DSTO

AR-010-606

Application of Game Theory to Tactical Development in Simulation Studies

Brian Hanlon

DSTO-TR-0706

19981112 029

APPROVED FOR PUBLIC RELEASE

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

Application of Game Theory to Tactical Development in Simulation Studies

Brian Hanlon

Air Operations Division
Aeronautical and Maritime Research Laboratory

DSTO-TR-0706

ABSTRACT

Sophisticated simulation models provide powerful tools with which to study the development and outcomes of highly interactive scenarios involving multiple players. The effectiveness of such outcomes, however, is strongly dependent on the set of tactics available to the players involved. Game Theory provides a framework in which optimal tactics can be developed in adversarial domains. Rather than constructing complete mathematical solutions, this report investigates how broad analysis within the scope of Game Theory can be used to provide insight into an operational scenario. When such an insight is gained into the general properties of an optimal solution the knowledge acquired can be applied as inputs to relevant simulation models. In this way simulation tools can be more effectively brought to bear on complex real world problems. This approach is investigated through the analysis of a simple tactical scenario.

RELEASE LIMITATION

Approved for public release

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

Published by

DSTO Aeronautical and Maritime Research Laboratory
PO Box 4331
Melbourne Victoria 3001 Australia

Telephone: (03) 9626 7000
Fax: (03) 9626 7999

AR-010-606
August 1998

APPROVED FOR PUBLIC RELEASE

Application of Game Theory to Tactical Development in Simulation Studies

Executive Summary

Computer based simulation is a powerful means by which complicated war gaming at all levels can be studied. Indeed, as the physical attributes and technologies of sensors and weapon systems can all be incorporated within such simulations, a faithful representation of a given scenario can be provided to the researcher. However, while simulation provides useful results for given inputs it is less suited to the study of optimal employment of the platforms and systems. While it is certainly possible to test changes in tactical application within the simulation framework there is no systematic way that optimal tactics can be developed, nor indeed is any real notion of what constitutes `optimal' provided.

It is thus of interest to investigate what means are available to complement simulation studies and provide a framework for determining optimal application of sensors *and* platforms. Such a framework is provided by the theory of Games. This is a mathematical theory of broad scope which has been actively pursued in military operational analysis for many years. Its sphere of application lies in determining optimal strategies for interacting players, with, perhaps, mutually opposed goals. While many fascinating results and insights can be gained from its exclusive application it suffers from the usual shortcomings that all purely mathematical approaches must when dealing with real world problems: the complication of accurately accommodating the many parameters inherent to sensors and platforms as well as the vagueness that can accompany incomplete and fuzzy information sources.

Rather than mathematically model such attributes and information structures in detail, it is generally possible to construct simpler mathematical representations, particularly when operational level results are of primary interest. Such simplified representations allow Game Theory techniques to be realistically brought to bear on meaningful tactical problems. In this way, insight can be provided to the researcher on possible novel tactical applications of sensor technologies and platforms, suggesting tactics which can be tested with more detailed simulation tools. That optimal solutions to adversarial problems can be provided through a Game theoretic analysis gives the researcher greater confidence that the ultimate outcome will be close to the optimal solution that can be achieved.

In this report the application of broad concepts from Game Theory to this goal is discussed. To illustrate its possible application a simple engagement scenario between a fighter type aircraft and a surveillance aircraft is investigated. The important sensor contribution to this tactical picture is incorporated by introducing a simple representation of the differing degrees of situational awareness between the adversaries, mimicking what may actually occur between such aircraft in close combat. The extension of such ideas to more realistic situations is also addressed.

Brian Hanlon
Air Operations Division

Brian Hanlon graduated from The University of Melbourne with a Ph.D. degree in mathematical physics in 1994. He undertook post-doctoral research at the National High Energy Research Centre of Japan in the field of computational studies of quantum gravity before joining Air Operations Division in 1995. Since then he has been active in operational effectiveness studies for the P-3C Orion upgrade program as well as contributing to sensor model development for AEW&C operational analysis. He remains a research associate with The University of Melbourne.

Contents

1. Introduction

The capabilities of modern computer hardware and software have made detailed simulation an indispensable component of military modelling and operational analysis. However, while crucial to gaining insight into tactical outcomes, such simulations are poorly suited to the development of novel tactics. While it is certainly possible to test changes in tactical application within the simulation framework there is no systematic way that optimal tactics can be developed, nor indeed is any real notion of what constitutes `optimal' provided.

In this regard, analysis external to a simulation is warranted in order that the space of possible tactical choices is fully explored. This can be a daunting task, which can be further frustrated by an unwarranted bias towards existing tactical doctrine. To investigate counter-intuitive tactics an objective procedure needs to be followed. Game Theory provides such an objective framework.

Rather than attempt to apply the mathematics of Game Theory in detail to the many degrees of freedom of platforms and sensors, a complementary approach with simulation which best utilises the strengths of each appears to provide the most promising path. By considering simplified operational level models of platforms and sensors, Game Theory analysis can be realistically brought to bear on meaningful tactical problems. In this way, insight can be provided to the researcher on possible novel tactical applications, suggesting tactics which can be tested with more detailed simulation tools. That optimal solutions to adversarial problems can be provided through a Game theoretic analysis gives the researcher greater confidence that the ultimate outcome will be close to the optimal solution that can be achieved.

In this report the application of broad Game Theory analysis to simulation modelling will be explored. To help motivate such an approach a particular military application will be investigated. While other flavours of Game Theory will be discussed this example will make use of *Differential Games* as expounded by Isaacs [1]. A pursuit and evasion game representing an agile pursuer P, such as a fighter aircraft, and a slower evader E, such as a surveillance aircraft, is considered. An extension on traditional Game Theory analysis is made in that the two participants are endowed with differing sensor capabilities. That is, there is an explicit asymmetry in the data gathering abilities of the participants, with E being the superior. Such accords with what could be expected in a real life engagement of this kind. In this way the application of Game theory to simulation studies incorporating platforms *and* sensors is considered with the aim of investigating, *at a broad level*, the possible novel tactics which could result.

