JPRS 83983

27 July 1983

USSR Report

ELECTRONICS AND ELECTRICAL ENGINEERING

No. 110

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CONTENTS

ANTENNAS AND PROPAGATION

UDC 621.396.96'06

ALLOWANCE FOR ANTENNA ERROR IN DETERMINING RADAR TARGET SCATTERING MATRIX

Kiev IZVESTIYA VYSSHIKH UCHEBNYKH ZAVEDENIY: RADIOELEKTRONIKA in Russian Vol 26 No 3, Mar 83 (manuscript received after revision 10 Mar 82) pp 74-75

[Article by B.A. Atayants and V.V. Yezerskiy]

[Text] The target scattering matrix [1]

$$\dot{S} = \begin{bmatrix} \dot{S}_{11} & \dot{S}_{12} \\ \dot{S}_{21} & \dot{S}_{22} \end{bmatrix} \tag{1}$$

in any polarization basis

$$\vec{e}_1 = \vec{e}(\psi_{pb}, \tau_{pb}), \quad \vec{e}_2 = \vec{e}(\psi_{pb} + \pi/2, -\tau_{pb}), \tag{2}$$

where ψ_{pb} and τ_{pb} -- basis angle of orientation and ellipticity, respectively -- is done with antennas having two orthogonal polarizations which coincide with the unit vectors of the basis (2). However, because of the finite accuracy with which antennas are fabricated their actual polarization differs from the required figure. It is important to investigate the connection between discrepancies in measurements and discrepancies in the polarization parameters of the antennas, and to find a method to allow for errors.

The voltage at the input of the receiving antenna can be found from the formula [2]

$$\dot{V} = \tilde{\dot{A}}\dot{S}\dot{B}, \tag{3}$$

where the $\tilde{}$ designates transposition, and the column matrices $\dot{A}$ and $\dot{B}$ describe the receiving and transmitting antennas. The elements of matrices

$\dot{A}$ and $\dot{B}$ are the projections of the radio wave formed by the antennas in the radiating mode onto the unit vectors $\vec{e}_1$ and $\vec{e}_2$ of polarization basis (2).

In the general case the errors in the antenna orientation and ellipticity angles differ for different polarizations. We shall designate the actual antenna polarizations (ψ'_A, τ'_A), (ψ''_A, τ''_A), (ψ'_B, τ'_B) and (ψ''_B, τ''_B), respectively, for the receiving and transmitting antennas. Then

$$\begin{aligned}\Delta\psi'_A = \psi'_A - \psi_{pb},\ \Delta\tau'_A = \tau'_A - \tau_{pb},\ \Delta\psi''_A = \psi''_A - \psi_{pb} - \pi/2,\quad \Delta\tau''_A = \tau''_A + \tau_{pb},\\ \Delta\psi'_B = \psi'_B - \psi_{pb},\ \Delta\tau'_B = \tau'_B - \tau_{pb},\ \Delta\psi''_B = \psi''_B - \psi_{pb} - \pi/2,\quad \Delta\tau''_B = \tau''_B + \tau_{pb}.\end{aligned} \tag{4}$$

Knowing the results of measuring the voltage at the output of the receiving antenna $\dot{V}_{11}$, $\dot{V}_{12}$, $\dot{V}_{22}$ for the three different antenna polarization combinations, we can obtain from (3) the following system of equations with respect to the unknown $\dot{S}_{11}$, $\dot{S}_{12}$, $\dot{S}_{22}$ (assuming $S_{12}=S_{21}$):

$$\begin{aligned}\dot{V}_{11} = \dot{a}'_1\dot{b}'_1\dot{S}_{11} + (\dot{a}'_1\dot{b}'_2 + \dot{a}'_2\dot{b}'_1)\,\dot{S}_{12} + \dot{a}'_2\dot{b}'_2\dot{S}_{22},\quad \dot{V}_{12} = \dot{a}'_1\dot{b}''_1\dot{S}_{11} + (\dot{a}'_1\dot{b}''_2 + \dot{a}'_2\dot{b}''_1)\,\dot{S}_{12} + \dot{a}'_2\dot{b}''_2\dot{S}_{22},\\ \dot{V}_{22} = \dot{a}''_1\dot{b}''_1\dot{S}_{11} + (\dot{a}''_1\dot{b}''_2 + \dot{a}''_2\dot{b}''_1)\,\dot{S}_{12} + \dot{a}''_2\dot{b}''_2\dot{S}_{22}.\end{aligned} \tag{5}$$

The coefficients of $\dot{S}_{11}$, $\dot{S}_{12}$ and $\dot{S}_{22}$ of this system can be found by using the unitary matrix obtained in [3] to calculate the transformations of the elements of column matrices A and B:

$$\begin{aligned}\dot{a}'_1 = \cos\Delta\psi'_A\cos\Delta\tau'_A + j\sin\Delta\psi'_A\sin(2\tau_{pb} + \Delta\tau'_A),\quad \dot{a}'_2 = \sin\Delta\psi'_A\cos(2\tau_{pb} + \Delta\tau'_A) -\\ - j\cos\Delta\psi'_A\sin\Delta\tau'_A,\quad \dot{a}''_1 = -\sin\Delta\psi''_A\cos(\Delta\tau''_A - 2\tau_{pb}) + j\cos\Delta\psi''_A\sin\Delta\tau''_A,\\ \dot{a}''_2 = \cos\Delta\psi''_A\cos\Delta\tau''_A + j\sin\Delta\psi''_A\sin(\Delta\tau''_A - 2\tau_{pb}).\end{aligned} \tag{6}$$

$\dot{b}'_1$, $\dot{b}'_2$, $\dot{b}''_1$, $\dot{b}''_2$ are described by analogy with (6), substituting the subscript B for A.

By solving (5) for the unknowns $\dot{S}_{11}$, $\dot{S}_{12}$, $\dot{S}_{22}$, we obtain the following in general form:

$$\dot{S}_{kn} = \dot{X}_{kn}\dot{V}_{11} + \dot{Y}_{kn}\dot{V}_{12} + \dot{Z}_{kn}\dot{V}_{22}, \qquad (7)$$

$$k, n = 1, 2,\ \dot{X}_{kn} = (-1)^{k+n}\dot{b}''_{3-k}\dot{b}''_{3-n}/[(\dot{b}'_1\dot{b}''_2 - \dot{b}''_1\dot{b}'_2)(\dot{a}'_1\dot{b}''_2 - \dot{a}'_2\dot{b}''_1)],$$

where

$$\dot{Y}_{kn} = (-1)^{k+n-1}\{\dot{b}'_{3-n}\dot{b}''_{3-k}/[(\dot{b}'_1\dot{b}''_2 - \dot{b}''_1\dot{b}'_2)(\dot{a}_1\dot{b}''_2 - \dot{a}'_2\dot{b}''_2)] + \dot{a}''_{3-n}\dot{a}'_{3-k}/[(\dot{a}'_1\dot{a}''_2 - \dot{a}''_1\dot{a}'_2) \times$$
$$\times (\dot{a}'_1\dot{b}''_2 - \dot{a}'_2\dot{b}''_1)]\}, \quad \dot{Z}_{kn} = (-1)^{k+n}\dot{a}'_{3-k}\dot{a}'_{3-n}/[(\dot{a}'_1\dot{a}''_2 - \dot{a}''_1\dot{a}'_2)(\dot{a}'_1\dot{b}''_2 - \dot{a}'_2\dot{b}''_1)]. \qquad (8)$$

These expressions are not valid when the determinant of system of equations (5) is zero, which can occur in one of these three situations:

$$\dot{a}'_1\dot{a}''_2 - \dot{a}'_2\dot{a}''_1 = 0, \qquad \dot{b}'_1\dot{b}''_2 - \dot{b}'_2\dot{b}''_1 = 0, \qquad \dot{a}'_1\dot{b}''_2 - \dot{a}'_2\dot{b}''_1 = 0. \qquad (9)$$

It is easy to demonstrate that one of these conditions can be satisfied in accordance with one of these practical cases: 1) $\tau'_A = \tau''_A = \pm 45°$, i.e., both polarizations of the receiving antenna are circular and coincide; 2) $\tau'_B = \tau''_B = \pm 45°$, i.e., both polarizations of the transmitting antenna are circular and coincide; 3) $\tau'_A = \tau''_B = \pm 45°$ or $\tau''_A = \tau'_B = \pm 45°$, i.e., $\dot{V}_{12}$ or $\dot{V}_{21}$ is measured with the receiving and transmitting antennas having coinciding circular polarizations.

It is clear that these situations contradict the original statement of the problem of measuring the scattering matrix by antennas with different polarizations, and indicate that two of the three equations in system (5) become linearly dependent. Therefore, it is impossible for conditions (9) to be satisfied in practice.

Expressions (7) and (8) thus make it possible to calculate the scattering matrix elements in any polarization basis $(\vec{e}_1, \vec{e}_2)$ by the results of three measurements of the complex voltages by antennas with polarizations which are different from those required.

Let us look at three cases which are of practical importance.

