Second International Conference organized by
Association Française d'Approximation

# CURVES AND SURFACES

## ABSTRACTS

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Second International Conference organized by
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# CURVES AND SURFACES

## ABSTRACTS

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CHAMONIX MONT-BLANC
FRANCE

June 10-16, 1993

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## Association Française d'Approximation (A.F.A.)

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# CONTENTS

# UNE METHODE DE CONSTRUCTION D'ELEMENTS FINIS RATIONNELS DE CLASSE $\mathcal{C}^k$ DANS $\mathbb{R}^2$

**K. Nicolas AGBEVE**

*(Université de Nantes)*
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Soient $\Omega$ un ouvert borné de $\mathbb{R}^2$ et $S = \{a_i, i = 1, \cdots, n\}$ un ensemble de points isolés de $\bar{\Omega}$. Notons $\mathcal{T}$ une triangulation basée sur $S$, c'est à dire dont les sommets des simplexes sont dans $S$.

Pour construire une fonction de classe $\mathcal{C}^k$ avec des données d'interpolation sur les points de $S$, on peut utiliser des méthodes globales, ( Duchon, Bouhamidi,...), des méthodes radiales, (Dyn & Levin, Powel,...) et aussi des fonctions splines polynomiales.

Si les deux premières méthodes ont le désavantage d'être difficiles à mettre au point, la dernière elle, engendre des coûts élevés à l'utilisation.

Le but visé ici, est la construction des éléments finis de même régularité que celle que l'on peut avoir avec les méthodes de splines polynomiales mais en utilisant des fonctions rationnelles pour réduire le nombre de degrés de liberté. La méthode consiste à chercher un élément fini sous la forme d'un interpolant $\mathcal{P}$ qui s'écrit : $\mathcal{P} = \mathcal{C} + \mathcal{R}$ où $\mathcal{C}$ est une fonctionnelle polynomiale pure et $\mathcal{R}$ une fonctionnelle rationnelle pure c'est à dire irréductible. L'interpolant $\mathcal{R}$ apparaîtra alors comme un correcteur de l'interpolant $\mathcal{C}$ , donc doit conserver les propriétés de $\mathcal{C}$.

## RATE OF CONVERGENCE OF INTERPOLATING MEANS WITH EXPONENTIAL-TYPE WEIGHTS

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Given a set $S_n \subset [a,b] \subset \mathbb{R}$ of irregularly-spaced points, we consider the interpolation of a function $f \in C^r[a,b]$ by the weighted arithmetic mean

$$\mu \equiv \mu(x;T,S_n) = \sum_{k=1}^{n} T_k(x;f,r)\, w_k(x;S_n),$$

that is a positive linear functional. In order to increase the degree of approximation of $\mu$ to $f$, we use in the operator $\mu$ , instead of the value $f(x_k)$, $(k = 1,2,..,n)$, of the interpolated function, the truncated Taylor expansion of $f$ up to the derivative of order $r$ about the point $x_k$

$$T_k \equiv T_k(x;f,r) = \sum_{j=0}^{r} \frac{f^{(j)}(x_k)}{j!}(x-x_k)^j .$$

The weights multiplying the Taylor expansions are

$$w_k \equiv w_k(x,S_n) = \frac{|x-x_k|^{-q} exp(-\beta|x-x_k|)}{\sum_{h=1}^{n} |x-x_h|^{-q} exp(-\beta|x-x_h|)},$$

with $q$ and $\beta$ nonnegative real numbers . They satisfy the properties

$$w_k(x;S_n) \geq 0\,, \qquad \sum_{k=1}^{n} w_k(x;S_n) = 1,$$

depend on the data points and are approximately exponentially decreasing with the distance. These weights are interesting, both theoretically and practically, and include noteworthy particular cases.

At first we prove that the derivatives $D^s w_k$, $(s = 0,1,...r)$, of the weights vanish at the interpolation nodes. As a conseguence, we obtain that the derivatives $D^s\mu$ of the interpolating operator and the corresponding derivatives $D^s f$ of the interpolated function are equal at the nodes.

We then examine the degree of approximation of $\mu$ to $f$, giving upper bounds for the difference $|D^s\mu - D^s f|$ by means of the modulus of continuity of $D^s f$.

Title:

# A Vector Spline Quasi-Interpolation.

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**Abstract:**

We present a new vector quasi-interpolant based on a vector spline function introduced by the authors ( J. Approx. Th., Vol. 67, No. 1, Oct. 91). Following a recent approach used in quasi-interpolation (C. Rabut, Thèse d'Etat, 90), the elementary matrix approximant is obtained by convolution of a discretized version of the partial differential operator, which caracterize the vector spline function, and the exact fundamental solution associated to the operator. Some general regularity properties of the elementary matrix are given along with results on convergence of the quasi-interpolant. Specific features related to the vector structure ot the problem are showed. Finally numerical exemples of the elementary matrix are given.

**2nd INTERNATIONAL CONFERENCE ON CURVES AND SURFACES**
**Chamonix Mont-Blanc, June 10-16, 1993**

Title of Submitted Paper:

# Separable and parallel approximation methods for data whose ordinates lie on or near a family of lines or curves.

Authors: **I J Anderson, M G Cox, S Harbour and J C Mason**

Speaker: **J C Mason**

Address: **Applied and Computational Mathematics Group,**
**RMCS (Cranfield), Shrivenham, Swindon SN6 8LA, England**

ABSTRACT:

Structured data, such as data whose ordinates lie on or near a set of parallel lines or a family of curves, should be fitted in such a way as to exploit the data structure - even when the ordinates are irregularly spaced and in different positions on each line/curve. Existing algorithms of this type, which effectively separate the variables, include the 1963 algorithm of C W Clenshaw and J G Hayes for Chebyshev polynomial approximation of lines of data, the 1990 algorithm of J C Mason and R P Bennell which effectively extends this to a procedure for curves of data, and the 1993 algorithm (to appear in "Algorithms for Approximation 3" - Proceedings of 1992 NATO Symposium, Oxford) of I J Anderson, M G Cox, and J C Mason which iteratively interpolates a B-spline through data near to lines. These algorithms are now being extended in two principal ways: first, B-splines are being fitted in a least squares sense to (overspecified) data on or near lines/curves, and second, parallel processing procedures are being investigated and implemented for exploiting the separability of the data/algorithms. It is relatively straightforward to envisage the principles of parallel algorithms for these problems, but it is more important and difficult to carry this out in practice and to assess the communications and organisational overheads for specific parallel computer platforms.

# Smoothing Noisy Data with Coiflets

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ABSTRACT

This paper is concerned with an orthogonal wavelet series regression estimator of an unknown smooth regression function observed with noise on a bounded interval. The method is based on applying results of the recently developed theory of wavelets and uses the specific asymptotic interpolating properties of the wavelet approximation generated by a particular wavelet basis, Daubechie's coiflets. Conditions are given for the estimator to attain optimal convergence rates in the integrated mean square sense as the sample size increases to infinity. Moreover, the estimator is shown to be pointwise consistent and asymptotically normal. The numerical implementation of the estimation procedure relies on the discrete wavelet transform and the algorithm for smoothing a noisy sample of size $n$ requires order $\mathcal{O}(n)$ operations. The general theory is illustrated with simulated and real examples and a comparison with other nonparametric smoothers is made.

*Keywords and phrases*: Nonparametric regression, curve smoothing, wavelets, multiresolution analysis, splines.

# QUASI-INTERPOLANTS, BOX-SPLINES AND (QUASI-)WAVELETS [P(D)]-manifolds.

by Marc ATTEIA,

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Let $\zeta = (\zeta_1, ..., \zeta_n) \in \mathbb{R}^n$ and $P_{\ell_j}(\zeta)$, $1 \leq \ell \leq m$, $1 \leq j \leq q$, be a real polynomial in the variables $\zeta_1, ..., \zeta_n$ with constant coefficients. We set $P(\zeta) = (P_{\ell_j}(\zeta))$.

In [1], with convenient assumptions, we have proved the existence of a spline manifold $\sigma$ relating to $P(D)$ (where $D = \left(\dfrac{\partial}{\partial x_n}, ..., \dfrac{\partial}{\partial x_n}\right)$) and that $\sigma$ in characterized by a hilbertian (semi-)kernel $\mathcal{L}$ relating to ${}^tPP$.

In this lecture, we define and study quasi-interpolants (resp. box-splines) manifolds and (quasi-)wavelets manifolds, when $P$ in homogeneous. They are obtained by approximating in a convenient way the formula ${}^tPP\mathcal{L} = \delta$ ($\delta$ is the Dirac functional) on a sequence of regular meshes of $\mathbb{R}^n$.

We give some examples.

[1] Spline manifolds, Curves and surfaces, Chamonix 1990, pp. 1-4, Academic Press N.Y. 1990

# Affine Classification of Bézier Cubics

M. L. Baart

Potchefstroom University for C.H.E.

**Abstract:**

Cubics occupy an intermediate position in the hierarchy of algebraic curves. They allow considerably more freedom than conics, but limit the types of complications that can occur for higher order curves. We review some relevant algebraic properties of cubics in general, and polynomial cubics in particular. Of these properties, those that remain invariant under affine transformations are of particular interest in applications in computer graphics. These affine-invariant properties can be used to find a practical classification of planar polynomial cubics. As an application, a complete partition of the parameter-plane of such Bézier cubics are presented. In conclusion we consider additional classes introduced by allowing rational parametrization.

**References:**

1. Baart, M.L.; McLeod, R.J.Y. Rational curves and isoparametric transformations. *J. Comp. & Appl. Math.* 18 (1987), 121–128.
2. Farin, G. From conics to NURBS. *IEEE Comp. Graphics & Appls* (Sept.1992), 78–86.
3. Manocha, D.; Canny, J.F. Detecting cusps and inflection points in curves. *Computer Aided Geometric Design* 9 (1992), 1–24.

# "WAVELET DECOMPOSITION IN COMPUTERIZED TOMOGRAPHY

L. Bacchelli Montefusco*, C. Guerrini*

* Dipartimento di Matematica Universita' di Bologna

## ABSTRACT

The problem of reconstructing a function from its line integrals has arisen independently in many scientific fields, astrophysics, geophysics, and particularly in medicine. In practice there is an unknown compact-supported function $f$ defined in $R^2$ , representing relative tissue density, a finite number of measurements are given approximating integrals of $f$ along lines and we have to reconstruct $f$ from these data.

Two approaches have been used to solve this problem, the first one deals with methods aiming to approximate the inverse of the transformation defined by the line integrals or Radon Transform; these type of methods are colled transform methods and one of them, convolution-backprojection, has been universally adopted for medical purposes. The sencond approach consists of discretizing the model at an early stage of the process, leading to a finite dimensional algebraic system of linear equations for which a solution has to be choosen using some iterative methods . Both these methods involve large amount of data when dealing with practical applications.

In this work we have studied and experimented the use of orthogonal and biortogonal wavelet and wavelet packet bases with the aim to construct efficient parallel algorithms for the solution of these reconstruction problems. Numerical results obtained on the distributed memory system iPSC/860 hypercube are presented.

## REFERENCES

[1] G. T. Herman, Image reconstruction from projection, the fundamentals of computerized tomography, Academic Press, 1980.

[2]C. Guerrini, G. Spaletta, An image reconstruction algorithm in tomography: a version for the CRAY X-MP vector computer, Comput. & Graphics vol.13,n.3, (1989) pp.367-372 .

[3] I. Daubechies, Orthonormal bases of compactly supported Wavelets, Comm. Pure and Appl. Math., n.XLI (1988).

[4] C. K. Chui and J. Z. Wang, On compactly supported spline wavelets and duality principle, CAT Report 213, May 1990.

[5] C. K. Chui, Non orthogonal wavelet packets, CAT Report 261 Dec 1991.

# WAVELET PACKET DECOMPOSITION FOR PARALLEL NUMERICAL ALGORITHMS

L. Bacchelli Montefusco*, L. Puccio**

* Dipartimento di Matematica Universita' di Bologna
** Dipartimento di Matematica Universit di Messina

## ABSTRACT

Wavelet decomposition has recently been used to convert dense matrices of a certain type into higly sparse matrices, with consequent reduction of the computational cost applying them to arbitrary vectors [1], [2]. Moreover, efficient nonstandard matrix multiplication algorithms have been obtained by doing a wavelet packet dec omposition of the matrices and setting all coefficients to zero whose absolute values are less than a fixed tollerance [3]. In this paper we take advantage of the typical block-structure presented by the wavelet packet decomposed matrices in order to construct parallel numerical algorithms whose hight efficiency is mostly due to the strong diagonal dominance of the trasformed matrices and the same structure of the different blocks. An extensive experimentation taken on a distributed memory multiprocessor iPSC/860 is presented.

## REFERENCES

[1] G. Beylkin, R. Coifman, and V. Rokhlin, Fast wavelet trasforms and numerical algorithms: I, Comm. Pure Appl. Math. 44 (1991), 141-183.
[2] F. Keinert, Biorthogonal wavelets for fast matrix computations, preprint.
[3] M.V. Wickerhauser, Nonstandard matrix multiplication, available via anonymus ftp from ceres.math.yale.edu, 1990.

**A Unimodality Property and B-splines**

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Given a point on a B-spline curve, one sometimes wishes to find which control vertices most affect that point. The question arises: if we find the control vertex with maximum influence, will vertices with indices near the index of the vertex of maximum influence affect the curve more than vertices with indices farther away? More specifically, given a sequence of B-splines $\{N_j\}$, and a domain value $t$, if $k$ is an index such that $N_k(t) \geq N_j(t)$ for all $j$, is it true that $N_j(t) \leq N_{j+1}(t)$ for all $j < k$, and $N_j(t) \geq N_{j+1}(t)$ for all $j \geq k$? This talk will show that this property does hold for many types of B-splines, but not for all.

# Object Oriented Spline Software

Richard Bartels
Department of Computer Science
University of Waterloo, Canada

**Abstract**

Object oriented programming provides software with a facility for imitating the mathematician's tool of theoretical abstraction. The communality of a collection of related entities can be described in terms of a *base class*, which may be added to an object oriented language as a new data type. Each concrete entity of the collection can be described as a *derived class* by specifying whatever features distinguish it from the base, and the entity can be added to the language as a further data type. Algorithms written in terms of the base type can be applied to any of the derived types.

This talk will introduce the concepts of object oriented programming and illustrate the use of its facilities for abstraction by presenting class designs for spline bases, spline functions (curves and surfaces), and spline knot insertion. Some examples of the use of these classes, of their further extension, or of alternative approaches to class design will be given in associated talks scheduled for the Workshop on Software Infrastructure for Computer Aided Design.

# Workshop on Software Infrastructure for Computer Aided Design

Richard Bartels
Guenther Greiner and Philipp Slusallek
David Forsey
Bruce Hickey and Hans-Peter Seidel
Alan Vermeulen

**Abstract**

This workshop will cover some issues on the design, implementation, and use of software for spline curves, surfaces, and finite elements. Techniques for object oriented programming will be a primary focus of the talks. Bartels will cover the basic concepts of object oriented programming, specifically its capacity to support the abstraction of algorithms and data types, and will offer basic examples in the design of classes for spline bases, refinement, and curves and surfaces. Greiner and Slusallek will show how the abstraction of surface software facilitates the implementation of surface blending. Hickey and Seidel will cover the use of object oriented classes and symbolic algebra to build a general system for connection matrix splines. Vermeulen will cover the use of splines and object oriented techniques to tackle the geometry and finite element analysis of physical structures.

## An Algorithm for Bivariate

## Simultaneous Polynomial Approximation

**G. Baszenski**
**Dortmund, Germany**

We consider the problem of approximating a smooth function f(x,y) on the square [-1,1] x [-1,1] by a polynomial p(x,y) such that derivatives of f(x,y) are approximated by derivatives of p(x,y). By blending interpolation methods we construct a preapproximation based on the boundary values of f(x,y). The remainder function is then interpolated on a Chebyshev node grid. Test computation results show that this procedure yields an approximant with high convergence orders for the function values and derivatives.

# On polynomial functions defining the $G^2$ geometrical continuity between two (SBR).

J.L. Bauchat
L.C.R.M.A.O.*- E.N.S.A.M. - C.E.R. de Lille
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This talk develops works about the $G^1$ and $G^2$ geometrical continuity between rational surfaces put into $(SBR)$ form.([1] , [2] , [3]) .
The study of $G^1$ and $G^2$ geometrical continuity between two surfaces which are $\Pi\Omega$ - projections of polynomial surfaces , leads to necessary and sufficient conditions pertaining to the parametrizations $\mathcal{L}(u,v)$ and $\mathcal{R}(u,v)$ of the surfaces "above".They are expressed by polynomial functions $\{f_1, f_2, f_3, f_4\}$, for the $G^1$ continuity , and $\{g_1, g_2, g_3, g_4\}$ , for the $G^2$ continuity, giving relations between the last rows of the massic net controlling $S_l$ and the first of the massic net of $S_r$.
An example deals with the $G^2$ geometrical connection between two quadrics: the obtained functions show how the massic vectors of $S_l$ and $S_r$ are interrelated.
Then we analyse the constaints to be satisfied by the coefficients of the polynomial functions $\{f_1, f_2, f_3, f_4\}$ in order to guarantee the $G^1$ geometrical continuity between a given $(SBR)$ surface $S_l$ and a new $(SBR)$ surface to be construct . The results are illustrated by an example.

[1] Fiorot , J.C. , P. Jeannin , *Courbes et Surfaces rationnelles . Applications à la C.A.O.* , Coll. R.M.A. 12 , Masson , Paris 1989.
[2] Bauchat , J.L. , Etude du raccordement géométrique $G^2$ de surfaces rationnelles mises sous forme $(SBR)$, Thèse Université des Sciences et Technologies de Lille , Laboratoire A.N.O. , mars 1992.
[3] Bauchat,J.L. , J.C. Fiorot , P. Jeannin , $G^1$ and $G^2$ continuity of $(SBR)$ surfaces in *Curves and Surfaces* , P.J. Laurent , A. le Méhauté and L.L. Schumaker (eds) , Academic Press , New York 1991.

*Laboratoire de conception et de réalisation mécaniques assistées par ordinateur

# CONSTRUCTION OF ITERATION FUNCTIONS FOR THE SIMULTANEOUS COMPUTATION OF THE SOLUTIONS OF SYSTEMS OF ALGEBRAIC EQUATIONS

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The material of this talk is part of a thesis advised by J.C. Yakoubsohn ([2]). Our purpose was to generalize to the multivariate case the well-known Weierstrass' iteration function ([5]), for the simultaneous computation of the roots of a monic polynomial $P$ of degree $d$ in the case of simple roots ; this function is attractive not only for its local quadratic convergence but above all because numerical experimentation incited a few authors to advance the conjecture that it converges with almost every given starting point ([4]).
The unified approach of the methods of simultaneous computation of roots of generalized polynomials developed by A. Frommer ([3]) particularly well fitted with our aim.
Our contribution to this approach in the univariate case ([1]) essentially relied on three points :

- the first one is the use of spaces $V$ containing the functions of the studied system whose dimension $D$ may be greater than $d+1$, by focussing our attention on the part played by the kernel of a linear functionnal $\Lambda$.

- the second one is the introduction of interpolation basis of this kernel (depending on the iteration points ) to construct in practice the iteration function, in the case in which, from a theoretical point of view, basis are supposed to be independent of these points.

- the third one is the extension of the theoretical construction of iteration functions with basis depending on the iteration points.

Here, we show how the two first points are the keys to construct iteration functions for the simultaneous computation of the solutions of a system of algebraic equations $(S)$ $f^{\lambda} = 0$ $\lambda = 1, \ldots, n$, in the case of zero-dimensional sets of solutions.
We obtain in particular, to compute the $d$ solutions of $(S)$, a « generalized Weierstrass' iteration function » from $\mathbb{C}^{nd}$ into $\mathbb{C}^{nd}$ in the form $\mathcal{I} = (\mathcal{I}_1, \ldots, \mathcal{I}_d)$ with :

$$\mathcal{I}_i(\tau) = \mathcal{I}_i(\tau^{(1)}, \ldots, \tau^{(d)}) = \tau^{(i)} - \left[J^{(i)}(\Upsilon(\tau))\right]^{-1} \cdot \mathbb{F}(\tau^{(i)})$$

where $\mathbb{F}(\tau^{(i)})$ is the transposed vector of $(f^1(\tau^{(i)}), \ldots, f^n(\tau^{(i)}))$, and $J^{(i)}(\Upsilon(\tau))$ the Jacobian matrix evaluated at the point $\tau^{(i)}$ of a system $\Upsilon(\tau)$ of $n$ equations and $n$ unknowns, depending on $\tau$ and identical to $(S)$ at the solution point.
We sum up the first experimentations and show how to improve convergence results in order to obtain a conjecture of global convergence in the real case.
We end applying one of these algorithms to the solution of a problem of robotic.

## REFERENCES

[1] Bellido A.-M. : Construction de fonctions d'itération pour le calcul simultané des racines d'une équation $f(s) = 0$, *C.R.A.S. Paris, t.* , 1992.

[2] Bellido A.-M. : Doctoral dissertation, Université Paul Sabatier, Toulouse, France, 1992.

[3] Frommer A. : A Unified Approach to Methods for the simultaneous Computation of All Zeros of Generalized Polynomials, *Numer.Math.*, Vol.54, pp.105-116, 1988.

[4] Pasquini L.,Trigiante D. : A Globally Method for Simultaneous Finding Polynomial Roots, *Math. Comp.*, Vol.44, No.169, pp.135-149, 1985.

[5] Weierstrass K. : Neuer Beweis des Fundamentalsatzes der Algebra. *Mathematische Werke*, tome III, Mayer u. Müller Berlin, pp.251-269, 1903.

**3D Generalized Voronoi Diagram**

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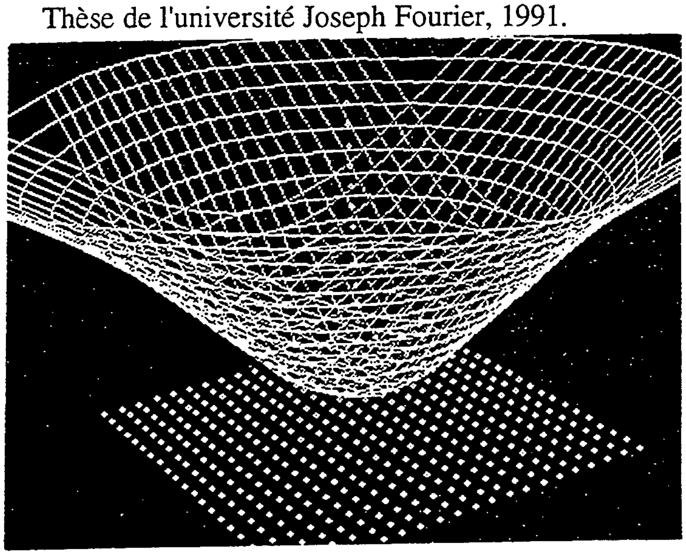
**Abstract:** This paper describes a study of the 3D Generalized Voronoi Diagram for polyhedra (3D GVD) as a set of quadric surfaces and planes. The separator between two distinct objects $Obj_1$ and $Obj_2$ is defined as the set of points equidistant to $Obj_1$ and $Obj_2$. The 3D GVD is the union of such separators. Two main research axis will be presented. The first axis is a theoritical study of the equations of separators between 3D polyhedral objects. This study will be illustrated with a first theorem. The second axis involves presentation of an algorithm computing an approximated solution of the 3D GVD based on uses of 3D Voronoi Diagram for Ponctual seeds. The convergence of this approach is provided in a second theorem.

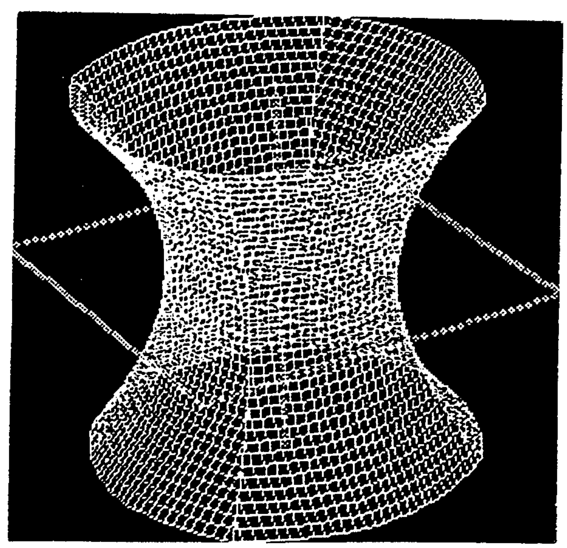
**Theorem 1:** Let S be a set of polyhedral objects, the separators between the objects of S are composed of following quadric surfaces: Paraboloid of revolution: $x^2 + y^2 + z = 1$, Hyperbolic paraboloid: $x^2 - y^2 = z$, Parabolic surface: $x^2 = z$, Quadric which equation is $x^2 + y^2 - z^2 = 0$ and Plane: $a.x + b.y + c.z + d = 0$.

A second result concerns computation of the 3D GVD of a set of polyhedric volumes. Our approach is based on discretization of the polyhedral surfaces and utilisation of the 3D Voronoi Diagram for Points. The result obtained with this algorithm is an approximation of the 3D GVD of the set of objects and the proof of this algorithm is based on the following theorem.

**Theorem 2:** The discretisation of the 3D GVD by the 3D Voronoi Diagram for Points converges toward the 3D GVD when the step of discretisation converges towards zero.

**Reference:** Hu H.T., Diagramme de Voronoï Généralisé Pour un ensemble de Polygones, Thèse de l'université Joseph Fourier, 1991.





3D Voronoi Diagram for one point and a square.

3D Voronoi Diagram for the border of a square and one segment.

# Spline under tension, interpolants of scattered data

A. Bouhamidi and A.Le Méhauté

**Abstract**

We recall briefly some properties of $L^{m,\ell,s}$–spline and we show how these functions enable us to characterize thin plate spline under tension. Some examples are presented, showing the effect of the parameter of tension.