In modern military operations, considerations on situational awareness require confronting the significant technological capabilities of surveillance and weapons systems as well as issues surrounding data fusion, command and control and human factors. Clearly, this is a daunting, if interesting, task. Consequently, in this report a, somewhat contrived, simpler game will be considered to illustrate the possible utility that Game theory may play in this arena, with interest focussed on a within visual range encounter[1].

2. Concepts from Differential Game Theory

2.1 The General Theory of Differential Games

The theory of games has been part of military operational analysis almost from its inception [2]. Essentially, Game Theory is concerned with determining the optimal choice of strategy, from a given set of strategies, for players competing to optimise some pay-off. In this sense, Game Theory is often analytically more difficult than standard operational analysis, where consideration must now be given to the simultaneous optimisation of strategy choice for several *interacting* participants, the optimal choice of strategy for any given participant being intimately related to the strategy choice of others. This is to be contrasted with the theory of one person dynamic optimisation germane to operational analysis, where optimal outcomes are determined against constant backgrounds. To apply Game Theory, use is often made of the game matrix to set out the pay-offs for particular strategy choices of the players. The application of the game matrix is useful for formulating problems, particularly when the strategy spaces are discrete and small in number.

Use of a game matrix is a particular instance of a game represented in *strategic* form, that is the players, the strategy spaces and the pay-offs are all specified and the players choose their actions simultaneously. To incorporate a dynamic element application must be made of *extensive* form games which dictate the order in which players move and the information available to players at each move. In this way strategies become contingent plans rather than un-contingent actions [3]. Differential Game Theory treats games in which players have lengthy sequences of such contingent strategy choices, being either discrete or continuous.

Differential Game Theory arose from the study of pursuit and evasion problems between players moving under simple kinematic laws, where generically the pursuer is denoted as P and the evader as E. Restricted to the case of single player games, such analysis recovers the concepts of operational analysis and control theory. Indeed, the same general structure employed in differential games can be found in the theory of control. Principally, this consists in firstly determining what constitutes the state and control variables. The game plays out in the space defined by the state variables, $\mathcal{E}$, while the control variables can be manipulated by the players to achieve certain state values. The choice of values for the control variables thus constitutes the choice of strategy.

[1] Since the scenario investigated here is a tactical one, rather than touching on the broader characteristics of warfare, the question of preferences and utilities is avoided and the problem will remain one of a zero-sum-game [4].

For zero sum[2] differential games, the general mathematical framework and method of solution has been fully described by Isaacs[3] [1]. The solution technique encompasses two essential elements: (i) a local solution which utilises the machinery of differential equations and (ii) a determination of the singular surfaces which delineate these local regions. The possible types of such singular surfaces is numerous and to date defies any systematic categorisation. In addition, the nature and location of such surfaces which occur in a given game must be determined from the characteristics of that particular game since no general theory of such entities exists.

The local solution technique, however, is amenable to a general treatment and centres on the concept of the value of the game, denoted as V. For two player zero sum games the value of the game is the pay-off achieved by one player, and the loss incurred by the other, at the end of the game when the players play optimally. Consistent with games of pursuit, where time until capture could be the pay-off, P seeks to minimise the value and E to maximise it. The pay-off may change with time over the duration of the game and may include a contribution from the point in state space where the game terminates. The points in state space where the game terminates constitute a surface in the playing space known as the terminal surface, $\mathscr{C}$. Denoting by ϕ the control variables of P and ψ those of E the value of the game can be represented as [1]:

$$\int G(x,\phi,\psi)dt + H$$

where x represents the state space variables, G varies over the paths followed by P and E in state space and H is the contribution to the pay-off where the game terminates on $\mathscr{C}$. Note that the value will generally be a function of the current state variables $V = V(x)$ since the current state of any given game could correspond to the initial state of a different game for which the value would generally be different. It can be shown that $V(x)$ satisfies a first order partial differential equation, known as the main equation [1]:

$$\min_{\varphi}\max_{\psi}\sum_{j}\left[V_j f_j(x,\phi,\psi) + G(x,\phi,\psi)\right] = 0$$

where V_j and f_j are the partial derivatives of the value and kinematic equations with respect to the state variables, respectively. From this it is possible to extract the full set of ordinary differential equations required for a solution.

[2] That is, what one player gains in pay-off the other loses

[3] It is interesting to note that to prove that the particular solutions produced by this methodology are indeed optimal is often laborious and more difficult than the production of the original solution.

2.2 Differential Games of Kind and the Barrier Concept

The forgoing discussion is primarily associated with games of *degree*. That is, there is available a quantity, the value, which the opponents wish to minimise or maximise. Such assumes that the value of the game actually exists, for example that capture by P of E actually occurs. The question of whether a particular event will or will not occur comes under the aegis of games of *kind*. Such games can be subsumed into the general mathematical framework of differential games by assigning numerical values for the value in each case: +1 if the event occurs and -1 otherwise.

Since now there is only a simple fixed pay-off, optimal strategies are may no longer be unique. For example, E need only just escape rather than, say, escape in the least time since the pay-off remains the same. To avoid addressing all such possibilities application is made of the *barrier concept*. A barrier is a particular kind of singular surface in the state space which, at the terminal surface, separates the region where the event occurs, such as capture, and does not occur, such as escape. In this sense meeting the terminal surface along the barrier corresponds to a neutral outcome, and thus a pay-off of 0. Should P and E begin a game on this surface, sub-optimal play by either will allow the other to either force capture or escape. Consequently, the strategy selections along the barrier are unique and calculable.

Application of the barrier concept allows for a detailed appraisal of a given game without the need for articulating the specific solution for all starting positions for a given pay-off function. This has been utilised to investigate fighter combat problems and draw out the essential dynamics of within visual range engagements. Such an approach will be adopted here, where an extension will be made to also accommodate differing degrees of situational awareness between the players.

