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Reliability Assessment Test (RAT) Program Life Data Analysis Methods

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Two life data analysis methods, which will be employed in the Global Positioning System Block IIR Rubidium Atomic Frequency Standard Assessment Test Program will be discussed. The analysis methods will provide information regarding the population characteristics.

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PURPOSE

The purpose of this report is to discuss two life data analysis methods which will be employed in the Global Positioning System (GPS) Block IIR Rubidium Atomic Frequency Standard (RAFS), Reliability Assessment Test program. The intent here is not to perform analysis of the currently available RAFS life data but to describe recommendations for the analysis methodology. Methods for parameter modeling or analysis of the RAFS performance (accuracy, stability, or drift) will not be considered. The procedures proposed suggest a method for inferring the expected useful life of the Block IIR RAFS. It is expected that there will be enhancements to, or alternative analysis methods proposed as Reliability Assessment Test (RAT) RAFS lifetime data are accumulated. In addition, the analysis results could be used to support RAFS risk analysis. Risk analyses are the calculations that predict present and future risk. Present risk is the expected number of failures from the current population of RAFS, while future risk is a prediction of the number of RAFS failures over some future period of time. Predictions could be made for common or multiple failure modes. This will be possible if the appropriate life distribution and its parameters can be established and that the Block IIR RAFS becomes the primary space borne atomic reference in the GPS satellite constellation.

RAT LIFE DATA ANALYSIS

An objective of the RAFS RAT is to perform lifetime data analysis of the Block IIR RAFS. The results are to be used to develop estimates of the RAFS mean life based on actual life time observations. Lifetime is defined as the period from the beginning of on-orbit operations to the time where a particular failure mode has caused the clock to be considered unfit for use. Unfitness for use does not require a catastrophic failure. Degradation in performance below an acceptable level will be identified as a failure. The RAT's period of performance is only three years. Therefore, it is expected that there will be no, or at best, very few failures observed. In the case where there are no failures it will be impossible to estimate the life distribution parameters. In the case of few failures, parameter estimates would be suspect and subject to considerable uncertainty. The precision or accuracy of the estimates are directly related to the number of life times (actual failures) observed. It is anticipated that actual failure data will be limited. Therefore, the approaches are based on a no or few failure situation. The approaches are designed to establish lower bound confidence levels, rather than point estimates of the RAFS's mean-life (θ) or characteristic life (η). The lower bound confidence level is the probability

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that a given parameter lies above a lower bound. One point should be made regarding the precision of the lower bounds and a specified confidence level; they are inversely related. Figures (1) to (5) illustrate this. The RAFS life data analysis methods are based on procedures where the times-to-failures are Exponential (one parameter) and Weibull (two parameter) distributed. The procedures assume that all RAFS operating times (both the ground test and in orbit) will be considered part of the analysis program.

LIFE TIME ANALYSIS PROBABILITY DISTRIBUTIONS

Exponential:

The Exponential distribution is the most commonly used distribution in time-to-failure analysis of electronic systems. Some of the reasons for its popularity are:

1. Simplicity; data analysis methods are simpler than many other distributions.
2. It has been sanctified by years of research, usage, and technical publications.
3. Most government contracts specify the use of Military Handbook 217 which is based on the Exponential distribution.

Weibull:

The Weibull distribution is also a widely used distribution for life testing applications. The Weibull distribution describes the life characteristics of non-repairable parts and components. Depending on its shape (β) and scale (η) parameters, it can fit many sets of lifetime data. Some contemporary reliability and life data analysis authorities advocate the use of the Weibull over the Exponential. The use of the Weibull is gaining in popularity partly because of computer statistical analysis software.

WEIBULL DISTRIBUTION CHARACTERISTICS

Most people in the defense industry are familiar with Exponential life data applications. However, many are unfamiliar with use of the Weibull distribution or its applications in life data analysis. The Block IIR prime contractor (LMMC) has requested that Weibull analysis be considered as an adjunct to any analysis based on Exponential distribution.

