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13. ABSTRACT (Maximum 200 words) The design, development, and tests of a high frequency thermoacoustic refrigerator are presented. It was developed to operate at 5000 Hz with a piezoelectric driver and air at 1 atmosphere. The stack is unusual in that it is made of random fibers which can provide an enormous surface area for interaction, with the sound field. At each end of the stack heat was transferred using copper heat exchangers. A temperature difference of 50° C was produced for sound levels of 160 dB at the stack. The cooling power of this device was in the range of 0.4 to 1 watt. High frequency operation leads to a large critical temperature gradient across the stack of 125° C/cm, which opens up the possibility of producing very low temperatures in suitable geometries. The impressive performance of our device demonstrates its potential for refrigeration of small objects, such as high speed electronic components and biological samples. The device is compact (3.5 cm long), simple, and it can be interfaced to microelectronics for heat removal applications.			
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**FINAL REPORT**

**HIGH FREQUENCY THERMOACOUSTIC REFRIGERATOR**

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**March 12, 1999**

### Abstract

This report presents the design, development, and tests of a high frequency thermoacoustic refrigerator. It was developed to operate at 5000 Hz with a piezoelectric driver and air at 1 atmosphere. The device is compact even though it was operated with a cavity tuned to two half-waves of the drive frequency. The stack is unusual in that it is made up of random fibers which can provide an enormous surface area for interaction with the sound field. At each end of the stack heat was transferred to copper heat exchangers. Substantial cooling was observed with this refrigerator. Temperature difference of 50° C were produced for sound levels of 160 dB at the stack. The cooling power of this device was between 0.4 and 1.0 watt. Such limits are attributed to a thermal boundary resistance at the stack-heat exchanger interfaces and to the short stack. Experiments showed that limitations in the stack length could be overcome by using multiple stacks; this approach could be extended to many stages of refrigeration. High frequency operation leads to a large critical temperature gradient 125° C/cm in our case, which opens up the possibility of producing very low temperatures in a suitable geometry. The compactness of our device is well-suited for high pressure operation and for rapid cool-down operation. The impressive performance of our device demonstrates its use to a variety of refrigeration applications ranging from the cooling of biological samples to cooling of components in high speed electronics. Further developments of this device would be the use of gas mixtures at high pressure which it can take mechanically due to its small size.

## High Frequency Thermoacoustic Refrigerator

### I. INTRODUCTION

Since the discovery by Merkli and Thomann [1] that cooling can be produced by the thermoacoustic effect in a resonance tube, research has concentrated on developing the effect for practical applications. One approach was to increase the audio pumping rate. Whilst the experiments of Merkli and Thomann used frequencies of around 100 Hz, Wheatley and collaborators [2] successfully raised the operating frequency to around 500 Hz and achieved impressive cooling rates in their refrigerator. This triggered a wide range of very promising thermoacoustic refrigerators. Here results will be presented on a refrigerator developed for operation around 5,000 Hz. The report brings out the design, the fabrication and tests on its performance.

Some of the reasons for using high frequency pumping in a thermoacoustic refrigerator are:

- the device will be compact as its dimensions scale with the wavelength of the audio drive.
- quick response and fast equilibration rate for small devices.
- it is a convenient frequency range for piezoelectric drivers, which are light, efficient, and cheap.
- some parts, such as heat exchangers, can be fabricated using photolithography and their film technology.
- power density of device can be raised.
- many applications require small compact refrigerators.

An important element in the operation of a thermoacoustic refrigerator is the special thermal interaction of the sound field with a plate or a series of plates known as the stack. It is a weak thermal interaction characterized by a time constant given by  $\omega\tau \approx 1$  where  $\omega$  is the audio pump frequency and  $\tau$  is the thermal relaxation time for a thin layer of gas to interact thermally with a plate or stack. The amount of gas interacting with the stack is determined by a thermal penetration depth  $\delta_k$  given by:

$$\delta_k = (2\kappa/\omega)^{1/2} \quad (1)$$

By increasing  $\omega$  the condition of weakly coupling is met by a reduction of  $\delta_k$  and hence of  $\tau$ . Here  $\kappa$  is the thermal diffusivity. The work of pumping heat up a temperature gradient as in a refrigerator is performed by essentially the gas within the penetration depth. The amount of this gas will have an important dependence on the frequency of the audio drive.

It may seem that conduction of heat is a slow process and hence that this would limit the operation of a high frequency refrigerator. Such objection can be set aside, at least for some of the frequencies, because heat conduction is not a slow process when small distances and masses are in question [3, 4]. This fact will be applied to the high frequency refrigerator presented here.

### II. GOALS

In this project the main goals were:

- to develop a high frequency thermoacoustic refrigerator. The high frequency refers to a pumping rate of 5 kHz which was chosen arbitrarily. It is about a factor of 10 above the frequencies used by Wheatley and collaborators; it falls within a frequency

range where relatively cheap hi-fi equipment can be used, and it falls within a frequency range where piezoelectric drivers are efficient.

- to develop a device which can be adapted for miniaturization.
- to study the performance of a small refrigerator and the features peculiar to its compactness.
- to develop a simple, compact, and cheap device which can be used for cooling small electronic components and small biological systems.
- to explore the field of thermoacoustics at high frequencies with emphasis on new concepts and new materials.

### III APPROACH

Because the frequency range for our refrigerator is an order of magnitude higher than in conventional thermoacoustic refrigerators, the approach and some of the components are different from the 500 Hz range refrigerators. In fact, in some of the features, high frequency operation leads to a simplification.

