REPORT DOCUMENTATION P

AFRL-SR-BL-TR-98-

0554

oved
704-0188

Public reporting burden for this collection of information is estimated to average 1 hour pe... gathering and maintaining the data needed, and completing and reviewing the collection o... collection of information, including suggestions for reducing this burden, to Washington H... Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management an...
...g existing data sources, any other aspect of this Reports, 1215 Jefferson ... DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	June 30, 1998	Final technical, 4/1/95-3/31/98

4. TITLE AND SUBTITLE
Control Systems Analysis and Design via Matrix Inequalities and Interior Point Methods

5. FUNDING NUMBERS
G: F49620-95-1-0318

6. AUTHOR(S)
Principal Investigator: Stephen P. Boyd

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Stanford University
857 Serra St., Room 260
Stanford, CA 94305-4125

8. PERFORMING ORGANIZATION REPORT NUMBER
SPO 14560

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
110 Duncan Aven. Room B115
Bolling Air Force Base, DC 20332-8080

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

19980803 099

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
No limitations.

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

During the contract period we made considerable progress, developing new families of convex optimization problems for use in control engineering, forging new areas of control applications, and improvements in algorithms. Two new families of convex optimization problems studied are determinant maximization with LMI contraints, and second-order cone programming. Interior-point code has been developed and tested for both of these, that are already widely used and cited. We have also developed a preliminary second (sparse) version of our original semidefinite programming code SP, and new sophisticated methods of global optimization used for nonconvex programming have been successfully extended to the BMI problem. We have pursued a number of applications including a new method for FIR filter design using spectral factorization and convex optimization, robust open-loop model predictive control using second-order programming, VLSI circuit synthesis via semidefinite programming, and low-authority control via convex optimization. Previous work on developing fast algorithms for carrier phase GPS has also continued and the methods have been applied with success to spacecraft formation flying.

14. SUBJECT TERMS
semidefinite programming, determinant maximization, second-order cone programming, FIR filter design, model predictive control, bilinear matrix inequalities, VLSI circuit synthesis

15. NUMBER OF PAGES
8

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

Final Technical Report
for AFOSR Grant F49620-95-1-0318

Control System Analysis and Design via Matrix Inequalities and Interior-Point Methods

Principal Investigator: Stephen P. Boyd, Stanford University

Period Covered by Report: April 1, 1995, through March 31, 1998
Date of Report Submission: June 30, 1998
Report Submitted to: Dr. Marc Jacobs, Contracting Officer, AFOSR

Sponsoring Program: Stanford University
Sponsored Projects Office
857 Serra St.
Stanford, CA 94305-4125

During the contract period we made considerable progress, developing new families of convex optimization problems for use in control engineering, forging new areas of control applications, and improvements in algorithms. At the same time, control engineering methods based on LMIs and SDP, the bases for which were developed during the previous AFOSR contract, have been very widely adopted and used.

Research Summary

Detailed descriptions of our research accomplishments have been published in archival journals or books, or submitted to journals and made available at Professor Boyd's web site,

`http://www-isl/people/boyd/`

Several software packages developed under this research contract have also been made available via WWW. Therefore this comprehensive summary of the work and accomplishments will mostly refer to papers or software that are available elsewhere.

Our survey paper on semidefinite programming, [VB96], appeared in SIAM Review and has been extremely widely read (and cited) by researchers in control, statistics, applied mathematics, and many other areas, as well as engineers in many areas of control engineering. Other papers related to various theoretical and computational aspects of SDP are [VB98, BV96, VB95].

