A PRACTICAL PROBABILISTIC ANALYSIS METHOD FOR HYDROCODE-BASED LETHALITY ASSESSMENT

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Abstract

A new approach for practical hydrocode-based lethality assessment is proposed. The approach couples hydrocode impact analysis with more efficient probabilistic analysis methods originally developed by the structural reliability community. The probabilistic methods are based upon advanced reliability methods which utilize reasonable assumptions together with efficient iteration and convergence algorithms to obtain an approximation of the boundary in engagement space that separates those intercepts that negate the threat from those that do not. Once this is accomplished, the lethality is uniquely determined. A specific implementation of the proposed methods is used to determine the lethality for four different threat warhead damage levels in a simulated 2-D intercept space. These approximate results are then compared with a 30,000-sample Monte Carlo solution in order to independently assess the accuracy of the methodology. The error in the calculated lethality obtained using the proposed method ranged from a maximum of 1.9% to a minimum of 0.01% while requiring 16 or fewer impact analyses for each lethality assessment. Additionally, the proposed approach identifies a *critical engagement* that is useful for providing guidance in improving the lethality and for selecting the most meaningful impact conditions for small scale or rocket sled tests.

Introduction

The Navy Theater-Wide ballistic missile defense system, the STANDARD Missile-3 (SM-3), is currently in development. The initial deployment of the missile will rely on kinetic energy to defeat its targets. Selection of the optimal firing doctrine for the missile requires the most accurate assessment of weapon system effectiveness possible.

Weapon system effectiveness is divided into three top-level components: (1) the probability of detecting the threat, (2) the probability of hitting the threat, and (3) the probability of negating the threat given a hit. This last term is referred to as *lethality*. Lethality assessment requires a computational tool for simulating the anticipated impact events and a probabilistic method for integrating the results of the individual intercepts into a comprehensive measure of the probability that given all possible intercept geometries and velocities, the threat is negated.

The computational tools currently used to assess high- and hypervelocity impact events may be divided into two general categories. On the one hand there are the fast-running empirical codes referred to as "engineering codes." The Parametric Endo- Exo-atmospheric Lethality Simulation (PEELS) is one example of an engineering code. A completely different approach to modeling the impact events of interest is provided by the "hydrocodes." These codes tend to be very computation-intensive as they attempt to model the actual impact physics of one body colliding with another. The CTH and SPHINX codes are two examples of commonly used hydrocodes. Small scale impact tests performed in support of the SM-3 program indicate that hydrocodes can achieve greater accuracy in modeling the damage incurred in the tests.

Lethality assessment has traditionally only been performed using the engineering codes with hydrocodes playing more of a support role. The use of fast-running codes is compatible with the traditional random sampling methods used for the probabilistic analysis portion of lethality assessment. These methods allow for a very precise statistical assessment; however, the value of this estimate is undermined to some degree by the over-simplifications made in modeling the physics of the impact event.

The Navy is cognizant of the balance between the accuracy of the impact simulation tool on the one hand and the precision of the statistical estimate on the other. In what may be termed a risk reduction effort, alternative probabilistic methods are being investigated to determine whether the more accurate, but more computation-intensive, hydrocodes can be

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used to provide higher quality lethality estimates than can be generated using the traditional approach.

While it is true that standard random sampling methods of probabilistic analysis typically require tens or hundreds of thousands of impact simulations to properly assess very high or very low probability events, there are other methods available which employ reasonable assumptions and provide more limited information that require orders of magnitude fewer "samples." A certain class of these methods based upon the use of advanced reliability methods are currently being investigated for possible use in SM-3 lethality assessment. The significant reduction in the computational effort required for the more limited, but essential information provided by these methods is well-suited to support practical lethality assessment using hydrocodes as the only impact analysis tool.

System and Analysis Requirements

Weapon system effectiveness is measured by the Navy in terms of the probability of negation (P_N) metric. Negation of a threat missile; i.e., substantially impairing the mission of the threat missile, requires that a certain level of damage be achieved as a result of an interceptor/threat impact. The level of damage required is unique for each payload and is determined based on the nature of the protected asset and the associated ground effects.

Contributors to P_N are the probability the ship will detect and track the threat properly so as to provide the necessary engagement support (P_{ES}) and the probability that the missile will accept the target tracking information provided by the ship and then properly execute its own tracking and maneuver tasks required to hit the target (P_H). The final top-level contributor to P_N is the probability that given a hit, the interceptor will cause sufficient damage to negate the threat. This term is identified as $P_{D|H}$ and is the measure of the interceptor lethality. The overall negation requirement may then be stated as

$$P_N \leq P_{ES} \cdot P_H \cdot P_{D|H}, \qquad (1)$$

where the independently assessed contributors on the right-hand side of the equation must collectively meet or exceed the P_N requirement.

Given the objective level of P_N currently assigned to the SM-3 and allowing for less than perfect performance of the various systems responsible for the P_{ES} and P_H terms, the level of lethality required if the SM-3 is to meet its performance requirement can be quite high. For illustrative purposes, assume the range to be

$$P_{D|H} > 0.95. \qquad (2)$$

In this case, in order to demonstrate that the SM-3 satisfies the performance requirement, analysts would be required to show, through simulation, that the lethality lies in the range suggested by Eq. (2). This has significant implications for the probabilistic method selected to synthesize the individual hydrocode analyses into the lethality estimate. Generally speaking, the computational effort required for a probabilistic analysis is highest when estimating probabilities in the tails of a distribution. The range indicated by Eq. (2) is in one tail of the threat warhead damage level distribution. This is an important consideration when selecting a probabilistic method suitable for hydrocode-based lethality assessment.