1. $\Delta\psi'_A = \Delta\psi''_A$, $\Delta\tau'_A = -\Delta\tau''_A$, $\Delta\psi'_B = \Delta\psi''_B$, $\Delta\tau'_B = -\Delta\tau''_B$, i.e., the antenna errors are the same for both polarizations and differ for the receiving

and transmitting antennas. It then follows from (5) that

$$\dot{a}'_1 = \dot{a}^{*}_{2}, \qquad \dot{a}'_2 = -\dot{a}^{*}_{1}, \qquad \dot{b}'_1 = \dot{b}^{*}_{2}, \qquad \dot{b}'_2 = -\dot{b}^{*}_{1}, \tag{10}$$

where * designates complex conjugation.

2. $\Delta\psi'_A = \Delta\psi'_B$, $\Delta\psi''_A = \Delta\psi''_B$, $\Delta\tau'_A = \Delta\tau'_B$, $\Delta\tau''_A = \Delta\tau''_B$, i.e., the same antenna is used for receiving and transmitting. Then

$$\dot{a}'_1 = \dot{b}'_1, \qquad \dot{a}'_2 = \dot{b}'_2, \qquad \dot{a}''_1 = \dot{b}''_1, \qquad \dot{a}''_2 = \dot{b}''_2. \tag{11}$$

3. $\Delta\psi'_A = \Delta\psi''_A = \Delta\psi'_B = \Delta\tau''_B$, $\Delta\tau'_A = -\Delta\tau''_A = \Delta\tau'_B = -\Delta\tau''_B$, i.e., the conditions of the first two cases are satisfied, which yields

$$\dot{a}'_1 = \dot{b}'_1 = \dot{a}^{*}_{2} = \dot{b}^{*}_{2}, \qquad \dot{a}'_2 = \dot{b}'_2 = -\dot{a}^{*}_{1} = -\dot{b}^{*}_{1}. \tag{12}$$

Consequently, the linear relationship (7) holds between the measurements of $\dot{V}_{11}$, $\dot{V}_{12}$, $\dot{V}_{22}$ and the scattering matrix elements $\dot{S}_{11}$, $\dot{S}_{12}$, $\dot{S}_{22}$ in the polarization basis $(\vec{e}_1, \vec{e}_2)$. The coefficients which enter into expressions (7) are in general functions of the eight complex numbers $\dot{a}'_1$, $\dot{a}'_2$, $\dot{a}''_1$, $\dot{a}''_2$ $\dot{b}'_1$, $\dot{b}'_2$, $\dot{b}''_1$, $\dot{b}''_2$ (8), and they are expressed through the antenna polarization parameter errors (6).

Expressions (8) can be simplified in certain cases by reducing the number of unknown variables to four (10), (11), or even to two (12).

Obviously, these findings allow the a priori known measurement antenna parameters ψ'_A, τ'_A, ψ''_A, τ''_A, ψ'_B, τ'_B, ψ''_B, τ''_B and the results of the three measurements of $\dot{V}_{11}$, $\dot{V}_{12}$ and $\dot{V}_{22}$ to calculate the elements of the scattering matrix in any polarization basis.

This analysis is valid for single-position radar location, i.e., when the equality $\dot{S}_{12} = \dot{S}_{21}$ is satisfied. Violation of this condition makes it necessary to make four measurements of $\dot{V}_{11}$, $\dot{V}_{12}$, $\dot{V}_{21}$ and $\dot{V}_{22}$ for the four combinations of polarizations of the receiving and transmitting antennas and, accoringly, to solve a system of four linear equations instead of (5).

The final expression for the sought $\dot{S}_{kn}$ will contain four terms, rather than the three in (7).

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CSO: 8144/1267

COMMUNICATIONS

UDC 621.391.8

CROSS SPECTRAL-CORRELATION PROCESSING OF SIGNALS IN DIFFERENT ORTHOGONAL BASES

Kiev IZVESTIYA VYSSHIKH UCHEBNYKH ZAVEDENIY: RADIOELEKTRONIKA in Russian Vol 26, No 3, Mar 83 (manuscript received after revision 28 Apr 82) pp 3-7

[Article by N.V. Bebikh and A.I. Denisov]

[Text] This study investigates aspects of cross spectral-correlation transformations in different orthogonal bases. A method is examined for computing the correlation function by the method of spectral-correlation transformation and of computing the spectra and correlation functions of certain signals; the structural diagram of a spectral-correlation converter is presented as well.

Systems of non-sinusoidal functions have recently been used effectively in modern radio electronics. These include systems of Walsh, Haar and other functions which are convenient for digital processing by computer, as well as systems of Laguerre, Hermite and Karunen-Loew functions for investigating transient attenuating processes, etc. The choice of a system of basis functions is determined by the mean square approximation error, the amount of computational labor involved, the noise tolerance of the system of basis functions and the ease of hardware implementation. In order to provide an operational analysis of signals under study in the most suitable orthogonal basis it is helpful to have the capability to switch quickly between different systems of basis functions.

The connection between spectral coefficients in two different systems of functions is demonstrated in general form in [1].

Let us represent the investigated signal by a Walsh series and define the Walsh-Fourier cross-transition matrix $[M_{WF}]$: $[M_{WF}]=1/N[\overline{S}_F]x[W]^T$, where $[\overline{S}_F]$ -- matrix of partial sums of trigonometric functions; $[W]^T$ -- transposed Walsh matrix, with dimension NxN, $N=2^n$, where n -- order of system of Walsh functions.

The elements of matrix $[\overline{S}_F]$ are determined by the following formulas: for the odd matrix rows

$$\bar{S}_{km} = \frac{1}{2\pi} \int_{2\pi(m-1)/N}^{2\pi m/N} \sin kx dx,$$

where k=1,2,..., N/2 -- number of odd row in matrix; for even matrix rows

$$\bar{S}_{lm} = \frac{1}{2\pi} \int_{2\pi(m-1)/N}^{2\pi m/N} \cos lx dx,$$

where k=1,2,..., N/2 -- number of odd row of matrix; ℓ=1,2,..., N/2 -- number of even row of matrix; m=1,2,..., N -- number of matrix column; $N=2^n$ -- number of digitization intervals.

The Fourier spectrum of the signal in question can then be computed by the formula

$$a_{Fj} = \sum_{\substack{i=1 \\ j=1}}^{N} a_{Wi} M_{ij},$$

where a_{Fj} -- spectral Fourier coefficient; a_{Wi} -- spectral Walsh coefficient; M_{ij} -- Walsh-Fourier cross-transition coefficients; or, in matrix form, $[A_F] = [M_{WF}] \times [A_W]$, where $[A_F]$ -- column vector of spectral Fourier coefficient; $[A_W]$ -- column vector of spectral Walsh coefficient; $[M_{WF}]$ -- Walsh-Fourier cross-transition matrix.

The number of operations required to make the cross transition between spectral Walsh and Fourier coefficients can be defined by the recursive formula

$$L_{WF} = 1 + \sum_{1}^{n} 2^{n-1} (2^{n-1} - 1),$$

where n=1,2,... -- order of system of Walsh functions; $N=2^n$ -- number of spectral coefficients.

The Walsh-Haar cross-transition matrix $[M_{WH}]$ and Harr-Fourier matrix $[M_{HF}]$ can be calculated analogously:

$$[M_{WH}] = 1/N\,[\bar{S}_H] \times [W]^T,\ [M_{HF}] = 1/N\,[\bar{S}_F] \times [H]^T,$$

where $[\bar{S}_H]$ -- matrix of partial sums of Harr functions; $[H]^T$ -- transpose of normalized Haar functions matrix, with dimension NxN.

The number of operations required for the Walsh-Haar cross transition can be calculated by the formula

$$L_{WH} = 1 + \sum_{1}^{n} 2^{n-1}(2^n - 1).$$

The number of operations for the Haar-Fourier cross transition is defined as $L_{HF}=2(L_{n-1}-1)+(2^{n-1}-1)(2^{n+1}-1)$, n=2,3..., for n=1, $L_{HF}=1$.

The total number of operations required to compute the spectral coefficients in Walsh, Fourier and Haar bases with the help of the cross-transition matrices $[M_{WF}]$ and $[M_{WH}]$ will be: $L_{tot_W}=L_1+L_2+L_3+L_4$, where $L_1=2^n$ -- computation of partial sums of signal in question; $L_2=n2^n$ -- calculation of Walsh coefficients with help of fast Walsh transform; $L_3=L_{WF}$ -- calculation of Fourier coefficients with help of Walsh-Fourier cross transition matrix; $L_4=L_{WH}$ -- calculation of Haar coefficients with help of Walsh-Haar transition matrix. For N=16 $L_{tot_W}=16+64+71+156=307$.

When a system of Haar functions is used as the initial system, the total number of calculations will be $L_{tot_H}=L_1+L_2+L_3+L_4$, where $L_1=2^n$ -- calculation of partial sums of signal in question; $L_2=2(2^n-1)$ -- calculation of Haar spectrum with help of fast Haar transform; $L_3=L_{HW}$ -- calculation of Walsh coefficients with help of matrix $[M_{HW}]$; $L_4=L_{HF}$ -- calculation of Fourier coefficients by matrix. For N=16 $L_{tot_H}=16+30+71+329=446$.