3. A Particular Tactical Application

3.1 The Tactical Scenario

Consider an engagement between a fighter type aircraft and a surveillance aircraft. In a Beyond Visual Range (BVR) encounter it would be anticipated that the novel application of the sensor and weapon systems will play the dominant role in deciding the engagement. This is an issue of paramount interest and possibly amenable to the application of Game Theory, albeit incorporating some advanced concepts. Rather than address such issues, appeal is here made to some more traditional aspects of differential games by considering a Within Visual Range (WVR) encounter as a means of demonstrating the possible utility of Game Theory to broad high level analysis.

Scenarios of this kind have already been investigated operationally by the US Navy [5]. That such a situation could arise follows from some of the operational constraints that surveillance aircraft must sometimes work under. For example, it may be paramount that a particular area remain under surveillance for broader strategic reasons. Of course, it is unlikely that *any* aircraft could survive an undetected BVR attack or an attack by two or more fighter aircraft. The type of situation, then, in which the application of imaginative WVR tactics could be beneficial is when a fighter aircraft opportunistically encounters a surveillance aircraft and attempts to undertake a lone attack, confident that a slower patrol aircraft would be no match [5]. That such confidence may be misplaced centres on the differing sensor performance. Surveillance aircraft can have impressive all round visual capability and, unlike fighter aircraft, the aircrew to constantly occupy the observation stations and the on-board command and control to assimilate and act on the information provided [5].

The actual pursuit-evasion to be investigated here is presumed to occur within a two dimensional plane. Clearly this introduces a great deal of mathematical simplification but is also not far removed from how an actual scenario may play out. It is likely that E would fly close to the ground (or sea level) since this will cause ground or ocean returns to degrade P's radar performance should P attack from above. Further, owing to P's greater speed an attack from above will also entail the possibility that P may collide with the surface if sufficient distance is lacking after a firing solution is made. In this way a two dimensional scenario may be quickly forced on the participants, as also is the dependence on vision as the primary sensor.

While fighter aircraft have a definite manoeuvring advantage they generally have short low altitude endurance and a limited range. It is thus possible that E need only evade for a short time to survive an encounter. There can thus be some intrinsic constraints on P's behaviour. In the situation addressed here a guns only attack will be considered. This could arise if an opportunistic encounter occurred after the primary mission of the fighter had been completed and its missile store depleted.

To operationally model the differing visual capabilities of the aircraft the visual detection range will be represented by a "cookie cutter" range around the aircraft, as depicted in Figure 1. Within that range, complete information on the adversary's state is known while outside the cookie cutter range there is no available information. The visual sensor superiority of the surveillance aircraft will then be represented by that aircraft having a greater effective visual range.

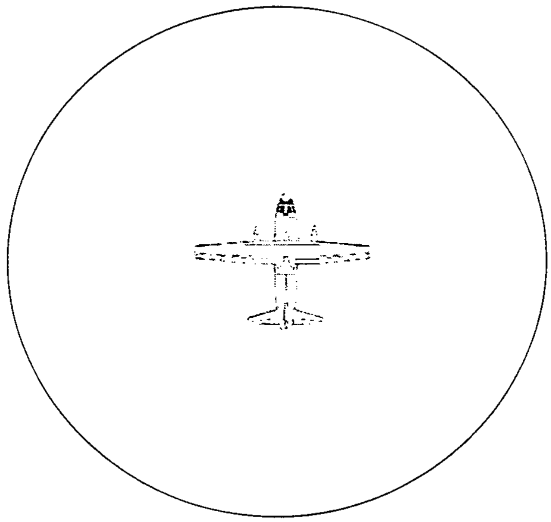

Figure 1. Aircraft with accompanying visual sensor range within which complete information on the adversary is available. (Not to scale).

This defines the notion of incomplete information to be applied in this report. It is important to note that this differs from other definitions of incomplete information or uncertainty in that there is no fuzziness in the information (ie. no measurement errors and no notion of obtaining partial information), no cost in obtaining the information and no delay time in receiving it. All these issues would need to be addressed if a broader appraisal of the sensor capability, for example in a BVR scenario, was being undertaken.

3.2 The Two Car Game

While three dimensional air combat problems, usually focussing on fighter vs fighter encounters, have been investigated [6], the planar game first investigated by Isaacs [1], known as the Two Car Game, remains as a useful differential game in which to study many interesting scenarios. In this game the participants move with fixed speeds and specified maximum turn rates. This allows simple modelling of the differing turn capabilities of the players but obviously does not account for the frequent accelerations and decelerations which would be encountered in reality. The coplanar geometry is represented in Figure 2, where application will be made of the reduced coordinate set relative to the pursuer, P.

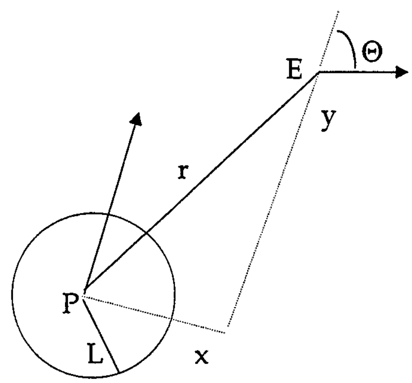

Figure 2. Relative geometry for the Two Car Game, here Θ is the relative heading.

Here, L represents the all aspect weapons range, or in game theoretic terms the capture radius, and defines the terminal surface, $\mathscr{C}$. Most applications of the Two Car Game to the air combat problem have focussed on the necessary requirements to ensure capture of E by P [7, 8, 9] by applying the barrier concept of Isaacs [1]. At the terminal surface, the points separating capture (E can be forced inside the all aspect weapons range) and escape (P cannot force E inside the all aspect weapons range) will clearly correspond to the points on the terminal surface, $\mathscr{C}$, where the radial velocity of E with respect to P is zero[4]. At these points the barrier will make contact with the terminal surface. Indeed, these points give the boundary conditions from which the barrier surface can be constructed. Given the symmetry of the terminal surface, the locus of these points will correspond to two complementary sets on either side of the capture region (note that we are dealing with a three dimensional state space comprising x,y and Θ so that a locus of points will be expected as Θ varies). It is if the barrier surfaces which arise from these initial (or more correctly terminal) conditions meet that the barrier will delineate regions of state space in which capture can always be ensured or frustrated. Even if no closed barrier surface is found, so that capture is always possible, important information on the general tactical scenario can still be inferred.

