ANALYTIC INVESTIGATION OF THE PROCESSES

IN A FERRITE CORE

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By Ye. I. Klepfer and G. M. Tikhomirov

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1. DERIVATION OF THE EQUATION OF THE HYSTERESIS
LOOP

The relation between the magnetic field strength
in a space occupied by any given medium and the magnetic
induction is expressed by the well known equation

$$B = \mu H \qquad (1)$$

The magnetic permeability μ is a physical quan-
tity which characterizes the magnetic properties of the
medium.

As is known, equation (1) can be expanded as
follows:

$$B = \mu_0 H + 4\pi I \qquad (2)$$

Here μ_0 is the magnetic permeability of vacuum, I is the
magnetic strength which characterizes the addition to the
fundamental magnetic process that has been introduced by
the medium. For a vacuum we always have $I = 0$. In any me-
dium $I \neq 0$. In the case of iron and ferromagnetic mate-
rials $I > 0$ and is relatively much more important.

According to current views of the nature of ferro-

magnetism, a ferrous magnet represents the sum of elementary small magnets which can rotate about their axes without encountering any resistance or analogous friction from neighboring elementary magnets, but which at the same time act magnetically on one another. In the normal state, during the absence of an external magnetic field, all the small magnets separate into a series of diverse closed groups which possess relative stability of domains. Upon magnetization of the body by an external field these groups gradually fall apart and form stable new groupings as indicated by the external field. This process continues until all of the small elementary magnets are oriented in the field, i. e., until saturation.
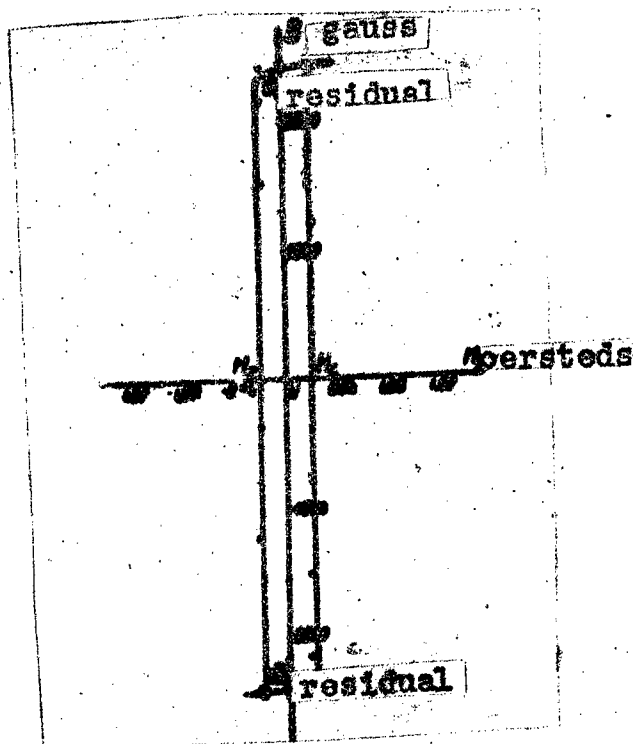
Fig. 1. "Static" hysteresis loop of ferrite

Dimensions: $K = 65$ (7x4x2.1 mm); $K = 29.75$;
$\blacksquare = 0.5$; $\blacksquare = 0.866$; $H_m = 0.54$ oersteds;
$H_c = 0.27$ oersteds; $B_m = 2440$ gauss
$B_1 = 2400$ gauss; $\mu_0 = 1$.
The solid line shows the experimentally obtained curve. The computed points are shown by dots.

During this process the intensity of magnetization I reaches a maximum and any further increase in magnetic induction will take place, as in a vacuum, at the expense of $\mu_0 H$ in equation (2).

After the external magnetic field ceases to act, the small elementary magnets form new groupings. The predominant orientation of the small magnets imposed by the external field is preserved to a definite extent in the new groupings. This determines the residual magnetism of the body B_{res} (Fig. 1). In order to eliminate the residual magnetism it is necessary to apply an external field strength H in the opposite direction.