We developed a new family of optimization problems involving determinant maximization with LMI constraints (abbreviated CP in the book[BEFB94]). These problems arise in many problems of system and control, especially those involving extremal ellipsoids, maximum entropy, central $\mathbf{H}_\infty$ controllers, and so on. We wrote a paper on the topic, (which will appear in SIMAX) that covers the theory, algorithms, and details many applications[VBW98]. We developed, tested, and released a complete software package for solving maxdet problems (MAXDET), and incorporated this new solver into SDPSOL, a new parser/solver for solving problems involving matrix inequalities (and now, determinant maximization) [WB96b, WB96a, WVB96]. MAXDET has been used, by us and others, for a wide variety of problems ranging from controller synthesis via V–K iteration, to obstacle and collision detection in robotics [RB96, RB97], to optimal frequency-domain experiment design[JBK$^+$96]. Like linear matrix inequalities and semidefinite programming, maxdet programming will become another standard tool for control engineers. The paper and code have been widely used and cited [WBV96, VBW98].

We developed a preliminary second (sparse) version of our original semidefinite programming[VB96] code SP, and made it available via WWW. The more efficient handling of the large sparse problems arising in control allowed the code to be used in several new control applications at universities and companies, e.g., Kevin Wise's group at McDonnell Douglas. Perhaps the best news, however, from the point of view of computational algorithms for semidefinite programming and linear matrix inequalities, is the extraordinary on-going effort in the optimization community to develop extremely high performance SDP codes. The interest in the optimization community is due, in part, to work (by many researchers) on LMIs and SDPs in control. The next generation of codes are just now becoming available and will give yet another large efficiency gain. One of our main goals — wide and common use of LMIs and SDPs in control — is already starting to be achieved.

We also completed our work on another new family of convex optimization problems that arise in control — second-order cone problems (SOCP). A major paper on this topic was prepared and accepted by Linear Algebra and Applications [LVBL97]. In this paper we work out most of the theory, and introduce a large number of new applications in control and other engineering fields. We developed and tested an interior-point code (SOCP[LVB97]) for this problem, and released it via WWW. This code is also already widely used and cited. Moreover other packages for convex programming (e.g., SDPPACK[AHN+97]) have incorporated SOCP as a new basic problem, citing our paper.

We pursued a number of applications, for example, a new method for FIR filter design using spectral factorization and convex optimization[WBV96, WBV97, WPB96], which we will continue next year. For model predictive control, we introduced the idea of using robust open-loop programming via SOCP to develop robust control laws that incorporate ellipsoidal confidence bounds. This work was reported in two plenary lectures (in Banff and in Seoul) and appears in the paper[BCH97].

We have extended new sophisticated methods of global optimization used for nonconvex quadratic programming to the BMI problem: primal-relaxed dual global optimization (see, e.g., [FV93]). Initial results are very encouraging, and were reported in the European Control Conference[BVB97, BG96].

We have also developed a new method, based on linear matrix inequalities, in VLSI circuit synthesis[VBE96, VBE97, VBE98]. These are essentially *control problems* with time constants at or below the nanosecond level. The "controller" parameters are the widths of wires and transistors in the circuit, and speed of response translates directly into achievable clock rate, i.e., processing speed. Other important objectives include average power and total area. From the mathematical point of view the most important feature is that the open- and closed-loop systems are symmetric — i.e., governed by diffusion dynamics. For such systems, complete (controller) parameter design is possible via LMIs and SDP. There are many (commercial) codes that do some types of wire and transistor sizing, but our methods handle several problems that can not be handled by any previous method. These include interconnects with loops, as in, e.g., clock meshes, busses, and so on. While some problems of interest are small enough to be handled by existing SDP codes, some are very large, and will require new methods that exploit the problem structure. This means that any effort in this area — exploiting problem structure in SDP — will benefit not only control and combinatorial optimization, but also, it seems, VLSI design. Stanford's Office of Technology Licensing is currently preparing several patent applications for these new methods.

