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13. ABSTRACT *(Maximum 200 words)*

This grant was for research in methods for optimizing parameters in complex simulation models. The principal method investigated was that of sample-path optimization (SPO). During the period of the grant, progress was achieved in extending SPO to the solution of stochastic variational inequalities, and in supporting areas. Results were documented in two papers prepared for publication and in one completed Ph.D. dissertation.

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Final Technical Report

Planning Under Uncertainty: Methods and Applications

AFOSR Grant No. F49620-97-1-0283

University of Wisconsin–Madison

1 Objective

The principal objective of this research project (as originally proposed for a three-year effort) was to investigate methods for optimizing parameters in complex simulation models. These models may be of two types.

- In the first type the underlying model is a deterministic optimization, but each instance of the optimization contains stochastic elements whose distribution depends on the parameters mentioned above, and therefore the real problem is that of choosing these parameters to optimize an expectation.

- In the second type the underlying model is an existing stochastic simulation model containing parameters, with no optimization capability. The model may be very large and complex. One is interested in using the output of runs of this model to choose parameter values that improve the expectation of some measure of merit such as cost to perform a mission, or time required for a task at a given cost.

One example of the first kind of model is a project network consisting of subtasks interrelated by precedence constraints. The completion time of the project depends on the durations of the various subtasks. If these durations are stochastic, then so is the project completion time. One may have the possibility of investing a limited amount of money in tightening the distributions of subtask durations, or in reducing their expectations, and the question is then how to allocate this limited budget so as to obtain the greatest possible reduction in overall expected project completion time.

Another example of a problem of the first type might be a transportation network in which availability of a particular mode of transportation (for example, a particular type of aircraft) may be stochastic. If the availabilities are known, the problem of choosing the best transportation plan is a deterministic network optimization problem, in which some measure of performance such as cost, time, etc. is to be optimized. By investing in facilities, maintenance, extra equipment, etc., one may make the availability distributions more favorable. How should one invest a limited budget to obtain the best expected performance from the transportation system?

An example of the second type of model is a simulation of a very complex system, such as a transfer line in manufacturing, or the operation of a system of airfields over time. Here the dynamics of the system are so complicated that one cannot represent it by an underlying optimization model, as in the examples above. However, the simulation model can produce a satisfactory representation of the system's behavior, given the resources (fuel, parking capacity, etc.) available. If one has the possibility of obtaining limited additional resources, how should these be invested (that is, how should one change the parameters that are regarded as fixed by the simulation) so as to obtain the greatest improvement in the expected performance of the model?

The main approaches originally proposed were in three areas:

- Combining the method of sample-path optimization (SPO) with underlying deterministic optimization models, especially convex models such as network flow representations of transportation, management, or other types.
- Enhancing the effectiveness of SPO for use with complex stochastic simulation models by automating the derivative computations that currently have to be done through infinitesimal perturbation analysis (IPA) "add-ons."
- Increasing the usefulness of SPO output by developing effective methods for computing confidence regions for optimizers, particularly in the case in which the underlying performance function is nonsmooth.

2 Overview of Research Accomplished

The grant resulting from this proposal was for a period of nine months, and the PI decided to concentrate for that period on the first of the above objectives. Progress was achieved in that area; in fact, it was found that results could be obtained not only for optimization problems but also for the more general category of variational inequalities. The solution of stochastic variational inequalities by simulation appears appears to be a new area, and as described below the research team was successful in presenting and justifying an apparently effective new algorithm.

The progress achieved during the nine-month period of the grant was documented in two papers and one Ph.D. dissertation, listed below. These works acknowledged AFOSR support.

In the rest of this section we briefly describe this progress and provide citations to the works listed in the next section. All are available either in the published literature, in press, or (in the case of works in the publication process) from the principal investigator. References in brackets refer to numbered publications listed in Section 3.

The method of sample-path optimization was successfully extended to variational inequalities containing stochastic elements. The problem addressed is that of solving

$$0 \in E\{f(x, \omega)\} + N_C(x),$$

where C is a polyhedral convex set in $\mathbf{R}^n$, $x \in \mathbf{R}^n$, and ω is a random element. The symbol $N_C(x)$ denotes the *normal cone* to C at the point x, and E denotes expectation. This formalism includes a wide variety of stochastic problems including, but not limited to, problems of stochastic optimization. A proof of convergence of the method was obtained, and the algorithm was illustrated on an energy planning problem containing 175 variables (that is, in the above $n = 175$). These results are described in [P1] and [P3].

Over the past several years the method of sample-path optimization has been developed, justified, and applied to several classes of problems with AFOSR support. The Computer Sciences Technical Section of the Institute for Operations Research and the Management Sciences (INFORMS) invited the PI to present a survey lecture at their meeting in January 1998 describing this method and illustrating its effectiveness. The writeup of that lecture was completed in 1997 and appeared by invitation in the proceedings of the conference [P2].

3 Works Resulting from Research Activity

The following scientific works acknowledge support from Grant F49620-97-1-0283.

[P1] G. Gürkan, A. Y. Özge, and S. M. Robinson, Sample-path solution of stochastic variational inequalities. Submitted to *Mathematical Programming* and now undergoing minor revision for resubmission.

[P2] G. Gürkan, A. Y. Özge, and S. M. Robinson, Sample-path solutions for simulation optimization problems and stochastic variational inequalities, in: D. L. Woodruff, ed., *Advances in Computational and Stochastic Optimization, Logic Programming, and Heuristic Search: Interfaces in Computer Science and Operations Research* (Kluwer Academic Publishers, Boston, Dordrecht, and London, 1998) 169–188.

[P3] A. Y. Özge, *Sample-Path Solution of Stochastic Variational Inequalities and Simulation Optimization Problems,* Ph.D. Dissertation, Department of Industrial Engineering, University of Wisconsin-Madison, 1997.

4 Participating Professionals

The following professional personnel received salary support from Grant F49620-97-1-0283.

- Stephen M. Robinson, Professor
- A. Yonca Özge, Research Assistant

5 Degrees Awarded

A. Yonca Özge, Research Assistant, received the Ph.D. (Industrial Engineering) from the University of Wisconsin-Madison in 1997.

6 Inventions and Patent Disclosures

During the work under this grant, there were no inventions that appeared to have any patent possibilities. Other (non-patentable) discoveries are contained in the papers reported above.

7 Other Information

During the period of this grant the PI received the George B. Dantzig Prize from the Society for Industrial and Applied Mathematics (SIAM) and the Mathematical Programming Society. The citation for the Prize was:

> The Mathematical Programming Society and The Society for Industrial and Applied Mathematics present the George B. Dantzig Prize to Stephen M. Robinson
>
> For his work on the solution, stability, and sensitivity of nonlinear optimization problems and, in particular, for introducing generalized equations and the strong-regularity condition into optimization. His work has had a major impact on the analysis and development of algorithms for systems of nonlinear equations, nonlinear complementarity, variational inequalities, and nonlinearly constrained optimization.

Further information about any of the activities reported above, or other aspects of this research program, can be obtained from the principal investigator, Stephen M. Robinson, at the Department of Industrial Engineering, University of Wisconsin–Madison, 1513 University Avenue, Madison, WI 53706-1572, telephone (608) 263-6862, fax (608) 262-8454, email `smrobins@facstaff.wisc.edu`.