As demonstrated by Cockayne [10] if P has a greater speed and lateral acceleration it is possible for P to achieve capture even with a zero capture radius - ie. point capture is possible. For the tactical situation considered in this paper both these requirements are met so that closed barriers would not be anticipated. Thus it should always be possible for P to capture E, albeit with some manoeuvring.

However, our tactical situation has changed in that complete information on the state of the game is possible only within the visual detection range. For sufficiently small

[4] At $\mathscr{C}$, the barrier separates the useable from the non-useable part of the terminal surface, these being defined by the regions on $\mathscr{C}$ where the radial distance is instantaneously decreasing or increasing under optimal play by E and P respectively.

visual ranges it may become necessary for P to allow E to escape from within visual range to set up a possible capture. This can be demonstrated by explicit consideration of the barrier for a specific set of parameters. In the reduced coordinate set there are three state variables, the position (x,y) of E relative to P and the relative heading of E with respect to P, Θ. The corresponding platform parameters will be set as: w_p=205m/s, w_e=103m/s, R_P=800m and R_E=360m, corresponding to P and E's speed and minimum turn radius respectively. This choice of values gives E a slight advantage in angular turn rate [5]. Applying Isaacs' solution [1] it is possible to extract the barrier cross sections for particular choices of relative heading, being careful to correctly account for the singular behaviours on the barrier surfaces (ie. contributions from paths in which E utilises a straight run as part of its optimal strategy on the barrier). Taking a 700m visual range[5] and 100m capture radius for P and plotting the barrier cross sections out to the maximum visual range gives the results:

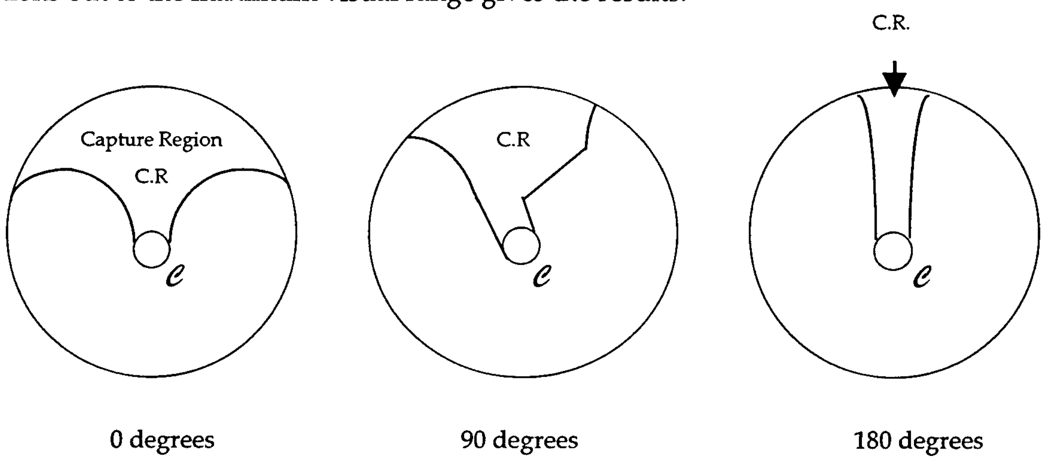

Figure 3. Barrier cross sections from the terminal surface, $\mathcal{C}$, to the limit of P's visual range at Θ = 0, 90 and 180 degrees respectively. Here, P is travelling up the page with the region, defined by the barriers and the visual range, within which P can ensure capture of E under any opposition from E shown as the Capture Region, C.R.

It is important to note that for any given pursuit the relative heading will be continuously varying throughout the encounter. As such, the paths on the barrier surface will vary in x,y and Θ. The barrier cross sections, therefore, *do not* correspond to paths which would be followed but rather, each point on a given cross section corresponds to a particular path with a given (x,y) for that particular Θ. The interesting behaviour at $\Theta = 90$ degrees is the result of that barrier having contributions from paths for which some part of that path's history involves a singular surface and consequently application of a straight run by E in addition to turning hard left or right, the later being the only contributions for the other cross sections given in Figure 3.

[5] It is worth noting that small visual ranges often occur due to environmental effects such as haze.

From Figure 3 it follows that if E should come within P's visual range and P can keep E immediately ahead, then capture can be assured without P having to manoeuvre in such a way as to allow E to venture outside P's visual range. Otherwise, a swerve type manoeuvre by P may be required if E is to be captured, allowing E to move beyond P's visual range; a definite disadvantage for P. That is, rather than, say, P being able to turn hard right to bring E into contact with that part of the capture region where capture can be forced upon E (the useable part of the capture region) it may be necessary for P to first turn left and travel *away* from E before making a right hand turn back toward E. That such a manoeuvre could be required follows from the nature of the barrier surface. To achieve capture, P must get E *around* the barrier and not simply to the barrier. To force this situation P may need to undertake this *swerve* operation.

From E's perspective, it is clearly advantageous, given that straying into P's visual range is unavoidable, to enter P's sensor range towards the rear of P. Furthermore, independent of where the visual range is first encroached, it also appears advantageous for E to seek a relative heading of around 180 degrees as this constricts the barriers. Given that E could, in such a situation, ensure encroachment outside the direct capture region defined by the barriers, and given that P knows that it cannot capture E without losing sensor contact, it becomes of interest to know to what degree E can exploit the visual range asymmetry. To this end, it is first necessary to determine to what degree escape from P's visual range can be made problematic.