As a starting point the refrigerator was developed with the following main features:

- working fluid: air at 1 atmosphere.
- driver: piezoelectric device.
- resonator: cylindrical.
- stack: random fibers
- heat exchangers: copper

This approach was chosen for its simplicity realizing that it is far from ideal. It is a first approximation. As the development progressed changes were made to improve the performance. An analysis will now be presented of each component so as to show the advantages of our approach as well as some of the limitations. Fig. 1 shows the basic set-up.

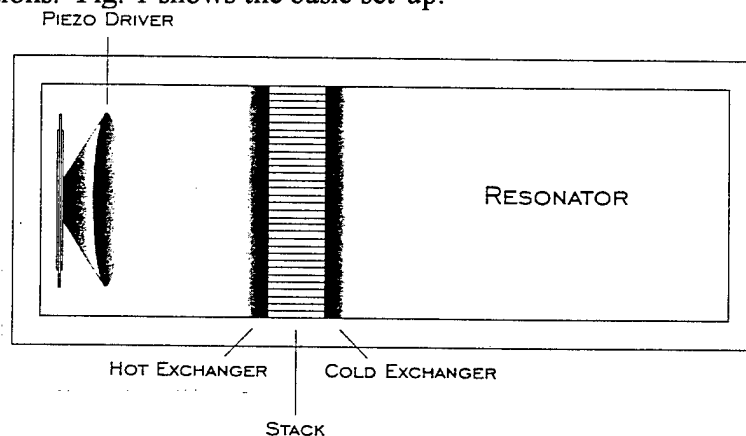


Fig. 1 Basic high-frequency refrigerator.

#### A. ACOUSTIC DRIVER

We selected a piezoelectric driver of the bimorph type, an example of one being the Motorola KSN 1046, horn-loaded for better impedance matching. This model was chosen mainly because of its high sensitivity and broad frequency response. Some of its characteristics are:

- mass: 1.3 g

- sensitivity: ~ 95 dB/watt/m but it can vary by a few decibels, depending on the unit.
- frequency response: 4-27 kHz.

Fig. 2 shows the frequency response as provided by the manufacturer: there are large variations with each unit. The horn cone has a maximum diameter of 4 cm. This driver efficiency can be as high as 70-90%, depending on the load.

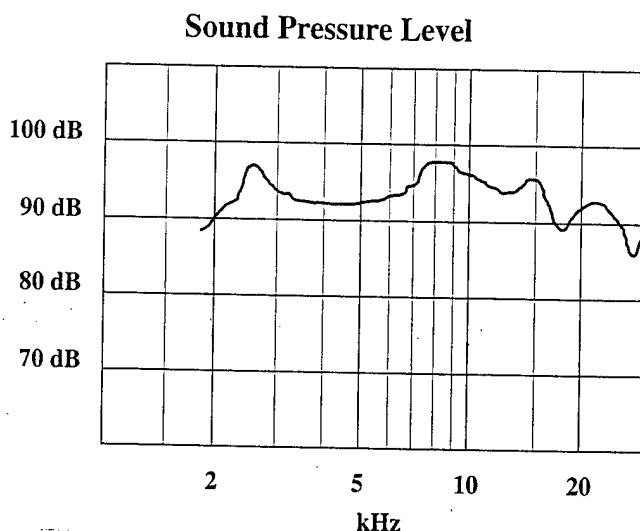


Fig. 2. Typical frequency response of bimorph driver.

This type of driver has ideal characteristics for use in a high frequency refrigerator. Dissipation power losses are very small since a piezoelectric is a capacitor (ours has a capacitance  $C$  of 145 nano Farads) whose losses come from the hysteresis behavior of the dielectric. Compared to an electromagnetic driver whose voice coil has typically ~ 8 ohms resistance, its series equivalent resistance representing losses is:

$$R_e = (6.28 f C \tan \delta)^{-1}$$

being of order 1-10 kilo-ohms, depending on the material. Here  $\tan \delta$  is the dissipation factor. Since dissipation power is  $V^2/R_e$  it is much smaller for the piezo driver than for the regular electromagnetic driver. The piezoelectric driver is a voltage device while an electromagnetic driver is a current device.

In a bimorph, two piezoelectric discs are bonded together on each side of a brass shim; they change lengths in opposite direction with applied voltage and this causes a large bending action which is coupled to a cone diaphragm. This device behaves like a bimetallic strip which flexes upon heating. Fig. 3 shows a bimorph driver.

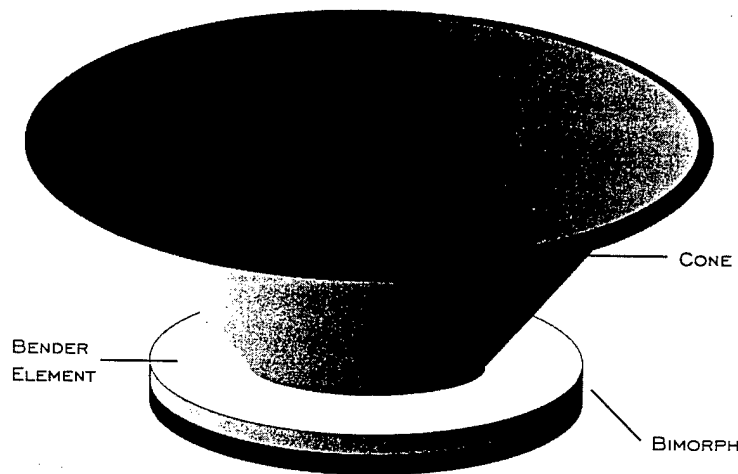


Fig. 3. Bimorph Driver.