We have also continued previous work [HB95, HB96] on developing fast signal processing algorithms for the global positioning system (GPS). A key problem in differential carrier phase GPS is the detection of the number of carrier signal cycles between the receiver and the satellites when the carrier signal is initially phase locked. The carrier phase measurements are linear in these unknown integer valued carrier signal cycles, and the problem becomes one of estimating integer parameters in a linear model. We have developed a theoretical framework for integer parameter estimation in linear models and have derived computationally efficient methods for maximum likelihood estimation and verification. These methods are based on a celebrated algorithm in number theory due to A. K. Lenstra, H. W. Lenstra, and L. Lovász [LLL82]. Previous methods (see, e.g., [Teu94, Teu95, KT96, HWLC97, SB97, Teu97, PS96]) for integer parameter estimation and verification were mainly based on heuristics and a precise mathematical justification was lacking. Our latest results are given in the paper [HB98a]. These results have been succesfully applied in spacecraft formation flying using GPS sensing as reported in [CRA+97] which was awarded as the best student paper in the ION GPS-97 conference. The proposed methods for integer parameter estimation in linear models are general and can be applied to applications beyond GPS. These include radar imaging, magnetic resonance imaging (MRI), and communications. The paper [MTHBC98] describes the applications of these methods in communication over multi-input/multi-output channels.

We have also developed a new method for low-authority controller design. The premise in low-authority control (LAC) is that the actuators have limited authority and cannot modify the system dynamics by a large amount. The main use of LAC is in lightly damped large structures with many elastic modes, where LAC is used to provide a small amount of damping in a wide range of modes for maximum robustness. Our LAC design method is based on convex programming (LP, SDP, SOCP) and can therefore deal with very large-scale problems. It is possible to formulate many different design specifications such as eigenvalue-placement, robustness, disturbance rejection, limits on feedback gains, static structural constraints, etc. Also, our method gives a powerful heuristic for solving the actuator/sensor selection and controller topology design problem via ℓ_1 norm minimization. Currently, we are finishing a paper on the subject where we cover the theory, algorithms, and applications of LAC design. A paper on preliminary results has already been submitted to the 1998 IEEE Conference on Decision and Control.

Other related work, not supported by AFOSR, includes work on CMOS amplifier design [HBL97a, HBL97b], development of a new course on convex optimization with engineering applications [BV95].

References

[AHN+97] F. Alizadeh, J. P. Haeberly, M. V. Nayakkankuppam, M. L. Overton, and S. Schmieta. SDPPACK *User's Guide, Version 0.9 Beta.* NYU, June 1997.

[BCH97] S. Boyd, C. Crusius, and A. Hansson. Control applications of nonlinear convex programming. *Process Control*, 1997. Special issue for papers presented at the 1997 IFAC Conference on Advanced Process Control, June 1997, Banff.

[BEFB94] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics.* SIAM, Philadelphia, PA, June 1994.

[BG96] E. Beran and K. Grigoriadis. A combined alternating and semidefinite programming algorithm for low-order control design. In *Proceedings of the 13th World Congress of IFAC*, volume C, pages 85–90, 1996.

[Boy98] S. Boyd. Entropy and random feedback. In V. Blondel, E. Sontag, M. Vidyasagar, and J. Willems, editors, *Open Problems in Mathematical Systems Theory and Control.* Springer-Verlag, 1998.

[BV95] S. Boyd and L. Vandenberghe. Crcd program: Convex optimization for engineering analysis and design. In *Proc. American Control Conf.*, pages 1069–1071, June 1995.

[BV96] S. Boyd and L. Vandenberghe. Semidefinite programming relaxations of non-convex problems in control and combinatorial optimization. In *Mathematical Engineering: A Kailath Festschrift*, chapter 1, pages 1–10. Kluwer, 1996.

[BVB97] E. Beran, L. Vandenberghe, and S. Boyd. A global BMI algorithm based on the generalized Benders decomposition. In *Proceedings of the European Control Conference*, July 1997.

[CRA+97] T. Corazzini, A. Robertson, J. C. Adams, A. Hassibi, and J. P. How. GPS sensing for spacecraft formation flying. In *Proc. of the Institute of Navigation GPS-97 Conf.*, Kansas City, MO, September 1997.

