

**NAVAL POSTGRADUATE SCHOOL  
Monterey, California**



**THESIS**

**AN EVALUATION OF MARKOV CHAIN MODELING FOR  
F/A-18 AIRCRAFT READINESS**

by

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September 1998

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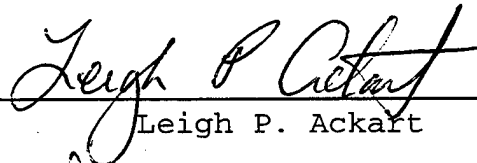
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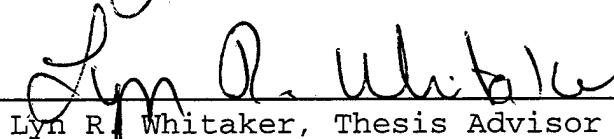
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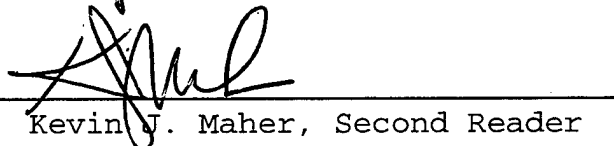
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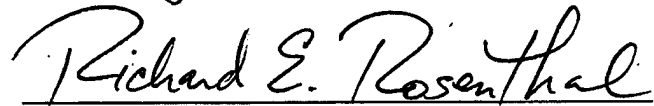
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## ABSTRACT

During its 1998 deployment the USS INDEPENDENCE (CV 62) and Carrier Air Wing Five operated under the control of Commander, Task Force 50 (CTF-50). To balance resources and readiness, CTF-50 asked the following question: "How many days can the USS INDEPENDENCE go without "off ship" logistics support before the number of Mission Capable aircraft can be expected to fall below Chief of Naval Operations readiness goals?" This thesis develops a Markov chain model to answer this question. Explanatory variables for this model include sorties flown, cannibalization rate and frequency of "off ship" logistics support. Using data from INDEPENDENCE, this thesis analyzes aviation readiness by estimating the number of F/A-18 aircraft capable of performing at least one of its intended missions.

Both non-linear Markov models and Generalized Linear Models are employed to estimate the effect of the operating environment on the number of mission capable aircraft available. The analysis demonstrates how the Markov approach captures the cyclic nature of aircraft operations and maintenance. Specifically, it is shown that INDEPENDENCE can expect to operate five to eight days without "off ship" logistics support before F/A-18 MC rates fall below CNO readiness goals. Recommendations for further studies are included.



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## LIST OF ACRONYMS

AIMD	Aviation Intermediate Maintenance Department
AWP	Awaiting Parts
CNA	Center for Naval Analyses
CNO	Chief of Naval Operations
COD	Carrier Onboard Delivery
CTF	Carrier Task Force
FH	Flight Hours
FMC	Full Mission Capable
GLM	Generalized Linear Model
MC	Mission Capable
nls	Non-linear Least Squares
NMC	Not Mission Capable
NMCM	Not Mission Capable Maintenance
NMCS	Not Mission Capable Supply
NPS	Naval Postgraduate School
OLS	Ordinary Least Squares
PMC	Partial Mission Capable
SF	Sorties Flown
RSE	Residual Squared Error
RSS	Residual Sum of Squares



## EXECUTIVE SUMMARY

Keeping a carrier air wing "Mission Capable" requires a complex integrated logistics support system composed of various functions which include both supply and maintenance support elements. While most aircraft can be repaired without "off ship" assistance, occasionally components fail and the maintenance organization is unable to repair the aircraft because it lacks the appropriate onboard spare. In these cases, the material is requisitioned from an "off ship" source and shipped to a shore based logistics site located near the operating area of the carrier. The material is then delivered to the ship through a process known as Carrier Onboard Delivery (COD).

USS INDEPENDENCE (CV 62) and Carrier Air Wing Five operated under the control of Commander, Task Force 50 (CTF- 50) during its 1998 deployment to the Persian Gulf. In an effort to balance operational tempo with available resources, CTF-50 hypothesized that reducing the frequency of COD flights, would result in achieving resource conservation. At the same time, reducing the number of CODs would have an adverse impact on the air-strike readiness. Without CODs, the maintenance communities will have to wait longer for critical "off ship" requisitions. To balance resources and readiness, CTF-50 asked the following question: How many days can INDEPENDENCE go without a COD before the number of available Mission Capable (MC) / Full Mission Capable (FMC) aircraft can be expected to fall below Chief of Naval Operations (CNO) readiness goals? CTF-50 and INDEPENDENCE asked the Naval Postgraduate School (NPS) to provide assistance in analyzing the effect of changes in the frequency of COD

service on carrier based aircraft readiness. This thesis develops a non-linear model based on a modified Markov chain to support its findings.

The Markov approach finds a significant link between the frequency of COD service and cannibalization opportunity and the rate at which aircraft are repaired. Additionally, it shows a significant link between aircraft utilization rate and the rate an aircraft fails. Finally, the model estimates that INDEPENDENCE can expect to go five to eight days without “off ship” logistics support before aircraft readiness falls below the CNO goal.

While the model includes a term to capture cannibalization, its effect is not as large as expected. That is, cannibalization did not significantly reduce the effect of a long-term COD deprivation. The effect modeled here can most likely be attributed to increased pressure on maintenance personnel to maintain MC rates above CNO goals as the number of NMC aircraft increases. In a wartime scenario, or a period where a prolonged outage of “off ship” logistic support is anticipated, cannibalizations would increase and maintenance personnel would be more aggressive in the repair effort, which would likely extend the number of days readiness rates would remain above CNO goals.

Areas for further research include refinement of the data collection to provide better model predictions, simulation of the Markov model to improve the estimates of carrier based aviation readiness during a prolonged deprivation of “off ship” logistics support, and application of the Markov model to a second deployed carrier to demonstrate the portability of the Markov approach.

## **I. INTRODUCTION**

Keeping a carrier air wing "Mission Capable" requires a complex integrated logistics support system composed of various functions which include both supply support and maintenance support. These logistics support functions can be classified as either "on ship" or "off ship" elements. "On ship" elements include organizational maintenance personnel, Aviation Intermediate Maintenance Department (AIMD) and an extensive inventory of spares, tools and test equipment. "Off ship" elements include depot level maintenance activities, stock points, and transportation assets.

Because carriers operate forward deployed, the carrier air wing must be able to operate for extended periods of time without support from shore based activities. While most aircraft can be repaired without "off ship" support, there are times when components fail, and the maintenance organization is unable to repair the aircraft because it lacks an appropriate onboard spare. In these cases, the material is requisitioned from "off ship" logistics activities and shipped to a shore based logistics site located near the operating area of the carrier. The material is then delivered to the ship through a process known as Carrier Onboard Delivery (COD).

### **A. BACKGROUND**

USS INDEPENDENCE (CV 62) and Carrier Air Wing Five operated under the control of Commander, Task Force 50 (CTF- 50) during its 1998 deployment to the Persian Gulf. In an effort to balance operational tempo with available resources, CTF-50 hypothesized that reducing the frequency of COD flights, would result in achieving



resource conservation. At the same time, reducing the number of CODs would have an adverse impact on the air-strike readiness. Without CODs, the maintenance communities will have to wait longer for critical "off ship" requisitions. To balance resources and readiness, CTF-50 asked the following question: How many days can INDEPENDENCE go without a COD before the number of available Mission Capable (MC) / Full Mission Capable (FMC) aircraft can be expected to fall below Chief of Naval Operations (CNO) readiness goals of 78% MC and 61% FMC, OPNAVINST 4790.2F (1995).