The concept of spline curve under tension was introduced by Schweikert and later D. Franke investigates a scheme for the representation of surfaces which incorporates the same idea of tension. The method of Franke presents an elegant aspect but, unlike Duchon's thin plates and other radial basis functions methods,it is lacking in some mathematical theory and presents also a problem of a numerical matter, because it involves Bessel functions that need to be approximate for each entry of a full matrix. Using another differential operator, and the associated reproducing kernel, we are able to construct surface spline under tension as a particular case of $L^{m,\ell,s}$–spline, a generalization of thin plates splines. We recall briefly some properties of these functions and show how it works on some examples The data, with scattered data points are taken from Franke and have respectively 23 points, 33 points and 130 points.

All of the examples illustrate the behavior of the surfaces for various values of the tension parameter. We note that as the value tension is increased, the overshoot becomes less, and finally disappears.

The same idea, restricting to the univariate case, give a theoretical framework (Hilbert spaces and reproducing kernels) for spline curves under tension.

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# SINGULARITY DETECTION FROM NOISED DATA WITH WAVELETS

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We have studied the problem of detecting the locations of irregularity points of a function $y = f(x)$ on the base of a data set corrupted by noise:

$$y_i = f_i + e_i \qquad i = 1, \ldots, n$$

$$e_i \quad r.v. \quad idd \quad with \quad E(e_i) = 0 \quad \sigma^2_{e_i} = \sigma^2$$

The problem is particulary complex because it is difficult to distinguish discontinuities of the function from those caused by the noise.

In literature there are some papers about the matter. In particular we have considered [3] where Lee has tried to solve the problem using the Gabor window together with a statistical test on the error behaviour while, in more recent papers [1] and [2], the authors have tried to detect that points, studying the evolution of the wavelet transform across scales.

We present a method that intends to detect such irregularity points, using the wavelet transform together with a statistical study of the error spectrum.

## References

[1] Di Nunzio P. *Applicazioni delle ondine all'analisi delle irregolarità delle funzioni*, Thesis a.a 90/91.

[2] Mallat S. and Hwang W. L. *Singularity Detection and Processing with Wavelets*, IEEE Transactions on Information Theory, Vol.38, No.2, March 1992.

[3] Lee D. *Coping with Discontinuities in Computer Vision: Their Detection, Classification and Measurement*, IEEE Transaction on Pattern Analysis and Machine Intelligence, Vol.12, No.4, April 1990.

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# An introduction to Padé approximations

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This talk is an introduction to Padé approximations.

Let $f$ be a formal power series. A Padé approximant is a rational function whose power series expansion agrees with that of $f$ as far as possible. After reviewing some of their properties, the problem of their recursive computation will be treated by introducing formal orthogonal polynomials. Then, their convergence will be studied.

Padé approximants have very many applications ranging from the approximation of functions to the inversion of the Laplace transform and to the construction of A-acceptable approximations to the exponential function. Some of these applications will be discussed and a practical method for estimating the error will be given.

Padé approximants have been generalized in several directions including the multivariate case, series in a non-commutative algebra, Padé-type and partial Padé approximants, vector Padé approximation and so on. Some of these generalizations will be discussed.

## References

[1] C. Brezinski, J. van Iseghem, Padé Approximations, in *Handbook of Numerical Analysis*, P.G. Ciarlet and J.L. Lions eds., North-Holland, Amsterdam, vol. III, 1993.

# Scattered Data Interpolation over the Sphere

*by Jeff Brown*

In 1980 Robin Sibson developed a scattered data interpolant for euclidean spaces which has local support, i.e. the interpolated function value at a point **p** depends only on the function values at data sites near **p**. Let $\{\mathbf{p}_i\}$ be the data sites. Sibson's interpolant is based on a set of coordinate functions $s_i$ which satisfy the identities

$$\sum s_i = 1 \qquad\qquad \mathbf{p} = \sum s_i(\mathbf{p})\, \mathbf{p}_i.$$

The interpolant is defined throughout the convex hull of the data sites and is $C^1$ except at the data sites, where it is $C^0$. (In the planar case, the speaker has extrapolated Sibson's coordinates to the entire plane in such a way that they are still $C^1$ almost everywhere and Sibson's identities are preserved.)

The basic Sibson coordinates have been used as building blocks for other interpolation schemes by Sibson and by Farin.

The speaker will define coordinate systems for points scattered on the sphere which are analogous to Sibson's coordinates in the plane. These coordinates provide an interpolant over the sphere which has local support and is $C^1$, except at the data sites, where it is $C^0$. As in the planar case, the local support property makes these coordinate functions good candidates for use in other interpolation schemes.

# Box Spline Interpolation and Solution of the Poisson Problem

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The Fourier technique is very convenient to describe cardinal interpolation by spline functions generated by integer translates of a box spline (see K. Jetter, in: Approximation Theory VII, Academic Press 1992, pp. 131–161). Concerning numerical applications the case of periodic interpolation based on periodization of the cardinal spline function is more important than the cardinal case. Even the point of Fourier analytical view shows the close connection between cardinal and periodic interpolation.

In this talk we construct solutions of the Poisson equation on the unit square by box spline translates. Our method is based on new applications of bivariate periodic spline interpolation and attenuation factors (see H. G. ter Morsche, in: Topics in Multivariate Approximation, Academic Press 1987, pp. 165–174; M. Gutknecht, Numer. Math. **51** (1987), pp. 615–629).
We obtain solutions of the Poisson problem in form of Fourier series by two different methods. Both procedures can be considered as a generalization of fast Poisson solvers. By our first method we derive a kind of eigenfunction expansion. The second approach is a collocation method.

The Fourier representation of our solutions provides very efficient numerical algorithms based on FFT-techniques. Numerical test results will be presented.

# An energetic paradigm for multimodal images fusion

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In many applications, and especially in biomedicine, the analysis and the interpretation of a three-dimensional scene can rarely be performed using a single source of information. In such applications, merging the informations given by various sensors can provide a powerful alternative. A good example of this procedure is the fusion of scanner pre-operative 3D images and X-ray intra-operative 2D images which allows to infer, during the intervention, a 3D representation of the surgical scene.

The matching of two 3D models of a same structure or the registration of a 3D model with a set of 3D points has been widely studied in the literature and is today pretty well solved. On the contrary, the estimation of the pose of a smooth object by matching 2D images with a 3D a priori model of this object remains a widely open problem.

We show that this problem can be solved by an iterative procedure based on a mechanical model. We will first suppose that the contour of the object has been extracted in all the 2D images (at least two), called "dynamic" images. In the "dynamic" pose of the object, all the projection lines (i.e. the lines joining a pixel to the focal point of its image) are tangent to the surface of the 3D model. For complex objects, we can reasonably suppose that in a large neighborhood, this dynamic attitude is the only one leading to such a geometric configuration. Then, in a given pose, we consider, for any projection line, the spring of minimal length making this line tangent to the surface S. S being globally attracted to the springs, the distance from S to the set of lines will decrease. Based on this intuitive idea, we have formalized an iterative procedure in which the estimation of the dynamic pose of the object is described as the evolution of a 3D model in a spring-type forces field. A dual approach is to consider the potential energy of the set of lines with respect to the surface. Indeed, this energy defines a criterion of goodness-of-fit of a pose estimation which can be minimized to infer the dynamic attitude.

Both approaches rely on the computation of the 3D Euclidean distance in the neighborhood of the surface. For this purpose, we have defined several procedures for computing 3D distance maps, either based on dynamic programing or adaptative hierarchical data structures (octrees).

Simulations have showed an accuracy of 1 degree in rotation and 1 mm in translation. Biomedical experiments have been successfully run both with synthetic vertebras and volunteers (merging of MRI data and X-rays).

For non-segmented 2D images, we define the notion of matching credibility. Based on the merging of the informations provided by various procedures of image analysis and by the knowledge of the context, a credibility field is introduced which defines some kind of probability for a projection line to be a contour line (i.e. based on a contour pixel). In a first strategy, we define the notion of matching credibility (or attitude credibility) as a function of the credibilities of the projection lines tangent to the surface.
Then the attitude of the object is evaluated by maximizing this credibility function with respect to the 6 attitude parameters. In a second strategy, this credibility field is merged with the energetic model to define a new "energy" function which can be minimized either by classical minimization procedures or by Kalman filtering.

# Elastic Curves on the Sphere

**Guido Brunnett**
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Naval Postgraduate School

**Peter E. Crouch**
Center for Systems Science and Engineering
Arizona State University

This talk deals with the derivation of equations suitable for the computation of elastic curves on the sphere. To this end equations for the main invariants of spherical elastic curves are given. A new method for solving geometrically constraint differential equations is used to compute the curves for given initial values. A classification of the fundamental forms of the curves is presented.

## Recent Advances in the Theory of Radial Basis Functions:

### Interpolation and Least-squares Approximations

Martin Buhmann

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In this talk we shall review recent results about approximations to multivariable functions from radial basis function spaces. These are spaces spanned by radially symmetric functions $\phi(\|\cdot - x_j\|_2) : \mathcal{R}^n \mapsto \mathcal{R}$, where the "centres" $x_j$ are given. The radial basis function $\phi$ is prescribed too. Particularly well-known and useful examples include $\phi(r) = r^2 \log r$, the thin plate spline, and $\phi(r) = \sqrt{r^2 + c^2}$, the multiquadric. We will focus on two approaches to approximating functions from those spaces: interpolation at the centres and least-squares approximations, and we will give a review of several results that have been found during the last two years, say.

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ABSTRACT

## APPROXIMATION WITH AESTHETIC CONSTRAINTS

In this paper we show how to compute a best approximating $C^1$- and piecewise $C^2$-spline-curve, *e.g.*, a circular spline (with sufficiently small segments), subject to constraints for the curvature to be positive, montone, and convex in a certain discretized sense, with respect to certain intrinsic parametrizations. A family of such conditions is proposed which includes ordinary convexity and logarithmic convexity. Logarithmically convex curvature is particularly well suited for automobile design lines. With these constraints, a procedure is developed to approximate data by a discrete list of points for which the existence of an interpolating circular spline with positive, monotone and approximately logarithmically convex curvature can be proved. Some of this is joint work, going back to the 'sixties at GM Research Labs, *cf.* H. G. Burchard, J. A. Ayers, W. H. Frey, N. S. Sapidis, *Approximation with Aesthetic Constraints*, Report GMR-7814, General Motors Res. Labs. (1992), to appear in: **Designing Fair Curves and Surfaces**, N. S. Sapidis, editor, S.I.A.M, Philadelphia, PA, (1993).

# CONSTRUCTION OF TRIANGULAR AND RATIONAL PIECEWISE $C^k$ SURFACES

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First we will give a necessary and sufficient condition so that the projections on a three-dimension space, $L = \pi\omega(\mathcal{L})$ and $R = \pi\omega(\mathcal{R})$, of two surfaces L and R in a four-dimension space, may join with $C^k$-continuity.

Then we will show how these conditions can be explained in terms of massic vectors in the case of triangular rational surfaces.

And eventually we will give an efficient method to construct triangular and rational piecewise $C^k$ surfaces.

## Références:

[1].Canonne,J.C.,J.C. Fiorot , Une condition nécessaire et suffisante de raccordement $C^k$ de surfaces rationnelles,C.R. Acad.Sci.Paris, **312**,série I,1991,p. 897-901.

[2].Canonne,J.C.,J.C. Fiorot,Une condition suffisante de raccordement de surfaces rationnelles triangulaires,C.R. Acad.Sci.Paris, **313**,série I,1991,p. 541-546.

[3].Farin,G. , Triangular Bernstein-Bezier patches, Computer-Aided Geom. Design , **3** ,1986,83-127.

[4].Fiorot,J.C. and P.Jeannin, *Courbes et Surfaces Rationnelles : Applications à la CAO*,Coll.RMA **12** ,Masson,Paris,1989.

[5].Fiorot,J.C. and P.Jeannin, *Courbes Splines Rationnelles* . ,Coll.RMA **24** ,Masson,Paris,1992.

## Monotonicity preserving representations.

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**Abstract:** In Computer Aided Geometric Design, shape preserving properties are crucial for curve and surface design. In this paper we deal with shape preserving properties related with monotonicity. In this sense, a basis of blending functions for constructing a parametric curve from its control polygon preserves monotonicity if the following property holds: If the projections of the control points into a line is a sequence of ordered points, then the projected parametrical curve is given by a monotone function. Furthermore we shall impose that the orientation is preserved. Our goal can be achieved using algebraic tools: A basis of blending functions is monotonicity preserving if and only if the corresponding collocation matrices transform monotone increasing vectors into monotone increasing vectors. We characterize these matrices. In [Karlin], the stochastic transformations preserving monotonicity were characterized in terms of all the minors of order 1 and 2 of the matrix. Here we obtain a simpler test to prove that a matrix transforms a monotone vector into a monotone vector. We also characterize other related classes of matrices. Finally, we study the particular case when the matrix is totally positive and stochastic, which corresponds in Computer Aided Geometric Design to transformations with good shape preserving properties (see [Goodman] and [Carnicer and Peña]). We prove that these representations always preserve monotonicity.

[Carnicer and Peña] Carnicer, J. M.; Peña, J. M.: Shape preserving representations and optimality of the Bernstein basis, to appear in Advances in Computational Mathematics.

[Goodman] Goodman, T.N.T.: Shape preserving representations in *Mathematical Methods in CAD*, T. Lyche and L. L. Schumaker eds., Academic Press, 1989, pp. 333-357.

[Karlin] Karlin, S.: *Total Positivity* Stanford University Press, 1968.

## 3D curve reconstruction from noisy projections

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The data is composed of a small number of projections (three in our application) of a 3D curve. Indeed, these projections are 2D images obtained from aligned sources: for each source, the curve projection is not a curve but is degraded into a noisy grey level image spread over it, [1].

Our approach is of "active curve" type, [2,3,4,5]. Let $\omega$ be the generic curve and $g$ the data. This approach consists in designing an energy $U(\omega, g)$ that measures the compatibility between $\omega$ and $g$, then to compute iteratively: $\min_\omega U(\omega, g)$ starting from an initial curve $\omega(0)$. For most of the applications, the active curve $\omega(t)$ obtained at each iteration converges to a local minimum of $U$, highly dependent of $\omega(0)$. Here, we have to avoid this drawback since we suppose there is a single possible curve hidden in $g$.

Our reconstruction task may be summarized as follows. $\omega$ designates a smooth curved and $\omega + \delta$ is a deformation of $\omega$ by $\delta$. $\omega$ is a parametrized spline function: $\omega = B\theta$ where $B$ is the $B$-spline matrix based on a discretization of $[0, 1]$. $\delta$ is a 3D random vector with a Gibbs distribution. Let $\mathcal{P}^\ell$ be the $\ell$th projection. If $\omega^* = B\theta^*$ is the thrue unknown smooth curve, then we define:

$$U_{\theta^*}(\omega, \delta, g) = \sum_\ell U^\ell_{\theta^*}(\mathcal{P}^\ell(\omega + \delta), \delta, g)$$

$$U^\ell_{\theta^*}(.) = ||\mathcal{P}^\ell(\omega + \delta) - \mathcal{P}^\ell(B\theta^*)||^2 + ||\xi^\ell(\omega + \delta) - \mathcal{P}^\ell(B\theta^*)||^2$$

where $\xi^\ell$ are "attracting" points computed as: $\xi^\ell_i = \sum_s s g^\ell_s d(s - \omega_i - \delta_i)$, $d$ being a distance.

Our goal is to estimate $\theta^*$. We proposed an iterative algorithm in the spirit of a stochastic EM algorithm, [6]. If $\theta'$ is the estimate input in the current iteration, then the new estimate is given by:

$$\min_\theta E[U_\theta(\omega, \delta, g)]$$

$$E[.] = \sum_{\omega, \delta} U_\theta(\omega, \delta, g) P(\delta \mid \omega, g, \theta') P(\omega \mid g, \theta')$$

$P(\omega \mid g, \theta') = \mathbf{1}_{\omega = B\theta'}$ and $P(\delta \mid \omega, g, \theta')$ is a Gibbs distribution whose energy is a weighted sum of a smoothness term and a local distance between $\omega + \delta$ and $g$. The implemented algorithm alternates a deterministic phase D and a stochastic phase S. During the phase D, we compute iteratively $\min_\theta U_\theta(\omega', 0, g)$. During the phase S, given $\omega'$, we compute by a Monte Carlo method $\min_\theta \sum_\delta U_\theta(\omega', \delta, g) P(\delta \mid \omega', g, \theta')$, using simulations of $P(\delta \mid \omega', g, \theta')$.

[1] R. Azencott, B. Chalmond, F. Coldefy, *Poc. ICPR*, The Hague, 1992.
[2] D. Munford, J. Shah, *Proc. CVPR*, San francisco 1985.
[3] U. Grenander, D. Keenan, *J. Appl. Statis.*, 1989.
[4] F. Leitner, I. Marque, S. Lavallée, P. Cinquin, *Curves and Surfaces*, 1991.
[5] D. Terzopoulos, D. Metaxas, *IEEE PAMI*, 1991.
[6] B. Chalmond, *Pattern Recogn.*, 1989.

# Dyadic Affine Decompositions and Functional Wavelet Transforms

**Charles K. Chui**

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Decompostion of continuous functions can be accomplished by considering the difference of consecutive interpolation operators.

In this presentation, we will discuss the structure of such decomposition spaces, including the formulation of bases and their duals, which lead to the notion of the functional wavelet transform (FWT). This transform retains some of the most important properties of the integral wavelet transform of Grossmann and Morlet, such as the property of vanishing moments which has significant applications to engineering problems.

## Reducing the degree of Bézier curves

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**Abstract:**

Various methods for reducing the degree of Bézier curves have been introduced, for example by Watkins and Worsey in 1988. We found that all of these methods lead to discontinuity if you want to reduce the degree of curves consisting of two or more segments of coplanar cubic Bézier curves. Watkins and Worsey suggested that this problem can be eliminated be keeping the endpoints of the segments fixed. This guarantees $C^0$ continuity, but it is more difficult to retain $C^1$ (or more spesifically $G^1$) continuity. We suggest two methods for solving this problem, and discuss the generalization of these methods to curves of higher degree.

**References:**

1. Farin, G. *Curves and surfaces for computer-aided geometric design*, Academic Press, USA (1988)
2. Gregory, J.A. 'Geometric continuity' in: *Mathematical methods in CAGD*, Academic Press, New York (1989), pp 353 - 371
3. Watkins, M.A. & Worsey, A.J. 'Degree reduction of Bézier curves', *Computer-aided design*, Vol 20 No 7 (1988), pp 398 - 405

# NEW METHODS FOR 2-D SHAPE-PRESERVING INTERPOLATION

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It is well known to any researcher working in Numerical Analysis or in Computer Aided Geometric Design that in many practical applications it is often necessary to produce curves or surfaces subject to constraints of various kind. As a typical example, we can consider the modeling of a phisical phenomena, described by an (unknown) increasing function, using a set of experimental data. In this case we must use shape-preserving interpolation techniques, in which the curve assumes given values at the interpolation points and possesses the same shape (in this case monotonicity) of the data .

The problems concerning shape-preserving interpolation (existence, characterization and algorithmical construction) have been widely studied since the seventies and a great amount of papers has been published. However a closer inspection reveals that only few papers are concerned with 2-D shape-preserving interpolation and many important problems have not yet been solved both for the "simple" case of data given on a rectangular grid (it is not clear, for example, how to obtain viable schemes for reproducing the convexity and concavity of the data) and for the interesting one of non regular -scattered- data (for which specifical methods have not yet been devised).

The aim of this talk is thus to present new methods which can be successfully used in shape-preserving 2-D interpolation.

**Tentative title:**

# Smoothness of refinable functions obtained by convolution products

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**Abstract**

Based on a given self-similar lattice tiling, we construct a multiresolution analysis and the associated wavelet basis. A smooth generator is obtained by the convolution product of the characteristic functions of some specific self-similar tiles. It turns out that there is an intimate connection between stationary subdivision and the concept of self-similar lattice tilings. Employing some results from stationary subdivision combined with a judicious choice of representers of the equivalence classes induced by the underlying scaling matrices we are able to derive sharp estimates for the smoothness of the generator.

### Local decompositions of refinable spaces and some applications

Wolfgang Dahmen

**Abstract:** Multiscale techniques have become more and more important in many areas of mathematical theory and applications. They are usually based on appropriate sequences of nested approximation spaces. Decomposing each such space into a direct sum of its predecessor and some complement facilitates successively updating current approximations by including more and more details on finer scales. Specifically, choosing orthogonal complements, leads to wavelet decompositions. However, in particular in the multivariate case, orthogonality interferes with locality of corresponding bases. On the other hand, in many applications it is of crucial importance to work with compactly supported basis functions while orthogonality is not so essential and in some cases not even desirable. This talk is concerned with determining all possible decompositions of refinable spaces which are in a certain sense local and stable. Some of the results to be mentioned were obtained in collaboration with J.M. Carnicer and J.M. Pena. It turns out that these decompositions can be conveniently characterized in terms of bi-infinite matrix relations in that any such decomposition can be generated from an arbitrary fixed one through certain matrix operations. In particular, this allows one to identify decompositions induced by possibly nonorthogonal projectors. This has important applications, for instance, in multilevel preconditioning linear systems arising from Galerkin discretizations of elliptic boundary value problems as well as for matrix compression techniques in connection with the numerical solution of integral or pseudodifferential equations.

# Approximation of stochastic processes with wavelets

J.P. D'Ales

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**Abstract**

The purpose of this work consists in building approximations of non-stationnary random processes using wavelet bases. The case of stationnary processes has already been studied by A.Cohen, J.Froment, and J.Istas. We are dealing here with transient processes, and particularly with piecewise stationnary processes on a time interval $[0, T]$.

To build the approximation, one performs a wavelet decomposition of the signal using the well known Fast Wavelet Transform algorithm of S.Mallat. One then keeps the best wavelet coefficients, and discards the others. Here "best" means larger in amplitude. Finally the approximation is obtained by applying the reconstruction algorithm to the shrinked wavelet transform. This confers the property of spatial adaptation to our approximation: the asymptotic performance is the same for stationnary and piecewise stationnary processes. That means that the presence of isolated discontinuities in a smooth signal does not alter the order of the approximation. Moreover we use the orthonormal wavelet basis adapted on the interval of A.Cohen, I.Daubechies and P.Vial. Thus the "edge effects" are totally supressed and the quality of our approximation does not decrease near the extremities of the signal.

The comparison has been made with the same type of approximations using other orthonormal bases (Fourier, DCT). One shows that the wavelet approximation outperforms the Fourier approximation in the case of transient signals, but not in the case of stationnary processes. Some numerical experiments are being processed and should be available at the time of conference.

# A SURFACE-SURFACE INTERSECTION ALGORITHM WITH A FAST CLIPPING TECHNIQUE

**Marc DANIEL, Alain NICOLAS**

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We present a new approach for surface-surface intersection. An hybrid method suitable for B-splines or Nurbs surfaces has been developped. It is decomposed in two main steps : localization and intersections computations.

The localization is achieved through a fast clipping technique based on the local scheme and the convex hull modeling properties. The position of a surface patch defined over one parametric rectangle depends on the position of a set of neighbouring control points. So, the study of the respective positions of one surface control points and the second surface minmax box allows for defining elementary parametric rectangles on the first surface where there are potential intersections. The algorithm is organized to produce this result very quickly. Connecting the neighbouring rectangles provides, for the first surface, free form areas with potential intersections. Each area can be bounded with a minmax box defined with its associated control points. With these minmax boxes, we can similarly compute the corresponding free form areas for the second surface. This clipping process is an iterative one : it is repeated until the different areas on both surfaces remain unchanged. The convergence is quickly obtained.

Each set of corresponding areas is independently proceeded. It is first checked if there is a degeneracy (tangency or partially identical surfaces) or a closed curve. This test[1] studies and compares normal surface bounding cones. It has been extended for degeneracies detection ; the difference being the existence of an intersection point with colinear normal vectors in the case of degeneracies. If a closed curve is detected (degeneracies are separatly computed using a net of isoparametric curves on both surface and a robust curve-curve intersection algorithm), the area of each surface is split into four new areas with the aim to only obtain opened curve segments on each part. This splitting involves surface subdivisions only if the area is defined over one elementary parametric rectangle. Otherwise, the new areas are immediatly deduced from the separation of the parametric domain. The process for closed curves elimination is a recursive one. Finally, only opened intersection curves can exist on each couple of areas. They necessary begin on area boundary curves (isoparametric curves) and are computed with a marching method. The opened curve segments are eventually connected together to produce the existing closed curves.

(1) T.W Sederberg, H. N. Christiansen, S. Katz, "Improved test for closed loops in surface intersections". Computer Aided Design, vol. 21, n° 8, pp. 505-508, 1989

# Zoonoidal surfaces[1].

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In the euclidian space $E = \mathbb{R}^n$, a zonotope is a convex centrally symmetric set, defined as the Minkowski sum of segments. The closure, in the Hausdorff metric, of the set of zonotopes in $E$, is a set of convex bodies so called zonoids. Some papers deal with the general theory of zonoids and approximation by sequences of zonotopes.[1,2]

The zonoids arise naturally in many contexts. The motivation of our study is the geometric management of mixtures which issues from actual applications in oil industry.[3,4] In the geometric model we use, any product can be viewed as a vector in $\mathbb{R}^n_+$, one component for each species, and the mixing procedure corresponds to vector addition in the vector space $\mathbb{R}^n$. Given a set of basic products $\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_p$, a product $\boldsymbol{m}$ is feasible if and only if $\boldsymbol{m}$ lies in the set $Z(\boldsymbol{n}_1, \boldsymbol{n}_2, \ldots, \boldsymbol{n}_p) = \left\{\sum_{1\leq j\leq p} \lambda^j \boldsymbol{n}_j,\ 0\leq\lambda^j\leq 1\ (1\leq j\leq p)\right\}$ which is a zonotope.

