0.003	0.002	0.000
10	0.002	0.000	0.000	0.001	0.000	0.000	0.004	0.000	0.001	0.000
15	0.000	0.003	0.003	0.002	0.004	0.005	0.001	0.004	0.005	0.004
20	0.002	0.004	0.001	0.003	0.002	0.004	0.005	0.003	0.004	0.003
25	0.005	0.001	0.004	0.004	0.002	0.003	0.004	0.005	0.003	0.006
30	0.003	0.005	0.003	0.005	0.005	0.006	0.006	0.004	0.005	0.005
35	0.012	0.015	0.007	0.008	0.005	0.007	0.008	0.003	0.008	0.006
40	0.023	0.011	0.007	0.007	0.007	0.008	0.006	0.008	0.010	0.007
45	0.012	0.015	0.011	0.014	0.011	0.013	0.001	0.006	0.003	0.006
50	0.016	0.012	0.008	0.009	0.007	0.015	0.014	0.010	0.004	0.008

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.001	0.003	0.001	0.003	0.000	0.002	0.000	0.001	0.002
10	0.003	0.000	0.001	0.000	0.001	0.002	0.004	0.000	0.003	0.000
15	0.003	0.003	0.000	0.003	0.003	0.007	0.003	0.005	0.003	0.005
20	0.005	0.004	0.004	0.000	0.002	0.005	0.003	0.004	0.000	0.000
25	0.004	0.007	0.005	0.004	0.004	0.004	0.005	0.007	0.002	0.004
30	0.002	0.005	0.003	0.002	0.001	0.009	0.004	0.003	0.005	0.003
35	0.000	0.007	0.005	0.006	0.005	0.006	0.007	0.004	0.006	0.005
40	0.008	0.013	0.006	0.007	0.006	0.007	0.005	0.010	0.005	0.010
45	0.009	0.008	0.010	0.006	0.007	0.008	0.006	0.006	0.004	0.009
50	0.013	0.010	0.008	0.008	0.012	0.010	0.011	0.008	0.010	0.009

Table B.37 Q-Statistic Lower Tail Critical Values: $\beta = 1.0$

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.281	1.296	1.307	1.316	1.324	1.333	1.340	1.348	1.354	1.361
10	1.373	1.417	1.447	1.470	1.491	1.508	1.523	1.538	1.551	1.563
15	1.541	1.586	1.620	1.645	1.668	1.689	1.705	1.721	1.735	1.748
20	1.646	1.699	1.736	1.762	1.787	1.807	1.824	1.841	1.855	1.869
25	1.724	1.776	1.807	1.834	1.856	1.878	1.896	1.911	1.927	1.941
30	1.784	1.837	1.873	1.898	1.921	1.939	1.958	1.974	1.988	2.002
35	1.843	1.900	1.935	1.962	1.986	2.006	2.023	2.038	2.054	2.066
40	1.889	1.943	1.981	2.008	2.031	2.051	2.069	2.085	2.100	2.114
45	1.933	1.984	2.021	2.049	2.070	2.091	2.107	2.123	2.138	2.150
50	1.966	2.016	2.053	2.080	2.101	2.119	2.135	2.150	2.165	2.178

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.367	1.373	1.379	1.385	1.390	1.396	1.402	1.407	1.413	1.418
10	1.575	1.585	1.595	1.606	1.615	1.624	1.633	1.641	1.649	1.657
15	1.759	1.771	1.783	1.793	1.803	1.812	1.822	1.831	1.840	1.849
20	1.883	1.896	1.908	1.919	1.930	1.941	1.951	1.961	1.971	1.981
25	1.954	1.966	1.978	1.990	2.000	2.011	2.021	2.031	2.040	2.050
30	2.014	2.027	2.038	2.050	2.061	2.071	2.081	2.091	2.100	2.109
35	2.079	2.091	2.104	2.115	2.126	2.136	2.146	2.156	2.165	2.173
40	2.127	2.139	2.151	2.162	2.172	2.182	2.192	2.201	2.210	2.220
45	2.163	2.176	2.187	2.198	2.208	2.217	2.225	2.234	2.242	2.251
50	2.190	2.203	2.214	2.225	2.233	2.243	2.251	2.260	2.269	2.277

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.429	1.438	1.448	1.458	1.467	1.476	1.486	1.495	1.504	1.512
10	1.672	1.687	1.701	1.714	1.727	1.739	1.752	1.763	1.775	1.787
15	1.866	1.882	1.897	1.912	1.927	1.942	1.956	1.970	1.984	1.996
20	1.999	2.018	2.035	2.052	2.067	2.083	2.098	2.113	2.127	2.141
25	2.067	2.084	2.100	2.116	2.132	2.147	2.161	2.174	2.188	2.202
30	2.127	2.143	2.158	2.174	2.189	2.203	2.217	2.230	2.243	2.257
35	2.190	2.206	2.221	2.237	2.251	2.264	2.277	2.290	2.303	2.315
40	2.237	2.254	2.268	2.283	2.298	2.312	2.325	2.337	2.350	2.362
45	2.266	2.281	2.296	2.310	2.324	2.337	2.350	2.362	2.375	2.387
50	2.293	2.307	2.322	2.336	2.349	2.362	2.374	2.387	2.400	2.411

Table B.38 Q-Statistic Lower Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.002	0.001	0.001	0.001	0.000	0.001	0.002	0.004	0.000	0.001
10	0.003	0.002	0.003	0.000	0.002	0.002	0.001	0.002	0.000	0.005
15	0.003	0.005	0.004	0.003	0.004	0.004	0.005	0.006	0.002	0.007
20	0.000	0.005	0.006	0.005	0.003	0.000	0.004	0.008	0.000	0.005
25	0.006	0.004	0.003	0.007	0.002	0.002	0.005	0.005	0.003	0.008
30	0.004	0.008	0.004	0.004	0.003	0.005	0.004	0.004	0.007	0.005
35	0.010	0.014	0.007	0.009	0.007	0.011	0.008	0.007	0.004	0.007
40	0.009	0.010	0.009	0.010	0.009	0.007	0.007	0.010	0.008	0.008
45	0.013	0.011	0.011	0.014	0.008	0.007	0.008	0.006	0.009	0.014
50	0.024	0.018	0.013	0.014	0.012	0.012	0.009	0.015	0.028	0.022

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.001	0.001	0.000	0.001	0.002	0.000	0.000	0.001	0.001	0.000
10	0.003	0.002	0.001	0.002	0.001	0.000	0.000	0.002	0.000	0.001
15	0.003	0.005	0.002	0.002	0.003	0.003	0.004	0.004	0.003	0.002
20	0.000	0.004	0.002	0.003	0.001	0.003	0.002	0.003	0.004	0.003
25	0.004	0.003	0.004	0.004	0.002	0.003	0.002	0.006	0.003	0.004
30	0.003	0.005	0.005	0.003	0.003	0.005	0.003	0.004	0.005	0.005
35	0.009	0.006	0.006	0.005	0.005	0.010	0.009	0.002	0.006	0.010
40	0.014	0.007	0.008	0.006	0.008	0.009	0.005	0.008	0.005	0.008
45	0.008	0.009	0.007	0.009	0.006	0.009	0.010	0.006	0.005	0.006
50	0.012	0.013	0.011	0.008	0.009	0.011	0.006	0.010	0.008	0.007

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.001	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.000	0.002
10	0.002	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.000
15	0.002	0.001	0.004	0.002	0.003	0.002	0.001	0.007	0.001	0.000
20	0.003	0.002	0.003	0.003	0.004	0.003	0.003	0.004	0.004	0.002
25	0.000	0.004	0.004	0.003	0.003	0.004	0.005	0.003	0.003	0.004
30	0.002	0.003	0.005	0.002	0.002	0.003	0.002	0.005	0.002	0.005
35	0.005	0.003	0.004	0.004	0.004	0.006	0.005	0.004	0.005	0.008
40	0.003	0.002	0.009	0.006	0.005	0.007	0.008	0.008	0.007	0.005
45	0.008	0.007	0.008	0.007	0.006	0.009	0.007	0.006	0.005	0.007
50	0.009	0.011	0.009	0.008	0.009	0.008	0.009	0.010	0.007	0.004

Table B.39 Q-Statistic Upper Tail Critical Values: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.435	2.407	2.388	2.371	2.356	2.342	2.329	2.317	2.305	2.294
10	3.704	3.556	3.459	3.384	3.322	3.268	3.220	3.177	3.139	3.104
15	4.398	4.167	4.014	3.903	3.809	3.740	3.676	3.621	3.569	3.522
20	4.852	4.536	4.366	4.243	4.149	4.067	3.996	3.934	3.875	3.825
25	4.478	4.231	4.091	3.986	3.905	3.838	3.780	3.728	3.683	3.642
30	4.303	4.100	3.974	3.885	3.814	3.754	3.698	3.653	3.615	3.578
35	4.246	4.045	3.929	3.844	3.770	3.714	3.667	3.629	3.589	3.556
40	4.214	4.028	3.921	3.837	3.779	3.726	3.683	3.64	3.607	3.575
45	4.102	3.927	3.831	3.757	3.699	3.654	3.614	3.576	3.543	3.515
50	4.010	3.864	3.766	3.700	3.647	3.603	3.563	3.532	3.503	3.476

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.282	2.272	2.262	2.253	2.244	2.235	2.225	2.216	2.209	2.200
10	3.072	3.041	3.011	2.986	2.960	2.935	2.913	2.891	2.872	2.852
15	3.481	3.442	3.407	3.375	3.346	3.316	3.288	3.262	3.240	3.217
20	3.777	3.735	3.694	3.658	3.625	3.592	3.561	3.533	3.506	3.479
25	3.603	3.566	3.533	3.504	3.476	3.449	3.424	3.400	3.377	3.356
30	3.544	3.514	3.486	3.461	3.436	3.412	3.389	3.367	3.347	3.327
35	3.526	3.499	3.473	3.448	3.425	3.402	3.381	3.363	3.344	3.327
40	3.543	3.516	3.489	3.466	3.444	3.424	3.404	3.386	3.367	3.349
45	3.488	3.464	3.440	3.419	3.399	3.380	3.362	3.344	3.328	3.311
50	3.450	3.428	3.406	3.385	3.366	3.348	3.332	3.316	3.301	3.286

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.184	2.169	2.155	2.140	2.126	2.113	2.010	2.087	2.075	2.063
10	2.815	2.781	2.748	2.718	2.689	2.663	2.637	2.611	2.588	2.565
15	3.172	3.132	3.093	3.057	3.023	2.992	2.964	2.934	2.906	2.880
20	3.430	3.383	3.341	3.299	3.260	3.227	3.194	3.162	3.133	3.103
25	3.315	3.278	3.244	3.210	3.180	3.152	3.124	3.098	3.074	3.049
30	3.292	3.258	3.227	3.197	3.169	3.141	3.117	3.093	3.070	3.049
35	3.293	3.260	3.232	3.205	3.180	3.156	3.133	3.110	3.089	3.069
40	3.316	3.285	3.257	3.229	3.204	3.182	3.159	3.138	3.117	3.098
45	3.281	3.253	3.227	3.202	3.178	3.156	3.136	3.116	3.098	3.079
50	3.258	3.233	3.210	3.187	3.164	3.143	3.124	3.105	3.087	3.070

Table B.40 Q-Statistic Upper Tail Standard Deviations: $\beta = 1.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.001	0.002	0.000	0.003	0.001	0.003	0.000	0.001	0.000
10	0.000	0.003	0.004	0.002	0.001	0.000	0.001	0.000	0.000	0.003
15	0.000	0.000	0.003	0.005	0.002	0.000	0.004	0.004	0.004	0.005
20	0.005	0.003	0.003	0.007	0.003	0.003	0.000	0.005	0.003	0.003
25	0.003	0.005	0.004	0.009	0.001	0.002	0.003	0.003	0.005	0.002
30	0.004	0.002	0.005	0.004	0.004	0.004	0.005	0.007	0.007	0.004
35	0.005	0.006	0.007	0.006	0.005	0.006	0.005	0.004	0.007	0.000
40	0.010	0.005	0.005	0.007	0.006	0.007	0.006	0.010	0.013	0.008
45	0.009	0.004	0.006	0.008	0.007	0.006	0.010	0.006	0.008	0.009
50	0.009	0.010	0.011	0.010	0.012	0.008	0.008	0.008	0.010	0.013

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001
10	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001
15	0.004	0.005	0.001	0.003	0.003	0.005	0.002	0.001	0.002	0.000
20	0.005	0.006	0.003	0.006	0.003	0.007	0.002	0.006	0.004	0.005
25	0.004	0.006	0.004	0.002	0.003	0.004	0.003	0.003	0.003	0.003
30	0.003	0.005	0.006	0.006	0.007	0.005	0.006	0.007	0.006	0.008
35	0.005	0.006	0.003	0.006	0.003	0.007	0.002	0.006	0.004	0.005
40	0.004	0.010	0.008	0.008	0.007	0.011	0.008	0.009	0.008	0.008
45	0.008	0.013	0.007	0.009	0.005	0.011	0.005	0.009	0.007	0.007
50	0.009	0.011	0.009	0.011	0.008	0.013	0.009	0.010	0.011	0.010

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.003
10	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
15	0.003	0.003	0.003	0.001	0.001	0.002	0.001	0.002	0.001	0.002
20	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.001
25	0.000	0.001	0.001	0.001	0.000	0.001	0.001	0.000	0.000	0.000
30	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.003	0.002	0.002
35	0.002	0.000	0.000	0.002	0.002	0.001	0.001	0.002	0.002	0.002
40	0.005	0.003	0.004	0.003	0.001	0.001	0.001	0.001	0.001	0.001
45	0.004	0.005	0.008	0.006	0.006	0.003	0.004	0.005	0.004	0.004
50	0.008	0.006	0.010	0.006	0.005	0.005	0.006	0.007	0.006	0.005

Table B.41 Q-Statistic Lower Tail Critical Values: $\beta = 1.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.286	1.301	1.312	1.323	1.332	1.341	1.348	1.356	1.363	1.369
10	1.384	1.426	1.456	1.478	1.498	1.513	1.528	1.542	1.553	1.564
15	1.551	1.598	1.625	1.649	1.668	1.684	1.699	1.712	1.725	1.736
20	1.646	1.696	1.726	1.752	1.772	1.789	1.804	1.818	1.831	1.843
25	1.717	1.759	1.787	1.811	1.830	1.847	1.861	1.874	1.885	1.896
30	1.764	1.811	1.840	1.862	1.880	1.895	1.910	1.922	1.933	1.944
35	1.814	1.859	1.890	1.912	1.931	1.946	1.960	1.974	1.985	1.996
40	1.853	1.895	1.925	1.949	1.968	1.983	1.998	2.011	2.022	2.033
45	1.885	1.930	1.957	1.978	1.995	2.011	2.025	2.036	2.047	2.058
50	1.912	1.953	1.980	2.001	2.018	2.032	2.045	2.057	2.068	2.078

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.375	1.382	1.387	1.393	1.398	1.404	1.409	1.415	1.420	1.425
10	1.575	1.585	1.594	1.603	1.611	1.619	1.626	1.634	1.641	1.648
15	1.747	1.758	1.767	1.776	1.785	1.793	1.801	1.809	1.816	1.823
20	1.853	1.863	1.873	1.884	1.894	1.902	1.911	1.919	1.927	1.934
25	1.906	1.916	1.927	1.937	1.946	1.954	1.961	1.969	1.977	1.984
30	1.955	1.964	1.974	1.982	1.991	2.000	2.008	2.015	2.022	2.029
35	2.006	2.015	2.023	2.032	2.040	2.048	2.056	2.063	2.070	2.077
40	2.042	2.052	2.060	2.069	2.077	2.084	2.093	2.099	2.106	2.112
45	2.066	2.076	2.084	2.093	2.100	2.107	2.114	2.120	2.127	2.133
50	2.086	2.095	2.103	2.111	2.119	2.126	2.132	2.138	2.145	2.151

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.434	1.444	1.453	1.462	1.471	1.480	1.488	1.497	1.505	1.514
10	1.662	1.674	1.687	1.699	1.710	1.721	1.732	1.742	1.753	1.763
15	1.837	1.851	1.863	1.876	1.889	1.900	1.911	1.923	1.934	1.945
20	1.949	1.963	1.976	1.989	2.002	2.015	2.027	2.038	2.049	2.060
25	1.998	2.011	2.024	2.036	2.047	2.059	2.069	2.080	2.091	2.101
30	2.043	2.055	2.067	2.078	2.089	2.101	2.111	2.121	2.131	2.140
35	2.090	2.102	2.114	2.125	2.135	2.145	2.156	2.165	2.175	2.185
40	2.125	2.137	2.149	2.160	2.171	2.181	2.190	2.200	2.208	2.218
45	2.145	2.156	2.167	2.178	2.188	2.198	2.207	2.216	2.224	2.234
50	2.163	2.175	2.185	2.195	2.205	2.215	2.224	2.232	2.241	2.249

Table B.42 Q-Statistic Lower Tail Standard Deviations: $\beta = 1.5$

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.000	0.001	0.000
10	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
15	0.004	0.006	0.002	0.006	0.002	0.006	0.004	0.006	0.002	0.005
20	0.005	0.001	0.003	0.003	0.002	0.000	0.002	0.005	0.002	0.004
25	0.004	0.003	0.003	0.002	0.003	0.003	0.003	0.006	0.003	0.006
30	0.005	0.007	0.007	0.006	0.006	0.008	0.006	0.005	0.004	0.005
35	0.007	0.006	0.003	0.006	0.002	0.005	0.004	0.006	0.002	0.007
40	0.011	0.009	0.007	0.008	0.008	0.008	0.008	0.010	0.008	0.008
45	0.011	0.009	0.005	0.009	0.005	0.007	0.007	0.013	0.005	0.009
50	0.013	0.010	0.008	0.011	0.009	0.010	0.011	0.011	0.009	0.010

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.003	0.001	0.002	0.002	0.002	0.003	0.004	0.002	0.004	0.002
10	0.003	0.001	0.002	0.000	0.001	0.003	0.001	0.002	0.004	0.001
15	0.001	0.004	0.003	0.005	0.003	0.004	0.005	0.004	0.001	0.003
20	0.004	0.002	0.006	0.004	0.002	0.002	0.005	0.002	0.005	0.003
25	0.002	0.002	0.003	0.002	0.002	0.001	0.002	0.002	0.002	0.003
30	0.002	0.002	0.004	0.004	0.003	0.002	0.003	0.004	0.003	0.004
35	0.001	0.004	0.002	0.003	0.001	0.000	0.003	0.004	0.000	0.003
40	0.001	0.004	0.002	0.003	0.003	0.003	0.002	0.003	0.001	0.004
45	0.009	0.003	0.006	0.002	0.004	0.006	0.008	0.005	0.007	0.004
50	0.010	0.009	0.010	0.008	0.007	0.009	0.009	0.011	0.009	0.010

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.000	0.000	0.001	0.001	0.001	0.004	0.002	0.001	0.001	0.001
10	0.001	0.003	0.003	0.003	0.002	0.003	0.003	0.002	0.002	0.006
15	0.001	0.000	0.004	0.000	0.001	0.001	0.001	0.001	0.000	0.001
20	0.003	0.002	0.004	0.002	0.001	0.002	0.001	0.002	0.003	0.002
25	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.002	0.003	0.004
30	0.002	0.001	0.001	0.002	0.003	0.003	0.002	0.002	0.002	0.001
35	0.003	0.005	0.004	0.005	0.004	0.004	0.004	0.004	0.004	0.004
40	0.003	0.003	0.003	0.000	0.002	0.004	0.003	0.003	0.001	0.004
45	0.005	0.005	0.006	0.005	0.006	0.008	0.005	0.006	0.004	0.005
50	0.006	0.005	0.005	0.008	0.002	0.006	0.006	0.004	0.008	0.007

Table B.43 Q-Statistic Upper Tail Critical Values: $\beta = 1.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.417	2.383	2.360	2.341	2.322	2.307	2.292	2.278	2.264	2.251
10	3.342	3.205	3.119	3.055	3.002	2.956	2.917	2.881	2.848	2.818
15	3.739	3.554	3.435	3.351	3.288	3.231	3.183	3.142	3.105	3.073
20	3.951	3.742	3.616	3.536	3.470	3.411	3.363	3.321	3.280	3.247
25	3.667	3.503	3.410	3.339	3.281	3.233	3.193	3.159	3.130	3.101
30	3.537	3.403	3.318	3.258	3.214	3.172	3.139	3.110	3.082	3.058
35	3.496	3.366	3.291	3.233	3.186	3.150	3.115	3.086	3.062	3.041
40	3.469	3.356	3.280	3.228	3.186	3.149	3.120	3.092	3.068	3.047
45	3.386	3.278	3.212	3.164	3.125	3.095	3.069	3.047	3.025	3.005
50	3.328	3.228	3.169	3.126	3.094	3.066	3.041	3.018	2.998	2.980

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.239	2.229	2.218	2.208	2.198	2.188	2.179	2.170	2.161	2.153
10	2.791	2.768	2.745	2.724	2.705	2.686	2.669	2.652	2.635	2.620
15	3.045	3.018	2.994	2.970	2.949	2.927	2.908	2.890	2.872	2.855
20	3.215	3.185	3.159	3.135	3.112	3.090	3.069	3.050	3.031	3.012
25	3.076	3.054	3.033	3.013	2.994	2.977	2.961	2.944	2.930	2.915
30	3.036	3.015	2.996	2.977	2.962	2.946	2.932	2.918	2.904	2.891
35	3.021	3.001	2.983	2.967	2.952	2.938	2.924	2.911	2.898	2.885
40	3.028	3.011	2.993	2.977	2.962	2.949	2.936	2.923	2.911	2.898
45	2.987	2.972	2.957	2.943	2.929	2.916	2.904	2.892	2.880	2.870
50	2.963	2.947	2.933	2.919	2.906	2.894	2.881	2.871	2.861	2.851

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.137	2.121	2.107	2.092	2.079	2.065	2.052	2.039	2.027	2.016
10	2.590	2.564	2.539	2.514	2.492	2.472	2.452	2.433	2.413	2.396
15	2.823	2.793	2.766	2.741	2.717	2.695	2.672	2.652	2.632	2.614
20	2.978	2.947	2.918	2.892	2.865	2.842	2.820	2.799	2.777	2.757
25	2.888	2.864	2.839	2.818	2.797	2.778	2.760	2.743	2.726	2.710
30	2.866	2.843	2.821	2.801	2.781	2.764	2.747	2.731	2.716	2.701
35	2.862	2.841	2.822	2.804	2.787	2.771	2.756	2.741	2.727	2.714
40	2.876	2.857	2.837	2.819	2.803	2.787	2.772	2.757	2.743	2.731
45	2.851	2.831	2.814	2.798	2.782	2.768	2.754	2.741	2.727	2.715
50	2.832	2.815	2.799	2.783	2.769	2.755	2.742	2.729	2.717	2.706

Table B.44 Q-Statistic Upper Tail Standard Deviations: $\beta = 1.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.001
10	0.000	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001
15	0.002	0.004	0.001	0.003	0.003	0.002	0.003	0.005	0.003	0.003
20	0.002	0.001	0.003	0.002	0.002	0.003	0.004	0.004	0.002	0.002
25	0.004	0.003	0.005	0.004	0.005	0.003	0.002	0.003	0.003	0.005
30	0.003	0.002	0.002	0.003	0.003	0.003	0.003	0.001	0.003	0.004
35	0.003	0.005	0.004	0.008	0.002	0.002	0.008	0.002	0.005	0.002
40	0.006	0.004	0.010	0.005	0.003	0.005	0.006	0.002	0.006	0.003
45	0.007	0.009	0.009	0.014	0.008	0.009	0.012	0.004	0.012	0.008
50	0.009	0.007	0.006	0.012	0.010	0.009	0.011	0.003	0.011	0.010

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.000	0.002	0.004	0.002	0.000	0.002	0.002	0.000	0.000	0.003
10	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.002	0.004	0.002
15	0.002	0.002	0.001	0.002	0.003	0.000	0.002	0.002	0.003	0.002
20	0.003	0.004	0.002	0.001	0.004	0.004	0.001	0.003	0.002	0.005
25	0.005	0.003	0.004	0.004	0.006	0.002	0.003	0.001	0.004	0.003
30	0.004	0.004	0.001	0.005	0.007	0.005	0.003	0.003	0.002	0.004
35	0.005	0.005	0.003	0.004	0.004	0.002	0.005	0.005	0.005	0.004
40	0.004	0.004	0.004	0.005	0.005	0.003	0.007	0.003	0.004	0.006
45	0.005	0.005	0.008	0.009	0.006	0.004	0.008	0.005	0.005	0.007
50	0.006	0.008	0.004	0.011	0.009	0.006	0.004	0.004	0.004	0.005

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.001	0.000	0.003	0.002	0.002	0.002	0.003	0.002	0.001	0.002
10	0.001	0.001	0.001	0.002	0.001	0.002	0.003	0.002	0.001	0.001
15	0.003	0.003	0.002	0.002	0.003	0.002	0.002	0.000	0.004	0.003
20	0.003	0.002	0.005	0.004	0.003	0.004	0.004	0.004	0.002	0.003
25	0.002	0.002	0.003	0.003	0.002	0.003	0.002	0.003	0.002	0.002
30	0.004	0.003	0.005	0.005	0.003	0.005	0.005	0.006	0.003	0.004
35	0.003	0.001	0.003	0.003	0.003	0.002	0.001	0.003	0.001	0.003
40	0.004	0.005	0.007	0.006	0.004	0.005	0.004	0.003	0.004	0.006
45	0.003	0.004	0.003	0.004	0.004	0.006	0.005	0.003	0.005	0.005
50	0.003	0.002	0.005	0.004	0.002	0.004	0.006	0.005	0.002	0.002

Table B.45 Q-Statistic Lower Tail Critical Values: $\beta = 2.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.288	1.303	1.315	1.326	1.336	1.345	1.353	1.360	1.367	1.374
10	1.393	1.436	1.465	1.489	1.506	1.522	1.536	1.549	1.561	1.572
15	1.565	1.607	1.638	1.660	1.679	1.695	1.710	1.722	1.734	1.745
20	1.661	1.709	1.740	1.762	1.781	1.798	1.812	1.826	1.839	1.851
25	1.725	1.771	1.797	1.818	1.837	1.853	1.867	1.879	1.890	1.901
30	1.776	1.817	1.844	1.868	1.884	1.899	1.913	1.925	1.937	1.947
35	1.821	1.865	1.892	1.916	1.933	1.947	1.960	1.972	1.982	1.993
40	1.860	1.902	1.930	1.950	1.968	1.982	1.995	2.007	2.018	2.028
45	1.890	1.930	1.957	1.977	1.991	2.006	2.018	2.029	2.039	2.048
50	1.915	1.953	1.977	1.997	2.013	2.025	2.037	2.048	2.058	2.070

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.379	1.386	1.392	1.398	1.404	1.409	1.414	1.419	1.424	1.429
10	1.583	1.593	1.602	1.610	1.618	1.626	1.633	1.640	1.647	1.654
15	1.756	1.766	1.775	1.783	1.792	1.799	1.807	1.814	1.821	1.828
20	1.862	1.872	1.881	1.890	1.899	1.907	1.915	1.922	1.930	1.937
25	1.911	1.921	1.929	1.938	1.946	1.953	1.961	1.969	1.975	1.982
30	1.957	1.965	1.973	1.981	1.989	1.996	2.003	2.010	2.017	2.023
35	2.002	2.011	2.019	2.026	2.033	2.041	2.048	2.054	2.061	2.067
40	2.037	2.045	2.053	2.060	2.068	2.074	2.081	2.087	2.094	2.100
45	2.057	2.066	2.073	2.080	2.086	2.093	2.099	2.105	2.111	2.117
50	2.075	2.083	2.090	2.096	2.103	2.109	2.116	2.121	2.127	2.132

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.439	1.448	1.457	1.466	1.475	1.483	1.492	1.500	1.508	1.516
10	1.666	1.679	1.691	1.703	1.714	1.724	1.734	1.745	1.755	1.765
15	1.842	1.855	1.867	1.878	1.889	1.901	1.912	1.922	1.932	1.943
20	1.951	1.963	1.976	1.988	2.000	2.011	2.022	2.032	2.043	2.053
25	1.995	2.006	2.018	2.029	2.039	2.049	2.060	2.069	2.079	2.088
30	2.035	2.046	2.057	2.067	2.077	2.087	2.096	2.106	2.115	2.123
35	2.078	2.089	2.100	2.110	2.119	2.128	2.138	2.146	2.155	2.163
40	2.111	2.122	2.132	2.142	2.151	2.160	2.169	2.177	2.185	2.193
45	2.128	2.138	2.147	2.157	2.166	2.175	2.183	2.191	2.199	2.206
50	2.143	2.153	2.163	2.172	2.181	2.189	2.197	2.205	2.213	2.220

Table B.46 Q-Statistic Lower Tail Standard Deviations: $\beta = 2.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.002	0.001	0.000	0.000	0.001	0.002	0.003	0.004	0.002	0.002
10	0.002	0.005	0.000	0.000	0.004	0.003	0.006	0.006	0.007	0.004
15	0.002	0.003	0.005	0.005	0.002	0.002	0.002	0.007	0.003	0.002
20	0.003	0.005	0.006	0.006	0.004	0.003	0.005	0.003	0.004	0.006
25	0.003	0.003	0.003	0.003	0.003	0.005	0.005	0.005	0.005	0.003
30	0.005	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.003	0.004
35	0.004	0.006	0.007	0.007	0.006	0.006	0.006	0.009	0.006	0.004
40	0.006	0.005	0.006	0.006	0.004	0.005	0.005	0.008	0.004	0.006
45	0.006	0.006	0.004	0.004	0.007	0.007	0.007	0.006	0.005	0.006
50	0.006	0.005	0.009	0.009	0.005	0.005	0.006	0.008	0.006	0.006

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.001	0.000	0.003	0.002	0.002	0.002	0.003	0.002	0.001	0.002
10	0.001	0.001	0.001	0.002	0.001	0.002	0.003	0.002	0.001	0.001
15	0.003	0.005	0.003	0.003	0.005	0.005	0.003	0.003	0.003	0.005
20	0.002	0.003	0.002	0.005	0.002	0.007	0.002	0.003	0.005	0.002
25	0.004	0.003	0.005	0.003	0.004	0.003	0.003	0.003	0.003	0.003
30	0.006	0.004	0.004	0.007	0.006	0.005	0.003	0.003	0.007	0.004
35	0.004	0.006	0.004	0.007	0.006	0.009	0.006	0.006	0.007	0.006
40	0.006	0.004	0.006	0.006	0.005	0.008	0.004	0.005	0.006	0.005
45	0.006	0.006	0.006	0.009	0.005	0.008	0.005	0.005	0.009	0.006
50	0.006	0.005	0.006	0.004	0.007	0.006	0.007	0.006	0.004	0.007

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.001	0.000
10	0.001	0.002	0.001	0.001	0.001	0.001	0.003	0.003	0.001	0.002
15	0.001	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.001	0.001
20	0.000	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.001
25	0.002	0.003	0.002	0.002	0.002	0.002	0.000	0.003	0.002	0.001
30	0.001	0.004	0.002	0.003	0.002	0.002	0.002	0.002	0.003	0.002
35	0.002	0.002	0.001	0.000	0.000	0.002	0.000	0.000	0.000	0.001
40	0.002	0.002	0.000	0.002	0.002	0.002	0.004	0.003	0.000	0.002
45	0.004	0.002	0.003	0.005	0.003	0.003	0.004	0.003	0.003	0.003
50	0.005	0.003	0.004	0.004	0.004	0.005	0.004	0.004	0.005	0.003

Table B.47 Q-Statistic Upper Tail Critical Values: $\beta = 2.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.412	2.378	2.352	2.332	2.313	2.296	2.280	2.266	2.252	2.239
10	3.217	3.089	3.004	2.944	2.896	2.858	2.821	2.788	2.758	2.733
15	3.511	3.340	3.245	3.175	3.118	3.071	3.030	2.996	2.965	2.936
20	3.649	3.481	3.382	3.309	3.251	3.206	3.165	3.131	3.098	3.070
25	3.406	3.271	3.192	3.138	3.087	3.050	3.016	2.988	2.963	2.942
30	3.299	3.192	3.120	3.065	3.028	2.997	2.972	2.947	2.925	2.904
35	3.259	3.158	3.095	3.050	3.009	2.977	2.952	2.929	2.908	2.889
40	3.247	3.146	3.085	3.041	3.009	2.979	2.954	2.932	2.913	2.895
45	3.169	3.085	3.030	2.990	2.957	2.932	2.910	2.891	2.874	2.858
50	3.120	3.042	2.996	2.960	2.930	2.906	2.886	2.868	2.852	2.837

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.227	2.217	2.206	2.195	2.184	2.175	2.165	2.156	2.148	2.139
10	2.710	2.689	2.667	2.648	2.631	2.613	2.596	2.580	2.566	2.553
15	2.911	2.890	2.868	2.849	2.829	2.813	2.798	2.781	2.766	2.750
20	3.045	3.021	2.999	2.977	2.958	2.940	2.923	2.906	2.891	2.875
25	2.921	2.903	2.886	2.869	2.854	2.840	2.825	2.812	2.798	2.787
30	2.885	2.869	2.853	2.838	2.824	2.810	2.797	2.786	2.775	2.764
35	2.873	2.856	2.841	2.829	2.815	2.803	2.792	2.781	2.771	2.762
40	2.879	2.864	2.850	2.837	2.825	2.813	2.803	2.792	2.781	2.772
45	2.843	2.830	2.817	2.805	2.795	2.784	2.774	2.765	2.755	2.747
50	2.824	2.812	2.799	2.788	2.777	2.766	2.757	2.747	2.739	2.730

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.122	2.107	2.093	2.078	2.064	2.052	2.039	2.027	2.015	2.004
10	2.527	2.502	2.480	2.459	2.439	2.420	2.402	2.385	2.368	2.352
15	2.722	2.698	2.674	2.652	2.631	2.612	2.593	2.574	2.557	2.542
20	2.848	2.824	2.798	2.776	2.755	2.736	2.716	2.697	2.680	2.663
25	2.765	2.745	2.726	2.707	2.692	2.676	2.660	2.645	2.632	2.618
30	2.744	2.725	2.707	2.690	2.674	2.660	2.646	2.632	2.619	2.607
35	2.743	2.726	2.709	2.694	2.679	2.666	2.653	2.641	2.629	2.618
40	2.753	2.736	2.721	2.707	2.692	2.679	2.667	2.656	2.644	2.632
45	2.730	2.715	2.701	2.688	2.675	2.662	2.651	2.640	2.628	2.617
50	2.715	2.700	2.686	2.672	2.660	2.648	2.638	2.628	2.618	2.609

Table B.48 Q-Statistic Upper Tail Standard Deviations: $\beta = 2.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.000	0.001	0.003	0.001	0.000	0.002	0.000	0.003	0.002
10	0.002	0.004	0.002	0.000	0.003	0.004	0.003	0.002	0.001	0.003
15	0.001	0.002	0.001	0.002	0.000	0.001	0.002	0.001	0.002	0.005
20	0.004	0.004	0.003	0.002	0.001	0.003	0.004	0.004	0.005	0.005
25	0.003	0.003	0.005	0.003	0.003	0.002	0.003	0.003	0.002	0.003
30	0.004	0.002	0.002	0.002	0.006	0.001	0.003	0.001	0.004	0.003
35	0.002	0.003	0.002	0.004	0.004	0.004	0.003	0.002	0.003	0.003
40	0.004	0.004	0.005	0.005	0.006	0.002	0.005	0.005	0.010	0.004
45	0.011	0.009	0.006	0.003	0.008	0.003	0.007	0.007	0.008	0.008
50	0.008	0.007	0.011	0.009	0.011	0.005	0.009	0.010	0.008	0.010

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.000	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.001
10	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.002
15	0.001	0.003	0.001	0.001	0.003	0.002	0.002	0.001	0.002	0.001
20	0.002	0.002	0.002	0.002	0.003	0.002	0.001	0.002	0.002	0.002
25	0.002	0.000	0.001	0.002	0.000	0.001	0.002	0.002	0.002	0.001
30	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.003	0.003
35	0.002	0.000	0.001	0.002	0.000	0.001	0.002	0.002	0.002	0.001
40	0.002	0.002	0.003	0.002	0.003	0.002	0.003	0.002	0.002	0.002
45	0.003	0.004	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.004
50	0.005	0.005	0.005	0.005	0.006	0.004	0.003	0.005	0.004	0.004

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.002	0.001	0.002	0.002	0.000	0.003	0.002	0.000	0.001	0.003
10	0.001	0.000	0.002	0.002	0.000	0.002	0.001	0.001	0.002	0.001
15	0.001	0.000	0.002	0.002	0.002	0.004	0.001	0.001	0.001	0.002
20	0.001	0.004	0.001	0.002	0.001	0.002	0.002	0.002	0.002	0.003
25	0.001	0.003	0.000	0.002	0.002	0.002	0.002	0.002	0.002	0.004
30	0.002	0.003	0.004	0.002	0.003	0.002	0.002	0.003	0.002	0.003
35	0.002	0.001	0.003	0.003	0.001	0.001	0.002	0.003	0.002	0.002
40	0.003	0.003	0.000	0.001	0.002	0.001	0.002	0.001	0.003	0.003
45	0.005	0.004	0.002	0.005	0.004	0.003	0.005	0.001	0.002	0.003
50	0.004	0.005	0.003	0.004	0.003	0.004	0.004	0.003	0.005	0.004

Table B.49 Q-Statistic Lower Tail Critical Values: $\beta = 2.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.288	1.304	1.317	1.328	1.338	1.347	1.355	1.362	1.369	1.376
10	1.401	1.442	1.473	1.494	1.512	1.528	1.543	1.556	1.568	1.579
15	1.574	1.617	1.646	1.669	1.688	1.705	1.719	1.732	1.744	1.755
20	1.671	1.719	1.750	1.773	1.792	1.808	1.824	1.838	1.850	1.862
25	1.734	1.780	1.809	1.830	1.848	1.864	1.877	1.889	1.901	1.911
30	1.786	1.827	1.855	1.878	1.896	1.911	1.925	1.937	1.948	1.959
35	1.832	1.877	1.906	1.927	1.943	1.958	1.970	1.981	1.992	2.001
40	1.872	1.915	1.941	1.961	1.979	1.993	2.005	2.016	2.027	2.036
45	1.900	1.941	1.966	1.987	2.002	2.016	2.027	2.038	2.048	2.057
50	1.925	1.964	1.988	2.006	2.022	2.034	2.046	2.056	2.066	2.074

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.382	1.388	1.395	1.400	1.406	1.411	1.417	1.422	1.427	1.432
10	1.589	1.598	1.607	1.616	1.624	1.631	1.638	1.645	1.653	1.660
15	1.765	1.775	1.784	1.793	1.801	1.808	1.816	1.823	1.830	1.837
20	1.872	1.882	1.892	1.901	1.910	1.918	1.925	1.933	1.940	1.947
25	1.921	1.931	1.940	1.947	1.955	1.963	1.971	1.977	1.984	1.991
30	1.968	1.976	1.984	1.992	1.999	2.005	2.012	2.019	2.025	2.032
35	2.011	2.020	2.028	2.035	2.042	2.049	2.056	2.062	2.068	2.074
40	2.046	2.054	2.062	2.069	2.076	2.083	2.090	2.096	2.102	2.107
45	2.065	2.073	2.080	2.087	2.094	2.100	2.106	2.112	2.118	2.123
50	2.082	2.090	2.097	2.103	2.110	2.116	2.122	2.128	2.133	2.139

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.442	1.451	1.460	1.469	1.478	1.486	1.494	1.503	1.511	1.519
10	1.673	1.685	1.698	1.708	1.720	1.730	1.740	1.750	1.760	1.770
15	1.851	1.863	1.875	1.887	1.898	1.909	1.920	1.930	1.940	1.949
20	1.961	1.973	1.986	1.997	2.008	2.019	2.030	2.041	2.051	2.061
25	2.004	2.015	2.027	2.037	2.048	2.058	2.067	2.077	2.086	2.095
30	2.044	2.054	2.065	2.075	2.084	2.094	2.103	2.112	2.120	2.128
35	2.086	2.096	2.106	2.116	2.126	2.135	2.144	2.152	2.160	2.168
40	2.118	2.129	2.138	2.148	2.157	2.165	2.174	2.182	2.190	2.197
45	2.134	2.144	2.153	2.162	2.170	2.179	2.187	2.195	2.202	2.209
50	2.148	2.158	2.167	2.176	2.184	2.193	2.201	2.208	2.215	2.222

Table B.50 Q-Statistic Lower Tail Standard Deviations: $\beta = 2.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.000	0.001	0.002	0.003	0.002	0.001	0.002	0.002	0.000
10	0.002	0.001	0.000	0.003	0.002	0.000	0.003	0.001	0.000	0.001
15	0.002	0.001	0.002	0.004	0.001	0.001	0.002	0.000	0.001	0.002
20	0.002	0.003	0.003	0.005	0.003	0.003	0.003	0.005	0.002	0.003
25	0.005	0.003	0.004	0.004	0.006	0.004	0.005	0.003	0.004	0.002
30	0.004	0.004	0.002	0.003	0.002	0.002	0.002	0.003	0.003	0.004
35	0.003	0.003	0.003	0.003	0.004	0.004	0.003	0.005	0.003	0.004
40	0.004	0.004	0.002	0.003	0.005	0.002	0.000	0.003	0.004	0.004
45	0.003	0.004	0.003	0.003	0.002	0.003	0.002	0.003	0.003	0.004
50	0.006	0.005	0.006	0.007	0.008	0.006	0.006	0.006	0.009	0.006

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.002	0.001	0.003
10	0.002	0.003	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.000
15	0.001	0.002	0.001	0.002	0.003	0.003	0.002	0.001	0.000	0.002
20	0.003	0.001	0.001	0.003	0.000	0.003	0.003	0.002	0.002	0.001
25	0.002	0.001	0.003	0.000	0.002	0.001	0.003	0.001	0.002	0.003
30	0.002	0.002	0.001	0.000	0.004	0.002	0.002	0.004	0.002	0.004
35	0.005	0.001	0.003	0.002	0.005	0.002	0.004	0.001	0.001	0.003
40	0.003	0.003	0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.002
45	0.005	0.007	0.005	0.002	0.003	0.003	0.005	0.004	0.005	0.003
50	0.004	0.006	0.006	0.006	0.007	0.005	0.004	0.005	0.005	0.006

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.001	0.002
10	0.000	0.002	0.000	0.005	0.000	0.001	0.001	0.000	0.000	0.002
15	0.001	0.003	0.002	0.001	0.001	0.004	0.003	0.002	0.002	0.003
20	0.001	0.002	0.001	0.004	0.002	0.002	0.001	0.001	0.001	0.002
25	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.003	0.002	0.004
30	0.004	0.002	0.002	0.002	0.002	0.004	0.002	0.002	0.001	0.003
35	0.001	0.003	0.001	0.004	0.000	0.003	0.002	0.002	0.002	0.005
40	0.003	0.001	0.003	0.001	0.002	0.002	0.001	0.002	0.003	0.000
45	0.005	0.004	0.006	0.005	0.006	0.005	0.004	0.004	0.004	0.004
50	0.004	0.003	0.002	0.003	0.004	0.003	0.001	0.005	0.003	0.002

Table B.51 Q-Statistic Upper Tail Critical Values: $\beta = 2.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.411	2.376	2.351	2.329	2.310	2.293	2.276	2.262	2.248	2.236
10	3.180	3.049	2.969	2.910	2.865	2.824	2.791	2.760	2.732	2.709
15	3.421	3.278	3.186	3.119	3.065	3.019	2.982	2.951	2.923	2.898
20	3.542	3.393	3.301	3.235	3.184	3.137	3.100	3.069	3.043	3.016
25	3.319	3.191	3.123	3.071	3.028	2.996	2.965	2.937	2.914	2.894
30	3.221	3.121	3.057	3.009	2.973	2.944	2.918	2.896	2.875	2.858
35	3.176	3.086	3.030	2.989	2.955	2.927	2.903	2.883	2.863	2.845
40	3.172	3.080	3.023	2.984	2.954	2.927	2.904	2.885	2.868	2.851
45	3.101	3.023	2.975	2.937	2.908	2.883	2.864	2.846	2.830	2.816
50	3.057	2.987	2.942	2.908	2.883	2.861	2.843	2.825	2.809	2.797

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.224	2.213	2.202	2.192	2.181	2.172	2.162	2.153	2.144	2.136
10	2.685	2.666	2.647	2.629	2.611	2.595	2.580	2.565	2.550	2.535
15	2.874	2.852	2.833	2.815	2.798	2.781	2.765	2.750	2.736	2.722
20	2.993	2.971	2.953	2.933	2.914	2.896	2.881	2.866	2.851	2.838
25	2.875	2.857	2.840	2.827	2.814	2.801	2.788	2.777	2.765	2.755
30	2.843	2.827	2.812	2.797	2.785	2.773	2.762	2.750	2.739	2.729
35	2.830	2.815	2.802	2.790	2.779	2.769	2.758	2.748	2.738	2.729
40	2.836	2.823	2.811	2.799	2.787	2.777	2.767	2.757	2.747	2.738
45	2.803	2.790	2.778	2.767	2.758	2.747	2.738	2.731	2.722	2.714
50	2.785	2.773	2.762	2.751	2.741	2.731	2.722	2.714	2.706	2.698

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.118	2.103	2.089	2.075	2.061	2.047	2.036	2.024	2.013	2.001
10	2.510	2.487	2.465	2.446	2.427	2.408	2.391	2.374	2.359	2.343
15	2.696	2.672	2.650	2.630	2.611	2.593	2.575	2.558	2.542	2.527
20	2.812	2.788	2.767	2.747	2.728	2.708	2.690	2.672	2.656	2.641
25	2.733	2.715	2.696	2.680	2.664	2.649	2.636	2.622	2.609	2.597
30	2.710	2.693	2.677	2.662	2.647	2.633	2.620	2.608	2.597	2.585
35	2.711	2.696	2.681	2.667	2.653	2.640	2.628	2.617	2.606	2.596
40	2.722	2.706	2.692	2.678	2.666	2.654	2.643	2.631	2.621	2.610
45	2.699	2.685	2.672	2.659	2.648	2.636	2.626	2.615	2.606	2.596
50	2.684	2.670	2.657	2.646	2.635	2.624	2.614	2.604	2.596	2.587

Table B.52 Q-Statistic Upper Tail Standard Deviations: $\beta = 2.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.001	0.000	0.002	0.001	0.000	0.000	0.002	0.002	0.002	0.003
10	0.002	0.001	0.000	0.001	0.001	0.002	0.001	0.002	0.001	0.001
15	0.001	0.002	0.003	0.002	0.003	0.001	0.002	0.001	0.003	0.000
20	0.002	0.003	0.001	0.003	0.002	0.003	0.003	0.003	0.003	0.002
25	0.002	0.004	0.004	0.003	0.003	0.003	0.005	0.002	0.003	0.004
30	0.003	0.002	0.003	0.001	0.002	0.004	0.004	0.004	0.003	0.002
35	0.004	0.003	0.004	0.005	0.002	0.002	0.004	0.003	0.004	0.003
40	0.004	0.006	0.006	0.005	0.004	0.007	0.007	0.005	0.005	0.006
45	0.005	0.005	0.003	0.005	0.005	0.006	0.006	0.004	0.004	0.005
50	0.006	0.004	0.005	0.007	0.003	0.009	0.005	0.005	0.003	0.004

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.000	0.001	0.002	0.002	0.000	0.000	0.002	0.001	0.002	0.002
10	0.002	0.002	0.003	0.001	0.002	0.000	0.001	0.000	0.000	0.001
15	0.002	0.001	0.000	0.001	0.001	0.001	0.001	0.002	0.001	0.000
20	0.001	0.003	0.004	0.002	0.002	0.002	0.003	0.002	0.003	0.002
25	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.001	0.004	0.000
30	0.003	0.002	0.003	0.002	0.004	0.001	0.003	0.004	0.003	0.003
35	0.005	0.003	0.003	0.004	0.003	0.003	0.004	0.003	0.003	0.003
40	0.002	0.002	0.003	0.003	0.004	0.004	0.005	0.003	0.005	0.001
45	0.004	0.001	0.004	0.004	0.002	0.002	0.002	0.002	0.002	0.002
50	0.006	0.005	0.006	0.006	0.006	0.009	0.006	0.007	0.007	0.005

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.000	0.001	0.002	0.002	0.000	0.003	0.002	0.000	0.001
10	0.002	0.000	0.002	0.001	0.000	0.001	0.002	0.000	0.002	0.002
15	0.001	0.001	0.001	0.001	0.001	0.002	0.000	0.002	0.001	0.000
20	0.002	0.001	0.003	0.003	0.004	0.001	0.003	0.003	0.002	0.002
25	0.004	0.003	0.002	0.002	0.002	0.001	0.002	0.002	0.004	0.000
30	0.004	0.004	0.003	0.001	0.003	0.001	0.003	0.002	0.005	0.002
35	0.002	0.002	0.002	0.002	0.002	0.002	0.004	0.001	0.003	0.003
40	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.005	0.004	0.003
45	0.002	0.002	0.004	0.002	0.004	0.003	0.004	0.004	0.002	0.001
50	0.006	0.007	0.006	0.005	0.006	0.009	0.006	0.006	0.006	0.005

Table B.53 Q-Statistic Lower Tail Critical Values: $\beta = 3.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.289	1.305	1.318	1.329	1.339	1.348	1.356	1.364	1.371	1.377
10	1.404	1.446	1.478	1.498	1.516	1.533	1.547	1.561	1.573	1.583
15	1.579	1.623	1.652	1.676	1.695	1.711	1.725	1.739	1.752	1.763
20	1.678	1.726	1.758	1.782	1.802	1.818	1.833	1.847	1.859	1.870
25	1.743	1.788	1.818	1.840	1.857	1.873	1.886	1.898	1.910	1.921
30	1.795	1.837	1.866	1.888	1.907	1.922	1.935	1.947	1.959	1.969
35	1.842	1.887	1.916	1.938	1.954	1.968	1.981	1.992	2.002	2.012
40	1.884	1.926	1.952	1.972	1.989	2.004	2.016	2.028	2.038	2.048
45	1.911	1.951	1.978	1.997	2.014	2.027	2.038	2.049	2.059	2.068
50	1.936	1.974	1.999	2.017	2.033	2.046	2.057	2.067	2.077	2.085

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.384	1.390	1.396	1.402	1.408	1.413	1.418	1.424	1.429	1.434
10	1.593	1.603	1.612	1.620	1.628	1.636	1.643	1.650	1.657	1.665
15	1.772	1.782	1.791	1.800	1.808	1.816	1.824	1.831	1.839	1.846
20	1.881	1.891	1.901	1.911	1.920	1.928	1.936	1.943	1.951	1.958
25	1.931	1.940	1.949	1.958	1.965	1.973	1.980	1.987	1.994	2.001
30	1.978	1.987	1.995	2.003	2.010	2.017	2.023	2.030	2.036	2.042
35	2.022	2.030	2.038	2.045	2.052	2.059	2.066	2.073	2.079	2.084
40	2.057	2.065	2.073	2.080	2.087	2.094	2.100	2.107	2.112	2.118
45	2.076	2.084	2.091	2.098	2.105	2.111	2.117	2.123	2.129	2.135
50	2.093	2.101	2.108	2.115	2.121	2.127	2.133	2.139	2.144	2.150

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.443	1.452	1.461	1.471	1.479	1.488	1.496	1.505	1.513	1.521
10	1.678	1.691	1.703	1.714	1.725	1.735	1.746	1.756	1.766	1.775
15	1.859	1.871	1.884	1.895	1.906	1.917	1.928	1.937	1.947	1.957
20	1.971	1.984	1.996	2.008	2.019	2.030	2.041	2.051	2.062	2.072
25	2.013	2.025	2.037	2.048	2.059	2.068	2.078	2.087	2.097	2.105
30	2.054	2.065	2.076	2.085	2.095	2.104	2.113	2.122	2.130	2.138
35	2.096	2.107	2.118	2.128	2.138	2.147	2.155	2.163	2.171	2.179
40	2.129	2.140	2.149	2.159	2.168	2.177	2.185	2.193	2.201	2.208
45	2.145	2.155	2.164	2.173	2.182	2.190	2.198	2.205	2.213	2.219
50	2.160	2.169	2.178	2.187	2.195	2.203	2.211	2.218	2.225	2.232

Table B.54 Q-Statistic Lower Tail Standard Deviations: $\beta = 3.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.000	0.001	0.000	0.002	0.002	0.003	0.002	0.002	0.001	0.001
10	0.000	0.003	0.003	0.003	0.003	0.002	0.003	0.003	0.002	0.002
15	0.002	0.002	0.000	0.001	0.001	0.001	0.000	0.000	0.002	0.001
20	0.002	0.002	0.002	0.003	0.004	0.004	0.002	0.004	0.002	0.001
25	0.002	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.002
30	0.000	0.003	0.003	0.003	0.003	0.002	0.003	0.003	0.002	0.002
35	0.002	0.002	0.003	0.002	0.003	0.004	0.003	0.002	0.003	0.001
40	0.004	0.002	0.002	0.004	0.002	0.000	0.001	0.002	0.001	0.002
45	0.006	0.004	0.007	0.006	0.004	0.005	0.005	0.006	0.005	0.008
50	0.003	0.005	0.004	0.005	0.004	0.003	0.003	0.004	0.005	0.005

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.001	0.001	0.002	0.003	0.001	0.001	0.002	0.002	0.001	0.001
10	0.001	0.000	0.002	0.002	0.000	0.001	0.001	0.000	0.000	0.000
15	0.001	0.000	0.001	0.001	0.001	0.002	0.001	0.000	0.001	0.003
20	0.002	0.002	0.002	0.002	0.000	0.004	0.003	0.002	0.002	0.001
25	0.003	0.002	0.002	0.003	0.000	0.001	0.003	0.003	0.000	0.002
30	0.003	0.003	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.00
35	0.002	0.000	0.001	0.002	0.002	0.002	0.000	0.002	0.002	0.002
40	0.002	0.000	0.003	0.002	0.002	0.002	0.002	0.000	0.002	0.003
45	0.004	0.005	0.004	0.005	0.004	0.004	0.005	0.006	0.004	0.005
50	0.005	0.002	0.003	0.003	0.002	0.003	0.003	0.004	0.003	0.004

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.002	0.004	0.003	0.002	0.002	0.000	0.002	0.001	0.002
10	0.002	0.004	0.002	0.000	0.001	0.001	0.002	0.002	0.001	0.001
15	0.002	0.003	0.003	0.001	0.000	0.002	0.002	0.000	0.002	0.001
20	0.001	0.001	0.003	0.003	0.001	0.002	0.000	0.002	0.003	0.003
25	0.002	0.002	0.001	0.000	0.001	0.001	0.002	0.000	0.001	0.002
30	0.002	0.002	0.003	0.004	0.002	0.002	0.001	0.001	0.001	0.002
35	0.002	0.002	0.003	0.003	0.003	0.002	0.004	0.003	0.002	0.002
40	0.003	0.001	0.002	0.001	0.003	0.002	0.003	0.001	0.002	0.002
45	0.004	0.004	0.004	0.005	0.003	0.004	0.003	0.003	0.004	0.005
50	0.005	0.003	0.003	0.004	0.001	0.005	0.002	0.004	0.005	0.002

Table B.55 Q-Statistic Upper Tail Critical Values: $\beta = 3.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.410	2.377	2.351	2.329	2.309	2.292	2.275	2.261	2.248	2.235
10	3.170	3.040	2.965	2.905	2.859	2.819	2.785	2.756	2.729	2.706
15	3.397	3.260	3.175	3.109	3.053	3.012	2.979	2.948	2.920	2.891
20	3.509	3.370	3.282	3.218	3.170	3.129	3.092	3.060	3.032	3.008
25	3.293	3.172	3.106	3.057	3.017	2.986	2.957	2.933	2.909	2.888
30	3.202	3.107	3.048	2.998	2.966	2.935	2.910	2.889	2.869	2.852
35	3.158	3.068	3.017	2.979	2.947	2.920	2.896	2.876	2.859	2.842
40	3.156	3.067	3.014	2.976	2.945	2.920	2.899	2.880	2.863	2.848
45	3.086	3.011	2.964	2.929	2.902	2.880	2.859	2.842	2.826	2.811
50	3.046	2.976	2.934	2.903	2.877	2.856	2.839	2.821	2.807	2.793

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.223	2.212	2.202	2.192	2.182	2.171	2.161	2.152	2.143	2.135
10	2.684	2.665	2.646	2.628	2.610	2.593	2.578	2.564	2.550	2.537
15	2.869	2.848	2.829	2.812	2.794	2.778	2.763	2.749	2.734	2.721
20	2.984	2.963	2.944	2.926	2.909	2.893	2.878	2.864	2.850	2.837
25	2.870	2.852	2.837	2.824	2.811	2.799	2.786	2.775	2.764	2.753
30	2.837	2.822	2.807	2.794	2.782	2.769	2.758	2.748	2.738	2.728
35	2.826	2.813	2.799	2.788	2.777	2.766	2.756	2.747	2.738	2.729
40	2.833	2.820	2.807	2.796	2.785	2.774	2.764	2.756	2.747	2.739
45	2.799	2.788	2.776	2.766	2.756	2.747	2.738	2.730	2.722	2.713
50	2.781	2.770	2.759	2.749	2.740	2.730	2.722	2.714	2.706	2.698

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.119	2.103	2.089	2.075	2.061	2.048	2.036	2.024	2.013	2.001
10	2.512	2.489	2.467	2.447	2.428	2.411	2.393	2.377	2.361	2.345
15	2.696	2.674	2.652	2.632	2.613	2.595	2.578	2.562	2.547	2.532
20	2.811	2.787	2.766	2.746	2.727	2.709	2.691	2.675	2.660	2.645
25	2.734	2.715	2.697	2.681	2.666	2.651	2.637	2.625	2.611	2.599
30	2.710	2.693	2.676	2.661	2.647	2.634	2.622	2.611	2.599	2.588
35	2.712	2.696	2.682	2.668	2.655	2.643	2.630	2.619	2.609	2.599
40	2.723	2.708	2.693	2.680	2.668	2.655	2.645	2.634	2.624	2.614
45	2.699	2.686	2.673	2.661	2.649	2.638	2.628	2.618	2.609	2.600
50	2.684	2.672	2.660	2.648	2.637	2.627	2.618	2.608	2.599	2.590

Table B.56 Q-Statistic Upper Tail Standard Deviations: $\beta = 3.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.001	0.000	0.002	0.002	0.000	0.003	0.003	0.001	0.002
10	0.003	0.003	0.001	0.002	0.000	0.001	0.003	0.001	0.003	0.002
15	0.001	0.000	0.002	0.002	0.002	0.001	0.002	0.004	0.001	0.001
20	0.001	0.003	0.002	0.002	0.000	0.002	0.004	0.002	0.002	0.002
25	0.002	0.003	0.002	0.003	0.004	0.003	0.003	0.002	0.002	0.002
30	0.002	0.001	0.003	0.001	0.003	0.003	0.002	0.001	0.002	0.002
35	0.001	0.004	0.002	0.001	0.001	0.002	0.003	0.002	0.002	0.002
40	0.001	0.000	0.002	0.000	0.002	0.001	0.001	0.002	0.002	0.001
45	0.004	0.005	0.004	0.003	0.003	0.003	0.004	0.004	0.005	0.004
50	0.005	0.004	0.005	0.004	0.002	0.001	0.003	0.003	0.002	0.005

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.002	0.001	0.000	0.001	0.000	0.001	0.001	0.000	0.001	0.001
10	0.002	0.000	0.000	0.002	0.002	0.000	0.004	0.001	0.002	0.001
15	0.000	0.003	0.002	0.001	0.002	0.003	0.001	0.003	0.004	0.002
20	0.001	0.002	0.003	0.002	0.000	0.003	0.002	0.001	0.001	0.000
25	0.001	0.004	0.000	0.002	0.003	0.003	0.002	0.002	0.003	0.000
30	0.002	0.003	0.002	0.004	0.002	0.003	0.003	0.001	0.002	0.003
35	0.004	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.001	0.002
40	0.001	0.002	0.003	0.002	0.001	0.002	0.003	0.003	0.001	0.002
45	0.003	0.003	0.004	0.004	0.003	0.005	0.004	0.003	0.005	0.006
50	0.006	0.005	0.003	0.006	0.005	0.005	0.007	0.005	0.005	0.004

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.001	0.000	0.002	0.002	0.001	0.001	0.002	0.002	0.002
10	0.002	0.002	0.003	0.003	0.002	0.002	0.003	0.002	0.002	0.002
15	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.001
20	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002
25	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.000	0.000	0.000
30	0.002	0.002	0.003	0.001	0.002	0.002	0.002	0.002	0.002	0.002
35	0.003	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
40	0.003	0.001	0.003	0.002	0.001	0.002	0.001	0.002	0.001	0.001
45	0.005	0.004	0.006	0.003	0.005	0.004	0.005	0.004	0.005	0.005
50	0.004	0.004	0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.003

Table B.57 Q-Statistic Lower Tail Critical Values: $\beta = 3.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.289	1.305	1.318	1.330	1.340	1.349	1.357	1.365	1.372	1.378
10	1.407	1.450	1.480	1.501	1.520	1.535	1.551	1.564	1.576	1.587
15	1.583	1.627	1.657	1.680	1.700	1.716	1.731	1.744	1.757	1.768
20	1.683	1.732	1.764	1.788	1.808	1.825	1.840	1.854	1.866	1.877
25	1.750	1.794	1.824	1.847	1.866	1.880	1.894	1.907	1.918	1.928
30	1.801	1.845	1.874	1.897	1.915	1.931	1.945	1.957	1.968	1.977
35	1.850	1.897	1.925	1.946	1.962	1.977	1.990	2.001	2.012	2.022
40	1.891	1.935	1.961	1.982	1.999	2.013	2.026	2.038	2.049	2.059
45	1.920	1.960	1.987	2.007	2.024	2.036	2.049	2.059	2.069	2.078
50	1.943	1.984	2.009	2.027	2.042	2.056	2.067	2.077	2.087	2.095

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.385	1.391	1.397	1.403	1.409	1.414	1.420	1.425	1.430	1.435
10	1.597	1.606	1.615	1.624	1.632	1.639	1.647	1.654	1.662	1.669
15	1.778	1.788	1.797	1.806	1.814	1.822	1.830	1.838	1.845	1.852
20	1.889	1.899	1.909	1.919	1.927	1.936	1.944	1.952	1.959	1.966
25	1.938	1.948	1.957	1.966	1.974	1.982	1.989	1.996	2.003	2.010
30	1.987	1.996	2.004	2.012	2.020	2.027	2.033	2.040	2.046	2.052
35	2.030	2.039	2.047	2.055	2.063	2.069	2.076	2.082	2.088	2.095
40	2.067	2.076	2.084	2.091	2.097	2.104	2.111	2.117	2.123	2.129
45	2.086	2.095	2.102	2.109	2.116	2.122	2.129	2.134	2.140	2.146
50	2.104	2.111	2.119	2.126	2.132	2.138	2.145	2.150	2.156	2.161

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.444	1.454	1.463	1.472	1.481	1.489	1.498	1.506	1.514	1.522
10	1.682	1.695	1.707	1.718	1.729	1.740	1.751	1.761	1.771	1.780
15	1.865	1.878	1.890	1.902	1.913	1.924	1.934	1.944	1.955	1.965
20	1.979	1.992	2.005	2.017	2.029	2.040	2.050	2.061	2.071	2.082
25	2.022	2.034	2.046	2.058	2.068	2.078	2.088	2.098	2.107	2.116
30	2.064	2.076	2.086	2.095	2.105	2.114	2.123	2.132	2.140	2.148
35	2.107	2.118	2.129	2.139	2.148	2.157	2.167	2.175	2.182	2.190
40	2.140	2.151	2.161	2.170	2.179	2.188	2.196	2.205	2.212	2.220
45	2.156	2.166	2.175	2.184	2.193	2.201	2.209	2.217	2.224	2.231
50	2.172	2.181	2.190	2.198	2.207	2.215	2.222	2.230	2.237	2.244

Table B.58 Q-Statistic Lower Tail Standard Deviations: $\beta = 3.5$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.001	0.002	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
10	0.000	0.001	0.001	0.002	0.003	0.002	0.001	0.002	0.003	0.003
15	0.000	0.002	0.001	0.001	0.004	0.000	0.000	0.000	0.001	0.001
20	0.002	0.000	0.003	0.005	0.003	0.001	0.000	0.000	0.001	0.001
25	0.000	0.002	0.002	0.003	0.002	0.003	0.002	0.001	0.004	0.000
30	0.002	0.003	0.002	0.000	0.001	0.001	0.001	0.002	0.003	0.000
35	0.003	0.003	0.003	0.002	0.002	0.003	0.003	0.003	0.003	0.002
40	0.004	0.005	0.004	0.003	0.004	0.024	0.002	0.003	0.004	0.003
45	0.005	0.004	0.005	0.005	0.005	0.004	0.002	0.006	0.008	0.004
50	0.002	0.005	0.004	0.007	0.003	0.008	0.004	0.007	0.006	0.006

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.002	0.002	0.002	0.002	0.001	0.003	0.002	0.002	0.002	0.002
10	0.000	0.000	0.001	0.001	0.000	0.001	0.001	0.000	0.001	0.001
15	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003
20	0.002	0.002	0.001	0.001	0.002	0.000	0.002	0.002	0.001	0.000
25	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002
30	0.001	0.001	0.002	0.001	0.002	0.003	0.001	0.002	0.001	0.003
35	0.002	0.002	0.002	0.002	0.001	0.003	0.002	0.002	0.002	0.002
40	0.002	0.002	0.002	0.003	0.003	0.003	0.002	0.002	0.002	0.002
45	0.005	0.005	0.004	0.005	0.003	0.006	0.005	0.004	0.004	0.005
50	0.003	0.003	0.003	0.003	0.004	0.003	0.003	0.003	0.004	0.004

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.002	0.001	0.003	0.003	0.002	0.000	0.000	0.001	0.002	0.002
10	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.003	0.002	0.002
15	0.002	0.002	0.001	0.001	0.002	0.000	0.002	0.000	0.001	0.002
20	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.001
25	0.002	0.002	0.003	0.002	0.004	0.003	0.003	0.003	0.002	0.002
30	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.004	0.001	0.002
35	0.002	0.002	0.003	0.002	0.004	0.003	0.003	0.003	0.001	0.002
40	0.003	0.002	0.003	0.002	0.001	0.001	0.002	0.001	0.002	0.002
45	0.004	0.005	0.004	0.004	0.003	0.003	0.003	0.005	0.004	0.004
50	0.005	0.002	0.003	0.003	0.002	0.004	0.001	0.004	0.005	0.005

Table B.59 Q-Statistic Upper Tail Critical Values: $\beta = 3.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.411	2.378	2.352	2.329	2.310	2.292	2.276	2.262	2.248	2.235
10	3.175	3.047	2.970	2.910	2.862	2.825	2.792	2.760	2.735	2.712
15	3.404	3.266	3.186	3.114	3.063	3.023	2.989	2.956	2.928	2.901
20	3.519	3.376	3.290	3.227	3.178	3.138	3.103	3.072	3.043	3.017
25	3.297	3.181	3.113	3.066	3.028	2.996	2.967	2.944	2.921	2.901
30	3.212	3.113	3.056	3.012	2.977	2.947	2.921	2.900	2.880	2.863
35	3.164	3.079	3.024	2.987	2.958	2.931	2.908	2.888	2.870	2.852
40	3.165	3.076	3.025	2.986	2.958	2.932	2.910	2.891	2.874	2.859
45	3.093	3.019	2.974	2.942	2.914	2.891	2.871	2.853	2.838	2.825
50	3.056	2.986	2.945	2.915	2.889	2.868	2.849	2.832	2.818	2.804

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.223	2.213	2.203	2.192	2.182	2.172	2.162	2.153	2.144	2.136
10	2.690	2.671	2.650	2.633	2.614	2.599	2.584	2.570	2.557	2.544
15	2.879	2.858	2.838	2.822	2.803	2.788	2.772	2.758	2.744	2.731
20	2.996	2.974	2.956	2.937	2.919	2.904	2.889	2.875	2.860	2.847
25	2.882	2.866	2.851	2.837	2.823	2.811	2.798	2.787	2.776	2.765
30	2.846	2.831	2.818	2.805	2.793	2.781	2.770	2.758	2.749	2.740
35	2.838	2.824	2.811	2.800	2.789	2.779	2.769	2.759	2.750	2.742
40	2.844	2.832	2.819	2.809	2.798	2.787	2.778	2.768	2.760	2.752
45	2.811	2.800	2.789	2.778	2.768	2.759	2.750	2.741	2.733	2.725
50	2.793	2.782	2.771	2.761	2.751	2.743	2.734	2.726	2.718	2.711

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.120	2.105	2.090	2.076	2.062	2.049	2.037	2.025	2.014	2.003
10	2.519	2.496	2.475	2.454	2.435	2.416	2.400	2.383	2.367	2.351
15	2.705	2.683	2.662	2.642	2.624	2.606	2.589	2.572	2.557	2.543
20	2.823	2.800	2.778	2.757	2.739	2.721	2.704	2.689	2.673	2.658
25	2.745	2.727	2.709	2.693	2.678	2.663	2.649	2.636	2.623	2.611
30	2.721	2.704	2.688	2.673	2.659	2.646	2.634	2.622	2.611	2.600
35	2.725	2.709	2.694	2.680	2.667	2.655	2.643	2.633	2.62	2.611
40	2.735	2.720	2.706	2.690	2.680	2.669	2.658	2.647	2.636	2.627
45	2.711	2.698	2.685	2.672	2.661	2.651	2.641	2.630	2.621	2.612
50	2.697	2.684	2.672	2.661	2.650	2.640	2.630	2.621	2.612	2.603

Table B.60 Q-Statistic Upper Tail Standard Deviations: $\beta = 3.5$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.002	0.000	0.001	0.002	0.003	0.002	0.001	0.002	0.000	0.000
10	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.000	0.002	0.001
15	0.003	0.003	0.002	0.005	0.004	0.003	0.002	0.003	0.003	0.004
20	0.003	0.002	0.004	0.004	0.003	0.001	0.003	0.002	0.004	0.002
25	0.004	0.002	0.004	0.004	0.004	0.005	0.003	0.004	0.002	0.003
30	0.003	0.003	0.001	0.002	0.000	0.002	0.001	0.003	0.001	0.002
35	0.003	0.002	0.002	0.003	0.002	0.003	0.003	0.001	0.003	0.003
40	0.004	0.005	0.004	0.005	0.002	0.006	0.005	0.003	0.006	0.005
45	0.004	0.005	0.001	0.005	0.003	0.006	0.005	0.003	0.006	0.005
50	0.003	0.004	0.005	0.007	0.005	0.005	0.006	0.007	0.009	0.003

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.000	0.000	0.001	0.000	0.001	0.001	0.000	0.000	0.001
10	0.000	0.002	0.002	0.001	0.002	0.000	0.002	0.002	0.002	0.000
15	0.003	0.002	0.002	0.002	0.002	0.004	0.004	0.001	0.004	0.002
20	0.002	0.003	0.004	0.003	0.004	0.002	0.002	0.002	0.002	0.002
25	0.002	0.001	0.001	0.001	0.001	0.002	0.001	0.000	0.001	0.000
30	0.002	0.001	0.001	0.000	0.003	0.003	0.003	0.002	0.002	0.002
35	0.003	0.002	0.002	0.002	0.002	0.004	0.002	0.002	0.002	0.003
40	0.002	0.003	0.001	0.000	0.001	0.003	0.001	0.002	0.001	0.004
45	0.005	0.004	0.004	0.006	0.003	0.005	0.003	0.004	0.003	0.004
50	0.006	0.004	0.005	0.004	0.005	0.006	0.005	0.005	0.005	0.004

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.000	0.000	0.002	0.002	0.002	0.001	0.001	0.003	0.002	0.003
10	0.002	0.002	0.001	0.002	0.000	0.003	0.002	0.004	0.002	0.002
15	0.003	0.003	0.002	0.002	0.004	0.003	0.002	0.003	0.002	0.002
20	0.001	0.003	0.002	0.003	0.003	0.001	0.002	0.002	0.002	0.001
25	0.002	0.001	0.001	0.002	0.002	0.000	0.001	0.002	0.001	0.004
30	0.000	0.001	0.001	0.002	0.002	0.000	0.002	0.001	0.001	0.002
35	0.002	0.001	0.003	0.001	0.000	0.003	0.003	0.003	0.002	0.001
40	0.001	0.002	0.001	0.002	0.001	0.004	0.002	0.003	0.002	0.002
45	0.003	0.003	0.004	0.004	0.003	0.005	0.005	0.004	0.004	0.004
50	0.004	0.001	0.005	0.005	0.002	0.004	0.002	0.003	0.005	0.003

Table B.61 Q-Statistic Lower Tail Critical Values: $\beta = 4.0$.

Sample Size	Significance Level (α)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	1.289	1.305	1.319	1.331	1.341	1.350	1.358	1.365	1.372	1.379
10	1.409	1.453	1.482	1.503	1.523	1.538	1.553	1.565	1.579	1.589
15	1.585	1.631	1.660	1.684	1.704	1.720	1.735	1.749	1.761	1.772
20	1.687	1.737	1.768	1.793	1.814	1.831	1.845	1.859	1.872	1.884
25	1.755	1.799	1.828	1.853	1.872	1.886	1.900	1.912	1.925	1.935
30	1.807	1.851	1.881	1.904	1.922	1.938	1.952	1.964	1.975	1.985
35	1.856	1.903	1.931	1.952	1.969	1.984	1.998	2.009	2.020	2.030
40	1.900	1.942	1.968	1.990	2.007	2.022	2.035	2.047	2.058	2.067
45	1.928	1.968	1.996	2.015	2.032	2.045	2.057	2.068	2.078	2.087
50	1.950	1.991	2.016	2.035	2.051	2.064	2.076	2.086	2.096	2.106

Sample Size	Significance Level (α)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	1.386	1.392	1.398	1.403	1.409	1.415	1.420	1.426	1.431	1.435
10	1.599	1.609	1.618	1.626	1.635	1.642	1.650	1.657	1.665	1.672
15	1.783	1.793	1.802	1.810	1.819	1.827	1.835	1.843	1.850	1.857
20	1.895	1.905	1.915	1.925	1.934	1.943	1.951	1.958	1.965	1.973
25	1.946	1.955	1.964	1.973	1.981	1.989	1.996	2.004	2.011	2.017
30	1.994	2.004	2.012	2.020	2.027	2.035	2.041	2.048	2.055	2.061
35	2.039	2.047	2.055	2.064	2.071	2.079	2.085	2.091	2.098	2.104
40	2.077	2.086	2.093	2.100	2.106	2.114	2.120	2.127	2.132	2.138
45	2.095	2.103	2.111	2.118	2.125	2.132	2.138	2.145	2.150	2.156
50	2.113	2.121	2.128	2.136	2.142	2.149	2.155	2.161	2.166	2.172

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	1.445	1.455	1.464	1.473	1.482	1.491	1.499	1.507	1.515	1.523
10	1.686	1.698	1.710	1.722	1.733	1.744	1.755	1.765	1.775	1.784
15	1.871	1.884	1.896	1.908	1.919	1.930	1.940	1.951	1.961	1.971
20	1.986	1.999	2.012	2.025	2.035	2.047	2.058	2.070	2.080	2.091
25	2.030	2.042	2.054	2.065	2.076	2.087	2.097	2.107	2.116	2.125
30	2.073	2.085	2.095	2.104	2.114	2.123	2.133	2.141	2.150	2.158
35	2.116	2.128	2.139	2.148	2.157	2.167	2.176	2.184	2.192	2.200
40	2.150	2.161	2.171	2.180	2.190	2.198	2.207	2.215	2.223	2.231
45	2.166	2.177	2.186	2.195	2.203	2.212	2.220	2.227	2.235	2.243
50	2.182	2.192	2.201	2.209	2.218	2.226	2.234	2.241	2.249	2.255

Table B.62 Q-Statistic Lower Tail Standard Deviations: $\beta = 4.0$.

Sample Size	Significance Level (α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.001
10	0.001	0.002	0.002	0.002	0.001	0.002	0.000	0.002	0.000	0.000
15	0.003	0.001	0.001	0.000	0.000	0.002	0.000	0.000	0.004	0.000
20	0.003	0.002	0.002	0.000	0.002	0.001	0.003	0.002	0.003	0.003
25	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.002	0.001
30	0.002	0.000	0.000	0.002	0.001	0.001	0.000	0.001	0.003	0.000
35	0.003	0.001	0.001	0.003	0.001	0.002	0.002	0.002	0.002	0.002
40	0.003	0.001	0.001	0.004	0.003	0.002	0.002	0.002	0.002	0.002
45	0.007	0.006	0.006	0.005	0.004	0.006	0.003	0.004	0.004	0.003
50	0.004	0.004	0.004	0.003	0.004	0.004	0.004	0.003	0.004	0.004

Sample Size	Significance Level (α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.000
10	0.001	0.002	0.003	0.000	0.001	0.000	0.001	0.000	0.001	0.002
15	0.002	0.003	0.002	0.001	0.002	0.002	0.002	0.001	0.001	0.002
20	0.002	0.003	0.000	0.002	0.002	0.002	0.002	0.002	0.001	0.000
25	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
30	0.002	0.003	0.000	0.004	0.002	0.003	0.002	0.000	0.001	0.003
35	0.001	0.002	0.000	0.002	0.000	0.002	0.001	0.002	0.001	0.002
40	0.000	0.002	0.000	0.001	0.000	0.000	0.000	0.001	0.001	0.000
45	0.004	0.002	0.004	0.003	0.002	0.004	0.004	0.002	0.003	0.003
50	0.003	0.002	0.003	0.004	0.003	0.002	0.004	0.003	0.002	0.003

Sample Size	Significance Level (α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	0.001	0.002	0.002	0.001	0.000	0.001	0.002	0.001	0.002	0.001
10	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001
15	0.002	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.002	0.002
20	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000
25	0.000	0.002	0.001	0.001	0.000	0.003	0.001	0.001	0.001	0.002
30	0.003	0.003	0.003	0.004	0.003	0.003	0.002	0.004	0.004	0.002
35	0.003	0.000	0.002	0.003	0.002	0.000	0.000	0.001	0.002	0.001
40	0.002	0.000	0.002	0.003	0.002	0.002	0.002	0.000	0.002	0.002
45	0.000	0.002	0.000	0.002	0.001	0.002	0.001	0.001	0.001	0.000
50	0.000	0.002	0.003	0.003	0.004	0.002	0.003	0.003	0.001	0.003

Table B.63 Q-Statistic Upper Tail Critical Values: $\beta = 4.0$.

Sample Size	Significance Level (1-α)									
	0.995	0.990	0.985	0.980	0.975	0.970	0.965	0.960	0.955	0.950
5	2.412	2.379	2.353	2.329	2.310	2.293	2.277	2.263	2.248	2.236
10	3.187	3.059	2.979	2.921	2.872	2.835	2.800	2.771	2.744	2.721
15	3.424	3.281	3.199	3.128	3.079	3.039	3.004	2.970	2.942	2.918
20	3.537	3.395	3.311	3.246	3.198	3.155	3.122	3.091	3.062	3.038
25	3.324	3.201	3.132	3.084	3.046	3.014	2.986	2.961	2.939	2.920
30	3.233	3.134	3.072	3.032	2.996	2.966	2.940	2.919	2.898	2.879
35	3.184	3.094	3.042	3.005	2.975	2.949	2.926	2.906	2.887	2.871
40	3.183	3.097	3.043	3.007	2.976	2.951	2.928	2.909	2.891	2.877
45	3.111	3.037	2.993	2.960	2.932	2.909	2.889	2.872	2.855	2.842
50	3.078	3.006	2.964	2.933	2.907	2.885	2.866	2.850	2.836	2.822

Sample Size	Significance Level (1-α)									
	0.945	0.940	0.935	0.930	0.925	0.920	0.915	0.910	0.905	0.900
5	2.225	2.214	2.204	2.193	2.183	2.173	2.163	2.154	2.146	2.138
10	2.698	2.678	2.660	2.642	2.623	2.608	2.593	2.580	2.567	2.553
15	2.894	2.873	2.854	2.836	2.819	2.802	2.786	2.771	2.758	2.745
20	3.015	2.993	2.972	2.954	2.937	2.921	2.906	2.890	2.877	2.864
25	2.902	2.885	2.870	2.855	2.842	2.828	2.816	2.804	2.793	2.782
30	2.863	2.848	2.834	2.821	2.809	2.797	2.786	2.775	2.765	2.755
35	2.856	2.842	2.829	2.818	2.806	2.796	2.786	2.776	2.767	2.758
40	2.863	2.850	2.838	2.827	2.816	2.805	2.795	2.785	2.777	2.768
45	2.830	2.818	2.806	2.794	2.785	2.776	2.767	2.758	2.750	2.742
50	2.809	2.799	2.789	2.779	2.769	2.760	2.751	2.743	2.735	2.728

Sample Size	Significance Level (1-α)									
	0.890	0.880	0.870	0.860	0.850	0.840	0.830	0.820	0.810	0.800
5	2.122	2.107	2.091	2.078	2.064	2.051	2.039	2.027	2.016	2.004
10	2.528	2.504	2.484	2.463	2.443	2.425	2.407	2.390	2.374	2.358
15	2.720	2.696	2.675	2.655	2.636	2.619	2.601	2.585	2.569	2.555
20	2.840	2.816	2.794	2.774	2.756	2.738	2.721	2.704	2.689	2.675
25	2.761	2.742	2.726	2.709	2.693	2.678	2.665	2.651	2.638	2.625
30	2.736	2.720	2.704	2.689	2.676	2.662	2.649	2.637	2.626	2.615
35	2.741	2.725	2.710	2.697	2.683	2.671	2.660	2.648	2.637	2.626
40	2.752	2.737	2.723	2.709	2.697	2.684	2.674	2.663	2.653	2.643
45	2.728	2.715	2.702	2.689	2.677	2.666	2.656	2.647	2.637	2.628
50	2.714	2.701	2.689	2.677	2.666	2.656	2.646	2.637	2.628	2.619

Table B.64 Q-Statistic Upper Tail Standard Deviations: $\beta = 4.0$.

Sample Size	Significance Level ($1-\alpha$)									
	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
5	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.001	0.000	0.001
10	0.001	0.001	0.003	0.001	0.000	0.001	0.003	0.002	0.002	0.000
15	0.003	0.003	0.002	0.002	0.003	0.003	0.003	0.004	0.002	0.002
20	0.002	0.002	0.003	0.003	0.002	0.004	0.002	0.003	0.001	0.004
25	0.002	0.003	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003
30	0.001	0.001	0.000	0.002	0.000	0.001	0.000	0.002	0.000	0.002
35	0.002	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.003
40	0.002	0.002	0.000	0.000	0.000	0.002	0.002	0.000	0.001	0.001
45	0.004	0.004	0.004	0.003	0.006	0.004	0.004	0.004	0.004	0.005
50	0.005	0.005	0.005	0.005	0.005	0.005	0.010	0.006	0.005	0.006

Sample Size	Significance Level ($1-\alpha$)									
	0.055	0.060	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100
5	0.003	0.000	0.000	0.001	0.003	0.001	0.002	0.002	0.002	0.002
10	0.003	0.002	0.001	0.002	0.002	0.004	0.001	0.002	0.002	0.001
15	0.004	0.002	0.002	0.002	0.002	0.003	0.001	0.002	0.002	0.000
20	0.002	0.001	0.002	0.001	0.004	0.000	0.001	0.001	0.002	0.002
25	0.003	0.003	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.004
30	0.002	0.003	0.001	0.002	0.001	0.001	0.002	0.002	0.003	0.003
35	0.001	0.001	0.000	0.002	0.002	0.000	0.001	0.001	0.002	0.002
40	0.003	0.001	0.004	0.002	0.003	0.004	0.005	0.005	0.005	0.002
45	0.003	0.001	0.002	0.003	0.001	0.003	0.003	0.002	0.001	0.000
50	0.004	0.003	0.003	0.005	0.004	0.005	0.004	0.004	0.004	0.003

Sample Size	Significance Level (α)									
	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
5	0.003	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.00	0.002
10	0.000	0.002	0.002	0.001	0.002	0.000	0.002	0.002	0.002	0.002
15	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.001	0.001	0.001
20	0.002	0.002	0.004	0.002	0.002	0.003	0.002	0.002	0.001	0.000
25	0.000	0.000	0.001	0.002	0.000	0.001	0.002	0.001	0.000	0.000
30	0.000	0.002	0.002	0.003	0.000	0.002	0.002	0.002	0.003	0.002
35	0.002	0.000	0.001	0.002	0.003	0.003	0.003	0.003	0.002	0.000
40	0.000	0.001	0.002	0.001	0.000	0.001	0.001	0.001	0.003	0.001
45	0.005	0.004	0.004	0.004	0.006	0.005	0.005	0.004	0.005	0.004
50	0.002	0.003	0.003	0.003	0.004	0.003	0.003	0.005	0.004	0.002

Appendix C. Attained Significance Level Tables

C.1 Attained Significance Level Tables for H_o = Weibull($\beta = 0.5$).

Table C.1 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.034	0.044	0.054	0.064	0.074	0.083	0.094	0.103	0.113	0.124	0.134	0.144	0.153	0.163	0.173	0.183	0.193	0.204
0.02	0.025	0.032	0.039	0.049	0.059	0.069	0.078	0.088	0.099	0.108	0.118	0.129	0.138	0.148	0.158	0.168	0.178	0.188	0.197	0.209
0.03	0.035	0.041	0.048	0.055	0.065	0.074	0.084	0.093	0.104	0.113	0.123	0.134	0.143	0.153	0.163	0.173	0.182	0.193	0.202	0.213
0.04	0.044	0.051	0.056	0.063	0.070	0.078	0.087	0.097	0.107	0.116	0.126	3.000	0.146	0.156	0.165	0.175	0.185	0.195	0.204	0.215
0.05	0.054	0.060	0.065	0.071	0.077	0.084	0.092	0.101	0.111	0.120	0.130	0.140	0.150	0.159	0.168	0.178	0.188	0.198	0.207	0.218
0.06	0.064	0.070	0.075	0.080	0.086	0.092	0.099	0.106	0.115	0.124	0.134	0.144	0.153	0.163	0.172	0.182	0.191	0.201	0.210	0.221
0.07	0.074	0.079	0.084	0.089	0.095	0.100	0.107	0.113	0.120	0.128	0.138	0.148	0.157	0.167	0.176	0.185	0.194	0.204	0.213	0.224
0.08	0.084	0.089	0.094	0.099	0.105	0.110	0.116	0.121	0.128	0.135	0.143	0.152	0.161	0.171	0.180	0.189	0.198	0.208	0.217	0.227
0.09	0.094	0.099	0.104	0.109	0.114	0.119	0.124	0.130	0.136	0.142	0.149	0.157	0.165	0.174	0.183	0.193	0.201	0.211	0.220	0.231
0.10	0.103	0.108	0.113	0.118	0.123	0.127	0.133	0.138	0.144	0.150	0.156	0.163	0.170	0.178	0.187	0.196	0.205	0.215	0.223	0.234
0.11	0.113	0.118	0.123	0.127	0.132	0.137	0.142	0.147	0.152	0.158	0.164	0.170	0.176	0.184	0.191	0.200	0.209	0.218	0.227	0.237
0.12	0.123	0.128	0.132	0.136	0.141	0.145	0.151	0.155	0.161	0.166	0.171	0.177	0.183	0.190	0.196	0.204	0.213	0.222	0.231	0.241
0.13	0.132	0.137	0.142	0.146	0.150	0.155	0.160	0.164	0.169	0.174	0.180	0.186	0.191	0.197	0.203	0.210	0.217	0.226	0.234	0.244
0.14	0.143	0.148	0.152	0.156	0.160	0.165	0.169	0.174	0.179	0.184	0.189	0.194	0.199	0.205	0.210	0.216	0.223	0.230	0.238	0.248
0.15	0.153	0.157	0.161	0.165	0.170	0.174	0.178	0.183	0.187	0.192	0.197	0.202	0.207	0.213	0.218	0.223	0.229	0.236	0.243	0.251
0.16	0.163	0.167	0.171	0.175	0.179	0.183	0.188	0.192	0.197	0.201	0.206	0.211	0.216	0.221	0.226	0.231	0.237	0.243	0.249	0.256
0.17	0.173	0.178	0.182	0.186	0.190	0.194	0.198	0.202	0.207	0.211	0.216	0.221	0.225	0.230	0.234	0.240	0.245	0.251	0.256	0.263
0.18	0.184	0.188	0.192	0.196	0.200	0.203	0.208	0.212	0.216	0.220	0.225	0.230	0.234	0.239	0.243	0.248	0.253	0.258	0.264	0.270
0.19	0.193	0.197	0.201	0.205	0.209	0.212	0.217	0.220	0.225	0.229	0.233	0.238	0.242	0.247	0.251	0.255	0.260	0.265	0.270	0.276
0.20	0.203	0.207	0.211	0.214	0.218	0.222	0.226	0.230	0.234	0.238	0.242	0.247	0.251	0.255	0.259	0.263	0.268	0.273	0.278	0.283

Table C.2 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.015	0.024	0.034	0.043	0.053	0.063	0.073	0.083	0.093	0.103	0.113	0.123	0.132	0.142	0.151	0.161	0.171	0.180	0.190	0.199
0.02	0.023	0.029	0.037	0.046	0.056	0.065	0.075	0.085	0.095	0.104	0.114	0.124	0.134	0.143	0.153	0.163	0.172	0.181	0.191	0.200
0.03	0.033	0.036	0.042	0.049	0.058	0.067	0.077	0.087	0.096	0.106	0.115	0.125	0.135	0.144	0.153	0.163	0.173	0.182	0.191	0.201
0.04	0.042	0.044	0.049	0.054	0.061	0.070	0.079	0.089	0.098	0.107	0.117	0.127	0.136	0.145	0.155	0.164	0.174	0.183	0.192	0.201
0.05	0.052	0.054	0.057	0.061	0.066	0.073	0.081	0.090	0.099	0.109	0.118	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.193	0.202
0.06	0.060	0.062	0.064	0.068	0.072	0.078	0.084	0.093	0.101	0.110	0.120	0.129	0.138	0.148	0.157	0.166	0.176	0.185	0.194	0.203
0.07	0.070	0.071	0.072	0.075	0.079	0.084	0.089	0.096	0.104	0.112	0.121	0.131	0.140	0.149	0.158	0.167	0.177	0.186	0.195	0.204
0.08	0.079	0.080	0.081	0.084	0.087	0.091	0.095	0.101	0.107	0.115	0.123	0.132	0.141	0.150	0.159	0.168	0.178	0.186	0.195	0.205
0.09	0.089	0.089	0.090	0.092	0.095	0.098	0.101	0.106	0.112	0.118	0.126	0.134	0.143	0.152	0.160	0.170	0.179	0.187	0.196	0.205
0.10	0.100	0.100	0.100	0.102	0.104	0.106	0.109	0.113	0.118	0.124	0.130	0.137	0.145	0.153	0.162	0.171	0.180	0.189	0.198	0.207
0.11	0.110	0.110	0.111	0.112	0.113	0.115	0.118	0.121	0.125	0.130	0.135	0.141	0.148	0.156	0.164	0.173	0.182	0.190	0.199	0.208
0.12	0.121	0.121	0.121	0.122	0.123	0.125	0.126	0.129	0.132	0.136	0.141	0.147	0.153	0.160	0.167	0.175	0.183	0.192	0.200	0.209
0.13	0.131	0.131	0.131	0.131	0.132	0.133	0.135	0.137	0.140	0.143	0.147	0.152	0.157	0.163	0.170	0.177	0.185	0.193	0.202	0.210
0.14	0.141	0.141	0.141	0.141	0.142	0.143	0.144	0.146	0.148	0.151	0.154	0.158	0.163	0.168	0.174	0.181	0.188	0.196	0.204	0.212
0.15	0.151	0.151	0.151	0.151	0.151	0.152	0.153	0.155	0.156	0.159	0.162	0.165	0.169	0.174	0.179	0.185	0.191	0.198	0.206	0.214
0.16	0.161	0.161	0.161	0.161	0.161	0.162	0.163	0.164	0.166	0.168	0.170	0.173	0.177	0.181	0.185	0.190	0.196	0.202	0.209	0.216
0.17	0.171	0.171	0.171	0.171	0.171	0.172	0.172	0.173	0.174	0.176	0.178	0.181	0.184	0.187	0.191	0.196	0.201	0.206	0.212	0.219
0.18	0.180	0.180	0.180	0.180	0.181	0.181	0.181	0.182	0.183	0.184	0.186	0.189	0.191	0.194	0.197	0.201	0.206	0.211	0.216	0.222
0.19	0.191	0.191	0.191	0.191	0.191	0.191	0.191	0.192	0.192	0.194	0.195	0.197	0.199	0.202	0.205	0.208	0.212	0.216	0.221	0.227
0.20	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.201	0.201	0.202	0.203	0.205	0.207	0.210	0.212	0.215	0.218	0.222	0.226	0.231

Table C.3 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.013	0.021	0.031	0.041	0.051	0.062	0.071	0.081	0.090	0.102	0.112	0.122	0.132	0.142	0.152	0.163	0.173	0.183	0.193	0.203
0.02	0.021	0.025	0.033	0.042	0.052	0.063	0.072	0.081	0.091	0.102	0.112	0.123	0.133	0.143	0.153	0.163	0.173	0.184	0.193	0.203
0.03	0.030	0.033	0.038	0.045	0.054	0.064	0.073	0.082	0.092	0.103	0.113	0.123	0.133	0.143	0.153	0.163	0.174	0.184	0.193	0.203
0.04	0.040	0.042	0.045	0.050	0.057	0.066	0.074	0.083	0.093	0.103	0.114	0.124	0.133	0.143	0.153	0.164	0.174	0.184	0.194	0.203
0.05	0.050	0.051	0.053	0.057	0.062	0.069	0.076	0.085	0.094	0.104	0.114	0.124	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.203
0.06	0.061	0.061	0.062	0.065	0.069	0.074	0.080	0.087	0.095	0.105	0.115	0.125	0.135	0.144	0.154	0.164	0.175	0.185	0.194	0.203
0.07	0.071	0.071	0.072	0.074	0.077	0.081	0.086	0.092	0.098	0.108	0.117	0.126	0.136	0.145	0.155	0.165	0.175	0.185	0.194	0.204
0.08	0.081	0.081	0.081	0.083	0.085	0.088	0.092	0.097	0.102	0.110	0.119	0.128	0.137	0.146	0.155	0.165	0.175	0.185	0.195	0.204
0.09	0.091	0.091	0.091	0.092	0.094	0.096	0.099	0.103	0.108	0.115	0.122	0.130	0.138	0.147	0.156	0.166	0.176	0.186	0.195	0.204
0.10	0.101	0.101	0.101	0.102	0.103	0.105	0.107	0.110	0.114	0.120	0.126	0.133	0.141	0.149	0.157	0.167	0.177	0.186	0.196	0.205
0.11	0.112	0.112	0.112	0.112	0.113	0.114	0.116	0.118	0.122	0.126	0.132	0.138	0.144	0.151	0.159	0.168	0.178	0.187	0.196	0.205
0.12	0.122	0.122	0.122	0.122	0.123	0.124	0.125	0.127	0.130	0.133	0.138	0.143	0.148	0.155	0.162	0.171	0.179	0.189	0.197	0.206
0.13	0.133	0.133	0.133	0.133	0.134	0.134	0.135	0.137	0.139	0.142	0.145	0.149	0.154	0.160	0.166	0.174	0.182	0.190	0.198	0.207
0.14	0.143	0.143	0.143	0.143	0.144	0.144	0.145	0.146	0.147	0.150	0.152	0.156	0.160	0.165	0.171	0.178	0.185	0.193	0.200	0.208
0.15	0.153	0.153	0.153	0.153	0.153	0.153	0.154	0.155	0.156	0.157	0.160	0.163	0.167	0.171	0.176	0.182	0.189	0.196	0.203	0.210
0.16	0.163	0.163	0.163	0.163	0.163	0.164	0.164	0.164	0.165	0.167	0.169	0.171	0.174	0.177	0.182	0.187	0.193	0.200	0.206	0.213
0.17	0.173	0.173	0.173	0.173	0.173	0.173	0.174	0.174	0.175	0.176	0.177	0.179	0.181	0.185	0.188	0.193	0.198	0.204	0.210	0.217
0.18	0.183	0.183	0.183	0.183	0.183	0.183	0.183	0.183	0.183	0.184	0.185	0.187	0.189	0.192	0.195	0.199	0.203	0.209	0.214	0.220
0.19	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.194	0.194	0.195	0.196	0.198	0.200	0.203	0.206	0.210	0.215	0.220	0.225
0.20	0.203	0.203	0.203	0.203	0.203	0.203	0.203	0.203	0.203	0.203	0.204	0.205	0.206	0.208	0.210	0.213	0.217	0.221	0.225	0.230

Table C.4 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.013	0.022	0.032	0.041	0.051	0.061	0.071	0.081	0.091	0.101	0.111	0.120	0.130	0.140	0.149	0.159	0.169	0.179	0.189	0.199
0.02	0.021	0.026	0.034	0.042	0.051	0.061	0.071	0.082	0.091	0.102	0.111	0.120	0.130	0.140	0.149	0.159	0.169	0.179	0.189	0.199
0.03	0.031	0.034	0.038	0.045	0.053	0.062	0.072	0.082	0.092	0.102	0.111	0.121	0.130	0.140	0.149	0.159	0.169	0.179	0.189	0.199
0.04	0.041	0.042	0.045	0.050	0.056	0.064	0.073	0.083	0.092	0.102	0.112	0.121	0.131	0.140	0.149	0.159	0.169	0.179	0.189	0.199
0.05	0.051	0.052	0.053	0.056	0.061	0.068	0.075	0.084	0.093	0.103	0.112	0.121	0.131	0.140	0.149	0.159	0.169	0.179	0.189	0.199
0.06	0.061	0.061	0.062	0.064	0.067	0.073	0.079	0.087	0.095	0.104	0.113	0.122	0.131	0.141	0.150	0.159	0.169	0.179	0.189	0.199
0.07	0.071	0.071	0.072	0.073	0.075	0.079	0.084	0.090	0.097	0.105	0.114	0.122	0.132	0.141	0.150	0.160	0.169	0.179	0.189	0.199
0.08	0.081	0.081	0.081	0.082	0.083	0.086	0.090	0.095	0.101	0.109	0.116	0.124	0.133	0.142	0.150	0.160	0.169	0.180	0.189	0.199
0.09	0.090	0.090	0.090	0.091	0.092	0.094	0.097	0.101	0.106	0.112	0.119	0.126	0.134	0.143	0.151	0.160	0.170	0.180	0.189	0.199
0.10	0.100	0.100	0.100	0.100	0.101	0.102	0.104	0.108	0.112	0.117	0.123	0.129	0.137	0.144	0.152	0.161	0.170	0.180	0.190	0.200
0.11	0.110	0.110	0.110	0.110	0.110	0.111	0.113	0.116	0.119	0.124	0.128	0.134	0.140	0.147	0.155	0.163	0.171	0.181	0.190	0.200
0.12	0.120	0.120	0.120	0.120	0.120	0.121	0.122	0.124	0.127	0.131	0.135	0.139	0.145	0.151	0.158	0.165	0.173	0.183	0.191	0.201
0.13	0.130	0.130	0.130	0.130	0.130	0.131	0.131	0.133	0.135	0.138	0.141	0.145	0.150	0.156	0.162	0.168	0.176	0.184	0.193	0.202
0.14	0.140	0.140	0.140	0.140	0.140	0.141	0.141	0.142	0.144	0.146	0.149	0.152	0.156	0.161	0.166	0.172	0.179	0.187	0.195	0.203
0.15	0.150	0.150	0.150	0.150	0.150	0.150	0.151	0.151	0.152	0.154	0.156	0.159	0.163	0.167	0.171	0.177	0.183	0.190	0.198	0.205
0.16	0.161	0.161	0.161	0.161	0.161	0.161	0.161	0.161	0.162	0.163	0.165	0.167	0.170	0.174	0.178	0.182	0.188	0.195	0.201	0.208
0.17	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.171	0.172	0.173	0.174	0.176	0.178	0.181	0.184	0.188	0.193	0.199	0.205	0.212
0.18	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.181	0.181	0.182	0.183	0.184	0.186	0.188	0.191	0.195	0.199	0.205	0.210	0.216
0.19	0.191	0.191	0.191	0.191	0.191	0.191	0.191	0.191	0.191	0.192	0.192	0.193	0.195	0.197	0.199	0.202	0.206	0.211	0.216	0.221
0.20	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.202	0.202	0.203	0.204	0.206	0.208	0.210	0.214	0.218	0.222	0.227

Table C.5 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.014	0.022	0.031	0.041	0.050	0.060	0.069	0.079	0.089	0.100	0.110	0.120	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.02	0.022	0.026	0.033	0.041	0.051	0.060	0.070	0.079	0.090	0.100	0.110	0.120	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.03	0.031	0.033	0.037	0.044	0.052	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.04	0.040	0.041	0.044	0.049	0.055	0.062	0.071	0.080	0.090	0.100	0.110	0.120	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.05	0.050	0.051	0.052	0.055	0.060	0.066	0.073	0.081	0.091	0.100	0.110	0.121	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.06	0.060	0.061	0.061	0.063	0.067	0.071	0.077	0.084	0.093	0.102	0.111	0.121	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.07	0.070	0.071	0.071	0.072	0.075	0.078	0.083	0.088	0.096	0.104	0.112	0.122	0.132	0.141	0.151	0.160	0.170	0.180	0.190	0.200
0.08	0.080	0.080	0.080	0.081	0.083	0.085	0.089	0.093	0.099	0.106	0.114	0.123	0.133	0.142	0.151	0.160	0.170	0.180	0.190	0.200
0.09	0.089	0.090	0.090	0.090	0.092	0.093	0.096	0.100	0.105	0.110	0.117	0.125	0.134	0.143	0.152	0.161	0.171	0.180	0.190	0.201
0.10	0.100	0.100	0.100	0.101	0.102	0.103	0.105	0.108	0.112	0.116	0.122	0.129	0.137	0.146	0.154	0.162	0.172	0.181	0.191	0.201
0.11	0.110	0.110	0.110	0.111	0.111	0.112	0.114	0.116	0.119	0.123	0.127	0.134	0.141	0.148	0.156	0.164	0.173	0.182	0.192	0.202
0.12	0.120	0.120	0.120	0.121	0.121	0.122	0.123	0.125	0.127	0.130	0.134	0.139	0.145	0.152	0.159	0.166	0.175	0.183	0.193	0.202
0.13	0.131	0.131	0.131	0.131	0.132	0.132	0.133	0.134	0.136	0.138	0.142	0.146	0.151	0.157	0.163	0.169	0.177	0.185	0.194	0.204
0.14	0.141	0.141	0.141	0.141	0.142	0.142	0.143	0.144	0.145	0.147	0.150	0.153	0.158	0.162	0.167	0.173	0.181	0.188	0.197	0.205
0.15	0.152	0.152	0.152	0.152	0.152	0.152	0.153	0.154	0.155	0.156	0.158	0.161	0.165	0.169	0.173	0.178	0.185	0.192	0.200	0.208
0.16	0.162	0.162	0.162	0.162	0.162	0.162	0.162	0.163	0.164	0.165	0.167	0.170	0.173	0.176	0.180	0.184	0.190	0.196	0.203	0.211
0.17	0.171	0.171	0.171	0.171	0.171	0.171	0.172	0.172	0.173	0.174	0.176	0.178	0.180	0.183	0.186	0.190	0.195	0.200	0.207	0.214
0.18	0.181	0.181	0.181	0.181	0.182	0.182	0.182	0.182	0.183	0.184	0.185	0.187	0.190	0.192	0.194	0.197	0.201	0.206	0.212	0.218
0.19	0.191	0.191	0.191	0.192	0.192	0.192	0.192	0.192	0.193	0.194	0.195	0.197	0.198	0.200	0.202	0.205	0.208	0.212	0.217	0.223
0.20	0.201	0.201	0.201	0.201	0.201	0.201	0.201	0.202	0.202	0.203	0.204	0.205	0.207	0.209	0.210	0.213	0.215	0.219	0.223	0.228

Table C.6 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.012	0.020	0.029	0.039	0.049	0.059	0.069	0.078	0.088	0.099	0.109	0.118	0.128	0.137	0.147	0.157	0.167	0.177	0.187	0.197
0.02	0.019	0.024	0.031	0.040	0.050	0.059	0.069	0.078	0.088	0.099	0.109	0.118	0.128	0.137	0.147	0.157	0.167	0.177	0.187	0.197
0.03	0.029	0.031	0.036	0.043	0.051	0.060	0.069	0.079	0.088	0.099	0.109	0.118	0.128	0.137	0.147	0.157	0.167	0.177	0.187	0.197
0.04	0.039	0.040	0.043	0.047	0.054	0.062	0.071	0.080	0.089	0.099	0.109	0.118	0.128	0.137	0.147	0.157	0.167	0.177	0.187	0.197
0.05	0.049	0.050	0.052	0.055	0.060	0.066	0.074	0.082	0.090	0.100	0.110	0.119	0.129	0.138	0.147	0.157	0.167	0.177	0.187	0.197
0.06	0.060	0.060	0.062	0.064	0.067	0.071	0.078	0.085	0.093	0.102	0.111	0.120	0.130	0.138	0.148	0.157	0.167	0.177	0.187	0.197
0.07	0.070	0.070	0.072	0.073	0.075	0.079	0.083	0.089	0.096	0.105	0.113	0.122	0.131	0.139	0.149	0.158	0.167	0.178	0.187	0.197
0.08	0.080	0.080	0.081	0.082	0.084	0.086	0.090	0.095	0.100	0.108	0.116	0.124	0.133	0.141	0.150	0.159	0.168	0.178	0.188	0.198
0.09	0.090	0.090	0.091	0.092	0.093	0.095	0.098	0.102	0.106	0.112	0.119	0.127	0.135	0.143	0.152	0.161	0.170	0.179	0.189	0.198
0.10	0.099	0.099	0.100	0.101	0.102	0.104	0.106	0.109	0.113	0.118	0.124	0.130	0.138	0.145	0.154	0.162	0.171	0.181	0.190	0.199
0.11	0.110	0.110	0.111	0.112	0.113	0.114	0.116	0.118	0.121	0.125	0.130	0.135	0.142	0.149	0.157	0.165	0.173	0.183	0.191	0.201
0.12	0.120	0.120	0.120	0.121	0.122	0.123	0.125	0.127	0.129	0.132	0.136	0.141	0.146	0.152	0.160	0.167	0.176	0.185	0.193	0.202
0.13	0.130	0.130	0.130	0.131	0.132	0.133	0.134	0.136	0.138	0.140	0.144	0.147	0.152	0.157	0.164	0.171	0.178	0.187	0.195	0.204
0.14	0.139	0.139	0.140	0.140	0.141	0.142	0.143	0.145	0.146	0.148	0.151	0.154	0.158	0.163	0.168	0.175	0.182	0.190	0.198	0.206
0.15	0.149	0.149	0.150	0.150	0.151	0.152	0.153	0.154	0.156	0.157	0.160	0.163	0.166	0.169	0.174	0.180	0.186	0.194	0.201	0.209
0.16	0.159	0.159	0.160	0.160	0.161	0.161	0.162	0.164	0.165	0.167	0.169	0.171	0.174	0.177	0.181	0.186	0.191	0.198	0.205	0.212
0.17	0.168	0.168	0.169	0.169	0.170	0.170	0.171	0.173	0.174	0.175	0.177	0.179	0.181	0.184	0.188	0.191	0.196	0.202	0.209	0.216
0.18	0.179	0.179	0.179	0.179	0.180	0.180	0.181	0.182	0.184	0.185	0.186	0.188	0.190	0.193	0.196	0.199	0.203	0.208	0.214	0.220
0.19	0.189	0.189	0.189	0.190	0.190	0.191	0.191	0.192	0.193	0.195	0.196	0.198	0.199	0.202	0.204	0.207	0.210	0.215	0.220	0.225
0.20	0.199	0.199	0.200	0.200	0.200	0.201	0.202	0.202	0.203	0.204	0.206	0.207	0.209	0.211	0.213	0.215	0.218	0.222	0.226	0.231

Table C.7 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 35.

	Q-Statistic G.O.F. Test Significance Level																			
Skewness G.O.F. Test Significance Level	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.013	0.022	0.031	0.041	0.051	0.061	0.071	0.081	0.091	0.102	0.112	0.122	0.134	0.143	0.153	0.163	0.174	0.184	0.194	0.204
0.02	0.022	0.026	0.034	0.043	0.052	0.061	0.072	0.081	0.092	0.102	0.112	0.123	0.134	0.143	0.153	0.163	0.174	0.184	0.194	0.204
0.03	0.032	0.035	0.039	0.046	0.055	0.063	0.073	0.082	0.092	0.102	0.112	0.123	0.134	0.143	0.153	0.163	0.174	0.184	0.194	0.204
0.04	0.042	0.044	0.047	0.052	0.058	0.066	0.075	0.084	0.094	0.103	0.113	0.124	0.134	0.144	0.154	0.164	0.174	0.185	0.194	0.204
0.05	0.052	0.053	0.056	0.059	0.064	0.070	0.078	0.087	0.096	0.105	0.115	0.125	0.135	0.145	0.154	0.164	0.175	0.185	0.195	0.205
0.06	0.062	0.063	0.065	0.068	0.071	0.076	0.083	0.090	0.099	0.107	0.117	0.127	0.137	0.146	0.155	0.165	0.175	0.186	0.195	0.205
0.07	0.073	0.074	0.076	0.078	0.080	0.084	0.089	0.095	0.103	0.111	0.120	0.129	0.139	0.148	0.157	0.167	0.177	0.187	0.196	0.206
0.08	0.082	0.083	0.085	0.087	0.089	0.092	0.096	0.101	0.107	0.115	0.123	0.132	0.142	0.151	0.159	0.168	0.178	0.188	0.198	0.207
0.09	0.091	0.093	0.094	0.096	0.098	0.100	0.104	0.108	0.113	0.119	0.127	0.135	0.145	0.153	0.162	0.171	0.180	0.190	0.199	0.209
0.10	0.102	0.103	0.104	0.106	0.107	0.110	0.113	0.116	0.120	0.125	0.132	0.139	0.148	0.156	0.164	0.173	0.183	0.192	0.201	0.210
0.11	0.112	0.113	0.114	0.115	0.117	0.119	0.122	0.124	0.128	0.132	0.138	0.144	0.152	0.160	0.167	0.176	0.185	0.194	0.203	0.212
0.12	0.121	0.122	0.123	0.125	0.126	0.128	0.130	0.133	0.136	0.140	0.144	0.150	0.157	0.164	0.171	0.179	0.188	0.197	0.205	0.214
0.13	0.132	0.133	0.134	0.135	0.137	0.138	0.141	0.143	0.146	0.149	0.153	0.157	0.163	0.169	0.176	0.183	0.192	0.200	0.208	0.217
0.14	0.142	0.143	0.144	0.145	0.147	0.148	0.150	0.152	0.155	0.158	0.161	0.165	0.170	0.175	0.181	0.188	0.195	0.203	0.212	0.220
0.15	0.152	0.153	0.154	0.155	0.156	0.158	0.160	0.161	0.164	0.166	0.169	0.173	0.177	0.182	0.187	0.193	0.200	0.207	0.215	0.223
0.16	0.162	0.162	0.163	0.164	0.165	0.167	0.169	0.171	0.173	0.175	0.178	0.181	0.185	0.189	0.193	0.198	0.205	0.212	0.219	0.226
0.17	0.172	0.173	0.174	0.175	0.176	0.177	0.179	0.180	0.183	0.185	0.187	0.190	0.194	0.197	0.201	0.205	0.211	0.217	0.223	0.230
0.18	0.183	0.184	0.184	0.185	0.186	0.187	0.189	0.191	0.193	0.195	0.197	0.200	0.203	0.206	0.209	0.213	0.218	0.224	0.229	0.235
0.19	0.192	0.193	0.194	0.195	0.195	0.197	0.198	0.200	0.202	0.204	0.206	0.208	0.211	0.214	0.217	0.220	0.225	0.230	0.235	0.241
0.20	0.203	0.203	0.204	0.205	0.206	0.207	0.208	0.210	0.211	0.213	0.215	0.218	0.220	0.223	0.225	0.229	0.232	0.237	0.241	0.246

Table C.8 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 40.

	Q-Statistic G.O.F. Test Significance Level																			
Skewness G.O.F. Test Significance Level	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.013	0.022	0.031	0.041	0.051	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.02	0.022	0.027	0.035	0.043	0.052	0.062	0.071	0.081	0.090	0.100	0.110	0.121	0.131	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.03	0.032	0.035	0.040	0.047	0.055	0.064	0.073	0.082	0.091	0.101	0.111	0.121	0.131	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.04	0.042	0.045	0.048	0.053	0.060	0.068	0.076	0.085	0.094	0.103	0.113	0.123	0.132	0.142	0.152	0.162	0.172	0.181	0.192	0.202
0.05	0.052	0.054	0.057	0.060	0.065	0.072	0.080	0.088	0.096	0.106	0.115	0.125	0.134	0.144	0.153	0.163	0.173	0.182	0.192	0.202
0.06	0.062	0.064	0.066	0.069	0.073	0.079	0.085	0.092	0.100	0.109	0.118	0.127	0.137	0.146	0.155	0.165	0.174	0.184	0.194	0.203
0.07	0.072	0.074	0.076	0.079	0.082	0.086	0.091	0.098	0.104	0.112	0.121	0.130	0.139	0.148	0.158	0.167	0.176	0.185	0.195	0.205
0.08	0.082	0.083	0.085	0.088	0.091	0.094	0.098	0.104	0.110	0.117	0.125	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.197	0.206
0.09	0.091	0.093	0.095	0.097	0.100	0.103	0.106	0.111	0.116	0.122	0.129	0.137	0.146	0.154	0.163	0.172	0.181	0.190	0.199	0.208
0.10	0.101	0.103	0.105	0.107	0.109	0.112	0.115	0.119	0.123	0.129	0.135	0.142	0.150	0.158	0.166	0.175	0.184	0.193	0.202	0.211
0.11	0.111	0.113	0.114	0.117	0.119	0.121	0.124	0.127	0.131	0.135	0.141	0.147	0.154	0.162	0.170	0.178	0.187	0.195	0.204	0.213
0.12	0.121	0.123	0.125	0.127	0.129	0.131	0.133	0.136	0.140	0.144	0.149	0.154	0.160	0.167	0.174	0.182	0.190	0.198	0.207	0.216
0.13	0.132	0.133	0.135	0.136	0.139	0.141	0.143	0.146	0.149	0.152	0.157	0.161	0.167	0.173	0.179	0.186	0.194	0.202	0.211	0.219
0.14	0.141	0.143	0.144	0.146	0.148	0.150	0.152	0.155	0.158	0.161	0.165	0.169	0.174	0.179	0.185	0.191	0.199	0.206	0.214	0.222
0.15	0.151	0.152	0.153	0.155	0.157	0.159	0.161	0.164	0.166	0.169	0.173	0.177	0.181	0.186	0.191	0.197	0.204	0.211	0.218	0.226
0.16	0.161	0.162	0.163	0.165	0.167	0.169	0.170	0.173	0.175	0.178	0.181	0.185	0.189	0.193	0.198	0.203	0.209	0.216	0.223	0.230
0.17	0.170	0.171	0.173	0.174	0.176	0.178	0.180	0.182	0.184	0.187	0.190	0.193	0.197	0.201	0.205	0.210	0.215	0.221	0.228	0.235
0.18	0.180	0.181	0.182	0.184	0.185	0.187	0.189	0.191	0.193	0.196	0.199	0.202	0.205	0.209	0.213	0.217	0.222	0.227	0.234	0.240
0.19	0.190	0.191	0.193	0.194	0.196	0.197	0.199	0.201	0.203	0.206	0.209	0.211	0.214	0.218	0.222	0.225	0.230	0.235	0.240	0.246
0.20	0.201	0.202	0.203	0.204	0.206	0.207	0.209	0.211	0.213	0.215	0.218	0.221	0.224	0.227	0.230	0.234	0.238	0.242	0.247	0.253

Table C.9 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.014	0.023	0.031	0.041	0.051	0.060	0.071	0.081	0.091	0.101	0.111	0.120	0.129	0.139	0.149	0.160	0.169	0.179	0.190	0.201
0.02	0.023	0.028	0.035	0.044	0.053	0.062	0.072	0.082	0.092	0.101	0.111	0.121	0.130	0.139	0.150	0.160	0.169	0.179	0.190	0.201
0.03	0.033	0.036	0.040	0.048	0.056	0.064	0.074	0.084	0.094	0.103	0.113	0.122	0.131	0.140	0.150	0.160	0.170	0.180	0.190	0.201
0.04	0.042	0.044	0.048	0.053	0.060	0.068	0.077	0.087	0.096	0.105	0.115	0.124	0.132	0.141	0.152	0.161	0.171	0.181	0.191	0.202
0.05	0.051	0.054	0.057	0.061	0.066	0.072	0.081	0.090	0.099	0.108	0.117	0.126	0.134	0.143	0.153	0.163	0.172	0.182	0.192	0.203
0.06	0.061	0.063	0.066	0.069	0.073	0.078	0.085	0.094	0.102	0.110	0.119	0.128	0.136	0.145	0.155	0.165	0.173	0.183	0.193	0.204
0.07	0.072	0.074	0.076	0.079	0.083	0.087	0.092	0.099	0.106	0.114	0.123	0.131	0.139	0.148	0.158	0.167	0.176	0.185	0.195	0.206
0.08	0.082	0.084	0.086	0.089	0.092	0.096	0.100	0.105	0.112	0.119	0.127	0.134	0.142	0.151	0.160	0.170	0.178	0.188	0.197	0.208
0.09	0.092	0.094	0.096	0.098	0.101	0.104	0.108	0.113	0.118	0.124	0.131	0.138	0.146	0.154	0.163	0.173	0.181	0.190	0.200	0.210
0.10	0.101	0.103	0.105	0.107	0.110	0.113	0.116	0.120	0.125	0.130	0.136	0.143	0.150	0.157	0.166	0.175	0.184	0.193	0.202	0.213
0.11	0.110	0.112	0.114	0.116	0.119	0.121	0.125	0.128	0.132	0.137	0.142	0.148	0.154	0.162	0.170	0.179	0.187	0.196	0.205	0.215
0.12	0.120	0.122	0.124	0.126	0.128	0.131	0.134	0.137	0.141	0.145	0.149	0.155	0.160	0.167	0.175	0.183	0.190	0.199	0.208	0.218
0.13	0.130	0.132	0.133	0.136	0.138	0.140	0.143	0.146	0.149	0.153	0.157	0.162	0.167	0.173	0.180	0.187	0.194	0.203	0.211	0.221
0.14	0.140	0.141	0.143	0.145	0.147	0.149	0.152	0.155	0.158	0.161	0.165	0.169	0.174	0.179	0.185	0.192	0.199	0.206	0.214	0.224
0.15	0.150	0.151	0.153	0.155	0.157	0.159	0.162	0.164	0.167	0.171	0.174	0.178	0.182	0.187	0.192	0.198	0.204	0.211	0.219	0.228
0.16	0.160	0.162	0.163	0.165	0.167	0.169	0.171	0.174	0.176	0.179	0.183	0.186	0.190	0.195	0.200	0.205	0.210	0.217	0.224	0.232
0.17	0.171	0.172	0.174	0.175	0.177	0.179	0.181	0.184	0.186	0.189	0.192	0.195	0.199	0.203	0.207	0.212	0.217	0.223	0.230	0.238
0.18	0.181	0.182	0.183	0.185	0.187	0.189	0.191	0.193	0.196	0.198	0.201	0.204	0.208	0.211	0.215	0.220	0.224	0.230	0.236	0.243
0.19	0.191	0.192	0.193	0.195	0.196	0.198	0.200	0.203	0.205	0.207	0.210	0.213	0.216	0.220	0.223	0.228	0.232	0.237	0.242	0.249
0.20	0.201	0.202	0.203	0.205	0.206	0.208	0.210	0.212	0.214	0.217	0.219	0.222	0.225	0.229	0.232	0.236	0.240	0.245	0.249	0.256

Table C.10 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 0.5; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.014	0.022	0.032	0.042	0.052	0.062	0.072	0.082	0.092	0.102	0.112	0.122	0.133	0.142	0.152	0.162	0.171	0.181	0.190	0.200
0.02	0.023	0.027	0.035	0.044	0.054	0.064	0.073	0.083	0.093	0.103	0.112	0.123	0.134	0.142	0.152	0.162	0.172	0.181	0.190	0.200
0.03	0.032	0.036	0.041	0.048	0.057	0.066	0.076	0.085	0.095	0.104	0.114	0.124	0.135	0.144	0.153	0.163	0.172	0.182	0.190	0.200
0.04	0.042	0.045	0.049	0.055	0.061	0.070	0.079	0.088	0.098	0.107	0.116	0.126	0.136	0.145	0.154	0.164	0.173	0.182	0.191	0.201
0.05	0.052	0.055	0.058	0.063	0.068	0.075	0.083	0.091	0.101	0.109	0.118	0.128	0.138	0.147	0.156	0.165	0.175	0.184	0.192	0.202
0.06	0.063	0.065	0.068	0.072	0.076	0.081	0.088	0.095	0.104	0.112	0.121	0.131	0.141	0.149	0.158	0.167	0.177	0.186	0.194	0.203
0.07	0.073	0.075	0.078	0.081	0.085	0.089	0.095	0.101	0.108	0.116	0.125	0.134	0.144	0.152	0.161	0.170	0.179	0.188	0.196	0.205
0.08	0.082	0.084	0.087	0.090	0.093	0.097	0.102	0.107	0.114	0.121	0.129	0.137	0.147	0.155	0.164	0.172	0.181	0.190	0.198	0.207
0.09	0.093	0.095	0.098	0.101	0.104	0.107	0.111	0.116	0.121	0.127	0.134	0.142	0.151	0.159	0.167	0.176	0.185	0.193	0.201	0.210
0.10	0.103	0.105	0.107	0.110	0.113	0.116	0.120	0.124	0.129	0.134	0.140	0.147	0.156	0.163	0.171	0.179	0.188	0.196	0.204	0.213
0.11	0.113	0.114	0.117	0.119	0.122	0.125	0.129	0.133	0.137	0.141	0.147	0.153	0.161	0.167	0.175	0.183	0.191	0.199	0.207	0.215
0.12	0.122	0.124	0.126	0.128	0.131	0.134	0.137	0.141	0.145	0.149	0.154	0.160	0.166	0.172	0.180	0.187	0.195	0.203	0.210	0.219
0.13	0.132	0.134	0.136	0.138	0.141	0.144	0.147	0.150	0.154	0.157	0.162	0.167	0.173	0.179	0.185	0.192	0.199	0.207	0.214	0.222
0.14	0.142	0.144	0.145	0.148	0.150	0.153	0.156	0.159	0.162	0.166	0.170	0.175	0.180	0.185	0.191	0.197	0.204	0.211	0.218	0.226
0.15	0.152	0.154	0.156	0.158	0.160	0.162	0.165	0.168	0.171	0.175	0.179	0.183	0.188	0.193	0.198	0.203	0.209	0.216	0.223	0.230
0.16	0.162	0.164	0.166	0.167	0.169	0.172	0.174	0.177	0.180	0.184	0.187	0.191	0.196	0.200	0.205	0.210	0.216	0.222	0.228	0.235
0.17	0.172	0.174	0.175	0.177	0.179	0.181	0.184	0.186	0.189	0.193	0.196	0.200	0.205	0.208	0.213	0.218	0.223	0.229	0.234	0.241
0.18	0.183	0.184	0.185	0.187	0.189	0.191	0.193	0.196	0.199	0.202	0.205	0.209	0.213	0.217	0.221	0.225	0.230	0.235	0.240	0.247
0.19	0.192	0.194	0.195	0.197	0.198	0.200	0.203	0.205	0.208	0.211	0.214	0.217	0.221	0.225	0.229	0.233	0.237	0.242	0.247	0.253
0.20	0.202	0.203	0.205	0.206	0.208	0.210	0.212	0.214	0.217	0.220	0.222	0.226	0.230	0.233	0.237	0.241	0.245	0.250	0.254	0.259

C.2 Attained Significance Level Tables for H_o = Weibull(β = 1.0).