There are two approaches, probability plotting (graphical) and statistical, for performing Weibull life data analysis. Probability plotting is the technique predominately used because of its simplicity and applicability to a situation where only scant data is available. Additionally, software is readily available for doing probability plotting of Weibull distributed data. Figure 1 is a Weibull probability plot using Weibull analysis software. The abscissa is the order times to failure and the ordinate is $-\ln[1-F(t)]$, where F(t) is the probability of occurrence. The data analyzed was derived from data known to be Weibull distributed.

Statistical analysis technique followed development of the graphical technique. The use of statistical procedures to estimate distribution parameters adds precision to the analysis and removes the subjectivity which occurs when reading probability plots. There are some drawbacks with the statistical analysis approach. It requires a much broader appreciation of mathematical statistics and probably won't provide much insight into what the RAT is trying to accomplish. As a result, statistical analysis techniques will not be initially considered, but may be used as failure data is accumulated.

As previously stated, the Weibull distribution is extremely flexible and analysis can be performed with small data sets. Inference can be made from an analysis of a few data points. The following information can be obtained directly from a probability plot:

1. The estimates of the shape parameter β or sometimes called the slope.
2. The estimates of the population characteristic life (η) of which 63.2% of the population will have a life less than or equal to.
3. The percentage of the population that is expected to fail by some time (t_i).
4. The Weibull mean (or Mean-Time-To-Failure (MTTF)) can be estimated based on the following functional relationship between the Weibull mean, shape, and scale parameters, $MTTF=\eta[\Gamma(1+1/\beta)]$.

In figure 1 the characteristic life can be read directly from the abscissa, at the point, were -ln[1-F(t)] equals one (1) on the ordinate intersects with the fitted line. With a little bit of algebra the probability of occurrence, F(t) can be computed as $1-e^{-1}$ or .632. To estimate what percentage (x) of a population is predicted to have a life less than or equal to some time (t) take the value (y) on the ordinate, which is equal to -ln[1-x]. Then read the life time on the abscissa where (y) intersects the fitted line. Of course, the reverse of this computation can be performed based on some specified life time and solving for F(t). Beta can be determined by taking the slope of the fitted line. However, the software will do this and provide estimates of (η) and (β) in the analysis summary.

THE EXPONENTIALLY DISTRIBUTED RAFS LIFE DATA ANALYSIS METHODOLOGY

The point estimates of the RAFS's MTTF will not be established but rather, instead, a lower confidence bound on the MTTF will be. This approach is based on the work by two statistical researchers Benjamin Epstein and Milton Sobel and is described in their landmark paper "Life Testing." Epstein and Sobel showed that if the time-to-failure (t) is exponentially distributed the random variable $\frac{2r\theta_{est}}{\theta_{true}}$ is chi-squared distributed. θ_{est} is the MTTF based on a sample, while θ_{true} is the MTTF of the population. α is the level of significance. With the probability statement being

$$P\left[\chi^2_{1-\frac{\alpha}{2},2r} \leq \frac{2r\theta_{est}}{\theta_{true}} \leq \chi^2_{\frac{\alpha}{2},2r}\right] = 1-\alpha$$

and using some algebra the lower one-sided confidence bound for a time censor life test is

$P\left[\frac{2T_a}{\chi^2_{\alpha,2r+2}} \leq \theta\right] = 1-\alpha$. r equals the number of observed failures and setting θ equals to