In this paper, we consider the three dimensional euclidian space and we deal with particular zonoids defined by :

$$Z(f) = \left\{\boldsymbol{m} = \int_I \lambda(t) f(t)\, dt,\ \lambda \in L_\infty(I),\ 0\leq\lambda(t)\leq 1\ \left(t\in I\right)\right\},$$

with $I$ a compact interval in $\mathbb{R}$ and $f$ a function in $\mathcal{C}^0\left[I, \mathbb{R}^3\right]$. For example, a zonoid in this previous form can be viewed as the set of feasible mixtures by a distillation process.

We name the boundary of a zonoid as a "zonoidal surface" and we study some structural properties of this surface. Using a generalization of the projective diagram of a zonotope, we obtain the decomposition of the zonoidal surface into a set of convex regular patches. In the particular case of a "convex" projective diagram, the boundary of the zonoid is decomposable into one or two patches and we obtain simple parametric equations of these patches. This basic case is used in a general framework leading to the parametric equations of the patches for a large class of zonoidal surfaces. Then, we can show how the regularity of these patches is related to the regularity properties of the function $f$. At last, we present numerical methods for visualizing a zonoid, computing plane sections of a zonoid, and other related problems.

**References.**

1. E.D. Bolker, "A class of convex bodies", *Trans. Amer. Math. Soc.*, Vol. 145, Nov. 1969.

2. J. Bourgain, J. Lindenstrauss, V. Milman, "Approximation of zonoids by zonotopes", *Acta Math.*, 162, (1989).

3. D. Girard et P. Valentin, "Zonotopes and Mixtures management", *New Methods in optimization and their industrial uses", International Series of Numerical mathematics*, Birkhäuser Verlag, 1988.

4. N. Szafran, "Zonoèdres : de la géométrie algorithmique à la théorie de la séparation", *Thèse de l'Université Joseph Fourier*, Grenoble, Octobre 1991.

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# SPLINES ET COORDONNÉES POLAIRES

La Mécanique ou la Physique sont résolument Métriques. Doit on alors opposer la Géométrie métrique à la Géométrie affine des B-Splines, et peut on aussi continuer à ignorer superbement la présence d'un centre attractif dominant, par une définition par B-Splines de la trajectoire d'un mobile spatial ?

Cas élémentaire des coniques : Optique Géométrique ou orbites planétaires ; Définition symmétrique d'un arc par ses éléments caractéristiques _ Théorèmes de Poncelet : Système de deux miroirs conjugués. Aplanétisme.

I Théorème du moment cinétique et loi des Aires II Potentiel et Force vive Recherche de la continuité avec la définition "Blossoming" de la Parabole.

Difficultés rencontrées _ Vers une généralisation cohérente ou plusieurs à la Carte.

Compatibilité de l'interpolation "de type Spline" avec les coordonnées polaires.

EXPOSÉ POUR LA CONFÉRENCE

"Courbes et Surfaces" Chamonix 10-16 Juin 1992

proposé par :

M^r de CASTELJAU Paul

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## An exclusion-bissection algorithm for computing the intersection of curves and surfaces

Jean-Pierre DEDIEU

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The exclusion-bissection algorithm describded in this talk computes the intersection of n algebraic hypersurfaces in $R^n$ with an accuracy r in $O(Log(1/r))$ steps.

# Fair Surface Design using

# Recursive Subdivision Surfaces

**Tony DeRose**
University of Washington

There has been considerable recent activity in the area of fair surface design. Although several new schemes offer impressive improvements, they still suffer from extreme computation time, or are limited in the topological types of the surfaces they can represent.

In this talk I'll argue that recursive subdivision surfaces, such as the ones developed by Doo-Sabin and Catmull-Clark, offer potential advantages when attempting to overcome the above deficiencies. In particular, I'll describe techniques for efficiently computing fair subdivision surfaces that interpolate the vertices of a given polyhedron.

Joint work with: **Michael Kass** and **Mark Halstead** of Apple Computer, **Joe Warren** of Rice University, and **Michael Lounsbery** of the University of Washington.

## Abstract

*Image Processing and Hyperbolic Wavelets*

Ronald A. DeVore, University of South Carolina

Wavelets have had successful application to several areas including image processing and surface compression. We shall review some of these applications and point out some deficiencies of the usual wavelets. We will then introduce hyperbolic wavelets, discuss some of their properties, and apply them to generic problems in image processing.

Z. Ditzian

Title of talk

"Multivariate Bernstein polynomial approximation"

Abstract

Relation between Bernstein polynomial approximation and a K-functional defined with the aid of a weakly elliptic operator, is established.

# Spline Conversion, Existing Solutions, and Open Problems

Tor Dokken and Tom Lyche

SINTEF SI and The University of Oslo

January 29, 1993

**Abstract**

There is a great need in industry for methods to convert between different representation formats. Such conversion involves much more than transformation between different spline formats. Consider for example the geometry received from PC based CAD-systems. This is often represented by chains of straight lines representing a more complex shape. Another problem is conversion of trimmed surfaces. The transformed surface is often not usable for further design. Examples like these show that the standard conversion tools in use today are not sufficient to fulfill industrial needs. In our talk we will give an overview of some of the conversion problems facing industry, describe alternative solutions, and list some open problems still existing.

# SUBDIVISION SCHEMES

**Nira Dyn**

School of Mathematical Sciences
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In this talk various classes of subdivision schemes for the generation of curves and surfaces will be reviewed: the uniform stationary schemes and their extensions to non-stationary and non-uniform schemes and to integral subdivision schemes. Typical examples of these classes will be presented. Conditions for strong and weak convergence of subdivision schemes will be discussed.

The relation to multi-resolution analysis and to wavelets theory with the analysis of a specific example of $C_0^\infty$ wavelets will be reviewed.

# A Stepwise Algorithm for Converting B-Splines

Matthias Eck

T.H. Darmstadt, Germany

In this talk a method is proposed for converting B-spline curves of order $k$ (resp. B-spline surfaces of order $(k,l)$) approximatively into B-spline curves of any lower order $\bar{k}$ (resp. B-spline surfaces of any lower order $(\bar{k},\bar{l})$). Further, it is required for practical purposes mainly that the resulting B-splines do not consist of too many spline segments.

In contrast to most other conversion methods described in the literature and treating the above polynomial problem, our algorithm carries out the conversion stepwise. Here, in each step either the order of one particular segment is lowered by one or two neighbouring segments are merged together to only one segment. Then these two major steps are repeated in a specific order until the target order of the B-spline is reached. Further, one can hope that the final number of segments is low as the method works carefully in this regard.

Both major steps are based on recent results of the author. So the first step is nearly identical with a degree reduction scheme for polynomial Bézier curves and surfaces whereby certain continuity conditions at the boundaries are preserved. Further, the second step is very close to a knot removal scheme for polynomial B-splines.

Summarized, the main advantages of these two methods are:

1. The new control points are obtained by simple and explicitly known constructions from the given control points only. Here it turns out that the bivariate case is just a two-fold application of the curve construction, firstly, to all rows and, secondly, to all columns of the control net. Altogether, these calculations are very fast.

2. Upper bounds of the maximal approximation error for all $t$ are apriori known in each step. These bounds can be used to carry out the conversion within a given pointwise tolerance. Here, of course, we have to subdivide spline segments where the error tolerance is exceeded.

# Knot Removal for B-Spline Curves

Matthias Eck & Jan Hadenfeld

T.H. Darmstadt, Germany

The approximative problem of knot removal is well known and has been discussed by several authors before (e.g. Lyche, Mørken '90). Here, it is seen as a procedure for representing a given B-spline curve $\mathbf{X}$ of order $k$, defined over the knot vector $T = (t_0, t_1, \ldots, t_{n+k})$, as good as possible by another B-spline curve $\bar{\mathbf{X}}$ of same order $k$ but defined over a shorter knot vector $\bar{T} = (\bar{t}_0, \bar{t}_1, \ldots, \bar{t}_{m+k})$, being a subset of the original knot vector $T$.

In this talk we mainly consider the special case $m = n - 1$ which means the removal of only one knot. This restriction is necessary here since we are interested in explicit solutions for the new curve $\bar{\mathbf{X}}$.

Doing so, we interpret knot removal as an inverse procedure of knot insertion. This idea leads us to a very simple and local construction of the new control points from the ones of the given curve. Now, this construction contains some additional degrees of freedom which are determined explicitly in several ways for a general knot vector $T$ as well as for general order $k$ by minimizing different error functions between $\mathbf{X}$ and $\bar{\mathbf{X}}$.

The error functions, we used, are either "continuous" norms, considering the difference for all parameter values, or "discrete" norms, considering only the difference between the old and new control polygon.

All methods are illustrated with several examples which also show that good results can be obtained if we remove more than one knot by applying our constuctive algorithm for one knot after another.

# Shape from Motion

**Michael Eichhard** **Knut Möller**

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January 27, 1993

Abstract

The need to model surfaces is of great importance in Computer Aided Design (CAD) and in a different setting in control applications. While CAD research is dominated by searching for efficient representations and manipulations of surfaces, control is concerned with modelling dynamic effects. Often in optimal control a prediction problem needs to be solved (e.g. grasping a rolling ball with a robotmanipulator relies on a prediction of the ball trajectory, knowledge about the manipulator dynamics etc.). In control surface representations are mainly needed for

1. environmental modelling (prediction purposes) and
2. as so called decision planes supporting fast decision making in real-time control systems.

Usually the construction of surface modells in CAD is based on Bezier-representations which provide a powerful tool for approximating curves and surfaces. These representations have a number of attractive features e.g. quasi-locality, differentiability, compactness etc. But (from the control point of view) unfortunately the CAD-designer is interactively involved in manipulating representations until they fit his needs. In learning control a different problem has to be faced. From possibly noisy observations (in our case of a moving object) an efficient representation of a surface needs to be derived. We were particularly interested how representations can be utilized for control applications.

In the talk we will give a specification of the problem followed by a description of how to include dynamics into the process of generating a surface representation. Finally the approach is compared with a number of different other approximation schemes including fashionable "neural" learning methods.

# Best constrained approximations of planar curves by Bézier curves

Eberhard F. Eisele

In CAD systems methods for good approximation of a given curve by a certain class of spline curves are frequently needed. For instance, a method for approximate conversion of spline representations is required for exchanging data between different CAD systems or the problem arises to approximate offset curves by spline curves of a certain kind.

Given a parametric planar curve $\mathbf{p}$ and any Bézier curve $\mathbf{x}$ of degree $n$ such that $\mathbf{p}$ and $\mathbf{x}$ have contact of fixed order $k$ at the common end points, we use the normal vector field of $\mathbf{p}$ to measure the distance of corresponding points of $\mathbf{p}$ and $\mathbf{x}$. Then the Bézier curve $\mathbf{x}$ is called a best approximation to $\mathbf{p}$ if the maximum norm $\| \rho_{\mathbf{x}} \|_\infty$ of this distance (or deviation) function $\rho_{\mathbf{x}}$ is minimal for $\mathbf{x}$ and the best approximation $\mathbf{x}$ has been characterized by an alternation property of $\rho_{\mathbf{x}}$ in [Degen '92] and [Eisele '92].

This paper addresses to the problem of characterizing a best approximation $\mathbf{x}$ satisfying certain constraints. We consider the cases that either each $\mathbf{x}$ interpolates to $\mathbf{p}$ at certain prescribed points, or that each deviation function $\rho_{\mathbf{x}}$ has a restricted range; in particular the second case includes one-sided Chebyshev approximation. In both cases we characterize a best constrained approximation $\mathbf{x}$ by an alternation property of $\rho_{\mathbf{x}}$, applying results of Chebyshev approximation with side conditions. Finally, Remes type algorithms can be used for the computation of best constrained approximations.

## References


[1] Degen, W. L. F. (1992): Best approximations of parametric curves by splines. In: *Mathematical Methods in CAGD and Image Processing*, T. Lyche and L. L. Schumaker, editors. Academic Press, Boston.

[2] Eisele, E. F. (1992): Chebyshev approximation of plane curves by splines, *J. of Approx. Theory*, to appear.

# A Geometry Based Approach to the Generation of Unstructured Surface Grids

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We introduce a method to generate unstructured triangular surface grids, that uses the intrinsic geometry of the surface to define the size and the shape of the triangular elements. The method is based on the Advancing Front Technique, but instead of using a background grid, a transformation tensor, $T$, is defined using the principal curvatures, and the principal directions of curvature. These properties are intrinsic to the surface, and not dependent on the particular parametrisation used, the resulting grid is also independent of the parametrisation.

Given a parametrically defined surface, $x(\xi, \eta)$, the two principal curvatures and the two principal directions of curvature are defined by two generalised eigenvalues, $\kappa_{1,2}$, and unit eigenvectors, $\epsilon_{1,2}$, of the system:

$$D\epsilon = \kappa G \epsilon,$$

where $G$ and $D$ are the first and second fundamental matrices of the surface, respectively. The target triangle to be generated at $x$ may now be defined as a triangle inscribed in an ellipse centered at $x$, where the two axes of the ellipse are aligned to $\epsilon_{1,2}$, and their length is proportional to $\kappa_{1,2}$. We refer to the constant of proportionality, $\delta$, as the geometry resolution, and it is globally defined for the surface.

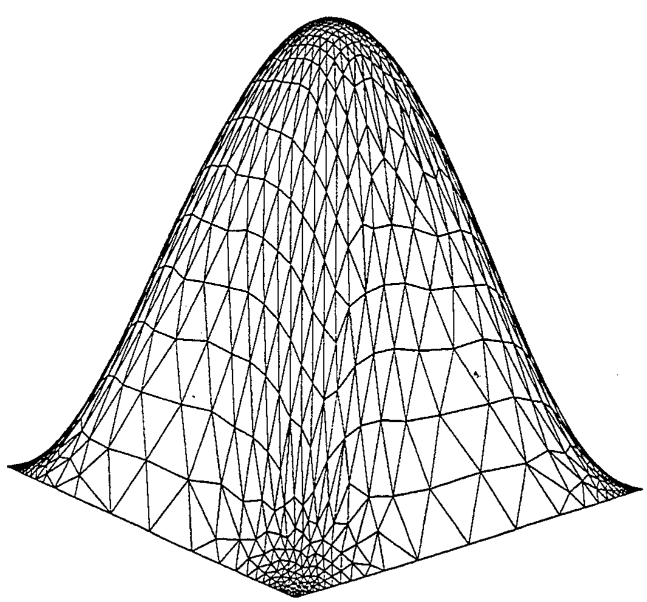
The transformation tensor, $T$, defines a mapping from the parameter space of the surface to a control space, $\mathcal{R}^{2*}$, in which the triangulation algorithm (AFT) strives to generate equilateral triangles of unit size. Given the above, we can now define the tensor $T$ as:

$$T = \begin{bmatrix} \epsilon_{2_\eta}/C/M_1 & -\epsilon_{1_\eta}/C/M_2 \\ -\epsilon_{2_\xi}/C/M_1 & \epsilon_{1_\xi}/C/M_2 \end{bmatrix},$$

where

$$C = \|\epsilon_1 \times \epsilon_2\|, \qquad M_{1,2} = S_{1,2}/\,\epsilon_{1,2} G \epsilon_{1,2}, \qquad S_{1,2} = \delta/\,\kappa_{1,2}.$$

An equilateral triangle of unit size in $\mathcal{R}^{2*}$ will thus have the desired size and shape on the surface.



*A sample grid on the surface $x(\xi, \eta) = (\xi, \eta, \sin\xi \sin\eta)$.*

# Projective cubic B-splines

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Abstract: We will consider piecewise cubics in a projective environment. The concept of differentiability will be based on that of osculants, which are certain blossoms.

We will show how to construct B-spline-like curves with these tools, and show how they relate to standard rational B-splines.

The talk will include a projective treatment of conics in the Bernstein-Bézier form.

# ALGORITHMS INTENDED TO FIND A SIMPLICIAL SURFACE FROM A SET OF 3D DATA POINTS

Chantal FAVARDIN

This study consists in elaborating algorithms based on geometric criteria, that allow to reconstruct a connected surface $\mathcal{S}$, from a discrete unorganized representation. More precisely, we want to determine, from the only knowledge of the coordinates of $n$ $\mathbb{R}^3$-points $M_i^n = (x_i, y_i)$, $i = 1...n$ belonging to $\mathcal{S}$, a polyhedral surface with triangular faces (named simplicial surface), whose vertices are the points $M_i^n$, and that uniformly converges to the initial surface, when the numbers of data points $n$, tends to infinity. Many practical applications, such as pattern recognition, computer vision, medical imagery, where the objects are represented by a set of data points only known by their coordinates, aroused interest in this type of study, that also can constitute a preliminary stage for data interpolation problems. Two types of approach were considered up to now. The first one assumes that the data are divided up into planar crossed sections and so have an initial structure ([2], [4]). These methods consist in determining a triangulation between two adjacent crossed sections. The second one assumes that the points have no initial structure ([1],[3],[5]) and that only the coordinates of the points are known. This is the approach that we consider here. We present two new algorithms to solve this problem.

The first one consists in maximizing the dihedral angle between two adjacent faces, and concerns closed convex surfaces. More precisely, let us assume that the algorithm has already built a simplicial surface, whose vertices, taken among the points $M_i^n$, are noted $P_1^n, ..., P_k^n$. We denote by $P_{b_1}^n, ..., P_{b_l}^n$ the vertices belonging to the boundary of this simplicial surface, (two points with consecutive suffixes are connected). To extend this polyhedral surface, we determine for each $j \in \{1, ..., l\}$ a point $P^n$ defined by:

- $U_k = (\{M_i^n,\ 1 \leq i \leq n\} \backslash \{P_i^n | 1 \leq i \leq k\}) \cup \{P_{b_1}^n, ..., P_{b_l}^n\}$
- $P^n \in U_k$ and maximizes the convex dihedral angle between the triangles $(P_{b_j}^n P_{b_{j+1}}^n I^n)$ and $(P_{b_j}^n P_{b_{j+1}}^n M_i^n)$ for all $M_i^n \in U_k$ (the point $I^n$ designates the point with which the points $P_{b_j}^n$ and $P_{b_{j-1}}^n$ form a triangle of the simplicial surface built so far.

The algorithm stops when the set of points belonging to the boundary of the polyhedral surface is empty.

We prove that the criterion enables to obtain a triangulation of the convex hull of the points $M_i^n$, and that this triangulation is the only one if four points (or more) coplanar, are never cocyclic. So this criterion fits quite the case of points belonging to a closed convex surface.

The second algorithm enables to adapt this criterion to the case of points belonging to a closed non-convex surface. The point $P^n$ is chosen in a neighborhood of $P_{b_j}^n$ and $P_{b_{j+1}}^n$. An insertion stage is also added. The case of surface with boundary curves is solved by an heuristic. The expected time complexity of these different algorithms, that are illustrated by various examples, is in $O(n \log n)$.


1. J.D. Boissonnat,Geometric structures for three-dimensional shape representation, ACM Trans. on Graph., Vol.3, No.4, (Oct. 1984), pp.266-286.
2. J.D. Boissonnat, Shape reconstruction from planar cross sections, Computer Vison Graphics and Image Processing, Vol 44, No 1, (Oct 1988), pp.1-29.
3. H. Hoppe and al. Surface reconstruction from unorganized points, Computer Graphics Vol. 26, No.2, July 1992, SIGGRAPH'92 Conference Proceedings.
4. E. Keppel, Approximating complex surfaces by triangulation of contour lines, IBM J. Res. Develop. (Jan. 1975), pp.2-10.
5. J.O'Rourke, Polyedra of minimal Area as 3D object Models, in Proceeding of the international joint conference on artificial intelligence 81 (Vancouver, Canada) 1981, pp.664-666.


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# Interpolating by conic splines

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Polynomial spline curves do not allow to interpolate given points of the plane and their derivatives. Conic splines i.e. piecewise conic curves solve this $C^1$ Hermite problem. This interpolating problem can consider points at infinity. Conditions are given so that each conic segment does not include points at infinity except possibly at its interpolating end-points.

# PLANAR RATIONAL CUBICS AS BR-CUBICS

J.C. Fiorot, P. Jeannin and S.Taleb

The concept of *massic vectors* enables to represent *all* rational parametric curves under B-rational form, BR-form in abrev: [Fiorot & Jeannin'89,'91,'92]. A BR-curve is determined by a *massic polygon*, likewise a Bézier curve is determined by a control polygon. As particular cases, the BR-form includes polynomial Bézier curves, and rational Bézier curves [Farin'90].

In BR-form, a rational cubic is determined *via* Bernstein polynomials by four massic vectors.

In this work, we treat the problem of detecting some geometric elements such as cusps, loops, inflection points, asymptotes and parabolic points on the BR-cubic.

Conditions for the occurence of cusps, loops, and inflection points are investigated. This leads to a projective classification of planar rational cubics. This study gives a new approach and another numerical point of view than the algebraic aspect given in [Patterson'88], [Walker'50], in order to use them in computer science, CAGD and CAM.

We also determine conditions for asymptotes and parabolic points in terms of massic vectors.

### Some References

1. Boehm W. (1982), On cubics: A survey, Comput. Graph. and Image Proc. 19, 201-226.
2. Farin G.E. (1990), *Curves and Surfaces for Computer Aided Design*, Second Edition, Academic Press, New-York.
3. Fiorot J.C. and Jeannin P. (1989), *Courbes et Surfaces Rationnelles. Applications à la CAO*, RMA 12, Masson, Paris.
4. Fiorot J.C. and Jeannin P. (1991), B-Rational curves and surfaces, N-Rational splines, in: P.J. Laurent, A. Le Méhauté and L.L. Schumaker, eds., *Curves and Surfaces*, Academic Press, New York, 181–184.
5. Fiorot J.C. and Jeannin P. (1992), *Rational Curves and Surfaces. Applications to CAD*, J. Wiley and Sons, Chichester.
6. Forrest A. (1980), The twisted cubic curve: A computer aided geometric design approach, Comput. Aided Design 12, 165-172.
7. Patterson R.R. (1988), Parametric cubics as algebraic curves, Computer-Aided Geom. Design 5, 139-159.
8. Stone M.C., De Rose T.D. (1989), A geometric characterization of parametric cubic curves, ACM Trans. Graph. 8, 147–163.
9. Taleb S. (1991), Incidence du paramétrage sur le contrôle des courbes rationnelles, Thèse Université des Sciences et Technologies de Lille (France).
10. Walker R.J. (1950), *Algebraic Curves*, Princeton University Press, Princeton.
11. Wang C.Y. (1981), Shape classification of the parametric cubic curve and parametric B-spline cubic curve, Comput. Aided Design 13, 199-206.

# A GENERALIZATION OF BÉZIER CURVES AND SURFACES

I. GÂNSCĂ*, Gh. COMAN and L. ŢÂMBULEA**

**Abstract**: As it is known a Bézier curve (or surface) is defined by using the Bernstein basis

$b_{n,k}(t) = \binom{n}{k} t^k (1-t)^{n-k}$, $k=\overline{0,n}$, $t \in [0,1]$ *and a set of points* $P_k$, $k=\overline{0,n}$,

*(or* $P_{ij}$, $i=\overline{0,m}$, $j=\overline{0,n}$, *for a surface)*

In this paper we change the Bernstein basis by the following basis (D.D. Stancu)

$$w_{n,r,k}(t) = \begin{cases} \binom{n-r}{k} t^k (1-t)^{n-r-k+1}, & 0 \le k < r \\ \binom{n-r}{k} t^k (1-t)^{n-r-k+1} + \binom{n-r}{k-r} t^{k-r+1} (1-t)^{n-k}, & r \le k \le n-r \\ \binom{n-r}{k-r} t^{k-r+1} (1-t)^{n-k}, & n-r < k \le n, \end{cases}$$

$k=\overline{0,n}$, $t \in [0,1]$,

where r is a non-negative integer parameter conditioned by the inegality $2r \le n$, and obtain curves and surfaces that generalize the Bézier curves and surfaces, because, for $r=0$ and $r=1$ results $w_{n,r,k}(t) = b_{n,k}(t)$, $\forall t \in [0,1]$. Taking in view that $w_{n,r,k} \ge 0$, $t \in [0,1]$, $k=\overline{0,n}$ and $\sum_{k=i}^{n} w_{n,r,k}(t) = 1$, we have a new partition of the interval [0,1].

One shows that the well known de Casteljau algorithm is applicable, but, in this case, for the variable points $Q_k(t) = (1-t)P_k + tP_{r+k}$, $k=\overline{0,n-r}$, $t \in [0,1]$. For $r=0$ results $Q_k(t) = P_k$ and for $r=1$ we obtain $Q_k(t) = (1-t)P_k + tP_{k+1}$, $k=\overline{0,n-1}$, that is, the points $Q_k(t)$ rezult from the given points $P_k$, $k=\overline{0,n}$, after the first step of the Casteljau algorithm.

Also, the paper contains formulas for derivatives and some figures with curves and surfaces which are compared with the standard Bézier curves and surfaces corresponding to the same set of given points.

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## Corner cutting algorithms and totally positive matrices.

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Corner cutting algorithms play an important role in Computer Aided Geometric Design. Many authors have shown their conection with Total Positivity. In [3] Goodman and Micchelli defined precisely these algorithms and proved that there exists a corner cutting algorithm transforming a control polygon into another one with the same number of vertices if and only if the matrix which relates both polygons is totally positive and stochastic. In fact they gave a detailed matricial description of such algorithms. Goodman and Said [4] proved that there are corner cutting algorithms which transform the control polygon associated to a Ball basis of polynomials of a curve $\gamma$ into the Bezier control polygon of $\gamma$. They conjectured that the same could happen for the control polygon associated to any totally positive basis of polynomials. The conjecture was recently proved by Carnicer and Peña [1].

In this paper, we use the results to appear in [2] and prove that under an extra (but quite natural) condition to be added to the ones in [2] the corner cutting algorithms which are studied there can be carried out in a unique way. More precisely, for a given nonsingular totally positive stochastic matrix there exists a unique factorization which describes a corner cutting algorithm.