The losses of energy incured during the magnetization cycle (hysteresis losses) are explained by the setting into oscillation of the small elementary magnets during regrouping and these losses produce an increase in the kinetic energy of the molecules, i. e., a heat effect.

According to the above, the intensity of magnetization I determines that increase in magnetic induction of the medium which arises as a result of the predominant orientation of the small elementary magnets by the external field.

Each small elementary magnet has a very definite magnetic moment.

$$M = mL, \qquad\qquad (3)$$

where m is the magnetic mass of one pole;
L is the distance between the poles of the small magnet, taken as a vector.

We note that representation of the magnetic mass and the distance between the poles of a small elementary magnet is only a convention. Equation (3) emphasizes the fact that each elementary magnet creates a magnetic field whose absolute quantity is independent of external condition but whose direction depends upon the physical state of the body, the nature of the external field and the degree of interaction with surrounding small magnets. All these elementary small magnets have a differently composed magnetic moment in the direction in which the exter-

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nal magnetic field strength H is acting. Let φ be the angle of inclination of the axes of the elementary small magnet to the direction of H.

We will accept the fact that the probability of a given small elementary magnet having a $\cos\varphi = \eta$ is equal to

$$P(\eta_1, \eta_1 < \eta_2) = \frac{2}{\sqrt{2\pi\sigma}} \int_{\eta_1}^{\eta_2} e^{-\frac{(x-a)^2}{2\sigma^2}}\, dx \tag{4}$$

i. e., the distribution of elementary magnets along the cosine is taken as normal since such a distribution is encountered in practice each time there is a variation in the observed effect or magnitude (in our case due to the action of a large number of forces which are mutually additive, almost independent, and small in comparison to the total. Such a summation is usually based on the fundamental law which determines the average effect and leads to a "normal" distribution curve. The normal law is observed, i. g., in the distribution of the velocity components of gas molecules, in the dispersion of the coordinates of particles undergoing Brownian movements, and in the variability of plant organisms. Let us assume that it also acts in the case we are considering.

In place of the exact equation (4) we will introduce an approximation substituting probability for relative frequency of the event (frequency).

$$\frac{n}{N} \approx \frac{2}{\sqrt{2\pi\sigma}} \int_{\eta_1}^{\eta_2} e^{-\frac{(x-a)^2}{2\sigma^2}}\, dx. \tag{5}$$

Here n is the number of elementary magnets inclined toward H at an angle φ where $\eta_1 < \cos\varphi < \eta_2$

and N is the total number of elementary magnets.

Multiplying the left and right sides of equation (5) by mN we will obtain an approximate value for the increase in magnetization strength that corresponds to the rotation of part of the elementary magnets to a certain angle

$$\Delta I = nm \approx \frac{2Nm}{V\overline{2\pi\sigma}} \int_{x_0}^{x_0} e^{-\frac{(x-a)^2}{2\sigma^2}} dx. \tag{6}$$

This same rotation is a result of the action of the external magnetic field H, i. e., the angle of rotation is a function of H or $\eta = f(H)$. Carrying out the transformation in equation (6) and introducing the designation $k = \frac{1}{\sigma}$ we obtain

$$\Delta I \approx \frac{2I_m}{V\overline{2\pi}} \int_{k(\eta_1-a)}^{k(\eta_2-a)} e^{-\frac{z^2}{2}} dz, \tag{7}$$

where $I_m = mN$.
If we assume that

$$\eta = f(H) = \frac{H}{H_m}\cos\varphi \mp \sin\varphi \sqrt{1-\left(\frac{H}{H_m}\right)^2}, \tag{8}$$

where $\frac{H_i}{H_m} = \sin\varepsilon$, then

$$I \approx \frac{2I_m}{V\overline{2\pi}} \int_0^{k\left[\frac{H}{H_m}\cos\varphi + \sin\varphi\sqrt{1-\left(\frac{H}{H_m}\right)^2}\right]} e^{-\frac{z^2}{2}} dz. \tag{9}$$

k is expressed in such a fashion that the distribution curve in the section outside the interval $-1 < \eta < 1$ differs little from zero. Substituting (9) in (2) we obtain the more general relationship

$$B = \mu_0 H + 4\pi \overline{2\pi} I_m \int_0^z e^{-\frac{z^2}{2}} dz \cdot k\left|\frac{H}{H_m}\cos\varphi + \sin\varphi \sqrt{1 - \left(\frac{H}{H_m}\right)^2}\right| \tag{9.5}$$