3.3 The Escape Game

In this sub-game, P strives to maintain E within visual range. Reciprocally, it is presumed that it is advantageous for E to escape from P's visual range. Such is consistent with the strictures of a zero-sum-game. With these goals in mind the roles of P and E are reversed, ie. E strives for termination and P seeks to frustrate it. Furthermore, the state space, $\mathcal{E}$ is now the interior region of P's visual range and the terminal surface defined by the limits of this visual range. The kinematic equations are as in the pursuit game and are given by:

$$\frac{dx}{dt} = -\frac{w_P}{R_P} y\phi + w_E \sin\Theta$$

$$\frac{dy}{dt} = \frac{w_P}{R_P} x\phi - w_P + w_E \cos\Theta$$

$$\frac{d\Theta}{dt} = -\frac{w_P}{R_P}\phi + \frac{w_E}{R_E}\psi$$

where ϕ and ψ are P and E's control respectively, and $-1 \le \phi, \psi \le 1$. The essence of the game is contained in the Main Equation, which, with E and P's roles reversed, becomes:

$$\min_{\psi}\max_{\phi}[-\phi\frac{w_P}{R_p}A + w_E(\nu_x\sin\Theta + \nu_y\cos\Theta) + \frac{w_E}{R_E}\nu_\Theta\psi - w_P\nu_y] = 0$$

where $A = \nu_x y - \nu_y x + \nu_\Theta$ and ν_x, ν_y and ν_Θ are the components of the normal vector to the barrier surface. Interpreted as a game with terminal pay-off, this normal vector is proportional to the gradient of the value of the game. The choice of control variables which minimise and maximise the Main Equation are:

$$\overline{\phi} = -\operatorname{sgn} A = -\sigma_1 \quad \overline{\psi} = -\operatorname{sgn}\nu_\Theta = -\sigma_2$$

where compatibility with Isaacs' original treatment of the Two Car Game has been retained [1]. Parameterising the terminal surface as:

$$x = R\sin s_1$$
$$y = R\cos s_1$$
$$\Theta = s_2$$

(where R =700m in our case) the boundary of the useable part of the terminal surface follows as in [1] and is given by:

$$\sin s_1 = \pm\frac{w_P - w_E\cos s_2}{W} \quad \cos s_1 = \pm\frac{w_E\sin s_2}{W}$$

where $W = \sqrt{w_P^2 + w_E^2 - 2w_P w_E\cos s_2}$. Unlike [1] the useable part of the terminal surface is defined by the region where, under optimal play by both participants, the radial distance, r, is increasing. Furthermore, the choice of normal vector at the terminal surface must account for the state space now being defined by the region interior to that defined by the visual range of P. Consequently, the normal vector components at the terminal surface are chosen to be:

$$\nu_x = -\sin s_1 \quad \nu_y = -\cos s_1 \quad \nu_\Theta = 0$$

The remainder of the analysis follows as in [1]. Indeed , it follows that on the boundary of the useable part of the terminal surface $\sigma_1 = \mp 1$, so that P should execute a hard right (left) on the right (left) barrier. The choice of optimal control for E is also recovered, as are the nature of the Universal and Dispersal curves.

It is thus possible to consider the escape barrier cross sections as was done for the pursuit game. It is found that a solution within the state space exists only for small values of relative bearing. The case at a relative bearing of 0 degrees is shown in Figure 4:

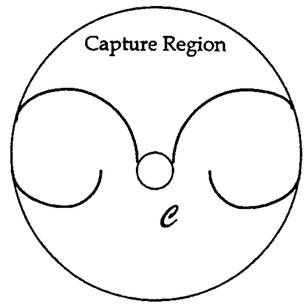

Figure 4. Barrier cross sections for the pursuit-evasion and escape games at Θ = 0 degrees. The useable part for the escape game is given by the lower region bounded by the evasive barriers. Again, P is travelling up the page.

Cross sections determined at relative bearings of 90 and 180 degrees do not yield escape barriers within the playing space. Consequently, some more careful manoeuvring by E to escape from the visual range of P is required only for small relative headings. At other headings there is no impediment to reaching the escape zone, remembering that the escape zone itself will alter with heading. Once again, it is advantageous for E to seek larger relative headings. In any case, it will always be possible for E to escape from the visual range of P.

3.4 Strategy Beyond P's Sensor Range

How should E play in order to frustrate ultimate capture after inevitably escaping from P's sensor range? There is by no means a simple nor definitive answer to this question since reliance must now be placed upon models of inference and incomplete information which are far less subject to objective logical prescriptions. For this reason there exists a plethora of possible models of rational reasoning at various levels of sophistication and, it would be anticipated, many possible solutions.

As indicated above, the notion of incomplete information adopted in this paper is very simple. It would be anticipated that in more sophisticated models (such as with BVR modelling) that the information structures involved in the analysis would be more involved and subtle. For this reason the specific case of Bayesian analysis will be employed. A more involved analysis does not appear warranted by the sophistication of the information structure under consideration. Nevertheless, the general philosophy of our analytical approach prevails; the derived results should provide some insight into the general structure of the problem and provide direction toward possible solutions to be explored through simulation[6].

[6] A possible general framework for information management and decision making is to make use of the construction of a pignistic probability function (pignis being "bet" in Latin) over a set of beliefs entertained at a credal level [12]. That is, probability functions are used to quantify beliefs only when a decision is made, allowing beliefs to be maintained without any revealing behaviour manifestations. At the credal (i.e. credibility) level, beliefs can be quantified by other models which have been proposed in the past as superior candidates to a simple Bayesian analysis, such as Dempster-Shafer models. Maintaining a probability function for decisions also insures no "coherency" problems. The important point is that a probability function can be constructed for any credibility function quantifying beliefs at the credal level [12]. In this way, different belief models can be explored for a given problem and the most appropriate utilised within one general framework.

To explore the possible strategies for E consideration will be given to a specific case from which some general inferences will be extracted, extending use of the "Method of the Explicit Policy" as espoused by Isaacs [1]. As mentioned above, a unique solution for E is not anticipated. Nevertheless, by incorporating the structure derived from the Game Theory analysis it should be possible to explore some novel, perhaps optimal, strategies for E which need not be obvious otherwise.