[1] Carnicer, J.M. and J.M. Peña : Shape preserving representations and optimality of the Bernstein basis. To appear in *Advances in Computational Mathematics* (1992).
[2] Gasca, M. and J.M. Peña : A matricial description of Neville elimination, with applications to total positivity. To appear in *Linear Alg. and Applic.*
[3] Goodman, T.N.T., Micchelli C.A.: Corner cutting algorithms for the Bezier representation of free form curves. *Linear Alg. Appl.* 99 (1988).
[4] Goodman, T.N.T., Said, H.B.: Shape preserving properties of the generalized Ball basis. *Computer Aided Geometric Design* 8 (1991).

# Characterizations of the set of rational parametric curves with rational offsets

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It is well-known that generally the offsets of a rational parametric curve are not rational. We denote by $I_{\mathcal{R}}$ the set of the rational curves with rational offsets.We give a geometrical characterization of the curves of $I_{\mathcal{R}}$ and the explicit form of their parametrization.The arc length of a curve of $I_{\mathcal{R}}$ is an integral of a rational function.We specify the subset $\mathcal{A}_{\mathcal{R}}$ of $I_{\mathcal{R}}$ consisting of the rational curves with rational arc length:We show that it is the set of the caustics of the rational parametric curves.

# Composite Curve Estimation for Human Body Modelling

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We will report on the state of our research on the analysis of human movement from digital image sequences. In this work, the human body is modelled as a set of articulated objects that are considered representative for the overall shape and the task is to estimate the parameters of these objects from features detected in the image sequence. For the sake of mathematical and numerical tractability the model is restricted to two-dimensional planar ellipses and applied to static images (although intended for a future extension to sequences). As feature data we use the (sparse) contour points detected in the grey level images. Thus, a set of ellipses is fitted to the contour points. As the error measure for the fitting of one curve we use the sum of squares of the deviations $\epsilon_i = f(\underline{x}_i; \underline{p})$ of the implicit relation for the ellipse at contour points $\underline{x}_i$, with $\underline{p}$ representing the position, orientation, and size of the curve. The segmentation problem of identifying which contour point belongs to which ellipse, a major problem when fitting multiple curves to a data set, is integrated in the optimization task by applying a so-called fuzzy representation. In this representation, a normalized membership vector is associated with each feature point, with the components of this vector indicating the importance (weight) of the relation between the data point and each of the curves. The error measure for a set of ellipses is the sum of squares of deviations, weighted by the membership coefficients. The spatial relations between the individual ellipses are expressed as nonlinear constraints on the resulting optimization problem. A hierarchical decomposition allows to choose the level of detail of the representation and helps reducing the risk of being trapped in one of the numerous local minima of the nonlinearly-constrained optimization problem. We will report results on both artificial and real data sets.

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# Invariant Approximation of planar star-shaped contours

F. Ghorbel and V. Burdin

**Abstract:**

Euclidean invariance is widely recognized as an important property of curve and surface shape description algorithms for computer-aided design, pattern recognition and image analysis.

The Euclidean transformations are defined by combining rotations and translations. Euclidean invariance is equivalent to axis-independence. This simply means that applying the curve algorithm to the transformed data set is equivalent to applying the algorithm to the original data set and transforming the resultant curve or surface.

In a series of papers, authors have investigated the general functional form of curve algorithms that are invariant under certain transformation groups. In this paper, we use the radial curve representation with arc length parameterisation in order to obtain axis-independence for any planar curve algorithms. However, we have a restriction, we only consider a particular class of planar shapes called star-shaped curves.

We present a study of the polar arc length parameterization and we show that the radial function which is invariant with respect to translation and rotation, is sufficient to represent completely the star-shaped curves. We obtain an invariant curve algorithm by approximating this radial function rather than cartesian coordinates.

In many applications in pattern recognition or image analysis, this parameterisation can be justified by its adaptation to a wde range of shapes (for example in medical and biological imaging). Finally, we present the advantages and the limits of this method by giving results concerning a particular class of planar contours.

# Polynomial Approximation of Spheres

Sigurd Glærum
University of Oslo

**Abstract.** A quadrilateral approximation scheme for spherical surfaces is presented. The scheme is based on a simple generalization of circle approximations. It will be shown that the spherical aproximant retain many of the properties of the circle approximations. For instance, if the circle approximation is of order k, the spherical approximation will be of order k in each parameter. The geometric continuity between two neighboring spherical approximations is also closely linked to the geometric continuity between two neighboring circle approximations. The problem of dealing with the singularities that occur as a result of using quadrilateral patches is briefly considered.

# Dual Polynomial Bases

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## Abstract

Blossoming, the de Boor-Fix dual functionals, Marsden's identity, the Oslo algorithm, recursive procedures for evaluation and differentiation: these are topics usually associated only with B-splines. But all these formulas have local polynomial interpretations. The goal of this talk is to extend these formulas and techniques to totally arbitrary polynomial, and locally linearly independent piecewise polynomial, bases. These extensions help to unify the theory of univariate polynomials and splines, and they also provide some additional perspective on the special status of the B-spline and Bernstein bases.

We shall also develop necessary and sufficient criteria for extending the blossoming form of the dual functionals to multivariate polynomial bases. We close with a few open questions for future research.

# Interpolation for solutions of the Helmholtz equation

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Using Fourier analysis techniques we construct a Hilbert space whose elements are solutions of the Helmholtz equation. A characterization of the corresponding reproducing kernel is given explicitly. Thus, by minimizing the corresponding norm, we construct an interpolant that passes through the data points on a line. These values sample the solution of the two-dimensional Helmholtz equation. The interpolant, —that for any number of prescribed data points is a solution of the equation— is given by Bessel functions of order zero and one. Since these functions are bell-shaped, the corresponding linear system of equations has a good condition number. This analysis is relevant for the $4\pi$-optical sampling problem by sensors on a screen and for holographic design.

MSC: 41A05, 41A29, 35J05, 78A40

# Curvature of rational quadratic splines

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Abstract

Monotonicity of curvature is investigated for a rational quadratic segment in Bézier form. In particular it is shown that for fixed end-points and tangent directions, provided that the tangents do not turn through more than a right-angle, there is a specific interval of weights for which the curvature is monotone. An algorithm is then given for minimising the number of local extrema of curvature of a piecewise rational curve with specified values and tangent directions at knots.

## B-Spline Knot-Lines Elimination and $C^r$ Continuity Conditions

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Universidad de Chile

Following the work of Dahmen-Micchelli-Seidel [1], where a new scheme for multivariate B-spline is proposed, some insight of its geometrical nature is presented. This scheme for multivariate B-spline provide automaticaly $C^{k-1}$ continuity with piecewise polynomials of degree $k$ but the cost to be payed is a complex structure of the piecewise polynomial spline. Moreover, too much flexibility is provided around de edges of the triangulation. This excesive flexibility can produce irregular surfaces.

An analogy between those B-splines and triangular Bezier patches is given. The affine conditions between the poles of two adjacents Bezier patches for $C^r$ continuity are replaced by similar conditions between the poles of the B-splines. This time the conditions do not produce additional continuity but some knot-lines are eliminated. The "knot-lines elimination conditions" can be use to reduce the flexibility of the scheme.

[1] W.Dahmen, C.A. Micchelli and H.-P. Seidel, *Blossoming begets B-splines built better by B-patches. Math. Comp.*, 59, (1992)

Applications of Constrained Polynomials to Curve and Surface Approximation.

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Abstract:

This talk will describe two families of constrained polynomials and their applications to problems in the approximation of parametric curves and surfaces arising in the exchange of geometric product data.

A generalisation of the classical Chebyshev economisation method provides a method of degree reduction for polynomial functions with minimum error in the sense of the uniform norm. The new polynomial basis provides this degree reduction facility whilst at the same time maintaining the value of the function and selected derivatives at the end points. The application of this method has been extended from simple polynomial functions to parametric polynomial curves and bi-parametric polynomial surfaces. The method provides upper bounds for the approximation errors. With this approach the degree reduction is achieved in a sequential manner reducing the degree by one at each step of the computation. The ability to maintain the values of end points and derivatives makes it possible to subdivide the curve or surface and obtain polynomial spline approximations of greater accuracy.

The constrained orthogonal polynomials also provide an optimum approximation combined with the ability to maintain continuity conditions at the end points of parametric curves and along the boundaries of bi-parametric surfaces. In this case the approximation is optimal in the least squares sense and also provides an easy method of computing the mean square error in the resulting approximation. At the core of the computational method is the evaluation of integrals of products of the curve or surface to be approximated and the basis functions. This means that although the technique can be applied to degree reduction problems it can also be applied to any more general approximation problem in which the curve or surface to be approximated is sufficiently well defined for the integrals to be evaluated. The method has been applied to obtain polynomial approximations to rational curves and surfaces, to approximate offset curves and surfaces, to approximate procedurally defined surfaces and to obtain a single span high degree parametric polynomial approximation to a spline curve. A generalisation of the method makes it possible to a bi-parametric polynomial approximation to a trimmed portion of a surface.

**Semiregular B-spline surfaces**

**Jens Gravesen**
**Mathematical Institute**
**The Technical University of Denmark**

A B-spline surface can be considered as the surface swept out by a B-spline curve with a fixed knot sequence and with the control polygon moving along a B-spline curve. If the knot sequence is allowed to vary, we obtain a new type of surface called a semiregular B-spline surface. This type of surface has some advantages over the ordinary (regular) B-spline surface with respect to its interpolating properties and the possibility to describe surfaces with singularities.

# Blending functions and scattered data interpolation using surfaces with minimal curvature

Günther Greiner
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Universität Erlangen, Germany

**Abstract**

Finding a blend (i.e. a rather smooth transition between primary surfaces) as well as scattered data interpolation finally leads to the following problem: One has to construct a surface which satisfies certain (linear) constraints and is nice looking (e.g. does not oscillate too rapidly). In case of blending, the constaints are boundary conditions (determined by the primary surfaces). In the other case interpolation gives the constraints.

We present a method based on variational principles to construct these surfaces. The basic idea is as follows. In a (sufficiently large) class of tensor-spline surfaces one has to determine the surface which satisfies the constraints and in addition minimizes a certain functional. The functional will be chosen such that a mixture of mean curvature and surface area become minimal. We describe the procedure in detail and compare it with other approaches.

USenglish

# Design of Blending-Surfaces in an Object-Oriented Framework

Günther Greiner[†]
Philipp Slusallek[†]

**Abstract:**

In computer graphics and geometric modeling one generally faces the problem of integrating a variety of curve and surface types into a single program. Blending surfaces are especially difficult, because by their definition they need to interface to all other supported surface types. Object-oriented design offers the opportunity of using the inherent hierarchical structure of curves and surfaces to solve this problem.
This paper presents an object-oriented framework together with its C++ implementation that starts from an abstract class of general differentiable curves and surfaces and in turn refines this design to curves and surfaces that are explicitly given in either parametric or implicit form. The standard curve and surface types are derived from these abstract classes.
An approach to design blending surfaces in this framework is presented. Also, examples that visualize the differential geometry of curves and surfaces illustrate the approach.

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**Abstract**

"Convex interpolation with minimal energy"

Christoph Henninger - Karl Scherer

The following problem is considered: let $P_0, \ldots, P_{N+1}$ be data points in $\mathbf{R}^2$ such that the polygon passing through the $P_i$ is convex. Then determine a curve $F(t) : t \in [a, b] \to \mathbf{R}^2$ such that the functional

$$J(F) := \int_a^b \|F"(t)\|^2 dt \qquad (\|\| = Euclidean)$$

is minimized under the constraints

$$F(t_i) = P_i, 0 \leq i \leq N+1; \quad \kappa(F) := F' \otimes F" \leq 0 \quad .$$

Here $\otimes$ denotes the cross product. The nodes satisfy $a = t_0 < t_1 < \ldots < t_{N+1} = b$.

It is shown that the above problem does have always a solution. Every solution is $C^2$ - continuous and piecewise cubic in each segment $(t_i, t_{i+1})$ with at most two extra knots. Criteria are given which allow to determine their number and positions exactly. Based on this characterization an algorithm for determining a soltuion is constructed.

# Connection Matrix Splines

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and
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**Abstract**

Connection matrix splines are a generalization of geometrically continuous splines that are based on arbitrary connection matrices. The `CMSBasis` library implements cubic connection matrix spline curves with arbitrary knot sequences and arbitrary connection matrices in C++ as part of the Waterloo spline library. The hierarchy in this implementation is

$$\underbrace{\texttt{Curve} \rightarrow \texttt{LCCurve} \rightarrow \texttt{BCurve}}_{\text{Part of the Curve library}} \rightarrow \underbrace{\texttt{CMSCurve} \rightarrow \texttt{ExplCMSCurve}}_{\text{Part of the }\texttt{CMSBasis}\text{ library}}.$$

Underlying each `ExplCMSCurve` is a `ExplCMSBasis`, which has the following ancestry

$$\underbrace{\texttt{FuncBasis} \rightarrow \texttt{LCBasis} \rightarrow \texttt{BBasis}}_{\text{Part of the Curve library}} \rightarrow \underbrace{\texttt{CMSBasis} \rightarrow \texttt{ExplCMSBasis}}_{\text{Part of the }\texttt{CMSBasis}\text{ library}}.$$

The `Expl` prefix to the classes indicates that all the knot/connection matrix information is explicitly stored, and thus variable.

# A Local Convexity Preserving Interpolation Method

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**Abstract**

We consider the problem of interpolating scattered 3D data by a geometrically smooth surface. Assuming that data points, corresponding normals and a triangulation are given, we propose a parametric local method which constructs a smooth surface, interpolating the given data, with the additional property that it is convex if the data (and the triangulation thereof) are. First we construct boundary curves and define normals on them. The next thing to do is to fill in each patch, which we shall perform by a kind of blending. This results in a smooth convex surface, which is non–polynomial. An analysis of the parametric method is given and some examples are presented.

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# Mesh Optimization

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We present a method for solving the following problem: Given a set of data points scattered in three dimensions and an initial triangular mesh $M_0$, produce a mesh $M$, of the same topological type as $M_0$, that fits the data well and has a small number of vertices. Our approach is to minimize an energy function that explicitly models the competing desires of conciseness of representation and fidelity to the data.

We show that mesh optimization can be effectively used in at least two applications: surface reconstruction from unorganized points, and mesh simplification (the reduction of the number of vertices in an initially dense mesh of triangles).

## Conversion of (trimmed) integral and rational B-Spline surfaces

Josef Hoschek, Franz-Josef Schneider
TH Darmstadt

May be given a set of (trimmed) integral or rational Bézier tensor Product surfaces of degree (n,m) or a set of integral or rational B-Spline surfaces of order (k,l). These surfaces shall be converted to rational or integral B-Spline surfaces with respect to a required order $(k^*,l^*)$ and a minimal number of patches with respect to a given (distance) error tolerance.

First generic new boundary curves (knots in the knot vector) are introduced with respect to the curvature of the parametric lines of the given set of surface patches, then we choose a suitable set of points on the given surfaces and approximate these points with the discrete norm and additionally parameter correction.

The trimming curves are proposed to be given in the parametric domain, the error tolerance for the given and the approximated trimming curves is measured on the surfaces. This approximation is also combined with parameter correction.

If the given set of B-Spline patches has $C^1$- or $C^2$-continuity, this continuity class can be hold. The method can also be used for merging of sets of Bézier surfaces and data reduction.

Conference on CURVES AND SURFACES
Chamonix-Mont-Blanc, June 10 – 16, 1993

## Riesz Bases in Scattered Data Approximation

Kurt Jetter, Universität Duisburg

Stable $L_2$-Approximation with translates $\varphi(\cdot - x_k), k \in I$, of a basis function $\varphi : \mathrm{R}^d \to \mathrm{R}$, where $X = \{x_k; k \in \mathrm{Z}^d\} \subseteq \mathrm{R}^d$ is a set of knots and $I \subseteq \mathrm{Z}^d$ is a *finite* index set, requires that the translates $\varphi(\cdot - x_k), k \in \mathrm{Z}^d$, are a Riesz basis for $S_\varphi = \mathrm{clos}_{L^2} S_\varphi^0$, where

$$S_\varphi^0 := \{\sum_{k \in I} c_k \varphi(\cdot - x_k); I \subseteq \mathrm{Z}^d, \mathrm{card(I)} < \infty\}.$$

Equivalently, in terms of the auto-correlation function $\phi = \varphi * \varphi^*$ ( with $\varphi^*(x) = \overline{\varphi}(-x)$ the involution of $\varphi$ ), one asks for uniform lower and upper bounds $0 < \lambda \leq \Lambda$ for the spectrum of the collocation matrices

$$A_I = (\phi(x_\ell - x_k))_{k,l \in I}.$$

The talks aims at reporting on recent results along these lines when $\varphi$ or $\phi$ are given by preconditioned versions of radial basis functions which originate from univariate functions in the way $\phi(x) = g(\|x\|^2)$ and $g$ is an m-th order completely monotone function on the positive real numbers. In some aspects, results from nonharmonic Fourier analysis are useful.

# A Geometrical Approach to Interpolation on Quadric Surfaces

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Quadric surfaces like spheres, hyperboloids of one or two sheets, elliptic and hyperbolic paraboloids are often used in industrial applications. The paper presents a geometrical approach to the interpolation of given points by rational Bézier- and B-spline-curves and -surfaces on quadrics. Based on a result from number theory, a generalization of the stereographic projection on the unit sphere has been introduced in [Dietz & Hoschek & Jüttler'93]. This paper studies the properties of this generalized projection.

The generalized stereographic projection is shown to be the composition of the usual stereographic projection with a second map. The second map is called hyperbolic projection. Its properties are presented and some connections to kinematics and to advanced geometry are outlined. The properties of the generalized stereographic projection result immediately from those of the hyperbolic projection. The extension of the results to arbitrary nondegenerated quadric surfaces is given.

As a first application, the generalized stereographic projection is applied to the construction of rational Bézier surface patches on the unit sphere from given boundaries. Furthermore, interpolation with rational curves on quadrics will be shown to be a linear problem. More detailled constructions of curves and surfaces on quadrics will be given in a paper by R. Dietz to be presented at the 3. SIAM-Conference on Geometric Design (Tempe, November 93).

## References

- Dietz, R., Hoschek, J. and Jüttler, B. (1993): An Algebraic Approach to Curves and Surfaces on the Sphere and on Other Quadrics, accepted for publication in *Computer Aided Geometric Design*

# Decomposed Surface Editing

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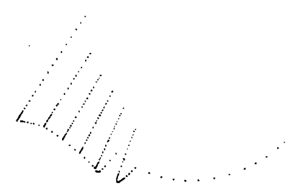
February 3, 1993

**Abstract**

Decomposition is an important tool for representation and manipulation of geometry, with applications in a variety of areas. A main idea is to split a model into a sequence of components with common properties, such that each component can be handled separately. The decomposition could for instance be based on the level of details in the representation. This gives a better flexibility for representation and manipulation. Our idea is to use the principle in reverse, by combining geometrical components into a complete form. An example would be to add local features on a global underlying surface.

In this paper we look at a problem often met in CAD industry. A master surface has to be modified for a specific use. For this problem we have developed a new surface editing scheme based on decomposition. We call it the 'face lift' scheme. From the basic surface and scattered points on the modified surface this scheme generates the difference between the existing surface and the desired result. This 'face lift' surface should also satisfy certain smoothness constraints. The 'face lift' surface is constructed by use of scattered data techniques and tensor product B-spline approximation. The illustration shows 'face lift' of a turbine blade.



**Face Lift Data**

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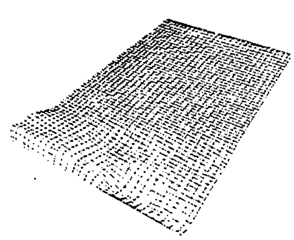

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**Basic Surface** | **Difference Surface** | **Modified Surface**

# GENERIC PROPERTIES OF THE SET OF CROSS-SECTIONS AND THE SET OF ORTHOGONAL PROJECTIONS OF A SMOOTH SURFACE

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We give a complete classification and description of the generic local properties of the family of cross-sections of a surface and the family of orthogonal projections of a surface. A smooth 2-manifold being given, we look for properties that are generic in the space of its embeddings in 3-space. These properties will single out qualitative types of smooth mappings mainly depending on types of singularities.

In the case of cross-sections, the position of a cross-sectional plane depends on 3 parameters. Given an embedded surface and a line in space, the set of its cross-sections by the planes which are normal to the direction can be studied through the orthogonal projection of the surface on the line. The generic properties of that object are a classical result of Morse theory. To account for the whole family of cross-sections, one makes this object depend on the direction parameter, to be taken as a point on a sphere (we neglect antipodal identification and consider oriented directions to avoid introducing 2-dimensional projective space). The locus of points in that control space for which Morse theory does not apply (unstability points, or bifurcation points) is generically made of smooth curves with cusps and possibly crossings. These curves are he images through the Gauss map of the parabolic points of the surface under study. Their cusps correspond to the "godrons" of the surface; observing them requires complete control on the cross-sectional plane (their codimension is 2).The qualitative patterns of the 1-family of parallel cross-sections at each of such bifurcation control points and their unfolding when the control direction is perturbed are described. The bifurcation diagram can be computed from the equation of the surface. Applications to cross-sectional medical imaging and surface coding are briefly presented.

The case of projections is more complex and we only summarize the results. Given a single orthogonal projection, its generic properties are a consequence of the classical work of Whitney on singularities of mappings. As in the case of sections, to get the whole family one makes the projection depend on the oriented direction, i.e. a point on a sphere. The observable qualitative changes in the projection as the direction is moved are now due to two classes of local surface configuration : parabolic points and swallow-tail points. In control space, one describes curves associated either to beek-to beek or lip projection patterns (parabolic class) or to swallow-tail projection pattern; these curves correspond to curves on the surfaceunder study. Codimension 2 patterns include the "godron" (merging parabolic and swallow-tail points), the "gouttiere" (transition between beek-to-beek and lip points), and the butterfly (swallowtail class). The bifurcation diagram can be computed from the equations of the surface. Applications to projection medical imaging (standard radiology) are presented.

REFERENCES:

Y.L.KERGOSIEN, R. THOM: "Sur les points paraboliques des surfaces", C.R.Acad. SC. Paris, t. 290 (21 avril 1980), Serie A-705-710.

Y.L. KERGOSIEN: "La famille des projectons orthogonales d'une surface et ses singularites", C.R. Acad. Sc. Paris, t. 292 (1er juin 1981), Serie I-929-932.

Y.L. KERGOSIEN: "Generic sign systems in medical imaging", IEEE CG&A, Vol 11, No 5, pp. 46-65 (Sep. 1991).

Y.L. KERGOSIEN: "Topology and visualization : from generic singularities to combinatorial shape modelling". In : Modern Geometric Methods for Computing, Kunii T. ed. Springer 1992, pp. 31-54.

## Multiresolution Approximation in Statistic

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It is very usual in Statistics to reduce a non parametric problem (such as estimating the common density of n independent random variables) to a simpler one of finite dimension with the help of some classical results in approximation theory. In this talk, we'll try to show that in some specific problems , it is absolutely necessary to use an approximation taking into account different levels of resolution at the same time. And we would like to emphasize that in such cases the wavelets methods appeared to be much more accurate than the classical ones ( Kernel approximations, Fourier series,... 0 We'll particularly illustrate this phenomena in two special cases :

We'll first consider the density estimation problem when there is no artificial link between the a priori knowledge on the density and the measure of the loss of an arbitrary procedure. In that case we'll show that the wavelet framework allows us to construct an estimation procedure which is nearly optimal almost without any a priori knowledge.

The second example consists in estimating a non linear function of the density of integral type. We'll particularly focuse on the critical case occuring for densities of low regularity. In this case, we'll see that the Haar basis, with its very specific properties (both of wavelet and martingale types) appears to be very important.

# Finding Shortest Paths on Graph Surfaces

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## Abstract

This paper presents a new algorithm for determining minimal length paths between two points or regions on a three dimensional surface.

The numerical implementation is based on finding equal distance contours from a given point or area.

These contours are calculated as zero sets of a bivariate function designed to evolve so as to track the equal distance curves on the given surface.

The algorithm produces all minimal length paths between the source and the destination areas on the surface given as height values on a rectangular grid.

Complexity and accuracy are governed by the grid resolution and the distance step size in the iterative scheme.

Using the distance maps from three areas we also solve the Steiners'problem (for three points) on a surface.

**Key Words:**
Curve evolution, Equal distance contours, Geodesic path,
Partial differential equations, Digital implementation,
Numerical algorithms.

# The Construction of TPBC(Triplicated Piecewise Bezier Cubic)-Curve as PC-Graphics Tool


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## ABSTRACT


The degree of Bezier curve constructed with a set of discrete plane points increases in proportion to the number of data points if we define a whole curve as one Bezier curve. Thus, in computer graphics and computer-aided geometric design, Bezier curves require higher computational complexity.

To avoid this problem, it is necessary to construct a modified Bezier curve which is a piecewise low degree polynomial curve. For this purpose, we bind a set of given data points with 4 data points overlapped in turn and construct a cubic Bezier curve on each subinterval, and compute a linear combination of these cubic Bezier curves to generate a smooth curve. Our resulting curve is cubic on the whole interval and is an approximate curve while keeping the characteristics of Bezier curve. In fact, we construct a new cubic curve which fits very closely with the Bezier curve, which is called as a Triplicated Piecewise Bezier Cubic(TPBC)-Curve.

# Polygonalization Of Real Algebraic Surfaces

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I would like to submit the following abstract to the conference

## Curves and Surfaces

Chamonix Mont-Blanc, France, June 10-16, 1993

An algorithm is presented for polygonizing real algebraic surfaces. The algorithm is based on an adaptive marching cube technique combined with a Bernstein-Bézier-representation of the polynomials. Using the Bernstein-Bézier-representation, an estimation of the maximum curvature of the surface in a tetrahedral region can be achieved. This estimation allows to control the adaptive subdivision in an easy and efficient way.

The Bernstein-Bézier-representation is also useful to detect those regions of the space, which do not contain any parts of the surfaces. This property provides a possibility to overcome one problem of the marching cube technique: it fails in cases, where the surfaces consist of several not connected components, or where components of the surfaces are only connected at singular points.

We discuss this algorithm in detail and compare it to existing methods. Several examples will illustrate the approach.