Table C.11 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.045	0.055	0.064	0.074	0.084	0.094	0.104	0.114	0.124	0.134	0.144	0.154	0.164	0.173	0.183	0.193	0.203
0.02	0.028	0.036	0.044	0.053	0.061	0.070	0.080	0.089	0.099	0.109	0.119	0.129	0.138	0.148	0.158	0.168	0.178	0.188	0.198	0.207
0.03	0.037	0.045	0.053	0.061	0.069	0.078	0.086	0.095	0.104	0.114	0.123	0.133	0.143	0.153	0.163	0.173	0.182	0.192	0.202	0.212
0.04	0.047	0.054	0.062	0.070	0.078	0.086	0.094	0.103	0.111	0.120	0.129	0.139	0.148	0.158	0.168	0.177	0.187	0.197	0.207	0.217
0.05	0.057	0.064	0.072	0.079	0.087	0.095	0.103	0.111	0.119	0.127	0.136	0.145	0.154	0.163	0.173	0.182	0.192	0.202	0.211	0.221
0.06	0.067	0.074	0.081	0.088	0.096	0.103	0.111	0.119	0.127	0.135	0.143	0.152	0.160	0.169	0.178	0.188	0.197	0.206	0.216	0.226
0.07	0.077	0.084	0.091	0.098	0.105	0.113	0.120	0.128	0.136	0.143	0.151	0.159	0.168	0.177	0.185	0.194	0.203	0.212	0.221	0.231
0.08	0.087	0.093	0.100	0.107	0.114	0.122	0.129	0.137	0.144	0.152	0.160	0.167	0.176	0.184	0.192	0.201	0.209	0.218	0.227	0.236
0.09	0.096	0.103	0.110	0.117	0.124	0.131	0.138	0.146	0.153	0.161	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.225	0.234	0.243
0.10	0.106	0.113	0.120	0.127	0.133	0.140	0.148	0.155	0.162	0.169	0.177	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.241	0.249
0.11	0.116	0.123	0.130	0.136	0.143	0.150	0.157	0.164	0.171	0.178	0.186	0.193	0.201	0.208	0.216	0.224	0.231	0.239	0.248	0.256
0.12	0.126	0.133	0.139	0.146	0.153	0.159	0.166	0.173	0.180	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.246	0.255	0.263
0.13	0.136	0.142	0.149	0.155	0.161	0.168	0.175	0.181	0.188	0.195	0.202	0.209	0.216	0.224	0.231	0.238	0.245	0.253	0.261	0.269
0.14	0.146	0.152	0.158	0.164	0.171	0.177	0.183	0.190	0.196	0.203	0.210	0.217	0.224	0.231	0.238	0.245	0.252	0.260	0.267	0.275
0.15	0.156	0.161	0.168	0.174	0.180	0.186	0.192	0.199	0.205	0.212	0.218	0.225	0.232	0.239	0.246	0.252	0.259	0.267	0.274	0.282
0.16	0.165	0.171	0.177	0.183	0.189	0.195	0.202	0.208	0.214	0.220	0.227	0.233	0.240	0.247	0.253	0.260	0.267	0.274	0.281	0.288
0.17	0.175	0.181	0.187	0.193	0.199	0.205	0.211	0.217	0.223	0.229	0.235	0.242	0.248	0.255	0.261	0.268	0.275	0.281	0.288	0.295
0.18	0.185	0.191	0.197	0.202	0.208	0.214	0.220	0.226	0.232	0.238	0.244	0.250	0.257	0.263	0.269	0.276	0.282	0.289	0.296	0.303
0.19	0.195	0.200	0.206	0.212	0.217	0.223	0.229	0.235	0.241	0.247	0.253	0.259	0.265	0.271	0.277	0.284	0.290	0.296	0.303	0.310
0.20	0.205	0.210	0.216	0.221	0.227	0.232	0.238	0.244	0.249	0.256	0.262	0.267	0.273	0.279	0.286	0.292	0.298	0.304	0.311	0.317

Table C.12 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.035	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.204
0.02	0.025	0.032	0.041	0.050	0.060	0.070	0.079	0.089	0.099	0.109	0.119	0.129	0.139	0.149	0.159	0.169	0.178	0.188	0.198	0.208
0.03	0.035	0.041	0.048	0.056	0.065	0.074	0.084	0.094	0.104	0.113	0.123	0.133	0.143	0.153	0.163	0.172	0.182	0.192	0.202	0.212
0.04	0.045	0.050	0.056	0.063	0.071	0.080	0.089	0.098	0.108	0.117	0.127	0.137	0.147	0.156	0.166	0.176	0.186	0.195	0.205	0.215
0.05	0.055	0.060	0.065	0.071	0.078	0.086	0.094	0.103	0.112	0.121	0.131	0.141	0.150	0.160	0.170	0.179	0.189	0.199	0.208	0.218
0.06	0.065	0.069	0.074	0.080	0.086	0.093	0.100	0.108	0.117	0.126	0.135	0.144	0.154	0.163	0.173	0.183	0.192	0.202	0.211	0.221
0.07	0.075	0.079	0.084	0.089	0.094	0.101	0.107	0.115	0.123	0.131	0.139	0.148	0.158	0.167	0.176	0.186	0.195	0.205	0.214	0.224
0.08	0.084	0.089	0.093	0.098	0.103	0.108	0.115	0.121	0.129	0.136	0.145	0.153	0.162	0.171	0.180	0.189	0.199	0.208	0.218	0.227
0.09	0.094	0.098	0.102	0.107	0.112	0.117	0.123	0.129	0.135	0.142	0.150	0.158	0.166	0.175	0.184	0.193	0.202	0.211	0.221	0.230
0.10	0.104	0.108	0.112	0.116	0.121	0.125	0.131	0.136	0.143	0.149	0.156	0.164	0.172	0.180	0.188	0.197	0.206	0.215	0.224	0.233
0.11	0.113	0.117	0.121	0.125	0.129	0.134	0.139	0.144	0.150	0.156	0.162	0.170	0.177	0.185	0.193	0.201	0.210	0.218	0.227	0.236
0.12	0.123	0.126	0.130	0.134	0.138	0.143	0.147	0.152	0.158	0.163	0.169	0.176	0.183	0.190	0.197	0.205	0.214	0.222	0.231	0.239
0.13	0.133	0.136	0.140	0.143	0.147	0.151	0.156	0.161	0.166	0.171	0.176	0.183	0.189	0.196	0.203	0.210	0.218	0.226	0.234	0.243
0.14	0.142	0.146	0.149	0.153	0.156	0.160	0.165	0.169	0.174	0.179	0.184	0.190	0.196	0.202	0.209	0.216	0.223	0.231	0.239	0.247
0.15	0.152	0.155	0.158	0.162	0.165	0.169	0.173	0.177	0.182	0.187	0.191	0.197	0.203	0.209	0.215	0.222	0.229	0.236	0.243	0.251
0.16	0.162	0.165	0.168	0.171	0.175	0.178	0.182	0.186	0.190	0.195	0.199	0.204	0.210	0.215	0.221	0.227	0.234	0.241	0.248	0.256
0.17	0.172	0.174	0.177	0.181	0.184	0.187	0.191	0.195	0.199	0.203	0.207	0.212	0.217	0.223	0.228	0.234	0.240	0.247	0.253	0.261
0.18	0.182	0.184	0.187	0.190	0.193	0.196	0.200	0.203	0.207	0.211	0.215	0.220	0.225	0.230	0.235	0.241	0.247	0.253	0.259	0.266
0.19	0.191	0.194	0.196	0.199	0.202	0.205	0.209	0.212	0.216	0.220	0.224	0.228	0.233	0.238	0.243	0.248	0.253	0.259	0.265	0.272
0.20	0.201	0.203	0.206	0.209	0.211	0.214	0.218	0.221	0.225	0.228	0.232	0.236	0.241	0.245	0.250	0.255	0.260	0.266	0.272	0.278

Table C.13 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.034	0.044	0.054	0.064	0.074	0.084	0.094	0.104	0.114	0.123	0.133	0.143	0.153	0.163	0.173	0.183	0.193	0.203
0.02	0.025	0.031	0.039	0.048	0.058	0.068	0.077	0.087	0.097	0.106	0.116	0.126	0.136	0.146	0.156	0.165	0.175	0.185	0.195	0.205
0.03	0.034	0.039	0.045	0.053	0.062	0.071	0.081	0.090	0.100	0.110	0.119	0.129	0.139	0.148	0.158	0.168	0.178	0.187	0.197	0.207
0.04	0.044	0.048	0.053	0.059	0.067	0.075	0.084	0.094	0.103	0.113	0.122	0.132	0.142	0.151	0.161	0.170	0.180	0.190	0.199	0.209
0.05	0.053	0.057	0.062	0.067	0.073	0.081	0.089	0.097	0.106	0.116	0.125	0.135	0.144	0.154	0.163	0.173	0.182	0.192	0.202	0.211
0.06	0.063	0.066	0.070	0.075	0.080	0.087	0.094	0.101	0.110	0.119	0.128	0.138	0.147	0.156	0.166	0.175	0.185	0.194	0.204	0.213
0.07	0.072	0.076	0.079	0.084	0.088	0.094	0.100	0.106	0.114	0.123	0.131	0.141	0.150	0.159	0.168	0.178	0.187	0.196	0.206	0.215
0.08	0.082	0.085	0.088	0.092	0.097	0.101	0.107	0.112	0.120	0.127	0.136	0.144	0.153	0.162	0.171	0.180	0.190	0.199	0.208	0.218
0.09	0.092	0.094	0.097	0.101	0.105	0.109	0.114	0.119	0.125	0.132	0.140	0.148	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.220
0.10	0.102	0.104	0.107	0.110	0.114	0.118	0.122	0.127	0.132	0.138	0.145	0.152	0.160	0.169	0.177	0.186	0.195	0.204	0.213	0.222
0.11	0.111	0.114	0.116	0.119	0.123	0.126	0.130	0.134	0.139	0.145	0.151	0.158	0.165	0.173	0.181	0.189	0.198	0.207	0.216	0.225
0.12	0.121	0.123	0.126	0.128	0.131	0.135	0.138	0.142	0.147	0.152	0.157	0.163	0.170	0.177	0.185	0.193	0.201	0.210	0.218	0.227
0.13	0.131	0.133	0.135	0.138	0.141	0.144	0.147	0.151	0.155	0.159	0.165	0.170	0.176	0.183	0.190	0.197	0.205	0.213	0.221	0.230
0.14	0.141	0.143	0.145	0.147	0.150	0.153	0.156	0.159	0.163	0.167	0.172	0.177	0.182	0.188	0.195	0.202	0.209	0.217	0.225	0.233
0.15	0.151	0.152	0.154	0.156	0.159	0.162	0.165	0.168	0.171	0.175	0.179	0.184	0.189	0.195	0.201	0.207	0.214	0.221	0.228	0.236
0.16	0.161	0.162	0.164	0.166	0.168	0.171	0.174	0.176	0.180	0.183	0.187	0.191	0.196	0.201	0.207	0.212	0.219	0.225	0.232	0.240
0.17	0.171	0.172	0.173	0.175	0.178	0.180	0.183	0.185	0.188	0.192	0.195	0.199	0.203	0.208	0.213	0.218	0.224	0.230	0.237	0.244
0.18	0.180	0.182	0.183	0.185	0.187	0.189	0.192	0.194	0.197	0.200	0.204	0.207	0.211	0.215	0.220	0.225	0.230	0.236	0.242	0.249
0.19	0.190	0.191	0.193	0.194	0.196	0.198	0.201	0.203	0.206	0.209	0.212	0.215	0.219	0.223	0.227	0.231	0.236	0.242	0.248	0.254
0.20	0.200	0.201	0.202	0.204	0.206	0.208	0.210	0.212	0.215	0.218	0.221	0.224	0.227	0.231	0.235	0.239	0.243	0.248	0.254	0.259

Table C.14 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.015	0.024	0.034	0.044	0.054	0.063	0.073	0.083	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.162	0.172	0.182	0.192	0.202
0.02	0.024	0.030	0.038	0.047	0.057	0.066	0.076	0.085	0.095	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.183	0.193	0.203
0.03	0.033	0.038	0.044	0.051	0.060	0.069	0.079	0.088	0.098	0.107	0.117	0.127	0.136	0.146	0.156	0.166	0.175	0.185	0.195	0.205
0.04	0.043	0.047	0.051	0.057	0.065	0.073	0.082	0.091	0.101	0.110	0.119	0.129	0.139	0.148	0.158	0.168	0.177	0.187	0.197	0.206
0.05	0.053	0.056	0.060	0.065	0.071	0.078	0.086	0.095	0.104	0.113	0.122	0.132	0.141	0.151	0.160	0.170	0.179	0.189	0.199	0.208
0.06	0.062	0.065	0.068	0.073	0.078	0.084	0.091	0.098	0.107	0.116	0.125	0.134	0.144	0.153	0.162	0.172	0.181	0.191	0.200	0.210
0.07	0.072	0.074	0.078	0.081	0.086	0.090	0.096	0.103	0.111	0.119	0.128	0.137	0.146	0.156	0.165	0.174	0.183	0.193	0.202	0.212
0.08	0.082	0.084	0.087	0.090	0.094	0.098	0.103	0.109	0.116	0.123	0.131	0.140	0.149	0.158	0.167	0.176	0.185	0.195	0.204	0.214
0.09	0.091	0.093	0.096	0.099	0.102	0.106	0.110	0.115	0.121	0.128	0.135	0.143	0.152	0.161	0.169	0.178	0.188	0.197	0.206	0.216
0.10	0.101	0.103	0.105	0.108	0.111	0.114	0.118	0.122	0.128	0.134	0.140	0.148	0.156	0.164	0.172	0.181	0.190	0.199	0.208	0.218
0.11	0.111	0.113	0.115	0.117	0.120	0.123	0.126	0.130	0.135	0.140	0.146	0.153	0.160	0.168	0.176	0.184	0.193	0.202	0.211	0.220
0.12	0.121	0.122	0.124	0.126	0.129	0.132	0.135	0.138	0.142	0.147	0.152	0.158	0.165	0.172	0.179	0.187	0.196	0.204	0.213	0.222
0.13	0.131	0.132	0.133	0.136	0.138	0.141	0.143	0.147	0.150	0.154	0.159	0.164	0.170	0.176	0.183	0.191	0.199	0.207	0.216	0.224
0.14	0.140	0.142	0.143	0.145	0.147	0.150	0.152	0.155	0.158	0.162	0.166	0.171	0.176	0.182	0.188	0.195	0.202	0.210	0.218	0.227
0.15	0.150	0.151	0.153	0.154	0.157	0.159	0.161	0.164	0.167	0.170	0.174	0.178	0.183	0.188	0.193	0.200	0.206	0.213	0.221	0.230
0.16	0.160	0.161	0.162	0.164	0.166	0.168	0.170	0.173	0.175	0.178	0.182	0.186	0.190	0.194	0.199	0.205	0.211	0.218	0.225	0.233
0.17	0.170	0.171	0.172	0.174	0.175	0.177	0.179	0.182	0.184	0.187	0.190	0.194	0.197	0.201	0.206	0.211	0.216	0.222	0.229	0.237
0.18	0.180	0.181	0.182	0.183	0.185	0.187	0.189	0.191	0.193	0.196	0.199	0.202	0.205	0.209	0.213	0.217	0.222	0.228	0.234	0.241
0.19	0.190	0.191	0.191	0.193	0.194	0.196	0.198	0.200	0.202	0.204	0.207	0.210	0.213	0.216	0.220	0.224	0.229	0.234	0.240	0.246
0.20	0.200	0.201	0.201	0.202	0.204	0.205	0.207	0.209	0.211	0.213	0.216	0.219	0.221	0.224	0.228	0.232	0.236	0.240	0.246	0.251

Table C.15 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level (alpha)																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.015	0.024	0.033	0.043	0.053	0.063	0.072	0.082	0.092	0.102	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.02	0.024	0.030	0.038	0.046	0.056	0.065	0.075	0.085	0.094	0.104	0.114	0.123	0.133	0.143	0.153	0.162	0.172	0.182	0.192	0.202
0.03	0.033	0.038	0.044	0.051	0.060	0.068	0.078	0.087	0.096	0.106	0.116	0.125	0.135	0.145	0.154	0.164	0.174	0.183	0.193	0.203
0.04	0.042	0.046	0.051	0.057	0.065	0.072	0.081	0.090	0.099	0.108	0.118	0.127	0.137	0.146	0.156	0.166	0.175	0.185	0.194	0.204
0.05	0.052	0.055	0.059	0.064	0.070	0.077	0.085	0.093	0.102	0.111	0.120	0.129	0.139	0.148	0.158	0.167	0.177	0.186	0.196	0.206
0.06	0.062	0.064	0.068	0.072	0.077	0.083	0.090	0.098	0.106	0.114	0.123	0.132	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207
0.07	0.071	0.073	0.076	0.080	0.084	0.090	0.096	0.102	0.110	0.118	0.126	0.134	0.143	0.152	0.162	0.171	0.180	0.189	0.199	0.208
0.08	0.081	0.083	0.085	0.088	0.092	0.097	0.102	0.108	0.115	0.122	0.130	0.138	0.146	0.155	0.164	0.173	0.182	0.191	0.200	0.210
0.09	0.091	0.092	0.095	0.097	0.101	0.104	0.109	0.114	0.120	0.127	0.134	0.142	0.150	0.158	0.167	0.175	0.184	0.193	0.202	0.212
0.10	0.101	0.102	0.104	0.106	0.109	0.112	0.116	0.121	0.126	0.132	0.139	0.146	0.154	0.161	0.170	0.178	0.187	0.195	0.204	0.213
0.11	0.111	0.112	0.113	0.115	0.118	0.121	0.124	0.128	0.133	0.138	0.144	0.151	0.158	0.165	0.173	0.181	0.189	0.198	0.206	0.215
0.12	0.120	0.121	0.123	0.125	0.127	0.129	0.133	0.136	0.140	0.145	0.150	0.157	0.163	0.170	0.177	0.185	0.192	0.201	0.209	0.218
0.13	0.130	0.131	0.132	0.134	0.136	0.138	0.141	0.144	0.148	0.152	0.157	0.162	0.168	0.175	0.181	0.189	0.196	0.204	0.212	0.220
0.14	0.140	0.141	0.142	0.143	0.145	0.147	0.150	0.152	0.156	0.159	0.164	0.169	0.174	0.180	0.186	0.193	0.200	0.207	0.215	0.223
0.15	0.150	0.151	0.152	0.153	0.154	0.156	0.159	0.161	0.164	0.167	0.171	0.176	0.180	0.186	0.192	0.198	0.204	0.211	0.219	0.227
0.16	0.160	0.161	0.161	0.163	0.164	0.166	0.168	0.170	0.172	0.175	0.179	0.183	0.187	0.192	0.197	0.203	0.209	0.216	0.223	0.230
0.17	0.170	0.171	0.171	0.172	0.173	0.175	0.177	0.179	0.181	0.184	0.187	0.190	0.194	0.199	0.204	0.209	0.215	0.221	0.227	0.234
0.18	0.180	0.180	0.181	0.182	0.183	0.184	0.186	0.188	0.190	0.192	0.195	0.198	0.202	0.206	0.210	0.215	0.220	0.226	0.232	0.239
0.19	0.190	0.190	0.191	0.192	0.193	0.194	0.195	0.197	0.199	0.201	0.204	0.207	0.210	0.213	0.217	0.222	0.227	0.232	0.237	0.243
0.20	0.200	0.200	0.201	0.201	0.202	0.203	0.205	0.206	0.208	0.210	0.212	0.215	0.218	0.221	0.224	0.228	0.233	0.237	0.242	0.248

Table C.16 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.024	0.033	0.042	0.052	0.062	0.072	0.082	0.091	0.101	0.111	0.121	0.131	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.02	0.024	0.030	0.038	0.046	0.055	0.064	0.074	0.083	0.093	0.103	0.112	0.122	0.132	0.142	0.152	0.162	0.171	0.181	0.191	0.201
0.03	0.033	0.038	0.044	0.051	0.059	0.068	0.076	0.086	0.095	0.104	0.114	0.124	0.133	0.143	0.153	0.163	0.172	0.182	0.192	0.202
0.04	0.042	0.046	0.051	0.057	0.064	0.072	0.080	0.089	0.098	0.107	0.116	0.125	0.135	0.144	0.154	0.164	0.173	0.183	0.193	0.203
0.05	0.051	0.055	0.059	0.064	0.071	0.078	0.085	0.093	0.102	0.110	0.119	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.204
0.06	0.061	0.064	0.067	0.072	0.078	0.084	0.091	0.098	0.106	0.114	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.186	0.195	0.205
0.07	0.071	0.073	0.076	0.080	0.085	0.090	0.096	0.103	0.111	0.118	0.126	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.197	0.206
0.08	0.080	0.082	0.085	0.088	0.092	0.097	0.103	0.109	0.116	0.123	0.130	0.138	0.146	0.154	0.163	0.171	0.180	0.189	0.199	0.208
0.09	0.090	0.092	0.094	0.096	0.100	0.105	0.110	0.115	0.122	0.128	0.135	0.142	0.150	0.158	0.166	0.175	0.183	0.192	0.201	0.210
0.10	0.100	0.101	0.103	0.105	0.108	0.112	0.117	0.122	0.128	0.134	0.140	0.147	0.155	0.162	0.170	0.178	0.186	0.195	0.203	0.212
0.11	0.110	0.111	0.113	0.114	0.117	0.120	0.124	0.129	0.134	0.140	0.146	0.152	0.159	0.166	0.174	0.181	0.189	0.197	0.206	0.215
0.12	0.120	0.121	0.122	0.124	0.126	0.129	0.132	0.136	0.141	0.146	0.152	0.158	0.164	0.171	0.178	0.186	0.193	0.201	0.209	0.218
0.13	0.130	0.131	0.132	0.133	0.135	0.138	0.140	0.144	0.148	0.153	0.158	0.164	0.170	0.176	0.183	0.190	0.197	0.204	0.212	0.220
0.14	0.140	0.141	0.141	0.143	0.144	0.146	0.149	0.152	0.156	0.160	0.165	0.170	0.175	0.182	0.188	0.194	0.201	0.208	0.216	0.224
0.15	0.150	0.150	0.151	0.152	0.154	0.155	0.157	0.160	0.163	0.167	0.171	0.176	0.181	0.187	0.193	0.199	0.206	0.213	0.220	0.227
0.16	0.160	0.160	0.161	0.162	0.163	0.165	0.166	0.168	0.171	0.175	0.179	0.183	0.188	0.193	0.199	0.204	0.211	0.217	0.224	0.231
0.17	0.170	0.170	0.171	0.171	0.172	0.174	0.175	0.177	0.179	0.183	0.186	0.190	0.194	0.199	0.204	0.210	0.216	0.222	0.228	0.235
0.18	0.180	0.180	0.181	0.181	0.182	0.183	0.184	0.186	0.188	0.191	0.194	0.197	0.201	0.206	0.210	0.215	0.221	0.227	0.233	0.239
0.19	0.190	0.190	0.191	0.191	0.192	0.193	0.194	0.195	0.197	0.199	0.202	0.205	0.208	0.212	0.217	0.221	0.227	0.232	0.238	0.244
0.20	0.200	0.200	0.200	0.201	0.202	0.202	0.203	0.205	0.206	0.208	0.210	0.213	0.216	0.220	0.224	0.228	0.233	0.238	0.243	0.249

Table C.17 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.024	0.033	0.042	0.052	0.061	0.071	0.081	0.091	0.101	0.111	0.121	0.131	0.141	0.151	0.160	0.170	0.180	0.190	0.200
0.02	0.024	0.031	0.038	0.047	0.056	0.065	0.074	0.083	0.093	0.102	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.03	0.033	0.039	0.045	0.053	0.061	0.069	0.077	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.152	0.162	0.172	0.182	0.192	0.201
0.04	0.042	0.047	0.052	0.059	0.066	0.074	0.082	0.090	0.099	0.107	0.117	0.126	0.135	0.144	0.154	0.163	0.173	0.183	0.192	0.202
0.05	0.051	0.055	0.060	0.066	0.072	0.080	0.087	0.095	0.103	0.111	0.120	0.128	0.137	0.147	0.156	0.165	0.175	0.184	0.194	0.203
0.06	0.061	0.064	0.068	0.073	0.079	0.085	0.092	0.100	0.107	0.115	0.123	0.132	0.140	0.149	0.158	0.167	0.177	0.186	0.195	0.205
0.07	0.070	0.072	0.076	0.081	0.086	0.092	0.098	0.105	0.112	0.120	0.128	0.136	0.144	0.152	0.161	0.170	0.179	0.188	0.197	0.206
0.08	0.080	0.082	0.085	0.089	0.093	0.099	0.105	0.111	0.117	0.124	0.132	0.140	0.148	0.156	0.164	0.173	0.181	0.190	0.199	0.208
0.09	0.090	0.091	0.094	0.097	0.101	0.106	0.112	0.117	0.123	0.130	0.137	0.145	0.152	0.160	0.168	0.176	0.184	0.193	0.202	0.211
0.10	0.100	0.101	0.103	0.106	0.109	0.113	0.118	0.124	0.129	0.135	0.142	0.149	0.156	0.164	0.171	0.179	0.187	0.196	0.204	0.213
0.11	0.110	0.111	0.112	0.114	0.117	0.121	0.126	0.130	0.136	0.141	0.148	0.154	0.161	0.168	0.176	0.183	0.191	0.199	0.207	0.216
0.12	0.120	0.120	0.122	0.123	0.126	0.129	0.133	0.138	0.143	0.148	0.154	0.160	0.167	0.173	0.180	0.188	0.195	0.203	0.211	0.219
0.13	0.130	0.130	0.131	0.133	0.135	0.138	0.141	0.145	0.149	0.154	0.160	0.166	0.172	0.178	0.185	0.192	0.199	0.207	0.214	0.222
0.14	0.140	0.140	0.141	0.142	0.144	0.146	0.149	0.153	0.157	0.161	0.166	0.172	0.177	0.184	0.190	0.197	0.204	0.211	0.218	0.226
0.15	0.150	0.150	0.151	0.152	0.153	0.155	0.157	0.160	0.164	0.168	0.173	0.178	0.184	0.189	0.195	0.202	0.208	0.215	0.222	0.230
0.16	0.160	0.160	0.160	0.161	0.162	0.164	0.166	0.169	0.172	0.176	0.180	0.185	0.190	0.195	0.201	0.207	0.213	0.220	0.227	0.234
0.17	0.170	0.170	0.170	0.171	0.172	0.173	0.175	0.177	0.180	0.183	0.187	0.192	0.196	0.201	0.207	0.212	0.218	0.225	0.231	0.238
0.18	0.180	0.180	0.180	0.181	0.182	0.182	0.184	0.186	0.188	0.191	0.195	0.199	0.203	0.208	0.212	0.218	0.223	0.229	0.236	0.243
0.19	0.190	0.190	0.190	0.191	0.191	0.192	0.193	0.195	0.197	0.200	0.203	0.206	0.210	0.214	0.219	0.224	0.229	0.235	0.241	0.247
0.20	0.200	0.200	0.200	0.200	0.201	0.202	0.203	0.204	0.206	0.208	0.211	0.214	0.217	0.221	0.226	0.230	0.235	0.241	0.246	0.253

Table C.18 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.025	0.034	0.043	0.052	0.062	0.071	0.081	0.091	0.101	0.111	0.121	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.200
0.02	0.025	0.032	0.040	0.048	0.056	0.065	0.075	0.084	0.093	0.103	0.112	0.122	0.132	0.142	0.151	0.161	0.171	0.181	0.191	0.201
0.03	0.034	0.039	0.046	0.054	0.062	0.070	0.079	0.088	0.096	0.106	0.115	0.124	0.134	0.143	0.153	0.163	0.172	0.182	0.192	0.201
0.04	0.043	0.048	0.054	0.061	0.068	0.076	0.084	0.092	0.100	0.109	0.118	0.127	0.136	0.146	0.155	0.164	0.174	0.183	0.193	0.202
0.05	0.052	0.056	0.062	0.068	0.074	0.081	0.089	0.097	0.105	0.113	0.122	0.130	0.139	0.148	0.157	0.166	0.176	0.185	0.194	0.204
0.06	0.061	0.065	0.070	0.075	0.081	0.087	0.094	0.102	0.109	0.117	0.126	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.196	0.206
0.07	0.071	0.074	0.078	0.083	0.088	0.094	0.100	0.107	0.114	0.122	0.130	0.138	0.146	0.155	0.163	0.172	0.181	0.190	0.199	0.208
0.08	0.081	0.083	0.086	0.090	0.095	0.101	0.106	0.113	0.119	0.127	0.134	0.142	0.150	0.158	0.167	0.175	0.183	0.192	0.201	0.210
0.09	0.090	0.092	0.095	0.098	0.103	0.108	0.113	0.119	0.125	0.132	0.139	0.147	0.154	0.162	0.170	0.178	0.186	0.195	0.203	0.212
0.10	0.100	0.101	0.104	0.107	0.110	0.115	0.120	0.125	0.131	0.138	0.144	0.151	0.159	0.166	0.174	0.182	0.190	0.198	0.206	0.215
0.11	0.110	0.111	0.113	0.115	0.119	0.123	0.127	0.132	0.138	0.144	0.150	0.157	0.164	0.171	0.178	0.186	0.194	0.202	0.210	0.218
0.12	0.120	0.121	0.122	0.124	0.127	0.131	0.135	0.139	0.144	0.150	0.156	0.162	0.169	0.176	0.183	0.190	0.198	0.205	0.213	0.221
0.13	0.130	0.130	0.132	0.133	0.135	0.139	0.142	0.147	0.151	0.156	0.162	0.168	0.174	0.181	0.188	0.195	0.202	0.209	0.217	0.225
0.14	0.140	0.140	0.141	0.142	0.144	0.147	0.150	0.154	0.158	0.163	0.169	0.174	0.180	0.186	0.193	0.200	0.207	0.214	0.221	0.229
0.15	0.150	0.150	0.151	0.152	0.153	0.156	0.158	0.162	0.166	0.170	0.175	0.180	0.186	0.192	0.198	0.204	0.211	0.218	0.225	0.232
0.16	0.160	0.160	0.161	0.161	0.163	0.164	0.167	0.170	0.173	0.177	0.182	0.187	0.192	0.198	0.204	0.210	0.216	0.223	0.230	0.237
0.17	0.170	0.170	0.170	0.171	0.172	0.174	0.176	0.178	0.181	0.185	0.189	0.194	0.199	0.204	0.209	0.215	0.221	0.228	0.234	0.241
0.18	0.180	0.180	0.180	0.181	0.182	0.183	0.184	0.187	0.189	0.193	0.197	0.201	0.205	0.210	0.215	0.221	0.227	0.233	0.239	0.246
0.19	0.190	0.190	0.190	0.191	0.191	0.192	0.194	0.196	0.198	0.201	0.204	0.208	0.212	0.217	0.222	0.227	0.233	0.239	0.245	0.251
0.20	0.200	0.200	0.200	0.200	0.201	0.202	0.203	0.204	0.206	0.209	0.212	0.215	0.219	0.224	0.228	0.233	0.238	0.244	0.250	0.256

Table C.19 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.034	0.043	0.052	0.062	0.071	0.081	0.091	0.101	0.111	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
0.02	0.025	0.032	0.039	0.048	0.056	0.065	0.074	0.084	0.093	0.102	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.03	0.033	0.039	0.046	0.053	0.061	0.069	0.078	0.087	0.096	0.105	0.114	0.124	0.133	0.143	0.152	0.162	0.172	0.182	0.191	0.201
0.04	0.042	0.047	0.053	0.059	0.066	0.074	0.082	0.091	0.099	0.108	0.117	0.126	0.136	0.145	0.154	0.164	0.173	0.183	0.193	0.202
0.05	0.052	0.056	0.060	0.066	0.073	0.080	0.088	0.096	0.104	0.112	0.121	0.130	0.139	0.148	0.157	0.166	0.175	0.185	0.194	0.204
0.06	0.061	0.064	0.068	0.073	0.079	0.086	0.093	0.101	0.108	0.116	0.124	0.133	0.142	0.150	0.159	0.168	0.178	0.187	0.196	0.205
0.07	0.071	0.073	0.076	0.081	0.086	0.092	0.099	0.106	0.114	0.121	0.129	0.137	0.145	0.154	0.163	0.171	0.180	0.189	0.198	0.208
0.08	0.080	0.082	0.085	0.089	0.094	0.099	0.105	0.112	0.119	0.126	0.134	0.141	0.149	0.158	0.166	0.175	0.183	0.192	0.201	0.210
0.09	0.090	0.091	0.094	0.097	0.101	0.107	0.112	0.118	0.125	0.132	0.139	0.146	0.154	0.162	0.170	0.178	0.187	0.195	0.204	0.212
0.10	0.100	0.101	0.103	0.106	0.109	0.114	0.119	0.125	0.131	0.138	0.144	0.151	0.159	0.166	0.174	0.182	0.190	0.198	0.207	0.215
0.11	0.110	0.111	0.112	0.114	0.118	0.122	0.126	0.132	0.137	0.144	0.150	0.157	0.164	0.171	0.178	0.186	0.194	0.202	0.210	0.218
0.12	0.120	0.120	0.122	0.124	0.126	0.130	0.134	0.139	0.144	0.150	0.156	0.162	0.169	0.176	0.183	0.190	0.198	0.206	0.213	0.221
0.13	0.130	0.130	0.131	0.133	0.135	0.138	0.141	0.146	0.151	0.156	0.162	0.168	0.174	0.181	0.188	0.195	0.202	0.209	0.217	0.225
0.14	0.140	0.140	0.141	0.142	0.144	0.146	0.149	0.154	0.158	0.163	0.168	0.174	0.180	0.186	0.193	0.199	0.207	0.214	0.221	0.229
0.15	0.150	0.150	0.150	0.151	0.153	0.155	0.158	0.161	0.165	0.170	0.175	0.180	0.186	0.191	0.198	0.204	0.211	0.218	0.225	0.233
0.16	0.160	0.160	0.160	0.161	0.162	0.164	0.166	0.169	0.173	0.177	0.182	0.187	0.192	0.197	0.203	0.210	0.216	0.223	0.230	0.237
0.17	0.170	0.170	0.170	0.171	0.171	0.173	0.175	0.178	0.181	0.185	0.189	0.193	0.198	0.203	0.209	0.215	0.221	0.228	0.234	0.241
0.18	0.180	0.180	0.180	0.180	0.181	0.182	0.184	0.186	0.189	0.192	0.196	0.200	0.205	0.210	0.215	0.221	0.227	0.233	0.239	0.246
0.19	0.190	0.190	0.190	0.190	0.191	0.192	0.193	0.195	0.197	0.200	0.204	0.208	0.212	0.216	0.221	0.227	0.232	0.238	0.244	0.251
0.20	0.200	0.200	0.200	0.200	0.201	0.201	0.202	0.204	0.206	0.208	0.211	0.215	0.219	0.223	0.228	0.232	0.238	0.244	0.249	0.256

Table C.20 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.0; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.033	0.042	0.052	0.061	0.071	0.081	0.091	0.101	0.111	0.121	0.130	0.140	0.150	0.160	0.170	0.180	0.190	0.200
0.02	0.024	0.031	0.039	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.181	0.191	0.201
0.03	0.033	0.039	0.045	0.053	0.061	0.069	0.078	0.087	0.096	0.105	0.115	0.124	0.133	0.143	0.153	0.162	0.172	0.182	0.192	0.201
0.04	0.042	0.047	0.053	0.059	0.067	0.074	0.083	0.091	0.100	0.109	0.117	0.126	0.136	0.145	0.155	0.164	0.174	0.183	0.193	0.203
0.05	0.051	0.055	0.060	0.066	0.073	0.080	0.087	0.095	0.104	0.112	0.121	0.129	0.139	0.148	0.157	0.166	0.175	0.185	0.194	0.204
0.06	0.061	0.064	0.068	0.073	0.079	0.086	0.093	0.100	0.108	0.116	0.125	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.206
0.07	0.070	0.073	0.076	0.081	0.087	0.093	0.099	0.106	0.114	0.122	0.129	0.137	0.146	0.155	0.163	0.172	0.181	0.190	0.199	0.208
0.08	0.080	0.082	0.085	0.089	0.094	0.100	0.106	0.112	0.119	0.126	0.134	0.141	0.150	0.158	0.167	0.175	0.183	0.192	0.201	0.210
0.09	0.090	0.091	0.094	0.098	0.102	0.107	0.112	0.118	0.125	0.132	0.139	0.146	0.154	0.162	0.170	0.178	0.187	0.195	0.204	0.212
0.10	0.100	0.101	0.103	0.106	0.110	0.114	0.119	0.125	0.131	0.137	0.144	0.151	0.158	0.166	0.174	0.182	0.190	0.198	0.207	0.215
0.11	0.110	0.111	0.112	0.115	0.118	0.122	0.126	0.131	0.137	0.143	0.150	0.156	0.163	0.171	0.178	0.186	0.194	0.202	0.210	0.218
0.12	0.120	0.120	0.122	0.123	0.126	0.129	0.134	0.138	0.144	0.149	0.155	0.162	0.169	0.176	0.183	0.190	0.198	0.205	0.213	0.222
0.13	0.130	0.130	0.131	0.133	0.135	0.138	0.142	0.146	0.151	0.156	0.162	0.167	0.174	0.181	0.188	0.195	0.202	0.209	0.217	0.225
0.14	0.140	0.140	0.141	0.142	0.144	0.146	0.150	0.153	0.158	0.163	0.168	0.174	0.180	0.186	0.193	0.199	0.206	0.214	0.221	0.229
0.15	0.150	0.150	0.150	0.151	0.153	0.155	0.158	0.161	0.165	0.170	0.175	0.180	0.186	0.192	0.198	0.204	0.211	0.218	0.225	0.233
0.16	0.160	0.160	0.160	0.161	0.162	0.164	0.166	0.169	0.173	0.177	0.182	0.187	0.192	0.198	0.204	0.210	0.216	0.223	0.230	0.237
0.17	0.170	0.170	0.170	0.171	0.172	0.173	0.175	0.178	0.181	0.185	0.189	0.194	0.199	0.204	0.210	0.215	0.222	0.228	0.234	0.241
0.18	0.180	0.180	0.180	0.181	0.181	0.183	0.184	0.186	0.189	0.193	0.196	0.201	0.205	0.211	0.216	0.221	0.227	0.233	0.239	0.246
0.19	0.190	0.190	0.190	0.190	0.191	0.192	0.193	0.195	0.197	0.200	0.204	0.208	0.212	0.217	0.222	0.227	0.232	0.238	0.244	0.250
0.20	0.200	0.200	0.200	0.200	0.201	0.201	0.203	0.204	0.206	0.209	0.212	0.215	0.219	0.224	0.228	0.233	0.238	0.244	0.249	0.256

C.3 Attained Significance Level Tables for H_o = Weibull(β = 1.5).

Table C.21 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.113	0.123	0.133	0.142	0.152	0.162	0.172	0.182	0.192	0.202
0.02	0.028	0.037	0.046	0.054	0.063	0.072	0.081	0.090	0.099	0.108	0.118	0.127	0.137	0.146	0.156	0.166	0.176	0.185	0.195	0.205
0.03	0.038	0.047	0.055	0.063	0.072	0.080	0.089	0.098	0.106	0.115	0.124	0.133	0.143	0.152	0.161	0.171	0.180	0.190	0.199	0.209
0.04	0.048	0.056	0.064	0.072	0.081	0.089	0.097	0.106	0.114	0.123	0.132	0.140	0.149	0.158	0.167	0.176	0.186	0.195	0.204	0.214
0.05	0.058	0.066	0.074	0.082	0.090	0.098	0.106	0.114	0.122	0.131	0.139	0.148	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.219
0.06	0.067	0.075	0.083	0.091	0.099	0.107	0.115	0.123	0.131	0.139	0.147	0.156	0.164	0.173	0.181	0.190	0.199	0.207	0.216	0.225
0.07	0.077	0.085	0.093	0.100	0.108	0.116	0.124	0.132	0.139	0.148	0.156	0.164	0.172	0.180	0.189	0.197	0.206	0.215	0.223	0.232
0.08	0.087	0.095	0.102	0.110	0.118	0.125	0.133	0.140	0.148	0.156	0.164	0.172	0.180	0.188	0.197	0.205	0.213	0.222	0.230	0.239
0.09	0.097	0.105	0.112	0.120	0.127	0.135	0.142	0.150	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.221	0.230	0.238	0.246
0.10	0.107	0.114	0.122	0.129	0.137	0.144	0.151	0.159	0.166	0.174	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.254
0.11	0.117	0.124	0.132	0.139	0.146	0.154	0.161	0.168	0.176	0.183	0.191	0.199	0.207	0.214	0.222	0.230	0.238	0.246	0.254	0.262
0.12	0.127	0.134	0.141	0.149	0.156	0.163	0.170	0.178	0.185	0.192	0.200	0.208	0.216	0.223	0.231	0.239	0.247	0.255	0.262	0.270
0.13	0.137	0.144	0.151	0.158	0.166	0.173	0.180	0.187	0.194	0.202	0.210	0.217	0.225	0.232	0.240	0.247	0.255	0.263	0.271	0.279
0.14	0.147	0.154	0.161	0.168	0.175	0.182	0.189	0.196	0.204	0.211	0.219	0.226	0.234	0.241	0.249	0.256	0.264	0.272	0.279	0.287
0.15	0.157	0.164	0.171	0.178	0.185	0.192	0.199	0.206	0.213	0.220	0.228	0.235	0.243	0.250	0.258	0.265	0.273	0.280	0.288	0.295
0.16	0.167	0.174	0.181	0.188	0.194	0.202	0.208	0.215	0.223	0.230	0.237	0.244	0.252	0.259	0.266	0.274	0.281	0.289	0.296	0.304
0.17	0.177	0.184	0.190	0.197	0.204	0.211	0.218	0.225	0.232	0.239	0.246	0.254	0.261	0.268	0.275	0.283	0.290	0.297	0.305	0.312
0.18	0.186	0.193	0.200	0.207	0.214	0.221	0.228	0.235	0.241	0.249	0.256	0.263	0.270	0.277	0.285	0.292	0.299	0.306	0.314	0.321
0.19	0.196	0.203	0.210	0.217	0.224	0.231	0.237	0.244	0.251	0.258	0.265	0.272	0.279	0.287	0.294	0.301	0.308	0.315	0.323	0.330
0.20	0.206	0.213	0.220	0.227	0.233	0.240	0.247	0.254	0.261	0.268	0.275	0.282	0.289	0.296	0.303	0.310	0.317	0.324	0.331	0.338

Table C.22 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.035	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.034	0.043	0.051	0.061	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.139	0.149	0.159	0.169	0.179	0.189	0.199	0.209
0.03	0.036	0.044	0.051	0.059	0.068	0.077	0.086	0.096	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204	0.213
0.04	0.046	0.053	0.060	0.068	0.076	0.084	0.093	0.102	0.111	0.120	0.130	0.140	0.149	0.159	0.169	0.179	0.188	0.198	0.208	0.218
0.05	0.056	0.062	0.069	0.077	0.084	0.092	0.101	0.109	0.118	0.127	0.136	0.146	0.155	0.164	0.174	0.184	0.193	0.203	0.213	0.222
0.06	0.066	0.072	0.079	0.086	0.093	0.101	0.109	0.117	0.125	0.134	0.143	0.152	0.161	0.170	0.179	0.189	0.198	0.208	0.217	0.227
0.07	0.076	0.082	0.088	0.095	0.102	0.109	0.117	0.125	0.133	0.141	0.150	0.159	0.167	0.176	0.185	0.194	0.204	0.213	0.222	0.232
0.08	0.086	0.092	0.098	0.104	0.111	0.118	0.125	0.133	0.141	0.149	0.157	0.165	0.174	0.182	0.191	0.200	0.209	0.218	0.227	0.236
0.09	0.096	0.101	0.107	0.114	0.120	0.127	0.134	0.141	0.149	0.156	0.164	0.172	0.180	0.189	0.197	0.206	0.214	0.223	0.232	0.241
0.10	0.105	0.111	0.117	0.123	0.130	0.136	0.143	0.150	0.157	0.164	0.172	0.180	0.188	0.196	0.204	0.212	0.221	0.229	0.238	0.247
0.11	0.115	0.121	0.127	0.133	0.139	0.145	0.152	0.159	0.165	0.173	0.180	0.188	0.195	0.203	0.211	0.219	0.227	0.235	0.244	0.252
0.12	0.125	0.131	0.137	0.142	0.148	0.155	0.161	0.168	0.174	0.181	0.188	0.196	0.203	0.211	0.218	0.226	0.234	0.242	0.250	0.259
0.13	0.135	0.141	0.146	0.152	0.158	0.164	0.170	0.176	0.183	0.190	0.197	0.204	0.211	0.218	0.226	0.233	0.241	0.249	0.257	0.265
0.14	0.145	0.151	0.156	0.161	0.167	0.173	0.179	0.185	0.192	0.198	0.205	0.212	0.219	0.226	0.233	0.241	0.248	0.256	0.264	0.271
0.15	0.155	0.160	0.166	0.171	0.177	0.182	0.188	0.194	0.200	0.207	0.213	0.220	0.227	0.234	0.241	0.248	0.255	0.263	0.270	0.278
0.16	0.165	0.170	0.175	0.181	0.186	0.192	0.197	0.203	0.209	0.216	0.222	0.229	0.235	0.242	0.248	0.255	0.263	0.270	0.277	0.285
0.17	0.175	0.180	0.185	0.190	0.196	0.201	0.207	0.212	0.218	0.224	0.230	0.237	0.243	0.250	0.256	0.263	0.270	0.277	0.285	0.292
0.18	0.185	0.190	0.195	0.200	0.205	0.210	0.216	0.221	0.227	0.233	0.239	0.245	0.252	0.258	0.264	0.271	0.278	0.285	0.292	0.299
0.19	0.195	0.199	0.204	0.209	0.214	0.220	0.225	0.230	0.236	0.242	0.248	0.254	0.260	0.266	0.272	0.279	0.285	0.292	0.299	0.306
0.20	0.205	0.209	0.214	0.219	0.224	0.229	0.234	0.239	0.245	0.250	0.256	0.262	0.268	0.274	0.280	0.287	0.293	0.300	0.307	0.313

Table C.23 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.035	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.026	0.033	0.041	0.051	0.060	0.070	0.080	0.090	0.100	0.109	0.119	0.129	0.139	0.149	0.159	0.169	0.179	0.189	0.199	0.209
0.03	0.036	0.042	0.049	0.057	0.066	0.075	0.085	0.094	0.104	0.114	0.124	0.134	0.143	0.153	0.163	0.173	0.183	0.193	0.203	0.213
0.04	0.045	0.051	0.058	0.065	0.073	0.082	0.091	0.100	0.109	0.118	0.128	0.138	0.148	0.157	0.167	0.177	0.187	0.197	0.206	0.216
0.05	0.055	0.061	0.067	0.074	0.081	0.089	0.097	0.106	0.114	0.123	0.133	0.142	0.152	0.161	0.171	0.181	0.191	0.200	0.210	0.220
0.06	0.065	0.070	0.076	0.082	0.089	0.097	0.104	0.112	0.121	0.129	0.138	0.147	0.157	0.166	0.175	0.185	0.195	0.204	0.214	0.223
0.07	0.075	0.080	0.086	0.092	0.098	0.105	0.112	0.120	0.127	0.136	0.144	0.153	0.162	0.171	0.180	0.189	0.199	0.208	0.218	0.227
0.08	0.085	0.090	0.095	0.101	0.107	0.113	0.120	0.127	0.135	0.142	0.150	0.159	0.167	0.176	0.185	0.194	0.203	0.212	0.222	0.231
0.09	0.094	0.099	0.104	0.110	0.116	0.122	0.128	0.135	0.142	0.149	0.157	0.165	0.174	0.182	0.190	0.199	0.208	0.217	0.226	0.235
0.10	0.104	0.109	0.114	0.119	0.125	0.131	0.137	0.143	0.150	0.157	0.165	0.172	0.180	0.188	0.196	0.205	0.213	0.222	0.231	0.240
0.11	0.114	0.119	0.123	0.128	0.134	0.140	0.146	0.152	0.158	0.165	0.172	0.179	0.187	0.195	0.202	0.211	0.219	0.227	0.236	0.245
0.12	0.124	0.128	0.133	0.138	0.143	0.149	0.154	0.160	0.167	0.173	0.180	0.187	0.194	0.202	0.209	0.217	0.225	0.233	0.241	0.250
0.13	0.134	0.138	0.142	0.147	0.152	0.157	0.163	0.169	0.175	0.181	0.188	0.195	0.202	0.209	0.216	0.223	0.231	0.239	0.247	0.255
0.14	0.144	0.148	0.152	0.156	0.161	0.166	0.172	0.177	0.183	0.189	0.196	0.202	0.209	0.216	0.223	0.230	0.237	0.245	0.253	0.261
0.15	0.154	0.157	0.161	0.166	0.170	0.175	0.181	0.186	0.192	0.198	0.204	0.210	0.217	0.223	0.230	0.237	0.244	0.251	0.259	0.267
0.16	0.163	0.167	0.171	0.175	0.180	0.185	0.190	0.195	0.200	0.206	0.212	0.218	0.224	0.231	0.237	0.244	0.251	0.258	0.266	0.273
0.17	0.173	0.177	0.180	0.185	0.189	0.194	0.199	0.204	0.209	0.214	0.220	0.226	0.232	0.238	0.245	0.251	0.258	0.265	0.272	0.279
0.18	0.183	0.186	0.190	0.194	0.198	0.203	0.208	0.212	0.218	0.223	0.228	0.234	0.240	0.246	0.252	0.258	0.265	0.272	0.279	0.286
0.19	0.193	0.196	0.200	0.203	0.208	0.212	0.217	0.221	0.226	0.232	0.237	0.242	0.248	0.254	0.260	0.266	0.272	0.279	0.286	0.293
0.20	0.203	0.206	0.209	0.213	0.217	0.221	0.226	0.230	0.235	0.240	0.245	0.251	0.256	0.262	0.268	0.274	0.280	0.286	0.293	0.299

Table C.24 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.016	0.025	0.035	0.045	0.055	0.065	0.075	0.085	0.095	0.104	0.114	0.124	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.025	0.032	0.041	0.050	0.059	0.069	0.079	0.089	0.099	0.109	0.118	0.128	0.138	0.148	0.158	0.168	0.178	0.188	0.197	0.207
0.03	0.035	0.041	0.048	0.056	0.065	0.074	0.084	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.161	0.171	0.181	0.191	0.201	0.211
0.04	0.045	0.050	0.057	0.064	0.072	0.080	0.089	0.098	0.107	0.117	0.127	0.136	0.146	0.155	0.165	0.175	0.185	0.194	0.204	0.214
0.05	0.055	0.060	0.066	0.072	0.079	0.087	0.095	0.103	0.112	0.121	0.131	0.140	0.150	0.159	0.169	0.178	0.188	0.198	0.207	0.217
0.06	0.064	0.069	0.075	0.081	0.087	0.094	0.102	0.109	0.118	0.127	0.136	0.144	0.154	0.163	0.172	0.182	0.191	0.201	0.211	0.220
0.07	0.074	0.079	0.084	0.090	0.096	0.102	0.109	0.116	0.124	0.133	0.141	0.150	0.159	0:168	0.177	0.186	0.195	0.205	0.214	0.224
0.08	0.084	0.088	0.093	0.099	0.104	0.111	0.117	0.124	0.131	0.139	0.147	0.155	0.164	0.172	0.181	0.190	0.199	0.209	0.218	0.227
0.09	0.094	0.098	0.103	0.108	0.113	0.119	0.125	0.132	0.139	0.146	0.154	0.161	0.170	0.178	0.186	0.195	0.204	0.213	0.222	0.231
0.10	0.104	0.108	0.112	0.117	0.122	0.128	0.134	0.140	0.146	0.153	0.161	0.168	0.176	0.184	0.192	0.200	0.209	0.217	0.226	0.235
0.11	0.114	0.117	0.122	0.126	0.131	0.137	0.142	0.148	0.154	0.161	0.168	0.175	0.182	0.190	0.198	0.206	0.214	0.222	0.231	0.240
0.12	0.123	0.127	0.131	0.136	0.140	0.145	0.151	0.157	0.162	0.169	0.175	0.182	0.189	0.196	0.204	0.212	0.219	0.228	0.236	0.245
0.13	0.133	0.137	0.141	0.145	0.149	0.154	0.160	0.165	0.171	0.177	0.183	0.190	0.196	0.203	0.211	0.218	0.225	0.233	0.241	0.250
0.14	0.143	0.146	0.150	0.154	0.158	0.163	0.168	0.173	0.179	0.184	0.191	0.197	0.203	0.210	0.217	0.224	0.231	0.239	0.247	0.255
0.15	0.153	0.156	0.160	0.164	0.167	0.172	0.177	0.182	0.187	0.193	0.198	0.204	0.211	0.217	0.224	0.231	0.238	0.245	0.253	0.260
0.16	0.163	0.166	0.169	0.173	0.177	0.181	0.186	0.190	0.196	0.201	0.206	0.212	0.218	0.225	0.231	0.237	0.244	0.251	0.259	0.266
0.17	0.173	0.176	0.179	0.183	0.186	0.190	0.195	0.199	0.204	0.209	0.215	0.220	0.226	0.232	0.238	0.245	0.251	0.258	0.265	0.272
0.18	0.183	0.185	0.189	0.192	0.195	0.200	0.204	0.208	0.213	0.218	0.223	0.229	0.234	0.240	0.246	0.252	0.258	0.265	0.272	0.279
0.19	0.192	0.195	0.198	0.201	0.205	0.209	0.213	0.217	0.222	0.227	0.232	0.237	0.242	0.248	0.253	0.259	0.265	0.272	0.278	0.285
0.20	0.202	0.205	0.208	0.211	0.214	0.218	0.222	0.226	0.231	0.235	0.240	0.245	0.250	0.256	0.261	0.267	0.272	0.279	0.285	0.291

Table C.25 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.025	0.035	0.045	0.055	0.065	0.074	0.084	0.094	0.104	0.114	0.124	0.134	0.144	0.154	0.164	0.174	0.183	0.193	0.203
0.02	0.025	0.032	0.041	0.050	0.059	0.069	0.079	0.088	0.098	0.108	0.118	0.128	0.137	0.147	0.157	0.167	0.177	0.187	0.196	0.206
0.03	0.035	0.041	0.048	0.056	0.065	0.074	0.083	0.093	0.102	0.112	0.122	0.131	0.141	0.151	0.160	0.170	0.180	0.190	0.199	0.209
0.04	0.045	0.050	0.056	0.064	0.072	0.080	0.089	0.097	0.107	0.116	0.126	0.135	0.145	0.154	0.164	0.173	0.183	0.193	0.202	0.212
0.05	0.054	0.059	0.065	0.072	0.079	0.087	0.094	0.103	0.112	0.121	0.130	0.139	0.148	0.158	0.167	0.177	0.186	0.196	0.205	0.215
0.06	0.064	0.069	0.074	0.080	0.087	0.094	0.101	0.108	0.117	0.125	0.134	0.143	0.152	0.161	0.171	0.180	0.189	0.199	0.208	0.218
0.07	0.074	0.078	0.083	0.089	0.095	0.101	0.108	0.115	0.123	0.131	0.139	0.148	0.157	0.165	0.175	0.184	0.193	0.202	0.212	0.221
0.08	0.084	0.088	0.092	0.098	0.103	0.109	0.116	0.122	0.130	0.137	0.145	0.153	0.162	0.170	0.179	0.188	0.197	0.206	0.215	0.225
0.09	0.093	0.097	0.102	0.107	0.112	0.118	0.124	0.130	0.137	0.144	0.151	0.159	0.167	0.175	0.184	0.192	0.201	0.210	0.219	0.228
0.10	0.103	0.107	0.111	0.116	0.121	0.126	0.132	0.138	0.144	0.151	0.158	0.165	0.173	0.181	0.189	0.197	0.205	0.214	0.223	0.232
0.11	0.113	0.117	0.120	0.125	0.130	0.135	0.140	0.145	0.152	0.158	0.165	0.172	0.179	0.186	0.194	0.202	0.210	0.218	0.227	0.236
0.12	0.123	0.126	0.130	0.134	0.139	0.143	0.148	0.154	0.160	0.166	0.172	0.179	0.186	0.193	0.200	0.208	0.216	0.224	0.232	0.240
0.13	0.133	0.136	0.139	0.143	0.148	0.152	0.157	0.162	0.168	0.173	0.179	0.186	0.193	0.199	0.206	0.214	0.221	0.229	0.237	0.245
0.14	0.142	0.145	0.149	0.153	0.157	0.161	0.166	0.171	0.176	0.181	0.187	0.193	0.200	0.206	0.213	0.220	0.227	0.234	0.242	0.249
0.15	0.152	0.155	0.158	0.162	0.166	0.170	0.174	0.179	0.184	0.189	0.195	0.201	0.207	0.213	0.219	0.226	0.233	0.240	0.247	0.255
0.16	0.162	0.165	0.168	0.171	0.175	0.179	0.183	0.188	0.192	0.197	0.203	0.208	0.214	0.220	0.226	0.233	0.239	0.246	0.253	0.260
0.17	0.172	0.175	0.177	0.181	0.184	0.188	0.192	0.196	0.201	0.206	0.211	0.216	0.222	0.227	0.233	0.239	0.246	0.252	0.259	0.266
0.18	0.182	0.184	0.187	0.190	0.193	0.197	0.201	0.205	0.209	0.214	0.219	0.224	0.229	0.235	0.240	0.246	0.253	0.259	0.265	0.272
0.19	0.192	0.194	0.197	0.200	0.203	0.206	0.210	0.214	0.218	0.223	0.227	0.232	0.237	0.242	0.248	0.254	0.260	0.266	0.272	0.279
0.20	0.202	0.204	0.206	0.209	0.212	0.215	0.219	0.223	0.227	0.231	0.236	0.241	0.246	0.250	0.256	0.261	0.267	0.273	0.279	0.285

Table C.26 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.035	0.045	0.054	0.064	0.074	0.084	0.094	0.104	0.114	0.123	0.133	0.143	0.153	0.163	0.173	0.183	0.193	0.203
0.02	0.026	0.033	0.041	0.050	0.059	0.069	0.078	0.088	0.098	0.107	0.117	0.127	0.137	0.146	0.156	0.166	0.176	0.186	0.196	0.205
0.03	0.035	0.041	0.049	0.057	0.065	0.074	0.083	0.092	0.102	0.111	0.121	0.130	0.140	0.150	0.159	0.169	0.179	0.188	0.198	0.208
0.04	0.044	0.050	0.056	0.064	0.071	0.080	0.088	0.097	0.106	0.115	0.124	0.134	0.143	0.153	0.163	0.172	0.182	0.191	0.201	0.211
0.05	0.054	0.059	0.065	0.071	0.078	0.086	0.094	0.102	0.111	0.120	0.129	0.138	0.147	0.157	0.166	0.175	0.185	0.194	0.204	0.213
0.06	0.064	0.068	0.073	0.079	0.086	0.093	0.100	0.108	0.117	0.125	0.134	0.142	0.151	0.160	0.169	0.179	0.188	0.197	0.207	0.216
0.07	0.073	0.078	0.082	0.088	0.094	0.100	0.107	0.114	0.122	0.130	0.138	0.147	0.155	0.164	0.173	0.182	0.192	0.201	0.210	0.219
0.08	0.083	0.087	0.091	0.096	0.102	0.108	0.115	0.121	0.129	0.136	0.144	0.152	0.160	0.169	0.178	0.187	0.195	0.204	0.213	0.223
0.09	0.093	0.096	0.101	0.105	0.110	0.116	0.122	0.128	0.135	0.142	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.217	0.226
0.10	0.103	0.106	0.110	0.114	0.119	0.125	0.130	0.136	0.142	0.149	0.156	0.164	0.171	0.179	0.187	0.195	0.204	0.212	0.221	0.230
0.11	0.112	0.116	0.119	0.124	0.128	0.133	0.138	0.144	0.150	0.156	0.163	0.170	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.234
0.12	0.122	0.125	0.129	0.133	0.137	0.142	0.147	0.152	0.158	0.164	0.170	0.177	0.184	0.191	0.198	0.206	0.214	0.221	0.229	0.238
0.13	0.132	0.135	0.138	0.142	0.146	0.151	0.155	0.160	0.166	0.171	0.177	0.184	0.190	0.197	0.204	0.211	0.219	0.226	0.234	0.242
0.14	0.142	0.145	0.148	0.151	0.155	0.159	0.164	0.169	0.174	0.179	0.185	0.191	0.197	0.204	0.211	0.217	0.225	0.232	0.239	0.247
0.15	0.152	0.154	0.157	0.161	0.165	0.168	0.173	0.177	0.182	0.187	0.192	0.198	0.204	0.211	0.217	0.224	0.231	0.237	0.245	0.252
0.16	0.162	0.164	0.167	0.170	0.174	0.178	0.181	0.186	0.190	0.195	0.200	0.206	0.212	0.218	0.224	0.230	0.237	0.243	0.250	0.258
0.17	0.172	0.174	0.177	0.180	0.183	0.187	0.190	0.194	0.199	0.203	0.208	0.214	0.219	0.225	0.231	0.237	0.243	0.250	0.256	0.263
0.18	0.182	0.184	0.186	0.189	0.192	0.196	0.200	0.203	0.207	0.212	0.217	0.221	0.227	0.232	0.238	0.244	0.250	0.256	0.262	0.269
0.19	0.191	0.193	0.196	0.199	0.202	0.205	0.209	0.212	0.216	0.220	0.225	0.230	0.234	0.240	0.245	0.251	0.256	0.263	0.269	0.275
0.20	0.201	0.203	0.206	0.208	0.211	0.214	0.218	0.221	0.225	0.229	0.233	0.238	0.243	0.248	0.253	0.258	0.264	0.270	0.276	0.282

Table C.27 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level: 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.035	0.044	0.054	0.064	0.074	0.084	0.094	0.103	0.113	0.123	0.133	0.143	0.153	0.163	0.173	0.182	0.192	0.202
0.02	0.026	0.033	0.042	0.050	0.059	0.069	0.078	0.088	0.097	0.107	0.117	0.126	0.136	0.146	0.155	0.165	0.175	0.185	0.195	0.205
0.03	0.035	0.041	0.049	0.057	0.065	0.074	0.083	0.092	0.102	0.111	0.121	0.130	0.140	0.149	0.159	0.168	0.178	0.188	0.197	0.207
0.04	0.044	0.050	0.056	0.064	0.072	0.080	0.088	0.097	0.106	0.115	0.125	0.134	0.143	0.152	0.162	0.171	0.181	0.190	0.200	0.209
0.05	0.054	0.058	0.065	0.071	0.079	0.086	0.094	0.103	0.112	0.120	0.129	0.138	0.147	0.156	0.165	0.175	0.184	0.193	0.203	0.212
0.06	0.063	0.068	0.073	0.079	0.086	0.093	0.101	0.109	0.117	0.126	0.134	0.143	0.152	0.160	0.169	0.178	0.187	0.197	0.206	0.215
0.07	0.073	0.077	0.082	0.087	0.094	0.100	0.107	0.115	0.123	0.131	0.139	0.147	0.156	0.165	0.173	0.182	0.191	0.200	0.209	0.218
0.08	0.083	0.086	0.091	0.096	0.102	0.108	0.115	0.122	0.129	0.137	0.145	0.153	0.161	0.169	0.178	0.186	0.195	0.204	0.213	0.222
0.09	0.092	0.096	0.100	0.105	0.110	0.116	0.122	0.129	0.136	0.143	0.151	0.158	0.166	0.174	0.183	0.191	0.199	0.208	0.217	0.226
0.10	0.102	0.105	0.109	0.113	0.118	0.124	0.130	0.136	0.143	0.150	0.157	0.164	0.172	0.180	0.187	0.195	0.204	0.212	0.221	0.229
0.11	0.112	0.115	0.118	0.122	0.127	0.132	0.138	0.144	0.150	0.157	0.164	0.171	0.178	0.185	0.193	0.201	0.209	0.217	0.225	0.233
0.12	0.122	0.125	0.128	0.132	0.136	0.141	0.146	0.152	0.158	0.164	0.171	0.177	0.184	0.191	0.198	0.206	0.214	0.222	0.230	0.238
0.13	0.132	0.134	0.137	0.141	0.145	0.150	0.154	0.160	0.165	0.171	0.178	0.184	0.191	0.197	0.204	0.211	0.219	0.227	0.235	0.242
0.14	0.142	0.144	0.147	0.150	0.154	0.158	0.163	0.168	0.173	0.179	0.185	0.191	0.197	0.203	0.210	0.217	0.224	0.232	0.239	0.247
0.15	0.151	0.153	0.156	0.159	0.163	0.167	0.171	0.176	0.181	0.187	0.192	0.198	0.204	0.210	0.216	0.223	0.230	0.237	0.245	0.252
0.16	0.161	0.163	0.166	0.169	0.172	0.176	0.180	0.185	0.190	0.195	0.200	0.205	0.211	0.217	0.223	0.229	0.236	0.243	0.250	0.257
0.17	0.171	0.173	0.175	0.178	0.182	0.185	0.189	0.193	0.198	0.203	0.208	0.213	0.218	0.224	0.230	0.236	0.242	0.249	0.256	0.263
0.18	0.181	0.183	0.185	0.188	0.191	0.194	0.198	0.202	0.206	0.211	0.216	0.220	0.226	0.231	0.237	0.243	0.249	0.255	0.262	0.269
0.19	0.191	0.193	0.195	0.197	0.200	0.204	0.207	0.211	0.215	0.219	0.224	0.229	0.234	0.239	0.244	0.250	0.255	0.262	0.268	0.275
0.20	0.201	0.202	0.204	0.207	0.210	0.213	0.216	0.220	0.224	0.228	0.232	0.236	0.241	0.246	0.251	0.257	0.262	0.268	0.275	0.281

Table C.28 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level: 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.026	0.036	0.045	0.054	0.064	0.074	0.084	0.093	0.103	0.113	0.123	0.133	0.143	0.153	0.163	0.172	0.182	0.192	0.202
0.02	0.026	0.034	0.042	0.051	0.060	0.069	0.079	0.088	0.097	0.107	0.116	0.126	0.136	0.146	0.155	0.165	0.175	0.185	0.195	0.204
0.03	0.035	0.042	0.050	0.058	0.066	0.075	0.084	0.093	0.102	0.111	0.121	0.130	0.140	0.149	0.159	0.168	0.178	0.187	0.197	0.207
0.04	0.044	0.050	0.057	0.065	0.073	0.081	0.090	0.098	0.107	0.116	0.125	0.134	0.144	0.153	0.162	0.172	0.181	0.191	0.200	0.210
0.05	0.054	0.059	0.065	0.072	0.080	0.088	0.096	0.104	0.112	0.121	0.130	0.139	0.148	0.157	0.166	0.175	0.184	0.194	0.203	0.212
0.06	0.063	0.068	0.074	0.080	0.087	0.094	0.102	0.110	0.118	0.126	0.135	0.143	0.152	0.161	0.170	0.179	0.188	0.197	0.206	0.216
0.07	0.073	0.077	0.083	0.088	0.095	0.102	0.109	0.116	0.124	0.132	0.140	0.149	0.157	0.166	0.175	0.183	0.192	0.201	0.210	0.219
0.08	0.083	0.087	0.091	0.097	0.102	0.109	0.115	0.122	0.130	0.138	0.146	0.154	0.162	0.171	0.179	0.188	0.196	0.205	0.214	0.223
0.09	0.092	0.096	0.100	0.105	0.111	0.117	0.123	0.129	0.137	0.144	0.152	0.159	0.168	0.176	0.184	0.192	0.201	0.209	0.218	0.227
0.10	0.102	0.105	0.110	0.114	0.119	0.124	0.130	0.137	0.143	0.150	0.158	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.222	0.230
0.11	0.112	0.115	0.119	0.123	0.128	0.133	0.138	0.144	0.151	0.157	0.164	0.172	0.179	0.186	0.194	0.202	0.210	0.218	0.226	0.235
0.12	0.122	0.125	0.128	0.132	0.136	0.141	0.146	0.152	0.158	0.164	0.171	0.178	0.185	0.192	0.200	0.207	0.215	0.223	0.231	0.239
0.13	0.132	0.134	0.138	0.141	0.145	0.150	0.154	0.160	0.166	0.172	0.178	0.184	0.191	0.198	0.206	0.213	0.220	0.228	0.236	0.244
0.14	0.142	0.144	0.147	0.150	0.154	0.158	0.163	0.168	0.173	0.179	0.185	0.191	0.198	0.204	0.211	0.218	0.225	0.233	0.241	0.248
0.15	0.151	0.154	0.157	0.160	0.163	0.167	0.171	0.176	0.181	0.187	0.192	0.199	0.205	0.211	0.218	0.224	0.232	0.239	0.246	0.254
0.16	0.161	0.163	0.166	0.169	0.172	0.176	0.180	0.185	0.190	0.195	0.200	0.206	0.212	0.218	0.224	0.231	0.237	0.244	0.251	0.259
0.17	0.171	0.173	0.176	0.178	0.181	0.185	0.189	0.193	0.198	0.203	0.208	0.213	0.219	0.225	0.231	0.237	0.243	0.250	0.257	0.264
0.18	0.181	0.183	0.185	0.188	0.190	0.194	0.197	0.202	0.206	0.211	0.215	0.221	0.226	0.232	0.238	0.244	0.250	0.256	0.263	0.270
0.19	0.191	0.193	0.195	0.197	0.200	0.203	0.206	0.210	0.214	0.219	0.223	0.229	0.234	0.239	0.245	0.250	0.257	0.263	0.269	0.276
0.20	0.201	0.202	0.204	0.207	0.209	0.212	0.215	0.219	0.223	0.227	0.231	0.236	0.241	0.247	0.252	0.258	0.263	0.270	0.276	0.282

Table C.29 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.036	0.045	0.055	0.064	0.074	0.084	0.093	0.103	0.113	0.123	0.133	0.143	0.152	0.162	0.172	0.182	0.192	0.202
0.02	0.026	0.034	0.042	0.051	0.060	0.069	0.078	0.088	0.097	0.107	0.116	0.126	0.136	0.145	0.155	0.165	0.175	0.184	0.194	0.204
0.03	0.035	0.042	0.049	0.057	0.066	0.074	0.084	0.093	0.102	0.111	0.120	0.130	0.139	0.149	0.158	0.168	0.177	0.187	0.197	0.206
0.04	0.044	0.050	0.057	0.064	0.072	0.080	0.089	0.098	0.107	0.116	0.125	0.134	0.143	0.152	0.161	0.171	0.180	0.190	0.199	0.209
0.05	0.053	0.059	0.065	0.071	0.079	0.087	0.095	0.103	0.112	0.120	0.129	0.138	0.147	0.156	0.165	0.174	0.184	0.193	0.202	0.212
0.06	0.063	0.067	0.073	0.079	0.086	0.093	0.101	0.109	0.117	0.126	0.134	0.143	0.152	0.160	0.169	0.178	0.187	0.196	0.206	0.215
0.07	0.073	0.077	0.082	0.087	0.094	0.100	0.108	0.116	0.123	0.131	0.140	0.148	0.156	0.165	0.173	0.182	0.191	0.200	0.209	0.218
0.08	0.083	0.086	0.090	0.096	0.102	0.108	0.115	0.122	0.129	0.137	0.145	0.153	0.161	0.170	0.178	0.186	0.195	0.204	0.213	0.222
0.09	0.092	0.096	0.099	0.104	0.110	0.115	0.122	0.129	0.136	0.143	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.217	0.225
0.10	0.102	0.105	0.109	0.113	0.118	0.123	0.129	0.136	0.142	0.149	0.157	0.164	0.172	0.180	0.187	0.195	0.204	0.212	0.221	0.229
0.11	0.112	0.115	0.118	0.122	0.127	0.132	0.137	0.143	0.149	0.156	0.163	0.170	0.178	0.185	0.193	0.200	0.208	0.217	0.225	0.234
0.12	0.122	0.124	0.127	0.131	0.135	0.140	0.145	0.151	0.157	0.163	0.170	0.176	0.184	0.191	0.198	0.206	0.213	0.222	0.230	0.238
0.13	0.131	0.134	0.137	0.140	0.144	0.149	0.153	0.159	0.164	0.170	0.176	0.183	0.190	0.197	0.204	0.211	0.218	0.226	0.234	0.242
0.14	0.141	0.144	0.146	0.149	0.153	0.157	0.162	0.167	0.172	0.178	0.184	0.190	0.197	0.203	0.210	0.217	0.224	0.232	0.240	0.247
0.15	0.151	0.153	0.156	0.159	0.162	0.166	0.171	0.175	0.180	0.186	0.191	0.197	0.204	0.210	0.216	0.223	0.230	0.237	0.245	0.252
0.16	0.161	0.163	0.165	0.168	0.172	0.175	0.179	0.184	0.188	0.193	0.199	0.205	0.211	0.217	0.223	0.229	0.236	0.243	0.250	0.258
0.17	0.171	0.173	0.175	0.177	0.181	0.184	0.188	0.192	0.197	0.201	0.206	0.212	0.217	0.223	0.229	0.235	0.242	0.249	0.256	0.263
0.18	0.181	0.183	0.184	0.187	0.190	0.193	0.197	0.201	0.205	0.209	0.214	0.219	0.225	0.230	0.236	0.242	0.248	0.255	0.262	0.268
0.19	0.191	0.192	0.194	0.196	0.199	0.202	0.206	0.210	0.214	0.218	0.222	0.227	0.232	0.238	0.243	0.249	0.255	0.261	0.268	0.274
0.20	0.201	0.202	0.204	0.206	0.209	0.212	0.215	0.218	0.222	0.226	0.231	0.235	0.240	0.245	0.250	0.256	0.262	0.268	0.274	0.280

Table C.30 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 1.5; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.035	0.045	0.054	0.064	0.074	0.083	0.093	0.103	0.113	0.123	0.132	0.142	0.152	0.162	0.172	0.182	0.192	0.202
0.02	0.026	0.033	0.042	0.051	0.060	0.069	0.078	0.087	0.097	0.106	0.116	0.126	0.135	0.145	0.155	0.164	0.174	0.184	0.194	0.204
0.03	0.035	0.041	0.049	0.057	0.066	0.074	0.083	0.092	0.101	0.111	0.120	0.129	0.139	0.148	0.158	0.167	0.177	0.187	0.196	0.206
0.04	0.044	0.050	0.057	0.064	0.072	0.081	0.089	0.098	0.106	0.115	0.124	0.134	0.143	0.152	0.161	0.171	0.180	0.190	0.199	0.209
0.05	0.053	0.058	0.065	0.072	0.079	0.087	0.095	0.103	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.174	0.183	0.193	0.202	0.211
0.06	0.063	0.067	0.073	0.079	0.086	0.093	0.101	0.109	0.117	0.125	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.215
0.07	0.072	0.076	0.081	0.087	0.093	0.100	0.108	0.115	0.123	0.131	0.139	0.147	0.156	0.164	0.173	0.182	0.191	0.200	0.209	0.218
0.08	0.082	0.086	0.090	0.096	0.101	0.108	0.115	0.122	0.129	0.137	0.144	0.153	0.161	0.169	0.178	0.186	0.195	0.204	0.213	0.221
0.09	0.092	0.095	0.099	0.104	0.109	0.115	0.122	0.129	0.135	0.143	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.217	0.225
0.10	0.102	0.105	0.109	0.113	0.118	0.124	0.129	0.136	0.142	0.149	0.156	0.164	0.171	0.179	0.187	0.195	0.204	0.212	0.221	0.229
0.11	0.112	0.114	0.118	0.122	0.126	0.132	0.137	0.143	0.149	0.156	0.163	0.170	0.177	0.185	0.193	0.200	0.209	0.217	0.225	0.233
0.12	0.121	0.124	0.127	0.131	0.135	0.140	0.145	0.151	0.156	0.163	0.169	0.176	0.183	0.191	0.198	0.205	0.213	0.221	0.229	0.238
0.13	0.131	0.134	0.137	0.140	0.144	0.148	0.153	0.158	0.164	0.170	0.176	0.183	0.189	0.196	0.204	0.211	0.219	0.226	0.234	0.242
0.14	0.141	0.143	0.146	0.149	0.153	0.157	0.162	0.166	0.171	0.177	0.183	0.189	0.196	0.203	0.209	0.217	0.224	0.231	0.239	0.247
0.15	0.151	0.153	0.156	0.159	0.162	0.166	0.170	0.174	0.179	0.185	0.190	0.196	0.202	0.209	0.216	0.222	0.230	0.237	0.244	0.252
0.16	0.161	0.163	0.165	0.168	0.171	0.175	0.179	0.183	0.187	0.192	0.197	0.203	0.209	0.216	0.222	0.229	0.236	0.243	0.250	0.257
0.17	0.171	0.173	0.175	0.177	0.180	0.184	0.187	0.191	0.196	0.200	0.205	0.211	0.217	0.223	0.229	0.235	0.242	0.249	0.256	0.263
0.18	0.181	0.182	0.184	0.187	0.189	0.193	0.196	0.200	0.204	0.208	0.213	0.218	0.224	0.230	0.236	0.242	0.248	0.255	0.261	0.268
0.19	0.191	0.192	0.194	0.196	0.199	0.202	0.205	0.208	0.212	0.216	0.221	0.226	0.231	0.237	0.243	0.248	0.254	0.261	0.267	0.274
0.20	0.201	0.202	0.204	0.206	0.208	0.211	0.214	0.217	0.221	0.225	0.229	0.234	0.239	0.244	0.250	0.255	0.261	0.267	0.273	0.280

C.4 Attained Significance Level Tables for H_o = Weibull(β = 2.0).

Table C.31 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.132	0.142	0.151	0.161	0.171	0.181	0.191	0.201
0.02	0.029	0.037	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.109	0.118	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.204
0.03	0.038	0.047	0.056	0.064	0.073	0.081	0.090	0.099	0.107	0.116	0.125	0.134	0.143	0.152	0.161	0.171	0.180	0.189	0.198	0.208
0.04	0.048	0.056	0.065	0.073	0.082	0.090	0.098	0.107	0.115	0.124	0.133	0.141	0.150	0.159	0.168	0.177	0.186	0.195	0.204	0.213
0.05	0.058	0.066	0.074	0.082	0.091	0.099	0.107	0.115	0.124	0.132	0.141	0.149	0.158	0.167	0.175	0.184	0.193	0.202	0.210	0.219
0.06	0.068	0.076	0.084	0.092	0.100	0.108	0.116	0.124	0.132	0.141	0.149	0.157	0.166	0.174	0.183	0.191	0.200	0.209	0.217	0.226
0.07	0.078	0.085	0.094	0.101	0.109	0.117	0.125	0.133	0.141	0.149	0.158	0.166	0.174	0.183	0.191	0.199	0.208	0.216	0.225	0.233
0.08	0.088	0.095	0.103	0.111	0.119	0.126	0.134	0.142	0.150	0.158	0.166	0.174	0.183	0.191	0.199	0.207	0.216	0.224	0.232	0.241
0.09	0.097	0.105	0.113	0.121	0.128	0.136	0.144	0.151	0.159	0.167	0.175	0.183	0.191	0.199	0.207	0.215	0.224	0.232	0.240	0.248
0.10	0.107	0.115	0.123	0.130	0.138	0.145	0.153	0.161	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240	0.248	0.256
0.11	0.117	0.125	0.132	0.140	0.147	0.155	0.162	0.170	0.178	0.185	0.193	0.201	0.209	0.217	0.225	0.233	0.241	0.248	0.257	0.265
0.12	0.127	0.135	0.142	0.149	0.157	0.164	0.172	0.179	0.187	0.194	0.202	0.210	0.218	0.226	0.233	0.241	0.249	0.257	0.265	0.273
0.13	0.137	0.145	0.152	0.159	0.167	0.174	0.181	0.189	0.196	0.204	0.211	0.219	0.227	0.234	0.242	0.250	0.258	0.265	0.273	0.281
0.14	0.147	0.154	0.162	0.169	0.176	0.184	0.191	0.198	0.206	0.213	0.221	0.228	0.236	0.243	0.251	0.259	0.266	0.274	0.282	0.289
0.15	0.157	0.164	0.171	0.179	0.186	0.193	0.200	0.208	0.215	0.222	0.230	0.237	0.245	0.252	0.260	0.267	0.275	0.282	0.290	0.298
0.16	0.167	0.174	0.181	0.188	0.196	0.203	0.210	0.217	0.224	0.232	0.239	0.246	0.254	0.261	0.269	0.276	0.284	0.291	0.299	0.306
0.17	0.177	0.184	0.191	0.198	0.205	0.212	0.219	0.227	0.234	0.241	0.248	0.256	0.263	0.271	0.278	0.285	0.293	0.300	0.308	0.315
0.18	0.187	0.194	0.201	0.208	0.215	0.222	0.229	0.236	0.243	0.250	0.258	0.265	0.272	0.280	0.287	0.294	0.302	0.309	0.317	0.324
0.19	0.197	0.204	0.211	0.218	0.225	0.232	0.239	0.246	0.253	0.260	0.267	0.274	0.282	0.289	0.296	0.303	0.311	0.318	0.325	0.333
0.20	0.207	0.214	0.220	0.227	0.234	0.241	0.248	0.255	0.262	0.269	0.276	0.284	0.291	0.298	0.305	0.312	0.320	0.327	0.334	0.341

Table C.32 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.045	0.055	0.065	0.074	0.084	0.094	0.104	0.114	0.124	0.134	0.144	0.153	0.163	0.173	0.183	0.193	0.203
0.02	0.028	0.036	0.044	0.053	0.062	0.071	0.081	0.090	0.100	0.109	0.119	0.129	0.138	0.148	0.158	0.168	0.178	0.187	0.197	0.207
0.03	0.037	0.045	0.053	0.062	0.070	0.079	0.088	0.097	0.106	0.116	0.125	0.135	0.144	0.154	0.163	0.173	0.182	0.192	0.202	0.212
0.04	0.047	0.055	0.062	0.070	0.079	0.087	0.096	0.105	0.114	0.123	0.132	0.141	0.150	0.160	0.169	0.179	0.188	0.197	0.207	0.217
0.05	0.057	0.064	0.072	0.080	0.088	0.096	0.104	0.113	0.122	0.130	0.139	0.148	0.157	0.166	0.176	0.185	0.194	0.203	0.213	0.223
0.06	0.067	0.074	0.081	0.089	0.097	0.105	0.113	0.121	0.130	0.139	0.147	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.219	0.229
0.07	0.077	0.084	0.091	0.098	0.106	0.114	0.122	0.130	0.138	0.147	0.155	0.164	0.172	0.181	0.190	0.199	0.208	0.217	0.226	0.235
0.08	0.087	0.094	0.101	0.108	0.116	0.123	0.131	0.139	0.147	0.155	0.164	0.172	0.180	0.189	0.198	0.207	0.215	0.224	0.233	0.242
0.09	0.097	0.103	0.110	0.118	0.125	0.133	0.140	0.148	0.156	0.164	0.172	0.180	0.189	0.197	0.206	0.214	0.223	0.232	0.240	0.249
0.10	0.106	0.113	0.120	0.127	0.134	0.142	0.149	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.214	0.222	0.231	0.239	0.248	0.256
0.11	0.116	0.123	0.130	0.137	0.144	0.151	0.159	0.166	0.174	0.182	0.189	0.197	0.205	0.214	0.222	0.230	0.238	0.247	0.255	0.264
0.12	0.126	0.133	0.140	0.146	0.153	0.161	0.168	0.175	0.183	0.191	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.255	0.263	0.271
0.13	0.136	0.143	0.149	0.156	0.163	0.170	0.177	0.184	0.192	0.200	0.207	0.215	0.222	0.231	0.238	0.246	0.254	0.263	0.271	0.279
0.14	0.146	0.153	0.159	0.166	0.173	0.180	0.187	0.194	0.201	0.208	0.216	0.223	0.231	0.239	0.246	0.254	0.262	0.270	0.278	0.286
0.15	0.156	0.163	0.169	0.176	0.182	0.189	0.196	0.203	0.210	0.217	0.225	0.232	0.240	0.247	0.255	0.263	0.271	0.278	0.286	0.294
0.16	0.166	0.172	0.179	0.185	0.192	0.199	0.206	0.212	0.219	0.227	0.234	0.241	0.248	0.256	0.263	0.271	0.279	0.286	0.294	0.302
0.17	0.176	0.182	0.189	0.195	0.202	0.208	0.215	0.222	0.229	0.236	0.243	0.250	0.257	0.265	0.272	0.279	0.287	0.295	0.302	0.310
0.18	0.186	0.192	0.198	0.205	0.211	0.218	0.224	0.231	0.238	0.245	0.252	0.259	0.266	0.274	0.281	0.288	0.295	0.303	0.310	0.318
0.19	0.196	0.202	0.208	0.215	0.221	0.227	0.234	0.240	0.247	0.254	0.261	0.268	0.275	0.282	0.289	0.297	0.304	0.312	0.319	0.326
0.20	0.206	0.212	0.218	0.224	0.231	0.237	0.243	0.250	0.257	0.263	0.270	0.277	0.284	0.291	0.298	0.305	0.312	0.320	0.327	0.334

Table C.33 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.026	0.036	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.035	0.044	0.052	0.062	0.071	0.081	0.09	0.1	0.11	0.12	0.13	0.139	0.149	0.159	0.169	0.179	0.189	0.199	0.209
0.03	0.037	0.044	0.052	0.061	0.069	0.078	0.087	0.097	0.106	0.116	0.126	0.135	0.145	0.155	0.164	0.174	0.184	0.194	0.204	0.213
0.04	0.046	0.054	0.061	0.07	0.078	0.087	0.095	0.104	0.113	0.123	0.132	0.142	0.151	0.16	0.17	0.18	0.189	0.199	0.208	0.218
0.05	0.056	0.063	0.071	0.079	0.087	0.095	0.104	0.112	0.121	0.13	0.139	0.148	0.158	0.167	0.176	0.186	0.195	0.204	0.214	0.223
0.06	0.066	0.073	0.08	0.088	0.096	0.104	0.112	0.12	0.129	0.138	0.146	0.155	0.165	0.174	0.183	0.192	0.201	0.21	0.22	0.229
0.07	0.076	0.083	0.09	0.097	0.105	0.113	0.121	0.129	0.137	0.146	0.154	0.163	0.172	0.181	0.19	0.199	0.208	0.217	0.226	0.235
0.08	0.086	0.092	0.099	0.107	0.114	0.122	0.129	0.137	0.145	0.154	0.162	0.171	0.179	0.188	0.197	0.205	0.214	0.223	0.232	0.241
0.09	0.096	0.102	0.109	0.116	0.123	0.13	0.138	0.146	0.154	0.162	0.17	0.178	0.187	0.195	0.204	0.212	0.221	0.23	0.238	0.247
0.10	0.106	0.112	0.119	0.125	0.132	0.14	0.147	0.154	0.162	0.17	0.178	0.186	0.195	0.203	0.211	0.219	0.228	0.236	0.245	0.254
0.11	0.116	0.122	0.128	0.135	0.142	0.149	0.156	0.163	0.171	0.179	0.186	0.195	0.203	0.211	0.219	0.227	0.235	0.244	0.252	0.261
0.12	0.126	0.131	0.138	0.144	0.151	0.158	0.165	0.172	0.18	0.187	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.268
0.13	0.136	0.141	0.148	0.154	0.161	0.167	0.174	0.181	0.188	0.196	0.203	0.211	0.219	0.227	0.235	0.242	0.25	0.258	0.267	0.275
0.14	0.145	0.151	0.157	0.163	0.17	0.177	0.183	0.19	0.197	0.205	0.212	0.22	0.227	0.235	0.243	0.25	0.258	0.266	0.274	0.282
0.15	0.155	0.161	0.167	0.173	0.18	0.186	0.193	0.199	0.206	0.214	0.221	0.228	0.236	0.243	0.251	0.258	0.266	0.273	0.281	0.289
0.16	0.165	0.171	0.177	0.183	0.189	0.195	0.202	0.209	0.215	0.222	0.229	0.237	0.244	0.251	0.259	0.266	0.274	0.281	0.289	0.297
0.17	0.175	0.18	0.186	0.192	0.199	0.205	0.211	0.218	0.224	0.231	0.238	0.245	0.252	0.26	0.267	0.274	0.281	0.289	0.297	0.304
0.18	0.185	0.19	0.196	0.202	0.208	0.214	0.22	0.227	0.233	0.24	0.247	0.254	0.261	0.268	0.275	0.282	0.289	0.297	0.304	0.312
0.19	0.195	0.2	0.206	0.211	0.217	0.223	0.23	0.236	0.242	0.249	0.256	0.263	0.269	0.276	0.284	0.291	0.298	0.305	0.312	0.319
0.20	0.205	0.21	0.215	0.221	0.227	0.233	0.239	0.245	0.252	0.258	0.265	0.271	0.278	0.285	0.292	0.299	0.306	0.313	0.32	0.327

Table C.34 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.017	0.026	0.036	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.027	0.035	0.043	0.052	0.062	0.071	0.081	0.090	0.100	0.110	0.120	0.130	0.140	0.150	0.159	0.169	0.179	0.189	0.199	0.209
0.03	0.036	0.044	0.052	0.060	0.069	0.078	0.087	0.097	0.106	0.116	0.125	0.135	0.145	0.155	0.164	0.174	0.184	0.194	0.203	0.213
0.04	0.046	0.053	0.061	0.069	0.078	0.086	0.095	0.104	0.113	0.122	0.132	0.141	0.151	0.160	0.170	0.179	0.189	0.199	0.208	0.218
0.05	0.056	0.063	0.070	0.078	0.086	0.094	0.102	0.111	0.120	0.129	0.138	0.147	0.157	0.166	0.175	0.185	0.194	0.204	0.213	0.223
0.06	0.066	0.072	0.080	0.087	0.095	0.103	0.111	0.119	0.128	0.136	0.145	0.154	0.163	0.172	0.182	0.191	0.200	0.209	0.219	0.228
0.07	0.076	0.082	0.089	0.096	0.104	0.111	0.119	0.127	0.135	0.144	0.152	0.161	0.170	0.179	0.188	0.197	0.206	0.215	0.224	0.234
0.08	0.086	0.091	0.098	0.105	0.113	0.120	0.127	0.135	0.143	0.152	0.160	0.168	0.177	0.186	0.195	0.203	0.212	0.221	0.230	0.239
0.09	0.095	0.101	0.108	0.115	0.122	0.129	0.136	0.144	0.152	0.160	0.168	0.176	0.184	0.193	0.202	0.210	0.219	0.228	0.236	0.245
0.10	0.105	0.111	0.117	0.124	0.131	0.138	0.145	0.152	0.160	0.168	0.176	0.184	0.192	0.201	0.209	0.217	0.226	0.234	0.243	0.252
0.11	0.115	0.121	0.127	0.134	0.140	0.147	0.154	0.161	0.169	0.177	0.184	0.192	0.200	0.208	0.217	0.225	0.233	0.241	0.250	0.258
0.12	0.125	0.131	0.137	0.143	0.150	0.156	0.163	0.170	0.178	0.185	0.193	0.200	0.208	0.216	0.224	0.232	0.240	0.248	0.257	0.265
0.13	0.135	0.140	0.146	0.153	0.159	0.166	0.172	0.179	0.186	0.194	0.201	0.208	0.216	0.224	0.232	0.240	0.248	0.256	0.264	0.272
0.14	0.145	0.150	0.156	0.162	0.168	0.175	0.181	0.188	0.195	0.202	0.209	0.217	0.224	0.232	0.240	0.248	0.255	0.263	0.271	0.279
0.15	0.155	0.160	0.166	0.172	0.178	0.184	0.190	0.197	0.204	0.211	0.218	0.225	0.232	0.240	0.248	0.255	0.263	0.270	0.278	0.286
0.16	0.165	0.170	0.175	0.181	0.187	0.193	0.200	0.206	0.212	0.220	0.226	0.233	0.241	0.248	0.255	0.263	0.270	0.277	0.285	0.293
0.17	0.175	0.180	0.185	0.191	0.197	0.203	0.209	0.215	0.221	0.228	0.235	0.242	0.249	0.256	0.264	0.271	0.278	0.285	0.293	0.300
0.18	0.185	0.189	0.195	0.200	0.206	0.212	0.218	0.224	0.230	0.237	0.244	0.250	0.257	0.264	0.271	0.278	0.286	0.293	0.300	0.307
0.19	0.195	0.199	0.204	0.210	0.215	0.221	0.227	0.233	0.239	0.246	0.252	0.259	0.265	0.272	0.279	0.286	0.293	0.300	0.307	0.315
0.20	0.204	0.209	0.214	0.219	0.225	0.231	0.236	0.242	0.248	0.255	0.261	0.267	0.274	0.281	0.288	0.294	0.301	0.308	0.315	0.322

Table C.35 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.026	0.036	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.194	0.204
0.02	0.027	0.035	0.043	0.052	0.062	0.071	0.081	0.090	0.100	0.110	0.120	0.130	0.139	0.149	0.159	0.169	0.179	0.189	0.199	0.209
0.03	0.036	0.044	0.052	0.060	0.069	0.078	0.087	0.097	0.106	0.116	0.125	0.135	0.144	0.154	0.164	0.174	0.183	0.193	0.203	0.213
0.04	0.046	0.053	0.061	0.069	0.077	0.086	0.095	0.103	0.113	0.122	0.131	0.141	0.150	0.159	0.169	0.179	0.188	0.198	0.208	0.217
0.05	0.056	0.063	0.070	0.078	0.086	0.094	0.102	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.175	0.184	0.194	0.203	0.212	0.222
0.06	0.066	0.072	0.079	0.086	0.094	0.102	0.111	0.119	0.127	0.136	0.145	0.154	0.163	0.172	0.181	0.190	0.199	0.208	0.218	0.227
0.07	0.076	0.082	0.089	0.095	0.103	0.111	0.119	0.127	0.135	0.143	0.152	0.161	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232
0.08	0.085	0.092	0.098	0.105	0.112	0.120	0.127	0.135	0.143	0.151	0.160	0.168	0.177	0.185	0.194	0.202	0.211	0.220	0.229	0.238
0.09	0.095	0.101	0.108	0.114	0.121	0.128	0.136	0.144	0.151	0.159	0.167	0.176	0.184	0.192	0.200	0.209	0.218	0.226	0.235	0.244
0.10	0.105	0.111	0.117	0.123	0.130	0.137	0.145	0.152	0.160	0.167	0.175	0.183	0.191	0.199	0.207	0.216	0.224	0.233	0.241	0.250
0.11	0.115	0.121	0.127	0.133	0.139	0.146	0.153	0.161	0.168	0.175	0.183	0.191	0.199	0.207	0.215	0.223	0.231	0.239	0.248	0.256
0.12	0.125	0.130	0.136	0.142	0.149	0.155	0.162	0.169	0.177	0.184	0.191	0.199	0.207	0.214	0.222	0.230	0.238	0.246	0.254	0.263
0.13	0.135	0.140	0.146	0.151	0.158	0.165	0.171	0.178	0.185	0.192	0.200	0.207	0.215	0.222	0.230	0.238	0.245	0.253	0.261	0.269
0.14	0.145	0.150	0.155	0.161	0.167	0.174	0.180	0.187	0.194	0.201	0.208	0.215	0.222	0.230	0.237	0.245	0.253	0.260	0.268	0.276
0.15	0.155	0.160	0.165	0.170	0.176	0.183	0.189	0.196	0.203	0.209	0.216	0.223	0.230	0.238	0.245	0.252	0.260	0.267	0.275	0.283
0.16	0.164	0.169	0.174	0.180	0.186	0.192	0.198	0.205	0.211	0.218	0.225	0.232	0.238	0.245	0.253	0.260	0.267	0.275	0.282	0.290
0.17	0.174	0.179	0.184	0.189	0.195	0.201	0.207	0.214	0.220	0.227	0.233	0.240	0.247	0.253	0.260	0.268	0.275	0.282	0.290	0.297
0.18	0.184	0.189	0.194	0.199	0.204	0.210	0.217	0.223	0.229	0.235	0.242	0.248	0.255	0.262	0.268	0.275	0.282	0.290	0.297	0.304
0.19	0.194	0.199	0.203	0.208	0.214	0.220	0.226	0.232	0.238	0.244	0.250	0.257	0.263	0.270	0.276	0.283	0.290	0.297	0.304	0.312
0.20	0.204	0.208	0.213	0.218	0.223	0.229	0.235	0.241	0.247	0.253	0.259	0.265	0.271	0.278	0.284	0.291	0.298	0.305	0.312	0.319

Table C.36 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.046	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.035	0.044	0.053	0.062	0.071	0.081	0.091	0.100	0.110	0.120	0.130	0.139	0.149	0.159	0.169	0.179	0.189	0.199	0.208
0.03	0.037	0.044	0.052	0.061	0.070	0.079	0.088	0.097	0.106	0.116	0.125	0.135	0.144	0.154	0.164	0.173	0.183	0.193	0.203	0.212
0.04	0.046	0.053	0.061	0.069	0.078	0.086	0.095	0.104	0.113	0.122	0.131	0.141	0.150	0.159	0.169	0.179	0.188	0.198	0.207	0.217
0.05	0.056	0.063	0.070	0.078	0.086	0.094	0.102	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.174	0.184	0.193	0.203	0.212	0.221
0.06	0.066	0.072	0.079	0.086	0.094	0.102	0.110	0.118	0.127	0.136	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.208	0.217	0.226
0.07	0.075	0.082	0.088	0.096	0.103	0.111	0.118	0.126	0.134	0.143	0.151	0.160	0.168	0.177	0.186	0.195	0.204	0.213	0.222	0.232
0.08	0.085	0.091	0.098	0.105	0.112	0.119	0.127	0.134	0.142	0.151	0.159	0.167	0.175	0.184	0.193	0.201	0.210	0.219	0.228	0.237
0.09	0.095	0.101	0.107	0.114	0.121	0.128	0.135	0.143	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.216	0.225	0.234	0.243
0.10	0.105	0.110	0.117	0.123	0.130	0.137	0.144	0.151	0.158	0.166	0.174	0.181	0.189	0.198	0.206	0.214	0.223	0.231	0.240	0.248
0.11	0.115	0.120	0.126	0.132	0.139	0.145	0.152	0.160	0.167	0.174	0.182	0.189	0.197	0.205	0.213	0.221	0.230	0.238	0.246	0.255
0.12	0.125	0.130	0.136	0.142	0.148	0.154	0.161	0.168	0.175	0.182	0.190	0.197	0.204	0.212	0.220	0.228	0.236	0.244	0.253	0.261
0.13	0.134	0.140	0.145	0.151	0.157	0.163	0.170	0.176	0.183	0.190	0.197	0.205	0.212	0.220	0.227	0.235	0.243	0.251	0.259	0.267
0.14	0.144	0.149	0.155	0.161	0.166	0.172	0.179	0.185	0.192	0.199	0.206	0.213	0.220	0.227	0.235	0.242	0.250	0.258	0.266	0.274
0.15	0.154	0.159	0.164	0.170	0.176	0.181	0.188	0.194	0.200	0.207	0.214	0.221	0.228	0.235	0.242	0.250	0.257	0.265	0.273	0.280
0.16	0.164	0.169	0.174	0.179	0.185	0.190	0.197	0.203	0.209	0.216	0.222	0.229	0.235	0.242	0.250	0.257	0.264	0.272	0.279	0.287
0.17	0.174	0.178	0.183	0.189	0.194	0.200	0.206	0.212	0.218	0.224	0.230	0.237	0.244	0.250	0.257	0.265	0.272	0.279	0.287	0.294
0.18	0.184	0.188	0.193	0.198	0.204	0.209	0.215	0.221	0.226	0.233	0.239	0.245	0.252	0.258	0.265	0.272	0.279	0.287	0.294	0.301
0.19	0.194	0.198	0.203	0.208	0.213	0.218	0.224	0.230	0.235	0.241	0.247	0.254	0.260	0.267	0.273	0.280	0.287	0.294	0.301	0.309
0.20	0.204	0.208	0.212	0.217	0.222	0.228	0.233	0.239	0.244	0.250	0.256	0.262	0.268	0.275	0.281	0.288	0.295	0.302	0.309	0.316

Table C.37 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.046	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.035	0.043	0.052	0.062	0.071	0.081	0.090	0.100	0.110	0.119	0.129	0.139	0.149	0.159	0.169	0.178	0.188	0.198	0.208
0.03	0.036	0.044	0.052	0.060	0.069	0.078	0.087	0.097	0.106	0.115	0.125	0.134	0.144	0.154	0.163	0.173	0.183	0.193	0.202	0.212
0.04	0.046	0.053	0.061	0.069	0.077	0.086	0.094	0.103	0.112	0.122	0.131	0.140	0.149	0.159	0.168	0.178	0.188	0.197	0.207	0.216
0.05	0.056	0.062	0.069	0.077	0.085	0.093	0.102	0.111	0.119	0.128	0.137	0.146	0.155	0.165	0.174	0.183	0.193	0.202	0.211	0.221
0.06	0.065	0.072	0.079	0.086	0.094	0.102	0.110	0.118	0.127	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.216	0.226
0.07	0.075	0.081	0.088	0.095	0.102	0.110	0.118	0.126	0.134	0.142	0.151	0.159	0.168	0.177	0.186	0.195	0.204	0.213	0.222	0.231
0.08	0.085	0.091	0.097	0.104	0.111	0.118	0.126	0.134	0.142	0.150	0.158	0.166	0.175	0.183	0.192	0.201	0.210	0.218	0.227	0.236
0.09	0.095	0.100	0.106	0.113	0.120	0.127	0.134	0.142	0.149	0.157	0.165	0.173	0.182	0.190	0.199	0.207	0.216	0.225	0.233	0.242
0.10	0.105	0.110	0.116	0.122	0.129	0.135	0.143	0.150	0.158	0.165	0.173	0.181	0.189	0.197	0.206	0.214	0.223	0.231	0.240	0.248
0.11	0.114	0.120	0.125	0.131	0.138	0.144	0.151	0.159	0.166	0.174	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.238	0.246	0.254
0.12	0.124	0.129	0.135	0.141	0.147	0.153	0.160	0.167	0.174	0.182	0.189	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.261
0.13	0.134	0.139	0.144	0.150	0.156	0.162	0.169	0.176	0.183	0.190	0.197	0.204	0.212	0.219	0.227	0.235	0.243	0.251	0.259	0.267
0.14	0.144	0.149	0.154	0.159	0.165	0.171	0.178	0.185	0.191	0.198	0.205	0.212	0.219	0.227	0.235	0.242	0.250	0.258	0.266	0.273
0.15	0.154	0.159	0.163	0.169	0.175	0.180	0.187	0.193	0.200	0.207	0.213	0.220	0.227	0.235	0.242	0.250	0.257	0.265	0.273	0.280
0.16	0.164	0.168	0.173	0.178	0.184	0.189	0.196	0.202	0.208	0.215	0.222	0.228	0.235	0.242	0.250	0.257	0.265	0.272	0.280	0.287
0.17	0.173	0.178	0.182	0.187	0.193	0.198	0.204	0.210	0.217	0.223	0.229	0.236	0.243	0.250	0.257	0.264	0.272	0.279	0.286	0.294
0.18	0.183	0.188	0.192	0.197	0.202	0.207	0.213	0.219	0.225	0.232	0.238	0.244	0.251	0.258	0.265	0.272	0.279	0.286	0.294	0.301
0.19	0.193	0.197	0.201	0.206	0.211	0.217	0.222	0.228	0.234	0.240	0.246	0.252	0.259	0.265	0.272	0.279	0.286	0.293	0.301	0.308
0.20	0.203	0.207	0.211	0.216	0.221	0.226	0.231	0.237	0.243	0.249	0.255	0.261	0.267	0.274	0.280	0.287	0.294	0.301	0.308	0.315

Table C.38 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.037	0.046	0.056	0.066	0.075	0.085	0.095	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.035	0.044	0.053	0.062	0.072	0.081	0.091	0.100	0.110	0.120	0.129	0.139	0.149	0.159	0.168	0.178	0.188	0.198	0.208
0.03	0.037	0.044	0.052	0.061	0.070	0.079	0.088	0.097	0.106	0.116	0.125	0.135	0.144	0.154	0.164	0.173	0.183	0.192	0.202	0.212
0.04	0.046	0.053	0.061	0.069	0.078	0.086	0.095	0.104	0.113	0.122	0.132	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207	0.217
0.05	0.056	0.063	0.070	0.078	0.086	0.094	0.103	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.175	0.184	0.193	0.202	0.212	0.221
0.06	0.066	0.072	0.079	0.087	0.095	0.102	0.110	0.119	0.127	0.136	0.145	0.153	0.162	0.171	0.180	0.189	0.199	0.208	0.217	0.226
0.07	0.075	0.082	0.088	0.095	0.103	0.110	0.118	0.126	0.134	0.143	0.151	0.160	0.169	0.177	0.186	0.195	0.204	0.213	0.222	0.231
0.08	0.085	0.091	0.097	0.104	0.112	0.119	0.126	0.134	0.142	0.150	0.158	0.167	0.175	0.184	0.192	0.201	0.210	0.219	0.227	0.236
0.09	0.095	0.101	0.107	0.113	0.120	0.128	0.135	0.142	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.216	0.225	0.233	0.242
0.10	0.105	0.110	0.116	0.122	0.129	0.136	0.143	0.151	0.158	0.166	0.174	0.182	0.190	0.198	0.206	0.214	0.223	0.231	0.240	0.248
0.11	0.115	0.120	0.125	0.132	0.138	0.145	0.152	0.159	0.166	0.174	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.237	0.246	0.254
0.12	0.124	0.130	0.135	0.141	0.147	0.154	0.160	0.167	0.174	0.182	0.189	0.197	0.205	0.212	0.220	0.228	0.236	0.244	0.252	0.261
0.13	0.134	0.139	0.144	0.150	0.156	0.163	0.169	0.176	0.183	0.190	0.197	0.205	0.212	0.220	0.227	0.235	0.243	0.251	0.259	0.267
0.14	0.144	0.149	0.154	0.160	0.166	0.172	0.178	0.184	0.191	0.198	0.205	0.213	0.220	0.227	0.235	0.242	0.250	0.258	0.266	0.273
0.15	0.154	0.159	0.163	0.169	0.175	0.181	0.187	0.193	0.200	0.206	0.214	0.221	0.228	0.235	0.242	0.250	0.257	0.265	0.272	0.280
0.16	0.164	0.168	0.173	0.178	0.184	0.190	0.195	0.202	0.208	0.215	0.222	0.228	0.235	0.243	0.250	0.257	0.264	0.272	0.279	0.287
0.17	0.174	0.178	0.183	0.188	0.193	0.199	0.204	0.210	0.217	0.223	0.230	0.237	0.244	0.250	0.257	0.264	0.272	0.279	0.286	0.294
0.18	0.184	0.188	0.192	0.197	0.202	0.208	0.213	0.219	0.225	0.232	0.238	0.245	0.251	0.258	0.265	0.272	0.279	0.286	0.293	0.301
0.19	0.193	0.197	0.202	0.206	0.212	0.217	0.222	0.228	0.234	0.240	0.247	0.253	0.259	0.266	0.273	0.279	0.286	0.293	0.300	0.307
0.20	0.203	0.207	0.211	0.216	0.221	0.226	0.231	0.237	0.243	0.249	0.255	0.261	0.268	0.274	0.281	0.287	0.294	0.301	0.308	0.314

Table C.39 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.046	0.056	0.066	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.144	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.027	0.035	0.044	0.053	0.063	0.072	0.081	0.091	0.101	0.110	0.120	0.130	0.139	0.149	0.159	0.169	0.178	0.188	0.198	0.208
0.03	0.037	0.044	0.053	0.061	0.070	0.079	0.088	0.097	0.107	0.116	0.125	0.135	0.145	0.154	0.164	0.173	0.183	0.193	0.202	0.212
0.04	0.046	0.053	0.061	0.069	0.078	0.086	0.095	0.104	0.113	0.122	0.131	0.140	0.150	0.159	0.169	0.178	0.188	0.197	0.207	0.216
0.05	0.056	0.063	0.070	0.078	0.086	0.094	0.102	0.111	0.120	0.128	0.137	0.146	0.156	0.165	0.174	0.183	0.193	0.202	0.211	0.221
0.06	0.065	0.072	0.079	0.086	0.094	0.102	0.110	0.118	0.127	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.216	0.226
0.07	0.075	0.081	0.088	0.095	0.102	0.110	0.118	0.126	0.134	0.142	0.151	0.159	0.168	0.177	0.186	0.194	0.203	0.212	0.221	0.231
0.08	0.085	0.091	0.097	0.104	0.111	0.119	0.126	0.134	0.142	0.150	0.158	0.167	0.175	0.184	0.192	0.201	0.209	0.218	0.227	0.236
0.09	0.095	0.100	0.107	0.113	0.120	0.127	0.134	0.142	0.150	0.157	0.165	0.174	0.182	0.190	0.199	0.207	0.216	0.224	0.233	0.241
0.10	0.104	0.110	0.116	0.122	0.129	0.136	0.143	0.150	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.222	0.230	0.238	0.247
0.11	0.114	0.120	0.125	0.131	0.138	0.144	0.151	0.158	0.166	0.173	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.245	0.253
0.12	0.124	0.129	0.135	0.141	0.147	0.153	0.160	0.167	0.174	0.181	0.188	0.196	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259
0.13	0.134	0.139	0.144	0.150	0.156	0.162	0.168	0.175	0.182	0.189	0.196	0.203	0.211	0.218	0.226	0.233	0.241	0.249	0.257	0.265
0.14	0.144	0.148	0.154	0.159	0.165	0.171	0.177	0.184	0.190	0.197	0.204	0.211	0.218	0.226	0.233	0.240	0.248	0.256	0.264	0.272
0.15	0.154	0.158	0.163	0.168	0.174	0.180	0.186	0.192	0.199	0.205	0.212	0.219	0.226	0.233	0.240	0.248	0.255	0.263	0.271	0.278
0.16	0.164	0.168	0.173	0.178	0.183	0.189	0.195	0.201	0.207	0.214	0.220	0.227	0.234	0.241	0.248	0.255	0.263	0.270	0.278	0.285
0.17	0.173	0.178	0.182	0.187	0.193	0.198	0.204	0.210	0.216	0.222	0.229	0.235	0.242	0.249	0.256	0.263	0.270	0.277	0.284	0.292
0.18	0.183	0.187	0.192	0.197	0.202	0.207	0.213	0.219	0.225	0.231	0.237	0.243	0.250	0.257	0.263	0.270	0.277	0.284	0.292	0.299
0.19	0.193	0.197	0.201	0.206	0.211	0.216	0.222	0.227	0.233	0.239	0.245	0.251	0.258	0.264	0.271	0.278	0.285	0.291	0.299	0.306
0.20	0.203	0.207	0.211	0.215	0.220	0.225	0.231	0.236	0.242	0.247	0.253	0.260	0.266	0.272	0.279	0.285	0.292	0.299	0.306	0.313

Table C.40 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.0; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.037	0.046	0.056	0.065	0.075	0.085	0.095	0.105	0.114	0.124	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.203
0.02	0.027	0.035	0.044	0.053	0.063	0.072	0.081	0.091	0.100	0.110	0.120	0.129	0.139	0.149	0.158	0.168	0.178	0.188	0.197	0.207
0.03	0.036	0.044	0.052	0.061	0.070	0.079	0.088	0.097	0.106	0.116	0.125	0.134	0.144	0.153	0.163	0.172	0.182	0.192	0.201	0.211
0.04	0.046	0.053	0.061	0.069	0.077	0.086	0.095	0.104	0.113	0.122	0.131	0.140	0.150	0.159	0.168	0.177	0.187	0.197	0.206	0.215
0.05	0.056	0.062	0.070	0.078	0.085	0.094	0.102	0.111	0.119	0.128	0.137	0.146	0.155	0.164	0.173	0.182	0.192	0.201	0.211	0.220
0.06	0.065	0.072	0.079	0.086	0.094	0.102	0.110	0.118	0.126	0.135	0.144	0.153	0.161	0.170	0.179	0.188	0.197	0.206	0.215	0.225
0.07	0.075	0.081	0.088	0.095	0.102	0.110	0.118	0.126	0.134	0.142	0.151	0.159	0.168	0.176	0.185	0.194	0.203	0.212	0.221	0.230
0.08	0.085	0.091	0.097	0.104	0.111	0.118	0.126	0.134	0.141	0.150	0.158	0.166	0.174	0.183	0.191	0.200	0.209	0.217	0.226	0.235
0.09	0.095	0.100	0.106	0.113	0.120	0.127	0.134	0.142	0.149	0.157	0.165	0.173	0.181	0.189	0.198	0.206	0.215	0.223	0.232	0.241
0.10	0.104	0.110	0.116	0.122	0.129	0.136	0.143	0.150	0.157	0.165	0.173	0.181	0.188	0.196	0.204	0.213	0.221	0.229	0.238	0.246
0.11	0.114	0.119	0.125	0.131	0.138	0.144	0.151	0.158	0.165	0.173	0.180	0.188	0.196	0.203	0.211	0.219	0.227	0.236	0.244	0.252
0.12	0.124	0.129	0.134	0.140	0.146	0.153	0.160	0.166	0.173	0.180	0.188	0.195	0.203	0.210	0.218	0.226	0.234	0.242	0.250	0.258
0.13	0.134	0.138	0.144	0.149	0.155	0.162	0.168	0.175	0.181	0.188	0.195	0.203	0.210	0.217	0.225	0.233	0.241	0.248	0.257	0.265
0.14	0.144	0.148	0.153	0.159	0.164	0.170	0.177	0.183	0.190	0.197	0.203	0.211	0.218	0.225	0.233	0.240	0.248	0.255	0.263	0.271
0.15	0.154	0.158	0.163	0.168	0.173	0.179	0.186	0.192	0.198	0.205	0.212	0.218	0.226	0.233	0.240	0.247	0.255	0.262	0.270	0.278
0.16	0.163	0.167	0.172	0.178	0.183	0.188	0.194	0.200	0.207	0.213	0.220	0.226	0.233	0.240	0.247	0.254	0.262	0.269	0.277	0.284
0.17	0.173	0.177	0.182	0.187	0.192	0.197	0.203	0.209	0.215	0.221	0.228	0.234	0.241	0.248	0.255	0.262	0.269	0.276	0.284	0.291
0.18	0.183	0.187	0.191	0.196	0.201	0.206	0.212	0.218	0.224	0.230	0.236	0.242	0.249	0.256	0.262	0.269	0.276	0.283	0.291	0.298
0.19	0.193	0.197	0.201	0.206	0.210	0.215	0.221	0.226	0.232	0.238	0.244	0.250	0.257	0.263	0.270	0.277	0.283	0.291	0.298	0.305
0.20	0.203	0.206	0.211	0.215	0.220	0.225	0.230	0.235	0.241	0.247	0.253	0.259	0.265	0.271	0.278	0.284	0.291	0.298	0.305	0.312

C.5 Attained Significance Level Tables for H_o = Weibull(β = 2.5).

Table C.41 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.161	0.171	0.180	0.190	0.200
0.02	0.029	0.037	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.109	0.118	0.128	0.137	0.146	0.156	0.165	0.174	0.184	0.193	0.203
0.03	0.038	0.047	0.056	0.064	0.073	0.082	0.090	0.099	0.108	0.117	0.126	0.134	0.143	0.152	0.162	0.171	0.180	0.189	0.198	0.208
0.04	0.048	0.057	0.065	0.074	0.082	0.090	0.099	0.107	0.116	0.125	0.133	0.142	0.151	0.160	0.169	0.177	0.186	0.195	0.204	0.213
0.05	0.058	0.066	0.074	0.083	0.091	0.099	0.107	0.116	0.124	0.133	0.141	0.150	0.159	0.167	0.176	0.184	0.193	0.202	0.211	0.220
0.06	0.068	0.076	0.084	0.093	0.101	0.108	0.117	0.125	0.133	0.141	0.150	0.158	0.167	0.175	0.184	0.192	0.201	0.209	0.218	0.227
0.07	0.078	0.086	0.094	0.102	0.110	0.118	0.126	0.134	0.142	0.150	0.159	0.167	0.175	0.183	0.192	0.200	0.208	0.217	0.226	0.234
0.08	0.088	0.096	0.103	0.112	0.119	0.127	0.135	0.143	0.151	0.159	0.167	0.175	0.184	0.192	0.200	0.208	0.216	0.225	0.233	0.242
0.09	0.098	0.105	0.113	0.121	0.129	0.137	0.144	0.152	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.216	0.224	0.233	0.241	0.249
0.10	0.107	0.115	0.123	0.131	0.138	0.146	0.154	0.162	0.169	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.233	0.241	0.249	0.257
0.11	0.117	0.125	0.133	0.140	0.148	0.156	0.163	0.171	0.179	0.186	0.194	0.202	0.210	0.218	0.226	0.233	0.241	0.250	0.258	0.266
0.12	0.127	0.135	0.142	0.150	0.158	0.165	0.172	0.180	0.188	0.195	0.203	0.211	0.219	0.227	0.234	0.242	0.250	0.258	0.266	0.274
0.13	0.137	0.145	0.152	0.160	0.167	0.175	0.182	0.190	0.197	0.205	0.212	0.220	0.228	0.235	0.243	0.251	0.258	0.266	0.274	0.282
0.14	0.147	0.155	0.162	0.169	0.177	0.184	0.191	0.199	0.206	0.214	0.221	0.229	0.237	0.244	0.252	0.259	0.267	0.275	0.283	0.290
0.15	0.157	0.164	0.172	0.179	0.186	0.194	0.201	0.208	0.216	0.223	0.231	0.238	0.246	0.253	0.261	0.268	0.276	0.284	0.291	0.299
0.16	0.167	0.174	0.181	0.189	0.196	0.203	0.210	0.218	0.225	0.232	0.240	0.247	0.255	0.262	0.270	0.277	0.285	0.292	0.300	0.308
0.17	0.177	0.184	0.191	0.199	0.206	0.213	0.220	0.227	0.234	0.242	0.249	0.256	0.264	0.272	0.279	0.286	0.294	0.301	0.309	0.316
0.18	0.187	0.194	0.201	0.208	0.215	0.222	0.229	0.237	0.244	0.251	0.259	0.266	0.273	0.281	0.288	0.295	0.302	0.310	0.318	0.325
0.19	0.197	0.204	0.211	0.218	0.225	0.232	0.239	0.246	0.253	0.261	0.268	0.275	0.283	0.290	0.297	0.304	0.312	0.319	0.327	0.334
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.249	0.256	0.263	0.270	0.277	0.284	0.292	0.299	0.306	0.313	0.321	0.328	0.335	0.343

Table C.42 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.046	0.055	0.065	0.074	0.084	0.094	0.103	0.113	0.123	0.133	0.142	0.152	0.162	0.172	0.182	0.192	0.202
0.02	0.028	0.037	0.045	0.054	0.063	0.072	0.081	0.091	0.100	0.109	0.119	0.128	0.138	0.147	0.157	0.167	0.176	0.186	0.196	0.206
0.03	0.038	0.046	0.054	0.063	0.072	0.080	0.089	0.098	0.107	0.117	0.126	0.135	0.144	0.153	0.163	0.172	0.182	0.191	0.201	0.211
0.04	0.048	0.056	0.064	0.072	0.081	0.089	0.098	0.106	0.115	0.124	0.133	0.142	0.151	0.160	0.170	0.179	0.188	0.197	0.207	0.216
0.05	0.057	0.065	0.073	0.081	0.090	0.098	0.106	0.115	0.123	0.132	0.141	0.150	0.159	0.168	0.177	0.186	0.195	0.204	0.213	0.222
0.06	0.067	0.075	0.083	0.091	0.099	0.107	0.115	0.124	0.132	0.141	0.149	0.158	0.167	0.176	0.185	0.193	0.202	0.211	0.220	0.229
0.07	0.077	0.085	0.093	0.100	0.108	0.116	0.125	0.133	0.141	0.150	0.158	0.167	0.175	0.184	0.193	0.201	0.210	0.219	0.228	0.237
0.08	0.087	0.095	0.102	0.110	0.118	0.126	0.134	0.142	0.150	0.159	0.167	0.176	0.184	0.192	0.201	0.210	0.218	0.227	0.236	0.245
0.09	0.097	0.104	0.112	0.120	0.127	0.135	0.143	0.151	0.159	0.167	0.176	0.184	0.192	0.201	0.209	0.218	0.226	0.235	0.243	0.252
0.10	0.107	0.114	0.122	0.129	0.137	0.145	0.153	0.161	0.168	0.177	0.185	0.193	0.201	0.210	0.218	0.226	0.234	0.243	0.252	0.260
0.11	0.117	0.124	0.132	0.139	0.147	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.210	0.218	0.227	0.235	0.243	0.251	0.260	0.268
0.12	0.127	0.134	0.141	0.149	0.156	0.164	0.171	0.179	0.187	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.260	0.268	0.276
0.13	0.137	0.144	0.151	0.158	0.166	0.173	0.181	0.189	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.285
0.14	0.147	0.154	0.161	0.168	0.175	0.183	0.190	0.198	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.260	0.268	0.276	0.285	0.293
0.15	0.157	0.164	0.171	0.178	0.185	0.192	0.200	0.207	0.215	0.222	0.230	0.238	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301
0.16	0.167	0.173	0.181	0.188	0.195	0.202	0.209	0.217	0.224	0.232	0.239	0.247	0.255	0.262	0.270	0.278	0.285	0.293	0.301	0.309
0.17	0.177	0.183	0.190	0.197	0.204	0.212	0.219	0.226	0.233	0.241	0.248	0.256	0.264	0.271	0.279	0.287	0.294	0.302	0.310	0.318
0.18	0.187	0.193	0.200	0.207	0.214	0.221	0.228	0.236	0.243	0.250	0.258	0.265	0.273	0.280	0.288	0.295	0.303	0.311	0.318	0.326
0.19	0.197	0.203	0.210	0.217	0.224	0.231	0.238	0.245	0.252	0.260	0.267	0.275	0.282	0.289	0.297	0.304	0.312	0.319	0.327	0.335
0.20	0.207	0.213	0.220	0.227	0.234	0.241	0.248	0.255	0.262	0.269	0.276	0.284	0.291	0.299	0.306	0.313	0.321	0.328	0.336	0.343

Table C.43 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.036	0.046	0.055	0.065	0.075	0.085	0.095	0.104	0.114	0.124	0.134	0.144	0.154	0.163	0.173	0.183	0.193	0.203
0.02	0.028	0.036	0.045	0.054	0.063	0.073	0.082	0.092	0.101	0.110	0.120	0.130	0.139	0.149	0.159	0.168	0.178	0.188	0.198	0.208
0.03	0.037	0.046	0.054	0.063	0.072	0.081	0.090	0.099	0.109	0.118	0.127	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.203	0.213
0.04	0.047	0.055	0.063	0.072	0.081	0.089	0.098	0.107	0.116	0.126	0.135	0.144	0.153	0.163	0.172	0.181	0.191	0.200	0.210	0.219
0.05	0.057	0.065	0.073	0.081	0.090	0.098	0.107	0.116	0.125	0.134	0.143	0.152	0.161	0.170	0.180	0.189	0.198	0.207	0.217	0.226
0.06	0.067	0.075	0.083	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.224	0.233
0.07	0.077	0.084	0.092	0.100	0.109	0.117	0.125	0.134	0.142	0.151	0.160	0.168	0.177	0.186	0.195	0.204	0.213	0.222	0.231	0.240
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.134	0.143	0.151	0.160	0.168	0.177	0.185	0.194	0.203	0.212	0.220	0.229	0.238	0.247
0.09	0.097	0.104	0.112	0.119	0.127	0.135	0.144	0.152	0.160	0.168	0.177	0.185	0.194	0.202	0.211	0.220	0.228	0.237	0.246	0.255
0.10	0.107	0.114	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.186	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.254	0.263
0.11	0.117	0.124	0.131	0.139	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.203	0.211	0.219	0.228	0.236	0.244	0.253	0.262	0.270
0.12	0.127	0.133	0.141	0.148	0.156	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.219	0.228	0.236	0.244	0.253	0.261	0.270	0.278
0.13	0.136	0.143	0.150	0.158	0.165	0.173	0.181	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.261	0.269	0.277	0.286
0.14	0.146	0.153	0.160	0.167	0.175	0.182	0.190	0.198	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.294
0.15	0.156	0.163	0.170	0.177	0.184	0.192	0.199	0.207	0.214	0.222	0.230	0.238	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.302
0.16	0.166	0.173	0.180	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.247	0.254	0.262	0.270	0.278	0.286	0.294	0.302	0.310
0.17	0.176	0.183	0.190	0.197	0.204	0.211	0.218	0.226	0.233	0.240	0.248	0.256	0.263	0.271	0.279	0.286	0.294	0.302	0.310	0.318
0.18	0.186	0.193	0.199	0.206	0.213	0.220	0.228	0.235	0.242	0.250	0.257	0.264	0.272	0.280	0.287	0.295	0.303	0.310	0.318	0.326
0.19	0.196	0.202	0.209	0.216	0.223	0.230	0.237	0.244	0.251	0.259	0.266	0.273	0.281	0.288	0.296	0.303	0.311	0.319	0.327	0.334
0.20	0.206	0.212	0.219	0.226	0.232	0.239	0.246	0.253	0.261	0.268	0.275	0.282	0.290	0.297	0.305	0.312	0.320	0.327	0.335	0.342

Table C.44 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.037	0.046	0.056	0.066	0.076	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.028	0.036	0.045	0.055	0.064	0.073	0.083	0.092	0.102	0.111	0.121	0.131	0.140	0.150	0.160	0.170	0.179	0.189	0.199	0.209
0.03	0.037	0.046	0.054	0.063	0.072	0.081	0.090	0.100	0.109	0.118	0.128	0.137	0.147	0.157	0.166	0.176	0.185	0.195	0.205	0.215
0.04	0.047	0.055	0.064	0.072	0.081	0.090	0.099	0.108	0.117	0.126	0.135	0.145	0.154	0.163	0.173	0.182	0.192	0.201	0.211	0.221
0.05	0.057	0.065	0.073	0.081	0.090	0.099	0.107	0.116	0.125	0.134	0.143	0.152	0.161	0.171	0.180	0.189	0.199	0.208	0.217	0.227
0.06	0.067	0.074	0.083	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.178	0.188	0.197	0.206	0.215	0.224	0.234
0.07	0.077	0.084	0.092	0.100	0.108	0.117	0.125	0.134	0.142	0.151	0.160	0.169	0.177	0.186	0.195	0.204	0.213	0.222	0.232	0.241
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.134	0.143	0.151	0.159	0.168	0.177	0.186	0.194	0.203	0.212	0.221	0.230	0.239	0.248
0.09	0.097	0.104	0.111	0.119	0.127	0.135	0.143	0.151	0.160	0.168	0.177	0.185	0.194	0.203	0.211	0.220	0.229	0.238	0.246	0.255
0.10	0.106	0.113	0.121	0.129	0.136	0.144	0.152	0.160	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.237	0.245	0.254	0.263
0.11	0.116	0.123	0.131	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.211	0.219	0.227	0.236	0.245	0.253	0.262	0.270
0.12	0.126	0.133	0.141	0.148	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.219	0.227	0.236	0.244	0.252	0.261	0.269	0.278
0.13	0.136	0.143	0.150	0.157	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.260	0.269	0.277	0.286
0.14	0.146	0.153	0.160	0.167	0.174	0.182	0.189	0.197	0.205	0.213	0.221	0.229	0.236	0.244	0.252	0.260	0.269	0.277	0.285	0.294
0.15	0.156	0.163	0.170	0.177	0.184	0.191	0.199	0.206	0.214	0.222	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301
0.16	0.166	0.172	0.179	0.186	0.193	0.201	0.208	0.215	0.223	0.231	0.238	0.246	0.254	0.261	0.269	0.277	0.285	0.293	0.301	0.309
0.17	0.176	0.182	0.189	0.196	0.203	0.210	0.217	0.225	0.232	0.240	0.247	0.255	0.263	0.270	0.278	0.286	0.293	0.301	0.309	0.317
0.18	0.186	0.192	0.199	0.206	0.213	0.220	0.227	0.234	0.241	0.249	0.256	0.264	0.271	0.279	0.286	0.294	0.302	0.310	0.317	0.325
0.19	0.196	0.202	0.209	0.215	0.222	0.229	0.236	0.243	0.251	0.258	0.265	0.273	0.280	0.288	0.295	0.303	0.310	0.318	0.326	0.333
0.20	0.206	0.212	0.218	0.225	0.232	0.239	0.246	0.253	0.260	0.267	0.274	0.282	0.289	0.296	0.304	0.311	0.318	0.326	0.334	0.341

Table C.45 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.018	0.027	0.037	0.046	0.056	0.066	0.076	0.085	0.095	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.174	0.184	0.194	0.204
0.02	0.028	0.036	0.045	0.055	0.064	0.073	0.083	0.092	0.102	0.112	0.121	0.131	0.141	0.150	0.160	0.170	0.180	0.190	0.199	0.209
0.03	0.037	0.046	0.054	0.063	0.072	0.081	0.091	0.100	0.109	0.119	0.128	0.138	0.147	0.157	0.166	0.176	0.186	0.195	0.205	0.215
0.04	0.047	0.055	0.064	0.072	0.081	0.090	0.099	0.108	0.117	0.127	0.136	0.145	0.155	0.164	0.173	0.183	0.192	0.202	0.212	0.221
0.05	0.057	0.065	0.073	0.082	0.090	0.099	0.108	0.117	0.125	0.135	0.144	0.153	0.162	0.171	0.181	0.190	0.199	0.209	0.218	0.227
0.06	0.067	0.075	0.083	0.091	0.099	0.108	0.116	0.125	0.134	0.143	0.152	0.161	0.170	0.179	0.188	0.197	0.206	0.216	0.225	0.234
0.07	0.077	0.084	0.092	0.100	0.108	0.117	0.125	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.135	0.143	0.151	0.160	0.169	0.178	0.186	0.195	0.204	0.213	0.221	0.230	0.239	0.248
0.09	0.097	0.104	0.111	0.119	0.127	0.135	0.144	0.152	0.160	0.169	0.177	0.186	0.195	0.203	0.212	0.220	0.229	0.238	0.247	0.256
0.10	0.107	0.114	0.121	0.129	0.136	0.144	0.153	0.161	0.169	0.177	0.186	0.194	0.203	0.211	0.220	0.228	0.237	0.245	0.254	0.263
0.11	0.116	0.124	0.131	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.203	0.211	0.219	0.228	0.236	0.245	0.253	0.262	0.270
0.12	0.126	0.133	0.140	0.148	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.220	0.228	0.236	0.244	0.253	0.261	0.270	0.278
0.13	0.136	0.143	0.150	0.157	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.253	0.261	0.269	0.277	0.286
0.14	0.146	0.153	0.160	0.167	0.174	0.182	0.190	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.293
0.15	0.156	0.163	0.170	0.177	0.184	0.191	0.199	0.207	0.214	0.222	0.230	0.238	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301
0.16	0.166	0.173	0.179	0.186	0.193	0.200	0.208	0.216	0.223	0.231	0.238	0.246	0.254	0.261	0.269	0.277	0.285	0.293	0.301	0.309
0.17	0.176	0.182	0.189	0.196	0.203	0.210	0.217	0.225	0.232	0.240	0.247	0.255	0.262	0.270	0.278	0.285	0.293	0.301	0.309	0.317
0.18	0.186	0.192	0.199	0.205	0.212	0.219	0.227	0.234	0.241	0.249	0.256	0.264	0.271	0.279	0.286	0.294	0.301	0.309	0.317	0.325
0.19	0.196	0.202	0.209	0.215	0.222	0.229	0.236	0.243	0.251	0.258	0.265	0.273	0.280	0.287	0.295	0.302	0.310	0.317	0.325	0.333
0.20	0.206	0.212	0.218	0.225	0.231	0.238	0.245	0.252	0.260	0.267	0.274	0.281	0.289	0.296	0.303	0.310	0.318	0.325	0.333	0.340

Table C.46 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.057	0.066	0.076	0.086	0.096	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.194	0.204
0.02	0.028	0.037	0.046	0.055	0.065	0.074	0.083	0.093	0.103	0.112	0.122	0.132	0.141	0.151	0.161	0.171	0.180	0.190	0.200	0.210
0.03	0.038	0.046	0.055	0.064	0.073	0.082	0.091	0.101	0.110	0.119	0.129	0.139	0.148	0.158	0.167	0.177	0.187	0.196	0.206	0.215
0.04	0.048	0.056	0.064	0.073	0.082	0.091	0.100	0.109	0.118	0.127	0.137	0.146	0.155	0.165	0.174	0.184	0.193	0.203	0.212	0.222
0.05	0.057	0.065	0.074	0.082	0.091	0.099	0.108	0.117	0.126	0.135	0.144	0.154	0.163	0.172	0.181	0.191	0.200	0.209	0.219	0.228
0.06	0.067	0.075	0.083	0.091	0.100	0.108	0.117	0.126	0.134	0.143	0.152	0.161	0.170	0.180	0.189	0.198	0.207	0.216	0.226	0.235
0.07	0.077	0.084	0.093	0.101	0.109	0.117	0.126	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.135	0.143	0.151	0.160	0.168	0.177	0.186	0.195	0.204	0.212	0.221	0.230	0.239	0.248
0.09	0.097	0.104	0.112	0.119	0.127	0.135	0.144	0.152	0.160	0.168	0.177	0.185	0.194	0.203	0.211	0.220	0.229	0.238	0.247	0.255
0.10	0.107	0.114	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.237	0.245	0.254	0.263
0.11	0.117	0.123	0.131	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.211	0.219	0.227	0.236	0.244	0.253	0.262	0.270
0.12	0.127	0.133	0.141	0.148	0.156	0.163	0.171	0.179	0.186	0.194	0.202	0.211	0.219	0.227	0.235	0.244	0.252	0.260	0.269	0.278
0.13	0.136	0.143	0.150	0.158	0.165	0.173	0.180	0.188	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.252	0.260	0.268	0.277	0.285
0.14	0.146	0.153	0.160	0.167	0.174	0.182	0.189	0.197	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.285	0.293
0.15	0.156	0.163	0.170	0.177	0.184	0.191	0.199	0.206	0.213	0.221	0.229	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300
0.16	0.166	0.173	0.179	0.186	0.193	0.201	0.208	0.215	0.222	0.230	0.237	0.245	0.253	0.260	0.268	0.276	0.284	0.292	0.300	0.308
0.17	0.176	0.182	0.189	0.196	0.203	0.210	0.217	0.224	0.231	0.239	0.246	0.254	0.262	0.269	0.277	0.284	0.292	0.300	0.308	0.316
0.18	0.186	0.192	0.199	0.205	0.212	0.219	0.226	0.233	0.240	0.248	0.255	0.262	0.270	0.277	0.285	0.292	0.300	0.308	0.316	0.324
0.19	0.196	0.202	0.208	0.215	0.222	0.229	0.236	0.243	0.249	0.256	0.264	0.271	0.279	0.286	0.293	0.301	0.308	0.316	0.324	0.331
0.20	0.206	0.212	0.218	0.225	0.231	0.238	0.245	0.252	0.258	0.265	0.273	0.280	0.287	0.294	0.301	0.309	0.316	0.323	0.331	0.339

Table C.47 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 35.

	Q-Statistic G.O.F. Test Significance Level																			
Skewness G.O.F. Test Significance Level	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.056	0.066	0.076	0.086	0.096	0.106	0.115	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.028	0.037	0.046	0.055	0.064	0.074	0.083	0.093	0.102	0.112	0.122	0.131	0.141	0.151	0.161	0.170	0.180	0.190	0.200	0.210
0.03	0.038	0.046	0.055	0.063	0.073	0.082	0.091	0.100	0.110	0.119	0.129	0.138	0.148	0.157	0.167	0.177	0.186	0.196	0.206	0.215
0.04	0.047	0.055	0.064	0.072	0.081	0.090	0.099	0.108	0.118	0.127	0.136	0.145	0.155	0.164	0.174	0.183	0.193	0.202	0.212	0.221
0.05	0.057	0.065	0.073	0.082	0.090	0.099	0.108	0.117	0.126	0.135	0.144	0.153	0.162	0.172	0.181	0.190	0.199	0.209	0.218	0.228
0.06	0.067	0.074	0.082	0.091	0.099	0.108	0.117	0.125	0.134	0.143	0.152	0.161	0.170	0.179	0.188	0.197	0.207	0.216	0.225	0.234
0.07	0.077	0.084	0.092	0.100	0.108	0.117	0.125	0.134	0.143	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.242
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.134	0.143	0.151	0.160	0.168	0.177	0.186	0.195	0.204	0.213	0.222	0.230	0.240	0.249
0.09	0.097	0.104	0.111	0.119	0.127	0.135	0.143	0.151	0.160	0.168	0.177	0.185	0.194	0.203	0.211	0.220	0.229	0.238	0.247	0.256
0.10	0.106	0.113	0.121	0.128	0.136	0.144	0.152	0.160	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.254	0.263
0.11	0.116	0.123	0.130	0.138	0.146	0.153	0.161	0.169	0.177	0.186	0.194	0.202	0.210	0.219	0.227	0.236	0.244	0.253	0.262	0.270
0.12	0.126	0.133	0.140	0.147	0.155	0.163	0.170	0.178	0.186	0.194	0.202	0.210	0.219	0.227	0.235	0.244	0.252	0.260	0.269	0.278
0.13	0.136	0.143	0.150	0.157	0.164	0.172	0.179	0.187	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.260	0.268	0.277	0.285
0.14	0.146	0.152	0.159	0.166	0.173	0.181	0.188	0.196	0.204	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.268	0.276	0.284	0.293
0.15	0.156	0.162	0.169	0.176	0.183	0.190	0.198	0.205	0.213	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300
0.16	0.166	0.172	0.179	0.185	0.192	0.200	0.207	0.214	0.222	0.229	0.237	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300	0.308
0.17	0.176	0.182	0.188	0.195	0.202	0.209	0.216	0.223	0.230	0.238	0.245	0.253	0.261	0.268	0.276	0.284	0.292	0.300	0.308	0.316
0.18	0.185	0.192	0.198	0.204	0.211	0.218	0.225	0.232	0.239	0.247	0.254	0.262	0.269	0.277	0.284	0.292	0.300	0.308	0.316	0.324
0.19	0.195	0.201	0.207	0.214	0.221	0.228	0.235	0.241	0.248	0.256	0.263	0.270	0.278	0.285	0.293	0.300	0.308	0.316	0.323	0.331
0.20	0.205	0.211	0.217	0.224	0.230	0.237	0.244	0.251	0.258	0.265	0.272	0.279	0.286	0.294	0.301	0.309	0.316	0.324	0.331	0.339

Table C.48 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 40.

	Q-Statistic G.O.F. Test Significance Level																			
Skewness G.O.F. Test Significance Level	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.067	0.076	0.086	0.096	0.106	0.116	0.125	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.132	0.142	0.151	0.161	0.171	0.180	0.190	0.200	0.210
0.03	0.038	0.046	0.055	0.064	0.073	0.082	0.092	0.101	0.111	0.120	0.129	0.139	0.148	0.158	0.168	0.177	0.187	0.196	0.206	0.216
0.04	0.048	0.056	0.064	0.073	0.082	0.091	0.100	0.109	0.118	0.128	0.137	0.146	0.155	0.165	0.175	0.184	0.193	0.203	0.212	0.222
0.05	0.057	0.065	0.074	0.082	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.154	0.163	0.172	0.182	0.191	0.200	0.210	0.219	0.228
0.06	0.067	0.075	0.083	0.091	0.100	0.109	0.117	0.126	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.217	0.226	0.235
0.07	0.077	0.085	0.093	0.101	0.109	0.118	0.126	0.135	0.144	0.152	0.161	0.170	0.179	0.188	0.197	0.206	0.215	0.224	0.233	0.242
0.08	0.087	0.094	0.102	0.110	0.118	0.127	0.135	0.143	0.152	0.161	0.169	0.178	0.187	0.195	0.204	0.213	0.222	0.231	0.240	0.249
0.09	0.097	0.104	0.112	0.119	0.127	0.136	0.144	0.152	0.161	0.169	0.177	0.186	0.195	0.203	0.212	0.221	0.229	0.238	0.247	0.256
0.10	0.106	0.114	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.186	0.194	0.203	0.211	0.220	0.228	0.237	0.246	0.254	0.263
0.11	0.116	0.123	0.131	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.211	0.219	0.228	0.236	0.244	0.253	0.262	0.270
0.12	0.126	0.133	0.140	0.148	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.219	0.228	0.236	0.244	0.252	0.261	0.269	0.278
0.13	0.136	0.143	0.150	0.157	0.165	0.172	0.180	0.187	0.195	0.203	0.211	0.219	0.227	0.236	0.244	0.252	0.260	0.269	0.277	0.285
0.14	0.146	0.153	0.159	0.167	0.174	0.181	0.189	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.285	0.293
0.15	0.156	0.162	0.169	0.176	0.183	0.191	0.198	0.205	0.213	0.221	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300
0.16	0.166	0.172	0.179	0.186	0.193	0.200	0.207	0.214	0.222	0.229	0.237	0.245	0.253	0.260	0.268	0.276	0.284	0.292	0.300	0.308
0.17	0.176	0.182	0.188	0.195	0.202	0.209	0.216	0.223	0.231	0.238	0.246	0.253	0.261	0.269	0.277	0.284	0.292	0.300	0.308	0.316
0.18	0.186	0.192	0.198	0.205	0.212	0.219	0.226	0.233	0.240	0.247	0.255	0.262	0.270	0.277	0.285	0.293	0.300	0.308	0.316	0.323
0.19	0.195	0.202	0.208	0.215	0.221	0.228	0.235	0.242	0.249	0.256	0.264	0.271	0.278	0.286	0.293	0.301	0.308	0.316	0.324	0.331
0.20	0.205	0.211	0.218	0.224	0.231	0.237	0.244	0.251	0.258	0.265	0.272	0.279	0.287	0.294	0.302	0.309	0.316	0.324	0.332	0.339

Table C.49 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.057	0.066	0.076	0.086	0.096	0.106	0.116	0.126	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.161	0.171	0.181	0.191	0.200	0.210
0.03	0.038	0.046	0.055	0.064	0.073	0.082	0.092	0.101	0.111	0.120	0.130	0.139	0.149	0.158	0.168	0.177	0.187	0.197	0.206	0.216
0.04	0.048	0.056	0.064	0.073	0.082	0.091	0.100	0.109	0.119	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.203	0.213	0.222
0.05	0.057	0.065	0.074	0.082	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.154	0.163	0.172	0.182	0.191	0.200	0.210	0.219	0.228
0.06	0.067	0.075	0.083	0.091	0.100	0.109	0.117	0.126	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.216	0.226	0.235
0.07	0.077	0.085	0.093	0.101	0.109	0.117	0.126	0.135	0.143	0.152	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.242
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.135	0.143	0.152	0.161	0.169	0.177	0.186	0.195	0.204	0.213	0.222	0.230	0.239	0.249
0.09	0.097	0.104	0.112	0.119	0.127	0.136	0.144	0.152	0.161	0.169	0.177	0.186	0.194	0.203	0.212	0.221	0.229	0.238	0.247	0.256
0.10	0.106	0.114	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.211	0.220	0.228	0.237	0.245	0.254	0.263
0.11	0.116	0.123	0.131	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.210	0.219	0.227	0.236	0.244	0.253	0.261	0.270
0.12	0.126	0.133	0.140	0.148	0.155	0.163	0.171	0.178	0.186	0.194	0.202	0.210	0.218	0.227	0.235	0.243	0.251	0.260	0.268	0.277
0.13	0.136	0.143	0.150	0.157	0.164	0.172	0.180	0.187	0.195	0.203	0.210	0.218	0.227	0.235	0.243	0.251	0.259	0.268	0.276	0.284
0.14	0.146	0.153	0.159	0.167	0.174	0.181	0.189	0.196	0.204	0.212	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.292
0.15	0.156	0.162	0.169	0.176	0.183	0.191	0.198	0.205	0.213	0.220	0.228	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.291	0.299
0.16	0.166	0.172	0.179	0.186	0.193	0.200	0.207	0.214	0.222	0.229	0.236	0.244	0.252	0.259	0.267	0.275	0.283	0.291	0.298	0.307
0.17	0.176	0.182	0.188	0.195	0.202	0.209	0.216	0.223	0.231	0.238	0.245	0.252	0.260	0.268	0.275	0.283	0.290	0.298	0.306	0.314
0.18	0.185	0.192	0.198	0.205	0.211	0.218	0.225	0.232	0.239	0.247	0.254	0.261	0.268	0.276	0.283	0.291	0.298	0.306	0.314	0.322
0.19	0.195	0.201	0.208	0.214	0.221	0.228	0.234	0.241	0.248	0.255	0.262	0.269	0.277	0.284	0.292	0.299	0.306	0.314	0.321	0.329
0.20	0.205	0.211	0.217	0.224	0.230	0.237	0.243	0.250	0.257	0.264	0.271	0.278	0.286	0.293	0.300	0.307	0.315	0.322	0.330	0.337

Table C.50 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 2.5; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.067	0.076	0.086	0.096	0.106	0.116	0.126	0.136	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.028	0.037	0.046	0.056	0.065	0.074	0.084	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.161	0.171	0.181	0.191	0.200	0.210
0.03	0.038	0.046	0.055	0.064	0.073	0.083	0.092	0.101	0.111	0.120	0.130	0.139	0.149	0.158	0.168	0.177	0.187	0.197	0.206	0.216
0.04	0.048	0.056	0.065	0.073	0.082	0.091	0.100	0.109	0.119	0.128	0.137	0.147	0.156	0.165	0.175	0.184	0.193	0.203	0.212	0.222
0.05	0.057	0.065	0.074	0.082	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.154	0.163	0.172	0.182	0.191	0.200	0.209	0.219	0.228
0.06	0.067	0.075	0.083	0.091	0.100	0.108	0.117	0.126	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.216	0.225	0.235
0.07	0.077	0.085	0.093	0.101	0.109	0.117	0.126	0.135	0.143	0.152	0.161	0.170	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241
0.08	0.087	0.094	0.102	0.110	0.118	0.126	0.135	0.143	0.152	0.160	0.169	0.178	0.186	0.195	0.204	0.212	0.221	0.230	0.239	0.248
0.09	0.097	0.104	0.112	0.119	0.127	0.135	0.144	0.152	0.160	0.169	0.177	0.186	0.194	0.203	0.212	0.220	0.229	0.238	0.246	0.255
0.10	0.107	0.114	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.253	0.262
0.11	0.116	0.123	0.131	0.138	0.146	0.154	0.161	0.170	0.177	0.186	0.194	0.202	0.210	0.219	0.227	0.235	0.244	0.252	0.261	0.269
0.12	0.126	0.133	0.140	0.147	0.155	0.163	0.171	0.178	0.186	0.194	0.202	0.211	0.219	0.227	0.235	0.243	0.252	0.260	0.268	0.276
0.13	0.136	0.143	0.150	0.157	0.164	0.172	0.180	0.187	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.284
0.14	0.146	0.153	0.159	0.166	0.174	0.181	0.189	0.196	0.204	0.212	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.291
0.15	0.156	0.162	0.169	0.176	0.183	0.190	0.198	0.205	0.212	0.220	0.228	0.236	0.243	0.251	0.259	0.267	0.275	0.283	0.291	0.299
0.16	0.166	0.172	0.178	0.185	0.192	0.199	0.207	0.214	0.221	0.229	0.236	0.244	0.252	0.259	0.267	0.275	0.283	0.291	0.298	0.306
0.17	0.176	0.182	0.188	0.195	0.202	0.209	0.216	0.223	0.230	0.238	0.245	0.252	0.260	0.268	0.275	0.283	0.291	0.299	0.306	0.314
0.18	0.185	0.192	0.198	0.204	0.211	0.218	0.225	0.232	0.239	0.247	0.254	0.261	0.269	0.276	0.284	0.291	0.299	0.307	0.314	0.322
0.19	0.195	0.201	0.207	0.214	0.220	0.227	0.234	0.241	0.248	0.255	0.262	0.270	0.277	0.284	0.292	0.299	0.307	0.315	0.322	0.330
0.20	0.205	0.211	0.217	0.223	0.230	0.237	0.243	0.250	0.257	0.264	0.271	0.278	0.285	0.293	0.300	0.307	0.315	0.323	0.330	0.337

C.6 Attained Significance Level Tables for H_o = Weibull(β = 3.0).

