

# Chapter 1

## The DASCh Experience: How to Model a Supply Chain

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### 1.1. Introduction

Nonlinear dynamical systems have been a fertile field for the application of simulation techniques. Since the 1960's, System Dynamics has studied such problems by integrating systems of ordinary differential equations (ODE's) over time. More recently, increases in computer power have permitted the broad application of agent-based (or individual-based) modeling. In our work on supply chain modeling, we have found agent-based modeling to be more flexible than ODE models for basic exploration. One phenomenon we discovered, the inventory oscillator, can also be modeled in ODE's, an approach that permits more rapid manipulation in a spreadsheet environment. Further study permits derivation of a closed-form analytical model as well, which makes explicit a number of interesting structural features of the oscillator.

This paper does not pretend to enrich the repertoire of nontrivial behaviors known to complexity researchers. Mathematically, the behavior we observe is not particularly sophisticated: the inventory oscillator turns out to be computing a modulus function. Its intended contribution is twofold. First, and primarily, we seek to highlight the differences among agent-based, equation-based, and analytical system modeling, in terms of when they can be applied and the results one can expect to derive. The comparative simplicity of our system is what makes the analytical treatment possible at all. Second, manufacturing engineers find the potential for inventory fluctuation under stable boundary conditions counterintuitive and of great practical import. Its reducibility to the modulus function, far from making the results trivial, suggests that similar threshold nonlinearities may be responsible for other unexpected time-varying manufacturing measurements, and thus points the way to stabilize these important commercial systems.

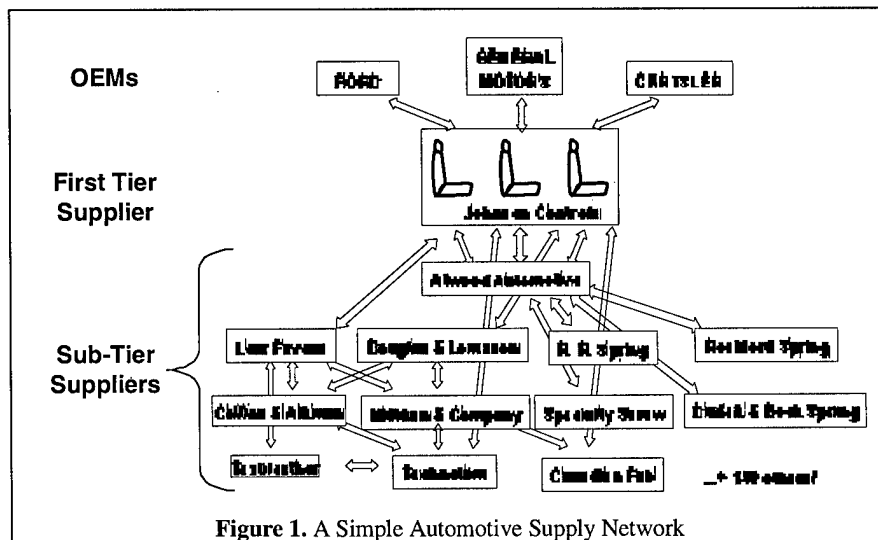
Section 2 of this paper describes the supply chain problem. Section 3 reports the three models that we constructed. Section 4 reviews the roles of each model and recommendations for their deployment. Section 5 summarizes our conclusions.

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## 1.2. The Supply Chain Challenge

Modern industrial strategists are developing the vision of the “virtual enterprise,” formed for a particular market opportunity from independent firms with well-defined core competencies [4]. The manufacturer of a complex product (the original equipment manufacturer, or “OEM”) may purchase half or even more of the content in the product from other firms. For example, an automotive manufacturer might buy seats from one company, brake systems from another, air conditioning from a third, and electrical systems from a fourth, and manufacture only the chassis, body, and powertrain in its own facilities. The suppliers of major subsystems (such as seats) in turn purchase much of their content from still other companies. As a result, the “production line” that turns raw materials into a vehicle is a “supply network” (more commonly though less precisely called a “supply chain”) of many different firms.

