Naval Research Laboratory
Stennis Space Center, MS 39529-5004

NRL/MR/7173--97-8069

Alternate Norms for the Cabrelli and Wiggins Blind Deconvolution Algorithms

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May 8, 1998

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REPORT DOCUMENTATION PAGE

Form Approved
OBM No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
May 8, 1998

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
Alternate Norms for the Cabrelli and Wiggins Blind Deconvolution Algorithms

5. FUNDING NUMBERS
Job Order No. 571611000
Program Element No. 0602314N
Project No.
Task No. UW-14-2-08
Accession No. DN164091

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Acoustics Division
Stennis Space Center, MS 39529-5004

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/MR/7173--97-8069

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
Two blind deconvolution algorithms are extended to include alternate norms and used to estimate transient source signatures. The inputs to the blind algorithms are received signals which have undergone propagation through a medium and may be difficult to recognize by a classifier. Both of the algorithms are based on an assumption of sparseness for the Green's or impulse response function. Simulations using model signals indicate that the results using alternate norms are better in some cases than the results using the original algorithm norms, meaning that the best source estimate is more similar to the true source, or that good source estimates are produced more consistently with varying filter length. No predictable pattern emerges to provide guidelines as to when each norm will work best when the Green's function consists of a series of alternating positive and negative spikes. However, if the Green's function consists of a series of spikes skewed to either the positive or negative amplitudes, then the odd-order alternate norms appear to work better than the original norms.

14. SUBJECT TERMS
acoustics, transients, classification, deconvolution, nongaussian, multipath

15. NUMBER OF PAGES
23

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

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I. Introduction

Two papers by Broadhead et al. (1996, 1997) have shown that the Wiggins blind deconvolution algorithm (Wiggins, 1978; Walden, 1985) can successfully be used to estimate a source signature in a realistic underwater acoustics application. More recently, Broadhead and Pflug (1998) have shown that the deconvolution algorithm developed by Cabrelli (1984) shows comparable results to the well-known Wiggins algorithm in the absence of noise, and superior results in the presence of noise. Both of the algorithms address the blind deconvolution problem, i.e., they attempt to recover either a source signal, $s(t)$, or an impulse response function, $h(t)$, from a received signal, $r(t)$. The received signal is related to the source signal and impulse response function by the process of convolution

$$r(\tau) = \int s(t)h(\tau - t)dt \ ,$$

which is denoted more conveniently as $r = s*h$. The problem is said to be "blind" when neither the source nor the impulse response function is known.

The Cabrelli and Wiggins algorithms are similar in that they seek a sparse representation for the impulse response function that can be used to achieve a source estimate, with the sparseness measured by different norms for the two methods. A sparse impulse response function is characterized by a few large amplitudes interspersed with a large number of small. Obviously, the success of these methods depends on how well the propagation path for a received signal satisfies the sparseness criteria. In many applications, ambient noise is a significant corrupting factor, and it is therefore important to consider how such an environment might affect a deconvolution algorithm, as done in Broadhead and Pflug (1998).

Alternate norms in the Wiggins algorithm have been investigated briefly by Broadhead et al. (1997) for underwater acoustics applications and by Nandi et al. (1997) for nondestructive laser testing of materials. Here, alternate norms are investigated for the more noise-resistant Cabrelli algorithm, with results using various norms for the Wiggins algorithm included for completeness.

II. Brief Review of Methods

A received signal, $x(t)$, can be written as the convolution of a source signal with an impulse response function, $x(t) = s(t)*h(t)$. The goal is to find a filter that, upon application to the received signal, produces a good estimate of the source signal or impulse response function, i.e., find $f(t)$ such that the impulse response estimate, $\hat{h}(t)$, is given by

$$\hat{h}(t) = f(t) * x(t) = f(t) * [s(t) * h(t)].$$

If $\hat{h}(t)$ is a good source estimate, i.e., $\hat{h}(t) * h^{-1}(t) \cong \delta(t)$, then

$$\begin{aligned} &\hat{h}(t) = f(t) * [s(t) * h(t)] \\ &\hat{h}(t) * h^{-1}(t) \cong f(t) * s(t) * \left[h(t) * h^{-1}(t)\right] \\ &\delta(t) \cong f(t) * s(t), \end{aligned}$$

and the source estimate is $\hat{s}(t) = f^{-1}(t)$.

A very brief review of the Cabrelli and Wiggins blind deconvolution methods are included here. Although the Wiggins method requires an iterative solution, and the Cabrelli method does not, computational intensities required to achieve solutions are similar.

A. Cabrelli Algorithm

The Cabrelli algorithm uses a measure of sparseness or simplicity called the D-norm, defined by

$$D(\mathbf{y}) = \max {|y_j|}\big/{\|\mathbf{y}\|},$$

for a real-valued vector $\mathbf{y}$ of length m, where

$$\|\mathbf{y}\| = \left(\sum_k y_k^2 \right)^{1/2}$$

is the Euclidean norm, which is related to the variance. Here, $\mathbf{y}_i = \mathbf{f}_i * \mathbf{x}$ is a vector of length $m = n + l - 1$, $\mathbf{x}$ is the input signal of length n, and $\mathbf{f}_i$ is a filter of fixed length l (to be determined). The blind deconvolution problem is solved by maximizing $D(\mathbf{y})$ over all nonzero filters $\mathbf{f}_i$, i.e., maximizing the sparseness of the impulse response function. Mathematically, $D(\mathbf{y})$ is maximized by differentiating $D(\mathbf{y})$ with respect to $\mathbf{f}_i$ for each i and equating to zero (finding the extremal point of $D(\mathbf{y})$). Differentiation leads to the matrix formulation

$$\left(\frac{y_i}{\|\mathbf{y}\|^2} \right) \mathbf{R} \cdot \mathbf{f} = \mathbf{x}^i$$

where $\mathbf{x}^{\mathrm{i}} = \left(x_j,\ x_{j+1},\ \ldots,\ x_{j+l-1} \right)^{\mathrm{t}}$ is an l-length subset of the n-length input channel $\mathbf{x}$, $\mathbf{R}$ is the Toeplitz autocorrelation matrix defined by

$$\mathbf{R} = \begin{pmatrix} \mathrm{r}_0 & \mathrm{r}_1 & \cdots & & \mathrm{r}_{l\text{-}1} \\ \mathrm{r}_1 & \mathrm{r}_0 & \cdots & & \mathrm{r}_{l\text{-}2} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \mathrm{r}_1 \\ \mathrm{r}_{l\text{-}1} & & \cdots & \mathrm{r}_1 & \mathrm{r}_0 \end{pmatrix},$$

and $\mathbf{f}_i$ is the set of filters found by calculating $\mathbf{f}_{\mathrm{i}} = \mathbf{R}^{-1}\mathbf{x}^{\mathrm{i}}$. Once the filters are found, they are convolved with the input ($\mathbf{y}_i = \mathbf{f}_i * \mathbf{x}$) to generate $m = n + l - 1$ *potential* impulse response function estimates. The i for which the D-norm of $\mathbf{y}_i$ is maximum indicates that the impulse response estimate and the corresponding source estimate are $\hat{h}(t) = y_i(t)$, and $\hat{s}(t) = f_i^{-1}(t)$.