Table C.51 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.160	0.170	0.180	0.190	0.200
0.02	0.029	0.037	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.109	0.119	0.128	0.137	0.146	0.156	0.165	0.174	0.184	0.194	0.203
0.03	0.038	0.047	0.056	0.064	0.073	0.082	0.090	0.099	0.108	0.117	0.126	0.135	0.144	0.153	0.161	0.171	0.180	0.189	0.198	0.208
0.04	0.048	0.057	0.065	0.074	0.082	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.177	0.186	0.195	0.205	0.213
0.05	0.058	0.066	0.075	0.083	0.091	0.100	0.108	0.116	0.125	0.133	0.142	0.150	0.159	0.167	0.176	0.185	0.193	0.202	0.211	0.220
0.06	0.068	0.076	0.084	0.092	0.101	0.109	0.117	0.125	0.133	0.142	0.150	0.159	0.167	0.176	0.184	0.192	0.201	0.210	0.219	0.227
0.07	0.078	0.086	0.094	0.102	0.110	0.118	0.126	0.134	0.142	0.151	0.159	0.167	0.175	0.184	0.192	0.200	0.209	0.217	0.226	0.235
0.08	0.088	0.096	0.104	0.112	0.120	0.128	0.135	0.143	0.151	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.217	0.225	0.234	0.242
0.09	0.098	0.105	0.113	0.121	0.129	0.137	0.145	0.153	0.160	0.168	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.233	0.242	0.250
0.10	0.107	0.115	0.123	0.131	0.139	0.146	0.154	0.162	0.169	0.177	0.186	0.194	0.201	0.209	0.217	0.225	0.233	0.241	0.250	0.258
0.11	0.117	0.125	0.133	0.141	0.148	0.156	0.163	0.171	0.179	0.187	0.195	0.203	0.210	0.218	0.226	0.234	0.242	0.250	0.258	0.266
0.12	0.127	0.135	0.142	0.150	0.158	0.165	0.173	0.180	0.188	0.196	0.204	0.212	0.219	0.227	0.235	0.242	0.250	0.258	0.266	0.274
0.13	0.137	0.145	0.152	0.160	0.167	0.175	0.182	0.190	0.197	0.205	0.213	0.221	0.228	0.236	0.243	0.251	0.259	0.267	0.275	0.283
0.14	0.147	0.155	0.162	0.170	0.177	0.185	0.192	0.199	0.207	0.214	0.222	0.230	0.237	0.245	0.252	0.260	0.268	0.275	0.283	0.291
0.15	0.157	0.164	0.172	0.179	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.246	0.254	0.261	0.269	0.277	0.284	0.292	0.300
0.16	0.167	0.174	0.181	0.189	0.196	0.204	0.211	0.218	0.225	0.233	0.241	0.248	0.256	0.263	0.270	0.278	0.285	0.293	0.301	0.308
0.17	0.177	0.184	0.191	0.199	0.206	0.213	0.220	0.228	0.235	0.242	0.250	0.257	0.265	0.272	0.279	0.287	0.294	0.302	0.309	0.317
0.18	0.187	0.194	0.201	0.208	0.216	0.223	0.230	0.237	0.244	0.252	0.259	0.266	0.274	0.281	0.288	0.296	0.303	0.311	0.318	0.326
0.19	0.197	0.204	0.211	0.218	0.225	0.232	0.239	0.247	0.254	0.261	0.268	0.276	0.283	0.290	0.297	0.305	0.312	0.319	0.327	0.334
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.249	0.256	0.263	0.270	0.278	0.285	0.292	0.299	0.306	0.314	0.321	0.328	0.336	0.343

Table C.52 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.162	0.172	0.181	0.191	0.201
0.02	0.028	0.037	0.046	0.054	0.063	0.073	0.082	0.091	0.100	0.110	0.119	0.128	0.138	0.147	0.157	0.166	0.176	0.185	0.195	0.205
0.03	0.038	0.046	0.055	0.063	0.072	0.081	0.090	0.099	0.108	0.117	0.126	0.135	0.144	0.154	0.163	0.172	0.182	0.191	0.201	0.210
0.04	0.048	0.056	0.064	0.073	0.081	0.090	0.099	0.107	0.116	0.125	0.134	0.143	0.152	0.161	0.170	0.179	0.188	0.197	0.207	0.216
0.05	0.058	0.066	0.074	0.082	0.090	0.099	0.108	0.116	0.125	0.133	0.142	0.151	0.160	0.169	0.177	0.186	0.195	0.204	0.214	0.223
0.06	0.068	0.076	0.084	0.092	0.100	0.108	0.117	0.125	0.133	0.142	0.151	0.159	0.168	0.177	0.185	0.194	0.203	0.212	0.221	0.230
0.07	0.078	0.085	0.093	0.101	0.109	0.118	0.126	0.134	0.142	0.151	0.159	0.168	0.176	0.185	0.194	0.202	0.211	0.220	0.229	0.238
0.08	0.088	0.095	0.103	0.111	0.119	0.127	0.135	0.143	0.151	0.160	0.168	0.176	0.185	0.194	0.202	0.211	0.219	0.228	0.237	0.245
0.09	0.097	0.105	0.113	0.120	0.128	0.136	0.144	0.152	0.160	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.227	0.236	0.245	0.253
0.10	0.107	0.115	0.122	0.130	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.194	0.203	0.211	0.219	0.228	0.236	0.245	0.253	0.262
0.11	0.117	0.125	0.132	0.140	0.148	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.212	0.220	0.228	0.236	0.244	0.253	0.261	0.270
0.12	0.127	0.135	0.142	0.149	0.157	0.165	0.173	0.180	0.188	0.196	0.204	0.212	0.221	0.229	0.237	0.245	0.253	0.261	0.270	0.278
0.13	0.137	0.144	0.152	0.159	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.221	0.230	0.238	0.245	0.254	0.262	0.270	0.278	0.287
0.14	0.147	0.154	0.162	0.169	0.177	0.184	0.192	0.199	0.207	0.215	0.223	0.231	0.239	0.247	0.254	0.262	0.270	0.279	0.287	0.295
0.15	0.157	0.164	0.171	0.179	0.186	0.194	0.201	0.209	0.217	0.224	0.232	0.240	0.248	0.256	0.263	0.271	0.279	0.287	0.296	0.304
0.16	0.167	0.174	0.181	0.189	0.196	0.203	0.211	0.218	0.226	0.234	0.241	0.249	0.257	0.265	0.272	0.280	0.288	0.296	0.304	0.312
0.17	0.177	0.184	0.191	0.198	0.206	0.213	0.220	0.228	0.235	0.243	0.250	0.258	0.266	0.274	0.281	0.289	0.297	0.305	0.313	0.320
0.18	0.187	0.194	0.201	0.208	0.215	0.223	0.230	0.237	0.245	0.252	0.260	0.267	0.275	0.283	0.290	0.298	0.306	0.314	0.321	0.329
0.19	0.197	0.204	0.211	0.218	0.225	0.232	0.240	0.247	0.254	0.262	0.269	0.277	0.284	0.292	0.299	0.307	0.315	0.322	0.330	0.338
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.249	0.257	0.264	0.271	0.278	0.286	0.293	0.301	0.308	0.316	0.324	0.331	0.339	0.347

Table C.53 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.056	0.065	0.075	0.085	0.094	0.104	0.114	0.124	0.133	0.143	0.153	0.163	0.173	0.183	0.193	0.202
0.02	0.028	0.037	0.046	0.055	0.064	0.073	0.083	0.092	0.101	0.111	0.120	0.130	0.140	0.149	0.159	0.169	0.178	0.188	0.197	0.207
0.03	0.038	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.109	0.119	0.128	0.138	0.147	0.157	0.166	0.175	0.185	0.194	0.204	0.213
0.04	0.048	0.056	0.065	0.073	0.082	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.155	0.164	0.173	0.183	0.192	0.201	0.211	0.220
0.05	0.058	0.066	0.074	0.083	0.091	0.100	0.109	0.118	0.126	0.135	0.145	0.154	0.163	0.172	0.181	0.190	0.199	0.209	0.218	0.227
0.06	0.068	0.076	0.084	0.092	0.101	0.109	0.118	0.127	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.207	0.216	0.225	0.234
0.07	0.077	0.085	0.094	0.102	0.110	0.119	0.127	0.136	0.144	0.153	0.162	0.171	0.179	0.188	0.197	0.206	0.215	0.224	0.233	0.242
0.08	0.087	0.095	0.103	0.111	0.119	0.128	0.136	0.145	0.153	0.162	0.170	0.179	0.188	0.197	0.206	0.214	0.223	0.232	0.241	0.250
0.09	0.097	0.105	0.113	0.121	0.129	0.137	0.146	0.154	0.162	0.171	0.179	0.188	0.197	0.205	0.214	0.223	0.231	0.240	0.249	0.258
0.10	0.107	0.115	0.123	0.131	0.139	0.147	0.155	0.163	0.172	0.180	0.189	0.197	0.206	0.214	0.223	0.231	0.240	0.249	0.257	0.266
0.11	0.117	0.125	0.133	0.140	0.148	0.156	0.165	0.173	0.181	0.189	0.198	0.206	0.214	0.223	0.231	0.240	0.248	0.257	0.266	0.274
0.12	0.127	0.135	0.142	0.150	0.158	0.166	0.174	0.182	0.190	0.198	0.207	0.215	0.223	0.232	0.240	0.249	0.257	0.265	0.274	0.282
0.13	0.137	0.144	0.152	0.160	0.168	0.176	0.183	0.191	0.199	0.208	0.216	0.224	0.232	0.241	0.249	0.257	0.265	0.274	0.282	0.291
0.14	0.147	0.154	0.162	0.170	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.233	0.241	0.250	0.258	0.266	0.274	0.283	0.291	0.299
0.15	0.157	0.164	0.172	0.179	0.187	0.195	0.202	0.210	0.218	0.226	0.234	0.242	0.250	0.259	0.267	0.275	0.283	0.291	0.299	0.307
0.16	0.167	0.174	0.182	0.189	0.197	0.204	0.212	0.220	0.228	0.236	0.243	0.251	0.259	0.267	0.275	0.284	0.291	0.300	0.308	0.316
0.17	0.177	0.184	0.191	0.199	0.206	0.214	0.222	0.229	0.237	0.245	0.253	0.260	0.268	0.276	0.284	0.292	0.300	0.308	0.316	0.324
0.18	0.187	0.194	0.201	0.208	0.216	0.223	0.231	0.239	0.246	0.254	0.262	0.269	0.277	0.285	0.293	0.301	0.309	0.317	0.325	0.333
0.19	0.197	0.204	0.211	0.218	0.225	0.233	0.240	0.248	0.256	0.263	0.271	0.278	0.286	0.294	0.302	0.309	0.317	0.325	0.333	0.341
0.20	0.207	0.214	0.221	0.228	0.235	0.243	0.250	0.257	0.265	0.272	0.280	0.288	0.295	0.303	0.311	0.318	0.326	0.334	0.342	0.350

Table C.54 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.057	0.066	0.076	0.086	0.095	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.203
0.02	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.160	0.170	0.180	0.189	0.199	0.209
0.03	0.038	0.047	0.056	0.065	0.074	0.083	0.092	0.101	0.111	0.120	0.129	0.139	0.148	0.158	0.167	0.176	0.186	0.196	0.205	0.215
0.04	0.048	0.056	0.065	0.074	0.083	0.092	0.101	0.110	0.119	0.128	0.138	0.147	0.156	0.165	0.175	0.184	0.193	0.203	0.212	0.222
0.05	0.058	0.066	0.075	0.083	0.092	0.101	0.110	0.119	0.128	0.137	0.146	0.155	0.164	0.173	0.183	0.192	0.201	0.210	0.220	0.229
0.06	0.068	0.076	0.084	0.093	0.101	0.110	0.119	0.128	0.137	0.146	0.155	0.164	0.173	0.182	0.191	0.200	0.209	0.218	0.227	0.237
0.07	0.077	0.086	0.094	0.102	0.111	0.119	0.128	0.137	0.146	0.155	0.163	0.172	0.181	0.190	0.199	0.208	0.217	0.226	0.235	0.245
0.08	0.087	0.095	0.104	0.112	0.120	0.129	0.137	0.146	0.155	0.163	0.172	0.181	0.190	0.198	0.207	0.216	0.225	0.234	0.243	0.252
0.09	0.097	0.105	0.113	0.121	0.130	0.138	0.146	0.155	0.164	0.172	0.181	0.190	0.198	0.207	0.216	0.225	0.233	0.242	0.251	0.260
0.10	0.107	0.115	0.123	0.131	0.139	0.147	0.156	0.164	0.173	0.181	0.190	0.199	0.207	0.216	0.224	0.233	0.242	0.250	0.259	0.268
0.11	0.117	0.125	0.133	0.141	0.149	0.157	0.165	0.174	0.182	0.191	0.199	0.207	0.216	0.224	0.233	0.241	0.250	0.259	0.267	0.276
0.12	0.127	0.135	0.143	0.150	0.159	0.167	0.175	0.183	0.191	0.200	0.208	0.216	0.225	0.233	0.241	0.250	0.259	0.267	0.276	0.284
0.13	0.137	0.145	0.152	0.160	0.168	0.176	0.184	0.192	0.201	0.209	0.217	0.225	0.234	0.242	0.250	0.258	0.267	0.275	0.284	0.292
0.14	0.147	0.154	0.162	0.170	0.178	0.185	0.193	0.202	0.210	0.218	0.226	0.234	0.242	0.250	0.259	0.267	0.275	0.284	0.292	0.300
0.15	0.157	0.164	0.172	0.180	0.187	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.276	0.284	0.292	0.300	0.309
0.16	0.167	0.174	0.182	0.189	0.197	0.205	0.212	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.293	0.301	0.309	0.317
0.17	0.177	0.184	0.192	0.199	0.207	0.214	0.222	0.230	0.238	0.246	0.254	0.261	0.269	0.277	0.285	0.293	0.301	0.309	0.317	0.326
0.18	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.247	0.255	0.263	0.270	0.278	0.286	0.294	0.302	0.310	0.318	0.326	0.334
0.19	0.197	0.204	0.211	0.219	0.226	0.233	0.241	0.249	0.256	0.264	0.272	0.280	0.287	0.295	0.303	0.311	0.319	0.326	0.334	0.342
0.20	0.207	0.214	0.221	0.228	0.236	0.243	0.250	0.258	0.266	0.273	0.281	0.289	0.296	0.304	0.311	0.319	0.327	0.335	0.342	0.350

Table C.55 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.066	0.076	0.086	0.096	0.105	0.115	0.125	0.135	0.145	0.155	0.165	0.174	0.184	0.194	0.204
0.02	0.029	0.037	0.047	0.056	0.065	0.075	0.084	0.094	0.103	0.113	0.122	0.132	0.142	0.151	0.161	0.171	0.181	0.190	0.200	0.210
0.03	0.038	0.047	0.056	0.065	0.074	0.083	0.093	0.102	0.111	0.121	0.130	0.140	0.149	0.159	0.168	0.178	0.188	0.197	0.207	0.216
0.04	0.048	0.057	0.065	0.074	0.083	0.092	0.101	0.111	0.120	0.129	0.138	0.148	0.157	0.167	0.176	0.185	0.195	0.204	0.214	0.223
0.05	0.058	0.066	0.075	0.084	0.093	0.102	0.110	0.119	0.128	0.138	0.147	0.156	0.165	0.175	0.184	0.193	0.203	0.212	0.221	0.231
0.06	0.068	0.076	0.085	0.093	0.102	0.111	0.120	0.128	0.137	0.146	0.155	0.165	0.174	0.183	0.192	0.201	0.211	0.220	0.229	0.238
0.07	0.078	0.086	0.094	0.103	0.111	0.120	0.129	0.137	0.146	0.155	0.164	0.173	0.182	0.191	0.200	0.209	0.218	0.228	0.237	0.246
0.08	0.088	0.096	0.104	0.112	0.121	0.129	0.138	0.147	0.155	0.164	0.173	0.182	0.191	0.200	0.209	0.218	0.227	0.236	0.245	0.254
0.09	0.098	0.105	0.114	0.122	0.130	0.139	0.147	0.156	0.165	0.173	0.182	0.191	0.200	0.208	0.217	0.226	0.235	0.244	0.253	0.262
0.10	0.107	0.115	0.123	0.132	0.140	0.148	0.157	0.165	0.174	0.182	0.191	0.200	0.208	0.217	0.226	0.235	0.243	0.252	0.261	0.270
0.11	0.117	0.125	0.133	0.141	0.150	0.158	0.166	0.175	0.183	0.192	0.200	0.209	0.217	0.226	0.235	0.243	0.252	0.261	0.269	0.278
0.12	0.127	0.135	0.143	0.151	0.159	0.167	0.176	0.184	0.192	0.201	0.209	0.218	0.226	0.235	0.243	0.252	0.260	0.269	0.278	0.286
0.13	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.193	0.201	0.210	0.218	0.227	0.235	0.243	0.252	0.260	0.269	0.277	0.286	0.294
0.14	0.147	0.155	0.162	0.170	0.178	0.187	0.195	0.203	0.211	0.219	0.227	0.236	0.244	0.252	0.261	0.269	0.277	0.286	0.294	0.303
0.15	0.157	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.261	0.269	0.277	0.285	0.294	0.302	0.310
0.16	0.167	0.174	0.182	0.190	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.278	0.286	0.294	0.302	0.310	0.318
0.17	0.177	0.184	0.192	0.199	0.207	0.215	0.223	0.230	0.238	0.246	0.254	0.262	0.270	0.278	0.286	0.294	0.302	0.310	0.319	0.327
0.18	0.187	0.194	0.201	0.209	0.217	0.225	0.232	0.240	0.248	0.255	0.263	0.271	0.279	0.287	0.295	0.303	0.311	0.319	0.327	0.335
0.19	0.197	0.204	0.211	0.219	0.226	0.234	0.242	0.249	0.257	0.265	0.272	0.280	0.288	0.296	0.304	0.312	0.320	0.327	0.335	0.343
0.20	0.207	0.214	0.221	0.229	0.236	0.244	0.251	0.259	0.266	0.274	0.282	0.289	0.297	0.305	0.313	0.321	0.328	0.336	0.344	0.352

Table C.56 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.048	0.057	0.067	0.077	0.086	0.096	0.106	0.116	0.126	0.135	0.145	0.155	0.165	0.175	0.185	0.195	0.204
0.02	0.029	0.038	0.047	0.056	0.066	0.075	0.085	0.094	0.104	0.114	0.123	0.133	0.143	0.152	0.162	0.172	0.181	0.191	0.201	0.210
0.03	0.039	0.047	0.056	0.066	0.075	0.084	0.094	0.103	0.112	0.122	0.131	0.141	0.150	0.160	0.169	0.179	0.188	0.198	0.208	0.217
0.04	0.048	0.057	0.066	0.075	0.084	0.093	0.102	0.112	0.121	0.130	0.140	0.149	0.158	0.168	0.177	0.186	0.196	0.205	0.215	0.224
0.05	0.058	0.067	0.075	0.084	0.093	0.102	0.112	0.121	0.130	0.139	0.148	0.157	0.167	0.176	0.185	0.194	0.204	0.213	0.223	0.232
0.06	0.068	0.077	0.085	0.094	0.103	0.112	0.121	0.130	0.139	0.148	0.157	0.166	0.175	0.184	0.193	0.202	0.212	0.221	0.230	0.240
0.07	0.078	0.086	0.095	0.103	0.112	0.121	0.130	0.139	0.148	0.157	0.166	0.174	0.183	0.193	0.202	0.211	0.220	0.229	0.238	0.247
0.08	0.088	0.096	0.104	0.113	0.122	0.130	0.139	0.148	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.219	0.228	0.237	0.246	0.255
0.09	0.098	0.106	0.114	0.122	0.131	0.140	0.148	0.157	0.165	0.174	0.183	0.192	0.200	0.209	0.218	0.227	0.236	0.245	0.254	0.263
0.10	0.108	0.116	0.124	0.132	0.141	0.149	0.157	0.166	0.174	0.183	0.192	0.200	0.209	0.218	0.226	0.235	0.244	0.253	0.262	0.271
0.11	0.118	0.126	0.133	0.142	0.150	0.158	0.167	0.175	0.183	0.192	0.201	0.209	0.218	0.226	0.235	0.244	0.252	0.261	0.270	0.278
0.12	0.128	0.135	0.143	0.151	0.160	0.168	0.176	0.184	0.193	0.201	0.209	0.218	0.226	0.235	0.243	0.252	0.261	0.269	0.278	0.286
0.13	0.138	0.145	0.153	0.161	0.169	0.177	0.185	0.193	0.202	0.210	0.218	0.227	0.235	0.244	0.252	0.260	0.269	0.278	0.286	0.294
0.14	0.147	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.219	0.227	0.236	0.244	0.252	0.260	0.269	0.277	0.286	0.294	0.302
0.15	0.157	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.245	0.253	0.261	0.269	0.277	0.286	0.294	0.302	0.310
0.16	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.221	0.229	0.237	0.245	0.253	0.262	0.270	0.278	0.286	0.294	0.302	0.311	0.319
0.17	0.177	0.185	0.192	0.200	0.208	0.215	0.223	0.231	0.239	0.247	0.255	0.263	0.270	0.278	0.287	0.295	0.303	0.311	0.319	0.327
0.18	0.187	0.194	0.202	0.209	0.217	0.225	0.233	0.240	0.248	0.256	0.264	0.272	0.279	0.287	0.295	0.303	0.311	0.319	0.327	0.335
0.19	0.197	0.204	0.212	0.219	0.227	0.234	0.242	0.249	0.257	0.265	0.273	0.281	0.288	0.296	0.304	0.312	0.320	0.328	0.335	0.343
0.20	0.207	0.214	0.221	0.229	0.236	0.244	0.251	0.259	0.267	0.274	0.282	0.290	0.297	0.305	0.313	0.321	0.328	0.336	0.344	0.351

Table C.57 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.048	0.057	0.067	0.077	0.086	0.096	0.106	0.116	0.126	0.136	0.145	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.029	0.038	0.047	0.056	0.066	0.075	0.085	0.094	0.104	0.114	0.123	0.133	0.143	0.152	0.162	0.172	0.182	0.191	0.201	0.211
0.03	0.038	0.047	0.056	0.066	0.075	0.084	0.094	0.103	0.112	0.122	0.131	0.141	0.150	0.160	0.169	0.179	0.189	0.198	0.208	0.218
0.04	0.048	0.057	0.066	0.075	0.084	0.093	0.102	0.112	0.121	0.130	0.140	0.149	0.158	0.168	0.177	0.187	0.196	0.206	0.215	0.225
0.05	0.058	0.066	0.075	0.084	0.093	0.102	0.111	0.120	0.129	0.139	0.148	0.157	0.166	0.176	0.185	0.194	0.204	0.213	0.223	0.232
0.06	0.068	0.076	0.085	0.094	0.102	0.111	0.120	0.129	0.138	0.147	0.156	0.166	0.175	0.184	0.193	0.202	0.212	0.221	0.230	0.240
0.07	0.078	0.086	0.094	0.103	0.112	0.121	0.129	0.138	0.147	0.156	0.165	0.174	0.183	0.192	0.201	0.211	0.220	0.229	0.238	0.247
0.08	0.088	0.096	0.104	0.113	0.121	0.130	0.139	0.147	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.219	0.228	0.237	0.246	0.255
0.09	0.098	0.105	0.114	0.122	0.131	0.139	0.148	0.157	0.165	0.174	0.183	0.191	0.200	0.209	0.218	0.227	0.236	0.245	0.254	0.263
0.10	0.108	0.115	0.124	0.132	0.140	0.149	0.157	0.166	0.174	0.183	0.191	0.200	0.209	0.217	0.226	0.235	0.244	0.253	0.262	0.270
0.11	0.117	0.125	0.133	0.142	0.150	0.158	0.167	0.175	0.183	0.192	0.200	0.209	0.218	0.226	0.235	0.244	0.253	0.261	0.270	0.279
0.12	0.127	0.135	0.143	0.151	0.159	0.168	0.176	0.184	0.193	0.201	0.209	0.218	0.226	0.235	0.243	0.252	0.261	0.269	0.278	0.286
0.13	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.210	0.218	0.227	0.235	0.244	0.252	0.261	0.269	0.278	0.286	0.295
0.14	0.147	0.155	0.163	0.170	0.178	0.187	0.195	0.203	0.211	0.219	0.227	0.236	0.244	0.252	0.261	0.269	0.277	0.286	0.294	0.303
0.15	0.157	0.165	0.172	0.180	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.245	0.253	0.261	0.269	0.278	0.286	0.294	0.303	0.311
0.16	0.167	0.174	0.182	0.190	0.198	0.206	0.213	0.221	0.230	0.237	0.245	0.253	0.262	0.270	0.278	0.286	0.295	0.303	0.311	0.319
0.17	0.177	0.184	0.192	0.199	0.207	0.215	0.223	0.231	0.239	0.247	0.254	0.262	0.270	0.279	0.287	0.295	0.303	0.311	0.319	0.327
0.18	0.187	0.194	0.202	0.209	0.217	0.224	0.232	0.240	0.248	0.255	0.263	0.271	0.279	0.287	0.295	0.303	0.311	0.319	0.327	0.335
0.19	0.197	0.204	0.211	0.219	0.226	0.234	0.242	0.249	0.257	0.265	0.272	0.280	0.288	0.296	0.304	0.312	0.320	0.328	0.336	0.344
0.20	0.207	0.214	0.221	0.228	0.236	0.243	0.251	0.259	0.266	0.274	0.281	0.289	0.297	0.305	0.313	0.321	0.328	0.336	0.344	0.352

Table C.58 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.067	0.077	0.087	0.097	0.106	0.116	0.126	0.136	0.146	0.156	0.166	0.175	0.185	0.195	0.205
0.02	0.029	0.038	0.047	0.057	0.066	0.076	0.085	0.095	0.105	0.114	0.124	0.134	0.143	0.153	0.163	0.172	0.182	0.192	0.202	0.211
0.03	0.039	0.047	0.057	0.066	0.075	0.085	0.094	0.103	0.113	0.122	0.132	0.141	0.151	0.161	0.170	0.180	0.189	0.199	0.209	0.218
0.04	0.048	0.057	0.066	0.075	0.084	0.094	0.103	0.112	0.122	0.131	0.140	0.150	0.159	0.168	0.178	0.187	0.197	0.206	0.216	0.225
0.05	0.058	0.067	0.076	0.085	0.094	0.103	0.112	0.121	0.131	0.140	0.149	0.158	0.167	0.177	0.186	0.195	0.205	0.214	0.224	0.233
0.06	0.068	0.077	0.085	0.094	0.103	0.112	0.121	0.130	0.140	0.149	0.158	0.167	0.176	0.185	0.194	0.204	0.213	0.222	0.231	0.241
0.07	0.078	0.086	0.095	0.104	0.113	0.122	0.131	0.139	0.148	0.158	0.167	0.175	0.185	0.194	0.203	0.212	0.221	0.230	0.239	0.248
0.08	0.088	0.096	0.105	0.113	0.122	0.131	0.140	0.149	0.157	0.166	0.175	0.184	0.193	0.202	0.211	0.220	0.229	0.238	0.247	0.256
0.09	0.098	0.106	0.114	0.123	0.132	0.140	0.149	0.158	0.166	0.175	0.184	0.193	0.202	0.210	0.219	0.228	0.237	0.246	0.255	0.264
0.10	0.108	0.116	0.124	0.133	0.141	0.150	0.158	0.167	0.175	0.184	0.193	0.202	0.210	0.219	0.228	0.237	0.245	0.254	0.263	0.272
0.11	0.118	0.125	0.134	0.142	0.151	0.159	0.168	0.176	0.185	0.193	0.202	0.210	0.219	0.228	0.236	0.245	0.254	0.262	0.271	0.280
0.12	0.128	0.135	0.143	0.152	0.160	0.168	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.253	0.262	0.270	0.279	0.288
0.13	0.137	0.145	0.153	0.161	0.170	0.178	0.186	0.195	0.203	0.211	0.220	0.228	0.236	0.245	0.253	0.262	0.270	0.279	0.287	0.296
0.14	0.147	0.155	0.163	0.171	0.179	0.187	0.196	0.204	0.212	0.220	0.229	0.237	0.245	0.254	0.262	0.270	0.278	0.287	0.295	0.304
0.15	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.238	0.246	0.254	0.262	0.271	0.279	0.287	0.295	0.304	0.312
0.16	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.247	0.254	0.263	0.271	0.279	0.287	0.295	0.304	0.312	0.320
0.17	0.177	0.185	0.192	0.200	0.208	0.216	0.224	0.232	0.239	0.247	0.255	0.263	0.271	0.280	0.288	0.296	0.303	0.312	0.320	0.328
0.18	0.187	0.194	0.202	0.210	0.218	0.225	0.233	0.241	0.249	0.256	0.264	0.272	0.280	0.288	0.296	0.304	0.312	0.320	0.328	0.336
0.19	0.197	0.204	0.212	0.219	0.227	0.235	0.242	0.250	0.258	0.266	0.273	0.281	0.289	0.297	0.305	0.312	0.320	0.328	0.336	0.344
0.20	0.207	0.214	0.222	0.229	0.237	0.244	0.252	0.259	0.267	0.275	0.282	0.290	0.298	0.305	0.313	0.321	0.329	0.337	0.344	0.352

Table C.59 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.067	0.077	0.087	0.097	0.106	0.116	0.126	0.136	0.146	0.156	0.166	0.175	0.185	0.195	0.205
0.02	0.029	0.038	0.047	0.057	0.066	0.076	0.085	0.095	0.105	0.114	0.124	0.134	0.143	0.153	0.163	0.173	0.182	0.192	0.202	0.212
0.03	0.039	0.048	0.057	0.066	0.075	0.085	0.094	0.104	0.113	0.123	0.132	0.142	0.151	0.161	0.171	0.180	0.190	0.199	0.209	0.219
0.04	0.048	0.057	0.066	0.075	0.085	0.094	0.103	0.112	0.122	0.131	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207	0.216	0.226
0.05	0.058	0.067	0.076	0.085	0.094	0.103	0.112	0.121	0.131	0.140	0.149	0.158	0.168	0.177	0.186	0.196	0.205	0.215	0.224	0.233
0.06	0.068	0.077	0.086	0.094	0.103	0.112	0.121	0.130	0.139	0.149	0.158	0.167	0.176	0.185	0.194	0.204	0.213	0.222	0.232	0.241
0.07	0.078	0.086	0.095	0.104	0.113	0.122	0.130	0.139	0.148	0.157	0.166	0.175	0.184	0.194	0.203	0.212	0.221	0.230	0.239	0.248
0.08	0.088	0.096	0.105	0.113	0.122	0.131	0.140	0.149	0.157	0.166	0.175	0.184	0.193	0.202	0.211	0.220	0.229	0.238	0.247	0.256
0.09	0.098	0.106	0.114	0.123	0.132	0.140	0.149	0.158	0.166	0.175	0.184	0.193	0.202	0.210	0.219	0.228	0.237	0.246	0.255	0.264
0.10	0.108	0.116	0.124	0.133	0.141	0.150	0.158	0.167	0.175	0.184	0.193	0.202	0.210	0.219	0.227	0.236	0.245	0.254	0.263	0.272
0.11	0.118	0.126	0.134	0.142	0.151	0.159	0.167	0.176	0.184	0.193	0.202	0.210	0.219	0.227	0.236	0.245	0.253	0.262	0.271	0.280
0.12	0.128	0.136	0.144	0.152	0.160	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.227	0.236	0.244	0.253	0.262	0.270	0.279	0.288
0.13	0.138	0.145	0.153	0.161	0.170	0.178	0.186	0.194	0.202	0.211	0.219	0.228	0.236	0.244	0.253	0.261	0.270	0.278	0.287	0.295
0.14	0.147	0.155	0.163	0.171	0.179	0.187	0.195	0.204	0.212	0.220	0.228	0.237	0.245	0.253	0.261	0.270	0.278	0.287	0.295	0.303
0.15	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.262	0.270	0.278	0.286	0.295	0.303	0.311
0.16	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.254	0.262	0.270	0.278	0.286	0.295	0.303	0.311	0.319
0.17	0.177	0.185	0.192	0.200	0.208	0.216	0.223	0.231	0.239	0.247	0.255	0.263	0.271	0.279	0.287	0.295	0.303	0.311	0.319	0.327
0.18	0.187	0.194	0.202	0.210	0.217	0.225	0.233	0.240	0.248	0.256	0.264	0.272	0.279	0.287	0.295	0.303	0.311	0.319	0.327	0.335
0.19	0.197	0.204	0.211	0.219	0.227	0.235	0.242	0.250	0.257	0.265	0.273	0.281	0.288	0.296	0.304	0.312	0.320	0.328	0.335	0.343
0.20	0.207	0.214	0.221	0.229	0.236	0.244	0.251	0.259	0.266	0.274	0.282	0.290	0.297	0.305	0.312	0.320	0.328	0.336	0.344	0.351

Table C.60 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.0; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.068	0.077	0.087	0.097	0.107	0.117	0.126	0.136	0.146	0.156	0.166	0.176	0.186	0.195	0.205
0.02	0.029	0.038	0.048	0.057	0.067	0.076	0.086	0.095	0.105	0.115	0.124	0.134	0.144	0.153	0.163	0.173	0.183	0.192	0.202	0.212
0.03	0.039	0.048	0.057	0.066	0.076	0.085	0.095	0.104	0.114	0.123	0.133	0.142	0.152	0.161	0.171	0.180	0.190	0.200	0.209	0.219
0.04	0.049	0.057	0.067	0.076	0.085	0.094	0.103	0.113	0.122	0.132	0.141	0.151	0.160	0.169	0.179	0.188	0.198	0.207	0.217	0.226
0.05	0.058	0.067	0.076	0.085	0.094	0.103	0.113	0.122	0.131	0.140	0.150	0.159	0.168	0.178	0.187	0.196	0.205	0.215	0.224	0.234
0.06	0.068	0.077	0.086	0.095	0.103	0.112	0.121	0.130	0.140	0.149	0.158	0.167	0.177	0.186	0.195	0.204	0.213	0.222	0.232	0.241
0.07	0.078	0.087	0.095	0.104	0.113	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.185	0.194	0.203	0.212	0.221	0.230	0.239	0.249
0.08	0.088	0.096	0.105	0.114	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.185	0.193	0.202	0.211	0.220	0.229	0.238	0.247	0.256
0.09	0.098	0.106	0.115	0.123	0.132	0.140	0.149	0.158	0.167	0.176	0.184	0.193	0.202	0.211	0.220	0.228	0.237	0.246	0.255	0.264
0.10	0.108	0.116	0.124	0.133	0.141	0.150	0.158	0.167	0.176	0.185	0.193	0.202	0.211	0.219	0.228	0.237	0.246	0.255	0.263	0.272
0.11	0.118	0.126	0.134	0.142	0.151	0.159	0.168	0.176	0.185	0.193	0.202	0.211	0.219	0.228	0.236	0.245	0.254	0.262	0.271	0.280
0.12	0.128	0.136	0.144	0.152	0.160	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.253	0.262	0.271	0.279	0.288
0.13	0.138	0.145	0.153	0.161	0.170	0.178	0.186	0.195	0.203	0.211	0.220	0.228	0.236	0.245	0.253	0.261	0.270	0.279	0.287	0.295
0.14	0.147	0.155	0.163	0.171	0.179	0.187	0.196	0.204	0.212	0.220	0.229	0.237	0.245	0.253	0.262	0.270	0.278	0.287	0.295	0.303
0.15	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.238	0.246	0.254	0.262	0.270	0.278	0.287	0.295	0.303	0.311
0.16	0.167	0.175	0.183	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.247	0.255	0.263	0.271	0.279	0.287	0.295	0.303	0.311	0.320
0.17	0.177	0.185	0.192	0.200	0.208	0.216	0.224	0.231	0.239	0.247	0.255	0.264	0.271	0.279	0.287	0.295	0.303	0.312	0.320	0.328
0.18	0.187	0.195	0.202	0.210	0.217	0.225	0.233	0.241	0.248	0.256	0.264	0.272	0.280	0.288	0.296	0.304	0.312	0.320	0.328	0.336
0.19	0.197	0.204	0.212	0.219	0.227	0.234	0.242	0.250	0.257	0.265	0.273	0.281	0.289	0.296	0.304	0.312	0.320	0.328	0.336	0.344
0.20	0.207	0.214	0.221	0.229	0.236	0.244	0.252	0.259	0.267	0.275	0.282	0.290	0.298	0.305	0.313	0.321	0.328	0.336	0.344	0.352

C.7 Attained Significance Level Tables for H_o = Weibull(β = 3.5).

Table C.61 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.160	0.170	0.180	0.190	0.200
0.02	0.029	0.038	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.110	0.119	0.128	0.137	0.146	0.156	0.165	0.175	0.184	0.194	0.203
0.03	0.038	0.047	0.056	0.064	0.073	0.082	0.090	0.099	0.108	0.117	0.126	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.208
0.04	0.048	0.057	0.065	0.074	0.082	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.177	0.186	0.195	0.204	0.213
0.05	0.058	0.066	0.075	0.083	0.091	0.100	0.108	0.116	0.125	0.133	0.142	0.150	0.159	0.168	0.176	0.185	0.193	0.202	0.211	0.220
0.06	0.068	0.076	0.084	0.092	0.101	0.109	0.117	0.125	0.134	0.142	0.150	0.159	0.167	0.176	0.184	0.193	0.201	0.210	0.219	0.227
0.07	0.078	0.086	0.094	0.102	0.110	0.118	0.126	0.134	0.143	0.151	0.159	0.167	0.176	0.184	0.192	0.201	0.209	0.218	0.226	0.235
0.08	0.088	0.096	0.104	0.112	0.120	0.128	0.135	0.143	0.152	0.160	0.168	0.176	0.184	0.192	0.200	0.208	0.217	0.225	0.234	0.242
0.09	0.098	0.105	0.113	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.234	0.242	0.250
0.10	0.108	0.115	0.123	0.131	0.139	0.147	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.210	0.218	0.225	0.234	0.242	0.250	0.258
0.11	0.117	0.125	0.133	0.141	0.148	0.156	0.164	0.171	0.179	0.187	0.195	0.203	0.211	0.218	0.226	0.234	0.242	0.250	0.258	0.266
0.12	0.127	0.135	0.142	0.150	0.158	0.166	0.173	0.181	0.188	0.196	0.204	0.212	0.219	0.227	0.235	0.243	0.251	0.258	0.267	0.275
0.13	0.137	0.145	0.152	0.160	0.168	0.175	0.182	0.190	0.197	0.205	0.213	0.221	0.228	0.236	0.244	0.251	0.259	0.267	0.275	0.283
0.14	0.147	0.155	0.162	0.170	0.177	0.185	0.192	0.199	0.207	0.215	0.222	0.230	0.237	0.245	0.253	0.260	0.268	0.276	0.283	0.291
0.15	0.157	0.164	0.172	0.179	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.247	0.254	0.261	0.269	0.277	0.284	0.292	0.300
0.16	0.167	0.174	0.182	0.189	0.197	0.204	0.211	0.218	0.226	0.233	0.241	0.248	0.256	0.263	0.270	0.278	0.286	0.293	0.301	0.308
0.17	0.177	0.184	0.191	0.199	0.206	0.213	0.220	0.228	0.235	0.242	0.250	0.257	0.265	0.272	0.280	0.287	0.294	0.302	0.310	0.317
0.18	0.187	0.194	0.201	0.208	0.216	0.223	0.230	0.237	0.244	0.252	0.259	0.267	0.274	0.281	0.288	0.296	0.303	0.311	0.318	0.326
0.19	0.197	0.204	0.211	0.218	0.225	0.232	0.239	0.247	0.254	0.261	0.268	0.276	0.283	0.290	0.297	0.305	0.312	0.320	0.327	0.334
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.249	0.256	0.263	0.270	0.278	0.285	0.292	0.299	0.307	0.314	0.321	0.328	0.336	0.343

Table C.62 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.055	0.065	0.074	0.084	0.093	0.103	0.113	0.122	0.132	0.142	0.152	0.161	0.171	0.181	0.191	0.201
0.02	0.028	0.037	0.046	0.055	0.064	0.073	0.082	0.091	0.100	0.109	0.119	0.128	0.138	0.147	0.157	0.166	0.176	0.185	0.195	0.204
0.03	0.038	0.046	0.055	0.064	0.072	0.081	0.090	0.099	0.108	0.117	0.126	0.135	0.145	0.154	0.163	0.172	0.182	0.191	0.200	0.210
0.04	0.048	0.056	0.065	0.073	0.081	0.090	0.099	0.108	0.116	0.125	0.134	0.143	0.152	0.161	0.170	0.179	0.188	0.197	0.207	0.216
0.05	0.058	0.066	0.074	0.082	0.091	0.099	0.108	0.116	0.125	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223
0.06	0.068	0.076	0.084	0.092	0.100	0.108	0.117	0.125	0.134	0.142	0.151	0.159	0.168	0.177	0.186	0.194	0.203	0.212	0.221	0.230
0.07	0.078	0.085	0.093	0.101	0.110	0.118	0.126	0.134	0.143	0.151	0.159	0.168	0.177	0.185	0.194	0.202	0.211	0.220	0.229	0.238
0.08	0.087	0.095	0.103	0.111	0.119	0.127	0.135	0.143	0.152	0.160	0.168	0.177	0.185	0.194	0.202	0.211	0.219	0.228	0.237	0.246
0.09	0.097	0.105	0.113	0.121	0.129	0.137	0.144	0.153	0.161	0.169	0.177	0.186	0.194	0.202	0.211	0.219	0.228	0.236	0.245	0.254
0.10	0.107	0.115	0.123	0.130	0.138	0.146	0.154	0.162	0.170	0.178	0.186	0.195	0.203	0.211	0.219	0.228	0.236	0.245	0.253	0.262
0.11	0.117	0.125	0.132	0.140	0.148	0.155	0.163	0.171	0.179	0.187	0.195	0.204	0.212	0.220	0.228	0.236	0.245	0.253	0.262	0.270
0.12	0.127	0.135	0.142	0.150	0.157	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.262	0.270	0.278
0.13	0.137	0.145	0.152	0.159	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.254	0.262	0.270	0.279	0.287
0.14	0.147	0.154	0.162	0.169	0.177	0.184	0.192	0.200	0.207	0.215	0.223	0.231	0.239	0.247	0.255	0.263	0.271	0.279	0.287	0.295
0.15	0.157	0.164	0.172	0.179	0.186	0.194	0.202	0.209	0.217	0.225	0.232	0.240	0.248	0.256	0.264	0.272	0.280	0.288	0.296	0.304
0.16	0.167	0.174	0.181	0.189	0.196	0.204	0.211	0.219	0.226	0.234	0.242	0.249	0.257	0.265	0.273	0.281	0.289	0.297	0.305	0.313
0.17	0.177	0.184	0.191	0.199	0.206	0.213	0.221	0.228	0.236	0.244	0.251	0.259	0.266	0.274	0.282	0.290	0.298	0.306	0.314	0.322
0.18	0.187	0.194	0.201	0.208	0.216	0.223	0.230	0.238	0.245	0.253	0.261	0.268	0.276	0.283	0.291	0.299	0.307	0.314	0.322	0.330
0.19	0.197	0.204	0.211	0.218	0.225	0.233	0.240	0.248	0.255	0.262	0.270	0.277	0.285	0.293	0.300	0.308	0.316	0.323	0.331	0.339
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.250	0.257	0.264	0.272	0.279	0.287	0.294	0.302	0.309	0.317	0.325	0.332	0.340	0.348

Table C.63 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.046	0.056	0.065	0.075	0.085	0.094	0.104	0.114	0.123	0.133	0.143	0.153	0.163	0.172	0.182	0.192	0.202
0.02	0.029	0.037	0.046	0.055	0.064	0.074	0.083	0.092	0.102	0.111	0.121	0.130	0.140	0.149	0.159	0.169	0.178	0.188	0.198	0.207
0.03	0.038	0.047	0.056	0.064	0.073	0.082	0.091	0.101	0.110	0.119	0.129	0.138	0.147	0.157	0.166	0.175	0.185	0.194	0.204	0.213
0.04	0.048	0.056	0.065	0.074	0.082	0.091	0.100	0.109	0.118	0.127	0.137	0.146	0.155	0.164	0.174	0.183	0.192	0.201	0.211	0.220
0.05	0.058	0.066	0.075	0.083	0.091	0.100	0.109	0.118	0.127	0.136	0.145	0.154	0.163	0.172	0.181	0.190	0.199	0.209	0.218	0.227
0.06	0.068	0.076	0.084	0.092	0.101	0.110	0.118	0.127	0.136	0.145	0.154	0.163	0.171	0.180	0.189	0.198	0.207	0.217	0.226	0.235
0.07	0.078	0.086	0.094	0.102	0.110	0.119	0.127	0.136	0.145	0.153	0.162	0.171	0.180	0.189	0.198	0.206	0.215	0.224	0.234	0.242
0.08	0.088	0.096	0.104	0.112	0.120	0.128	0.137	0.146	0.154	0.162	0.171	0.180	0.188	0.197	0.206	0.215	0.223	0.232	0.241	0.250
0.09	0.098	0.105	0.113	0.121	0.130	0.138	0.146	0.155	0.163	0.171	0.180	0.189	0.197	0.206	0.215	0.223	0.232	0.241	0.250	0.258
0.10	0.108	0.115	0.123	0.131	0.139	0.148	0.156	0.164	0.172	0.181	0.189	0.198	0.206	0.215	0.223	0.232	0.241	0.249	0.258	0.267
0.11	0.118	0.125	0.133	0.141	0.149	0.157	0.165	0.174	0.182	0.190	0.198	0.207	0.215	0.224	0.232	0.241	0.249	0.258	0.266	0.275
0.12	0.128	0.135	0.143	0.151	0.159	0.167	0.175	0.183	0.191	0.199	0.208	0.216	0.224	0.233	0.241	0.250	0.258	0.267	0.275	0.284
0.13	0.137	0.145	0.153	0.160	0.168	0.176	0.184	0.193	0.201	0.209	0.217	0.225	0.233	0.242	0.250	0.258	0.267	0.275	0.284	0.292
0.14	0.147	0.155	0.163	0.170	0.178	0.186	0.194	0.202	0.210	0.218	0.226	0.234	0.243	0.251	0.259	0.267	0.276	0.284	0.292	0.301
0.15	0.157	0.165	0.172	0.180	0.188	0.196	0.203	0.212	0.219	0.227	0.235	0.244	0.252	0.260	0.268	0.276	0.285	0.293	0.301	0.309
0.16	0.167	0.175	0.182	0.190	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301	0.310	0.318
0.17	0.177	0.185	0.192	0.199	0.207	0.215	0.223	0.231	0.238	0.246	0.254	0.262	0.270	0.278	0.286	0.294	0.302	0.310	0.318	0.326
0.18	0.187	0.194	0.202	0.209	0.217	0.225	0.232	0.240	0.248	0.255	0.263	0.271	0.279	0.287	0.295	0.303	0.311	0.319	0.327	0.335
0.19	0.197	0.204	0.212	0.219	0.227	0.234	0.242	0.250	0.257	0.265	0.272	0.280	0.288	0.296	0.304	0.312	0.320	0.328	0.335	0.343
0.20	0.207	0.214	0.222	0.229	0.236	0.244	0.251	0.259	0.267	0.274	0.282	0.290	0.297	0.305	0.313	0.321	0.329	0.336	0.344	0.352

Table C.64 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.066	0.076	0.086	0.096	0.105	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.183	0.193	0.203
0.02	0.029	0.038	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.160	0.170	0.180	0.189	0.199	0.209
0.03	0.038	0.047	0.056	0.065	0.074	0.083	0.093	0.102	0.111	0.121	0.130	0.139	0.149	0.158	0.168	0.177	0.187	0.196	0.206	0.215
0.04	0.048	0.057	0.066	0.075	0.084	0.093	0.102	0.111	0.120	0.129	0.138	0.148	0.157	0.166	0.176	0.185	0.194	0.204	0.213	0.222
0.05	0.058	0.067	0.075	0.084	0.093	0.102	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.174	0.184	0.193	0.202	0.211	0.221	0.230
0.06	0.068	0.076	0.085	0.094	0.103	0.111	0.120	0.129	0.138	0.147	0.156	0.165	0.174	0.183	0.192	0.201	0.211	0.220	0.229	0.238
0.07	0.078	0.086	0.095	0.103	0.112	0.120	0.129	0.138	0.147	0.156	0.164	0.173	0.182	0.191	0.201	0.210	0.219	0.228	0.237	0.246
0.08	0.088	0.096	0.104	0.113	0.122	0.130	0.139	0.147	0.156	0.165	0.173	0.182	0.191	0.200	0.209	0.218	0.227	0.236	0.245	0.254
0.09	0.098	0.106	0.114	0.122	0.131	0.139	0.148	0.156	0.165	0.174	0.182	0.191	0.200	0.209	0.218	0.227	0.235	0.244	0.253	0.262
0.10	0.108	0.116	0.124	0.132	0.141	0.149	0.157	0.166	0.174	0.183	0.191	0.200	0.209	0.217	0.226	0.235	0.244	0.253	0.261	0.270
0.11	0.118	0.125	0.134	0.142	0.150	0.158	0.167	0.175	0.184	0.192	0.200	0.209	0.218	0.226	0.235	0.244	0.252	0.261	0.270	0.278
0.12	0.127	0.135	0.143	0.151	0.160	0.168	0.176	0.184	0.193	0.201	0.210	0.218	0.226	0.235	0.244	0.252	0.261	0.270	0.278	0.287
0.13	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.210	0.219	0.227	0.235	0.244	0.252	0.261	0.269	0.278	0.287	0.295
0.14	0.147	0.155	0.163	0.171	0.179	0.187	0.195	0.203	0.211	0.220	0.228	0.236	0.244	0.253	0.261	0.270	0.278	0.287	0.295	0.303
0.15	0.157	0.165	0.173	0.181	0.189	0.196	0.204	0.212	0.221	0.229	0.237	0.245	0.253	0.262	0.270	0.278	0.287	0.295	0.303	0.312
0.16	0.167	0.175	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.254	0.262	0.270	0.279	0.287	0.295	0.304	0.312	0.320
0.17	0.177	0.185	0.192	0.200	0.208	0.216	0.224	0.231	0.239	0.247	0.255	0.263	0.271	0.279	0.288	0.296	0.304	0.312	0.320	0.328
0.18	0.187	0.194	0.202	0.210	0.218	0.225	0.233	0.241	0.249	0.257	0.264	0.272	0.280	0.288	0.296	0.304	0.312	0.321	0.329	0.337
0.19	0.197	0.204	0.212	0.220	0.227	0.235	0.243	0.250	0.258	0.266	0.274	0.281	0.289	0.297	0.305	0.313	0.321	0.329	0.337	0.345
0.20	0.207	0.214	0.222	0.229	0.237	0.244	0.252	0.260	0.267	0.275	0.283	0.291	0.298	0.306	0.314	0.322	0.330	0.338	0.346	0.353

Table C.65 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.067	0.076	0.086	0.096	0.106	0.115	0.125	0.135	0.145	0.155	0.164	0.174	0.184	0.194	0.204
0.02	0.029	0.038	0.047	0.056	0.066	0.075	0.085	0.094	0.104	0.113	0.123	0.133	0.142	0.152	0.162	0.171	0.181	0.191	0.200	0.210
0.03	0.039	0.048	0.057	0.066	0.075	0.084	0.094	0.103	0.112	0.122	0.131	0.141	0.150	0.159	0.169	0.178	0.188	0.197	0.207	0.217
0.04	0.049	0.057	0.066	0.075	0.084	0.093	0.102	0.112	0.121	0.130	0.139	0.149	0.158	0.167	0.177	0.186	0.196	0.205	0.214	0.224
0.05	0.058	0.067	0.076	0.085	0.093	0.103	0.112	0.121	0.130	0.139	0.148	0.157	0.167	0.176	0.185	0.194	0.204	0.213	0.222	0.232
0.06	0.068	0.077	0.085	0.094	0.103	0.112	0.121	0.130	0.139	0.148	0.157	0.166	0.175	0.184	0.193	0.203	0.212	0.221	0.230	0.239
0.07	0.078	0.086	0.095	0.104	0.113	0.121	0.130	0.139	0.148	0.157	0.166	0.175	0.184	0.193	0.202	0.211	0.220	0.229	0.238	0.248
0.08	0.088	0.096	0.105	0.114	0.122	0.131	0.140	0.148	0.157	0.166	0.175	0.184	0.193	0.202	0.211	0.220	0.229	0.238	0.247	0.256
0.09	0.098	0.106	0.115	0.123	0.132	0.140	0.149	0.158	0.166	0.175	0.184	0.193	0.201	0.210	0.219	0.228	0.237	0.246	0.255	0.264
0.10	0.108	0.116	0.124	0.133	0.141	0.150	0.158	0.167	0.176	0.184	0.193	0.202	0.210	0.219	0.228	0.237	0.246	0.254	0.263	0.272
0.11	0.118	0.126	0.134	0.143	0.151	0.159	0.168	0.176	0.185	0.194	0.202	0.211	0.219	0.228	0.237	0.245	0.254	0.263	0.272	0.281
0.12	0.128	0.136	0.144	0.152	0.161	0.169	0.177	0.186	0.194	0.203	0.211	0.220	0.228	0.237	0.246	0.254	0.263	0.271	0.280	0.289
0.13	0.138	0.146	0.154	0.162	0.170	0.179	0.187	0.195	0.203	0.212	0.220	0.229	0.237	0.246	0.254	0.263	0.271	0.280	0.288	0.297
0.14	0.148	0.156	0.164	0.172	0.180	0.188	0.196	0.205	0.213	0.221	0.229	0.238	0.246	0.255	0.263	0.271	0.280	0.288	0.297	0.305
0.15	0.158	0.165	0.173	0.182	0.190	0.198	0.206	0.214	0.222	0.230	0.239	0.247	0.255	0.264	0.272	0.280	0.288	0.297	0.305	0.314
0.16	0.168	0.175	0.183	0.191	0.199	0.207	0.215	0.223	0.231	0.240	0.248	0.256	0.264	0.272	0.281	0.289	0.297	0.305	0.314	0.322
0.17	0.178	0.185	0.193	0.201	0.209	0.217	0.225	0.233	0.241	0.249	0.257	0.265	0.273	0.281	0.289	0.297	0.305	0.314	0.322	0.330
0.18	0.188	0.195	0.203	0.211	0.219	0.227	0.234	0.242	0.250	0.258	0.266	0.274	0.282	0.290	0.298	0.306	0.314	0.322	0.330	0.339
0.19	0.197	0.205	0.213	0.221	0.228	0.236	0.244	0.252	0.259	0.267	0.275	0.283	0.291	0.299	0.307	0.315	0.323	0.331	0.339	0.347
0.20	0.207	0.215	0.222	0.230	0.238	0.246	0.253	0.261	0.269	0.276	0.284	0.292	0.300	0.308	0.316	0.323	0.331	0.339	0.347	0.355

Table C.66 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.067	0.077	0.087	0.096	0.106	0.116	0.126	0.135	0.145	0.155	0.165	0.175	0.185	0.194	0.204
0.02	0.029	0.038	0.048	0.057	0.066	0.076	0.085	0.095	0.105	0.114	0.124	0.133	0.143	0.153	0.162	0.172	0.182	0.192	0.201	0.211
0.03	0.039	0.048	0.057	0.066	0.076	0.085	0.094	0.104	0.113	0.123	0.132	0.142	0.151	0.161	0.170	0.180	0.189	0.199	0.208	0.218
0.04	0.049	0.058	0.067	0.076	0.085	0.094	0.103	0.113	0.122	0.131	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207	0.216	0.225
0.05	0.059	0.068	0.076	0.085	0.094	0.103	0.113	0.122	0.131	0.140	0.149	0.159	0.168	0.177	0.186	0.196	0.205	0.214	0.224	0.233
0.06	0.069	0.077	0.086	0.095	0.104	0.113	0.122	0.131	0.140	0.149	0.158	0.167	0.177	0.186	0.195	0.204	0.213	0.223	0.232	0.241
0.07	0.078	0.087	0.095	0.104	0.113	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.185	0.194	0.203	0.212	0.221	0.231	0.240	0.249
0.08	0.088	0.097	0.105	0.114	0.123	0.132	0.140	0.149	0.158	0.167	0.176	0.185	0.194	0.203	0.212	0.221	0.230	0.239	0.248	0.257
0.09	0.098	0.107	0.115	0.124	0.132	0.141	0.150	0.159	0.168	0.176	0.185	0.194	0.203	0.212	0.221	0.229	0.238	0.247	0.256	0.265
0.10	0.108	0.117	0.125	0.133	0.142	0.151	0.159	0.168	0.177	0.185	0.194	0.203	0.212	0.220	0.229	0.238	0.247	0.256	0.264	0.273
0.11	0.118	0.126	0.134	0.143	0.151	0.160	0.169	0.177	0.186	0.194	0.203	0.212	0.221	0.229	0.238	0.246	0.255	0.264	0.273	0.282
0.12	0.128	0.136	0.144	0.153	0.161	0.170	0.178	0.187	0.195	0.204	0.212	0.221	0.229	0.238	0.247	0.255	0.264	0.273	0.281	0.290
0.13	0.138	0.146	0.154	0.162	0.171	0.179	0.187	0.196	0.205	0.213	0.222	0.230	0.238	0.247	0.255	0.264	0.273	0.281	0.290	0.298
0.14	0.148	0.156	0.164	0.172	0.180	0.189	0.197	0.205	0.214	0.222	0.230	0.239	0.247	0.256	0.264	0.272	0.281	0.289	0.298	0.306
0.15	0.158	0.166	0.174	0.182	0.190	0.198	0.206	0.215	0.223	0.231	0.240	0.248	0.256	0.264	0.273	0.281	0.289	0.298	0.306	0.315
0.16	0.168	0.176	0.183	0.191	0.200	0.208	0.216	0.224	0.232	0.240	0.249	0.257	0.265	0.273	0.282	0.290	0.298	0.306	0.315	0.323
0.17	0.178	0.186	0.193	0.201	0.209	0.217	0.225	0.233	0.241	0.249	0.258	0.266	0.274	0.282	0.290	0.298	0.306	0.314	0.323	0.331
0.18	0.188	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.291	0.299	0.307	0.315	0.323	0.331	0.339
0.19	0.198	0.205	0.213	0.220	0.229	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300	0.308	0.315	0.324	0.332	0.340	0.348
0.20	0.208	0.215	0.222	0.230	0.238	0.246	0.254	0.262	0.270	0.277	0.285	0.293	0.301	0.309	0.317	0.324	0.332	0.340	0.348	0.356

Table C.67 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.068	0.077	0.087	0.097	0.106	0.116	0.126	0.136	0.146	0.156	0.165	0.175	0.185	0.195	0.205
0.02	0.029	0.038	0.048	0.057	0.067	0.076	0.086	0.095	0.105	0.114	0.124	0.134	0.143	0.153	0.163	0.172	0.182	0.192	0.201	0.211
0.03	0.039	0.048	0.057	0.066	0.076	0.085	0.095	0.104	0.113	0.123	0.132	0.142	0.151	0.161	0.170	0.180	0.190	0.199	0.209	0.218
0.04	0.049	0.058	0.067	0.076	0.085	0.094	0.104	0.113	0.122	0.132	0.141	0.150	0.160	0.169	0.179	0.188	0.197	0.207	0.216	0.226
0.05	0.059	0.067	0.076	0.085	0.094	0.104	0.113	0.122	0.131	0.140	0.150	0.159	0.168	0.178	0.187	0.196	0.205	0.215	0.224	0.234
0.06	0.069	0.077	0.086	0.095	0.104	0.113	0.122	0.131	0.140	0.149	0.158	0.168	0.177	0.186	0.195	0.204	0.214	0.223	0.232	0.241
0.07	0.078	0.087	0.096	0.105	0.113	0.122	0.131	0.140	0.149	0.158	0.167	0.176	0.186	0.195	0.204	0.213	0.222	0.231	0.240	0.249
0.08	0.088	0.097	0.105	0.114	0.123	0.132	0.141	0.150	0.158	0.167	0.176	0.185	0.194	0.203	0.212	0.221	0.230	0.239	0.248	0.258
0.09	0.098	0.107	0.115	0.124	0.132	0.141	0.150	0.159	0.168	0.176	0.185	0.194	0.203	0.212	0.221	0.230	0.238	0.247	0.256	0.266
0.10	0.108	0.116	0.125	0.133	0.142	0.151	0.159	0.168	0.177	0.185	0.194	0.203	0.212	0.220	0.229	0.238	0.247	0.256	0.265	0.274
0.11	0.118	0.126	0.135	0.143	0.152	0.160	0.169	0.177	0.186	0.195	0.203	0.212	0.221	0.229	0.238	0.247	0.255	0.264	0.273	0.282
0.12	0.128	0.136	0.144	0.153	0.161	0.170	0.178	0.187	0.195	0.204	0.212	0.221	0.229	0.238	0.247	0.255	0.264	0.273	0.281	0.290
0.13	0.138	0.146	0.154	0.162	0.171	0.179	0.187	0.196	0.204	0.213	0.221	0.230	0.238	0.247	0.255	0.264	0.272	0.281	0.290	0.298
0.14	0.148	0.156	0.164	0.172	0.180	0.188	0.197	0.205	0.214	0.222	0.230	0.239	0.247	0.255	0.264	0.273	0.281	0.289	0.298	0.306
0.15	0.158	0.166	0.174	0.182	0.190	0.198	0.206	0.215	0.223	0.231	0.239	0.248	0.256	0.264	0.273	0.281	0.290	0.298	0.307	0.315
0.16	0.168	0.175	0.184	0.192	0.200	0.208	0.216	0.224	0.232	0.240	0.249	0.257	0.265	0.273	0.282	0.290	0.298	0.307	0.315	0.323
0.17	0.178	0.185	0.193	0.201	0.209	0.217	0.225	0.234	0.242	0.250	0.258	0.266	0.274	0.282	0.291	0.299	0.307	0.315	0.323	0.332
0.18	0.188	0.195	0.203	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.291	0.299	0.307	0.315	0.324	0.332	0.340
0.19	0.198	0.205	0.213	0.221	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300	0.308	0.316	0.324	0.332	0.340	0.348
0.20	0.207	0.215	0.223	0.230	0.238	0.246	0.254	0.262	0.270	0.278	0.285	0.293	0.301	0.309	0.317	0.325	0.333	0.341	0.349	0.357

Table C.68 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.039	0.048	0.058	0.068	0.077	0.087	0.097	0.107	0.117	0.126	0.136	0.146	0.156	0.166	0.175	0.185	0.195	0.205
0.02	0.029	0.039	0.048	0.058	0.067	0.077	0.086	0.096	0.105	0.115	0.125	0.134	0.144	0.154	0.163	0.173	0.183	0.193	0.202	0.212
0.03	0.039	0.048	0.058	0.067	0.076	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.152	0.162	0.172	0.181	0.191	0.200	0.210	0.220
0.04	0.049	0.058	0.067	0.076	0.086	0.095	0.104	0.114	0.123	0.132	0.142	0.151	0.161	0.170	0.180	0.189	0.199	0.208	0.218	0.227
0.05	0.059	0.068	0.077	0.086	0.095	0.104	0.113	0.123	0.132	0.141	0.151	0.160	0.169	0.179	0.188	0.197	0.207	0.216	0.225	0.235
0.06	0.069	0.077	0.086	0.095	0.105	0.114	0.123	0.132	0.141	0.151	0.160	0.169	0.178	0.187	0.197	0.206	0.215	0.224	0.233	0.243
0.07	0.079	0.087	0.096	0.105	0.114	0.123	0.132	0.141	0.150	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.224	0.233	0.242	0.251
0.08	0.088	0.097	0.106	0.115	0.124	0.133	0.141	0.150	0.159	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241	0.250	0.259
0.09	0.098	0.107	0.115	0.124	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.204	0.213	0.222	0.231	0.240	0.249	0.258	0.267
0.10	0.108	0.117	0.125	0.134	0.143	0.152	0.160	0.169	0.178	0.187	0.195	0.204	0.213	0.222	0.231	0.240	0.249	0.257	0.266	0.275
0.11	0.118	0.126	0.135	0.144	0.152	0.161	0.170	0.178	0.187	0.196	0.204	0.213	0.222	0.231	0.240	0.248	0.257	0.266	0.275	0.283
0.12	0.128	0.136	0.145	0.153	0.162	0.171	0.179	0.188	0.196	0.205	0.214	0.222	0.231	0.240	0.249	0.257	0.266	0.274	0.283	0.292
0.13	0.138	0.146	0.155	0.163	0.172	0.180	0.188	0.197	0.205	0.214	0.223	0.231	0.240	0.249	0.257	0.266	0.274	0.283	0.291	0.300
0.14	0.148	0.156	0.164	0.173	0.181	0.190	0.198	0.206	0.215	0.223	0.232	0.240	0.249	0.257	0.266	0.274	0.283	0.291	0.300	0.308
0.15	0.158	0.166	0.174	0.182	0.191	0.199	0.207	0.216	0.224	0.232	0.241	0.249	0.258	0.266	0.275	0.283	0.291	0.300	0.308	0.316
0.16	0.168	0.176	0.184	0.192	0.200	0.209	0.217	0.225	0.233	0.242	0.250	0.258	0.266	0.275	0.283	0.291	0.300	0.308	0.316	0.324
0.17	0.178	0.186	0.194	0.202	0.210	0.218	0.226	0.234	0.242	0.251	0.259	0.267	0.275	0.283	0.292	0.300	0.308	0.317	0.325	0.333
0.18	0.188	0.196	0.204	0.211	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.301	0.309	0.317	0.325	0.333	0.341
0.19	0.198	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301	0.309	0.317	0.325	0.334	0.341	0.349
0.20	0.207	0.215	0.223	0.231	0.239	0.247	0.255	0.262	0.270	0.278	0.286	0.294	0.302	0.310	0.318	0.326	0.334	0.342	0.350	0.358

Table C.69 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 45.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.039	0.048	0.058	0.068	0.078	0.087	0.097	0.107	0.117	0.127	0.136	0.146	0.156	0.166	0.176	0.186	0.195	0.205
0.02	0.029	0.039	0.048	0.058	0.067	0.077	0.086	0.096	0.106	0.115	0.125	0.135	0.144	0.154	0.164	0.173	0.183	0.193	0.203	0.212
0.03	0.039	0.048	0.058	0.067	0.076	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.153	0.162	0.172	0.181	0.191	0.201	0.210	0.220
0.04	0.049	0.058	0.067	0.077	0.086	0.095	0.104	0.114	0.123	0.133	0.142	0.152	0.161	0.171	0.180	0.189	0.199	0.209	0.218	0.228
0.05	0.059	0.068	0.077	0.086	0.095	0.104	0.114	0.123	0.132	0.142	0.151	0.160	0.170	0.179	0.188	0.198	0.207	0.217	0.226	0.235
0.06	0.069	0.078	0.087	0.096	0.105	0.114	0.123	0.132	0.141	0.151	0.160	0.169	0.178	0.188	0.197	0.206	0.215	0.225	0.234	0.243
0.07	0.079	0.088	0.096	0.105	0.114	0.123	0.132	0.141	0.150	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.233	0.242	0.251
0.08	0.089	0.097	0.106	0.115	0.124	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241	0.250	0.259
0.09	0.098	0.107	0.116	0.124	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.204	0.213	0.222	0.231	0.240	0.249	0.258	0.267
0.10	0.108	0.117	0.125	0.134	0.143	0.152	0.160	0.169	0.178	0.187	0.196	0.205	0.213	0.222	0.231	0.240	0.248	0.257	0.266	0.275
0.11	0.118	0.127	0.135	0.144	0.152	0.161	0.170	0.178	0.187	0.196	0.205	0.213	0.222	0.231	0.240	0.248	0.257	0.266	0.275	0.283
0.12	0.128	0.137	0.145	0.153	0.162	0.171	0.179	0.188	0.196	0.205	0.214	0.222	0.231	0.239	0.248	0.257	0.265	0.274	0.283	0.291
0.13	0.138	0.146	0.155	0.163	0.172	0.180	0.189	0.197	0.205	0.214	0.223	0.231	0.240	0.248	0.257	0.265	0.274	0.282	0.291	0.300
0.14	0.148	0.156	0.165	0.173	0.181	0.190	0.198	0.206	0.215	0.223	0.232	0.240	0.249	0.257	0.265	0.274	0.282	0.291	0.299	0.308
0.15	0.158	0.166	0.174	0.183	0.191	0.199	0.207	0.216	0.224	0.232	0.241	0.249	0.257	0.266	0.274	0.282	0.291	0.299	0.307	0.316
0.16	0.168	0.176	0.184	0.192	0.200	0.209	0.217	0.225	0.233	0.241	0.250	0.258	0.266	0.274	0.283	0.291	0.299	0.307	0.316	0.324
0.17	0.178	0.186	0.194	0.202	0.210	0.218	0.226	0.234	0.242	0.251	0.259	0.267	0.275	0.283	0.291	0.300	0.308	0.316	0.324	0.332
0.18	0.188	0.196	0.204	0.211	0.220	0.228	0.236	0.244	0.252	0.260	0.268	0.276	0.284	0.292	0.300	0.308	0.316	0.324	0.332	0.340
0.19	0.198	0.206	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.269	0.277	0.285	0.293	0.301	0.309	0.317	0.325	0.333	0.341	0.349
0.20	0.208	0.215	0.223	0.231	0.239	0.247	0.255	0.262	0.270	0.278	0.286	0.294	0.302	0.309	0.318	0.325	0.333	0.341	0.349	0.357

Table C.70 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 3.5; Sample Size = 50.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.039	0.048	0.058	0.068	0.078	0.087	0.097	0.107	0.117	0.127	0.137	0.146	0.156	0.166	0.176	0.186	0.196	0.206
0.02	0.029	0.039	0.048	0.058	0.067	0.077	0.087	0.096	0.106	0.115	0.125	0.135	0.145	0.154	0.164	0.174	0.183	0.193	0.203	0.213
0.03	0.039	0.048	0.058	0.067	0.077	0.086	0.096	0.105	0.115	0.124	0.134	0.143	0.153	0.163	0.172	0.182	0.191	0.201	0.211	0.220
0.04	0.049	0.058	0.067	0.077	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.152	0.162	0.171	0.181	0.190	0.200	0.209	0.218	0.228
0.05	0.059	0.068	0.077	0.086	0.096	0.105	0.114	0.124	0.133	0.142	0.152	0.161	0.170	0.180	0.189	0.199	0.208	0.217	0.227	0.236
0.06	0.069	0.078	0.087	0.096	0.105	0.114	0.124	0.133	0.142	0.151	0.161	0.170	0.179	0.189	0.198	0.207	0.216	0.225	0.235	0.244
0.07	0.079	0.087	0.097	0.106	0.115	0.124	0.133	0.142	0.151	0.160	0.170	0.179	0.188	0.197	0.206	0.215	0.224	0.234	0.243	0.252
0.08	0.089	0.097	0.106	0.115	0.124	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.206	0.214	0.224	0.232	0.242	0.251	0.260
0.09	0.098	0.107	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241	0.250	0.259	0.268
0.10	0.108	0.117	0.126	0.134	0.143	0.152	0.161	0.170	0.178	0.187	0.196	0.205	0.214	0.223	0.232	0.241	0.249	0.258	0.267	0.276
0.11	0.118	0.127	0.135	0.144	0.153	0.161	0.170	0.179	0.188	0.197	0.205	0.214	0.223	0.232	0.240	0.249	0.258	0.266	0.275	0.284
0.12	0.128	0.137	0.145	0.154	0.162	0.171	0.180	0.188	0.197	0.206	0.214	0.223	0.232	0.240	0.249	0.257	0.266	0.275	0.283	0.292
0.13	0.138	0.146	0.155	0.163	0.172	0.180	0.189	0.198	0.206	0.215	0.223	0.232	0.240	0.249	0.257	0.266	0.274	0.283	0.292	0.300
0.14	0.148	0.156	0.165	0.173	0.181	0.190	0.198	0.207	0.215	0.224	0.232	0.241	0.249	0.258	0.266	0.274	0.283	0.291	0.300	0.308
0.15	0.158	0.166	0.175	0.183	0.191	0.199	0.208	0.216	0.225	0.233	0.241	0.250	0.258	0.267	0.275	0.283	0.291	0.300	0.308	0.317
0.16	0.168	0.176	0.184	0.193	0.201	0.209	0.217	0.226	0.234	0.242	0.250	0.259	0.267	0.275	0.283	0.292	0.300	0.308	0.316	0.325
0.17	0.178	0.186	0.194	0.202	0.210	0.218	0.227	0.235	0.243	0.251	0.259	0.268	0.276	0.284	0.292	0.300	0.308	0.316	0.324	0.333
0.18	0.188	0.196	0.204	0.212	0.220	0.228	0.236	0.244	0.252	0.261	0.268	0.277	0.285	0.293	0.301	0.309	0.317	0.325	0.333	0.341
0.19	0.198	0.206	0.214	0.221	0.229	0.237	0.245	0.253	0.262	0.270	0.277	0.285	0.294	0.302	0.309	0.317	0.325	0.333	0.341	0.349
0.20	0.208	0.215	0.223	0.231	0.239	0.247	0.255	0.263	0.271	0.279	0.287	0.295	0.302	0.310	0.318	0.326	0.334	0.342	0.349	0.357

C.8 Attained Significance Level Tables for H_o = Weibull($\beta = 4.0$).

Table C.71 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 5.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.037	0.047	0.056	0.065	0.074	0.084	0.093	0.103	0.112	0.122	0.131	0.141	0.151	0.161	0.170	0.180	0.190	0.200
0.02	0.029	0.038	0.046	0.055	0.065	0.073	0.082	0.091	0.100	0.110	0.119	0.128	0.137	0.147	0.156	0.165	0.175	0.184	0.194	0.203
0.03	0.038	0.047	0.056	0.064	0.073	0.082	0.091	0.099	0.108	0.117	0.126	0.135	0.144	0.153	0.162	0.171	0.180	0.189	0.198	0.208
0.04	0.048	0.057	0.065	0.074	0.082	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.160	0.169	0.178	0.186	0.195	0.204	0.213
0.05	0.058	0.066	0.075	0.083	0.092	0.100	0.108	0.116	0.125	0.133	0.142	0.151	0.159	0.168	0.176	0.185	0.194	0.202	0.211	0.220
0.06	0.068	0.076	0.084	0.093	0.101	0.109	0.117	0.125	0.134	0.142	0.150	0.159	0.167	0.176	0.184	0.193	0.201	0.210	0.219	0.227
0.07	0.078	0.086	0.094	0.102	0.110	0.118	0.126	0.134	0.143	0.151	0.159	0.167	0.176	0.184	0.192	0.201	0.209	0.218	0.226	0.235
0.08	0.088	0.096	0.104	0.112	0.120	0.128	0.136	0.144	0.152	0.160	0.168	0.176	0.184	0.192	0.201	0.209	0.217	0.226	0.234	0.242
0.09	0.098	0.105	0.113	0.121	0.129	0.137	0.145	0.153	0.161	0.169	0.177	0.185	0.193	0.201	0.209	0.217	0.225	0.234	0.242	0.250
0.10	0.108	0.115	0.123	0.131	0.139	0.147	0.154	0.162	0.170	0.178	0.186	0.194	0.202	0.210	0.218	0.226	0.234	0.242	0.250	0.258
0.11	0.117	0.125	0.133	0.141	0.148	0.156	0.164	0.171	0.179	0.187	0.195	0.203	0.211	0.218	0.226	0.234	0.242	0.250	0.258	0.266
0.12	0.127	0.135	0.142	0.150	0.158	0.166	0.173	0.181	0.188	0.196	0.204	0.212	0.219	0.227	0.235	0.243	0.251	0.259	0.266	0.274
0.13	0.137	0.145	0.152	0.160	0.168	0.175	0.183	0.190	0.198	0.205	0.213	0.221	0.229	0.236	0.244	0.252	0.259	0.267	0.275	0.283
0.14	0.147	0.155	0.162	0.170	0.177	0.185	0.192	0.199	0.207	0.215	0.222	0.230	0.237	0.245	0.253	0.260	0.268	0.276	0.283	0.291
0.15	0.157	0.164	0.172	0.179	0.187	0.194	0.201	0.209	0.216	0.224	0.231	0.239	0.247	0.254	0.261	0.269	0.277	0.284	0.292	0.300
0.16	0.167	0.174	0.181	0.189	0.196	0.204	0.211	0.218	0.226	0.233	0.241	0.248	0.256	0.263	0.270	0.278	0.286	0.293	0.301	0.308
0.17	0.177	0.184	0.191	0.199	0.206	0.213	0.220	0.228	0.235	0.242	0.250	0.257	0.265	0.272	0.279	0.287	0.294	0.302	0.309	0.317
0.18	0.187	0.194	0.201	0.208	0.216	0.223	0.230	0.237	0.245	0.252	0.259	0.267	0.274	0.281	0.289	0.296	0.303	0.311	0.318	0.326
0.19	0.197	0.204	0.211	0.218	0.225	0.233	0.239	0.247	0.254	0.261	0.269	0.276	0.283	0.290	0.298	0.305	0.312	0.320	0.327	0.334
0.20	0.207	0.214	0.221	0.228	0.235	0.242	0.249	0.256	0.263	0.271	0.278	0.285	0.292	0.300	0.307	0.314	0.321	0.329	0.336	0.343

Table C.72 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 10.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.046	0.056	0.065	0.075	0.084	0.094	0.103	0.113	0.122	0.131	0.141	0.151	0.161	0.170	0.179	0.189	0.199
0.02	0.028	0.037	0.046	0.054	0.063	0.072	0.082	0.091	0.100	0.109	0.119	0.127	0.137	0.146	0.156	0.165	0.174	0.182	0.192	0.203
0.03	0.038	0.047	0.056	0.064	0.072	0.081	0.090	0.099	0.108	0.117	0.126	0.135	0.144	0.152	0.162	0.171	0.180	0.188	0.198	0.208
0.04	0.047	0.056	0.065	0.072	0.081	0.089	0.098	0.107	0.116	0.125	0.134	0.142	0.151	0.159	0.169	0.178	0.187	0.195	0.204	0.214
0.05	0.058	0.066	0.075	0.082	0.091	0.099	0.108	0.116	0.125	0.134	0.142	0.151	0.159	0.168	0.177	0.186	0.194	0.202	0.212	0.221
0.06	0.068	0.076	0.084	0.092	0.100	0.108	0.117	0.125	0.134	0.142	0.151	0.159	0.168	0.176	0.185	0.194	0.202	0.210	0.219	0.229
0.07	0.078	0.086	0.094	0.101	0.110	0.117	0.126	0.134	0.143	0.151	0.160	0.168	0.176	0.184	0.194	0.202	0.210	0.218	0.227	0.236
0.08	0.088	0.096	0.104	0.112	0.120	0.128	0.136	0.144	0.153	0.161	0.169	0.177	0.185	0.194	0.203	0.211	0.219	0.227	0.236	0.245
0.09	0.098	0.106	0.114	0.121	0.129	0.137	0.145	0.153	0.162	0.170	0.178	0.186	0.194	0.202	0.211	0.219	0.227	0.235	0.244	0.253
0.10	0.109	0.117	0.124	0.131	0.139	0.147	0.155	0.163	0.172	0.180	0.188	0.195	0.204	0.211	0.220	0.228	0.236	0.244	0.252	0.261
0.11	0.118	0.126	0.134	0.141	0.149	0.156	0.165	0.172	0.181	0.189	0.197	0.204	0.212	0.220	0.229	0.237	0.245	0.252	0.261	0.270
0.12	0.129	0.137	0.144	0.151	0.159	0.166	0.174	0.182	0.190	0.198	0.206	0.214	0.222	0.229	0.238	0.246	0.254	0.261	0.269	0.278
0.13	0.139	0.147	0.154	0.161	0.169	0.176	0.184	0.192	0.200	0.208	0.216	0.223	0.231	0.239	0.248	0.255	0.263	0.270	0.279	0.287
0.14	0.149	0.156	0.164	0.171	0.178	0.186	0.194	0.201	0.209	0.217	0.225	0.232	0.240	0.248	0.256	0.264	0.272	0.279	0.287	0.296
0.15	0.159	0.167	0.174	0.181	0.188	0.196	0.204	0.211	0.219	0.227	0.235	0.242	0.250	0.257	0.266	0.273	0.281	0.288	0.296	0.305
0.16	0.169	0.176	0.184	0.190	0.198	0.205	0.213	0.221	0.229	0.236	0.244	0.251	0.259	0.266	0.275	0.282	0.290	0.297	0.305	0.313
0.17	0.179	0.186	0.194	0.200	0.208	0.215	0.223	0.230	0.238	0.246	0.253	0.260	0.268	0.275	0.284	0.291	0.298	0.305	0.314	0.322
0.18	0.189	0.196	0.204	0.210	0.218	0.225	0.233	0.240	0.248	0.255	0.263	0.270	0.278	0.285	0.293	0.301	0.308	0.315	0.323	0.331
0.19	0.200	0.207	0.214	0.221	0.228	0.236	0.243	0.250	0.258	0.266	0.273	0.280	0.288	0.295	0.303	0.311	0.318	0.324	0.332	0.341
0.20	0.210	0.217	0.225	0.231	0.238	0.245	0.253	0.260	0.268	0.276	0.283	0.290	0.297	0.304	0.313	0.320	0.327	0.334	0.341	0.350

Table C.73 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 15.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.067	0.076	0.087	0.096	0.106	0.117	0.127	0.136	0.146	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.029	0.038	0.046	0.056	0.066	0.075	0.084	0.094	0.103	0.113	0.123	0.133	0.142	0.152	0.161	0.171	0.180	0.190	0.200	0.209
0.03	0.039	0.048	0.056	0.065	0.075	0.083	0.092	0.102	0.111	0.121	0.131	0.140	0.150	0.159	0.168	0.177	0.187	0.197	0.206	0.215
0.04	0.048	0.057	0.065	0.074	0.084	0.092	0.101	0.110	0.120	0.129	0.139	0.148	0.157	0.166	0.175	0.184	0.194	0.203	0.213	0.222
0.05	0.058	0.067	0.075	0.084	0.093	0.102	0.110	0.120	0.128	0.137	0.147	0.157	0.166	0.175	0.183	0.192	0.201	0.211	0.220	0.229
0.06	0.068	0.076	0.084	0.093	0.102	0.110	0.119	0.128	0.137	0.146	0.155	0.165	0.174	0.182	0.191	0.200	0.209	0.218	0.227	0.236
0.07	0.078	0.086	0.094	0.103	0.112	0.120	0.128	0.137	0.146	0.155	0.164	0.173	0.182	0.191	0.199	0.208	0.217	0.226	0.235	0.244
0.08	0.087	0.095	0.103	0.112	0.121	0.129	0.137	0.146	0.155	0.163	0.173	0.182	0.191	0.199	0.207	0.216	0.225	0.234	0.243	0.252
0.09	0.097	0.105	0.112	0.121	0.130	0.137	0.146	0.154	0.163	0.172	0.181	0.190	0.199	0.207	0.215	0.224	0.233	0.242	0.251	0.259
0.10	0.106	0.114	0.122	0.131	0.139	0.147	0.155	0.164	0.172	0.181	0.190	0.199	0.207	0.216	0.224	0.232	0.241	0.250	0.259	0.267
0.11	0.116	0.124	0.132	0.140	0.149	0.156	0.164	0.173	0.181	0.190	0.199	0.208	0.216	0.224	0.232	0.241	0.249	0.258	0.267	0.276
0.12	0.126	0.134	0.141	0.150	0.158	0.166	0.174	0.182	0.191	0.199	0.208	0.217	0.225	0.233	0.241	0.250	0.258	0.267	0.275	0.284
0.13	0.136	0.144	0.151	0.159	0.168	0.175	0.183	0.192	0.200	0.208	0.217	0.226	0.234	0.242	0.250	0.258	0.266	0.275	0.284	0.292
0.14	0.146	0.154	0.161	0.169	0.177	0.185	0.193	0.201	0.209	0.217	0.226	0.235	0.243	0.251	0.259	0.267	0.275	0.284	0.292	0.300
0.15	0.156	0.163	0.171	0.179	0.187	0.194	0.202	0.210	0.218	0.226	0.235	0.244	0.252	0.260	0.267	0.276	0.284	0.292	0.300	0.309
0.16	0.165	0.173	0.180	0.188	0.196	0.204	0.211	0.220	0.228	0.236	0.244	0.253	0.261	0.269	0.276	0.284	0.292	0.301	0.309	0.137
0.17	0.175	0.183	0.190	0.198	0.206	0.213	0.221	0.229	0.237	0.245	0.253	0.262	0.269	0.277	0.285	0.293	0.301	0.309	0.317	0.325
0.18	0.186	0.193	0.200	0.208	0.216	0.223	0.231	0.239	0.247	0.255	0.264	0.272	0.279	0.287	0.295	0.303	0.310	0.319	0.327	0.335
0.19	0.196	0.204	0.211	0.218	0.226	0.234	0.241	0.249	0.257	0.265	0.273	0.281	0.289	0.297	0.304	0.312	0.319	0.328	0.336	0.344
0.20	0.206	0.214	0.221	0.228	0.236	0.243	0.251	0.259	0.266	0.274	0.283	0.291	0.298	0.306	0.313	0.321	0.328	0.337	0.344	0.352

Table C.74 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 20.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.039	0.048	0.058	0.068	0.077	0.086	0.096	0.105	0.115	0.125	0.135	0.145	0.155	0.164	0.174	0.184	0.193	0.203
0.02	0.029	0.039	0.048	0.057	0.067	0.076	0.085	0.094	0.103	0.113	0.122	0.132	0.143	0.152	0.162	0.171	0.18	0.19	0.199	0.209
0.03	0.039	0.048	0.057	0.067	0.076	0.085	0.093	0.103	0.112	0.121	0.13	0.14	0.15	0.159	0.169	0.178	0.187	0.197	0.206	0.216
0.04	0.048	0.058	0.066	0.076	0.085	0.094	0.102	0.111	0.12	0.129	0.138	0.148	0.158	0.167	0.176	0.185	0.194	0.204	0.213	0.223
0.05	0.058	0.067	0.076	0.085	0.094	0.103	0.111	0.12	0.129	0.138	0.147	0.157	0.166	0.175	0.184	0.193	0.202	0.212	0.221	0.23
0.06	0.068	0.077	0.085	0.094	0.104	0.112	0.12	0.129	0.137	0.147	0.155	0.165	0.174	0.183	0.192	0.201	0.21	0.219	0.228	0.238
0.07	0.078	0.087	0.096	0.104	0.113	0.122	0.13	0.138	0.147	0.156	0.164	0.174	0.183	0.192	0.201	0.21	0.219	0.228	0.237	0.246
0.08	0.088	0.097	0.106	0.114	0.123	0.132	0.139	0.148	0.156	0.165	0.174	0.183	0.192	0.201	0.21	0.219	0.227	0.236	0.245	0.254
0.09	0.099	0.108	0.116	0.125	0.134	0.142	0.149	0.158	0.166	0.175	0.183	0.193	0.202	0.21	0.219	0.228	0.236	0.245	0.254	0.263
0.10	0.108	0.117	0.125	0.134	0.143	0.151	0.159	0.167	0.175	0.184	0.192	0.201	0.21	0.219	0.228	0.236	0.244	0.253	0.262	0.271
0.11	0.118	0.127	0.135	0.144	0.152	0.16	0.168	0.176	0.184	0.193	0.201	0.21	0.219	0.227	0.236	0.244	0.253	0.261	0.27	0.279
0.12	0.128	0.137	0.145	0.154	0.162	0.17	0.177	0.185	0.194	0.202	0.21	0.219	0.228	0.237	0.245	0.253	0.262	0.27	0.278	0.287
0.13	0.138	0.147	0.155	0.163	0.172	0.18	0.187	0.195	0.203	0.212	0.219	0.228	0.237	0.246	0.254	0.262	0.27	0.279	0.287	0.296
0.14	0.149	0.157	0.165	0.173	0.182	0.19	0.197	0.205	0.213	0.221	0.229	0.238	0.247	0.255	0.263	0.271	0.279	0.288	0.296	0.304
0.15	0.158	0.167	0.175	0.183	0.191	0.199	0.206	0.214	0.222	0.23	0.238	0.247	0.256	0.264	0.272	0.28	0.288	0.296	0.304	0.313
0.16	0.167	0.176	0.184	0.192	0.2	0.208	0.215	0.223	0.23	0.239	0.246	0.255	0.264	0.272	0.28	0.288	0.296	0.304	0.312	0.32
0.17	0.178	0.186	0.194	0.202	0.21	0.218	0.225	0.232	0.24	0.248	0.256	0.265	0.273	0.281	0.289	0.297	0.305	0.313	0.321	0.329
0.18	0.188	0.196	0.204	0.212	0.22	0.227	0.234	0.242	0.25	0.258	0.265	0.274	0.282	0.29	0.298	0.306	0.314	0.322	0.329	0.337
0.19	0.197	0.206	0.213	0.221	0.229	0.237	0.244	0.251	0.259	0.267	0.274	0.283	0.291	0.299	0.307	0.314	0.322	0.33	0.337	0.346
0.20	0.207	0.215	0.223	0.231	0.238	0.246	0.253	0.26	0.268	0.276	0.283	0.291	0.3	0.307	0.315	0.323	0.33	0.338	0.345	0.354

Table C.75 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 25.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.028	0.038	0.047	0.057	0.066	0.076	0.085	0.096	0.105	0.114	0.124	0.133	0.143	0.153	0.163	0.173	0.183	0.194	0.203
0.02	0.028	0.038	0.047	0.056	0.066	0.075	0.084	0.093	0.104	0.113	0.122	0.131	0.140	0.150	0.160	0.170	0.179	0.190	0.200	0.209
0.03	0.038	0.047	0.056	0.065	0.075	0.083	0.092	0.102	0.112	0.121	0.130	0.139	0.148	0.158	0.167	0.177	0.186	0.196	0.206	0.216
0.04	0.048	0.057	0.066	0.075	0.084	0.093	0.102	0.111	0.121	0.130	0.139	0.148	0.157	0.166	0.176	0.185	0.194	0.204	0.214	0.223
0.05	0.058	0.067	0.076	0.084	0.093	0.102	0.111	0.120	0.130	0.138	0.147	0.156	0.165	0.175	0.184	0.193	0.202	0.212	0.222	0.231
0.06	0.068	0.077	0.085	0.094	0.103	0.111	0.120	0.129	0.139	0.147	0.156	0.165	0.174	0.183	0.192	0.201	0.210	0.220	0.230	0.239
0.07	0.078	0.087	0.095	0.104	0.113	0.121	0.129	0.138	0.148	0.156	0.165	0.174	0.182	0.192	0.201	0.210	0.219	0.229	0.238	0.247
0.08	0.088	0.097	0.105	0.113	0.122	0.130	0.139	0.148	0.157	0.165	0.174	0.183	0.191	0.200	0.209	0.218	0.227	0.237	0.246	0.255
0.09	0.098	0.106	0.115	0.123	0.132	0.140	0.148	0.157	0.166	0.174	0.183	0.191	0.200	0.209	0.218	0.227	0.236	0.245	0.254	0.263
0.10	0.108	0.116	0.125	0.133	0.141	0.149	0.158	0.166	0.176	0.184	0.192	0.200	0.209	0.218	0.227	0.236	0.244	0.254	0.263	0.271
0.11	0.118	0.126	0.134	0.143	0.151	0.159	0.167	0.176	0.185	0.193	0.201	0.210	0.218	0.227	0.235	0.244	0.253	0.262	0.271	0.280
0.12	0.128	0.136	0.144	0.153	0.161	0.169	0.177	0.185	0.194	0.202	0.211	0.219	0.227	0.236	0.244	0.253	0.262	0.271	0.280	0.288
0.13	0.138	0.146	0.154	0.163	0.171	0.178	0.187	0.195	0.204	0.212	0.220	0.228	0.236	0.245	0.253	0.262	0.271	0.280	0.289	0.297
0.14	0.148	0.156	0.164	0.172	0.180	0.188	0.196	0.204	0.213	0.221	0.229	0.237	0.245	0.254	0.262	0.270	0.279	0.288	0.297	0.305
0.15	0.158	0.165	0.173	0.181	0.189	0.197	0.205	0.213	0.222	0.230	0.238	0.246	0.254	0.262	0.270	0.279	0.287	0.296	0.305	0.313
0.16	0.168	0.175	0.183	0.191	0.199	0.207	0.215	0.223	0.231	0.239	0.247	0.255	0.263	0.271	0.279	0.288	0.296	0.305	0.313	0.321
0.17	0.177	0.185	0.193	0.201	0.209	0.216	0.224	0.232	0.241	0.248	0.256	0.264	0.272	0.280	0.288	0.296	0.304	0.313	0.322	0.330
0.18	0.187	0.195	0.202	0.210	0.218	0.226	0.233	0.241	0.250	0.257	0.265	0.273	0.281	0.289	0.297	0.305	0.313	0.322	0.330	0.338
0.19	0.197	0.205	0.212	0.220	0.228	0.235	0.243	0.251	0.259	0.267	0.275	0.282	0.290	0.298	0.306	0.314	0.322	0.330	0.339	0.347
0.20	0.207	0.215	0.222	0.230	0.238	0.245	0.253	0.260	0.269	0.276	0.284	0.291	0.299	0.307	0.315	0.323	0.331	0.339	0.347	0.355

Table C.76 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 30.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.039	0.048	0.058	0.068	0.078	0.088	0.098	0.108	0.118	0.128	0.138	0.148	0.158	0.168	0.177	0.188	0.198	0.208
0.02	0.028	0.038	0.047	0.057	0.066	0.076	0.086	0.096	0.106	0.116	0.126	0.135	0.145	0.155	0.164	0.175	0.184	0.194	0.204	0.214
0.03	0.039	0.048	0.057	0.066	0.075	0.085	0.095	0.104	0.114	0.124	0.134	0.143	0.153	0.163	0.172	0.183	0.192	0.202	0.211	0.221
0.04	0.049	0.057	0.067	0.076	0.085	0.095	0.104	0.113	0.123	0.133	0.143	0.152	0.161	0.171	0.180	0.191	0.199	0.209	0.219	0.229
0.05	0.058	0.067	0.076	0.084	0.094	0.103	0.113	0.122	0.131	0.141	0.151	0.160	0.169	0.179	0.188	0.198	0.207	0.217	0.226	0.236
0.06	0.068	0.077	0.086	0.094	0.103	0.113	0.122	0.131	0.141	0.150	0.160	0.169	0.178	0.188	0.197	0.207	0.215	0.225	0.234	0.244
0.07	0.078	0.087	0.096	0.104	0.113	0.123	0.132	0.141	0.150	0.160	0.169	0.178	0.187	0.197	0.205	0.215	0.224	0.233	0.243	0.252
0.08	0.088	0.097	0.105	0.114	0.123	0.132	0.141	0.150	0.159	0.169	0.178	0.187	0.196	0.205	0.214	0.224	0.232	0.242	0.251	0.260
0.09	0.098	0.107	0.115	0.124	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.223	0.233	0.241	0.250	0.259	0.268
0.10	0.108	0.116	0.125	0.133	0.142	0.151	0.160	0.169	0.178	0.187	0.196	0.205	0.213	0.223	0.231	0.241	0.249	0.258	0.267	0.276
0.11	0.118	0.126	0.135	0.143	0.152	0.161	0.170	0.178	0.187	0.196	0.205	0.214	0.222	0.232	0.240	0.250	0.258	0.267	0.276	0.284
0.12	0.128	0.136	0.145	0.153	0.162	0.170	0.179	0.188	0.197	0.206	0.214	0.223	0.231	0.241	0.249	0.258	0.266	0.276	0.284	0.293
0.13	0.139	0.147	0.155	0.163	0.172	0.180	0.189	0.197	0.206	0.215	0.224	0.232	0.241	0.250	0.258	0.267	0.275	0.284	0.293	0.301
0.14	0.148	0.156	0.165	0.173	0.181	0.190	0.198	0.207	0.215	0.224	0.233	0.241	0.249	0.258	0.267	0.276	0.283	0.292	0.301	0.309
0.15	0.158	0.166	0.174	0.182	0.190	0.199	0.208	0.216	0.224	0.233	0.242	0.250	0.258	0.267	0.275	0.284	0.292	0.301	0.309	0.317
0.16	0.168	0.176	0.184	0.192	0.200	0.209	0.217	0.225	0.234	0.242	0.251	0.259	0.267	0.276	0.284	0.293	0.300	0.309	0.317	0.326
0.17	0.177	0.185	0.193	0.201	0.209	0.218	0.226	0.234	0.243	0.251	0.259	0.268	0.275	0.284	0.292	0.301	0.308	0.317	0.325	0.334
0.18	0.187	0.195	0.202	0.210	0.218	0.227	0.235	0.243	0.251	0.260	0.268	0.276	0.284	0.293	0.301	0.309	0.317	0.325	0.333	0.342
0.19	0.197	0.205	0.213	0.221	0.229	0.237	0.245	0.253	0.261	0.270	0.278	0.286	0.294	0.302	0.310	0.319	0.326	0.334	0.342	0.350
0.20	0.207	0.215	0.223	0.230	0.238	0.246	0.255	0.262	0.270	0.279	0.287	0.295	0.302	0.311	0.318	0.327	0.334	0.343	0.351	0.358

Table C.77 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 35.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.030	0.040	0.049	0.058	0.067	0.077	0.086	0.096	0.106	0.116	0.125	0.135	0.145	0.154	0.164	0.174	0.184	0.194	0.204
0.02	0.029	0.039	0.049	0.058	0.066	0.076	0.086	0.095	0.104	0.113	0.123	0.132	0.142	0.152	0.161	0.171	0.181	0.190	0.200	0.211
0.03	0.039	0.048	0.058	0.067	0.075	0.085	0.094	0.103	0.112	0.122	0.131	0.140	0.150	0.160	0.169	0.179	0.188	0.198	0.208	0.218
0.04	0.049	0.058	0.068	0.076	0.085	0.094	0.104	0.112	0.121	0.130	0.140	0.149	0.158	0.168	0.177	0.187	0.196	0.205	0.215	0.225
0.05	0.058	0.068	0.077	0.086	0.094	0.103	0.113	0.121	0.130	0.139	0.149	0.157	0.167	0.176	0.186	0.195	0.204	0.213	0.223	0.233
0.06	0.067	0.077	0.086	0.095	0.103	0.112	0.121	0.130	0.139	0.147	0.157	0.165	0.175	0.184	0.193	0.203	0.212	0.221	0.230	0.240
0.07	0.077	0.086	0.096	0.104	0.112	0.121	0.130	0.139	0.148	0.156	0.166	0.174	0.183	0.193	0.202	0.211	0.220	0.229	0.238	0.248
0.08	0.088	0.097	0.106	0.115	0.123	0.131	0.141	0.149	0.158	0.166	0.175	0.184	0.193	0.202	0.211	0.220	0.229	0.238	0.247	0.257
0.09	0.097	0.106	0.116	0.124	0.132	0.141	0.150	0.158	0.167	0.175	0.184	0.192	0.201	0.211	0.219	0.229	0.237	0.246	0.255	0.265
0.10	0.108	0.117	0.126	0.134	0.142	0.150	0.159	0.168	0.176	0.185	0.193	0.202	0.211	0.220	0.228	0.238	0.246	0.255	0.264	0.273
0.11	0.117	0.126	0.135	0.143	0.151	0.159	0.168	0.176	0.185	0.193	0.202	0.210	0.219	0.228	0.237	0.246	0.254	0.263	0.272	0.281
0.12	0.127	0.136	0.145	0.153	0.161	0.169	0.178	0.186	0.194	0.203	0.211	0.219	0.228	0.237	0.246	0.254	0.263	0.271	0.280	0.289
0.13	0.137	0.146	0.155	0.163	0.170	0.179	0.188	0.195	0.204	0.212	0.220	0.228	0.237	0.246	0.254	0.263	0.271	0.280	0.289	0.298
0.14	0.147	0.155	0.164	0.172	0.180	0.188	0.197	0.204	0.212	0.220	0.229	0.237	0.245	0.254	0.263	0.271	0.279	0.288	0.297	0.306
0.15	0.157	0.165	0.174	0.182	0.189	0.198	0.206	0.214	0.222	0.230	0.238	0.246	0.255	0.263	0.272	0.280	0.288	0.297	0.305	0.314
0.16	0.167	0.175	0.184	0.192	0.199	0.207	0.216	0.223	0.231	0.239	0.247	0.255	0.264	0.272	0.280	0.289	0.297	0.305	0.314	0.322
0.17	0.177	0.185	0.194	0.201	0.209	0.217	0.225	0.233	0.241	0.248	0.257	0.264	0.273	0.281	0.289	0.298	0.306	0.314	0.322	0.331
0.18	0.186	0.194	0.203	0.211	0.218	0.226	0.234	0.242	0.249	0.257	0.265	0.273	0.281	0.289	0.298	0.306	0.314	0.322	0.330	0.339
0.19	0.196	0.205	0.213	0.221	0.228	0.236	0.244	0.251	0.259	0.266	0.274	0.282	0.290	0.299	0.306	0.315	0.322	0.330	0.339	0.347
0.20	0.206	0.214	0.223	0.230	0.237	0.245	0.253	0.261	0.268	0.276	0.283	0.291	0.299	0.307	0.315	0.323	0.331	0.339	0.347	0.355

Table C.78 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 40.

Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.058	0.067	0.077	0.088	0.097	0.107	0.117	0.127	0.137	0.146	0.155	0.165	0.175	0.185	0.195	0.205
0.02	0.030	0.039	0.048	0.058	0.068	0.077	0.087	0.097	0.106	0.116	0.126	0.136	0.145	0.154	0.164	0.173	0.183	0.193	0.202	0.213
0.03	0.040	0.049	0.058	0.068	0.077	0.086	0.096	0.106	0.115	0.125	0.135	0.144	0.154	0.163	0.172	0.181	0.191	0.201	0.210	0.220
0.04	0.049	0.058	0.067	0.077	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.153	0.162	0.171	0.180	0.189	0.199	0.208	0.217	0.228
0.05	0.059	0.068	0.077	0.086	0.096	0.104	0.114	0.124	0.133	0.142	0.152	0.161	0.170	0.179	0.188	0.198	0.207	0.216	0.226	0.236
0.06	0.068	0.076	0.086	0.095	0.104	0.113	0.122	0.132	0.141	0.150	0.160	0.169	0.178	0.187	0.196	0.205	0.214	0.224	0.233	0.243
0.07	0.077	0.086	0.095	0.104	0.114	0.122	0.132	0.141	0.150	0.159	0.169	0.178	0.187	0.195	0.204	0.213	0.222	0.232	0.240	0.250
0.08	0.087	0.096	0.105	0.114	0.123	0.131	0.141	0.150	0.159	0.168	0.177	0.186	0.195	0.204	0.212	0.221	0.230	0.240	0.249	0.258
0.09	0.097	0.105	0.114	0.123	0.132	0.141	0.150	0.159	0.168	0.177	0.186	0.195	0.204	0.212	0.221	0.230	0.238	0.248	0.257	0.266
0.10	0.106	0.115	0.123	0.132	0.141	0.150	0.159	0.168	0.177	0.185	0.195	0.204	0.212	0.221	0.229	0.238	0.247	0.256	0.265	0.274
0.11	0.116	0.124	0.133	0.142	0.150	0.159	0.168	0.177	0.185	0.194	0.203	0.212	0.221	0.229	0.237	0.246	0.255	0.264	0.273	0.282
0.12	0.126	0.134	0.143	0.152	0.160	0.169	0.177	0.186	0.195	0.204	0.213	0.221	0.230	0.238	0.246	0.255	0.264	0.273	0.281	0.290
0.13	0.136	0.144	0.153	0.162	0.170	0.178	0.187	0.196	0.205	0.213	0.222	0.231	0.239	0.247	0.255	0.264	0.272	0.281	0.290	0.299
0.14	0.146	0.154	0.162	0.171	0.179	0.187	0.196	0.205	0.213	0.222	0.231	0.239	0.247	0.255	0.263	0.272	0.280	0.289	0.298	0.307
0.15	0.155	0.163	0.171	0.180	0.188	0.196	0.205	0.214	0.222	0.230	0.239	0.248	0.256	0.264	0.272	0.280	0.288	0.297	0.305	0.314
0.16	0.165	0.173	0.181	0.190	0.198	0.206	0.215	0.223	0.231	0.240	0.248	0.257	0.265	0.272	0.280	0.289	0.297	0.306	0.314	0.323
0.17	0.175	0.182	0.190	0.199	0.207	0.215	0.224	0.232	0.240	0.248	0.257	0.265	0.273	0.281	0.289	0.297	0.305	0.314	0.322	0.331
0.18	0.184	0.192	0.200	0.208	0.217	0.224	0.233	0.241	0.249	0.257	0.266	0.274	0.282	0.290	0.297	0.305	0.314	0.322	0.330	0.339
0.19	0.194	0.201	0.209	0.218	0.226	0.233	0.242	0.250	0.258	0.266	0.274	0.283	0.290	0.298	0.306	0.314	0.322	0.330	0.338	0.347
0.20	0.204	0.211	0.219	0.227	0.235	0.243	0.251	0.259	0.267	0.275	0.283	0.291	0.299	0.307	0.314	0.322	0.330	0.338	0.346	0.355

Table C.79 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 45.

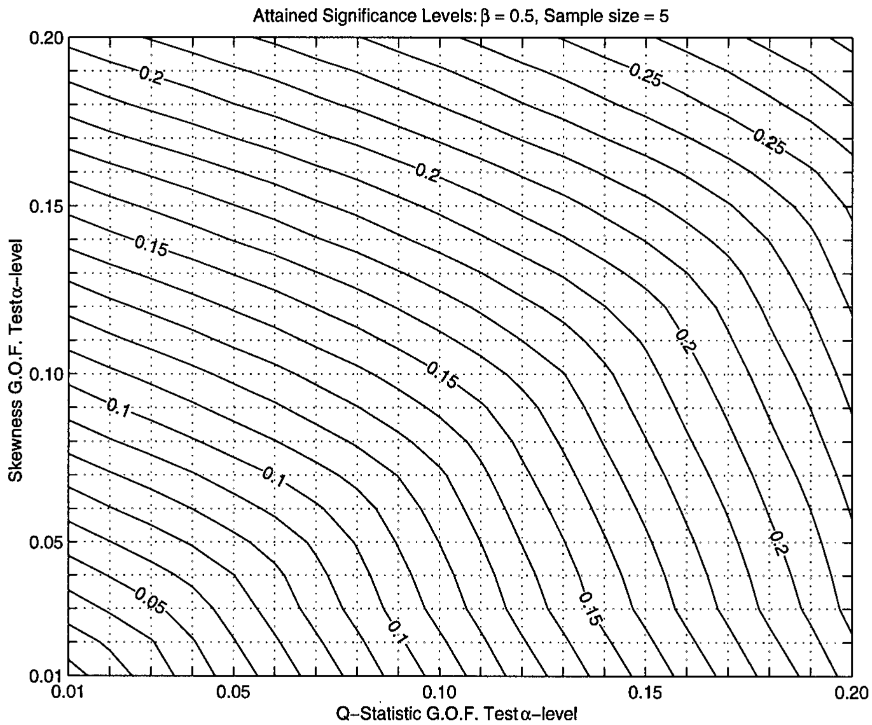
Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.020	0.030	0.039	0.048	0.059	0.068	0.078	0.087	0.098	0.107	0.117	0.126	0.136	0.146	0.156	0.166	0.177	0.186	0.196	0.206
0.02	0.030	0.039	0.049	0.058	0.068	0.077	0.087	0.096	0.106	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.204	0.213
0.03	0.040	0.049	0.058	0.067	0.077	0.086	0.096	0.105	0.115	0.124	0.133	0.143	0.152	0.162	0.172	0.181	0.192	0.201	0.211	0.220
0.04	0.049	0.058	0.067	0.076	0.086	0.095	0.104	0.113	0.123	0.132	0.142	0.151	0.160	0.170	0.179	0.189	0.199	0.209	0.218	0.227
0.05	0.059	0.068	0.077	0.085	0.095	0.104	0.113	0.122	0.132	0.141	0.150	0.159	0.168	0.178	0.188	0.197	0.207	0.216	0.226	0.235
0.06	0.068	0.077	0.086	0.094	0.104	0.113	0.122	0.131	0.141	0.149	0.159	0.167	0.177	0.186	0.196	0.205	0.215	0.224	0.234	0.242
0.07	0.078	0.087	0.096	0.104	0.114	0.122	0.131	0.140	0.150	0.159	0.168	0.176	0.186	0.195	0.204	0.214	0.224	0.233	0.242	0.251
0.08	0.089	0.097	0.106	0.114	0.124	0.132	0.141	0.150	0.160	0.168	0.177	0.186	0.195	0.204	0.213	0.223	0.233	0.241	0.251	0.259
0.09	0.098	0.107	0.116	0.124	0.133	0.142	0.151	0.159	0.169	0.177	0.186	0.195	0.204	0.213	0.222	0.231	0.241	0.250	0.259	0.267
0.10	0.109	0.117	0.126	0.134	0.143	0.151	0.160	0.169	0.179	0.187	0.196	0.204	0.213	0.222	0.231	0.240	0.250	0.258	0.267	0.276
0.11	0.119	0.127	0.135	0.144	0.153	0.161	0.170	0.178	0.188	0.196	0.205	0.213	0.222	0.231	0.240	0.249	0.258	0.267	0.276	0.284
0.12	0.129	0.137	0.146	0.154	0.163	0.171	0.180	0.188	0.197	0.205	0.214	0.222	0.231	0.240	0.249	0.257	0.267	0.275	0.284	0.292
0.13	0.139	0.147	0.156	0.164	0.172	0.180	0.189	0.197	0.207	0.215	0.223	0.231	0.240	0.249	0.257	0.266	0.276	0.284	0.293	0.301
0.14	0.149	0.157	0.165	0.173	0.182	0.190	0.198	0.206	0.216	0.224	0.232	0.240	0.249	0.257	0.266	0.275	0.284	0.292	0.301	0.309
0.15	0.159	0.167	0.175	0.183	0.191	0.199	0.208	0.216	0.225	0.233	0.241	0.249	0.258	0.266	0.275	0.283	0.293	0.301	0.309	0.317
0.16	0.169	0.177	0.185	0.192	0.201	0.209	0.217	0.225	0.234	0.242	0.250	0.258	0.266	0.275	0.283	0.292	0.301	0.309	0.317	0.325
0.17	0.178	0.186	0.194	0.202	0.210	0.218	0.226	0.234	0.243	0.251	0.259	0.267	0.275	0.284	0.292	0.300	0.309	0.317	0.325	0.333
0.18	0.188	0.196	0.204	0.211	0.220	0.227	0.235	0.243	0.252	0.260	0.268	0.275	0.284	0.292	0.300	0.308	0.317	0.325	0.333	0.341
0.19	0.197	0.205	0.213	0.221	0.229	0.236	0.244	0.252	0.261	0.268	0.276	0.284	0.292	0.300	0.308	0.317	0.325	0.333	0.341	0.349
0.20	0.208	0.215	0.223	0.230	0.239	0.246	0.254	0.262	0.270	0.278	0.286	0.293	0.301	0.309	0.317	0.326	0.334	0.342	0.350	0.357

Table C.80 Attained Significance Levels: Weibull Distribution Shape Parameter Value = 4.0; Sample Size = 50.

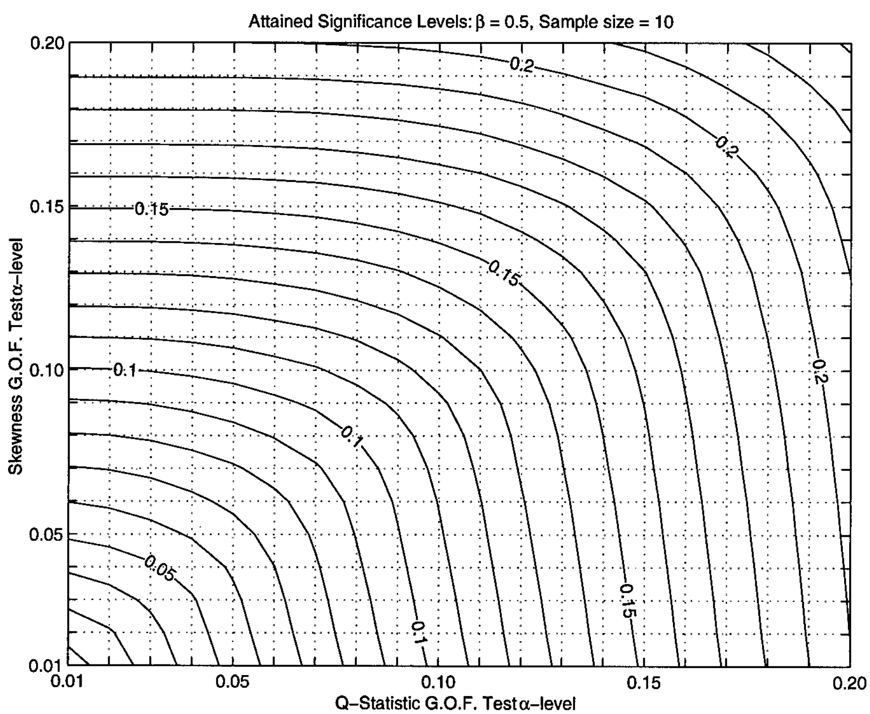
Skewness G.O.F. Test Significance Level	Q-Statistic G.O.F. Test Significance Level																			
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.01	0.019	0.029	0.038	0.048	0.057	0.066	0.075	0.085	0.096	0.106	0.115	0.125	0.134	0.144	0.154	0.164	0.174	0.184	0.194	0.203
0.02	0.029	0.039	0.048	0.057	0.066	0.076	0.084	0.094	0.104	0.115	0.124	0.133	0.142	0.152	0.162	0.172	0.182	0.191	0.201	0.210
0.03	0.039	0.048	0.058	0.066	0.076	0.085	0.093	0.103	0.113	0.123	0.132	0.141	0.150	0.160	0.170	0.180	0.189	0.199	0.208	0.218
0.04	0.049	0.058	0.067	0.076	0.085	0.094	0.102	0.112	0.122	0.132	0.141	0.150	0.159	0.168	0.178	0.188	0.197	0.207	0.216	0.225
0.05	0.059	0.068	0.077	0.086	0.095	0.103	0.112	0.121	0.131	0.141	0.150	0.159	0.167	0.177	0.186	0.197	0.206	0.215	0.224	0.233
0.06	0.069	0.078	0.087	0.095	0.104	0.113	0.121	0.130	0.140	0.150	0.159	0.168	0.176	0.186	0.195	0.205	0.214	0.223	0.232	0.241
0.07	0.079	0.088	0.097	0.105	0.114	0.123	0.131	0.140	0.150	0.159	0.168	0.177	0.185	0.195	0.204	0.214	0.223	0.232	0.241	0.250
0.08	0.090	0.098	0.107	0.115	0.124	0.132	0.141	0.150	0.159	0.169	0.177	0.186	0.194	0.204	0.213	0.223	0.231	0.240	0.249	0.258
0.09	0.100	0.108	0.117	0.125	0.134	0.142	0.150	0.159	0.168	0.178	0.186	0.195	0.203	0.212	0.221	0.231	0.240	0.249	0.258	0.267
0.10	0.110	0.118	0.127	0.135	0.143	0.152	0.160	0.169	0.178	0.187	0.196	0.204	0.212	0.221	0.230	0.240	0.249	0.258	0.266	0.275
0.11	0.120	0.128	0.137	0.145	0.153	0.161	0.169	0.178	0.187	0.197	0.205	0.213	0.221	0.230	0.239	0.249	0.257	0.266	0.275	0.283
0.12	0.129	0.138	0.146	0.154	0.162	0.170	0.178	0.187	0.196	0.205	0.214	0.222	0.230	0.239	0.247	0.257	0.265	0.274	0.283	0.291
0.13	0.140	0.148	0.156	0.164	0.173	0.181	0.189	0.197	0.206	0.215	0.223	0.232	0.240	0.248	0.257	0.266	0.275	0.283	0.292	0.300
0.14	0.150	0.158	0.167	0.175	0.183	0.191	0.198	0.207	0.216	0.225	0.233	0.241	0.249	0.257	0.266	0.275	0.283	0.292	0.300	0.309
0.15	0.160	0.169	0.177	0.184	0.193	0.200	0.208	0.216	0.225	0.234	0.242	0.250	0.258	0.266	0.275	0.284	0.292	0.301	0.309	0.317
0.16	0.170	0.178	0.186	0.194	0.202	0.209	0.217	0.225	0.234	0.243	0.251	0.259	0.266	0.275	0.283	0.292	0.300	0.309	0.317	0.325
0.17	0.179	0.187	0.195	0.203	0.211	0.219	0.226	0.234	0.243	0.252	0.260	0.267	0.275	0.283	0.292	0.300	0.309	0.317	0.325	0.333
0.18	0.190	0.198	0.205	0.213	0.221	0.229	0.236	0.244	0.253	0.261	0.269	0.277	0.284	0.292	0.301	0.310	0.317	0.326	0.334	0.342
0.19	0.199	0.207	0.215	0.222	0.230	0.238	0.245	0.253	0.262	0.270	0.278	0.286	0.293	0.301	0.309	0.318	0.326	0.334	0.342	0.350
0.20	0.209	0.217	0.224	0.232	0.240	0.247	0.254	0.262	0.271	0.279	0.287	0.294	0.302	0.310	0.318	0.327	0.334	0.342	0.350	0.358