**Enlèvement de nœuds et splines.**
**Courbes paramétriques.**

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On suppose donnés deux entiers naturels $n$ et $k$ avec $k \geq 2$. Soit $\mathbf{t} = (t_i)_{i=1}^{n+k+1}$ une suite croissante (strictement) de réels. On suppose que l'on dispose de $n+1$ vecteurs $(C_i)_{i=1}^{n+1}$ de $\mathbb{R}^d$, où $d$ est égal à 2 ou 3 dans la pratique—le cas des fonctions étant traité de la même manière . On considère la courbe spline définie par $\vec{\mathbf{s}}(x) = \sum_{i=1}^{n+1} C_i B_{i,k,\mathbf{t}}(x)$, où les $B_{i,k,\mathbf{t}}(x)$, pour $1 \leq i \leq n$, sont les B-splines d'ordre $k$ associées à $\mathbf{t}$.

Si on élimine un nœud quelconque $t_j$ avec $k+1 \leq j \leq n+1$ de $\mathbf{t}$, un nœud intérieur donc, pour former $\tau$, on cherche alors à déterminer une courbe $\vec{g}$ approchant $\vec{s}$ avec comme contrainte $\|\vec{g} - \vec{s}\| \leq \varepsilon$, où $\varepsilon > 0$ est une tolérance donnée et $\| \, . \, \|$ désigne une norme discrète à préciser. On cherche une courbe $\vec{h}(x) = \sum_{i=1}^{n} A_i B_{i,k,\tau}(x)$ définie à l'aide de $\tau$ et de $\vec{\mathbf{s}}$. Dans la base formée par les $B_{i,k,\mathbf{t}}(x)$ elle se met sous la forme

$$\vec{h}(x) = \sum_{i=1}^{n+1} \tilde{A}_i B_{i,k,\mathbf{t}}(x)$$

où les $\tilde{A}_i$ sont obtenus à l'aide de l'algorithme d'insertion de nœuds de Böehm en fonction des $(A_i)_{i=1}^{n}$ par

$$(*) \qquad \tilde{A}_i = \begin{cases} A_i & i \leq j-k \\ \alpha_i A_i + (1-\alpha_i) A_{i-1} & j-k+1 \leq i \leq j-1 \\ A_{i-1} & j \leq i \leq n+1 \end{cases}$$

où si $z = t_j$, on a $\alpha_i = (z - \tau_i)/(\tau_{i+k-1} - \tau_i)$.

On détermine les $A_i$ en identifiant les $\tilde{A}_i$ pour $i \leq j-k$ ou $j \leq i \leq n+1$ aux $C_i$ correspondant. Les autres $A_i$ restant sont obtenus par substitution dans $(*)$.

On peut ainsi trouver deux courbes $\vec{h}_1$ et $\vec{h}_2$, puis l'on retiendra celle qui est telle que l'erreur $\|h_i - \vec{s}\|$ est la plus petite, et que l'on note $\vec{h}$. Alors $t_j$ sera un nœud qui peut être éliminé si $\|\vec{h} - \vec{s}\| \leq \varepsilon$.

En itérant le procédé sur chacun des nœuds intérieurs $t_j$, on détermine un ensemble $\xi = (\xi_1, \ldots, \xi_q)$ de nœuds susceptibles d'être éliminés et on forme $\tau = \mathbf{t} \setminus \xi$, puis on trouve une courbe $\vec{f}(x) = \sum_{i=1}^{n+1-q} D_i B_{i,k,\tau}(x)$ telle que $\|\vec{f} - \vec{s}\| \leq \varepsilon$. On peut appliquer le procédé sur la courbe $\vec{f}$, et ainsi de suite , en comparant à chaque fois avec $\vec{\mathbf{s}}$, tant que l'opération n'altère pas $\vec{\mathbf{s}}$ de plus de la tolérance $\varepsilon$.

# MULTIVARIATE SPLINES CONSTRUCTED THROUGH PIECES OF POLYHARMONIC FUNCTIONS

Ognyan Iv. Kounchev

In the simplest case (the biharmonic), the **polysplines** are the solution to the following extremal problem

$$\int_D (\Delta f(x))^2 \, dx \longrightarrow \inf,$$

where the minimization is over a proper class of functions f such that $f(x) = g(x)$ for $x \in S = \bigcup_{j=1}^{N} T_j$, where $g(x)$ is a function defined on N closed smooth surfaces $T_j$ in $\bar{D}$, such that $T_j$ surrounds $T_{j-1}$, and $T_N = \partial D$. Also $(\partial/\partial n_N) f(x) = 0$, $x \in \partial D$, where $n_j$ denotes the inner unit normal vector to $T_j$.

The solution $s(x)$ to the above problem satisfies:

(i) $\Delta^2 s(x) = 0$ for $x \in D \backslash S$;

(ii) If we denote by $s_j(x)$ the restriction of $s(x)$ to the layer lying between $T_j$ and $T_{j-1}$, then $s_{j+1}(x) = s_j(x)$, $(\partial/\partial n_j) s_{j+1}(x) = (\partial/\partial n_j) s_j(x)$, $\Delta s_{j+1}(x) = \Delta s_j(x)$, all for $x \in T_j$, $j = 1,\dots,N-1$.

1. **Existence** and **uniqueness** of polysplines is proved in proper Sobolev-Slobodeckii spaces.

2. **Convergence** is proved when $N \longrightarrow \infty$ and the data function g is defined in the whole of the domain D.

3. Due to the Green's formulas the **radial basis** generated through the fundamental solution of the polyharmonic equation is dense in the polysplines defined above. Correspondingly, density results for such radial basis are proved through point 2.

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On approximation by bivariate incomplete polynomials

András Kroó

Let $K \subset \mathbb{R}^2$ be a star - like domain and consider polynomials of the form

$$* \qquad p(x,y) = \sum_{\theta n \le k+\ell \le n} a_{k\ell}\, x^k\, y^\ell\ ,$$

where $0 < \theta < 1$ is a fixed number. These polynomials are bivariate analogues of Lorentz' $\theta$-incomplete polynomials. The univariate theory implies that in order that $f \in C(K)$ can be approximated by these polynomials it is necessary that $f \equiv 0$ on $\theta^2 K$. In this paper we study the following question: can any $f \in C(K)$ such that $f \equiv 0$ on $\theta^2 K$ be approximated by polynomials of the form $(*)$? It turns out that if $K$ is a countable intersection of triangles with a vertex in the origin then the answer to the above question is affirmative. Thus the admissible domains include triangles and parallelograms, certain polygons and sections of discs. The proof is based on combining Müntz-type approximation with approximation by bivariate Bernstein polynomials on triangles.

We shall also discuss extension of these results to domains of higher dimension (countable intersections of $r$-dimensional simplices with a vertex at the origin).

# On the Fast Evaluation of Integrals of Wavelets and Box Splines

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**Abstract**

For solving partial differential equations by a Galerkin method, one has to compute the entries of mass and stiffness matrices which are integrals of (derivatives of) the trial functions. In the concept of refinable shift-invariant spaces, these expressions can be determined quickly and exactly (up to round-off) by solving a simple eigenvector problem. Here, we will focus on the algorithm and the application to biorthogonal wavelets and different two-dimensional box splines.

In addition, since integrals of box splines are again box splines, we will use the procedure for the fast evaluation of box splines, in particular, with many direction vectors.

MONOTONIZING PARAMETRIZATION FOR INTERPOLATIONAL SPLINES

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Let $P_i=(x_i,y_i)$, $i=0,1,\ldots,N$ be a sequence of pairwise different points in the plane XY. In order to draw a curve passing through these points in general case it is necessary to construct a mesh $\Delta$: $a=t_0<t_1<\ldots<t_N=b$ and to define a continuous vector-function $C(t)=(C_x(t),C_y(t))$, $t\in[a,b]$ such that $C_x(t_i)=x_i$, $C_y(t_i)=y_i$, $i=0,1,\ldots,N$, i.e. $C(t_i)=P_i$, $i=0,1,\ldots,N$. The form of the curve depends both on the knot selection and on the interpolation method.

In this paper, the notion of a parametrization correspondence with the given data has been introduced. We propose an algorithm of data dependent parametrization guaranteeing a preservation of the monotonicity properties for sufficiently arbitrary initial data. It is based on the proved invariance properties in a parametric space of n degree interpolating polynomials and splines. We establish a close connection of the suggested parametrization knots choice with sufficient conditions of monotonicity for cubic splines. The algorithm is generalized for the case of an multi-form surface approximation using NURBS. The given results of the numerical calculations exhibit our method as more preferable than generally accepted uniform and accumulated chord length parametrizations.

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# Computations of Curvatures Related to Surface-Surface Filleting/Chamfering.

Abstract

Fillet (and chamfer) constructions between two parametric surfaces usually employ tracing algorithms. Accuracy conditions require careful choice of the step size at every iteration of the algorithm. To determine the step size, one has to know exact curvature values for contact and spine curves in the object space as well as in parameter spaces of the surfaces.

The paper shows how to compute these values in cases of constant radius fillets and general fillets.

General fillets can be defined either locally, when every fillet arc radius depends only on local properties of the surfaces, or globally, when the radius depends on the position of the arc with respect to the entire fillet surface. Construction and multiple parametrizations of the corresponding Voronoi surface are instrumental in the derivations.

Similar derivations can be applied to construction of non-isosceles (elliptical, etc.) fillets and chamfers.

# LEAST-SQUARES OPTIMIZATION OF *THREAD* SURFACES


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In geology, faults may sometimes be considered as slipping surfaces between two (approximately) rigid blocks. Geometrically, we define these surfaces as follows: they are preserved by a (at least) single-parameter family of moves, and we call them *threads* by analogy with a screw and a nut. Besides, we need to optimize our subsurface models for oil exploration purposes. Therefore, we propose a method that optimizes a surface with respect to several least-squares criteria related to curvature, proximity to known points, and the "thread property".

This definition of threads is equivalent to following a characteristic property [suggested by C. Marle]: there exists at least one nonzero twistor $W$ such that $W(P) \in T_P\mathcal{S}$ at any point $P$ on the surface $\mathcal{S}$, with $T_P\mathcal{S}$ being the plane tangent to $\mathcal{S}$ at $P$.

We represent the unknown surface $\mathcal{S}$ parametrically by a function $(x, y, z) = \Phi(u, v)$, which we discretize using B-spline tensor-products. The optimization problem consists in minimizing the overall objective function $Q(\Phi, W) = Q_\varphi(\Phi, W) + Q_\alpha(\Phi, W)$, where twistor $W$ is an auxilliary unknown.

$Q_\varphi$ is the weighted *geometrical* objective function $Q_\varphi = W_{P\varphi}Q_{P\varphi} + W_C Q_C + W_T Q_T$. $Q_{P\varphi} = \sum_i \|(P_i, \Phi_i)_\perp\|^2$ measures the length of the normal part of the vectors that link known points $P_i$ and arbitrarily chosen points $\Phi_i = \Phi(u_i, v_i)$ of a parameterization $\Phi$ of the surface. $Q_C = \int_\mathcal{S} (\lambda_1^2 + \lambda_2^2) d\mathcal{S}$, where $\lambda_1$ and $\lambda_2$ are the principal curvatures at each point on the surface. $Q_T = \int_\mathcal{S} < N(P), W(P) >^2 d\mathcal{S}$, where $< N(P), W(P) >$ is the scalar product at $P$ of unit normal $N$ and twistor $W$.

$Q_\alpha = W_{P\alpha}Q_{P\alpha} + W_D Q_D + W_N Q_N$ is an *additional* objective function. Objective functions $Q_{P\alpha} = \sum_i \|(P_i, \Phi_i)_{tang}\|^2$ and $Q_D$ are necessary to constrain those of the degrees of freedom in the space of the parametric representations which change function $\Phi$ if they are perturbed, but preserve the surface itself. $Q_D$ is a smoothing term based on $\Phi$'s second derivatives. Twistor $W$ needs to be nonzero, because any surface would be a thread without this condition: this leads to a normalization objective function $Q_N = [(\int_\mathcal{S} \|W\|^2 d\mathcal{S})/(\int_\mathcal{S} d\mathcal{S}) - 1]^2$. A proper choice of weights $W_{P\varphi} \ldots$ enables these criteria to be adequately balanced.

The implementation of this optimization problem using the Gauss-Newton procedure leads to satisfactory numerical results.

# Approximating Reachable Sets by Extrapolation Methods

Curves, surfaces and higher dimensional manifolds, which are implicitly defined as submanifolds of reachable sets of controlled dynamical systems, comprise a challenging object of approximation methods. In this talk, our main interest lies in difference methods, especially in order of convergence results, for the discrete approximation of reachable sets with respect to Hausdorff distance.

We concentrate on a special approach for the numerical approximation of reachable sets of linear differential inclusions. This approach is based on the computation of Aumann's integral for set-valued mappings. It consists in exploiting ordinary quadrature formulae with nonnegative weights for the numerical approximation of the dual representation of Aumann's integral via its support functional. For set-valued integrands which are smooth in an appropriate sense, this approach yields integration methods of arbitrarily high order, e.g. extrapolation methods based on Romberg integration. For problems which are not smooth enough in that sense, this approach yields further insight into well-known order reduction phenomena.

Most important are adaptions of these extrapolation methods to linear differential inclusions. As a result, we get higher order methods for the discrete approximation of reachable sets of special smooth classes of linear control problems.

The talk finishes with visualizations of the numerical results for several model problems and with an outline of open questions and possible directions of future research.

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Convolution Kernels Based on Thin-Plate Splines

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A radial basis function is one of the form $\phi \circ \|\|$, where $\phi : \mathbf{R}^+ \to \mathbf{R}$ is continuous and $\|\|$ is the Euclidean norm. One method of constructing approximations to $f \in \mathbf{C}(\mathbf{R}^n)$ is *quasi-interpolation.* Here, we approximate

$$f(x) \approx \sum_{z \in \mathbf{Z}^n} f(hz)\psi\left(\frac{x}{h} - z\right),$$

for some dilation parameter $h$, where $\psi$ is a radial basis function. In order that this approximation converges we require that $\psi$ is integrable with integral equal to 1. In this paper we seek functions $\psi$ of the form

$$\psi(x) = \int_{\mathbf{R}} \frac{1}{\omega_{n-1}} \int_{S^{n-1}} \phi(\|x - tu\|) dS^{n-1}(u)\, d\mu(t),$$

where $\mu$ is some appropriate measure, $S^{n-1}$ is the unit $n$-sphere, $\omega_{n-1}$ its surface area, and $\phi$ is a function of the form

$$\phi(r) = \log r \sum_{k=0}^{\infty} a_k r^{\alpha - k}, \quad r > A > 0, \quad \alpha > -n.$$

We give conditions on the measure $\mu$ which ensure that $\psi$ is integrable and show that, under such conditions, $\psi$ has a non-zero integral only in a very specific set of circumstances.

## Vertex splines for smooth curve/surface interpolation with local tension

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ISRAEL

**Abstract.**

A new simple method for curve interpolation is presented. The method is based on vertex splines, it is local, and it also has local tension parameters. The method has a natural extension to surface interpolation of general (non-gridded) data. It is a simple k-smooth local surface interpolation scheme with a second order approximation order, and local tension parameters. A unique feature is the possibility of forcing discontinuities along edges, or at vertices.

## Approximation with thin plate splines and exponential radial basis functions

Will Light, University of Leicester

Over the last ten years there has been tremendous progress in the theoretical understanding of approximation by radial basis functions. These functions give methods of approximating either functions or data in $\mathbb{R}^n$ by linear combinations of functions with very simple structure. Let $x_1, \ldots, x_m$ be points in $\mathbb{R}^n$. Then the radial basis approximating subspace associated with this collection of points is the linear span of the set of functions

$$\{\phi(\|\cdot - x_i\|_2) : i = 1, 2, \ldots, m\}.$$

Here $\phi$ is a function from $\mathbb{R}^+$ to $\mathbb{R}$. Suitable choices for $\phi$ usually depend on the underlying dimension of the domain. For example, if $n$ is odd, then the multiquadrics $\phi(r) = \sqrt{r^2 + c}$, $c > 0$ are a good choice. If $n$ is even, then the thin plate splines $\phi(r) = r^2 \ln(r)$ perform well. (It should be observed that we are in the land where the thin plate spline was born, and that, as the first of the radial basis functions to be looked at in detail, it received curiously little attention at birth. However, it is a happy reality that the thin plate splines survived some apalling post-natal neglect, and have gone on to become a beautiful part of approximation theory!)

Practical experience has shown that radial basis functions are good for handling approximation problems where the data sampling is carried out at a 'scattered' or badly distributed set of points. They also perform well for high dimensional approximation problems. Once the dimension $n$ gets much above 3, it becomes very difficult to use many of the more sophisticated triangulation methods, but radial basis functions seem in some ways to be impervious to high dimensions.

The theoretical understanding, and some of the practical aspects of developing radial basis approximations are still lacking in several important features. In this talk we address two of these areas. The first concerns the approximation properties of radial basis functions when the associated points are evenly distributed in a compact domain. The main satisfactory treatment until now has been for evenly distributed points throughout the whole of $\mathbb{R}^n$, where the techniques of Fourier analysis can be brought to bear. We illustrate a line of thinking which allows quasi-interpolants to be constructed for compact domains, once the associated operator is known for the whole of $\mathbb{R}^n$. The second concerns approximation using functions which decay at infinity. A peculiar dichotomy seems to develop, brought on by the use of Fourier methods. If the radial basis function has growth at infinity, there is some chance of taking linear combinations which decay at infinity, and which satisfy the Strang-Fix conditions. These conditions guarantee that the associated quasi-interpolant reproduces polynomials, from whence come the desired error estimates by the familiar Taylor series arguments. If the radial basis function has decay at infinity, then this argument seems to fail. If time permits, we will illustrate two ways in which this apparent difficulty is overcome.

I appreciate that the audience will have a wide range of interests and backgrounds, and will try to make this interesting area of research accessible to all!

# Change of Basis Algorithms for Surfaces in CAGD

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**Abstract**

B-patches were introduced by Seidel and shown to agree with the multivariate B-splines on a certain region. Lineal polynomials, that is, polynomials formed by products of linear polynomials, were introduced by Cavaretta and Micchelli and shown to be dual to B-patches using a multivariate polynomial identity. This work establishes the duality between B-patches and lineal polynomials by generalizing the deBoor-Fix formula for curves to surfaces. This formula is then used to establish duality between several important multivariate bases for representing surfaces including the duality between Bézier and multinomial bases, Lagrange and power bases, and Newton and Newton dual bases.

Our purpose here is to establish the change of basis algorithms between any two of these bases. This is achieved by employing three basic techniques: homogenization, blossoming and the duality principle. The underlying theme is unification: unification of algorithms, unification of derivations, and unification of representations. This approach has the advantage of providing computational simplicity while at the same time revealing the beautiful geometric and algebraic structure of these various representations for surfaces.

Our approach provides a single consistent framework for many recursive algorithms, which are central to surface representations. We shall demonstrate that these algorithms including evaluation, subdivision, differentiation, degree raising and knot insertion algorithms are different manifestations of the change of basis algorithms. Although we present the change of basis algorithms for surfaces for the sake of simplicity, this theory extends to higher dimensional hypersurfaces in a straightforward way.

**Keywords:** algorithms, surfaces, CAGD, deBoor-Fix, basis change, evaluation, differentiation, knot insertion, degree elevation, subdivision, duality.

# Approximation of Circle Segments by Polynomial Curves

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**Abstract.** In a joint work with K. Mørken we consider approximating a segment of a circle by parametric curves $(x(t), y(t))$, where $x(t), y(t)$ are polynomials in $t$. For a segment of length $h$ it is known that for degree $n \leq 3$ there exist approximations with error $O(h^{2n})$. We extend this result to arbitrary $n$.

# MATHEMATICAL MODELLING OF FREE-FORM SURFACES FROM DIGITIZED DATA OF COORDINATE MEASURING MACHINES USING NON-UNIFORM RATIONAL B-SPLINE SURFACES

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## ABSTRACT

This paper introduces theoretical and practical approaches for mathematical modeling of free-form surfaces from digitized data obtained with three dimensional coordinate measuring machines (CMMs). Non-Uniform Rational B-Splines (NURBS) surfaces are used to represent the free-form shape. Least squares fitting techniques are applied to identify the parameters of a NURBS surface from discrete points.

The procedure begins with the parameterization of digitized points (or allocation of $u$ and $v$ parameters to each of the digitized points) using a first approximation of the free-form shape. The first approximation is called a base surface for mathematical modelling and is created from approximate boundary information of the underlying free-form surface. The parameterization is realized by projecting the digitized points to the base surface. The parameters of the projected points will be used as the parameters of the digitized points. After the parameterization of digitized data, the degree, the number of control points and the knot parameters are selected according to the complexity of the underlying shape. The selection of the above-mentioned parameters is flexible and can be made automatic via an iteration process. With the above information a set of simultaneous observation equations is constructed. The unknown variables in these observation equations are the three dimensional control points and the weights of these control points. Householder transformations is then applied to achieve the least squares solutions of the observation system.

Algorithms presented in this paper have been implemented on a VAX/VMS platform. A software package SHAPID (SHAPe IDentification) is developed for this purpose. At the present phase SHAPID is mainly composed of three functional modules, i.e., user and graphical interface, database management module and an interactive least square processing module. There are also some other supporting modules for input and output, entity manipulation, simple entity editing, etc. SHAPID has been integrated inside a commercially available CAD/CAM system UNIGRAPHICS (a product of EDS, formerly of McDonnell Douglas Corporation) for better graphical display.

This research work is sponsored by the Commission of the European Communities through a Brite/Euram project BE-4527, *PROBES: Surface Modelling and Quality Control System for Manual and Automatic Coordinate Measuring Machines*, and the Katholieke Universiteit Leuven through a doctoral scholarship.

# A Method for Summability of Lagrange Interpolation

### On P.L. Butzer's Problem

**Detlef H. Mache**

By using a technique from numerical integration and the new $\Theta$ - transformation (see [2]) we get a discretely defined method, which is a modification of the Lagrange projection. We will consider the modified summability methods of $L_n$ given by

$$(L_n^\star f)(x) := L_n^\star(z_{0,n},\cdots,z_{n,n};f,x) = \sum_{j=0}^{n} \beta_{j,n}\omega_j[f,T_j]_n T_j(x)\,,$$

with the generating polynomial (see [2])

$$b_n(x) = \kappa_n \frac{\left(1 - x\,\cos\frac{2\pi}{n+2}\right)\left(1-T_{n+2}(x)\right)}{(1-x)\left(x-\cos\frac{2\pi}{n+2}\right)^2} = \sum_{j=0}^{n}\beta_{j,n}\omega_j T_j(x)$$

where $\kappa_n = \frac{1}{\pi(n+2)}\sin^2\frac{\pi}{n+2}$ and

$$\beta_{j,n} = \frac{n-j+2}{n+2}\cos^2\frac{j\pi}{n+2} + \frac{\cos\frac{\pi}{n+2}}{(n+2)\sin\frac{\pi}{n+2}}\cos\frac{j\pi}{n+2}\sin\frac{j\pi}{n+2}\,.$$

For this **discrete linear positive approximation method** we have an instance of algebraic polynomials, for which we can prove estimates of Timan and Telyakovskii – Gopengauz type.

It is the aim of this present note to show that the local order of approximation by means of $(L_n^\star)$ is comparable with those furnished by the best approximation polynomials.

At the end of my explanations we get a conclusion to prove an problem of P.L.Butzer presented on a conference in Budapest [1].

## References

[1] BUTZER, P.L., Legendre Transform Methods in the Solution of Basic Problems in Algebraic Approximation, in: Functions, Series and Operators Vol.I (J.Szabados and B.Sz.-Nagy, eds.) North - Holland, Amsterdam (1983) 277 – 301.

[2] LUPAŞ, A. AND MACHE, D.H., The Degree of Approximation by a Class of Linear Positive Operators, (1992) (manuscript).

[3] MILLS, T.M. AND VARMA, A.K., On the Summability of Lagrange Interpolation, J. Appr. Theory 9, (1973) 349 – 356.

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Key words and phrases: Approximation by positive linear operators, discrete linear operators, Lagrange Interpolation, pointwise estimates, second order modulus of continuity, Lipschitz – type maximalfunction, Butzer's problem.

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# $\varepsilon$-$G^1$ Polynomial Approximations of Non-polynomial Surfaces

Stephen Mann

In CAGD, there are variety of surface representations that are mathematically complex or computationally expensive to evaluate. For example, the offset to a polynomial surface is a non-polynomial surface; S-patches are an n-sided surface patch of high rational degree. Additionally, it is sometimes desirable to have a parametric polynomial representation of an algebraic surface. Unfortunately, while parametric polynomial surfaces always have an algebraic representation, the converse is not always true. In general, there is no parametric polynomial representation for an algebraic surface.

In this talk, I will show how to use the cubic interpolant, a triangular, high-order-of-approximation cubic polynomial patch, to approximate these types of surfaces. In order to use this low degree surface patch, I relax the continuity conditions from $G^1$ to $\varepsilon$-$G^1$ (a surface is said to be $\varepsilon$-$G^1$ if the discontinuity in surface normal is bounded by $\varepsilon$). Despite the discontinuities in surface normals, the approximations are indistinguishable from the original surfaces.

# $C^1$ comonotone Hermite interpolation via parametric cubics

*Carla Manni - Universita' di Pisa*

A visually pleasing interpolating function should generally retain the intrinsic form that is implied by the given data points. That is if the data are locally monotone, then the interpolating function should be also. A great deal of research has been done in recent years in constructing comonotone interpolants.

In this paper we assume that a set of points $x_0 < x_1 < \ldots < x_n$ and the values of a function $f(x_i)$ are given, in addition we assume that the derivatives $f'(x_i)$ at the data points are known (given or computed): we seek to find a function, $s(x) \in C^1[x_0, x_n]$ interpolating $f$ and its first derivative at the data points, which is increasing (decreasing) in $[x_i, x_{i+1}]$ if the data are.

It is well known that this problem can not be solved by $C^1$ piecewise polynomials with preassigned degree since the derivatives of such a function $(s'(x_i), s'(x_{i+1}))$ must belong to a bounded domain of the plane in order to obtain monotonocity.