One measure of an Air Wing's overall readiness is indicated by its Mission Capable (MC) Rate. MC is defined as the sum of Partial Mission Capable (PMC) and Full Mission Capable (FMC) aircraft. In simple terms, an aircraft is considered Full Mission Capable (FMC) if it can perform all of its intended missions and Partial Mission Capable (PMC) if it can perform at least one of its intended missions. The rate is the percentage of aircraft onboard that are MC. When an aircraft is neither PMC nor FMC it is defined to be Not Mission Capable (NMC). This thesis uses changes in the estimated MC rates to judge the effect of changes in "off ship" logistics support on aircraft readiness.

## **B. RESEARCH METHODOLOGY**

Identifying potential variables and collecting appropriate data is the first step in determining the potential impact of a loss of "off ship" logistics support on MC/FMC rates. Working closely with INDEPENDENCE, over 180 observable operational and support characteristics were identified and recorded. Operational characteristics included

items such as MC/FMC counts and rates, flying hours, sorties flown, date of last COD, and scheduled date of next COD. The support characteristics include supply effectiveness, range and depth of repair parts, and the number of components "awaiting repair due to parts" (AWP). The data consists of observations collected onboard INDEPENDENCE while operating in the Persian Gulf from February 1998 to June 1998. Observations were recorded in a Microsoft Excel spreadsheet and electronically transmitted to NPS for analysis.

There are several potential methods available for predicting MC rates. One approach is to use classic regression where the number of MC aircraft is modeled as a function of several explanatory variables, each representing some aspect of the operating environment. This is the approach that most of the previous analysis efforts use. For example, Makcel (1987) deals in detail with "logit regression" modeling of aircraft MC/FMC rates. More recently, a set of studies on aircraft readiness were performed by the Center for Naval Analyses (CNA) and is given in Francis and Oi (1998), and two works by Junor, et al. (1997, 1998). All three works deal with forecasting monthly MC/FMC rates for all Navy-wide fighter and attack aircraft. Francis and Oi (1998) examine the volatility observed in MC/FMC rates concluding that the observed variance is not unlike that observed in some economic time series. Junor (1997, 1998) looks at the use of classic regression model for forecasting aircraft readiness. Moore (1998) extends Junor to Generalized Linear Models and Tree Diagrams. Both Junor and Moore conclude that the significant explanatory variables found using classic regression techniques are helpful in predicting the effect their policy changes may have on long-term aviation

readiness. However, they also conclude their usefulness in forecasting month to month aviation readiness is limited.

During the course of a deployment, aircraft cycle between MC and NMC. Junor (1997) proposes modeling these cycles with a modified Markov chain. In a Markov chain the movement from one state to another is governed by transition probabilities. Junor was able to model the behavior of FMC rates by making the transition probabilities dependent on measures of personnel readiness, sorties flown per aircraft and supply support.

This thesis uses a combination of both linear and non-linear models in examining operational and logistics support environment on readiness. A linear Markov model with stationary (constant) transition probabilities is used to confirm that the Markov modeling approach is sensible. Then, a sequence of non-linear Markov models with variable transition probabilities are fit which lead to the selection of the Markov model used in the analysis portion of this thesis. Finally, a sequence of Generalized Linear Models (GLM) is used as a reference to evaluate the performance of the Markov models. Empirical findings indicate that the Markov models fit and provide forecasts that are significantly better than the GLM.

### **C. SCOPE AND LIMITATIONS**

This thesis is limited to the study of carrier based aviation readiness for INDEPENDENCE's 1998 deployment to the Persian Gulf. The work focuses on the 33 F/A-18 fighter attack aircraft assigned to the INDEPENDENCE to validate the Markov

model approach. F/A-18's were chosen because they comprise a significant portion of the embarked aircraft and make up over 50% of the Air Wing's strike capability.

Current reporting requirements concentrate on aggregate counts by squadron. That is, the data collected consists of aggregate counts of MC/FMC aircraft by aircraft type (i.e., F/A-18, F-14, etc). Additionally, support characteristics like range and depth of spare parts on board and the number of "off ship" requisitions are aggregate measures without regard for aircraft type. For this thesis, it was decided early in the process to use aggregate measures to build a forecasting model in order to be consistent with current reporting practices.

One of the goals of this thesis is to provide forecasts for the number of aircraft an operational commander can expect to launch in support of operations. The number of MC aircraft was chosen because it meets this goal and simplifies the model significantly. Even if the model were modified to predict the numbers of FMC and PMC aircraft separately, the model would not be able to predict the capabilities individual PMC aircraft may have. Additionally, it is reasonable to assume that the onboard mission planners could support strike requirements generated from the estimate of the MC count by matching the specific capabilities of individual PMC to roles in the mission.

#### **D. THESIS ORGANIZATION**

This thesis is organized along the general lines of the model development. This involves three basic steps: (1) specifying plausible equations and probability distributions (models) to describe the main features of the underlying process that govern the MC

rates; (2) using the data to estimate parameters (coefficients) for models and deciding which is the most plausible and best fitting; and (3) using the resulting fitted model to forecast MC rates under a variety of conditions.

Chapter II develops the use of Markov chain models in a readiness application. First, a simple Markov chain model is developed without explanatory variables to establish the underlying concepts and the approach's intuitive appeal. Then, a section is dedicated to methods for incorporating explanatory variables into the model.

Chapter III discusses the fitting of a sequence of Non-linear Markov models and a family of Generalized Linear Models (GLM) to estimate the effect of changes in the frequency of COD service on the expected number of MC aircraft. Empirical results observed during the fitting process are presented and a comparison of Markov models and GLM is discussed.

Chapter IV uses the Markov model built in Chapter III to estimate the effects of different operating and support environment on MC counts. The Markov model is also used to forecast aircraft readiness during a prolonged absence of "off ship" logistics support.

Chapter V presents conclusions and recommendations, and identifies areas for further research.

## **II. MODEL DESCRIPTION**

The first step in forecasting the number of days INDEPENDENCE can go without COD service before MC rate falls below the CNO goal is to model the number of Mission Capable (FMC and PMC) aircraft on a particular day as a function of the number of MC aircraft on the previous day along with any other explanatory variables which capture the current operating conditions on the ship. These explanatory variables can include sorties flown (SF), "off ship" logistics support, or range and depth of onboard repair parts.

One approach is to use classic regression where the number of MC aircraft is the response variable and the number of MC aircraft for the previous day, number of sorties flown, etc. are explanatory variables. This approach, however, fails to capture the structure of the underlying process. This chapter explains how the use of a modified Markov chain can add realism to the model that is absent in classic regression. This model was first proposed by Junor, et al. (1997) in a similar study, which sought to forecast Navy-wide readiness for fighter and attack aircraft on a month to month basis.

### **A. CONCEPTUAL MODEL**

At the beginning of the flying day or 1200 local, whichever occurs first, the number of MC aircraft is determined and reported. In the simplest terms, the material condition of an aircraft can be defined by one of two qualitative states, MC or NMC. Thus, the material condition of an individual aircraft can be modeled as a binary random variable, i.e., it assumes the value of one if the aircraft is MC or zero if the aircraft is NMC.

For simplicity consider that at noon each day technicians observe a series of Bernoulli trials (coin flips) which determine the state of each aircraft. For the aircraft that started the operating period in a MC state there is a probability,  $P_1$ , that the coin will turn up heads, meaning the aircraft will remain MC (a success), and a corresponding probability  $(1-P_1)$  that the aircraft will fail and transition from MC to NMC. Similarly a probability  $P_2$  can be used to govern the “successful” repair of an NMC aircraft, meaning it transitions from NMC to MC. After testing each plane, the technicians report the number of successful trials, which translates to the number of MC planes onboard.