Figure 1 illustrates part of a simple supply network [1, 3]. Johnson Controls supplies seating systems to Ford, General Motors, and Chrysler, and purchases components and subassemblies either directly or indirectly from over 150 other companies, some of which also supply one another. Product design and production schedule must be managed across all these firms to produce quality vehicles on time and at reasonable cost. Historically, this vision has been frustrated by unexpected behavior of the supply network, such as large swings in orders and inventories and unreliable information. Our research explores these problems from a dynamical systems perspective.

## 1.3. Three Models

We have modeled one aspect of supply chain behavior using three different approaches. Our initial agent-based model exhibited internal inventory oscillations

under stable conditions at the chain's boundaries. We replicated much of this behavior in an equation-based model using ODE's. Then we developed an analytical model in which we could prove certain empirically observed characteristics of the oscillator.

### 1.3.1. Agent-Based Model

The DASCh project (Dynamical Analysis of Supply Chains) [5, 6] includes three species of agents. *Company agents* represent the different firms that trade with one another in a supply network. They consume inputs from their suppliers and transform them into outputs that they send to their customers. *PPIC agents* model the Production Planning and Inventory Control algorithms used by company agents to determine what inputs to order from their suppliers, based on the orders they have received from their customers. These PPIC agents currently support a simple material requirements planning (MRP) model.<sup>1</sup> *Shipping agents* model the delay and uncertainty involved in the movement of both material and information between trading partners.

The initial DASCh experiments involve a supply chain with four company agents (Figure 2: a boundary supplier, a boundary consumer, and two intermediate firms producing a product with neither assembly nor disassembly). Each intermediate company agent has a PPIC agent. Shipping agents move both material and information among company agents.

We expected this simple structure to exhibit relatively uninteresting behavior, on which the impact of successive modifications could be studied. In fact, it shows a range of interesting behaviors in terms of the variability in orders and inventories of the various company agents.

We found four different behaviors in the model: amplification of variance in the order stream as one moves away from the customer, induction of spurious correlations in the order stream, persistence of disturbances long after a single change in orders has been made, and generation of variation in inventory levels in the system when the boundary conditions are

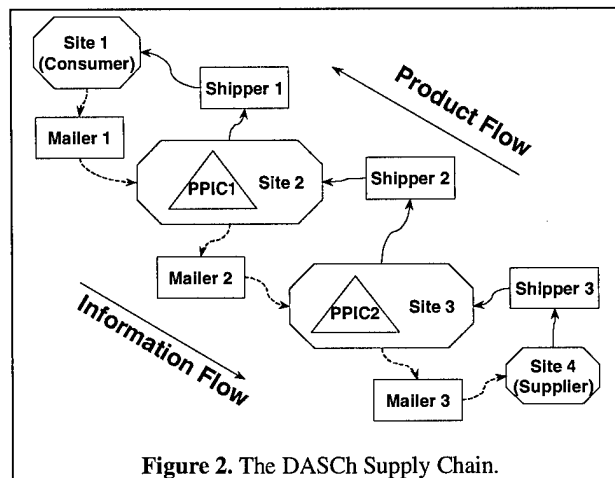


Figure 2. The DASCh Supply Chain.

<sup>1</sup> The basic MRP algorithm includes developing a forecast of future demand based either on past demand or on customer forecast (depending on location in the hourglass), estimating inventory changes through time due to processing, deliveries, and shipments, determining when inventory is in danger of falling below specified levels, and placing orders to replenish inventory early enough to allow for estimated delivery times of suppliers.

held constant. Details of these behaviors are discussed in [6].

This report focuses on the last effect, the generation of inventory variation. Even when top-level demand is constant and bottom-level supply is completely reliable, intermediate sites can generate complex oscillations in inventory levels, including phase locking and multiperiodicity, as a result of capacity limitations.

The consumer has a steady demand with no superimposed noise. The bottom-level supplier makes every shipment exactly when promised, exactly in the amount promised. Batch sizes are 1, but we impose a capacity limit on sites 2 and 3: at each time step they can process only 100 parts, a threshold nonlinearity. As long as the consumer's demand is below the capacity of the producers, the system quickly stabilizes to constant ordering levels and inventory throughout the chain. When the consumer demand exceeds the capacity of the producers, inventory levels in those sites begin to oscillate. The basic dynamic is that filling orders draws down inventory to make up a shortfall in production. When inventory falls too low, the current order is backlogged and the current production run provides a new inventory.