Appendix D. Contour Plots for Attained Significance Levels

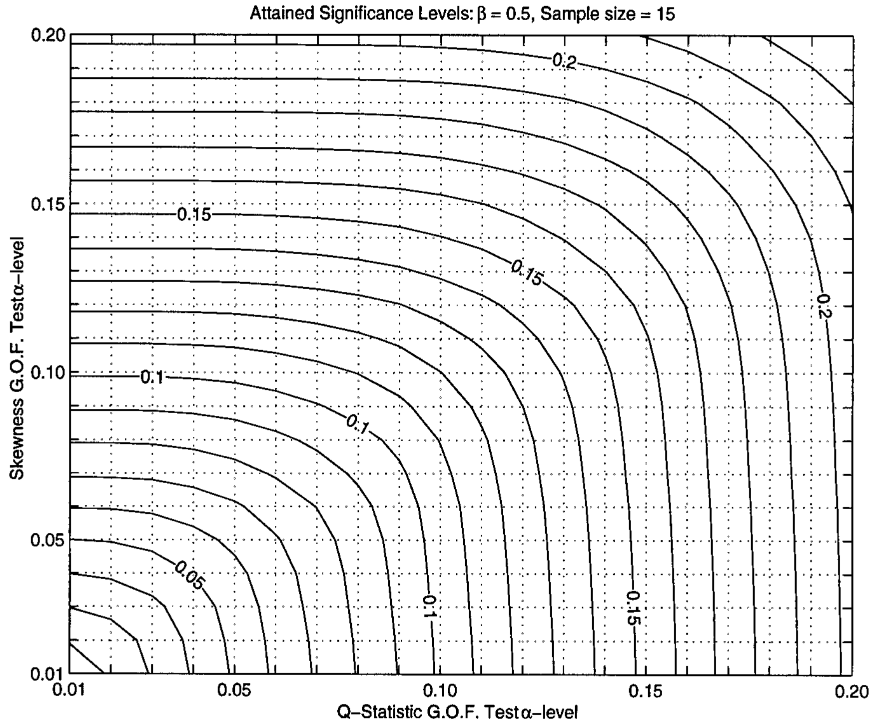
D.1 Weibull Shape $\beta = 0.5$



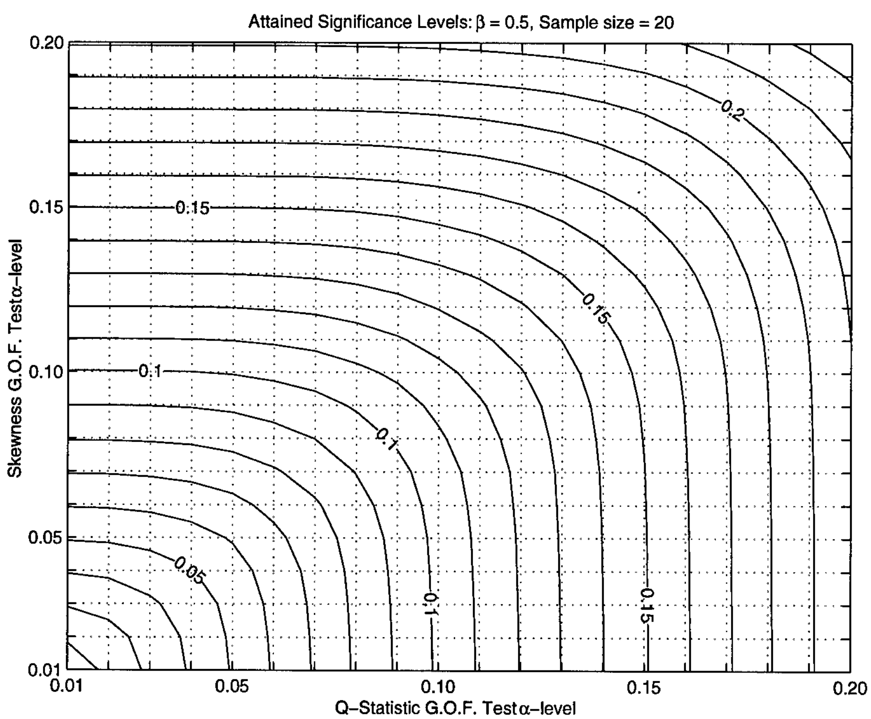
(a) Sample Size = 5



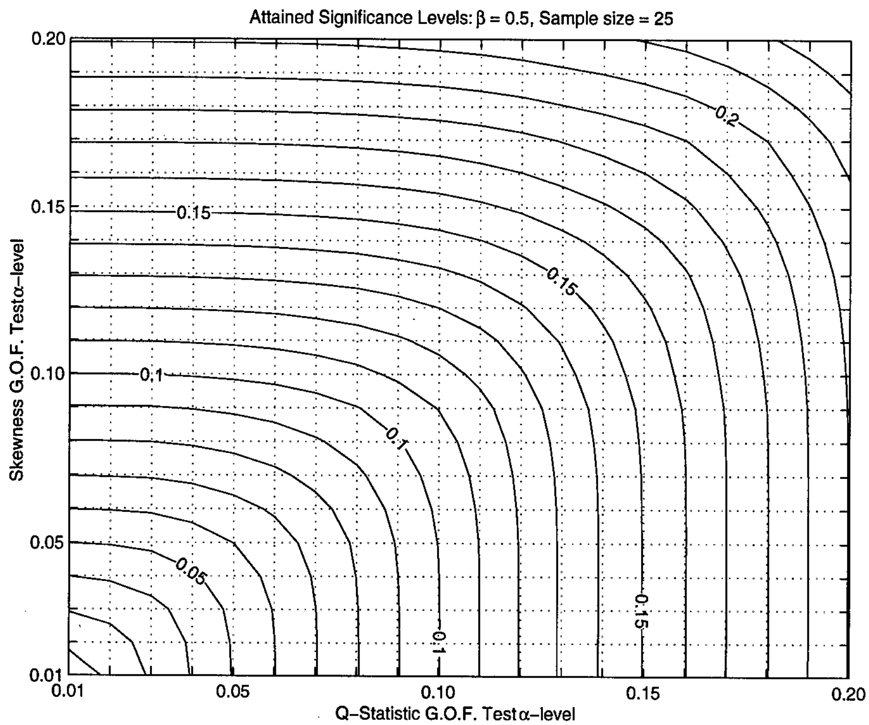
(b) Sample Size = 10



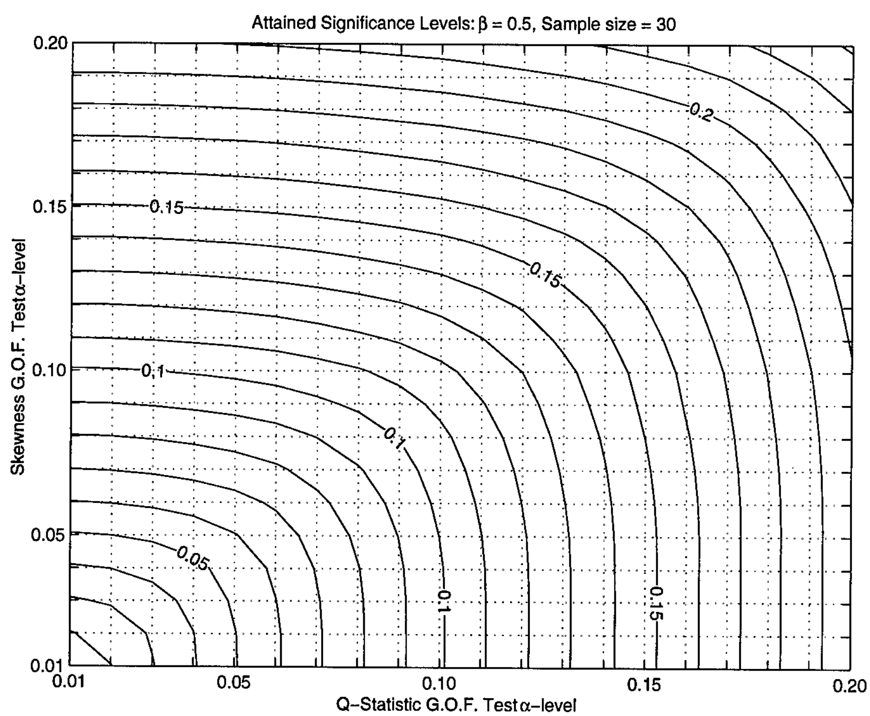
(c) Sample Size = 15



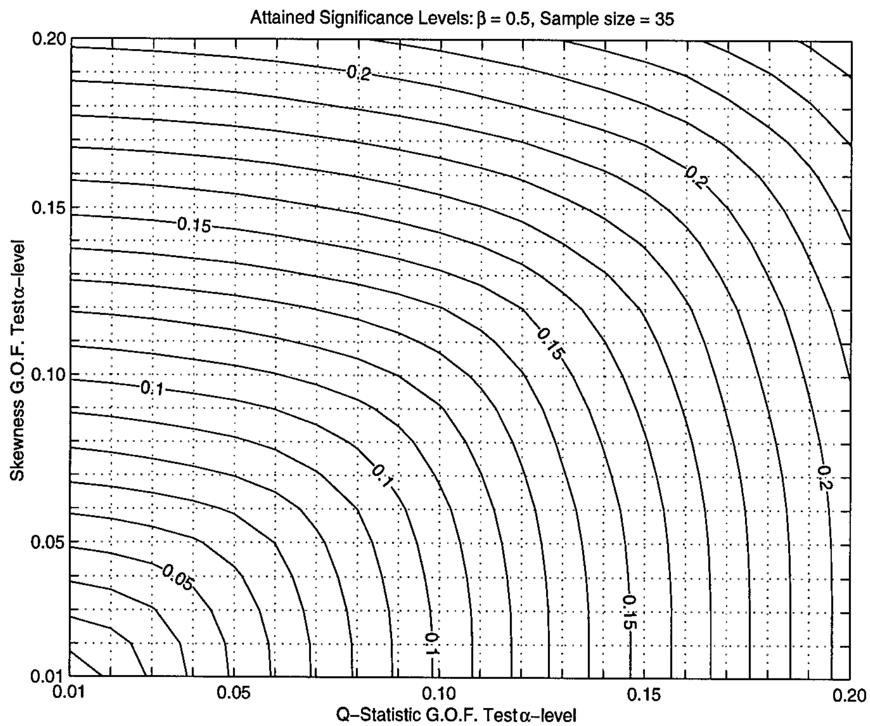
(d) Sample Size = 20



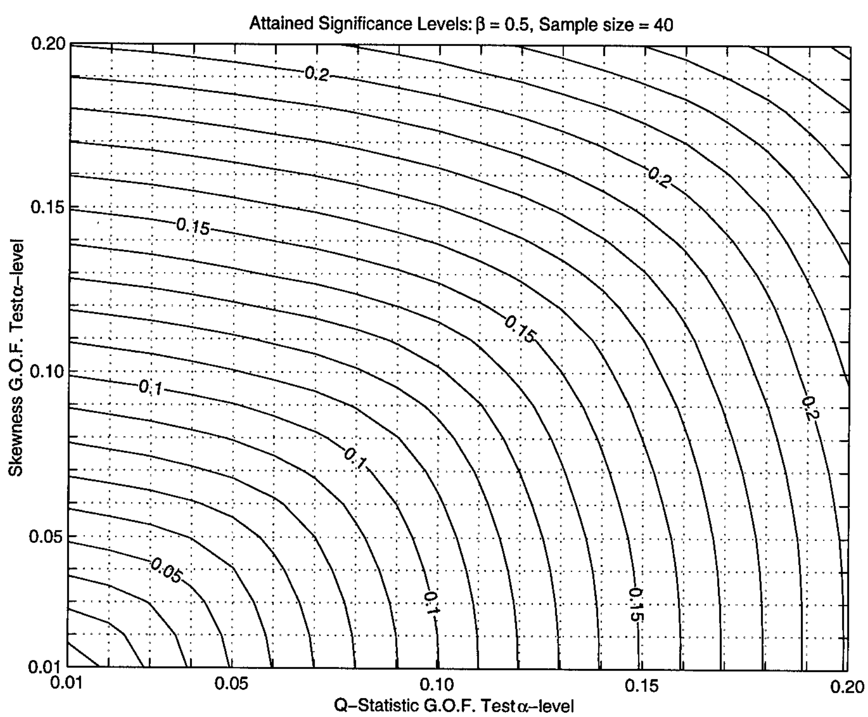
(e) Sample Size = 25



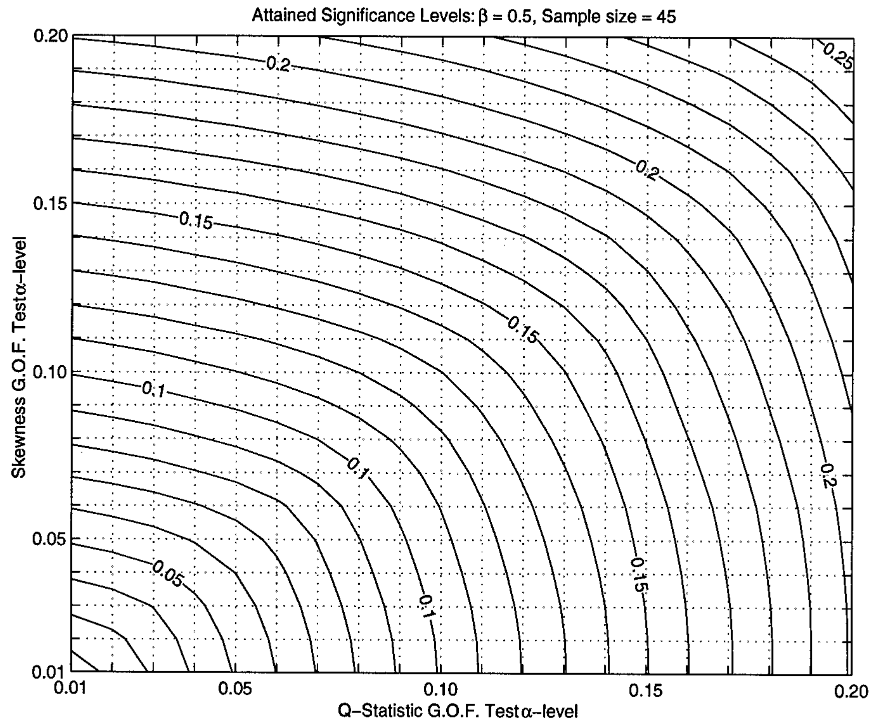
(f) Sample Size = 30



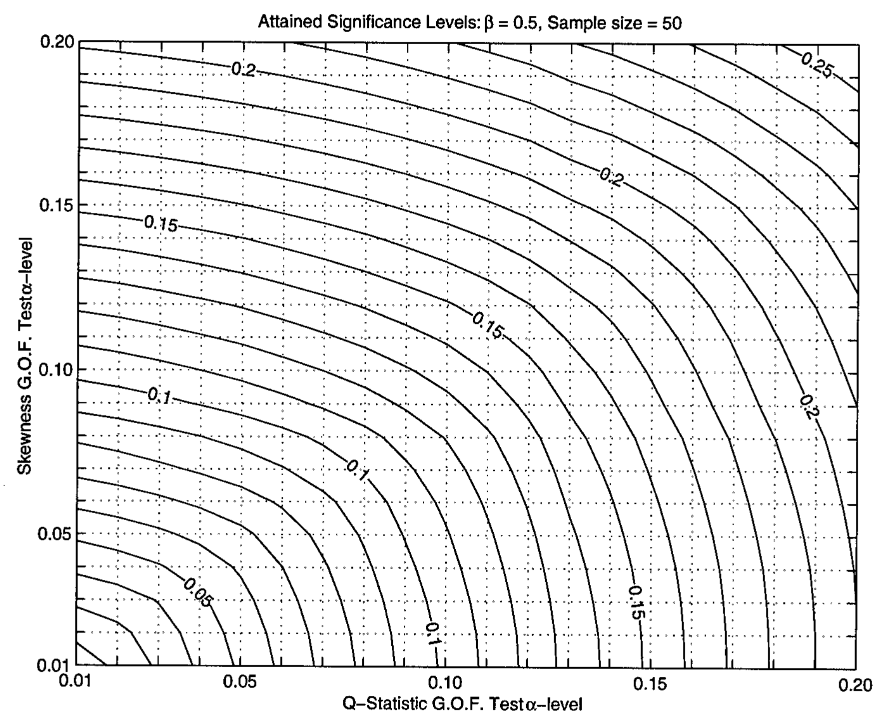
(g) Sample Size = 35



(h) Sample Size = 40

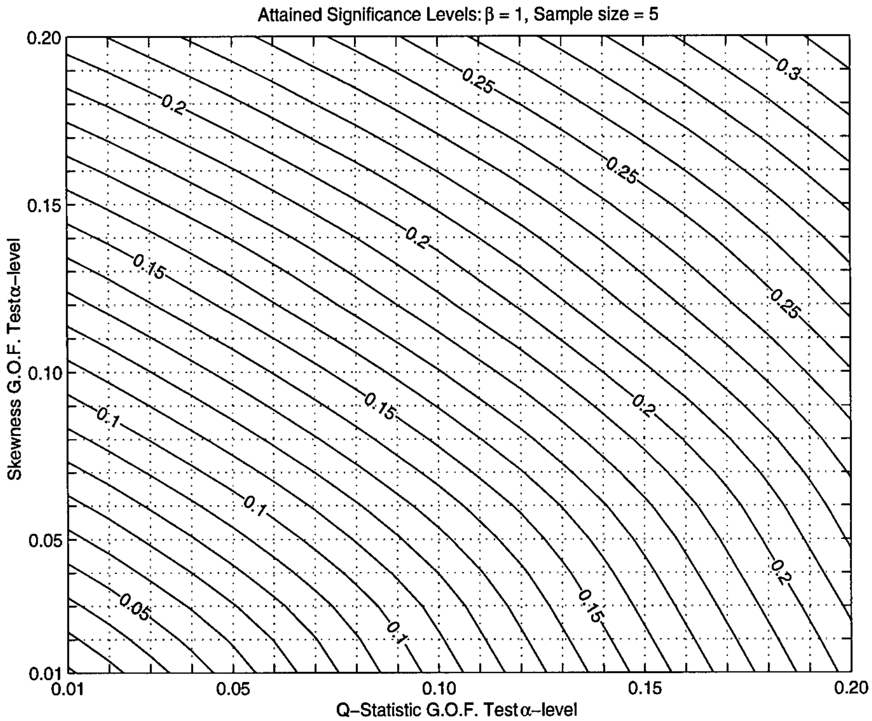


(i) Sample Size = 45

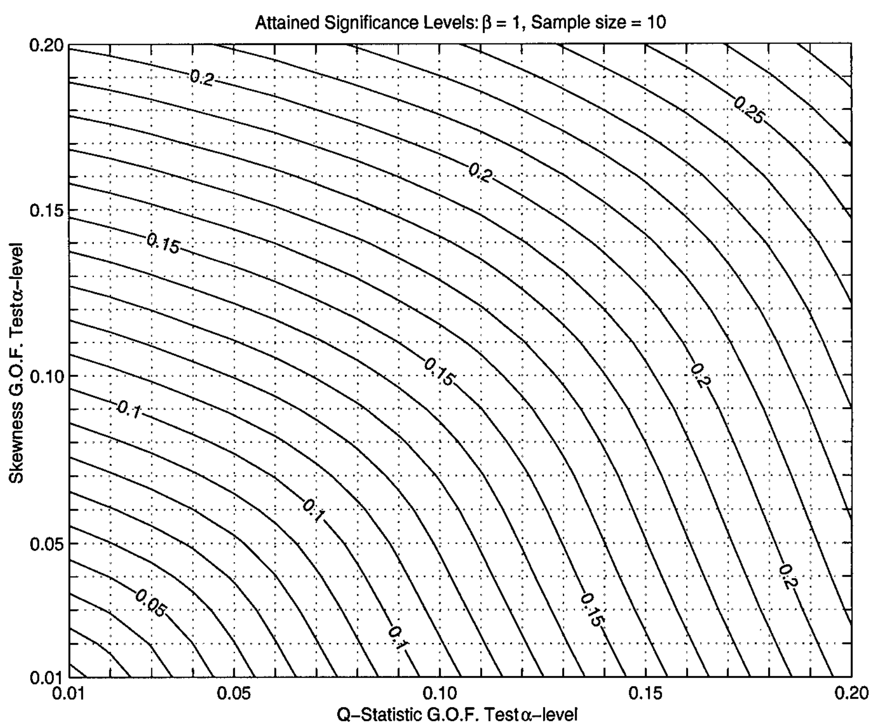


(j) Sample Size = 50

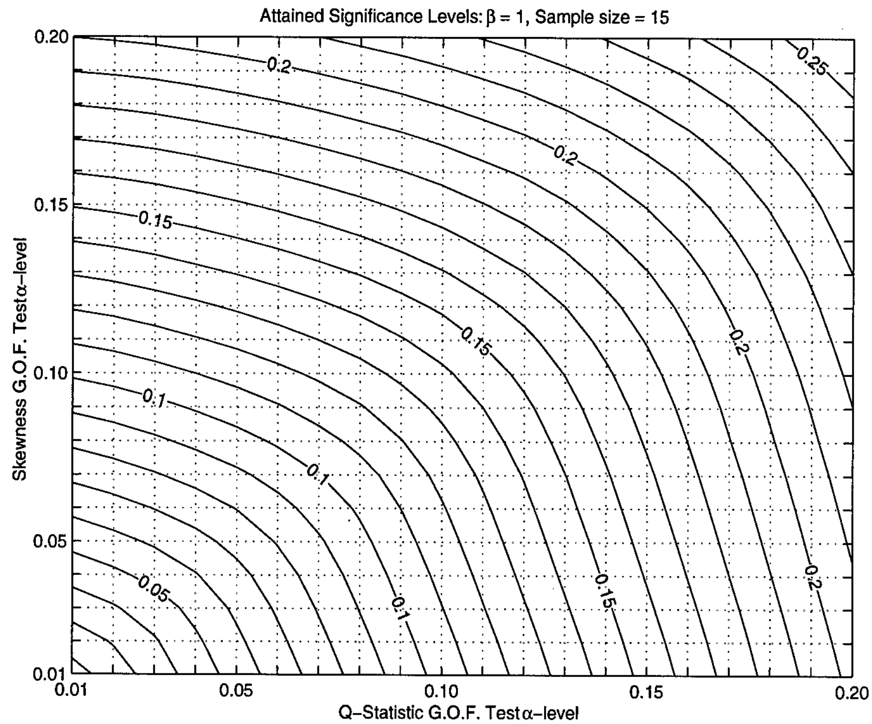
D.2 Weibull Shape $\beta = 1.0$



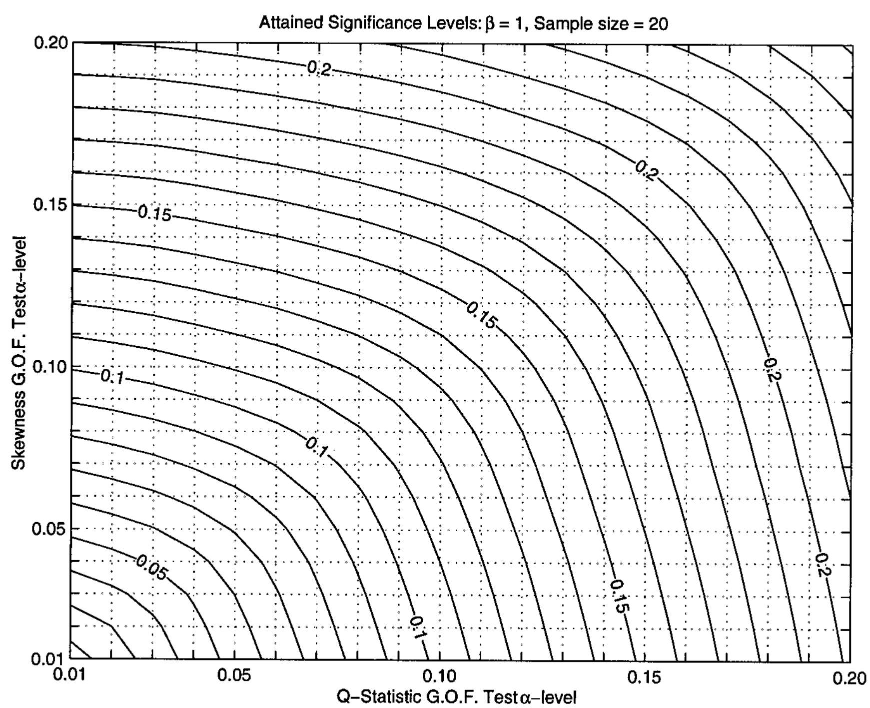
(a) Sample Size = 5



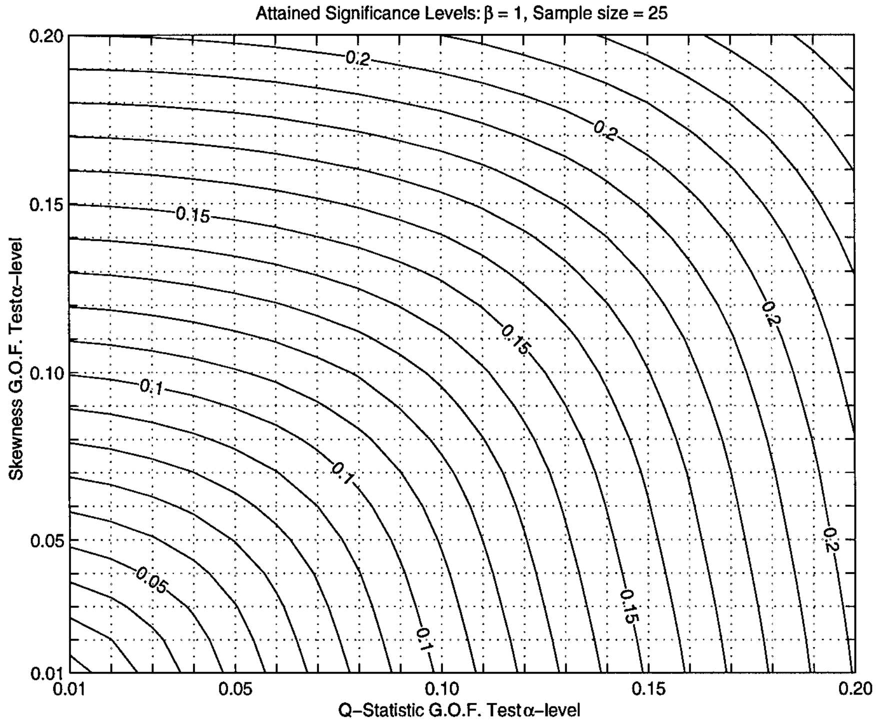
(b) Sample Size = 10



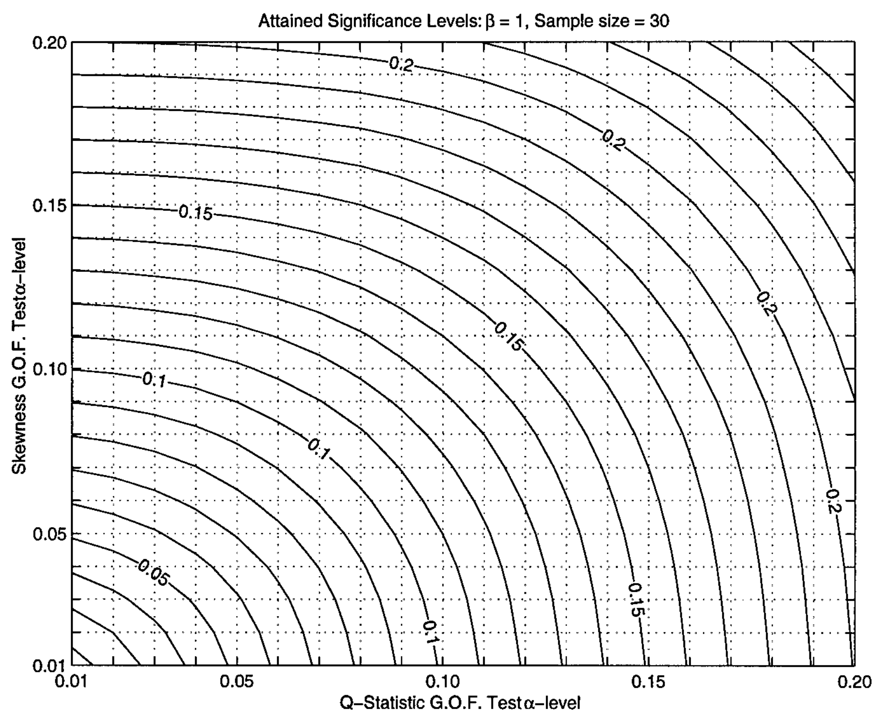
(c) Sample Size = 15



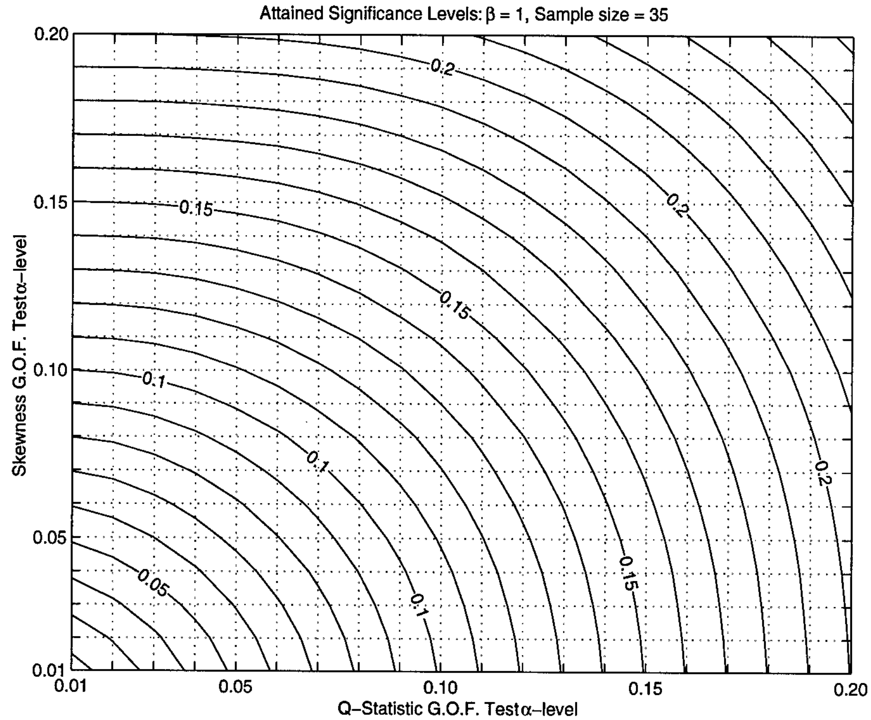
(d) Sample Size = 20



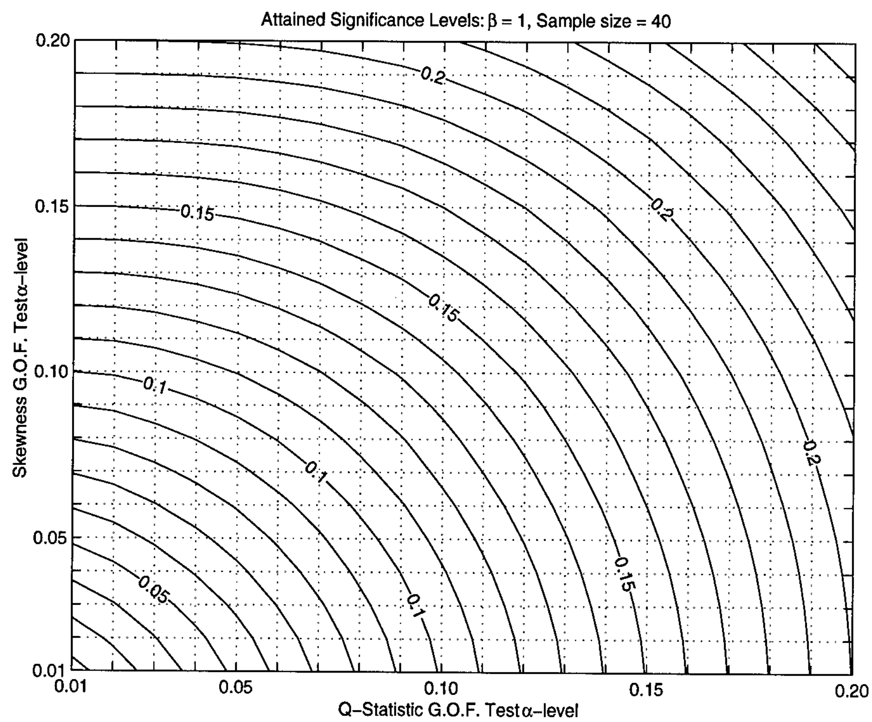
(e) Sample Size = 25



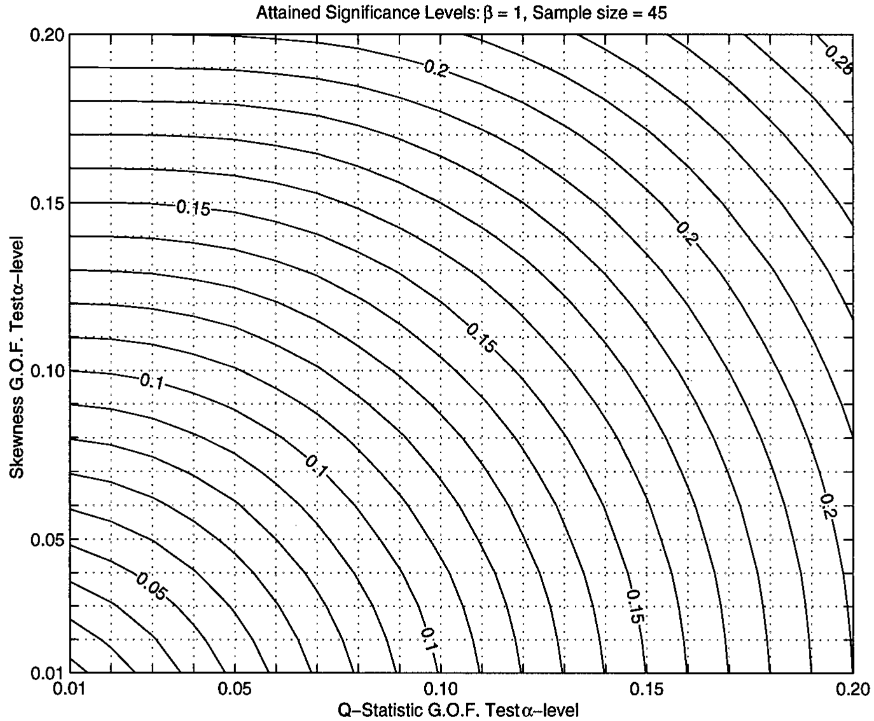
(f) Sample Size = 30



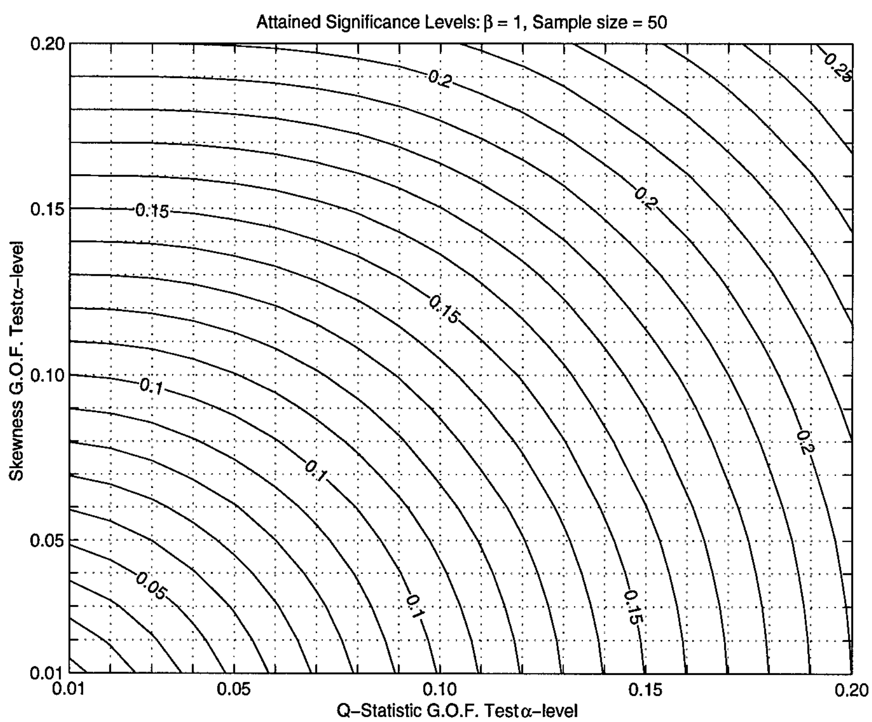
(g) Sample Size = 35



(h) Sample Size = 40

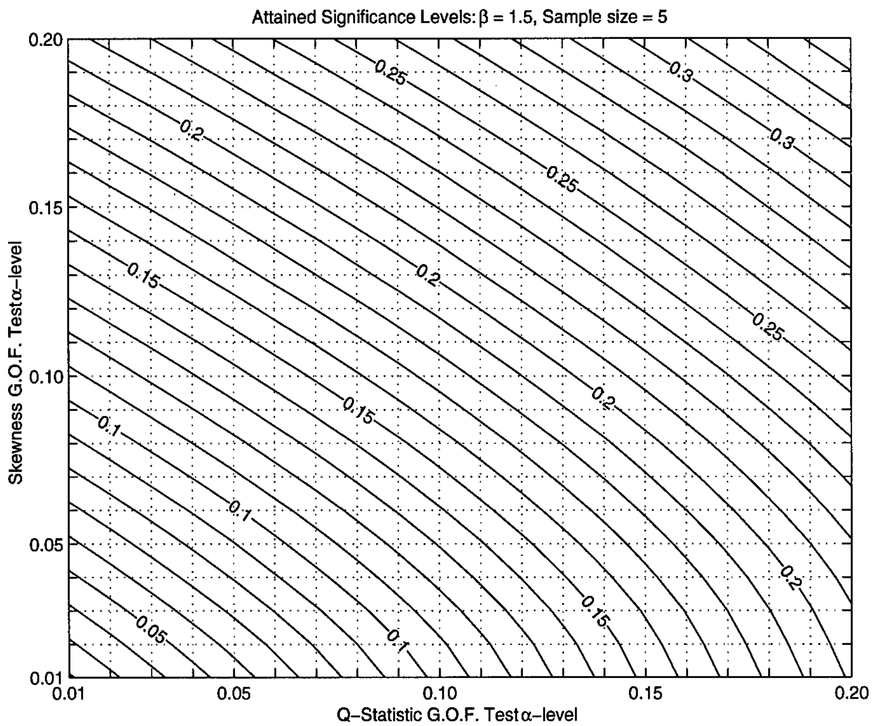


(i) Sample Size = 45

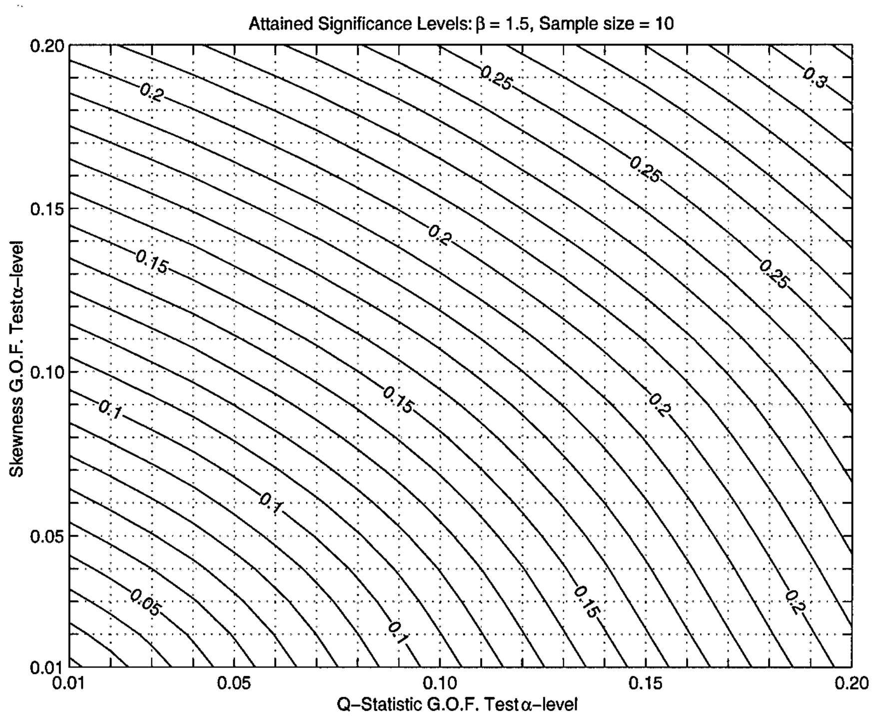


(j) Sample S ize = 50

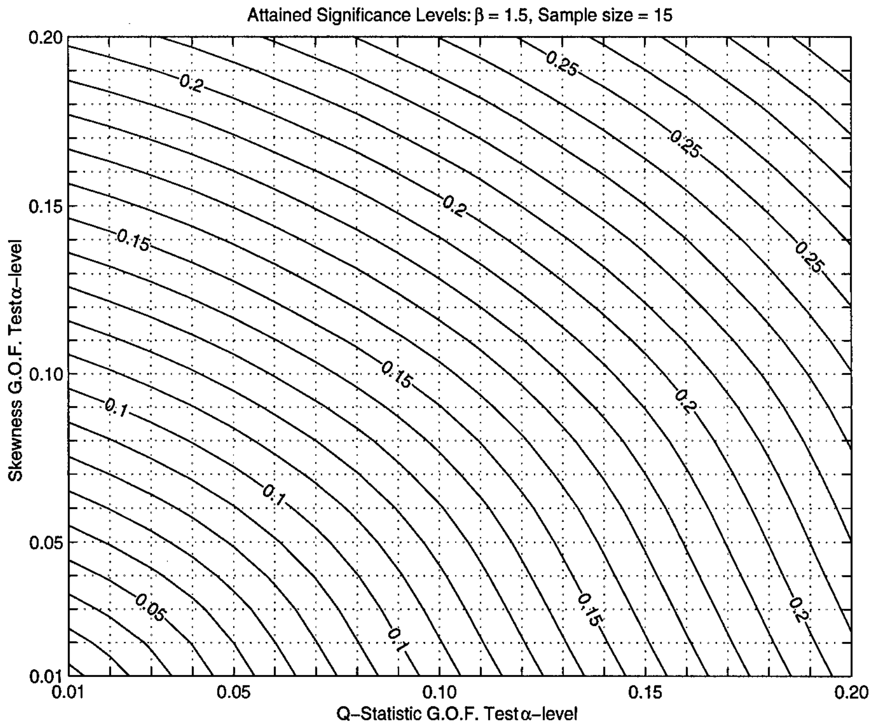
D.3 Weibull Shape $\beta = 1.5$



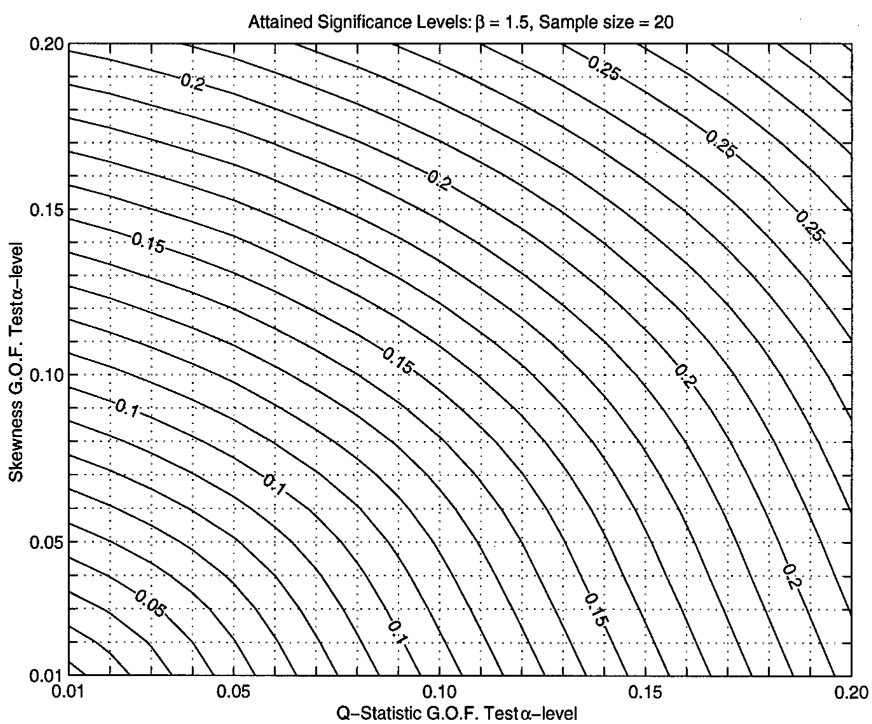
(a) Sample Size = 5



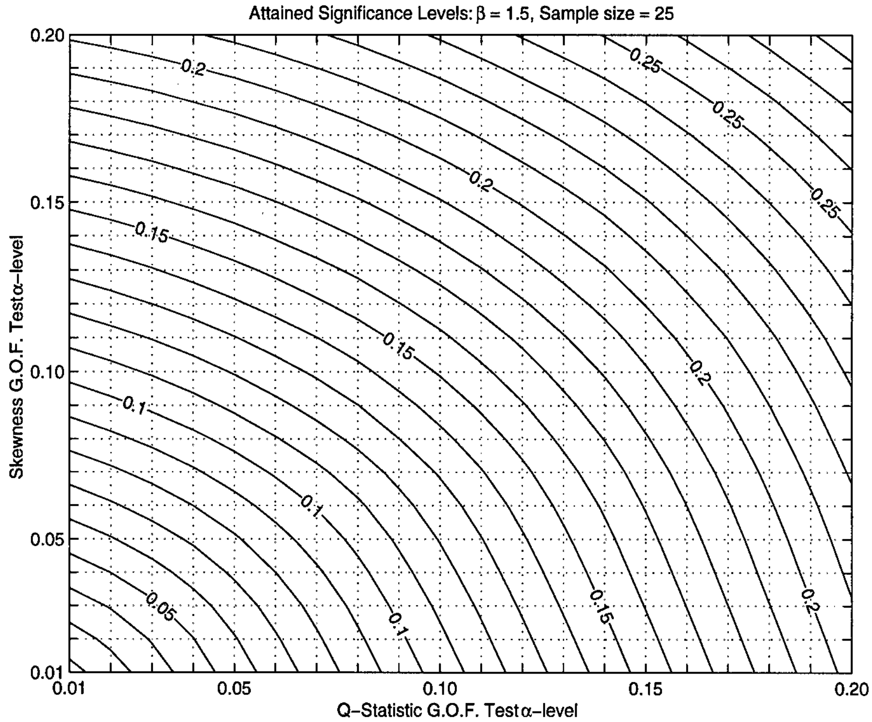
(b) Sample Size = 10



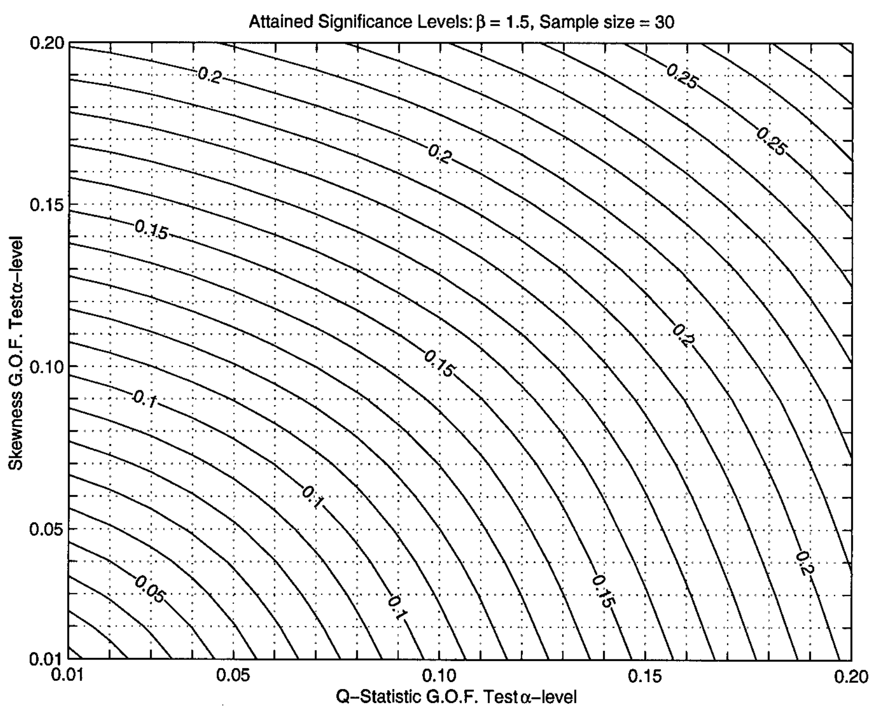
(c) Sample Size = 15



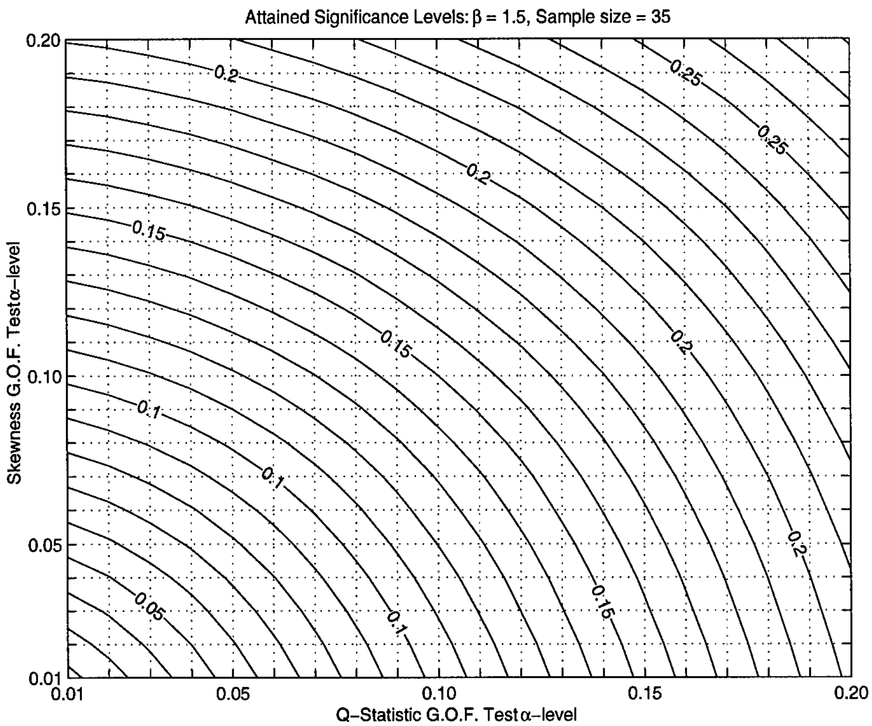
(d) Sample Size = 20



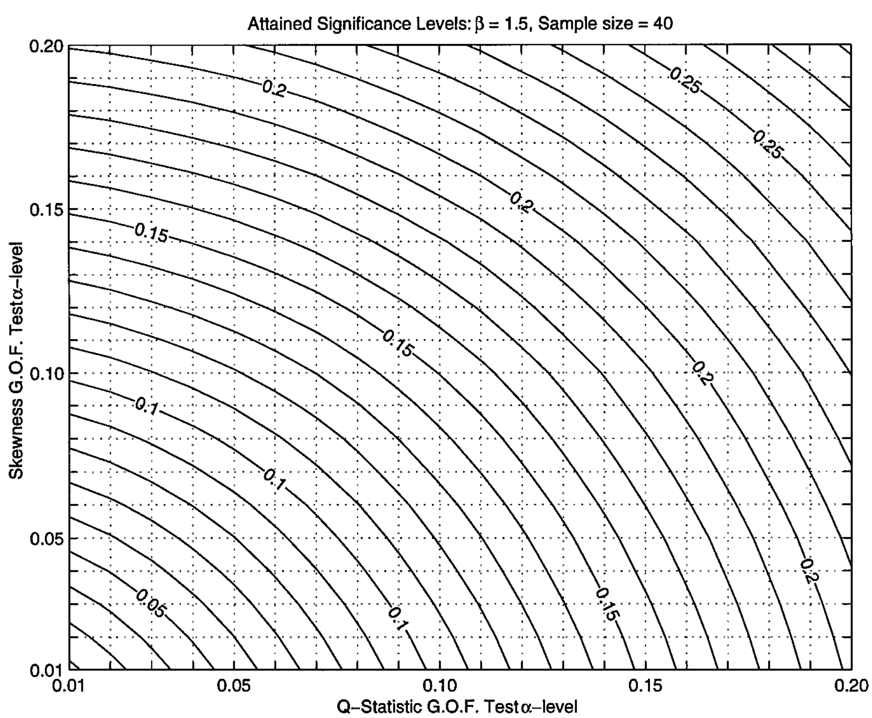
(e) Sample Size = 25



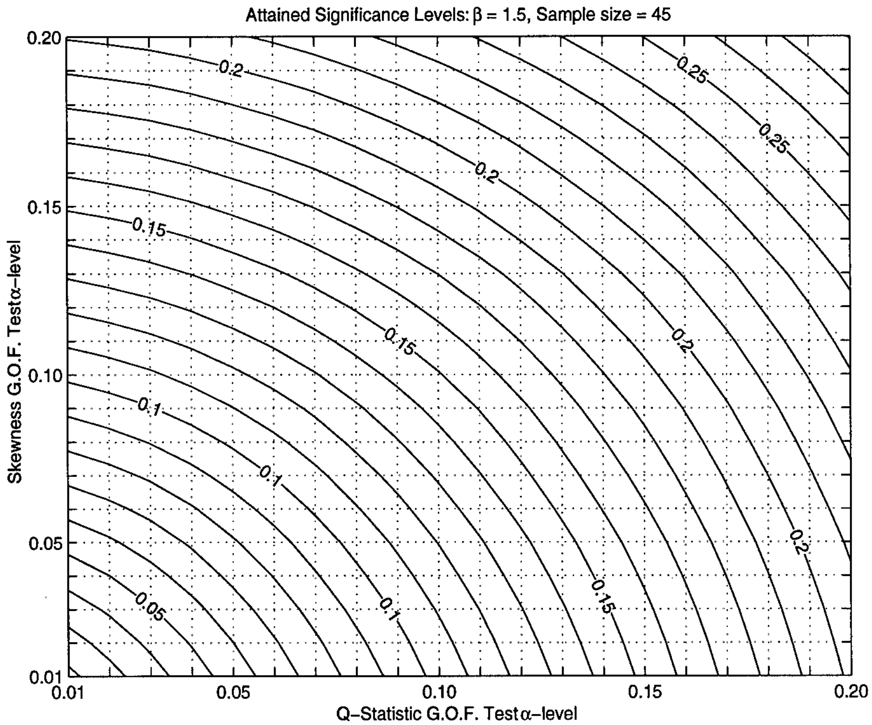
(f) Sample Size = 30



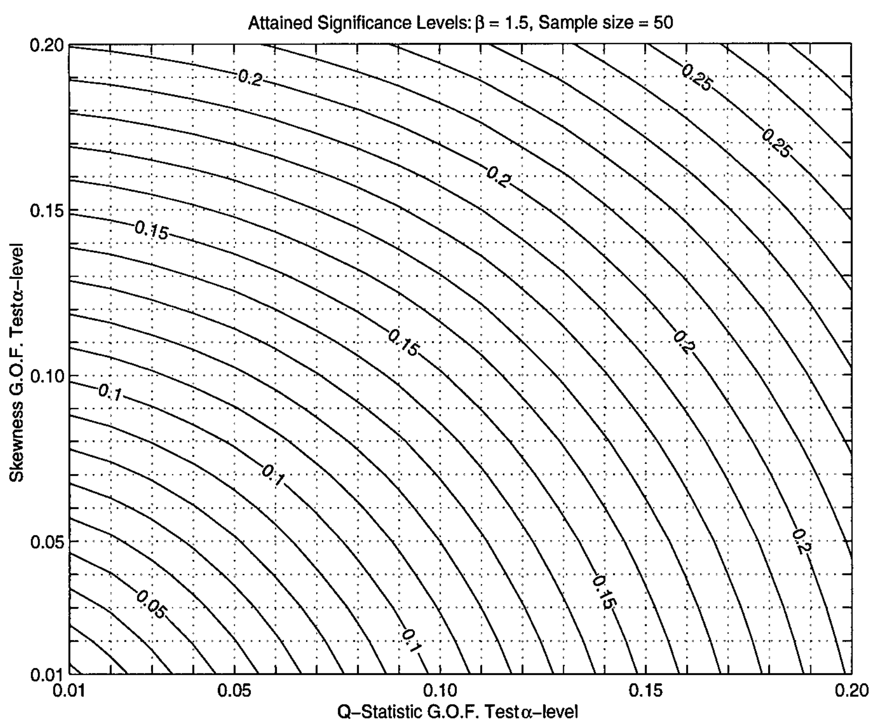
(g) Sample Size = 35



(h) Sample Size = 40

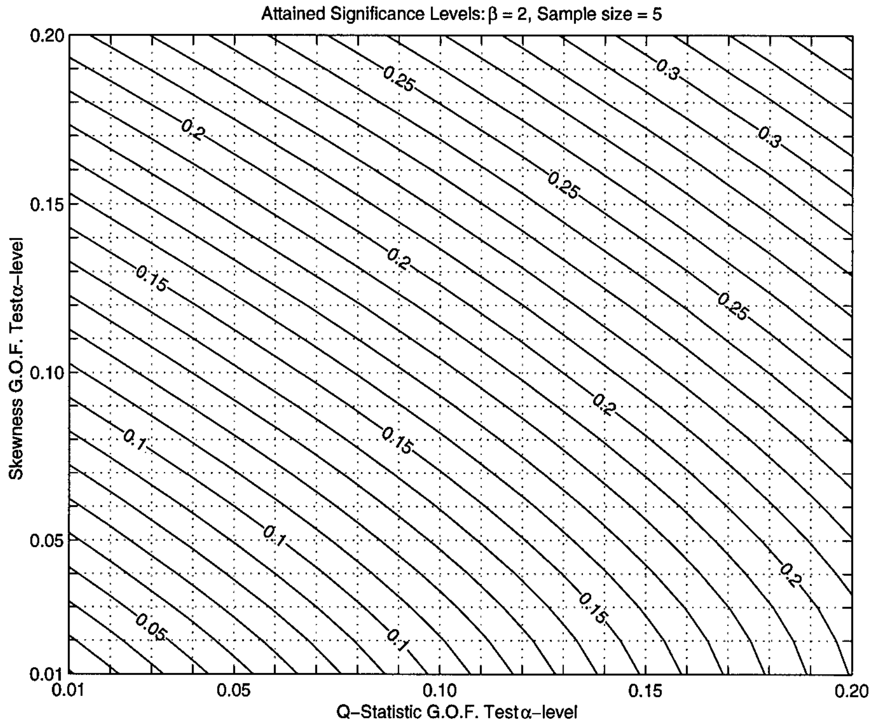


(i) Sample Size = 45

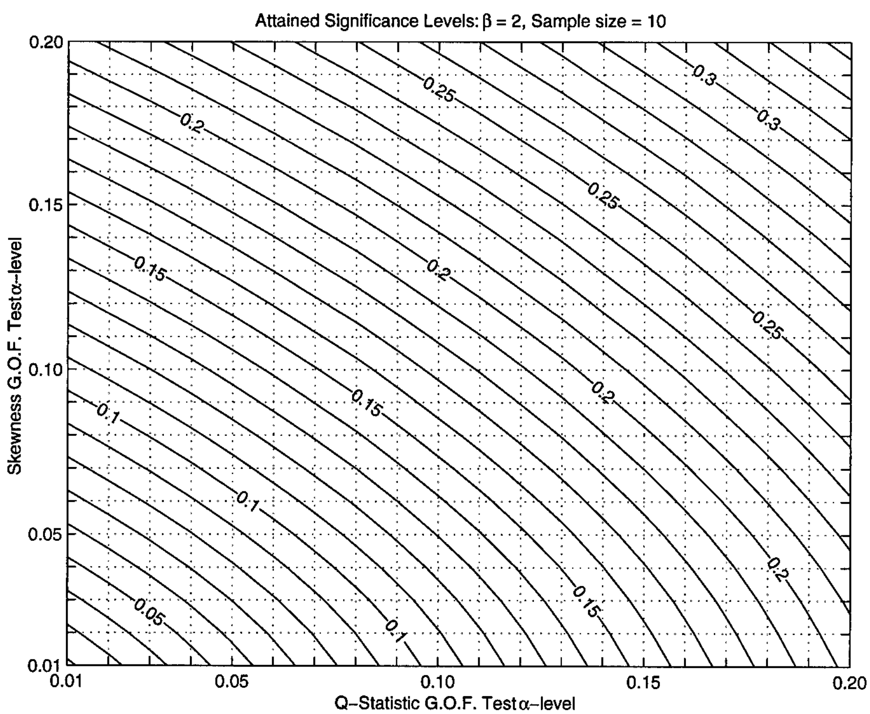


(j) Sample Size = 50

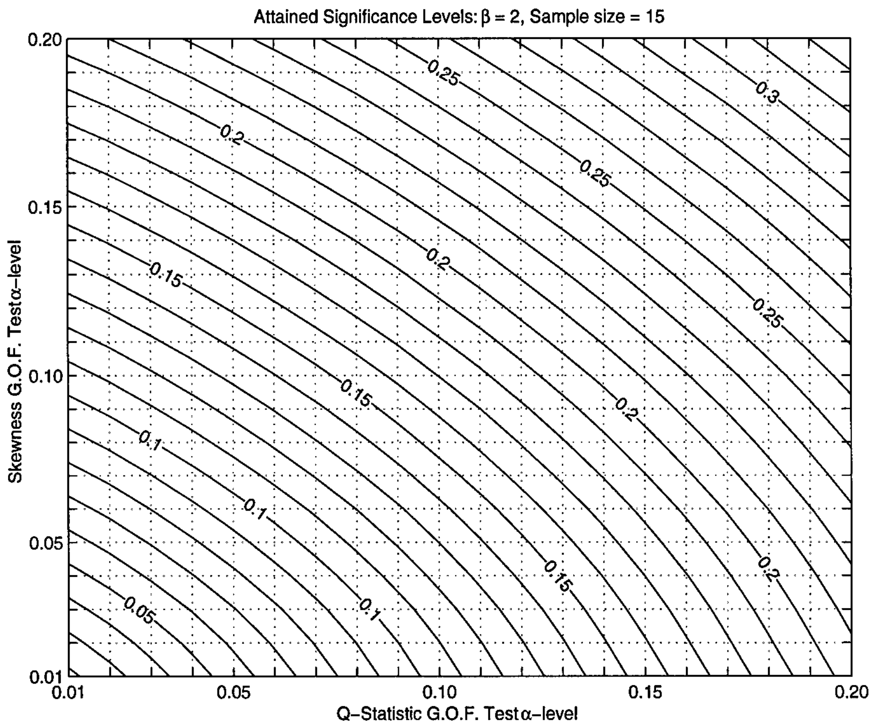
D.4 Weibull Shape $\beta = 2.0$



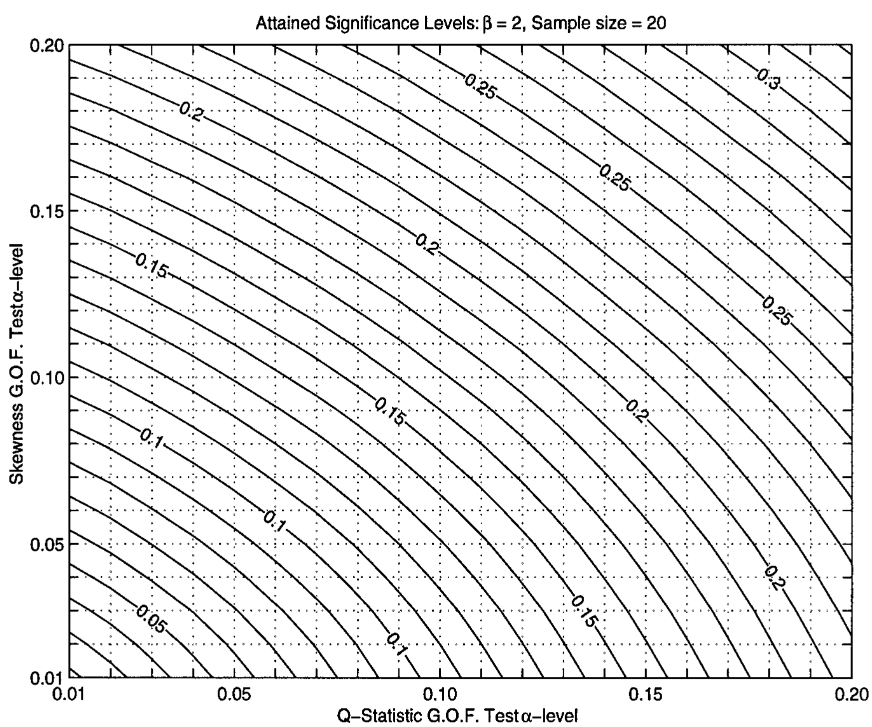
(a) Sample Size = 5



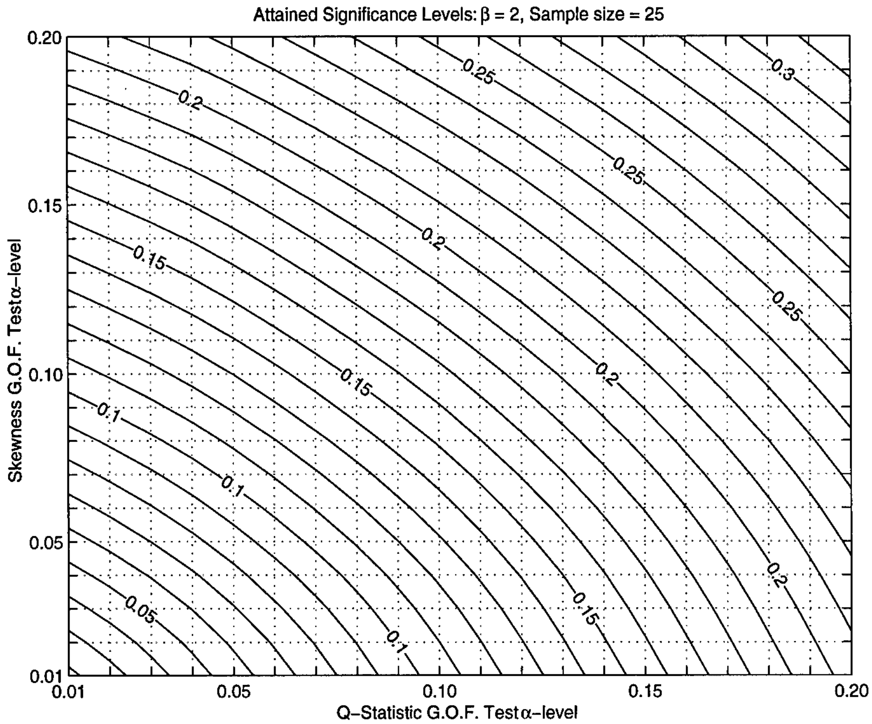
(b) Sample Size = 10



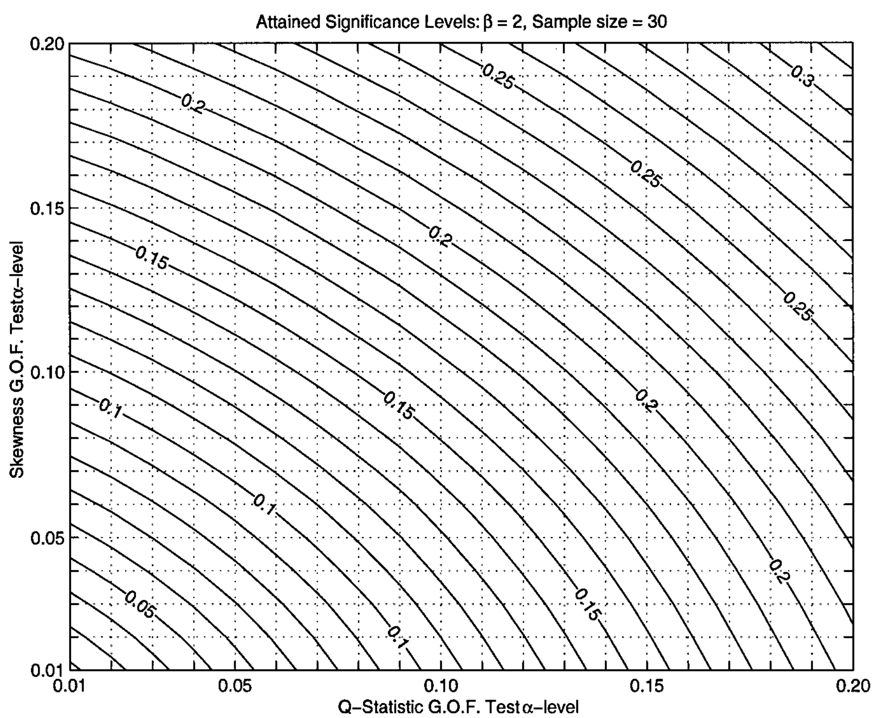
(c) Sample Size = 15



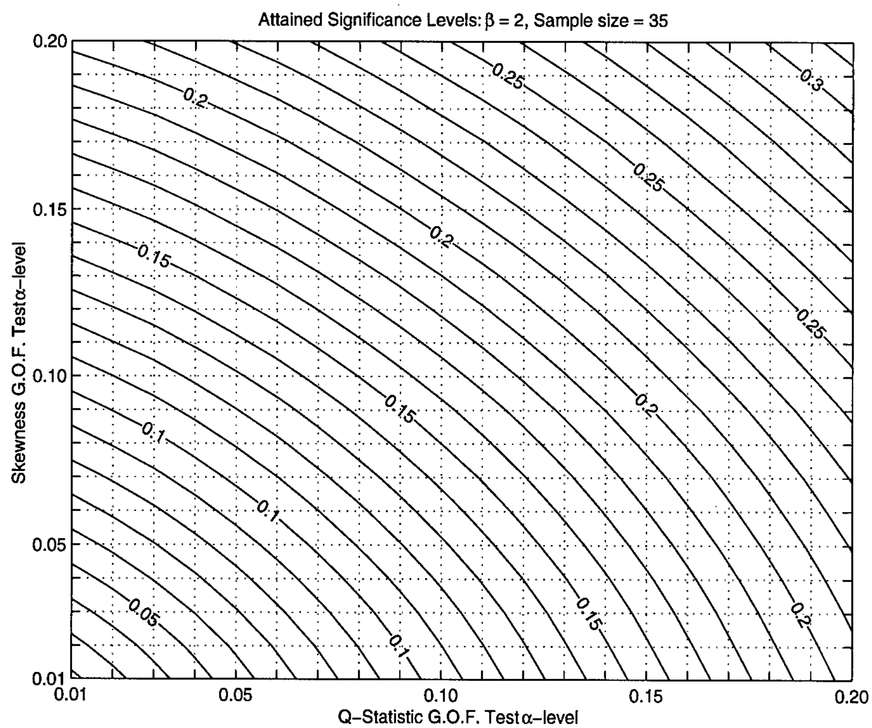
(d) Sample Size = 20



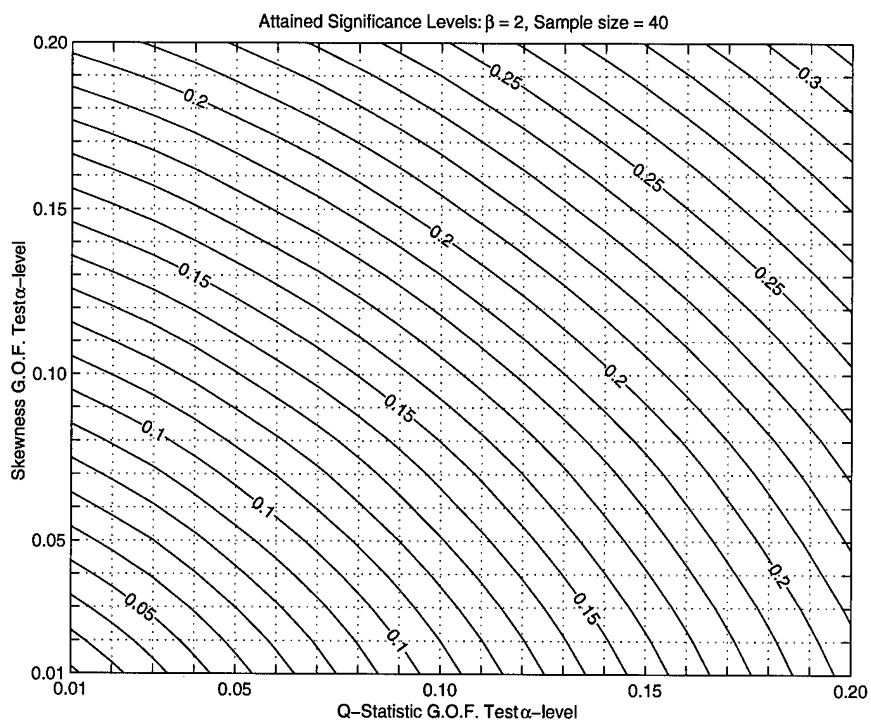
(e) Sample Size = 25



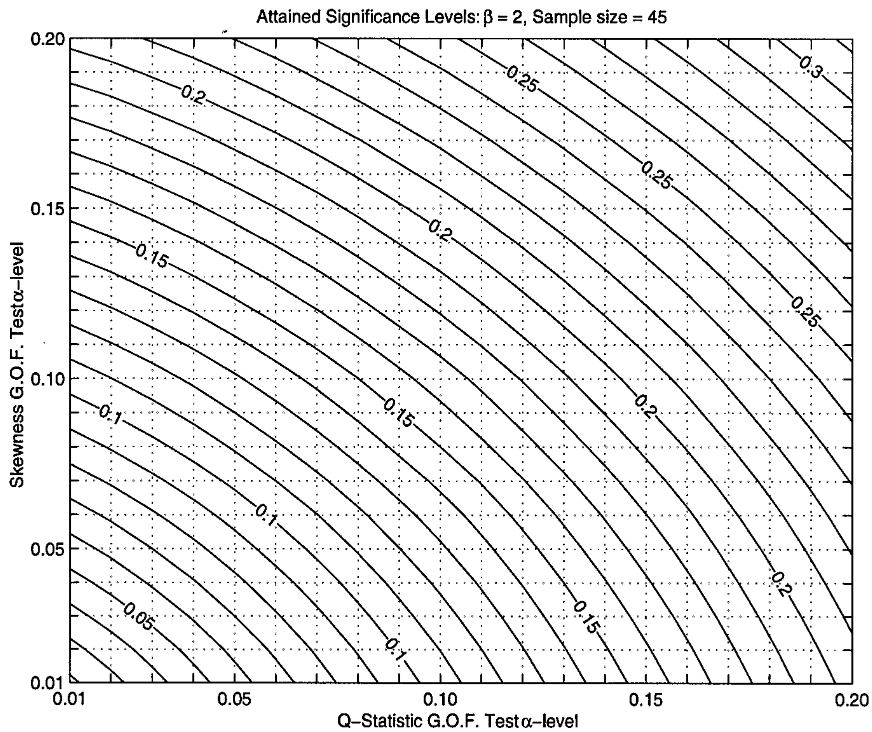
(f) Sample Size = 30



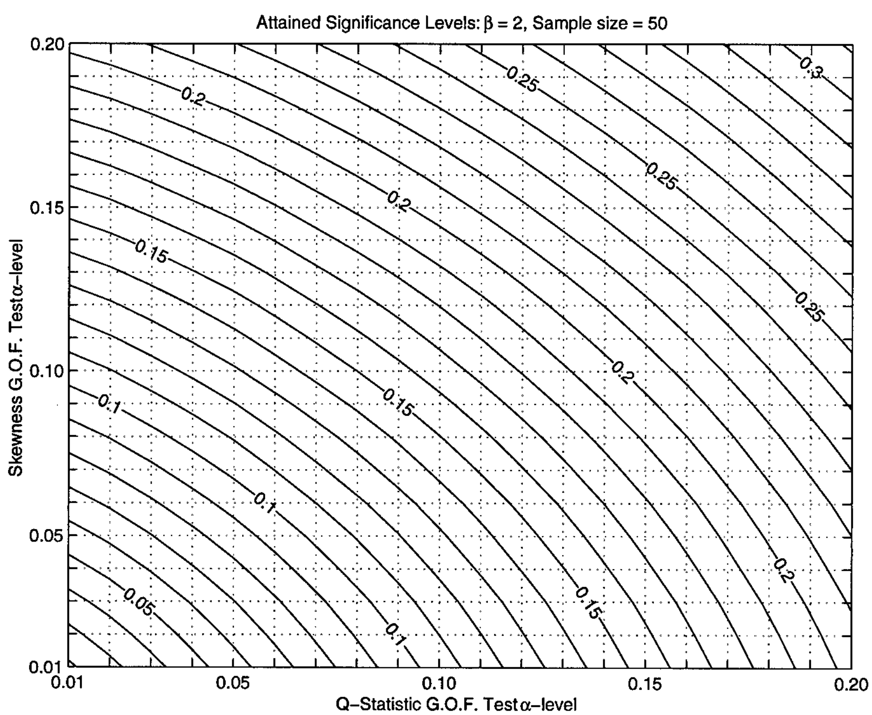
(g) Sample Size = 35



(h) Sample Size = 40

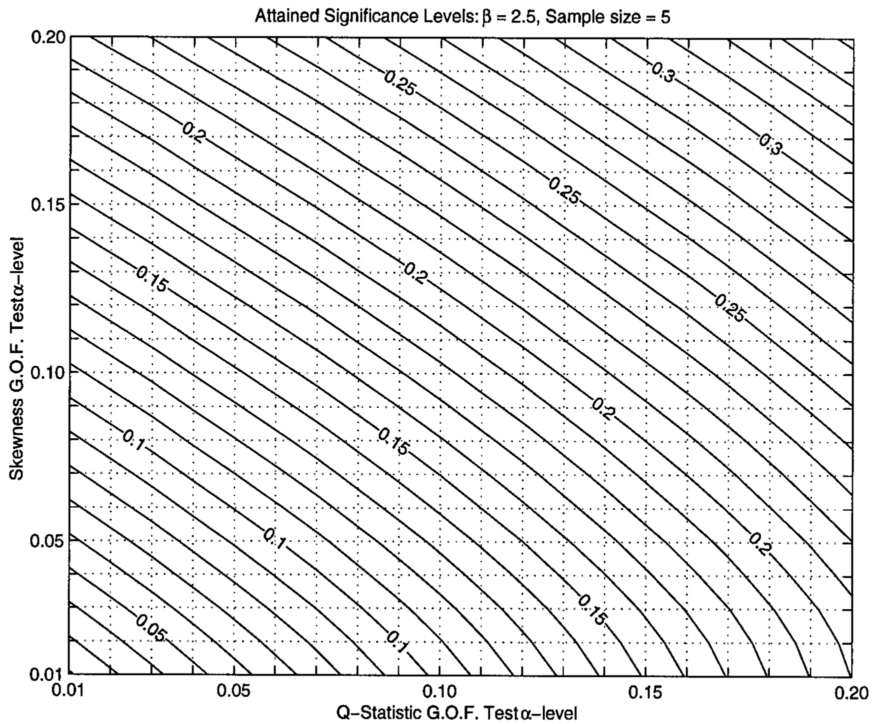


(i) Sample Size = 45

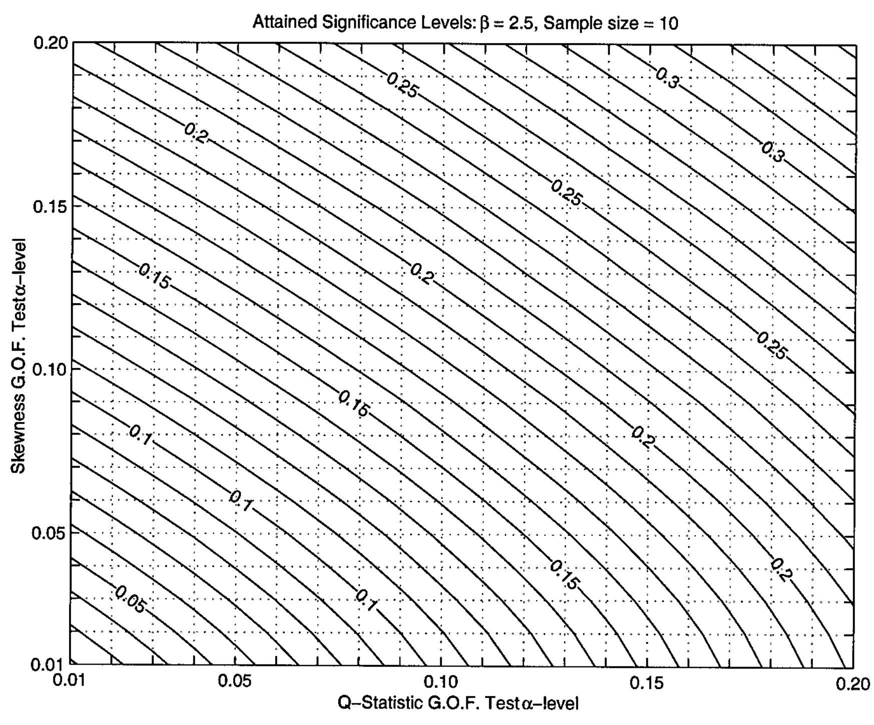


(j) Sample Size = 50

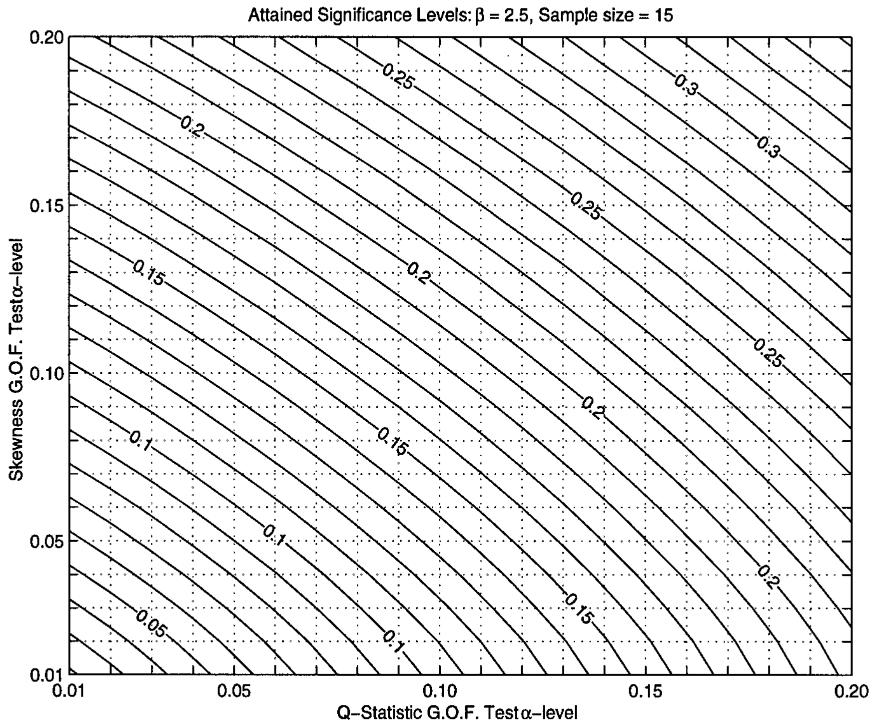
D.5 Weibull Shape $\beta = 2.5$



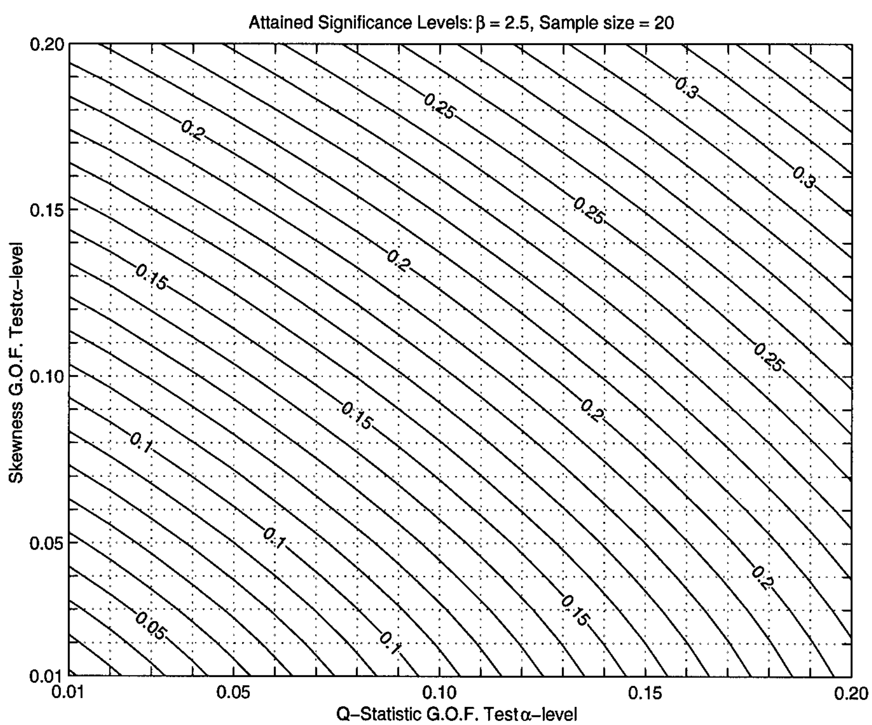
(a) Sample Size = 5



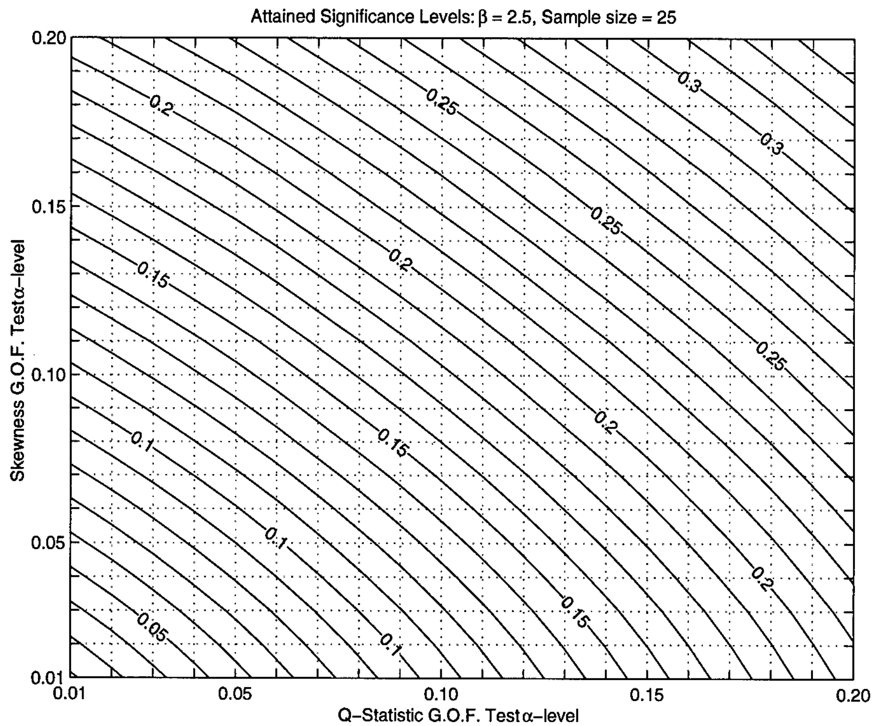
(b) Sample Size = 10



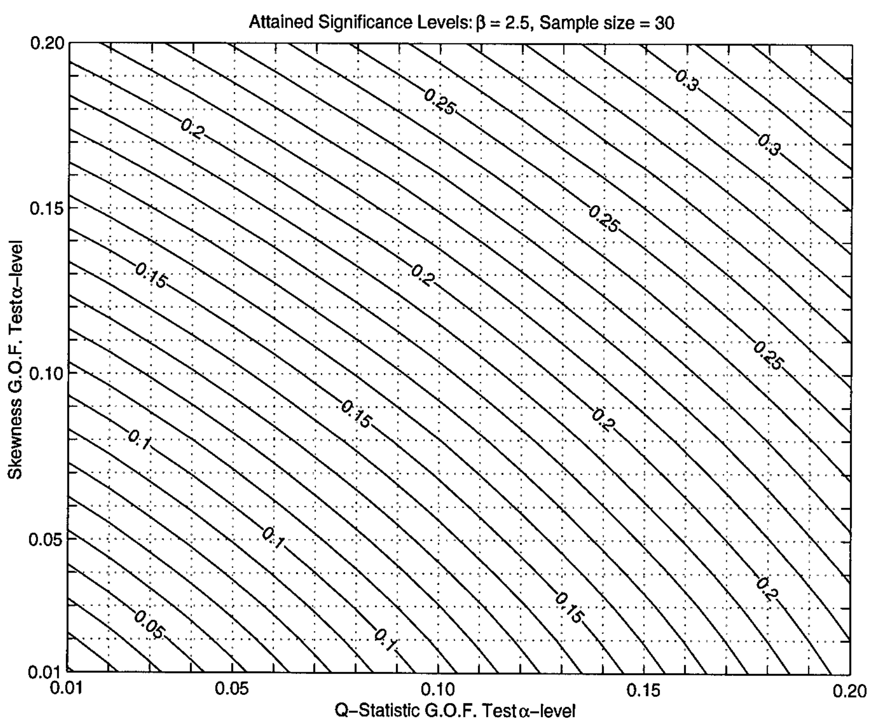
(c) Sample Size = 15



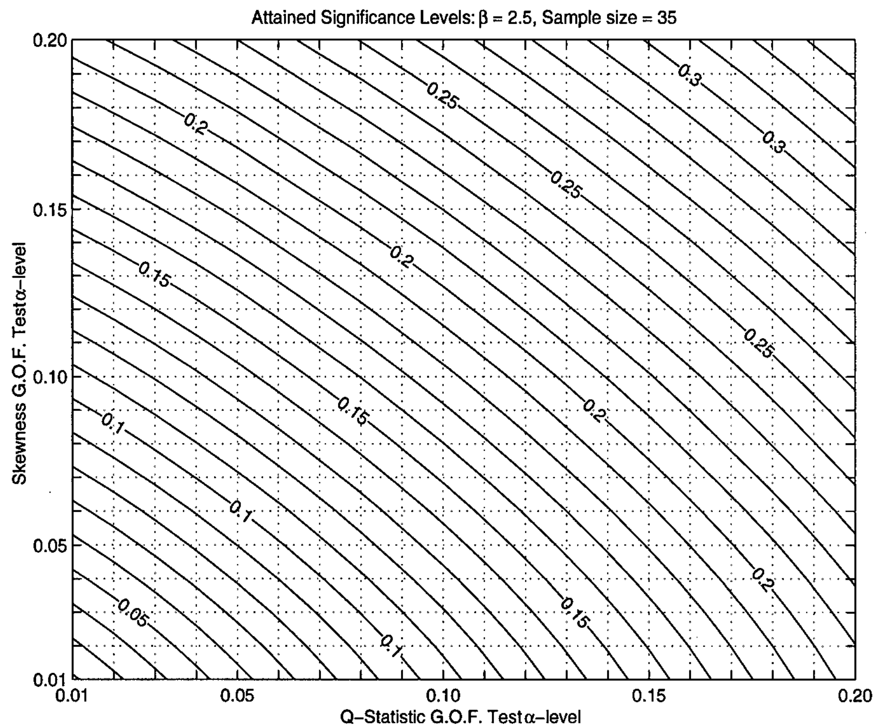
(d) Sample Size = 20



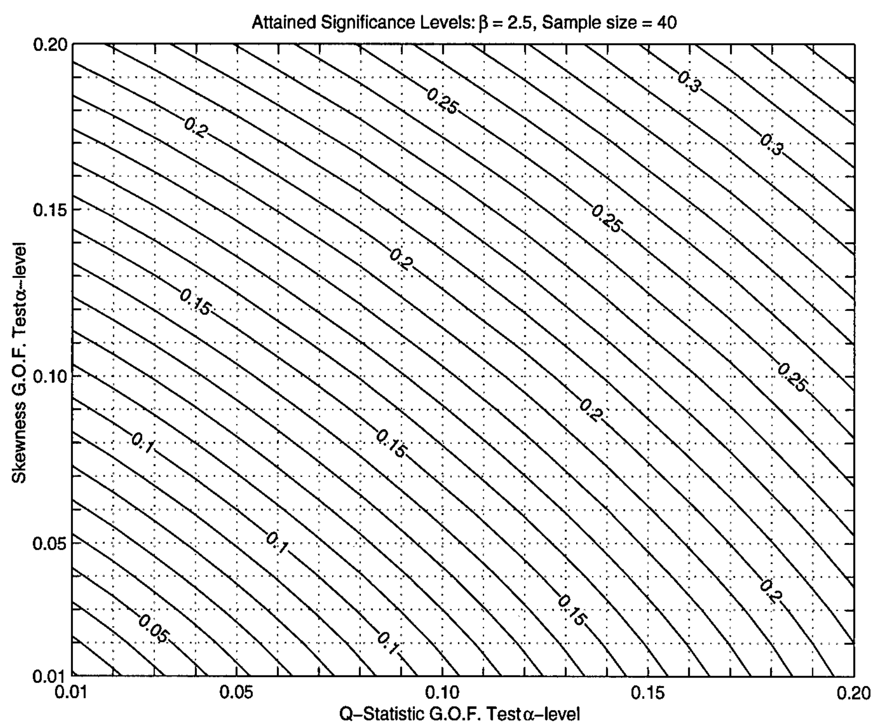
(e) Sample Size = 25



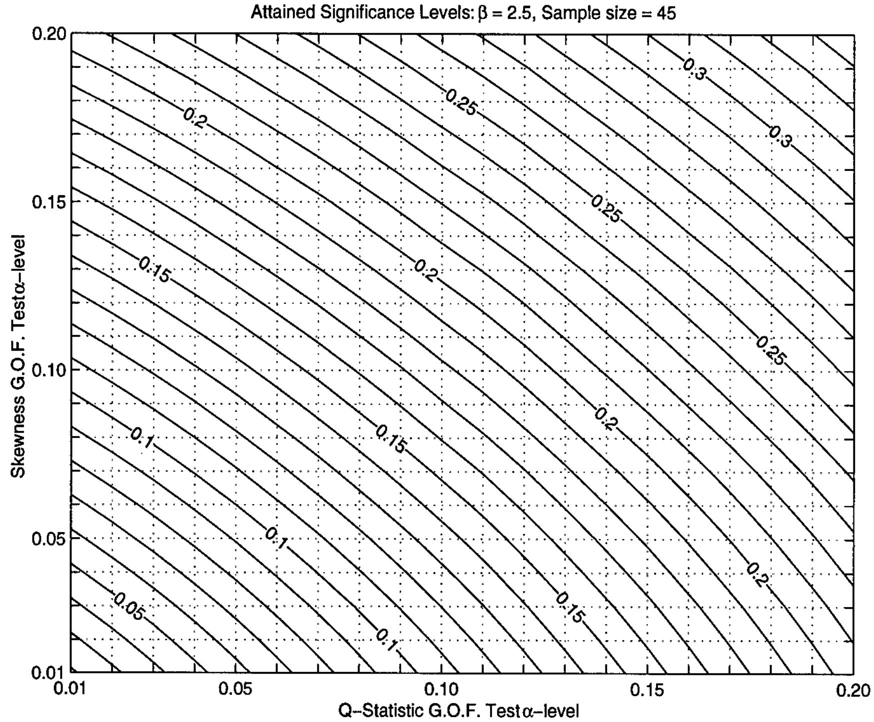
(f) Sample Size = 30



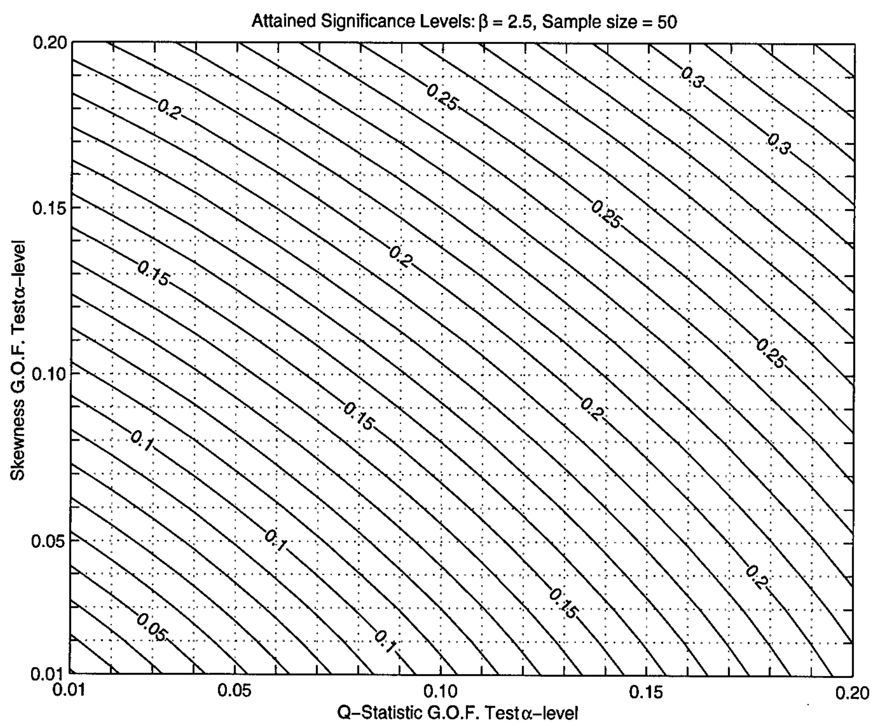
(g) Sample Size = 35



(h) Sample Size = 40

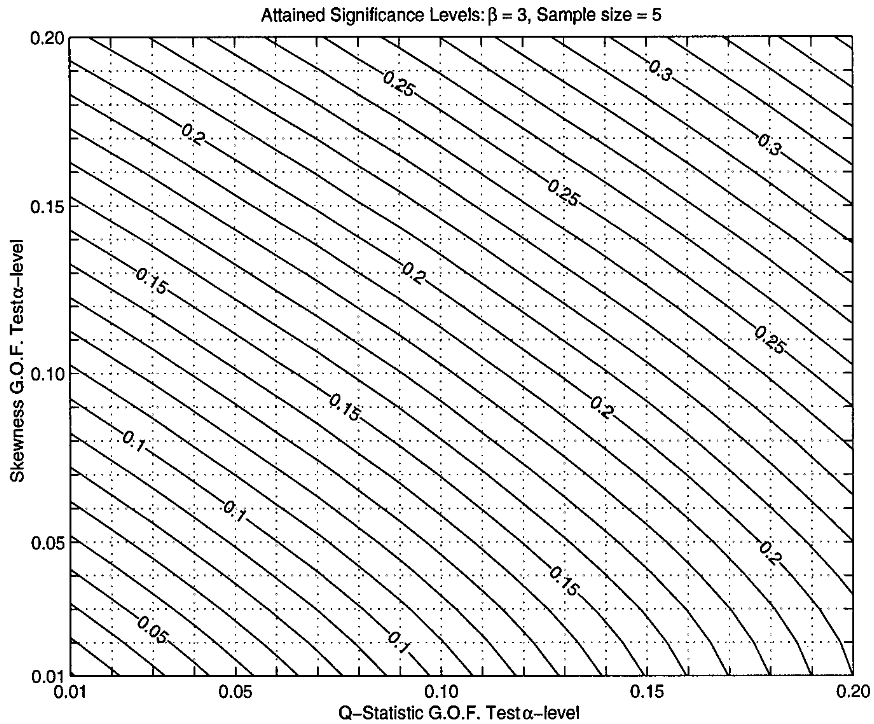


(i) Sample Size = 45

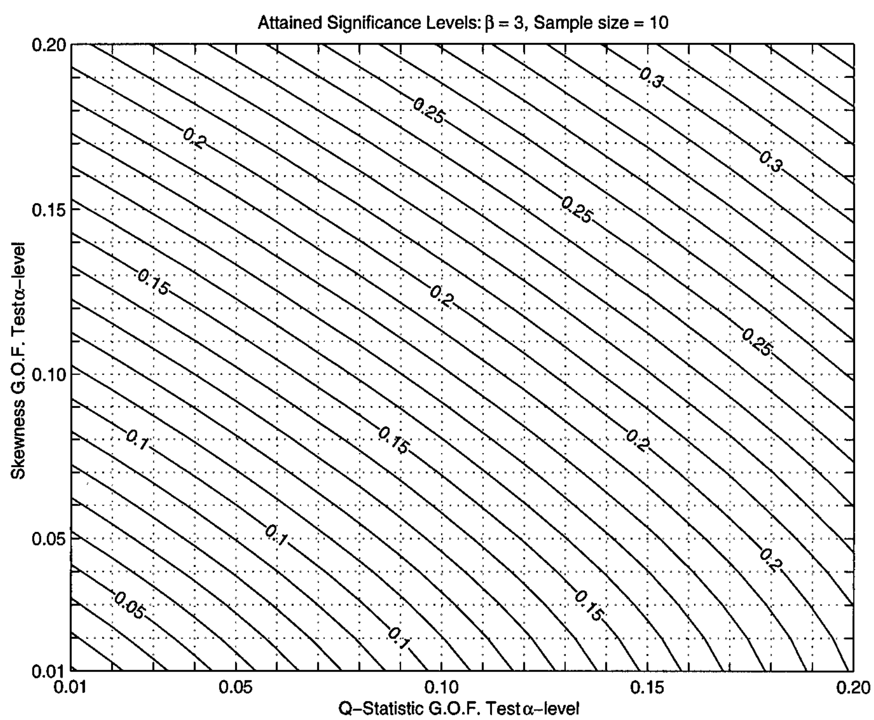


(j) Sample Size = 50

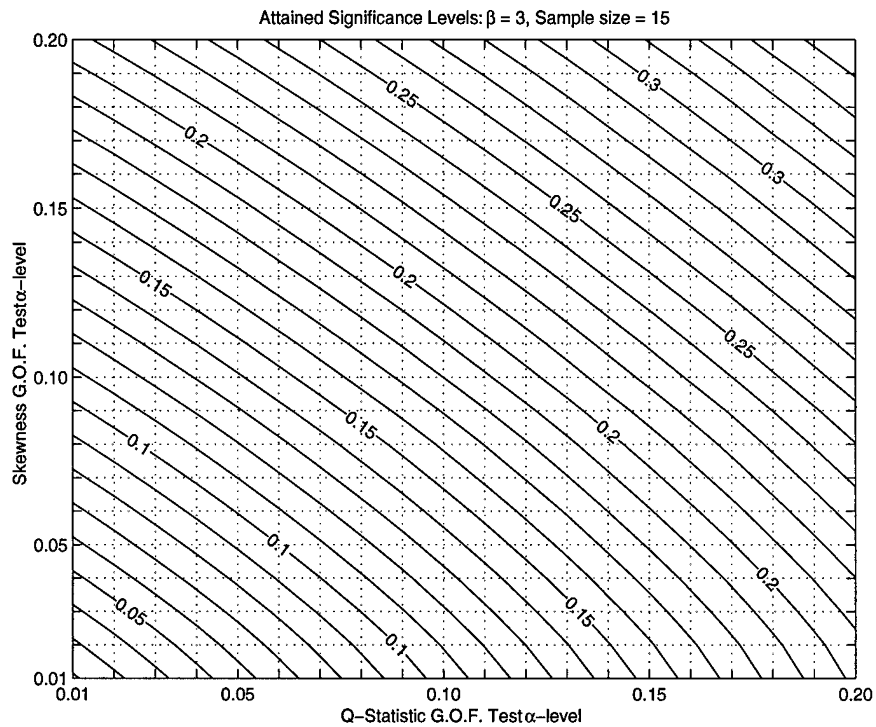
D.6 Weibull Shape $\beta = 3.0$



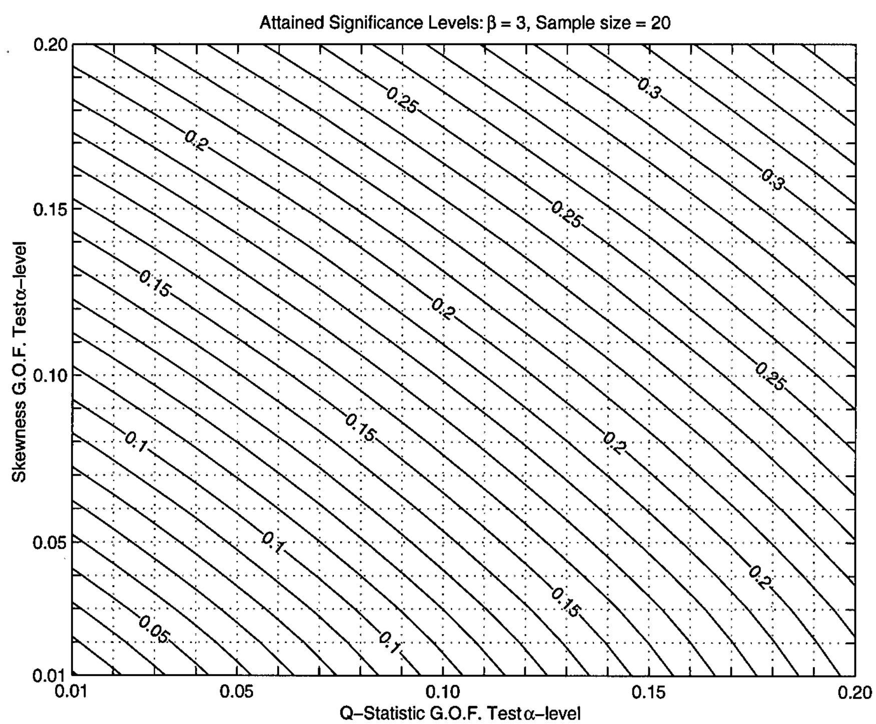
(a) Sample Size = 5



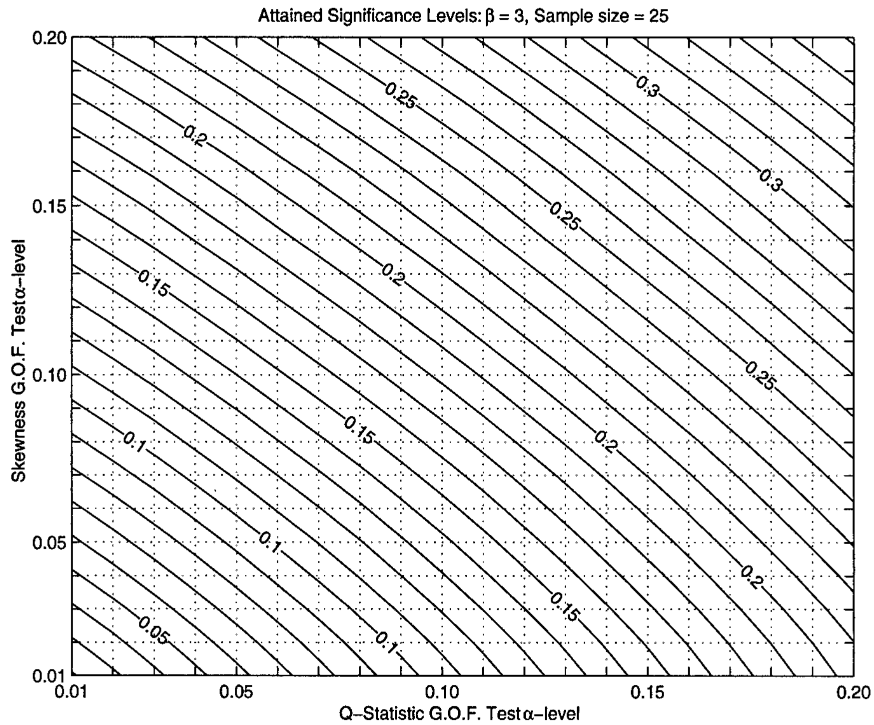
(b) Sample Size = 10



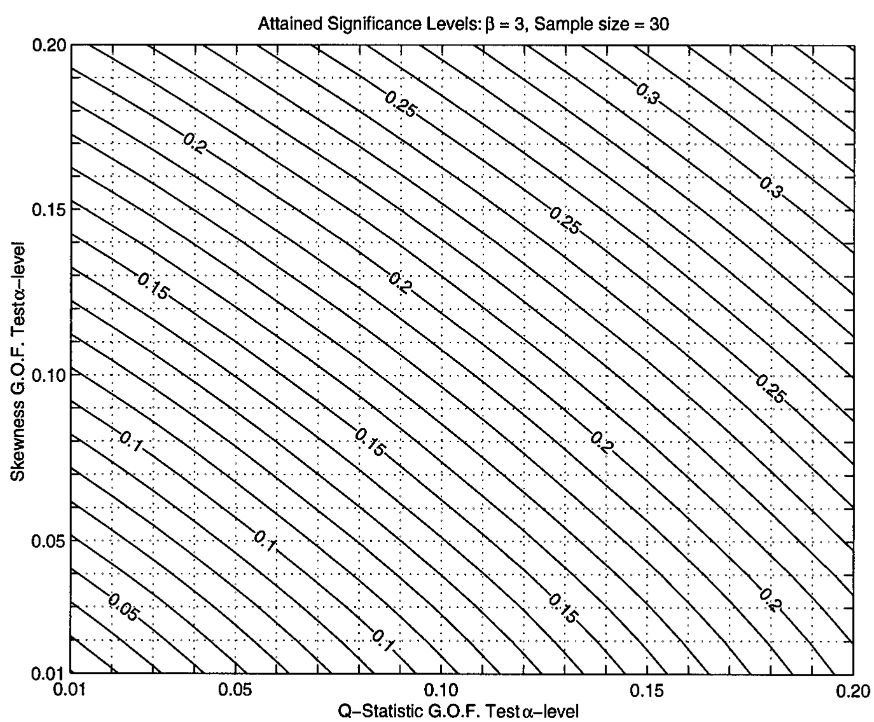
(c) Sample Size = 15



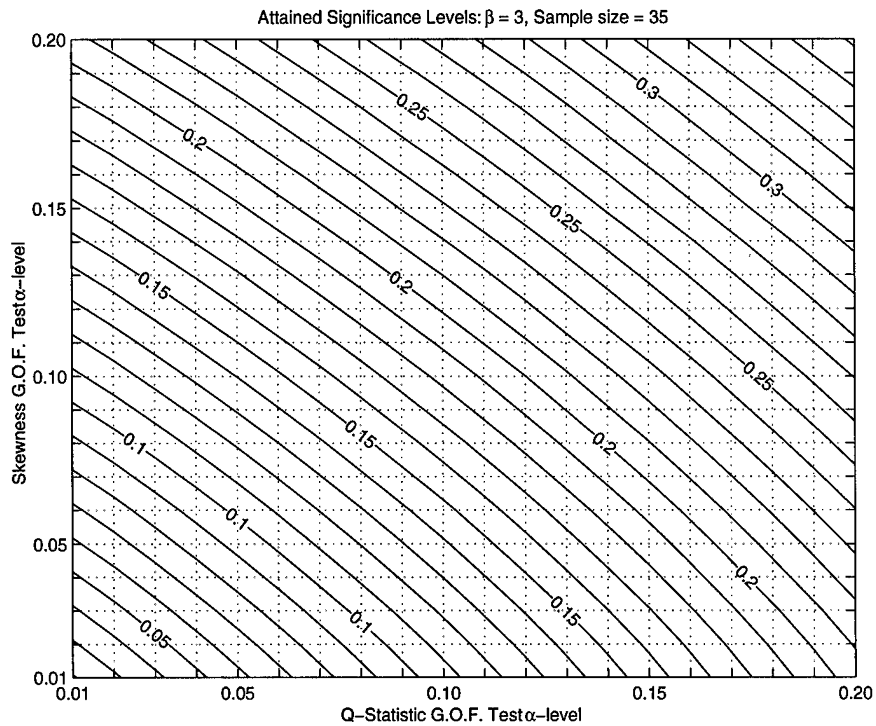
(d) Sample Size = 20



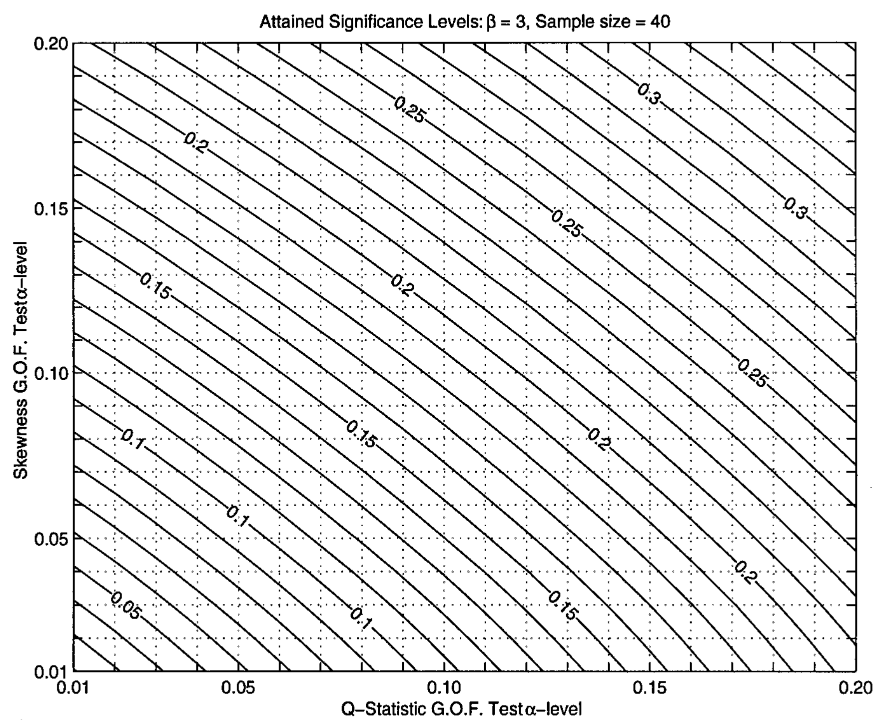
(e) Sample Size = 25



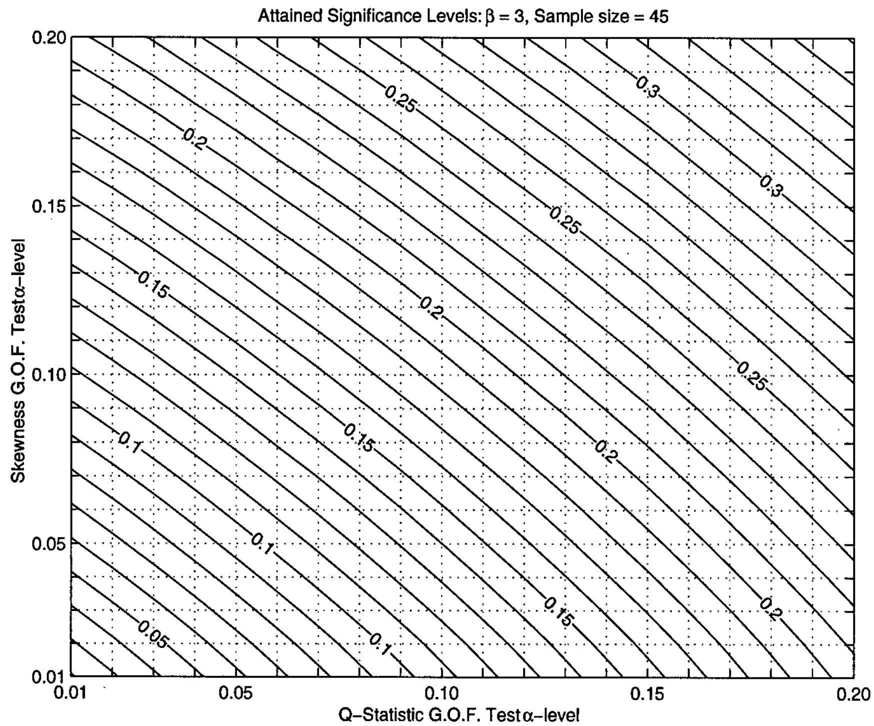
(f) Sample Size = 30



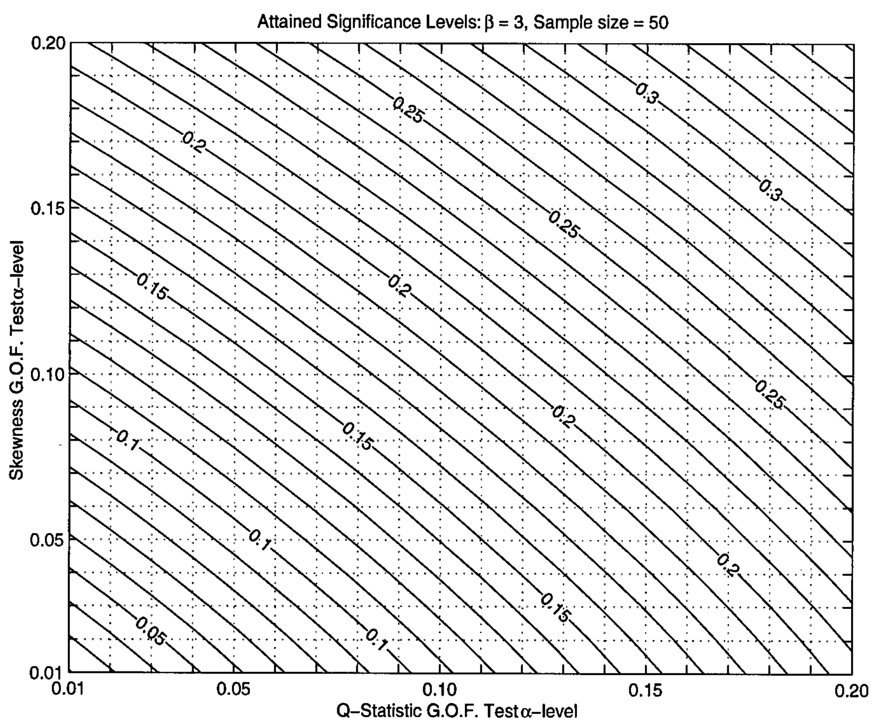
(g) Sample Size = 35



(h) Sample Size = 40

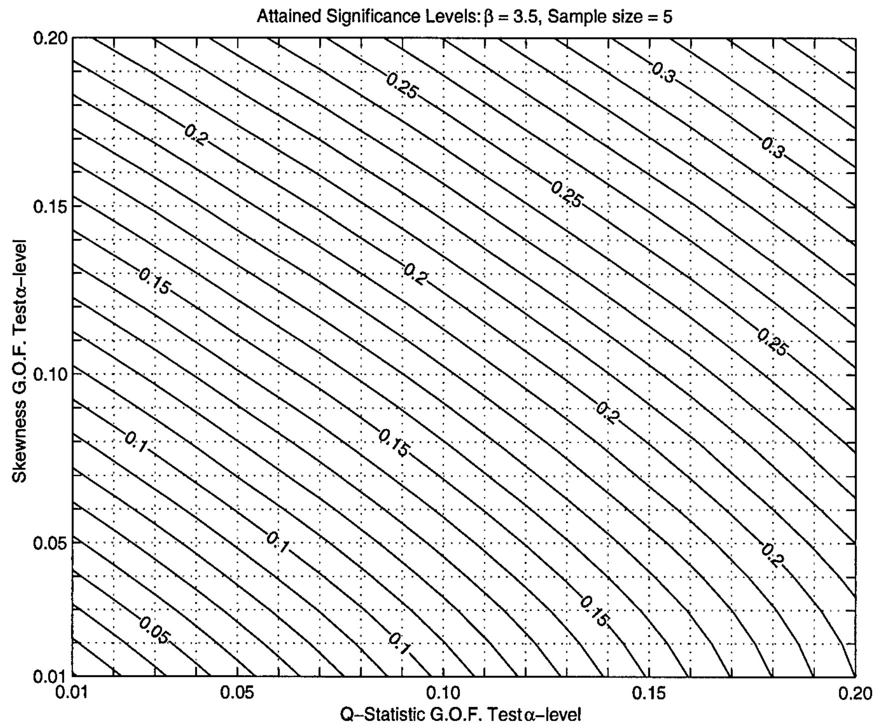


(i) Sample Size = 45

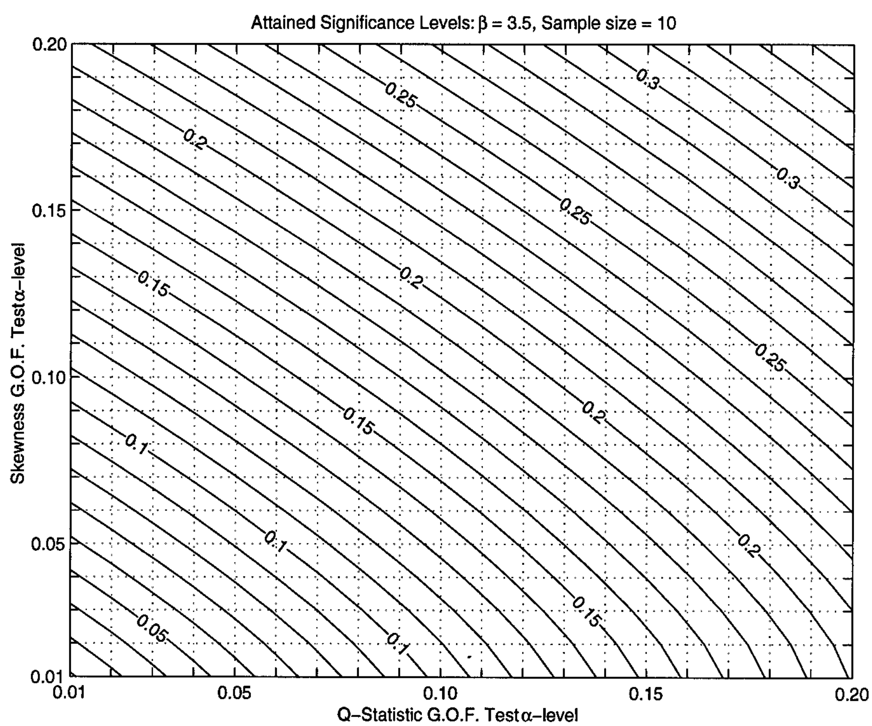


(j) Sample Size = 50

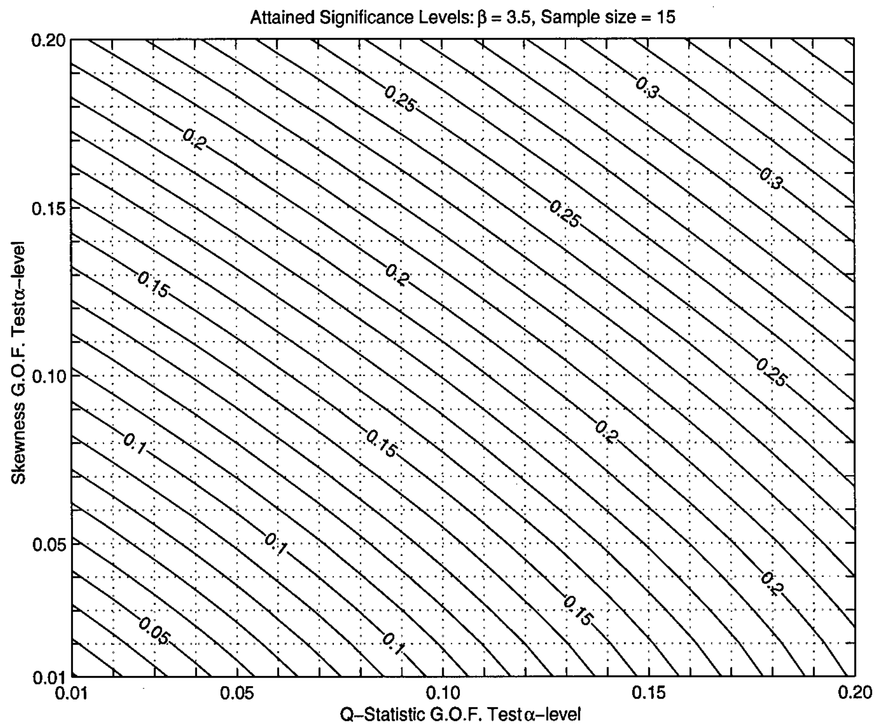
D.7 Weibull Shape $\beta = 3.5$



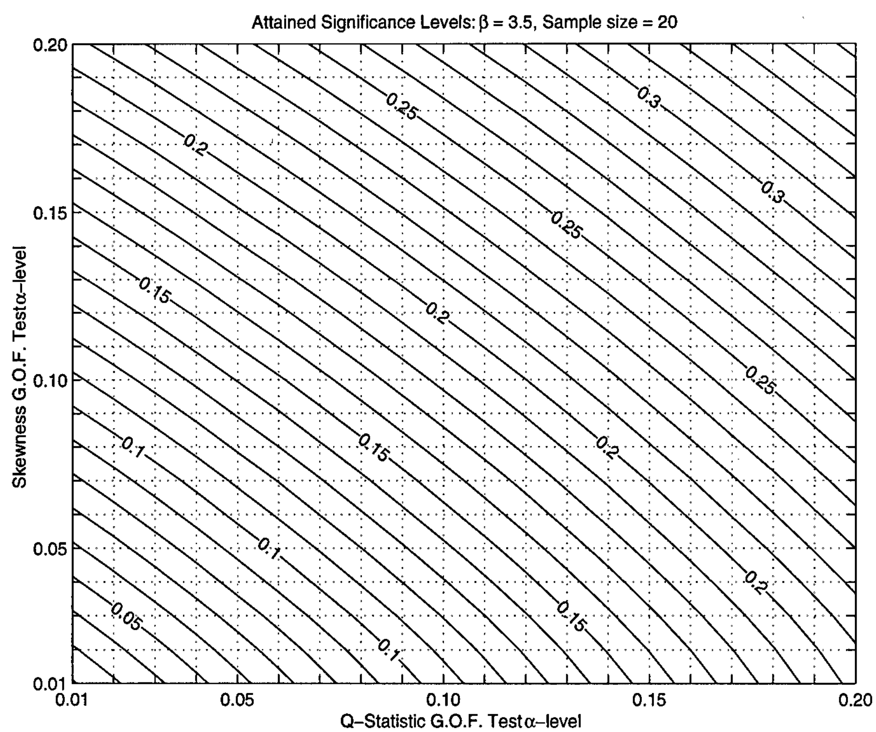
(a) Sample Size = 5



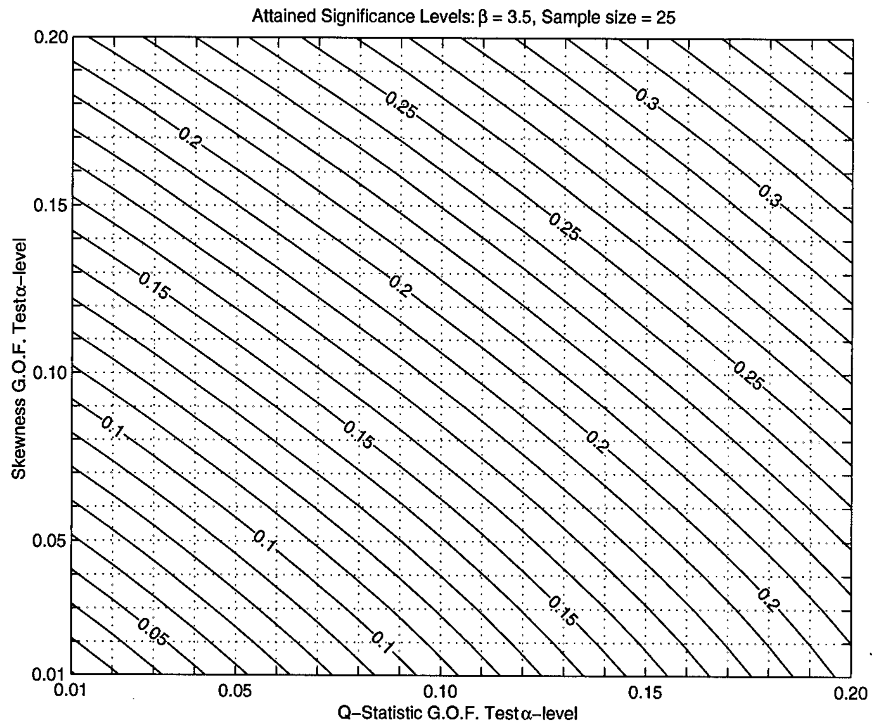
(b) Sample Size = 10



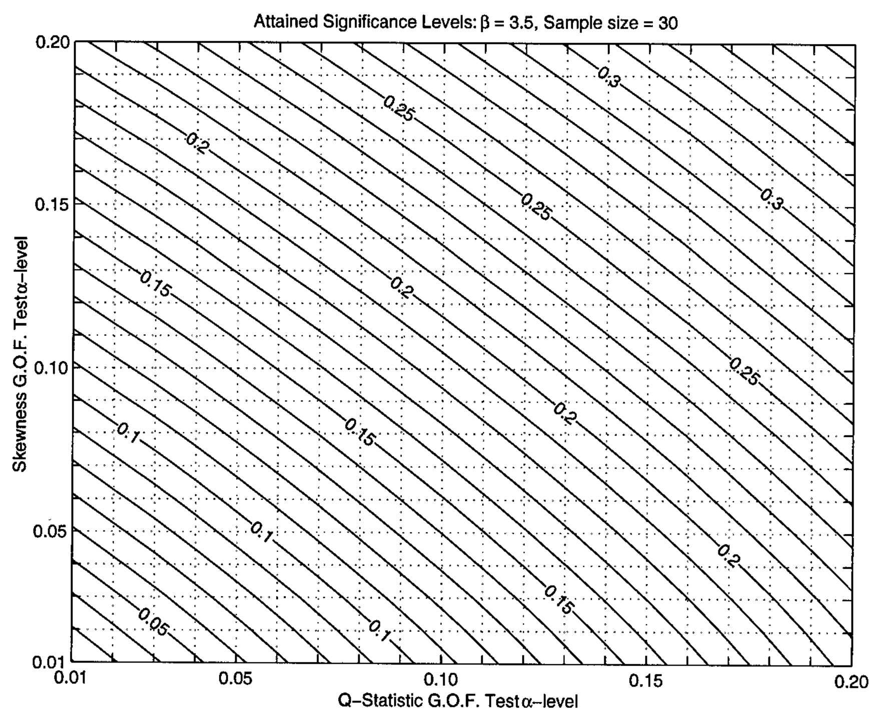
(c) Sample Size = 15



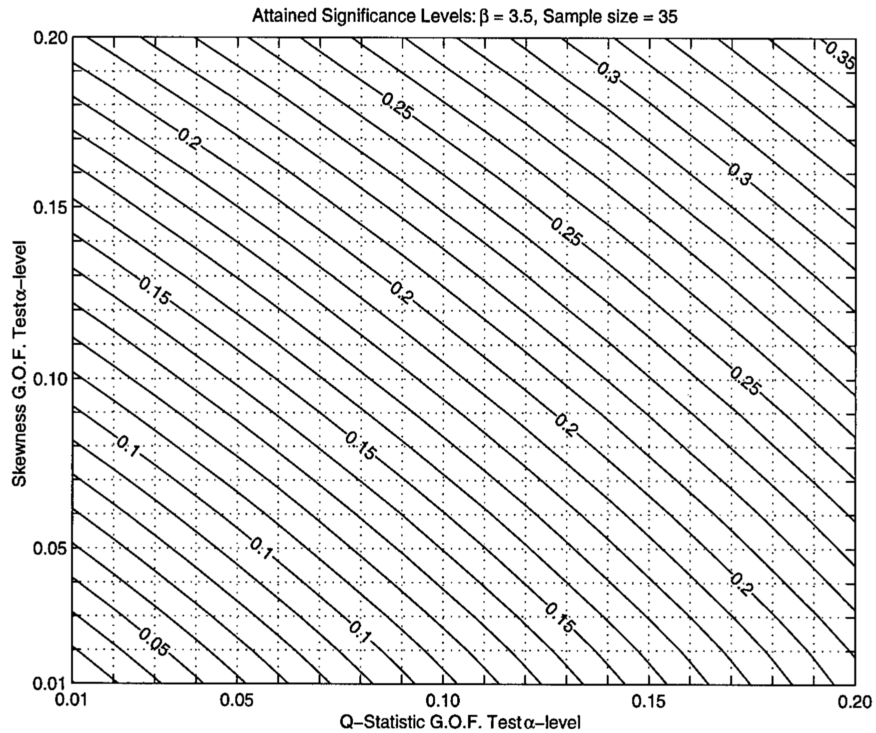
(d) Sample Size = 20



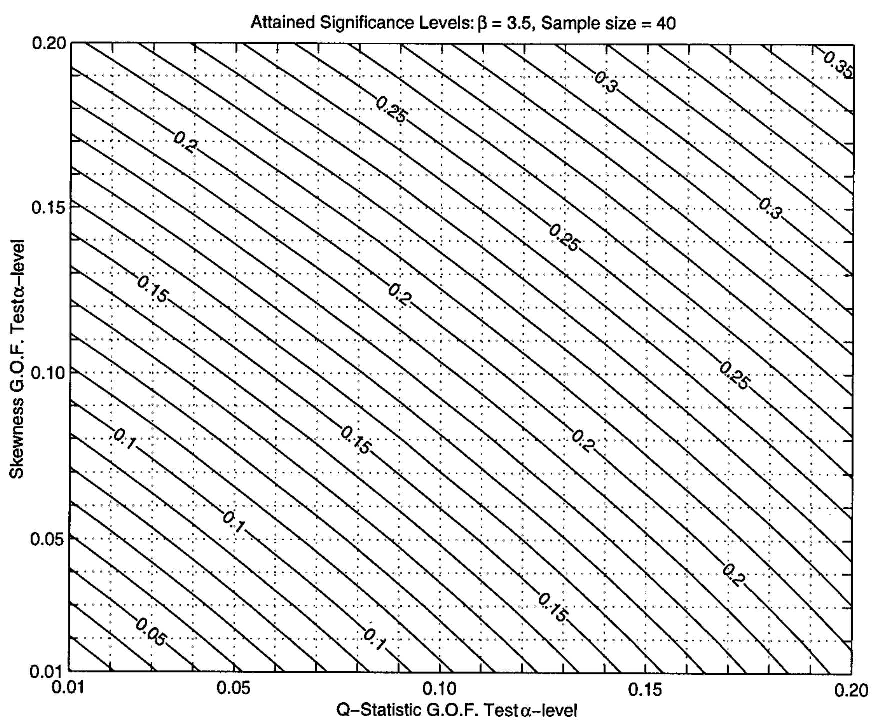
(e) Sample Size = 25



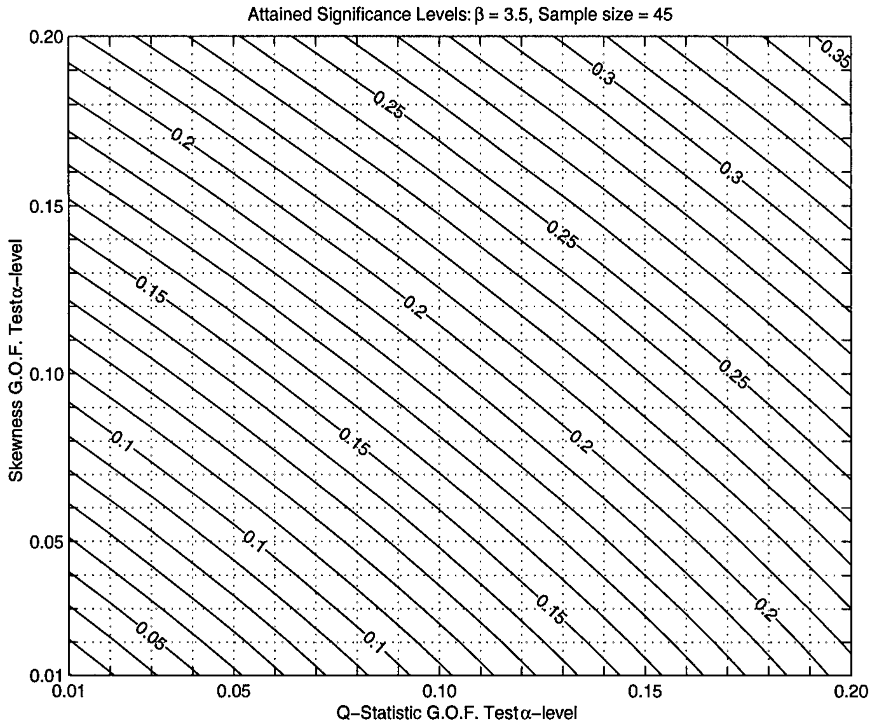
(f) Sample Size = 30



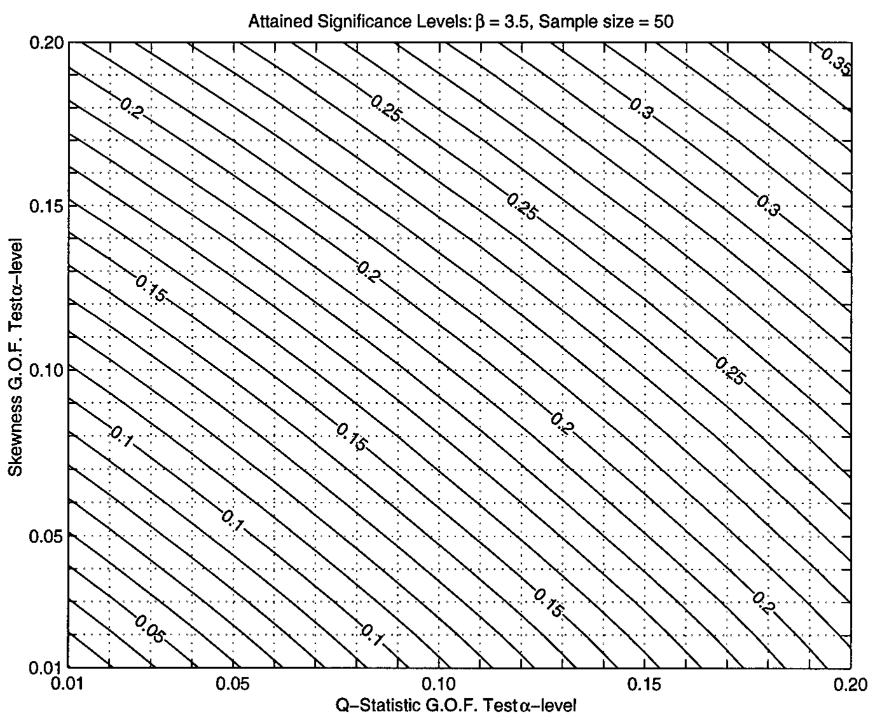
(g) Sample Size = 35



(h) Sample Size = 40

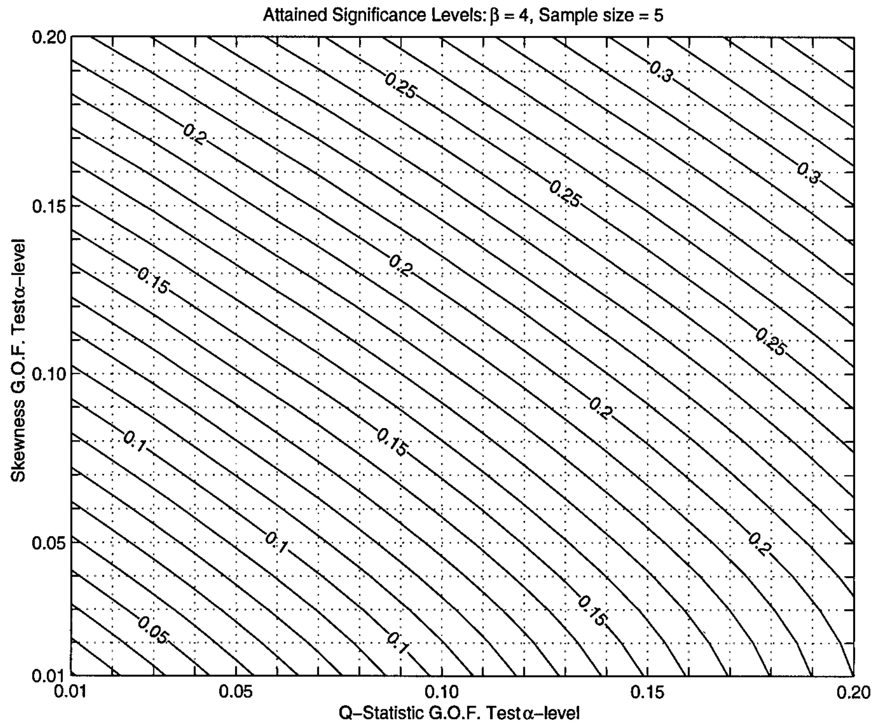


(i) Sample Size = 45

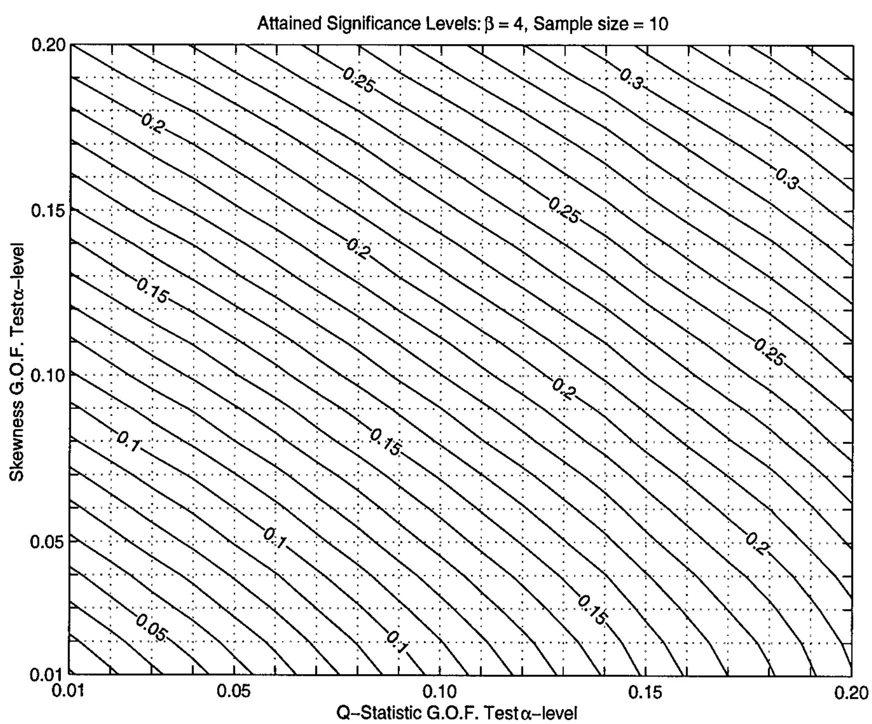


(j) Sample Size = 50

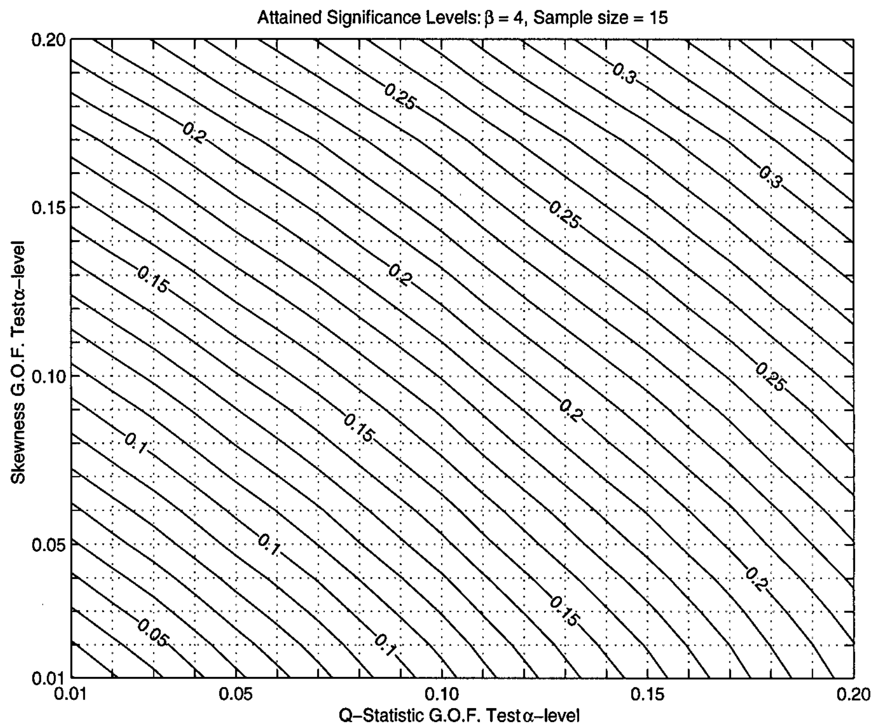
D.8 Weibull Shape $\beta = 4.0$



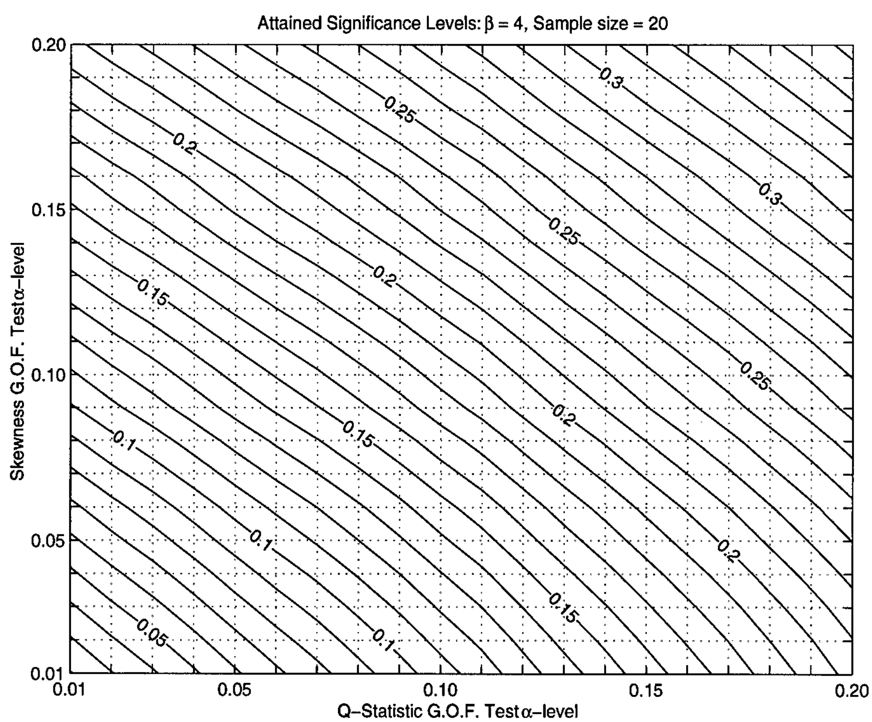
(a) Sample Size = 5



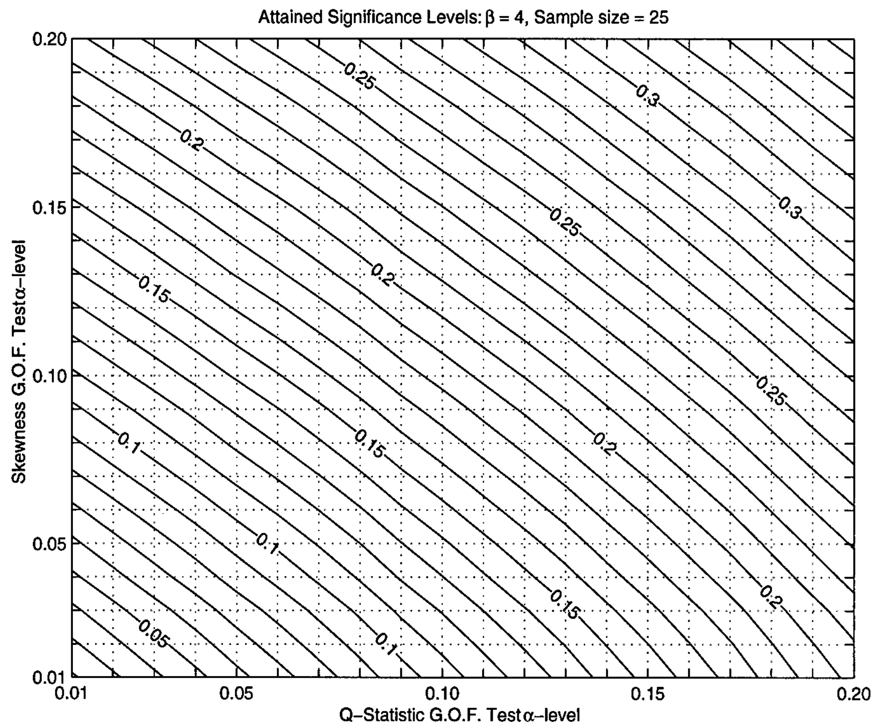
(b) Sample Size = 10



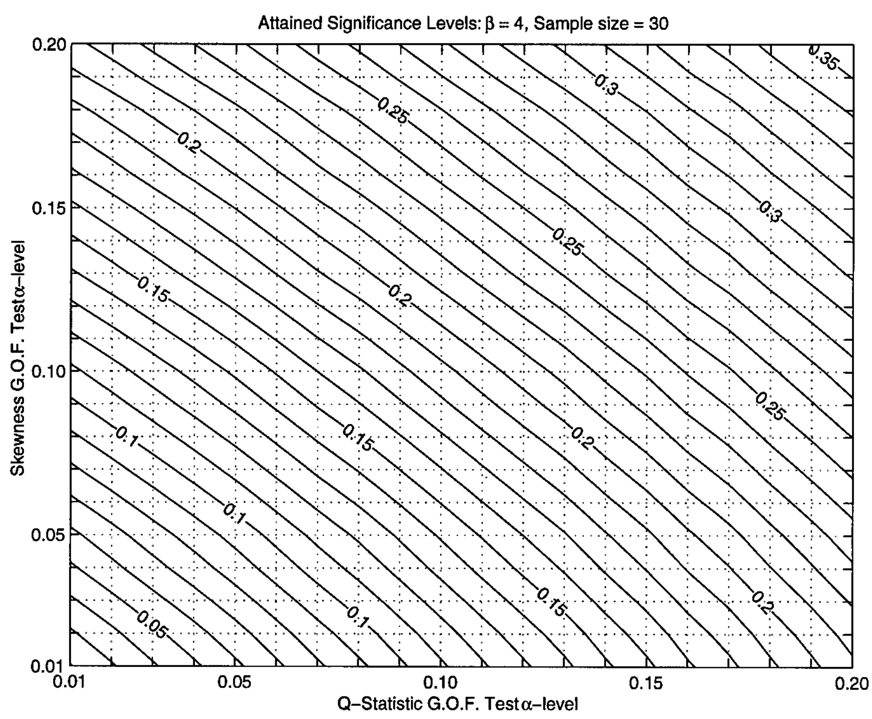
(c) Sample Size = 15



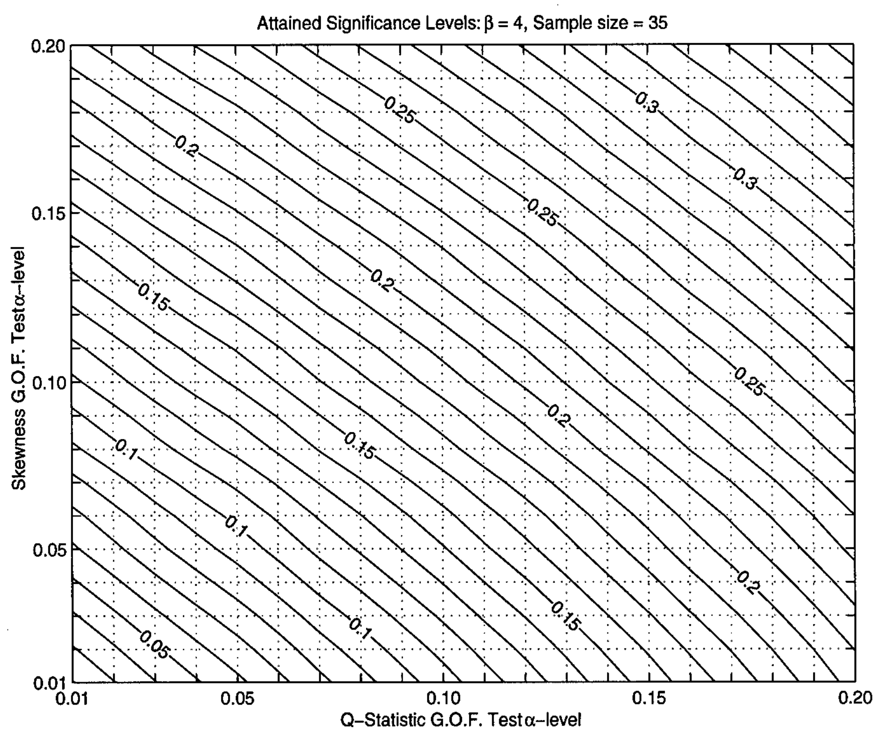
(d) Sample Size = 20



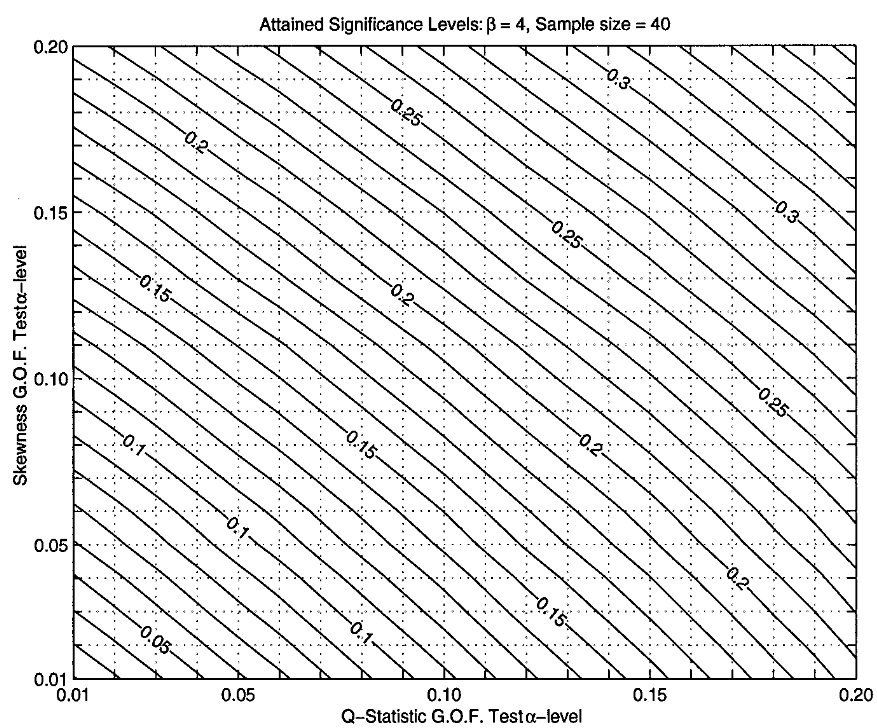
(e) Sample Size = 25



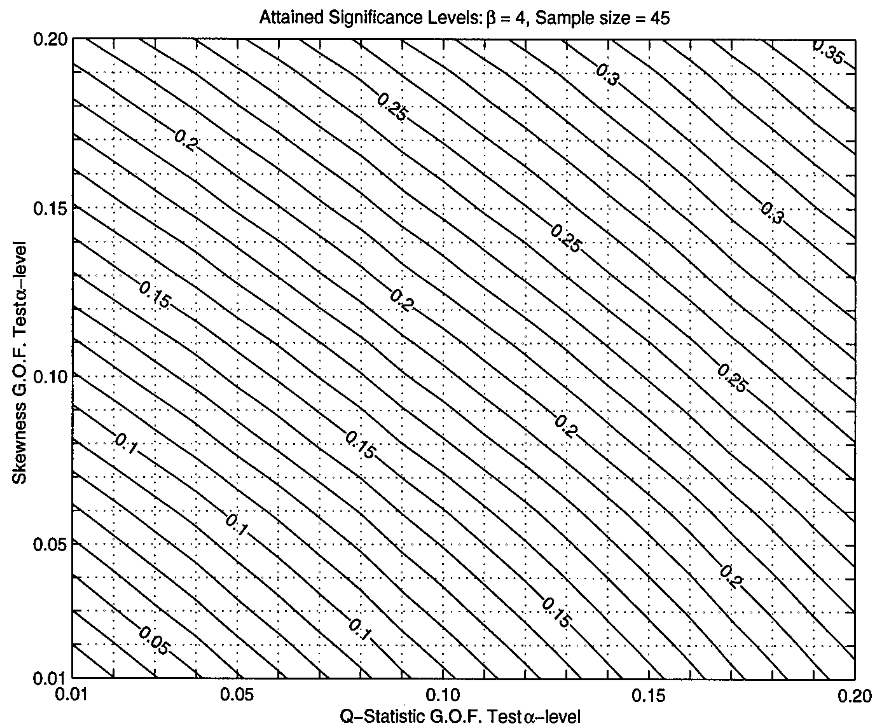
(f) Sample Size = 30



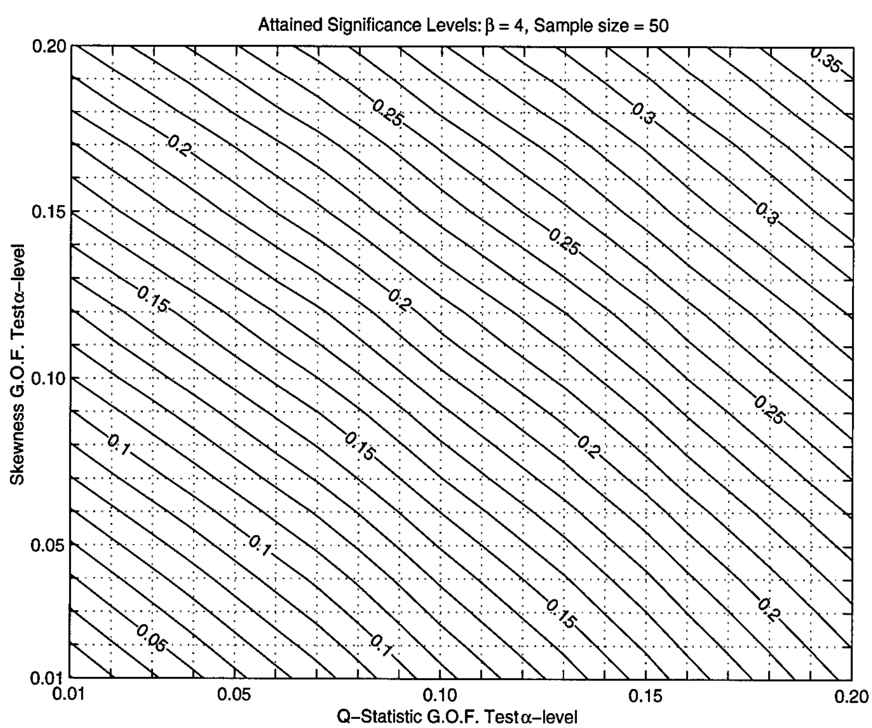
(g) Sample Size = 35



(h) Sample Size = 40



(i) Sample Size = 45



(j) Sample Size = 50

Appendix E. Additional Power Study Results

E.1 H_o: Weibull($\beta = 0.5$).