The simplest Markov model assumes that the probability an aircraft remains MC is  $P_1$  and the probability an NMC aircraft transitions to MC is  $P_2$ , and both are constant. In other words, the technician uses the same coin each day. By considering operational or support factors, one can envision a model in which  $P_1$  or  $P_2$  could be adjusted to capture the environment the aircraft operated in during the previous period. For example, if the number of sorties flown today was high,  $P_1$  could be adjusted down; this makes it more likely that an aircraft would fail and transition to the NMC state during the period.

## **B. MARKOV MODEL BASICS**

Let  $MC_t$  be the number of MC aircraft at time  $t$ , where  $t$  represents an integer number of days, and let  $N$  be the number of aircraft onboard. Since the INDEPENDENCE deploys with a fixed number of aircraft,  $N$  is constant. Then,  $\{MC_t, t = 0, 1, 2, \dots\}$  is a stochastic process which takes on the possible values  $\{0, 1, 2, \dots, N\}$ . If

$MC_t = i$ , then the process is said to be in state  $i$  at time  $t$ . Suppose that whenever the process is in state  $i$ , there is a fixed probability  $P_{i,j}$  that the next state will be  $j$  with

$$P_{i,j} = P[MC_{t+1} = j | MC_t = i]. \quad (1)$$

Such a stochastic process is called a Markov chain. One benefit of using a Markov chain for this application is the distribution of any future state  $MC_{t+1}$  depends only on the present state  $MC_t$ . (Ross, 1993)

Since probabilities  $P_{i,j}$  are non-negative and the process has to transition into some state it follows that

$$P_{i,j} \geq 0, i, j \geq 0 \quad \text{and} \quad \sum_{j=0}^N P_{i,j} = 1, i = 0,1,2,\dots,N. \quad (2)$$

Now let  $P$  denote the matrix of one step transition probabilities, such that

$$P = \begin{bmatrix} P_{0,0} & P_{0,1} & \cdots & P_{0,N} \\ P_{1,0} & P_{1,1} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ P_{N,0} & P_{N,1} & \cdots & P_{N,N} \end{bmatrix}. \quad (3)$$

For example, consider the four-state Markov chain depicted in Figure 1 with  $N = 3$  aircraft. When  $MC_t$  is 1, there is one MC aircraft and two NMC aircraft. There are two possible manners in which the process can get from state 1 to state 2, either, repair one of the two NCM aircraft while the one MC aircraft remains MC, or repair both NCM aircraft while the MC aircraft fails and transitions from MC to NMC. It is evident how quickly the number of calculations required to compute individual  $P_{i,j}$  increase as the number of aircraft is increased. In this thesis, the 33 F/A-18 aircraft assigned to INDEPENDENCE



are considered. The result is a 34 by 34 matrix  $P$  where each row defines the conditional distribution for  $MC_{t+1}$  given  $MC_t = i$ .

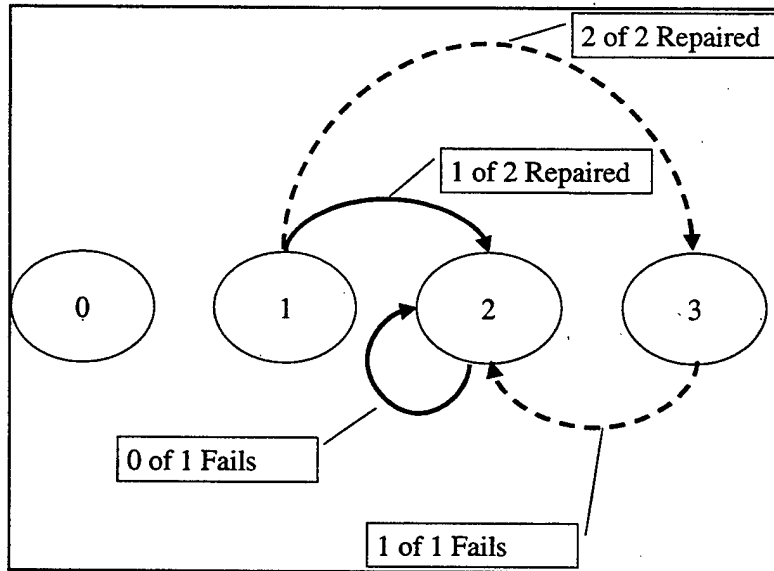


Figure 1: Transition Paths from State 1 to State 2

Because the Markov chain described here is irreducible and ergodic, there exist limiting probabilities, denoted  $\Pi_j$ , that the process will be in state  $j = 0, 1, 2, \dots, N$  at some future time. The  $\Pi_j$  are independent of the starting state  $i$  and can be shown to equal the long run proportion of time the process will be in state  $j$ . These limiting probabilities define the unconditional distribution of  $MC_t$  (Ross, 1993)

From an operational standpoint, commanders are interested in short-term forecasts for the expected number of MC aircraft available. In the long-term, policy makers are interested in estimating the impact policy changes will have on overall aircraft readiness. These Markov models allow the examination of both short-term and the long-term effects of changes in operational or logistics support factors on aircraft readiness.

### C. BASIC MARKOV MODEL

This section develops a Markov approach to modeling aircraft readiness and demonstrates how this approach accounts for the observed behavior of the actual data. Aircraft readiness can be modeled directly as a Markov process by generating a P matrix consisting of  $P_{ij}$ 's which are functions of  $P_1$  and  $P_2$ . Rather than focus on these transition probabilities for this model, it is simpler to describe the model in terms of the expected values. Concentrating on the expected values has the additional advantage that it is more readily interpretable for operators. The work in this section follows Junor (1997).

Given  $MC_t$  for day  $t$ , the number of MC aircraft for the next day  $MC_{t+1}$  is a random variable, which is the sum of two Binomial random variables. Let  $X_{t+1}$  represent the number of aircraft that started the period  $t$  in an MC state and remained that way (i.e., did not break), and let  $Y_{t+1}$ , represent the number of aircraft that transition from NMC to MC during time  $t$  (i.e., are repaired). Given  $MC_t$ ,  $X_{t+1}$  is modeled as a Binomial random variable with  $MC_t$  trials and probability  $P_1$  and  $Y_{t+1}$  is modeled as a Binomial random variable with  $N-MC_t$  trials and probability  $P_2$ . This model assumes that the outcome for each aircraft is independent of the outcome for any other aircraft on a given day. In addition, if the repair for NMC aircraft is independent of the event that an MC aircraft remains MC,  $X_{t+1}$  and  $Y_{t+1}$ , conditioned on  $MC_t$ , can also be modeled as independent. The relationship between the variables is expressed as

$$MC_{t+1} = Y_{t+1} + X_{t+1}, \quad (4)$$

where  $X_{t+1}$  given  $MC_t$  is Binomial( $MC_t, P_1$ ) and  $Y_{t+1}$  given  $MC_t$  is Binomial( $N-MC_t, P_2$ ).

Then the expected value of  $MC_{t+1}$ , given  $MC_t$  is

$$E[MC_{t+1} | MC_t] = P_1 * MC_t + P_2 * (N - MC_t). \quad (5)$$

Equation (5) gives the mean of the conditional distribution for  $MC_{t+1}$ , which is used to forecast for the number of MC aircraft in the next period. While this value is important to planners in the short term, the mean of the unconditional distribution for  $MC_t$  will provide more insight into the effect that changes in  $P_1$  and  $P_2$  have on long-term MC rates. The unconditional mean represents the stationary or equilibrium value about which the process would vary regardless of the starting point. Let  $MC_{ss}$  represent the mean of the random variable  $MC_t$ . Then by defining  $MC_{ss} = E[MC_{t+1}] = E[MC_{t+2}] \dots$  and substituting, Equation (5) can be manipulated to read

$$MC_{ss} = \frac{NP_2}{1 - P_1 + P_2}. \quad (6)$$

Of course the conditional distribution at a given  $t$  may be quite different than the mean of the unconditional steady state distribution, but this is perfectly consistent with a process being stationary (Chatfield, 1975). This unconditional steady state mean is used to evaluate long-term impact of changes on aircraft readiness.

