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THE INFLUENCE OF THE DENSITY OF WATER ON DEVIATIONS OF THE SEA LEVEL FROM THE SURFACE OF THE GEOID

[Following is a translation of an article by N. K. Khanaychenko in Meteorologiya i Gidrologiya (Meteorology and Hydrology), No. 3, 1960, pages 35-37.]

The sea level continuously changes under the influence of very different causes. Wind, waves, currents, atmospheric pressure and many other factors constantly cause changes in the vertical position of the level surface at any point of the world ocean. Besides changes caused by aperiodic forces, the sea level is subjected to constant periodic oscillations under the action of tidal forces exerted by the moon and the sun.

A representation of the level surface close to the geoid or parallel to it is based on values obtained by averaging numerous factually observed instantaneous heights of the sea level over a sufficiently long period of time.

In 1893 already, S. D. Ryl'ke [1] discovered the existence of a sloping level surface in the Baltic and Black Seas. Witting [4] made a map of the sloping level surface of the Baltic Sea according to which the surface of the Baltic Sea is inclined from North and East towards Southwest; furthermore, the magnitude of the level fall between the Nevskaya Bay and the Big Belt and Eresunn Straits is 20-25 cm. A level fall of approximately the same magnitude has been obtained on the basis of the latest state investigations.

Both Ryl'ke and Witting thought that such sloping of the level surface could occur because of differences in the degree of salinity or density of water. Another cause would undoubtedly be the average perennial atmospheric pressure distribution which caused a static variation of the level height.

The Baltic Sea connected to the North, Norwegian and Barents Seas and to the Atlantic Ocean through the Danish Strait can be considered as being a simple system of connected vessels filled with a liquid.

For the equilibrium of the liquid columns in two connected vessels to be maintained, the external pressure on the surface of the liquid must be equal to the internal pressure of the liquid per unit area.

As is well known, the internal pressure of a liquid is determined by the fundamental formula of hydrostatics,

$$p = \rho g h, \tag{1}$$

where p is the internal pressure, ρ the density of the liquid, g the acceleration of gravity, h the height of the liquid column.

We must obviously have p1 = p2 = p3 = ... = pn in order to maintain ocean water in equilibrium at various points or

$$\rho_1 g_1 h_1 = \rho_2 g_2 h_2 = \rho_3 g_3 h_3 = \ldots = \rho_n g_n h_n.$$

However, since the equation p1 = p2, etc. can be obeyed for various values of ρ and g, the height of liquid columns or, in our case, the heights of the average level at various points may be quite different.

Changes in the acceleration of gravity occur as a function of local latitude, whereas the quantity ρ may change within rather wide limits (between 1.000 and 1.028) even over a relatively small area. Therefore the actual average level of the world ocean may not coincide with the trace of the geoid at various points assuming a uniform external pressure over the whole surface of the ocean and equal internal pressures.

In order to determine the slope of the level surface of the Baltic Sea in terms of density, the height of the liquid column determining the limits of hydrostatic action must be found.

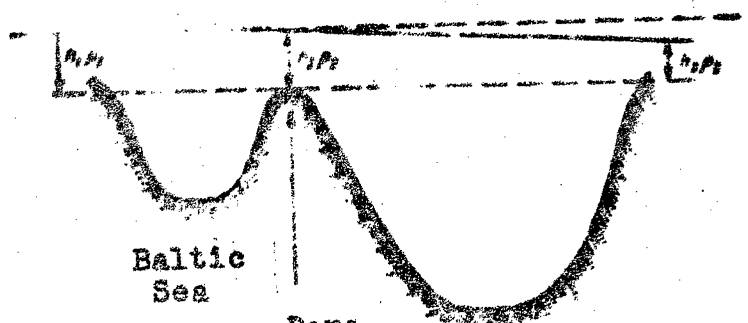

Figure 1. A Diagram of Open Connected Vessels Like the Baltic and Norwegian Seas.