[FV93] C. A. Floudas and V. Visweswaran. A primal-relaxed dual global optimization approach. *Journal of Optimization Theory and Applications*, 78(2):178–225, 1993.

[HB95] A. Hassibi and S. Boyd. Integer parameter estimation in linear models with GPS applications. Technical report, Information Systems Laboratory, Stanford University, Stanford, February 1995.

[HB96] A. Hassibi and S. Boyd. Integer parameter estimation in linear models with GPS applications. In *Proc. IEEE Conf. on Decision and Control*, Kobe, Japan, 1996.

[HB97] A. Hansson and S. Boyd. Optimal temperature profiles for post-exposure bake of phot-resist. To be presented at SPIE's 23rd Annual International Symposium on Microlithography, 1997.

[HB98a] A. Hassibi and S. Boyd. Integer parameter estimation in linear models with applications to GPS. *To Appear in IEEE Trans. on Signal Proc.*, 1998.

[HB98b] A. Hassibi and S. Boyd. Quadratic stabilization and control of piecewise-linear systems. In *Proc. American Control Conf.*, Philadelphia, PA, 1998.

[HBL97a] M. Hershenson, S. Boyd, and T. H. Lee. CMOS operational amplifier design and optimization via geometric programming. In *Proceedings of the First International Workshop on Design of Mixed-mode Integrated Circuits and Applications.*, pages 15–18, 1997.

[HBL97b] M. Hershenson, S. Boyd, and T. H. Lee. Optimal design of a CMOS op-amp via geometric programming. *Submitted to IEEE Transactions on Computer-Aided Design*, November 1997.

[HBVL97] A. Hansson, S. Boyd, L. Vandenberghe, and M. Lobo. Optimal linear static control with moment and yield objectives. In *Proceedings of the 37th IEEE Conference on Decision and Control*, San Diego, California, 1997. Submitted for possible presentation.

[HHB98] H. A. Hindi, B. Hassibi, and S. P. Boyd. Multiobjective $\mathcal{H}_2/\mathcal{H}_\infty$-optimal control via finite dimensional Π-parametrization and semidefinite programming. June 1998.

[HWLC97] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins. *Global Positioning System, Theory and Practice.* Springer-Verlag, 4th edition, 1997.

[JBK+96] G. B. Jávorzky, S. Boyd, I. Kollár, L. Vandenberghe, and S.-P. Wu. Optimal excitation signal design for frequency domain system identification using semidefinite programming. In *Proceedings of the 8th IMEKO TC4 Symposium on New Measurement and Calibration Methods of Electrical Quantities and Instruments*, pages 192–197, Budapest, Hungary, 1996.

[KKB94] M. G. Kabuli, R. L. Kosut, and S. Boyd. Improving static performance robustness of thermal processes. In *Proceedings of the 33rd IEEE Conference on Decision and Control*, pages 62–66, Orlando, Florida, 1994.

[KT96] A. Kleusberg and P. J. G. Teunissen. *GPS for Geodesy.* Lecture Notes in Earth Sciences. Springer-Verlag, 1996.

[LLL82] A. K. Lenstra, H. W. Lenstra, Jr., and L. Lovász. Factoring polynomials with rational coefficients. *Mathematische Annalen*, 261:515–534, 1982.

[LVB97] M. S. Lobo, L. Vandenberghe, and S. Boyd. SOCP*: Software for Second-Order Cone Programming.* Information Systems Laboratory, Stanford University, 1997.

[LVBL97] M. S. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret. Applications of second-order cone programming. *Linear Algebra and Appl.*, 1997. To appear.

[MTHBC98] A. Maleki-Tehrani, A. Hassibi, S. Boyd, and J. Cioffi. An implementation of discrete multi-tone over slowly time-varying multiple-input/multiple-output channels. *To appear in IEEE GLOBECOM*, 1998.

[PS96] B. W. Parkinson and J. J. Spilker, Jr. *Global positioning system : theory and applications.* American Institute of Aeronautics and Astronautics, Washington, DC, 1996.