B. Wiggins Algorithm

As in the Cabrelli algorithm, the goal of the Wiggins algorithm is to find the filter $\mathbf{f}_i$, that, upon convolution with the input signal $\mathbf{x}$, produces an estimate of the impulse response function and source signal ($\hat{h}(t) = f(t) * x(t)$ and $\hat{s}(t) = f^{-1}(t)$). Instead of the D-norm, Wiggins uses the Varimax norm, or V-norm, as a measure of sparseness,

$$V(\mathbf{y}) = \sum_k y_k^4 \Big/ \left(\sum_k y_k^2 \right)^2$$

Differentiating $V(\mathbf{y})$ with respect to the filter coefficients and equating to zero, a set of equations is obtained which can be rewritten in matrix form as

$$\mathbf{R(f)} \cdot \mathbf{f} = \mathbf{g(f)},$$

where $\mathbf{R(f)}$ is the Toeplitz autocorrelation matrix, and $\mathbf{g(f)}$ is a column vector proportional to $\mathbf{y}^3 * \mathbf{x}$. Choosing an initial filter $\mathbf{f}^0 = (0,...,0,1,0,...,0)$, an iterative algorithm can be generated by taking

$$\mathbf{f}^{n+1} = \left\{ \mathbf{R}(\mathbf{f}^n) \right\}^{-1} \mathbf{g}(\mathbf{f}^n).$$

A convergence criteria on the estimated source or filter is used to terminate the iterations.

III. n-th Order Norms for Deconvolution

It is shown in Broadhead and Pflug (1998) and Nandi et al. (1997) that the Wiggins algorithm can easily be extended to used alternate norms, i.e., the n-th order normalized moment instead of the fourth-order normalized moment, or V-norm. The Cabrelli algorithm can similarly be modified by simply substituting alternate norms for the D-norm in the implementation of the originally-derived formulation. Unlike the formally-derived extension of the Wiggins algorithm included in Broadhead and Pflug, the extension of the Cabrelli algorithm to n-th order norms is heuristic, based on the observation that the D-norm tends to echo the signal kurtosis, and that therefore norms based on moments (fourth and otherwise) might be useful.

IV. Simulated Data

Two source signals and two impulse response functions are used to compare the performance of n-th order norms in the Cabrelli and Wiggins deconvolution algorithms. The first source (s_1) is a pulse-type signal and the second source (s_2) is an exponentially-damped sinusoid, as shown in Fig. 1. The first impulse response function (h_1) is a series of five positive spikes with overall skewness equal to 11.4 and kurtosis equal to 136.5. Since a skewed signal is by definition also nonGaussian, the first impulse response has by necessity both significant skew and kurtosis. The second impulse response function (h_2) is a series of five positive and negative spikes with overall skewness equal to 0.9 and kurtosis equal to 81.8. That is, h_2 is fairly symmetric, but nonGaussian. The second impulse response is representative of multipath propagation that might occur in an underwater acoustic waveguide with air at the surface, for example. In contrast, the first might represent propagation in an ocean environment with a smooth ice cover. These two impulse response functions provide an opportunity to judge the relationship between the normalized moments of an impulse response function and the order of the norm used in the deconvolution algorithms.

Each signal is convolved with each impulse response function to create a set of simulated received signals for input into the deconvolution algorithms. These input signals are shown in Fig. 2. The spikes in h_1 and h_2 are spaced closely enough that multipath arrivals of the signal, especially s_2, are not spatially resolved.

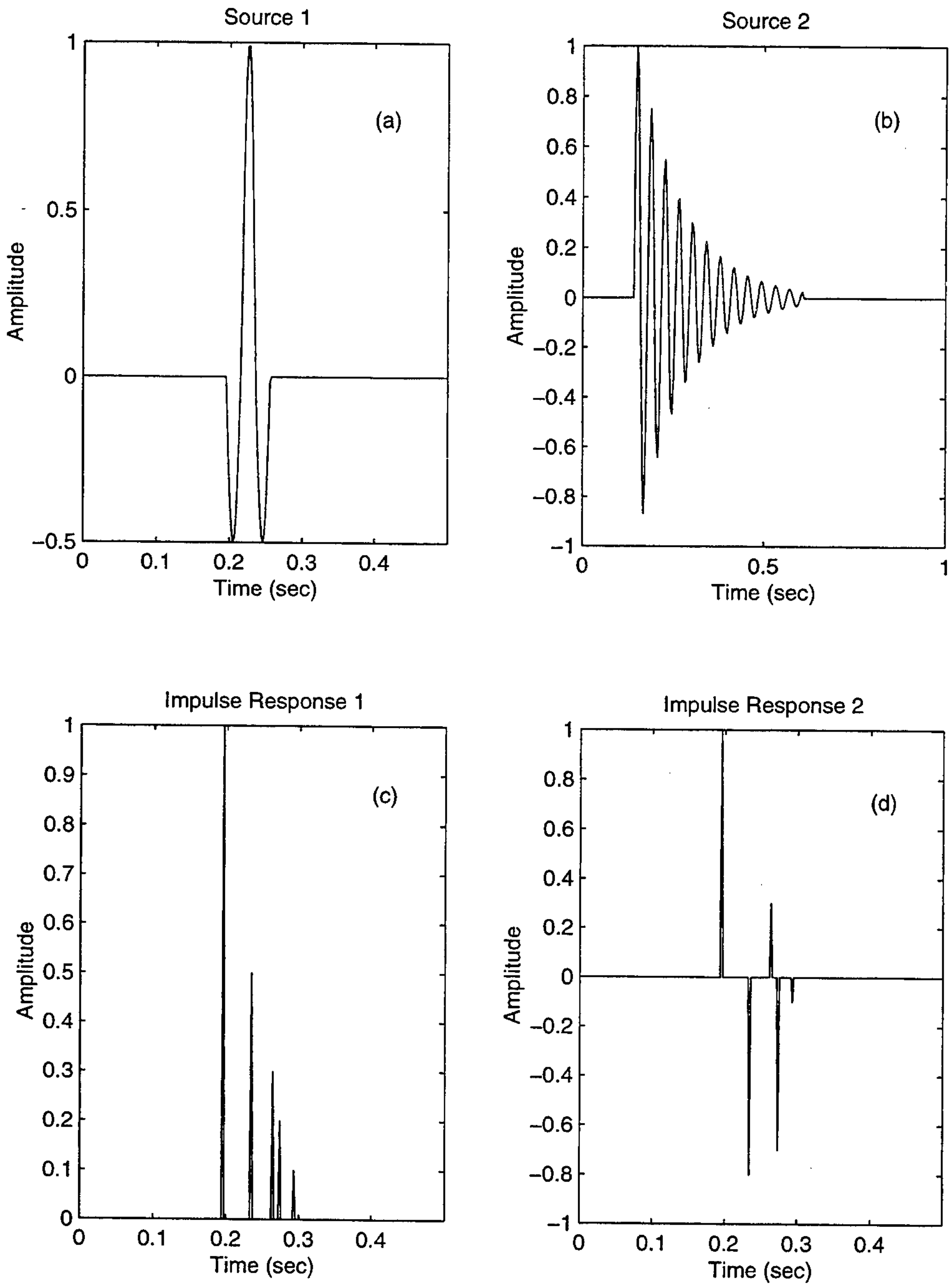

Figure 1. The (a) pulse-type source signal, the (b) damped sinusoid source signal, and (c)-(d) two impulse response functions.