In the model we have considered as an example the limit of hydrostatic action is the sea depth (h) at the ridge separating the hollow of the Baltic Sea from the ocean. The shallowest ridge is the Dars Ridge separating the southern part of the Baltic Sea, the Arkonskaya hollow in particular, from the Danish Straits. Schulz [3] indicates that the limiting depth at that ridge does not exceed 18 m while depths exceed 20 m in the deepest Danish Strait, the Big Belt, and its approaches.

The limit depth of hydrostatic action can thus be taken as being equal to 18 m.

Table 1

	Nevskaya Bay $\varphi = 60°00'$	Dars Ridge $\varphi = 54°27'$
Height of sea level above the surface of the ridge (h)	?	18.0
Average density of water in depth (ρ)	1.0011	1.010
Acceleration of gravity (g)	9.8191	9.8146

The average density of water ρ, or rather σ_t* at the Dars Ridge according to observations made from the Gedser Rev beacon ($\varphi = 54^\circ 27'$ and $\lambda = 12^\circ 11'$) during 1954-1956 [2] was equal to 1.00998. At the same time the perennial density of water in the Nevskaya Bay was equal to 1.0011.

Our initial data are recorded in Table 1.

Starting from equation (1) and the equilibrium of water in the ocean, we obtain

$$\rho_1 g_1 h_1 = \rho_2 g_2 h_2;$$

$$\sigma_{t_1} g_1 h_1 = \sigma_{t_2} g_2 h_2;$$

$$h_1 = \frac{\sigma_{t_2} g_2 h_2}{\sigma_{t_1} g_1}.$$

(quantities indexed with 1 refer to the Nevskaya Bay, those indexed with 2 to the Dars Ridge), where

$$h_1 = \frac{1,010 \times 9,8146 \cdot 18,0}{1,0011 \times 9,8191} = 18,15.$$

Hence, because of the difference in the density of water, the level surface of the Baltic Sea between the Dars Ridge and the Nevskaya Bay is raised by $\Delta n = n_2 - n_1$, i.e., by 0.15 m. The excess height of the level in the Nevskaya Bay over the level at the Dars Ridge is 20 m according to Vitting's map. Such an agreement has to be accepted since the average density of water at the Dars Ridge is computed by us from observational data spanning three years only. During the same period the monthly average density varies between 1.008 and 1.013. A drop in density down to 1.000 likewise occurs in the Nevskaya Bay during periods of maximum inflow of fresh waters. Hence, the slope of the level

*For oceanic conditions we have taken magnitude σ_t, in which water temperature has been taken into consideration, instead of laboratory density ρ.

surface of the Baltic Sea and, therefore, the excess height of the average level in the Nevskaya Bay over the average level at the Dars Ridge may oscillate within relatively wid limits, between 12 and 23 cm.

On the other hand, we should keep in mind that variations of the level height in the open part of the sea represented by isolines are obtained by interpolating shore data, and may therefore only be higher than the actual ones

We tried to compute the difference in level heights of the Baltic and Barents Seas so as not to limit ourselves to the above mentioned example. In the case of the Baltic and Barents Seas the limit of hydrostatic action is h = = 18 m at that same Dars Ridge.

The solution of this problem is even more rational since the average level height obtained over the Barents Sea from a series of observations spanning twenty years is connected by high quality surveying with the zero of the Kronshtadt tide-gauge, i.e., with the average level in the Baltic Sea (Nevskaya Bay).

Since the excess in level height of the Nevskaya Bay over the Dars Ridge (Δh_1) has been obtained by us beforehand, we need only solve the second part of the problem in order to obtain the excess level height of the Barents Sea over the Dars Ridge.

Let us take the following initial data in order to solve that problem. For the Baltic Sea: $h_2 = 18.0$; $\sigma_{t_2} = 1.010$; $g_2 = 9.8146$. For the Barents Sea ($\varphi = 69°19'$) $h_3 = ?$; $\sigma_{t_3} = 1.0267$; $g_3 = 9.8257$.