#### **D. SERIAL CORRELATION**

Observations of a random variable recorded with respect to time are called a time series. For example,  $MC_t$  is a time series. One phenomenon frequently observed in time series random variables is known as serial correlation. That is the tendency of

observations to run on one side of the mean or the other. Serial correlation, however, is not limited to successive observations of a random variable. Monthly retail sales figures, for example, often exhibit correlation in a seasonal pattern with period twelve. The number of periods between correlated observations is referred to as the lag. Serial correlation presents a problem for classic regression models as the observations of a random variable are assumed to be independent. (Hamilton, 1992)

Autocorrelation is a measure of the serial correlation present in a time series. Figure 2 is an autocorrelation plot of F/A-18 and F-14 MC counts observed by INDEPENDENCE. The autocorrelation function of S-Plus provides an estimate of the amount of correlation between observations of a random variable at various lags. The horizontal band about zero provides an interval such that falling outside implies rejecting the null hypothesis (at the 5% level) that the true autocorrelation is zero in a two tailed test. The plots indicate that serial correlation is present in both F/A-18 and F-14 MC counts.

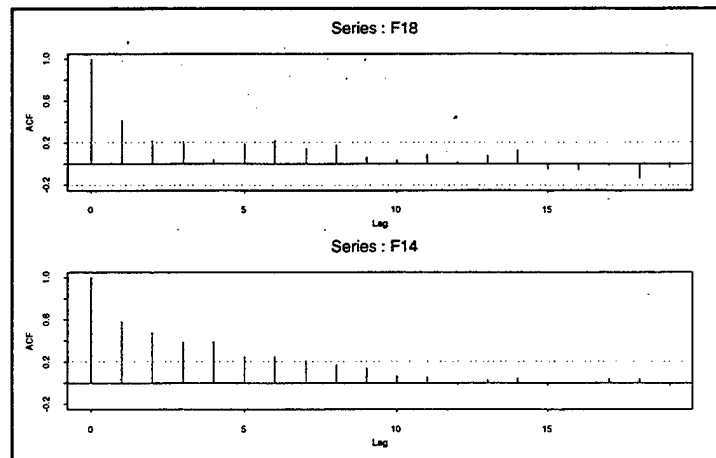


Figure 2: Autocorrelation Plot of F/A 18 and F-14 MC Counts

Although autocorrelation presents a problem in classic regression, time series regression accounts for autocorrelation by including lagged values of the dependent variable as an explanatory variable. Starting with Equation (5) define  $\alpha = P_2 - P_1$ . Then

$$E[MC_{t+1} | MC_t] = MC_{ss} - \alpha (MC_t - MC_{ss}), \quad (7)$$

where Equation (4) is in the form of a first order autoregressive process denoted AR(1), which is sometimes referred to as the Markov process (Chatfield, 1996).

Three consequences of the Markov modeling approach are now apparent: (1) a link has been established between the actual underlying process governing aircraft MC rates and a Markov chain; (2) estimates of the probabilities  $P_1$  and  $P_2$  can be obtained using Equation (5) as a regression equation; and (3) the model accounts for the autocorrelation observed in actual MC counts in a way classic regression cannot.

#### **E. INCORPORATING EXPLANATORY VARIABLES**

The Markov model introduced in Section C does not account for changes in the operational environment. These changes such as frequency of COD service, number of sorties flown etc., affect the entering and exiting probabilities  $P_1$  and  $P_2$ . The expected value of  $MC_{t+1}$  is a function of the entering and exiting probabilities,  $P_1$  and  $P_2$ . Rather than modeling the expected number of  $MC_{t+1}$  as a direct function of the explanatory variables, we will treat  $P_1$  and  $P_2$  as functions of the explanatory variables.

The form of the relationship between  $P_1$ ,  $P_2$  and the explanatory variables must consider that  $P_1$  and  $P_2$  are probabilities. Probabilities can only range between zero and one. A simple linear relationship would allow  $P_1$  or  $P_2$  to take on values outside this

range. To overcome this problem, a more complicated non-linear (logistic) relationship is chosen. Specifically, let  $x_1, x_2, \dots, x_p$  represent  $p$  explanatory variables such as, sorties flown, "off ship" logistic support, flight hours, etc. Then

$$P_1 = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_p x_p}}{(1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_p x_p})} \text{ and} \quad (8)$$

$$P_2 = \frac{e^{\phi_0 + \phi_1 x_1 + \dots + \phi_p x_p}}{(1 + e^{\phi_0 + \phi_1 x_1 + \dots + \phi_p x_p})} \quad (9)$$

where  $\beta_0, \dots, \beta_p$  and  $\phi_0, \dots, \phi_p$  are parameters or coefficients of the explanatory variables and  $0 \leq P_1, P_2 \leq 1$  for all possible values of the explanatory variables. Note that intercepts ( $\beta_0, \phi_0$ ) are included to allow  $P_1$  or  $P_2$  to equal a value other than 0.5 in cases where all the explanatory variables included in the function for a particular transition probability are zero.

In classic regression, the dependent variable, in this case  $MC_{t+1}$ , is modeled using a single linear combination consisting of coefficients and explanatory variables, which weigh the effect of operational and/or support characteristics to produce forecasts. In the Markov approach, which combines Equations (3), (8) and (9), the dependent variable is modeled as the sum of two distinct functions that include linear combinations of the explanatory variables. This allows the impact of a single operational or support element to affect either the failure rate or repair rate of aircraft. It also introduces greater realism to the model that classic regression techniques are unable to provide.



### III. FITTING MARKOV CHAIN MODELS

Chapter II develops the general form of the equations used in Markov models and explains how these equations capture the main features of the underlying process. This chapter describes how the data collected onboard INDEPENDENCE is used to estimate or fit the parameters (coefficients) for the Markov models.

Additionally, a family of Generalized Linear Models (GLM) is fit to serve as a baseline for the Markov models. Empirical results observed during the fitting process are presented and a comparison of Markov models and GLM is discussed.

#### A. ESTIMATION DETAILS

The equation for the Markov model with explanatory variables developed in Chapter II is non-linear in the parameters. The parameters for these non-linear models are estimated using the non-linear least-squares (nls) function provided with the statistical software package S-Plus. The nls algorithm uses iterative Gauss-Newton approach to find the parameter estimates that minimize the residual sum of squares (RSS). Detailed information on the nls algorithm is available in the S-PLUS Guide to Statistics (1997).

#### B. MODEL SELECTION CRITERION

The fit of competing models is evaluated by comparing the residual standard error (RSE) of each model:

$$RSE = \sqrt{\frac{RSS}{n-K}}, \quad (10)$$



where  $n$  is the sample size,  $K$  is the number of estimated parameters and  $RSS$  is the residual sum of squares. The residual standard error measures the scatter or spread of the data around a regression line – hence the goodness of fit (e.g., Hamilton (1992)). Additionally, RSE provides a measuring stick to judge the improvement in fit made by adding more parameters to a model.

In classic linear regression where the dependent variables are normally distributed, exact tests and confidence intervals are available to check the significance of individual parameters. Low significance implies that the particular explanatory variable has little or no predictive value in the model and therefore should be removed.

In the case of non-linear models or cases where the dependent variables are not normally distributed, approximate tests or confidence intervals are used to check the significance of individual parameters. As recommended by Venables and Ripley (1994), the “profile t” function in S-Plus is used to compute an approximate 90% confidence interval for each parameter estimate in the non-linear model. Confidence intervals that include zero imply that zero is a plausible value for the parameter and the explanatory variable should be eliminated from the model. Further details of the “profile t” function and the associated function to generate confidence intervals are available in S-Plus 4 Guide to Statistics (1997).