Consider the initial situation for the encounter depicted in Figure 5

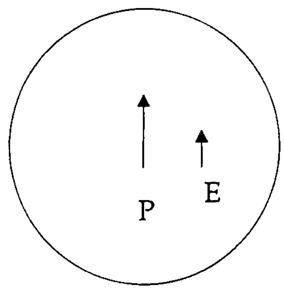

Figure 5. Initial configuration for pursuit-evasion game.

As demonstrated in the above analysis, for P to bring E into contact with the useable part of the terminal surface P must undertake a swerve type manoeuvre[7]. As with the homicidal chauffer game[8] [1], optimal play by E entails first following P before diverting for a getaway. Such a tactic forces a larger swerve upon P, increasing the time until ultimate capture. For the situation here, a similar tactic would maintain E within P's sensor range for an extended period of time. It would appear more advantageous for E to deny situational awareness to P and instead optimally play to deny this (it will be assumed that E can maintain visual contact at all times for simplicity). For the situation depicted in Figure 5. this would imply that E should turn hard right, while for P this would allow a smaller swerve to be used.

For particular starting positions of E relative to P there will always exist strategies for E which allow E to just skirt around P's visual sensor range. Indeed, particular solutions will always exist for specific starting positions. However, it is of interest to know what general ideas can be extracted, particularly if constraints are imposed. Furthermore, it is of interest to know which strategies reduce the risk of capture should P undertake an unexpected course of action.

With E playing optimally to escape from P's sensor range, P loses sensor contact with E very early in the engagement. P must thus make some decision as to E's initial subsequent tactics based upon prior beliefs. Rather than consider the continuum of possible choices that E may make of its control variable, consideration will be given to

[7] This is not necessarily true for all starting positions behind the barrier. If E is sufficiently rearward of P then P could bring E in to the useable part of the capture region by simply turning hard right, although situational awareness would still be lost.

[8] This game is a simpler version of the Two Car Game in which a swerve manoeuvre also occurs.

the following three possibilities: E maintains a hard right turn, E initiates a hard left turn or E flees by first utilising a hard right turn and then continuing without turning. Reduction to this subset greatly simplifies analysis and is not far removed from Game Theory type solutions which invariably utilise a choice of control variables at the extremes of their range, except perhaps on singular surfaces for which no application of a control degree of freedom is a common optimal application. Taking P to believe that E will not employ novel tactics (as assumed in actual trials [5]) the fleeing solution would be expected to feature most prominently in P's prior belief of E's course of action. To this end, a conservative choice of prior probabilities could be 1/4, 1/4 and 1/2 respectively. Presuming P to follow the course of maximum likelihood, and presuming E flees, the ensuing engagement would appear as in Figure 6:

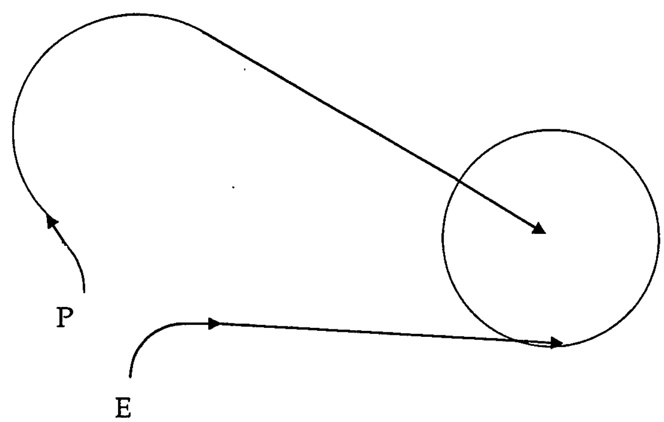

Figure 6. Possible form of engagement if E attempts to flee from P. When E enters P's sensor range (as indicated) it presents a small relative bearing angle, exposing itself to an open set of barriers. Note that the diagram is not to scale.

Clearly, it would be advantageous for E, given that it maintains complete information, to undertake a further, or continue with, a hard right hand turn to avoid P's sensor envelope. An initial hard left turn also may have been a viable option, although for certain starting positions it runs the risk that E may venture back into P's sensor range, so betraying the tactic. In particular, should P choose not to simply follow the path of maximum believed likelihood but instead mix strategies in accordance with the prior probabilities [1] there would be a reasonable chance that P would indeed pursue E under the belief that the left hand turn option had been followed. For E, it would not be possible to decide which option had been chosen by P until P began to emerge from the swerve manoeuvre. These considerations lead into the realm of adversarial planning, which would need to be employed in a more detailed analysis [11].

Taking E to have instead used the hard right hand turn option and P acting under the belief of E fleeing, upon P not intercepting E at the appropriate time simple Bayesian updating would give equal prominence to the posterior belief that E had initially

chosen the hard right or left option (for the initial prior probabilities chosen). Should P choose the subsequent course of action correctly, and should E have subsequently chosen some fleeing option then:
(i) E may quickly stray beyond its pre-assigned task area.
(ii) There is a heightened chance that, without evasive manoeuvring , P may regain sensor contact from the rear of E.
This later observation is crucial since from the Game Theory analysis it is known that the pursuit barrier allows for easiest capture for small relative bearing angles.