Table E.1 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 0.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.020	0.051
	0.049	0.030	0.030	0.048
	0.050	0.020	0.040	0.049
15	0.050	0.050	0.010	0.050
	0.049	0.040	0.040	0.050
	0.050	0.010	0.050	0.051
25	0.049	0.050	0.010	0.050
	0.047	0.040	0.040	0.049
	0.050	0.010	0.050	0.050
50	0.052	0.050	0.010	0.052
	0.049	0.040	0.030	0.049
	0.051	0.010	0.050	0.052

Table E.2 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 1.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.078	0.040	0.020	0.051
	0.070	0.030	0.030	0.048
	0.056	0.020	0.040	0.049
15	0.240	0.050	0.010	0.050
	0.217	0.040	0.040	0.050
	0.132	0.010	0.050	0.051
25	0.360	0.050	0.010	0.050
	0.329	0.040	0.040	0.049
	0.220	0.010	0.050	0.050
50	0.580	0.050	0.010	0.052
	0.553	0.040	0.030	0.049
	0.477	0.010	0.050	0.052

Table E.3 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.135	0.040	0.020	0.051
	0.121	0.030	0.030	0.048
	0.094	0.020	0.040	0.049
15	0.538	0.050	0.010	0.050
	0.503	0.040	0.040	0.050
	0.323	0.010	0.050	0.051
25	0.758	0.050	0.010	0.050
	0.729	0.040	0.040	0.049
	0.567	0.010	0.050	0.050
50	0.946	0.050	0.010	0.052
	0.937	0.040	0.030	0.049
	0.880	0.010	0.050	0.052

Table E.4 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.188	0.040	0.020	0.051
	0.169	0.030	0.030	0.048
	0.133	0.020	0.040	0.049
15	0.746	0.050	0.010	0.050
	0.717	0.040	0.040	0.050
	0.529	0.010	0.050	0.051
25	0.926	0.050	0.010	0.050
	0.913	0.040	0.040	0.049
	0.814	0.010	0.050	0.050
50	0.999	0.050	0.010	0.052
	0.997	0.040	0.030	0.049
	0.990	0.010	0.050	0.052

Table E.5 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.225	0.040	0.020	0.051
	0.203	0.030	0.030	0.048
	0.159	0.020	0.040	0.049
15	0.850	0.050	0.010	0.050
	0.827	0.040	0.040	0.050
	0.671	0.010	0.050	0.051
25	0.976	0.050	0.010	0.050
	0.970	0.040	0.040	0.049
	0.921	0.010	0.050	0.050
50	1.000	0.050	0.010	0.052
	1.000	0.040	0.030	0.049
	0.999	0.010	0.050	0.052

Table E.6 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.258	0.040	0.020	0.051
	0.233	0.030	0.030	0.048
	0.183	0.020	0.040	0.049
15	0.907	0.050	0.010	0.050
	0.890	0.040	0.040	0.050
	0.763	0.010	0.050	0.051
25	0.991	0.050	0.010	0.050
	0.988	0.040	0.040	0.049
	0.965	0.010	0.050	0.050
50	1.000	0.050	0.010	0.052
	1.000	0.040	0.030	0.049
	1.000	0.010	0.050	0.052

Table E.7 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 3.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.283	0.040	0.020	0.051
	0.258	0.030	0.030	0.048
	0.206	0.020	0.040	0.049
15	0.937	0.050	0.010	0.050
	0.924	0.040	0.040	0.050
	0.823	0.010	0.050	0.051
25	0.996	0.050	0.010	0.050
	0.995	0.040	0.040	0.049
	0.982	0.010	0.050	0.050
50	1.000	0.050	0.010	0.052
	1.000	0.040	0.030	0.049
	1.000	0.010	0.050	0.052

Table E.8 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.303	0.040	0.020	0.051
	0.277	0.030	0.030	0.048
	0.221	0.020	0.040	0.049
15	0.950	0.050	0.010	0.050
	0.941	0.040	0.040	0.050
	0.857	0.010	0.050	0.051
25	0.998	0.050	0.010	0.050
	0.997	0.040	0.040	0.049
	0.990	0.010	0.050	0.050
50	1.000	0.050	0.010	0.052
	1.000	0.040	0.030	0.049
	1.000	0.010	0.050	0.052

E.2 H_o: Weibull(β = 1.0).

Table E.9 Sequential G.O.F. Test Power: H_o: Weibull(β = 1.0); H_a: Weibull(β = 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.171	0.040	0.010	0.047
	0.167	0.030	0.030	0.053
	0.153	0.020	0.040	0.053
15	0.256	0.050	0.010	0.053
	0.242	0.040	0.020	0.048
	0.277	0.010	0.050	0.054
25	0.326	0.050	0.010	0.052
	0.380	0.030	0.040	0.051
	0.400	0.010	0.050	0.053
50	0.569	0.050	0.010	0.051
	0.662	0.030	0.040	0.053
	0.679	0.010	0.050	0.052

Table E.10 Sequential G.O.F. Test Power: H_o: Weibull(β = 1.0); H_a: Weibull(β = 1.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.050	0.040	0.010	0.047
	0.051	0.030	0.030	0.053
	0.048	0.020	0.040	0.053
15	0.122	0.050	0.010	0.053
	0.109	0.040	0.020	0.048
	0.081	0.010	0.050	0.054
25	0.189	0.050	0.010	0.052
	0.167	0.030	0.040	0.051
	0.144	0.010	0.050	0.053
50	0.336	0.050	0.010	0.051
	0.303	0.030	0.040	0.053
	0.268	0.010	0.050	0.052

Table E.11 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.086	0.040	0.010	0.047
	0.081	0.030	0.030	0.053
	0.068	0.020	0.040	0.053
15	0.415	0.050	0.010	0.053
	0.381	0.040	0.020	0.048
	0.292	0.020	0.040	0.048
25	0.663	0.050	0.010	0.052
	0.631	0.040	0.030	0.051
	0.592	0.030	0.040	0.051
50	0.929	0.050	0.010	0.051
	0.918	0.040	0.030	0.053
	0.899	0.030	0.040	0.053

Table E.12 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.099	0.040	0.010	0.047
	0.089	0.030	0.030	0.053
	0.074	0.020	0.040	0.053
15	0.524	0.050	0.010	0.053
	0.489	0.040	0.020	0.048
	0.394	0.020	0.040	0.048
25	0.790	0.050	0.010	0.052
	0.761	0.040	0.030	0.051
	0.725	0.030	0.040	0.051
50	0.981	0.050	0.010	0.051
	0.975	0.040	0.030	0.053
	0.968	0.030	0.040	0.053

Table E.13 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.123	0.040	0.010	0.047
	0.108	0.030	0.030	0.053
	0.088	0.020	0.040	0.053
15	0.663	0.050	0.010	0.053
	0.631	0.040	0.020	0.048
	0.539	0.020	0.040	0.048
25	0.905	0.050	0.010	0.052
	0.888	0.040	0.030	0.051
	0.867	0.030	0.040	0.051
50	0.998	0.050	0.010	0.051
	0.997	0.040	0.030	0.053
	0.996	0.030	0.040	0.053

Table E.14 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.0$); H_a: $\chi^2(4)$.

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
20	0.108	0.040	0.010	0.047
	0.097	0.030	0.030	0.053
	0.084	0.020	0.040	0.053
30	0.131	0.050	0.010	0.053
	0.115	0.040	0.020	0.048
	0.099	0.020	0.040	0.048
50	0.175	0.050	0.010	0.052
	0.155	0.040	0.030	0.051

E.3 H_o: Weibull(β = 1.5).

Table E.15 Sequential G.O.F. Test Power: H_0: Weibull(β= 1.5); H_a: Weibull(β= 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.270	0.040	0.010	0.048
	0.252	0.030	0.020	0.047
	0.236	0.020	0.040	0.054
15	0.505	0.040	0.020	0.051
	0.463	0.030	0.030	0.049
	0.429	0.020	0.040	0.051
25	0.698	0.040	0.020	0.050
	0.662	0.030	0.030	0.048
	0.626	0.020	0.040	0.050
50	0.927	0.040	0.020	0.050
	0.907	0.030	0.030	0.049
	0.894	0.020	0.040	0.051

Table E.16 Sequential G.O.F. Test Power: H_0: Weibull(β= 1.5); H_a: Weibull(β= 1.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.076	0.040	0.010	0.048
	0.072	0.030	0.020	0.047
	0.074	0.020	0.040	0.054
15	0.129	0.040	0.020	0.051
	0.115	0.030	0.030	0.049
	0.106	0.020	0.040	0.051
25	0.167	0.040	0.020	0.050
	0.147	0.030	0.030	0.048
	0.132	0.020	0.040	0.050
50	0.267	0.040	0.020	0.050
	0.238	0.030	0.030	0.049
	0.221	0.020	0.040	0.051

Table E.17 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.055	0.020	0.040	0.054
15	0.052	0.040	0.020	0.051
	0.050	0.030	0.030	0.049
	0.051	0.020	0.040	0.051
25	0.051	0.040	0.020	0.050
	0.049	0.030	0.030	0.048
	0.051	0.020	0.040	0.050
50	0.049	0.040	0.020	0.050
	0.048	0.030	0.030	0.049
	0.049	0.020	0.040	0.051

Table E.18 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.052	0.020	0.040	0.054
15	0.081	0.040	0.020	0.051
	0.074	0.030	0.030	0.049
	0.068	0.020	0.040	0.051
25	0.109	0.040	0.020	0.050
	0.096	0.030	0.030	0.048
	0.083	0.020	0.040	0.050
50	0.179	0.040	0.020	0.050
	0.156	0.030	0.030	0.049
	0.129	0.020	0.040	0.051

Table E.19 Sequential G.O.F. Test Power: H_0: Weibull(β = 1.5); H_a: Weibull(β = 2.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.055	0.040	0.010	0.048
	0.051	0.030	0.020	0.047
	0.055	0.020	0.040	0.054
15	0.147	0.040	0.020	0.051
	0.129	0.030	0.030	0.049
	0.110	0.020	0.040	0.051
25	0.237	0.040	0.020	0.050
	0.208	0.030	0.030	0.048
	0.176	0.020	0.040	0.050
50	0.454	0.040	0.020	0.050
	0.413	0.030	0.030	0.049
	0.363	0.020	0.040	0.051

Table E.20 Sequential G.O.F. Test Power: H_0: Weibull(β = 1.5); H_a: Weibull(β = 3.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.062	0.040	0.010	0.048
	0.058	0.030	0.020	0.047
	0.058	0.020	0.040	0.054
15	0.219	0.040	0.020	0.051
	0.194	0.030	0.030	0.049
	0.164	0.020	0.040	0.051
25	0.381	0.040	0.020	0.050
	0.340	0.030	0.030	0.048
	0.295	0.020	0.040	0.050
50	0.688	0.040	0.020	0.050
	0.651	0.030	0.030	0.049
	0.599	0.020	0.040	0.051

Table E.21 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 3.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.069	0.040	0.010	0.047
	0.063	0.030	0.030	0.053
	0.061	0.020	0.040	0.053
15	0.287	0.050	0.010	0.053
	0.258	0.040	0.020	0.048
	0.221	0.020	0.040	0.048
25	0.498	0.050	0.010	0.052
	0.457	0.040	0.030	0.051
	0.407	0.030	0.040	0.051
50	0.827	0.050	0.010	0.051
	0.799	0.040	0.030	0.053
	0.760	0.040	0.030	0.053

Table E.22 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.074	0.040	0.010	0.048
	0.066	0.030	0.020	0.047
	0.065	0.020	0.040	0.054
15	0.341	0.040	0.020	0.051
	0.309	0.030	0.030	0.049
	0.268	0.020	0.040	0.051
25	0.587	0.040	0.020	0.050
	0.546	0.030	0.030	0.048
	0.493	0.020	0.040	0.050
50	0.900	0.040	0.020	0.050
	0.881	0.030	0.030	0.049
	0.852	0.020	0.040	0.051

E.4 H_o: Weibull(β = 2.0).

Table E.23 Sequential G.O.F. Test Power: H_0: Weibull(β = 2.0); H_a: Weibull(β = 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.328	0.040	0.010	0.048
	0.311	0.030	0.020	0.047
	0.283	0.020	0.030	0.046
15	0.720	0.040	0.020	0.054
	0.685	0.030	0.030	0.052
	0.632	0.020	0.040	0.052
25	0.907	0.040	0.020	0.053
	0.888	0.030	0.030	0.052
	0.853	0.020	0.040	0.052
50	0.997	0.040	0.020	0.053
	0.995	0.030	0.030	0.052
	0.992	0.020	0.040	0.053

Table E.24 Sequential G.O.F. Test Power: H_0: Weibull(β = 2.0); H_a: Weibull(β = 1.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.105	0.040	0.010	0.048
	0.096	0.030	0.020	0.047
	0.086	0.020	0.030	0.046
15	0.268	0.040	0.020	0.054
	0.241	0.030	0.030	0.052
	0.207	0.020	0.040	0.052
25	0.409	0.040	0.020	0.053
	0.370	0.030	0.030	0.052
	0.320	0.020	0.040	0.052
50	0.684	0.040	0.020	0.053
	0.639	0.030	0.030	0.052
	0.574	0.020	0.040	0.053

Table E.25 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.053	0.040	0.010	0.048
	0.051	0.030	0.020	0.047
	0.050	0.020	0.030	0.046
15	0.090	0.040	0.020	0.054
	0.084	0.030	0.030	0.052
	0.079	0.020	0.040	0.052
25	0.117	0.040	0.020	0.053
	0.105	0.030	0.030	0.052
	0.094	0.020	0.040	0.052
50	0.174	0.040	0.020	0.053
	0.152	0.030	0.030	0.052
	0.128	0.020	0.040	0.053

Table E.26 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.047	0.040	0.010	0.048
	0.045	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.055	0.040	0.020	0.054
	0.054	0.030	0.030	0.052
	0.054	0.020	0.040	0.052
25	0.053	0.040	0.020	0.053
	0.050	0.030	0.030	0.052
	0.051	0.020	0.040	0.052
50	0.055	0.040	0.020	0.053
	0.053	0.030	0.030	0.052
	0.053	0.020	0.040	0.053

Table E.27 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.048	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.073	0.040	0.020	0.054
	0.068	0.030	0.030	0.052
	0.064	0.020	0.040	0.052
25	0.085	0.040	0.020	0.053
	0.076	0.030	0.030	0.052
	0.070	0.020	0.040	0.052
50	0.128	0.040	0.020	0.053
	0.114	0.030	0.030	0.052
	0.099	0.020	0.040	0.053

Table E.28 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.050	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.046	0.020	0.030	0.046
15	0.106	0.040	0.020	0.054
	0.096	0.030	0.030	0.052
	0.086	0.020	0.040	0.052
25	0.158	0.040	0.020	0.053
	0.138	0.030	0.030	0.052
	0.119	0.020	0.040	0.052
50	0.294	0.040	0.020	0.053
	0.263	0.030	0.030	0.052
	0.223	0.020	0.040	0.053

Table E.29 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 3.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.056	0.040	0.010	0.048
	0.052	0.030	0.020	0.047
	0.049	0.020	0.030	0.046
15	0.151	0.040	0.020	0.054
	0.134	0.030	0.030	0.052
	0.118	0.020	0.040	0.052
25	0.238	0.040	0.020	0.053
	0.212	0.030	0.030	0.052
	0.184	0.020	0.040	0.052
50	0.465	0.040	0.020	0.053
	0.426	0.030	0.030	0.052
	0.375	0.020	0.040	0.053

Table E.30 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.057	0.040	0.010	0.048
	0.052	0.030	0.020	0.047
	0.049	0.020	0.030	0.046
15	0.190	0.040	0.020	0.054
	0.171	0.030	0.030	0.052
	0.148	0.020	0.040	0.052
25	0.321	0.040	0.020	0.053
	0.288	0.030	0.030	0.052
	0.251	0.020	0.040	0.052
50	0.607	0.040	0.020	0.053
	0.569	0.030	0.030	0.052
	0.518	0.020	0.040	0.053

E.5 H_o: Weibull(β = 2.5).

Table E.31 Sequential G.O.F. Test Power: H_o: Weibull(β = 2.5); H_a: Weibull(β = 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.855	0.040	0.010	0.048
	0.832	0.030	0.020	0.047
	0.792	0.020	0.030	0.046
15	0.927	0.040	0.010	0.047
	0.910	0.030	0.030	0.054
	0.883	0.020	0.040	0.054
25	0.989	0.040	0.010	0.047
	0.985	0.030	0.030	0.054
	0.977	0.020	0.040	0.054
50	1.000	0.040	0.010	0.048
	1.000	0.030	0.020	0.046
	1.000	0.020	0.030	0.046

Table E.32 Sequential G.O.F. Test Power: H_o: Weibull(β = 2.5); H_a: Weibull(β = 1.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.128	0.040	0.010	0.048
	0.116	0.030	0.020	0.047
	0.104	0.020	0.030	0.046
15	0.393	0.040	0.010	0.047
	0.366	0.030	0.030	0.054
	0.324	0.020	0.040	0.054
25	0.620	0.040	0.010	0.047
	0.585	0.030	0.030	0.054
	0.530	0.020	0.040	0.054
50	0.910	0.040	0.010	0.048
	0.888	0.030	0.020	0.046
	0.852	0.020	0.030	0.046

Table E.33 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.065	0.040	0.010	0.048
	0.060	0.030	0.020	0.047
	0.057	0.020	0.030	0.046
15	0.147	0.040	0.010	0.047
	0.139	0.030	0.030	0.054
	0.122	0.020	0.040	0.054
25	0.229	0.040	0.010	0.047
	0.212	0.030	0.030	0.054
	0.182	0.020	0.040	0.054
50	0.431	0.040	0.010	0.048
	0.391	0.030	0.020	0.046
	0.338	0.020	0.030	0.046

Table E.34 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.010	0.048
	0.051	0.030	0.020	0.047
	0.050	0.020	0.030	0.046
15	0.062	0.040	0.010	0.047
	0.067	0.030	0.030	0.054
	0.064	0.020	0.040	0.054
25	0.078	0.040	0.010	0.047
	0.081	0.030	0.030	0.054
	0.075	0.020	0.040	0.054
50	0.112	0.040	0.010	0.048
	0.100	0.030	0.020	0.046
	0.087	0.020	0.030	0.046

Table E.35 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.047	0.020	0.030	0.046
15	0.047	0.040	0.010	0.047
	0.055	0.030	0.030	0.054
	0.056	0.020	0.040	0.054
25	0.049	0.040	0.010	0.047
	0.056	0.030	0.030	0.054
	0.056	0.020	0.040	0.054
50	0.046	0.040	0.010	0.048
	0.045	0.030	0.020	0.046
	0.045	0.020	0.030	0.046

Table E.36 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.048	0.030	0.020	0.047
	0.046	0.020	0.030	0.046
15	0.060	0.040	0.010	0.047
	0.064	0.030	0.030	0.054
	0.063	0.020	0.040	0.054
25	0.069	0.040	0.010	0.047
	0.073	0.030	0.030	0.054
	0.068	0.020	0.040	0.054
50	0.092	0.040	0.010	0.048
	0.083	0.030	0.020	0.046
	0.073	0.020	0.030	0.046

Table E.37 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 3.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.010	0.048
	0.049	0.030	0.020	0.047
	0.047	0.020	0.030	0.046
15	0.081	0.040	0.010	0.047
	0.083	0.030	0.030	0.054
	0.077	0.020	0.040	0.054
25	0.113	0.040	0.010	0.047
	0.110	0.030	0.030	0.054
	0.097	0.020	0.040	0.054
50	0.194	0.040	0.010	0.048
	0.173	0.030	0.020	0.046
	0.148	0.020	0.030	0.046

Table E.38 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.010	0.048
	0.048	0.030	0.020	0.047
	0.046	0.020	0.030	0.046
15	0.104	0.040	0.010	0.047
	0.103	0.030	0.030	0.054
	0.094	0.020	0.040	0.054
25	0.164	0.040	0.010	0.047
	0.156	0.030	0.030	0.054
	0.135	0.020	0.040	0.054
50	0.310	0.040	0.010	0.048
	0.281	0.030	0.020	0.046
	0.243	0.020	0.030	0.046

E.6 H_o: Weibull(β = 3.0).

Table E.39 Sequential G.O.F. Test Power: H_o: Weibull(β= 3.0); H_a: Weibull(β= 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.384	0.040	0.010	0.048
	0.362	0.030	0.020	0.047
	0.331	0.020	0.030	0.046
15	0.888	0.040	0.010	0.048
	0.867	0.030	0.020	0.046
	0.835	0.020	0.030	0.046
25	0.991	0.040	0.010	0.048
	0.988	0.030	0.020	0.047
	0.981	0.020	0.030	0.047
50	1.000	0.040	0.010	0.049
	1.000	0.030	0.020	0.048
	1.000	0.020	0.030	0.048

Table E.40 Sequential G.O.F. Test Power: H_o: Weibull(β= 3.0); H_a: Weibull(β= 1.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.145	0.040	0.010	0.048
	0.130	0.030	0.020	0.047
	0.113	0.020	0.030	0.046
15	0.500	0.040	0.010	0.048
	0.462	0.030	0.020	0.046
	0.415	0.020	0.030	0.046
25	0.763	0.040	0.010	0.048
	0.729	0.030	0.020	0.047
	0.684	0.020	0.030	0.047
50	0.979	0.040	0.010	0.049
	0.974	0.030	0.020	0.048
	0.960	0.020	0.030	0.048

Table E.41 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.073	0.040	0.010	0.048
	0.067	0.030	0.020	0.047
	0.062	0.020	0.030	0.046
15	0.210	0.040	0.010	0.048
	0.189	0.030	0.020	0.046
	0.164	0.020	0.030	0.046
25	0.365	0.040	0.010	0.048
	0.330	0.030	0.020	0.047
	0.286	0.020	0.030	0.047
50	0.668	0.040	0.010	0.049
	0.629	0.030	0.020	0.048
	0.573	0.020	0.030	0.048

Table E.42 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.053	0.040	0.010	0.048
	0.051	0.030	0.020	0.047
	0.043	0.020	0.030	0.046
15	0.092	0.040	0.010	0.048
	0.085	0.030	0.020	0.046
	0.076	0.020	0.030	0.046
25	0.139	0.040	0.010	0.048
	0.124	0.030	0.020	0.047
	0.109	0.020	0.030	0.047
50	0.263	0.040	0.010	0.049
	0.235	0.030	0.020	0.048
	0.198	0.020	0.030	0.048

Table E.43 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.048	0.040	0.010	0.048
	0.046	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.055	0.040	0.010	0.048
	0.051	0.030	0.020	0.046
	0.050	0.020	0.030	0.046
25	0.064	0.040	0.010	0.048
	0.059	0.030	0.020	0.047
	0.055	0.020	0.030	0.047
50	0.083	0.040	0.010	0.049
	0.074	0.030	0.020	0.048
	0.067	0.020	0.030	0.048

Table E.44 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.047	0.040	0.010	0.048
	0.045	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.046
	0.046	0.020	0.030	0.046
25	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.045	0.020	0.030	0.047
50	0.048	0.040	0.010	0.049
	0.047	0.030	0.020	0.048
	0.047	0.020	0.030	0.048

Table E.45 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 3.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.047	0.040	0.010	0.048
	0.046	0.030	0.020	0.047
	0.044	0.020	0.030	0.046
15	0.056	0.040	0.010	0.048
	0.053	0.030	0.020	0.046
	0.050	0.020	0.030	0.046
25	0.062	0.040	0.010	0.048
	0.059	0.030	0.020	0.047
	0.054	0.020	0.030	0.047
50	0.077	0.040	0.010	0.049
	0.073	0.030	0.020	0.048
	0.067	0.020	0.030	0.048

Table E.46 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.067	0.040	0.010	0.048
	0.062	0.030	0.020	0.046
	0.058	0.020	0.030	0.046
25	0.090	0.040	0.010	0.048
	0.083	0.030	0.020	0.047
	0.073	0.020	0.030	0.047
50	0.138	0.040	0.010	0.049
	0.126	0.030	0.020	0.048
	0.112	0.020	0.030	0.048

E.7 H_o: Weibull(β = 3.5).

Table E.47 Sequential G.O.F. Test Power: H_o: Weibull(β= 3.5); H_a: Weibull(β= 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.322	0.040	0.010	0.048
	0.305	0.030	0.030	0.047
	0.289	0.020	0.040	0.046
15	0.948	0.050	0.010	0.048
	0.931	0.040	0.020	0.047
	0.924	0.020	0.040	0.046
25	0.996	0.050	0.010	0.049
	0.992	0.040	0.030	0.048
	0.985	0.030	0.040	0.047
50	1.000	0.050	0.010	0.049
	1.000	0.040	0.030	0.048
	1.000	0.040	0.030	0.048

Table E.48 Sequential G.O.F. Test Power: H_o: Weibull(β= 3.5); H_a: Weibull(β= 1.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.081	0.040	0.010	0.048
	0.075	0.030	0.020	0.047
	0.066	0.020	0.030	0.046
15	0.268	0.040	0.010	0.048
	0.241	0.030	0.020	0.046
	0.207	0.020	0.030	0.046
25	0.478	0.040	0.010	0.048
	0.441	0.030	0.020	0.047
	0.388	0.020	0.030	0.047
50	0.820	0.040	0.010	0.049
	0.790	0.030	0.020	0.048
	0.743	0.020	0.030	0.048

Table E.49 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.048	0.030	0.020	0.047
	0.047	0.020	0.030	0.046
15	0.068	0.040	0.010	0.048
	0.064	0.030	0.020	0.046
	0.059	0.020	0.030	0.046
25	0.100	0.040	0.010	0.048
	0.091	0.030	0.020	0.047
	0.080	0.020	0.030	0.047
50	0.163	0.040	0.010	0.049
	0.145	0.030	0.020	0.048
	0.122	0.020	0.030	0.048

Table E.50 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.046	0.020	0.030	0.046
15	0.051	0.040	0.010	0.048
	0.050	0.030	0.020	0.046
	0.049	0.020	0.030	0.046
25	0.058	0.040	0.010	0.048
	0.055	0.030	0.020	0.047
	0.054	0.020	0.030	0.047
50	0.065	0.040	0.010	0.049
	0.061	0.030	0.020	0.048
	0.056	0.020	0.030	0.048

Table E.51 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.050	0.040	0.010	0.048
	0.048	0.030	0.020	0.047
	0.046	0.020	0.030	0.046
15	0.055	0.040	0.010	0.048
	0.052	0.030	0.020	0.046
	0.052	0.020	0.030	0.046
25	0.063	0.040	0.010	0.048
	0.059	0.030	0.020	0.047
	0.055	0.020	0.030	0.047
50	0.068	0.040	0.010	0.049
	0.065	0.030	0.020	0.048
	0.061	0.020	0.030	0.048

E.8 H_o: Weibull(β = 4.0).

Table E.52 Sequential G.O.F. Test Power: H_0: Weibull(β = 4.0); H_a: Weibull(β = 0.5).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.414	0.040	0.010	0.048
	0.392	0.030	0.020	0.047
	0.356	0.020	0.030	0.046
15	0.946	0.040	0.010	0.048
	0.932	0.030	0.020	0.048
	0.906	0.020	0.030	0.046
25	0.999	0.040	0.010	0.048
	0.998	0.030	0.020	0.047
	0.996	0.020	0.030	0.047
50	1.000	0.040	0.010	0.049
	1.000	0.030	0.020	0.048
	1.000	0.020	0.030	0.048

Table E.53 Sequential G.O.F. Test Power: H_0: Weibull(β = 4.0); H_a: Weibull(β = 1.0).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.162	0.040	0.010	0.048
	0.147	0.030	0.020	0.047
	0.125	0.020	0.030	0.046
15	0.633	0.040	0.010	0.048
	0.596	0.030	0.020	0.048
	0.537	0.020	0.030	0.046
25	0.892	0.040	0.010	0.048
	0.872	0.030	0.020	0.047
	0.838	0.020	0.030	0.047
50	0.998	0.040	0.010	0.049
	0.998	0.030	0.020	0.048
	0.997	0.020	0.030	0.048

Table E.54 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 1.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.085	0.040	0.010	0.048
	0.077	0.030	0.020	0.047
	0.068	0.020	0.030	0.046
15	0.321	0.040	0.010	0.048
	0.289	0.030	0.020	0.048
	0.244	0.020	0.030	0.046
25	0.562	0.040	0.010	0.048
	0.522	0.030	0.020	0.047
	0.468	0.020	0.030	0.047
50	0.897	0.040	0.010	0.049
	0.879	0.030	0.020	0.048
	0.844	0.020	0.030	0.048

Table E.55 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 2.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.059	0.040	0.010	0.048
	0.055	0.030	0.020	0.047
	0.052	0.020	0.030	0.046
15	0.158	0.040	0.010	0.048
	0.140	0.030	0.020	0.048
	0.118	0.020	0.030	0.046
25	0.278	0.040	0.010	0.048
	0.247	0.030	0.020	0.047
	0.210	0.020	0.030	0.047
50	0.555	0.040	0.010	0.049
	0.517	0.030	0.020	0.048
	0.459	0.020	0.030	0.048

Table E.56 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 2.5$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.051	0.040	0.010	0.048
	0.049	0.030	0.020	0.047
	0.047	0.020	0.030	0.046
15	0.087	0.040	0.010	0.048
	0.079	0.030	0.020	0.048
	0.069	0.020	0.030	0.046
25	0.131	0.040	0.010	0.048
	0.116	0.030	0.020	0.047
	0.101	0.020	0.030	0.047
50	0.256	0.040	0.010	0.049
	0.230	0.030	0.020	0.048
	0.194	0.020	0.030	0.048

Table E.57 Sequential G.O.F. Test Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 3.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.048	0.040	0.010	0.048
	0.046	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.058	0.040	0.010	0.048
	0.055	0.030	0.020	0.048
	0.052	0.020	0.030	0.046
25	0.073	0.040	0.010	0.048
	0.066	0.030	0.020	0.047
	0.061	0.020	0.030	0.047
50	0.109	0.040	0.010	0.049
	0.098	0.030	0.020	0.048
	0.085	0.020	0.030	0.048

Table E.58 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 3.5$).

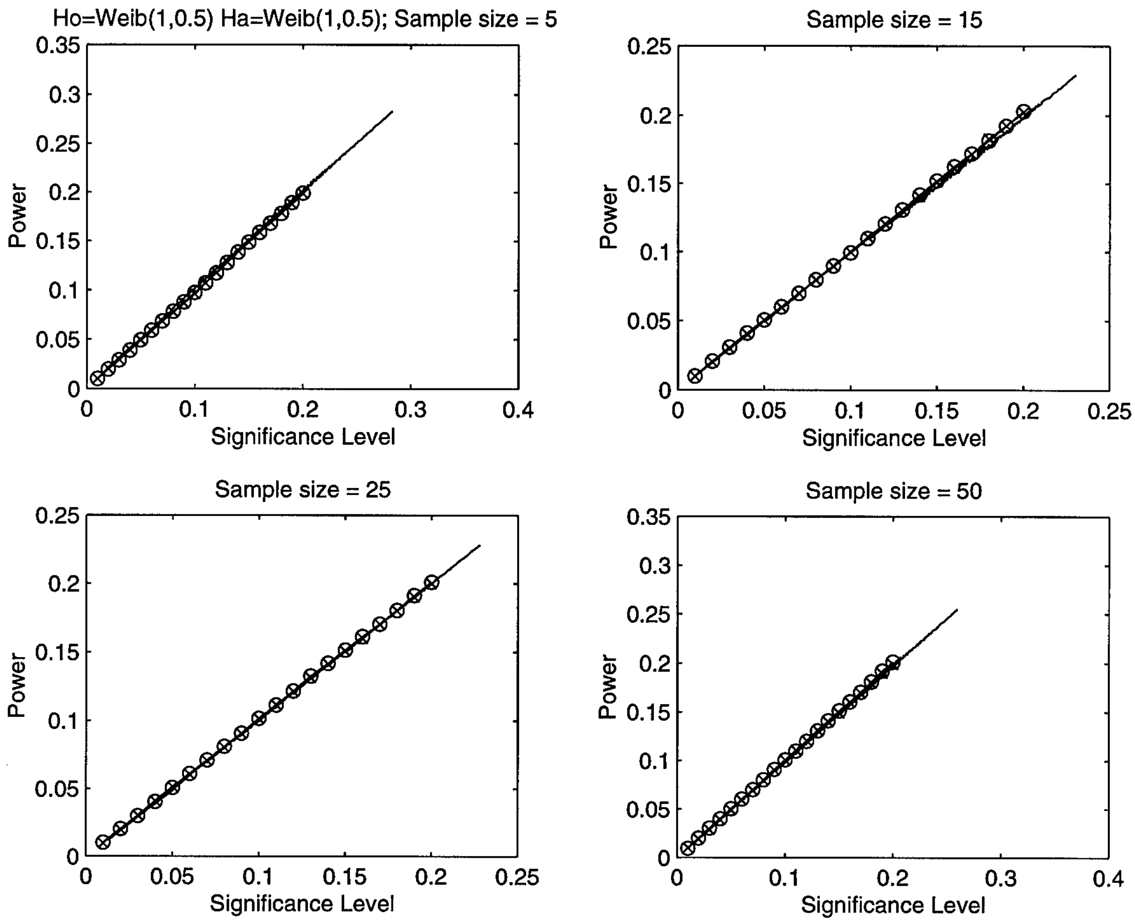
Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.047	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.044	0.020	0.030	0.046
15	0.048	0.040	0.010	0.048
	0.046	0.030	0.020	0.048
	0.046	0.020	0.030	0.046
25	0.052	0.040	0.010	0.048
	0.050	0.030	0.020	0.047
	0.049	0.020	0.030	0.047
50	0.057	0.040	0.010	0.049
	0.055	0.030	0.020	0.048
	0.053	0.020	0.030	0.048

Table E.59 Sequential G.O.F. Test Power: H_0: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 4.0$).

Sample Size	Sequential G.O.F. Test Power	Significance Level		
		$\sqrt{b_1}$ Test	Q-Statistic Test	Attained
5	0.048	0.040	0.010	0.048
	0.046	0.030	0.020	0.047
	0.045	0.020	0.030	0.046
15	0.048	0.040	0.010	0.048
	0.048	0.030	0.020	0.048
	0.046	0.020	0.030	0.046
25	0.049	0.040	0.010	0.048
	0.047	0.030	0.020	0.047
	0.047	0.020	0.030	0.047
50	0.048	0.040	0.010	0.049
	0.048	0.030	0.020	0.048
	0.049	0.020	0.030	0.048

Appendix F. Sequential G.O.F. Test Power vs. Individual G.O.F. Test Power

F.1 H_o: *Weibull*($\beta = 0.5$).

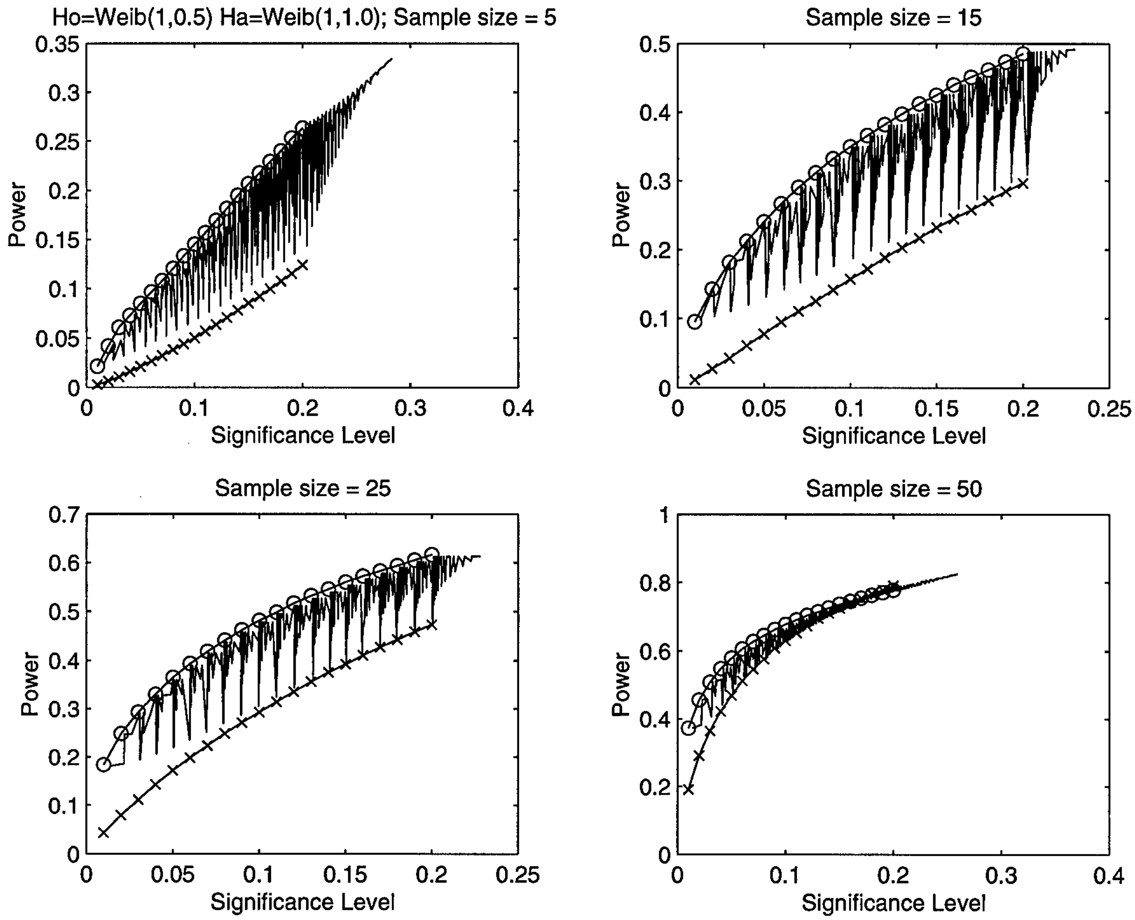


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.1 Individual vs. Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 0.5$).

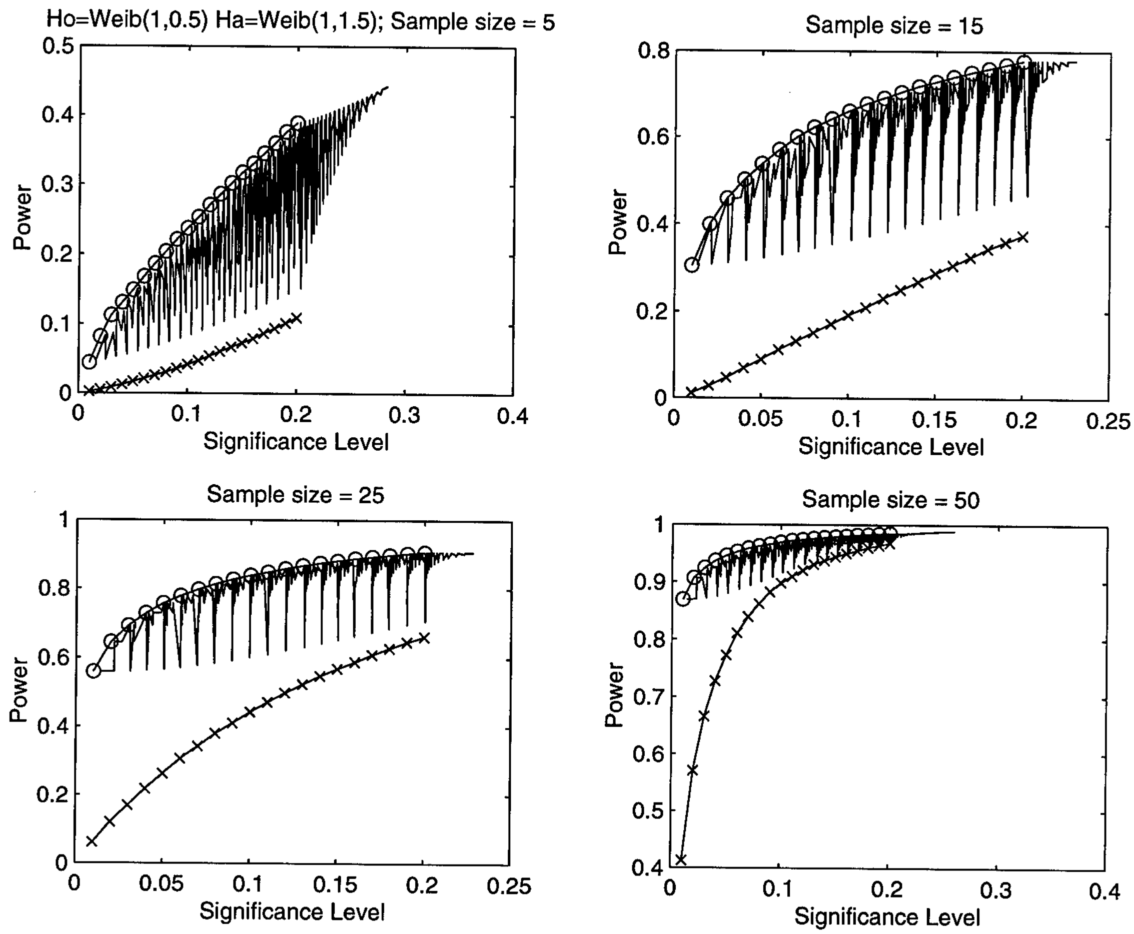


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.2 Individual vs. Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 1.0$).

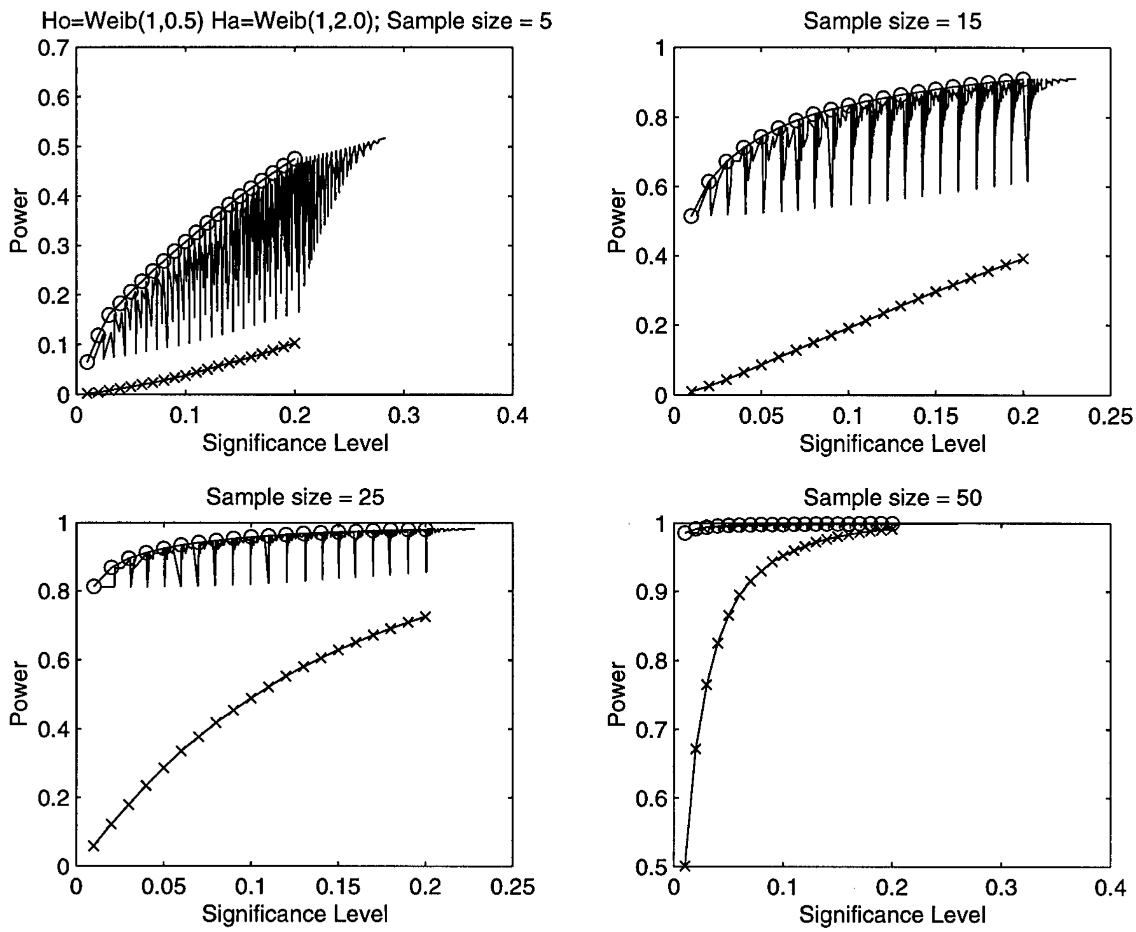


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.3 Individual vs. Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 1.5$).



— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.4 Individual vs. Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 2.0$).

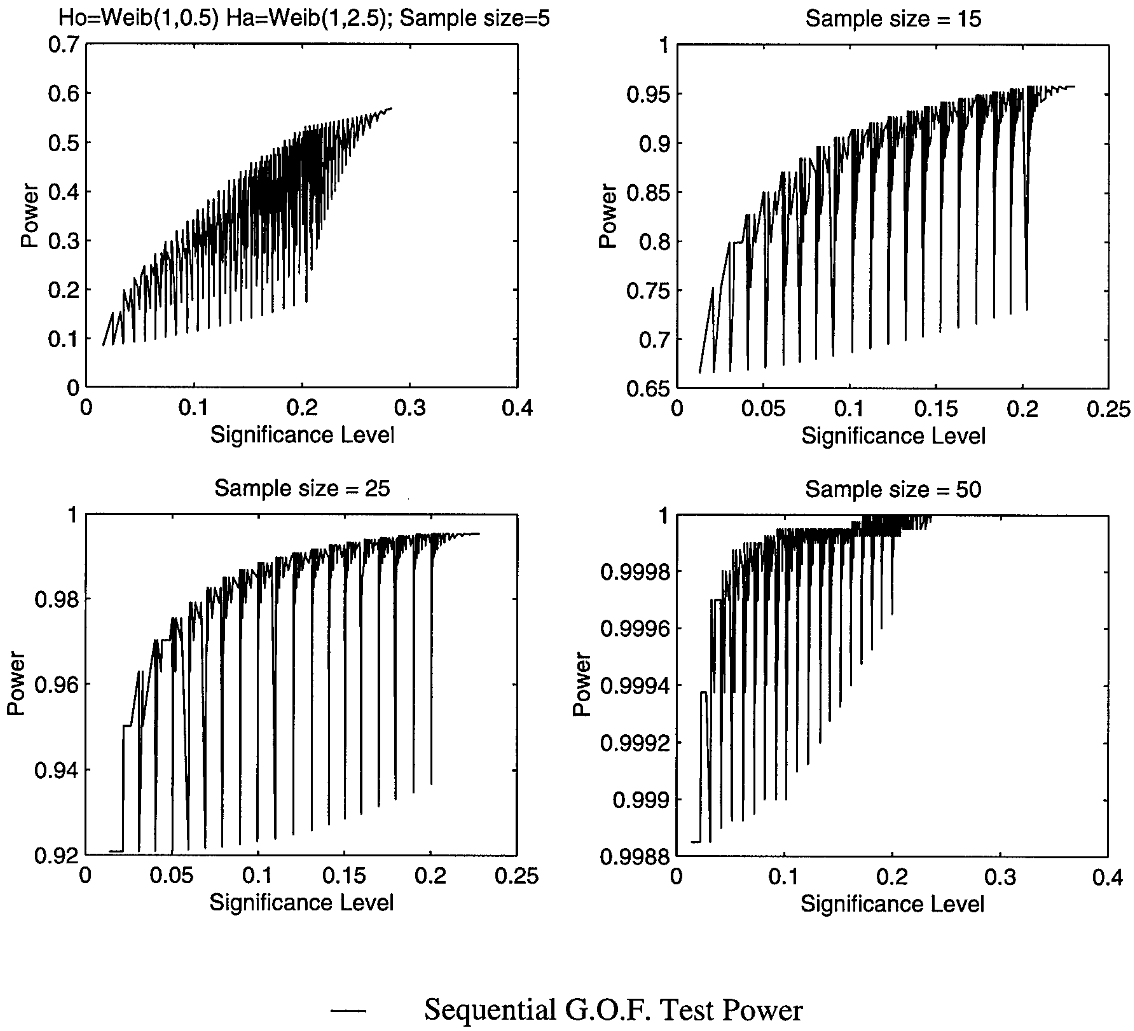


Figure F.5 Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 2.5$).

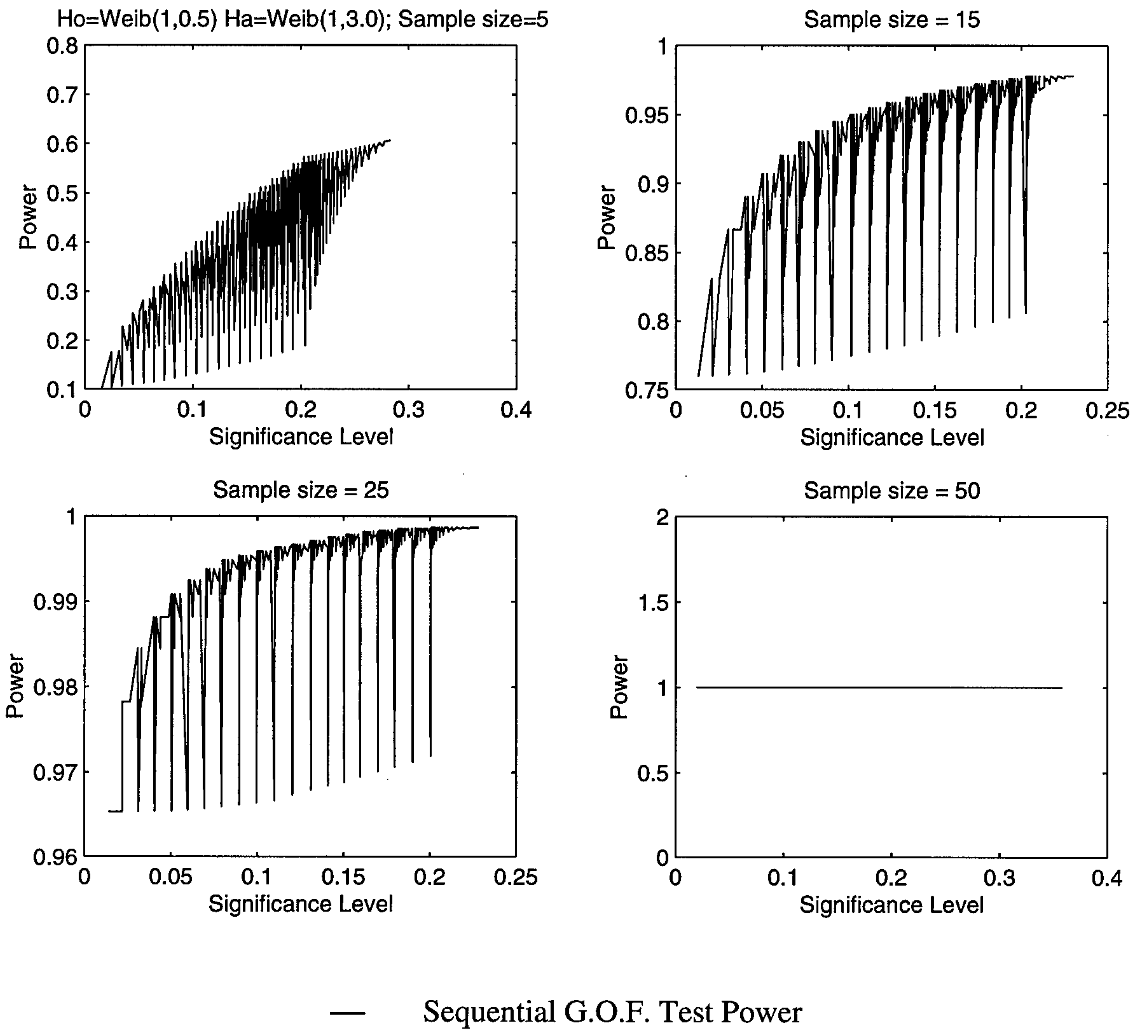
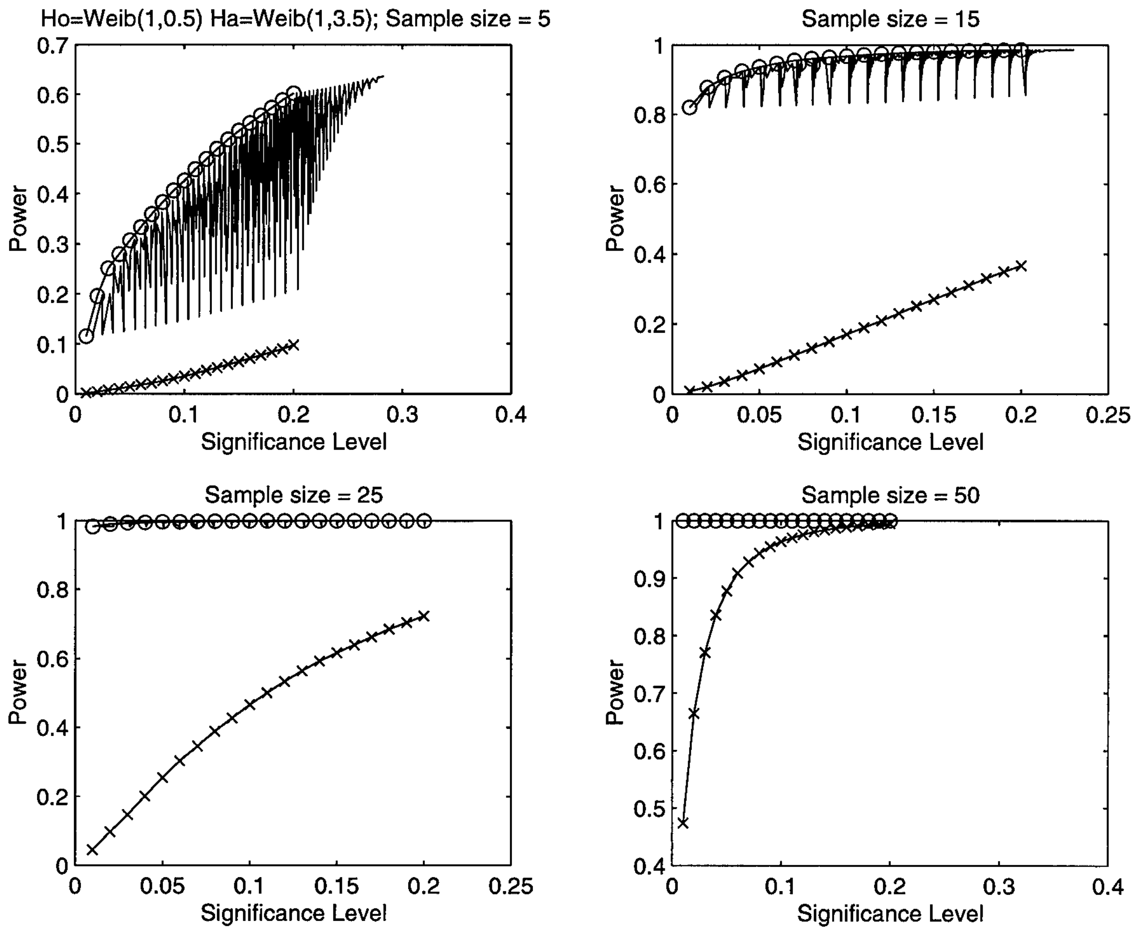


Figure F.6 Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 3.0$).

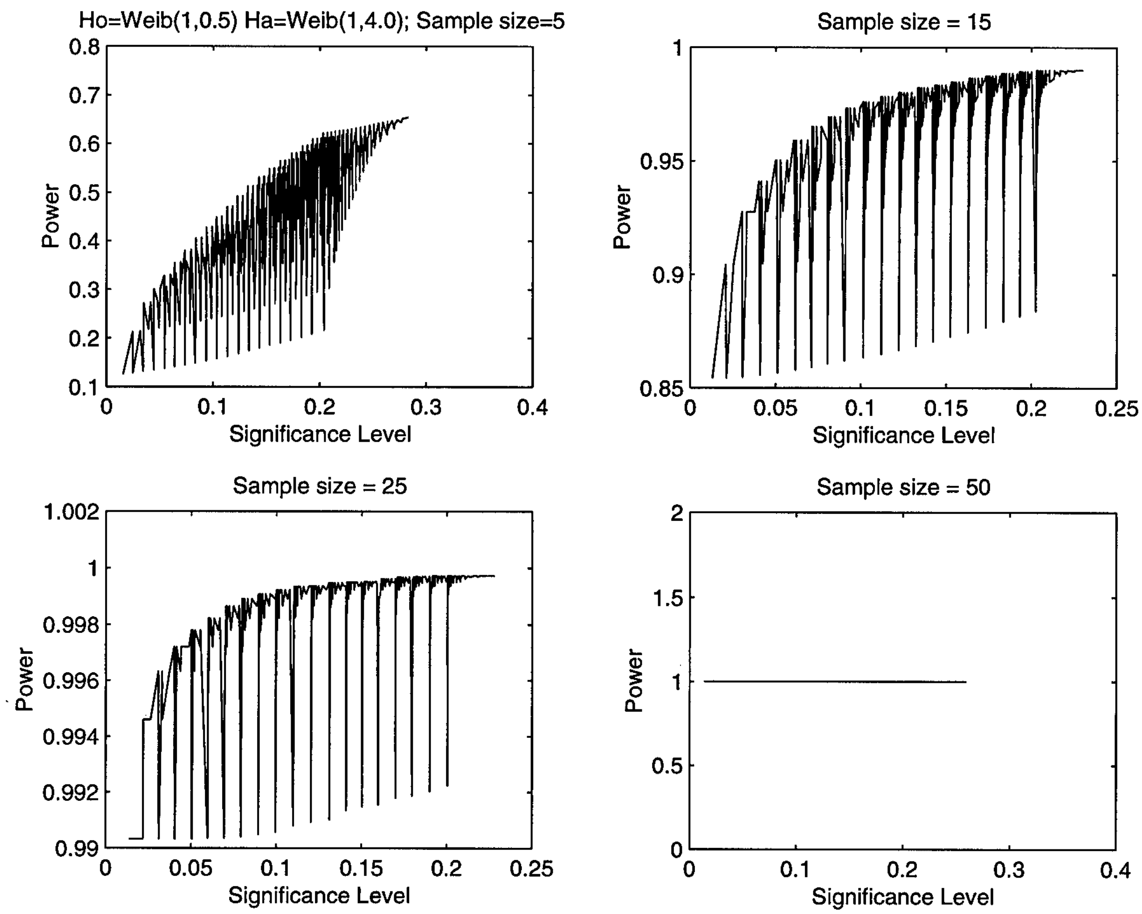


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

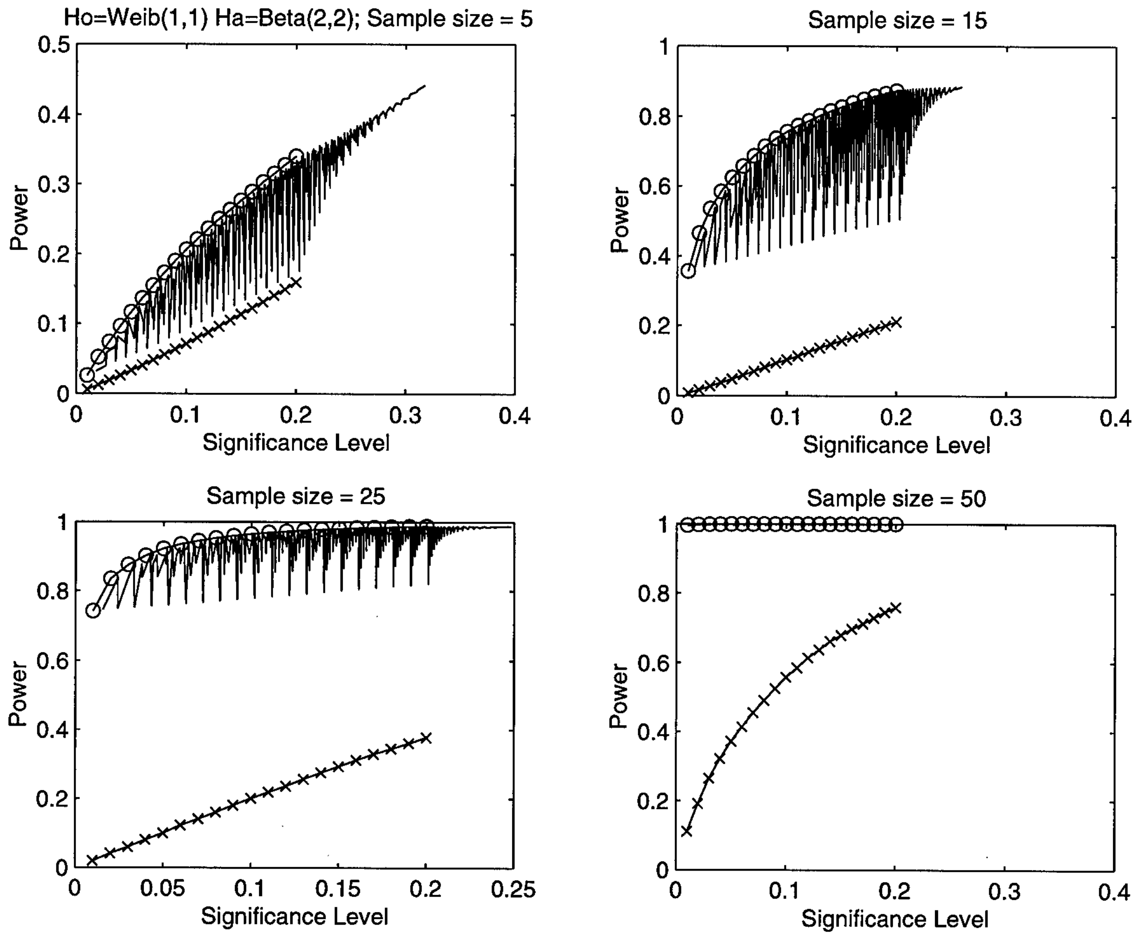
Figure F.7 Individual vs. Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

Figure F.8 Sequential Power: H_o: Weibull($\beta = 0.5$); H_a: Weibull($\beta = 4.0$).

F.2 $\quad$ H_o: *Weibull*($\beta = 1.0$).

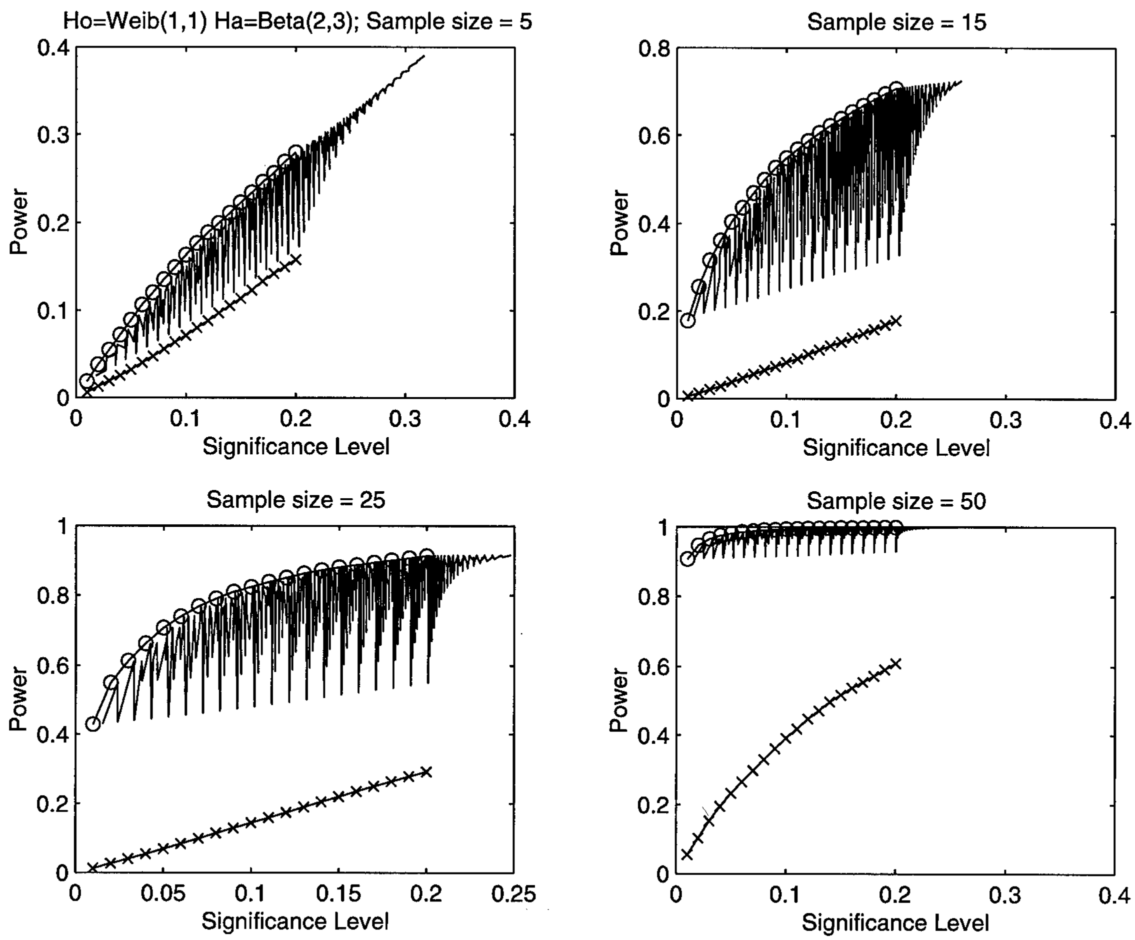


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.9 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Beta(2,2).

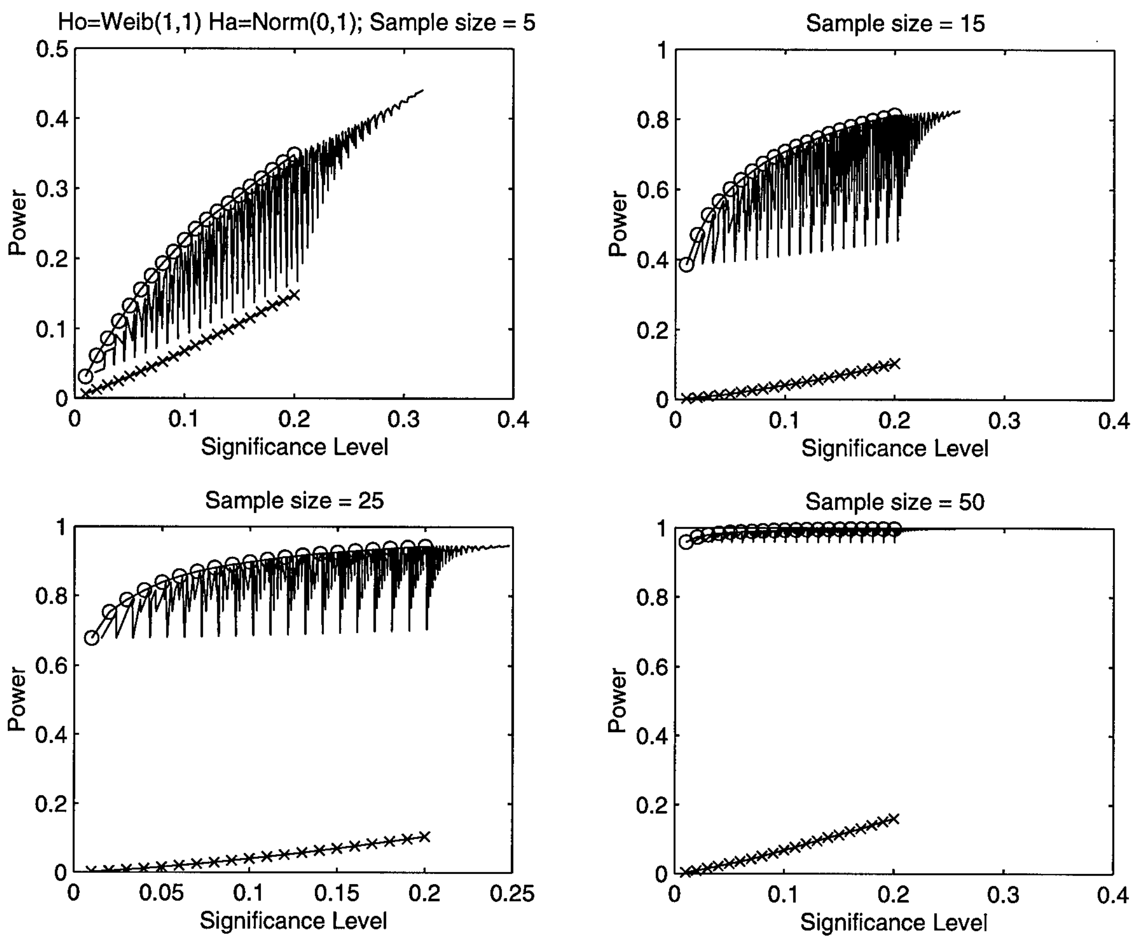


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.10 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Beta(2,3).

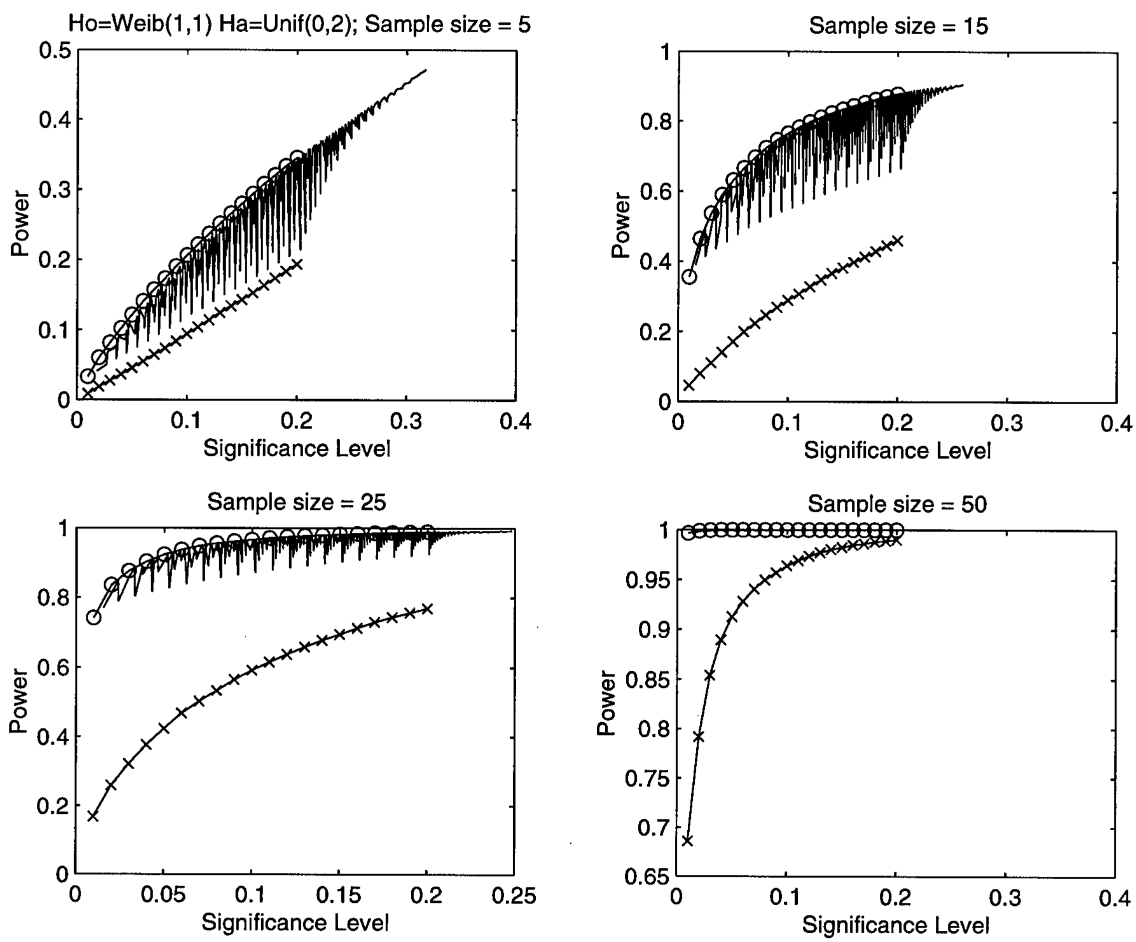


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.11 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Normal(0,1).

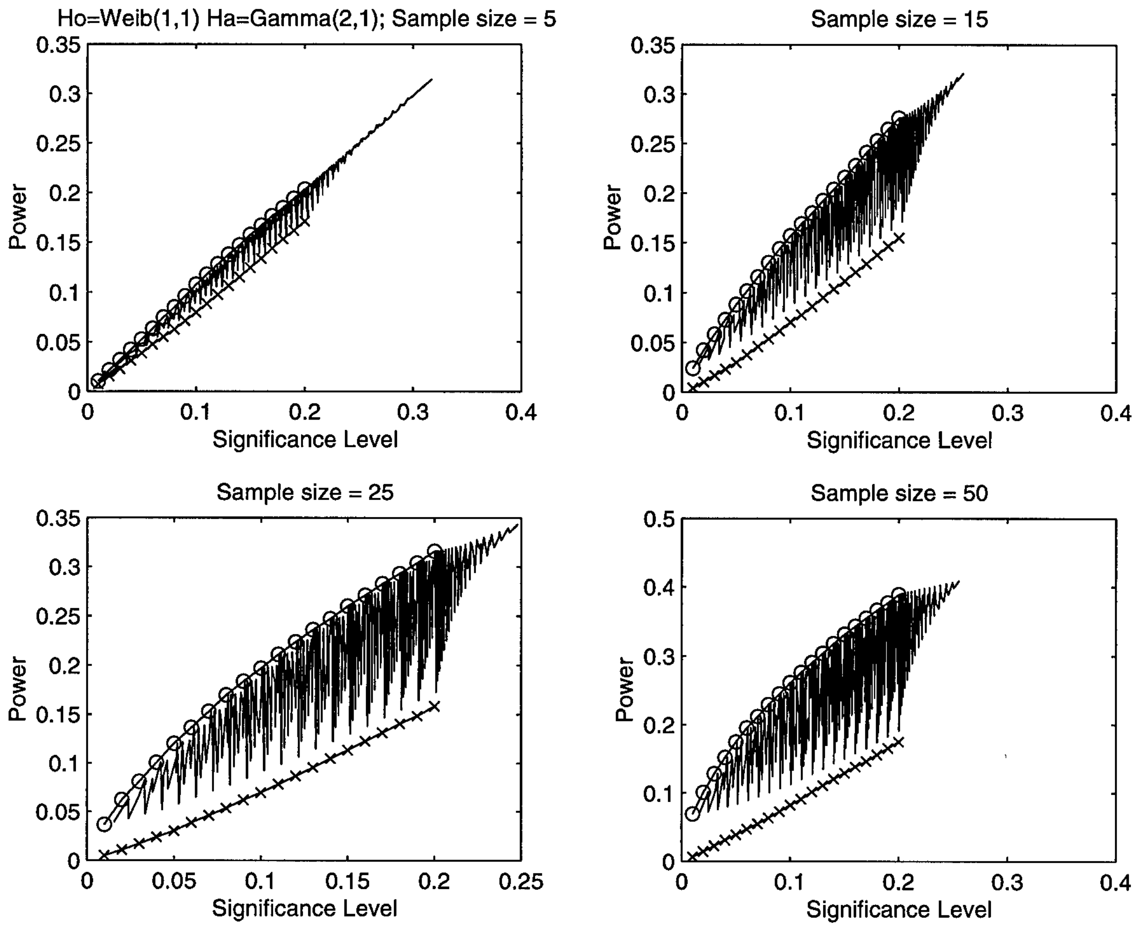


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.12 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Uniform(0,2).

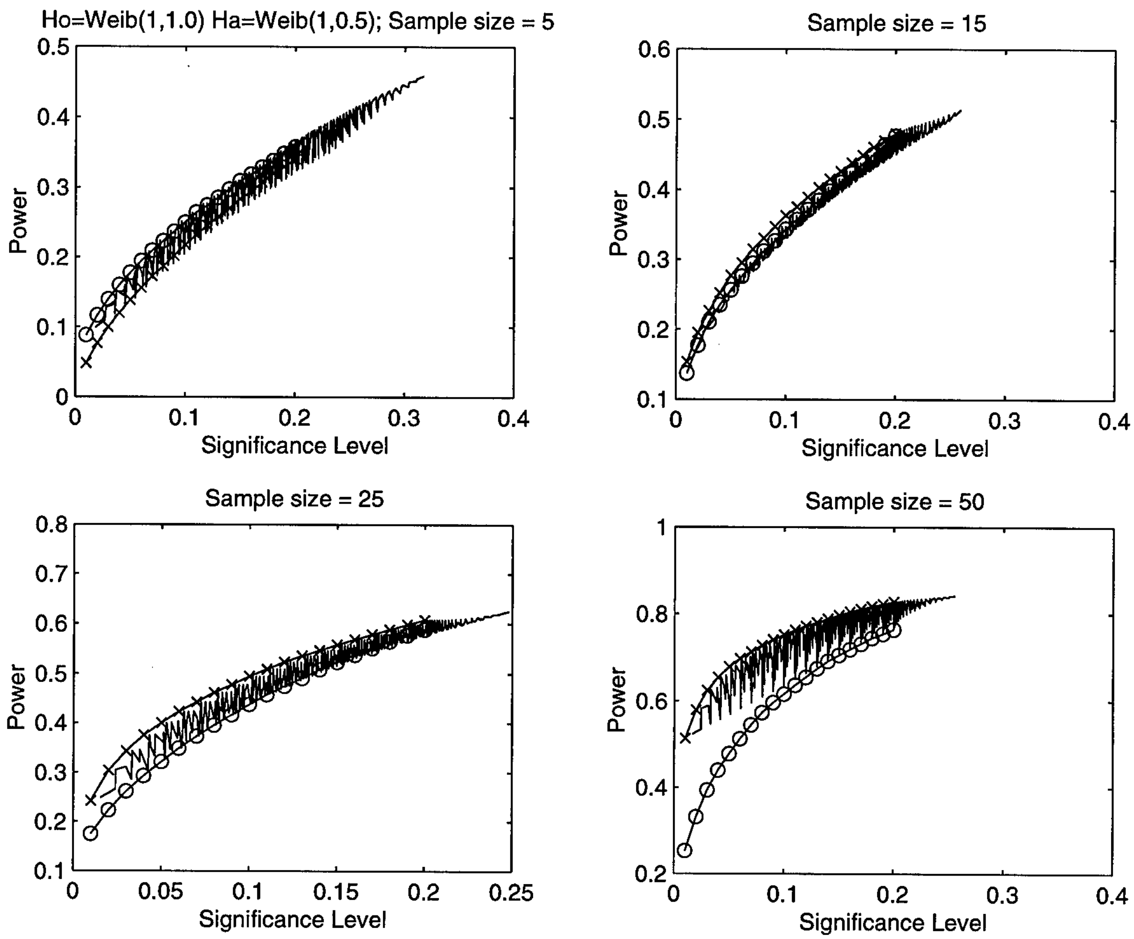


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.13 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Gamma(2,1).

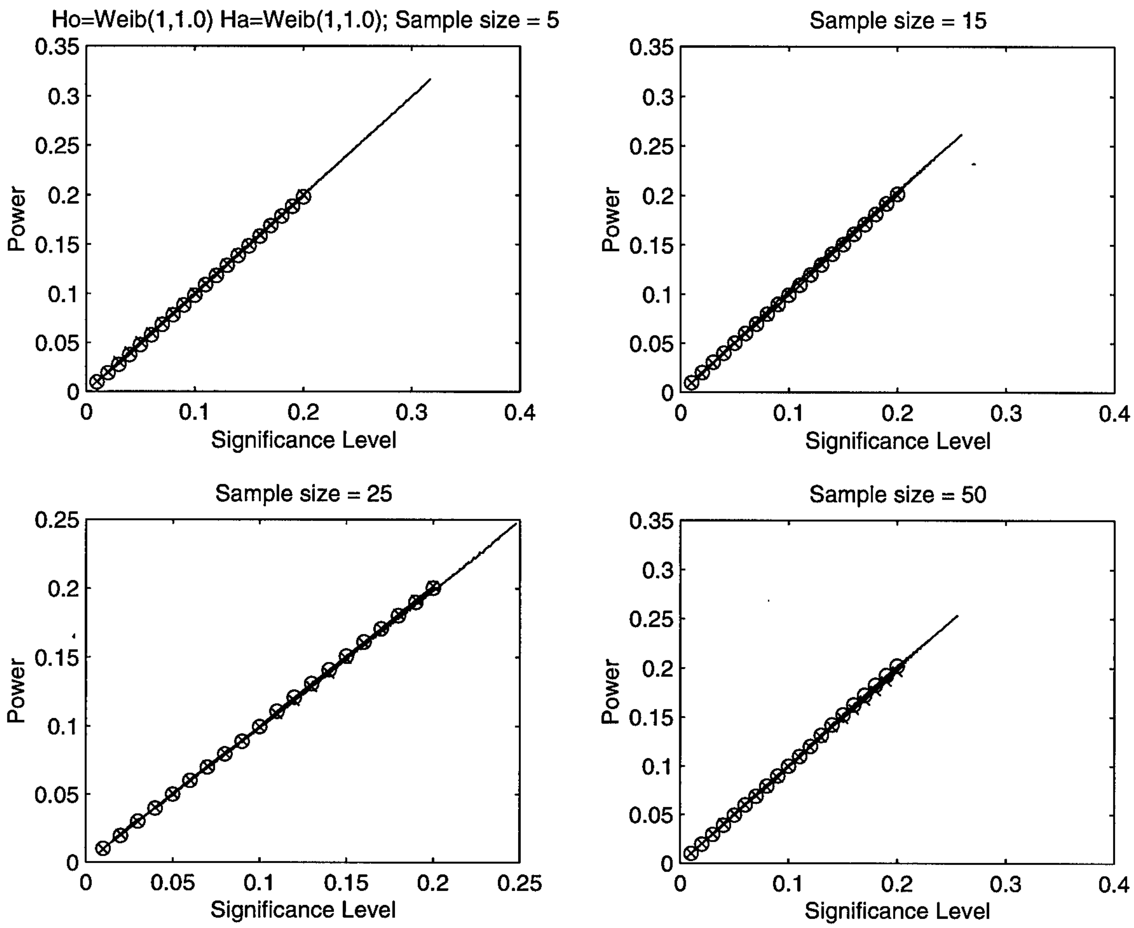


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.14 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 0.5$).

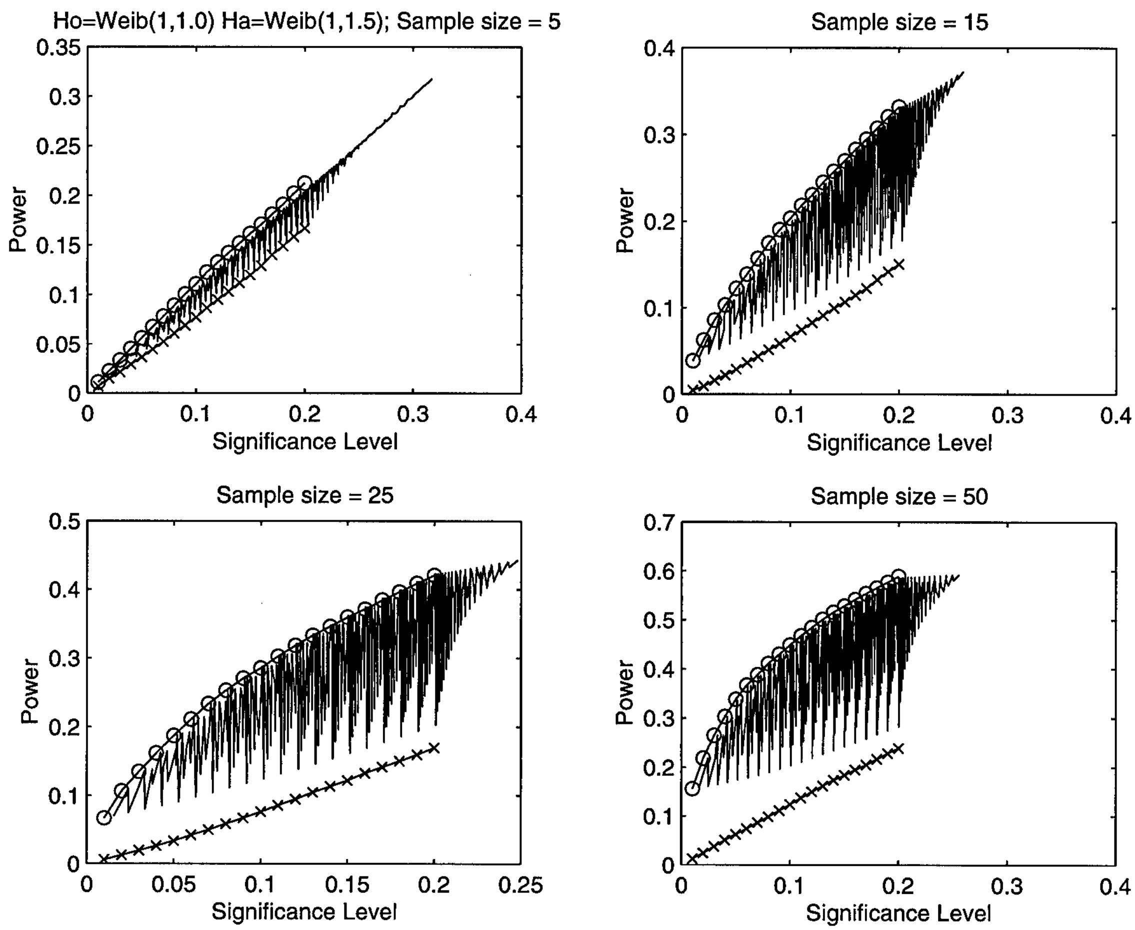


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.15 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 1.0$).

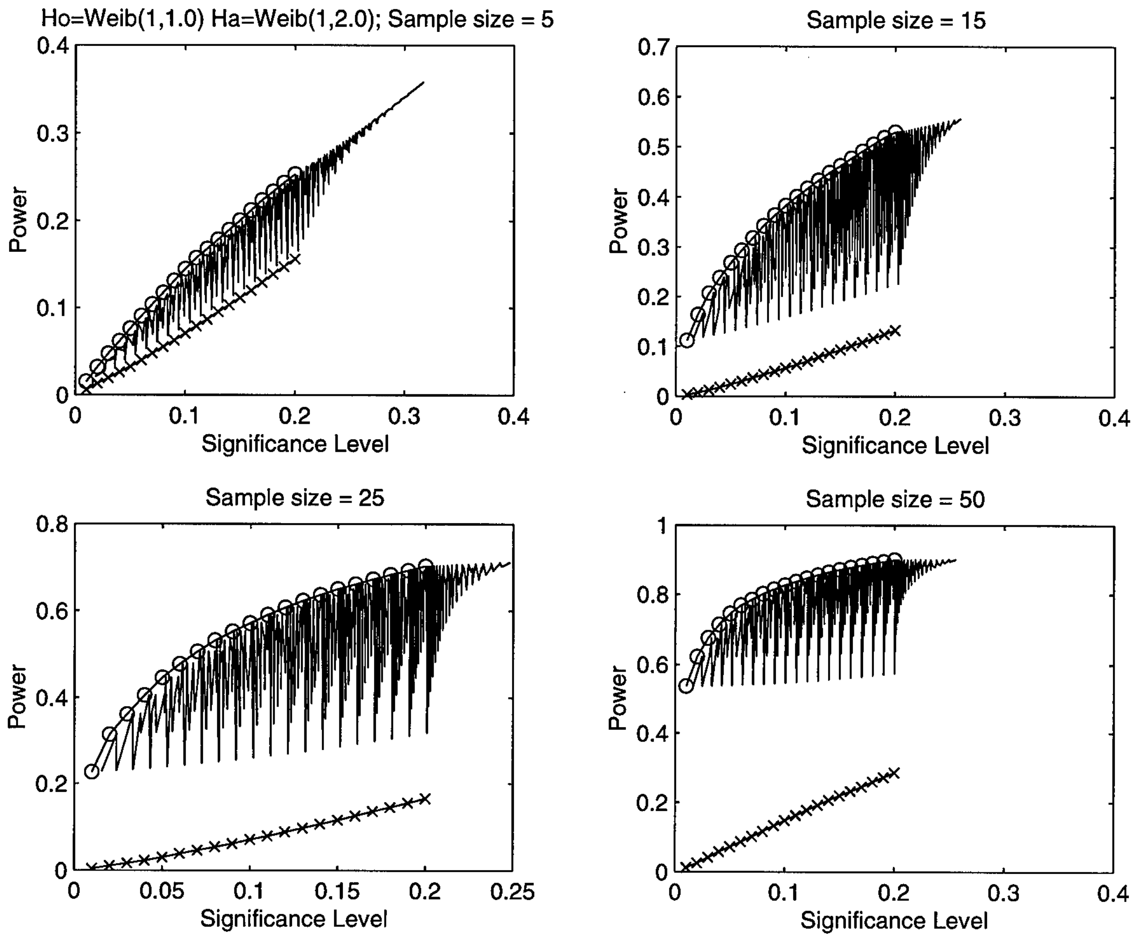


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.16 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 1.5$).



— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.17 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 2.0$).

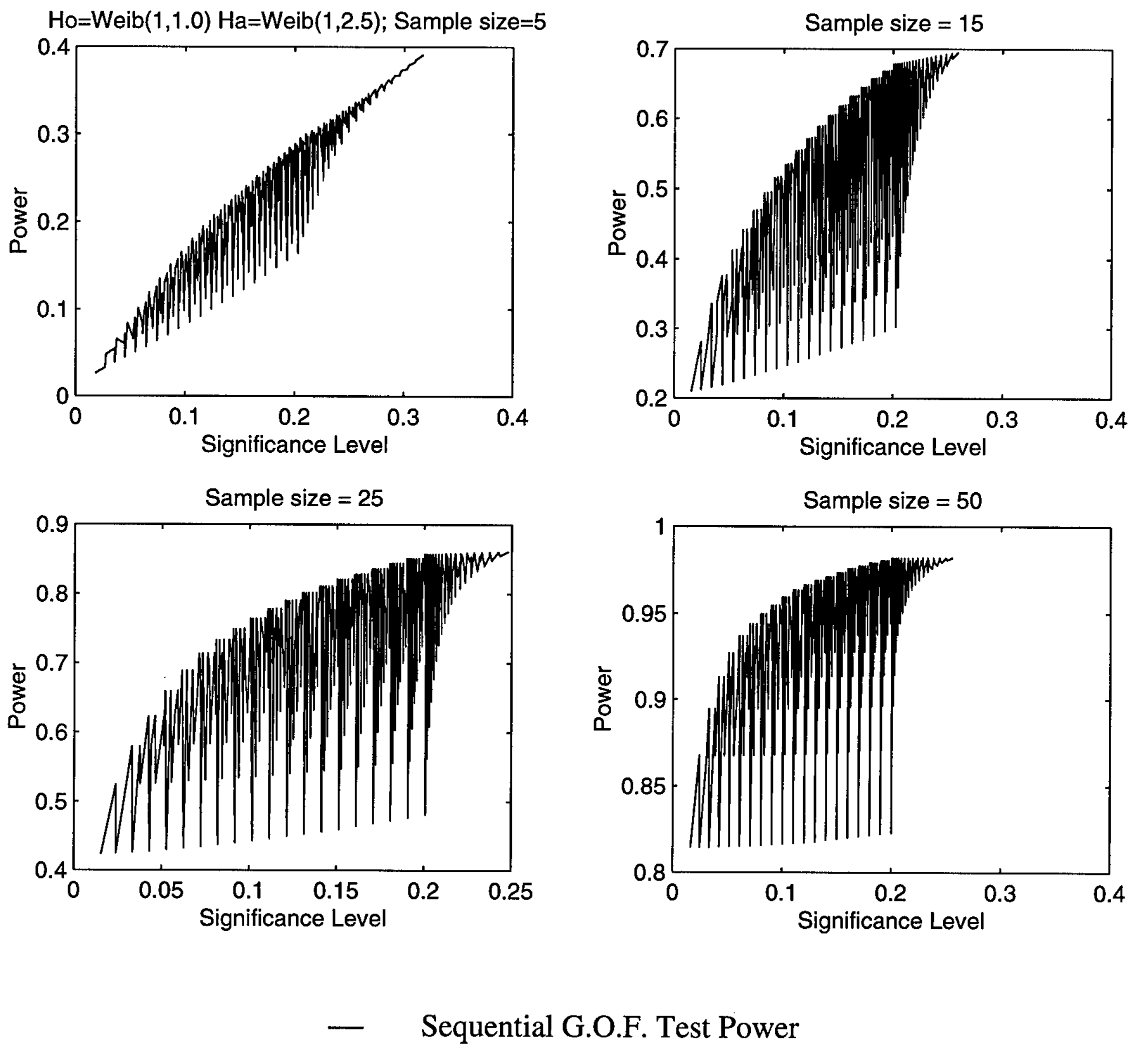


Figure F.18 Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 2.5$).

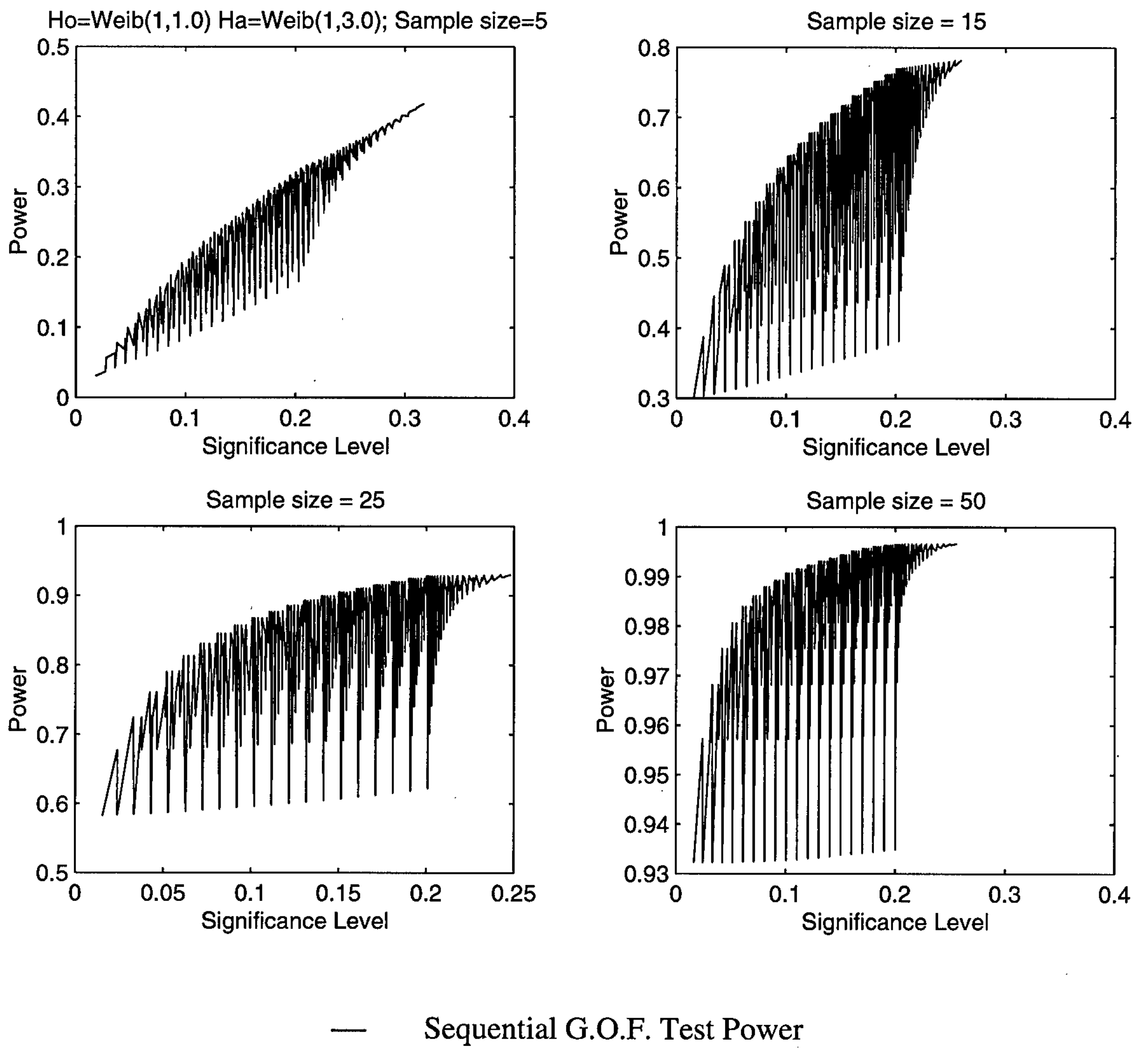
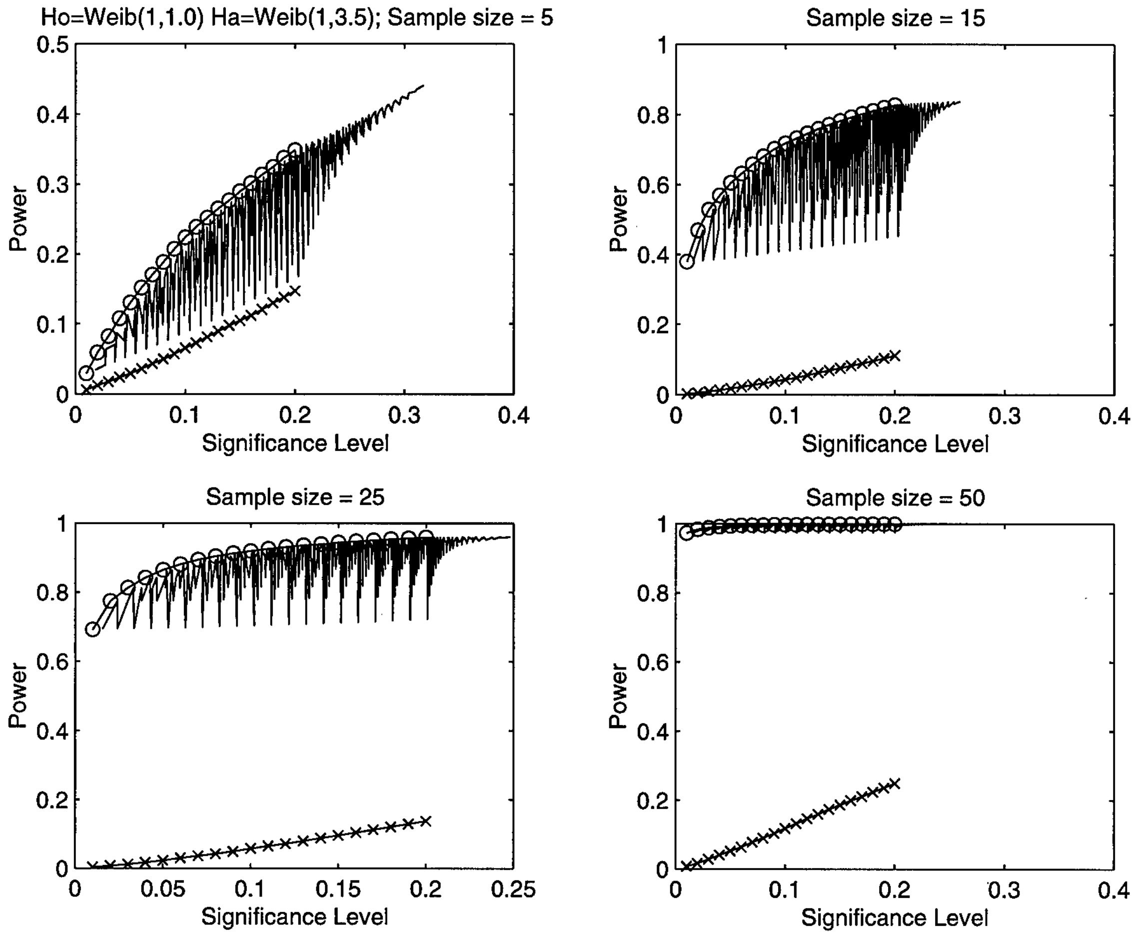


Figure F.19 Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 3.0$).

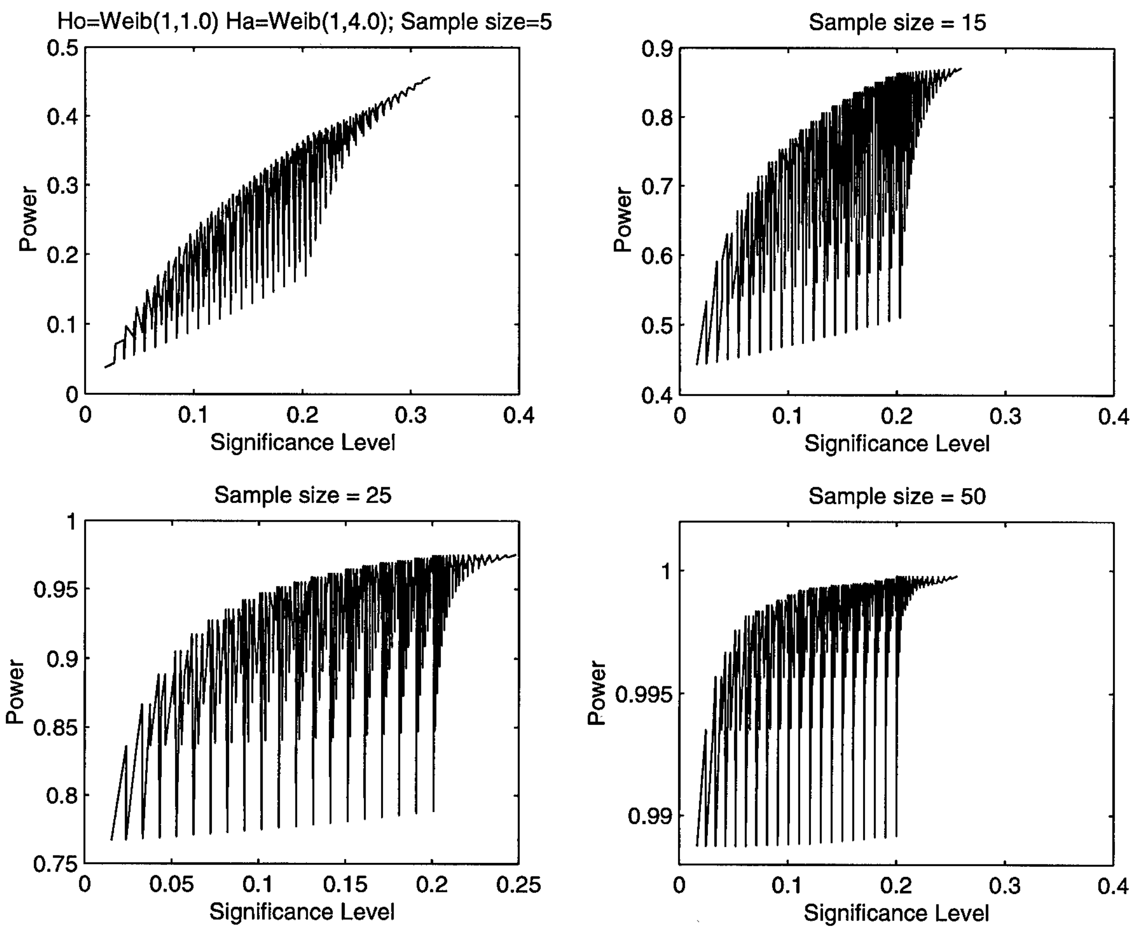


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

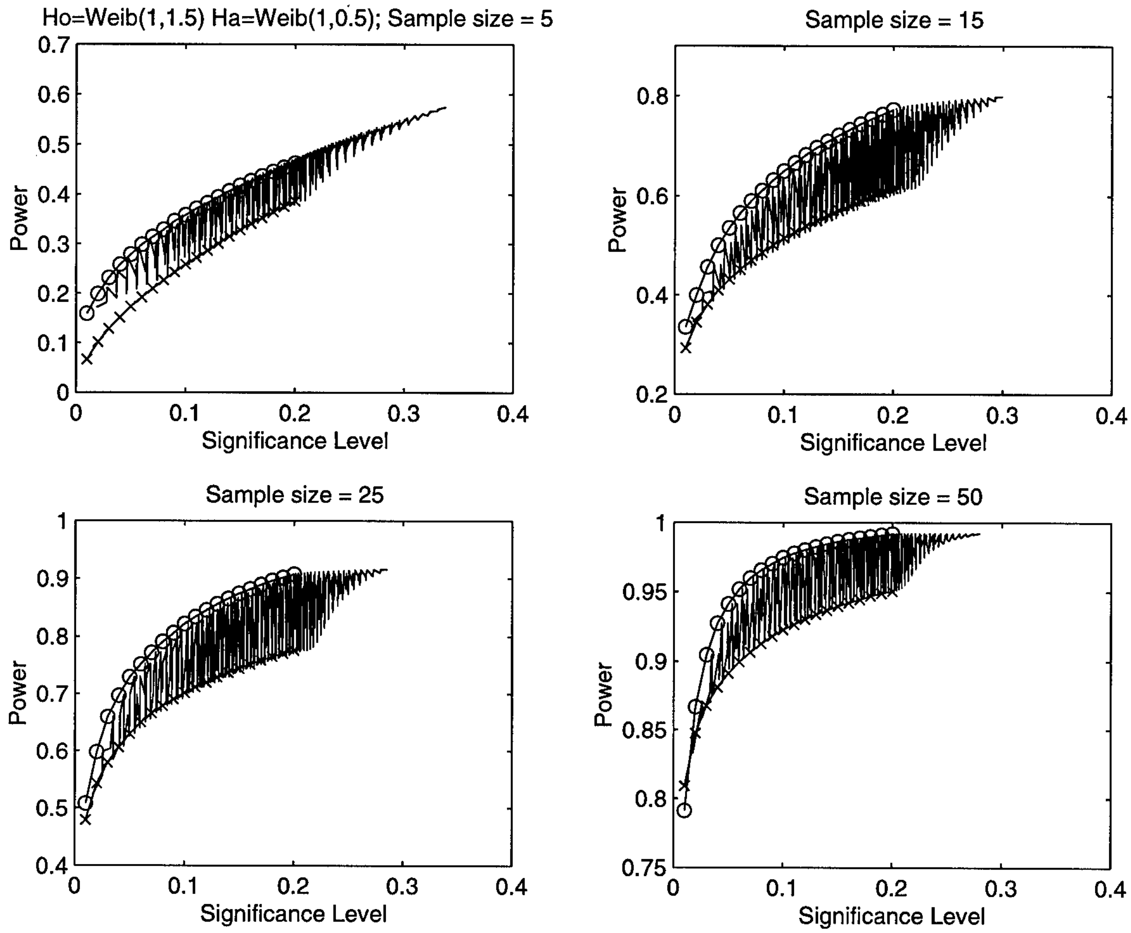
Figure F.20 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

Figure F.21 Sequential Power: H_o: Weibull($\beta = 1.0$); H_a: Weibull($\beta = 4.0$).

F.3 $\quad H_o$*: Weibull*($\beta = 1.5$).

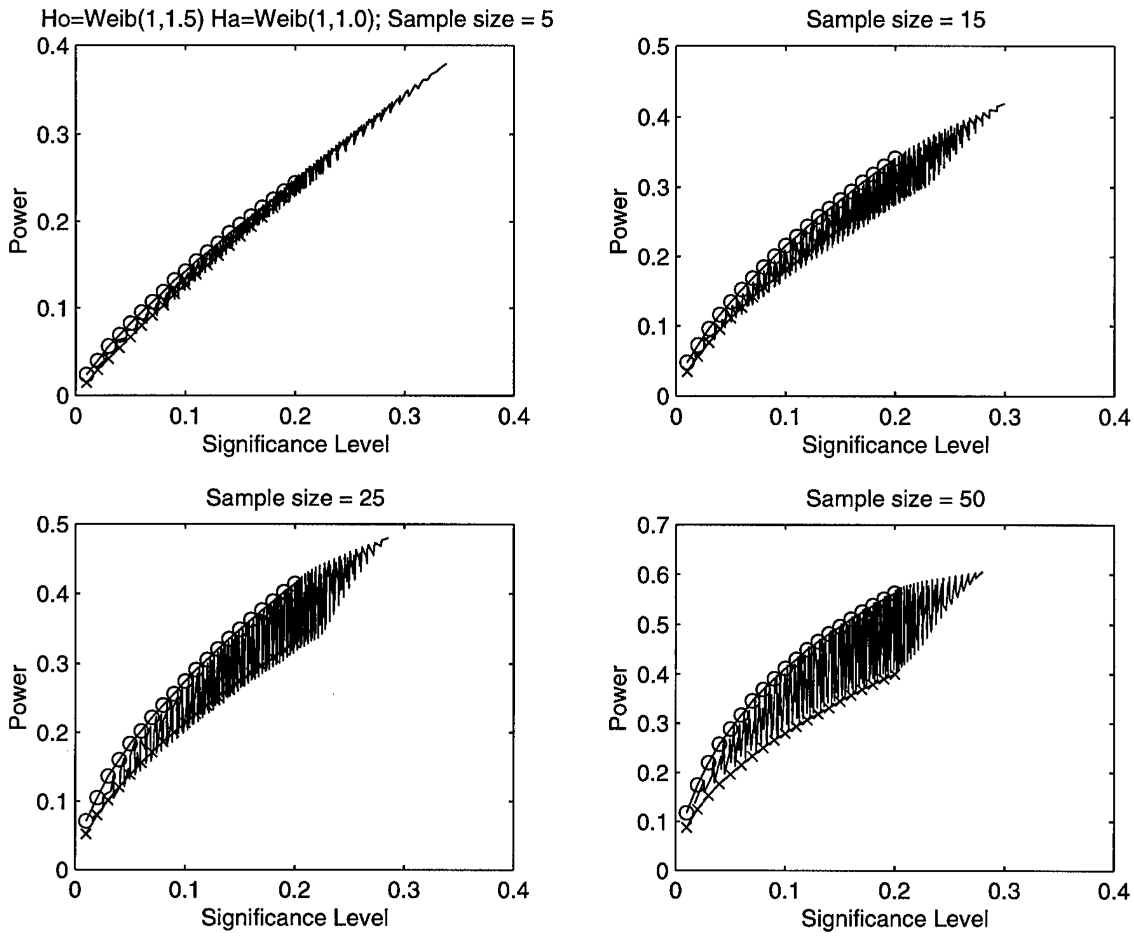


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.22 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 0.5$).

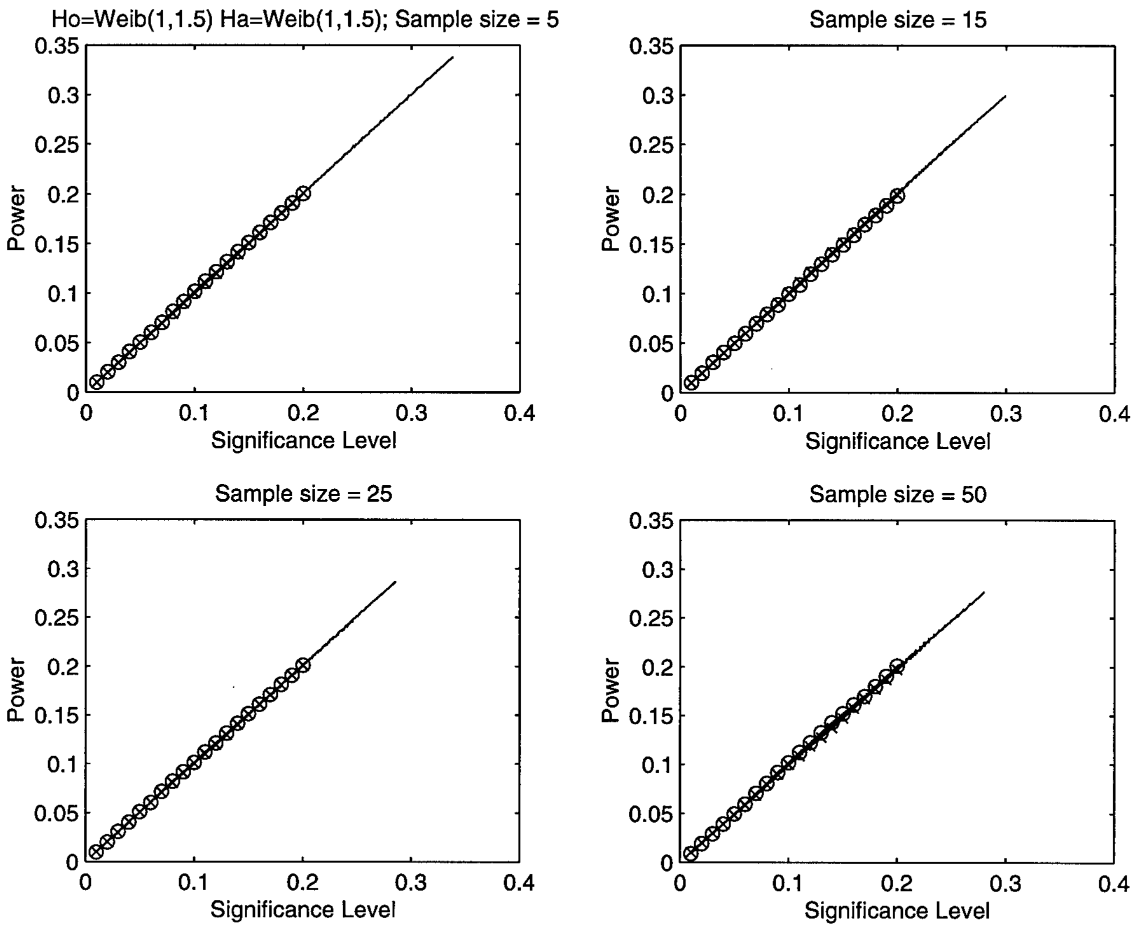


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.23 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 1.0$).

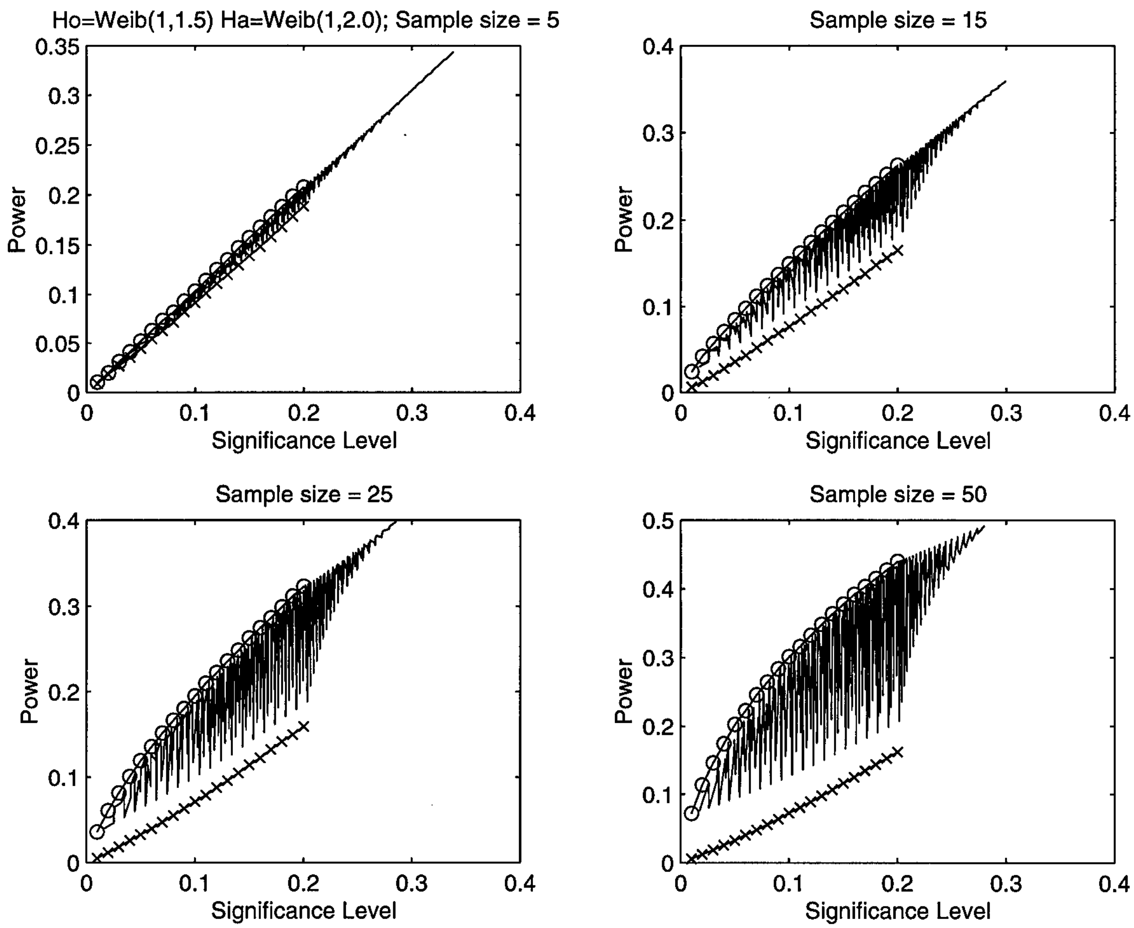


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.24 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 1.5$).



— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.25 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 2.0$).

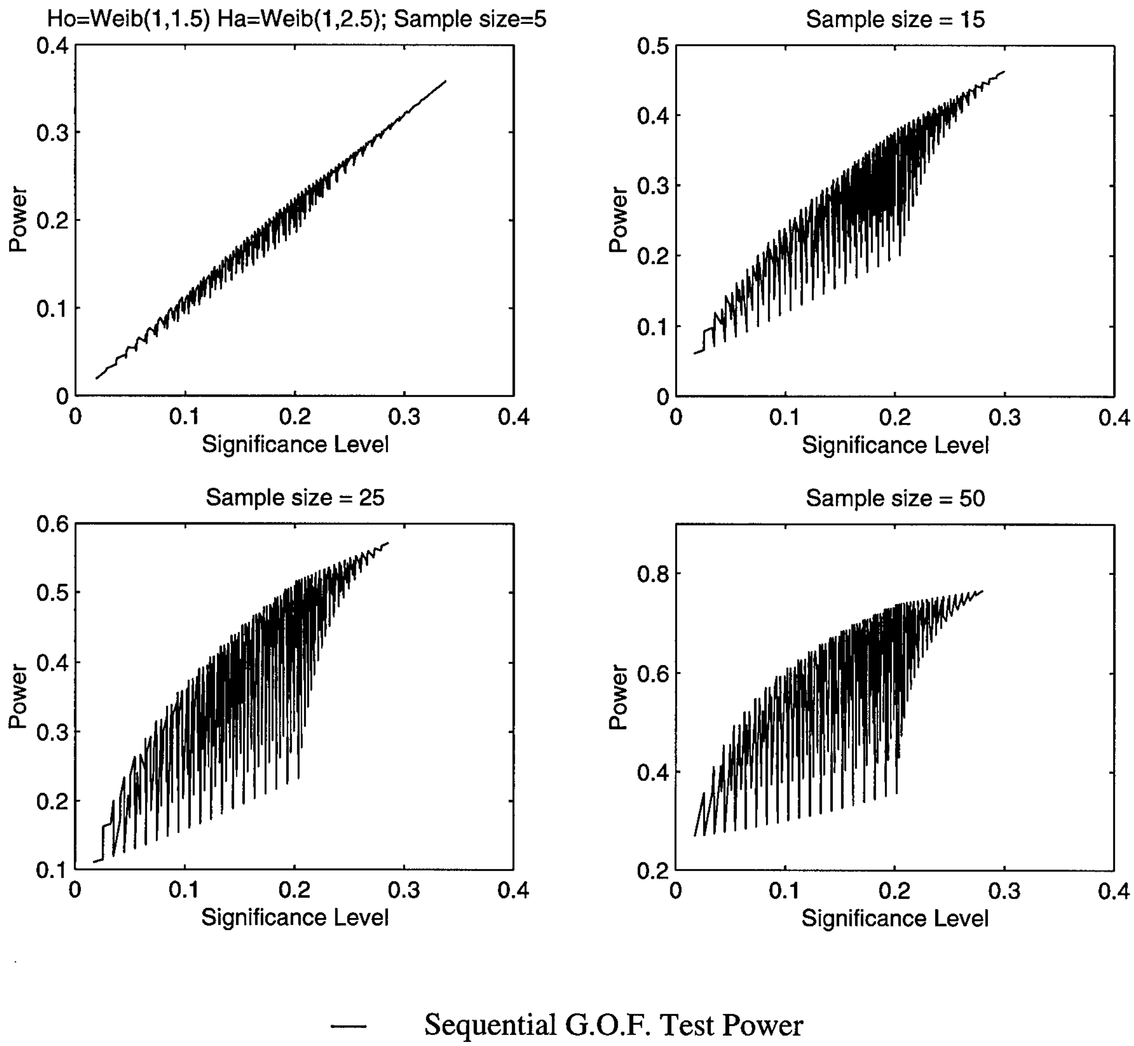


Figure F.26 Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 2.5$).