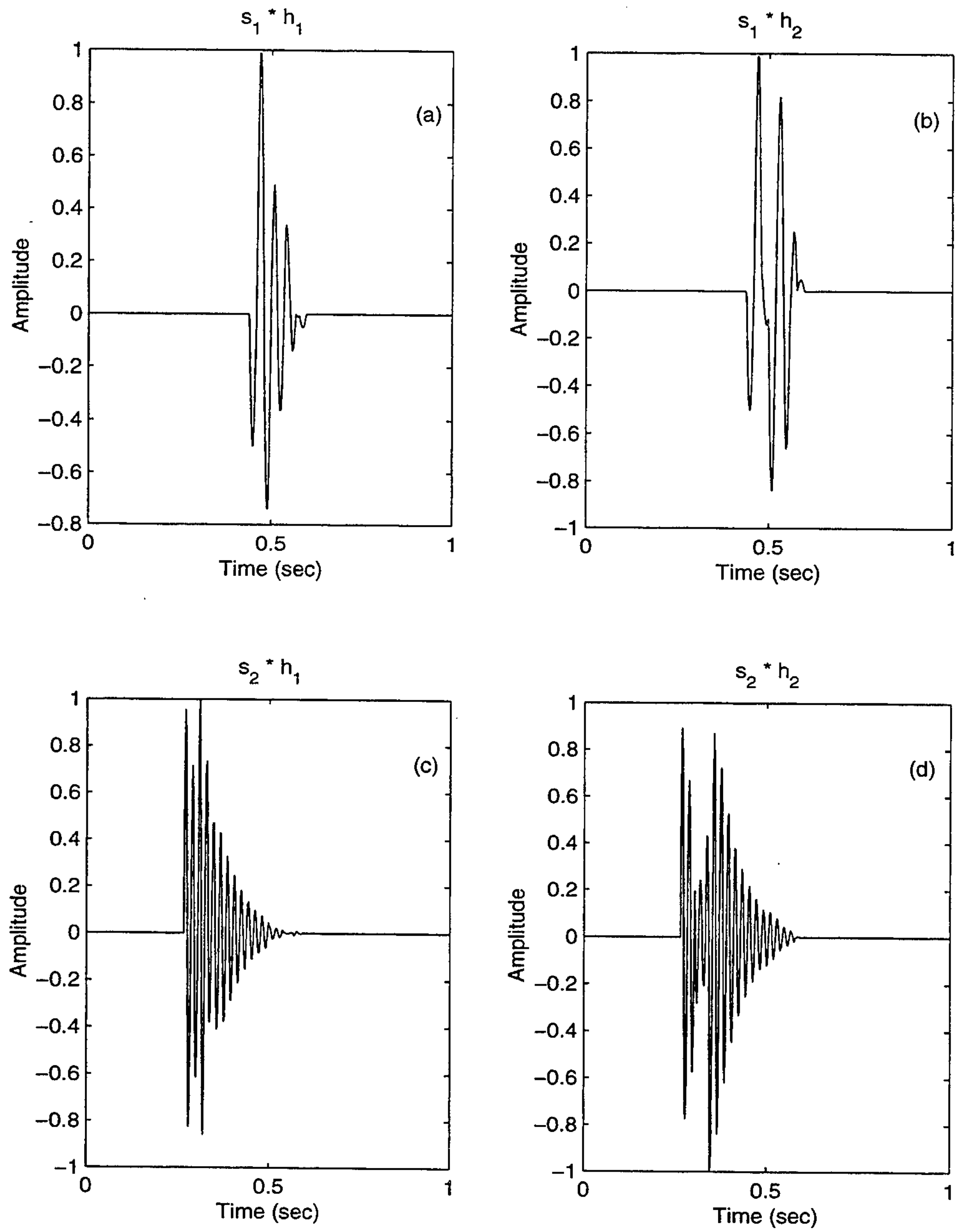

Figure 2. Input signals for the deconvolution algorithms, where s_1 and s_2 denote the first and second source signals, h_1 and h_2 denote the first and second impulse response functions, and '$*$' denotes the convolution operation. The four input signals are (a) $s_1 * h_1$, (b) $s_1 * h_2$, (c) $s_2 * h_1$, and (d) $s_2 * h_2$.

V. Algorithm Parameters and Performance Evaluation

For the Cabrelli and Wiggins algorithms, the 3rd, 4th, 5th, and 6th order normalized moments are evaluated, with the normalized moments defined by

$$m_n = \frac{\sum (y - \bar{y})^n}{N \|y\|^n}$$

for a signal sampled with N points and mean $\bar{y}$. For the Wiggins algorithm with n = 4, this is essentially equivalent to the V-norm, and is referred to as such.

Filter lengths from 1 to 50 are tested. For most of the cases tested here, at least one of these filter lengths is sufficient to produce a good source estimate. In practice, the filter length is unknown, but a set of filter lengths could routinely be tested to produce a set of possible solutions, which would then have to be evaluated in some systematic manner.

To evaluate algorithm performance, a rather simple, but effective, measure is used. The correlation coefficient (cc) between the source estimate, $\hat{s}(t)$ and the true source, $s(t)$ given by

$$\mathrm{cc} = \frac{\max \left| \sum s(t) \hat{s}(\tau - t) \Delta t \right|}{\left(\sum s^2(t) \Delta t \right)^{1/2} \left(\sum \hat{s}^2(t) \Delta t \right)^{1/2}}.$$

This quantity is bounded between zero and one, with a value of one indicating that the source estimate is equal to the true source.

The Wiggins algorithm requires a convergence criteria for the iterative solution to the nonlinear system of equations. This is chosen to be either the point at which the correlation coefficient between the current and previous source estimate is 0.9999, or at 100 iterations. When the goal is to estimate the impulse response function rather than the source signal, the criteria may more appropriately be placed on the estimated impulse response function. Although prewhitening is sometimes required for inversion of the autocorrelation matrix, it was not needed in these simulations. However, restricting the input and output signals to an estimate of the source signal passband was required in one case to achieve successful results with the Wiggins algorithm. This information would generally be available for high signal-to-noise ratio signals, and is not overly restrictive.

VI. Deconvolution Results

This section contains the results of the Cabrelli and Wiggins algorithm performance. No noise is included in the simulations, although, as shown in Broadhead and Pflug (1998) and claimed in Cabrelli (1984), the Cabrelli method appears to be more robust to additive noise than the Wiggins algorithm.

The results are shown in two forms. The first form is a figure depicting the correlation coefficient between the source estimate and the true source at each filter length. It is generally desirable that good source estimates be produced for many filter lengths, as the required filter length will be unknown in practice. The second form of results is a figure depicting the best source estimate over filter length for each method and norm. This allows a visual evaluation of the results and provides a guideline for determining the significance of the correlation coefficients. For comparison, the correlation coefficients between the unprocessed input signals and the true sources are given in the Table.