Given that P does choose a viable course of action, E remains restricted in its field of operation and that E wishes to forestall a possible encounter from the rear, a course of action which presents itself is for E to *deliberately* enter P's sensor range as near to a head on approach as circumstances and avoidance of capture will allow. Such a tactic takes advantage of the constriction of the barriers for near head on relative bearings. Once within P's sensor range another swerve type manoeuvre would be forced upon P, which could be exacerbated by E turning toward P, more akin to the homicidal chauffer game. In any case a similar cycle would repeat itself in which a similar strategy could be employed by E, or P may not, on a second attempt, make such fruitful decisions. The crucial point is that such a tactic leaves less to chance, that is less to P's lack of appropriate choice and more in control of E. Simply fleeing from P, with an area constraint and particularly if P has made a fortuitous early manoeuvre, runs the risk that capture may occur from the rear owing to P's greater agility. A possible engagement could look like that depicted in Figure 7:

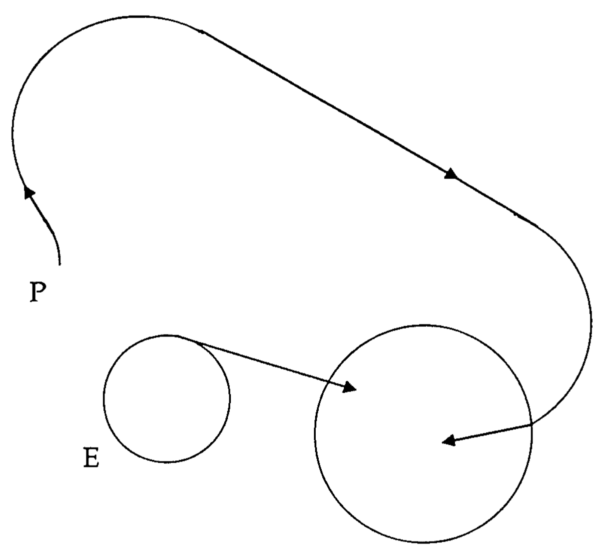

Figure 7. Representation of an engagement where E deliberately re-enters P's sensor range at a near to head on relative bearing. Note that here E has turned through a complete circle before moving towards P. (Not to scale)

While intrinsically unexpected a tactic of this kind makes use of the knowledge that, as fighter aircraft have limited range, two to three passes may be all that P can manage before having to disengage. This would appear to be an even more limiting constraint on P than the area constraint imposed on E since it also precludes P from learning the detailed tactics being undertaken by E in time to formulate a more successful strategy. It is worth noting that should P have mixed strategies and continued on a hard right turn that E, with complete information, could still turn in time to meet P in a similar manner.

Interestingly, the tactic of flying towards P has strong parallels with unorthodox tactics already trialed [5]. It was observed that exploiting the rapid rate of approach of E to P set up by such a tactic can deny P sufficient time to establish a firing solution on E. This could be incorporated in a Game Theory model by constraining the useable part of the terminal surface to allow capture only from the rear. Such could lead to some further interesting tactical development, even in the complete information game.

4. Extensions to more Interesting Scenarios

In addition to the general analysis conducted here, there is great scope to apply Game theoretic ideas to specific sub-problems of more direct interest. Indeed, the application of sub-games is a path often taken in developing Game Theory solutions to detailed problems. For instance, one area of current development involves a fighter combat problem with interest focussed on developments after a missile has been fired. The players reduce to a missile (the pursuer) and the evading aircraft[9] ie. the pursuing aircraft no longer plays a role in this part of the game. By incorporating the kinematic attributes of the missile, an objective study can be carried out on optimal evasive tactics which can then be further refined by particular simulation tools such as SWARMM[10] or Battle Model[11]. By restricting to such sub-games, the problems remain tractable to broad mathematical analysis.

Another interesting area for Game theoretic analysis is provided by the application of sensor technologies in BVR encounters. Rather than the simple "cookie cutter" approach adopted here, application could be made of the considerable literature which already exists on incomplete information games [3]. Specifically, for extensive form games the information content available to players can be accurately modelled and particular models of inference brought to bare to mimic decision making and planning. In this way, more accurate analysis could be applied to the optimal application of sensor technologies. For example, it would be interesting to optimise the cooperative application of radar and electronic support measures by investigating the trade-off

[9] This work is under development with David McIlroy of Air Operations Division.

[10] SWARMM, Smart Whole Air Mission Model: An agent-oriented simulation system developed by Air Operations Division and used for simulating the dynamics of whole air missions.

[11] Battle Model is the next generation mission level simulation environment under development at Air Operations Division, providing for flexible integration of sensor and agent models, including those originally residing in SWARMM.

between gathering intelligence and denying situational awareness to an adversary. This could lead to some very interesting and non-intuitive sensor employment strategies.

5. Conclusion

It is often presupposed that a mathematical analysis of combat problems is burdensome and complicated. However, as demonstrated here, the degree of mathematical difficulty need only correspond to the depth one wishes to pursue the analysis. In Game Theory terms, the greatest mathematical subtleties are usually concerned with determining the pay-off. However, as with our example, no such determination was needed since sufficient information on the dynamics of the problem was provided by restricting to a game of kind.

While the application of the barrier concept is not novel, the inclusion of situational awareness as an intimate part of such broad analysis can provide for new directions in research with practical consequences to tactical development in simulation studies.

6. References

[1] R. Isaacs, *Differential Games,* Wiley, New York, 1965.

[2] J. Von Neumann and O. Morgenstern, *The Theory of Games and Economic Behaviour,* Princeton University Press, Princeton 1944.

[3] Drew Fudenberg and Jean Tirole, *Game Theory,* MIT Press, 1993.

[4] See the introduction to Carl Von Clausewitz, *On War,* by Anatol Rapoport, Penguin Books, 1982.

[5] Lieutenant M. T. Ackerman (USN), *Nobody's Hapless Lumbering Grape,* United States Naval Institute Proceedings, Jan. 1988, p 101.

[6] See some of the articles in *New Trends in Dynamic Games and Applications,* Annals of the International Society of Dynamic Games 3, Birkhauser, 1995.

[7] J. V. Breakwell and A. W. Merz, *Minimum Required Capture Radius in a Coplanar Model of the Aerial Combat Problem,* AIAA Journal 15, p 1089, 1977.

[8] H. J. Kelley and L. Lefton, *Estimation of Weapon-Radius vs Maneuverability Tradeoff for Air-to-Air Combat,* AIAA Journal 15, p 145, 1977.

[9] G. J. Olsder and J. V. Breakwell, *Role Determination in an Aerial Dogfight,* Int. Journal of Game Theory 3, p 47, 1975.