So the problem can be faced via piecewise polynomials with fixed degree adding extra knots, via piecewise polynomials having degree dependig on the data or via piecewise rational functions.

In this paper we situate the problem in the general setting of parametric curves: we propose a solution constructed considering the graph of the function $x \to s(x)$ as the support of a bidimensional parametric curve $(x(t), y(t))$ where $x(t)$ is strictly increasing.

This point of view is rather new; in the papers following this approach the shape constraints are essentially obtained in three ways: considering the variation diminishing properties of the Bernstein polynomials, so that only sufficient conditions are considered, minimizing suitable functionals, which implies global schemes, or, finally, varing heuristically the parametrization of the curve.

The method proposed here constructs the required interpolant $s(x(t)),\ x \in [x_i, x_{i+1}]$, as a single valued parametric curve having cubic components. The comonotonicity of the curve is controlled by the amplitude of the tangent vector at the extrema of each subinterval; this amplitude is determined by using the well known necessary and sufficient conditions for the derivatives of monotone cubics.

The method yelds a second order $L_\infty$ approximation to a $C^2$ monotone function, while yelds a fourth order approximation to a $C^4$ function with no vanishing first derivative.

Finally the proposed method is local: a change in the data at the point $x_i$ reflects only in $[x_{i-1}, x_{i+1}]$.

# Evaluating Surface Intersections in Lower Dimension

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Evaluating the intersection of parametric and algebraic surfaces is a recurring operation in geometric and solid modeling. This problem has been studied extensively in the literature and the main approaches are based on subdivision, analytic and tracing methods. For rational and algebraic surfaces, the intersection problem can be formulated using algebraic sets and in the last few years a great deal of attention has been paid to evaluating these curves using marching methods. Robust algorithms for marching methods need to know start points on each component, singular points on the intersection curve and optimum step sizes as we are tracing along the curve. When it comes to tracing, we can choose to trace the curve in the higher dimensional space (spanned by the parameters) or its projection in the lower dimensional space, obtained by eliminating variables in the system of algebraic equations [1]. Earlier the lower dimensional approach was considered infeasible due to efficiency and numerical problems. In our earlier work, we had shown the lower dimensional formulation of the intersection can be represented as the determinant of a matrix [2]. A simple tracing algorithm proceeds by substituting the numerical values in the matrix and computing the resulting determinant using Gaussian elimination and using a modification of Gaussian elimination to compute the partials of the function (for tangent direction computation) [2].

In this talk, we highlight a new algorithm for tracing the intersection curve using the matrix formulation. The projection of the intersection curve is represented as the determinant of a matrix, $M(u,v)$. Given this formulation, we use the property that only those points, $(u_1, v_1)$ correspond to the intersection curve such that $M(u_1, v_1)$ is singular. The resulting algorithm for evaluating the intersection curve is based on eigendecompositions and singular value decompositions, as opposed to determinant computation, which improves its accuracy. Furthermore, at each stage of the tracing step, we reduce the problem to computing the nearest eigenvalue and eigenvector using inverse power iteration. We also make use of the structure of the matrices in performing these power iterations. A major advantage of inverse power iterations lies in the fact that its convergence is well understood as opposed to applying local methods (like Newton's method) to solve a system of polynomial equations. As a result, we are able to develop a robust tracing method using matrix computations and geometric reasoning in a lower dimensional space. We describe the implementation of this algorithm, its performance on a number of examples and our experiences in tracing curves with singularities.

[1] C. Hoffmann, "A dimensionality paradigm for surface interrogations", Computer Aided Geometric Design, vol. 7, pp. 517-532, 1990.

[2] D. Manocha and J. Canny, "A new approach for surface intersection", International Journal of Computational Geometry and Applications, vol. 1, number 4, pp. 491-519, 1991 (special issue of solid modeling).

CONVERGENCE OF NON-STATIONARY NON-NEGATIVE SUBDIVISION

Avraham A. Melkman[1]

**Abstract:** The first part of this talk is based on joint work with Alfred Cavaretta.

Following Buhmann& Micchelli we view subdivision as the iteration $v^{n+1} = Av^n$, $v^0 = v$, with $v \in \ell_\infty(\mathbb{Z})$. Here $A = \{a_{i,j}\}_{i,j=-\infty}^{\infty}$ is a bi-infinite matrix which, in the terminology of Dahmen& Micchelli, is two-slanted, i.e. for some $\ell \geq 1$

$$a_{i,j} = 0 \quad \text{unless} \quad 0 \leq i - 2j \leq \ell.$$

The usual stationary subdivision is obtained in case $a_{i,j} = a_{i+2,j+1}$. More generally, call $A$ *2k-stationary* if

$$a_{i+2k,j+k} = a_{i,j}, \text{ all } i, j.$$

This is the stationarity associated with subdivision using a $k \times k$ matrix mask.

When the subdivision coefficients are non-negative the convergence properties of the scheme do not depend on the actual values of its coefficients but rather on its *stencil*, $\sigma(A)$, defined to have entries $(\sigma(A))_{i,j} = 1$ if $a_{i,j} > 0$, and 0 otherwise.

We prove that a non-negative subdivision scheme whose stencil is $2k$-stationary converges if and only if there exist an $N$ and $c > 0$ such that every $\ell$ consecutive rows of $A^N$ contain a column all of whose elements are bounded from below by $c$.

Specializing to the case of stationary subdivision, this theorem leads to a simple necessary and sufficient condition for convergence couched only in terms of the support of the coefficients of the mask. To be precise, let $a_{i,j} = a_{i-2j}$ with $a_0 a_\ell \neq 0$. The condition reads then as follows: if $\ell$ is even then $a_i > 0$ for at least one odd integer $i$, while for $\ell$ odd $a_i > 0$ for at least one even $i$ and one odd $i$, $0 < i < \ell$.

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**Curves and Surfaces**
**CHAMONIX, France, June 10-16, 1993**

# Knot selection for quadratic splines

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**Résumé :** Soit $X_n = \{ a=x_0 < x_1 < \dots x_n = b \}$ une subdivision de $I = [a,b]$ et S l'espace des splines quadratiques $C^1$ ayant comme suite de noeuds $X_n \cup X'_n$, où $X'_n = \{ x'_i = (1-\alpha_i)\, x_i + \alpha_i x_{i+1}, 0<\alpha_i<1, 0 \le i \le n-1 \}$. On s'intéresse au problème d'Hermite unisolvent classique : trouver $v \in S$ vérifiant $v(x_i) = y_i$ et $v'(x_i) = y'_i$, avec des données quelconques $\{ y_i, y'_i, 0 \le i \le n \}$, ou au problème de Lagrange associé quand les $y'_i$ sont évaluées en fonction des données $y_i$. En pratique, on choisit souvent $\alpha'_i = \frac{1}{2}$, pour les noeuds intermédiaires, mais ce choix n'est pas toujours optimal. Nous tentons de définir des choix optimaux des $\alpha_i$ associés à certains critères définis sur v, par exemple minimisation de certaines semi-normes, de la courbure, de la longueur de la courbe, des normes des opérateurs d'interpolation, des constantes d'erreur. Un problème lié est le choix de $X'_n$ donnant de bonnes approximations des $y'_i$ lorsque les $y_i$ sont les seules données. On donne des exemples numériques pour illustrer les différents choix de noeuds $X'_n$.

**Abstract :** Let $X_n = \{ a=x_0 < x_1 < \dots x_n = b \}$ be some partition of $I = [a,b]$ and S be the space of $C^1$ quadratic splines with knot sequence $X_n \cup X'_n$, where $X'_n = \{ x'_i = (1-\alpha_i)\, x_i + \alpha_i x_{i+1}, 0 < \alpha_i < 1, 0 \le i \le n-1 \}$. We are interested in the classical unisolvent Hermite problem : find $v \in S$ satisfying $v(x_i) = y_i$ and $v'(x_i) = y'_i$, for arbitrary data $\{ y_i, y'_i, 0 \le i \le n \}$ or the associated Lagrange problem when the derivatives are not given, but computed in some way from the data $\{y_i\}$. In practice, one often chooses $\alpha_i = \frac{1}{2}$, but this choice is not always optimal. We try to define "optimal" choices of $\alpha_i$'s depending on the criteria to be satisfied by v, e.g. minimizing some semi-norms on v, the curvature, the length of the curve, the norms of the interpolation operator, the error constants. A related problem is the choice of $X'_n$ giving good approximate values of $\{y'_i\}$ when only the data $\{y_i\}$ are given. Numerical examples are given to illustrate the different knot selections $X'_n$.

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## On the spectral radius of the subdivision operator

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The spectral radius of a subdivision operator is fundamental in determining both convergence of stationary subdivision schemes and smoothness of the associated refinable function. In this talk we present some results on the problem of determining the spectral radius of a subdivision operator.

# Computation of the Monotone Cubic Spline Interpolant

by

Charles A. Micchelli and Florencio I. Utreras

*In this work we apply the theory developed in two preceding papers to build a computational scheme for computing the Best Interpolating Monotone Cubic Spline, i.e., the function* $u$ *in* $H^2[0,1]$ *which is monotone, interpolates non decreasing data, and minimizes the seminorm* $\int_0^1 [u''(t)]^2 dt$*. We present a numerical example to illustrate the properties of the method.*

**SMOOTHING NOISY DATA BY LOCAL KRIGING WITH NUGGET EFFECTS**

by

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**Abstract** - This paper deals with the problem of smoothing noisy data. Given N observed data $z(u_i) = P(u_i) + \epsilon(u_i)$, $i = 1,\ldots,N$, where $u_i \in \Omega$, $\Omega \subseteq \mathbb{R}^n$, $\epsilon(u_i)$ are independent normal random errors with zero mean and known or unknown variance, P is only known to be smooth. We want to estimate P in a reproducing kernel Hilbert space so that it minimizes the positive functional:

$$\Phi(P_\lambda) = \sum_{i=1}^{i=N} \frac{[z(u_i) - P_\lambda(u_i)]^2}{N\lambda} + \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} b_i\, b_j\, K(u_i, u_j)$$

where K(.,.) is the reproducing kernel (generalized covariance), $b_i$ and $b_j$ are scalars, $\lambda$ is a positive number, and $N\lambda$ is the nugget effect to be chosen in such a way that the solution $P_\lambda$ preserves smoothness and fidelity to the data. This problem was solved in the 70's and there exist softwares which are capable of findind the optimum parameter $\lambda$ which minimizes the Wahba's generalized cross-validation function $GCV(\lambda)$. In this paper, I propose a method of choosing a first guess of $\lambda$. When the variance of the errors is unknown, this choice is based upon the gross variogram of the data at the vicinity of the origin. Then, cross-validation method is used to find the optimum smoothing parameter. The method is applied in the local kriging scheme developed earlier in order to smooth scattered data on the line, in the plane, in the three-dimensional space, or on the unit sphere.

# Shape Functions Construction by Different Polynomial Approximation in the Finite Element Method *

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### Abstract

In the analysis of a Finite Element Method (FEM) given an element, we can describe his shape by a set of elementary functions known as *shape functions*. The approches in describing these functions are quite differents. In spaces $\mathbb{R}^2$ or $\mathbb{R}^3$, they generally are a product of lagrangian polynomials if the coordinate system can be chosen with the axes parallel to the sides of the element, otherwise a system of *barycentric coordinate* (sometimes called *area coordinates*) could be introduced.
The aim of this paper is the description and the representation of shape functions when the element has triangular or tetrahedal shape (the simplest). The representation has been done by using two algorithmic shemes: *Neville-Aitken* and *De Casteljau.* For these schemes we have deduced very important properties.
A completely new approach seems their description as *attractor* of a suitable *Iterated Functions System (IFS).* This idea becomes from the fractal interpolation. The main benefit of this technique is the compacteness of their internal representation on a computer and the possibility of a very fast reconstruction.
A comparison of different behaviours using known procedures is made and an analysis of the approximation errors is provided.

*This work has been done with the support of CNR within *"Progetto Strategico Sistema Lagunare Veneziano"* and 40% of MURST on the project *"Analisi Numerica e Matematica Computazionale"*

# High Order Approximation by Parametric Polynomials

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**Abstract.** In this joint work with Tom Lyche, we study approximation of smooth parametric curves by polynomial curves, i.e., each component of the approximating curve is a polynomial. It is known that in special cases, the approximation order for plane curves is $2n$, where $n$ is the polynomial degree. Here we attempt to generalize these results.

# Nonnegativity Preserving Interpolation by Powell-Sabin Splines

Bernd Mulansky
Technical University of Dresden

Given a set $D = \{x^i\} \subset \mathbb{R}^2$ of scattered data sites and associated *nonnegative* data values $F = \{f_i\} \subset \mathbb{R}$, the objective is to find a *nonnegative* interpolating $C^1$ function.

In order to define the so-called Powell-Sabin splines, each triangle (macrotriangle) of a suitable, possibly data-dependent, triangulation of the data sites is splitted into six micro-triangles. Every piecewise quadratic $C^1$ function on the refined triangulation is uniquely determined by its values and gradients at the vertices of the macro-triangulation, as can be easily shown using the Bernstein-Bezier representation of the quadratic pieces on the microtriangles.

The simple sufficient nonnegativity conditions, i.e., the nonnegativity of all B-ordinates, result in a solvable system of linear inequalities for the gradients, which is separated with respect to the vertices. Introducing a suitable objective functional, the construction of a nonnegative Powell-Sabin interpolant reduces to the solution of independent quadratic optimization problems in two variables, namely the partial derivatives, for each vertex.

The proposed method is illustrated by a few numerical examples.

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**Abstract** for *Curves and Surfaces* Conference
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Progress in Data Dependent Triangulations

Edmond Nadler

The study of *data dependent triangulations*, initiated by Dyn, Levin and Rippa in a 1990 paper in IMA J.N.A., is concerned with the choice of good or best triangulations with a given set of data points $v_i \in \mathbb{R}^2$ as vertices, and also dependent upon a vector $f$ of data given at the points $v_i$. This contrasts with more traditional methods, where only the points $v_i$ in the plane are taken into account, and the triangulation of choice is often the well known Delaunay triangulation.

These problems are looked at in the setting of *scattered data interpolation with piecewise polynomials* defined over a triangulation. Given a triangulation $\triangle := \{T_i\}_{i=1}^t$ of a set of distinct and noncolinear data points $V := \{v_i\}_{i=1}^n$, let $S_d^r(\triangle) := \{s \in C^r(Q) : s|_{T_i} \in \mathcal{P}_d\}$, with $Q := \cup_{i=1}^t T_i$ and $\mathcal{P}_d$ the space of bivariate polynomials of degree $d$. The scattered data interpolant is a function $s \in S_d^r(\triangle)$ which *interpolates* the data vector $f$. I focus on the the simplest case of $S_1^0$, the continous piecewise linear functions, which has plenty of interest, and for which the appropriate data vector $f$ is simply a function value at each point $v_i$, and for which there is obviously a unique interpolant.

In previous work, the data dependent triangulations have been obtained by minimizing different criteria measuring the roughness of the interpolating function. In the present work, possibilities are explored for using *other* types of criteria, such as *shape preservation*, and ones based on the results of the author's thesis (1985) for the optimal shape of triangles for local best approximation.

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# Convexity Preserving Interpolation of Scattered Data

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The purpose of the talk is to discuss some problems associated with smooth convexity preserving interpolation of scattered data in $\mathbb{R}^2$ and $\mathbb{R}^3$. An interpolation method is proposed which utilizes convex parametric Powell–Sabin patches.

# THE ITERATIVE SOLUTION OF A NONLINEAR INVERSE PROBLEM FROM INDUSTRY: DESIGN OF REFLECTORS

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**Abstract:**

We report about a project that has been done for and in coorporation with an Austrian company which designs and manufactures lightings for industrial illumination purposes. The lightings consist of a lamp with known light distribution and a reflector. Our task was to develop a mathematical model and an algorithm (resulting in computer software) for the problem of designing the shape of the reflector in such a way that the illumination distribution on the unit sphere centered at the lamp, obtained by the reflected outgoing light rays which are all thought to emanate from the lamp, behaves in a prescribed way. In the algorithm for this so called "inverse far field problem", the reflector should be described in a way that could be directly used for CAM purposes in that company.

The derivation of the model for the "inverse far field problem", is on the one hand based on the reflection condition, and on the other hand on the so called "balance condition". This condition means that the amount of light leaving the lamp in any solid angle increment $d\Omega_i$ should equal the amount of light reflected into the corresponding angle increment $d\Omega_o$ in the illuminated sphere.

For deriving a numerical algorithm, we have to describe the reflector by a finite number of variables. Since for CAM purposes, the company interpolates by splines, we have chosen a bicubic B–spline representation for the reflectors. The coefficients of this B–spline representation are determined in such a way that the balance condition holds best possible in the least squares sense. The resulting constrained minimization problem is solved iteratively with a projected conjugate gradient method.

Finally, we present a numerical example showing that the algorithm works in practice.

Restricted Splines with Modified Moments

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In the last years the so-called histo-splines gained considerable attention. Roughly speaking these splines are used to approximate histograms, which are in mathematical terms step functions. Normally, the splines are constructed in such a fashion, that the area of the underlying step function is stepwise preserved. There are several papers by Jochen W. Schmidt and co-authors (Dresden) on this subject. In many cases the histosplines represent counting results which are non-negative or population statistics which are increasing. Therefore it is natural to try to include these conditions in the construction of the approximating splines. One can try to construct splines of this type directly by choosing a certain class of splines.

We will investigate the mentioned problem a little differently and more generally. We shall treat the following

**Problem.** Given a mesh with $N > 1$ mesh points on the real line:

$$\Delta: \quad a = t_1 < t_1 < \cdots < t_N = b,$$

and two fixed integers $k, n \geq 0$. In addition there are $N-1$ real numbers $\alpha_j$, $j = 1, 2, \ldots, N-1$ and a fixed polynomial $p \in \Pi_n$ given. With these data we want to find functions $x \in W_2^k[a,b]$ ($x \in C^k[a,b]$ is essentially the same) which minimize

$$f(x) := \int_a^b \left(x^{(k)}(t)\right)^2 dt, \tag{1}$$

subject to

$$\lambda_j(x) := \int_{t_j}^{t_j+1} p(t)x(t)dt = \alpha_j, \; j = 1, 2, \ldots, N-1. \tag{2}$$

The functional $\lambda_j$ will be called *modified n-th moment* in $[t_j, t_{j+1}]$. If we choose $p_n(x) = x^n$ we obtain the ordinary moments, including the area. In a first step we shall derive necessary conditions for the stated problem and in a second step we will include non-negativity restrictions. The latter problem will consist of finding some additional mesh points with certain conditions.

# Local Surface Approximations from Unorganized Points

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A method for constructing local smooth approximations to a surface from a triangulated set of unorganized points in 3-space is described. For a given vertex of the triangulation, a local model surface is determined which interpolates a set of neighbors of the vertex. The necessary correspondence between a neighborhood of the given vertex and the parameter plane is established through an approximation to the inverse exponential map. The application of bivariate scattered data methods then yields a parametric representation of the local model surface. The model surface can be used for the estimation of geometric measures such as curvature elements as well as input for the construction of triangular surface patches.

## $G^2$-continuous cubic algebraic splines and their efficient display

Algebraic splines are built with arcs of algebraic curves (i.e. given by implicit polynomial equations) and have been introduced by Sederberg in the 80's. The spline is $G^2$-continuous if at each joint the adjacent arcs have the same curvature. A method is described to construct a $G^2$-continuous cubic algebraic spline consisting of $n$ convex segments that interpolates $n+1$ joint points. The full spline depends on the interpolation points and five homogeneous parameters. Each arc of the spline fits a triangle, two of whose sides and vertices are the lines tangent to the arc at its endpoints. Our spline is characterized by the fact that the implicit equation of each of its segments, when expressed in terms of the barycentric coordinates with respect to its corresponding triangle, has always the same expression:

$$F(s,t,u) = as^2u + bsu^2 - cst^2 - dt^2u + estu = 0.$$

where $s, t, u$ are the barycentric coordinates. These triangles compose the control polygon of the spline. The spline may be modified globally to follow more faithfully or fit more tightly the control polygon. The five homogeneous parameters, which determine the full spline are expressed in terms of shape handles that control this behavior. To track the spline we determine the barycentric coordinates of the points of each segment. Since all segments have the same equation, it is enough to track only one. The tracking algorithm uses the special knowledge one has on the behavior of one cubic segment: assuming we start at the endpoint with barycentric coordinates $(s,t,u) = (1,0,0)$ then as we march along the curve the $t$-coordinate increases up to a certain value $t = t_0$ and then decreases monotoneously to $t = 0$, till the second endpoint $(0,0,1)$ is reached. To display the spline the actual cartesian coordinates for each segment have to be computed.

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# B-Spline Approximation for CAD/CAM and Visualization

N. M. Patrikalakis * L. Bardis †
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To facilitate the exchange of geometric data between different geometric modeling systems, the approximate conversion of high degree B-spline or procedurally defined geometries to lower degree representations is required. High degree representations arise from fitting high degree polynomial functions to point data, from intersections, from lofting high degree curves, and from trimming low degree surfaces. Procedural definitions are produced, for example, by offsetting, sweeping, trimming or blending. In this lecture, we will review the algorithms we have developed to approximate high order and procedurally defined continuous univariate and bivariate functions with B-splines in an efficient manner. Examples for the approximation of surface curves, and generalized cylinder, offset and blending surfaces illustrate the method. These approximation algorithms are based on an iterative scheme in which the approximation error is estimated by adaptive sampling and is reduced through adaptive knot addition. We will also discuss local bounds for the approximation error as a motivation for these algorithms. For the approximation of high order B-splines, the effectiveness of a global error bound is discussed.

The representation of functions describing a measured physical property (e.g. via sparsely scattered or densely defined point data) requires methods for the creation of B-spline approximations that capture inherent features of the data. Such representations, for example, facilitate archiving, data storage reduction, visualization and other more general interrogation. Since the measuring devices or sensors used to produce such functions are of finite precision, there is a need to represent the function values together with their uncertainty and the uncertainty in the independent variables. In this lecture, we will discuss the concept of enveloping or interval B-splines. This concept allows the representation of both data and associated uncertainty. We will present an algorithm for the creation of interval B-splines based on minimization with linear constraints. We will illustrate the method using geophysical ocean data.

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# DIFFERENTIABILITY CONDITIONS OF SELF-AFFINE NON-POLYNOMIAL CURVES

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Self-affine property is usually associated to "wild" curves. However, all parametric polynomial curves own the self-affinity property that is closely related to the subdivision principle natural to the De Casteljau's algorithm [1]. By an adequate generalization of the De Casteljau's subdivision scheme a bigger class of (possibly non-polynomial) parametric self-affine curves may be built satisfying the usual properties of variation diminishing and invariance to affine transforms shared by polynomial curves. Careful choices of the parameter of the subdivision procedure allow the definition of non-polynomial curves of class $C^k$ from a set of at least 2k+1 points. If less of 2k+1 control points is given then the construction scheme under the $C^k$ condition reduces to the usual De Casteljau's subdivision method producing a polynomial curve of degree 2k.

Reference

1. Prautzch, H. and Micchelli, C.: Computing curves invariant under halving. CAGD, 4(1-2):133-140, 1987.

## Spline meshes for smooth blending of free form surfaces

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**Abstract**

In the manner of B-splines, but without the regularity restrictions on the control mesh, the algorithm presented here smoothes a mesh of points into a parametrized $C^1$ surface of low degree. Admissible input meshes may outline an open or closed free-form shape of arbitrary genus. There are no restrictions on the number of neighbors to a point. Blending ratios control the position of patches and the change of the surface normal. The user may choose freely from piecewise cubics and bicubics for the surface parametrization.

# General form of generalised B-spline functions on a simple quadrilateral mesh after k-th succesive subdivision of a cube

by

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## Abstract

*Goodman has suggested an approach to generate geometrically smooth closed surfaces by defining biquadratic splines whose domains are on the faces of a cube. Upon a regular subdivision of a cube, a simple quadrilateral mesh (SQM) of 24 faces is obtained. Further subdivision will result in a new SQM of 96 faces. Following the idea of Goodman, the author has constructed biquadratic splines on 24-faces SQM and piecewise tensor product polynomial functions of degree at most three on 96-faces SQM, both with minimal support. Even though the mesh is successively subdivided, the same continuity conditions across mesh lines are always kept fixed.*

*Upon k-th arbitrary regular subdivision of a cube, a new SQM of* $6(2^{2k})$ *faces will be obtained. In this paper, we will construct the general forms of generalised B-spline functions on this SQM with small support and sum to 1. These basis functions are useful for designing, approximating and interpolating closed surfaces. Because the same continuity conditions across mesh lines are always kept fixed, data reduction can then be used on closed surfaces defined by these basis functions.*

# Orthogonal periodic splines and spline wavelets

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Let $N_m$ be the cardinal B-spline of order $m$ and $P_m$ the $N$-periodization of $N_m$. Then the space $S_m^N(\boldsymbol{Z})$ of all $N$-periodic spline functions of order $m$ is the set of all linear combinations of shifts $P_m(\cdot - j)$ $(j = 0, ..., N-1)$.

Using Euler–Frobenius polynomials we shall introduce an orthogonal basis of $S_m^N(\boldsymbol{Z})$. With the help of this basis it can be shown that there is a close connection between $S_m^N(\boldsymbol{Z})$ and the space of trigonometric polynomials $T_m :=$ span$\{\exp(2\pi ij \cdot /N); j = 0, ..., N-1\}$.
In contrast to other approaches we shall prefer a representation which is based on the consequent application of the symbol of spline interpolation. In this way some known results of M. Kamada, K. Toraichi and R. Mory (J. Approx. Theory **55** (1988), 27 – 34) and S.L. Lee and W.S. Tang (Proc. Edinburgh Math. Soc. (2) **34** (1991), 363 – 382) can be simplified and improved.