Another consideration is the maximum practical amount of time that can be allocated to the lethality assessment of a given interceptor/threat pair. Given current computational capabilities and the required hydrocode model complexity, the maximum number of hydrocode analyses that can practically be performed for a single lethality assessment is estimated to be approximately thirty.

Hence, if lethality assessment relying solely on hydrocode analysis is to be practical, the probabilistic method selected should be capable of (1) producing good quality lethality estimates within the range indicated by Eq. (2), while (2) requiring fewer than approximately 30 individual hydrocode analyses. This is a non-trivial statistical estimation task, and excludes most, if not all, of the random sampling methods.

Proposed Approach for Lethality Assessment

The need to estimate probabilities well into the tails of a distribution while relying on computationally-intensive analysis codes is a common one. The structural reliability community, for example, is frequently confronted with the need to demonstrate component reliabilities in excess of 0.999, but the complexity of the component often requires that computation-intensive finite element methods be used to evaluate the stress state. The persistent need for more efficient probabilistic analysis methods has driven this community to develop advanced probabilistic methods[1]—the first- and second-order reliability methods (FORM/SORM)—which employ reasonable assumptions in order to avoid the high computational costs typically associated with random sampling methods.

FORM/SORM make use of exact solutions or highly accurate approximations for the probability given the boundary between "failure" and "safety" in random variable space. If the boundary (also referred to as the limit-state function) is a linear or quadratic function of the random variables, the associated probability of failure—the proportion of failures to the total number—is exact. Both FORM and SORM provide an exact answer when the boundary is linear. SORM provides an exact answer when the boundary is quadratic. An approximate, but still very accurate estimate of the probability can be obtained if the boundary is *nearly* linear or quadratic near the most-probable point[2] (MPP)—the point on the boundary where the joint probability density is maximum. Deviations from the linear or quadratic forms far away from the MPP introduce very little error since the contribution to the overall probability is very small in this region.

These methods are well suited for application in lethality assessment. In this context, the random variable space is the set of variables that describe all possible intercepts between a given interceptor and threat missile. At a minimum these include: (1) closing velocity, (2) strike angle, (3) axial miss-hit, and (4) lateral miss-hit. The "safety" and "failure" regions may be identified as "threat negation" and "threat non-negation" regions respectively. The negation boundary is the limit-state function that separates the two regions and is the set of all intercepts that result in just enough threat warhead damage to negate the threat. The minimum damage levels required to negate each distinct threat/payload combination are assumed to be known.

Finally, the MPP is the one intercept in the set of intercepts that comprise the negation boundary for which the intercept joint probability density is a maximum. In other words, of all the possible intercepts that produce a threat warhead damage level equal to the negation requirement, the MPP is *the* intercept in that set most likely to occur. This intercept condition has special meaning and is referred to frequently hereafter as the *critical engagement.*

The concepts discussed above are illustrated in Fig. 1. For the purposes of illustration, the 4-D intercept space is reduced to a 2-D space where only closing velocity, V, and strike angle, θ, are represented. A small, but representative assortment of intercepts is shown. In this reduced space, the negation boundary is a curve separating those intercepts that result in threat negation from those that fail to negate the threat. The critical engagement, or MPP, is shown on this curve where the density of intercepts is greatest (i.e., where the intercept joint probability density is a maximum). Since each intercept represents a hit, the lethality is merely the ratio of the number of intercepts that result in threat negation to the total number of intercepts. As the number of intercepts considered increases, so too does the accuracy of the ratio and, hence, the accuracy of the lethality estimate. This is the foundation of traditional random sampling methods. The MPP-based methods may be used to directly obtain an estimate of this ratio while avoiding the significant computational costs of many randomly selected impact analyses.

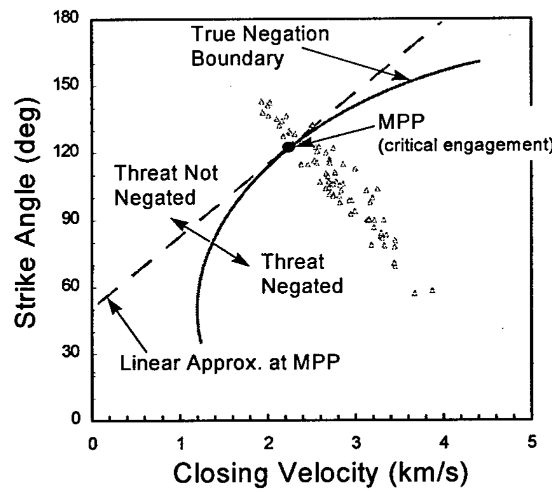

Fig. 1 Intercept Space and Negation Boundary.

In the application of the MPP-based methods, the primary task is to obtain a linear or quadratic approximation of the negation boundary at the critical engagement. This requires that the critical engagement first be located. In these methods, hydrocode analyses are selected so as to locate the critical engagement using iteration and geometric arguments[1] to govern convergence as guidance for determining which hydrocode analyses to perform.