If the spectral coefficients are calculated in Walsh, Fourier or Haar bases without using cross transition matrices and with the help of fast

transforms, the total number of operations will be $L_{tot}=L_1+L_2+L_3+L_4$, where $L_1=2^n$ -- calculation of partial sums of signal in question; $L_2=n2^n$ -- calculation of Fourier coefficients with help of FFT algorithm (number of complex multiplications followed by addition and subtraction); $L_3=n2^n$ -- calculation of Walsh coefficients with help of fast Walsh transform (number of additions and subtractions); $L_4=2(2^n-1)$ -- calculation of Haar coefficients with help of fast Haar transform (number of additions and subtractions).

The use of cross transition matrices to calculate spectra in different bases reduces the amount of computation, with the greatest gain being achieved when a system of Walsh functions is employed as the initial basis system. When implementing the device in hardware, in order to determine the spectral coefficients in different orthogonal bases it is best (in terms of hardware costs) to determine the spectral coefficients in one basis (e.g., a Walsh function basis), and then to switch to other function bases with the help of cross-transition matrices, which will consist of weighted resistance units and adders in terms of hardware.

Figure 1 shows the structural diagram of the spectrum converter. The spectral coefficients from the output of Walsh spectrum analyzer 1 are input to the inputs of the weight coefficient units -- Fourier 3, Haar 4 and Laguerre 5, the outputs of which are connected to switch 7. A signal from control system 8 causes the switch to connect the proper weight coefficient unit to adder unit 9, which outputs the spectral coefficients in the proper orthogonal basis in parallel. Converter 10 is used to obtain the serial spectrum.

This device permits spectral-correlation conversion of signals. In order to do this, synthesized signal distribution unit 2 and multiplier unit 6 must also be used. The correlation function $K(\tau)$ in this case is computed as follows:

$$K(\tau)=\sum_{\substack{j=1\\m=0}}^{N} S_j S_{j+m},$$

where $m=0,1,2,\ldots, N-1$; $j=1,2,3,\ldots,N$; $\overline{S}_j$ -- value of synthesized signal in digitization interval; $\tau=m/N$ -- shift parameter.

Since Walsh functions are piecewise constant over the digitization interval 1/N, it is sufficient to determine the values of the correlation function

at m/N points, assuming the changes between those points to be linear [2]. Distribution unit 2 connects to the inputs of multipliers 6 the required values of the synthesized signal, which are known in each digitization interval, as a function of the value of the shift parameter τ. The order of the switching is assigned by converter control system 8. With this method of obtaining the correlation functions there is no need to provide a time offset τ between the multiplied signals (like in ordinary correlators, where, e.g. frequency-dependent delay lines are used for this). In the present case the time offset is assigned by simply changing the combination of pairs of factors from the values of the synthesized signal known in each digitization interval.

When implementing spectral-correlation transformations by computer, it is best to represent the expression for the correlation function as follows:

$$K_{xy}(\tau) = \frac{1}{T}\int_0^T x(t)\, y(t+\tau)\, dt = \frac{1}{T}\sum_{j=1}^{N} a_j \sum_{i=1}^{N} a_i \int_0^T \mathrm{Wal}(j, t)\, \mathrm{Wal}(i, t+\tau)\, dt,$$

where a_i -- spectral coefficients of expansion of signal x(t) into Walsh series; a_j -- spectral coefficients of expanding the signal y(t) into a Walsh series.

The internal integral represents the cross-correlation function of the actual Walsh functions $K_W(\tau)$, and is independent of the type of signals being investigated; therefore, it can be calculated in advance for different values of τ.

Then the expression for the correlation function can be obtained in this form:

$$K_{xy}(\tau) = \sum_{j=1}^{N} a_j \sum_{i=1}^{N} a_i K_W(\tau).$$

If we have the pre-calculated matrices $[K_W(\tau)]$ for various values of τ and the column vectors of the spectral coefficients of the signals in question $[A_i]$, $[A_j]$, we can calculate the correlation function by the matrix method in accordance with this algorithm:

1. Calculation of column vector of inner sum: $[\sigma_i(\tau)]=[K_W(\tau_i)]x[A_i]$.

2. Formation of auxiliary matrix $[\sigma(\tau)]$

$$[\sigma(\tau)] = \begin{bmatrix} \sigma_1(\tau_1) & \sigma_2(\tau_1) & \dots \sigma_j(\tau_1) \\ \sigma_1(\tau_2) & \sigma_2(\tau_2) & \dots \sigma_j(\tau_2) \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \sigma_1(\tau_m) & \sigma_2(\tau_m) & \dots \sigma_j(\tau_m) \end{bmatrix},$$

where j=1,2,...,N -- number of matrix row; m=1,2,...,N -- number of matrix column.

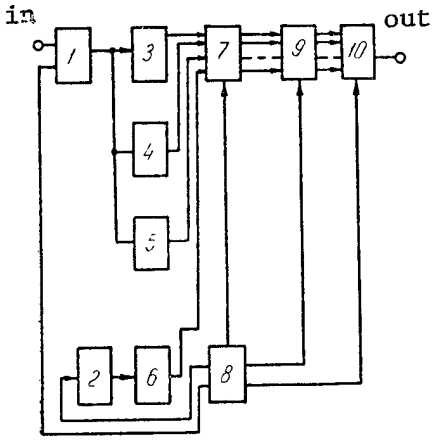

Fig. 1

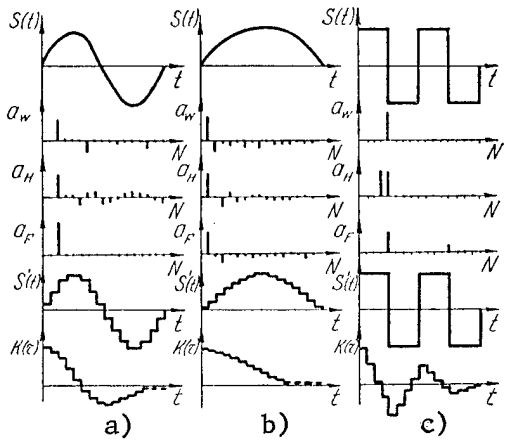

Fig. 2

3. Determination of correlation function matrix

$$[K(\tau)] = [\sigma(\tau)] \times [A_j].$$

In the present case the matrices $[K_W(\tau)]$ are calculated directly during computation of the correlation function; the intermediate results of computing $[K_W(\tau)]$ for different values of τ are not stored in memory, but are used immediately to calculate the current values of the inner sums of $[\sigma_i(\tau)]$, and each subsequent matrix $[K_W(\tau)]$ is written in the location of the preceding one, which reduces the computer memory size.

A program for calculating the correlation function by the spectral-correlation transformation method has been written in "Assembler-BASS" language and has been implemented on an "Elektronika NTs03-T" microcomputer for N=16. A standard matrix multiplication program and matrix row calculation subprogram were employed.

The results of calculating the spectral-correlation characteristics of some functions in Walsh, Haar and Fourier bases are shown in the form of time diagrams in Fig. 2, where S(t) -- investigated signal; a_W, a_H, a_F -- Walsh, Haar and Fourier spectra, respectively; S'(t) -- synthesized signal; $K(\tau)$ -- correlation function.

Analysis of the findings makes it possible to assess the effectiveness of using a particular function basis for signal processing (by minimum mean square error, number of non-zero expansion coefficients, correlation coefficient, etc.,) i.e., it permits adaptation to the signals being investigated. This increases the effectiveness of quick analysis of complex signals and of synthesizing optimal correcting and control effects in automatic information processing and control systems.

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6900
CSO: 8144/1267

UDC 621.391.2

POWER SPECTRUM COMPUTATION ALGORITHMS FOR PULSED RANDOM PROCESSES

Kiev IZVESITYA VYSSHIKH UCHEBNYKH ZAVEDENIY: RADIOELEKTRONIKA in Russian Vol 26 No 3, Mar 83 (manuscript received after revision 14 Jun 82) pp 7-12

[Article by T.G. Pletneva, G.N. Rozorinov and S.D. Eydel'man]

[Text] This study solves the problem of computing the power spectrum of a pulsed random process with determinate timing intervals in finite closed form. A computer program is described for calculating the power spectra of signals controlled by a regular Markov chain. A decomposition method is proposed which can be used to compute the spectra of digital signals formed from packets of varying length.

Two-level digital signals with an assigned timing frequency are employed widely in modern communications, hydro-acoustic and computer equipment [1].

When deciding which of the existing digital signals is most suitable for effective transmission through a given propagation medium, it is important to be able to find the power spectrum of the signal. Knowledge of the power spectrum makes it possible to determine the amount of influence between bits within symbols.

The limited bandwidth of a real data transmission channel causes the occurrence of high frequency and low frequency signal distortions. These distortions can be reduced by selecting signals which have the desired spectrum shape [2].

Two-level digital signals, which are mappings of fixed-length binary sequences (block codes), have attracted a great deal of attention recently in digital data transmission tasks. Each symbol in the input binary sequence is transmitted by a pulse with duration T, or by a group of pulses (a "packet"). In this case the sequence of states S_n of the digital signal is modeled well by a process controlled by a finite ergodic Markov chain [3]. Signals formed from "packets" of different lengths are also important for practical purposes. The study [3] solves the problem of computing the power spectra of signals formed from "packets" of the same length which are used, e.g., in digital magnetic recording technology.