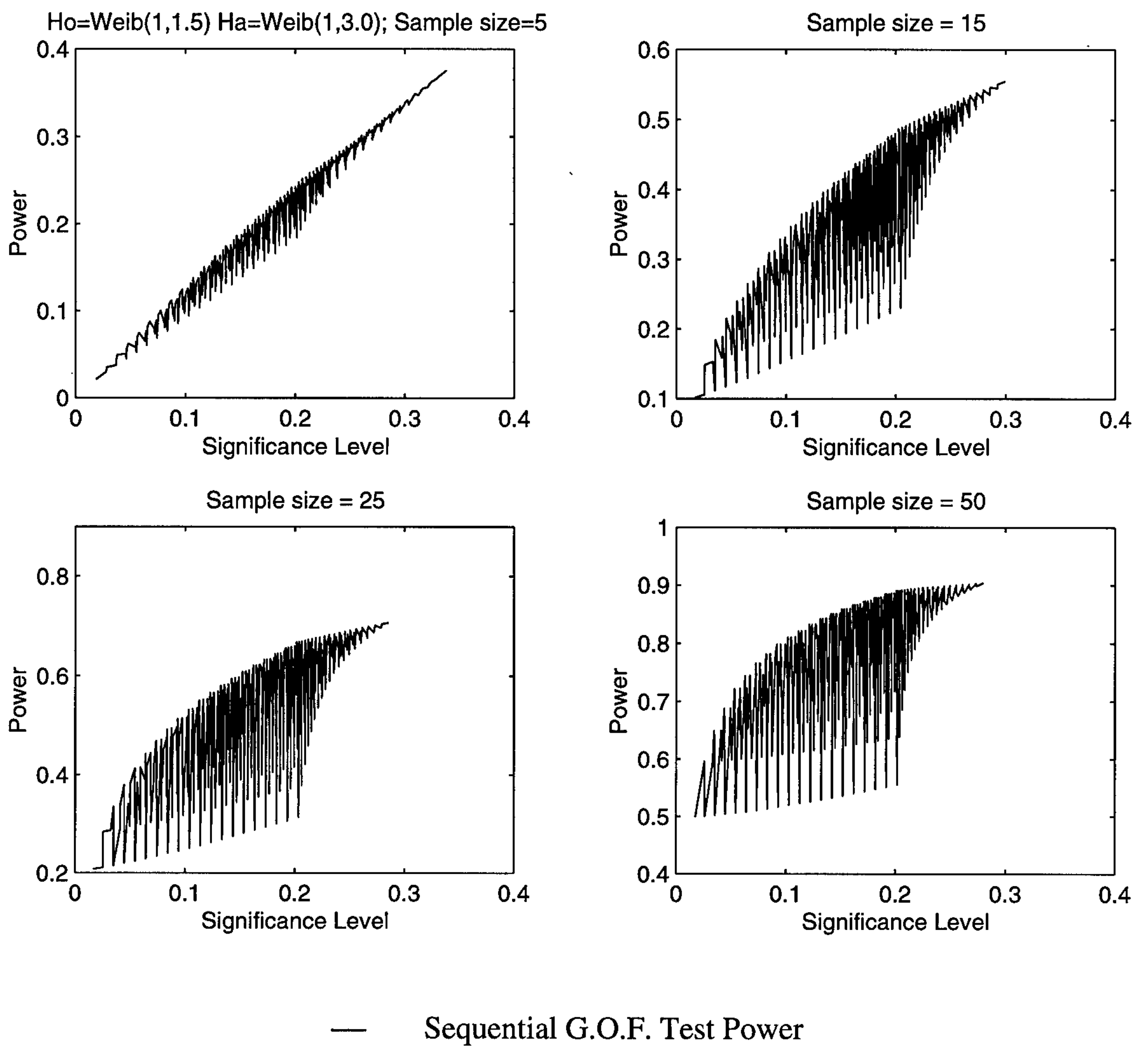
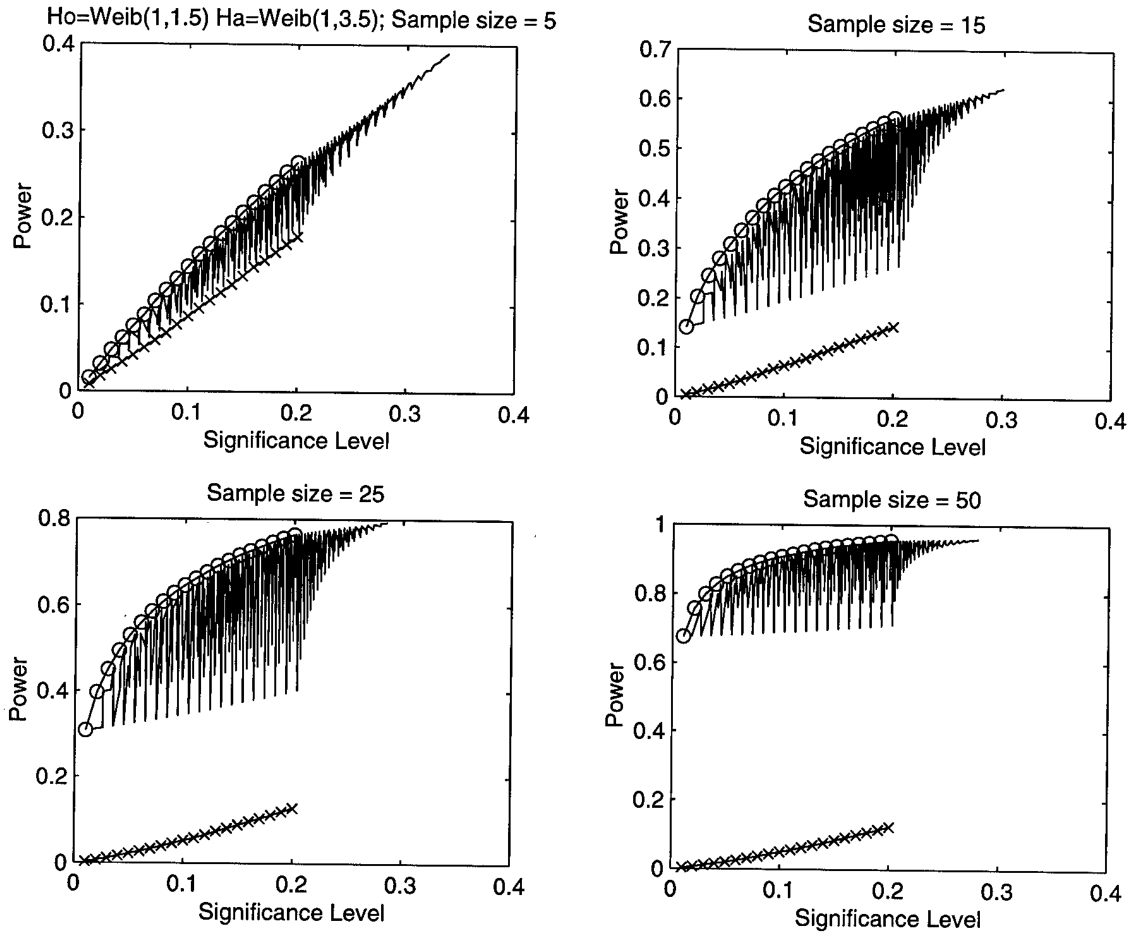


Figure F.27 Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 3.0$).

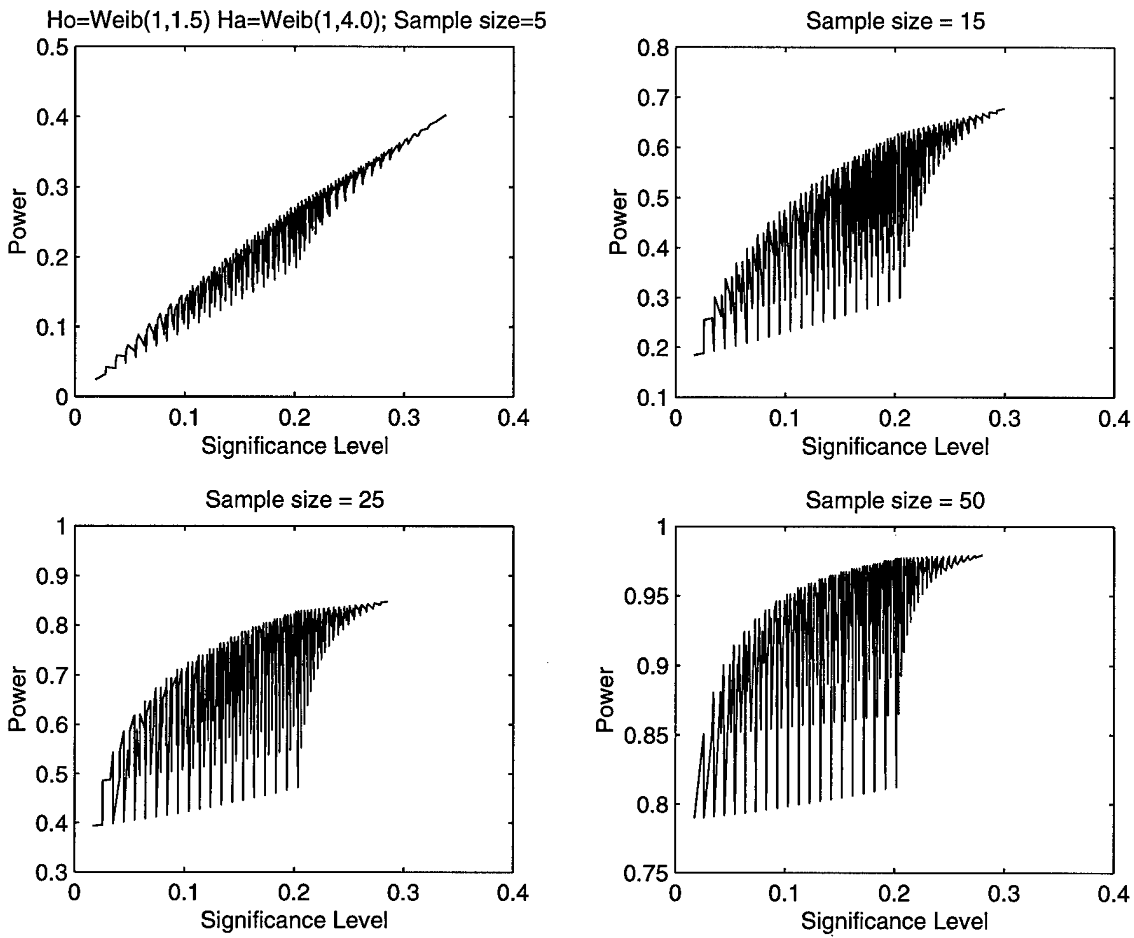


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

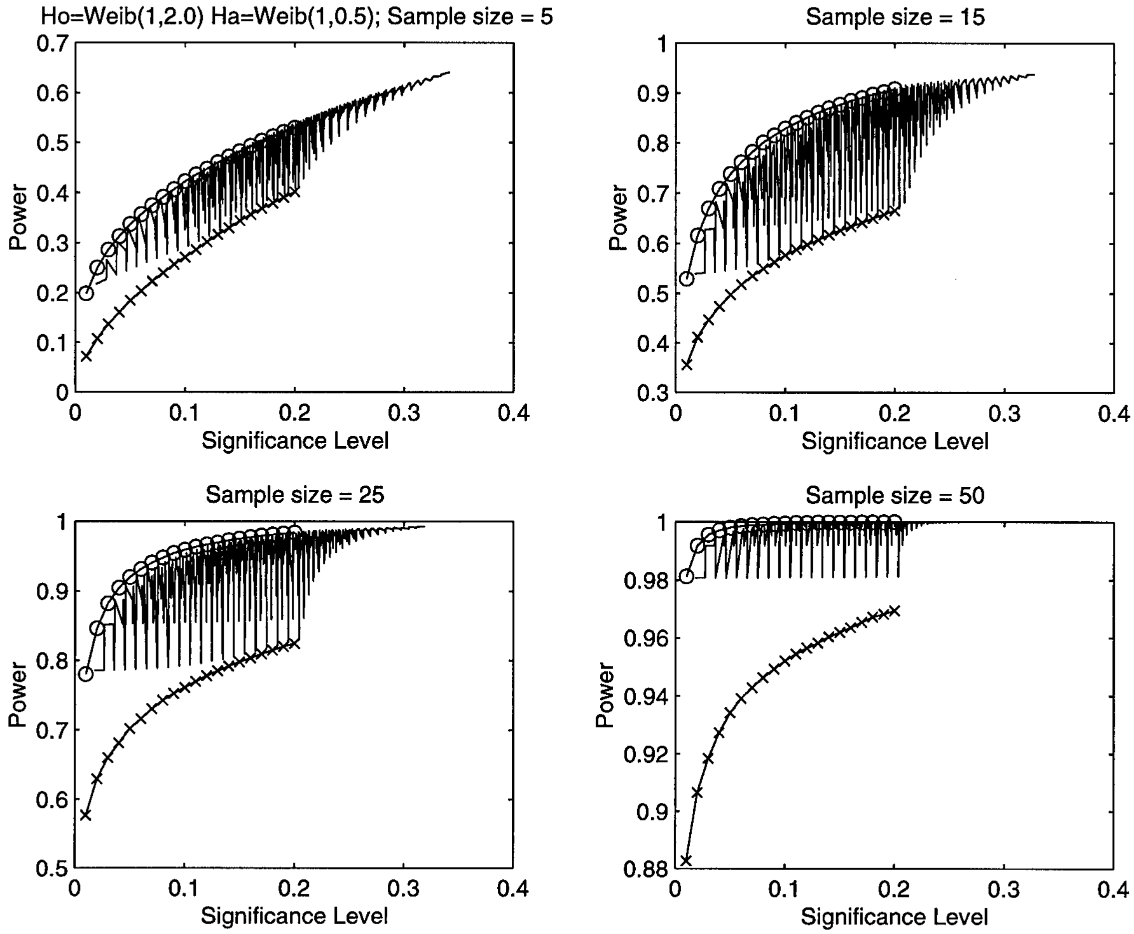
Figure F.28 Individual vs. Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

Figure F.29 Sequential Power: H_o: Weibull($\beta = 1.5$); H_a: Weibull($\beta = 4.0$).

F.4 H_o: *Weibull*(β = 2.0).

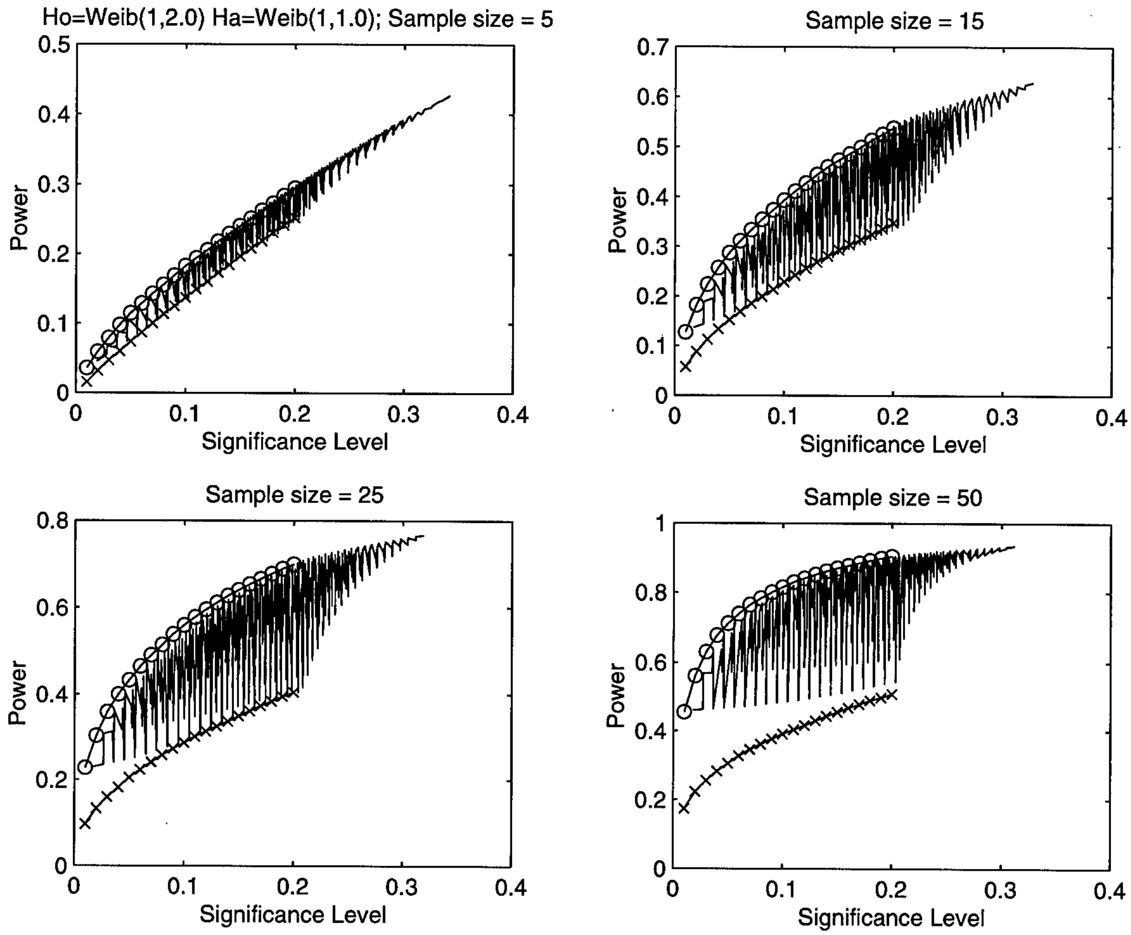


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.30 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 0.5$).

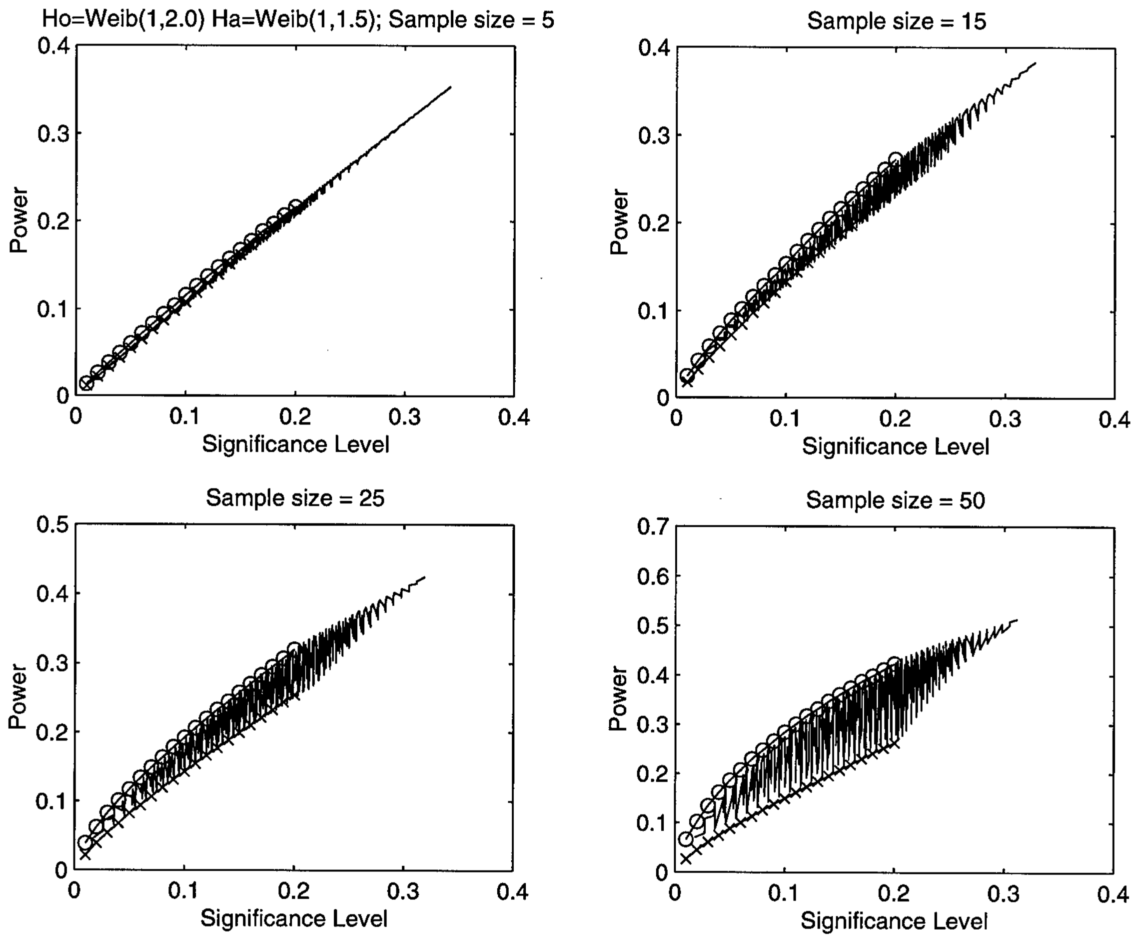


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.31 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 1.0$).

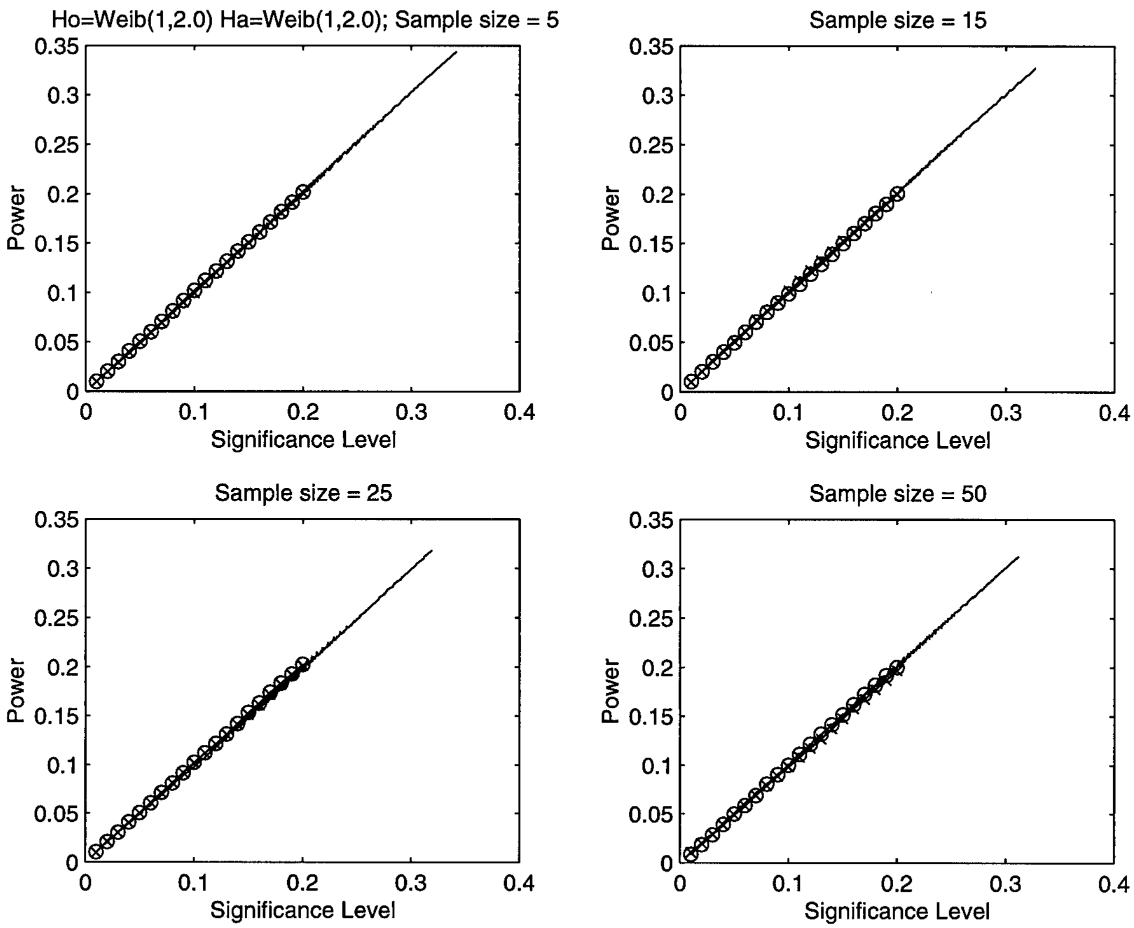


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.32 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 1.5$).



— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.33 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 2.0$).

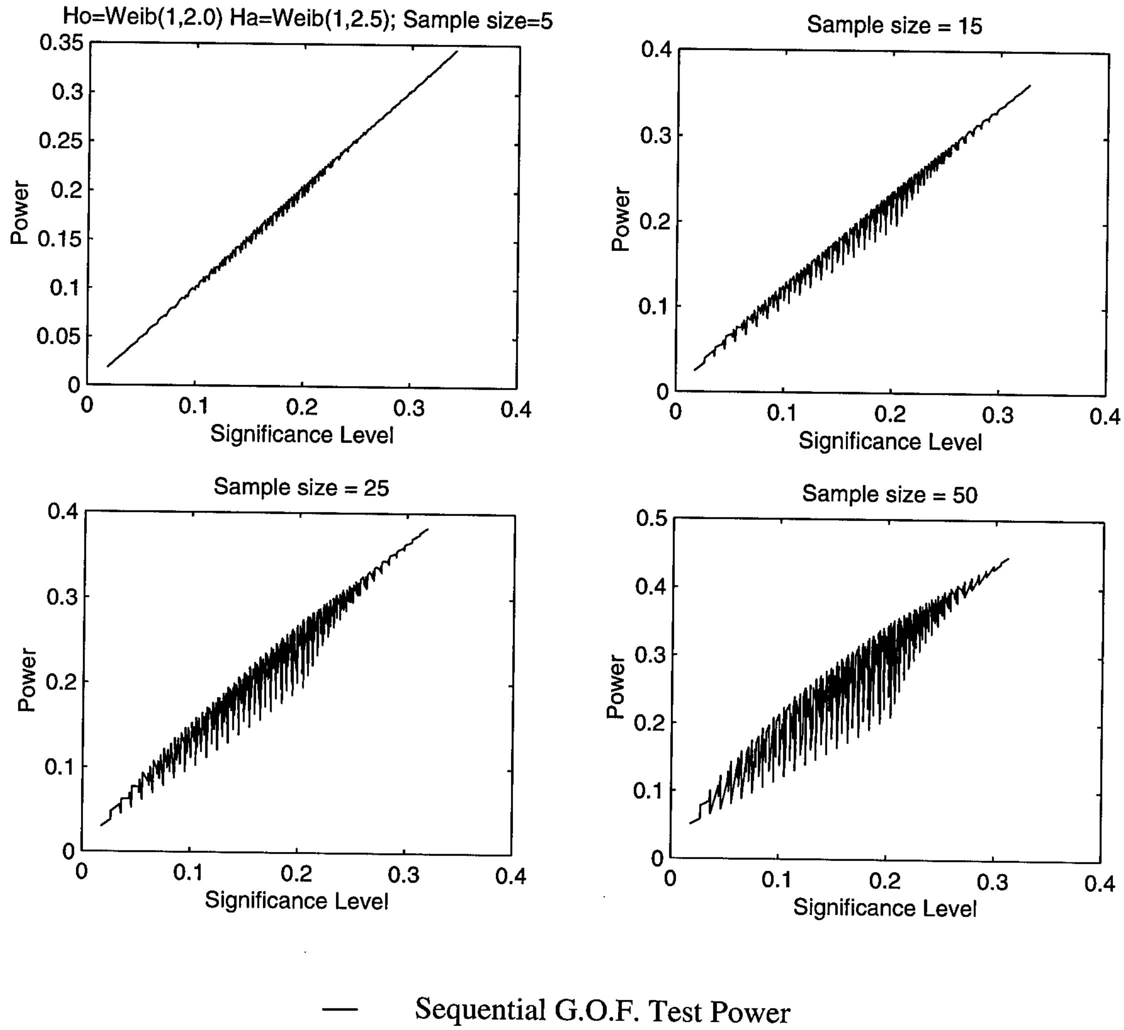


Figure F.34 Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 2.5$).

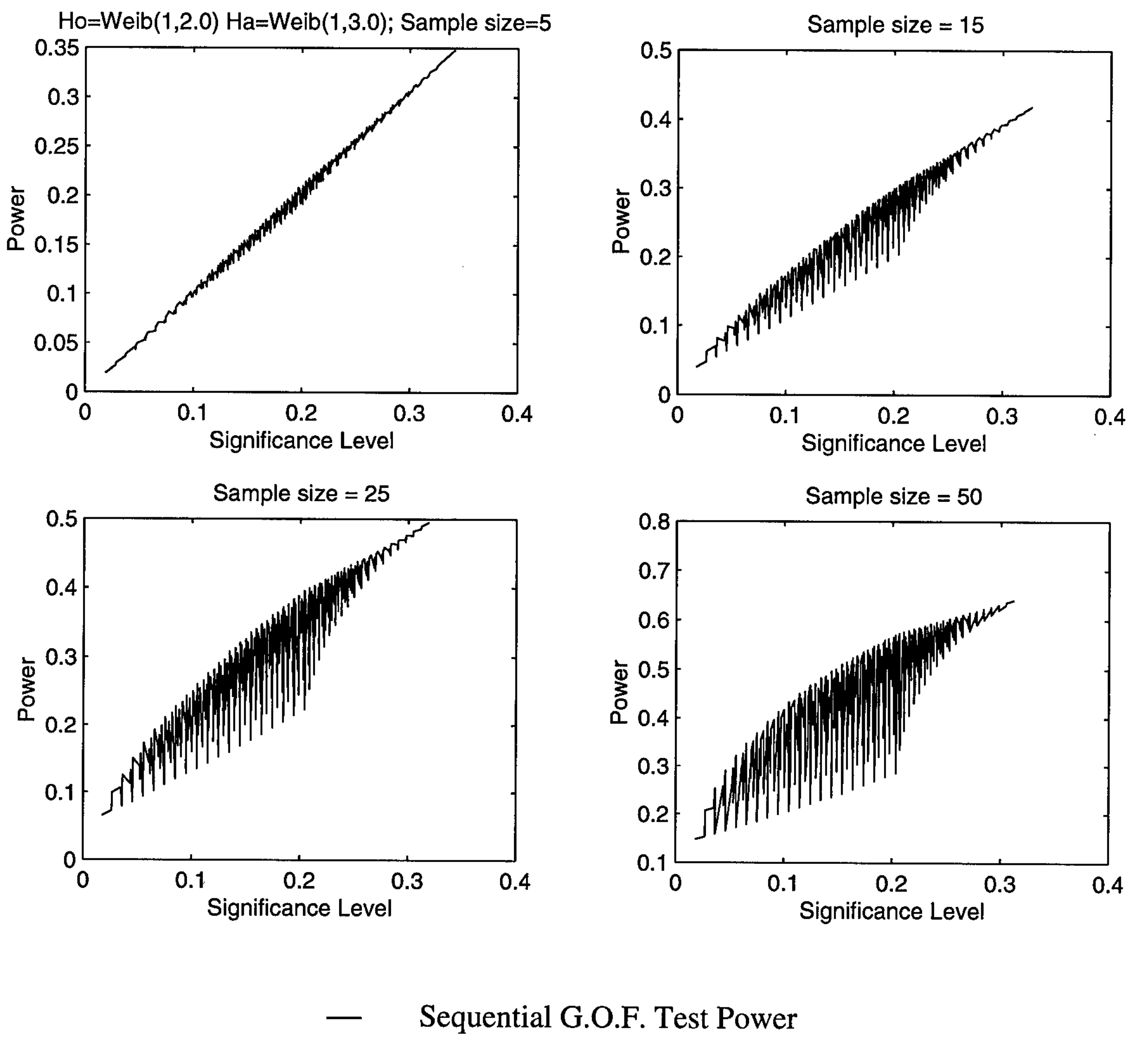
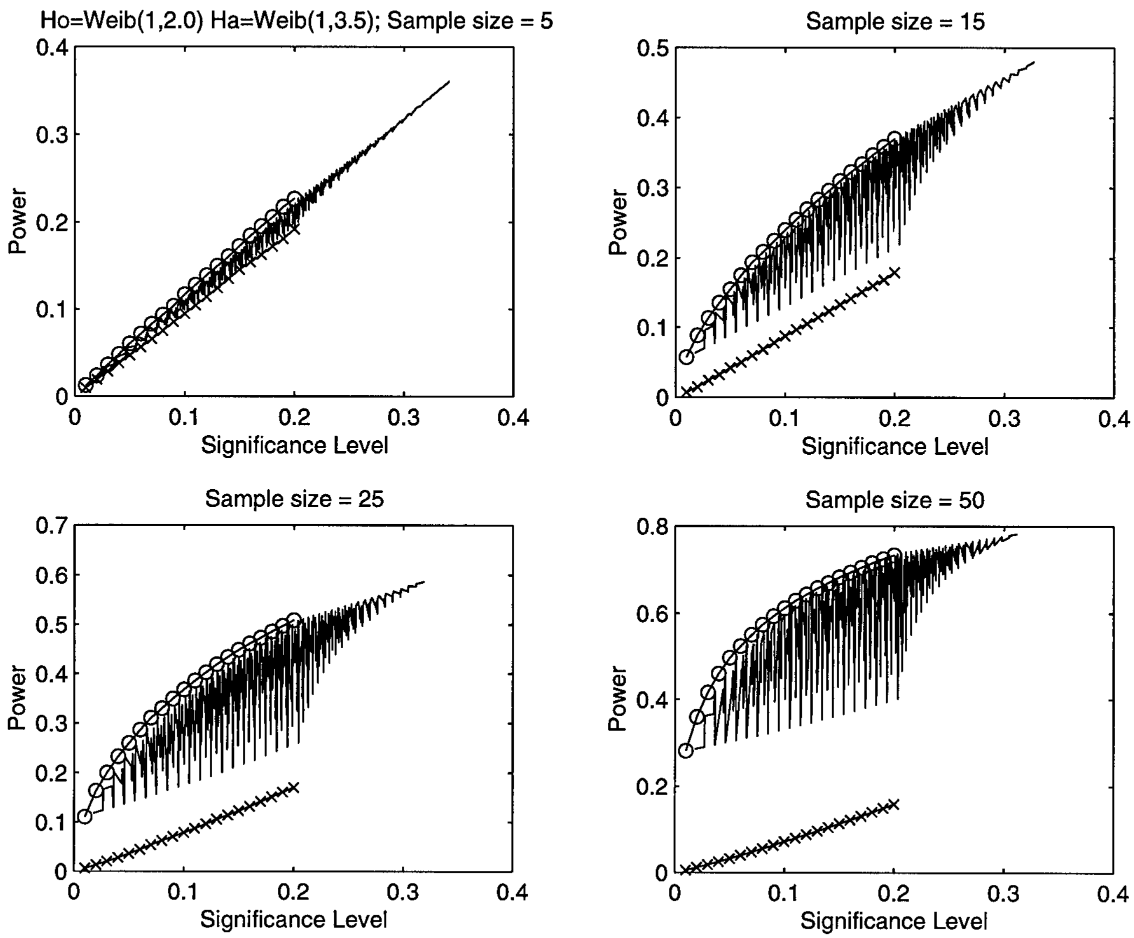


Figure F.35 Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 3.0$).

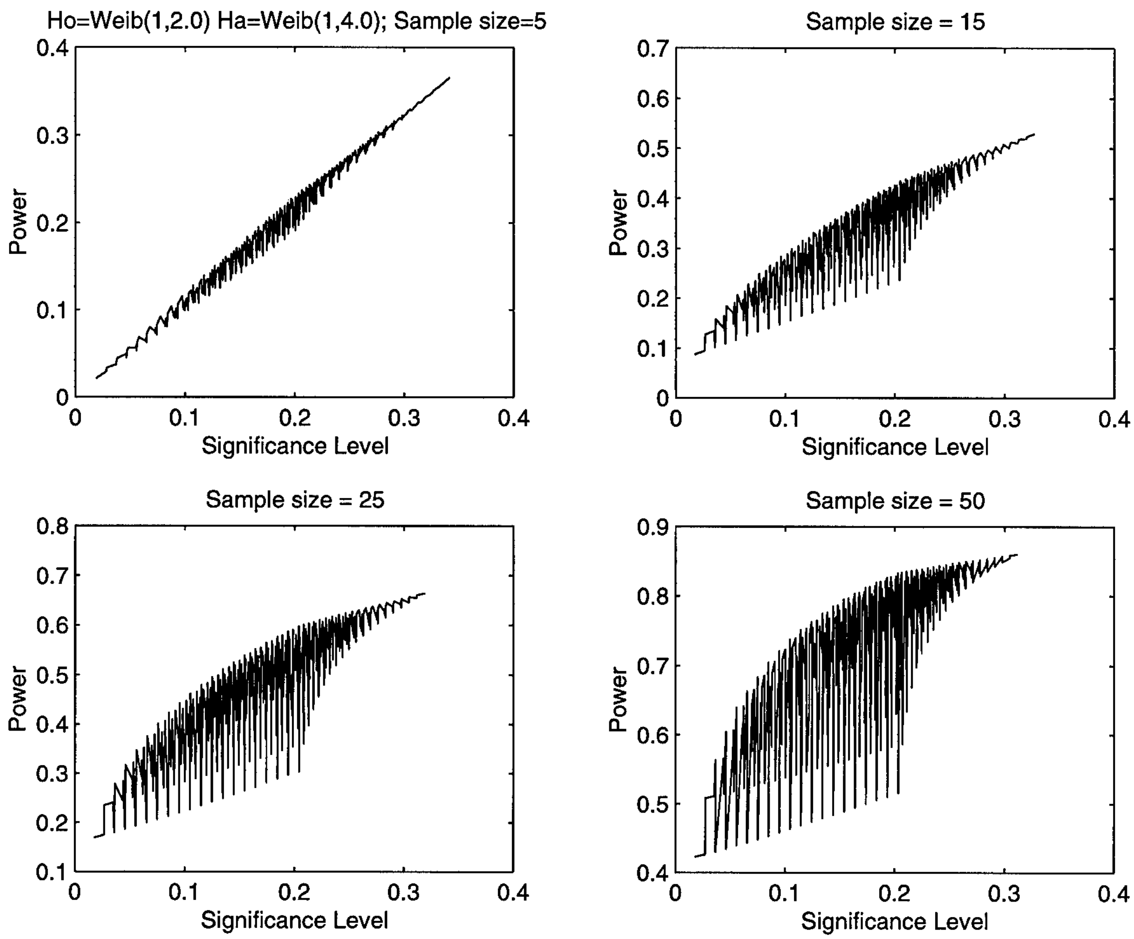


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.36 Individual vs. Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

Figure F.37 Sequential Power: H_o: Weibull($\beta = 2.0$); H_a: Weibull($\beta = 4.0$).

F.5 H_o: Weibull($\beta = 2.5$).

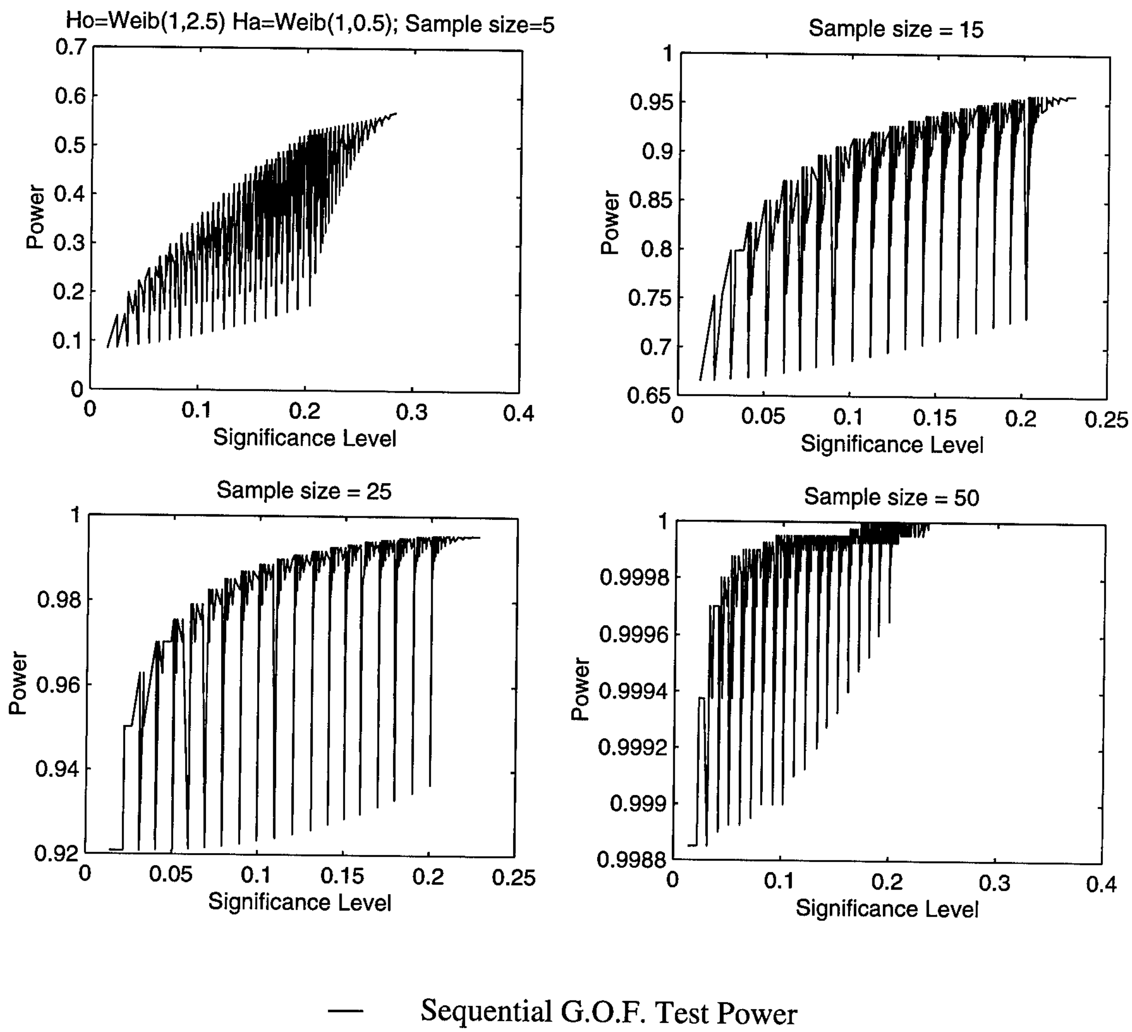
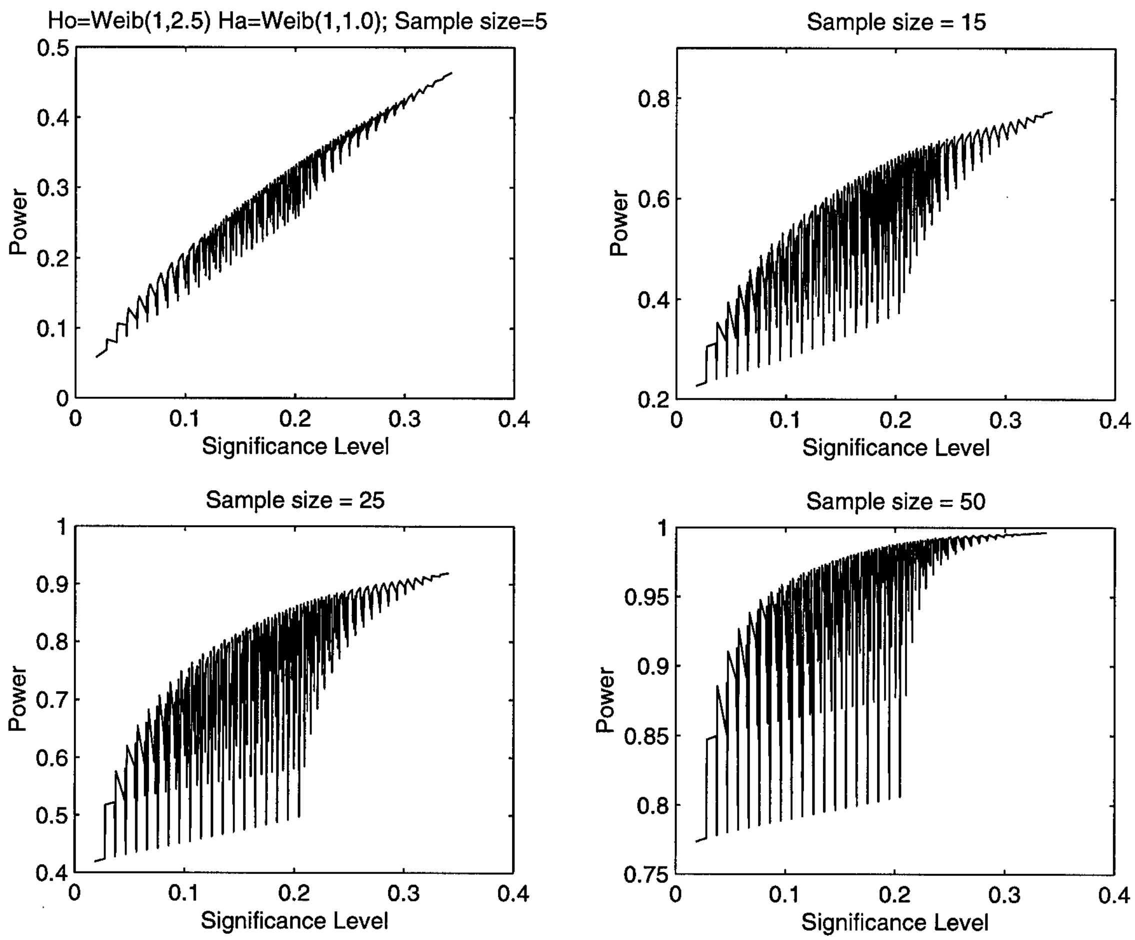
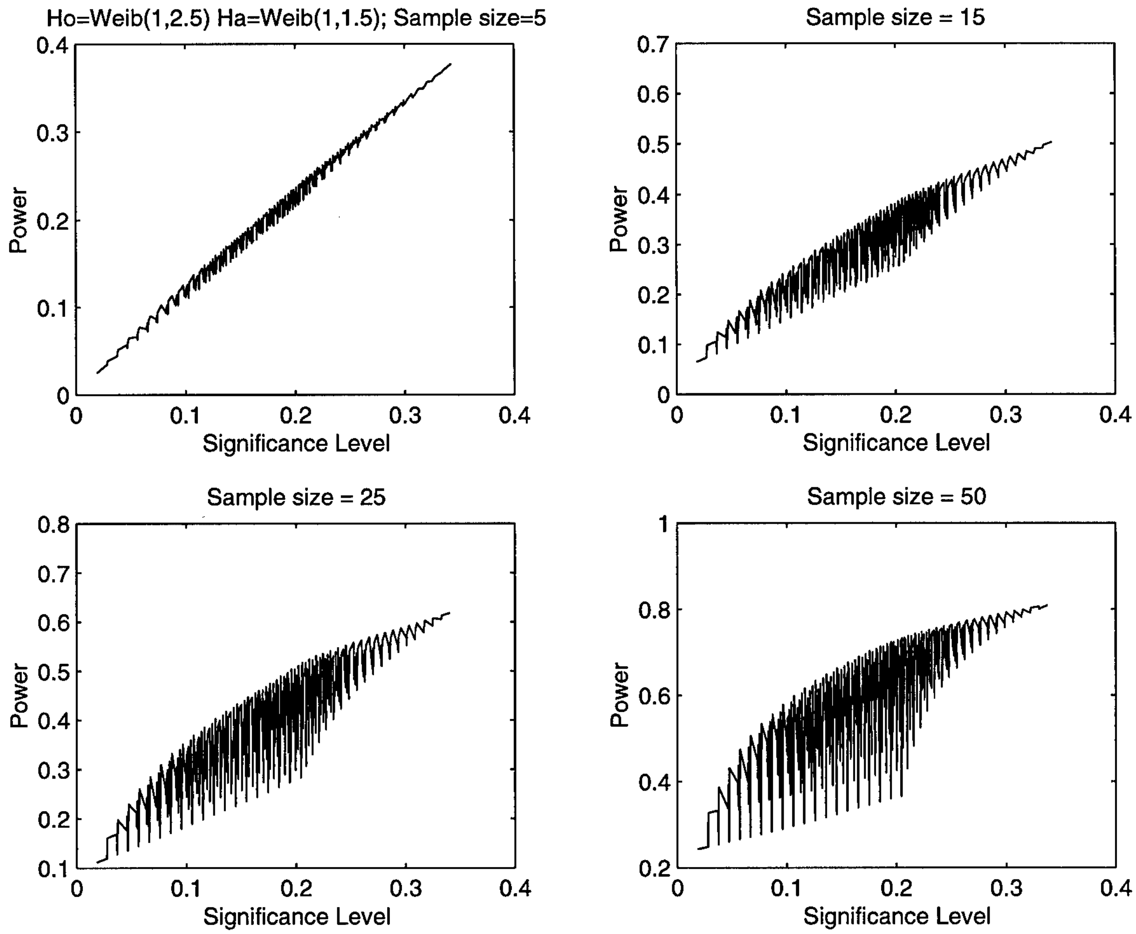


Figure F.38 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 0.5$).



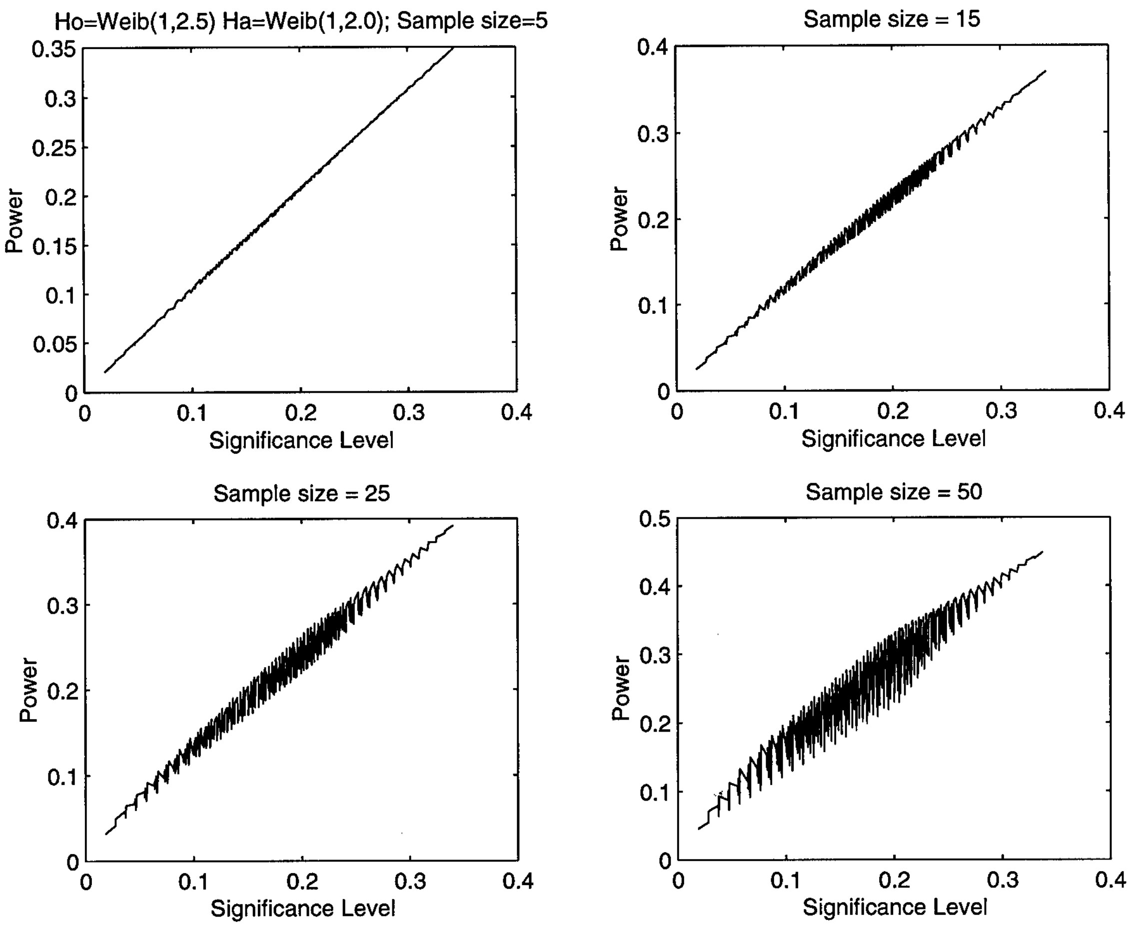
— Sequential G.O.F. Test Power

Figure F.39 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 1.0$).



— Sequential G.O.F. Test Power

Figure F.40 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 1.5$).



— Sequential G.O.F. Test Power

Figure F.41 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 2.0$).

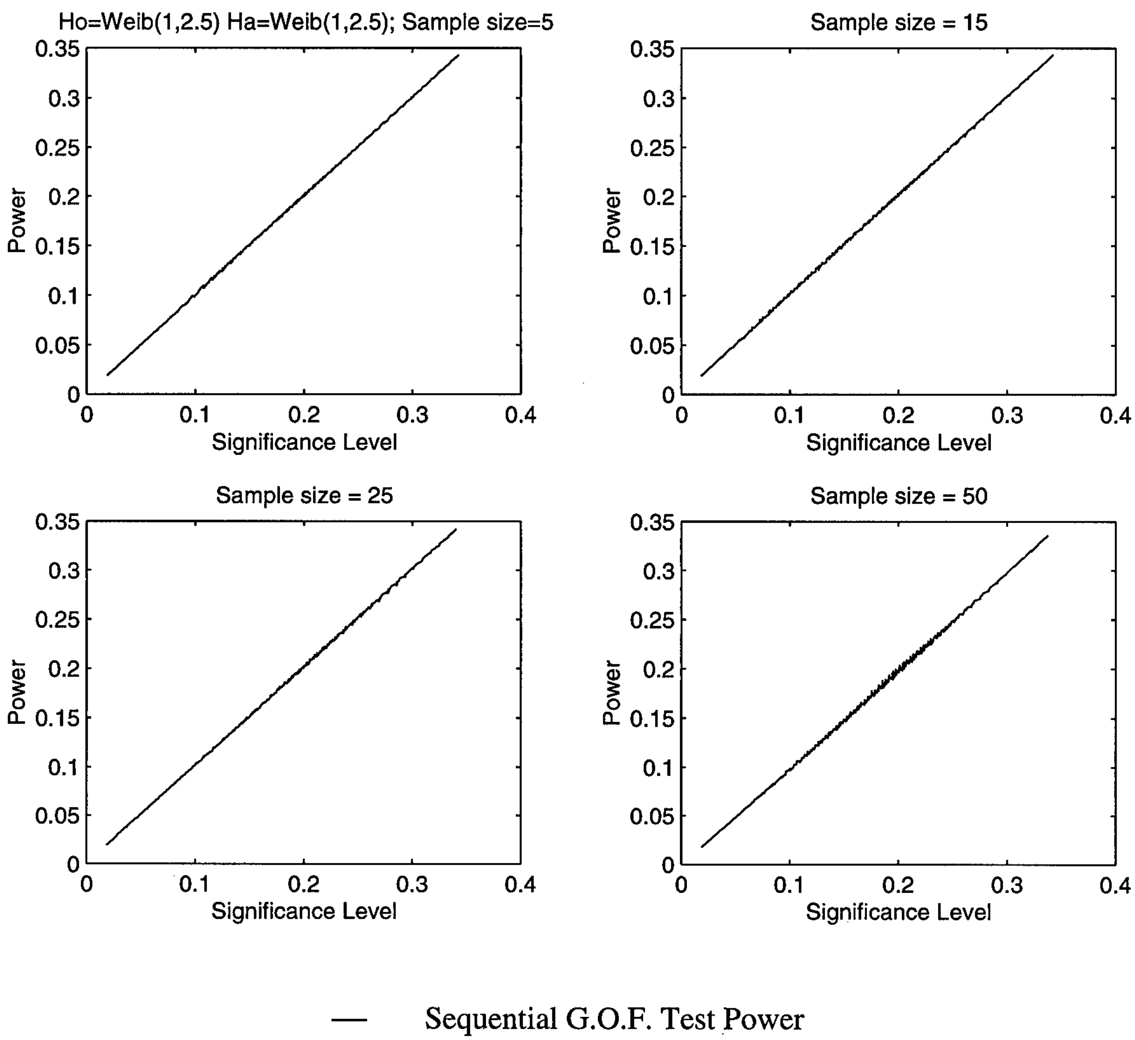
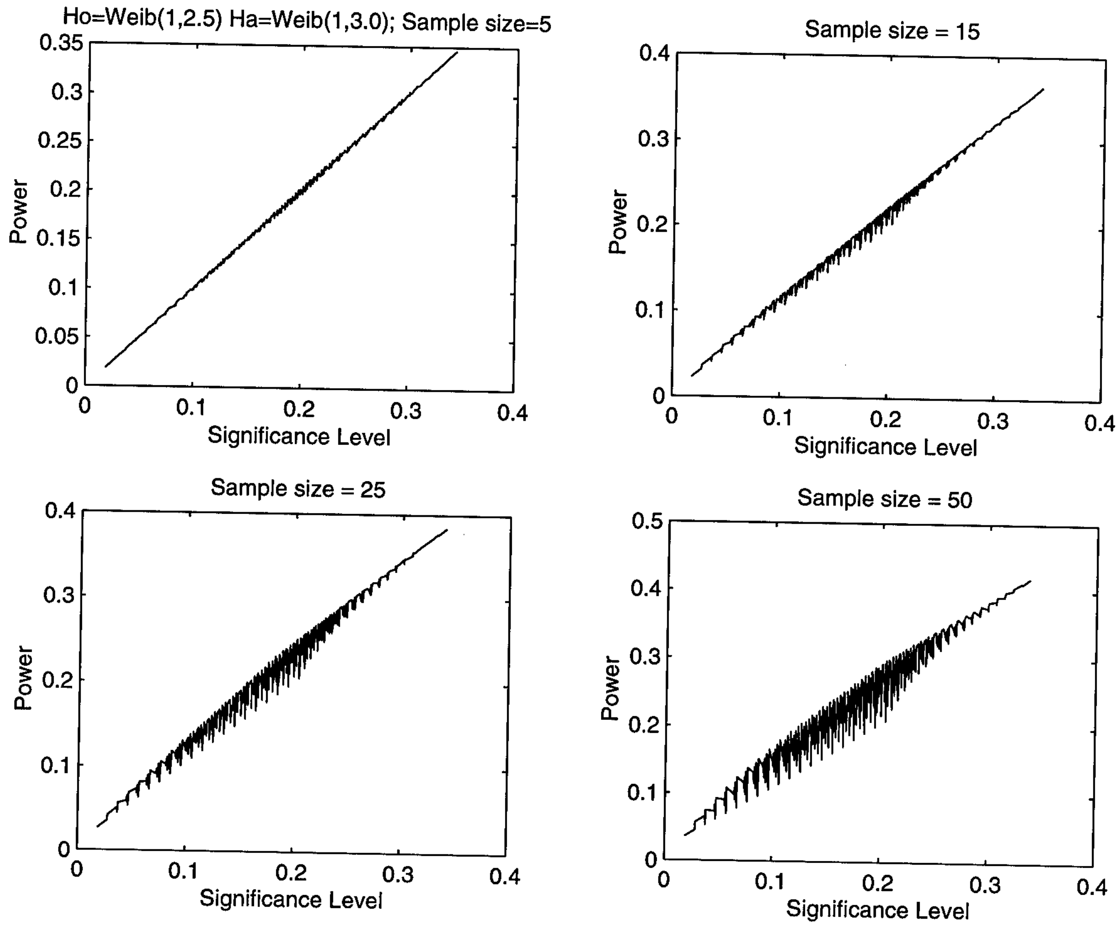


Figure F.42 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 2.5$).



— Sequential G.O.F. Test Power

Figure F.43 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 3.0$).

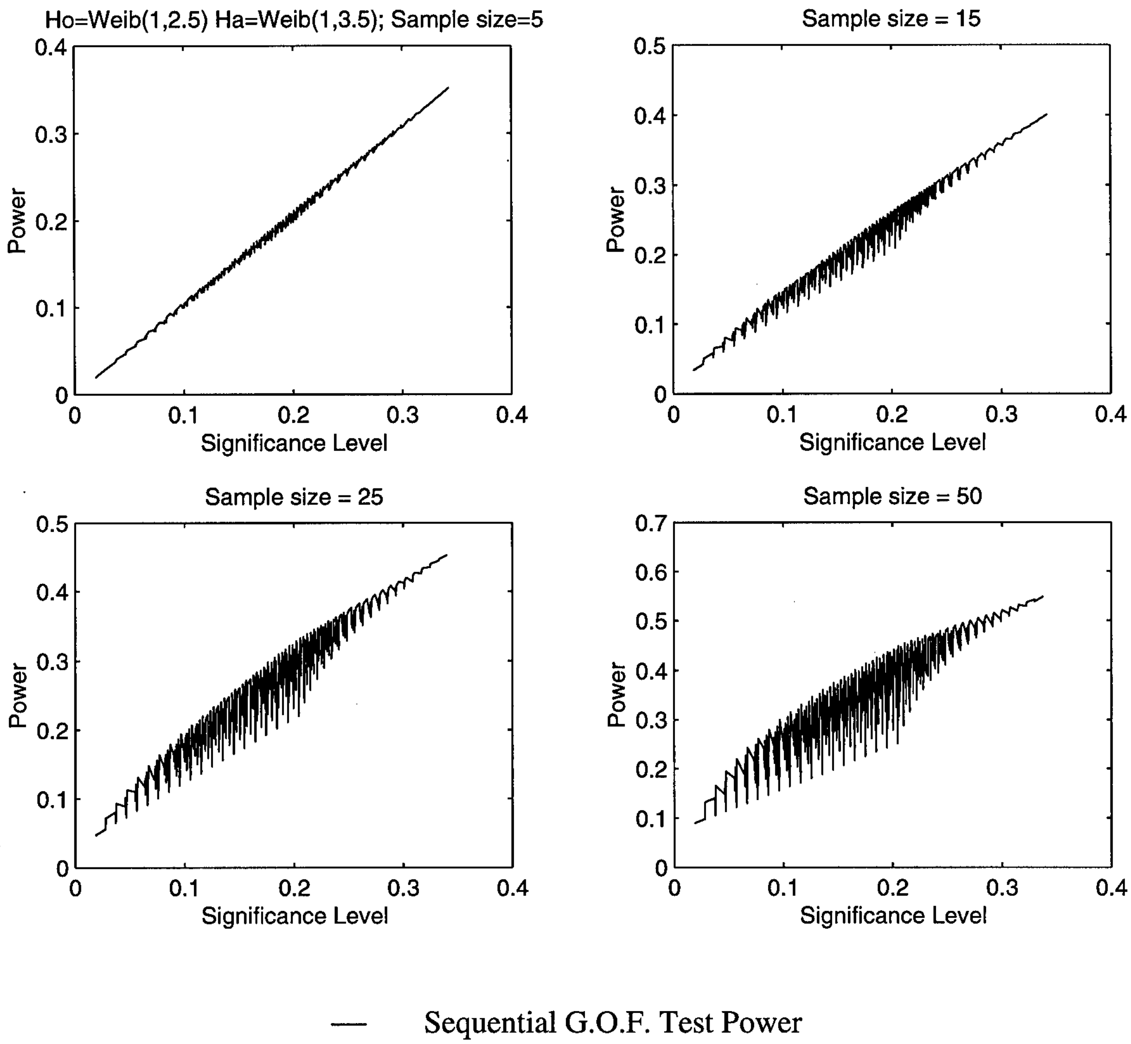


Figure F.44 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 3.5$).

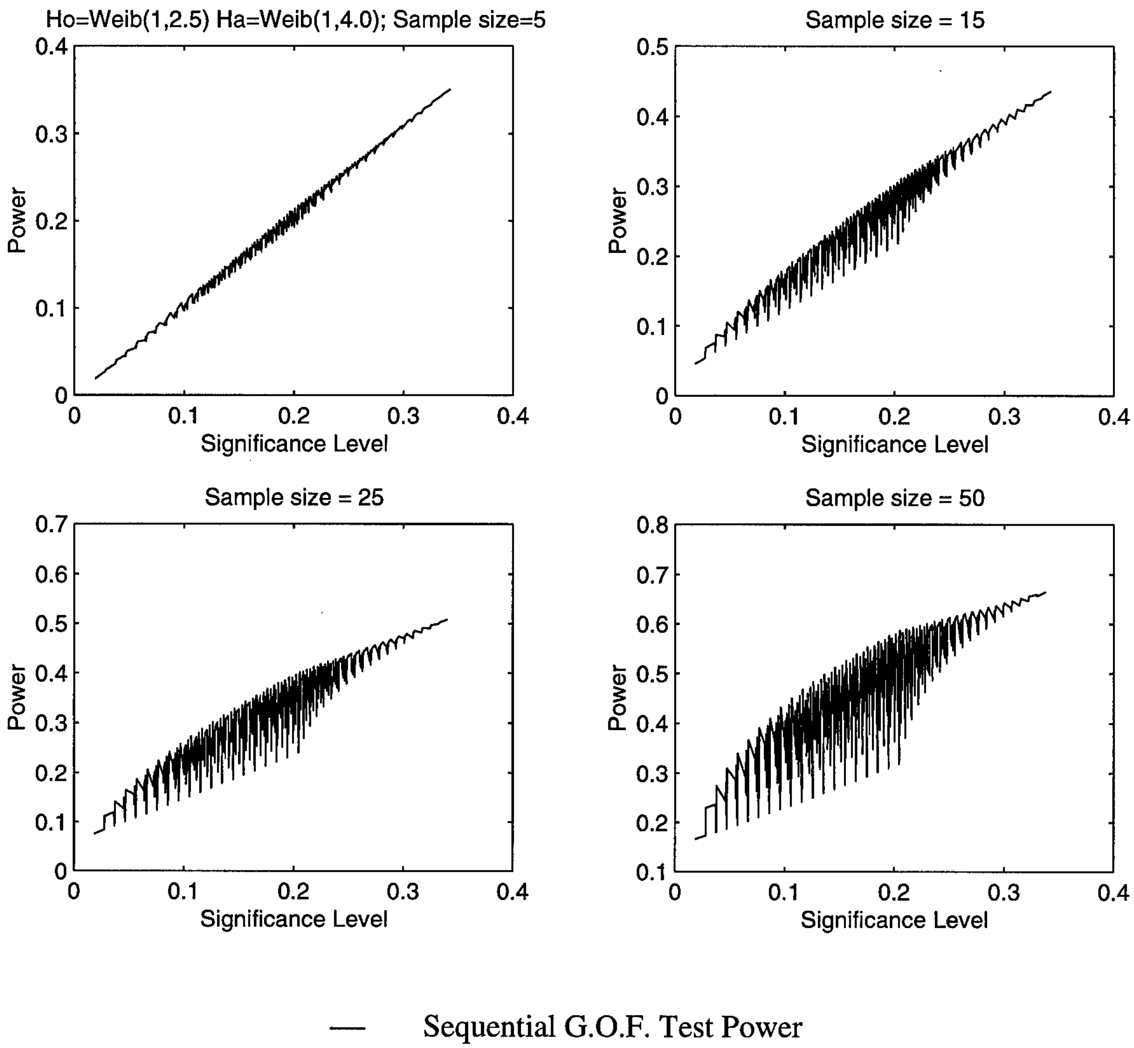
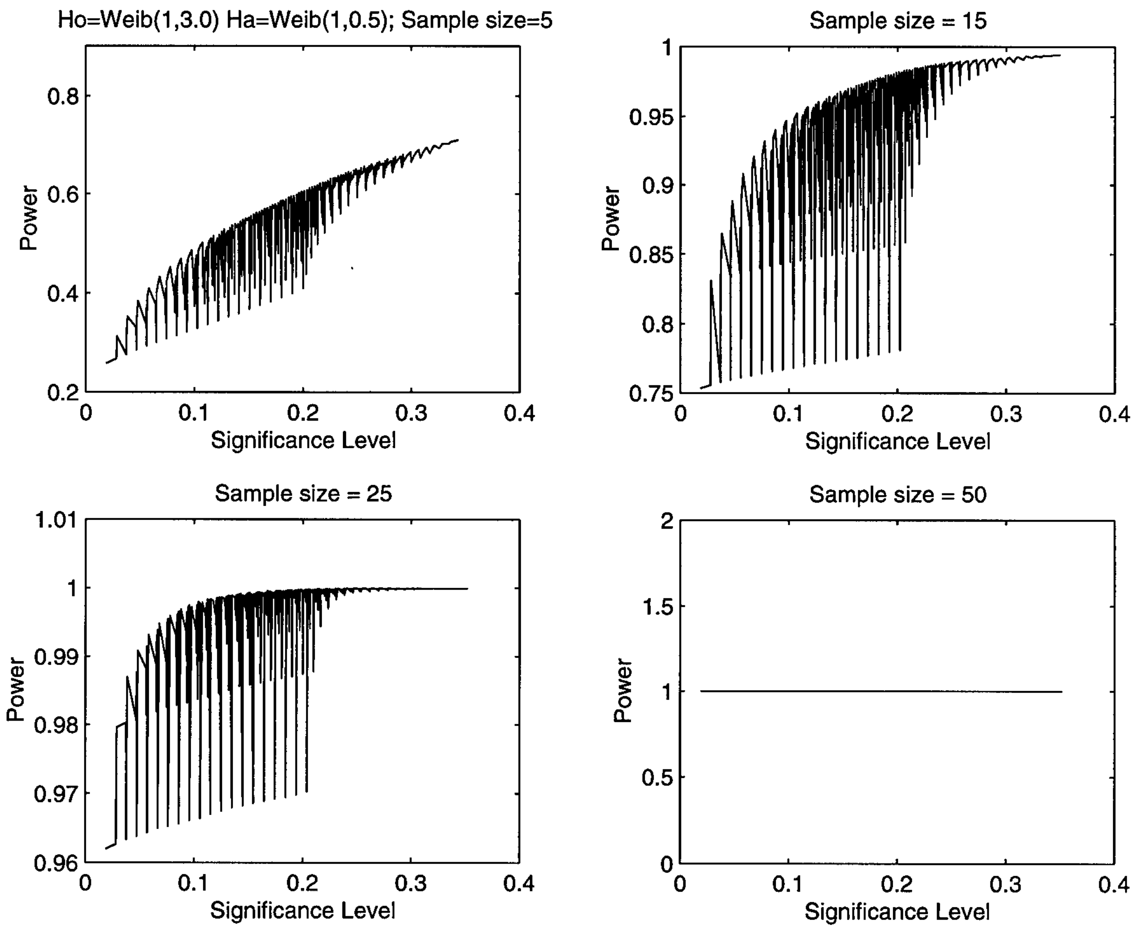


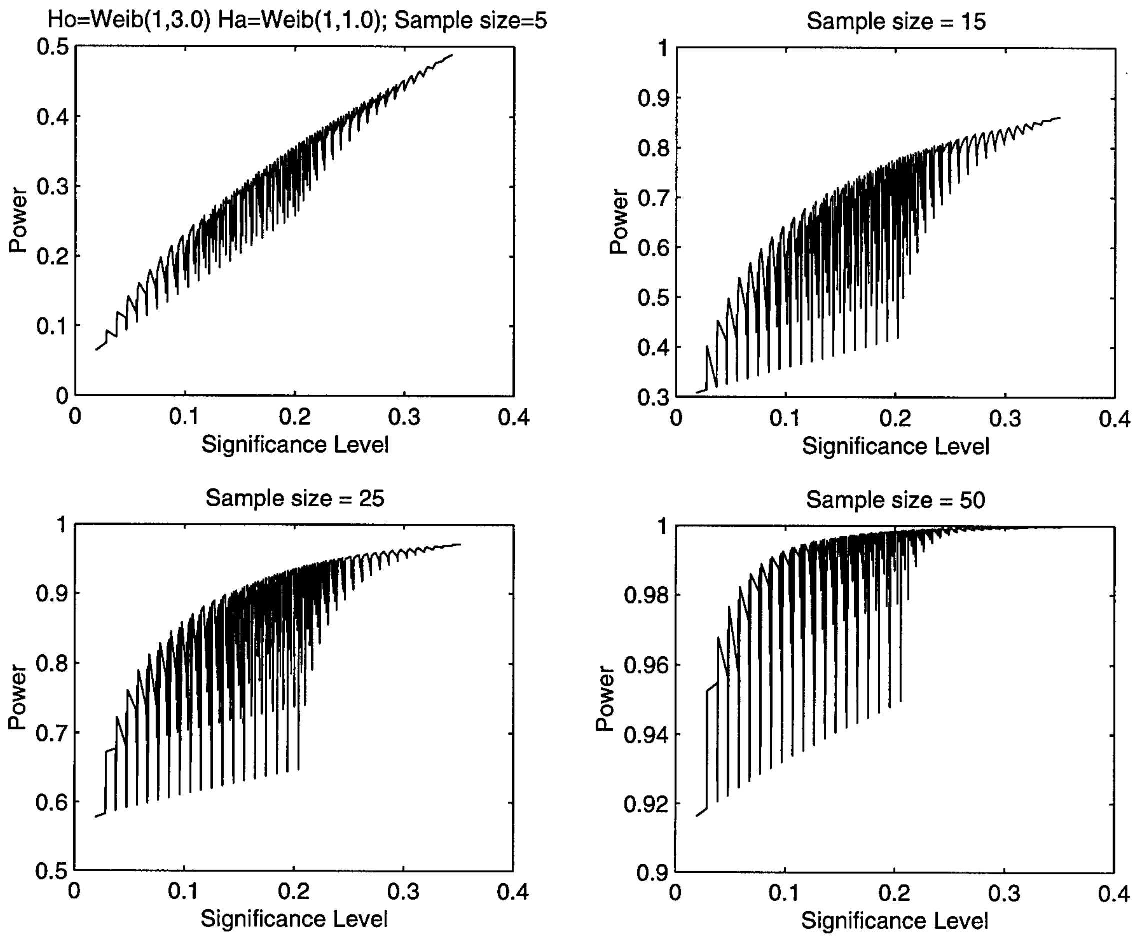
Figure F.45 Sequential Power: H_o: Weibull($\beta = 2.5$); H_a: Weibull($\beta = 4.0$).

F.6 H_o: Weibull($\beta = 3.0$).



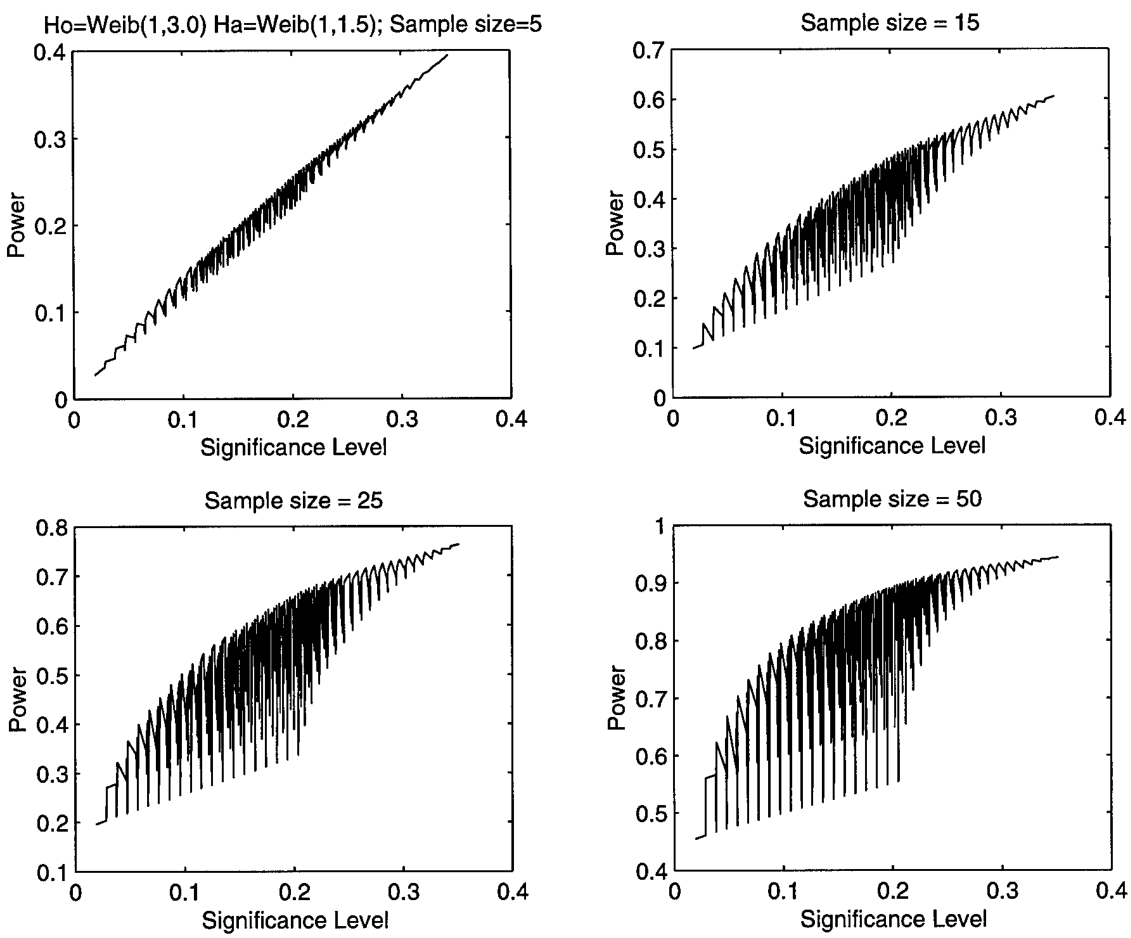
— Sequential G.O.F. Test Power

Figure F.46 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 0.5$).



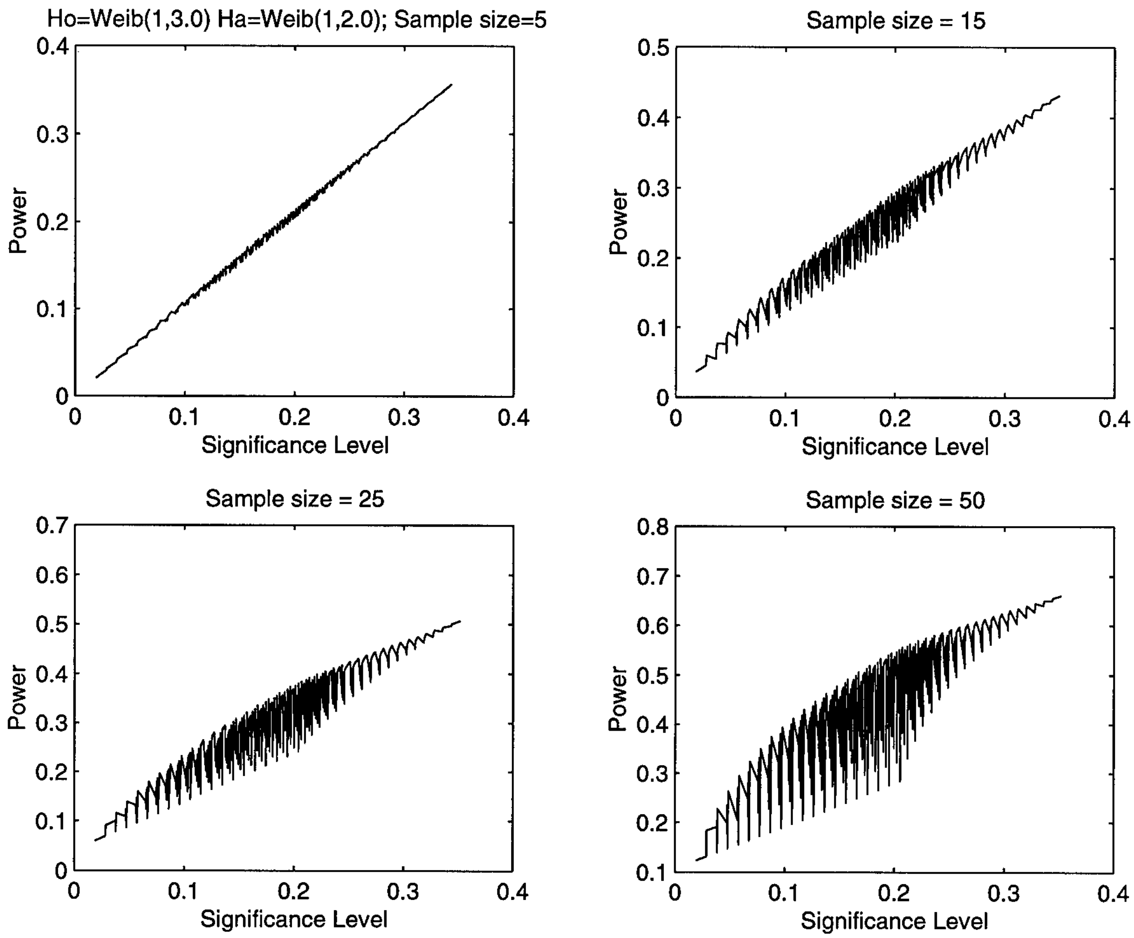
— Sequential G.O.F. Test Power

Figure F.47 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 1.0$).



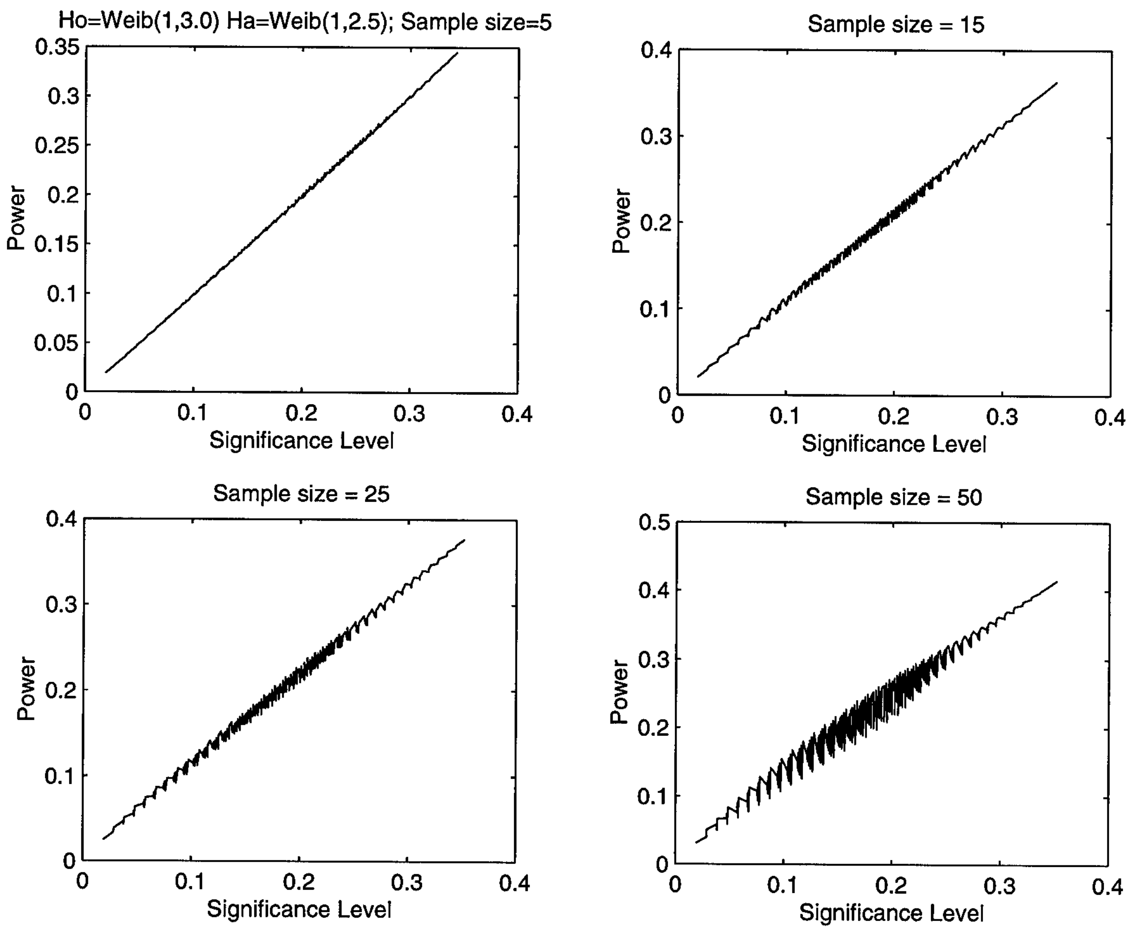
— Sequential G.O.F. Test Power

Figure F.48 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 1.5$).



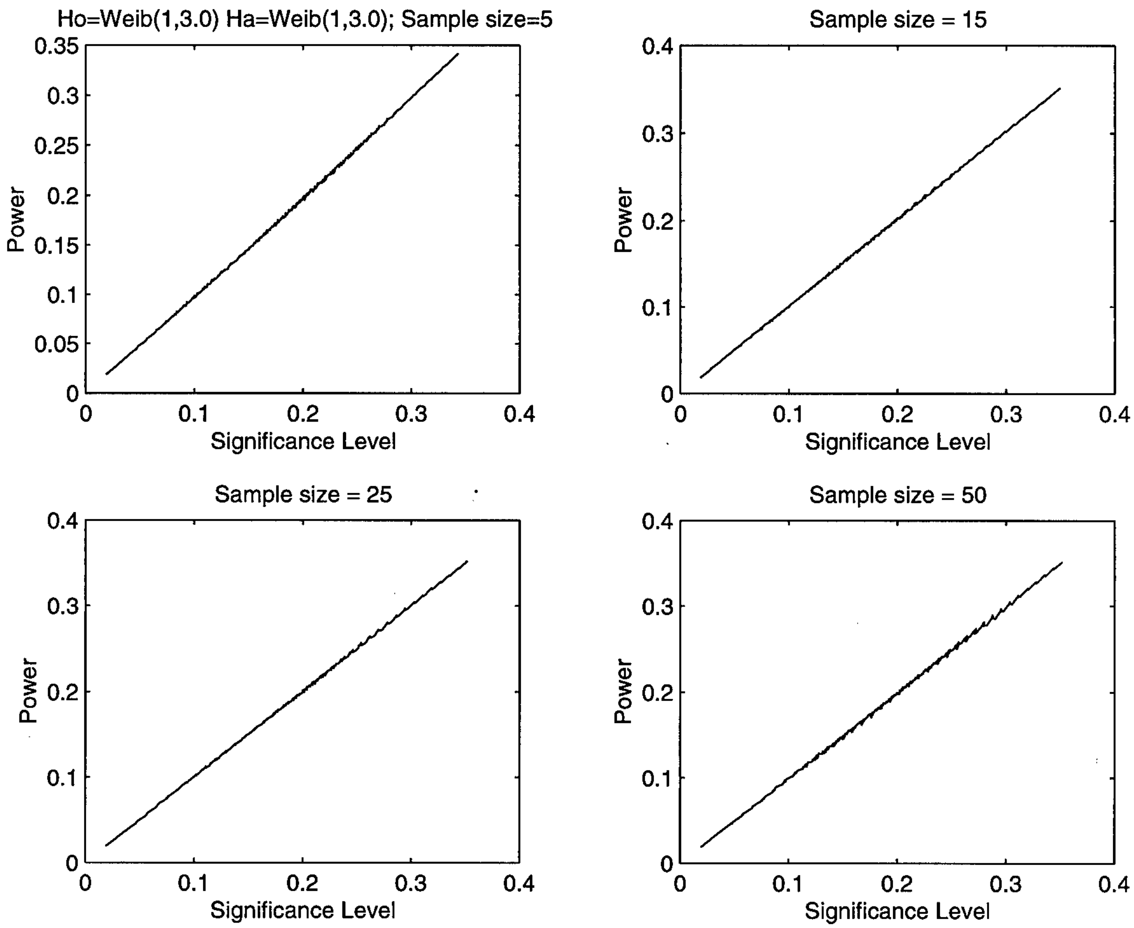
— Sequential G.O.F. Test Power

Figure F.49 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 2.0$).



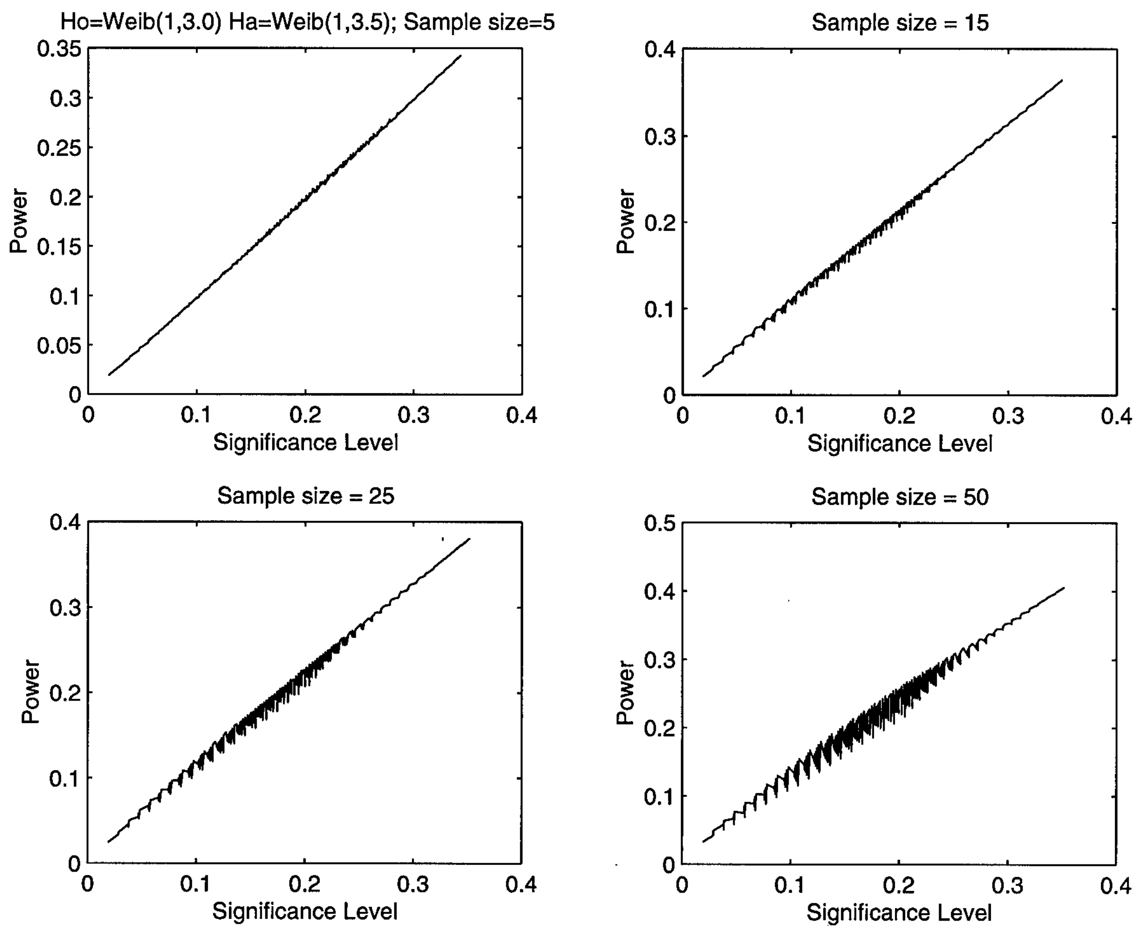
— Sequential G.O.F. Test Power

Figure F.50 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 2.5$).



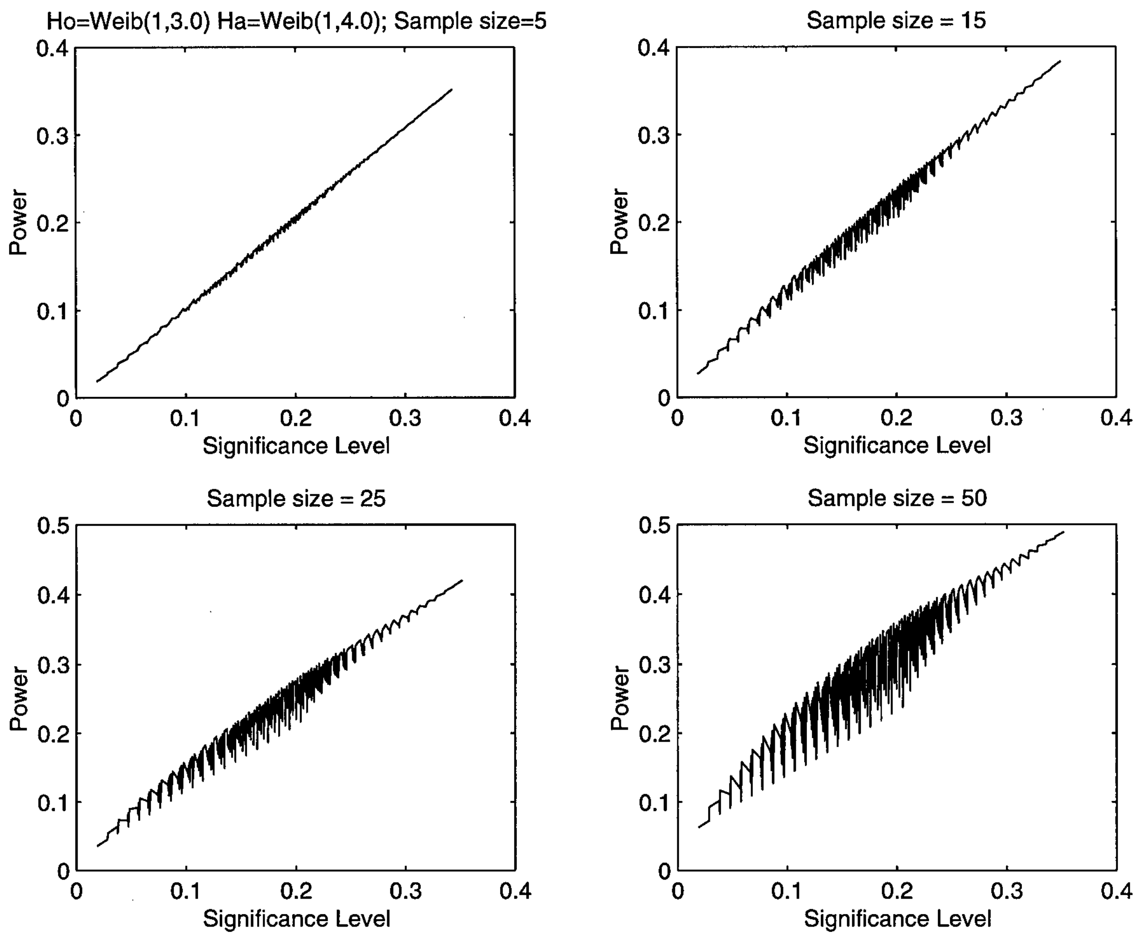
— Sequential G.O.F. Test Power

Figure F.51 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 3.0$).



— Sequential G.O.F. Test Power

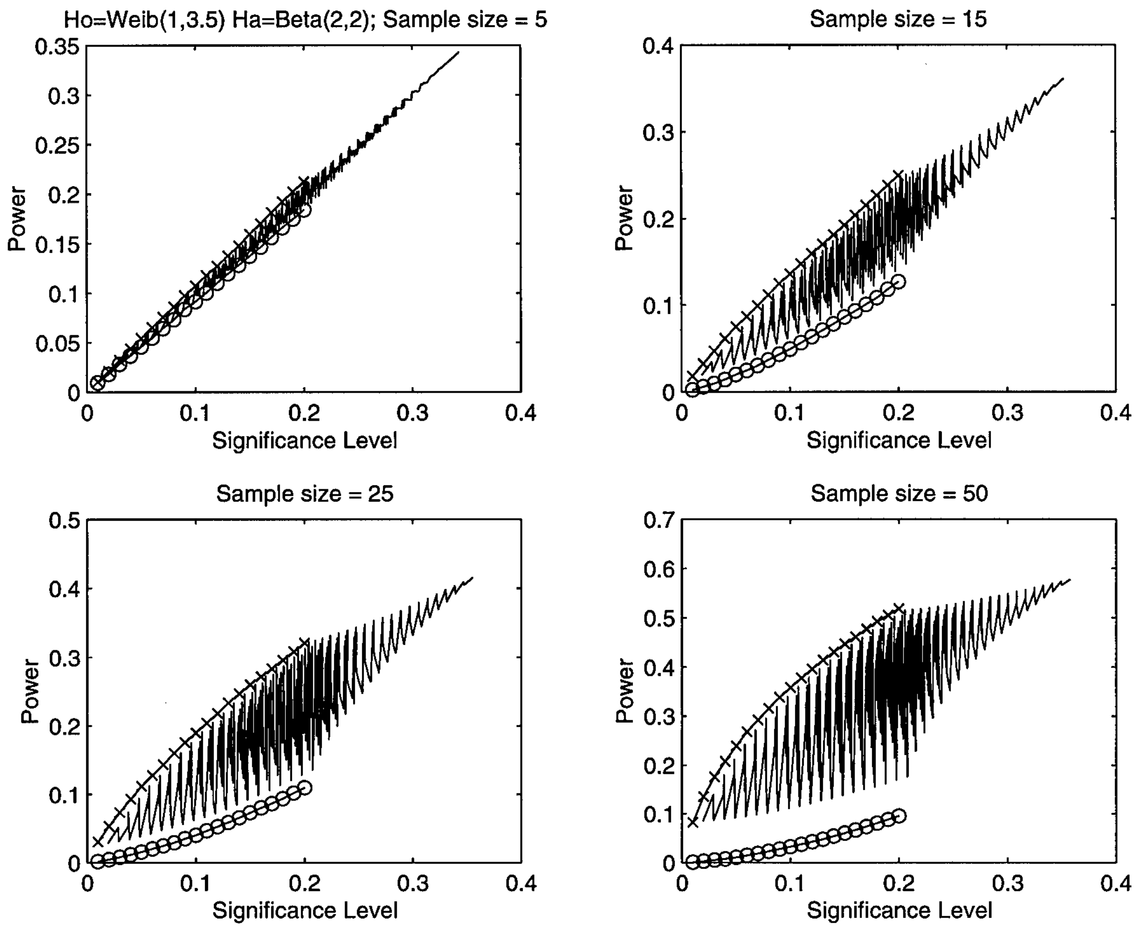
Figure F.52 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

Figure F.53 Sequential Power: H_o: Weibull($\beta = 3.0$); H_a: Weibull($\beta = 4.0$).

F.7 H_o: *Weibull*($\beta = 3.5$).

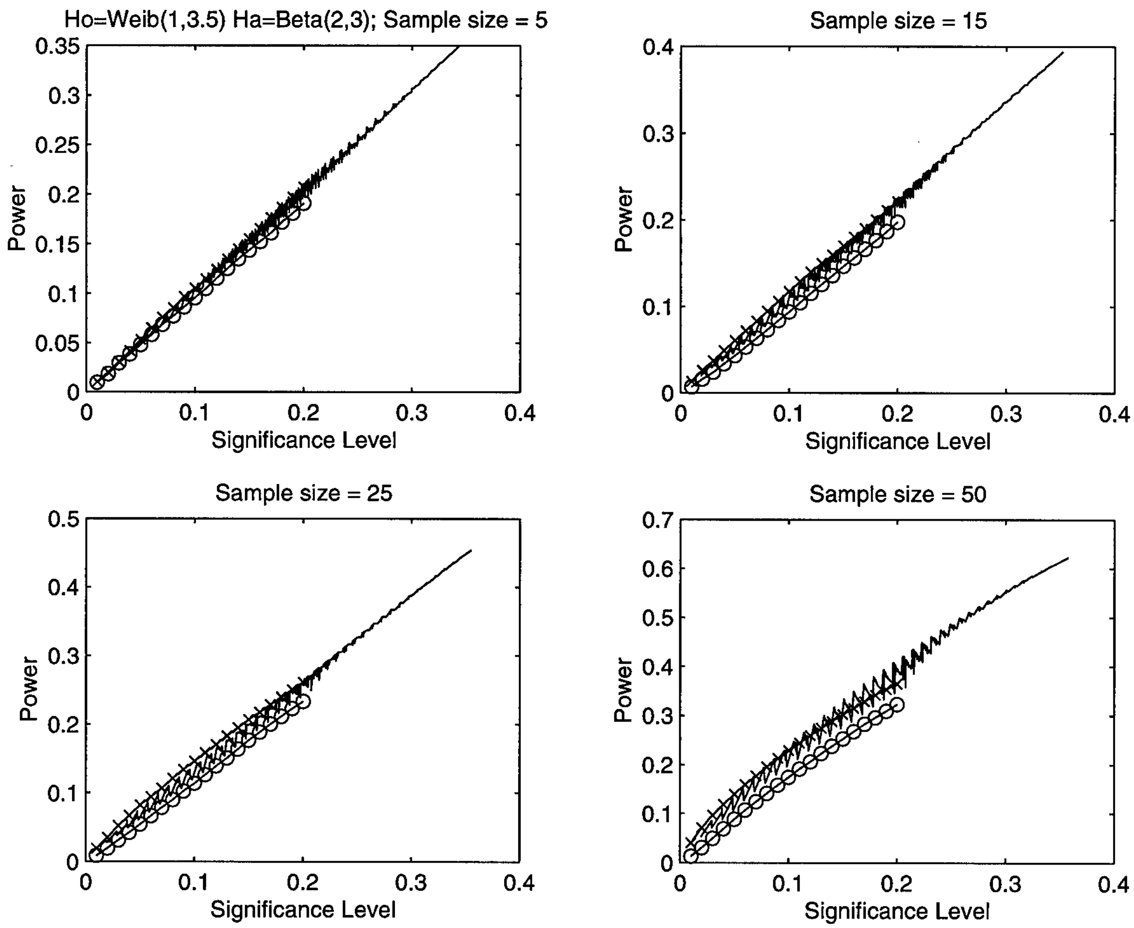


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.54 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Beta(2,2).

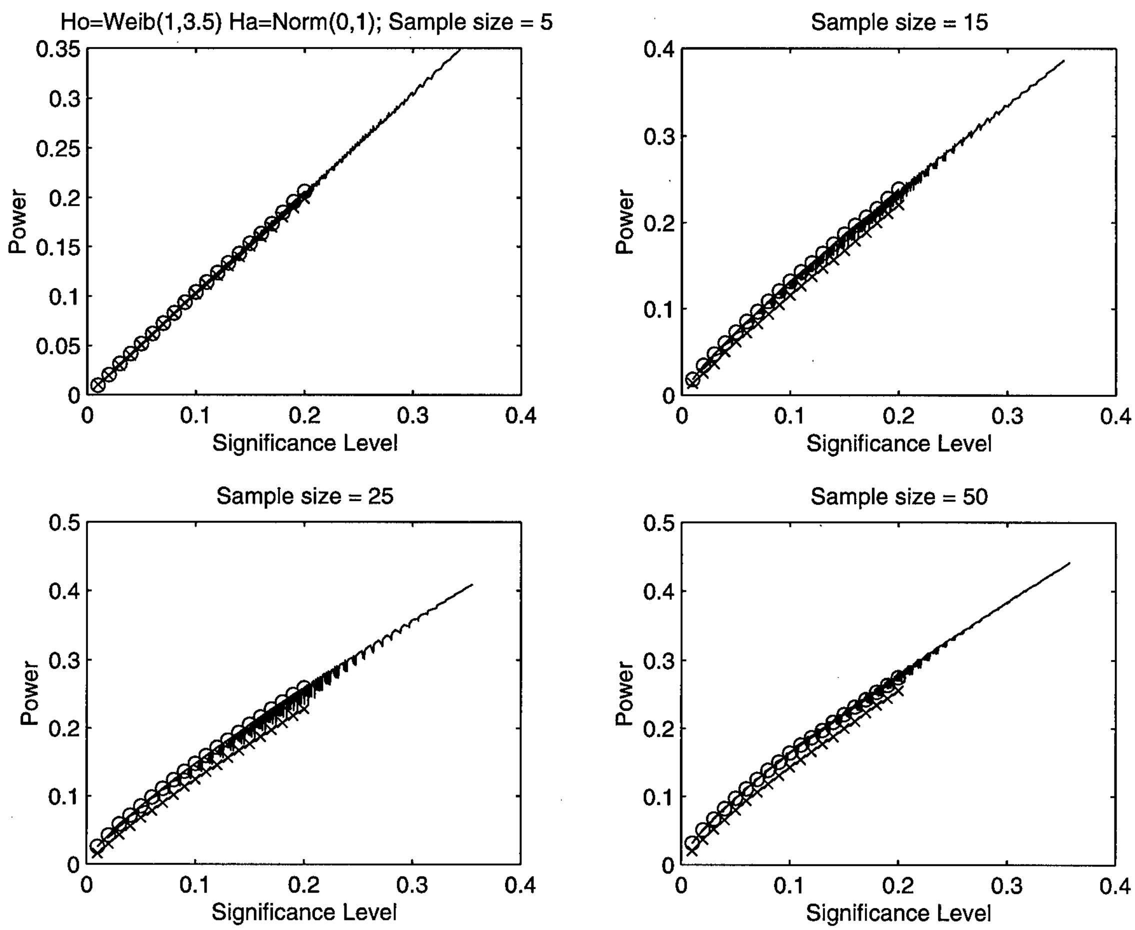


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.55 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Beta(2,3).

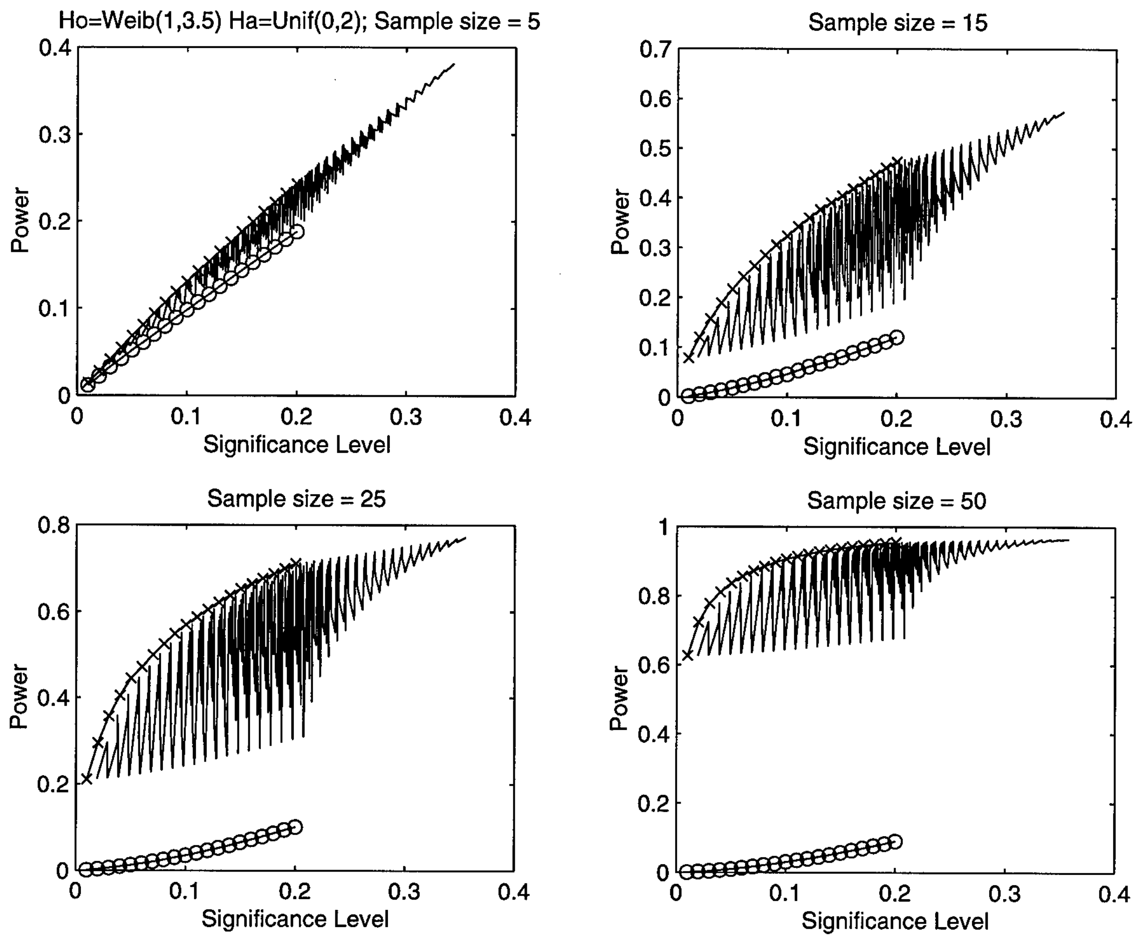


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.56 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Normal(0,1).

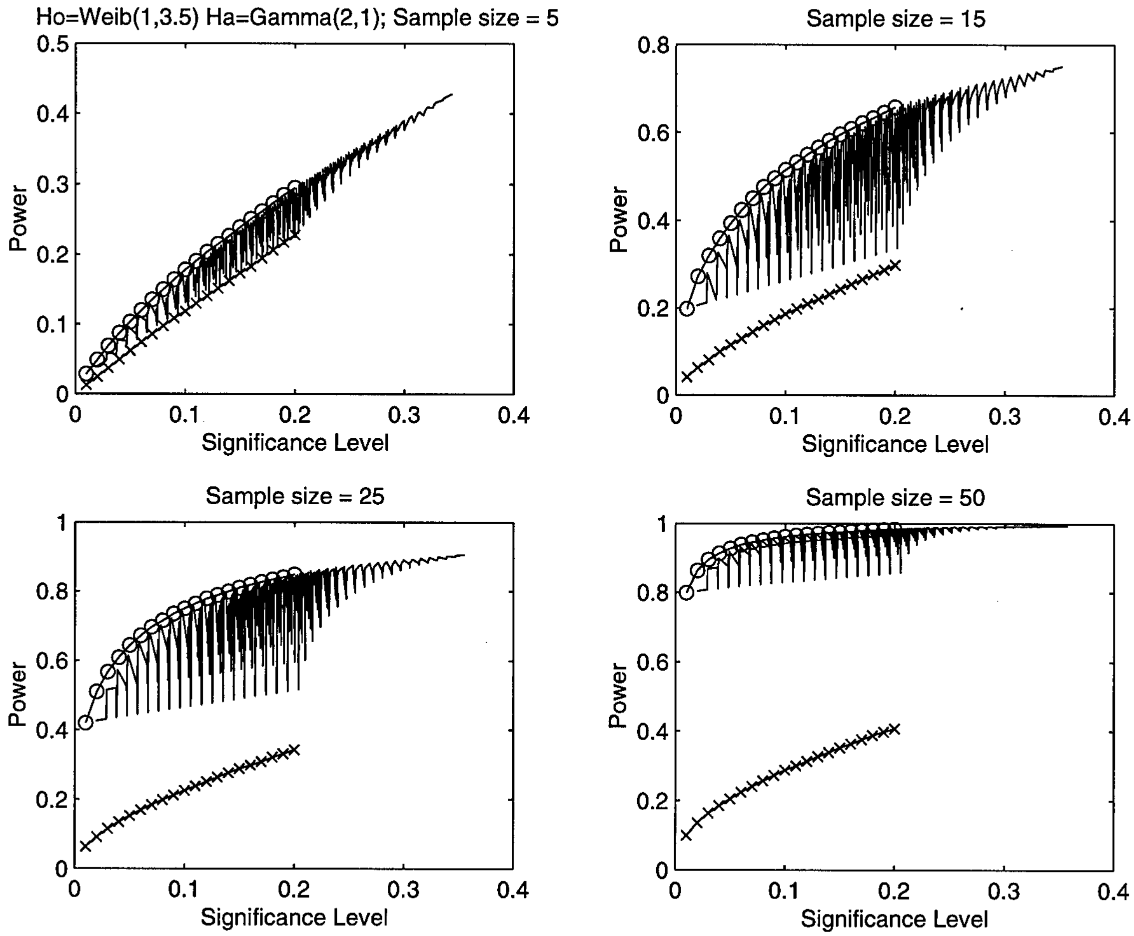


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.57 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Uniform(0,2).

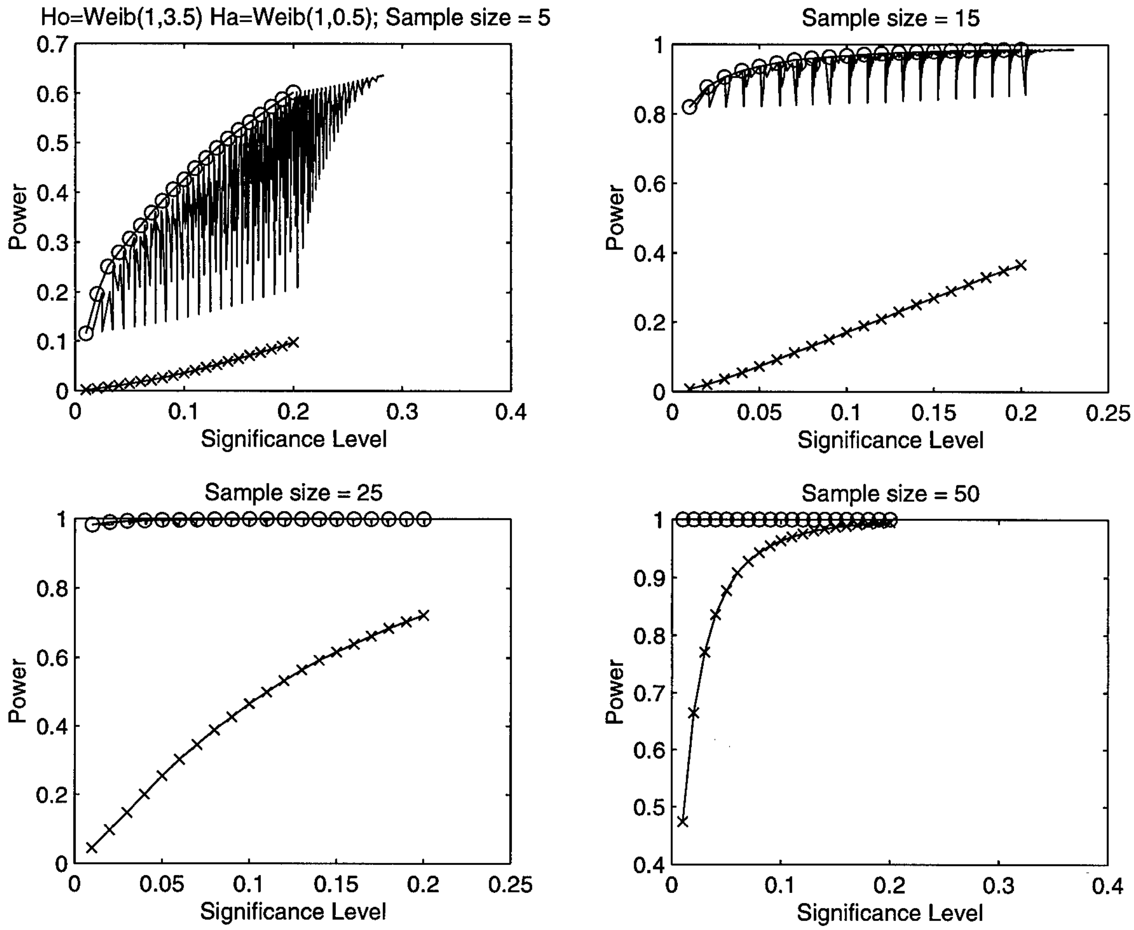


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.58 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Gamma(2,1).

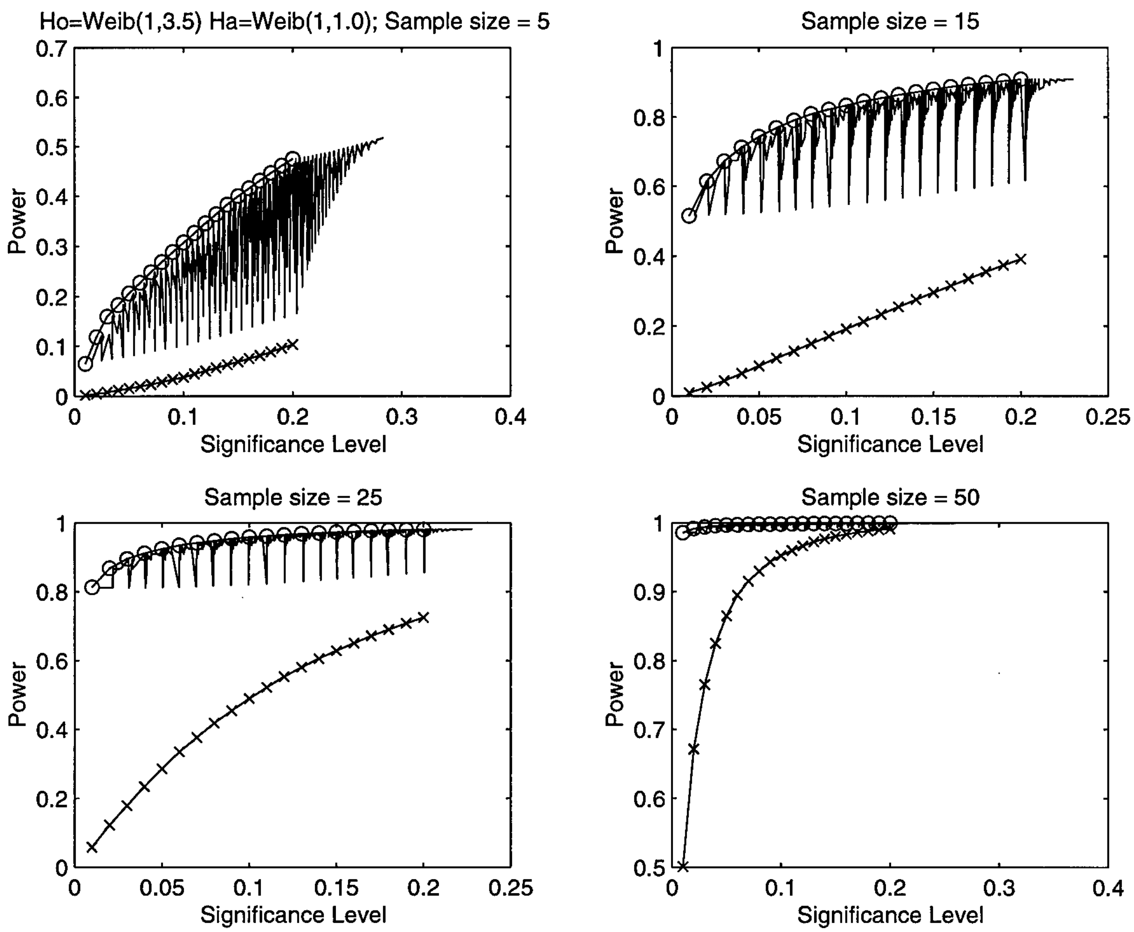


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.59 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 0.5$).

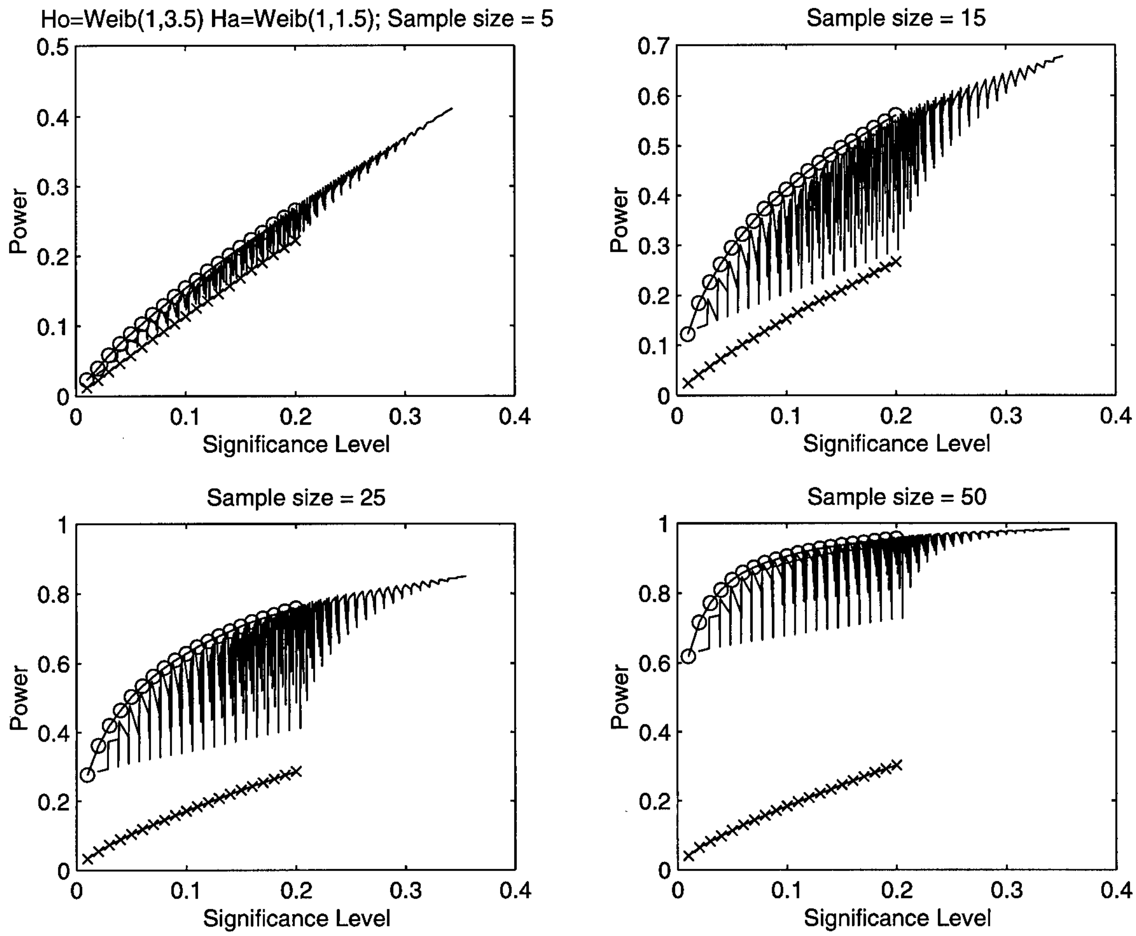


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.60 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 1.0$).

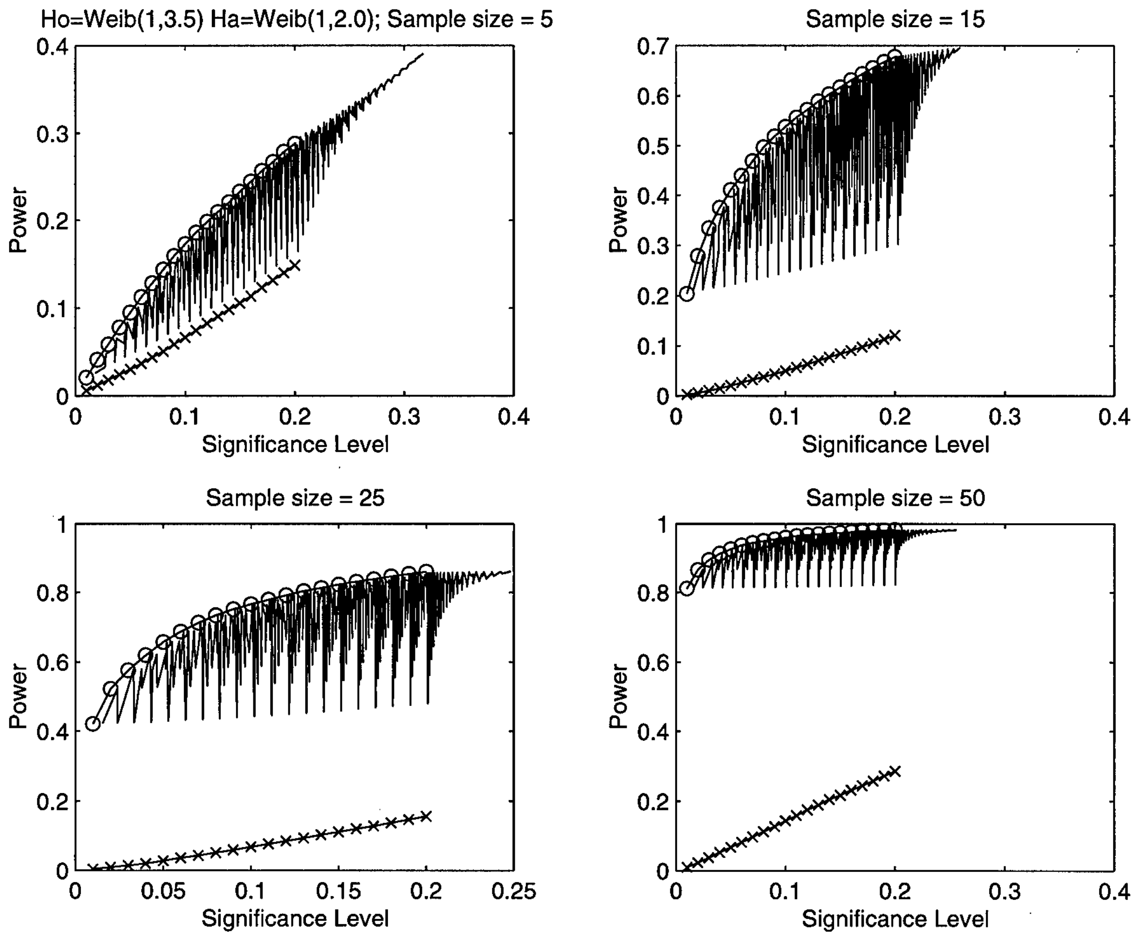


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

Figure F.61 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 1.5$).

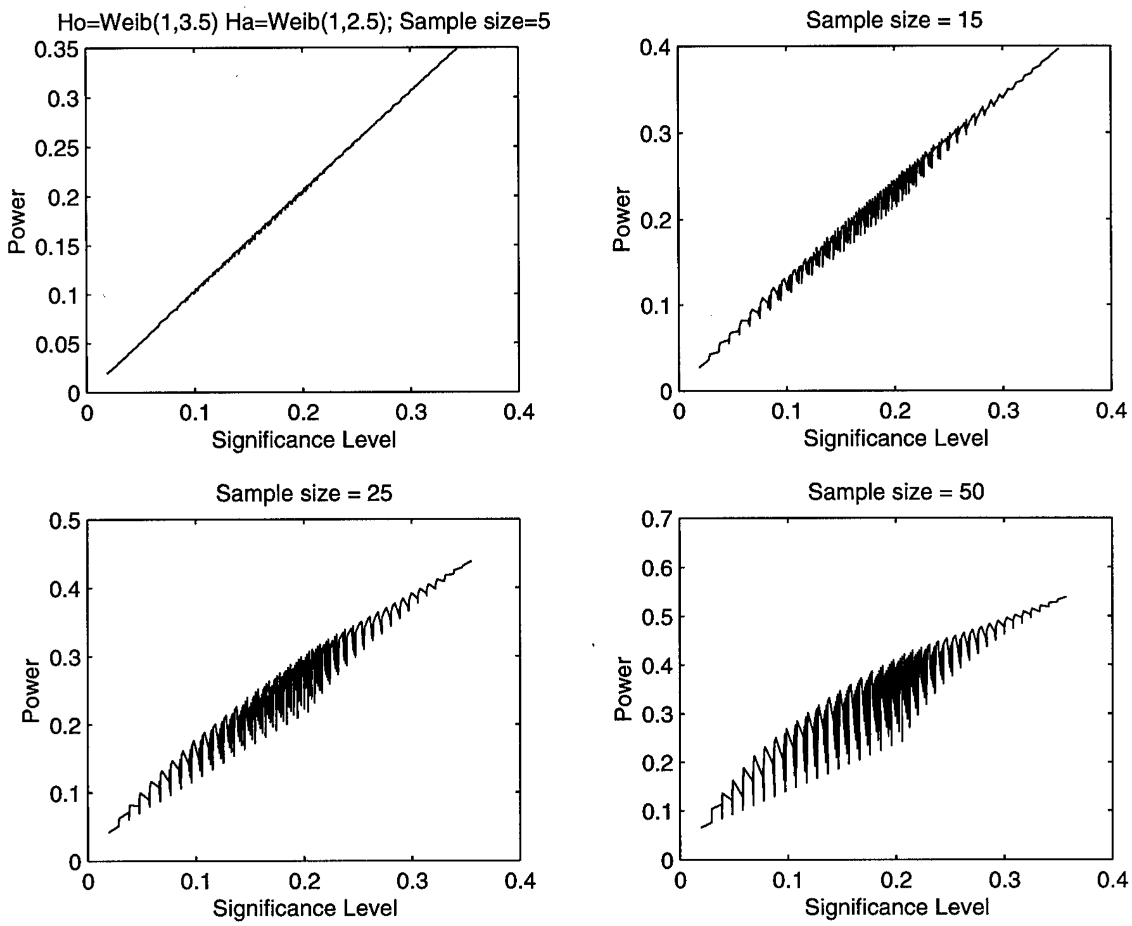


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

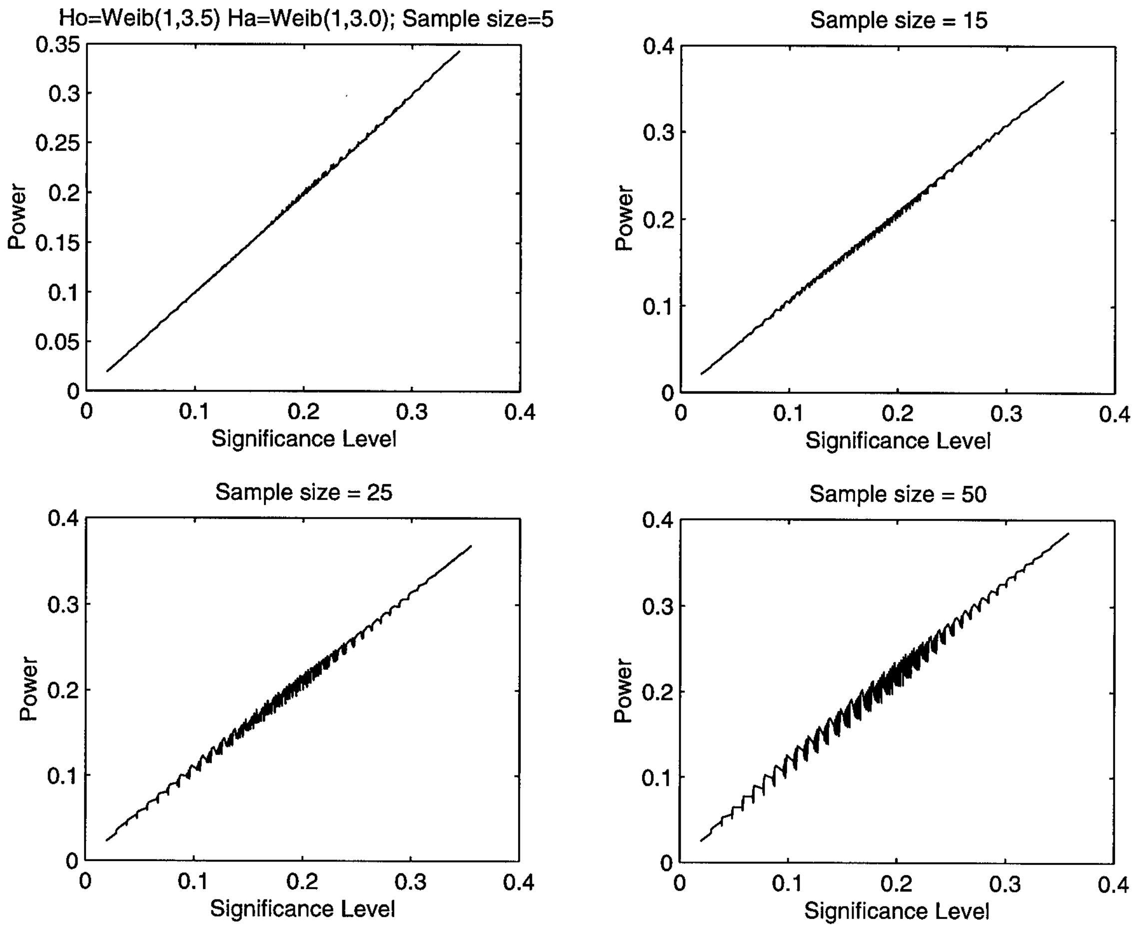
x x Q-Statistic G.O.F. Test Power

Figure F.62 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 2.0$).



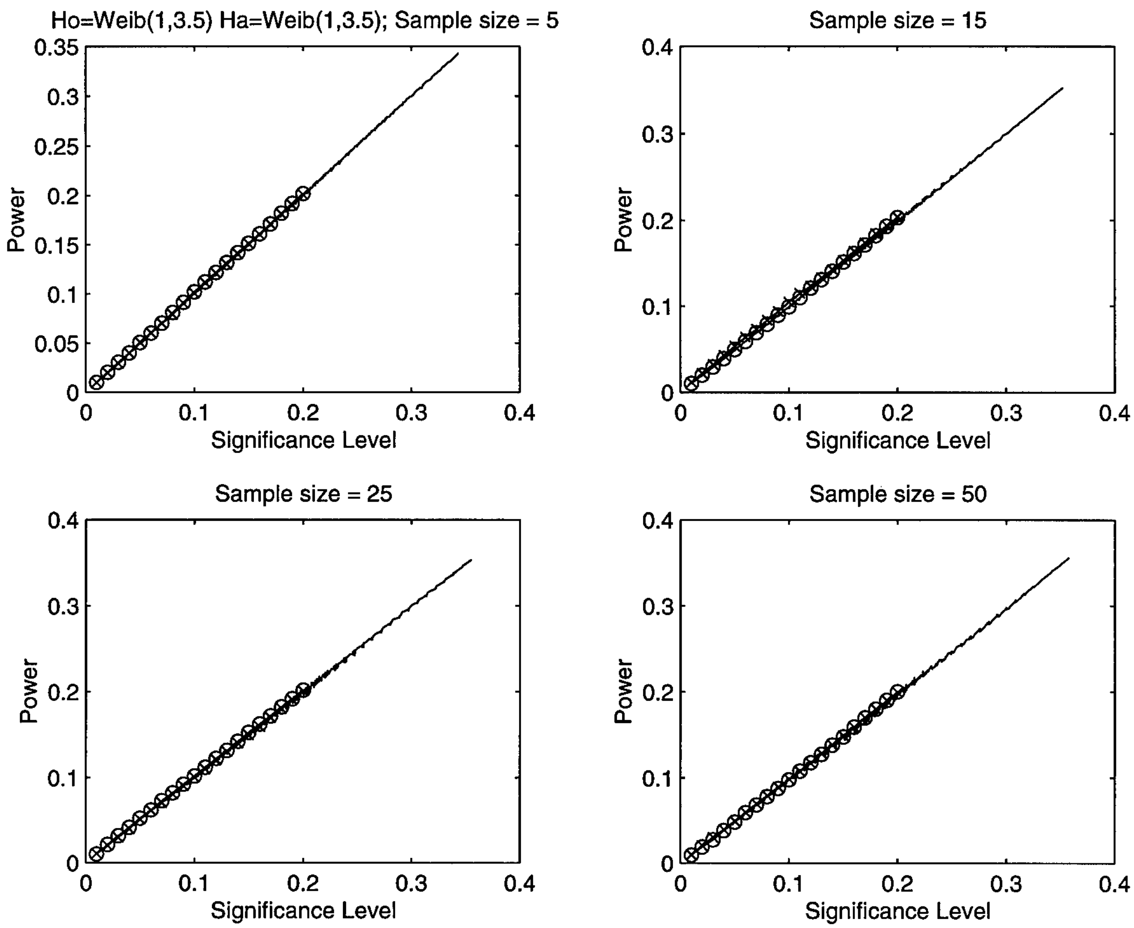
— Sequential G.O.F. Test Power

Figure F.63 Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 2.5$).



— Sequential G.O.F. Test Power

Figure F.64 Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 3.0$).