the mission required MTTF in years. The required total accumulated operating time (T_a) at the 100%(1-α) lower confidence bound can be computed as MTTF*.5 $\chi^2_{\alpha,\ 2(r+1)}$. The degrees of freedom for a time censor test (test terminated at a specific time) is 2(r+1). The r+1 rather than r degrees of freedom are based on the imminent failure assumption. For example, with a specified MTTF of 7.5 years and assuming zero failures, the total test time required for a 80% lower confidence bound would be 7.5*.5$\chi^2_{.2,\,2}$ or 12.07078 years total accumulated operating time. Additionally, using the equation for computing T_a and the specified MTTF, the level of confidence can be determined on the lower bound given the life data. For example, at the end of three years we have the following: three units with three years; one unit has two years and one unit has one year for a total of 12 years (3*3+2+1). Then $\frac{2T_a}{7.5} \leq \chi^2_{\alpha,\,2} = 3.2$. 3.2 on the chi-squared tables at 2 degrees of freedom is an α of .2 or a 1-α of .8, which is approximately 80% confidence lower bound on the MTTF of 7.5 years. Figure 2 provides a plot of the confidence level verses MTTF estimates. Notice the inverse relationship. The estimates of the MTTF decreases as the confidence level increases. Figure 3 provides plots of the total accumulated operating time required to establish the one-sided lower bound % confidence for different selected MTTFs. Note the direct relationship. For a given MTTF, as the confidence level increases the required total accumulated operating time increases. One of the benefits of this approach is its simplicity and straightforwardness. Computations are only a function of total accumulate operating time.

There are some drawbacks associated with the use of the exponential distribution. One drawback is that the inference procedures lack robustness. The decision to propose a Exponentially distributed time-to-failure analysis method was based on conclusions resulting from various technical journals. The journals revealed that while there is some disagreement as to whether or not the assumption of exponentiality of time-to-failure for electronics component may be questionable, all authors basically agreed that time-to-failure for systems tend to be exponentially distributed.

WEIBULL DISTRIBUTED RAFS LIFE-DATA ANALYSIS METHODOLOGY

The proposed analysis procedures, based on the Weibull distribution, were developed by Z. Huang and A. Porter of the Rockwell International, Rocketdyne Division. The analysis method was published in the paper "Lower Bound On Reliability for Weibull Distribution When Shape Parameter Is Not Estimated Accurately". The method provides techniques for performing life-data analysis with no or limited life-time (failure) data. The consequences of life-time analysis under these situations were already commented on. Traditionally, under such conditions the analyst assumes β is known and η and other quantities are estimated from the data. This approach would lead to a lot of questions and would be difficult to justify. An alternative method developed by Huang and Porter attempts to overcome the drawbacks of the "Assumed (β) Method". The approach provides methods where under general conditions, reliability and allowable life limits (tolerance limit) will have unique global minima over a range of β. The use of the allowable life limit lower bound method is proposed for RAFS life time data analysis. The method demonstrates that for a given reliability (assumed to be known and constant) the life limit experiences a minimum value as a function of β. The life limit reaches a minimum at some value of β. The method is based on the following statistical fact. If the time to failure (t) is Weibull distributed then t^β is exponential distributed with the MTTF parameter $\theta = \eta^\beta$. To calculate a chi-squared lower confidence limit use

$$P\left[\frac{2\sum_{i=1}^{n} t_i^\beta}{\eta^\beta} \le \chi_{\alpha,2r+2}\right] = 1-\alpha$$ and an estimate of the characteristic life

η is $\left(\frac{\sum_{i=1}^{n} t_i^\beta}{.5\chi_{\alpha,2r+2}}\right)^{\frac{1}{\beta}}$. t_i is the i-th life time, 1-α is the confidence level, and n is the number of observations. Put this result into the Weibull estimated reliability (R), equation

$$R(\beta) = e^{\left(-\frac{T^\beta}{\eta^\beta}\right)} = e^{\left(-\frac{(5\chi^2_{\alpha,2r+2})T^\beta}{\sum_{i=1}^{n} t_i^\beta}\right)}$$ and solve for $T = \left(\frac{-\ln(R)\sum_{i=1}^{n} t_i^\beta}{.5\chi_{\alpha,2r+2}}\right)^{\frac{1}{\beta}}$. The result is the Allowable Life Limit Lower Bound.

The use of the Huang/Porter Allowable Life Limit Lower Bound method is conditioned on the following inequality $k < \frac{\chi^2_{\alpha,\,2r+r}}{-\ln(R)} < n$. k is the number of observations which are equal to the maximum of all of the observations (i.e. $t_1 = t_2 = t_3 = \ldots\ldots t_n = t_{max}$). This condition will be violated at least initially because all the operating times will be approximately the same at least for the first three units (ground test and first flight units). Based on the method's condition, figure 4 shows how the confidence level, number of observations, and the value of (k) are directly related.