Input Signal	Correlation Coefficient
$s_1 * h_1$	0.8834
$s_1 * h_2$	0.7247
$s_2 * h_1$	0.9466
$s_2 * h_2$	0.7961

Table. Correlation Coefficients for the Original Input Signals (Before Deconvolution)

A. Signal 1

The correlation coefficients versus filter length for the Cabrelli algorithm are given in Fig. 3. In this and similar figures, the correlation coefficients between the unprocessed signals and the true sources from the Table are depicted by horizontal dashed black lines. When the first impulse response is used to create a received signal for input into the deconvolution routine ($s_1 * h_1$), the D-norm, 3rd, 4th, 5th, and 6th order norms produce good source estimates for many filter lengths. In this case, good solutions are those with correlation coefficients of 0.8960 and higher, and are only slightly better than the correlation coefficient for the unprocessed signal, which is 0.8834. Only the 3rd moment norm produces a significantly improved source estimate with cc = 0.9496 (see Fig. 4), and only for one filter length. The Wiggins algorithm with each norm produces at least one source estimate that is better than the input signal, as shown by the results in Figs. 5 and 6, with the best estimate given by the 5th order norm, cc = 0.9506. Most of the good solutions are only slight improvements, however, and they occur at fewer filter lengths in the Wiggins results compared to the Cabrelli results.

For the second impulse response, ($s_1 * h_2$), the Cabrelli and Wiggins algorithms produce improved source estimates over several filter lengths. In both algorithms, the 5th order norm gives the best solution with cc = 0.8941 for the Cabrelli algorithm and cc = 0.8827 for the Wiggins algorithm. These are significant improvements over the unprocessed input signal correlation coefficient of 0.7247. Note, however, that the D-norm and the Cabrelli algorithm gives produces improved source estimates at more filter lengths than the V-norm and the Wiggins algorithm, and the best source estimate from the D-norm has cc = 0.8706, which is comparable to the best source estimate from the V-norm with cc = 0.8787.

B. Signal 2

Results for the second test signal are shown in Figs. 7 and 8 for the Cabrelli algorithm and in Figs. 9 and 10 for the Wiggins algorithm. The $s_2 * h_1$ input signal is very similar to the true source, having cc = 0.9466. Even so, both the Cabrelli and Wiggins algorithms with all order norms produce many source estimates over the filter lengths. All order norms in the Cabrelli algorithm produce best source estimates with cc = 0.9971, and all the best source estimates are above cc = 0.9963 for the Wiggins algorithm.

The more significant distortions in the $s_2 * h_2$ inputs signal does not prevent either algorithm from producing excellent source estimates, but they occur over only about one-half of the filter lengths tested. Also, for the Cabrelli algorithm, only the D-norm, 4th and 6th moment norms produces significantly improved source estimates, with cc = 0.9951 for each, compared to the input signal correlation coefficient of 0.7961. The 3rd and 5th moment norms produce source estimates that appear to be reversed. Since there is no obvious reason why this should occur in either the Cabrelli or Wiggins algorithms, these occurrences are assumed at this point to be arbitrary. However, we note that if these source estimates were reversed, the correlation coefficients would increase, from 0.7855 to 0.9322 for the 3rd order norm and from 0.7814 to 0.9586 for the 5th order norm. Whether this is likely to occur for other signal types is as yet unknown. In contrast to the Cabrelli results, all the norms in the Wiggins algorithm produce good source estimates, and have correlation coefficients greater than 0.9966.

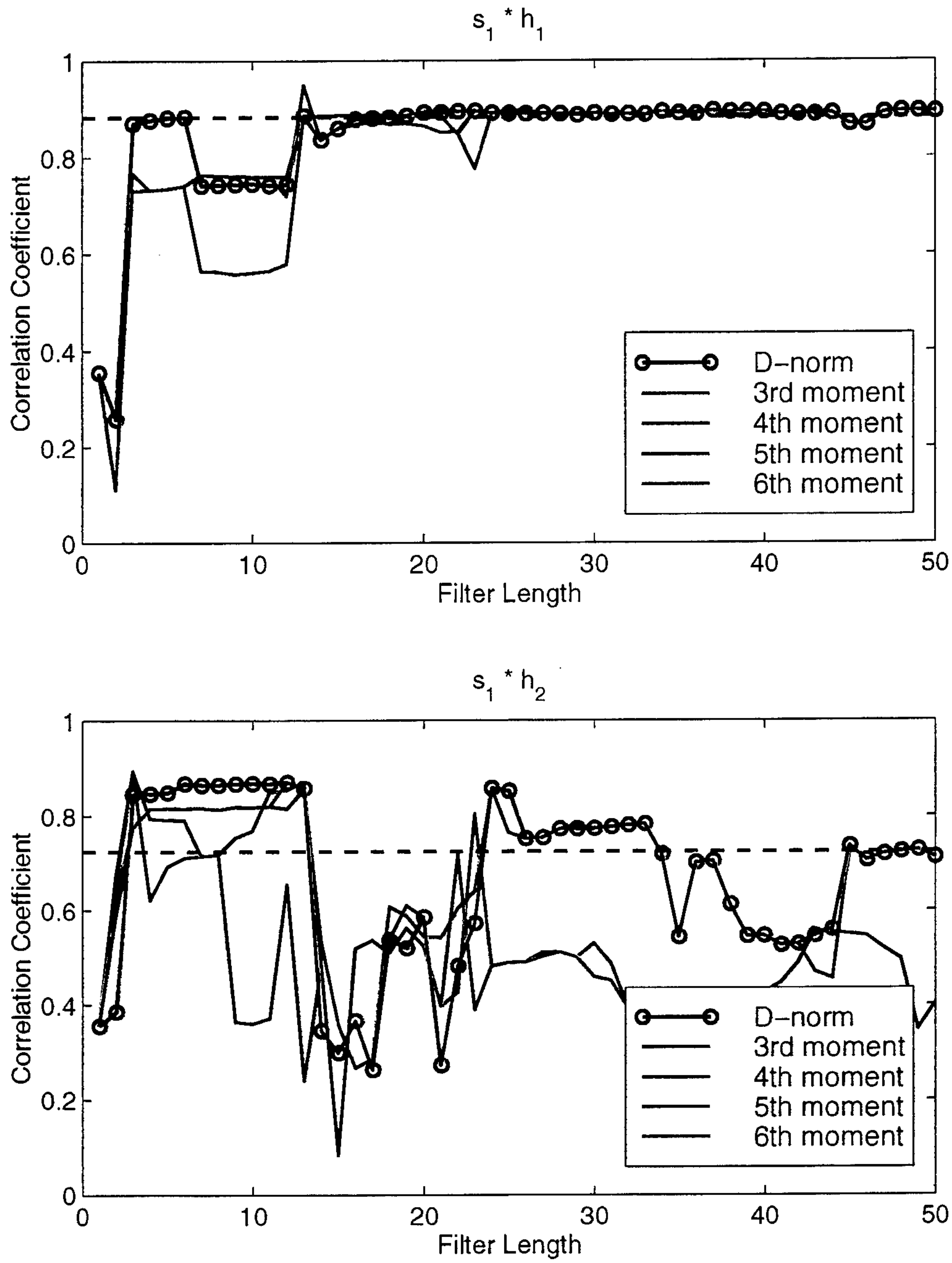

Figure 3. Correlation coefficients between the estimated pulse source, s_1, and the true source versus filter length using the Cabrelli algorithm for input signals (a) $s_1 * h_1$, and (b) $s_1 * h_2$.