Furthermore, our results can be applied to cardinal spline wavelets and $N$-periodic spline wavelets obtained by $N$-periodization.
We shall give best possible explicit lower and upper Riesz bounds for the spline basis as well as for the wavelet basis. These bounds are very important for finding numerically stable algorithms for the computation of spline wavelets. The properties of the Euler–Frobenius polynomials will play an important role in our procedure.
Our results improve known results by C.K. Chui („An Introduction to Wavelets“, Academic Press, 1992).

# REGULARITY ANALYSIS OF NON UNIFORM DATA

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**ABSTRACT**

A wavelet which is the $k^{th}$ derivative of a smoothing function is well fitted to characterize function singularities of order <k through multiscale wavelet analysis. A particular class of wavelet, derivatives of B-splines, leads to fast and efficient algorithms for contours detection of 1-D and 2-D signals by a discrete dyadic finite scale analysis (Mallat and Zhong).
For this class, the scale function $\phi$ is a B-spline of order $(a\text{-}k)$ if the wavelet $\psi$ is the $k^{th}$ derivative of a B-spline of order $a$. We express the multiscale analysis algorithm in terms of B-splines derivation formulas for the wavelet transform and by using a knots insertion scheme for the change of scale. This formulation allows us to generalize the algorithm for non uniformly sampled data. Under some assumptions on spacing, we obtain convergent subdivision schemes. This algorithm is used to detect the singularities of a function.

**RÉSUMÉ**

L'analyse multiéchelle en transformée en ondelettes est bien adaptée à la caractérisation des singularités d'ordre <k d'une fonction si l'on prend comme ondelette $\psi$ la dérivée $k^{ième}$ d'une fonction de lissage $\theta$. Une classe particulère d'ondelette, dérivées de B-splines, conduit à un algorithme rapide et efficace pour la détection des singularités dans des signaux 1-D ou 2-D par une analyse en ondelette dyadique discrète (Mallat et Zhong).
Pour cette classe particulière, la fonction d'échelle $\phi$ est une B-spline d'ordre $(a\text{-}k)$ si l'ondelette $\psi$ est une spline dérivée $k^{ième}$ d'une B-spline d'ordre $a$. On formalise l'algorithme d'analyse multiéchelle en exprimant la transformée en ondelettes à l'aide des formules de dérivation des B-splines et le passage d'une échelle à l'autre par un schéma d'insertion de noeuds. Cette formulation permet de généraliser l'algorithme au cas des données échantillonnées irrégulièrement. Pour une certaine régularité du pas, on obtient des schémas de subdivision convergents. On utilise ces résultats pour détecter les singularités d'une fonction dans le cas de données non équidistantes.

Reference : S. Mallat and S. Zhong, "Characterization of Signals from Multiscale Edges", IEEE T. on PAMI, July 92, vol 14 n°7 pp710--732.

## Special Functional Forms for Rational Curves and Surfaces

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An important part of geometric design is the use of the degrees of freedom provided by NURBS for achieving special functional properties or advantages in manufacturing. We present solutions to several problems arising in applications.

For the mathematical description of milling procedures and the representation of thick plates, offset curves and surfaces are important. However, offsets of rational curves or surfaces are in general not rational and therefore need to be approximated. In an attempt to avoid this approximation which is particularly subtle for surfaces, we derive an explicit representation of all rational curves and surfaces with rational offsets. It based on the dual representation and on rational parameterizations of the circle and the sphere. There is still enough flexibility in the obtained class of curves and surfaces to make these elements candidates for use in free–form design.

As an example for the use of rational curves with rational offsets we show an algorithm for the construction of NURBS representations of curves of constant width. Such curves are desirable profiles in certain cam mechanisms. Special rational surfaces with rational offsets such as tubular surfaces are useful as blending surfaces.

In several applications, the manufacturing process requires developable surfaces. Unlike previous approaches, we consider developable rational surfaces in their dual representation. This allows us to apply curve methods. Particularly, an algorithm for connecting two curves with a developable rational surface is presented. It is well-known that even if both curves are rational the connecting surface is in general not rational and therefore this is an approximation problem.

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# Trigonometric Interpolation and Wavelet Decompositions

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The aim of this talk is to describe decomposition and reconstruction algorithms for nested spaces of trigonometric polynomials in a suitable matrix/vector form. The scaling functions of these spaces are defined as fundamental polynomials of Lagrange interpolation. The interpolatory conditions and the construction of dual functions are crucial for the approach presented here. It turns out that the use of de la Vallée Poussin kernels leads to the desired results. The investigations are based on the works of A. A. Privalov on orthogonal bases of the space $C_{2\pi}$ and a paper by C.K. Chui and H.N. Mhaskar on trigonometric wavelets. This strategy is especially important in applications as signal analysis whenever a separation into frequency bands or octaves - and not just single sine/cosine frequencies - is necessary.

# BIORTHOGONAL WAVELET PACKETS FOR PARALLEL SOLUTION OF INTEGRAL EQUATIONS

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## ABSTRACT

It is well known that the numerical solution of an integral equation is a very ill-posed problem. Classical tools to obtain an approximated solution are given by the Truncated Singular Value Decomposition method and the Tikhonov regularization method [5]. Parallel version of both these methods have been already considered [4],[6].
In this work we look for an approximated solution of integral equations belonging to the spline space $V_j$, generated by the $m^{th}$ order B-spline $N_m(x)$ whit knots at $2^{-j}k$, $k \in Z$. This space can be decomposed as the direct sum of wavelet packet spaces $U_n$ [3]. As consequence of this decomposition we obtain a block partitioning of the matrix of the linear system arising after discretization of the integral operator [1],[2]. Based on this special structure, we have realized a parallel algorithm, well suited for distributed memory multiprocessor, which achieves very good efficiency for the low cost of the communication and syncronization overhead. Numerical results are presented.

## REFERENCES

[1] L. Bacchelli Montefusco, D. Lazzaro, Parallel preconditioners based on orthogonal wave packet bases, in preparation.
[2] L. Bacchelli Montefusco, L. Puccio, Wavelet packet decomposition for parallel numerical algorithms, in preparation.
[3] C.K. Chui, Non-orthogonal wavelet packets, CAT Report 261, dec.91.
[4] D.J. Evans, A.M.S. Rahma, The numerical solution of Fredholm integral equations on parallel computers, Parallel Computing, vol.10, (1989), 193-205.
[5] L. Puccio, D. Lazzaro, Analisi di metodi numerici per la soluzione di problemi mal posti, Rapporto tecnico n.1/58, Progetto Finalizzato Sistemi Informatici e Calcolo Parallelo, (1991).
[6] L. Puccio D. Lazzaro, Polinomial splines for image restoration on shared memory multiprocessor, Parallel Computing: Problems, Methods and Applications, P. Messina and A. Murli (Eds.) Elsevier (1992).

Spline wavelets on a bounded interval – Part I

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In two talks, some detailed results will be presented concerning a multiresolution analysis of spline wavelets which is not defined on the whole real axis but only on a bounded interval.

In Part I, the basic ideas will be addressed, including a comparison to other approaches for a bounded interval such as the ones by Meyer and Cohen-Daubechies-Jawerth. Special boundary scaling functions and wavelets are constructed using B-splines with multiple knots. To this end, the so-called Chui-Wang B-wavelet approach is suitably modified. As spline wavelets are only semi-orthogonal, it is also necessary to consider the corresponding dual scaling functions and wavelets.

The construction of the decomposition and reconstruction sequences and related algorithmic and implementational questions will be adressed in the second part of this presentation.

# Spline wavelets on a bounded interval – Part II

Ewald Quak and Norman Weyrich*
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Using the tools described in "Spline wavelets on a bounded interval – Part I" we compute cubic spline wavelets on the interval $[0,1]$ as well as the dual scaling functions and the dual wavelets. We describe the usual decomposition and reconstruction algorithms. Additionally, we give an improved version of the decomposition algorithm which works in $O(j)$ operations on each level $j$ and without knowing the explicit formulas of the dual functions. Numerical examples show that the boundary effects in the decomposition can be significantly reduced by using special boundary wavelets instead of truncating the wavelets defined on the whole real axis.

**Which criteria to evaluate the quality of a quasi-interpolant ?**

*Christophe Rabut*

This talk is a reflexion on the main properties we demand to a quasi-interpolant; we give and compare the interest of the most used criteria, and explain the importance of the "interpolating power of $\varphi$ " to evaluate how far from the data $(j, y_j)_{j\in\mathbb{Z}^d}$ the function $\sum_{j\in\mathbb{Z}^d} y_j\,\varphi(\bullet - j)$ is.

We propose to say that a function $\varphi$ is a quasi-interpolant if and only if

a. $\varphi$ is $\mathbb{P}_1$-reproducing, i.e. $\forall p\in\mathbb{P}_1\ ,\ \sum_{j\in\mathbb{Z}^d} p(j)\,\varphi(\bullet - j) = p$

b. $\|1-\widetilde{\varphi}\|_2 < 1$

($\mathbb{P}_k$ is the set of all polynomials of degree at most $k$, and $\widetilde{\varphi}$ is the "transfer function" (or the "symbol function") of the function $\varphi$, i.e. the function defined by $\forall\omega\in\mathbb{R}^d\ ,\ \widetilde{\varphi}(\omega) = \sum_{j\in\mathbb{Z}^d} \varphi(j)\,e^{-2i\pi j\omega}$ .

Furthermore, we say that the quality of a quasi-interpolant $\varphi$ may be evaluated by the following four criteria (in the order of importance) :

i. $\|1-\widetilde{\varphi}\|_2$ is low (the closer to zero $\|1-\widetilde{\varphi}\|_2$ is, the better the quasi-interpolant is).

ii. $\forall\omega\in\mathbb{R}^d\ ,\ \widehat{\varphi}(\omega)\geq 0$ ($\widehat{\varphi}$ is the Fourier transform of $\varphi$ ).

iii. $\varphi$ is $\mathbb{P}_n$-reproducing for high value of $n$ (the bigger is $n$, the better the quasi-interpolant is).

iv. $\varphi$ is $\mathbb{P}_{n'}$-interpolating for high value of $n'$ (the bigger is $n'$, the better the quasi-interpolant is) (a function $\varphi$ is said to be $\mathbb{P}_{n'}$-interpolating if and only if $\forall p\in\mathbb{P}_{n'}\ ,\ \forall k\in\mathbb{Z}^d\ ,\ \sum_{j\in\mathbb{Z}^d} p(j)\,\varphi(k-j) = p(k)$ ).

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# BIFURCATION PHENOMENON IN A TOOL PATH COMPUTATION

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This paper examines a problem arising in Numerical Control. The case studied is specific: a toroidal cutter, with vertical axis, moves in simultaneous contact with a horizontal cylinder and with a vertical plane. The cutter climbs the cylinder until it reaches the ridge, at which point it must translate horizontally through a distance equal to the major radius of the torus before descending. Without this translation gouging of the cylindrical surface will occur. Two geometrical analysis of this situation are given, using different representations of the torus. The first analysis provides physical contacts, but features a discontinous trajectory at the top of the cylinder. The second analysis shows that the discontinuity comes from a bifurcation phenomenon. The bifurcation theory provides both the nature and the shape of the singularity. Thanks to this information, the complete and continous trajectory can be computed.

# Hölder Regularity of Subdivision Schemes and Wavelets

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**Abstract.** We study special functions obtained as limits of subdivision schemes in one dimension. Convergent dyadic subdivision schemes are used for designing regular or fractal curves, which yield compactly supported wavelet bases under additional conditions on the subdivision masks. A "discrete" approach is undertaken to study the existence and Hölder regularity of limit functions of dyadic or "$p/q$-adic" subdivision schemes—the latter is a flexible "rational" generalization which has already found application in digital signal processing.

Sharp regularity estimates are derived, which are proven to be *optimal* in the dyadic case; in constrast with earlier ones, they can be easily implemented on a computer and include, as a special case, exact Sobolev regularity estimates.

The main difficulty of the rational case, as compared to the dyadic one, is the lack of shift invariance: This results in an infinite number of different limit functions instead of shifted copies of a single one. In this case, a direct application of former ideas becomes impossible. However, our "discrete" approach allows us to extend the results of Daubechies and Lagarias (on Hölder regularity estimates based on infinite products of matrices) to the rational case.

## Riesz Bases and Frames
## for
## Shift-Invariant and Affinely-Invariant Subspaces of $L_2(\mathbf{R}^d)$

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Frames for principal shift-invariant and finitely generated shift-invariant subspaces of $L_2$ are studied first. The characterization of frames is done via the decomposition of the space into spectral "fibers", and imposes no smoothness or decay conditions on the generators. The results draw interesting connections between frames and tight frames on the one hand, and the notions of *quasi-stable* and *quasi-orthogonal* generating sets (as introduced by de Boor, DeVore and Ron in [1]) on the other hand. The characterization is then extended to infinitely generated shift-invariant frames (and Riesz bases).

The results apply directly to Weyl-Heisenberg frames, but, seems, however, to only reproduce (essentially) known results about this relatively accessible case.

The wavelet (=:affine) case is approached then via the idea of *one-sided* frames. It provides a characterization of affine Riesz bases (generated by one, finitely many, or countably many, functions) for $L_2(\mathbf{R}^d)$. We believe that result to be new even for the case $d = 1$, and, in fact, know of no complete characterization of Riesz bases even for univariate wavelet Riesz bases generated by a single function. Necessary conditions and sufficient conditions are easily derived from the above mentioned characterizations. The approach also results in a similar characterization of wavelet *frames*.

The entire work is joint with **Zuowei Shen**.

[1] C. de Boor, R. DeVore and A. Ron, The structure of shift invariant spaces and applications to approximation theory, CMS-TSR 92–08, University of Wisconsin-Madison, February 1992, J. Functional Anal., to appear.

## Interpolation with an Arc Length Constraint

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Theory and algorithms will be presented for the generation of a parametric curve $\mathbf{f}$ into $R^2$ or $R^3$ which interpolates a finite set of data points $P_0, P_1, \ldots, P_n$, preserves the shape of the data if possible, and attains a specified arc length $L$. Generally, unit tangent vectors $\mathbf{u}_0$ and $\mathbf{u}_n$ are specified at the two end points $P_0$ and $P_n$. We also allow for specification of tangent directions at all of the data points. Of course, we must have $L \geq \sum_{k=0}^{n-1} \mid P_k P_{k+1} \mid$ in order for such an $\mathbf{f}$ to exist since the shortest curve interpolating the data is the piecewise linear curve connecting the consecutive data points. Furthermore, if $L$ is too large, the shape indicated by the data may not be preserved since inflections or loops may be forced on $\mathbf{f}$.

This work extends and uses recent work by Roulier and by Roulier and Piper on length specification for the case of two data points and unit tangent vectors. In this work, the arc length is reduced to a function $L(\alpha)$ of a single variable $\alpha$. Conditions are given in which $L(\alpha)$ is strictly concave upward on the real line and tends to $+\infty$ as $\alpha \longrightarrow \pm\infty$. The goal then, is to find a value of $\alpha$ so that $L(\alpha) = L$. Once such an $\alpha$ is found, $\mathbf{f}$ is determined by that $\alpha$.

The form of the function $L(\alpha)$ allows the use of the secant method to rapidly produce an acceptable value of $\alpha$. An algorithm will be presented which implements these ideas for our case. No user input of starting values for the secant method is required. Several examples will be given.

II$^e$ congrés international "Courbes et Surfaces" - Chamonix - 10,16 juin 1993

# Reconstruction de Courbes

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Etant donné un ensemble de $N$ points dans le plan que nous supposerons appartenir à une courbe de ce plan, nous proposons ici une méthode de reconstruction d'un ordre sur les points conforme à celui unduit par un paramétrage naturel de la courbe, tel que celui de l'abscisse curviligne par exemple.

L'intérêt pour ce genre de problème à été suscité par de nombreuses applications en reconnaissance de formes ou en imagerie. Une approche de reconstruction basée sur des critères de proximité à été développée par Dedieu et Favardin [DF 91] et consiste, pour un point à une étape donnée, à rechercher le point suivant le plus proche sur la courbe.

Nous présentons une approche différente non basée sur une simple proximité, mais sur le fait que le cercle osculateur en un point $P$ d'une courbe permet d'ordonner localement les points dans un voisinage de $P$ ([B-G]).

Ainsi, la méthode que nous employons dans le cas de courbes sans auto-intersections procède en trois étapes principales, qui sont la découpe de la courbe en arcs ouverts au moyen d'un quadtree, puis l'approximation par un cercle de l'arc contenu dans chaque feuille de l'arbre construit précédement, ceci s'accompagnant de la projection des points initiaux sur le cercle calculé dans le but de les ordonner, et enfin une dernière étape consistant à raccorder entre eux les différents arcs ainsi calculés pour obtenir le contour final ordonné de la totalité de la courbe.

Nous examinons donc ici les problèmes liés au calcul d'un cercle approximant un nuage de points, et présentons la méthode de minimisation avec contraintes que nous avons retenue, ainsi qu'une interprétation géometrique de ces calculs. Nous étudions de plus les propriétés de convergence locale vers la courbe, ainsi que les bornes sur le rayon du cercle approximant en fonction des propriétés de courbure de la courbe. Nous examinons aussi dans quelles limites l'ordre des points projetés sur le cercle correspond à l'ordre induit par le paramétrage sur la courbe.

Nous exposerons d'autre part les choix algorithmiques que nous avons faits pour résoudre le problème du raccord des arcs de courbe, ainsi que la méthode retenue pour traiter le problème des auto-intersections.

La complexité de l'algorithme sera enfin étudiée, ainsi que son comportement et sa robustesse dans le cas de jeux de données perturbées.

## Bibliographie

**[B-G]** M. Berger et B. Gostiaux, *Géométrie différentielle.* Armand Colin.

**[DF 91]** J.-P. Dedieu et C. Favardin, *How to draw a curve using geometrical data.*
Curves & Surfaces, P.J. Laurent, A. Le Méhauté and L.L. Schumaker (eds.) pp. 135-138,1991.

Curves and Surfaces
CHAMONIX, France, June 10-16, 1993

# Lagrange and Hermite interpolation by bivariate quadratic splines on nonuniform type 2 triangulations

Paul SABLONNIERE, Rennes, France
Fatima JEEAWOCK-ZEDEK, Lille, France

Let $X_1 = \{ x_i, 0 \le i \le 2m \}$ (resp. $Y_1 = \{ y_j, 0 \le j \le 2n \}$) and $X_0 = \{ x_{2i}, 0 \le i \le m \}$ (resp. $Y_0 = \{ y_{2j}, 0 \le j \le n \}$) be two embedded partitions of $I = [x_0, x_{2m}]$ (resp. $J = [y_0, y_{2n}]$). Let $\tau$ be the triangulation of $\Omega = I \times J$ obtained by drawing diagonals in rectangles whose set of vertices is $X_1 \times Y_1$ and let $\mathcal{A} = X_0 \times Y_0$. Let $LS_2^1(\Omega,\tau)$ be the subspace of quadratic splines $S \in S_2^1(\Omega,\tau)$ whose partial derivative $D^{01}S$ (resp. $D^{10}S$) is piecewise linear and continuous on partition $X_0$ (resp. $Y_0$) of horizontal (resp. vertical) meshlines determined by $Y_0$ (resp. $X_0$). We prove that $\dim LS_2^1(\Omega,\tau) = 3(m+1)(n+1)$ and that each element S of this space is uniquely defined by $\{ S(A), DS(A), A \in \mathcal{A} \}$. With $f \in C^1(\Omega)$, we associate its Hermite interpolant Hf on $\mathcal{A}$. By replacing partial derivatives by finite differences, we construct a Lagrange interpolant $\mathcal{L}f = \Sigma f(A)L_A$ of f, where $L_A$ is a Lagrange quadratic spline with compact support. For $f \in C^3(\Omega)$, we compute constants in error bounds for the norms of $Hf - f$ and $\mathcal{L}f - f$ (and their derivatives). We also compute the $L^\infty$- norm of $\mathcal{L}$. All these quantities depending heavily on subpartitions $X_1 \subset X_0$ (resp. $Y_1 \subset Y_0$), we try to define a **reasonable choice** of these subsets in terms of the characteristic parameters of partitions $X_0$ and $Y_0$.

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# The Ubiquitous Ellipse

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**Abstract**

We discuss three different affine invariant evolution processes for smoothing planar curves. The first one is derived from a *geometric heat-type flow*, both the initial and the smoothed curves being continuous. The second smoothing process is obtained from a discretization of this affine heat equation. In this case the curves are represented by planar *polygons*. The third process is based on *B-spline* approximations. For this process, the initial curve is a planar polygon, and the smoothed curves are continuous. We show that, in the limit, all three affine invariant smoothing processes collapse any initial curve into an *elliptic point*.

# Axial convexity - a well shaped shape property

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Motivated by the non-preservation of convexity by multivariate Bernstein polynomials, the notion of axial convexity was introduced in order to have a reasonable generalization of classical convexity to the multivariate setting that is more compatible with Bernstein polynomials and Bézier patches, respectively.

Indeed, axial convexity, defined by plain geometric arguments, is perceptable by the human eye and in this sense is called a "shape property". It provides various properties that are well-known and appreciated for the case of classical convexity, hence, it seems to be an appropriate generalization of convexity to the multivariate case. More precisely, one can show that, if resonably defined, axial convexity of Bézier nets is preserved by degree elevation and causes the resulting Bézier patch to be axially convex, too; moreover, axially convex functions have monotonous convergence of the associated Bernstein polynomials and satisfy a Jensen inequality as well as a maximum principle.

On the other hand, this notion enables the design of surfaces that e.g. contain saddle-points (and hence are ``visibly" non-convex) having nevertheless a describable convex behavior.

# PARTIAL DIFFERENTIAL EQUATION TECHNIQUES FOR 3D SURFACE DETERMINATION

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**Abstract**:

Current methods for 3D-surface reconstruction of medical images (MNR, CAT ...) are essentially bidimensional; segmentation is performed for each section and the surface is generated and visualized by a spline-model.

In this paper we present two direct methods to detect the surface:

1) We present a model of anisotropic diffussion equation proposed by Alvarez-Lions-Morel, which detect the object and by classical thersholding we detect their contour.

2) The second one is a new model of active contour (snakes) based on a partial differential equation proposed by Caselles-Catte-Coll-Dibos. ¿From this we obtain the surface of the object as limit of the evolution of an initial surface containing the object.

To visualize the detected surface we construct a model wire-frame from the contours of the object and render it with the classical techniques of computer graphics.

We present results on MNR images of brain tumours.

# Polynomial Reproduction Properties of Radial Basis Function Interpolants

**Robert Schaback, Göttingen**

For radial basis function interpolation of scattered data in a bounded region of $d$-space, the approximative reproduction of high-degree polynomials is studied. On the theoretical side, error estimates and convergence orders are proven, while on the practical side some numerical experiments are reported. The results roughly imply that all currently available radial basis functions will reproduce polynomials of arbitrary degree with arbitrary precision on arbitrary compact sets, provided that sufficiently many well-distributed centers are used. However, the attainable uniform convergence orders on compact sets will strongly depend on the given radial basis function and the degree of the given polynomial.

# Lifting of Curves and Surfaces in CAGD


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The lifting of curves and surfaces has become of great interest for instance in the automobile industry. Car bodies are designed using B-Spline curves and surfaces today. Manipulating control points is a widely used tool to alter the shape of a B-Spline surface. However in many applications the designer would rather like to manipulate a point on the surface itself. A direct manipulation could be the displacement of a single point on the surface into the ambient space. After the designer has defined such a displacement, he wants the original surface to follow this displacement. This operation is called *LIFTING*.

The first part of the talk will present solutions for the lifting of a single curve resp. surface, where it is assumed that $G^1$ continuity should be retained at the boundary. The question of how to preserve the shape of a curve resp. surface during the lifting process will also be addressed. In this context shape preservation is understood in the sense that for instance convex curves resp. surfaces remain convex after lifting is carried out.

The second part of the talk is dedicated to the case, where the effect of lifting a single surface stretches out to neighboring surfaces. In this case lifting will introduce gaps between adjacent patches. The treatment of how to close these gaps includes also trimmed surfaces, which are most common in the design of a car body. The talk gives an overview on the different types of boundary adjustments and the methods associated with them. Basically adjusting a parameterline boundary to a given curve in 3-D space must be treated entirely differently than adjusting a trimming curve boundary. The solutions are in most cases approximate ones. One reason for this is the fact that representing a trimming curve exactly would require a very high polynomial degree.

The talk concludes with some examples discussing the several types of boundary adjustments.

# Variation Diminution and Blossoming for Curves and Surfaces

**Gerd Schmeltz**

The variation diminishing property of some curve representation schemes has proven useful in many ways. This talk explores an idea to extend this notion to surfaces. While it is known that direct translations of the geometric definition result in properties not shared by Bézier surfaces (which seems inappropriate), another approach is used here: First, generalized blossoms are defined and shown to exist iff the underlying curve representation scheme is variation diminishing. Then analogous blossom structures for surfaces are formulated and investigated. They turn out to be far more more restrictive there than in the case of curves. It is shown that only rational Bézier triangles with positive weights have this property.

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**Interpolation with $C^1$-Splines under Positivity Constraints**

Jochen W. Schmidt, Dresden University of Technology

There are several problems of practical interest which lead to the interpolation of data sets under positivity constraints, and the interpolating functions should be of $C^1$-continuity at least. For positive interpolation polynomials are unsuited while the use of spline functions is much more promising. These, of course, should be as simple as possible. As it is well-known, positive interpolation with quadratic splines is not always successful if the spline knots are equal to the data abscissas. There exist data sets with positive ordinates such that all quadratic spline interpolants are not nonnegative everywhere. But we can offer particular rational quadratic $C^1$-splines such that positivity is always preserved provided the occuring rationality parameters are chosen accordingly. This approach is extended to the positive interpolation of gridded data in 2D as well as in 3D. The rational bi- and tri-quadratic splines used then are tensor products of the splines in 1D mentioned before.

# Approximation with Helix Splines

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Helix splines are spline curves with (piecewise) constant curvature and constant torsion. They are of interest for milling machines. A helix describes a simple motion, so helix splines can be useful for kinematics.