Being very light, this device is attractive for the present applications. One of its limitations is the maximum electrical power input that it can handle, being  $\sim 5$  watts before causing mechanical damage to the bimorph.

#### B. STACK

At first a conventional stack was tried; it consisted of a narrow strip of Mylar sheet to which nylon fishing line spacers were glued and then it was rolled into a cylindrical shape, as developed and described by T. Hofler [5]. This was unsuccessful here because for a stack length  $\Delta x$  of 0.5 cm it was hard to maintain uniform spacing but also it was difficult to make good thermal contact to a heat exchanger at each end of the stack. This was abandoned for a novel approach, which did not have the above problems.

We chose a random element stack using cotton wool. This is shown in Fig. 4. It was pressed so as to have a length  $\Delta x = 0.5$  cm. Some of characteristics of this stack are:

- density of cotton wool:  $0.08 \text{ g/cm}^3$
- thermal conductivity of each fiber:  $0.04 \text{ W/m } ^\circ\text{C}$
- average diameter of each fiber:  $12 \text{ }\mu\text{m}$
- number of fibers in stack, 3 cm in diameter:  $4 \times 10^6$
- effective total perimeter  $\Pi$  of fibers  $\sim 151 \text{ m}$
- effective cross-sectional area for heat pumping,  $\Pi\delta_k$ :  $7.5 \times 10^{-2} \text{ m}^2$
- total active area of stack exposed to sound field:  $7.5 \times 10^3 \text{ cm}^2$ .

The above quantities are maximum values, as in some cases, randomness of the fibers was not taken into account.

PLASTIC  
PLATES



PLATES

COTTON  
WOOL



5 MM THICK

Fig. 4. Random Fiber Stack.

C. HEAT EXCHANGERS

Two types of heat exchangers were used, both were made of copper.

- a photolithographically prepared heat exchanger with square holes,  $0.5 \text{ mm} \times 0.5 \text{ mm}$  and solid to solid spacers,  $0.8 \text{ mm} \times 0.8 \text{ mm}$ . This exchanger provided a sound transparency of 25%. Its diameter was 3.4 cm and it was fabricated from copper sheet 0.3 mm thick. Fig. 5 shows the heat exchanger. The cold and the hot heat exchangers had the same geometry.

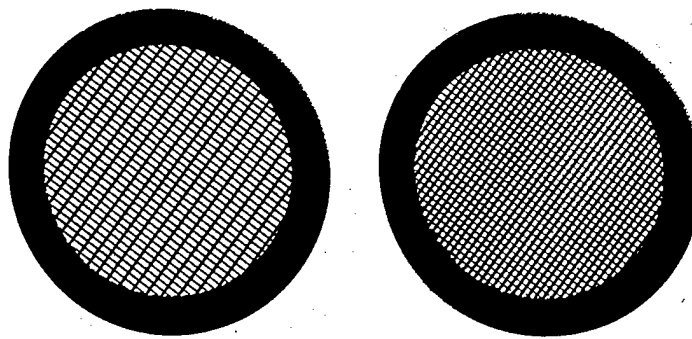


Fig. 5. Photolithographically prepared copper heat exchanger.

- a copper screen, flattened by a press, with square holes  $0.8 \text{ mm} \times 0.8 \text{ mm}$  and 1.2 mm for adjacent wire to wire distance. Its diameter was 3.4 cm and it had a thickness of 0.5 mm. Sound transparency was 44%. To improve heat transfer at the hot heat exchanger (since it handles more heat than the cold one) it was thermally anchored to a large, 0.5 cm thick, copper heat exchanger.



Although thin, the heat exchangers were developed to maintain heat flows of ~2 watts without creating a substantial  $\Delta T$  across the heat exchanger ( $\Delta T$  is less than  $0.1^\circ\text{C}$ ). The mass of each heat exchanger was 1.2 g. Mechanical contact was made between the heat exchangers and the cotton wool stack and this provided the thermal contact.

#### D. WORKING FLUID

As mentioned earlier, we started with air at 1 atmosphere as the working fluid, realizing all along that this is not the optimal working fluid. Important characteristics of air are:

- thermal conductivity:  $0.26 \text{ mW/cm}^\circ\text{C}$
- density at 1 atmosphere;  $20^\circ\text{C}$ :  $0.00121 \text{ g/cm}^3$
- viscosity at  $20^\circ\text{C}$ :  $18.1 \text{ } \mu\text{poise}$
- speed of sound at  $20^\circ\text{C}$ :  $344 \text{ m/sec}$
- thermal penetration depth at  $5 \text{ kHz}$ :  $0.05 \text{ mm}$
- viscous penetration depth at  $5 \text{ kHz}$ :  $0.035 \text{ mm}$
- Prandtl number:  $0.707$

Better performance is expected in a mixture of Argon and Helium.