For ferromagnetic materials under an external field
strength of $H = (0 \div 5) H_c$ the following expression is
valid:

$$B_m \approx 4\pi I_m \cdot k\left|\frac{H}{H_m}\cos\varphi \cdot \sin\varphi \sqrt{1 - \left(\frac{H}{H_m}\right)^2}\right| \tag{10}$$

when

$$B = \mu_0 H \cdot \frac{2B_m}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz \tag{10.5}$$

Equation (10) is the equation of a hysteresis loop. Knowing
parameters B_m, H_m, H_c, u_0 and k it is possible to find the
value of B for any H within the limits $-H_m$ to H_m. Fig. 1
shows the agreement of equation (10) with the experimental
curve.

2. STEP PROCESS IN A FERRITE CORE

As an example of the applicability of the equation
of the hysteresis loop (10) we will wxamine the step process
in a ferrite core, a diagram of which is shown in Fig. 2.
Starting from this arrangement it is possible to write the
following four equations:

$$E_1 = w_1 \frac{d\Phi}{dt} + R_1 i_1 \tag{11a}$$

$$0 = w_2 \frac{d\Phi}{dt} + R_2 i_2. \tag{11b}$$

$$B = \mu_0 H + \frac{2B_m}{\sqrt{2\pi}} \int\limits_0^{k\left[\frac{H}{H_m}\cos\varphi \mp \sin\varphi \sqrt{1-\left(\frac{H}{H_m}\right)^2}\right]} e^{-\frac{z^2}{2}} \, dz \tag{11c}$$
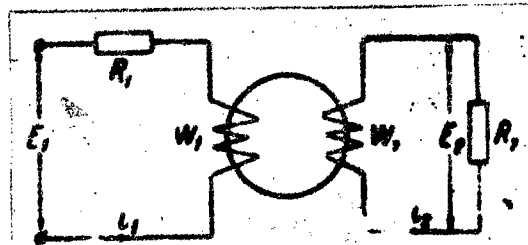
$$lH = 4\pi (i_1 w_1 - i_2 w_2). \tag{11d}$$

Fig. 2. Diagram of a ferrite core.

We will not consider the effect of eddy currents since in a ferrite they are practically equal to zero. We will eliminate from the given system of equations the currents i_1 and i_2. In order to do this we will write (11a) and (11b) in the form

$$\frac{E_1 w_1}{R_1} = \frac{w_1^2}{R_1}\frac{d\Phi}{dt} + i_1 w_1,$$

$$0 = \frac{w_2^2}{R_2}\frac{d\Phi}{dt} + i_2 w_2. \qquad (11e)$$

Substracting the upper equation from the lower we obtain

$$\frac{E_1 w_1}{R_1} = \left(\frac{w_1^2}{R_1} - \frac{w_2^2}{R_2}\right)s\frac{dB}{dH}\frac{dH}{dt} + \frac{lH}{4\pi} \qquad (11f)$$

where s is the cross section area of the ferrite ring;
     l is its average circumference. Substituting $l = 2\pi r$
     r is the radius of the average circumference,

$$\frac{dB}{dH} = \frac{2B_m k\left(\cos\varphi + \dfrac{H\cdot\sin\varphi}{H_m\sqrt{1-\left(\dfrac{H}{H_m}\right)^2}}\right)}{\sqrt{2\pi}\,H_m\,exp\,\dfrac{k^2}{2}\left[\dfrac{H}{H_m}\cos\varphi + \sin\varphi\sqrt{1-\left(\dfrac{H}{H_m}\right)^2}\right]^2} + \mu_0 \qquad (12)$$