### **C. THE FITTING PROCESS**

As a starting point,  $MC_{t+1}$  given  $MC_t$  is modeled as a function of  $MC_t$  with parameters  $P_1$  and  $P_2$ :

$$E[MC_{t+1} | MC_t] = P_1 * MC_t + P_2 * (N - MC_t). \quad (11)$$

Since this equation is linear, the parameters can be estimated using standard linear regression techniques. This model is chosen as a starting point to generate initial values for the intercepts of the more complicated non-linear models that follow. Close initial estimates for parameter in non-linear models improve the prospects for a successful fit. The initial estimates for the intercepts can be found by solving for  $\beta_0$  and  $\phi_0$  in Equations (8) and (9), respectively.

Since  $P_1$  and  $P_2$  are constant in this model a single  $P_{i,j}$  matrix describes the conditional distributions for  $MC_{t+1}$  given  $MC_t$ . Following the idea of a goodness-of-fit-test the model's conditional distribution can be compared with a discrete conditional distribution estimated empirically from the observed MC counts. From the INDEPENDENCE the proportion of observations for  $MC_{t+1}$  given  $MC_t$  is 28 and  $MC_{t+1}$  given  $MC_t$  is 29 are determined empirically. The given states ( $MC_t$  of 28 or 29) are selected to provide a sample of sufficient size to present results. Figure 3 shows the conditional distributions estimated from the Markov model with small circles and the observed proportions with horizontal lines. The shape of the observed proportions matches the shape of the conditional distribution of the Markov model.

Notice the shape of the observed data does not exhibit the variance predicted by the Markov model. The Markov model is the sum of the two Binomials. The variance of the sum of two Binomial random variables is the sum of the variances of the individual Binomial random variables (Larson, 1994) and is given by

$$\text{VAR}[MC_{t+1} | MC_t] = MC_t * P_1(1 - P_1) + (N - MC_t) * P_2(1 - P_2). \quad (12)$$

Using Equation (12), the estimated variance for  $MC_{t+1}$  given  $MC_t$  is 28 and  $MC_{t+1}$  given  $MC_t$  is 29 are 3.13 and 3.12, respectively. The observed variances are 1.16 and 1.67, based on 10 and 21 observations. This condition of observing a variance smaller than the estimate is called underdispersion and is not often found in practice. For the purposes of forecasting, underdispersion does not represent a problem for the predicted values, but it does mean that standard methods for calculating prediction intervals and confidence intervals will be conservative (too wide).

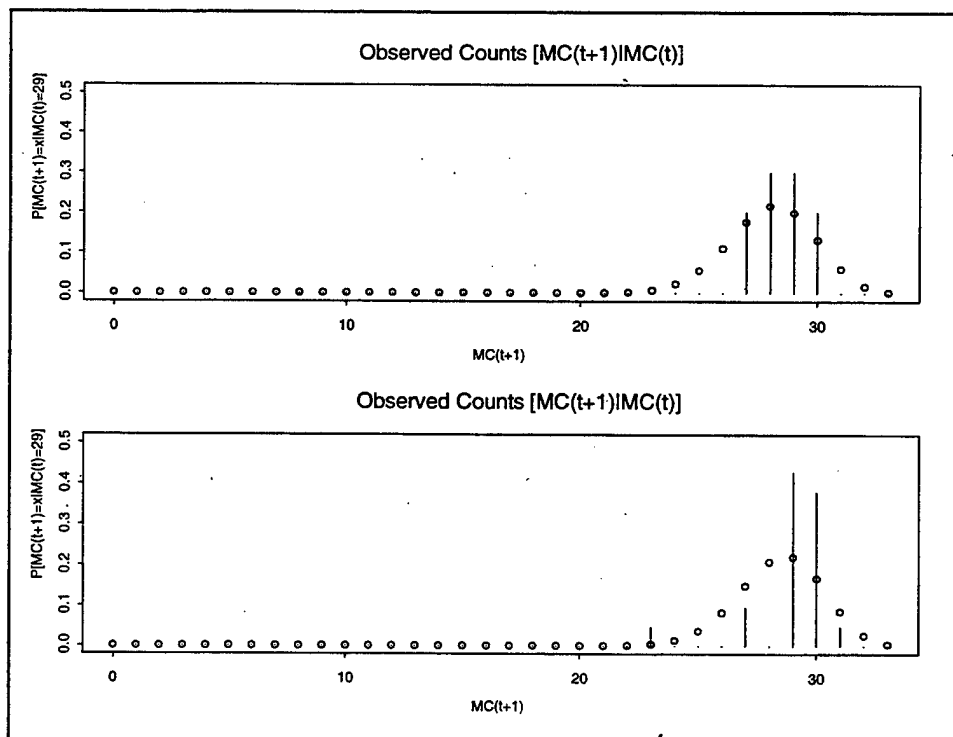


Figure 3: Conditional Distributions for the Markov Model

## 1. Stationary Transition Probabilities

Having fit a Markov model with a linear form. The linear parameters  $P_1$  and  $P_2$  in Equation (11) are replaced with the non-linear functions for  $P_1$  and  $P_2$ , Equations (8) and (9) respectively. This model estimates  $\hat{P}_1$  and  $\hat{P}_2$  for  $P_1$  and  $P_2$  as functions of  $\beta_0$  and  $\phi_0$ . Here,  $\hat{P}_1$  is 92.60%, which is reasonable for the probability that an aircraft remains MC. However,  $\hat{P}_2$  is 58.04%, which is higher than expected.

The summary of the model fit is presented in Table 1. A generalized likelihood ratio test is used to test the null hypothesis:  $E[MC_{t+1} | MC_t]$  is constant against the Markov model. The generalized ratio test gives a p-value of 0.0000 (The test statistic is 30.36 which under the null hypothesis has an approximate  $\chi^2$  with one degree of freedom distribution). Thus, we reject the null hypothesis and conclude the Markov model significantly improves the fit.

Variable Name	Parameter Estimate	Standard Error	90% CI	
			LL	UL
$\beta_0$ ( $P_1$ Intercept)	2.5280	0.1886	2.2450	2.8955
$\phi_0$ ( $P_2$ Intercept)	0.3233	0.4166	-0.3611	1.0972
Residual Sum of Squares (RSS)	78.9952			
Residual Standard Error (RSE)	1.0623			
Degrees of Freedom (n-k)	70			

Table 1: Markov Model with Stationary Transition Probabilities

Notice the confidence interval for intercept term,  $\hat{\phi}_0$ , includes zero, indicating the actual parameter may be zero. It was found that including an intercept term,  $\phi_0$ , in the function for  $P_2$  did not significantly improve the fit of the more complicated linear models that follow. Therefore, the remaining models described do not include an intercept term in the function for  $P_2$ .

## **2. "Off Ship" Logistic or COD Support**

As mentioned in Chapter 1, the goal of the thesis is to examine the impact of "off ship" logistics support on the MC rates for F/A-18 aircraft. It is a logical assumption that the availability of COD support affects the probability of repair,  $P_2$ . Several explanatory variables are used in trying to measure the impact of COD support. Most consist of some type of counter. Two are explained below.

The variable "logdays<sub>*t*</sub>" attempts to capture the quantity of material that may be on a particular COD in terms of "days of material". Material flows into theater of operation each day the carrier is deployed. This material is staged for the next COD flight to the ship. The value of "logdays<sub>*t*</sub>" is zero (i.e., no "off ship" logistics support) for any day without COD service. For each day a COD flight occurs it is assumed all material staged for the ship is sent to the ship. Therefore it is reasonable to represent the quantity of material on the COD by the number of days of logistics support backed up ashore. For example, "logdays<sub>*t*</sub>" is one if there was a COD flight yesterday and two if the last flight was two days ago and so on.