There is no random sampling involved. Much of the ongoing research with these methods is focused on techniques to minimize the number of response function evaluations (e.g., hydrocode analyses) required to identify the MPP.[2] Once the critical engagement is located and a linear or quadratic approximation to the limit-state function is found, the lethality is uniquely determined.

As discussed earlier, the error associated with the MPP methods arises in part as a result of the deviation of the linear (or quadratic) approximation from the true negation boundary. Consider Fig. 1, for example. Any intercept that falls within the region of intercept space lying between the true negation boundary and the linearized approximation would be identified as negated, whereas in reality the threat would not be negated. Hence, the lethality would be over-estimated in this example. Of course, this error may be very small if the identified region contains very few intercepts; that is, if the intercept joint probability density is very small.

The MPP-based methods produce only the probability of achieving a given threat warhead damage level. Traditional random sampling methods used in conjunction with engineering codes characterize the entire statistical variation in damage level. This is one advantage of these advanced probabilistic methods. They focus the impact analysis effort on the specific problem of interest; i.e., determining the lethality.

A comparison of the hypothetical results provided by (1) the traditional approach using the more approximate engineering codes, (2) the hypothetical results that would be obtained if hundreds of thousands of hydrocode analyses were possible, (3) the result provided by the proposed methodology, and (4) the true results that would be obtained from hundreds of thousands of actual intercepts are shown in Fig. 2. The three complete cumulative distribution functions (CDFs) represent a hypothetical comparison between the impact simulation codes and the true behavior under the assumption there is negligible error introduced by the traditional probabilistic method. The hypothetical hydrocode-based CDF is shown closest to the true CDF because hydrocodes provide a more accurate simulation of the impact physics. The only CDF that can actually be generated, however, is the one produced using the engineering impact analysis codes.

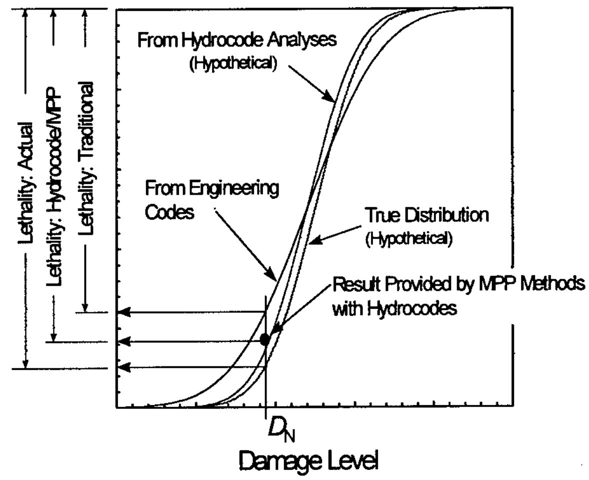

Fig. 2 Hypothetical Comparison of Methods.

However, if system lethality is the desired objective, only one point on each CDF is of any real practical interest: the probability associated with the negation requirement, identified as D_N in the figure. When coupled with hydrocode impact analysis, a successful application of the proposed probabilistic methods provides a very good estimate of the lethality that would be obtained from the hydrocode-based CDF if such a solution were possible. The corresponding lethality obtained from the traditional approach is also shown. The hydrocode/MPP-derived lethality estimate is shown as being closer to the hypothetical true value. The desire to develop a hydrocode-based lethality assessment capability is predicated on the proposition that the combined error in the lethality estimate due to the hydrocode and approximate probabilistic method is less than that introduced by the engineering code alone.

In addition to providing a direct route for calculating lethality, the MPP-based methods also provide valuable insight in the form of a random variable sensitivity analysis. This analysis is directly related to the location of the MPP in random variable space and provides a measure of the degree to which each random variable contributes to the probability. This information may be used in a variety of ways. First, the lethality community can provide guidance to the missile community at-large for improving the system lethality. The intercept variables that contribute most to the lethality are identified via the sensitivity analysis and this information may then be used to guide selection of the appropriate trajectory shaping scheme.

A second use of the sensitivity analysis is as an aid to reducing the number of hydrocode simulations required for follow-on lethality assessments. Consider, for example, that the sensitivity analysis indicates that one of the intercept variables contributes only very weakly to the lethality. Should a subsequent lethality assessment be required, this variable could be omitted from the early stages of the analysis thereby saving valuable analysis time.

Demonstration Problem

The concepts discussed in the preceding section are presented in this section through an example problem in which the lethality is determined using the proposed methodology. The solution obtained with the MPP-based method is compared to a solution obtained using the traditional Monte Carlo random sampling method. In order to generate the "exact" solution using the Monte Carlo method, hydrocode analysis is replaced by a simple mathematical function of the random variables. Also, in the interest of graphically presenting the solution process, the 4-D random variable space is reduced to the 2-D space shown in Fig. 3.