and
$$R = \frac{R_1 R_2}{R_2 w_1^2 - R_1 w_2^2}$$

we obtain the differential equation of a ferrite core

$$\frac{dH}{dt} = R\frac{\dfrac{E_1 w_1}{SR_1} - \dfrac{rH}{2S}}{\cos\varphi + \dfrac{H\cdot\sin\varphi}{H_m\sqrt{1-\left(\dfrac{H}{H_m}\right)}}}$$
$$\mu_0 + \frac{2B_m k}{\sqrt{2\pi}H_m}\,exp\,\dfrac{k^2}{2}\left[\dfrac{H}{H_m}\cos\varphi + \sin\varphi\sqrt{1-\left(\dfrac{H}{H_m}\right)^2}\right] \qquad (13)$$

In the case where $R_2 = \infty$, i. e., $i_2 = 0$, equation (11c) takes the form $1H = 4 \pi i_1 w_1$ and the equation of the ferrite core will be

$$\frac{dH}{dt} = \cfrac{\frac{L_1}{w_1 S} - \frac{K}{2S w_1^3} \cdot 1H}{2 B_m k \left(\cos z - \cfrac{H \cdot \sin \varphi}{H_m \sqrt{1 - \left(\frac{H}{H_m}\right)^2}} \right)} \cdot \cfrac{1}{\sqrt{2\pi} H_m \exp \frac{k^2}{2} \left| \frac{H}{H_m} \cos z - \sin \varphi \sqrt{1 - \left(\frac{H}{H_m}\right)^2} \right|}$$ (14)

It is necessary to observe that all arguments lead to the assumption that H remains exclusively within the limits $(-H_m < H < H_m)$. Correspondingly, the largest value of H cannot be arbitrary. When $H = H_{max}$, $\frac{dH}{dt} = 0$, and from this condition we find that

$$\frac{E_{10} w_1}{R_1} - \frac{r H_m}{2} = 0.$$ (15)

Equation (15) is Ohm's Law and describes the condition of the ferrite core in a steady state.

The voltage applied to winding w_1 may be any function of time. We will examine a partial case - the action of a square pulse, or more exactly, only its front. Since there is always in practice a certain time interval during the increase of voltage, we will represent it mathematically thus

$$E_1 = E_{10} (1 - e \text{ [illegible text]}$$

We will find a partial solution of differential equation (14) under initial conditions $t = 0$, $H = 0$ and the following parameters for a ferrite core: $K = 65$ ($7 \times 4 \times 2.1$) mm; $k = 29.75$; $\sin \varphi = 0.5$; $\cos \varphi = 0.866$; $H_m = 0.54$ orsteds; $H_c = 0.27$ oersteds; $B_m = 2440$ gauss; $B_{residual} = 2400$ gauss; $\mu_o = 1$. A static ferrite hysteresis loop is shown in Fig. 1.

Let us take

$$E_1 = 25(1 - e^{-4.107})v,$$

i. e., we will assume that after 0.1 microsecond the voltage in the primary winding w_1 attains a maximal value of $E_{1Q} = 25v$. For $w_1 = 10$ in equation (15) we will get $R = 84.18$ ohms. Carrying out numerical integration of differential equation (14) by the Runge-Kutta method we will find the relationship

$$H = F(t) \qquad (16)$$

for magnetization of the ferrite.

The results of the calculation are given in Table 1.

Table 1

Point number	1	2	3	4	5	6	7	8
$t \cdot 10^{-6}$	0,010	0,010	0,010	0,010	0,018	0,818	1.618	1.818
H oersteds	0,182	0,191	0,195	0,209	0,240	0,270	0,290	0,308
$\frac{dH}{dt}$	$5,93 \cdot 10^5$	$2,86 \cdot 10^5$	$1.44 \cdot 10^5$	$6,08 \cdot 10^6$	$5,53 \cdot 10^4$	$2,00 \cdot 10^4$	$4,22 \cdot 10^4$	$3,73 \cdot 10^4$
r	54.3	53,0	52,6	51,8	92,5	208,4	205,3	202,5

A graphically computed solution of the differential equation of a ferrite core is given in Fig. 3.

