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New higher order methods for linear programming have been developed that have led to significant improvement in performance.

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Final Technical Report on

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Summary Status of Effort

During the three years of this grant, research was focused on interior point algorithms for linear programming, linear and nonlinear complementarity problems, and general nonconvex nonlinear programming problems. In linear programming, the focus of research on this grant was on higher order methods, removal of dense columns, and testing of an algorithm for automatic infeasibility detection. This work, when coupled with algorithms for better matrix orderings and faster factorizations, led to significant improvement in performance of interior point methods for linear programming.

In linear complementarity, a globally convergent algorithm was devised, programmed and tested. It has proved very robust and extremely efficient, with extraordinarily low iteration counts even for problems with very large numbers of variables. A direct analogue of this algorithm was developed for nonlinear programming, again proven globally convergent under limited assumptions, and again programmed and tested. The algorithm is again robust, and compares favorably with other known algorithms on a standard test set. Also, the work shows the assumptions for convergence are necessary by providing examples of both linear and nonlinear complementarity problems which violate the assumptions and do not converge.

For nonlinear programming, an interior point algorithm which is a direct analogue of the linear and quadratic programming interior point algorithms has been developed. The method has been modified to include a merit function to assure progress toward a solution. The matrix of second partial derivatives is modified to guarantee a descent direction by adding, when necessary, a diagonal term. The method has been programmed and tested on several test sets, and has proved very robust and extremely efficient in comparison with two other standard codes.

Accomplishments/New Findings:

In linear programming, during the period of this research, this primal-dual predictor-corrector interior point algorithm was the focus of study. Research was directed toward improving performance of the algorithm through higher order methods derived from targeting, improved automatic handling of dense columns, better matrix orderings and factorizations, and most recently, a new method for handling free variables which eliminates the need to split free variables. This latter has led to a more stable algorithm, while the other modifications reduced running time on the Netlib test suite from 1694 seconds to 1190 seconds on a Silicon Graphics workstation. In addition, an infeasibility detecting algorithm was tested, and shown to be both efficient in detecting infeasibility and very robust on badly conditioned problems, but significantly slower than the standard algorithm on feasible, normally conditioned problems. This work is documented in [4].

In nonlinear programming, preliminary work was done on sensitivity analysis in quadratic programming using an interior point code. This work is continuing, and shows significant promise of being quite efficient for large problems. This work combines elements of simplex and interior point technologies, and should prove to be a very useful tool in determining the robustness of solutions to nonlinear programming problems.

The principal form of research in nonlinear programming has been the development of a primal-dual interior point algorithm for nonconvex nonlinear programming problems. Preliminary work, documented in [3], [4] showed that potentially such an algorithm could be very efficient, but that a merit function was needed to assure progress toward a true minimum of the objective function and that matrix modification was needed to assure a decent direction for this merit function. Work in the last year has developed and tested such an algorithm. This work, documented in [7], chooses as the merit function the classical Fiacco - McCormick penalty-barrier function. The paper shows how to choose the penalty parameter, and how to assure a descent direction. The method was programmed and tested extensively on both standard and nonstandard test problems. The method has proved to be very robust. The tests also show comparisons with MINOS and LANCELOT, and show that the method is extremely efficient compared to these well known algorithms. A code containing this algorithm is available from the website of Bob Vanderbei.

For complementarity problems, algorithms for both linear and nonlinear problems were developed, coded, and tested. These algorithms are documented in [5] and [6]. For linear complementarity, the algorithm is extremely efficient, with iteration counts remaining very small as problem sizes increased. The interior point code developed appears to be perhaps the most efficient linear complementarity code developed to date. The nonlinear code has also been extensively tested, and seems quite robust and comparable in efficiency to other state of the art codes. A major part of this research examines the problem of determining when a complementarity problem is infeasible using an infeasible interior point algorithm. Both the linear and nonlinear algorithms are proved convergent under a set of assumptions, and the research has demonstrated which assumption fails when the problem is infeasible. As this is a testable condition, it provides for automatic infeasibility detection for monotone problems. The algorithms have been applied successfully to a wide variety of problems, especially those arising in oligopolistic pricing models in economics, game theoretic models, and models arising from variational inequalities.