A second variable, called simply "cod<sub>t</sub>" is used to model the number of days since the last COD arrived onboard. When the ship receives a COD, the explanatory variable "cod<sub>t</sub>" is zero. The value of "cod<sub>t</sub>" is increased by one for each day without COD service.

Three models are fit with these two variables; one model with each explanatory variable by itself, and a third with both. The model with the variable "cod<sub>t</sub>" included in the function for P<sub>2</sub> with parameter  $\phi_1$  fits the best. The model summary for the best model is provided in Table 2.

Variable Name	Parameter Estimate	Standard Error	90% CI	
			LL	UL
$\beta_0$ (P <sub>1</sub> Intercept)	2.7464	0.0721	2.6268	2.8873
$\phi_1$ (P <sub>2</sub> , cod <sub>t</sub> )	-0.6811	0.3188	-1.3290	-0.2359
Residual Sum of Squares	72.2807			
Residual Standard Error	1.00195			
Degrees of Freedom	72			

Table 2: Markov Model with COD support

As mentioned earlier, the status of aircraft is determined at the start of the flying day or 1200 local. The COD often arrives well after the start of flight operations. From discussions with the Aviation Supply Officer onboard INDEPENDENCE, MC rates may not be accurate. This is due to the fact that if a part is scheduled to arrive on today's COD, the maintenance organization may actually classify a NMC plane as MC, anticipating a quick turnaround from the receipt of the part to returning the plane to a MC status. This practice was not a fixed policy on the ship and was determined on a case by case basis onboard. Since no records were kept of which days early credit for repair

actions was taken, the model was fit with both  $cod_t$  and  $cod_{t+1}$ , and it was found that the unlagged value, a COD today affects tomorrow's MC rate, was the better predictor. However, since daily COD support is the norm, there is little difference in the sequence of observed values and how they affect the fit of the model. It is recommended that future models check both lags during the model building process.

### **3. Aircraft Utilization Rate**

The probability of an MC aircraft being MC at the end of the operating period (24 hours) changes with the utilization rate for the aircraft. Utilization rate is a general term to denote the service conditions the aircraft was exposed to in the previous 24 hours. The two obvious choices for incorporating this effect of aircraft utilization are sorties flown (SF) and flight hours (FH). A sortie is one flying cycle for the aircraft and involves a takeoff and landing, both of which can be classified as shock events. It is assumed that the more takeoff and landings an aircraft is exposed to the more likely it is to fail. Many of the systems onboard an aircraft are only used when the aircraft is flying. It is assumed that failures occur either when the systems are started before flight operations or when the aircraft is flying. The longer the system is in use, the more likely it is to experience a failure. Two explanatory variables are defined, SF per MC aircraft and FH per MC aircraft. They are denoted  $SF_t$  and  $FH_t$ , respectively.

Overall it was found that models with  $SF_{t+1}$  performed better than those including  $FH_{t+1}$ . A model including both  $SF_{t+1}$  and  $FH_{t+1}$  did not perform well due to the high correlation between the variables. Another model with a random variable equal to the sum of  $SF_{t+1}$  and  $FH_{t+1}$  was fit in an effort to overcome the high correlation observed in

the two explanatory variables. However, this model did not significantly improve the fit.

A summary of the fit is provided in Table 3.

Variable Name	Parameter Estimate	Standard Error	90% CI	
			LL	UL
$\beta_0$ ( $P_1$ Intercept)	2.9746	0.2143	2.6558	3.3924
$\beta_1$ ( $P_1$ SF <sub>t+1</sub> )	-0.1712	0.1430	-0.4385	0.0527
$\phi_1$ ( $P_2$ , cod.)	-0.6996	0.3233	-1.3803	-0.2551
Residual Sum of Squares (RSS)	70.6477			
Residual Standard Error (RSE)	0.9975			
Degrees of Freedom (n-k)	71			

Table 3: Markov Model with COD support and SF

#### 4. Incorporating Cannibalization

Cannibalization is the practice of removing “good” components from one NMC aircraft to return another NMC aircraft to a MC status. While cannibalization can improve readiness, it is frowned upon in practice. Each NMC aircraft represents an opportunity to cannibalize good parts to repair other NMC aircraft. To capture the cannibalization opportunity explanatory variable  $CANN_t$  is added to the function for  $P_2$  where  $CANN_t$  is equal to the number of NMC aircraft on a given day. The fit of this model is described in detail in the next section.

Another hypothesis examines the possibility that the number of aircraft that start the day in a NMC status affects the number of aircraft that fail during the day. The idea behind the effect is twofold: cannibalizations may be used to return an aircraft to a MC status before it is reported as NMC, or as the readiness approaches the goal, maintenance



personnel are more aggressive in the repair effort. However, inclusion of the explanatory variable  $CANN_t$  in the function for  $P_1$  did not improve the fit.

#### D. THE FINAL MODEL

The final criteria are rather judgmental on the part of the author: "Does the model capture the underlying process?" Deciding this involves checking the parameter estimates to ensure changes in the explanatory variables have the expected impact.

The model selected captures three aspects of the underlying process: aircraft operating environment estimated by the number of sorties flown, "off ship" logistics support estimated by the number of days since the last "off ship" support, and cannibalization opportunities estimated by the number of down aircraft. The final model is

$$E[MC_{t+1} | MC_t] = \left\{ \frac{\exp(2.8686 - 0.2075 * SF_{t+1})}{(1 + \exp(2.8686 - 0.2075 * SF_{t+1}))} \right\} * MC_t + \left\{ \frac{\exp(-0.6924 * COD_t + 0.0856 * CANN_t)}{(1 + \exp(-0.6924 * COD_t + 0.0856 * CANN_t))} \right\} * NMC_t \quad (13)$$

Table 3 lists the model statistics including the 90% confidence levels for the parameters.

Variable Name	Parameter Estimate	Standard Error	90% CI	
			LL	UL
$\beta_0$ (P <sub>1</sub> Intercept)	2.8686	0.1970	2.5733	3.2458
$\beta_1$ (P <sub>1</sub> SF)	-0.2075	0.1261	-0.4392	-0.0058
$\phi_1$ (P <sub>2</sub> , cod)	-0.6924	0.2697	-1.2210	-0.2989
$\phi_2$ (P <sub>2</sub> , CANN)	0.0856	0.0476	0.0139	0.1711
Residual Sum of Squares (RSS)	66.7639			
Residual Standard Error (RSE)	0.9739			
Degrees of Freedom (n-k)	70			

Table 4: Final Markov Model

The extra parameter and the explanatory variable  $CANN_t$  improves the fit over the previous model and more importantly it captures the aspect of the underlying process set out as a goal for this thesis. A detailed interpretation of the model and parameter estimates is provided in Chapter IV.

#### E. GENERALIZED LINEAR MODEL

A family of Generalized Linear Models (GLM) is also fit to the F/A-18 MC counts to provide a baseline for comparison with the Markov model. The models are fit using the GLM function provided with S-Plus, with a Poisson response variable ( $MC_t$ ) and a log linear link function so that  $\log(E[MC_{t+1}|MC_t])$  are modeled as linear in the explanatory variables. While the response variable ( $MC_t$ ) is clearly not Poisson (mean does not equal the variance) it captures the counting aspect of the data and fits better than the alternative distributions available. Further details of the fitting algorithms are available in the S-Plus Guide to Statistics (1997). The fitting process follows the standard modeling techniques for GLMs (i.e., McCullagh and Nelder (1983)). The GLM

in Table 5 is fit with the same explanatory variables fit during the Markov modeling process.