Figure 3 shows the behavior when demand exceeds capacity by 10%. Site inventories oscillate out of phase with one another, in a sawtooth that rises rapidly and then drops off gradually. The inventory variation ranges from near-zero to the level of demand, much greater than the excess of demand over capacity.

Figure 4 shows the dynamics after increasing consumer demand to 150. The inventories follow a sawtooth of shorter period. Now one cycle's production of 100 can support only two orders, leading to a period-three oscillation. The inventories of sites 2 and 3, out of synch when  $\text{Demand/Capacity} = 110/100$ , become synchronized and in phase after a transition period.

The transition period is actually longer than appears from Figure 4. The increase from 110 to 150 takes place at time 133, but

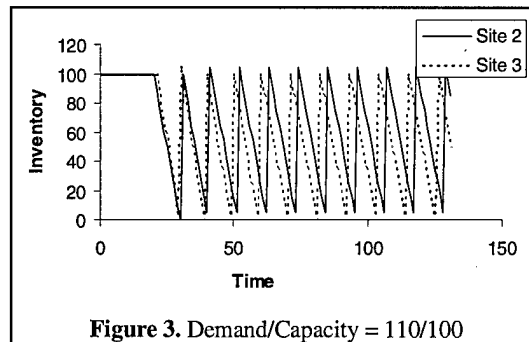


Figure 3. Demand/Capacity = 110/100

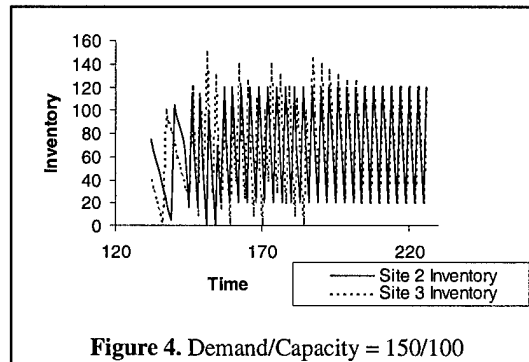


Figure 4. Demand/Capacity = 150/100

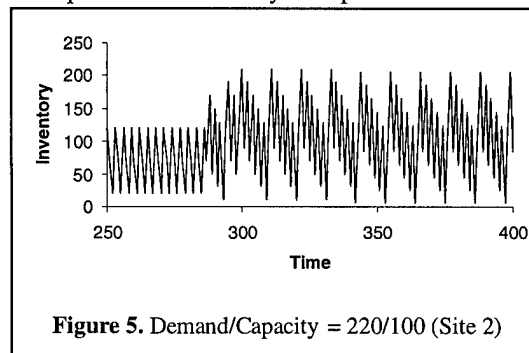


Figure 5. Demand/Capacity = 220/100 (Site 2)

the first evidence of it in site 2's dynamics appears at time 145. The delay is due to the backlog of over-capacity orders at the 110 level, which must be cleared before the new larger orders can be processed.

Figure 5 shows the result of increasing the overload even further. (Because of the increased detail in the dynamics, we show only the inventory level for site 2.) Now the consumer is ordering 220 units per time period. Again, backlogged orders at the previous level delay the appearance of the new dynamics: demand changes at time 228, but appears in the dynamics first at time 288, and the dynamics finally stabilize at time 300.

This degree of overload generates qualitatively new dynamical behavior. Instead of a single sawtooth, the inventories at sites 2 and 3 exhibit biperiodic oscillation, a broad sawtooth with a period of eleven, modulated with a period-two oscillation. This behavior is phenomenologically similar to bifurcations observed in nonlinear systems such as the logistic map, but does not lead to chaos in our model with the parameter settings used here. The occurrence of multiple frequencies is stimulated not by the absolute difference of demand over capacity, but by their incommensurability.