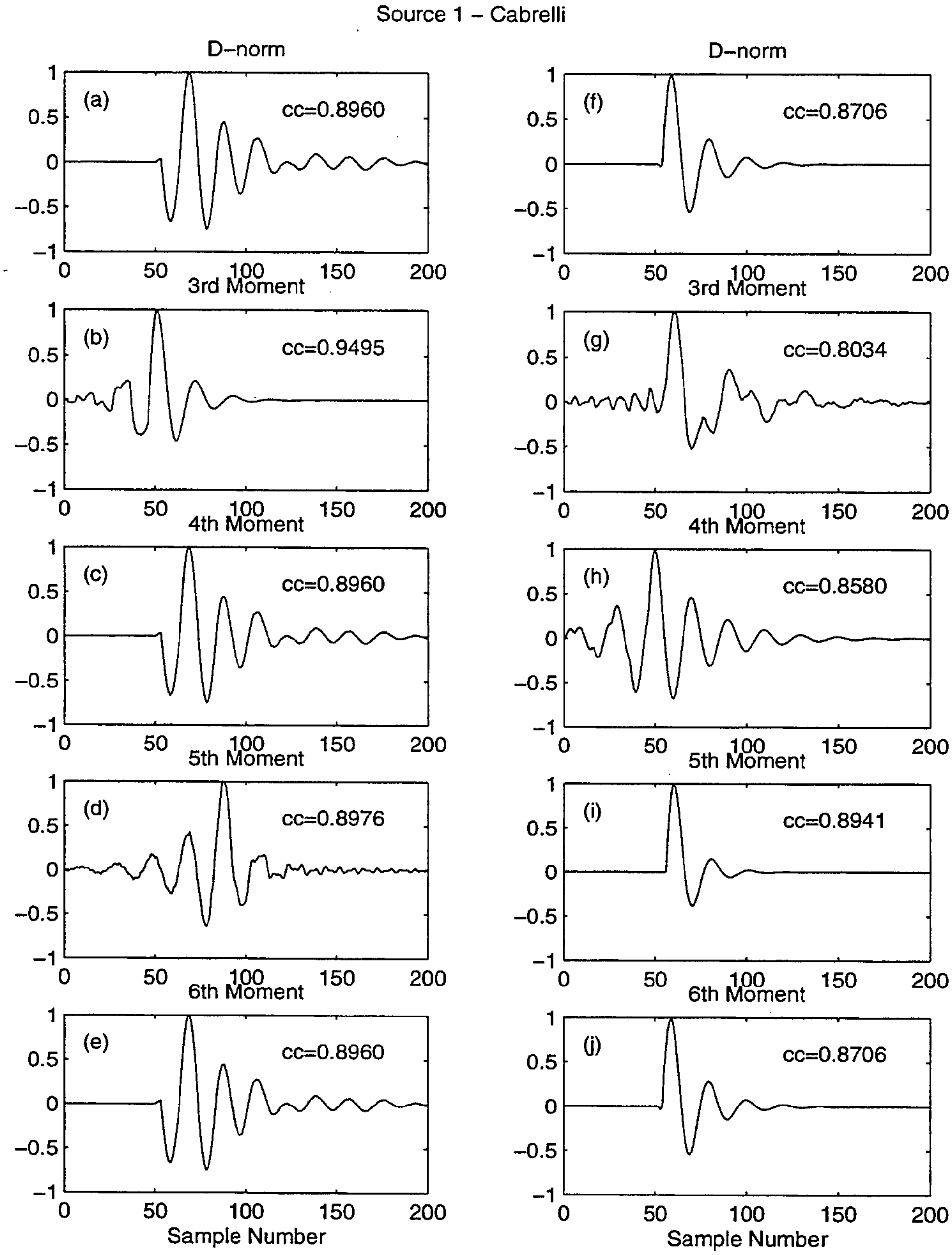

Figure 4. Best estimated source signals for input signals (a)-(e) $s_1 * h_1$, and (f)-(j) $s_1 * h_2$, resulting from the Cabrelli algorithm with various norms. The correlation coefficient (cc) between the source estimate and the true source is given within the plot.

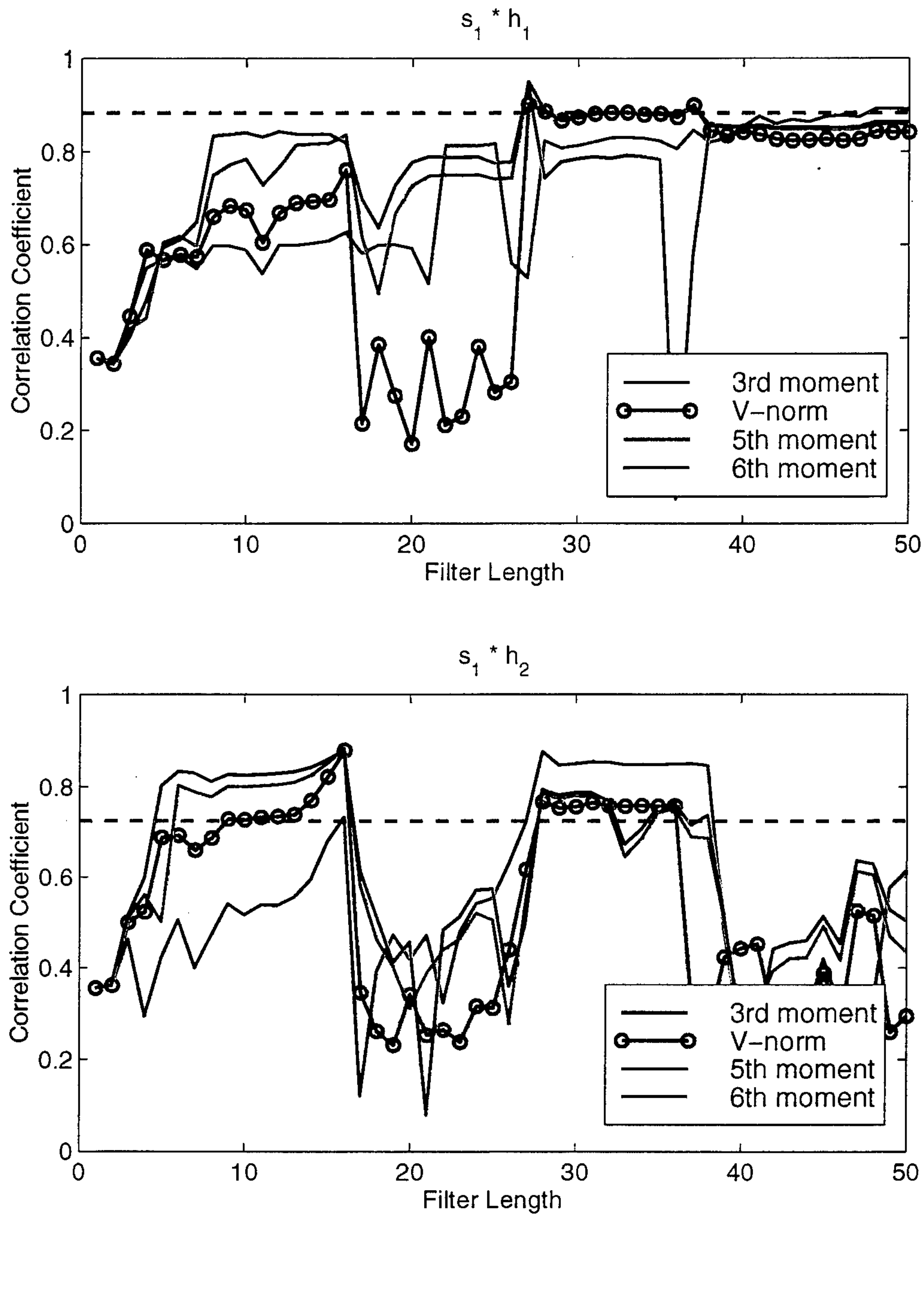

Figure 5. Correlation coefficients between the estimated pulse source, s_1, and the true source versus filter length using the Wiggins algorithm for input signals (a) $s_1 * h_1$, and (b) $s_1 * h_2$.