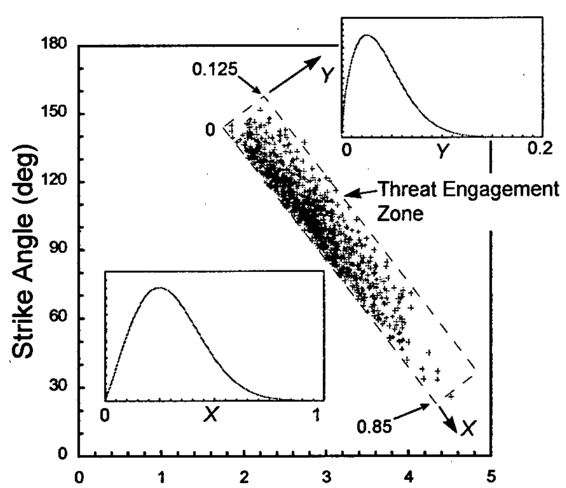

Fig. 3 Simulated Intercept Space.

While the distribution of the intercepts shown is hypothetical, the combinations of strike angle and closing velocity are representative of typical theater ballistic missile intercepts. Higher closing velocities are associated with smaller strike angles since a strike angle of 0° represents a head-on collision. Similarly, slower closing velocities are obtained for strike angles approaching a tail-chase condition (strike angle of 180°). The rectangular threat engagement zone identified in the figure is shown for reference purposes and encloses approximately 99.5% of all possible intercepts. For convenience in solving this particular problem, the engagement space is redefined in terms of the X and Y boundaries of the threat engagement zone. The probability density functions (PDFs) for both normalized directions are also shown in the figure. Note that the probability density is zero for both X and Y less than zero. Both PDFs are assumed to be given by the two-parameter Weibull distribution defined by

$$p(x,m,r) = \frac{m}{r}\left(\frac{x}{r}\right)^{m-1} e^{-(x/r)^m}, \qquad (3)$$

where the Weibull parameters, m and r, are defined for both PDFs in Table 1. The assortment of randomly selected intercepts shown in Fig. 3 is densest where both PDFs obtain their maximum values; i.e., where the intercept joint probability density is greatest. Of course, for a real lethality assessment, the population of intercepts would be first generated using high-fidelity flight simulation codes and modeled with the appropriate distributions afterwards.

Table 1. Weibull Distribution Parameters.

	X Distribution	*Y* Distribution
m	2.101	1.642
r	0.339	0.045

Since the random variables are defined most simplistically in terms of X and Y, it is also advantageous to perform the lethality assessment in this variable space as well, rather than in the V-θ space. The conversion from X-Y space to the V-θ space is accomplished with the transformation

$$\begin{Bmatrix} V \\ \theta \end{Bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 180 \end{bmatrix} \left[\begin{Bmatrix} 0.35 \\ 0.80 \end{Bmatrix} + \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} \right], \qquad (4)$$

where α is -0.915 radians, V is expressed in km/s, and θ is expressed in degrees. This transformation procedure is not part of the probabilistic methodology; it is only used in this particular problem to aid the solution. Without the transformation the explicit correlation between V and θ would have to be taken into account through different means. The transformation implemented here effectively decouples the two random variables.

Associated with each unique intercept defined by a particular V-θ (or X-Y) pair, is the threat warhead damage level that results from that intercept. In a real lethality assessment, a hydrocode simulation of each intercept would be used to determine the outcome, but for the purposes of this example problem, where the goal is to demonstrate the methodology and its accuracy relative to an "exact" answer, hydrocode simulation is replaced by the simple mathematical function describing the damage level as a function of V and θ:

$$D(V,\theta) = \left(\sum_{i=0}^{3} a_i V^i\right)\left(\sum_{i=0}^{3} b_i \theta^i\right). \qquad (5)$$

The constant coefficients a_i and b_i were selected to provide a normalized damage level between zero and unity over the range of interest and are defined in Table 2. Again, V is assumed to be expressed in units of km/s and θ in units of degrees.

Table 2. Damage Function Coefficients.

i	a_i	b_i
0	0.000	1.000
1	0.971	-4.105×10^{-3}
2	-0.300	9.127×10^{-5}
3	0.029	-5.518×10^{-7}

At this point in a real lethality assessment, appropriate statistical models have been identified to emulate the engagements determined from flight simulation studies and a hydrocode model of the threat and interceptor has been defined. The level of damage required to negate the payload of the threat missile, D_N, is known. The task remaining is to determine the probability of negating the threat given a hit; i.e., to determine the lethality of the interceptor missile.

Application of the MPP-based methods to solve for the lethality depends upon the specific method selected. The methods may be divided into two broad categories: (1) those that solve for the probability in "original" space[3,4] or (2) those that solve for the probability in "standard normal" space.[1] The first approach locates the MPP in original space; i.e., in terms of the original random variables, V and θ (or X and Y in this case). This approach avoids complexities that may arise due to the transformation to standard normal space, but introduces other complexities in calculating the probability once the MPP is found.[4]

The second approach requires that the original random variables be transformed into standard normal form in order to take advantage of the simple expression for the probability given the distance from the MPP to the origin in standard normal space. Standard normal refers to a normal distribution with a mean of zero and a standard deviation of unity. Any continuous non-normal distribution may be transformed into standard normal form through equivalency of their respective CDFs. Regardless of the specific nature of any distribution, its CDF is bounded between zero and unity. A variable in original space (often referred to as x-space) may be transformed into standard normal space (often referred to as u-space) by simply finding the u-space coordinate that shares the same CDF value as the x-space coordinate.