The present study proposes a method for solving the problem of calculating the power spectrum when the "packets" forming the signal are of different lengths.

Formulas for calculating power spectrum. The pulsed random process in question consists of "packets" with duration rT, and the forms of the "packets" are defined ($e_\mu=(\varepsilon_{1\mu}, \varepsilon_{2\mu},\ldots, \varepsilon_{q\mu})$). The process is controlled by a finite ergodic Markov chain with period d and transition probability matrix P. We shall designate $\vec{A}=(A_1,A_2,\ldots, A_q)$ the probability vector characterizing the stationary distribution of the Markov chain in question; $A_0 = \lim\limits_{n\to\infty} P^{dn}$, $\tilde{P}^s=P^{ds}-A_0$, $s=1,2,\ldots$; $\tilde{P}^0=I$ -- identity matrix; $\hat{A}=(A_i\delta_{ij})_{i,j=1,\ldots,q}$; δ_{ij} -- Kronecker's symbol; $\tilde{m}^{(s,\nu)}_{\mu\nu}=\tilde{P}^s P^\nu e^\mu \hat{A} e_\nu$; $\gamma=0,1,\ldots,$ $d-1$; $m^{(\gamma)}_{\mu\nu}=A_0P^\gamma e_\mu \hat{A} e_\nu$; $m^{(0)}_{\mu\nu}=Ae_\nu \hat{A} e_\nu$; $m=e_\mu \hat{A} e_\nu$; $a=\int\limits_0^T e^{-i\omega t}dt$; r -- number of pulses with duration T in "packet"; q -- number of "packets".

The formula derived in [3] for the averaged power spectrum of a random pulsed signal with "packets" of the same length controlled by an arbitrary finite ergodic Markov chain can be rewritten in the form

$$P(\omega)=\frac{4|a_1|^2}{rT}\sum_{\mu,\nu=1}^{r}\left\{\sum_{\gamma=0}^{d-1}P^\gamma\left[\mathrm{Re}\left(\left\{[I-(P^d-A_0)e^{idr\omega T}]^{-1}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.+\frac{\pi}{drT}\sum_{h=-\infty}^{\infty}\delta(\omega-2\pi h/drT)A_0\right\}e^{i(\gamma r+\mu-\nu)\omega T}\right)+\right.$$
$$\left.+\frac{\sin(dr/2-\gamma r-\mu+\nu)\omega T}{2\sin(dr\omega T/2)}A_0\right]-[I\cos(\mu-\nu)\omega T]/2\Big\}m_{\mu\nu}, \qquad (1)$$

where $[I-(P^d-A_0)e^{idr\omega T}]^{-1}$ -- the inverse matrix of $I-(P^d-A_0)e^{idr\omega T}$.

Formula (1) provides the solution to the problem of calculating the power spectrum in finite closed form. It is useful in that it allows standard

programs for calculating the inverse matrix to be employed in computing the power spectrum.

The next section examines a numerical algorithm for calculating the spectrum for the case of a random pulsed signal controlled by a regular Markov chain, which is the most important case in practical terms. In this case the formula appears as

$$P(\omega) = \frac{4|a|^2}{rT}\left\{\frac{1}{2}\sum_{\mu,\nu=1}^{r} m_{\mu\nu}\cos(\mu-\nu)\omega T + \right.$$
$$+\sum_{\mu,\nu=1}^{r}\sum_{s=1}^{\infty}\tilde{m}_{\mu\nu}^{(s)}\cos(sr+\mu-\nu)\omega T +$$
$$\left.+\sum_{\mu,\nu=1}^{r} m_{\mu\nu}^{(0)}\cos(\mu-\nu)\omega T\left[-\frac{1}{2}+\frac{\pi}{rT}\sum_{h=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi h}{rT}\right)\right]\right\}. \qquad (2)$$

We recall that the discrete component in (2) is lacking if $\sum_{j=1}^{q} A_j\varepsilon_{j\mu}=0,\ \forall\mu.$

In particular, this is the case when the matrix P is doubly stochastic and $\sum_{j=1}^{q}\varepsilon_{j\mu}=0,\ \forall\mu.$

Algorithm for calculating power spectra of signals controlled by regular Markov chain. Figure 1 shows the flowchart of the numerical algorithm for computing the spectrum in accordance with formula (2). The following notation is employed:

$$Q_1=\frac{1}{2}\sum_{\mu,\nu=1}^{r} m_{\mu\nu}\cos(\mu-\nu)\omega T;\quad Q_2=\sum_{\mu,\nu=1}^{r}\sum_{s=1}^{\infty}\tilde{m}_{\mu\nu}^{(s)}\cos(sr+\mu-\nu)\omega T;$$

$$Q_3=\sum_{\mu,\nu=1}^{r} m_{\mu\nu}^{(0)}\cos(\mu-\nu)\omega T\left[-\frac{1}{2}+\frac{\pi}{rT}\sum_{h=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi h}{rT}\right)\right].$$

The following is essential in the use of this algorithm. By using corollary (4.1.5.) from [4], we can demonstrate that the maximum number of required iterations S_m needed to ensure the required accuracy Δ of the spectrum computations is determined by the formula

$$S_M = N[\ln \Delta / \ln(1 - 2\varepsilon) + 1], \tag{3}$$

where N -- power of matrix of transitional probabilities P in which all elements are positive; ε -- smallest element of matrix P^N (actually, the number of iterations is always much smaller).

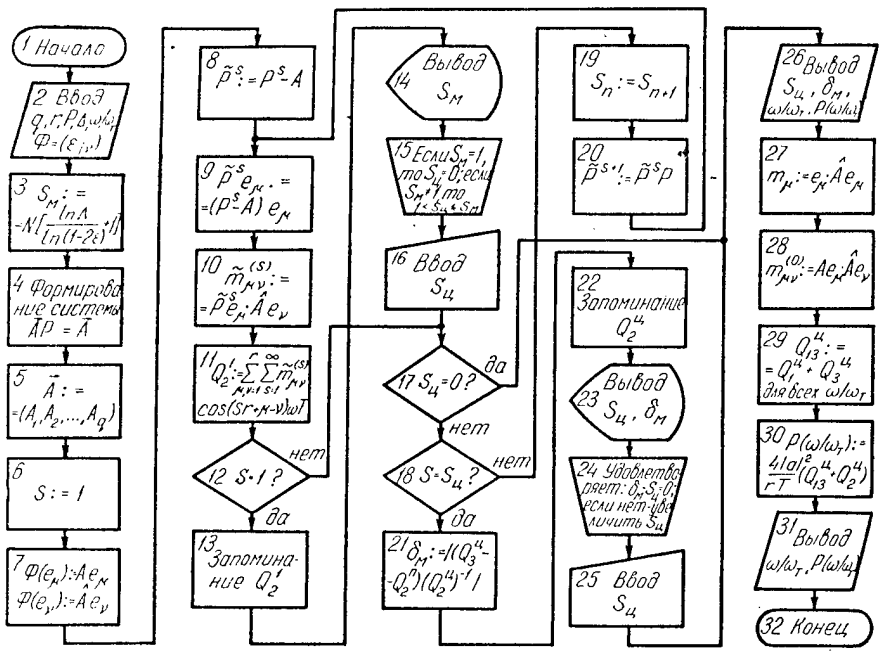

Fig. 1

Key:

1 -- start
2 -- input [quantity]
4 -- formation of system
13 -- store [quantity]
14 -- output [quantity]
15 -- if S_M=1, S_{ts}=0; if $S_M \neq 1$ $1<S_{ts}<S_M$
16 -- input S_{ts}
22 -- store [quantity]
23 -- output [quantity]

(continued)

Key to Fig. 1 (continued)

24 -- Is it the case that $\delta_M - S_{ts} = 0$; if not, increase S_{ts}

25 -- input S_{ts}

26 -- output [quantity]

29 -- $Q_{13}^{ts} := Q_1^{ts} + Q_3$ for all ω/ω_t

31 -- output [quantity]

32 -- end

After S_M is determined, actions are taken to find the final probability vector $\vec{A}$, i.e., the probabilistic solution of the algebraic system $\vec{A}P = \vec{A}$ is found (e.g., by the Gauss method with selection of the main element). We note that if P is a doubly stochastic matrix, then $\vec{A} = (1,1,\ldots,1)/q$. S=1 is then assigned and the second term Q_2 of formula (2) is computed. If $S_M = S = 1$, Q_2 is stored and then Q_1 and Q_3 are calculated, followed by the spectrum $P(\omega/\omega_T)$ for all given ω/ω_T.

If $S_M \neq 1$, an exhaustive search $1 < S_{ts} < S_m$ is made. $\tilde{P}^{S+1} = \tilde{P}^S \tilde{P}$ is first computed, and then the procedure described above is repeated cyclically.