Figure 5 provides plots illustrating the results obtained from analysis using the Huang/Porter method for the Allowable Life Limit Lower Bounds for a constant reliability of 80%. Five different confidence levels (50% to 90%) are provided to illustrate the impact different confidence levels will have on the analysis. The lower bound on the allowable life limit is inversely related to the confidence level. The plots show that for a given reliability and confidence level, the device could be operated for at least the minimum value (T) without failure. The worst-case values where β equals β_o are provided in the chart table. The data used for the plots was from an expanded Huang and Porter data set.

A concern with this method was that it assumes all test units will be subject to a common failure mode. In reality, the RAFS will probably exhibit competing failure modes. Huang in his paper

"Conditional Reliability Lower Bound for Weibull Distribution Without Estimating Shape Parameter" showed that the method is applicable to competing failure modes with the same shape parameter (β) but different scale parameters (η_i).

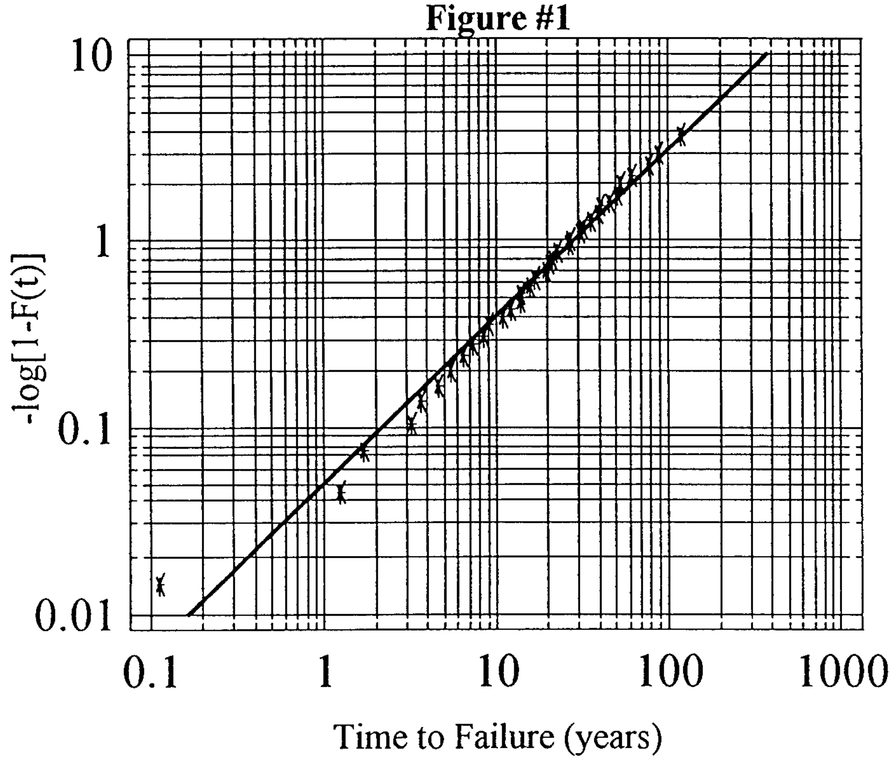
Competing failure modes is a situation where a unit can fail from m different independent failure modes. For example, suppose that a unit exhibits k modes of failure ($m_1, m_2, \ldots m_k$) and that the following random unit lifetime occurs. When the unit begins operation, each failure mode simultaneously generates a random lifetime that is independent of the other failure modes. Thus, in effect, k lifetimes have simultaneously begun. Failure of the unit occurs as soon as any one of the lifetimes is realized. In effect, the life length of the unit is equal to the minimum of the k random lifetimes.