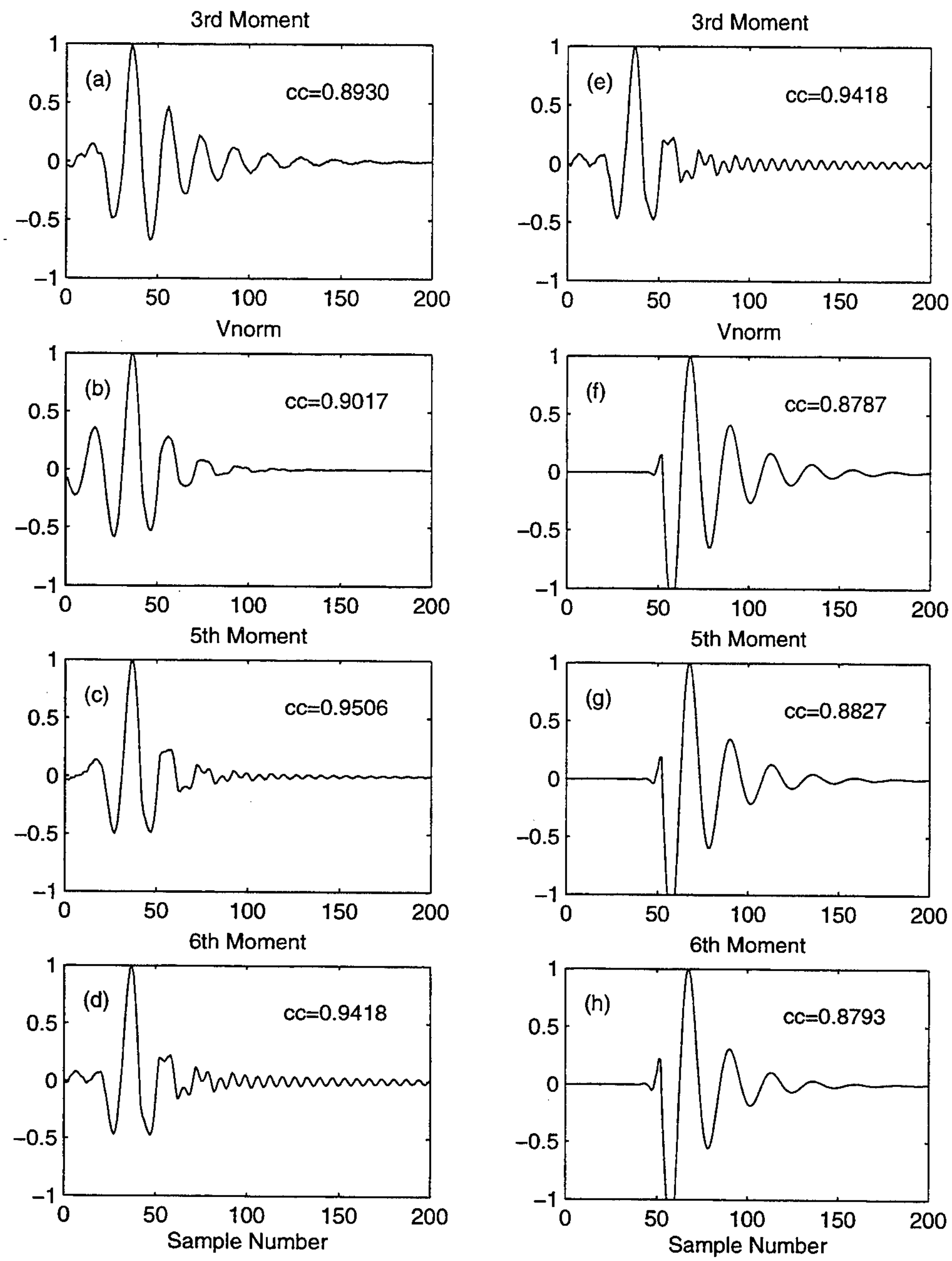

Figure 6. Best estimated source signals for input signals (a)-(d) $s_1 * h_1$, and (e)-(h) $s_1 * h_2$, resulting from the Wiggins algorithm with various norms. The correlation coefficient (cc) between the source estimate and the true source is given within the plot.