If the signal in question has a large transition probability matrix, S_{ts} may be large. In most practical cases the quantity S_{ts} can be bounded. Then the algorithm in question can be used to calculate the maximum error δ_M for a given S_{ts} in accordance with the formula

$$\delta_M = |(Q_2^{ts} - Q_2^p)/Q_2^{ts}|, \tag{4}$$

where Q_2^p -- value of Q_2 in previous iteration. If the error is not excessive, S_{ts} is increased no further.

In the below example of calculating the spectrum by the algorithm described above, the null members of the matrix P include those elements whose values do not exceed 10^{-4}. For $S_{ts}=1$ the calculation took 1.33 minutes, and for $S_{ts}=60$ -- 12.06 minutes.

Decomposition method in computing power spectra of signals formed from "packets" of different lengths. The above referred to the case in which the "packets" from which the signal was formed were of the same length rT. A whole group of signals which are promising for use in digital magnetic recording and which are obtained by means of group recoding of the input

binary information in which a particular number of zeros or ones may not occur in a row [5,6] do not have this property. These signals include the (0,1)-code or Gabor code [5], the (4,9)-code or Franaszek code, the M^2FM [6] and ChMP-1 signals (Fig. 2, a-d).

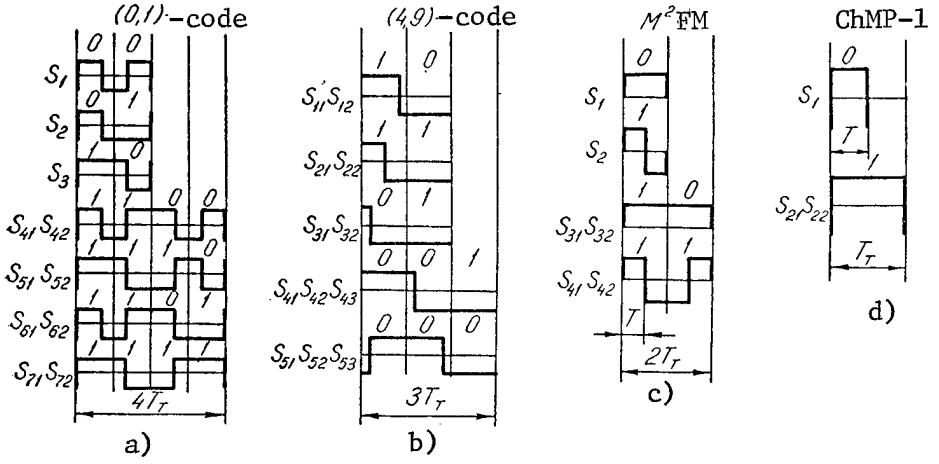

Fig. 2

The power spectra of these signals are successfully calculated by the decomposition method, which allows these signals to be put into correspondence with a certain Markov chain which controls the decomposed random process. The possibility of this decomposition follows from the fact that the causal connections which determine the structure of the process for the class of pulsed random processes with determinate timing intervals in question correspond precisely to the probabilistic connection between its elements.

The "packets" from which the signals in question are formed are thus divided into a definite number of "subpackets" of equal length, the probability of occurrence of which as part of its own "packet" is unity. The dimensions of the transition probability matrices, of course, are greater.

The application of the decomposition method is illustrated by the example of computing the averaged power spectrum of the signal whose structure is explained in Fig. 2b.

Example of calculating power spectrum. The (4,9)-code (Fig. 2b) is a promising one for use in high density digital magnetic recording. This signal has a rather complex structure. By using the decomposition method a (4,9)-code can be represented by 24 alternating "packets". The "packet" length is T_T, r=6 (T_T=6T), q=24, and the transition probability matrix

$$P=\left[\begin{array}{c|c} P_1 & P_2 \\ \hline P_2 & P_1 \end{array}\right].$$

The matrices e_1-e_6, P_1, P_2 are shown in Fig. 3.

$$\begin{aligned}
e_1 &= (1\ -1\ 1\ -1\ 1\ -1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ -1\ 1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ 1) \\
e_2 &= (1\ -1\ 1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ -1\ 1\ -1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ 1) \\
e_3 &= (1\ -1\ 1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ -1\ 1\ -1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ 1) \\
e_4 &= (1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ 1) \\
e_5 &= (1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ 1) \\
e_6 &= (-1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ 1\ -1\ 1\ 1\ -1\ 1\ 1)
\end{aligned}$$

$$P_1 = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/8 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/8 & 0 & 0
\end{bmatrix}
\qquad
P_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Fig. 3

It is easy to note that a transition, e.g., from "subpacket" S_{11} to "subpacket" S_{12}, has a probability of occurrence of unity, since both of them are part of the "packet" S_1, while the probability of a transition

from "subpacket" S_{12} (which is part of "packet" S_1) to "subpacket" S_{51} (which is part of "packet" S_5) is 1/8. We are easily convinced that the pulsed random process which is obtained is controlled by a regular Markov chain. In order to use formula (1) on a computer, the values of Δ and ω/ω_T must be added to the data above. We shall assign $\Delta=10^{-3}$, and $\omega/\omega_T=0-6$ in steps of 0.1.

The results of calculating the averaged power spectra are shown as plots in Fig. 4, where the values of $P(\omega)$, multiplied by the coefficient $\pi^2/2T_T$ are laid off along the ordinate, and the values of the normalized frequency ω/ω_T are laid off along the abscissa.

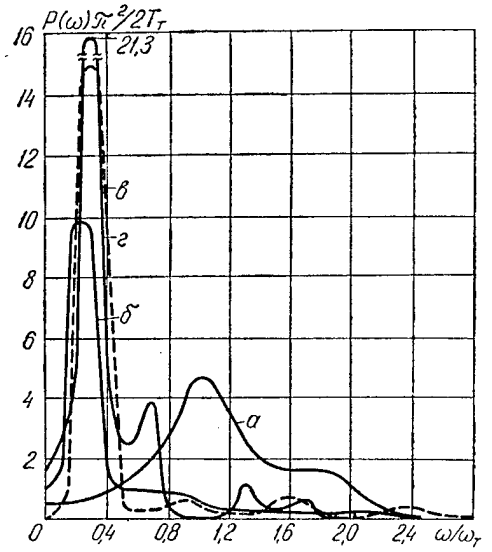

Fig. 4

Discussion of results. Conclusions. It is apparent from the plots in Fig. 4 that the maximum spectral energy of the (0,1)-code (curve a) lies near the normalized frequency $\omega/\omega_T=1$, i.e., in the region of the center frequencies of the passband of a magnetic record-play back channel. The constant component in the spectrum of the signal is small. These characteristics of the spectrum allow the (0,1)-code to be recommended for use when the recording density does not exceed 256 bits/mm.

The (4,9)-code (Fig. 4b) has a very useful spectral shape from the viewpoint of increasing recording density. The maximum spectral energy of the (4,9)-code lies in the vicinities of the normalized frequency $\omega/\omega_T=0.25$. However, the spectrum of the (4,9)-code contains a constant component twice as large as that in the previous case, which results in substantial distortions to the signal as it passes through the record-play back channel.

The M^2FM signal (Fig. 4c) differs from the others in Fig. 4 in that there is no constant component at all. The maximum value of the spectrum of the M^2FM signal occurs at the normalized frequency $\omega/\omega_T=0.3$. These characteristics of the M^2FM signal spectrum, which are helpful for increasing recording density, have resulted in its increasing use, especially for use with floppy disks.

It should be noted that the spectrum of an M^2FM signal as calculated by the author's method and the spectrum of the same signal cited in [7] are exactly the same.

Curve d in Fig. 4 reflects the spectrum of a ChMP-1 signal. The substantial level of the constant and extreme low-frequency components of the ChMP-1 signal, which are not passed by the magnetic record-play back channel, cause significant distortions. The smallest time intervals of a ChMP-1 signal which corresponds to data zeros are practically not reproducible. This makes it much harder to use the ChMP-1 and similar signals in high density digital magnetic recording.

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CSO: 8144/1267

UDC 621.391.14:519.2

USE OF DECORRELATIVE PROPERTIES OF DISCRETE SPECTRAL TRANSFORMATIONS IN MULTI-ALTERNATIVE RECOGNITION OF SIGNALS IN PRESENCE OF CORRELATED NOISE

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[Article by A.I. Rog and A.A. Sirota]

[Text] A method is presented for estimating the error probability in signal recognition against the background of correlated noise with incomplete description of the correlation connections of the features. The proposed method is used to substantiate the possibility of replacing optimal decorrelation transformation by partially decorrelative discrete spectral transformations.

The problems which arise in a number of practical applications, such as automatic recognition of pulsed signals of various forms: $S_i(t)=m_i(t)+N(t)$, $i=\overline{1,L}$, where $m_i(t)$ -- determinate signal component; $n(t)$ -- normally distributed noise with correlation function $\sigma^2R(\tau)$, involve using decision rules of the form [1,2]

$$k \Rightarrow \min [(\vec{x} - \vec{m}_i)^T M^{-1} (\vec{x} - \vec{m}_i)] \Rightarrow \vec{x} \in O_k. \quad (1)$$

In (1) $\vec{x}=(S(t_0),\ldots, S(t_{N-1}))$ -- vector of amplitude (feature) samples of signal $S(t)$ obtained in the time interval $[0,T]$; $\vec{m}_i$ -- vector of mathematical expectancies of features; M -- covariation matrix of features, $M=||a_{k\ell}||$, $a_{k\ell}=\sigma^2R(|k-\ell|T/N)$.