For the purposes of this demonstration problem, the most straightforward approach, the u-space method,[1] will be used. It is not clear at this point in the research effort which method, if either, provides a more efficient approach for lethality calculation. In addition to using the u-space approach, the most simplistic iteration algorithm is applied even though it is likely to require more "hydrocode" analyses than the more sophisticated algorithms.[2] Here, each evaluation of Eq. (5) is referred to as a "hydrocode" analysis. Clearly, the objective is to determine the lethality with as few damage function evaluations as possible.

An initial location of the MPP must first be assumed. A low-order approximation of the negation boundary is then obtained followed by a refinement of the location of the MPP. This process is repeated until no significant improvement in the MPP location is obtained, indicating the solution has converged. The distance from the origin to the MPP in standard normal space uniquely determines the probability on the basis of a negation boundary linearized at the critical engagement (the MPP).

In this most straightforward of applications, each iteration performed to locate the MPP requires N+1 "hydrocode" analyses, where N is the total number of random variables in the analysis. Here, $N = 2$. The N+1 impact analyses are comprised of one analysis at the assumed MPP followed by N additional analyses selected by perturbing each random variable in turn. For example, if the MPP is assumed to lie at (V_0, θ_0), then hydrocode analyses would be performed at (V_0, θ_0), ($V_0+\delta V$, θ_0), and (V_0, $\theta_0+\delta\theta$). This set of analyses provides the damage level at the assumed MPP plus the gradients in each random variable direction from which an updated location of the MPP can be obtained.

All iterations are carried out in standard normal space. Each updated estimate of the coordinates of the MPP is obtained in standard normal coordinates. Before the damage level associated with a new MPP can be obtained, the intercept must first be transformed back into original space. In this particular example, original space refers to the X-Y system. An additional transformation, Eq. (4), is required to determine the V-θ pair prior to substitution into Eq. (5).

The initial assumption for the location of the MPP is not too critical. Presumably, the closer the assumed location is to the actual location, the fewer the number of iterations required for solution convergence. The typical approach is to use the mean values for each of the random variables as the assumed starting point. This approach is adopted here. The damage level requirement is taken to be given by $D_N = 0.70$.

The solution for the lethality required four iterations before convergence was achieved. The evolution of the critical engagement is shown in Fig. 4. The threat engagement zone boundary is shown for reference. Each successive location of the critical engagement is labeled with a number indicating the order in which it was obtained. The solution process began at the 0th point. There are three (3) evaluations of Eq. (5) associated with each numbered point from 0 to 4. Each is identified with a "+" sign.

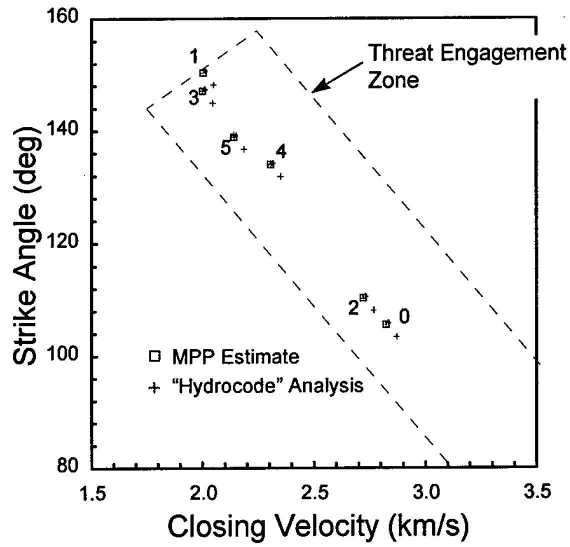

Fig. 4 Identfication of the Critical Engagement.

That the solution converged after the fourth iteration is evident from Fig. 5. Both the damage level and the estimated lethality are plotted for each updated location of the critical engagement. Not surprisingly, the damage level converges on the level required for threat negation. The u-space critical engagement lies at $u_X = -1.799$ and $u_Y = 0.337$. The distance to the origin in u-space, β, is $(u_X^2+u_Y^2)^{0.5}$ or 1.830 giving a FORM-based lethality of

$$P_{D|H} = \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = F(1.83) = 0.966 \ , \quad (6)$$

meaning that 96.6% of all engagements produce a threat warhead damage level equal to or in excess of 0.70. The function $F(\cdot)$ is the common standard normal CDF. The solution required 16 "hydrocode" analyses [i.e., 16 evaluations of Eq. (5)]. Fifteen evaluations were performed for iterations 0 through 4; i.e., $5(N+1)=15$ for $N = 2$. The N+1 evaluations performed at the 4th point provided the updated location for the critical engagement labeled "5" in Fig. 4. An additional evaluation of Eq. (5) at this location confirmed that the damage level was indeed equal to the negation requirement of 0.70. The probability, the lethality, was calculated as shown in Eq. (6). When transformed back into the more familiar V-θ space, the critical engagement is found to have a closing velocity of 2.14 km/s and a strike angle of 139.9°.