We will call the time during which the magnetic field intensity in the ferrite changes from $H = 0$ to $H = H_m$ the time of magnetization of the ferrite. In our case the time of magnetization amounted to 1.83 microseconds. The greatest part of the magnetization time is required for the rotation of the elementary magnets along the external field, i. e., during the transition of the magnetic state of the ferrite along the steepest part of the hysteresis loop in the region H_c.

It is interesting to find the relationship of the

induced e. m. f. in the secondary winding to the magnetization of the ferrite ore. In the general case where $R_2 = \infty$

$$E_2 = -\frac{d\Psi}{dt}$$

or

$$E_2 = -w_2 s \frac{dB}{dt}.$$

(17)

Multiplying the corresponding left and right sides of equations (12) and (14), we find

$$\frac{dB}{dt} = \frac{E_1}{w_1 s} - \frac{rR_1}{2sw_1^2} H,$$

hence

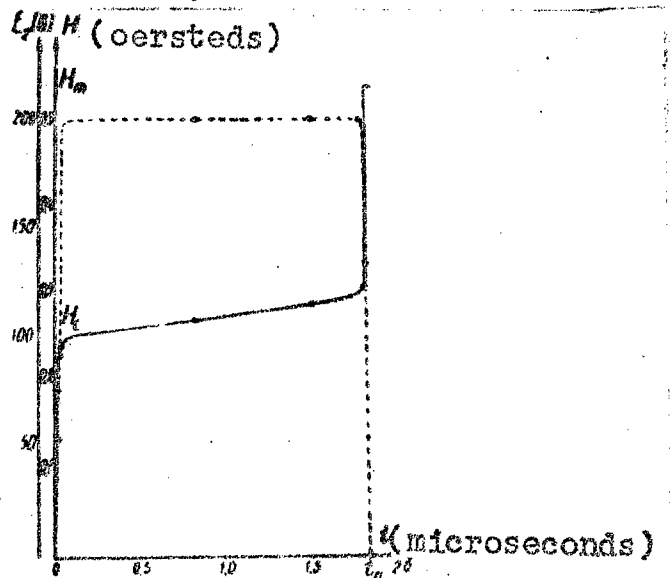$$E_2 = \frac{rR_1 w_2}{2w_1^2} H - \frac{w_2}{w_1} E_1.$$

(18)

Fig. 3. Graph of the step process in a ferrite core

Ferrite K = 65 (7x4x2.1); R_1 = 85 ohm, w_1 = 10,

$E_{10} = 250$, $R_2 = \infty$, $W_2 = 100$.
Solid line is the relationship $H(t)$.
Dotted line is $E_2(t)$.

The relationship $H = F(t)$ is known to us (Table 1).

It should be mentioned that upon numerical inte-
gration the expression $\frac{dB}{dt}$ is determined automatically.

The graph of the relationship $E_2 = E(t)$ is shown
in Fig. 3.

In order to check the results obtained, an experi-
mental determination was made of the magnetization time of
ferrite of the type $K = 65$ (7 x 4 x 2.1) mm. The outline
of the experiment corresponded to Fig. 2. The parameters of
the given ferrite core were selected to correspond to the
initial data of the calculation, i. e., $W_1 = 10$, $R_1 = 85$
ohms, $W_2 = 100$, $R_2 = \infty$, $E_{10} = 25v$. The duration of the
pulse front $\tau = 0.1$ microseconds.

Magnetization was carried out along winding W_1
from a type 26 I pulse generator.

The magnetization time was determined by the pulse
duration of the voltage induced in the second winding. All
measurements were done with a type CI-1 pulse synchroscope.

The magnetization time was determined for 10 ferrite
specimens rejected for use in a loop and amounted to 1.9 -
2.0 microseconds. The experimental results differed from
the calculations on the average of 5-10 percent. This makes
it possible to carry out a mathematical analysis of ferrite
circuits used in automation, telemechanics and computer
technology.

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