For the specific mixture  $\text{Ar}_{0.36}\text{He}_{0.64}$ , some of the characteristics which show more promise for the refrigerator are:

- thermal conductivity:  $0.09 \text{ W/m/K}$
- Prandtl number:  $0.351$
- speed of sound at  $20^\circ\text{C}$ :  $497 \text{ m/s}$

#### E. RESONATOR

A simple geometry was chosen for the resonator, that of a cylinder closed at both ends, with a drive at one end. Such tube resonator was used in 2 different ways, after modifications:

- (1) as a  $1/2$  wave resonator tuned to  $5000 \text{ Hz}$ .
- (2) as a double half-wave resonator tuned to  $5000 \text{ Hz}$  (i.e. the half-wave part was tuned to  $5000 \text{ Hz}$  and the resonator contained one full wave). This offers interesting advantages (one being a higher  $Q$ ) which will be discussed in the next sections.

Apart from the stack and heat exchangers, acoustic dissipation in the resonator is due mainly to viscosity and heat conduction losses on the surfaces of the resonator. In addition nonlinear dissipation caused by circulation effects and turbulence will come in at high sound intensities. Viscosity dissipation can be represented by a surface resistance  $R_s$  which is given by:

$$R_s = \frac{1}{2}(2\mu\rho\omega)^{\frac{1}{2}} \quad (3)$$

which at room temperature for air becomes  $R_s \approx 0.008 f^{\frac{1}{2}}$ . Here  $\mu$  is the coefficient of friction. Heat conduction losses, also for air at 1 atmosphere and room temperature can be written as:

$$H_s \approx (10^{-6}/\rho c)f^{\frac{1}{2}} \quad (4)$$

Such losses will have an immediate impact on the quality factor of the resonator [6] defined as:

$$Q = \omega_0 m / \rho c \theta \quad (5)$$

#### IV ACHIEVEMENTS

This section consists of 3 parts: experiments and tests, analysis of performance, and discussion of potential of approach.

##### A. EXPERIMENTS AND TESTS

The high-frequency refrigerator was developed to test the concepts of thermoacoustics applied here, to use the results as guideline for further improvements, and as a test unit for further developments in high frequency refrigeration. Basic tests of our device consisted of tuning the driver, adjusting the sound intensity level, and following the cooling with a thermocouple across the stack. Although the piezoelectric bimorph drivers are broad-band in frequency, a certain amount of tuning was necessary to take advantage of the small resonances with their increased sound intensity level. The operating frequency was between 4 and 5 kHz; the corresponding wavelength in air at 1 atmosphere ranged from 8 to 6.8 cm.

Hence a  $\frac{1}{2}$ -wave resonator at 5,000 Hz would be  $\sim 3.4$  cm long. This type of resonator provides the opportunity to make a compact refrigerator. We have concentrated our efforts mainly on the second type of resonator, the double half-wave resonator. It is tuned to  $\sim 5000$  Hz but it is twice as long as the  $\frac{1}{2}$  wave resonator since it contains 2 half-waves of the same wavelength as the  $\frac{1}{2}$  wave resonator. This is shown in Fig. 6 with the stack and heat exchangers positioned at the appropriate position with respect to the pressure standing wave in the resonator.

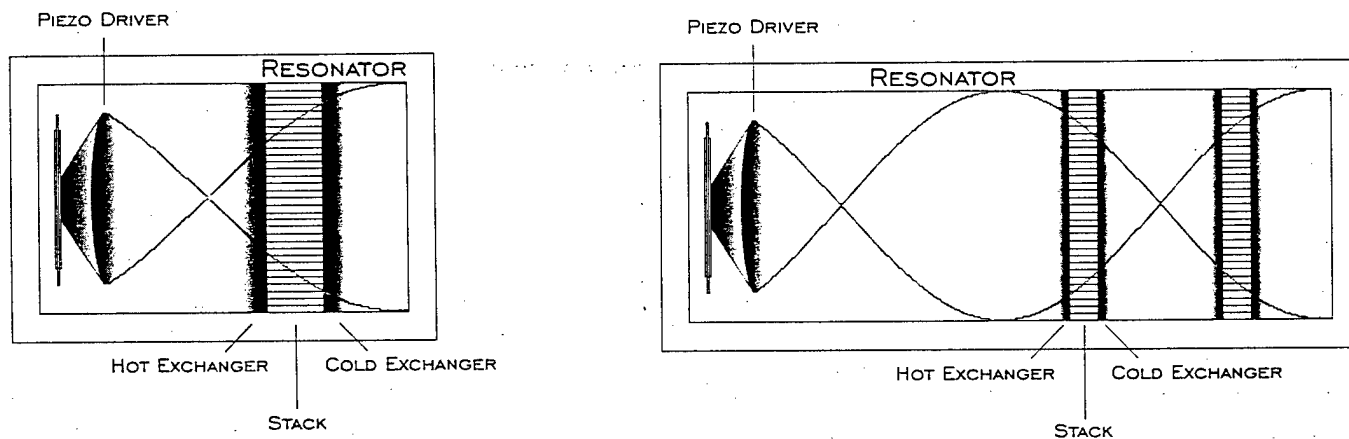


Fig. 6. Half-wave and double half-wave resonators.

The geometry of the double  $\frac{1}{2}$  wave resonator provides the option of having 2 or more stacks which could be eventually connected in tandem. Tests on such arrangement will be discussed later.