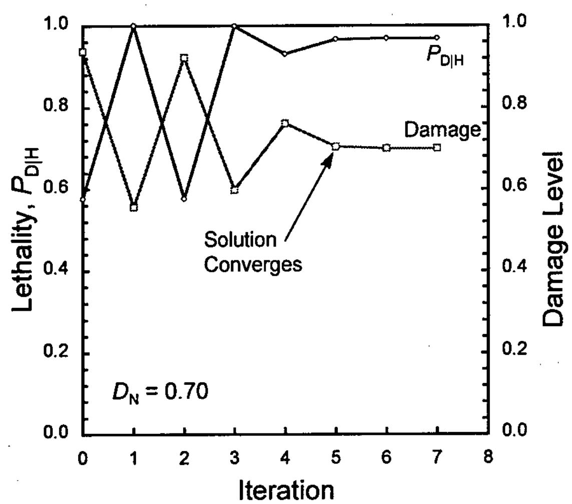

Fig. 5 Lethality Solution Convergence Behavior.

While a solution for the lethality was obtained using the proposed methodology, there is no direct evidence which attests to the accuracy of the solution. Convergence in and of itself does not guarantee accuracy. The assumptions and some of the possible sources of error were discussed in the preceding section. Methods for assessing solution accuracy are a current area of research in lethality applications of the MPP-based methods. These are not required for this example, however, since the insignificant time required for each evaluation of Eq. (5) allows for an "exact" solution using the traditional Monte Carlo methods.

A set of 30,000 randomly selected intercepts in X-Y space were drawn according to the distribution defined in Eq. (3) and the parameters defined in Table 1. These were then transformed into V-θ pairs using Eq. (4) and substituted into Eq. (5) to determine the corresponding damage levels. Of the 30,000 simulated intercepts, 29,010 were found to yield a damage level greater than or equal to the negation requirement of 0.70. Hence, the lethality determined from the Monte Carlo analysis is 0.967: a result virtually indistinguishable from that obtained using the proposed method.

As discussed in the previous section, the solution accuracy produced by the FORM solution is tied to the linear approximation of the negation boundary. A comparison of the approximate negation boundary with the true boundary [known only because Eq. (5) is an explicit function] is shown in Fig. 6 for $D_N = 0.70$. The final critical engagement shown in Fig. 4 is also identified in this figure. The threat engagement zone is again shown for reference.

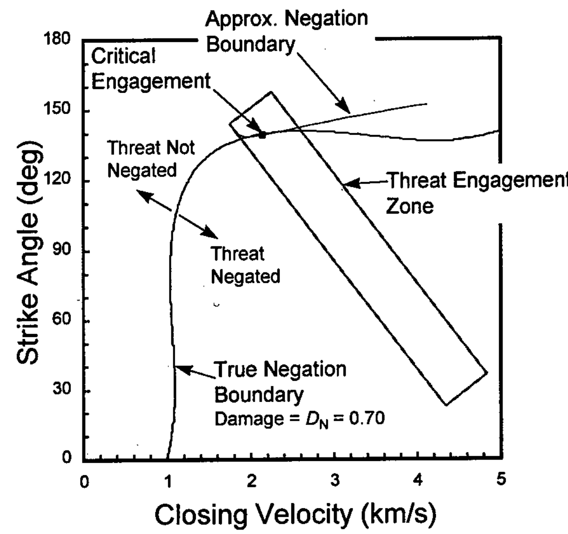

Fig. 6 Comparison of Approximate and True Negation Boundaries.

The difference between the approximate and the true negation boundaries is striking given the accuracy of the MPP-based (FORM) solution. To the left of the critical engagement, the approximate negation boundary bends sharply and effectively terminates in the upper left corner of the threat engagement zone, whereas the true boundary veers downward becoming virtually parallel to the strike angle axis. To the right of the critical engagement, the approximate boundary is essentially linear, but there is still a significant difference between the true boundary and the approximation. The discrepancy tends to increase with increasing distance from the critical engagement.

Given the large discrepancies observed between the two negation boundaries, why is the error in the calculated lethality so small? The answer lies in the distribution of the intercept joint probability density. There is a zero probability of an intercept occurring for $X < 0$ and $Y < 0$; hence, even though there is a significant difference between the approximate and true negation boundaries in this region, there is no error introduced in the lethality since no intercepts occur in that region of the intercept space. A similar argument can be made for the discrepancy observed to the right of the critical engagement. Here, the intercept joint probability density is not zero, but it is very small and continues to decrease with increasing distance from the critical engagement. The contribution to the lethality is negligible. Indeed, to the extent that there is error, the approximate negation boundary would tend to over-estimate the lethality since more of the engagement space lies in the negated region. In spite of this, however, the Monte Carlo solution gives a slightly higher lethality. This discrepancy is due to variance in both the MPP-based and finite-sample Monte Carlo solutions.

A final comment on the approximate negation boundary is in order. As discussed earlier, the lethality estimate is based on the assumption of a linearized negation boundary. Clearly, the approximate negation boundary shown in Fig. 6 is not linear. Recall, however, that the assumption of linearity applies in the transformed u-space. Some degree of curvature occurs when the linear negation boundary is transformed back into the x-space. If the random variables were normally distributed, rather than Weibull-based, the negation boundary would be linear in both the u- and x-spaces. While this is not the case here, some solution error can in general be expected to result from transformation between the u- and x-spaces. This is the motivation for the development of x-space methods that avoid the need for transformation into the u-space.