CONCLUSION

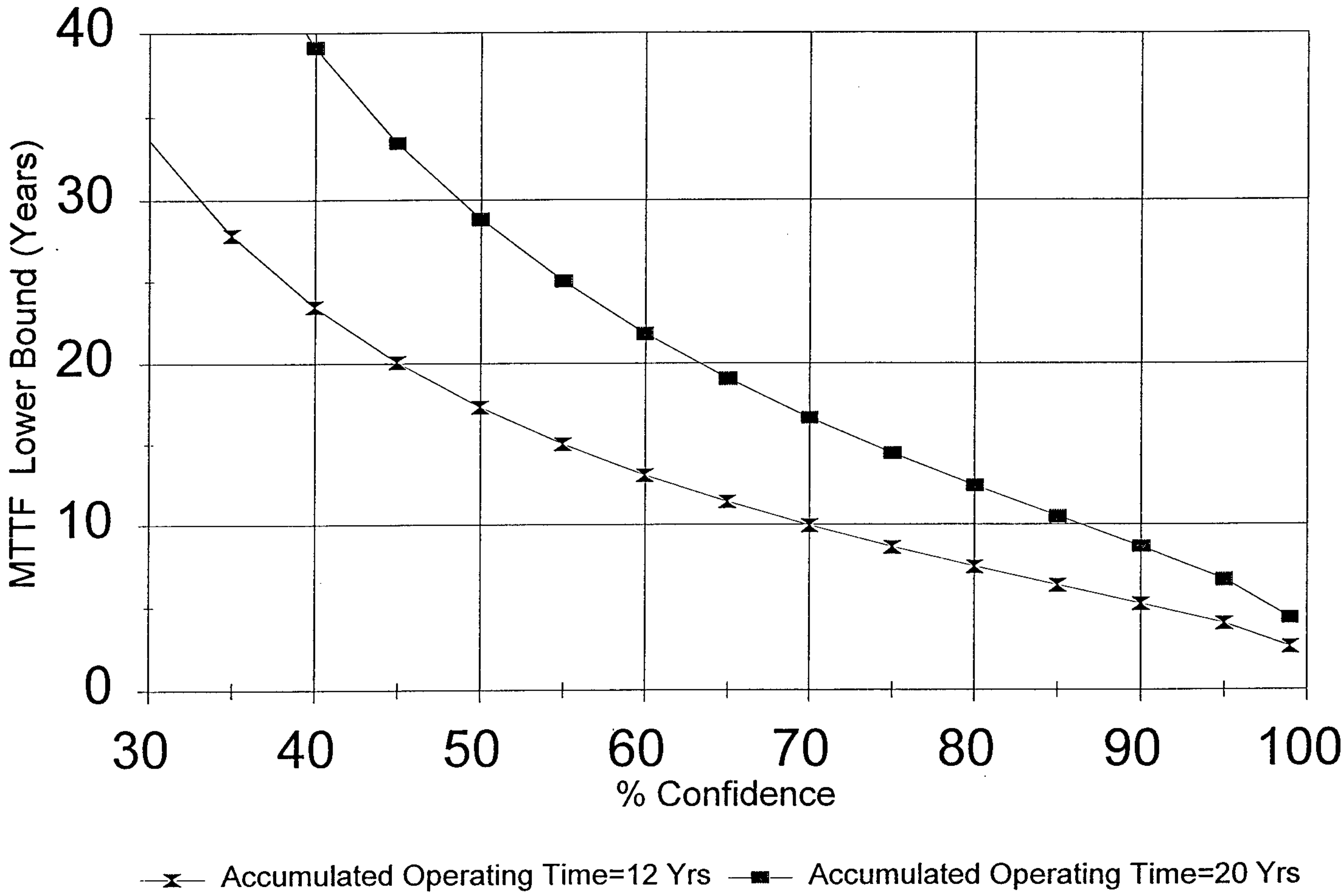
Eventually, a more traditional analysis methodology, based on the Weibull distributed time to failure, is expected to be the preferred method for the RAT. However, it is expected that all analysis will include lower confidence limit estimations of distribution parameters rather than point estimates. In the mean time, beginning at the completion of the first year of the RAT, the application of the methodology, based on the research of Epstein & Sobel, will be used and updated on six month intervals. The time to failure data is assumed to be exponentially distributed. The Huang /Porter Weibull distributed Method will not be used initially to support the RAT life analysis. It will be used when the sample population size increases to a point where the conditional requirements for the use of the Allowable Life-limit Lower Bound method are met. Research will continue for alternate analysis approaches. This on-going research will be done via searches through technical literature and state-of-the-art data analysis software packages.