##### (i) Tests with $\frac{1}{2}$ Wave Resonator.

As the temperature difference  $\Delta T$  across the stack is a function of the position

of the stack in the acoustic standing wave, a positioning device was incorporated in the device. This is shown in Fig. 7. It allowed us to study  $\Delta T$  across the stack

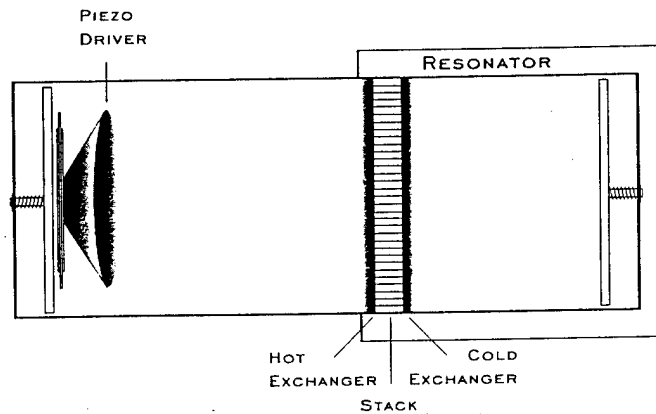


Fig. 7. Device for positioning the stack and heat exchangers.

as a function of its position. At some point, the temperature change due to the pressure change of the sound field is balanced out by the fluid displacement in a temperature gradient and this leads to a critical temperature gradient  $\Delta T_{crit}$ . It is defined as [7]:

$$\Delta T_{crit.} = \frac{\gamma - 1}{T_m \beta} \frac{T_m}{\lambda} \tan(x/\lambda) \quad (6)$$

where  $\gamma$  is the ratio of isobaric to isochoric specific heats,  $T_m$  is the mean temperature of the fluid,  $\lambda$  is the radian length,  $\beta$  is the thermal expansion coefficient, and  $x$  is the stack position relative to the pressure antinode. We have investigated the importance of the position of the stack relative to the acoustic standing wave; this is shown in Fig. 8, with the spatial dependence normalized to the sound radian length.

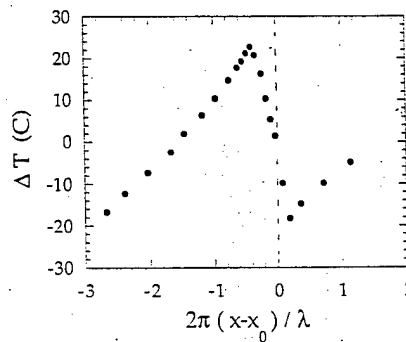


Fig. 8. Temperature difference across stack as a function of stack position in standing wave.

The results show how critical the positioning of the stack is and how the direction of the pressure gradient in the acoustic standing wave determines the sign and magnitude of  $\Delta T$ . In this case the stack length  $\Delta x$  was 0.5 cm.

Having established the position of maximum  $\Delta T$  the stack was fixed at that position so that the cooling could be studied as a function of sound power and time. Using calibrated miniature electret microphones in the body of the refrigerator the sound level was determined at the position of the stack. A typical sound intensity was 156 dB and this corresponds to  $0.4 \text{ W/cm}^2$ . For a 3 cm diameter stack this gives an input acoustic power level of  $\sim 2.5$  watts. At maximum power from the driver, the temperature difference  $\Delta T$  between the hot and the cold end of the stack reached typically  $39^\circ\text{C}$ . This would not last very long as the driver invariably cracked due to the large stress on the bimorph at such high intensity levels. Fig. 9 shows a typical cooling curve. Temperature differences were measured by a copper-constantan thermocouple.

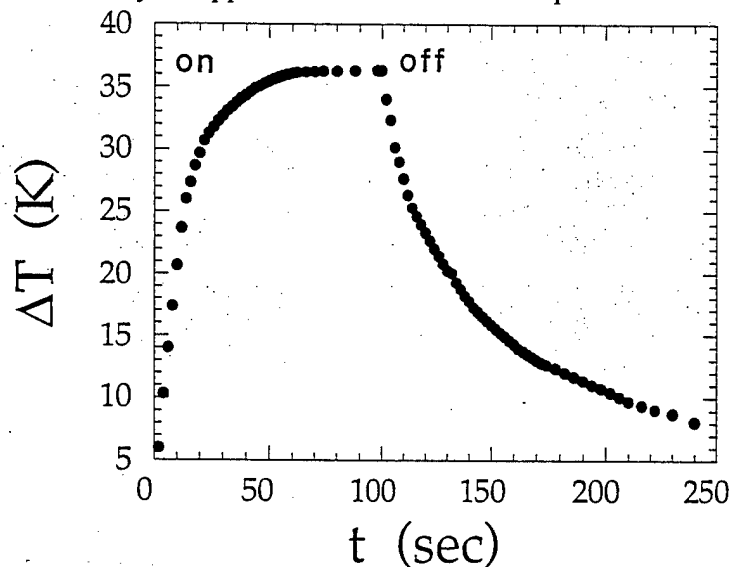


Fig. 9. Temperature difference across stack in the presence of intense sound at  $\sim 5000$  Hz.

When the sound power switched off, the temperature difference  $\Delta T$  across the stack decayed to zero with a characteristic time constant. The magnitude of the  $\Delta T$  achieved in each case depended on the quality of the driver and how well it was tuned for highest sound intensity. In a batch of drivers, the sound level sensitivity could vary by up to 10 dB.

(ii) Tests with double  $\frac{1}{2}$  wave resonator and 2 stacks.

Here we incorporated 2 stack-heat-exchanger units at appropriate positions in the double  $\frac{1}{2}$  wave resonator refrigerator. A thermocouple monitored the  $\Delta T$  across each stack-heat-exchanger unit as the sound level was raised and then maintained at the maximum level. At 160 dB sound level, one stack produced a  $\Delta T$  of  $25^\circ\text{C}$  while the other one produced  $35^\circ\text{C}$  at the same time. The difference is probably due to inaccurate positioning of one of the stack-exchanger units in the resonator. It is important to note that each unit was thermally isolated from the neighboring one. The results are interesting because the two units could be attached thermally in tandem for further overall temperature reduction.