Variable Name	Parameter Estimate	Standard Error	T value
$\beta_0$ (Intercept)	2.9667	0.5363	5.5320
$\beta_1$ ( $MC_t$ )	0.0147	0.01814	0.8122
$\beta_2$ ( $SF_{t+1}$ )	-0.0003	0.0002	-0.4311
$\beta_3$ ( $cod_t$ )	-0.0193	0.0066	-0.2466

Table 5: GLM with  $MC_t$ ,  $SF_{t+1}$  and  $cod_t$

A generalized likelihood ratio test is used to test the null hypothesis:

$$E[MC_{t+1} | MC_t] = \beta_0 \quad (14)$$

against the alternative model

$$E[MC_{t+1} | MC_t] = \beta_0 + \beta_1 MC_t + \beta_2 SF_{t+1} + \beta_3 cod_t. \quad (15)$$

The generalized ratio test gives a p-value of 0.1994 (The test statistic is 1.0029 which under the null hypothesis has an approximate  $\chi^2$  with three degrees of freedom distribution). Thus, we fail to reject the null hypothesis and conclude that the alternate model with explanatory variables  $MC_{t+1}$ ,  $SF_{t+1}$ , and  $cod_t$  does not significantly improve the fit over the null model. Parameter estimates for the alternative are given in Table 5. Difficulty in finding significant classic regression models that provide good forecasts is also found in both Junor (1998) and Moore (1998). For this data set, the Markov model not only performs better than classic regression models it is the only formulation tested that worked with the data provided.

#### IV. FORECASTING WITH MARKOV MODELS

This chapter begins with an interpretation of the parameter estimates for the Markov model and examines the effect of changes in the explanatory variables have on forecasted readiness. The Markov model is used to successfully forecast the final twelve days of the INDEPENDENCE's deployment. Finally, the Markov model is used to assess the effect of a prolonged deprivation of COD service on F/A-18 readiness.

##### A. REVIEW OF PARAMETER ESTIMATES

Sorties flown are included as an explanatory variable in the function for  $P_1$ . In Table 6, the impact of changing aircraft utilization rates is examined. Specifically, this involves varying the number of sorties flown per MC aircraft. As expected, as the number of SF increases, the probability that MC aircraft remain MC decreases. The final column examines the impact of the number of SF on  $MC_{ss}$ , the long run mean of the unconditional distribution of  $MC_{t+1}$ .

Value of "SF <sub>t+1</sub> "	$P_1$	$MC_{ss}$
0.0	0.9463	91.59
1.0	0.9347	89.95
2.0	0.9209	88.08
3.0	0.9044	85.95

Table 6: Impact of SF

In the final Markov model the function for  $P_2$  includes two explanatory variables,  $cod_t$  and  $CANN_t$ . To isolate the effect changes in one of the explanatory variables has on

$P_2$ , the other is held constant at the value observed most often in the INDEPENDENCE data set.

Table 7 presents the change in the  $P_2$  while the value of  $cod_t$  is varied and the value of  $CANN_t$  is held constant at four, which is approximately the average number of NMC aircraft observed on a day-by-day basis. COD support has a significant negative impact on the probability an aircraft is repaired. To provide a more intuitive measure of what a change in the probability of repairing an aircraft has, consider an individual aircraft. By definition each day at 1200 local NMC aircraft are subject to a Bernouli trial. The number of Bernoulli trials that must be conducted before the first success (a transition from NMC to MC) is a Negative Binomial random variable with parameter  $P_2$  and an expected value of  $1/P_2$  (Larson, 1994). Since the status of aircraft is determined only once each day there is a one to one conversion from the expected number of trials to the number of days a NMC aircraft can be expected to remain NMC. The model predicts that small increases in the average number of days between CODs will have a significant effect on aircraft repair turn around time.

Days since last COD	$P_2$	Days to Repair
0	0.5846	1.71
1	0.4135	2.14
2	0.2610	3.83
3	0.1502	6.65

Table 7: Impact of COD support

Table 8 examines the impact of the cannibalization opportunity represented by number of NMC aircraft. In this table, COD support is assumed and the value of  $cod_t$  is

held constant at zero, implying “daily COD support”. It is clear the availability of COD support outweighs the cannibalization opportunities presented by additional down aircraft. During the deployment, INDEPENDENCE never went more than four days without COD service. Frequent access to “off ship” logistics support reduces the need to cannibalize aircraft to maintain readiness above CNO goals. If the frequency of COD service is reduced, or COD service is interrupted for a prolonged period of time, it can be anticipated that the number of cannibalizations would increase and the magnitude of the parameter for  $cod_t$  would increase as well.

Number of NMC aircraft	$P_2$	Days to Repair
3	0.5637	1.77
4	0.5845	1.71
5	0.6052	1.65
6	0.6253	1.59

Table 8: Cannibalization

This Markov model does not capture onboard supply support elements. Much of this has to do with its one day forecast horizon. It can be argued that the aggregate measures normally monitored for onboard supply position do not change significantly overnight making inclusion of such variables of little use in short-term forecasting. However, a Markov model with longer planning horizons may be able to incorporate these measures. Inclusion of aggregate supply measures may never provide the resolution modelers seek in readiness models. It is more likely that the lack of specific components has a much greater predictive capacity than general measures such as range and depth of onboard spares.

One of the problems identified earlier in the INDEPENDENCE F/A-18 readiness data is underdispersion. Because the data is underdispersed there is not enough explainable variation to allow inclusion of as many explanatory variables as desired. One possible explanation for the underdispersion is the fact that not every MC aircraft flies on a given day. While some of the aircraft that do not fly transition to NMC as a result of discrepancies discovered during routine preventive maintenance, it is likely that some of the aircraft are not "tested" for failure at all. Thus, the parameter N in the Binomial distribution is too large, which leads the model to expect more variance than is present.

#### **B. FORECASTING**

The last twelve MC observations are retained to check the forecasting ability of the Markov model. Figure 4 starts with the MC counts for the 120 days INDEPENDENCE was in the Persian Gulf. The gaps in the counts are inport periods when MC counts are only recorded on Wednesdays. Fitted values are provided for the period used to estimate model parameters. Notice that fitted values start one day after the ship returns to sea because the model needs  $MC_t$  to predict  $MC_{t+1}$ . If the model were implemented onboard ship the  $MC_t$  would be known the day before the ship pulled out and fitted or forecasted values could be produced for those days.

Finally one-step forecasts from the Markov model are plotted against the observed counts for the last twelve days of the deployment. Overall the model performs very well, and the RSS for the 12 forecasts is less than 0.5 aircraft per day.