We approximate a given set of points $\mathbf{P}_k$ in $E^3$ by a helix spline with $n$ segments minimizing a least squares objective function. This nonlinear problem is solved with nonlinear optimization programs. In every step of an optimization program a helix spline is constructed from the unkowns. We discuss different approaches to define the unknowns.

To run the optimization program an initial guess is necessary. We calculate it by using a heuristic with differential geometric arguments which gives also an estimate for the needed number of segments $n$.

Since a helix is a transcendental curve it cannot be described by polynomial or rational curves. To deal with the problem in systems based on rational polynomials we introduce an approximating rational Bézier spline with small error (including small error in curvature and torsion) from which the *exact* data of the helix, i.e. axis direction, opening angle and pitch, can be reconstructed without error.

# A Solution to the Subdivision Problem for Triangular B-Splines

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**Abstract**

A new triangular B-spline scheme has recently been developed by Dahmen, Micchelli and Seidel. The scheme is based on blending functions and control points and allows to model $C^{n-1}$-continuous piecewise polynomial surfaces of degree $n$ over arbitrary triangulations of the parameter plane. The surface scheme exhibits both affine invariance, and the convex hull property, and the control points can be used to manipulate the shape of the surface locally [1]. Any piecewise polynomial can be represented by the new scheme [2]. A first test-implementation of the new scheme has succeeded in demonstrating the practical feasibility of its fundamental algorithms: Quadratic and cubic surfaces over arbitrary triangulations can be edited and rendered in real-time. Applications like the filling of polygonal holes, or the construction of smooth blends, demonstrate the potential of the new scheme when dealing with concrete design problems [3].

In this talk we provide a solution to the following *subdivision problem for triangular B-splines:* Let $T$ be a triangulation of $\mathbb{R}^2$ and let $\tilde{T}$ be an arbitrary subtriangulation. Is it possible to represent any triangular B-spline surface $F$ over $T$ as a B-spline surface over $\tilde{T}$?

We show that the answer to this problem is always positive (independent of the specific choice of knots assigned to every vertex (knot clouds)), and provide explicit formulas for the new control points $\tilde{\mathbf{c}}_{\tilde{I}}$ of $F$ over $\tilde{T}$ in terms of the old control points $\mathbf{c}_I$ of $F$ over $T$.

The resulting algorithm generalizes the well-known knot insertion algorithm for univariate B-splines to the multivariate setting.

[1] W. Dahmen, C.A. Micchelli, and H.-P. Seidel. Blossoming begets B-splines built better by B-patches. *Math. Comp.*, 59:97–115, 1992.

[2] H.-P. Seidel. Representing piecewise polynomials as linear combinations of multivariate B-splines. In T. Lyche and L. L. Schumaker, editors, *Curves and Surfaces*, pages 559–566. Academic Press, 1992.

[3] P. Fong and H.-P. Seidel. An implementation of multivariate B-spline surfaces over arbitrary triangulations. *Computer-Aided Geom. Design*, 10(1), 1993.

**Modeling of geological surfaces using finite elements**

Jörn Springer, GeoForschungsZentrum Potsdam

Many geological structures are described by surfaces which represent the boundaries between different geological units. These geological surfaces are known very incompletely and must therefore be reconstructed by use of interpolation methods.

In regions of undisturbed sedimentation almost horizontal surfaces develop which may be described by a continuous function $z = u(x,y)$, where z is the vertical coordinate (depth). However, tectonic stresses cause discontinuities (faults) and more complicated surfaces (folds) .

Modeling of general geological surfaces leads to the following mathematical problems:

1. From a set of given points $(x_i,y_i,z_i)$; $i = 1,...,n$, determine a function $u(x,y)$ with $u(x_i,y_i)=z_i$, $i = 1,...,n$. The resulting surface should be "as smooth as possible" to show not more features than are included in the data. This is the classical two-dimensional interpolation problem. For application on geological surfaces it should be possible to prescribe additionally the slope of the surface (i.e. the partial derivatives $u_x$ and $u_y$) at some or all of the data points.
2. The same interpolation problem is to be solved if discontinuities are allowed along given lines in the xy plane. The allowed discontinuities usually lead to a smoother surface in other regions.
3. In the presence of folds a one-valued function $u(x,y)$ may no longer suffice, a parameter representation of the surface must be used instead and the interpolation algorithm must be generalized.

To solve these problems we propose an algorithm using triangular finite elements. In problem 1 the triangle corners are identical with the data points, in case 2 additional points along the discontinuities are introduced. Among all functions satisfying the given constraints (u and possibly $u_x$ and $u_y$ at the data points) the sought function shall minimize an integral (potential energy of a thin elastic plate) .

For modeling a general surface in space (problem 3) beside the data points a connecting triangular mesh and normal vectors at each data point (computable from neighbour points) are necessary. Over each triangle the interpolation is done again by a finite element. To ensure a continuous and smooth surface over all triangles the algorithm is properly modified.

Some Quantitative Aspects of Wavelet Bases

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Different wavelet bases have been constructed for one-dimensional and multi-dimensional $L^2$-spaces. One typical requirement for such bases is the fast decay of the basis functions and of their Fourier transforms. Usually this is described in terms of polynomial or exponential decay. The decay properties are essential in order to obtain local information about the $L^2$-function $f$ by its wavelet coefficients at the same time as local information about its Fourier transform $\widehat{f}$.

A finer measure for the localization properties is the so-called *time-frequency window* associated with the wavelet function $\psi$. In this talk we give a quantitative analysis of the time-frequency window of univariate wavelets. The wavelets under consideration are defined by a positive definite scaling function which generates a multiresolution analysis. The method employed by us gives sharp bounds for orthogonal spline-wavelets of even order. The method also applies to "non-stationary" wavelets which can be obtained from a multiresolution of radial functions such as Hardy's multiquadrics.

# Periodic Spline–Wavelets

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In order to analyse periodic functions or functions defined on a bounded interval, it is necessary to develop the theory of periodic wavelets. In this talk we introduce $N$–periodic spline–wavelets analogously to the semiorthogonal cardinal spline–wavelets due to C.K. Chui (Introduction to Spline–Wavelets, Academic Press 1992). It is our aim to present a simple constructive approach to periodic wavelets.

Let $\varphi_j$ ($j = 0, 1, \ldots$) be the $N$–periodization of $N_m(2^j \cdot)$, where $N_m$ denotes the cardinal B–spline of order $m \geq 1$. Then for $j = 0, 1, \ldots$ we define following subspaces of the Hilbert space $L^2_N$

$$V_j := \operatorname{span}\{\varphi_j(\cdot - 2^{-j}k)\,;\, j = 0, \ldots, 2^j N - 1\}\,, \quad V_{-1} := \{0\}\,.$$

Then these subspaces form an $N$–periodic multiresolution analysis related to $N_m$. In particular, using Euler–Frobenius polynomials we are able to present rational optimal Riesz–bounds for the spline basis of $V_j$. More details on this topic and on the new technique of proof will be given in the talk of G. Plonka (Rostock).

We describe a new connection between reconstruction relations and decomposition relations based on a block circulant matrix and Fourier transform technique. The condition of this matrix is determined by the optimal lower Riesz–bound of the spline–basis of $V_0$. One can see that spline–wavelets of order $m \geq 6$ leads to numerical instabilities. Finally, we get the reconstruction algorithm and the decomposition algorithm. Applying FFT we present some numerical tests with periodic spline–wavelets.

If we extend our results to cardinal spline–wavelets, then we get some improvements of the known approach of C.K. Chui.

## Splines in a Topological Setting

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*Abstract*

We define a topological data structure, called *Handle-Edge*, and its corresponding structural operators, *Morse Operators*, which can deal, in the same environment, with surfaces embeded in $n$-dimensional spaces. The whole philosophy is object-oriented.

One of the main features of this structure is that it can handle surfaces represented in several forms like splines, implicit, etc. in the same setting allowing the manipulation and visualization of these surfaces in a unique computational sheme.

In this talk we will be describing how splines can profit from a topological environment by extending the Handle-Edge [1] data structure to this case.

Slides and video will be shown.

[1] G. Tavares, A. Castelo, H. Lopes - Handlebody Representation for Surfaces and Morse Operators. In J. Warren, Proc. on Curves and Surfaces in Computer Vision and Graphics III. SPIE (The International Society for Optical Engineering),

## Designing of a "progressive lens"

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**Abstract:**

We introduce a new method for designing a "progressive lens". This lens is represented by a sufficiently regular surface. On this surface we prescribe conditions on its principal curvatures in some regions (far vision region and near vision region) and other conditions on its principal directions of curvature in other regions ( nasal region and temporal region). The surface is described with tensor product B-splines of degre four. For its computation, we have to minimize a non-quadratic operator.

This minimization is then processed by an iterative method for which quick convergence is numerically tested.

# Approximation of Parametric Surfaces with Discontinuities by Discret Smoothing $D^m$-splines

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In this work, we consider the problem of fitting discontinuous parametric surfaces when the data points are part of the nodes of a curvilinear grid. This problem may appear in some fields, such as Geophysics or CAD, for modeling reverse faults or objects with cuts.

We state the problem in the following way: Let $S$ be a surface defined by a parameterization $f : D \to \mathbb{R}^3$. We suppose that $f$ is discontinuous (giving rise to faults or holes in the surface) and that $f$ belongs to the Sobolev space $H^m(D; \mathbb{R}^3)$, with $m \geq 2$. For any $n \in \mathbb{N}$, let $P_n$ be a given subset of the set of vertices of a curve network lying on $S$ that divides this surface into a finite number of rectangular (eventually triangular) patches. The objective is then to construct, for any $n \in \mathbb{N}$, an approximating surface from the set $P_n$.

First, we assign parameter values to the data points. For doing this, we fix a suitable planar domain $\overline{\Omega}$, tesselated into a grid of equal squares, and then we attach, in order, the points in $P_n$ to part of the vertices of these squares. We also associate the fault edges with a subset $\Phi$ of $\overline{\Omega}$. Next, we construct a parametric finite element space $V_n^3$, contained in the Sobolev space $H^m(\Omega; \mathbb{R}^3)$, in which searching for the parameterization of an approximating surface. This space is defined from the Bogner–Fox–Schmit generic finite element of class $C^k$, making some slight modifications in the basis functions in order to allow discontinuities on $\Phi$.

Then we consider the following minimization problem: find $\sigma_n$ such that

$$(\mathcal{P}) \qquad \begin{cases} \sigma_n \in V_n^3, \\ \forall v \in V_n^3, \; J_n(\sigma_n) \leq J_n(v), \end{cases}$$

$J_n$ being the functional defined on $V_n^3$ by

$$J_n(v) = \sum_{b \in B} \|v(b) - p_b\|^2 + \varepsilon_n |v|^2_{m,\Omega,\mathbb{R}^3},$$

where $B$ stands for the set of points in $\overline{\Omega}$ attached to data points, $p_b$ is the data point corresponding to $b \in B$, $\varepsilon_n$ is a given real positive number, and $| \ . \ |_{m,\Omega,\mathbb{R}^3}$ is the Sobolev semi–norm of order $m$ in the space $H^m(\Omega; \mathbb{R}^3)$.

It is easily shown that, under suitable hypotheses, problem $(\mathcal{P})$ has a unique solution, called *discret smoothing $D^m$-spline*, which is also the unique solution of an equivalent variational problem. From this one we finally obtain three linear systems having the same symmetric, positive definite matrix. These systems yield the coefficients of $\sigma_n$ as a linear combination of the basis functions of $V_n^3$.

The convergence of the method is studied and some numerical and graphical examples are also given.

# NATURAL SPLINES

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The concept of Natural Splines has been recently developped starting from the well known idea of natural neighbors created by R. Sibson in the early eighties. Redefining Sibson's interpolant using covering spheres and Bernstein polynomials some interesting results are found. The first one, a clear diferentiation of the field of definition of each interpolating function. The second, a natural way of upgrading the aproximating polynomials and a good geometrical interpretation of such upgrading. Finally, the redefinition provides the link with spline theory creating Natural Splines that is, splines based on natural neighbors whose field of definition are the same of Sibson's interpolant. It is shown that the combination of all this concepts becomes a useful tool in many fields, and that a similar reasoning can be used to find finite element aplications.

# Multiplication as a General Operation for Splines

Kenji Ueda

## Abstract

Degree raising and knot insertion are useful tools in making different B-spline curves and surfaces compatible. Multiplication, which is a basic B-spline function operation, can make different rational B-splines compatible to have a common weight function.

This talk presents a blossoming method for obtaining spline function products and show its equivalence to Mørken's algorithm based on discrete B-splines. This spline multiplication generalizes not only degree raising but also knot insertion and rational spline compatibility operation. The method is applied to surface design problems demonstrating its effect and added flexibility.

# Spline Finite Element Software


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Abstract:
The spline field approximation method is used to determine the load-deflection behaviour of a structure. For concreteness, we consider in this talk the non-linear load-deformation behaviour of a two dimensional bicycle wheel model. The deflection of each spoke, and the deflection of the rim, are approximated by B-spline functions; the control points for these splines are determined by solving equations obtained using the Rayleigh-Ritz procedure. In order that the spokes and the rim are joined properly, the spoke deflection splines are constrained so that the deflection of the spoke at the rim is equal to the rim deflection.

The software used to implement this application falls into two catagories: application independent software objects, and application dependent objects. Most of the application independent objects represent concepts from CAGD, such as splines, or concepts from general mathematics, such as non-linear equations. The application dependent objects serve largely as glue which holds the application together and allows the different pieces to communicate with each other. We'll show how these various software objects relate to each other, and discuss the potential for creating flexible code using object oriented design.

# WAVELET ANALYSIS OF REFINEMENT EQUATION SOLUTIONS

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A basis of smooth wavelets can be used to analyze the regularity of a compactly supported solution to a refinement equation. As a result of this procedure, it turns out that there is a simple relation between the exact Besov smoothness of the solution and the spectral radius of a subdivision operator associated with the the coefficients of the equation. This operator acts on all sequence spaces $\ell^p(\mathbb{Z})$, and the spectral radius depends on $p$.

Special choices of $p$ lead to both well known and some new criteria for continuity, Hölder continuity, Sobolev regularity, differentiability, integrability and absolute continuity. In some cases, estimates of the fractal (box counting) dimension of the graph of the refinement equation solution can be obtained.

We illustrate how the theory can be applied to give an elementary proof of the optimal continuity range of a one parameter family of 4-point dyadic interpolation schemes, considered both by Dubuc and Deslauriers as well as Dyn, Gregory and Levin. For a special choice of the parameter, it is known that the solution is almost everywhere twice differentiable but not two times continuously differentiable. This fact can also be obtained by coarse estimates of the spectral radii mentioned above.

Finally, we discuss possible extensions of the method to several dimensions.

**Spline conversion for present-day CAD data exchange problems**

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At present, data exchange between CAD systems still causes problems. Most CAD users are not interested what causes these problems but like to have them solved. To improve the data exchange process between a specific group of CAD systems namely surface modellers, spline conversion is essential.

The TNO CAD Centre is developing a so called 'Help Desk' which must enable companies to improve their data exchange process. This 'Help Desk' consists, amongst others, of a number of spline conversion routines and spline comparison routines. An overview will be given of the structure of this 'Help Desk'. The spline conversions which are essential for the improvement of the data exchange process will be presented. Further, a number of problems with the spline comparison methods will be discussed.

# Symmetric Tchebycheffian B–spline schemes

by M.G. Wagner and H. Pottmann

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Tchebycheffian B–spline schemes are a natural generalization of the polynomial B–spline scheme and possess essentially all properties which are considered important for CAGD. Recently, the importance of these free form schemes was shown by Lempel and Seroussi (cf. [1]). In this paper we use a more geometric approach. Based on the theory of extended Tchebycheffian spaces and the associated normal curves, one can define a blossoming method and a de Casteljau algorithm for the construction of the associated free form curves (cf. [2]).

We now determine all Tchebycheffian spline segments with the following symmetry property: Over each interval, the Tchebycheffian Bézier curve to the control points $\mathbf{b}_0 \ldots \mathbf{b}_m$ is the same as the curve to $\mathbf{b}_m \ldots \mathbf{b}_0$. The question is solved at hand of the corresponding normal curve which turns out to be an affine W–curve. The underlying function space is the null space of a self–adjoint linear differential operator. In case of order three one gets cubic spline curves, exponential spline segements in tension, and helix spline segments associated with the function space $\{1, t, \sin(t), \cos(t)\}$ as the only possible solutions.

[1] LEMPEL, A. and SEROUSSI, G. (1992), Systematic derivation of spline bases, Computer Aided Geometric Design 9, 349–363.

[2] POTTMANN, H. (1993), The geometry of Tchebycheffian splines, Computer Aided Geometric Design, to appear.

# Ondelettes et Interpolation de courbes et surfaces par des fonctions radiales

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**Résumé**

L'interpolation des courbes et surfaces par les fonctions radiales [2,5] passe par la résolution de systèmes linéaires très denses et de grandes tailles si le nombre de données à interpoler est grand, ce qui est souvent le cas. Le caractère dense et de grande taille du système sont les principales difficultés pour la résolution de ces systèmes linéaires, ce qui limite l'utilisation de cette méthode d'interpolation qui est reconnue comme l'une des meilleures [2].

Pour faire face à ces difficultés, nous proposons de rendre creuse la matrice du système en utilisant l'algorithme dû à Beylkin et *al.* [6] fondé sur une représentation de la matrice dans une base d'ondelettes [4,3,7].

Dans cet article, nous posons le problème général d'interpolation des surfaces par des fonctions radiales [5]. Un bref rappel sera fait d'une part sur les conditions d'existence et d'unicité de la solution [1] et d'autre part sur la formulation matricielle du système linéaire correspondant [2,5]. La mise en oeuvre de l'algorithme de Beylkin et *al.* montre que l'on peut rendre la matrice du système très creuse tout en conservant une bonne précision des résultats, pour différentes fonctions radiales [5] d'où l'accélération des calculs. Nous montrons : i) que les ondelettes splines [4] ne sont pas adaptées à ce type de problème ; ii) que les ondelettes à support compact [3] donnent de meilleurs résultats et enfin nous présentons des calculs obtenus avec des ondelettes biorthogonales [7].

## Références

[1] C. Micchelli, *Interpolation of scattered data: distance matrices and conditionnally positive definite functions*; Const. Approx., Vol. 2, pp. 11-12, 1986.

[2] R. Franke, *Recent advances in approximation of surfaces from scattered data*; Multivariate Approximation, C.K. Chui, L.L Schumaker and F. Utreras; Ed. Academic Press, pp. 79-98, 1987.

[3] I. Daubechies, *Orthonormal bases of compactly supported wavelets*; Comm. Pure Appl. Math., Vol. 41, pp. 909-996, Nov. 1988.

[4] P. G. Lemarié, *Nouvelle base d'ondelettes de $L^2(R^n)$*; Comm. Math. Pure Appli., Vol. 67, pp. 227-236, 1988.

[5] N. Dyn, *Interpolation and approximation of scattered data by radial and related functions*; Approximation Theory VI, C. K. Chui, L. L Schumaker and J. D. Wards: Ed. Academic Press, pp. 1-24, 1989.

[6] G. Beylkin, R. Coifman, V. Rokhlin, *Fast wavelet transforms and numerical algorithms I.*; Comm. Pure Appl. Math., Vol. 44, pp. 141-183, 1991.

[7] A. Cohen, I. Daubechies and J. C. Feauveau, *Biorthogonal Basis of Compactly Supported Wavelets*; Comm. Pure Appl. Math., Vol. 44, pp. 141-183, 1991.

# $C^n$ Local Control Interpolation of an Arbitrary Mesh

by

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## ABSTRACT

Given a mesh in $R^3$ constructed from curves with continuity $C^n$, the problem of finding a surface of continuity of some specified continuity $C^k$ which contains the given mesh has a long and venerable history and possesses a variety of different types of solutions. In the case that n and k equal 1 or 2 solutions have applications in Computer Aided Design situations in which designers first construct models constructed from curved meshes.

Solutions which satisfy the additional the constraint of "local control" - namely that modification of the surface by slightly changing one of the curves of the mesh should effect the resulting surface only in a local region of the change - have distinct benefit to the CAD community. Existing solutions to the general problem either do not satisfy the constraint of local control or impose constraints on the curves of the mesh - typically they are assumed to be Bezier splines.

The solution to be presented satisfies the local control condition, allows the curves of the mesh to be arbitrary, and is of relatively low degree in the case of a mesh consisting of piecewise polynomial curves. The solution employs the notion first introduced by Catmull and Rom that judiciously chosen "control functions" may be substituted for control vertices. In particular, the solution constructed may be considered as a B-spline averaging of surfaces which locally interpolate the given mesh.

# Multi-sided rational surface patches with independent boundary control

**Joe Warren**

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Various schemes have been proposed for creating rational surface patches with a variable number of sides. Although these schemes appear to be unrelated, they all produce surface parameterizations that have parameter values for which the numerator and denominator of the defining rational functions vanish simultaneously. These parameter values are base points of the rational parameterization.

This paper shows that any rational surface scheme permitting the independent control of non-adjacent boundary curves must produce parameterizations possessing base points. Locating these base points allows for a classification of possible multi-sided rational surface schemes.

Constrained surface fitting using Powell-Sabin splines

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We will consider the problem of smoothing a discrete set of data values corresponding to points arbitrarily scattered in a bounded area. Hereby data reduction is regarded as an important issue and the approximation should satisfy some additional conditions.

Although tensor product splines have already proven their usefulness while dealing with smoothing problems, in some cases they are not very appropriate.

Therefore we will consider in this talk smoothing with Powell-Sabin splines and we will particularly pay attention to the imposition of additional conditions such as convexity conditions, boundary conditions, ... We will comment on the translation of these conditions into a suitable set of constraints, on the special structure of the systems that originate from these constraints and on the solution of these systems taking account of their specific structure. Our talk will be illustrated with many practical examples.

# A NEW MULTIVARIATE BOX-SPLINE FORMULATION FOR IMAGE COMPRESSION AND RECONSTRUCTION

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**Abstract.** In joint work with Harvey Diamond and Louise Raphael, we present an alternative approach to data compression and reconstruction. Our algorithm begins by first calculating an intensity function of the images by using a finite sequence of approximations by multivariate box-splines at successively halved resolutions. The key is that these approximations are quasi-interpolants, which are nearly orthogonal projections. This fact ensures the local dependence of the image function approximation . This contrasts with customary spline and wavelet methods which employ exact orthogonal projections.

# Piecewise Cubic Interpolation Surfaces

Zhen-Xiang Xiong

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We have constructed a kind of multivariate piecewise interpolation polynomials of degree 2n-1 that is a kind of generalization from the univariate interpolation spline of degree 2n-1. It may also be called 'interpolation spline', but is different from that defined by Prof. de Boor. We have discussed in great detail about the bivariate cubic case. It can be used to fit and to design surfaces. Some results are as follows:

1. The system of equations and recurrence formulae for determining the unknown partial derivatives in its expression have been obtained under certain continuity requirments.

2. The property that its convexities can be determined by those at the knots has been proved.

3. A simple method for finding the intersection of two such surfaces has been derived.

4. Each of its rth partial derivatives appraoches the corresponding one of those of the fitted surface with the approximation order $4-r$ $(r=0,1,2,3)$.

This kind of cubic surfaces has been used to fit blue-print surfaces successfully. As a function of this kind of cubic surface, it has been used to solve the boundary problems of linear and quasi-linear P.D.E. successfully.

**Approximation of curves and surfaces by the exclusion algorithm.**

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In general way an exclusion algorithm permits to localize an variety of $R^n$.We here give the numerical results concerning curves and surfaces.We describe the two ingredients which are used:

1- the computation of the " best " exclusion function.

2- the geometrical organization of the algorithm.

We also explain how we represent curves and surfaces.

# The Optimal Error Bounds For The Cubic Spline Interpolation

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Abstract

Let $s_f(x)$ be the cubic spline interpolating to $f(x)$ on the equidistant knots of $[0, 1]$. The optimal error bound of the interpolation is defind by

$$C_{m,r} = \sup_{\substack{n \geq 1 \\ f \in C^m[0,1]}} \frac{\| s_f^{(r)}(x) - f^{(r)}(x) \|_\infty}{\| f^{(m)}(x) \|_\infty \, h^{m-r}}, \qquad 1 \leq m \leq 4, \quad 0 \leq r \leq \min\{m, 3\}.$$

We give the explicit expression of the kenel of the cubic spline interpolation, then exactly obtain the optimal error bounds for the cubic spline interpolation.

# B-Spline Patches for Surface Reconstruction

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This paper considers the problem of 3D surface reconstruction using B-spline patches in computer vision. This reconstruction is done by observing the occluding contours where the view lines gaze the surface. It has been shown that reconstruction of such a 3D surface is possible when the camera motion is known, except for the fully concave parts where no view line can be tangent to the surface.

Our original contributions with respect to these related works are mainly twofold. Firstly, we introduce a direct regularization on the 3D surface to be estimated instead of smoothing the contours in the 2D image. This is based on the regularized uniform bicubic B-spline patches. Secondly, we propose to globally recover the whole surface from small local patches. The others have only been content with the recovery of some local properties of the surface such as curvature estimation.

An image point of an observed occluding contour provides exactly three independent equations. Two of them state that the surface is reached by the view line associated with the image coordinates. The third equation expresses that the view line is tangent to the contour at the point which generates a plane tangent to the surface.

Therefore, each contour point gives a set of three non-linear equations, this leads to an over-determined system of non-linear equations for all observed points set in the image sequence. This system can then be solved by a non-linear least squares estimation, for example, by the Levenberg-Marquardt's algorithm which is currently implemented in our system. Parametrization of the surface patches has to be decided for implementing this approach. The choice we made is very close to the epipolar parametrization described by Blake and Cipolla. The starting values of the system is provided by the simple method of triangulation.

After such a patch is estimated, the problem remains of fusing these different patches obtainted with different motions. This is done by going back to a discrete representation with a net of 3D points.

We have experimented on some simulated smooth objects. The reconstruction process provided good results and its numerical behaviour was studied by adding different pixel noise to image points. We showed also how the different parameters should be conveniently selected according to available data through experiments. The experiments on real image sequence are undergoing.

# INDEX