[RB96] E. Rimon and S. Boyd. Obstacle collision detection using best ellipsoid fit. *Journal of Intelligent and Robotic Systems*, pages 1–22, December 1996.

[RB97] E. Rimon and S. P. Boyd. Obstacle collision detection using best ellipsoid fit. *Journal of Intelligent and Robotic Systems*, 18:105–126, 1997.

[SB97] G. Strang and K. Borre. *Linear Algebra, Geodesy, and GPS.* Wellesley-Cambridge Press, October 1997.

[Teu94] P. J. G. Teunissen. A new method for fast carrier phase ambiguity estimation. *Proc. IEEE Position, Location and Navigation Symp.*, pages 562–573, April 1994.

[Teu95] P. J. G. Teunissen. The invertible GPS ambiguity transformations. *Manuscripta Geodaetica*, 20(6):489–497, September 1995.

[Teu97] P. J. G. Teunissen. Closed form expressions for the volume of the GPS ambiguity search spaces. *Artificial Satellites*, 32(1):5–20, 1997.

[VB95] L. Vandenberghe and S. Boyd. A primal-dual potential reduction method for problems involving matrix inequalities. *Mathematical Programming*, 69(1):205–236, July 1995.

[VB96] L. Vandenberghe and S. Boyd. Semidefinite programming. *SIAM Review*, 38(1):49–95, March 1996.

[VB98] L. Vandenberghe and S. Boyd. Connections between semi-infinite and semidefinite programming. In R. Reemtsen and J.-J. Rueckmann, editors, *Semi-infinite programming*, chapter 8, pages 277–294. Kluwer, April 1998. Proceedings of the International Workshop on Semi-Infinite Programming.

[VBE96] L. Vandenberghe, S. Boyd, and A. El Gamal. Optimizing dominant time constant in RC circuits. Technical report, Information Systems Laboratory, Stanford University, November 1996.

[VBE97] L. Vandenberghe, S. Boyd, and A. El Gamal. Optimal wire and transistor sizing for circuits with non-tree topology. In *Proceedings of the 1997 IEEE/ACM International Conference on Computer Aided Design*, pages 252–259, 1997.

[VBE98] L. Vandenberghe, S. Boyd, and A. El Gamal. Optimizing dominant time constant in RC circuits. *IEEE Transactions on Computer-Aided Design*, 2(2):110–125, February 1998.

[VBW98] L. Vandenberghe, S. Boyd, and S.-P. Wu. Determinant maximization with linear matrix inequality constraints. *SIAM J. on Matrix Analysis and Applications*, April 1998. To appear.

[WB96a] S.-P. Wu and S. Boyd. Design and implementation of a parser/solver for sdps with matrix structure. In *Proc. IEEE Conf. on Computer Aided Control System Design*, 1996.

[WB96b] S.-P. Wu and S. Boyd. SDPSOL: *A Parser/Solver for Semidefinite Programming and Determinant Maximization Problems with Matrix Structure. User's Guide, Version Beta.* Stanford University, June 1996.

[WBV96] S.-P. Wu, S. Boyd, and L. Vandenberghe. FIR filter design via semidefinite programming and spectral factorization. In *Proc. IEEE Conf. on Decision and Control*, pages 271–276, 1996.

[WBV97] S.-P. Wu, S. Boyd, and L. Vandenberghe. FIR filter design via spectral factorization and convex optimization. In B. Datta, editor, *Applied and Computational Control, Signals and Circuits*, chapter 2, pages 51–81. Birkhauser, 1997.

[WPB96] S.-P. Wu, B. Putnam, and S. Boyd. FIR filter design based on minimizing perceptual spectral distance. 1996. Submitted to ICASSP 97.

[WVB96] S.-P. Wu, L. Vandenberghe, and S. Boyd. MAXDET: *Software for Determinant Maximization Problems. User's Guide, Alpha Version.* Stanford University, April 1996.