Weibull Probability Plot for Exponentially Distributed Time-To-Failure

Figure #1

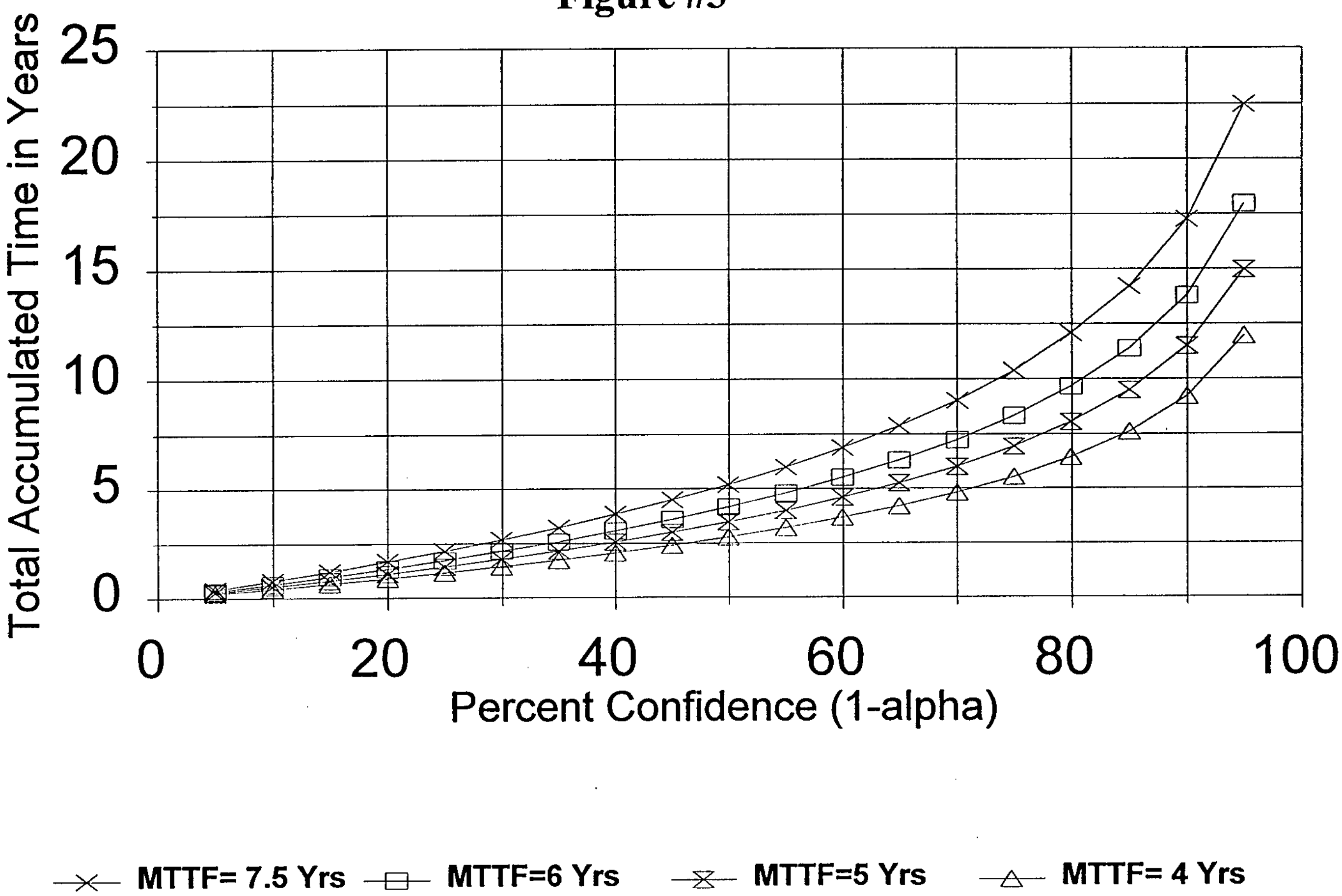


MTTF Lower Bound Vs Confidence level