Then $$h_3 = \frac{18.0 \times 1.010 \times 9,8146}{1.0267 \times 9,8257} = 17,68;$$

i.e., the excess level height at the Dars Ridge over the level of the Barents Sea is $\Delta h_2 = h_3 - h_2 = 0.32$ m.

The sum of the excess heights $\Delta h_1 + \Delta h_2$ will be equal to 0.47 m, i.e., the heights at which the average perennial level of the Barents Sea stays as calculated on the basis of density will be represented by a mark of -0.47 m above the zero of the Kronshtadt tide-gauge. The absolute level of the Barents Sea (for the twenty-year period) obtained by geodetic surveying from the zero of the Kronshtadt tide-gauge is -0.49 m (Figure 2).

Differences in level heights due to differences in the density of water have been computed by us assuming a uniform external pressure, however. The atmospheric pressure is not actually uniformly distributed over the earth's surface. For a period of twenty years the pressure over the Barents Sea has been 4 mb lower than in the vicinity of Kronshtadt on the average. The level at which the Barents Sea stays must therefore be higher than that of the Nevskaya Bay by a corresponding quantity (Δh_p). The level of the Barents Sea will then be 4 cm higher than the one computed above by us, i.e., $\Delta h_1 + \Delta h_2 + \Delta h_p = -0.47 + +0.04 = -0.43$ m because of the statistical change in the sea level due to atmospheric pressure.

The slope of the level surface over the area between the Nevskaya Bay and the Barents Sea as well as over the Baltic Sea cannot apparently stay constant because of seasonal and annual variations in the density of water in both seas.

We may also assume that variations in the density of water as well as other factors are one of the causes of average sea level height deviations from the average perennial level over a certain period both at Kronshtadt, in the Barents Sea and at any other sea level points of observation in general.

We must point out that only the influence of hdrostatic forces on the level surface is considered in this article proposed by us. The influence of dynamic factors, including those acting constantly like the Coriolis force, is not taken into account.

At the same time we must conclude that the density of water together with other hydrometeorological and astronomical factors determines the height of the sea level and, hence, the slope of the level surface of separate seas.

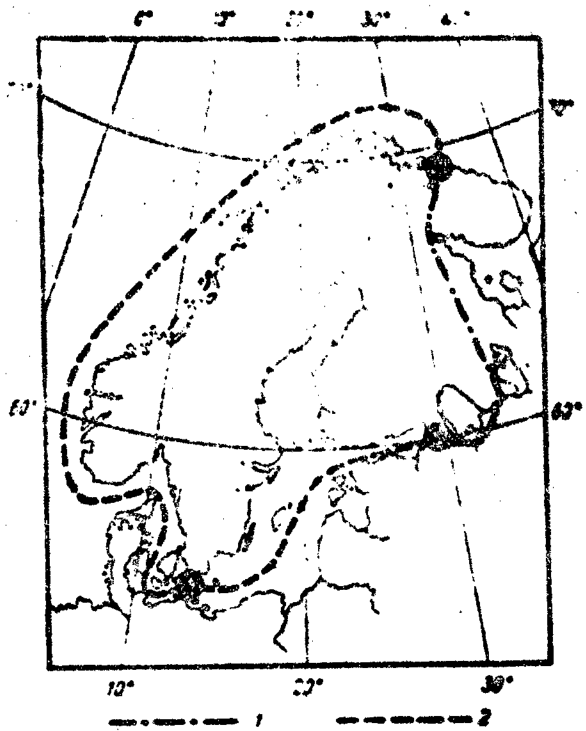

Figure 2. The location of points whose relative level height was computed by geodetic surveying (1) and by means of the hydrostatic pressure formula (2).

By taking the density of water and atmospheric pressure at every level point into account it will be possible to determine the magnitude of the deviation of the average perennial sea level from the surface of the geoid with sufficient accuracy and the degree of precision possessed by geodetic surveyings over large distances.

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