Note also that the location of the critical engagement depends strongly on the negation requirement since, by definition, it lies on the negation boundary. Three additional lethality assessments were performed using different negation requirements for each. Solutions for $D_N = 0.60$, 0.80, and 0.90 were generated. The number of "hydrocode" analyses required to converge to each solution and the lethality obtained from both the MPP-based method and the 30,000-sample Monte Carlo (M.C.) method are summarized in Table 3 for all four cases. The solution accuracy is quite good for all of the cases. A plot showing the location of the critical engagements obtained from each of the four solutions is shown in Fig. 7. The loci of critical engagements tends to march down the axis of the threat engagement zone as the negation requirement is increased.

Table 3 Summary of Lethality Calculations.

D_N	"Hydrocode" Analyses	Lethality (MPP)	Lethality (M.C.)
0.60	13	0.997	0.998
0.70	16	0.966	0.967
0.80	10	0.875	0.869
0.90	7	0.646	0.634

Using MPP-Based Sensitivities to Improve Lethality

Consider that the lethality assessment is complete and that when the system lethality is combined with the other contributors to the overall measure of system effectiveness, the total does not satisfy the requirement. In other words, the inequality specified in Eq. (1) is violated. What steps can be taken to improve the lethality component in order to satisfy Eq. (1)? The various sensitivity measures provided by the MPP-based methods are useful for answering this question.

Assume for the sake of discussion that only evolutionary, as opposed to revolutionary, changes are possible. A revolutionary change might be defined as a significant redesign of the interceptor kinetic warhead. An evolutionary change, on the other hand, is defined as a change in the guidance

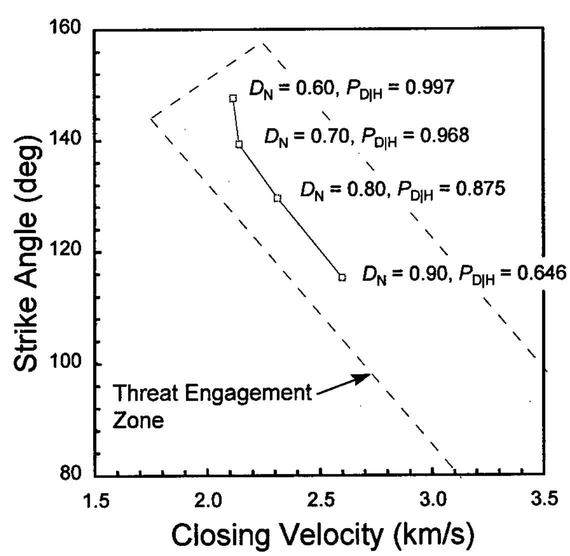

Fig. 7 Loci of Critical Engagements.

and control logic that governs end-game intercept conditions. In the context of the example in the preceding section, the true negation boundary shown in Fig. 6 would remain where it is, but the threat engagement zone and/or the PDFs shown in Fig. 3 would be altered somewhat. Trajectory shaping may therefore be used to "shift" the intercept joint probability density to a region of the engagement space where higher damage levels are obtainable. The sensitivity measures provided by the MPP-based methods are useful for guiding the trajectory shaping process.

Two general forms of sensitivity are common:[5] (1) the sensitivity of the lethality to each of the intercept variables and (2) the sensitivity of the lethality to individual distribution parameters of each of the intercept variables. Mathematically, these two sensitivity measures may be expressed in terms of the derivative of the lethality with respect to the desired quantity:

$$\frac{\partial P_{D|H}}{\partial x} = -\varphi(\beta)\frac{\partial \beta}{\partial x}, \tag{7}$$

where $\varphi(\cdot)$ is the standard normal PDF, and x represents either one of the intercept variables for case (1) or one of the distribution parameters of one of the intercept variables for case (2). Examples of intercept variable distribution parameters include the m and r parameters of the Weibull distribution defined in Eq. (3) or, alternatively, the mean and standard deviation. A closed-form expression for $\varphi(\beta)$ exists. The MPP-based methods numerically evaluate the $\partial\beta/\partial x$ term in Eq. (7).

The sensitivity of the lethality to a particular random variable may be most useful for improving lethality solution efficiency in subsequent analyses. If one or more of the intercept variables is found to play only a minor role, it can be held constant during the early stages of an analysis. Once a converged estimate for the location of the critical engagement is determined, the minor variable(s) may then be included and the critical engagement adjusted accordingly.

The sensitivity of lethality to the individual intercept distribution parameters may be most useful from the perspective of improving the lethality via trajectory shaping. It may well be that changes can be made to the guidance and control logic such that an increase in the mean of one of the variables can only be achieved at the expense of increasing its standard deviation. For example, a higher mean closing velocity may be possible, but in order to shift the mean, the spread of closing velocities increases. While the higher mean closing velocity may translate into increased damage potential, the increase in the variation may result in a reduction of the overall lethality. This sensitivity measure provides the guidance necessary to evaluate this form of performance trade-off.

Layered Lethality Assessment

The proposed probabilistic methods significantly reduce the number of impact analyses required to determine lethality, thereby allowing the use of more accurate, but computation-intensive impact analysis codes. They also provide valuable sensitivity information for optimizing subsequent lethality calculations or for improving the lethality. These are important "tactical" advantages offered by the methodology. There are also "strategic" improvements that are possible and should not be overlooked.