Total Accumulated Operating Time
Figure #2

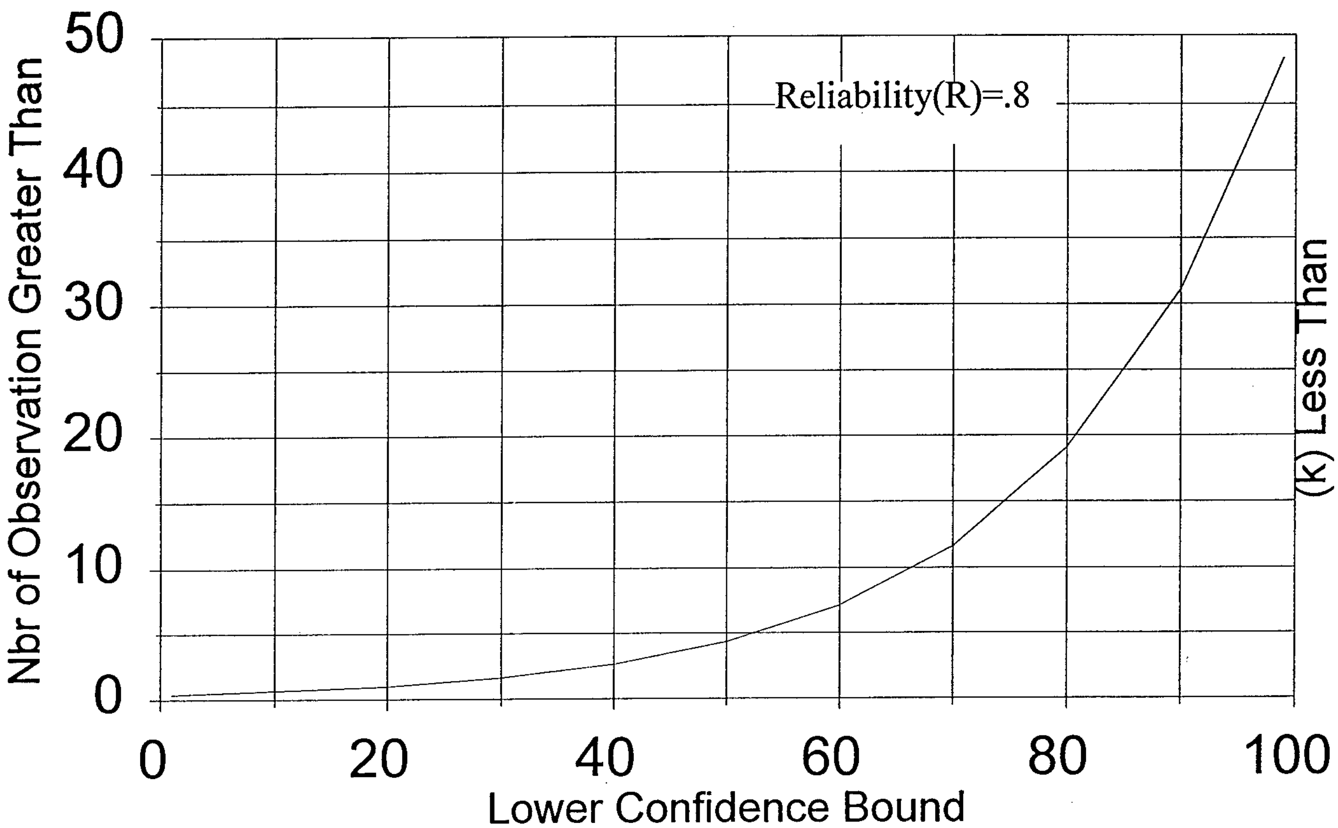


Accumulated Time Vs % Confidence for
a Specified MTTF with Zero Failures

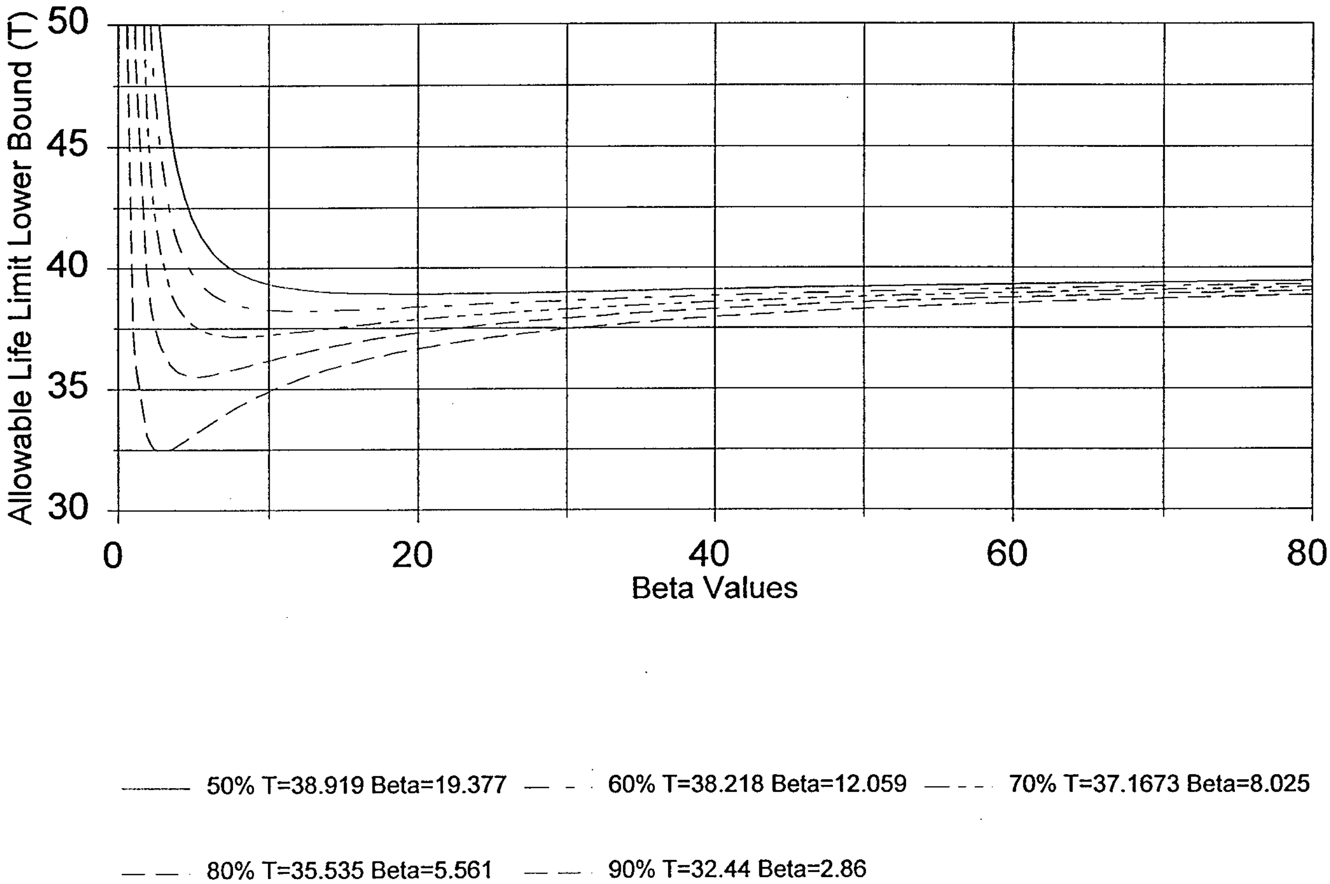
Figure #3



% Confidence Vs Nbr of Observations
Figure #4



50% to 90% Lower Confidence Bound
Figure #5



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