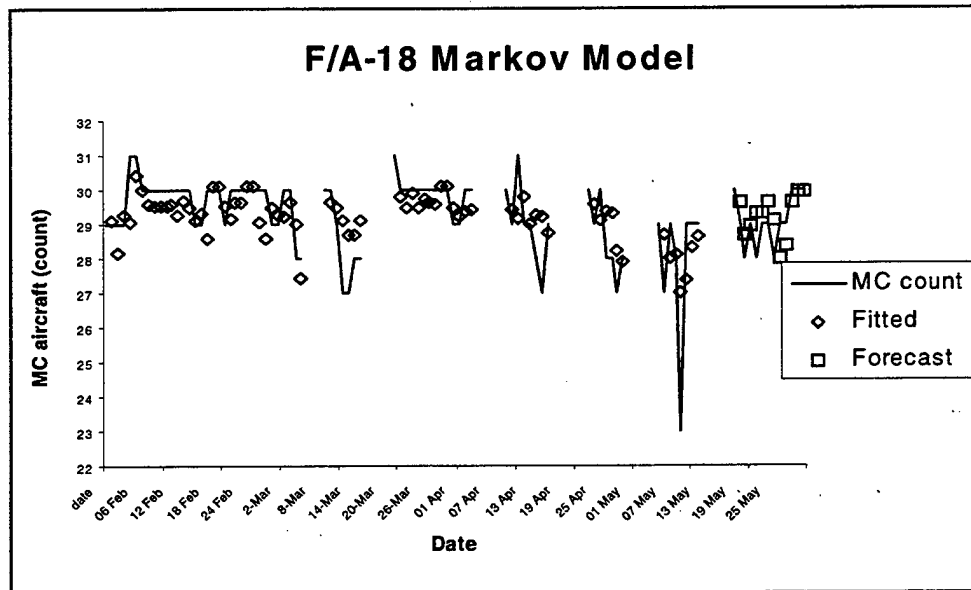


Figure 4: Fitted and Forecast MC Counts for the Markov Model

Of note on Figure 4, is May 10<sup>th</sup>. On that day the number of mission capable aircraft fell from 29 to 23. This change exceeds all others and warrants a closer examination. A review of the explanatory variables indicates nothing unusual about the day. However, discussion with INDEPENDENCE revealed the following: F/A-18 aircraft have sophisticated equipment that monitors the stress level placed on the aircraft during takeoff, flight, and landing. If certain stress levels are exceeded, the aircraft is subjected to a series of conditional inspections that must be completed before it can be returned to service. On May 9, three F/A-18 aircraft experienced what is called “hard landing,” that is, their sensors detected the aircraft impacted the deck too hard during landing. All three aircraft were reported NMC on May 10<sup>th</sup> while technicians completed the conditional inspections.



**C. PREDICTING THE IMPACT OF A LOSS OF "OFF SHIP" LOGISTICS SUPPORT**

Carrier Air Wings enjoy "off ship" logistics support an average of every 1.2 days while deployed. INDEPENDENCE averaged "off ship" logistics support every 1.38, with the longest period of independent operations being four days.

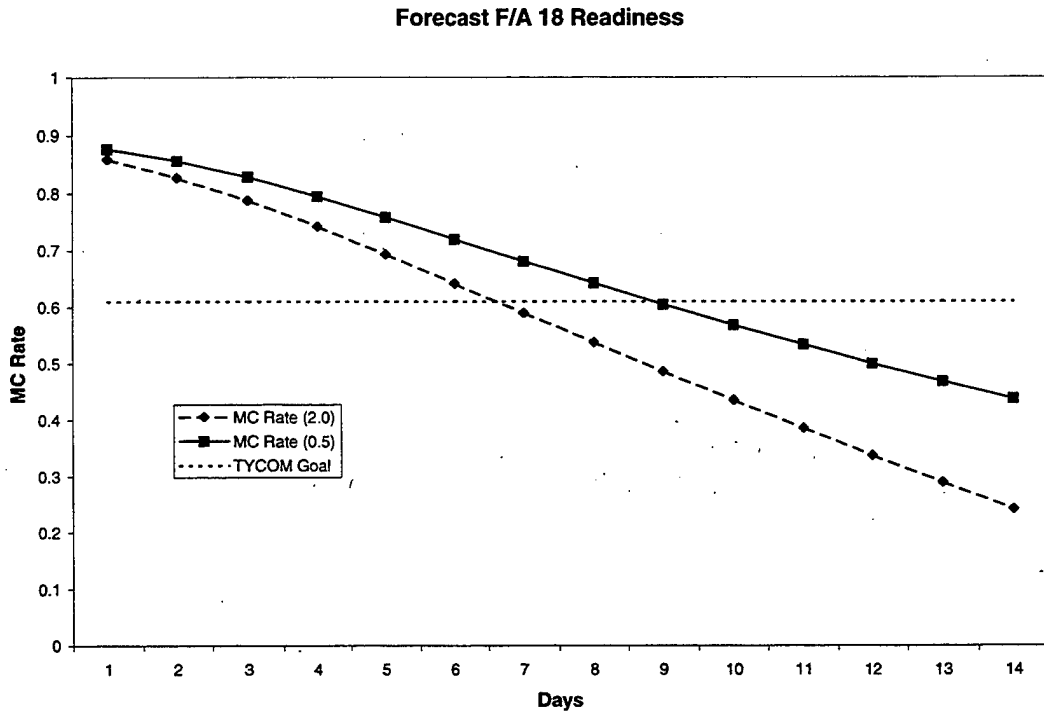


Figure 5: Forecasts for Prolonged Period of COD deprivation

To predict F/A-18 readiness in the absence of "off ship" logistics support,  $MC_{ss}$  is used to display the expected change in the unconditional mean of the process. The variable  $cod_t = 0, 1, 2, \dots, 15$  was used with two flying profiles, 15 and 58 sorties per day to represent both low and high aircraft utilization rates. The variable  $CANN_t$  was updated each day using the expected value from the previous day. Here the model predicts that the  $MC_{ss}$  rate falls below the CNO goal in five to eight days depending on aircraft

utilization rate. This result is in line with the intuition of personnel on INDEPENDENCE.

In the model, the impact of a prolonged deprivation of COD service virtually precludes repair of NCM aircraft after six days. This is not the case in practice, as aircraft would continue to be repaired at some reduced rate. One possible extension of the model would be to subdivide NMC aircraft into two categories: NMC supply (NMCS), i.e., an aircraft is down awaiting parts from an "off ship" source; and NMC maintenance (NMCM), i.e., an aircraft is down awaiting the attention of maintenance personnel. Because this model combines these two classifications, the explanatory variable COD affects the rate at which aircraft are returned from both NMCM and NMCS. In practice, cannibalization and innovative maintenance would allow more aircraft to be repaired onboard during the COD outage than is currently reflected in the model.



## V. CONCLUSIONS AND RECOMMENDATIONS

CTF-50 and INDEPENDENCE asked NPS to provide assistance in analyzing the effect of changes in the frequency of COD service on carrier based aircraft readiness. This thesis develops a non-linear model based on a modified Markov chain, which adds realism to the model that classic regression cannot. Empirical results indicate that significant parameters found with a Markov approach provide better forecasts than those obtained using classic regression techniques.

The Markov approach indicates a significant link between the frequency of COD service and cannibalization opportunity to the probability an aircraft is repaired. Additionally, it shows a significant link between aircraft utilization rate and the probability an aircraft fails. By appealing to stochastic queuing theory, aircraft mean time between reported failures and mean time to repair could be estimated.

The model estimates that INDEPENDENCE can expect to go five to eight days without "off ship" logistics support before F/A-18 readiness falls below the CNO goal. The number of days is sensitive to aircraft utilization rate with higher utilization rates reducing the estimated number of days.

While the model includes a term to capture cannibalization, its effect is not as large as expected. That is, cannibalization did not significantly reduce the effect of a long-term COD deprivation. The effect modeled here can most likely be attributed to increased pressure on maintenance personnel to maintain MC rates above CNO goals as the number of NMC aircraft increases. In a wartime scenario, or a period where a

prolonged outage of "off ship" logistic support is anticipated, cannibalizations would increase and maintenance personnel would be more aggressive in the repair effort, which would likely extend the number of days readiness rates would remain above CNO goals.

Areas for further research include refinement of the data collection to improve model predictions. This could include expansion of the basic Markov model to treat aircraft down for maintenance (NMCM) differently than aircraft down for parts (NMCS) which may well lead to more accurate estimates of aircraft readiness. Another potential area is the use of the Markov framework to simulate the effects of prolonged deprivation of "off ship" logistics support on carrier based aviation readiness. One interesting side effect of using a Markov model is the fact that the parameter estimates are easily transformed into transition probabilities; this lends itself to simulation.

The ability to transmit data over the Internet from the carrier to shore based activities enables near real time analysis of readiness. Establishing this data link enables shore based analysts to augment onboard analysis efforts thereby giving the deployed decision maker better information on which to make management decisions.

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