In spite of the significant time savings offered by these methods, the amount of time required to develop a hydrocode-based lethality assessment is still significant due to the long computation time required for each hydrocode analysis. It is therefore still not feasible to dispose of the engineering codes altogether as there are many aspects of assessing weapon system effectiveness that require this fast analysis capability. Instead, the proposed hydrocode-based lethality assessment capability should be integrated with existing tools and techniques. The capabilities and limitations offered by hydrocode-based lethality assessment, together with the unique insight offered by the critical engagement, present the lethality community with the opportunity to integrate the analysis and small scale testing into a "layered" lethality assessment effort.

In this layered approach, the initial phase of the investigation would revolve around the use of engineering codes. These would be used to determine first approximations of the defended areas for the particular threat/payload combinations. Associated with each defended area is a set of possible engagement conditions. The final defended area for a given threat/payload would be based on a lethality assessment using the traditional methods and the population of possible engagements. This is where lethality assessment has stopped in the past. Hydrocode analysis and small scale testing have been used to bolster the engineering codes. The methodology proposed here, however, allows for broader use of both the hydrocodes and the small scale testing.

Once the defended area is selected on the basis of the first-order (i.e., traditional) lethality assessment, the engagement space may be used in conjunction with the proposed hydrocode-based approach to obtain a more accurate lethality assessment. This offers the missile community at-large the opportunity to verify that the defended area is in fact realistic and to adjust it if required. This is the second layer.

The third and final layer begins once the second-order (hydrocode-based) lethality assessment is complete. A series of evolving estimates of the critical engagement will be identified as a natural byproduct of the application of these methods. If the second-order lethality assessment is reasonably accurate, small scale tests performed at each of the critical engagements obtained during the analysis (e.g., points 0 through 5 in Fig. 4) should produce target damage levels consistent with the results obtained during the analysis (e.g., the Damage Level vs. Iteration plot of Fig. 5). If a sufficient number of small scale tests cannot be performed, then the testing effort should be focused on the final critical engagement. If there is a region of engagement space where the hydrocode accuracy is essential, it is at the critical engagement.

A multi-layer approach to lethality assessment such as the one discussed here offers the missile community the opportunity to confirm, refine, and maximize the accuracy of missile system lethality—a task that is crucial for optimizing the firing doctrine of the missile system.

Summary and Conclusions

Traditional probabilistic methods currently used to assess lethality rely upon tens or hundreds of thousands of randomly selected intercept conditions for which a damage assessment is required. These methods preclude the direct use of the most accurate, physics-based impact analysis tools currently available—the hydrocodes—due to their long run-times. Instead, the current approach to lethality assessment relies on the more approximate engineering codes to assess the results of each interceptor/threat impact. The error in the assessed lethality due to the empirical modeling of the impact physics is unknown.

Alternate methods of probabilistic analysis are proposed in this paper for use in missile lethality assessment. These methods, adapted from the structural reliability community, significantly reduce the number of impact analyses required to assess lethality to a point where hydrocode-based lethality assessment now appears feasible. The use of these new probabilistic methods in conjunction with hydrocode impact analysis promises to yield more accurate lethality assessments than have been possible in the past.

The probabilistic methods proposed for use here are based upon the identification of the "most-probable point" and rely upon iteration and convergence algorithms to select the engagement conditions for which impact analyses are required. Reasonable assumptions and the limited, but significant, information the MPP-based methods provide are responsible for the significant reduction in the number of impact analyses required to obtain an estimate of the lethality.

A demonstration problem was presented in which the most straightforward application of the MPP-based methods was used to assess the lethality of a simulated two-dimensional intercept space. The error was found to be negligible in each of the four lethality assessments performed. No solution required more than 16 impact analyses. Application of the more sophisticated approaches to the full four-dimensional intercept space is expected to yield similar results. If this proves to be the case, accurate hydrocode-based lethality assessment is practical.

The proposed methods are also valuable for the additional insight they provide into system performance. These methods identify a *critical engagement* associated with a particular level of lethality. The critical engagement is fundamental in evaluating the influence of the various intercept variables or their probability distribution parameters on the calculated lethality. This information provides the guidance necessary to improve system lethality through trajectory shaping. Also, the critical engagement may be used to help select the most relevant impact conditions for small scale or rocket sled testing. This offers the possibility of a layered lethality assessment program which integrates the traditional approach, the proposed hydrocode-based approach, and experimental impact testing; all focused on providing the most accurate assessment of system lethality possible.

The example presented in this paper does indicate that a significant reduction in the number of impact analyses required to assess lethality is possible. However, the proposed method is sensitive to some degree to the shape of the negation boundary. Additional investigation is required before these methods can be recommended for general use. Specifically, the following issues are currently being studied:

- improved iteration and convergence algorithms;
- implementation of the x-space approach that avoids transformation to the u-space;
- methods for resolving solution difficulties that arise due to a highly curved negation boundary or multiple negation boundaries;
- methods for independently assessing solution accuracy.

A straightforward application of the proposed approach may work well for many of the realistic lethality problems that arise; however, the current areas of research will help to minimize the computational effort required and maximize the robustness of the approach as a whole.

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