### 1.3.2. Equation-Based Model

Following the pioneering work of Jay Forrester and the System Dynamics movement [2], virtually all simulation work to date on supply chains integrates a set of ordinary differential equations (ODE's) over time. It is customary in this community to represent these models graphically, using a notation that suggests a series of tanks connected by pipes with valves. The dynamics of our simple model can be represented by the following set of ODE's:

$$d(WIP3)/dt = orderRate - \min(capacity, WIP3/productionTime)$$

$$d(Finished3)/dt = \min(capacity, WIP3/productionTime) - \alpha$$

$$d(WIP2)/dt = \alpha - \min(capacity, WIP2/productionTime)$$

$$d(Finished2)/dt = \min(capacity, WIP2/productionTime) - \beta$$

where

$\alpha = orderRate$  if  $Finished3/orderPeriod + capacity > orderRate$ , otherwise 0;

$\beta = orderRate$  if  $Finished2/orderPeriod + capacity > orderRate$ , otherwise 0

$WIP\{2,3\}$  is work in process inventory at site 2 or 3, respectively;

$Finished\{2,3\}$  is finished goods inventory at site 2 or 3, respectively;

$orderRate$  is the rate of consumer orders to the chain;

$productionTime$  is the time needed at site 2 or site 3 to turn WIP to finished goods;

$capacity$  is the amount of WIP that site 2 or site 3 can turn into finished goods each time step.

This model does not support many of the behaviors in the agent-based

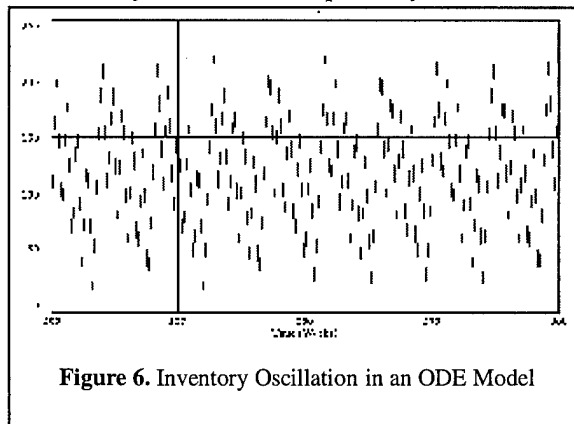


Figure 6. Inventory Oscillation in an ODE Model

model. In particular, amplification, correlation, and persistence of variation depend on the PPIC (Production Planning and Inventory Control) algorithm in DASCh, which is extremely difficult to capture in an ODE formalism [8]. However, the ODE model does demonstrate oscillations comparable to those in the DASCh model. For example, Figure 6 shows the biperiodic oscillations for Demand/Capacity = 220/100, generated by the VenSim® simulation environment. The system dynamics model shows the same periodicities as the agent-based model, though it does not show the transitional dynamics or phase locking behavior seen in Figure 4, because it has abstracted away the PPIC algorithm.

### 1.3.3. Analytical Model

If we further abstract away the dynamical behavior of production and shipping that generates the observed behavior, an even a simpler model is available. Since each time step generates new inventory of *capacity* and outstanding orders ship everything in excess of *order*, the inventory at the  $n$ th time step is just  $\text{mod}((n-1)*\text{capacity}, \text{order})$ , where  $\text{mod}()$  is the modulo function, the essence of a threshold nonlinearity. This level of abstraction permits us to prove a number of interesting relations among the  $\text{Inventory}(t)$  at a site, the constant *Demand* (order rate) from its customer, and its constant *Production* (capacity level). Critical derived quantities include  $D$  and  $P$  (the smallest integers such that  $D/P = \text{Demand}/\text{Production}$ ),  $I(t)$  ( $\text{Inventory}(t)$  in the same units as  $D$  and  $P$ ),  $H$  (the minimum of  $P$  and  $D - P$ ), *Period* (the minimum  $n$  such that  $I(t) = I(t+n)$ ), and *Sequence* (the shortest sequence of steps-to-next-local-maximum over the course of a single period). Many of these results are well-known characteristics of the modulo function. Proofs are available in [6]. For example:

**Attractor.**—If the system is initiated with  $\text{Inventory} \geq \text{Demand}$ , it will enter the region  $0 \leq \text{Inventory} < \text{Demand}$ , and then remain there.

**Scaling.**—If we multiply *Demand* and *Production* by the same integer factor, or if we divide out common integer factors, the  $\text{Inventory}(t)$  is multiplied or divided by the same integer factor, but *Sequence* and *Period* are unaffected. This result motivates the use of  $D$  and  $P$ , from which all common factors have been removed, as a unique representation of a given ratio  $\text{Demand}/\text{Production}$ .