Personnel Supported:

Principal Investigator:	David Shanno
Graduate Student:	Evangeline M. Simantiraki
Graduate Student:	Stefan Schmieta

Publications:

[1] "A globally convergent penalty-barrier algorithm for nonlinear programming," OPERATIONS RESEARCH PROCEEDINGS 1994, V. Derigs, A. Bachem, and A. Drexl, Editors, Springer-Verlag, 1995, 22-27 (with M.G. Breitfeld).

[2] "Computational experience with modified log-barrier methods for nonlinear programming," ANNALS OF OPERATIONS RESEARCH 62, 1996, 439-464, (with M.G. Breitfeld).

[3] "Implementing barrier methods for nonlinear programming," INTERIOR POINT METHODS FOR OPTIMIZATION, T. Terlaky, ed. Kluwer, 1996, 399-414 (with M.G. Breitfeld and E.M. Simantiraki).

[4] "Interior point methods for linear and nonlinear programming," THE STATE OF THE ART IN NUMERICAL ANALYSIS, I.A. Duff and G.A. Watsons, eds. Oxford, New York, 1997, 339-362 (with E.M. Simantiraki).

[5] "An infeasible-interior-point algorithm for solving mixed complementary problems," COMPLEMENTARITY AND VARIATIONAL PROBLEMS, STATE OF THE ART, M.C. Ferris and J.S. Pang, eds., SIAM, Philadelphia, 1997, 386-404 (with E.M. Simantiraki).

[6] "An infeasible-interior-point method for linear complementarily problems," SIAM JOURNAL ON OPTIMIZATION 7, 1997, 620-640 (with E.M. Simantiraki).

Submitted, not yet accepted:

[7] An interior point algorithm for nonconvex nonlinear programming, SOR-97-21, Statistics and Operations Research, Princeton University, 1997 (with R.J. Vanderbei).

Ph.D. Thesis:

E.M. Simantiraki, "Interior-Point Methods for Linear and Mixed Complementarity Problems", Rutgers University, 1996.

Interactional Transitions:

Presentation at meetings, conferences, etc.

Optimization Conference, Mathematics Research Institute, Oberwolfach, Germany, January 1995, 60 minute invited talk.

INFORMS National Meeting, Los Angeles, CA, Arpil 1995, 20 minute invited talk.

International Conference on Optimization, Braga, Portugal, July 1995, 60 minute invited talk.

INFORMS National meeting, New Orleans, LA., October, 1995, 20 minute invited talk.

International Conference on complementarity Problems, Johns Hopkins University, November, 1995, 45 minute invited talk.

INFORMS Computer Science Technical Section Conference, Dallas Texas, January 1996, 30 minute invited talk.

Institute for mathematics and Analysis, Conference on Progress in Numerical Analysis, York, England, March 1996, 45 minute invied talk.

CERFACS Conference on Linear Algebra in Optimization, Albi, France, April, 1996, 45 minute invited talk.

INFORMS national Meeting, Washington, D.C., May 1996, 20 minute invited talk.

International Symposium on Optimization and Computation, Hayama, Japan, August 1996, 45 minute invited talk.

Optimization Conference - Mathematics Research Institute, Oberwolfach, Germany, January, 1997, 30 minute invited talk.

Consultative: None

Transitions:

All research in linear and quadratic programming is incorporated into the CPLEX mathematical programming code. This is one of the most widely used commercial codes in existence. The complementarity codes are available via the NEOS server for complementarity problems. The nonlinear nonconvex code LOQO is available from the website of Robert Vanderbei.

New discoveries, inventions or patent disclosures: None

Honors/Awards: None in this period.