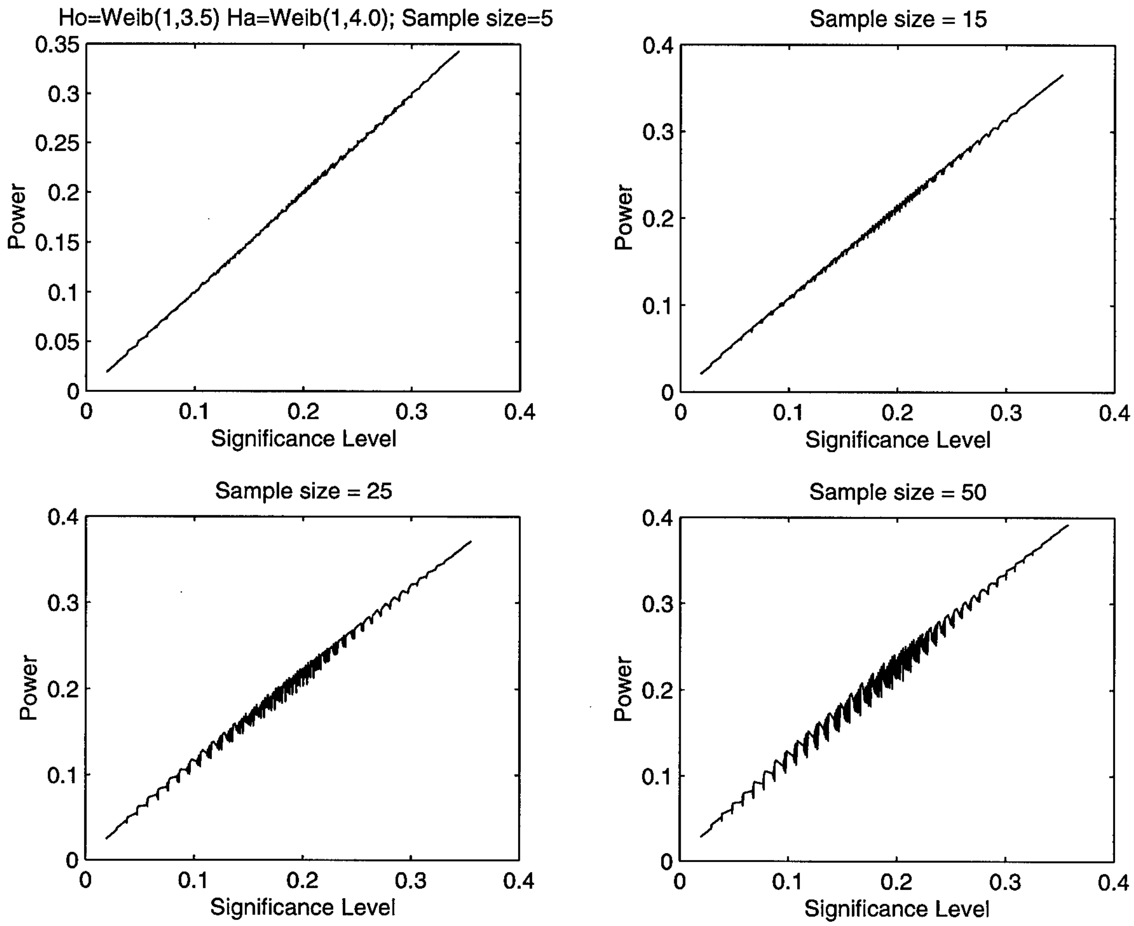


— Sequential G.O.F. Test Power

o o $\sqrt{b_1}$ G.O.F. Test Power

x x Q-Statistic G.O.F. Test Power

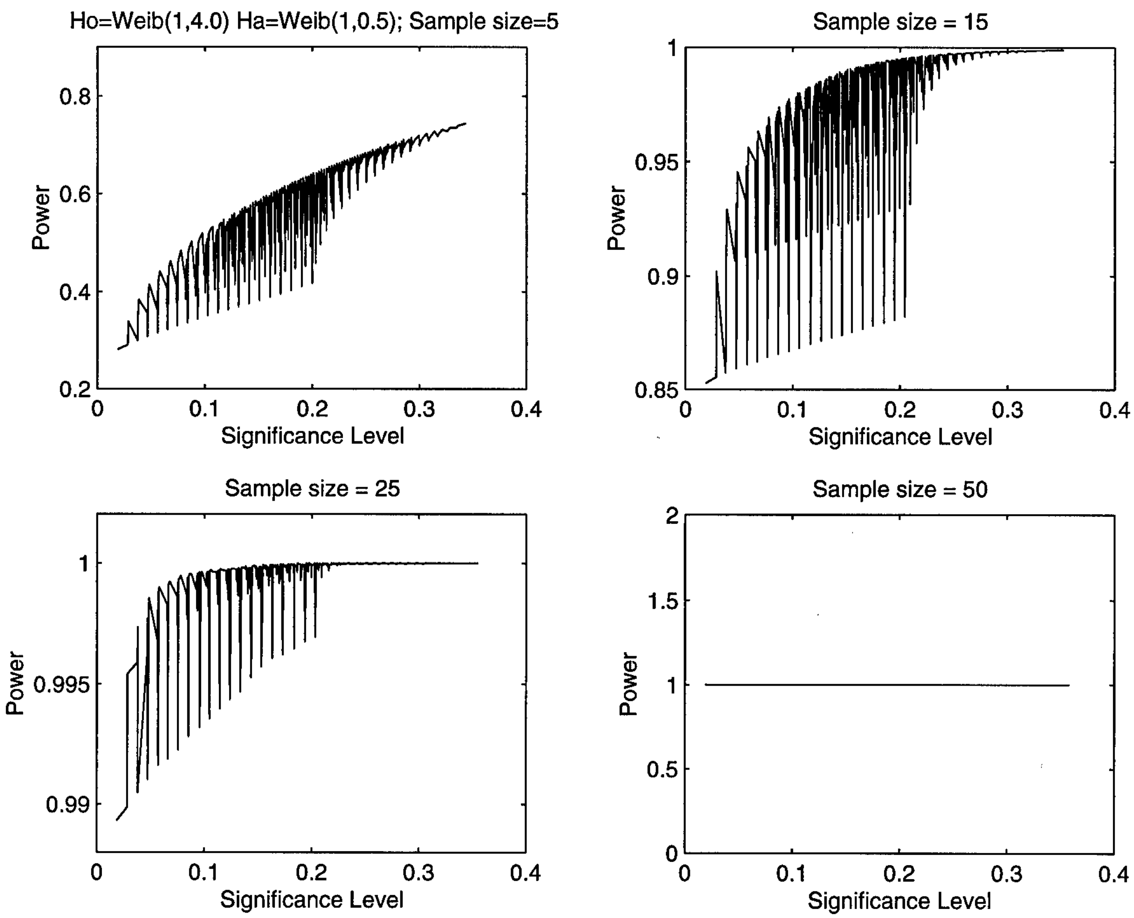
Figure F.65 Individual vs. Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 3.5$).



— Sequential G.O.F. Test Power

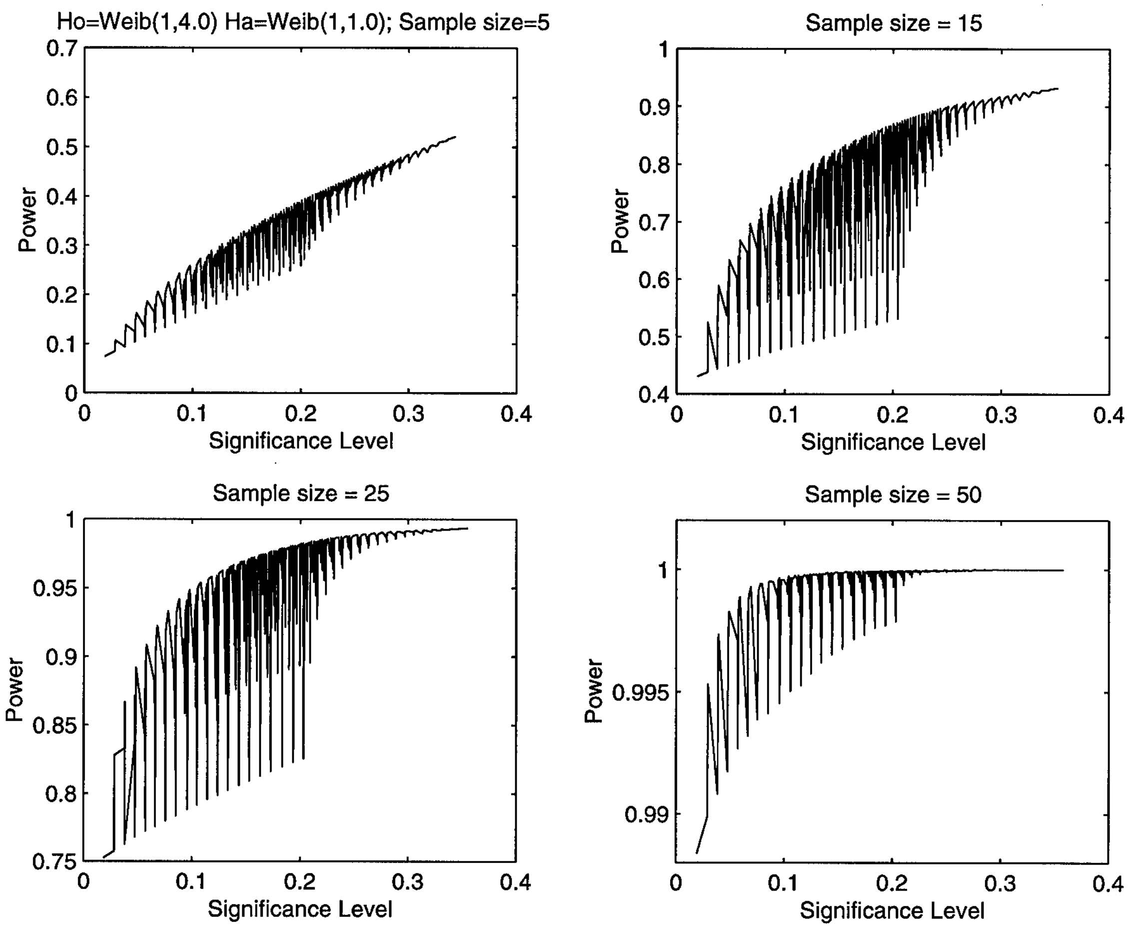
Figure F.66 Sequential Power: H_o: Weibull($\beta = 3.5$); H_a: Weibull($\beta = 4.0$).

F.8 H_o: *Weibull*($\beta = 4.0$).



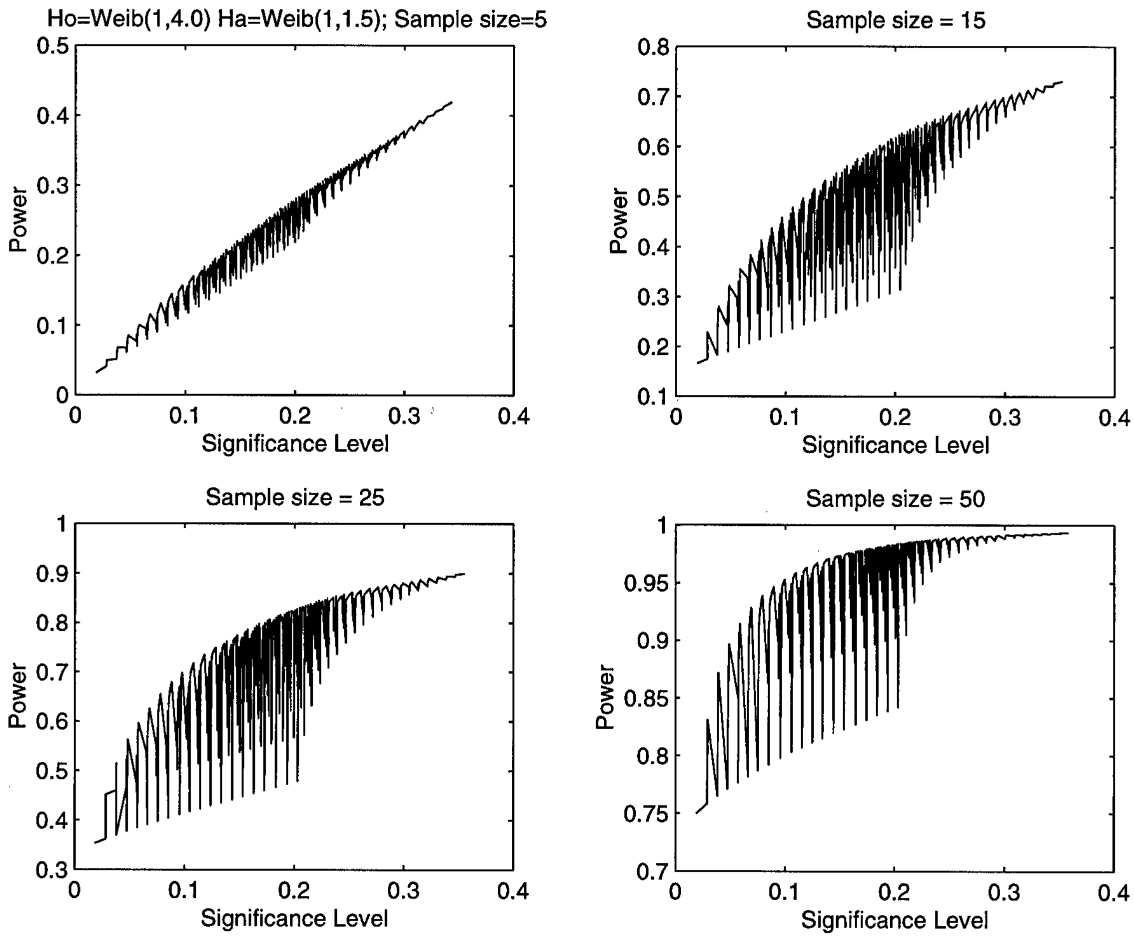
— Sequential G.O.F. Test Power

Figure F.67 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 0.5$).



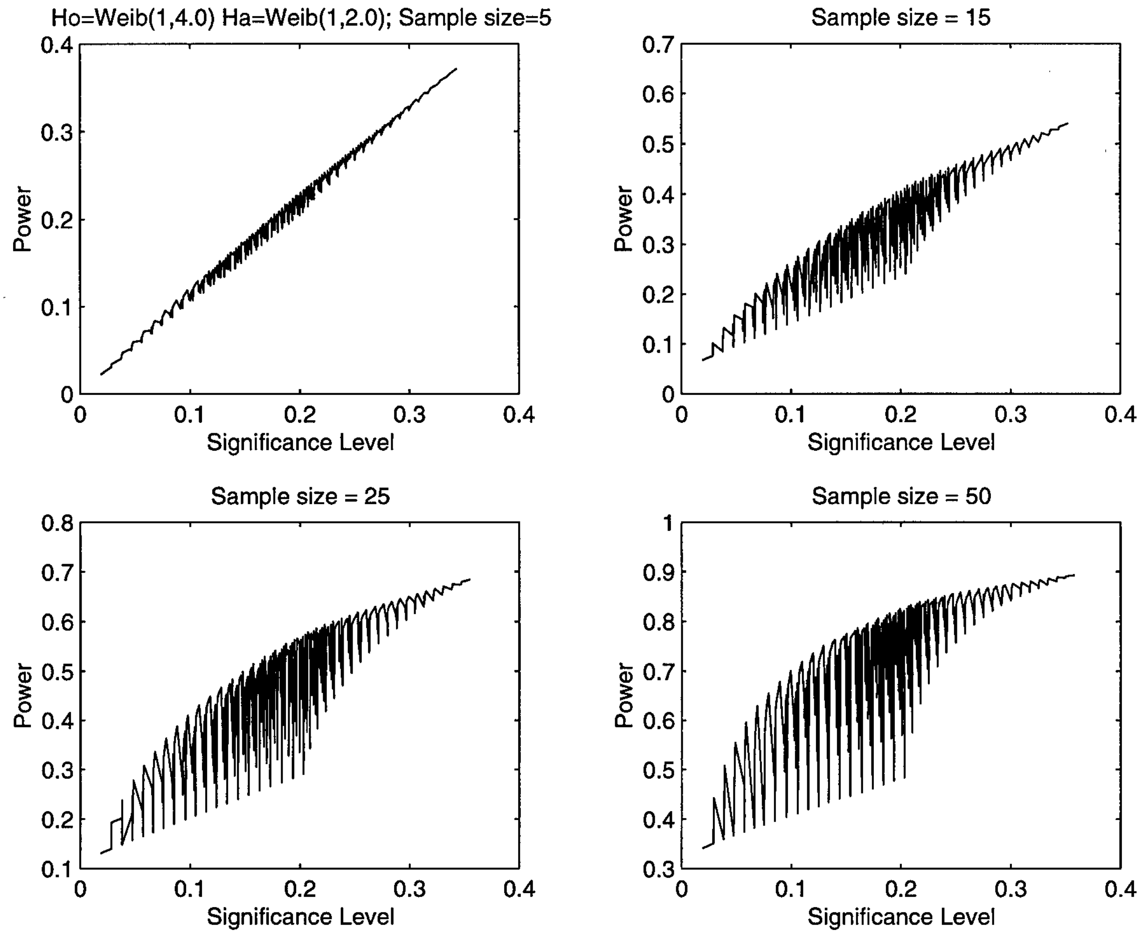
— Sequential G.O.F. Test Power

Figure F.68 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 1.0$).



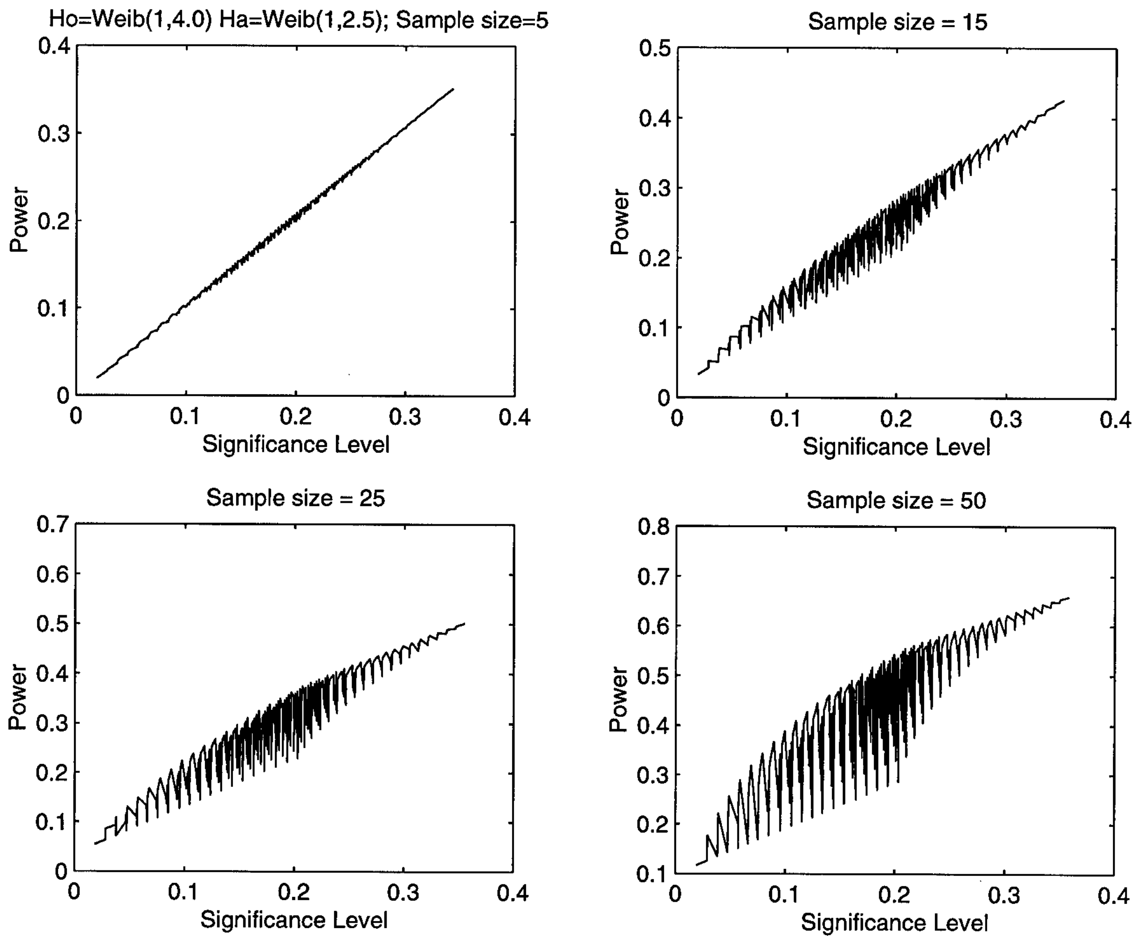
— Sequential G.O.F. Test Power

Figure F.69 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 1.5$).



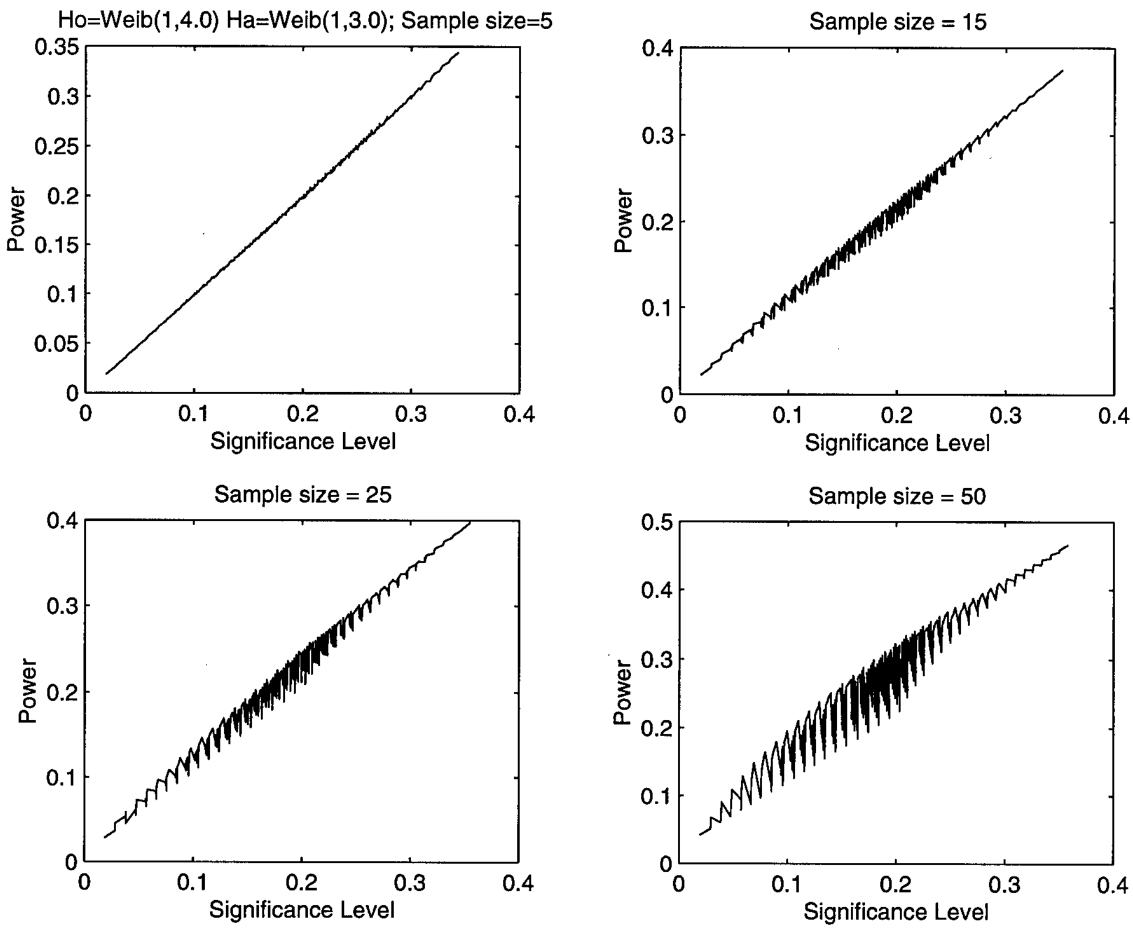
— Sequential G.O.F. Test Power

Figure F.70 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 2.0$).



— Sequential G.O.F. Test Power

Figure F.71 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 2.5$).



— Sequential G.O.F. Test Power

Figure F.72 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 3.0$).

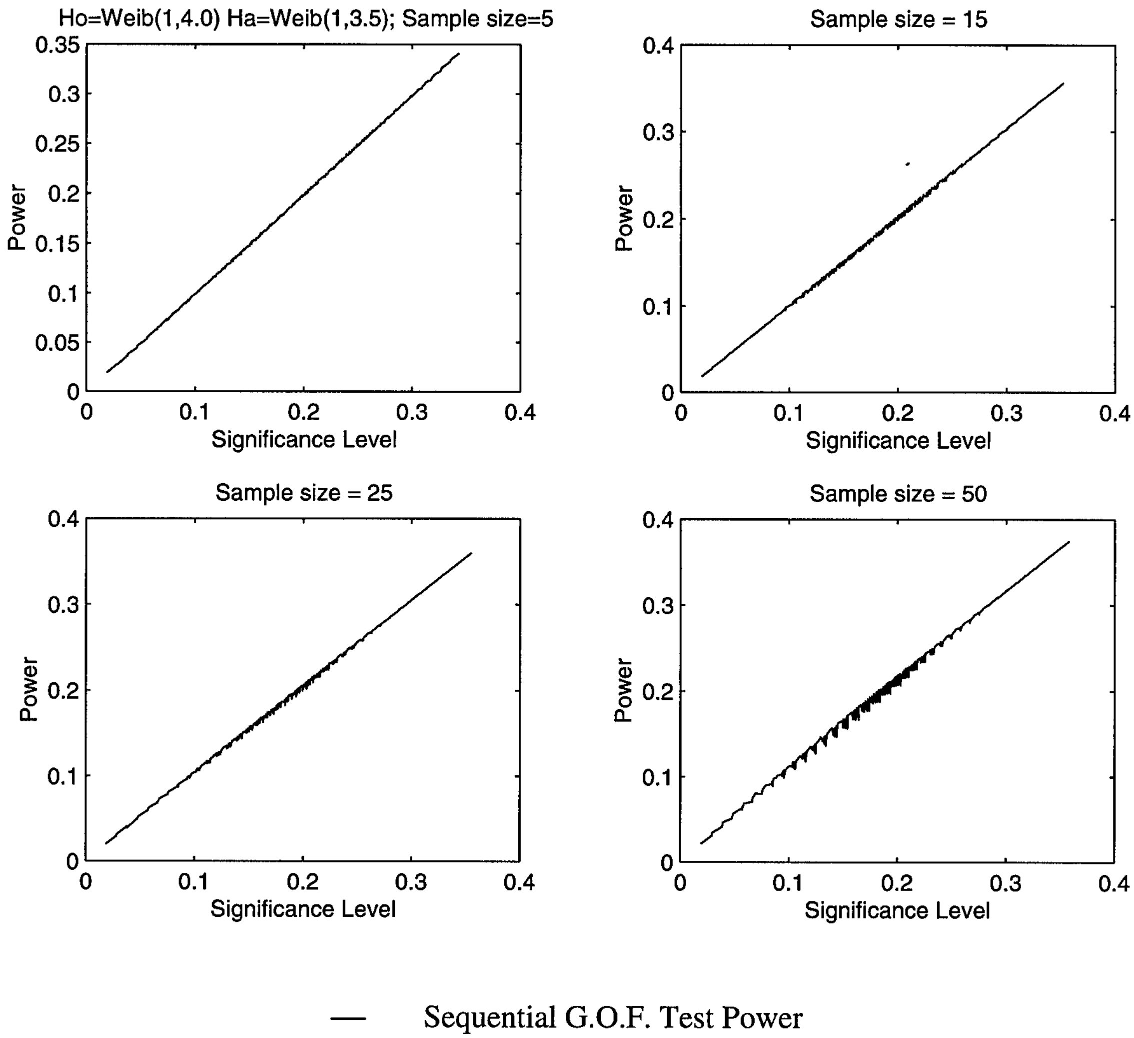


Figure F.73 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 3.5$).

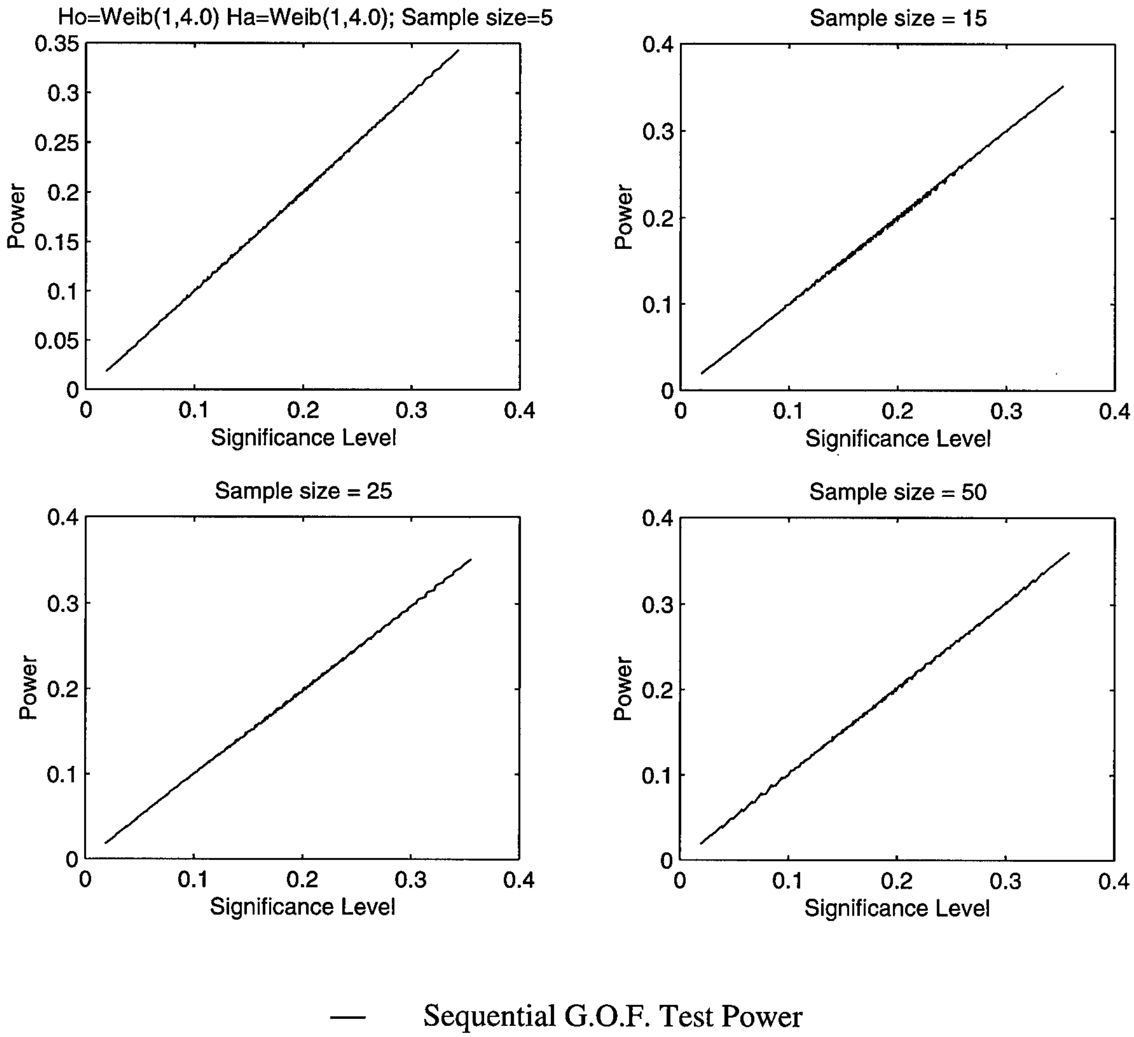


Figure F.74 Sequential Power: H_o: Weibull($\beta = 4.0$); H_a: Weibull($\beta = 4.0$).

Appendix G. Individual Two-Sided $\sqrt{b_1}$ and Q-Statistic G.O.F. Test Power Results

G.1 H_o: Weibull(β = 0.5).

G.1.1 $\sqrt{b_1}$ G.O.F. Test Results.

Table G.1 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: $\chi^2(1)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.032	0.113	0.186	0.252	0.310
30	0.048	0.152	0.242	0.310	0.371
50	0.096	0.235	0.335	0.410	0.473

Table G.2 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Double Exponential.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.107	0.202	0.273	0.331	0.383
30	0.132	0.229	0.296	0.353	0.403
50	0.174	0.276	0.349	0.405	0.458

Table G.3 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.099	0.188	0.255	0.311	0.361
30	0.130	0.222	0.286	0.342	0.390
50	0.175	0.275	0.347	0.402	0.456

Table G.4 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Cauchy(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.623	0.690	0.725	0.755	0.778
30	0.730	0.785	0.813	0.836	0.855
50	0.838	0.891	0.905	0.921	0.941

Table G.5 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = O.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.049	0.097	0.149	0.199
15	0.010	0.051	0.099	0.152	0.203
25	0.010	0.051	0.102	0.151	0.201
50	0.009	0.050	0.101	0.151	0.201

Table G.6 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β =1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.021	0.085	0.145	0.207	0.263
15	0.095	0.240	0.349	0.425	0.485
25	0.184	0.365	0.481	0.560	0.616
50	0.372	0.580	0.678	0.736	0.778

Table G.7 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β =O.5) ; H_a: Weibull(β =1.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.044	0.149	0.237	0.317	0.389
15	0.304	0.538	0.660	0.728	0.778
25	0.560	0.758	0.839	0.882	0.907
50	0.869	0.946	0.969	0.979	0.985

Table G.8 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.065	0.206	0.307	0.400	0.475
15	0.515	0.743	0.833	0.879	0.909
25	0.813	0.925	0.958	0.973	0.981
50	0.986	0.997	0.998	0.999	1.000

Table G.9 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.115	0.307	0.426	0.526	0.602
15	0.820	0.937	0.968	0.980	0.986
25	0.982	0.996	0.999	0.999	1.000
50	1.000	1.000	1.000	1.000	1.000

G.1.2 Q-Statistic G.O.F. Test Results

Table G.10 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: $\chi^2(1)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.020	0.081	0.151	0.210	0.269
30	0.034	0.128	0.218	0.291	0.358
50	0.073	0.228	0.351	0.445	0.520

Table G.11 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Double Exponential.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.101	0.183	0.251	0.305	0.357
30	0.150	0.256	0.328	0.384	0.433
50	0.261	0.380	0.456	0.511	0.554

Table G.12 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.110	0.199	0.268	0.324	0.375
30	0.164	0.274	0.349	0.405	0.455
50	0.283	0.407	0.485	0.540	0.584

Table G.13 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Cauchy(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.621	0.685	0.722	0.750	0.780
30	0.732	0.783	0.813	0.834	0.851
50	0.840	0.879	0.892	0.917	0.929

Table G.14 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = O.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.049	0.096	0.149	0.198
15	0.010	0.050	0.100	0.151	0.202
25	0.010	0.051	0.101	0.151	0.200
50	0.010	0.049	0.101	0.149	0.199

Table G.15 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.003	0.021	0.050	0.085	0.124
15	0.012	0.078	0.157	0.232	0.297
25	0.043	0.172	0.293	0.393	0.473
50	0.191	0.469	0.631	0.725	0.790

Table G.16 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.002	0.017	0.042	0.073	0.109
15	0.012	0.090	0.192	0.288	0.375
25	0.060	0.263	0.443	0.569	0.662
50	0.413	0.772	0.897	0.944	0.968

Table G.17 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.002	0.015	0.038	0.069	0.103
15	0.010	0.086	0.192	0.296	0.391
25	0.058	0.286	0.489	0.629	0.726
50	0.501	0.865	0.952	0.980	0.991

Table G.18 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5); H_a: Weibull(β = 3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.002	0.014	0.035	0.064	0.098
15	0.007	0.073	0.171	0.271	0.367
25	0.045	0.254	0.466	0.616	0.723
50	0.474	0.877	0.964	0.987	0.995

G.2 H_o: *Weibull*($\beta = 1.0$).

G.2.1 $\sqrt{b_1}$ G.O.F. Test Results.

Table G.19 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.026	0.116	0.206	0.277	0.340
15	0.356	0.624	0.756	0.827	0.873
25	0.742	0.922	0.965	0.981	0.988
50	0.996	1.000	1.000	1.000	1.000

Table G.20 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.019	0.089	0.163	0.223	0.279
15	0.179	0.404	0.548	0.637	0.706
25	0.429	0.707	0.824	0.881	0.913
50	0.907	0.980	0.993	0.996	0.998

Table G.21 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.053	0.107	0.157	0.203
15	0.024	0.088	0.157	0.216	0.276
25	0.037	0.120	0.197	0.260	0.316
50	0.069	0.175	0.261	0.332	0.389

Table G.22 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.031	0.133	0.226	0.291	0.348
15	0.385	0.601	0.709	0.770	0.812
25	0.678	0.839	0.898	0.926	0.943
50	0.959	0.986	0.993	0.996	0.997

Table G.23 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.033	0.121	0.206	0.280	0.345
15	0.355	0.632	0.765	0.834	0.878
25	0.741	0.924	0.966	0.982	0.989
50	0.997	1.000	1.000	1.000	1.000

Table G.24 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.088	0.177	0.250	0.309	0.357
15	0.137	0.256	0.344	0.411	0.477
25	0.175	0.322	0.438	0.523	0.589
50	0.253	0.478	0.616	0.706	0.764

Table G.25 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.048	0.098	0.148	0.198
15	0.010	0.050	0.099	0.150	0.201
25	0.010	0.050	0.100	0.151	0.200
50	0.010	0.049	0.100	0.152	0.202

Table G.26 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.056	0.111	0.162	0.212
15	0.039	0.123	0.204	0.270	0.332
25	0.067	0.187	0.285	0.360	0.421
50	0.156	0.338	0.449	0.529	0.589

Table G.27 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.015	0.076	0.145	0.200	0.253
15	0.112	0.268	0.383	0.463	0.530
25	0.226	0.447	0.572	0.650	0.703
50	0.537	0.745	0.826	0.872	0.899

Table G.28 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.029	0.130	0.224	0.290	0.348
15	0.379	0.605	0.718	0.782	0.827
25	0.692	0.865	0.919	0.944	0.959
50	0.973	0.994	0.997	0.998	0.999

Table G.29 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.029	0.090	0.153	0.208	0.265
30	0.028	0.096	0.164	0.225	0.282
50	0.032	0.110	0.192	0.263	0.323

Table G.30 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.030	0.108	0.178	0.243	0.301
30	0.042	0.128	0.207	0.271	0.329
50	0.065	0.173	0.257	0.328	0.390

Table G.31 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1$); H_a: Lognorm(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.072	0.158	0.230	0.293	0.350
30	0.093	0.200	0.285	0.354	0.402
50	0.136	0.268	0.372	0.442	0.498

Table G.32 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$); H_a: X Double Exponential.

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.341	0.470	0.565	0.628	0.675
30	0.437	0.612	0.699	0.754	0.794
50	0.626	0.790	0.858	0.894	0.916

Table G.33 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: X Logistic(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.360	0.494	0.594	0.659	0.707
30	0.454	0.637	0.728	0.784	0.824
50	0.642	0.811	0.879	0.914	0.935

Table G.34 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$); H_a: X Cauchy(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.784	0.853	0.898	0.916	0.935
30	0.893	0.952	0.971	0.978	0.982
50	0.986	0.995	0.997	0.999	1.000

G.2.2 Q-Statistic G.O.F. Test Results

Table G.35 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.033	0.072	0.113	0.159
15	0.008	0.047	0.103	0.158	0.211
25	0.020	0.100	0.200	0.294	0.378
50	0.111	0.372	0.558	0.680	0.759

Table G.36 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.032	0.071	0.114	0.158
15	0.006	0.038	0.083	0.129	0.179
25	0.013	0.068	0.144	0.219	0.292
50	0.055	0.233	0.392	0.517	0.609

Table G.37 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.039	0.080	0.124	0.171
15	0.004	0.030	0.070	0.112	0.155
25	0.005	0.030	0.070	0.113	0.158
50	0.007	0.039	0.082	0.129	0.175

Table G.38 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.031	0.068	0.107	0.148
15	0.002	0.016	0.041	0.070	0.104
25	0.002	0.016	0.041	0.070	0.104
50	0.003	0.029	0.068	0.113	0.160

Table G.39 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.046	0.095	0.143	0.193
15	0.047	0.172	0.288	0.381	0.460
25	0.168	0.424	0.592	0.695	0.769
50	0.686	0.913	0.964	0.982	0.990

Table G.40 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.049	0.139	0.217	0.283	0.340
15	0.154	0.277	0.363	0.426	0.480
25	0.242	0.400	0.494	0.558	0.607
50	0.513	0.678	0.752	0.804	0.827

Table G.41 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.051	0.100	0.150	0.200
15	0.010	0.050	0.100	0.150	0.200
25	0.010	0.050	0.100	0.149	0.202
50	0.010	0.051	0.099	0.149	0.197

Table G.42 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.037	0.077	0.120	0.167
15	0.005	0.029	0.067	0.107	0.150
25	0.006	0.033	0.076	0.122	0.170
50	0.012	0.063	0.124	0.184	0.239

Table G.43 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$); H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.033	0.071	0.111	0.156
15	0.003	0.024	0.057	0.093	0.133
25	0.004	0.030	0.071	0.116	0.165
50	0.012	0.072	0.148	0.219	0.285

Table G.44 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 3.5$).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
5	0.006	0.030	0.066	0.104	0.147
15	0.002	0.018	0.043	0.075	0.112
25	0.003	0.023	0.057	0.095	0.138
50	0.008	0.052	0.118	0.186	0.249

Table G.45 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: χ^2 (1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.034	0.106	0.171	0.229	0.285
30	0.041	0.119	0.191	0.253	0.313
50	0.058	0.159	0.236	0.304	0.362

Table G.46 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: χ^2 (4).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.004	0.031	0.069	0.110	0.152
30	0.005	0.032	0.068	0.111	0.156
50	0.007	0.040	0.084	0.130	0.175

Table G.47 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Lognorm(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.107	0.208	0.293	0.351	0.406
30	0.125	0.259	0.327	0.401	0.458
50	0.167	0.316	0.410	0.479	0.534

Table G.48 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: X Double Exponential.

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.394	0.520	0.592	0.642	0.680
30	0.599	0.712	0.770	0.805	0.832
50	0.825	0.896	0.923	0.939	0.949

Table G.49 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: X Logistic(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.419	0.549	0.622	0.670	0.707
30	0.630	0.741	0.795	0.828	0.852
50	0.851	0.913	0.937	0.950	0.959

Table G.50 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: X Cauchy(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.854	0.908	0.938	0.964	0.993
30	0.944	0.992	0.996	0.999	1.000
50	1.000	1.000	1.000	1.000	1.000

G.3 H_o: Weibull($\beta = 1.5$).

G.3.1 $\sqrt{b_1}$ G.O.F. Test Results.

Table G.51 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$); H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.020	0.075	0.134	0.188	0.240
30	0.023	0.084	0.145	0.202	0.255
50	0.032	0.104	0.171	0.233	0.289

Table G.52 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.159	0.279	0.358	0.418	0.463
15	0.335	0.535	0.651	0.721	0.772
25	0.508	0.729	0.822	0.874	0.908
50	0.791	0.941	0.975	0.986	0.992

Table G.53 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.024	0.082	0.142	0.196	0.244
15	0.048	0.134	0.216	0.282.	0.341
25	0.071	0.183	0.274	0.349	0.414
50	0.118	0.288	0.411	0.469	0.563

Table G.54 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.051	0.102	0.151	0.200
15	0.010	0.050	0.099	0.149	0.199
25	0.010	0.051	0.101	0.151	0.201
50	0.009	0.050	0.101	0.152	0.201

Table G.55 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.052	0.103	0.157	0.208
15	0.024	0.084	0.149	0.209	0.262
25	0.035	0.119	0.195	0.263	0.323
50	0.072	0.202	0.301	0.378	0.440

Table G.56 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.016	0.076	0.144	0.207	0.264
15	0.141	0.309	0.425	0.506	0.563
25	0.308	0.530	0.646	0.716	0.764
50	0.676	0.850	0.907	0.937	0.954

G.3.2 Q-Statistic G.O.F. Test Results.

Table G.57 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.018	0.068	0.124	0.176	0.230
30	0.023	0.080	0.139	0.196	0.248
50	0.027	0.093	0.158	0.218	0.272

Table G.58 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.067	0.173	0.259	0.328	0.386
15	0.293	0.431	0.515	0.571	0.615
25	0.479	0.630	0.702	0.744	0.776
50	0.809	0.891	0.923	0.940	0.951

Table G.59 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.014	0.067	0.127	0.183	0.239
15	0.035	0.111	0.181	0.241	0.298
25	0.052	0.139	0.214	0.275	0.330
50	0.088	0.197	0.280	0.345	0.400

Table G.60 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.050	0.100	0.150	0.201
15	0.010	0.050	0.101	0.151	0.200
25	0.010	0.051	0.100	0.150	0.201
50	0.010	0.049	0.100	0.147	0.197

Table G.61 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.045	0.091	0.139	0.189
15	0.006	0.036	0.069	0.120	0.165
25	0.005	0.032	0.071	0.114	0.160
50	0.006	0.032	0.072	0.116	0.162

Table G.62 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.042	0.087	0.134	0.179
15	0.004	0.027	0.064	0.102	0.144
25	0.003	0.022	0.052	0.089	0.128
50	0.003	0.020	0.050	0.084	0.123

G.4 H_o: Weibull($\beta = 2.0$)

G.4.1 $\sqrt{b_1}$ G.O.F. Test Results.

Table G.63 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.198	0.337	0.423	0.482	0.530
15	0.529	0.738	0.829	0.877	0.908
25	0.780	0.920	0.959	0.975	0.984
50	0.981	0.998	0.999	1.000	1.000

Table G.64 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.036	0.115	0.182	0.241	0.295
15	0.128	0.287	0.393	0.474	0.538
25	0.228	0.433	0.560	0.640	0.699
50	0.456	0.712	0.816	0.870	0.904

Table G.65 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.014	0.060	0.116	0.167	0.216
15	0.025	0.089	0.153	0.216	0.273
25	0.038	0.117	0.192	0.258	0.320
50	0.065	0.186	0.284	0.360	0.423

Table G.66 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.051	0.102	0.151	0.202
15	0.010	0.050	0.099	0.150	0.201
25	0.010	0.051	0.102	0.153	0.202
50	0.008	0.050	0.100	0.152	0.200

Table G.67 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.012	0.060	0.117	0.173	0.227
15	0.057	0.155	0.240	0.310	0.370
25	0.110	0.260	0.368	0.449	0.509
50	0.282	0.497	0.612	0.683	0.733

G.4.1 Q-Statistic G.O.F. Test Results.

Table G.68 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.072	0.184	0.271	0.344	0.402
15	0.356	0.497	0.576	0.625	0.665
25	0.577	0.702	0.761	0.798	0.824
50	0.883	0.934	0.952	0.962	0.969

Table G.69 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.016	0.074	0.137	0.196	0.252
15	0.058	0.152	0.229	0.293	0.348
25	0.096	0.205	0.286	0.349	0.405
50	0.173	0.306	0.391	0.456	0.509

Table G.70 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.055	0.108	0.162	0.213
15	0.018	0.072	0.133	0.188	0.240
25	0.022	0.082	0.142	0.199	0.254
50	0.026	0.088	0.149	0.207	0.263

Table G.71 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.050	0.099	0.150	0.201
15	0.010	0.051	0.102	0.152	0.200
25	0.010	0.050	0.102	0.151	0.201
50	0.011	0.049	0.098	0.148	0.197

Table G.72 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.047	0.094	0.146	0.192
15	0.007	0.042	0.087	0.133	0.179
25	0.006	0.036	0.078	0.123	0.170
50	0.006	0.032	0.072	0.113	0.158

G.5 H_o: Weibull($\beta = 3.5$)

G.5.1 $\sqrt{b_1}$ G.O.F. Test Results.

Table G.73 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.046	0.091	0.137	0.184
15	0.002	0.020	0.049	0.086	0.127
25	0.002	0.015	0.040	0.073	0.110
50	0.001	0.011	0.032	0.060	0.095

Table G.74 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.048	0.096	0.144	0.191
15	0.007	0.043	0.094	0.147	0.197
25	0.009	0.054	0.114	0.177	0.233
50	0.013	0.088	0.174	0.252	0.322

Table G.75 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.056	0.153	0.234	0.300	0.355
15	0.298	0.525	0.646	0.718	0.769
25	0.567	0.782	0.863	0.903	0.928
50	0.917	0.981	0.992	0.996	0.998

Table G.76 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.052	0.104	0.153	0.206
15	0.018	0.073	0.131	0.186	0.239
25	0.026	0.085	0.147	0.205	0.259
50	0.032	0.097	0.164	0.220	0.275

Table G.77 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.012	0.052	0.098	0.143	0.188
15	0.002	0.019	0.046	0.081	0.120
25	0.001	0.013	0.035	0.065	0.100
50	0.001	0.010	0.030	0.057	0.090

Table G.78 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.258	0.415	0.507	0.568	0.615
15	0.807	0.934	0.966	0.980	0.987
25	0.981	0.997	0.999	0.999	1.000
50	1.000	1.000	1.000	1.000	1.000

Table G.79 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.065	0.206	0.307	0.400	0.475
15	0.515	0.743	0.833	0.879	0.909
25	0.813	0.925	0.958	0.973	0.981
50	0.986	0.997	0.998	1.000	1.000

Table G.80 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.023	0.089	0.154	0.212	0.265
15	0.123	0.295	0.412	0.496	0.561
25	0.277	0.503	0.629	0.707	0.758
50	0.619	0.838	0.906	0.937	0.956

Table G.81 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.021	0.095	0.173	0.233	0.288
15	0.204	0.411	0.538	0.617	0.678
25	0.421	0.658	0.766	0.823	0.860
50	0.812	0.927	0.960	0.974	0.983

Table G.82 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.050	0.102	0.152	0.202
15	0.010	0.050	0.099	0.151	0.203
25	0.010	0.052	0.102	0.153	0.202
50	0.010	0.048	0.098	0.148	0.200

Table G.83 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.775	0.917	0.959	0.974	0.983
30	0.947	0.990	0.996	0.998	0.999
50	0.999	1.000	1.000	1.000	1.000

Table G.84 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.312	0.529	0.648	0.720	0.769
30	0.510	0.727	0.823	0.869	0.899
50	0.800	0.928	0.962	0.975	0.984

Table G.85 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Lognorm(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.762	0.896	0.942	0.956	0.970
30	0.930	0.980	0.991	0.995	0.998
50	0.998	1.000	1.000	1.000	1.000

Table G.86 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Double Exponential.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.924	0.966	0.979	0.985	0.988
30	0.990	0.997	0.998	0.999	0.999
50	1.000	1.000	1.000	1.000	1.000

Table G.87 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.957	0.987	0.994	0.996	0.998
30	0.997	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000

Table G.88 Power Study: $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Cauchy(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.986	0.994	0.997	0.998	0.998
30	0.999	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000

G.5.2 Q-Statistic G.O.F. Test Results

Table G.89 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.054	0.107	0.160	0.212
15	0.018	0.075	0.136	0.192	0.250
25	0.030	0.112	0.190	0.260	0.321
50	0.082	0.239	0.358	0.447	0.518

Table G.90 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.053	0.105	0.155	0.207
15	0.013	0.061	0.118	0.170	0.221
25	0.019	0.082	0.146	0.207	0.261
50	0.040	0.139	0.229	0.302	0.365

Table G.91 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.016	0.078	0.143	0.202	0.260
15	0.075	0.174	0.253	0.316	0.372
25	0.124	0.238	0.321	0.384	0.434
50	0.219	0.352	0.436	0.496	0.547

Table G.92 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.051	0.102	0.150	0.199
15	0.014	0.062	0.116	0.167	0.220
25	0.016	0.068	0.124	0.177	0.228
50	0.020	0.080	0.143	0.200	0.256

Table G.93 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.015	0.068	0.130	0.188	0.243
15	0.078	0.216	0.324	0.404	0.472
25	0.211	0.445	0.569	0.653	0.711
50	0.627	0.837	0.907	0.938	0.955

Table G.94 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.073	0.189	0.278	0.350	0.409
15	0.388	0.515	0.588	0.635	0.674
25	0.622	0.729	0.779	0.809	0.832
50	0.902	0.943	0.959	0.967	0.973

Table G.95 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.002	0.015	0.038	0.069	0.103
15	0.010	0.086	0.192	0.296	0.391
25	0.058	0.286	0.489	0.629	0.726
50	0.501	0.865	0.952	0.980	0.991

Table G.96 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.012	0.058	0.114	0.168	0.222
15	0.025	0.088	0.153	0.210	0.267
25	0.033	0.104	0.171	0.231	0.286
50	0.041	0.114	0.184	0.246	0.303

Table G.97 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.030	0.067	0.106	0.148
15	0.003	0.020	0.050	0.084	0.121
25	0.005	0.028	0.068	0.111	0.156
50	0.023	0.067	0.144	0.217	0.286

Table G.98 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.051	0.100	0.151	0.201
15	0.012	0.054	0.105	0.153	0.204
25	0.011	0.051	0.100	0.149	0.201
50	0.010	0.049	0.098	0.149	0.199

Table G.99 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.213	0.342	0.424	0.484	0.532
30	0.316	0.457	0.537	0.591	0.634
50	0.484	0.617	0.688	0.730	0.763

Table G.100 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.052	0.136	0.209	0.268	0.323
30	0.070	0.165	0.243	0.306	0.361
50	0.100	0.207	0.289	0.353	0.410

Table G.101 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Lognorm(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.510	0.650	0.723	0.760	0.791
30	0.718	0.820	0.861	0.883	0.901
50	0.913	0.955	0.972	0.977	0.982

Table G.102 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull(3.5) ; H_a: X Double Exponential.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.660	0.753	0.796	0.824	0.843
30	0.855	0.905	0.927	0.940	0.948
50	0.971	0.984	0.988	0.991	0.993

Table G.103 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.689	0.775	0.815	0.841	0.859
30	0.873	0.918	0.938	0.948	0.956
50	0.977	0.987	0.991	0.993	0.994

Table G.104 Power Study: Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$); H_a: X Cauchy(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.933	0.953	0.963	0.968	0.972
30	0.988	0.992	0.994	0.995	0.996
50	0.998	0.999	1.000	1.000	1.000

Appendix H. Individual One-Tailed $\sqrt{b_1}$ and Q-Statistic G.O.F. Test Power Results.

H.1 H_o: Weibull($\beta = 0.5$).

H.1.1 Lower Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.1 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: $\chi^2(1)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.058	0.184	0.299	0.388	0.467
30	0.080	0.242	0.364	0.460	0.540
50	0.140	0.336	0.471	0.566	0.638

Table H.2 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.042	0.141	0.255	0.350	0.415
15	0.147	0.353	0.486	0.578	0.649
25	0.245	0.477	0.608	0.692	0.752
50	0.458	0.680	0.783	0.840	0.876

Table H.3 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.082	0.237	0.386	0.495	0.558
15	0.398	0.663	0.778	0.842	0.883
25	0.645	0.838	0.907	0.940	0.959
50	0.909	0.970	0.985	0.991	0.994

Table H.4 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.119	0.306	0.475	0.584	0.643
15	0.617	0.833	0.906	0.940	0.960
25	0.867	0.959	0.981	0.989	0.993
50	0.991	0.998	0.999	1.000	1.000

Table H.5 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.196	0.426	0.600	0.703	0.754
15	0.875	0.966	0.985	0.992	0.996
25	0.989	0.999	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000

H.1.2 Upper Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.6 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 0.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.049	0.101	0.154	0.204
15	0.010	0.051	0.103	0.154	0.207
25	0.011	0.051	0.102	0.151	0.200
50	0.011	0.054	0.102	0.151	0.199

Table H.7 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.123	0.223	0.299	0.361	0.416
30	0.153	0.265	0.350	0.417	0.477
50	0.207	0.338	0.436	0.520	0.587

H.1.3 Lower Tail Q-Statistic G.O.F. Test Results.

Table H.8 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: $\chi^2(1)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.035	0.146	0.258	0.357	0.441
30	0.060	0.217	0.357	0.465	0.559
50	0.123	0.354	0.518	0.630	0.712

Table H.9 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.042	0.099	0.165	0.237
15	0.029	0.156	0.295	0.409	0.511
25	0.078	0.294	0.474	0.601	0.693
50	0.290	0.625	0.787	0.869	0.918

Table H.10 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 1.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.005	0.036	0.094	0.164	0.240
15	0.028	0.194	0.376	0.518	0.634
25	0.121	0.438	0.659	0.788	0.866
50	0.573	0.897	0.968	0.988	0.996

Table H.11 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = 2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.004	0.034	0.089	0.157	0.235
15	0.026	0.190	0.387	0.546	0.671
25	0.124	0.486	0.724	0.850	0.915
50	0.672	0.953	0.990	0.998	0.999

Table H.12 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_0: Weibull(β = O.5) ; H_a: Weibull(β = 3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.004	0.032	0.085	0.151	0.227
15	0.020	0.168	0.361	0.530	0.665
25	0.099	0.460	0.722	0.860	0.928
50	0.666	0.962	0.995	0.999	1.000

H.1.4 Upper Tail Q-Statistic G.O.F. Test Results.

Table H.13 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: Weibull(β = O.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.050	0.101	0.151	0.201
15	0.012	0.052	0.103	0.154	0.204
25	0.011	0.050	0.102	0.151	0.202
50	0.011	0.051	0.100	0.150	0.201

Table H.14 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = O.5) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.136	0.237	0.315	0.378	0.433
30	0.196	0.328	0.415	0.481	0.535
50	0.328	0.477	0.561	0.622	0.670

H.2 H_o: Weibull($\beta = 1.0$)

H.2.1 Lower Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.15 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.053	0.205	0.329	0.434	0.520
15	0.463	0.758	0.872	0.923	0.951
25	0.829	0.963	0.987	0.995	0.998
50	0.999	1.000	1.000	1.000	1.000

Table H.16 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.037	0.158	0.260	0.353	0.438
15	0.260	0.544	0.702	0.793	0.850
25	0.551	0.825	0.914	0.952	0.972
50	0.949	0.993	0.998	0.999	1.000

Table H.17 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.018	0.082	0.150	0.217	0.281
15	0.039	0.139	0.233	0.311	0.380
25	0.060	0.178	0.280	0.365	0.437
50	0.103	0.252	0.365	0.454	0.525

Table H.18 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.061	0.222	0.337	0.435	0.518
15	0.471	0.708	0.812	0.866	0.900
25	0.753	0.898	0.943	0.964	0.976
50	0.973	0.993	0.997	0.999	0.999

Table H.19 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.057	0.202	0.333	0.432	0.516
15	0.459	0.761	0.875	0.927	0.955
25	0.838	0.968	0.990	0.995	0.998
50	0.999	1.000	1.000	1.000	1.000

Table H.20 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.020	0.093	0.170	0.240	0.308
15	0.062	0.195	0.312	0.406	0.482
25	0.109	0.282	0.409	0.507	0.583
50	0.221	0.448	0.585	0.673	0.736

Table H.21 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.032	0.139	0.232	0.316	0.393
15	0.160	0.377	0.522	0.625	0.694
25	0.315	0.569	0.697	0.777	0.828
50	0.622	0.827	0.899	0.935	0.954

Table H.22 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.059	0.219	0.335	0.435	0.519
15	0.466	0.719	0.827	0.882	0.915
25	0.770	0.918	0.960	0.976	0.985
50	0.984	0.997	0.999	1.000	1.000

Table H.23 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.053	0.166	0.265	0.343	0.412
30	0.068	0.196	0.301	0.384	0.455
50	0.103	0.255	0.372	0.457	0.529

H.2.2 Upper Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.24 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.114	0.238	0.328	0.399	0.457
15	0.176	0.337	0.470	0.570	0.644
25	0.224	0.441	0.588	0.687	0.757
50	0.333	0.613	0.758	0.836	0.884

Table H.25 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.049	0.100	0.152	0.203
15	0.010	0.051	0.105	0.153	0.204
25	0.011	0.051	0.100	0.150	0.201
50	0.011	0.053	0.103	0.151	0.200

Table H.26 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.046	0.143	0.244	0.334	0.409
30	0.047	0.161	0.271	0.363	0.447
50	0.054	0.189	0.315	0.415	0.501

Table H.27 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Lognorm(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.094	0.210	0.311	0.387	0.448
30	0.125	0.272	0.381	0.460	0.537
50	0.176	0.365	0.486	0.571	0.641

Table H.28 Power Study: Upper Tail Skewness G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: X Logistic(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.407	0.596	0.706	0.775	0.820
30	0.530	0.734	0.826	0.875	0.907
50	0.717	0.882	0.936	0.959	0.972

H.2.3 Lower Tail Q-Statistic G.O.F. Test Results.

Table H.29 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.047	0.104	0.161	0.219
15	0.016	0.103	0.210	0.315	0.415
25	0.042	0.202	0.376	0.523	0.638
50	0.186	0.556	0.758	0.862	0.920

Table H.30 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.008	0.044	0.098	0.154	0.213
15	0.012	0.083	0.176	0.269	0.360
25	0.026	0.147	0.292	0.421	0.534
50	0.105	0.397	0.609	0.742	0.831

Table H.31 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.042	0.089	0.140	0.192
15	0.007	0.045	0.098	0.152	0.210
25	0.009	0.051	0.111	0.172	0.236
50	0.013	0.070	0.144	0.215	0.285

Table H.32 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 1$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.006	0.037	0.080	0.126	0.176
15	0.005	0.035	0.082	0.137	0.198
25	0.005	0.039	0.095	0.164	0.239
50	0.009	0.068	0.157	0.253	0.352

Table H.33 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.014	0.068	0.141	0.210	0.275
15	0.080	0.293	0.462	0.586	0.682
25	0.259	0.592	0.772	0.865	0.916
50	0.793	0.963	0.989	0.996	0.999

Table H.34 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.008	0.044	0.094	0.146	0.200
15	0.010	0.057	0.119	0.185	0.250
25	0.012	0.069	0.150	0.230	0.307
50	0.024	0.123	0.234	0.334	0.428

Table H.35 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.040	0.086	0.137	0.190
15	0.008	0.052	0.116	0.184	0.256
25	0.012	0.071	0.159	0.248	0.336
50	0.026	0.144	0.282	0.403	0.510

Table H.36 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.037	0.081	0.132	0.185
15	0.006	0.043	0.100	0.165	0.233
25	0.008	0.055	0.131	0.216	0.305
50	0.016	0.118	0.253	0.377	0.492

Table H.37 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.009	0.047	0.104	0.162	0.223
30	0.010	0.054	0.119	0.182	0.245
50	0.013	0.070	0.144	0.215	0.287

H.2.4 Upper Tail Q-Statistic G.O.F. Test Results.

Table H.38 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1) ; H_a: Weibull(β = 0.5).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
5	0.062	0.161	0.244	0.312	0.366
15	0.196	0.353	0.451	0.527	0.583
25	0.306	0.491	0.598	0.668	0.720
50	0.575	0.749	0.825	0.866	0.895

Table H.39 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1) ; H_a: Weibull(β = 1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
5	0.010	0.047	0.097	0.148	0.198
15	0.009	0.052	0.102	0.154	0.204
25	0.011	0.051	0.101	0.149	0.200
50	0.010	0.050	0.100	0.150	0.200

Table H.40 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1) ; H_a: χ^2 (1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.048	0.136	0.218	0.287	0.348
30	0.058	0.169	0.261	0.333	0.397
50	0.086	0.221	0.326	0.405	0.475

Table H.41 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1) ; H_a: Lognorm(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.120	0.224	0.316	0.390	0.455
30	0.144	0.275	0.379	0.459	0.528
50	0.186	0.354	0.469	0.554	0.623

Table H.42 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1) ; H_a: X Logistic(0,1).

Sample	Significance Level (α)				
Size	0.01	0.05	0.10	0.15	0.20
20	0.475	0.618	0.699	0.752	0.789
30	0.676	0.795	0.850	0.881	0.904
50	0.879	0.937	0.958	0.969	0.976

H.3 H_o: Weibull($\beta = 1.5$)

H.3.1 Lower Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.43 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.015	0.073	0.142	0.204	0.266
15	0.039	0.138	0.234	0.319	0.392
25	0.059	0.185	0.296	0.385	0.460
50	0.114	0.292	0.426	0.520	0.598

Table H.44 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.031	0.128	0.228	0.306	0.377
15	0.204	0.428	0.563	0.656	0.723
25	0.389	0.643	0.762	0.829	0.872
50	0.755	0.907	0.953	0.972	0.983

H.3.2 Upper Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.45 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.201	0.357	0.454	0.523	0.580
15	0.402	0.652	0.776	0.845	0.885
25	0.596	0.825	0.909	0.945	0.965
50	0.862	0.973	0.991	0.996	0.998

Table H.46 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.033	0.115	0.194	0.261	0.323
15	0.074	0.207	0.321	0.412	0.488
25	0.108	0.274	0.407	0.506	0.582
50	0.176	0.411	0.561	0.658	0.729

Table H.47 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.051	0.099	0.149	0.198
15	0.010	0.052	0.101	0.150	0.201
25	0.010	0.050	0.100	0.154	0.202
50	0.010	0.052	0.104	0.152	0.201

Table H.48 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: $\chi^2(4)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.031	0.107	0.180	0.252	0.311
30	0.034	0.121	0.205	0.277	0.342
50	0.047	0.154	0.251	0.330	0.397

H.3.3 Lower Tail Q-Statistic G.O.F. Test Results.

Table H.49 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1.5) ; H_a: Weibull(β = 2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.046	0.095	0.145	0.198
15	0.010	0.048	0.098	0.151	0.203
25	0.009	0.047	0.100	0.157	0.214
50	0.010	0.056	0.117	0.179	0.244

Table H.50 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 1.5) ; H_a: Weibull(β = 3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.042	0.088	0.135	0.187
15	0.007	0.038	0.081	0.129	0.179
25	0.005	0.036	0.078	0.128	0.181
50	0.006	0.040	0.089	0.147	0.209

H.3.4 Upper Tail Q-Statistic G.O.F. Test Results.

Table H.51 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.085	0.199	0.288	0.351	0.407
15	0.340	0.504	0.595	0.653	0.698
25	0.539	0.697	0.770	0.813	0.843
50	0.847	0.920	0.948	0.962	0.972

Table H.52 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.017	0.072	0.134	0.189	0.244
15	0.048	0.137	0.213	0.274	0.332
25	0.070	0.179	0.265	0.331	0.390
50	0.120	0.257	0.363	0.440	0.503

Table H.53 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.051	0.102	0.150	0.202
15	0.010	0.054	0.103	0.155	0.206
25	0.010	0.050	0.100	0.148	0.195
50	0.010	0.047	0.098	0.150	0.201

Table H.54 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_0: Weibull($\beta = 1.5$) ; H_a: $\chi^2(4)$.

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.026	0.091	0.160	0.221	0.277
30	0.032	0.108	0.183	0.248	0.310
50	0.043	0.130	0.214	0.288	0.350

H.4 H_o: Weibull($\beta = 2.0$)

H.4.1 Lower Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.55 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.018	0.092	0.168	0.237	0.300
15	0.085	0.232	0.355	0.446	0.521
25	0.160	0.360	0.494	0.587	0.658
50	0.359	0.610	0.734	0.804	0.851

H.4.2 Upper Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.56 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.246	0.417	0.517	0.587	0.645
15	0.622	0.833	0.910	0.943	0.963
25	0.850	0.959	0.984	0.992	0.996
50	0.991	0.999	1.000	1.000	1.000

Table H.57 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.055	0.168	0.262	0.337	0.405
15	0.184	0.395	0.538	0.630	0.701
25	0.304	0.560	0.697	0.780	0.833
50	0.565	0.815	0.901	0.939	0.961

Table H.58 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.019	0.081	0.149	0.211	0.271
15	0.042	0.143	0.238	0.318	0.390
25	0.059	0.184	0.298	0.390	0.466
50	0.103	0.283	0.417	0.516	0.595

Table H.59 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.050	0.101	0.149	0.202
15	0.011	0.050	0.103	0.153	0.202
25	0.010	0.050	0.099	0.150	0.201
50	0.010	0.051	0.102	0.152	0.200

H.4.3 Lower Tail Q-Statistic G.O.F. Test Results.

Table H.60 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.049	0.096	0.148	0.195
15	0.008	0.044	0.089	0.136	0.181
25	0.007	0.038	0.080	0.125	0.172
50	0.006	0.035	0.075	0.119	0.166

H.4.4 Upper Tail Q-Statistic G.O.F. Test Results.

Table H.61 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 0.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.088	0.211	0.299	0.365	0.417
15	0.407	0.556	0.636	0.688	0.727
25	0.633	0.761	0.818	0.850	0.875
50	0.902	0.950	0.967	0.975	0.981

Table H.62 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.018	0.075	0.138	0.197	0.253
15	0.079	0.182	0.261	0.323	0.378
25	0.123	0.250	0.339	0.403	0.456
50	0.218	0.376	0.476	0.547	0.599

Table H.63 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.012	0.056	0.109	0.162	0.215
15	0.021	0.081	0.139	0.193	0.246
25	0.027	0.091	0.156	0.210	0.264
50	0.034	0.110	0.181	0.245	0.301

Table H.64 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 2$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.051	0.102	0.152	0.199
15	0.010	0.051	0.102	0.153	0.203
25	0.010	0.049	0.099	0.148	0.197
50	0.010	0.051	0.101	0.152	0.201

H.5 H_o: Weibull($\beta = 3.5$)

H.5.1 Lower Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.65 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.048	0.092	0.139	0.187
15	0.004	0.028	0.069	0.114	0.163
25	0.002	0.022	0.057	0.101	0.149
50	0.003	0.023	0.060	0.106	0.158

Table H.66 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.053	0.105	0.155	0.207
15	0.019	0.075	0.134	0.188	0.241
25	0.028	0.087	0.148	0.209	0.260
50	0.032	0.097	0.166	0.222	0.277

Table H.67 Power Study: Lower Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.013	0.053	0.100	0.145	0.191
15	0.003	0.024	0.062	0.105	0.153
25	0.002	0.021	0.056	0.099	0.149
50	0.002	0.021	0.057	0.101	0.153

H.5.2 Upper Tail $\sqrt{b_1}$ G.O.F. Test Results.

Table H.68 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.013	0.065	0.126	0.187	0.246
15	0.015	0.086	0.179	0.261	0.337
25	0.019	0.108	0.220	0.321	0.411
50	0.030	0.170	0.317	0.436	0.534

Table H.69 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.098	0.200	0.312	0.402	0.513
15	0.399	0.622	0.729	0.808	0.847
25	0.538	0.839	0.912	0.937	0.974
50	0.921	0.989	0.998	1.000	1.000

Table H.70 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull(0.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.313	0.502	0.607	0.681	0.743
15	0.871	0.967	0.987	0.993	0.996
25	0.991	0.999	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000

Table H.71 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull(1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.087	0.296	0.396	0.473	0.546
15	0.689	0.825	0.880	0.908	0.953
25	0.879	0.956	0.970	0.983	0.996
50	0.990	0.999	1.000	1.000	1.000

Table H.72 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Weibull(1.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.036	0.138	0.233	0.310	0.381
15	0.188	0.413	0.563	0.659	0.726
25	0.366	0.630	0.759	0.832	0.877
50	0.716	0.901	0.952	0.972	0.983

Table H.73 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Weibull(2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.040	0.148	0.234	0.304	0.381
15	0.308	0.520	0.641	0.751	0.816
25	0.524	0.771	0.850	0.917	0.912
50	0.876	0.961	0.988	0.998	0.999

Table H.74 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Weibull(3.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.009	0.047	0.097	0.148	0.199
15	0.010	0.051	0.103	0.154	0.203
25	0.011	0.050	0.101	0.152	0.200
50	0.010	0.051	0.099	0.150	0.199

Table H.75 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.849	0.960	0.985	0.992	0.996
30	0.969	0.995	0.999	0.999	1.000
50	0.999	1.000	1.000	1.000	1.000

Table H.76 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.397	0.644	0.764	0.832	0.876
30	0.603	0.827	0.904	0.940	0.960
50	0.864	0.962	0.984	0.992	0.995

Table H.77 Power Study: Upper Tail $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.974	0.994	0.998	0.999	1.000
30	0.998	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000

H.5.3 Lower Tail Q-Statistic G.O.F. Test Results.

Table H.78 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Beta(2,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.013	0.062	0.121	0.177	0.232
15	0.032	0.127	0.221	0.301	0.374
25	0.052	0.181	0.305	0.406	0.490
50	0.137	0.360	0.516	0.619	0.698

Table H.79 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Beta(2,3).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.012	0.060	0.117	0.172	0.227
15	0.023	0.097	0.177	0.250	0.317
25	0.034	0.131	0.229	0.317	0.392
50	0.073	0.227	0.359	0.460	0.542

Table H.80 Power Study: Lower Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Uniform(0,2).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.021	0.091	0.164	0.231	0.294
15	0.121	0.323	0.465	0.562	0.638
25	0.297	0.576	0.720	0.800	0.853
50	0.725	0.903	0.955	0.974	0.984

H.5.4 Upper Tail Q-Statistic G.O.F. Test Results.

Table H.81 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Gamma(2,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.024	0.134	0.182	0.254	0.351
15	0.089	0.198	0.293	0.368	0.463
25	0.155	0.274	0.366	0.432	0.492
50	0.249	0.317	0.474	0.544	0.641

Table H.82 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Normal(O,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.011	0.053	0.105	0.156	0.207
15	0.017	0.073	0.136	0.194	0.249
25	0.023	0.090	0.158	0.221	0.282
50	0.032	0.117	0.199	0.270	0.335

Table H.83 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Weibull(0.5).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.088	0.213	0.301	0.361	0.412
15	0.433	0.574	0.647	0.693	0.727
25	0.667	0.776	0.825	0.855	0.877
50	0.918	0.956	0.970	0.977	0.982

Table H.84 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull(β = 3.5) ; H_a: Weibull(β = 1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.003	0.026	0.055	0.100	0.138
15	0.014	0.149	0.222	0.382	0.512
25	0.069	0.317	0.551	0.717	0.899
50	0.604	0.900	0.972	0.990	0.998

Table H.85 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 1.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.013	0.059	0.115	0.169	0.225
15	0.027	0.089	0.155	0.215	0.269
25	0.039	0.107	0.173	0.237	0.288
50	0.050	0.131	0.201	0.260	0.312

Table H.86 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 2$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.007	0.031	0.068	0.109	0.149
15	0.004	0.022	0.052	0.086	0.125
25	0.005	0.029	0.069	0.114	0.158
50	0.024	0.068	0.145	0.219	0.287

Table H.87 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: Weibull($\beta = 3.5$).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
5	0.010	0.052	0.100	0.149	0.198
15	0.010	0.051	0.101	0.151	0.198
25	0.010	0.051	0.101	0.152	0.200
50	0.011	0.051	0.102	0.153	0.202

Table H.88 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.247	0.383	0.468	0.526	0.572
30	0.362	0.512	0.594	0.649	0.690
50	0.538	0.679	0.748	0.789	0.820

Table H.89 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: χ^2 (4).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.067	0.155	0.225	0.284	0.334
30	0.090	0.197	0.279	0.345	0.397
50	0.130	0.259	0.349	0.418	0.475

Table H.90 Power Study: Upper Tail Q-Statistic G.O.F. Test — H_o: Weibull($\beta = 3.5$) ; H_a: X Logistic(0,1).

Sample Size	Significance Level (α)				
	0.01	0.05	0.10	0.15	0.20
20	0.721	0.810	0.852	0.877	0.894
30	0.889	0.932	0.950	0.960	0.967
50	0.982	0.991	0.994	0.996	0.997

Appendix I. One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ and Q-Statistic G.O.F. Tests.

I.1 H_o: Weibull(β = 0.5).

I.1.1 One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ G.O.F. Tests.

Table I.1 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(O.5); H_a: $\chi^2(1)$.

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.032	0.048	0.096
	One-Tailed	0.058	0.080	0.140
0.05	Two-Tailed	0.113	0.152	0.235
	One-Tailed	0.184	0.242	0.336
0.10	Two-Tailed	0.186	0.242	0.335
	One-Tailed	0.299	0.364	0.471
0.15	Two-Tailed	0.252	0.310	0.410
	One-Tailed	0.388	0.460	0.566
0.20	Two-Tailed	0.310	0.371	0.473
	One-Tailed	0.467	0.540	0.638

Table I.2 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(O.5); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.099	0.130	0.175
	One-Tailed	0.123	0.153	0.207
0.05	Two-Tailed	0.188	0.222	0.275
	One-Tailed	0.223	0.265	0.338
0.10	Two-Tailed	0.255	0.286	0.347
	One-Tailed	0.299	0.350	0.436
0.15	Two-Tailed	0.311	0.342	0.402
	One-Tailed	0.361	0.417	0.520
0.20	Two-Tailed	0.361	0.390	0.456
	One-Tailed	0.416	0.477	0.587

Table I.3 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.009
	One-Tailed	0.009	0.010	0.011	0.011
0.05	Two-Tailed	0.049	0.051	0.051	0.050
	One-Tailed	0.049	0.051	0.051	0.054
0.10	Two-Tailed	0.097	0.099	0.102	0.101
	One-Tailed	0.101	0.103	0.102	0.102
0.15	Two-Tailed	0.149	0.152	0.151	0.151
	One-Tailed	0.154	0.154	0.151	0.151
0.20	Two-Tailed	0.199	0.203	0.201	0.201
	One-Tailed	0.204	0.207	0.200	0.199

Table I.4 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.021	0.095	0.184	0.372
	One-Tailed	0.042	0.147	0.245	0.458
0.05	Two-Tailed	0.085	0.240	0.365	0.580
	One-Tailed	0.141	0.353	0.477	0.680
0.10	Two-Tailed	0.145	0.349	0.481	0.678
	One-Tailed	0.255	0.486	0.608	0.783
0.15	Two-Tailed	0.207	0.425	0.560	0.736
	One-Tailed	0.350	0.578	0.692	0.840
0.20	Two-Tailed	0.263	0.485	0.616	0.778
	One-Tailed	0.415	0.649	0.752	0.876

Table I.5 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.044	0.304	0.560	0.869
	One-Tailed	0.082	0.398	0.645	0.909
0.05	Two-Tailed	0.149	0.538	0.758	0.946
	One-Tailed	0.237	0.663	0.838	0.970
0.10	Two-Tailed	0.237	0.660	0.839	0.969
	One-Tailed	0.386	0.778	0.907	0.985
0.15	Two-Tailed	0.317	0.728	0.882	0.979
	One-Tailed	0.495	0.842	0.940	0.991
0.20	Two-Tailed	0.389	0.778	0.907	0.985
	One-Tailed	0.558	0.883	0.959	0.994

Table I.6 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(0.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.065	0.515	0.813	0.986
	One-Tailed	0.119	0.617	0.867	0.991
0.05	Two-Tailed	0.206	0.743	0.925	0.997
	One-Tailed	0.306	0.833	0.959	0.998
0.10	Two-Tailed	0.307	0.833	0.958	0.998
	One-Tailed	0.475	0.906	0.981	0.999
0.15	Two-Tailed	0.400	0.879	0.973	0.999
	One-Tailed	0.584	0.940	0.989	1.000
0.20	Two-Tailed	0.475	0.909	0.981	1.000
	One-Tailed	0.643	0.960	0.993	1.000

Table I.7 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(0.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.115	0.820	0.982	1.000
	One-Tailed	0.196	0.875	0.989	1.000
0.05	Two-Tailed	0.307	0.937	0.996	1.000
	One-Tailed	0.426	0.966	0.999	1.000
0.10	Two-Tailed	0.426	0.968	0.999	1.000
	One-Tailed	0.600	0.985	1.000	1.000
0.15	Two-Tailed	0.526	0.980	0.999	1.000
	One-Tailed	0.703	0.992	1.000	1.000
0.20	Two-Tailed	0.602	0.986	1.000	1.000
	One-Tailed	0.754	0.996	1.000	1.000

I.1.2 One- and Two-Tailed Power Comparisons for the Individual Q-Statistic G.O.F. Tests.

Table I.8 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: $\chi^2(1)$.

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.020	0.034	0.073
	One-Tailed	0.035	0.060	0.123
0.05	Two-Tailed	0.081	0.128	0.228
	One-Tailed	0.146	0.217	0.354
0.10	Two-Tailed	0.151	0.218	0.351
	One-Tailed	0.258	0.357	0.518
0.15	Two-Tailed	0.210	0.291	0.445
	One-Tailed	0.357	0.465	0.630
0.20	Two-Tailed	0.269	0.358	0.520
	One-Tailed	0.441	0.559	0.712

Table I.9 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.110	0.164	0.283
	One-Tailed	0.136	0.196	0.328
0.05	Two-Tailed	0.199	0.274	0.407
	One-Tailed	0.237	0.328	0.477
0.10	Two-Tailed	0.268	0.349	0.485
	One-Tailed	0.315	0.415	0.561
0.15	Two-Tailed	0.324	0.405	0.540
	One-Tailed	0.378	0.481	0.622
0.20	Two-Tailed	0.375	0.455	0.584
	One-Tailed	0.433	0.535	0.670

Table I.10 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.010
	One-Tailed	0.010	0.012	0.011	0.011
0.05	Two-Tailed	0.049	0.050	0.051	0.049
	One-Tailed	0.050	0.052	0.050	0.051
0.10	Two-Tailed	0.096	0.100	0.101	0.101
	One-Tailed	0.101	0.103	0.102	0.100
0.15	Two-Tailed	0.149	0.151	0.151	0.149
	One-Tailed	0.151	0.154	0.151	0.150
0.20	Two-Tailed	0.198	0.202	0.200	0.199
	One-Tailed	0.201	0.204	0.202	0.201

Table I.11 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.003	0.012	0.043	0.191
	One-Tailed	0.006	0.029	0.078	0.290
0.05	Two-Tailed	0.021	0.078	0.172	0.469
	One-Tailed	0.042	0.156	0.294	0.625
0.10	Two-Tailed	0.050	0.157	0.293	0.631
	One-Tailed	0.099	0.295	0.474	0.787
0.15	Two-Tailed	0.085	0.232	0.393	0.725
	One-Tailed	0.165	0.409	0.601	0.869
0.20	Two-Tailed	0.124	0.297	0.473	0.790
	One-Tailed	0.237	0.511	0.693	0.918

Table I.12 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.002	0.012	0.060	0.413
	One-Tailed	0.005	0.028	0.121	0.573
0.05	Two-Tailed	0.017	0.090	0.263	0.772
	One-Tailed	0.036	0.194	0.438	0.897
0.10	Two-Tailed	0.042	0.192	0.443	0.897
	One-Tailed	0.094	0.376	0.659	0.968
0.15	Two-Tailed	0.073	0.288	0.569	0.944
	One-Tailed	0.164	0.518	0.788	0.988
0.20	Two-Tailed	0.109	0.375	0.662	0.968
	One-Tailed	0.240	0.634	0.866	0.996

Table I.13 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.002	0.010	0.058	0.501
	One-Tailed	0.004	0.026	0.124	0.672
0.05	Two-Tailed	0.015	0.086	0.286	0.865
	One-Tailed	0.034	0.190	0.486	0.953
0.10	Two-Tailed	0.038	0.192	0.489	0.952
	One-Tailed	0.089	0.387	0.724	0.990
0.15	Two-Tailed	0.069	0.296	0.629	0.980
	One-Tailed	0.157	0.546	0.850	0.998
0.20	Two-Tailed	0.103	0.391	0.726	0.991
	One-Tailed	0.235	0.671	0.915	0.999

Table I.14 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(O.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.002	0.007	0.045	0.474
	One-Tailed	0.004	0.020	0.099	0.666
0.05	Two-Tailed	0.014	0.073	0.254	0.877
	One-Tailed	0.032	0.168	0.460	0.962
0.10	Two-Tailed	0.035	0.171	0.466	0.964
	One-Tailed	0.085	0.361	0.722	0.995
0.15	Two-Tailed	0.064	0.271	0.616	0.987
	One-Tailed	0.151	0.530	0.860	0.999
0.20	Two-Tailed	0.098	0.367	0.723	0.995
	One-Tailed	0.227	0.665	0.928	1.000

I.2 H_o: Weibull($\beta = 1.0$)

I.2.1 One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ G.O.F. Tests.

Table I.15 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Beta(2,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.026	0.356	0.742	0.996
	One-Tailed	0.053	0.463	0.829	0.999
0.05	Two-Tailed	0.116	0.624	0.922	1.000
	One-Tailed	0.205	0.758	0.963	1.000
0.10	Two-Tailed	0.206	0.756	0.965	1.000
	One-Tailed	0.329	0.872	0.987	1.000
0.15	Two-Tailed	0.277	0.827	0.981	1.000
	One-Tailed	0.434	0.923	0.995	1.000
0.20	Two-Tailed	0.340	0.873	0.988	1.000
	One-Tailed	0.520	0.951	0.998	1.000

Table I.16 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Beta(2,3).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.019	0.179	0.429	0.907
	One-Tailed	0.037	0.260	0.551	0.949
0.05	Two-Tailed	0.089	0.404	0.707	0.980
	One-Tailed	0.158	0.544	0.825	0.993
0.10	Two-Tailed	0.163	0.548	0.824	0.993
	One-Tailed	0.260	0.702	0.914	0.998
0.15	Two-Tailed	0.223	0.637	0.881	0.996
	One-Tailed	0.353	0.793	0.952	0.999
0.20	Two-Tailed	0.279	0.706	0.913	0.998
	One-Tailed	0.438	0.850	0.972	1.000

Table I.17 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Gamma(2,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.024	0.037	0.069
	One-Tailed	0.018	0.039	0.060	0.103
0.05	Two-Tailed	0.053	0.088	0.120	0.175
	One-Tailed	0.082	0.139	0.178	0.252
0.10	Two-Tailed	0.107	0.157	0.197	0.261
	One-Tailed	0.150	0.233	0.280	0.365
0.15	Two-Tailed	0.157	0.216	0.260	0.332
	One-Tailed	0.217	0.311	0.365	0.454
0.20	Two-Tailed	0.203	0.276	0.316	0.389
	One-Tailed	0.281	0.380	0.437	0.525

Table I.18 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Normal(O,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.031	0.385	0.678	0.959
	One-Tailed	0.061	0.471	0.753	0.973
0.05	Two-Tailed	0.133	0.601	0.839	0.986
	One-Tailed	0.222	0.708	0.898	0.993
0.10	Two-Tailed	0.226	0.709	0.898	0.993
	One-Tailed	0.337	0.812	0.943	0.997
0.15	Two-Tailed	0.291	0.770	0.926	0.996
	One-Tailed	0.435	0.866	0.964	0.999
0.20	Two-Tailed	0.348	0.812	0.943	0.997
	One-Tailed	0.518	0.900	0.976	0.999

Table I.19 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Uniform(0,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.033	0.355	0.741	0.997
	One-Tailed	0.057	0.459	0.838	0.999
0.05	Two-Tailed	0.121	0.632	0.924	1.000
	One-Tailed	0.202	0.761	0.968	1.000
0.10	Two-Tailed	0.206	0.765	0.966	1.000
	One-Tailed	0.333	0.875	0.990	1.000
0.15	Two-Tailed	0.280	0.834	0.982	1.000
	One-Tailed	0.432	0.927	0.995	1.000
0.20	Two-Tailed	0.345	0.878	0.989	1.000
	One-Tailed	0.516	0.955	0.998	1.000

Table I.20 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.088	0.137	0.175	0.253
	One-Tailed	0.114	0.176	0.224	0.333
0.05	Two-Tailed	0.177	0.256	0.322	0.478
	One-Tailed	0.238	0.337	0.441	0.613
0.10	Two-Tailed	0.250	0.344	0.438	0.616
	One-Tailed	0.328	0.470	0.588	0.758
0.15	Two-Tailed	0.309	0.411	0.523	0.706
	One-Tailed	0.399	0.570	0.687	0.836
0.20	Two-Tailed	0.357	0.477	0.589	0.764
	One-Tailed	0.457	0.644	0.757	0.884

Table I.21 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Weibull(1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.010
	One-Tailed	0.010	0.010	0.011	0.011
0.05	Two-Tailed	0.048	0.050	0.050	0.049
	One-Tailed	0.049	0.051	0.051	0.053
0.10	Two-Tailed	0.098	0.099	0.100	0.100
	One-Tailed	0.100	0.105	0.100	0.103
0.15	Two-Tailed	0.148	0.150	0.151	0.152
	One-Tailed	0.152	0.153	0.150	0.151
0.20	Two-Tailed	0.198	0.201	0.200	0.202
	One-Tailed	0.203	0.204	0.201	0.200

Table I.22 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.011	0.039	0.067	0.156
	One-Tailed	0.020	0.062	0.109	0.221
0.05	Two-Tailed	0.056	0.123	0.187	0.338
	One-Tailed	0.093	0.195	0.282	0.448
0.10	Two-Tailed	0.111	0.204	0.285	0.449
	One-Tailed	0.170	0.312	0.409	0.585
0.15	Two-Tailed	0.162	0.270	0.360	0.529
	One-Tailed	0.240	0.406	0.507	0.673
0.20	Two-Tailed	0.212	0.332	0.421	0.589
	One-Tailed	0.308	0.482	0.583	0.736

Table I.23 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.015	0.112	0.226	0.537
	One-Tailed	0.032	0.160	0.315	0.622
0.05	Two-Tailed	0.076	0.268	0.447	0.745
	One-Tailed	0.139	0.377	0.569	0.827
0.10	Two-Tailed	0.145	0.383	0.572	0.826
	One-Tailed	0.232	0.522	0.697	0.899
0.15	Two-Tailed	0.200	0.463	0.650	0.872
	One-Tailed	0.316	0.625	0.777	0.935
0.20	Two-Tailed	0.253	0.530	0.703	0.899
	One-Tailed	0.393	0.694	0.828	0.954

Table I.24 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.029	0.379	0.692	0.973
	One-Tailed	0.059	0.466	0.770	0.984
0.05	Two-Tailed	0.130	0.605	0.865	0.994
	One-Tailed	0.219	0.719	0.918	0.997
0.10	Two-Tailed	0.224	0.718	0.919	0.997
	One-Tailed	0.335	0.827	0.960	0.999
0.15	Two-Tailed	0.290	0.782	0.944	0.998
	One-Tailed	0.435	0.882	0.976	1.000
0.20	Two-Tailed	0.348	0.827	0.959	0.999
	One-Tailed	0.519	0.915	0.985	1.000

Table I.25 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: χ^2 (1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.029	0.028	0.032
	One-Tailed	0.046	0.047	0.054
0.05	Two-Tailed	0.090	0.096	0.110
	One-Tailed	0.143	0.161	0.189
0.10	Two-Tailed	0.153	0.164	0.192
	One-Tailed	0.244	0.271	0.315
0.15	Two-Tailed	0.208	0.225	0.263
	One-Tailed	0.334	0.363	0.415
0.20	Two-Tailed	0.265	0.282	0.323
	One-Tailed	0.409	0.447	0.501

Table I.26 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: χ^2 (4).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.030	0.042	0.065
	One-Tailed	0.053	0.068	0.103
0.05	Two-Tailed	0.108	0.128	0.173
	One-Tailed	0.166	0.196	0.255
0.10	Two-Tailed	0.178	0.207	0.257
	One-Tailed	0.265	0.301	0.372
0.15	Two-Tailed	0.243	0.271	0.328
	One-Tailed	0.343	0.384	0.457
0.20	Two-Tailed	0.301	0.329	0.390
	One-Tailed	0.412	0.455	0.529

Table I.27 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.360	0.454	0.642
	One-Tailed	0.407	0.530	0.717
0.05	Two-Tailed	0.494	0.637	0.811
	One-Tailed	0.596	0.734	0.882
0.10	Two-Tailed	0.594	0.728	0.879
	One-Tailed	0.706	0.826	0.936
0.15	Two-Tailed	0.659	0.784	0.914
	One-Tailed	0.775	0.875	0.959
0.20	Two-Tailed	0.707	0.824	0.935
	One-Tailed	0.820	0.907	0.972

Table I.28 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1); H_a: Lognorm(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.072	0.093	0.136
	One-Tailed	0.094	0.125	0.176
0.05	Two-Tailed	0.158	0.200	0.268
	One-Tailed	0.210	0.272	0.365
0.10	Two-Tailed	0.230	0.285	0.372
	One-Tailed	0.311	0.381	0.486
0.15	Two-Tailed	0.293	0.354	0.442
	One-Tailed	0.387	0.460	0.571
0.20	Two-Tailed	0.350	0.402	0.498
	One-Tailed	0.448	0.537	0.641

I.2.2 *One- and Two-Tailed Power Comparisons for the Individual Q-Statistic G.O.F. Tests.*

Table I.29 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Beta(2,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.008	0.020	0.111
	One-Tailed	0.009	0.016	0.042	0.186
0.05	Two-Tailed	0.033	0.047	0.100	0.372
	One-Tailed	0.047	0.103	0.202	0.556
0.10	Two-Tailed	0.072	0.103	0.200	0.558
	One-Tailed	0.104	0.210	0.376	0.758
0.15	Two-Tailed	0.113	0.158	0.294	0.680
	One-Tailed	0.161	0.315	0.523	0.862
0.20	Two-Tailed	0.159	0.211	0.378	0.759
	One-Tailed	0.219	0.415	0.638	0.920

Table I.30 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Beta(2,3).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.006	0.013	0.055
	One-Tailed	0.008	0.012	0.026	0.105
0.05	Two-Tailed	0.032	0.038	0.068	0.233
	One-Tailed	0.044	0.083	0.147	0.397
0.10	Two-Tailed	0.071	0.083	0.144	0.392
	One-Tailed	0.098	0.176	0.292	0.609
0.15	Two-Tailed	0.114	0.129	0.219	0.517
	One-Tailed	0.154	0.269	0.421	0.742
0.20	Two-Tailed	0.158	0.179	0.292	0.609
	One-Tailed	0.213	0.360	0.534	0.831

Table I.31 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Gamma(2,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.007	0.004	0.005	0.007
	One-Tailed	0.007	0.007	0.009	0.013
0.05	Two-Tailed	0.039	0.030	0.030	0.039
	One-Tailed	0.042	0.045	0.051	0.070
0.10	Two-Tailed	0.080	0.070	0.070	0.082
	One-Tailed	0.089	0.098	0.111	0.144
0.15	Two-Tailed	0.124	0.112	0.113	0.129
	One-Tailed	0.140	0.152	0.172	0.215
0.20	Two-Tailed	0.171	0.155	0.158	0.175
	One-Tailed	0.192	0.210	0.236	0.285

Table I.32 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Normal(O,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.002	0.002	0.003
	One-Tailed	0.006	0.005	0.005	0.009
0.05	Two-Tailed	0.031	0.016	0.016	0.029
	One-Tailed	0.037	0.035	0.039	0.068
0.10	Two-Tailed	0.068	0.041	0.041	0.068
	One-Tailed	0.080	0.082	0.095	0.157
0.15	Two-Tailed	0.107	0.070	0.070	0.113
	One-Tailed	0.126	0.137	0.164	0.253
0.20	Two-Tailed	0.148	0.104	0.104	0.160
	One-Tailed	0.176	0.198	0.239	0.352

Table I.33 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Uniform(0,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.009	0.047	0.168	0.686
	One-Tailed	0.014	0.080	0.259	0.793
0.05	Two-Tailed	0.046	0.172	0.424	0.913
	One-Tailed	0.068	0.293	0.592	0.963
0.10	Two-Tailed	0.095	0.288	0.592	0.964
	One-Tailed	0.141	0.462	0.772	0.989
0.15	Two-Tailed	0.143	0.381	0.695	0.982
	One-Tailed	0.210	0.586	0.865	0.996
0.20	Two-Tailed	0.193	0.460	0.769	0.990
	One-Tailed	0.275	0.682	0.916	0.999

Table I.34 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_0: Weibull(1); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.049	0.154	0.242	0.513
	One-Tailed	0.062	0.196	0.306	0.575
0.05	Two-Tailed	0.139	0.277	0.400	0.678
	One-Tailed	0.161	0.353	0.491	0.749
0.10	Two-Tailed	0.217	0.363	0.494	0.752
	One-Tailed	0.244	0.451	0.598	0.825
0.15	Two-Tailed	0.283	0.426	0.558	0.804
	One-Tailed	0.312	0.527	0.668	0.866
0.20	Two-Tailed	0.340	0.480	0.607	0.827
	One-Tailed	0.366	0.583	0.720	0.895

Table I.35 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_0: Weibull(1); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.010	0.010	0.010
	One-Tailed	0.010	0.009	0.011	0.010
0.05	Two-Tailed	0.051	0.050	0.050	0.051
	One-Tailed	0.047	0.052	0.051	0.050
0.10	Two-Tailed	0.100	0.100	0.100	0.099
	One-Tailed	0.097	0.102	0.101	0.100
0.15	Two-Tailed	0.150	0.150	0.149	0.149
	One-Tailed	0.148	0.154	0.149	0.150
0.20	Two-Tailed	0.200	0.200	0.202	0.197
	One-Tailed	0.198	0.204	0.200	0.200

Table I.36 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_0: Weibull(1); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.007	0.005	0.006	0.012
	One-Tailed	0.008	0.010	0.012	0.024
0.05	Two-Tailed	0.037	0.029	0.033	0.063
	One-Tailed	0.044	0.057	0.069	0.123
0.10	Two-Tailed	0.077	0.067	0.076	0.124
	One-Tailed	0.094	0.119	0.150	0.234
0.15	Two-Tailed	0.120	0.107	0.122	0.184
	One-Tailed	0.146	0.185	0.230	0.334
0.20	Two-Tailed	0.167	0.150	0.170	0.239
	One-Tailed	0.200	0.250	0.307	0.428

Table I.37 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.003	0.004	0.012
	One-Tailed	0.007	0.008	0.012	0.026
0.05	Two-Tailed	0.033	0.024	0.030	0.072
	One-Tailed	0.040	0.052	0.071	0.144
0.10	Two-Tailed	0.071	0.057	0.071	0.148
	One-Tailed	0.086	0.116	0.159	0.282
0.15	Two-Tailed	0.111	0.093	0.116	0.219
	One-Tailed	0.137	0.184	0.248	0.403
0.20	Two-Tailed	0.156	0.133	0.165	0.285
	One-Tailed	0.190	0.256	0.336	0.510

Table I.38 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.002	0.003	0.008
	One-Tailed	0.007	0.006	0.008	0.016
0.05	Two-Tailed	0.030	0.018	0.023	0.052
	One-Tailed	0.037	0.043	0.055	0.118
0.10	Two-Tailed	0.066	0.043	0.057	0.118
	One-Tailed	0.081	0.100	0.131	0.253
0.15	Two-Tailed	0.104	0.075	0.095	0.186
	One-Tailed	0.132	0.165	0.216	0.377
0.20	Two-Tailed	0.147	0.112	0.138	0.249
	One-Tailed	0.185	0.233	0.305	0.492

Table I.39 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: χ^2 (1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.034	0.041	0.058
	One-Tailed	0.048	0.058	0.086
0.05	Two-Tailed	0.106	0.119	0.159
	One-Tailed	0.136	0.169	0.221
0.10	Two-Tailed	0.171	0.191	0.236
	One-Tailed	0.218	0.261	0.326
0.15	Two-Tailed	0.229	0.253	0.304
	One-Tailed	0.287	0.333	0.405
0.20	Two-Tailed	0.285	0.313	0.362
	One-Tailed	0.348	0.397	0.475

Table I.40 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: χ^2 (4).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.004	0.005	0.007
	One-Tailed	0.009	0.010	0.013
0.05	Two-Tailed	0.031	0.032	0.040
	One-Tailed	0.047	0.054	0.070
0.10	Two-Tailed	0.069	0.068	0.084
	One-Tailed	0.104	0.119	0.144
0.15	Two-Tailed	0.110	0.111	0.130
	One-Tailed	0.162	0.182	0.215
0.20	Two-Tailed	0.152	0.156	0.175
	One-Tailed	0.223	0.245	0.287

Table I.41 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.419	0.630	0.851
	One-Tailed	0.475	0.676	0.879
0.05	Two-Tailed	0.549	0.741	0.913
	One-Tailed	0.618	0.795	0.937
0.10	Two-Tailed	0.622	0.795	0.937
	One-Tailed	0.699	0.850	0.958
0.15	Two-Tailed	0.670	0.828	0.950
	One-Tailed	0.752	0.881	0.969
0.20	Two-Tailed	0.707	0.852	0.959
	One-Tailed	0.789	0.904	0.976

Table I.42 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1); H_a: Lognorm(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.107	0.125	0.167
	One-Tailed	0.120	0.144	0.186
0.05	Two-Tailed	0.208	0.259	0.316
	One-Tailed	0.224	0.275	0.354
0.10	Two-Tailed	0.293	0.327	0.410
	One-Tailed	0.316	0.379	0.469
0.15	Two-Tailed	0.351	0.401	0.479
	One-Tailed	0.390	0.459	0.554
0.20	Two-Tailed	0.406	0.458	0.534
	One-Tailed	0.455	0.528	0.623

I.3 H_o: Weibull($\beta = 1.5$)

I.3.1 One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ G.O.F. Tests.

Table I.43 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.159	0.335	0.508	0.791
	One-Tailed	0.201	0.402	0.596	0.862
0.05	Two-Tailed	0.279	0.535	0.729	0.941
	One-Tailed	0.357	0.652	0.825	0.973
0.10	Two-Tailed	0.358	0.651	0.822	0.975
	One-Tailed	0.454	0.776	0.909	0.991
0.15	Two-Tailed	0.418	0.721	0.874	0.986
	One-Tailed	0.523	0.845	0.945	0.996
0.20	Two-Tailed	0.463	0.772	0.908	0.992
	One-Tailed	0.580	0.885	0.965	0.998

Table I.44 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.024	0.048	0.071	0.118
	One-Tailed	0.033	0.074	0.108	0.176
0.05	Two-Tailed	0.082	0.134	0.183	0.288
	One-Tailed	0.115	0.207	0.274	0.411
0.10	Two-Tailed	0.142	0.216	0.274	0.411
	One-Tailed	0.194	0.321	0.407	0.561
0.15	Two-Tailed	0.196	0.282	0.349	0.469
	One-Tailed	0.261	0.412	0.506	0.658
0.20	Two-Tailed	0.244	0.341	0.414	0.563
	One-Tailed	0.323	0.488	0.582	0.729

Table I.45 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.009
	One-Tailed	0.009	0.010	0.010	0.010
0.05	Two-Tailed	0.051	0.050	0.051	0.050
	One-Tailed	0.051	0.052	0.050	0.052
0.10	Two-Tailed	0.102	0.099	0.101	0.101
	One-Tailed	0.099	0.101	0.100	0.104
0.15	Two-Tailed	0.151	0.149	0.151	0.152
	One-Tailed	0.149	0.150	0.154	0.152
0.20	Two-Tailed	0.200	0.199	0.201	0.201
	One-Tailed	0.198	0.201	0.202	0.201

Table I.46 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.024	0.035	0.072
	One-Tailed	0.015	0.039	0.059	0.114
0.05	Two-Tailed	0.052	0.084	0.119	0.202
	One-Tailed	0.073	0.138	0.185	0.292
0.10	Two-Tailed	0.103	0.149	0.195	0.301
	One-Tailed	0.142	0.234	0.296	0.426
0.15	Two-Tailed	0.157	0.209	0.263	0.378
	One-Tailed	0.204	0.319	0.385	0.520
0.20	Two-Tailed	0.208	0.262	0.323	0.440
	One-Tailed	0.266	0.392	0.460	0.598

Table I.47 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.016	0.141	0.308	0.676
	One-Tailed	0.031	0.204	0.389	0.755
0.05	Two-Tailed	0.076	0.309	0.530	0.850
	One-Tailed	0.128	0.428	0.643	0.907
0.10	Two-Tailed	0.144	0.425	0.646	0.907
	One-Tailed	0.228	0.563	0.762	0.953
0.15	Two-Tailed	0.207	0.506	0.716	0.937
	One-Tailed	0.306	0.656	0.829	0.972
0.20	Two-Tailed	0.264	0.563	0.764	0.954
	One-Tailed	0.377	0.723	0.872	0.983

Table I.48 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(1.5); H_a: $\chi^2(4)$.

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.020	0.023	0.032
	One-Tailed	0.031	0.034	0.047
0.05	Two-Tailed	0.075	0.084	0.104
	One-Tailed	0.107	0.121	0.154
0.10	Two-Tailed	0.134	0.145	0.171
	One-Tailed	0.180	0.205	0.251
0.15	Two-Tailed	0.188	0.202	0.233
	One-Tailed	0.252	0.277	0.330
0.20	Two-Tailed	0.240	0.255	0.289
	One-Tailed	0.311	0.342	0.397

I.3.2 One- and Two-Tailed Power Comparisons for the Individual Q-Statistic G.O.F. Tests.

Table I.49 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.067	0.293	0.479	0.809
	One-Tailed	0.085	0.340	0.539	0.847
0.05	Two-Tailed	0.173	0.431	0.630	0.891
	One-Tailed	0.199	0.504	0.697	0.920
0.10	Two-Tailed	0.259	0.515	0.702	0.923
	One-Tailed	0.288	0.595	0.770	0.948
0.15	Two-Tailed	0.328	0.571	0.744	0.940
	One-Tailed	0.351	0.653	0.813	0.962
0.20	Two-Tailed	0.386	0.615	0.776	0.951
	One-Tailed	0.407	0.698	0.843	0.972

Table I.50 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.014	0.035	0.052	0.088
	One-Tailed	0.017	0.048	0.070	0.120
0.05	Two-Tailed	0.067	0.111	0.139	0.197
	One-Tailed	0.072	0.137	0.179	0.257
0.10	Two-Tailed	0.127	0.181	0.214	0.280
	One-Tailed	0.134	0.213	0.265	0.363
0.15	Two-Tailed	0.183	0.241	0.275	0.345
	One-Tailed	0.189	0.274	0.331	0.440
0.20	Two-Tailed	0.239	0.298	0.330	0.400
	One-Tailed	0.244	0.332	0.390	0.503

Table I.51 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.010
	One-Tailed	0.010	0.010	0.010	0.010
0.05	Two-Tailed	0.050	0.050	0.051	0.049
	One-Tailed	0.051	0.054	0.050	0.047
0.10	Two-Tailed	0.100	0.101	0.100	0.100
	One-Tailed	0.102	0.103	0.100	0.098
0.15	Two-Tailed	0.150	0.151	0.150	0.147
	One-Tailed	0.150	0.155	0.148	0.150
0.20	Two-Tailed	0.201	0.200	0.201	0.197
	One-Tailed	0.202	0.206	0.195	0.201

Table I.52 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.009	0.006	0.005	0.006
	One-Tailed	0.008	0.010	0.009	0.010
0.05	Two-Tailed	0.045	0.036	0.032	0.032
	One-Tailed	0.046	0.048	0.047	0.056
0.10	Two-Tailed	0.091	0.069	0.071	0.072
	One-Tailed	0.095	0.098	0.100	0.117
0.15	Two-Tailed	0.139	0.120	0.114	0.116
	One-Tailed	0.145	0.151	0.157	0.179
0.20	Two-Tailed	0.189	0.165	0.160	0.162
	One-Tailed	0.198	0.203	0.214	0.244

Table I.53 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.009	0.004	0.003	0.003
	One-Tailed	0.008	0.007	0.005	0.006
0.05	Two-Tailed	0.042	0.027	0.022	0.020
	One-Tailed	0.042	0.038	0.036	0.040
0.10	Two-Tailed	0.087	0.064	0.052	0.050
	One-Tailed	0.088	0.081	0.078	0.089
0.15	Two-Tailed	0.134	0.102	0.089	0.084
	One-Tailed	0.135	0.129	0.128	0.147
0.20	Two-Tailed	0.179	0.144	0.128	0.123
	One-Tailed	0.187	0.179	0.181	0.209

Table I.54 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(1.5); H_a: $\chi^2(4)$.

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.018	0.023	0.027
	One-Tailed	0.026	0.032	0.043
0.05	Two-Tailed	0.068	0.080	0.093
	One-Tailed	0.091	0.108	0.130
0.10	Two-Tailed	0.124	0.139	0.158
	One-Tailed	0.160	0.183	0.214
0.15	Two-Tailed	0.176	0.196	0.218
	One-Tailed	0.221	0.248	0.288
0.20	Two-Tailed	0.230	0.248	0.272
	One-Tailed	0.277	0.310	0.350

I.4 H_o: Weibull($\beta = 2.0$)

I.4.1 One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ G.O.F. Tests.

Table I.55 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(2); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.198	0.529	0.780	0.981
	One-Tailed	0.246	0.622	0.850	0.991
0.05	Two-Tailed	0.337	0.738	0.920	0.998
	One-Tailed	0.417	0.833	0.959	0.999
0.10	Two-Tailed	0.423	0.829	0.959	0.999
	One-Tailed	0.517	0.910	0.984	1.000
0.15	Two-Tailed	0.482	0.877	0.975	1.000
	One-Tailed	0.587	0.943	0.992	1.000
0.20	Two-Tailed	0.530	0.908	0.984	1.000
	One-Tailed	0.645	0.963	0.996	1.000

Table I.56 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(2); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.036	0.128	0.228	0.456
	One-Tailed	0.055	0.184	0.304	0.565
0.05	Two-Tailed	0.115	0.287	0.433	0.712
	One-Tailed	0.168	0.395	0.560	0.815
0.10	Two-Tailed	0.182	0.393	0.560	0.816
	One-Tailed	0.262	0.538	0.697	0.901
0.15	Two-Tailed	0.241	0.474	0.640	0.870
	One-Tailed	0.337	0.630	0.780	0.939
0.20	Two-Tailed	0.295	0.538	0.699	0.904
	One-Tailed	0.405	0.701	0.833	0.961

Table I.57 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(2); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.014	0.025	0.038	0.065
	One-Tailed	0.019	0.042	0.059	0.103
0.05	Two-Tailed	0.060	0.089	0.117	0.186
	One-Tailed	0.081	0.143	0.184	0.283
0.10	Two-Tailed	0.116	0.153	0.192	0.284
	One-Tailed	0.149	0.238	0.298	0.417
0.15	Two-Tailed	0.167	0.216	0.258	0.360
	One-Tailed	0.211	0.318	0.390	0.516
0.20	Two-Tailed	0.216	0.273	0.320	0.423
	One-Tailed	0.271	0.390	0.466	0.595

Table I.58 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(2); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.008
	One-Tailed	0.010	0.011	0.010	0.010
0.05	Two-Tailed	0.051	0.050	0.051	0.050
	One-Tailed	0.050	0.050	0.050	0.051
0.10	Two-Tailed	0.102	0.099	0.102	0.100
	One-Tailed	0.101	0.103	0.099	0.102
0.15	Two-Tailed	0.151	0.150	0.153	0.152
	One-Tailed	0.149	0.153	0.150	0.152
0.20	Two-Tailed	0.202	0.201	0.202	0.200
	One-Tailed	0.202	0.202	0.201	0.200

Table I.59 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(2); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.012	0.057	0.110	0.282
	One-Tailed	0.018	0.085	0.160	0.359
0.05	Two-Tailed	0.060	0.155	0.260	0.497
	One-Tailed	0.092	0.232	0.360	0.610
0.10	Two-Tailed	0.117	0.240	0.368	0.612
	One-Tailed	0.168	0.355	0.494	0.734
0.15	Two-Tailed	0.173	0.310	0.449	0.683
	One-Tailed	0.237	0.446	0.587	0.804
0.20	Two-Tailed	0.227	0.370	0.509	0.733
	One-Tailed	0.300	0.521	0.658	0.851

I.3.2 One- and Two-Tailed Power Comparisons for the Individual Q-Statistic G.O.F. Tests.

Table I.60 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(2); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.072	0.356	0.577	0.883
	One-Tailed	0.088	0.407	0.633	0.902
0.05	Two-Tailed	0.184	0.497	0.702	0.934
	One-Tailed	0.211	0.556	0.761	0.950
0.10	Two-Tailed	0.271	0.576	0.761	0.952
	One-Tailed	0.299	0.636	0.818	0.967
0.15	Two-Tailed	0.344	0.625	0.798	0.962
	One-Tailed	0.365	0.688	0.850	0.975
0.20	Two-Tailed	0.402	0.665	0.824	0.969
	One-Tailed	0.417	0.727	0.875	0.981

Table I.61 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(2); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.016	0.058	0.096	0.173
	One-Tailed	0.018	0.079	0.123	0.218
0.05	Two-Tailed	0.074	0.152	0.205	0.306
	One-Tailed	0.075	0.182	0.250	0.376
0.10	Two-Tailed	0.137	0.229	0.286	0.391
	One-Tailed	0.138	0.261	0.339	0.476
0.15	Two-Tailed	0.196	0.293	0.349	0.456
	One-Tailed	0.197	0.323	0.403	0.547
0.20	Two-Tailed	0.252	0.348	0.405	0.509
	One-Tailed	0.253	0.378	0.456	0.599

Table I.62 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(2); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.018	0.022	0.026
	One-Tailed	0.012	0.021	0.027	0.034
0.05	Two-Tailed	0.055	0.072	0.082	0.088
	One-Tailed	0.056	0.081	0.091	0.110
0.10	Two-Tailed	0.108	0.133	0.142	0.149
	One-Tailed	0.109	0.139	0.156	0.181
0.15	Two-Tailed	0.162	0.188	0.199	0.207
	One-Tailed	0.162	0.193	0.210	0.245
0.20	Two-Tailed	0.213	0.240	0.254	0.263
	One-Tailed	0.215	0.246	0.264	0.301

Table I.63 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(2); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.011
	One-Tailed	0.010	0.010	0.010	0.010
0.05	Two-Tailed	0.050	0.051	0.050	0.049
	One-Tailed	0.051	0.051	0.049	0.051
0.10	Two-Tailed	0.099	0.102	0.102	0.098
	One-Tailed	0.102	0.102	0.099	0.101
0.15	Two-Tailed	0.150	0.152	0.151	0.148
	One-Tailed	0.152	0.153	0.148	0.152
0.20	Two-Tailed	0.201	0.200	0.201	0.197
	One-Tailed	0.199	0.203	0.197	0.201

Table I.64 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(2); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.009	0.007	0.006	0.006
	One-Tailed	0.009	0.008	0.007	0.006
0.05	Two-Tailed	0.047	0.042	0.036	0.032
	One-Tailed	0.049	0.044	0.038	0.035
0.10	Two-Tailed	0.094	0.087	0.078	0.072
	One-Tailed	0.096	0.089	0.080	0.075
0.15	Two-Tailed	0.146	0.133	0.123	0.113
	One-Tailed	0.148	0.136	0.125	0.119
0.20	Two-Tailed	0.192	0.179	0.170	0.158
	One-Tailed	0.189	0.177	0.166	0.166

I.5 H_o: Weibull($\beta = 3.5$)

I.5.1 One- and Two-Tailed Power Comparisons for the Individual $\sqrt{b_1}$ G.O.F. Tests.

Table I.65 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Beta(2,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.009	0.002	0.002	0.001
	One-Tailed	0.009	0.004	0.002	0.003
0.05	Two-Tailed	0.046	0.020	0.015	0.011
	One-Tailed	0.048	0.028	0.022	0.023
0.10	Two-Tailed	0.091	0.049	0.040	0.032
	One-Tailed	0.092	0.069	0.057	0.060
0.15	Two-Tailed	0.137	0.086	0.073	0.060
	One-Tailed	0.139	0.114	0.101	0.106
0.20	Two-Tailed	0.184	0.127	0.110	0.095
	One-Tailed	0.187	0.163	0.149	0.158

Table I.66 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Beta(2,3).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.007	0.009	0.013
	One-Tailed	0.013	0.015	0.019	0.030
0.05	Two-Tailed	0.048	0.043	0.054	0.088
	One-Tailed	0.065	0.086	0.108	0.170
0.10	Two-Tailed	0.096	0.094	0.114	0.174
	One-Tailed	0.126	0.179	0.220	0.317
0.15	Two-Tailed	0.144	0.147	0.177	0.252
	One-Tailed	0.187	0.261	0.321	0.411
0.20	Two-Tailed	0.191	0.197	0.233	0.322
	One-Tailed	0.246	0.337	0.411	0.534

Table I.67 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(3.5); H_a: Gamma(2,1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.056	0.298	0.567	0.917
	One-Tailed	0.098	0.399	0.598	0.921
0.05	Two-Tailed	0.153	0.525	0.782	0.981
	One-Tailed	0.200	0.622	0.839	0.989
0.10	Two-Tailed	0.234	0.646	0.863	0.992
	One-Tailed	0.312	0.729	0.912	0.998
0.15	Two-Tailed	0.300	0.718	0.903	0.996
	One-Tailed	0.402	0.808	0.937	1.000
0.20	Two-Tailed	0.355	0.769	0.928	0.998
	One-Tailed	0.513	0.847	0.974	1.000

Table I.68 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(3.5); H_a: Normal(O,1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.010	0.018	0.026	0.032
	One-Tailed	0.010	0.019	0.028	0.032
0.05	Two-Tailed	0.052	0.073	0.085	0.097
	One-Tailed	0.053	0.075	0.087	0.097
0.10	Two-Tailed	0.104	0.131	0.147	0.164
	One-Tailed	0.105	0.134	0.148	0.166
0.15	Two-Tailed	0.153	0.186	0.205	0.220
	One-Tailed	0.155	0.188	0.209	0.222
0.20	Two-Tailed	0.206	0.239	0.259	0.275
	One-Tailed	0.207	0.241	0.260	0.277

Table I.69 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_0: Weibull(3.5); H_a: Uniform(0,2).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.012	0.002	0.001	0.001
	One-Tailed	0.013	0.003	0.002	0.002
0.05	Two-Tailed	0.052	0.019	0.013	0.010
	One-Tailed	0.053	0.024	0.021	0.021
0.10	Two-Tailed	0.098	0.046	0.035	0.030
	One-Tailed	0.100	0.062	0.056	0.057
0.15	Two-Tailed	0.143	0.081	0.065	0.057
	One-Tailed	0.145	0.105	0.099	0.101
0.20	Two-Tailed	0.188	0.120	0.100	0.090
	One-Tailed	0.191	0.153	0.149	0.153

Table I.70 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.258	0.807	0.981	1.000
	One-Tailed	0.313	0.871	0.991	1.000
0.05	Two-Tailed	0.415	0.934	0.997	1.000
	One-Tailed	0.502	0.967	0.999	1.000
0.10	Two-Tailed	0.507	0.966	0.999	1.000
	One-Tailed	0.607	0.987	1.000	1.000
0.15	Two-Tailed	0.568	0.980	0.999	1.000
	One-Tailed	0.681	0.993	1.000	1.000
0.20	Two-Tailed	0.615	0.987	1.000	1.000
	One-Tailed	0.743	0.996	1.000	1.000

Table I.71 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.065	0.515	0.813	0.986
	One-Tailed	0.087	0.689	0.879	0.990
0.05	Two-Tailed	0.206	0.743	0.925	0.997
	One-Tailed	0.296	0.825	0.956	0.999
0.10	Two-Tailed	0.307	0.833	0.958	0.998
	One-Tailed	0.396	0.880	0.970	1.000
0.15	Two-Tailed	0.400	0.879	0.973	1.000
	One-Tailed	0.473	0.908	0.983	1.000
0.20	Two-Tailed	0.475	0.909	0.981	1.000
	One-Tailed	0.546	0.953	0.996	1.000

Table I.72 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.023	0.123	0.277	0.619
	One-Tailed	0.036	0.188	0.366	0.716
0.05	Two-Tailed	0.089	0.295	0.503	0.838
	One-Tailed	0.138	0.413	0.630	0.901
0.10	Two-Tailed	0.154	0.412	0.629	0.906
	One-Tailed	0.233	0.563	0.759	0.952
0.15	Two-Tailed	0.212	0.496	0.707	0.937
	One-Tailed	0.310	0.659	0.832	0.972
0.20	Two-Tailed	0.265	0.561	0.780	0.956
	One-Tailed	0.381	0.726	0.877	0.983

Table I.73 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.021	0.204	0.421	0.812
	One-Tailed	0.040	0.308	0.524	0.876
0.05	Two-Tailed	0.095	0.411	0.658	0.927
	One-Tailed	0.148	0.520	0.771	0.961
0.10	Two-Tailed	0.173	0.538	0.766	0.960
	One-Tailed	0.234	0.641	0.850	0.988
0.15	Two-Tailed	0.233	0.617	0.823	0.974
	One-Tailed	0.304	0.751	0.917	0.998
0.20	Two-Tailed	0.288	0.678	0.860	0.983
	One-Tailed	0.381	0.816	0.912	0.999

Table I.74 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.010	0.010	0.010
	One-Tailed	0.009	0.010	0.011	0.010
0.05	Two-Tailed	0.050	0.050	0.052	0.048
	One-Tailed	0.047	0.051	0.050	0.051
0.10	Two-Tailed	0.102	0.099	0.102	0.098
	One-Tailed	0.097	0.103	0.101	0.099
0.15	Two-Tailed	0.152	0.151	0.153	0.148
	One-Tailed	0.148	0.154	0.152	0.150
0.20	Two-Tailed	0.202	0.203	0.202	0.200
	One-Tailed	0.199	0.203	0.200	0.199

Table I.75 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: χ^2 (1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.775	0.947	0.999
	One-Tailed	0.849	0.969	0.999
0.05	Two-Tailed	0.917	0.990	1.000
	One-Tailed	0.960	0.995	1.000
0.10	Two-Tailed	0.959	0.996	1.000
	One-Tailed	0.985	0.999	1.000
0.15	Two-Tailed	0.974	0.998	1.000
	One-Tailed	0.992	0.999	1.000
0.20	Two-Tailed	0.983	0.999	1.000
	One-Tailed	0.996	1.000	1.000

Table I.76 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: χ^2 (4).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.312	0.510	0.800
	One-Tailed	0.397	0.603	0.864
0.05	Two-Tailed	0.529	0.727	0.928
	One-Tailed	0.644	0.827	0.962
0.10	Two-Tailed	0.648	0.823	0.962
	One-Tailed	0.764	0.904	0.984
0.15	Two-Tailed	0.720	0.869	0.975
	One-Tailed	0.832	0.940	0.992
0.20	Two-Tailed	0.769	0.899	0.984
	One-Tailed	0.876	0.960	0.995

Table I.77 One- and Two-Tailed Power for $\sqrt{b_1}$ G.O.F. Test — H_o: Weibull(3.5); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.957	0.997	1.000
	One-Tailed	0.974	0.998	1.000
0.05	Two-Tailed	0.987	1.000	1.000
	One-Tailed	0.994	1.000	1.000
0.10	Two-Tailed	0.994	1.000	1.000
	One-Tailed	0.998	1.000	1.000
0.15	Two-Tailed	0.996	1.000	1.000
	One-Tailed	0.999	1.000	1.000
0.20	Two-Tailed	0.998	1.000	1.000
	One-Tailed	1.000	1.000	1.000

I.2.2 *One- and Two-Tailed Power Comparisons for the Individual Q-Statistic G.O.F. Tests.*

Table I.78 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_0: Weibull(3.5); H_a: Beta(2,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.018	0.030	0.082
	One-Tailed	0.013	0.032	0.052	0.137
0.05	Two-Tailed	0.054	0.075	0.112	0.239
	One-Tailed	0.062	0.127	0.181	0.360
0.10	Two-Tailed	0.107	0.136	0.190	0.358
	One-Tailed	0.121	0.221	0.305	0.516
0.15	Two-Tailed	0.160	0.192	0.260	0.447
	One-Tailed	0.177	0.301	0.406	0.619
0.20	Two-Tailed	0.212	0.250	0.321	0.518
	One-Tailed	0.232	0.374	0.490	0.698

Table I.79 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_0: Weibull(3.5); H_a: Beta(2,3).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.013	0.019	0.040
	One-Tailed	0.012	0.023	0.034	0.073
0.05	Two-Tailed	0.053	0.061	0.082	0.139
	One-Tailed	0.060	0.097	0.131	0.227
0.10	Two-Tailed	0.105	0.118	0.146	0.229
	One-Tailed	0.117	0.177	0.229	0.359
0.15	Two-Tailed	0.155	0.170	0.207	0.302
	One-Tailed	0.172	0.250	0.317	0.460
0.20	Two-Tailed	0.207	0.221	0.261	0.365
	One-Tailed	0.227	0.317	0.392	0.542

Table I.80 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Gamma(2,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.016	0.075	0.124	0.219
	One-Tailed	0.024	0.089	0.155	0.249
0.05	Two-Tailed	0.078	0.174	0.238	0.352
	One-Tailed	0.134	0.198	0.274	0.317
0.10	Two-Tailed	0.143	0.253	0.321	0.436
	One-Tailed	0.182	0.293	0.366	0.474
0.15	Two-Tailed	0.202	0.316	0.384	0.496
	One-Tailed	0.254	0.368	0.432	0.544
0.20	Two-Tailed	0.260	0.372	0.434	0.547
	One-Tailed	0.351	0.463	0.492	0.641

Table I.81 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Normal(O,1).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.011	0.014	0.016	0.020
	One-Tailed	0.011	0.017	0.023	0.032
0.05	Two-Tailed	0.051	0.062	0.068	0.080
	One-Tailed	0.053	0.073	0.090	0.117
0.10	Two-Tailed	0.102	0.116	0.124	0.143
	One-Tailed	0.105	0.136	0.158	0.199
0.15	Two-Tailed	0.150	0.167	0.177	0.200
	One-Tailed	0.156	0.194	0.221	0.270
0.20	Two-Tailed	0.199	0.220	0.228	0.256
	One-Tailed	0.207	0.249	0.282	0.335

Table I.82 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Uniform(0,2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.015	0.078	0.211	0.627
	One-Tailed	0.021	0.121	0.297	0.725
0.05	Two-Tailed	0.068	0.216	0.445	0.837
	One-Tailed	0.091	0.323	0.576	0.903
0.10	Two-Tailed	0.130	0.324	0.569	0.907
	One-Tailed	0.164	0.465	0.720	0.955
0.15	Two-Tailed	0.188	0.404	0.653	0.938
	One-Tailed	0.231	0.562	0.800	0.974
0.20	Two-Tailed	0.243	0.472	0.711	0.955
	One-Tailed	0.294	0.638	0.853	0.984

Table I.83 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(0.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.073	0.388	0.622	0.902
	One-Tailed	0.088	0.433	0.667	0.918
0.05	Two-Tailed	0.189	0.515	0.729	0.943
	One-Tailed	0.213	0.574	0.776	0.956
0.10	Two-Tailed	0.278	0.588	0.779	0.959
	One-Tailed	0.301	0.647	0.825	0.970
0.15	Two-Tailed	0.350	0.635	0.809	0.967
	One-Tailed	0.361	0.693	0.855	0.977
0.20	Two-Tailed	0.409	0.674	0.832	0.973
	One-Tailed	0.412	0.727	0.877	0.982

Table I.84 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(1).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.002	0.010	0.058	0.501
	One-Tailed	0.003	0.014	0.069	0.604
0.05	Two-Tailed	0.015	0.086	0.286	0.865
	One-Tailed	0.026	0.149	0.317	0.900
0.10	Two-Tailed	0.038	0.192	0.489	0.952
	One-Tailed	0.055	0.222	0.551	0.972
0.15	Two-Tailed	0.069	0.269	0.629	0.980
	One-Tailed	0.100	0.382	0.717	0.990
0.20	Two-Tailed	0.103	0.391	0.726	0.991
	One-Tailed	0.138	0.512	0.899	0.998

Table I.85 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(1.5).

Significance Level	Test Version	Sample Size 5	15	25	50
0.01	Two-Tailed	0.012	0.025	0.033	0.041
	One-Tailed	0.013	0.027	0.039	0.050
0.05	Two-Tailed	0.058	0.088	0.104	0.114
	One-Tailed	0.059	0.089	0.107	0.131
0.10	Two-Tailed	0.114	0.153	0.171	0.184
	One-Tailed	0.115	0.155	0.173	0.201
0.15	Two-Tailed	0.168	0.210	0.231	0.246
	One-Tailed	0.169	0.215	0.237	0.260
0.20	Two-Tailed	0.222	0.267	0.286	0.303
	One-Tailed	0.225	0.269	0.288	0.312

Table I.86 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(2).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.006	0.003	0.005	0.023
	One-Tailed	0.007	0.004	0.005	0.024
0.05	Two-Tailed	0.030	0.020	0.028	0.067
	One-Tailed	0.031	0.022	0.029	0.068
0.10	Two-Tailed	0.067	0.050	0.068	0.144
	One-Tailed	0.068	0.052	0.069	0.145
0.15	Two-Tailed	0.106	0.084	0.111	0.217
	One-Tailed	0.109	0.086	0.114	0.219
0.20	Two-Tailed	0.148	0.121	0.156	0.286
	One-Tailed	0.149	0.125	0.158	0.287

Table I.87 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: Weibull(3.5).

Significance Level	Test Version	Sample Size			
		5	15	25	50
0.01	Two-Tailed	0.010	0.012	0.011	0.010
	One-Tailed	0.010	0.010	0.010	0.011
0.05	Two-Tailed	0.051	0.054	0.051	0.049
	One-Tailed	0.052	0.051	0.051	0.051
0.10	Two-Tailed	0.100	0.105	0.100	0.098
	One-Tailed	0.100	0.101	0.101	0.102
0.15	Two-Tailed	0.151	0.153	0.149	0.149
	One-Tailed	0.149	0.151	0.152	0.153
0.20	Two-Tailed	0.201	0.204	0.201	0.199
	One-Tailed	0.198	0.198	0.200	0.202

Table I.88 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: χ^2 (1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.213	0.316	0.484
	One-Tailed	0.247	0.362	0.538
0.05	Two-Tailed	0.342	0.457	0.617
	One-Tailed	0.383	0.512	0.679
0.10	Two-Tailed	0.424	0.537	0.688
	One-Tailed	0.468	0.594	0.748
0.15	Two-Tailed	0.484	0.591	0.730
	One-Tailed	0.526	0.649	0.789
0.20	Two-Tailed	0.532	0.634	0.763
	One-Tailed	0.572	0.690	0.820

Table I.89 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: χ^2 (4).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.052	0.070	0.100
	One-Tailed	0.067	0.090	0.130
0.05	Two-Tailed	0.136	0.165	0.207
	One-Tailed	0.155	0.197	0.259
0.10	Two-Tailed	0.209	0.243	0.289
	One-Tailed	0.225	0.279	0.349
0.15	Two-Tailed	0.268	0.306	0.353
	One-Tailed	0.284	0.345	0.418
0.20	Two-Tailed	0.323	0.361	0.410
	One-Tailed	0.334	0.397	0.475

Table I.90 One- and Two-Tailed Power for Q-Statistic G.O.F. Test — H_o: Weibull(3.5); H_a: X Logistic(0,1).