The use of optimal decision rules with structure (1) involves definite difficulties which arise in estimating the matrix M and which also require a very great deal of time to implement the matrix transformations in (1) during the decision making stage. It becomes necessary to examine a special class of quasi-optimal recognition algorithms based on a certain

"disregard" of the correlation between features, which in the present case is manifested in that a matrix K which is slightly different from the actual matrix M is employed in (1).

Let us now examine the losses which occur when this class of quasi-optimal algorithms is employed. If the error probability of pairwise separation of signals for optimal algorithm (1) is defined by the familiar formular [2]

$$P_{ij}^0 = \Phi(\alpha_{ij}),\ \alpha_{ij} = 0{,}5\sqrt{(\vec{m}_i - \vec{m}_j)^T M^{-1}(\vec{m}_i - \vec{m}_j)},\quad \Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}dt,$$

we find this probability for the quasi-optimal algorithm from the distribution parameters of the logarithm of the likelihood ratio of the two signals [2] with matrix K substituted instead of matrix M:

$$\ln[l(\vec{x})] = \vec{x}^T K^{-1}(\vec{m}_i - \vec{m}_j) - \frac{1}{2}(\vec{m}_i + \vec{m}_j)^T K^{-1}(\vec{m}_i - \vec{m}_j).$$

It can be shown that

$$M_1\{[\ln l(\vec{x})]/O_k\} = \pm 0{,}5(\vec{m}_i - \vec{m}_j)^T K^{-1}(\vec{m}_i - \vec{m}_j);\qquad k = i, j;$$
$$D\{[\ln l(\vec{x})]/O_k\} = (\vec{m}_i - \vec{m}_j)^T K^{-1} M K^{-1}(\vec{m}_i - \vec{m}_j),$$

and the probability of error in pairwise separation in this case is

$$P_{ij}^k = \Phi(\beta_{ij}),\qquad \beta_{ij} = \frac{(\vec{m}_i - \vec{m}_j)^T K^{-1}(\vec{m}_i - \vec{m}_j)}{2\sqrt{(\vec{m}_i - \vec{m}_j)^T K^{-1} M K^{-1}(\vec{m}_i - \vec{m}_j)}}.$$

The relationship between the probabilities P_{ij}^0 and P_{ij}^k is determined by the relationship between the quantities α_{ij} and β_{ij} which can be represented in the form below by introducing the difference vector $\vec{a}=\vec{m}_i-\vec{m}_j$, the vector $\vec{b}=K^{-1/2}\vec{a}$ and the matrix $B=K^{-1/2}MK^{-1/2}$:

$$\alpha_{ij} = 0{,}5\sqrt{\vec{a}^T K^{-\frac{1}{2}} K^{\frac{1}{2}} M^{-1} K^{\frac{1}{2}} K^{-\frac{1}{2}}\vec{a}} = 0{,}5\sqrt{\vec{b}^T B^{-1}\vec{b}},$$
$$\beta_{ij} = \vec{a}^T K^{-\frac{1}{2}} K^{-\frac{1}{2}}\vec{a}/2\sqrt{\vec{a}^T K^{-\frac{1}{2}} K^{-\frac{1}{2}} M K^{-\frac{1}{2}} K^{-\frac{1}{2}}\vec{a}} = \vec{b}^T\vec{b}/2\sqrt{\vec{b}^T B\vec{b}}.$$

In order to estimate the losses occurring with the use of this class of quasi-optimal algorithms we examine the quantity $\Delta=\alpha_{ij}-\beta_{ij}<\alpha_{ij}[1-\min(\beta_{ij}/\alpha_{ij})]$ for a given error probability for optimal algorithm P^0_{ij} and corresponding value of α_{ij}.

Using Bekkenbakh and Kassel's inequality [3], we obtain

$$1 \leqslant \alpha_{ij}^2/\beta_{ij}^2 = (\vec{b}^T B\vec{b})(\vec{b}^T B^{-1}\vec{b})/(\vec{b}^T\vec{b})^2 \leqslant \frac{(\lambda_0+\lambda_{N-1})^2}{4\lambda_0\lambda_{N-1}}, \tag{2}$$

where λ_0, λ_{N-1} -- minimum and maximum eigenvalues of the matrix $B=K^{-1/2}MK^{-1/2}$. We obtain from (2) the following formula for the losses occurring with incomplete allowance for feature correlation in the quasi-optimal recognition algorithms:

$$\Delta P = \max P_{ij}^k - \Phi(\alpha_{ij}), \quad \max P_{ij}^k = \Phi\left(2\alpha_{ij}\frac{\sqrt{\lambda_0\lambda_{N-1}}}{\lambda_0+\lambda_{N-1}}\right). \tag{3}$$

Thus, for an algorithm which does not allow fully for sample correlation (K=I, where I -- identity matrix)

$$k \Rightarrow \min[(\vec{x}-\vec{m}_i)^T(\vec{x}-\vec{m}_i)] \Rightarrow \vec{x}\in O_k, \tag{4}$$

we can write the following expression for the maximum possible recognition error probability (P^0_{ij}=const):

$$P_{ij}^{k1} = \Phi\left(\alpha_{ij}\frac{2\sqrt{\varkappa_0\varkappa_{N-1}}}{\varkappa_0+\varkappa_{N-1}}\right), \qquad \alpha_{ij} = \Phi^{-1}(P_{ij}^0), \tag{5}$$

where $\varkappa_0$, $\varkappa_{N-1}$ -- eigenvalues of matrix M.

The difficulties involved in using information about the correlation of the initial features -- signal amplitude samples -- can be avoided by using the familiar decorrelation method. An optimal decorrelative transformation

is the expansion with respect to discrete eigenfunctions of the matrix M using a Karunen-Loew expansion [2]. The optimal decision rule in the space of the new uncorrelated features assigned by the vector $\vec{y}$ appears as

$$k \Rightarrow \min [(\vec{y} - \vec{n}_i)^T \Omega^{-1} (\vec{y} - \vec{n}_i)] \Rightarrow \vec{y} \in O_k, \tag{6}$$

where $\vec{y}=V\vec{x}$, $\Omega=VMV^T$, $\vec{n}_i=V\vec{m}_i$, V -- optimal decorrelative transformation (ODT) matrix. The main shortcoming of the ODT is the difficulty of finding the expansion functions and the excessive amount of time required to execute the ODT. It is therefore better to try to replace the ODT with some other orthogonal transformation, which may have slightly poorer decorrelative properties but which allows the transformation procedure to be speeded up and simplified significantly. Such transformations include Fourier, Walsh and Haar discrete spectral transformations (DST), for which "fast" transformation methods have been worked out. The idea of the ODT-DST replacement is based on certain facts of the similarity of the structure of the bases of orthogonal functions of the optimal decorrelative transformation and of spectral transformations assigned on a discrete finite interval [4,5]. For $N \to \infty$, for example, [5] notes that the Fourier transform approaches the Karunen-Loew transform asymptotically.

The problem of estimating the effectiveness of the use of an "approximate" decorrelative transformation instead of the ODT is equivalent to the problem of estimating the losses which occur with an incomplete description of the correlation connections of the signal samples in quasi-optimal recognition algorithms. Actually, replacing the ODT with some DST assigned by the matrix H actually consists of using in place of optimal algorithm (6) an analogous structure of a quasi-optimal algorithm implemented in the space of the spectral coefficients of the DST -- $\vec{z}$:

$$k \Rightarrow \min [(\vec{z} - \vec{l}_i)^T E^{-1} (\vec{z} - \vec{l}_i)] \Rightarrow \vec{z} \in O_k, \tag{7}$$

where $\vec{z}=H\vec{x}$, $\vec{l}_i=H\vec{m}_i$, E=diagP, $P=HMH^T$ -- covariation matrix of spectral coefficients.

Algorithm (7) does not make complete allowance for the correlation between the spectral coefficients, and is quasi-optimal with respect to an algorithm with structure (1); however, it is realized in a spectral coefficient space in which the covariation matrix of these coefficients P is replaced

by diagonal dispersion matrix E. We can therefore estimate the losses which occur when quasi-optimal algorithm (7) is used instead of optimal algorithms (1) and (6) on the basis of expression (3), which is fairly general. The expression for the maximum possible recognition error probability for algorithm (7) for a given value of P_{ij}^0 -- the error probability of algorithm (6) -- which determines the effectiveness of the ODT-DST substitution, looks like

$$P_{ij}^{k2} = \Phi\left(\alpha_{ij}\frac{2\sqrt{\chi_0\chi_{N-1}}}{\chi_0+\chi_{N-1}}\right), \quad \alpha_{ij} = \Phi^{-1}(P_{ij}^0), \tag{8}$$

where χ_0, χ_{N-1} -- eigenvalues of matrix $E^{-1/2}PE^{-1/2}$.