**Period.**—For any  $I(t)$  in the region  $0 \leq I < D$ , the system will return to the same inventory level at time  $t+D$ , so that  $\text{Period} = D$ . By the previous result,  $\text{Period} = D$  not only for systems in the  $(D, P, I)$  units, but for arbitrarily scaled  $(\text{Demand}, \text{Production}, \text{Inventory})$  units.

**Coverage.**—Between  $t$  and  $t + \text{Period}$ ,  $I$  assumes every value in the attracting range. This result holds only for the reduced units  $(D, P, I)$ , since it concerns units of parts produced. For systems in which *Demand* and *Production* have a common factor  $k$ , there will be bands of inventory values of width  $k$  that the system will never visit once it is in the attracting region.

**Length.**—The number of items in a *Sequence*, corresponding to the number of intermediate maxima between maxima of the same size (counting one of the ends), is  $H$ .

In addition, the pattern by which  $I(t)$  moves between local minima and local maxima in the attracting region, the proportion of long and short subperiods, and the

number of monotonic subsequences in the overall *Sequence* depend on  $H$  in ways defined more precisely in [6].

These results are consistent with a concise geometrical model of the dynamics, familiar to those acquainted with the behavior of the modulo function. The complete dynamics can be represented in a square of  $D$  units on a side. The left edge of the square corresponds to time  $t$ , the right edge to time  $t+D$ , the bottom to inventory  $0$ , and the top to inventory  $D$ . The system trajectories behave as though this square were formed into a two-torus. In our manufacturing domain,  $D$  and  $P$  are integer parameters, so  $D/P$  is rational by construction. However, the torus model supports irrational  $D/P$  as well. In this case, we would have quasiperiodicity, and the orbit on the torus would never retrace itself. Since the surface of a 2-torus is two-dimensional, this interpretation shows that the dynamics of the oscillator can be embedded in two dimensions. Thus in the limit of continuous time, and under the rules we explored, the oscillator can never go chaotic.

#### 1.4. The Right Tool for the Job

Each of the three modeling approaches offers distinctive contributions to our understanding of the dynamics of the inventory oscillator.

Each agent in the agent-based model maps directly to an entity in the problem domain. It is straightforward to represent the PPIC algorithm in such a model, so we did, and were able to discover a much wider range of interesting behaviors than in the ODE model, which lacks such an algorithm. Even for the oscillator, it supports some behaviors (transition effects and phase locking) that simpler models do not show. Elsewhere [7] we discuss in depth the advantages of agent-based models over the equation-based models of system dynamics. However, the agent-based model offers no *a priori* characterization of the relationships among the model observables.

The equation-based model makes these relationships explicit. However, its construction requires deciding in advance what observables to study, and demands that the relations among them be expressed in closed functional forms. The inventory oscillator lends itself to such expression. Other important features of the supply network (such as interacting PPIC algorithms) do not.

The analytical model offers a detailed characterization of the oscillator that is not available to either of the other approaches. It shows clearly why the oscillator cannot enter the formally chaotic regime without introducing some other complication. However, it is the most limited of the models. It depends on the reducibility of the dynamics to a simple function, it applies only to the oscillator, and then only to an abstraction in which common factors are removed from the values for demand and production.

#### 1.6. Conclusion

The three modeling methods explored in this paper can be compared in several ways. The agent model offers the most natural representation and greatest breadth of potential behavior, followed first by the equation-based model and then by the analytical model. However, the explicitness of the relationships among system

observables is greatest in the analytical model, followed by the equation-based model and then by the agent-based model. It is unlikely that we could have developed the analytical model without first of all discovering the oscillatory behavior in one of the other two models, and the ease of manipulation of the equation-based model in a spreadsheet form was a great help in testing hypotheses that led to the formulation of the theorems in the analytical model. The equation-based model, in turn, is only possible because these particular system observables and behaviors lend themselves to representation in closed functional forms. Other behaviors observed in the agent-based model could not be duplicated in the equation-based model, and would not have been discovered if we had begun with that form of model.

Thus our experience recommends that system modeling begin with a formalism as close as possible to the entities in the problem domain (that is, with an agent-based model). In some cases, experience with this model may permit the construction of a second, equation-based model that may be useful in generating large numbers of test cases quickly. Inspection of such results may (in simple cases) suggest analytical formalisms for specific behaviors.

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