(iii) Tests with the Double  $\frac{1}{2}$  Wave Resonator and 1 Stack

In this series of test only 1 stack-heat-exchanger unit was used. When tuned as to stack position and maximum sound power level, a  $\Delta T$  of  $50^\circ\text{C}$  could be reached, typically. The stack was located just before the last pressure antinode away from the driver. A better performance was achieved due to the higher  $Q$  of the resonator (compared to the single  $\frac{1}{2}$  wave resonator) as a substantially larger  $\Delta T$  was achieved. As mentioned before, the limiting factor in the achieved  $\Delta T$  was the performance of the driver. We used the drivers at their limit.

(iv) Tests of Double  $\frac{1}{2}$  wave resonator and 1 stack and a Load.

With the encouraging results obtained above, the thermoacoustic device that we have studied was incorporated into a refrigerator by anchoring the hot heat exchanger to room temperature and letting the cold heat exchanger drop in temperature. The standard heat exchanger at the hot side was thermally anchored to a large ribbed copper heat exchanger which was held at room temperature. The portion of the refrigerator from the hot exchanger down was surrounded by a vacuum jacket for thermal isolation. This allowed the cold heat exchanger to cool the portion of the resonator in contact with it ( $20\text{ cm}^3$  volume); however a  $\Delta T$  of  $50^\circ\text{C}$  could not be achieved. Instead a  $\Delta T$  of  $25^\circ\text{C}$  was reached and this deteriorated with time. A thermal bottleneck was encountered at the stack-hot heat exchanger interface and the temperature of the stack at the hot end rose above room temperature. Low heat transfer at interfaces limited the performance of our refrigerator.

(v) Refrigerator using gas mixtures and high pressure operation.

The experiments and tests with air as the working fluid provided us with basic information on a high frequency thermoacoustic refrigerator and how it could be improved. One direction still not addressed in this project was the optimal choice of working fluid. Since a limiting factor is the viscous boundary layer characterized by a viscous penetration depth  $\delta_v$ , it is important to choose a fluid where this depth is less than the thermal penetration  $\delta_k$ , i.e. a fluid with a low Prandtl number. Hence we chose a mixture of 64% He and 36% Ar whose Prandtl number is 0.3507 and where the speed of sound is 497 m/sec [8]. Compared to air this required a scaling factor of 1.4 in size to keep the resonance at the same frequency as for air. Tests were made with this mixture at 1 atmosphere. Preliminary results were encouraging as a  $\Delta T$  of  $30^\circ\text{C}$  across the stack was easily achieved for a stack of fixed position (and not necessarily located at optimal position).

All our tests were performed with working fluids at 1 atmosphere. Improved performance is expected when the fluid is at higher pressures because of scaling similitude principles [9] and because this provides superior impedance matching between the driver and the fluid. In fact, we are in an excellent situation for working at high pressure, since a small refrigerator is structurally strong enough to withstand very high pressures. Such approach remained in the planning stages.

## B. ANALYSIS OF PERFORMANCE

### (i) Maximum $\Delta T$

The maximum temperature difference that can be produced across a stack results from a competition between the temperature change due to an adiabatic pressure change of the working fluid and its displacement along the stack which has a temperature gradient. When the temperature rise due to an adiabatic compression is greater than the temperature rise due to the displacement along a temperature gradient of the stack, the engine works as a heat jump. Conversely, the engine works as a prime mover. The critical gradient  $\Delta T_{crit}$  given by Eq. 6 separates the 2 regimes. For our case  $\Delta T_{crit}=120^{\circ}\text{C}/\text{cm}$  and this determines the maximum  $\Delta T$  which can be achieved. For a stack of length  $\Delta x=0.5$ , cm as in our case, the maximum  $\Delta T$  which can be reached is  $60^{\circ}\text{C}$ . This is a fundamental limitation which can be overcome in 2 simple ways:

- use 2 stacks and corresponding heat exchangers inside a double  $\frac{1}{2}$  wave resonator and add serially the  $\Delta T$  of each. We have shown that this approach works. It has potential for future developments of thermoacoustic refrigeration.
- increase the stack length  $\Delta x$  by using a fluid where the speed of sound is higher than in air. With air at 1 atmosphere we were near the limit as far as  $\Delta T$  max is concerned.

### (ii) Heat transfer at stack-heat exchanger interface.

The gradual transport of heat along the stack during refrigeration operation ends when the symmetry is broken at each end and hence a heat exchanger is needed at each end to dispose of the heat or absorb it. At the cold end the interface has to transfer heat  $Q_c$  while at the hot end the heat transferred there is  $Q_c + W$ , where  $W$  is the work done on the system by sound. Since at the interface of stack-heat exchanger heat is transferred by thermal contact of the cotton wool fibers to the heat exchangers, the contact thermal resistance will limit the flow of heat. A contact thermal resistance  $R_{co}$  can be defined as:

$$R_{CO} = 1/h_{CO} A_e \quad (7)$$

$$\text{where } h_{CO} = 1.25 k_s (m/\sigma) (P/H) \quad (8)$$

with  $k_s$  being a harmonic mean thermal conductivity for the 2 solids in contact,  $\sigma$  is a measure of surface roughness of the 2 solids,  $m$  is related to angles of contact,  $P$  is the contact pressure and  $H$  is the microhardness of the softer solid. For a transistor casing and a nylon washer this resistance is  $2^{\circ}\text{C}/\text{W}$  while for transistor in contact with air it is  $5^{\circ}\text{C}/\text{W}$ . For our cotton wool to heat exchanger interface, the thermal resistance is estimated to be  $R_{CO} = 3.5\text{-}7^{\circ}\text{C}/\text{W}$ . For a total heat flow of 2 watts the interfaces can easily develop a  $\Delta T$  of  $7\text{-}15^{\circ}\text{C}$ . Moreover closer examination of our random stack showed that it was formed from 3 layers of cotton wool pressed together with the fiber alignment perpendicular to the axis of heat transport. This explains why a  $\Delta T$  of  $25^{\circ}\text{C}$  above the heat exchanger anchored to room temperature was observed at the hot end of the stack.

### (iii) Random Fiber Stack.

An important function of the stack is the storage of heat as it is being shuffled from one end of the stack to the other. This requires a large surface area; cotton wool is

exceptionally well-suited for this task. Our stack has an enormous surface area, 7450 cm<sup>2</sup>. It occupies 60% of the stack volume the rest being air. Actually such filling factor is too large and it should be reduced by a factor of ~2 to accommodate for the thermal penetration depth around each fiber.

Because our stack is so short,  $\Delta x = 0.5$  cm, thermal conduction losses across it are a limiting factor for large  $\Delta T$ . The total heat leak across air and the cotton wool fibers (thermal conductivity = 0.04 W/m/°C) in the stack is ~0.32 W for a  $\Delta T = 50^\circ\text{C}$ ; this is a significant fraction of the estimated 1.2 W cooling power of our refrigerator.

For short stacks, much as in our case, a random fiber approach provides improved performance over the fishing line spacers with mylar sheets and leads to simplicity in the construction of the stack.

#### (iv) Working Fluid.

The reason for choosing air at 1 atmosphere as working fluid was for simplicity and to demonstrate proof of concept. Since the speed of sound in air is low it determined the length  $\Delta x$  of the stack which limited the maximum  $\Delta T$  that could be achieved. A Prandtl number of 0.7 led to large viscous losses in the boundary layer next to the stack elements. However the low thermal conductivity of air, 0.26 mW/cm/°C, was useful for maintaining a large  $\Delta T$  across the stack.

The higher speed of sound in the He-Ar mixture will dictate a longer stack ( $\Delta x = 0.72$  cm) which will allow a maximum  $\Delta T$  of ~86°C to 90°C. Such improvement will have to be weighted against the heat losses by the gas across the stack, being larger since the mixture thermal conductivity is 0.6 mW/cm °C.

#### (v) Multiple Stacks.

This approach, demonstrated above, is important for at least 2 applications:

- the limiting  $\Delta T$  achieved across a stack is determined by the critical gradient and the length of the stack. By adding stacks in series thermally a larger overall  $\Delta T$  could be achieved, opening the way for very low temperature refrigeration using thermoacoustics.
- operation at high frequencies causes all the dimensions, including the stack, to be reduced. Ours is 0.5 cm for 5,000 Hz compared to the one of Wheatley et al, [2] and of Hofler [5] which is ~7 cm long. Increasing the frequency above 5,000 Hz would require even shorter stack lengths  $\Delta x$  which is impractical. However multiple stacks in series would overcome this problem, thus making it possible to go to the ultrasonic range.

#### (vi) High-frequency operation.

There are many reasons why high frequency operation of a thermoacoustic refrigerator can lead to important developments of this field. Gifford and Longworth in their classic paper [10] on Surface Heat Pumping point out that there is an optimum pulse rate for heat pumping which scales with the diffusivity of the gas and geometric factors. Swift [7] in his paper on Similitude in Thermoacoustics shows how to scale in size and frequency various systems. For example he shows how cutting the dimensions of a heat engine in half and doubling its mean pressure will leave temperature and pressure ratios unchanged, but cut heat and acoustic powers in half, and double the

pump frequency.

A more practical reason for going to high frequencies is that the coupling of a piezoelectric driver to air is more efficient at high frequencies since the acoustic impedance of air is higher; thus it provides a better match to the piezo. For audio a transformer is used in the form of 2 piezos face to face, i.e. the bimorph with parallel connection of its plates to reduce its impedance. Further improvements were achieved by cone loading the piezo. However, this cone will not be necessary when the pressure of the working fluid is raised. This brings out another advantage of high frequency operation (and smaller device): very high fluid pressures can be used before limitations of strength of materials come into effect since the surface area of our device is quite small ( $\sim 90 \text{ cm}^2$ ).

An important consideration for high frequency operation of this refrigerator is that large critical gradients  $\nabla T_{\text{crit}}$  can be attained. Since this parameter is essentially  $T_1/x_1$ , the temperature change  $T_1$  due to the acoustic pressure variation  $P_1$  and the displacement  $x_1$  in the sound wave; this leads to a large temperature change  $T_1$  with small displacement  $x_1$  since  $x_1 = u_1/\omega$  (where  $u_1$  is the particle speed in the sound field). Compression and expansion in a sound field causes a gas temperature oscillation which leads to a temperature difference between the gas and the stack. Such temperature difference causes a heat flow:

$$Q = \frac{-X}{2} \beta T_m P_1 x_1 \omega \quad (7)$$

where  $X$  relates to heat exchange. On the other hand, a temperature gradient along the stack causes a reverse heat flow from stack to gas:

$$Q = \frac{+X}{2} C_p \rho_m \nabla T_m x_1^2 \omega \quad (8)$$

where  $\rho_m$  is the density of the gas,  $C_p$  its specific heat,  $T_m$  is the ambient temperature, and  $\nabla T_m$  the temperature gradient [10]. This shows how a small  $x_1$  and large  $P_1$  lead to a large temperature difference across the stack and hence to a low minimal temperature.