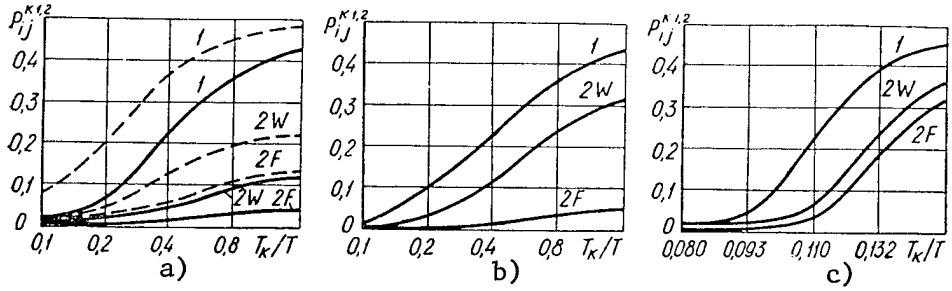

Fig. 1

Formula (8) was used as the basis for investigating the effectiveness of using Fourier and Walsh DST in quasi-optimal signal recognition algorithms. Figure 1 shows the probabilities of pairwise signal recognition errors for different quasi-optimal algorithms as a function of the ratio of the noise correlation time to the signal duration. For comparison, the error probability P_{ij}^{k1} was calculated simultaneously for quasi-optimal algorithm (4), which does not make complete allowance for sample correlation. Figure 1 a, b and c show P_{ij}^{k1} and P_{ij}^{k2} as a function of the ratio of the noise correlation time to the

signal duration -- T_k/T for different values of N and types of correlation function $R(\tau)$ for a value of $P_{ij}^0=0.01$ (the dotted line in Fig. 1a corresponds to N=32, and the solid line to N=16 for $R(\tau)=e^{-\alpha|\tau|}$; In Fig. 1 b and c for $R(\tau)=e^{-\alpha|\tau|}\cos\omega\tau$ and $R(\tau)=e^{-\alpha\tau^2}$, respectively). It follows from these relationships (1 -- probability P_{ij}^{k1}; 2F, 2W -- probability P_{ij}^{k2} computed for Fourier and Walsh transformations, respectively), that the losses which occur with the use of quasi-optimal algorithms with structure (7) in Fourier and Walsh coefficient space are significantly smaller than for quasi-optimal algorithms (4) realized in the sample space. The decorrelative properties of the DST are particularly strong for small values of T_k/T, when the quantity P_{ij}^{k2} is practically the same as P_{ij}^0. As follows, e.g., from Fig. 1a, by comparison with algorithm (6), based on the use of the ODT, signal recognition accuracy for noise described by an exponential correlation function is no more than 3-5% lower when quasi-optimal algorithms (7) with Fourier transformation are employed, and no more than 5-10% lower for Walsh transformation for values of $T_k/T\leq 0.4$.

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CSO: 8144/1267

UDC 621.391.81

APPROXIMATION OF FOURIER TRANSFORM KERNEL WITH HELP OF TWO-LEVEL FUNCTIONS

Kiev IZVESTIYA VYSSHIKH UCHEBNYKH ZAVEDENIY: RADIOELEKTRONIKA in Russian
Vol 26 No 3, Mar 83 (manuscript received after revision 5 Apr 82) pp 57-59

[Article by V.B. Dmitriyev-Zdorov and M.N. Surkov]

[Text] Spectrum analysis devices can be simplified significantly by approximating the transformation kernel with piecewise-constant functions with a small number of levels [1]. It is particularly attractive to do the approximation with the help of a periodic two-level signal: this makes it possible to increase dynamic range significantly and to simplify the analyzer structure. The spectrum of such a periodic two-level signal must contain a first harmonic, after which the series of harmonics (2,3,...,p) is lacking, and the amplitudes of the higher harmonics (p+1, p+2,...,) are arbitrary.

The objective of the present study is to synthesize a periodic digital signal whose spectrum does not contain harmonics 2, 3, ..., p. We shall represent the two-level signal which has the required spectral structure in the form

$$y(t)=\sum_{l=1}^{N}\xi_l h\{t-(l-1)t_u\},\qquad t\in(0,T),$$

where $h(\tau)=\begin{cases}1, \tau\in(0,t_i)\\ 0, \text{ for other } \tau\end{cases}$ -- describes the shape of rectangular pulses with duration t_i, the spacing of which in the periodic sequence of pulses y(t) with period $T=Nt_i$ is determined by the values of the coefficients

$$\xi_l=\{0,1\}. \tag{1}$$

We represent the complex amplitude of the ith harmonic as

$$C_i = \frac{1}{T}\int_0^T y(t)\,e^{-j2\pi it/N}dt = \frac{2}{N}\left(\frac{\sin(\pi i/N)}{\pi i/N}\right)\sum_{l=1}^{N}\xi_l e^{-j2\pi il/N}.$$

For $i<N$ the condition $C_i=0$ is equivalent to the requirement

$$\sum_{l=1}^{N}\xi_l e^{-j2\pi il/N} = 0. \qquad (2)$$

The simplest signals which correspond to the solutions of (2) are groups of m pulses spaced apart by N/(mi) pulse positions, i.e., by 1/m part of the period of the ith harmonic component (Fig. 1). The requirement of (2) is satisfied for the signal shown in Fig. 1, since the left part of (2) can be viewed as the sum of m vectors on a complex plane (Fig. 2) with norm of unity and arguments $2\pi n/m$, n=1,2,..., m (which can be obtained by assuming $l=nN/(mi)$ in (2)).

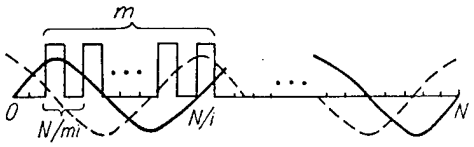

Fig. 1

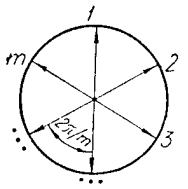

Fig. 2

We note that the condition $C_i=0$ is equivalent to having coefficients a_i and b_i of zero in the expansion of y(t) into a real Fourier series, since $2C_i = a_i - jb_i$ [2].

The other solutions of (2) can be obtained by linear combination of the simple solutions; however, (1) must not be violated when this is done.

Let U_i be a group of m pulses corresponding to the solutions of equation (2) for a given i. In a periodic signal this group repeats every N positions. In the spectrum $S_i(2\pi f_0 k)$ of such a periodic signal $U_i(t)$ the amplitude of the ith harmonic is zero: $S_i(2\pi f_0 i)=0$, where $f_0=1/T$ -- frequency of first harmonic of signal y(t). The convolution spectrum W(t) of such signals U_i, i=2,...,p

$$W(t) = U_2(t) * U_3(t) * \dots * U_p(t) \tag{3}$$

is equal to the product of the spectra of these signals $S_w(2\pi f_0 k) = \prod_{i=2}^{p} S_i(2\pi f_0 k)$

and, consequently, the amplitudes of the harmonic components i=2,...,p in it are zero. The convolution operation in (3) is the discrete convolution of periodic pulse sequences of the type: $A(j) = \sum_{i=1}^{N} B(i)\, C(i-j), \quad j = 1, \dots, N.$

The signal W(t) is also periodic, with period $T=Nt_i$. Obviously, the number of positions N within a period of this signal is a multiple of all factors of the type $m_i\ell$, i=2,...,p.

In general terms, this is the algorithm for constructing the sought signal W(t). It should be noted that the presence in the convolution (3) of a certain group U_i, as will be shown later, makes the amplitude of the harmonic numbered i equal to zero, as well as a series of higher harmonics. This makes it possible to reduce the number of groups in the right part of the convolution (3) and, consequently, to reduce significantly the number of positions N in the period of the signal W(t), which is extremely important for practical purposes.

Let us now examine the spectral structure of a T-periodic sequence of groups of m pulses spaced $t_0=T/(im)$ apart. Finding the frequencies of the harmonics which are lacking in the spectrum of this sequence involves solving this equation:

$$F(f)(X^{m-1} + X^{m-2} + \dots + X + 1) = 0, \quad \text{where } X=e^{j2\pi fT/(im)}$$

and corresponds to a delay operator by T/(im), while F(f) is determined by the shape of the pulses.

Obviously, the equation

$$X^{m-1}+X^{m-2}+\ldots+X+1=(X^m-1)/(X-1)=0, \tag{4}$$

is a special case of (2) and has the solutions $X_k=e^{j2\pi k/m}$, k=1,2,...,m-1. There corresponds to each of the solutions of (4) a series of frequencies which are found from the formula $X_k=X=e^{j2\pi fT/(im)}$. However, $X_k=e^{j2\pi k/m}$; consequently, $2\pi fT/(im)=2\pi k/m+2\pi r$, r=0, ±1, ±2,..., whence

$$f=(k+rm)\,i/T,\quad k=1,2\ldots,m-1,\ r=0,\pm1,\pm2,\ldots \tag{5}$$

We note that the factor (k+rm) in the right part of (5) can take on any integer values except for integer multiples of m. Consequently, the presence of a group U_i consisting of m pulses in the convolution (3) leads to compensation in the resulting signal W(t) of all of the harmonics whose numbers are multiples of i, except for those whose frequencies are represented in the form $f_\ell=f_0 im\ell$, $\ell=0,1,2,\ldots$. In order to construct a signal W(t) in which the amplitudes 2,3,...,p of the harmonics are zero it is necessary to perform a convolution of the type (3) which includes groups U_i of m pulses such that i runs through all of the primes and all of the integer powers of m not exceeding p. Let I(p,m) denote the set consisting of these values of i. As was noted earlier, the number of positions N within a period of the signal W(t) must be a multiple of all factors of the type im. Therefore, the minimum N is defined as N=mLCM {I(p,m)}, where LCM -- least common multiple.