Significance Level	Test Version	Sample Size		
		20	30	50
0.01	Two-Tailed	0.689	0.873	0.977
	One-Tailed	0.721	0.889	0.982
0.05	Two-Tailed	0.775	0.918	0.987
	One-Tailed	0.810	0.932	0.991
0.10	Two-Tailed	0.815	0.938	0.991
	One-Tailed	0.852	0.950	0.994
0.15	Two-Tailed	0.841	0.948	0.993
	One-Tailed	0.877	0.960	0.996
0.20	Two-Tailed	0.859	0.956	0.994
	One-Tailed	0.894	0.967	0.997

Appendix J. MATLAB Code

J.1 User-defined MATLAB function q = hoggq(y)

% ***
% Hoggq = Hogg's Q Statistic
% Q = hoggq(Y) returns the Q-Statistic developed by Robert V. Hogg. Hogg's Q-Statistic is a
% measure of tail thickness, similar to Kurtosis, but is a more robust discriminator between
% distributions. The statistic definition is $Q = \frac{[U(0.05) - L(0.05)]}{[U(0.5) - L(0.5)]}$, where U($\alpha$) = the average of
% the (α*n) largest order statistics and L(α) = the average of the (α*n) smallest order statistics,
% where n is the sample size. Partial order statistics are used if (α*n) is not an integer.
% For matrix Y, hoggq(Y) returns a row vector containing the Hogg-Q statistic of each column
% of Y.
%
% This user-defined MATLAB function q = hoggq(y) was writtten by Dr. John S. Crown, Air
% Force Institute of Technology, Department of Mathematics and Statistics, 2950 P ST, WPAFB
% OH 45433.
% Revision: 1.0 Date: 08/12/1998
% ***

```
[n,ncol] = size(y);
if n = = 1 & ncol = =1
 q=0;

else

 if n = = 1
  y = y';
  n = ncol;
  ncol = 1;
 end

 y = sort(y);
 j=1;
 while j < ncol+1
  x = y(:,j);
  frac05 = .05*n;
  frac5 = .5*n;
  fi05 = floor(frac05);
  fi5 = floor(frac5);
  if frac05 < 1
```

```
    u05bar = x(n);
    l05bar = x(1);

  else

    u05bar = (fi05*mean(x(n - fi05 +1:n)) + (frac05 - fi05)*x(n - fi05))/frac05;
    l05bar = (fi05*mean(x(1:fi05)) + (frac05 - fi05)*x(1 + fi05))/frac05;
  end

  u5bar = (fi5*mean(x(n - fi5+1:n)) + (frac5 - fi5)*x(n - fi5))/frac5;
  l5bar = (fi5*mean(x(1:fi5)) + (frac5 - fi5)*x(1 + fi5))/frac5;
  q(j) = (u05bar - l05bar)/(u5bar - l5bar);
  j = j+1;

 end

end
```

J-2 Critical Value Generator For Sample Skewness and Sample Q-Statistic

```
% ************************************************************************
% This MATLAB code generates the critical values for sample skewness and Q-Statistic of the
% Weibull samples of sizes n = 5(5)50 with known shape parameter value via Monte Carlo.
% simulation with a replication size of 100,000
%
% Tibet MEMIS   24 Aug 1998 (Modified from Clough's [26:H-1] critical value generator code)
% ************************************************************************

% Initialization of the parameter values for the 3-parameter Weibull distribution

delta = 0; % (Location Parameter)
theta = 1; % (Scale Parameter)
beta  = 1; % (Shape Parameter)              % Desired shape value is substituted here

% Conversion to a and b parameters used by the MATLAB built-in weibrnd function

a = 1/(theta^beta);
b = beta;

numsamples = 100000;          % The number of replications to generate of sample size n

                               % In order to calculate the median ranks,
icount = [1:numsamples];       % making of an array of indices (icount)


mrank = (icount - 0.3)./(icount + 0.4)        % Calculation of the median ranks array

% Beginning the main loop over the sample sizes. Generating numsamples of size n and finding
% sample skewness and Q-statistic for each one; saving the values in the arrays skew() and hq().


for n = 5:5:50
    sprintf('Starting sample size n =  %d\n', n)
    skew = zeros(l,numsamples);                    % Preallocating the vectors to expedite
    hq = zeros(l,numsamples);

        for i = 1:numsamples
          x = weibrnd(a, b, 1, n) + delta;
          sk =  skewness(x);
          hq = hoggq(x);
          skew(i) = sk;
          hq(i) = hq;
          if rem(i, 1000) = = 0 disp(i); end       %  For keeping track of the progress
```

```
        end % for

        disp('Finished Generating')

       % Sort the arrays in ascending order to facilitate interpolating critvals

     skew = sort(skew);
     hq = sort(hq);
     disp('sorted')

% Finding the upper and lower tail critical values for the given sample size n by interpolation.
% Step through alpha = 0.005(0.005)0 .10 and 0.10(0.01)0.20.

% Initialization of alpha values

     alpha = 0.005;
     step  = 0.005;

  while alpha < 21 0.20

      if alpha <= 0.10                  % Finding the column index for the critical arrays
            j = round(alpha*200);        % Mapping [0.005, 0.10] to [1, 20]
     else
            j = round((alpha+0.10)*100); % Mapping [0.11, 0.20] to [21, 30]
     end % if

     i = round(n/5);          % Finding the row index for critvals arrays

skewuptail(i, j)=interp1(mrank, skew,1-alpha);   % Interpolating to find the critical values and
skewlotail(i, j)=interp1(mrank, skew, alpha);     % saving them in arrays indexed by n
hquptail(i, j)=interp1(mrank, hq,1-alpha);
hqlotail(i, j)=interp1(mrank, hq, alpha)

     end % alpha

     if alpha >= 0.10
     step = 0.01;                    % Changing the step after 0.10
      end % if

     alpha = alpha + step;  % Incrementing alpha for the next level

     end % while

 sprintf('Finished with sample size %d\n', n)

     end % for
```

```
% End of loops that calculate the arrays of critical values

disp('Saving and Writing to Files')
save skewcrit  skewuptail  skewlotail;        % Saving the critvals to binary files for use
save hqcrit  hquptail  hqlotail               % in the future studies

format = '%2d     %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f
          %4.3f  %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f
          %4.3f  %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f
          %4.3f  %4.3f    %4.3f    %4.3f    %4.3f    %4.3f    %4.3f\n';

col = (5:5:50);          % Keeping track of sample sizes in output via vectoring

table = [col'skewuptail15];     % Sending the tables to 4 files  (upper/lower skewness/Q-Statistic)
file1 = fopen('skewup15.txt','w');
fprintf(file1,'Critical Values for Skewness -- Upper Tail\n');
fprintf(file1,'Shape = %d\n',beta);
fprintf(file1,'\n');
fprintf(file1,format,table');
fclose(file1);

table = [col'skewlotail15];
file2=fopen('skewlo15.txt','w');
fprintf(file2,'Critical Values for Skewness -- Lower Tail\n');
fprintf(file2,'Shape = %d\n',beta);
fprintf(file2,'\n');
fprintf(file2,format,table');
fclose(file2);

table=[col'hquptail15];
file3=fopen('hqup15.txt','w');
fprintf(file3,'Critical Values for hq -- Upper Tail\n');
fprintf(file3,'Shape = %d\n',beta);
fprintf(file3,'\n');
fprintf(file3,format,table');
fclose(file3);

table=[col'hqlotail15];
file4=fopen('hqlo15.txt','w');
fprintf(file4,'Critical Values for hq -- Lower Tail\n');
fprintf(file4,'Shap e =%d\n',beta);
fprintf(file4,'\n');
fprintf(file4,format,table');
fclose(file4);
```

J-3 Attained Significance Levels Generator

```
% **************************************************************************
% This MATLAB code computes the attained significance levels for a sequential a goodness-of-
% fit test for the Weibull distribution using two-tailed component skewness and Q-Statistic
% G.O.F. via  Monte Carlo simulation with a replication size of 100,000
%
% Tibet MEMIS   15 Sep 1998
% (Modified from Clough's [26:H-4] attained significance levels generator code)
% **************************************************************************

% Initialization of the parameter values for the 3-parameter Weibull distribution

delta = 0; % (Location Parameter)
theta = 1; % (Scale Parameter)
beta  = 1; % (Shape Parameter)              % Desired shape value is substituted here

% Conversion to a and b parameters used by the MATLAB built-in weibrnd function

a = 1/(theta^beta);
b = beta;

numsamples = 100000;      % The number of replications to generate of sample size n

load hqcrit15;      % Loading the created critical values vectors from critvals. (Need to change
load skewcrit15;    % names for different shape parameter values)
                    % This will load the following vectors:
                    % hqlotail       hquptail
                    % skewlotail     skewuptail

A = zeros(20,20,10);   % Initializing the counter array to all 0s

for n = 5:5:50
        sprintf('Starting sample size %d', n)

for i = 1:numsamples
        x = weibrnd(a, b,1, n) + delta;          % Generating 1xn vector of Weibull deviates
        sk = skewness(x);
        hq = hoggq(x);
        if rem(i,1000) = = 0 disp(i); end        % For keeping track of the progress

% Initializing the placeholders for this specific sample

        icurr = 1;      jcurr = 1;
        istop = 21;     jstop = 21;
```

```
% Conducting the Skewness Test (Test #1) at all alpha levels until a failure is encountered, then
% saving that point in istop. Alpha levels from 0.01 to 0.20 will be used here. The corresponding
% crit values for the two sided test are alpha/2 and 1 - (alpha/2) and correspond to columns 1 to
% 20 respectively in the upper and lower tail arrays loaded earlier. Columns 21-30 in these
% arrays are not needed at this time.

while icurr < istop
        if sk < skewlotail15(n/5,icurr)                    % fail lower tail
                istop = icurr;
        elseif sk > skewuptail15(n/5,icurr)                % fail upper tail
                istop = icurr;
        end % if

        icurr = icurr + 1;         % Increment icurr if passed or failed. If it failed, the resetting
                                   % of istop will force loop termination.
        end % while.
                                   % When the while loop ends, istop will equal the failure point
                                   % (1-20) or equal 21 if it passed all levels.

        % Now conduct the Q-Statistic Test (Test #2) similarly

while jcurr < jstop
        if hq < hqlotail15(n/5,jcurr)              % fail lower tail
                jstop = jcurr;
        elseif hq > hquptail15(n/5,jcurr)          % fail upper tail
                jstop = jcurr;
        end %if

        jcurr = jcurr+1

        end % while                        % At this point jstop will hold the fail point (1-20) if it
                                           % failed; If it passed all the jstop = 21

% Now figuring out which cells to increment in the counter array. Fail1 is an array of 0s and 1s
% indicating what levels the sample failed Test # 1; Fail2 is the similar array for Test  # 2.
% The Inc array is the union of the two and will be used to increment the main count array A.

Fail1 = zeros(20,20);
Fail2 = zeros(20,20);                      % Initialize them to all 0s
Inc  = zeros(20,20);

if istop < 21 Fail1(istop:20,:) = 1;end    % Filling in 1s where failed unless failed none
if jstop < 21 Fail2(:,jstop:20) = 1;end

Inc = Fail1 | Fail2;
```

```
A(:,:,n/5) = A(:,:,n/5) + Inc;

        end     % for loop (i) -- go back for next sample of size n

sprintf('Sample size is finished %d – switching to next. \n', n);

        end     % for loop (n) – now change sample sizes

% A currently has counts for all sample sizes (5:5:50)

A = A./numsamples;              % Dividing by numsamples to obtain alpha levels

% Now output to a file

save sigtable15 A;      % Saving A to a binary file for future use

file1 = fopen('sigtable15.txt','w');       % Saving to a text file

format = '%4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f\\\\ \\hline \n';

fprintf(file1,'Table for Shape = %d  \n', beta);
for n = 5:5:50
        fprintf(file1,'Significance Levels for Sample Size %2d \n',n);
        fprintf(file1,format,A(:,:,n/5)');
        fprintf(file1,'\n \n');
                end % for

fclose(file1);
disp('Finished and File Saved.')
```

J-4 Power Study Example

```
% ************************************************************************
% Power Study --------- Ho : Weibull(1,3.5)          Ha : Uniform(0,2)
% ========================================================================
% This MATLAB code computes the power of the sequential goodness-of-fit test for the Weibull
% distribution based on the two-tailed component skewness and Q-Statistic G.O.F. tests at each
% combination of individual significance levels between 0.01(0.01)0.20 and sample sizes 5, 15,
% 25, and 50 against the alternate distribution noted above. This particular study has been
% accomplished to compare power to that of Bush's [17] against this particular alternate
% distribution
% Tibet MEMIS   04 Oct 1998
% ************************************************************************

% Initialization of the parameter values for Uniform(0,2) distribution

a = 0;
b = 2;

numsamples = 40000;       % The number of replications to generate of sample size n

load hqcrit35;         % Loading the created critical values vectors generated for Ho :
                       % Weibull(1,3.5) from critvals. Simulation (Need to change
load skewcrit35;       % names for different shape parameter values)
                       % This will load the following vectors:
                       % hqlotail          hquptail
                       % skewlotail        skewuptail

A = zeros(20,20,10);   % Initializing the counter array to 0s

n=5;             % First sample size
step=10;         % Start with a step of 10 to do 5, 15, and 25

while n < 51   % Cycling until you step past sample size 50

        sprintf('Starting sample size %d',n)

for i = 1:numsamples
        x = unifrnd(a, b,1, n)           % Generating 1xn vector of Uniform(0,2) deviates
        sk = skewness(x);
        hq = hoggq(x);
        if rem(i,1000) = = 0 disp(i); end         % For keeping track of the progress

% Initializing the placeholders for this specific sample
```

```
        icurr = 1;        jcurr = 1;
        istop = 21;       jstop = 21;

% Conducting the Skewness Test (Test #1) at all alpha levels until a failure is encountered, then
% saving that point in istop. Alpha levels from 0.01 to 0.20 will be used here. The corresponding
% crit values for the two sided test are alpha/2 and 1 - (alpha/2) and correspond to columns 1 to
% 20 respectively in the upper and lower tail arrays loaded earlier. Columns 21-30 in these
% arrays are not needed at this time.

while icurr < istop
        if sk < skewlotail35(n/5,icurr)                 % fail lower tail
                istop = icurr;
        elseif sk > skewuptail35(n/5,icurr)             % fail upper tail
                istop = icurr;
        end % if

        icurr = icurr + 1;        % Incrementing icurr if passed or failed. If it failed, the resetting
                                  % of istop will force loop termination.
        end % while.
                                  % When the while loop ends, istop will equal the failure point
                                  % (1-20) or equal 21 if it passed all levels.

% Now conduct the Q-Statistic Test (Test #2) similarly

while jcurr < jstop
        if hq < hqlotail35(n/5,jcurr)                   % fail lower tail
                jstop = jcurr;
        elseif hq > hquptail35(n/5,jcurr)               % fail upper tail
                jstop = jcurr;
        end %if

        jcurr = jcurr+1

        end % while                   % At this point jstop will hold the fail point (1-20) if it
                                      % failed; If it passed all the jstop=21

% Now figuring out which cells to increment in the counter array. Fail1 is an array of 0s and 1s
% indicating what levels the sample failed Test # 1; Fail2 is the similar array for Test # 2.
% The Inc array is the union of the two and will be used to increment the main count array A.

Fail1 = zeros(20, 20);
Fail2 = zeros(20, 20);                    % Initialize them to all 0s
Inc  = zeros(20, 20);

if istop < 21 Fail1(istop:20,:) = 1;end   % Filling in 1s where failed unless failed none
```

```
if jstop < 21 Fail2(:,jstop:20) = 1;end

Inc = Fail1 | Fail2;
A(:,:,n/5) = A(:,:,n/5) + Inc;

        end     % for loop (i) -- go back for next sample of size n

sprintf('Sample size is finished %d – switching to next. \n', n);

        if n = = 25
                step = 25;      % Jump to 50 on the next iteration once you hit size 25
                end %if
        n = n + step;           % Increment n for the next sample size
        end                     % for loop (n) – now change sample sizes

% A currently has counts for all sample sizes (5:5:50)

A = A./numsamples;              % Divide by numsamples to obtain alpha levels

% Now output to a file

save pwrunif3502 A;             % Saving A to a binary file for future use

file1 = fopen('pwrunif3502.txt','w');  % Saving to a text file

cols = [0.01:0.01:0.2];         % labels for leftmost column

format = '%4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f\\\\ \\hline \n';

heading = '  & 0.01 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 & 0.10 & 0.11 &
0.12 & 0.13 & 0.14 & 0.15 & 0.16 & 0.17 & 0.18 & 0.19 & 0.20 \\\\ \\hline \\hline \n';

fprintf(file1,'Power Study: Ho: Weib(1,3.5)  Ha: Unif(0,2) \n');
for n = 5:5:50
fprintf(file1,'Powers for sample size %2d \n',n);
fprintf(file1,heading);
table=[cols',A(:,:,n/5)];
fprintf(file1,format,table');
fprintf(file1,'\n \n');
end

fclose(file1);
disp('Done: File saved.')
```

J-5 Individual Two-Tailed G.O.F. Test Power Study – Q-Statistic Example

```
% ************************************************************************
% Power Study Two-Tailed Q-Statistic Test Only- Ho : Weibull(1,0.5)     Ha : Chi-squared(1)
% ========================================================================
% This MATLAB code computes the power of the two-tailed individual skewness and Q-
% Statistic G.O.F. tests for the Weibull distribution tests at each combination of individual
% significance levels between 0.01(0.01)0.20 and sample sizes 5, 15, % 25, and 50 against the
% alternate distribution noted above.
% Tibet MEMIS   11 Nov 1998
% ************************************************************************

% Initialization of the parameter values for chi-squared(1) distribution

v = 1

numsamples = 40000;      % The number of replications to generate of sample size n

load hqcrit10;      % Loading the created critical values vectors generated for Ho :
                    % Weibull(1,3.5) from critvals. Simulation (Need to change
                    % names for different shape parameter values)
                    % This will load the following vectors:
                    % hqlotail        hquptail


A = zeros(20,20,10);

n = 20;         % First sample size
step=10;        % Start with a step of 10 to do 30

while n < 51    % Cycle until you step past sample size 50

    sprintf('Starting sample size %d',n)

    for i = 1:numsamples
        x  = chi2rnd(v,1,n);          % Generating 1xn vector of chi-squared(1) deviates
        hq = hoggq(x);

    if rem(i,1000) == 0 disp(i); end  % For keeping track of the progress

% Initializing the placeholders for this specific sample

    jcurr = 1;      jstop = 21;
```

% Conducting the Q-Statistic Test at all alpha levels until a failure is encountered, then
% saving that point in jstop. Alpha levels from 0.01 to 0.20 will be used here. The corresponding
% crit values for the two sided test are alpha/2 and 1 - (alpha/2) and correspond to columns 1 to
% 20 respectively in the upper and lower tail arrays loaded earlier. Columns 21-30 in these
% arrays are not needed at this time.

```
while jcurr < jstop
        if hq < hqlotail10(n/5,jcurr)
                jstop = jcurr;
        elseif hq > hquptail10(n/5,jcurr)
                jstop = jcurr;
        end % if

        jcurr = jcurr + 1;          % Increment icurr if passed or failed. If it failed, the resetting
                                    % of jstop will force loop termination.
        end % while.
                                    % When the while loop ends, jstop will equal the failure point
                                    % (1-20) or equal 21 if it passed all levels.

Fail1 = zeros(20,20);               % Initialize it to all 0s
Inc   = zeros(20,20);

if jstop < 21 Fail1(jstop:20,:) = 1; end         % Filling in 1s where failed unless failed none

Inc = Fail1;
A(:,:,n/5) = A(:,:,n/5) + Inc;

        end     % for loop (i) -- go back for next sample of size n

sprintf('Sample size is finished %d – switching to next. \n', n);

        if n = = 25
                step = 25;
                end                 % Jumping to 50 on the next iteration once you hit size 25

        n = n + step;               % Increment n for the next sample size
        end                         % for loop (n) – now change sample sizes

% A currently has counts for all sample sizes (5:5:50)

A = A./numsamples;                  % Divide by numsamples to obtain alpha levels

% Now output to a file

save pwrhqchi2051 A;                % Saving A to a binary file for future use
```

```
file1 = fopen('pwrhqchi2051.txt','w');          % Saving to a text file

cols = [0.01:0.01:0.2];          % labels for leftmost column

format = '%4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f & %4.4f &
          %4.4f & %4.4f\\\\ \\hline \n';

heading = '  & 0.01 & 0.02 & 0.03 & 0.04 & 0.05 & 0.06 & 0.07 & 0.08 & 0.09 & 0.10 & 0.11 &
0.12 & 0.13 & 0.14 & 0.15 & 0.16 & 0.17 & 0.18 & 0.19 & 0.20 \\\\ \\hline \\hline \n';

fprintf(file1,'Power Study: Ho: Weib(1,0.5)  Ha: hqChiSquare(1) \n');
for n=5:5:50
fprintf(file1,'Powers for sample size %2d \n',n);
fprintf(file1,heading);
table=[cols',A(:,:,n/5)];
fprintf(file1,format,table');
fprintf(file1,'\n \n');
end

fclose(file1);
disp('Finished: File saved.')
```

J-6 Individual One-Tailed G.O.F. Test Power Study – Q-Statistic Example

```
% ***********************************************************************
% Power Study: One Tailed Version -- Lower Tail Q-Statistic Test Only
% Ho: Weibull(1,l)     Ha: Normal(0,1)
% =======================================================================
% This MATLAB code computes the power of the G.O.F. test for the Weibull distribution based
% on the one-tailed component Q-Statistic G.O.F. test at individual significance levels between
% 0.01(0.01)0.20 and sample sizes 5, 15, 25, and 50 against the alternate distribution noted
% above. This particular study has been accomplished to compare power to that of Bush's [17]
% against this particular alternate distribution.
% Tibet MEMIS 28 Nov 98 (Modified from Clough's [26:H-10] code)
%***********************************************************************

  % Initialization of the Parameters for Ha: Normal(0,1)

mu = 0;
sigma = 1;

numsamples = 40000;          % number of samples of size n to generate

rand('seed',40042);          % Use a different one for each run

load hqcrits05;   % Load the critical values arrays generated for Ho:Weib(1,l)
                  % This will load the following hqlotail and hquptail arrays


A = zeros(10,20);     % Initializing the failure counter array to all 0s
                      % First index (rows) tracks sample sizes (5:5:50)
                      % Second index (cols) tracks alpha levels (.01:.01:.20)

n = 5;          % First sample size
step = 10;      % Starting with a step of 10 to do 5,15, and 25

while n < 51        % Cycling until you step past sample size 50

        sprintf('Starting sample size %d',n)

        for i = 1:numsamples
                x = normrnd(mu,sigma,l,n);      % Generating a lxn vector of  Normal deviates
                hq = hoggq(x);
                if rem(i,1000) == 0 disp(i); end        % Keeps track of the progress


% Initialize the placeholders for this particular sample
```

```
jcurr = 1;
jstop   21;
jindex    0;

% Added this for the one-tailed test to help reference the proper critical values -- Now, jindex =
% jcurr*2 if jcurr<= 10 and jindex = jcurr+10 if jcurr > 10. This is due to the fact that the critvals
% arrays columns &re indexed by 1-30 where the ith column is for the .01*i alpha level of the
% two tailed test meaning the actual value is the .01*i/2 crit value. For the one tailed test we
% want the actual .01*i critical value, so we must change the reference.

% Now conducting the one-tailed Q-Statistic Test

while jcurr < jstop

        if jcurr <= 10                      % This new block assigns jindex correctly
                jindex = jcurr*2;
                  else
                jindex = jcurr +10;
                  end;

        if kt < kurtlotail(n/5,jindex)         % fail lower tail
                jstop = jcurr;
                  end % if

               jcurr = jcurr + 1;

                 end % while

% At this point jstop will hold the fail point (1-20) if it failed; If it passed all then jstop = 21.

% Now figure out which cells to increment in the counter array. Fail is an array of 0s and 1s
% indicating what levels the sample failed the Kurtosis test. It is used to increment the
% counter array A.

Fail = zeros(1,20);                        % Initialize it to all Os

        if jstop < 21    Fail(1,jstop:20) = 1; end      % Switch to 1s for all failures

        A(n/5,:) = A(n/5,:) + Fail;

end % for loop (i) -- go back for next sample of size n

        sprintf('Finished sample size %d -- going to next.\n',n);

              if n = = 25
```

```
            step = 25;        % Once you hit size 25, jump to 50 on the next iteration

            end

         n = n + step;        % increment n for the next sample size

      end %  while loop (n) -- now change sample sizes

                                        % Now A has counts for all sample sizes (5:5:50)
A = A./ numsamples;                     % Dividing by numsamples to get alpha levels

% Now output to a file

save pwrhqlotailnorm1001 A;                        % Saving A to a binary file for future ref

filel   fopen('pwrhqnormlotail1001.txt','w');      % And save to a text file

format  = ' %2d & %4.3f & %4.3f & %4. 3f & %4.3f & %4.3f \\\\ \n ';

heading   'Size & 0.01 & 0.06 & 0.10 & 0.16 & 0.20 \\\\ \\hline \\hline \nl;

fprintf(filel,'Power Study: Lower Tail Q-Statistic Test -- Ho: Weib(l, l)   Ha: Normal(0,1) \n');
      fprintf(filel, heading);

   for n=5:5:50
         row=[n,A(n/5,[l 5 10 15 201)1;
         fprintf(filel,format,row);
         fprintf(filel,'\n \n');
      end % for

fclose(filel);
disp('Finished: File saved.')
```

J.7 Code for Power Plots

```
% ***************************************************************************
% Power Study: One Tailed Version -- Lower Tail Q-Statistic Test Only
% Ho: Weibull(1,l)     Ha: Normal(0,1)
% ===========================================================================
% This MATLAB code creates plots that compares the power of the sequential G.O.F. test with
% the individual two-sided G.O.F. tests against a specific alternate distribution for a given
% sample size.
% Tibet MEMIS 17 Jan 99 (Modified from Clough's [26:H-15] code)
%****************************************************************************

% ***************************** Sample Size = 5 ********************************

% ************ Data for the Power for the Sequential G.O.F. Test **********************

subplot(2,2,1)

load sigtable35;              % Loading the significance levels of the sample
Level=A(:,:,1);
clear A;

load pwrbeta3522;             % Loading the power for the same sample size
Power = A(:,:,1);
clear A;
Power=Power(:);               % Stringing out the arrays into vectors
Level=Level(:);
[sLevel i] = sort(Level);     % Sorting the levels and saving the indices to sort the
sPower = Power(i);            % corresponding powers pairwise

% ************ Data for the Power for the two-sided Q-Statistic G.O.F. Test **************

load pwrhqbeta3522;
Kpower = A(:,:,1);            % Adjusting first index acc to sample size
clear A;
Klevel = .01:.01:.20;         % Vector of corresponding significance levels

% ************ Data for the Power for the two-sided Skewness G.O.F. Test **************

load pwrskbeta3522;
SKpower = A(:,:,1);           % Adjusting first index acc to sample size
clear A;

plot(sLevel, sPower, Klevel, Kpower,'gx', Klevel, SKpower,'ro', Klevel, Kpower,'g', Klevel,
SKpower,'r')
title('Ho=Weib(1,3.5) Ha=Beta(2,2); Sample size = 5')
```

```
xlabel('Significance Level')
ylabel('Power')

% **************************** Sample Size = 15 *******************************

% ************ Data for the Power for the Sequential G.O.F. Test **********************

subplot(2,2,2)

load sigtable35;              % Loading the significance levels of the sample
Level=A(:,:,3);
clear A;

load pwrbeta3522;             % Loading the power for the same sample size
Power = A(:,:,3);
clear A;
Power=Power(:);               % Stringing out the arrays into vectors
Level=Level(:);
[sLevel i] = sort(Level);     % Sorting the levels and saving the indices to sort the
sPower = Power(i);            % corresponding powers pairwise

% ************ Data for the Power for the two-sided Q-Statistic G.O.F. Test **************

load pwrhqbeta3522;
Kpower = A(:,:,3);            % Adjusting first index acc to sample size
clear A;
Klevel = .01:.01:.20;         % Vector of corresponding significance levels

% ************ Data for the Power for the two-sided Skewness G.O.F. Test **************

load pwrskbeta3522;
SKpower = A(:,:,3);           % Adjusting first index acc to sample size
clear A;

plot(sLevel, sPower, Klevel, Kpower,'gx', Klevel, SKpower,'ro', Klevel, Kpower,'g', Klevel,
SKpower,'r')
title('Sample size = 15')
xlabel('Significance Level')
ylabel('Power')

% **************************** Sample Size = 25 *******************************

% ************ Data for the Power for the Sequential G.O.F. Test **********************
```

```
subplot(2,2,3)

load sigtable35;            % Loading the significance levels of the sample
Level=A(:,:,5);
clear A;

load pwrbeta3522;           % Loading the power for the same sample size
Power = A(:,:,5);
clear A;
Power=Power(:);             % Stringing out the arrays into vectors
Level=Level(:);
[sLevel i] = sort(Level);   % Sorting the levels and saving the indices to sort the
sPower = Power(i);          % corresponding powers pairwise

% ************ Data for the Power for the two-sided Q-Statistic G.O.F. Test **************

load pwrhqbeta3522;         % Adjusting first index acc to sample size
Kpower = A(:,:,5);
clear A;
Klevel = .01:.01:.20;       % Vector of corresponding significance levels

% ************ Data for the Power for the two-sided Skewness G.O.F. Test **************

load pwrskbeta3522;         % Adjusting first index acc to sample size
SKpower = A(:,:,5);
clear A;

plot(sLevel, sPower, Klevel, Kpower,'gx', Klevel, SKpower,'ro', Klevel, Kpower,'g', Klevel,
SKpower,'r')
title('Sample size = 25')
xlabel('Significance Level')
ylabel('Power')

% ***************************** Sample Size = 50 *******************************

% ************ Data for the Power for the Sequential G.O.F. Test *********************

subplot(2,2,4)

load sigtable35;            % Loading the significance levels of the sample
Level=A(:,:,10);
clear A;

load pwrbeta3522;           % Loading the power for the same sample size
Power = A(:,:,10);
```

```
clear A;
Power=Power(:);             % Stringing out the arrays into vectors
Level=Level(:);
[sLevel i] = sort(Level);   % Sorting the levels and saving the indices to sort the
sPower = Power(i);          % corresponding powers pairwise

% ************ Data for the Power for the two-sided Q-Statistic G.O.F. Test **************

load pwrhqbeta3522;         % Adjusting first index acc to sample size
Kpower = A(:,:,10);
clear A;
Klevel = .01:.01:.20;       % Vector of corresponding significance levels

% ************ Data for the Power for the two-sided Skewness G.O.F. Test **************

load pwrskbeta3522;         % Adjusting first index acc to sample size
SKpower = A(:,:,10);
clear A;

plot(sLevel, sPower, Klevel, Kpower,'gx', Klevel, SKpower,'ro', Klevel, Kpower,'g', Klevel,
SKpower,'r')
title('Sample size = 50')
xlabel('Significance Level')
ylabel('Power')
```

J.8 Contour Plots for the Attained Significance Level

```
%***************************************************************************
% This MATLAB code generates the contour plots of the Attained Significance Levels for the
% Sequential G.O.F. Test Procedure. This specific example was used for generating the contour
% plot for the attained significance levels for Weibull(1.5) at the sample size, n = 15. The
% generic coding  with the change in the Weibull shape value and sample sizes was used in this
% research.
% Tibet MEMIS 18 SEP 98 [Modified from Clough's Coding (13:H-13)].
%***************************************************************************

load sigtable15;                        % Loading the Weibull(1.5) attained significance levels

x = 0.01:.01:.20;                       % Creating the vectors for labeling the axes
y=x;
v = .01:.01:.35;                        % Creating the vectors for the % desired contour levels
S15 = A(:,:,3);                         % Picking off n = 15 layer
[s15label h]=contour(x, y, S15, v);     % Getting the labeling structure
clabel(s15label,h,'manual');
labels = ['0.01';
          '    ';
        '    ';
        '    ';
        '0.05';
        '    ';
        '    ';
        '    ';
        '    ';
        '0.10';
        '    ';
        '    ';
        '    ';
        '    ';
        '0.15';
        '    ';
        '    ';
        '    ';
        '    ';
        '0.20';];
set(gca,'XTick',x)                                % Adjusting the x-axis tick   % marks
set(gca,'YTick',y)                                % Adjusting the y-axis tick   % marks
set(gca,'XTickLabel',labels)                      % Relabeling the x-axis
set(gca,'YTickLabel',labels)                      % Relabeling the y-axis
title('Attained Significance Levels: \Beta= 1.5, Sample Size= 15');
grid;
```

```
xlabel('Q-Statistic G.O.F. Test  \alpha-level');
ylabel('Skewness G.O.F. Test  \alpha-level');
```

Appendix K. PDFs of Wozniak's Original and Transformed Alternate Distributions and the Random Deviate Generators for the Transformed Distributions.

K.1 Chi-square (χ^2) Distribution.

$$PDF \quad f(x;v) = \frac{1}{2^{\frac{v}{2}}\Gamma\left(\frac{v}{2}\right)} x^{\left(\frac{v-2}{2}\right)} e^{-\left(\frac{x}{2}\right)}, \quad x>0;$$

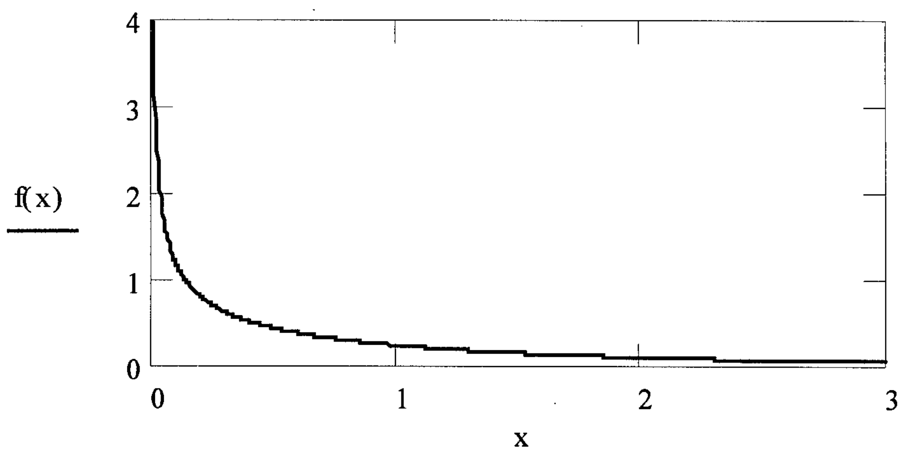


Figure K.1 χ^2 (1) PDF

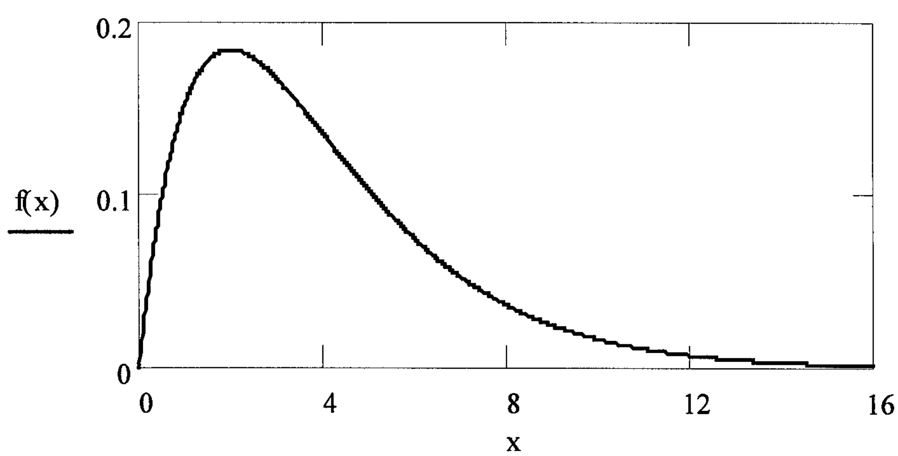


Figure K.2 χ^2 (4) PDF

K.2 Lognormal Distribution.

$$PDF \quad f(x;\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\left[\frac{1}{2}\left(\frac{\log x-\mu}{\sigma}\right)\right]}, \quad x>0; \quad \sigma>0$$

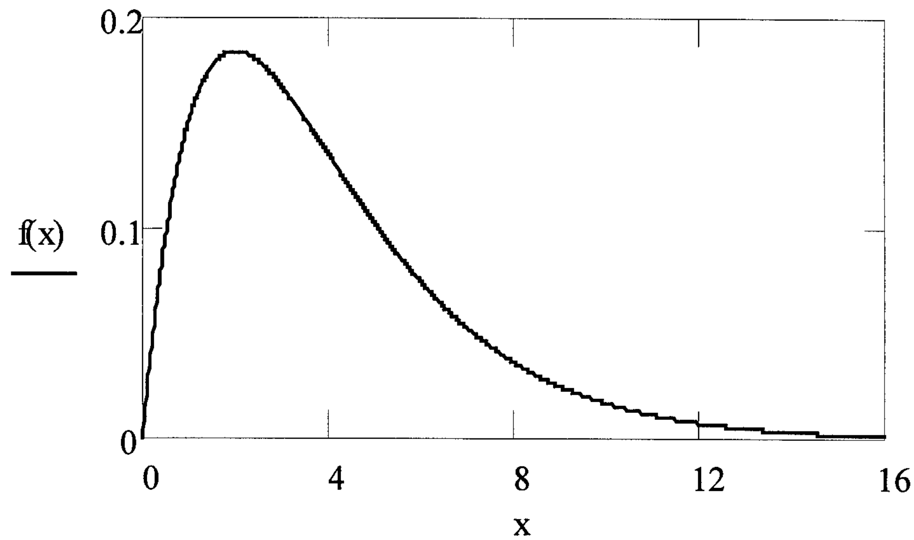


Figure K.3 Lognormal(0,1) PDF

K.3 Cauchy Distribution.

K.3.1 Cauchy Distribution.

$$PDF \quad f_y(y)=\frac{1}{\pi y}\left(\frac{\beta}{\beta^2+(y-\alpha)^2}\right), \quad -\infty<y<\infty; \quad \beta>0$$

K.3.2 Transformed Cauchy (X Cauchy) Distribution.

$$PDF \quad f_x(x)=\frac{1}{x\pi(1+\ln^2 x)}, \quad x>0$$

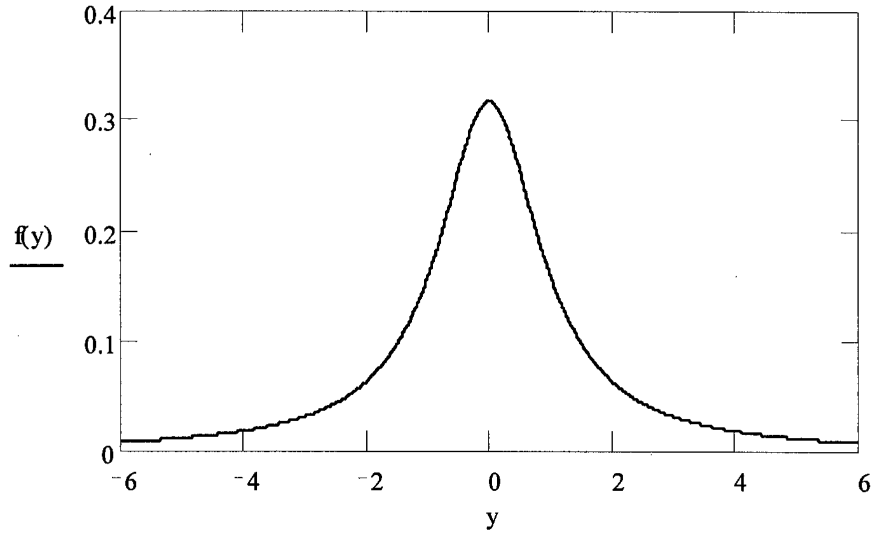


Figure K.4 Cauchy(0,1) PDF

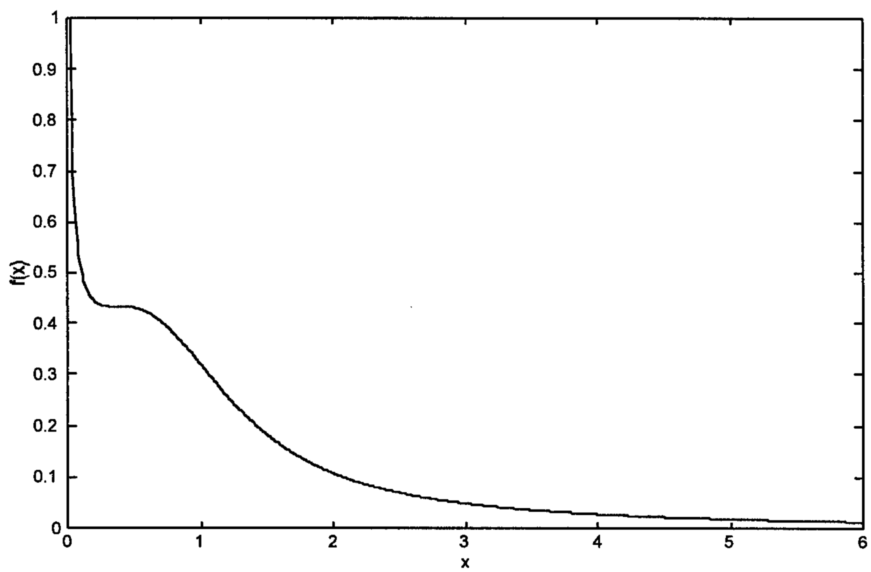


Figure K.5 Transformed Cauchy(0,1) PDF

K.3.3 Random Variate Generator for the Transformed Cauchy(0,1) Distribution.

$$X = e^{\left(\tan\left[\pi\left(U-0.5\right)\right]\right)},\quad \text{where U~Uniform(0,1).}$$

K.4 Logistic Distribution.

K.4.1 Logistic Distribution.

$$PDF \quad f_y(y) = \left(\frac{1}{\beta}\right) \frac{e^{-\left[\frac{(y-\alpha)}{\beta}\right]}}{\left(1+e^{-\left[\frac{(y-\alpha)}{\beta}\right]}\right)^2}, \qquad -\infty < y < \infty$$

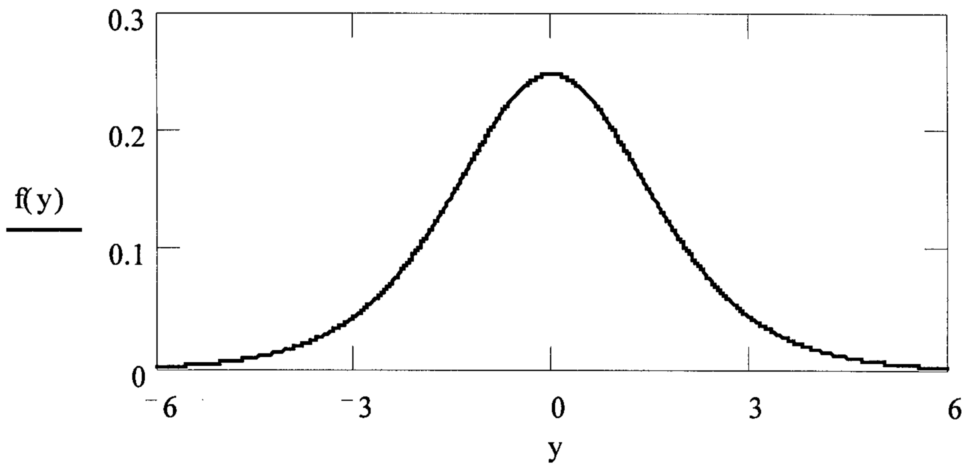


Figure K.6 Logistic(0,1) PDF

K.4.2 Transformed Logistic(0,1) Distribution.

$$PDF \quad f_x(x) = \frac{1}{(1+x)^2}, \qquad x > 0$$

K.4.3 Random Variate Generator for the Transformed Logistic(0,1) Distribution.

$$X = e^{\left[-\log\left(\frac{1}{U}-1\right)\right]} = \frac{U}{1-U}, \text{ where U~Uniform(0,1).}$$

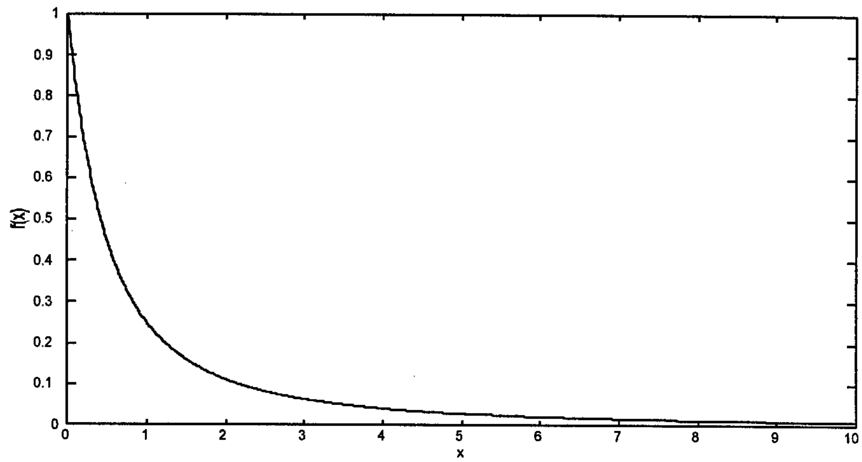


Figure K.7 Transformed Logistic(0,1) PDF

K.5 Double Exponential Distribution.

K.5.1 Double Exponential Distribution.

$$f_y(y) = \frac{1}{2} e^{-|y|}, \qquad -\infty < y < \infty$$

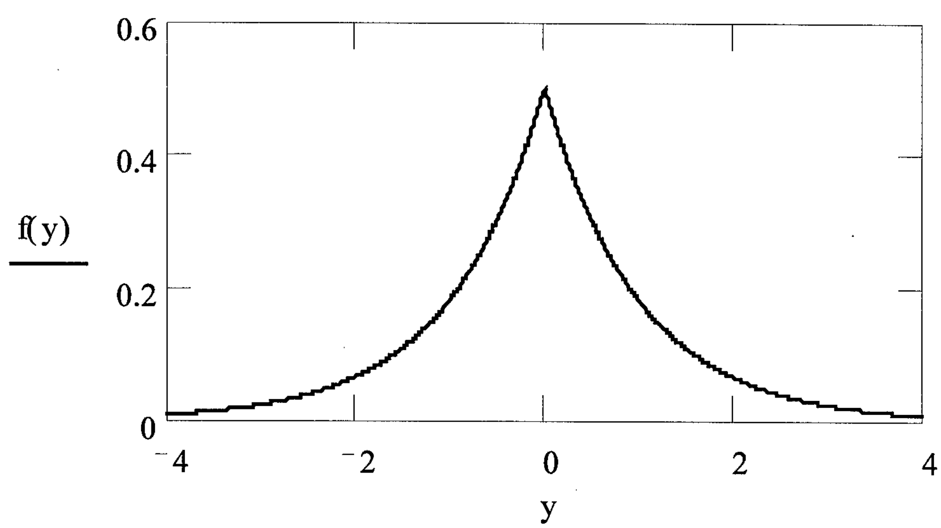


Figure K.8 Double Exponential PDF

K.5.2 Transformed Double Exponential Distribution.

$$f_x(x) = \begin{cases} 0 & if \quad x < 0 \\ \dfrac{1}{2} & if \quad 0 \le x \le 1 \\ \dfrac{1}{2x^2} & if \quad 1 < x \end{cases}$$

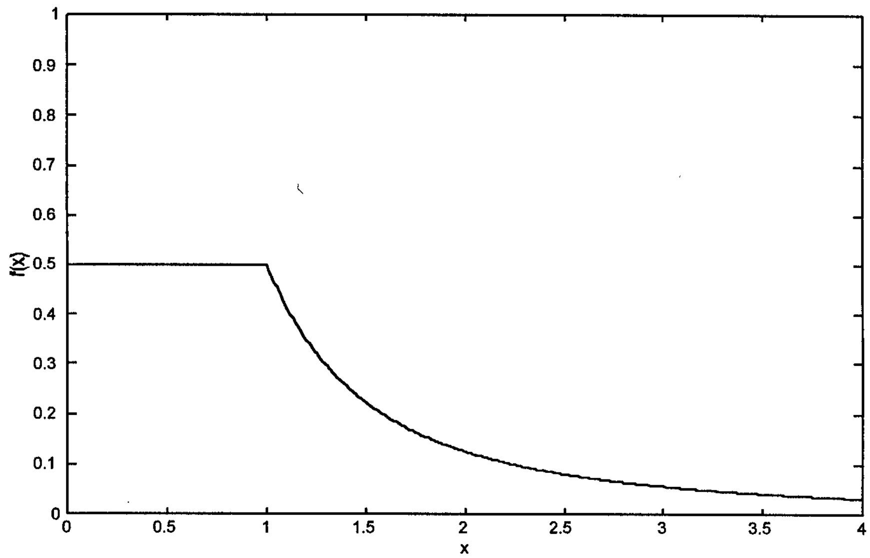
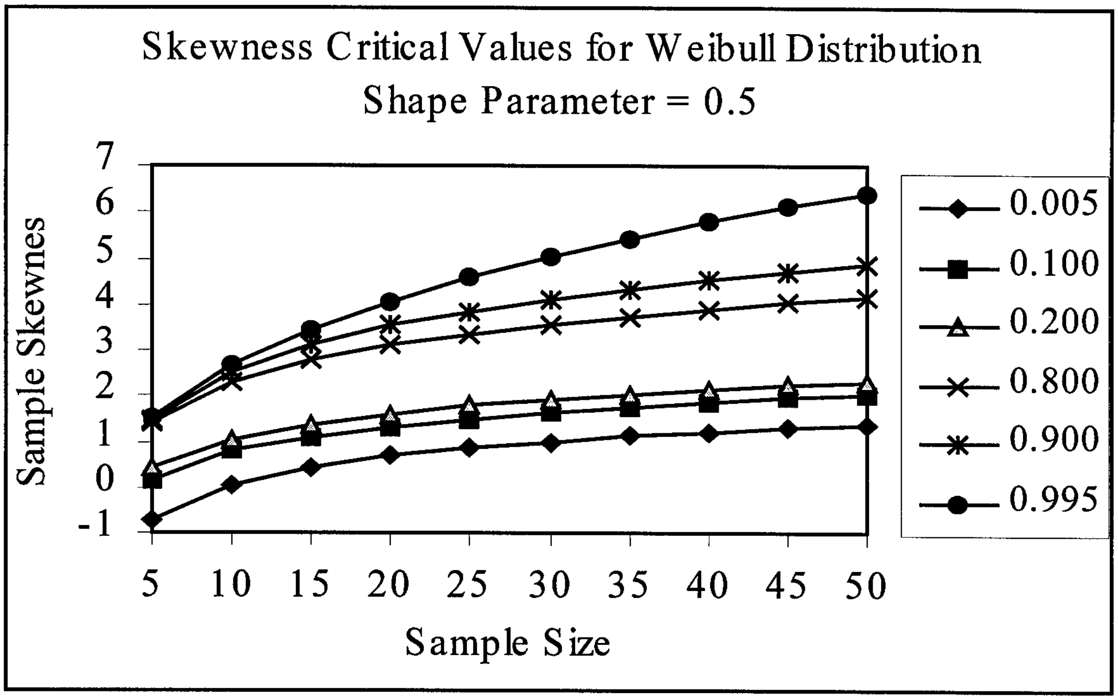


Figure K.9 Double Exponential PDF

K.5.3 Random Variate Generator for the Transformed Double Exponential Distribution.

$$f_x(x) = \begin{cases} 2U & if \quad U \le 0.5 \\ \dfrac{1}{2-2U} & if \quad U > 0.5 \end{cases}$$



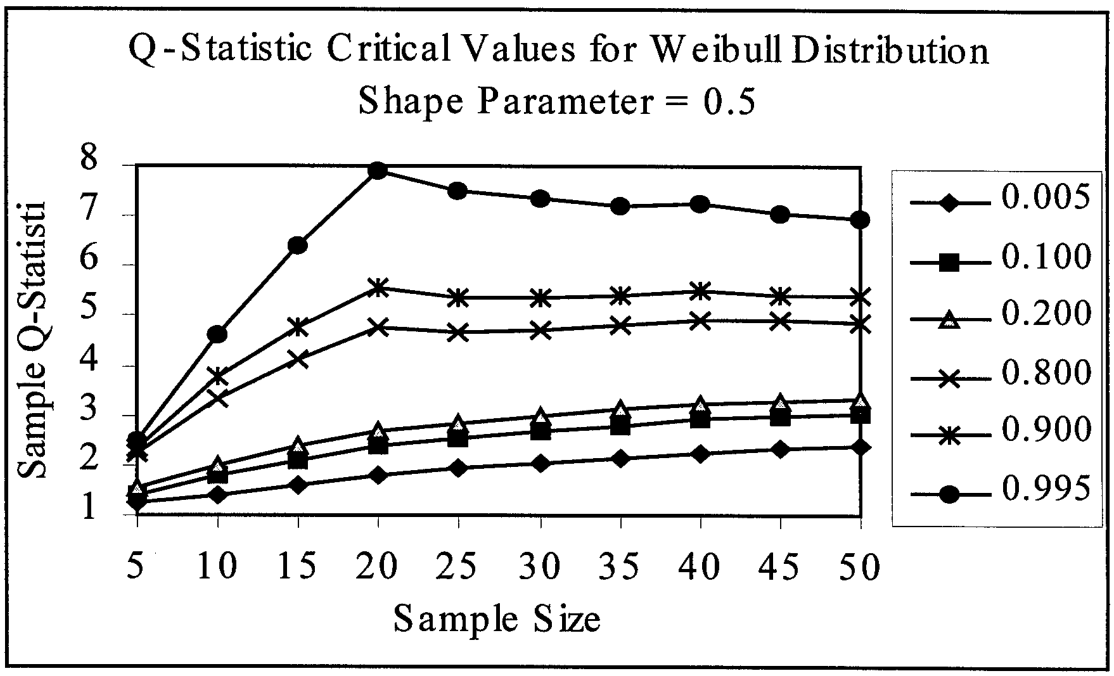
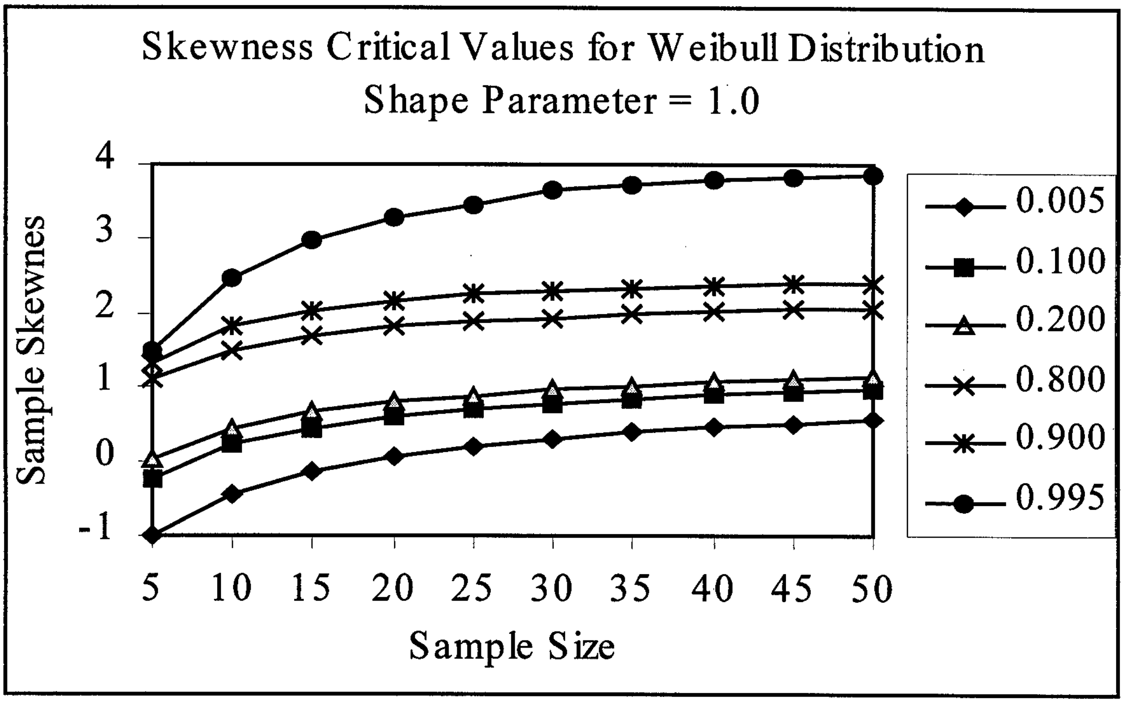


Figure L.1 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 0.5$); $n = 5(5)50$.



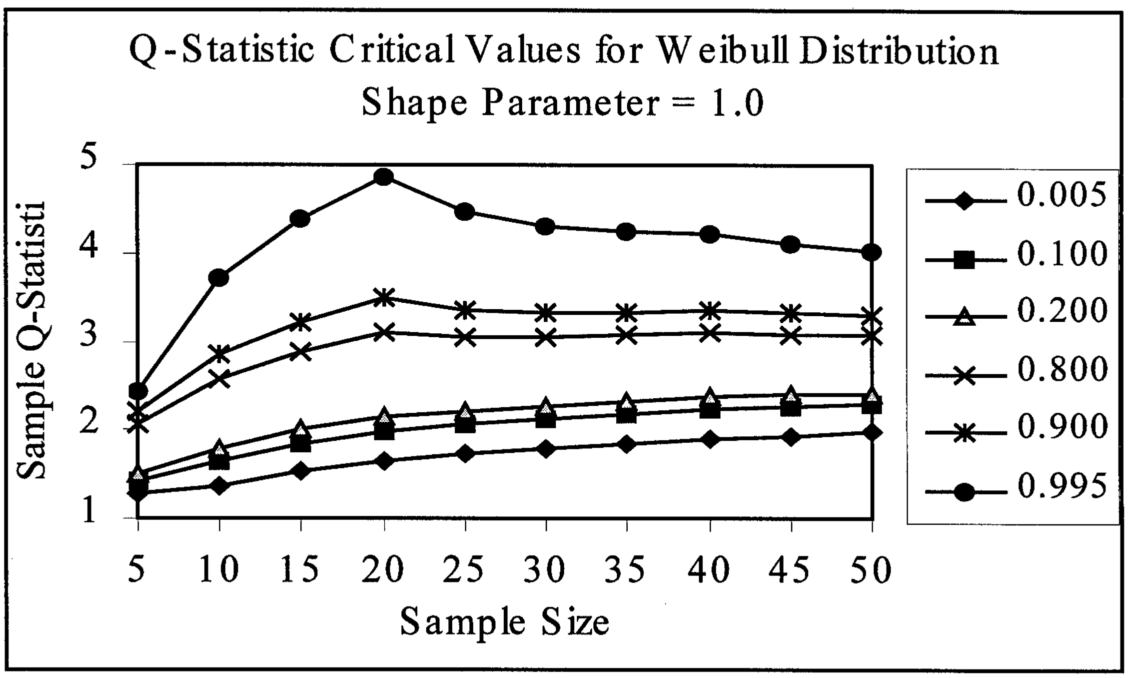
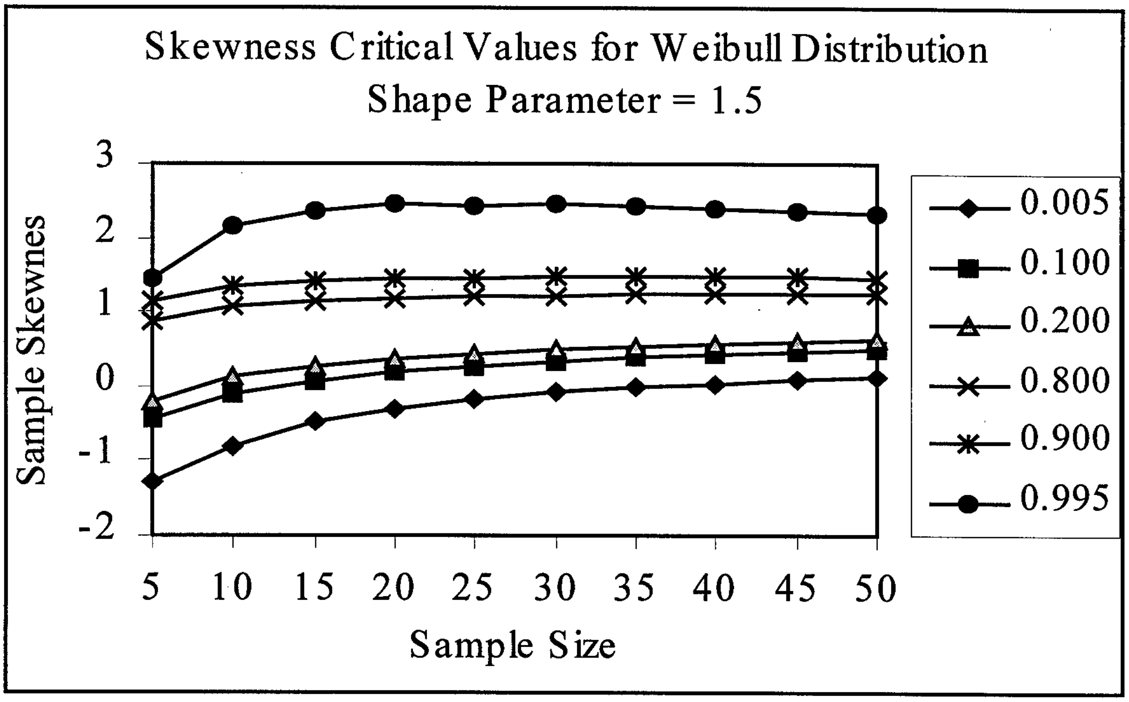


Figure L.2 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 1.0$); $n = 5(5)50$.



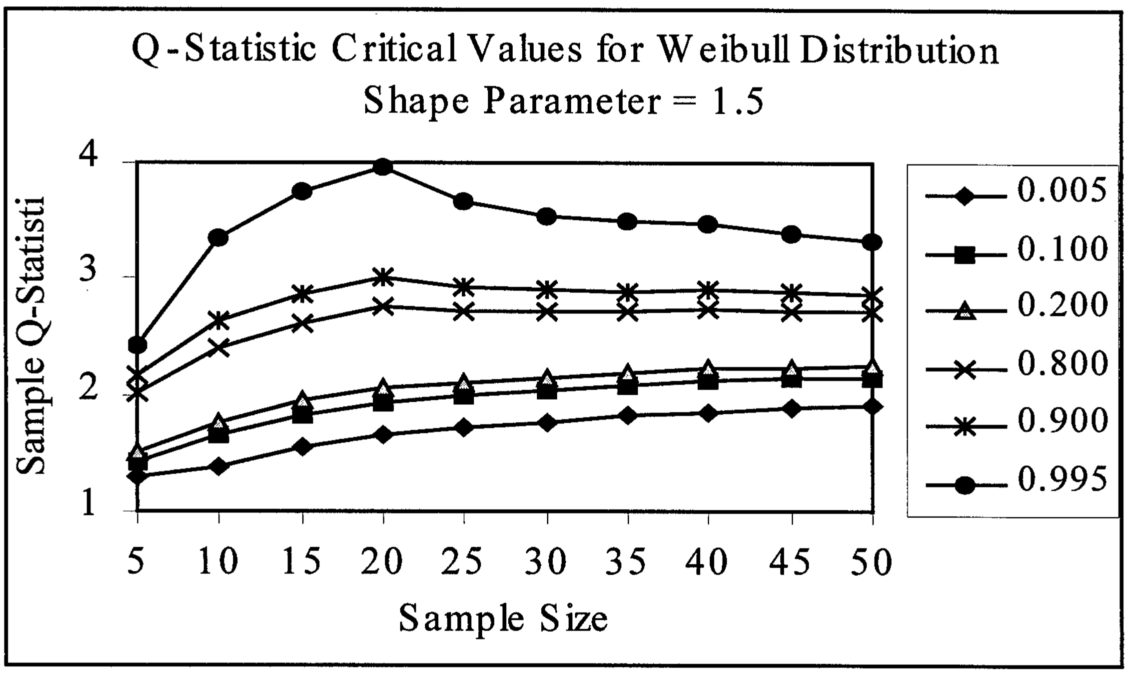
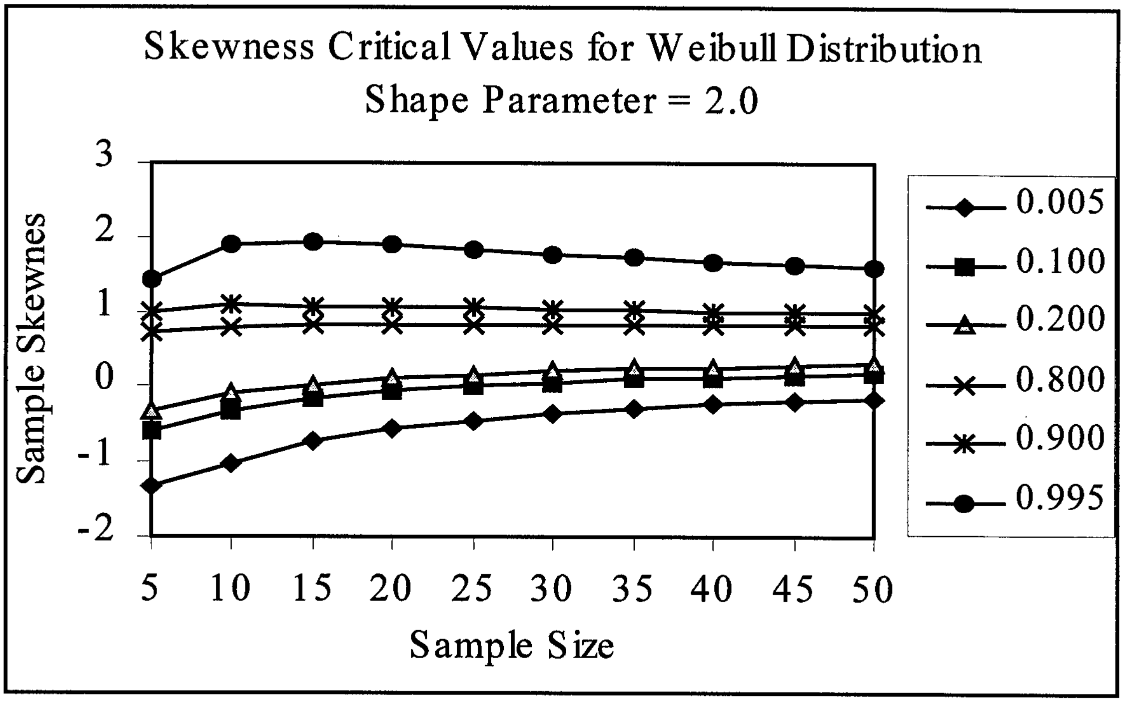


Figure L.3 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 1.5$); $n = 5(5)50$.



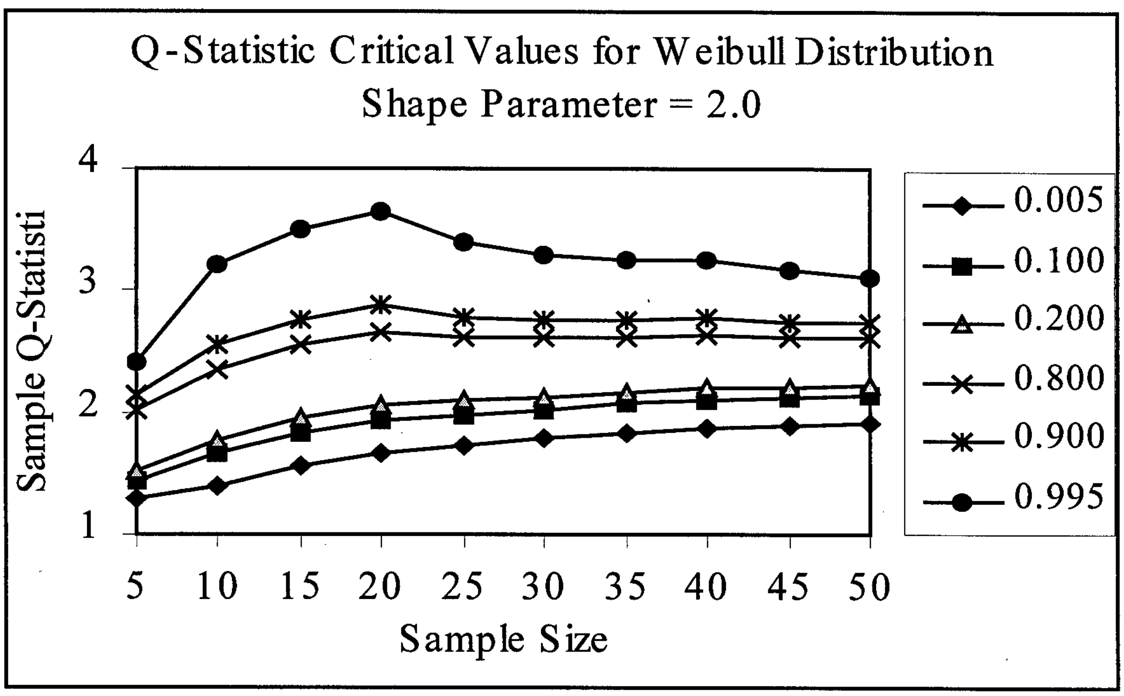
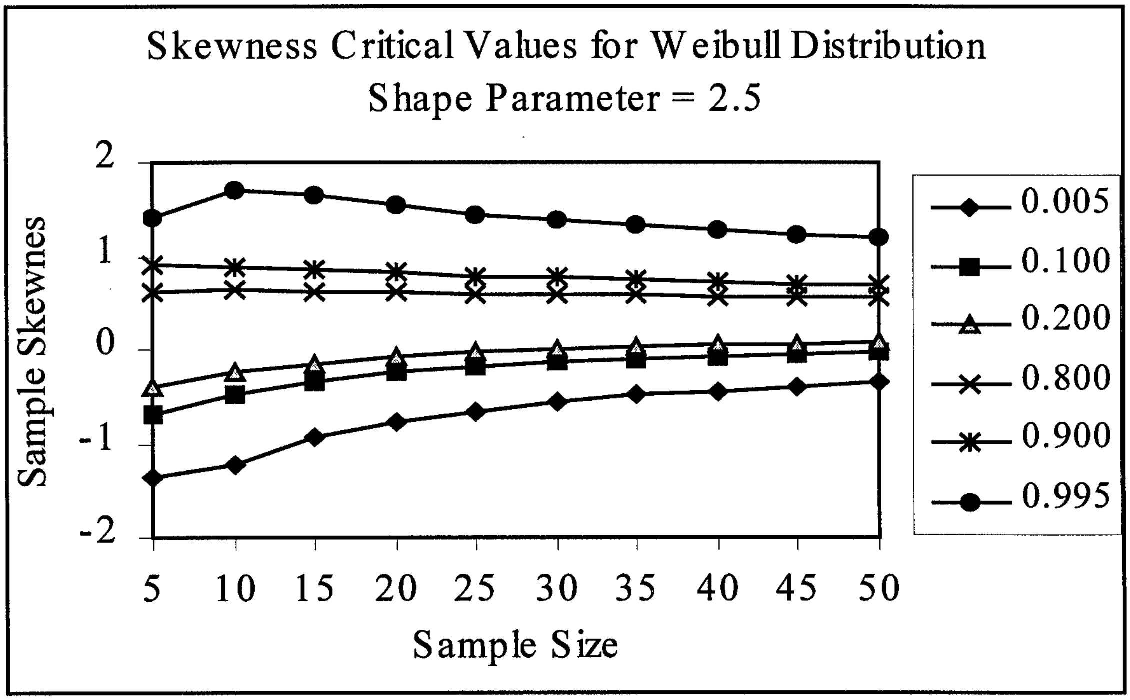


Figure L.4 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 2.0$); $n = 5(5)50$.



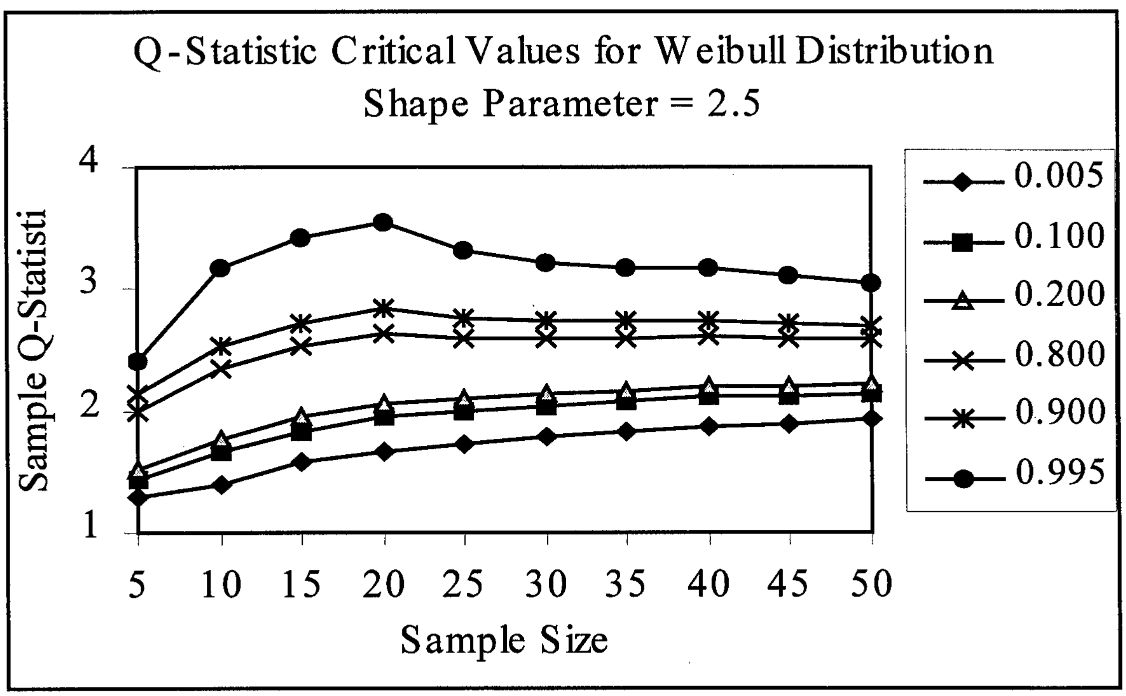
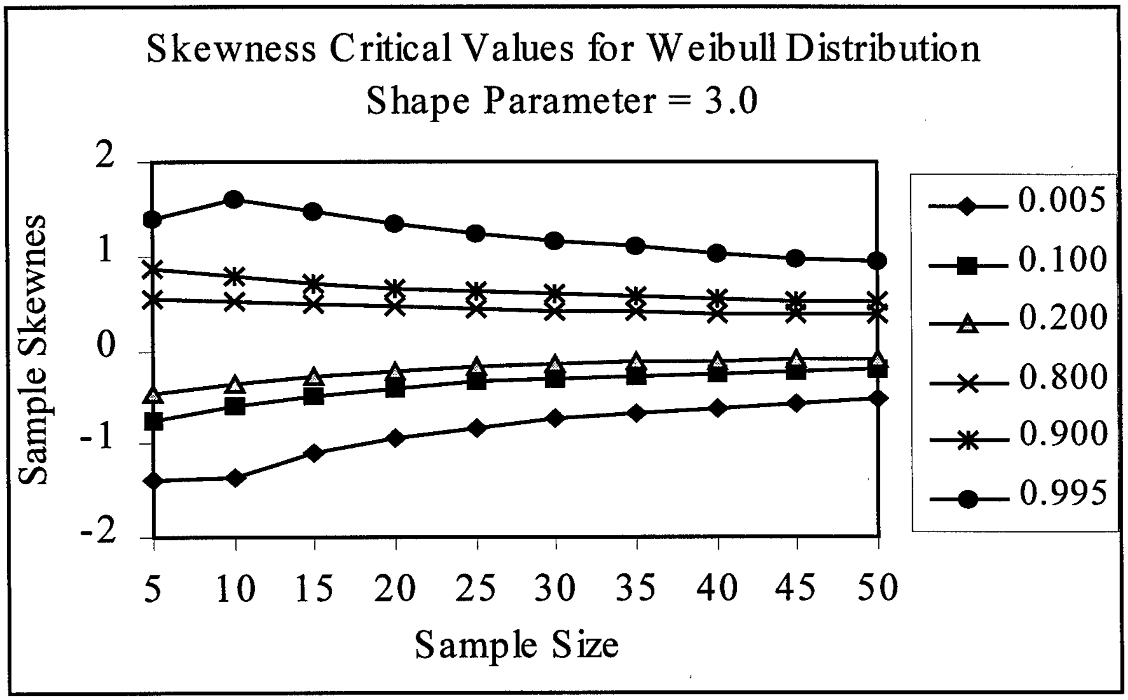


Figure L.5 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 2.5$); $n = 5(5)50$.



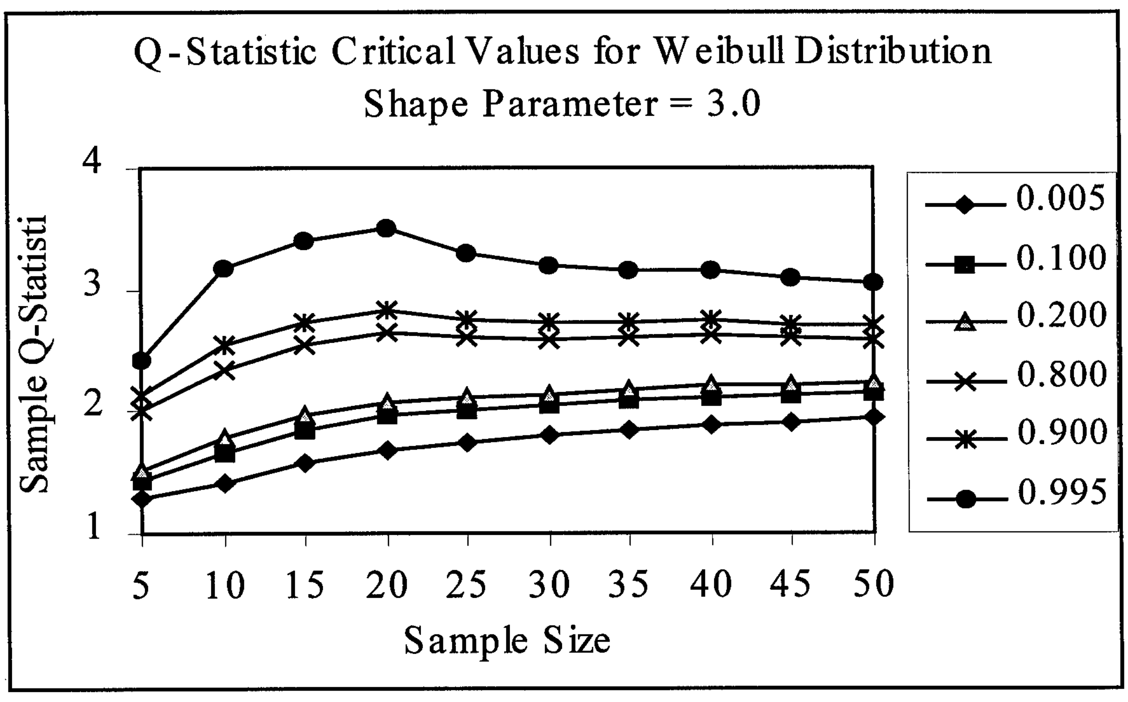
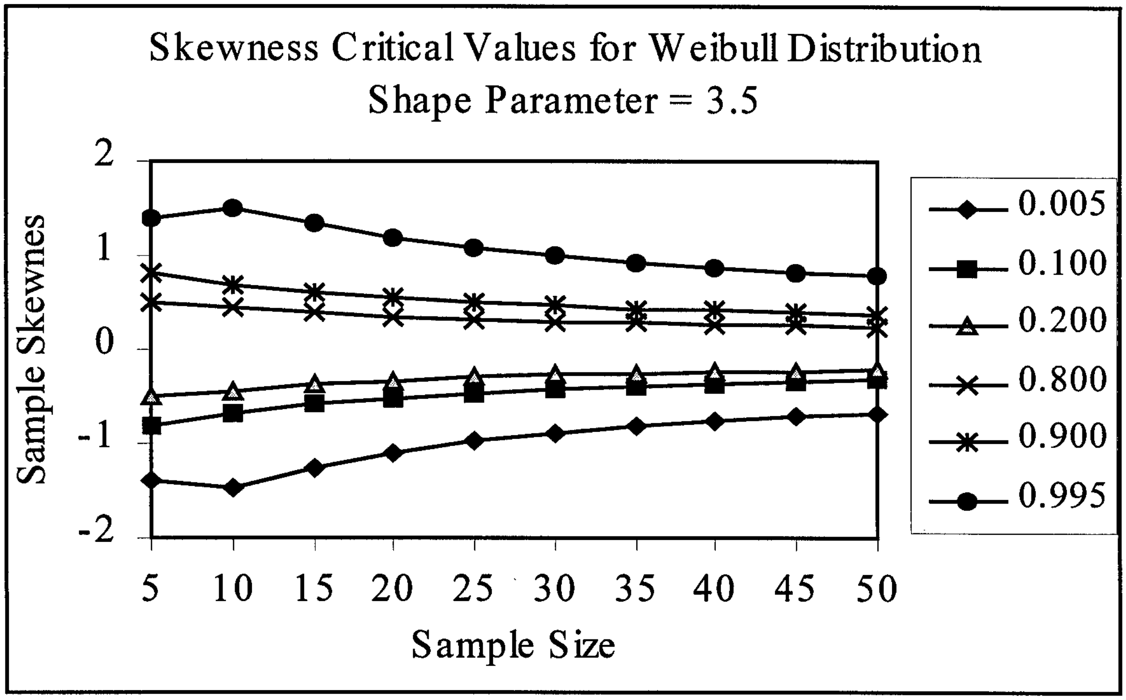


Figure L.6 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 3.0$); $n = 5(5)50$.



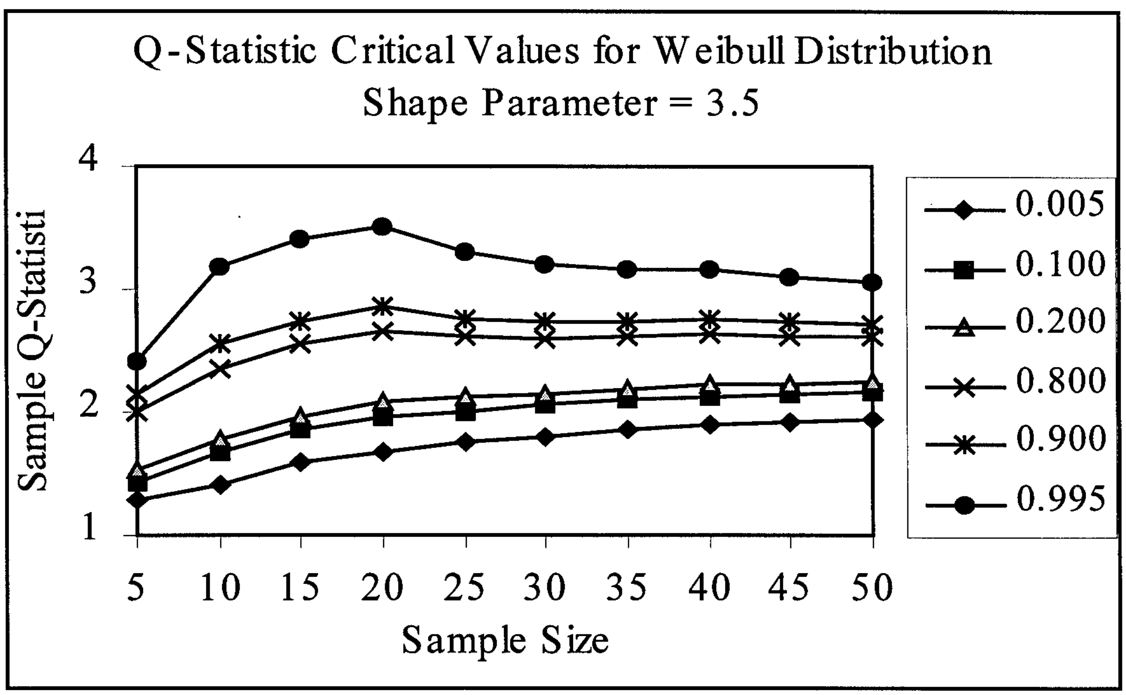
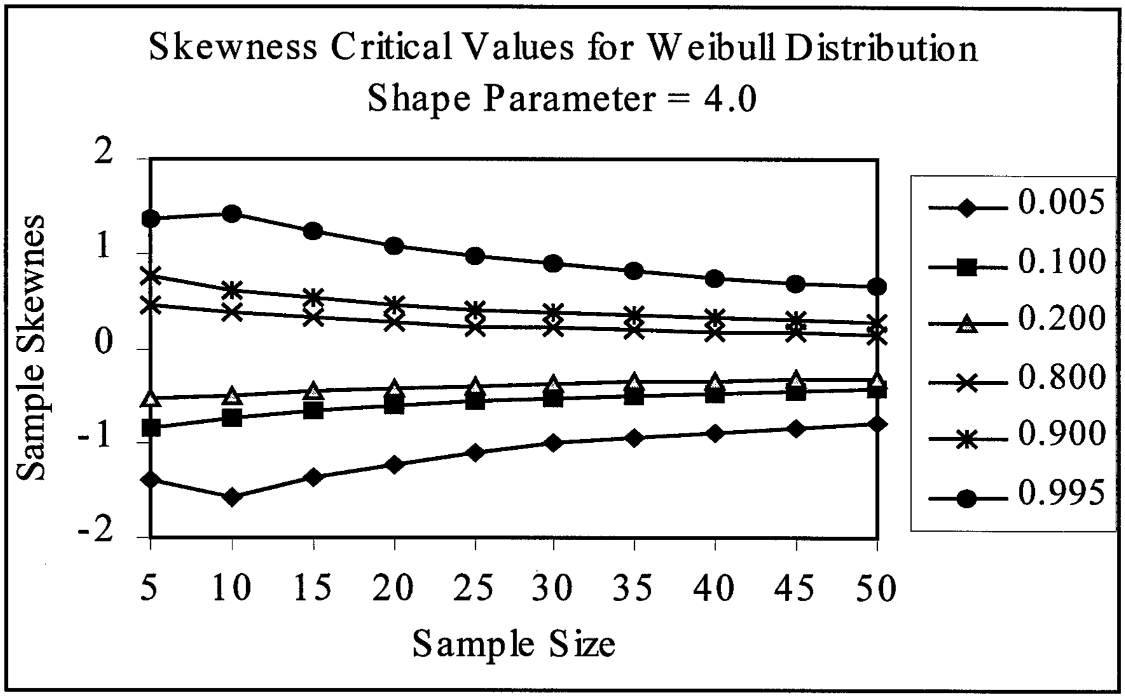


Figure L.7 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull($\beta = 3.5$); $n = 5(5)50$.



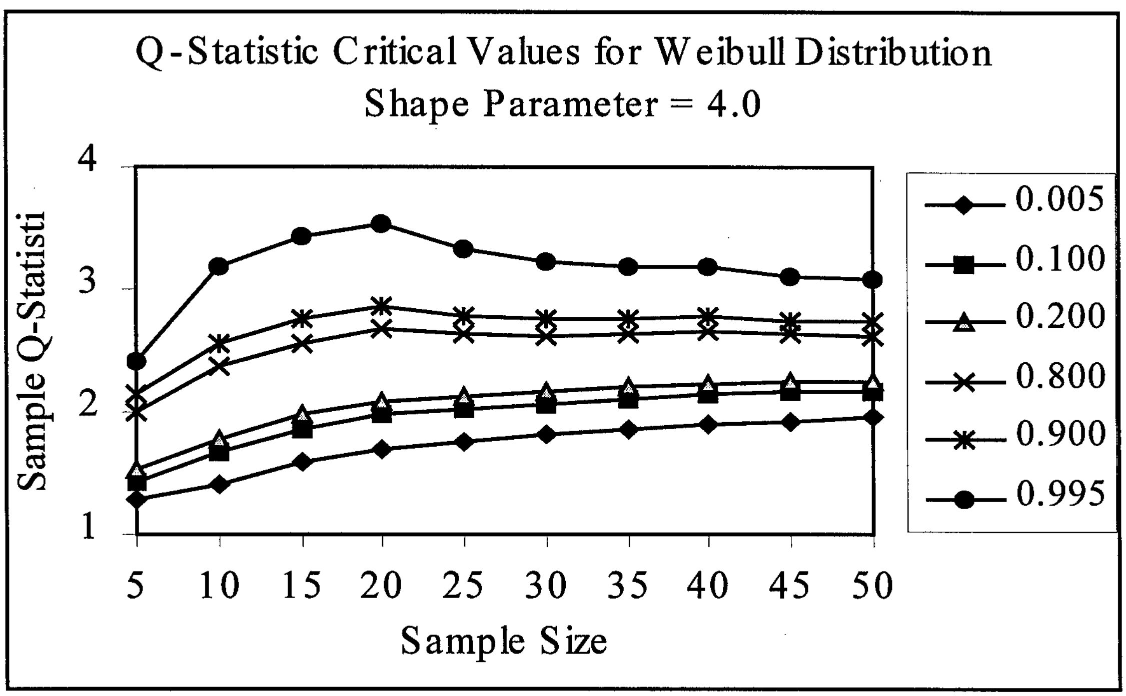
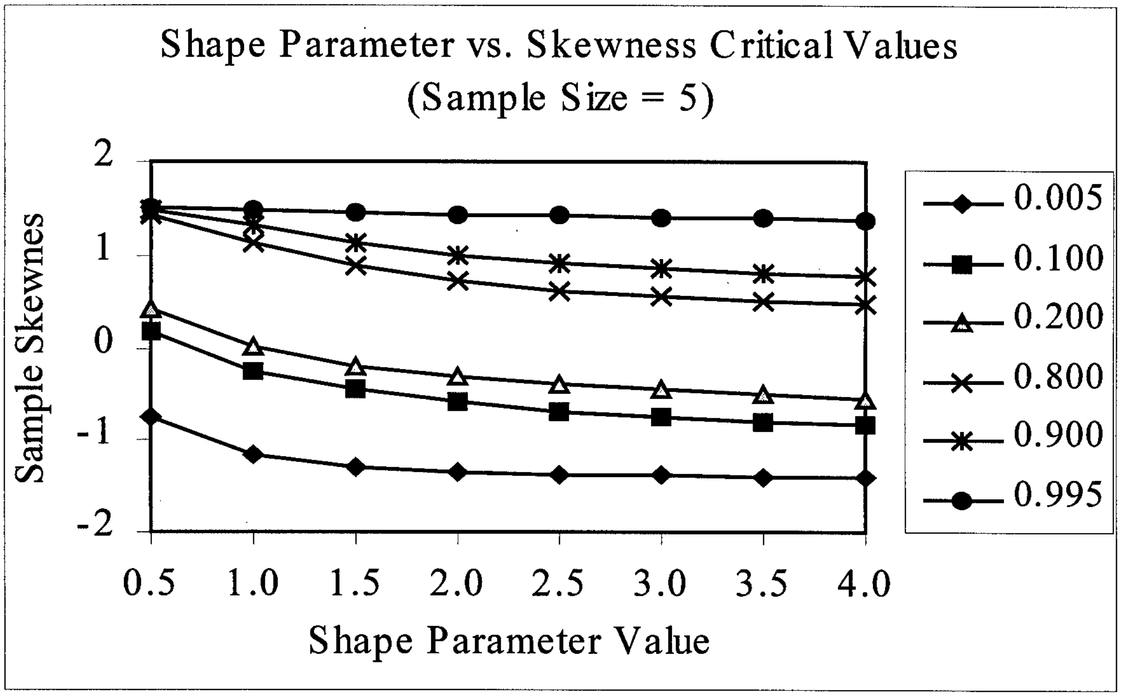


Figure L.8 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic G.O.F. tests for H_0 : Weibull(β = 4.0); n = 5(5)50.



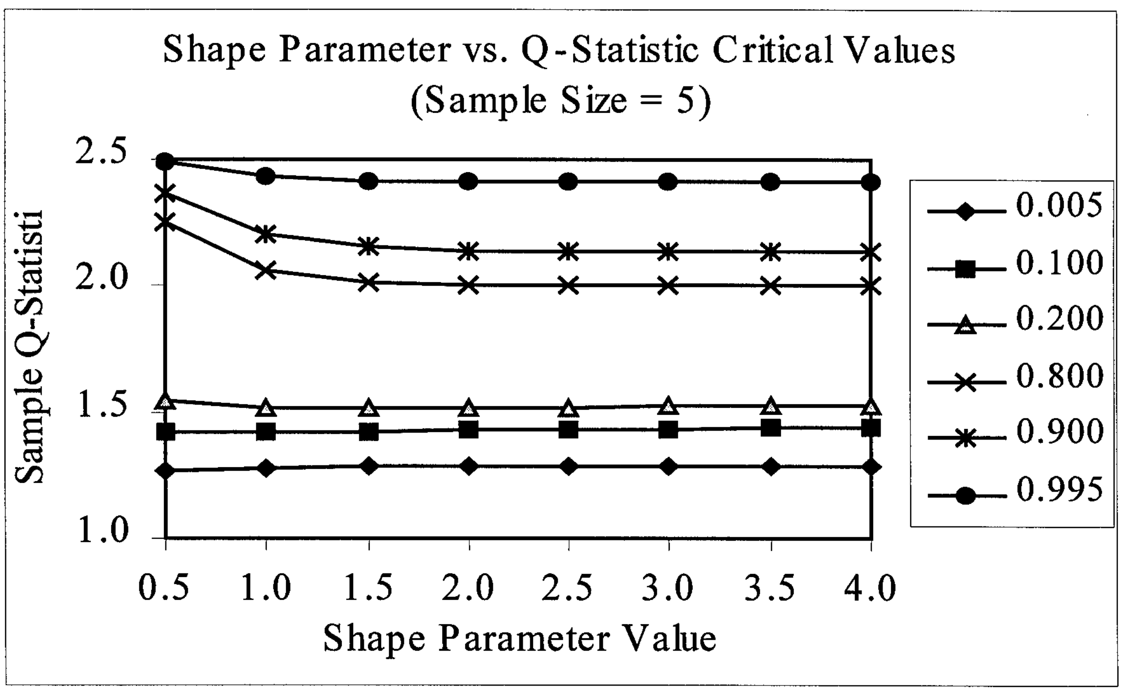
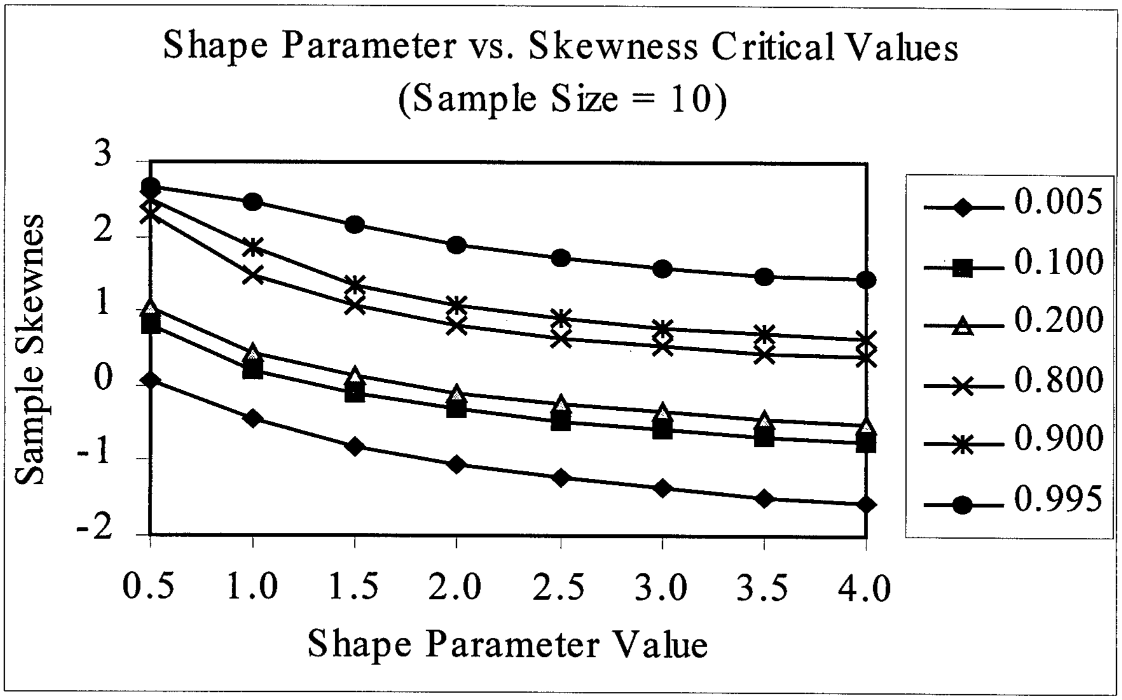


Figure M.1 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 5.



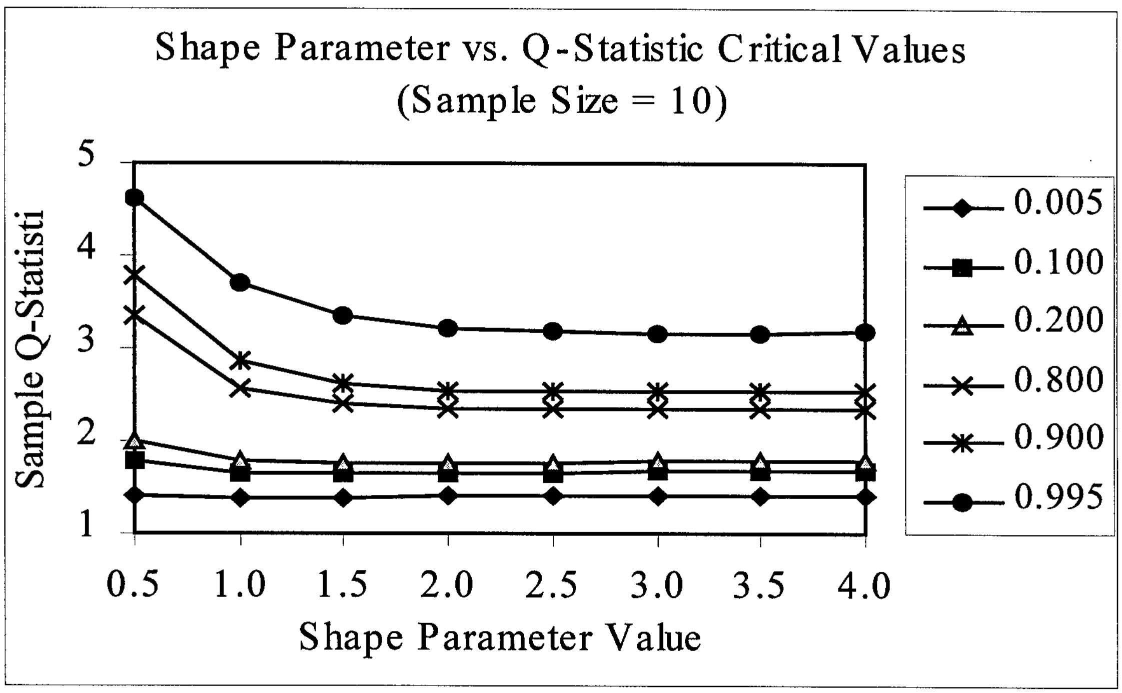
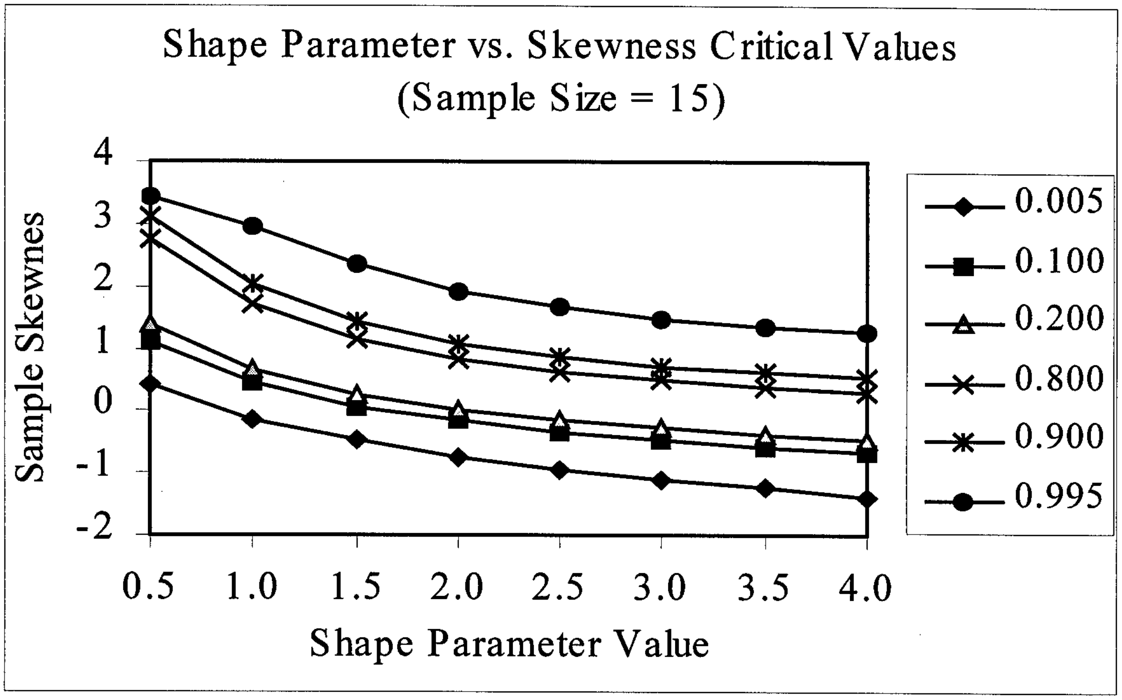


Figure M.2 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 10.



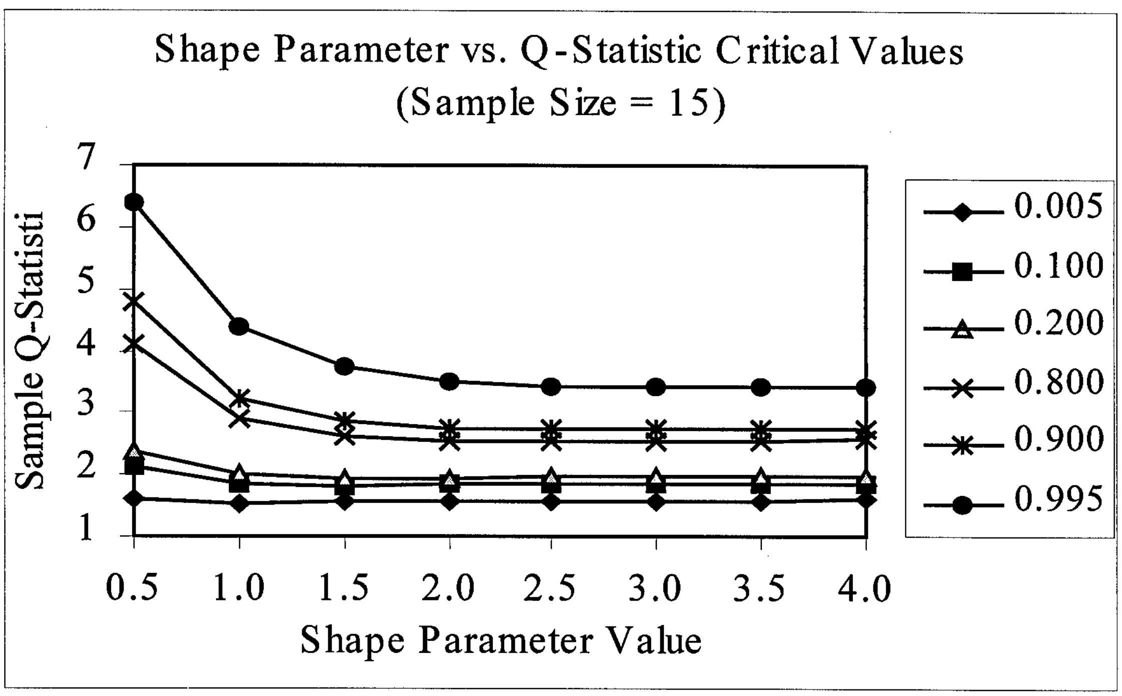
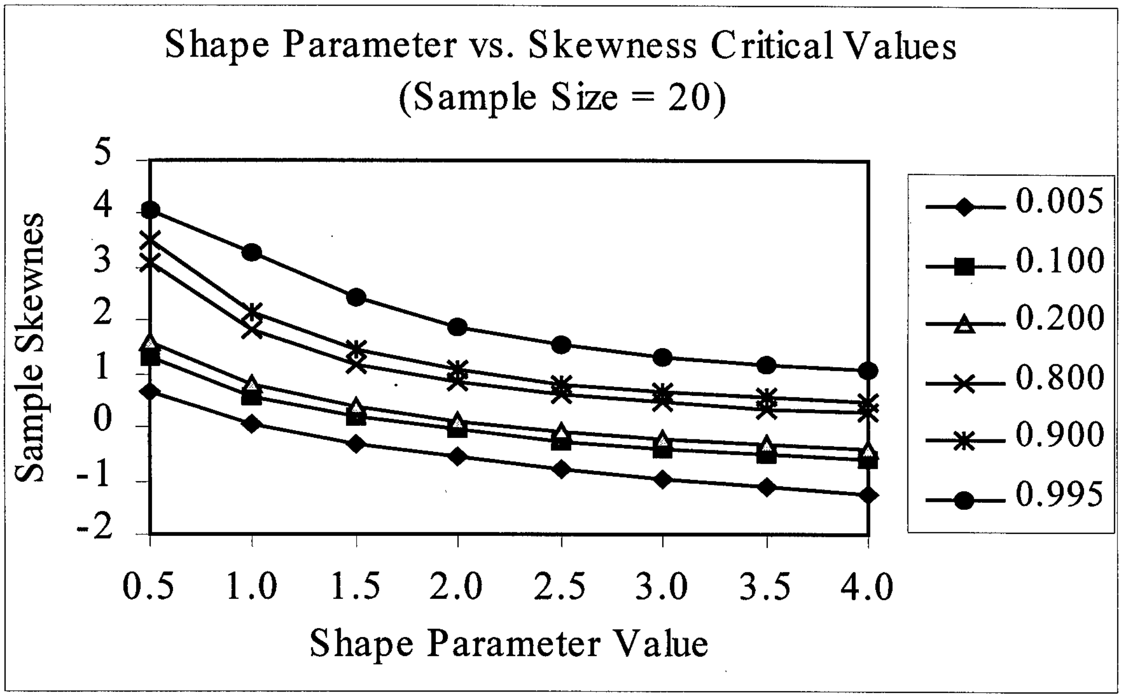


Figure M.3 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 15.



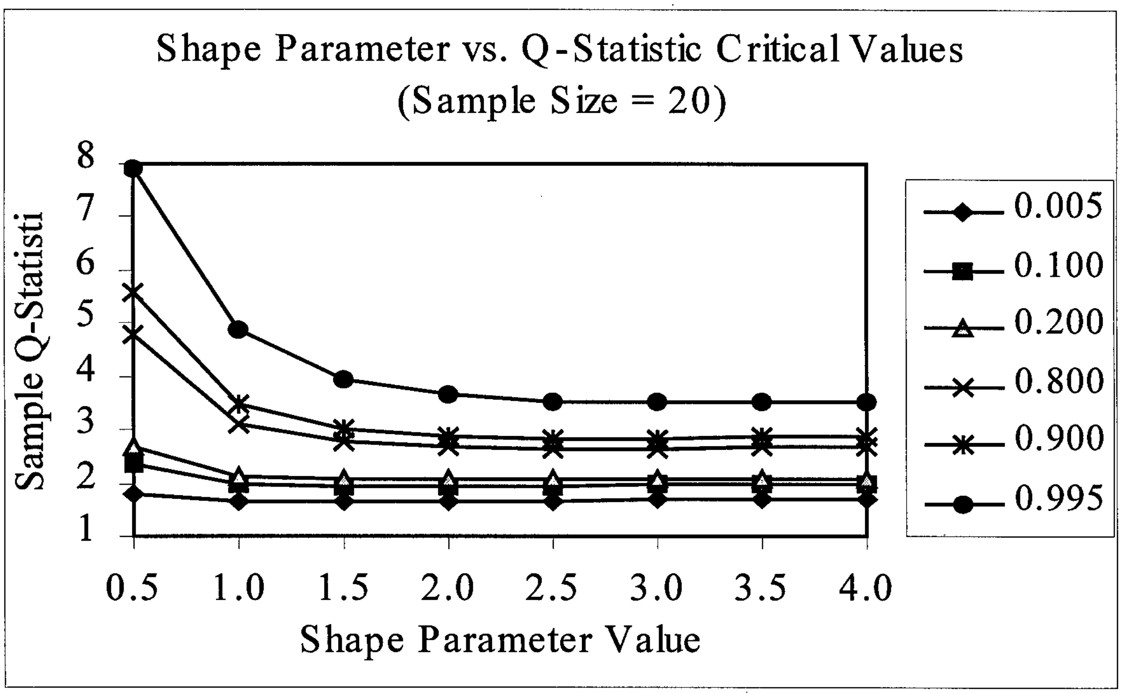
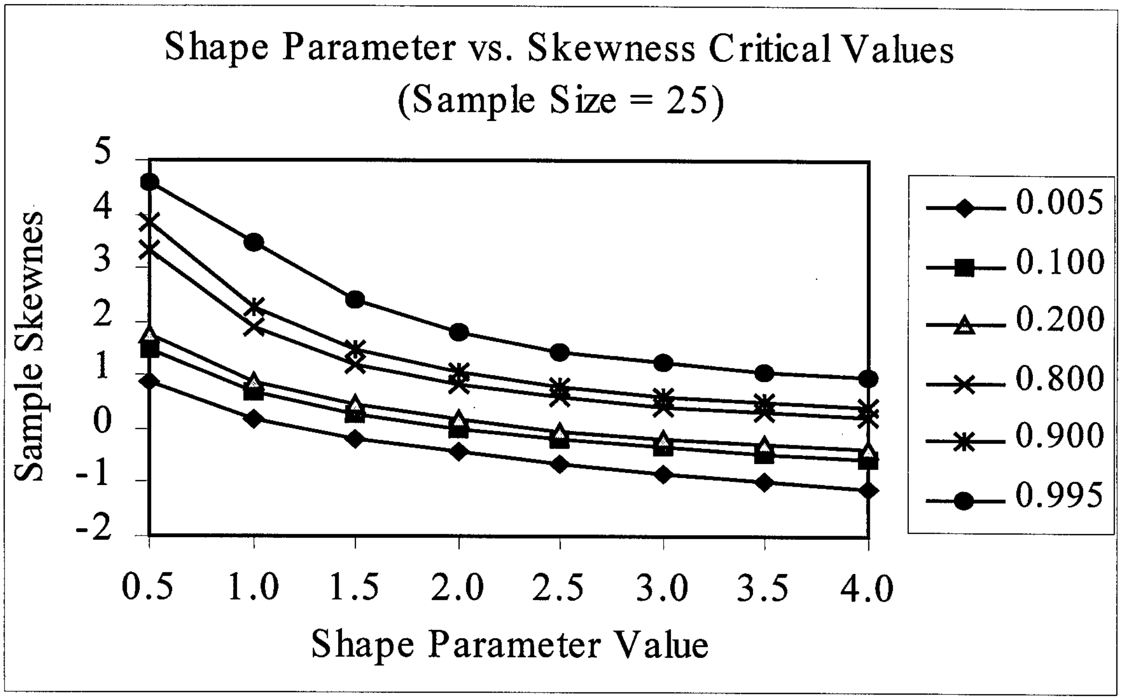


Figure M.4 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 20.



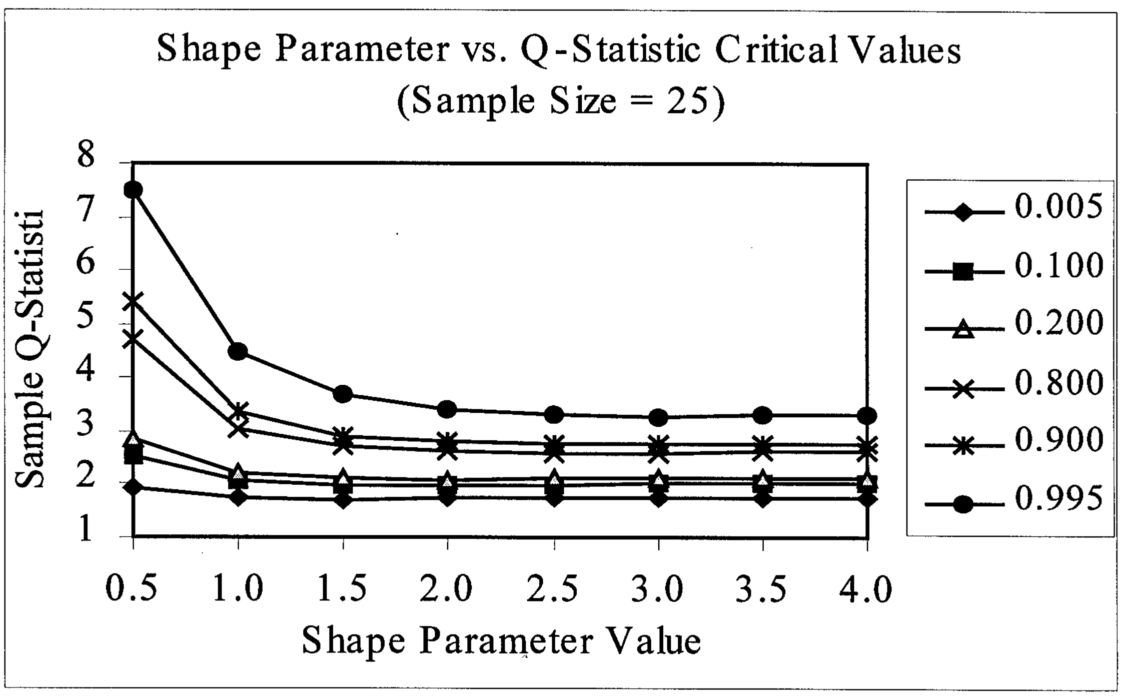
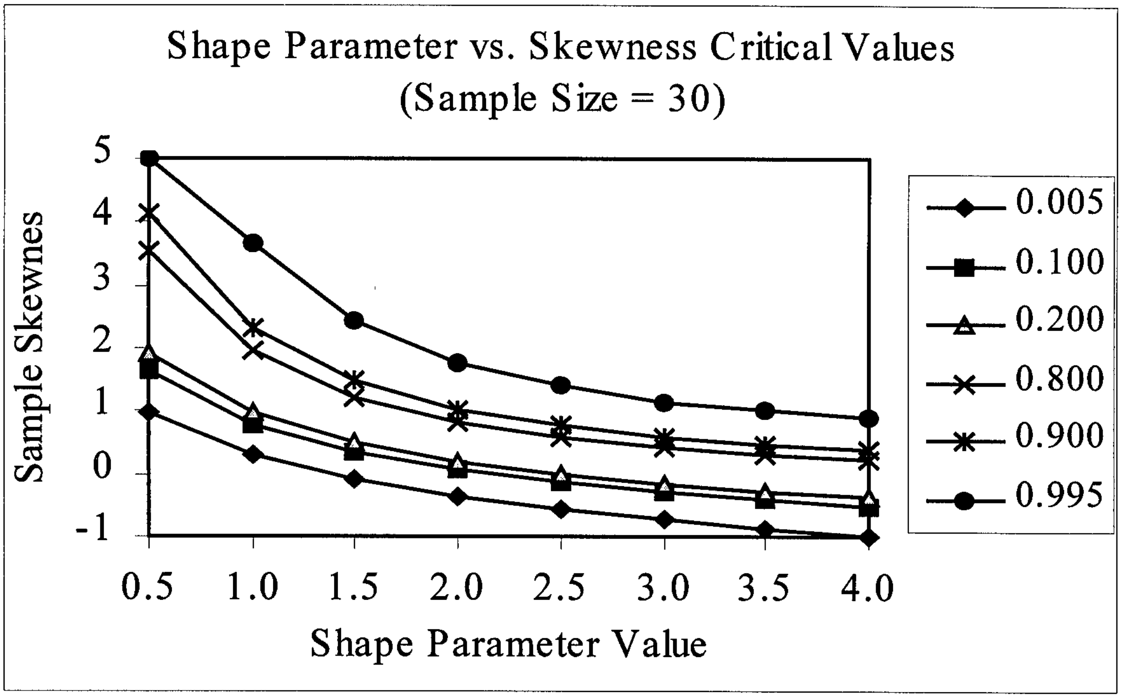


Figure M.5 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 25.



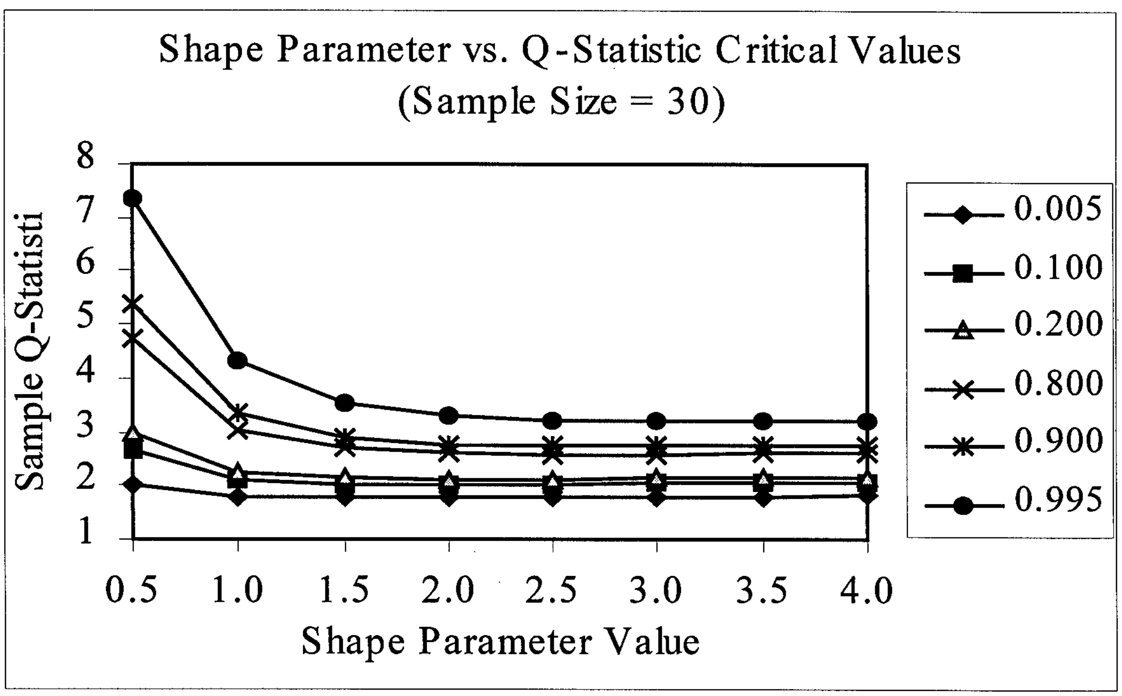
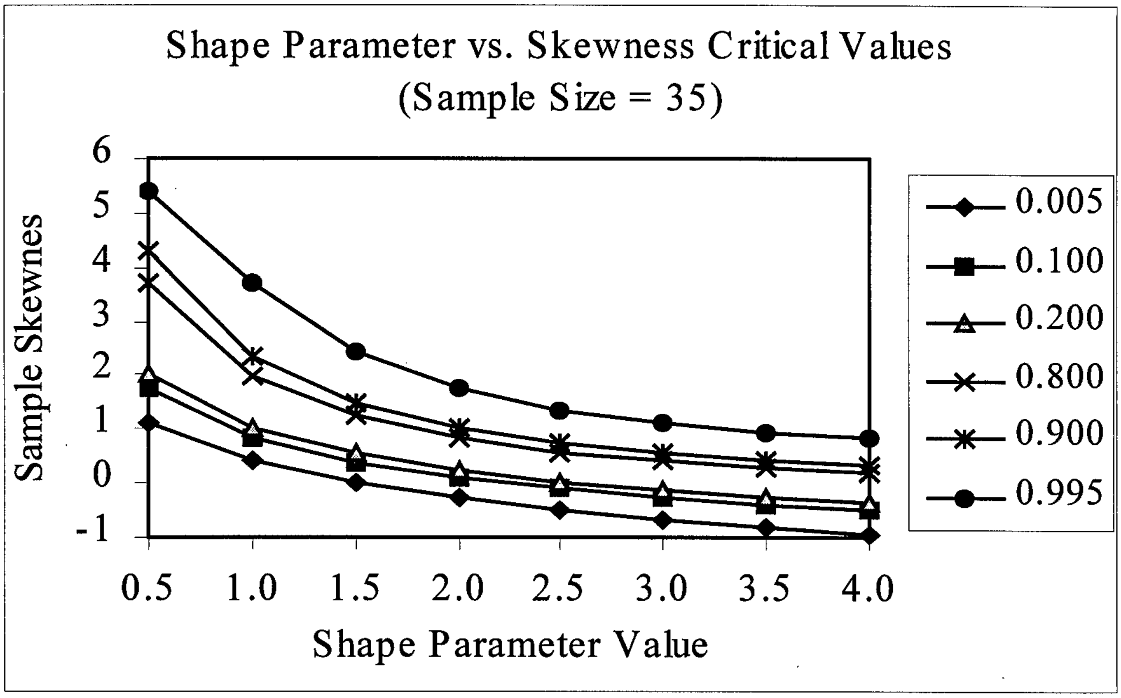


Figure M.6 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 30.



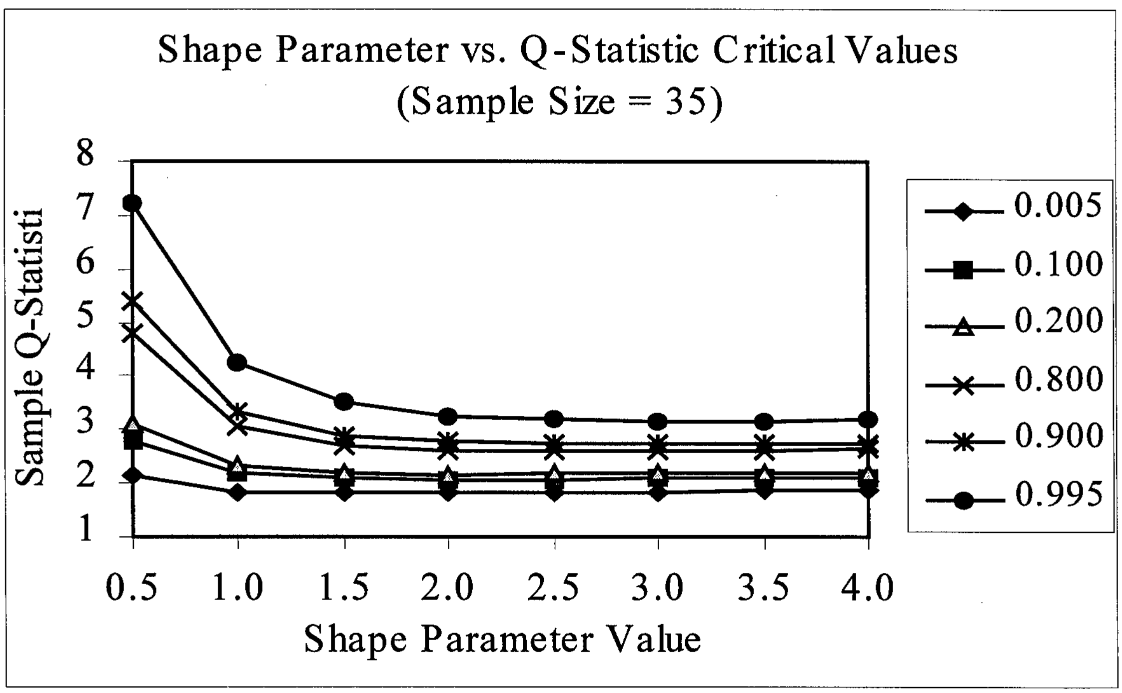
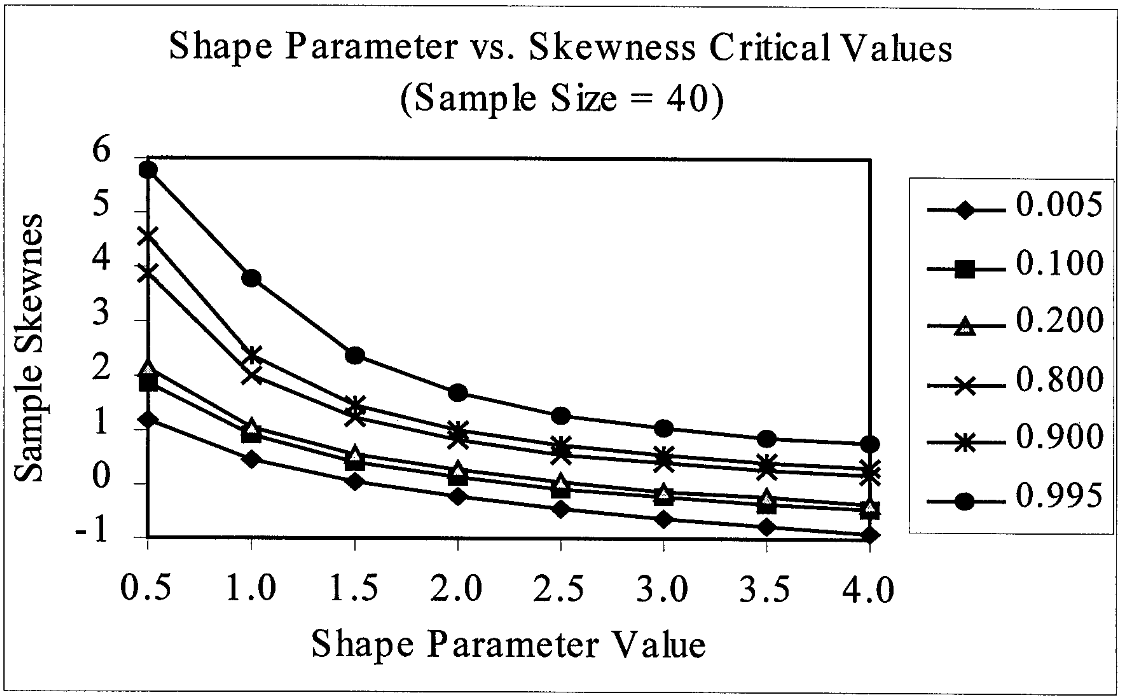


Figure M.7 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull($\beta = 0.5(0.5)4.0$); $n = 35$.



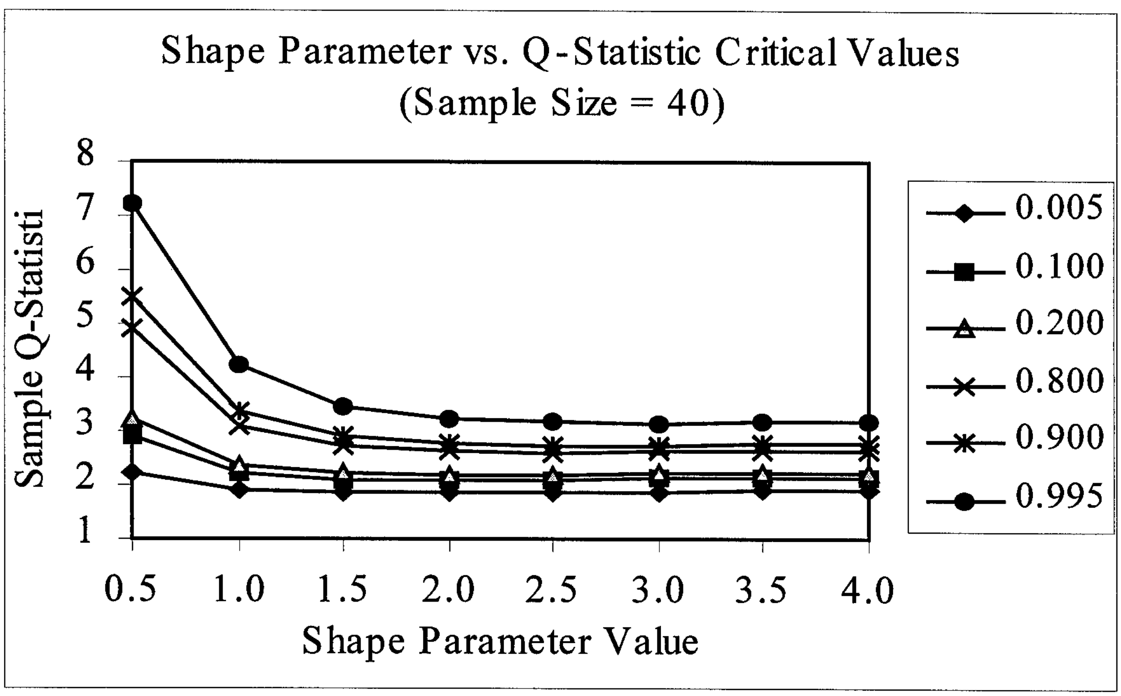
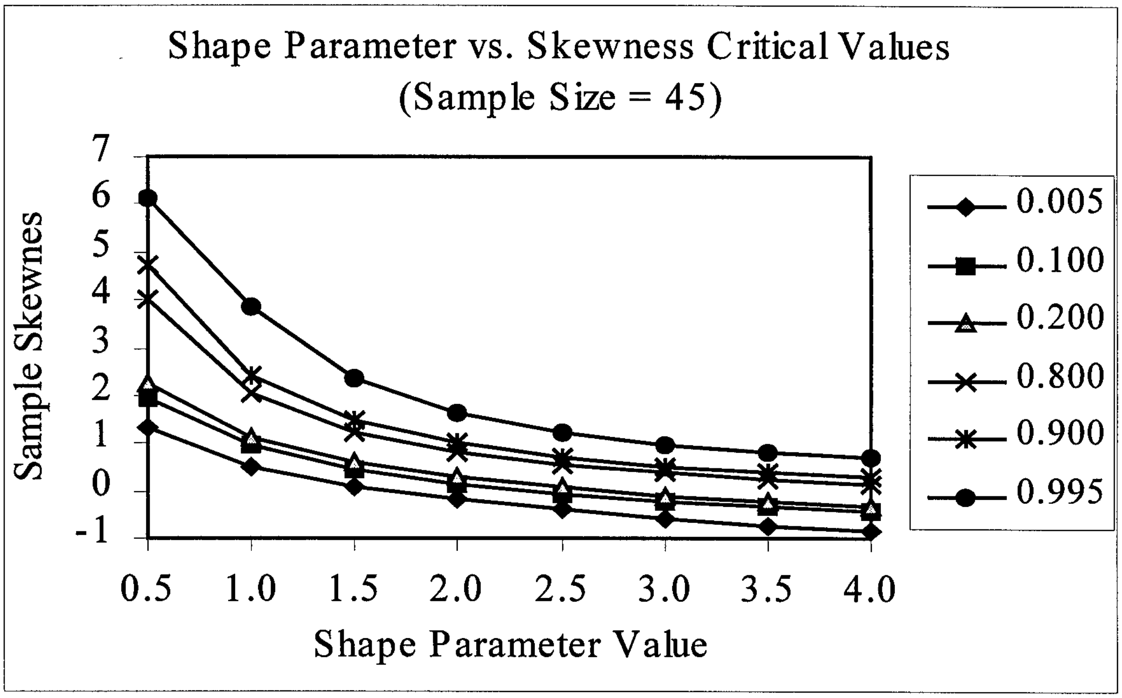


Figure M.8 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull($\beta = 0.5(0.5)4.0$); $n = 40$.



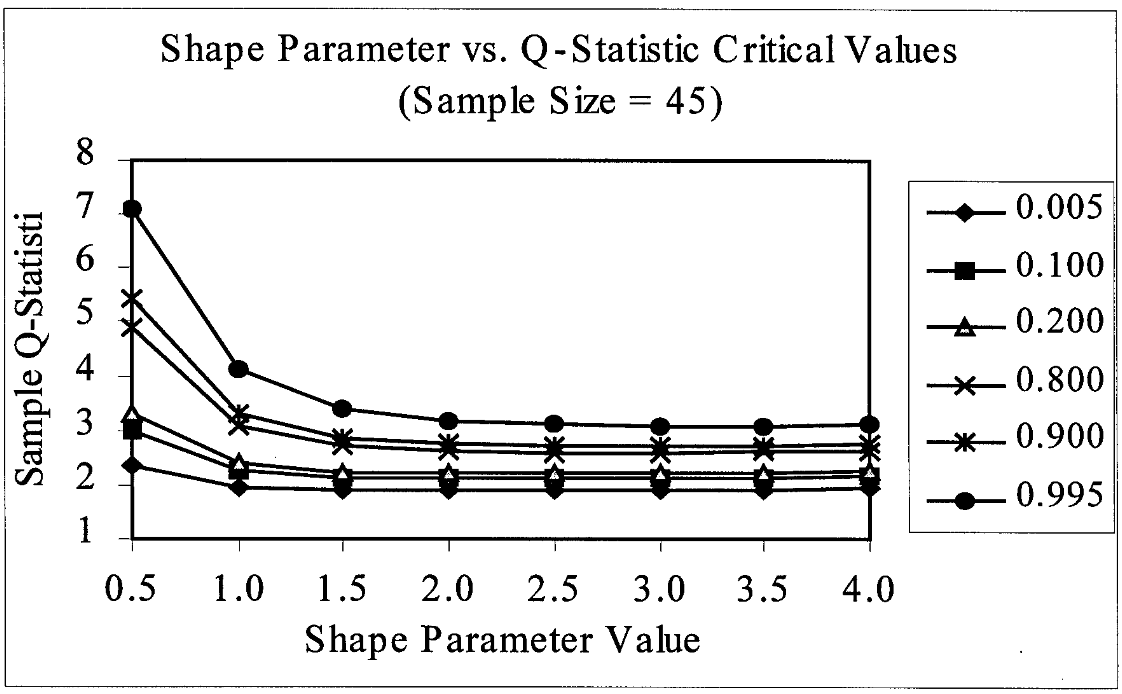
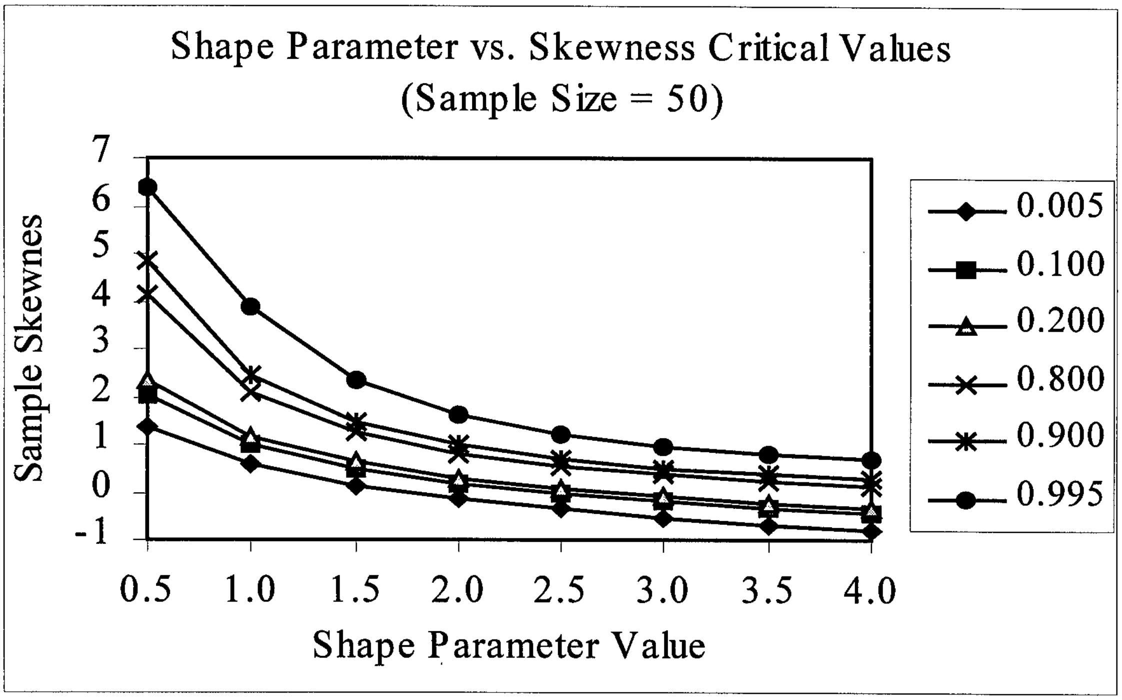


Figure M.9 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 45.



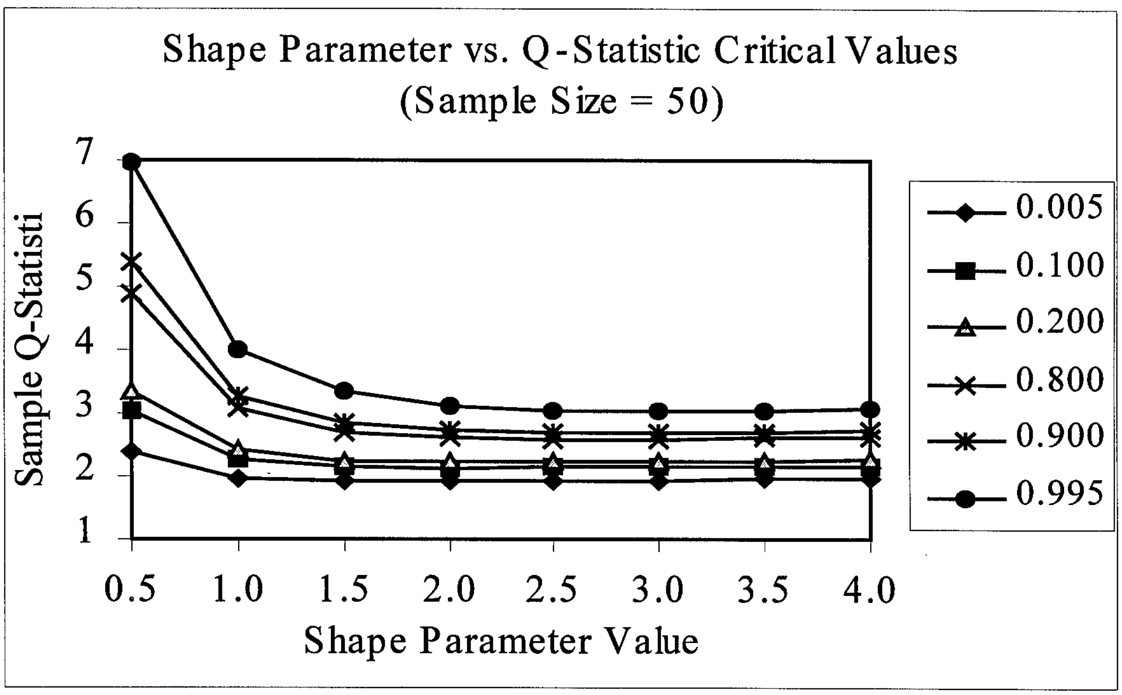


Figure M.10 Upper and Lower Tail Critical Values for $\sqrt{b_1}$ and Q-Statistic versus H_o : Weibull(β = 0.5(0.5)4.0); n = 50.

VITA

1ST LT. Tibet MEMIS was born on 01 July 1970 in Izmir, TURKIYE. Having finished his elementary and secondary education in beautiful harbor city Izmir, he attended Maltepe Military High School that is located in Guzelbahce, Izmir, TURKIYE from where he graduated in 1988. Upon graduation, he attended Turkish Air Force Academy (TUAFA) where he obtained a Bachelor of Science degree in Aeronautical Engineering on 30 August 1992. He was commissioned as a second lieutenant and started his undergraduate pilot training (UPT) at Cigli AFB, Izmir, TURKIYE. Having finished his elementary flight training in T-34 and flown several sorties in T-37, he got selected to go to Columbus AFB, Mississippi to complete his UPT. As a part of his flight training, he spent 8 months at Lackland AFB, Texas for the advanced English education. After finishing his education at Defense Language School on 01 AUGUST 1993, he attended the UPT training at Columbus AFB, Mississippi as a part of the class 94-13. Upon graduation, he was assigned to the combat readiness training in NF-5 at Konya AFB, Konya, TURKIYE. He, then, was assigned to Akinci AFB, Ankara, TURKIYE for his F-16 combat readiness training. Having gradudated from F-16 training, he was assigned to 192nd Squadron at Balikesir AFB, Balikesir, TURKIYE as an air defense wingman pilot in 1995. Then, he entered the Graduate Operations Research Program, School of Engineering, Air Force Institute of Technology (AFIT) in MAY, 1997.

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Sirinyer, IZMIR TURKIYE