High frequency operation also favors a high power density. The energy flux  $\dot{H}$  per unit volume is proportional to the pump frequency  $f$  being [7]:

$$\dot{H}/V \sim (f/2) T_m \beta P_1^2 / (1 + \epsilon_s) (\rho_m a^2) \quad (9)$$

where  $\epsilon_s$  is a parameter which compares the thermal properties of the gas ( $\rho_m C_p \delta_k$ ) to those of the stack ( $\rho_s C_p \delta_s$ ), and  $a$  is the speed of sound. Power densities of  $\sim 10 \text{ W/cm}^3$  can be achieved at  $\sim 5,000 \text{ Hz}$ .

Finally, high frequency operation for a resonant system leads to small total volume for the refrigerator. This is attractive from the point of view of applications: compactness and rapid cool-down are important factors.



Having presented the goals, the approach, and the experiments performed on a high frequency thermoacoustic refrigerator, it is important to put into evidence the quantitative aspects of this work and all the new things that have emerged from this research. This research relies on theoretical foundations laid down by Rott et al [11] and highlighted by Swift [7].

(i) DETAILS.

In this section some of the details and important qualities for the high frequency refrigerator will be listed and compared to those of a 500 Hz refrigerator discussed in ref. 7.

<u>High Freq. Refrigerator</u>	<u>Regular 500 Hz Refrigerator.</u>
medium: air	helium
f: 5,000 Hz	500 Hz
overall length: 10 cm	~40 cm
stack length $\Delta x$ : 0.5 cm	8 cm
normalized stack length $\Delta x/\lambda$ : 0.02-0.03	0.04
$\nabla T_{crit}$ : 120 K/cm	15 K/cm
stack surface area: 7,550 cm <sup>2</sup>	3920 cm <sup>2</sup>
total perimeter $\Pi$ : 151 m	4.9
thermal penetration depth $\delta_k$ : 0.04 mm	0.1 mm
acoustic displacement amplitude $x_1$ (typical): 0.2 mm	2 mm
$\Delta T$ : 50°C	90 K
total volume: 94 cm <sup>3</sup>	1200 cm <sup>3</sup>
sound pressure $p_1$ (at 160 dB): $2 \times 10^3$ pa	$2.9 \times 10^4$ pa (at ~180 dB)
ambient pressure $P_m$ : 1 atmosphere	10 atmospheres

For a ratio of actual temperature gradient to the critical gradient,  $\Gamma = \nabla T_m / \nabla T_{crit}$ , between 0.46 and 0.7 our cooling power has been typically 0.4-0.8 watt at a sound level of 160 dB at the stack. Slightly better performance was achieved with a square wave drive than with a sinusoidal drive.

From the design of the design of the high frequency thermoacoustic refrigerator and choice of components, high power capability or demonstrated by the low frequency refrigerator [12] was not expected. However unusual and new features have emerged from this research.

(ii) Special Features of High Frequency Thermoacoustic Refrigerator.

Research on the high frequency refrigerator has shown interesting features such as:

- large critical temperature gradients making it possible to get to very low temperatures.
- device is mainly a pressure device rather than a displacement device. It would be an ideal device as a second stage to a regular low frequency thermoacoustic refrigerator making it possible to achieve ultralow temperatures.
- offers the possibility to use multiple stacks and hence multiple stage devices. Such devices could be run in the ultrasonic range.
- well adapted to miniaturization.

- simple, cheap, compact.
- its small volume leads to a short cool-down time. In fact from this point of view many parallel high frequency refrigerators would perform better than a low frequency refrigerator.
- attractive, because of compactness and simplicity, for cooling:
  - biological samples
  - electronic devices
  - high speed electronics
  - infra-red detectors
- takes advantage of high efficiency of piezo drivers.
- smaller heat dissipation in piezo driver than in electromagnetic driver.
- high power density.
- large surface area stack.
- miniature heat exchangers.
- well-suited for high pressure operation.

(iii) Further Developments.

This refrigerator needs further developments in:

- heat transfer at stack-heat exchanger interface.
- application of gas mixtures.
- optimum acoustic drive frequency.
- limitations of refrigerator.

In fact, the above areas are part of the optimization steps discussed previously in the development of this refrigerator.

(iv) Personnel

- DeJuan Zhang: Research Associate Professor of Physics
- Thierry Klein: Postdoctoral Fellow
- Dan Dolan: graduate student, M.Sc.
- Orest G. Symko: Professor of Physics

(v) Publications and Lectures.

- High Frequency Thermoacoustic Refrigerator, O.G. Symko, D. Zheng, and T. Klein, in preparation.
- Lectures:
  - New Materials Workshop in Luxor, Egypt, 1997  
“Thermoacoustics”
  - University of J. Fourier, Grenoble, France 1994  
“Cooling with Sound”
  - E.T.H. Zurich, Switzerland, 1994  
“Thermoacoustic Refrigerator”
  - Brigham Young University, Utah, 1994  
“Thermoacoustic Refrigerator”

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