As m increases the number of groups U_i in convolution (3) becomes smaller. However, the resultant signal W(t) remains a two-level signal only for m=2 and m=3.

The highest suppressed frequency $f_h=f_0p$ is in principle unbounded. However, for large p(p>10) the required number of positions N in a signal period increases immeasurably.

Table 1 shows the minimum N=N(p) for the case in which the signal $W_p(t)$ is synthesized by convoluting simple two- and three-pulse signals

(for $m=m_2=2$ and $m=m_3=3$) and q -- number of pulses in period of $W_p(t)$.

TABLE 1

p	m_2/m_3	N_2/N_3	q_2/q_3	p	m_2/m_3	N_2/N_3	q_2/q_3
2	2/3	4/6	2/3	9	—/3	1680/1890	64/243
3	2/3	12/18	4/9	10	—/—	1680/1890	64/243
4	2/—	24/18	8/9	11	2/3	18480/20790	128/729
5	2/3	120/90	16/27	12	—/—	18480/20790	128/729
6	—/—	120/90	16/27	13	2/3	240240/270270	256/2187
7	2/3	840/630	32/81	14	—/—	240240/270270	256/2187
8	2/—	1680/630	64/81	15	—/—	240240/270270	256/2187
				16	2/—	480480/270270	512/2187

The class of two-level signals $W_p(t)$ can be used for spectrum analysis of signals occupying a limited frequency band $f_{min}<f<f_{max}$. It can be shown that there is no methodical analytical error if $p>f_{max}/f_{min}-1$, which confirms the advisability of adapting the synthesized signal $W_p(t)$ to the analyzed process. One feature of this approach, in contrast to universal algorithms (specifically the FFT [3,4] where the transformation kernel is not approximated) is the fact that by increasing the number of discrete samples of the analyzed signal (increasing the digitization frequency) it is possible to eliminate the multiplication operation completely. In order to obtain the expansion coefficients of the analyzed signal it is sufficient to add the values of the samples taken at the instance corresponding to non-zero levels of the digital signal $W_p(t)$. In calculating different expansion coefficients of the signal in question, the samples can be added independently and in parallel.

Increased speeds can be expected in analyzing signals with relatively narrow bandwidths, especially when the code word lengths in the computer are long. It should also be pointed out that it is possible in principle to filter the analyzed signal first in order to represent it as the sum of narrowband components, and then to apply the analysis algorithm in question.

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CSO: 8144/1267

UDC 621.391.27

EFFECTIVENESS OF DIGITAL TRANSMISSION EMPLOYING MODIFIED WAGNER METHOD TO CORRECT SINGLE ERRORS

Kiev IZVESTIYA VYSSHIKH UCHEBNYKH ZAVEDENIY: RADIOELEKTRONIKA in Russian Vol 26 No 3, Mar 83 (manuscript received after revision 23 Sep 82) pp 70-72

[Article by R.E. Gut, M.Ya. Lesman and A.S. Shutov]

[Text] Various decoding procedures which allow for the reliability of the decisions made about the signal elements transmitted are employed extensively in digital transmission systems. For transmission over radio channels these procedures are especially convenient when multi-channel modems are used (such as the Kineplex, MS [expansion not given], etc. [1]). In contrast to the approach usually taken [2], it is best to judge the symbol reliability according to the estimates of the signal/noise ratio provided by meters which are specially added to each of the modem channels.

The present study solves the problem of estimating the noise tolerance of parallel reception of signals coded with an (M, M-I) code and decoded by the Wagner method. The indicators which determine noise tolerance are the probabilities of correct reception of a code combination and a symbol in an arbitrary phased channel. Channel fading is assumed to be statistically uniform and independent.

It can be demonstrated that the expressions for the mean probabilities of correct reception of a code combination Q_1 and of a symbol in an arbitrary jth channel Q_2 in the case in question appear as

$$Q_1 = (1-\bar{p})^M + M\int_0^\infty\int_0^\infty p(h)\,f(S,h)\,dh\left\{\int_0^\infty [1-p(h)]\,F(S\mid h)\,dh\right\}^{M-1} dS, \qquad (1)$$

where $\bar{p}$ -- mean error probability during reception of elementary signal in any of M statistical uniform channels; p(h) -- current error probability in receiving elementary signal as function of actual signal/noise ratio; M -- number of symbols in code word (number of parallel channels); f(h) -- density function of signal/noise ratios; f(S,h) -- combined density function of estimate of signal/noise ratio and actual value of signal/noise

ratio; F(S|h) -- conditional distribution function of signal/noise ratio estimate assuming that the actual signal/noise ratio is h^2;

$$Q_2 = (1-\bar{p}) - \frac{1}{2}\int_0^\infty f(S)\left\{F^{M-1}(S) - \left[F(S) - 2\int_0^\infty p(h) f(h) F(S \mid h)\, dh\right]^{M-1}\right\} dS + \tag{2}$$

$$+ \int_0^\infty F^{M-1}(S) \int_0^\infty p(h) f(S, h)\, dh dS,$$

where f(S) and F(S) -- unconditional density function and distribution function of estimate of current value of signal/noise ratio, respectively.

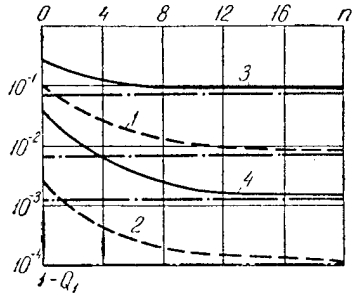

Fig. 1

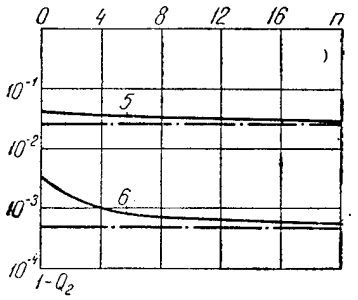

Fig. 2

Let us now specify the formulas derived above for the case of optimal incoherent reception of signals employing single phase-differential keying [2]. We shall assume that the channel fading is described by a Rayleigh distribution, and that the receiving location employs the devices for measuring the current signal/noise ratio which are described in [3], for which

$$f(S \mid h) = [1/\Gamma(\nu)]\,(4\nu h^2)^{\nu} S^{\nu-1} \exp(-4\nu h^2 S), \tag{3}$$

where 2ν -- size of sample used to form estimate of signal/noise ratio.

By substituting (3) in (1) and (2), we obtain the following analytical formulas:

$$Q_1 = \left[\frac{1+2h_0^2}{2(1+h_0^2)}\right]^M + \frac{\nu M}{2}\int_0^1 \frac{x^{\nu M}dx}{[1+h_0^2(1-x)]^{\nu+1}}\left[1-\frac{1}{2(1+h_0^2)[1+h_0^2(1-x)]^{\nu}}\right]^{M-1}; \tag{4}$$

$$Q_2 = \frac{1+2h_0^2}{2(1+h_0^2)} - \frac{1}{2M} + \frac{\nu}{2}\int_0^1 x^{\nu M-1}\left[1-\frac{1}{(1+h_0^2)[1+h_0^2(1-x)]^{\nu}}\right]^{M-1}dx + \\ + \frac{\nu}{2}\int_0^1 \frac{x^{\nu M-1}dx}{[1+h_0^2(1-x)]^{\nu+1}}. \tag{5}$$

Calculations were done in accordance with (4) and (5), some of the results of which are shown in Figs. 1 and 2. Figure 1 shows $(1-Q_1)$ during reception of a code combination as a function of the sample size ($n=2\nu$) for M=2 (dotted lines) and M=6 (solid line) and for two values of h_0^2=10 (curves 1 and 3) and h_0^2=100 (curves 2 and 4). Figure 2 shows the same for $(1-Q_2)$ with M=6 and h_0^2=10 (curve 5) and h_0^2=100 (curve 6). We note that when m=2 the probabilities Q_1 and Q_2 are the same. Furthermore, the straight dot-and-dash lines in Figures 1 and 2 show the values of the corresponding error probabilities during code combination and symbol reception which are achieved in a hypothetical system belonging to the class in question in which precise information is available about the actual signal/noise ratios in each channel.

Analysis of the findings permits the following conclusions:

1. The use of the modified Wagner procedure (considering the signal/noise ratio meters) allows the error probability for code combination and symbol reception in an arbitrary channel to be reduced by almost an order of magnitude, even in statistically uniform channels. This procedure should

provide even greater effectiveness when one of the channels is affected by spectrally concentrated noise.

2. The mean error probabilities for code combination ($1-Q_1$) and symbol ($1-Q_2$) reception in an arbitrary channel are practically the same as the corresponding figures for the hypothetical system with a sample size of $n=2_{\nu}=15-20$.

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