[10] E. Cockayne, *Plane Pursuit with Curvature Constraints,* SIAM J. Appl. Math 15, p 1511, 1967.

[11] P. E. Lehner and Shui Hu, *Adversarial Planning as Uncertainty Management,* IJCAI-91.

[12] P. Smets, *Constructing the Pignistic Probability Function in a Context of Uncertainty,* Uncertainty in Artificial Intelligence 5, p 29, 1990.

DISTRIBUTION LIST

Application of Game Theory to Tactical Development in Simulation Studies
Brian Hanlon

AUSTRALIA

DEFENCE ORGANISATION

Task Sponsor DSTO

S&T Program

Chief Defence Scientist
FAS Science Policy } shared copy
AS Science Corporate Management
Director General Science Policy Development
Counsellor Defence Science, London (Doc Data Sheet)
Counsellor Defence Science, Washington (Doc Data Sheet)
Scientific Adviser to MRDC Thailand (Doc Data Sheet)
Director General Scientific Advisers and Trials/Scientific Adviser Policy and Command (shared copy)
Navy Scientific Adviser (Doc Data Sheet and distribution list only)
Scientific Adviser - Army (Doc Data Sheet and distribution list only)
Air Force Scientific Adviser
Director Trials

Aeronautical and Maritime Research Laboratory
Director

Chief of Air Operations Division
Research Leader: Air Operations Analysis
Head Mission and Campaign Analysis
Head Operations & Performance Analysis
Task Manager: Simon Goss
Author: Brian Hanlon (2 copies)

DSTO Library

Library Fishermens Bend
Library Maribyrnong
Library Salisbury (2 copies)
Australian Archives
Library, MOD, Pyrmont (Doc Data sheet only)

Capability Development Division

Director General Maritime Development (Doc Data Sheet only)
Director General Land Development (Doc Data Sheet only)
Director General C3I Development (Doc Data Sheet only)

Army

ABCA Office, G-1-34, Russell Offices, Canberra (4 copies)
SO (Science), DJFHQ(L), MILPO Enoggera, Queensland 4051 (Doc Data Sheet only)

NAPOC QWG Engineer NBCD c/- DENGRS-A, HQ Engineer Centre Liverpool Military Area, NSW 2174 (Doc Data Sheet only)

Intelligence Program

DGSTA Defence Intelligence Organisation

Corporate Support Program (libraries)

OIC TRS, Defence Regional Library, Canberra
Officer in Charge, Document Exchange Centre (DEC), 1 copy
*US Defence Technical Information Center, 2 copies
*UK Defence Research Information Centre, 2 copies
*Canada Defence Scientific Information Service, 1 copy
*NZ Defence Information Centre, 1 copy
National Library of Australia, 1 copy

UNIVERSITIES AND COLLEGES

Australian Defence Force Academy
Library
Head of Aerospace and Mechanical Engineering
Deakin University, Serials Section (M list), Deakin University Library, Geelong, 3217
Senior Librarian, Hargrave Library, Monash University
Librarian, Flinders University

OTHER ORGANISATIONS

NASA (Canberra)
AGPS

OUTSIDE AUSTRALIA

ABSTRACTING AND INFORMATION ORGANISATIONS

INSPEC: Acquisitions Section Institution of Electrical Engineers
Library, Chemical Abstracts Reference Service
Engineering Societies Library, US
Materials Information, Cambridge Scientific Abstracts, US
Documents Librarian, The Center for Research Libraries, US

INFORMATION EXCHANGE AGREEMENT PARTNERS

Acquisitions Unit, Science Reference and Information Service, UK
Library - Exchange Desk, National Institute of Standards and Technology, US

SPARES (10 copies)

Total number of copies: 56

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION DOCUMENT CONTROL DATA						
				1. PRIVACY MARKING/CAVEAT (OF DOCUMENT)		
2. TITLE Application of Game Theory to Tactical Development in Simulation Studies			3. SECURITY CLASSIFICATION (FOR UNCLASSIFIED REPORTS THAT ARE LIMITED RELEASE USE (L) NEXT TO DOCUMENT CLASSIFICATION) Document (U) Title (U) Abstract (U)			
4. AUTHOR(S) Brian Hanlon			5. CORPORATE AUTHOR Aeronautical and Maritime Research Laboratory PO Box 4331 Melbourne Vic 3001 Australia			
6a. DSTO NUMBER DSTO-TR-0706	6b. AR NUMBER AR-010-606		6c. TYPE OF REPORT Technical Report		7. DOCUMENT DATE August 1998	
8. FILE NUMBER M1/9/539	9. TASK NUMBER DST 97/166	10. TASK SPONSOR DSTO		11. NO. OF PAGES 18		12. NO. OF REFERENCES 12
13. DOWNGRADING/DELIMITING INSTRUCTIONS none			14. RELEASE AUTHORITY Chief, Air Operations Division			

15. SECONDARY RELEASE STATEMENT OF THIS DOCUMENT

Approved for public release

OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH DOCUMENT EXCHANGE CENTRE, DIS NETWORK OFFICE, DEPT OF DEFENCE, CAMPBELL PARK OFFICES, CANBERRA ACT 2600

16. DELIBERATE ANNOUNCEMENT

No Limitations

17. CASUAL ANNOUNCEMENT Yes

18. DEFTEST DESCRIPTORS

Game theory, War games, Air combat simulation, Tactics

19. ABSTRACT

Sophisticated simulation models provide powerful tools with which to study the development and outcomes of highly interactive scenarios involving multiple players. The effectiveness of such outcomes, however, is strongly dependent on the set of tactics available to the players involved. Game Theory provides a framework in which optimal tactics can be developed in adversarial domains. Rather than constructing complete mathematical solutions, this report investigates how broad analysis within the scope of Game Theory can be used to provide insight into an operational scenario. When such an insight is gained into the general properties of an optimal solution the knowledge acquired can be applied as inputs to relevant simulation models. In this way simulation tools can be more effectively brought to bear on complex real world problems. This approach is investigated through the analysis of a simple tactical scenario.