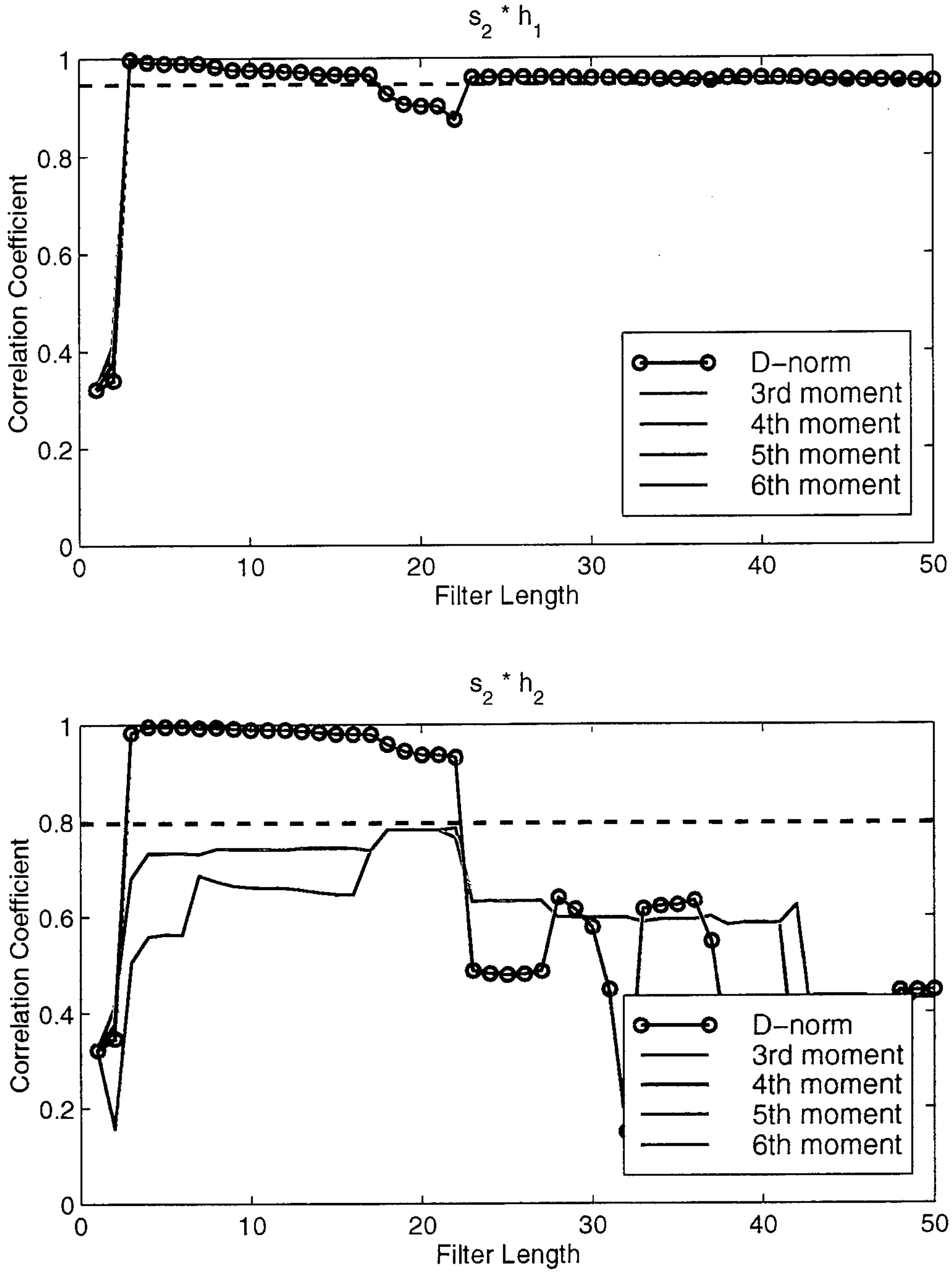

Figure 7. Correlation coefficients between the estimated damped sinusoid source, s_2, and the true source versus filter length using the Cabrelli algorithm for input signals (a) $s_2 * h_1$, and (b) $s_2 * h_2$.

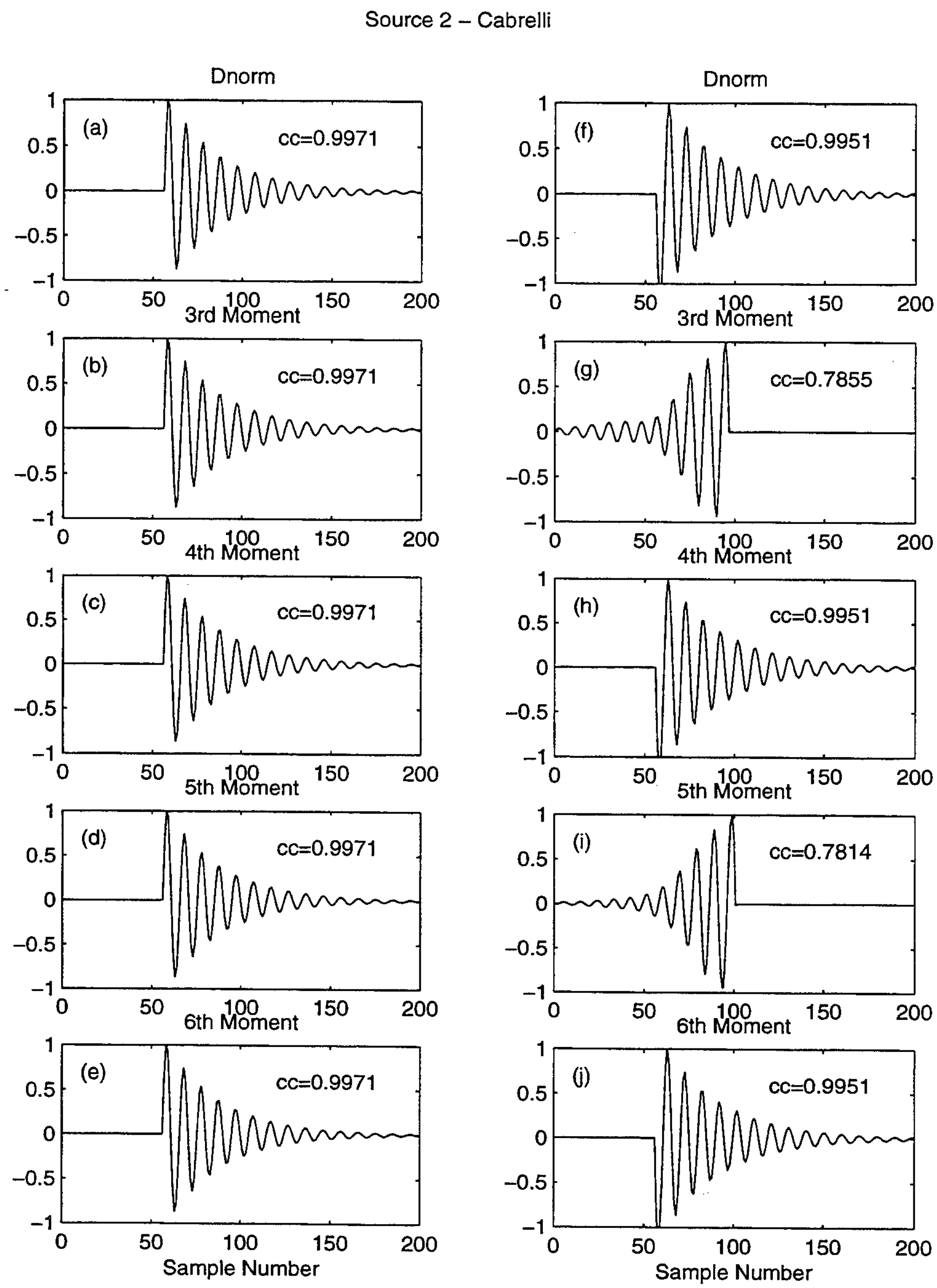

Figure 8. Best estimated source signals for input signals (a)-(e) $s_2 * h_1$, and (f)-(j) $s_2 * h_2$, resulting from the Cabrelli algorithm with various norms. The correlation coefficient (cc) between the source estimate and the true source is given within the plot.

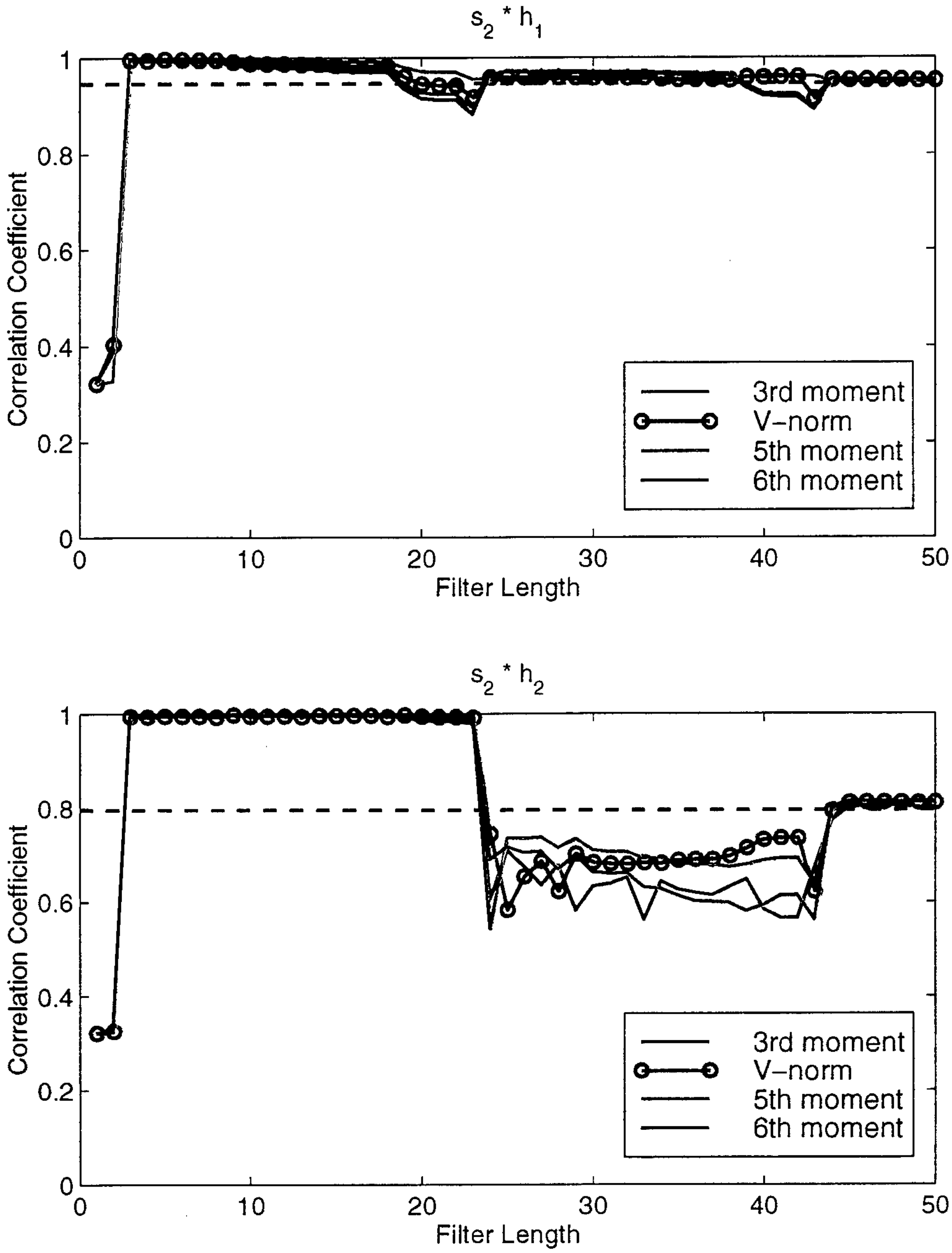

Figure 9. Correlation coefficients between the estimated damped sinusoid source, s_2, and the true source versus filter length using the Wiggins algorithm for input signals (a) $s_2 * h_1$, and (b) $s_2 * h_2$.

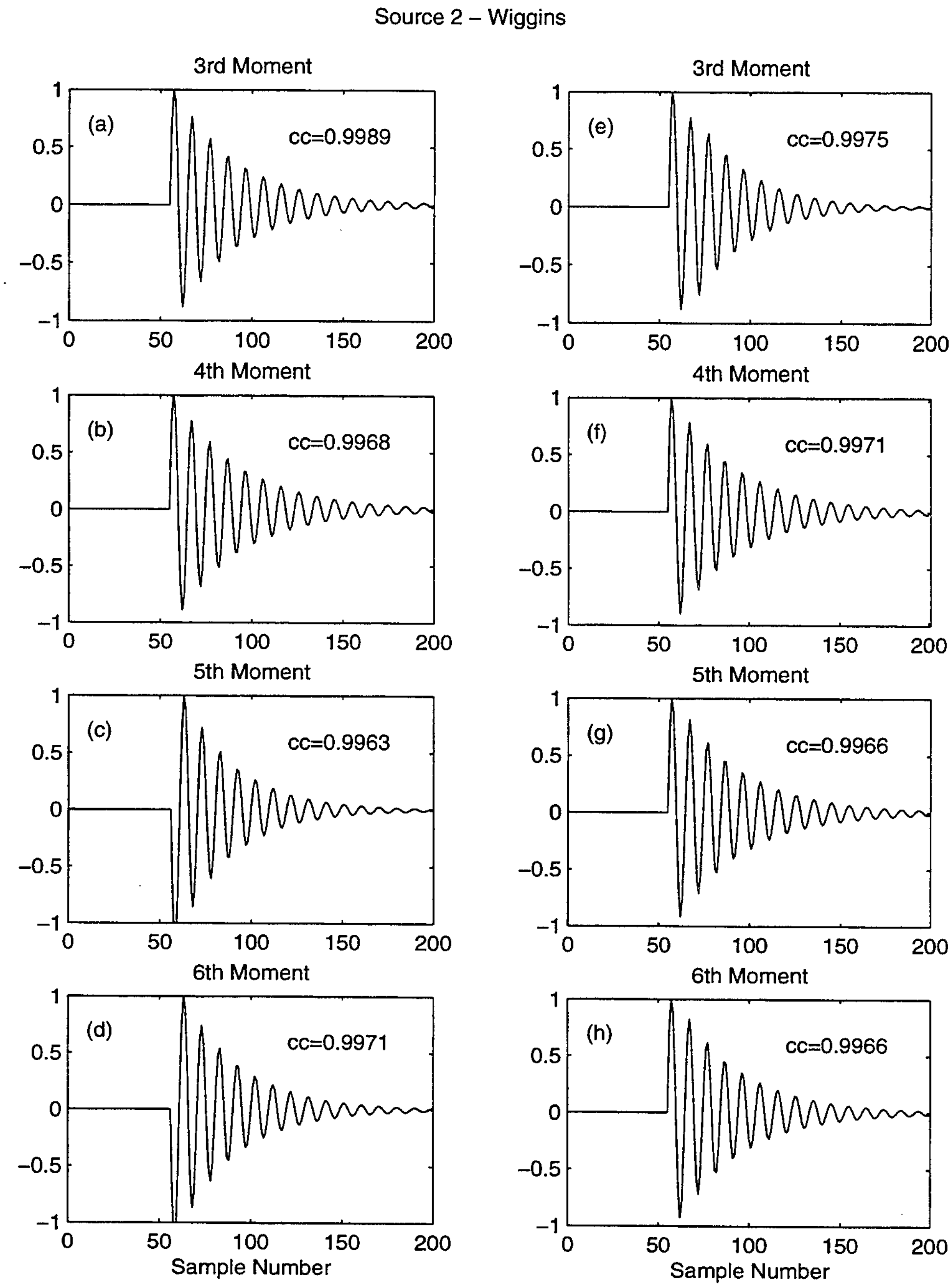

Figure 10. Best estimated source signals for input signals (a)-(d) $s_2 * h_1$, and (e)-(h) $s_2 * h_2$, resulting from the Wiggins algorithm with various norms. The correlation coefficient (cc) between the source estimate and the true source is given within the plot.

VI. Conclusions

From these results, it appears that there is generally little justification for using norms other than the D-norm for the Cabrelli algorithm, or the V-norm for the Wiggins algorithm. While the results using other norms are better in some cases, meaning that the best source estimate is more similar to the true source, or good source estimates are produced more consistently with varying filter length, no predictable pattern emerges to provide guidelines as to when which norm will work best in most cases. One could routinely use more than one norm for estimation, but the number of possible solutions then increases significantly, and unless one has an effective method of handling the large number of solutions generated by multiple norms and multiple filter lengths, there may be no improvement. The exception is an application in which the impulse response function is known to consist of all positive spikes, and the source is similar to the smooth, symmetric wavelet-type source (s_1) (what constitutes "similar" has not been determined). In this case, the 3rd and 5th order norms appear to work better in the Cabrelli algorithm than the D-norm or even order norms. Note that, excepting the cases in which the source estimate appears time-reversed, each norm and each algorithm produced a source estimate superior to the original input signal.

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Acknowledgments

This work was funded by the Office of Naval Research and the Naval Research Laboratory.