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Reliability Analysis of Networks Carrying Critical Mission Traffic

DANIEL C. LEE

*Command, Control, Communications, Computers, and Intelligence Branch
Space Systems Development Department*

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13. ABSTRACT (<i>Maximum 200 words</i>) This report suggests reliability analysis that takes into account the traffic accommodation (the network's ability to carry the required traffic) in the case of component failures. Such analysis is motivated by networks that have predetermined requirements of critical traffic flow such as in military surveillance, data processing, and dissemination. In the course of the discussion, this report suggests computational algorithms that determine whether the network can carry the required traffic. This report also presents new reliability measures that are constructed from the standpoint of network accommodation. Such reliability measures are determined by both the network topology and the traffic demand.			
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RELIABILITY ANALYSIS OF NETWORKS CARRYING CRITICAL MISSION TRAFFIC

1 Introduction

This report addresses network reliability issues for special-purpose networks which are used exclusively for a specified information transfer. Such networks can be found in special networks for military surveillance, data collection, processing, and dissemination. Such special networks typically have predetermined traffic requirements for critical operations, whereas traffic demands in commercial networks in general are random and up to the activities of the subscribers.

In analyzing the network reliability, the network is typically represented by graph $G = (V, E)$, where V denotes a set of nodes, and E denotes the set of links. Traditionally defined network reliability measures (e.g., edge connectivity, node connectivity, cut frequency vector, cohesion, source-to- K -terminal reliability, etc. [1]) are concerned with disconnection of nodes. For example, the reliability is measured by the minimum number of link failures that disconnects the graph (deterministic, worst-case setting) or the probability that the network is disconnected (probabilistic setting) [1]. Such network reliability measures have been studied extensively in the past. These existing measures are meaningful for general, commercial networks because they do not consider specific traffic demands from users.

This report is concerned that critical network operations still can fail even if network connectivity is preserved. For example, a certain link failure can keep the network connected but disable the critical mission that the network supports. In defining reliability measures for special-purpose networks, this report suggests that the concern be the ability to carry the critical mission rather than connectivity. Thus, the reliability measures are to be defined based on both the network topology and the specific traffic demand required for the critical mission.

2 Reliability Analysis

2.1 Traffic Accommodation

In defining a new reliability measure, we represent the network by a directed graph $G = (V, E)$ and link capacity c_j for each link $e_j \in E$. The directed graph G is also represented by the $|V| \times |E|$ node-arc incidence matrix [2], $A = [a_{ij}]$, where

$$a_{ij} = \begin{cases} +1 & \text{if link } e_j \text{ leaves node } i \\ -1 & \text{if link } e_j \text{ enters node } i \\ 0 & \text{otherwise} \end{cases}$$

We denote the traffic demand by a set of numbers $f_s, s = 1, 2, \dots, S$ where f_s represents the required data rate of session s . Each session is represented by an ordered pair of nodes; i.e., source node and destination node of the traffic. Given the network topology and the traffic demand, the natural question to raise is whether the network can accommodate all total traffic demand. Elaborating further, can we find a set of routes for each session so that the sum of all the traffic flow in each link is below the link capacity? If yes, we say that the network is *accommodating the traffic*.

We can answer the question of whether the network is accommodating the traffic demand by the following formulation. Denote by $|E|$ -dimensional vector $x^{(s)}$ the flow of traffic belonging to session s . The j -th component of this vector $x_j^{(s)}$ represents the flow of session s traffic through link $j \in E$. Then the conservation of flow at nodes gives the following constraint.

$$Ax^{(s)} = f_s d^{(s)}, \quad s = 1, 2, \dots, S$$

where $d^{(s)}$ is a $|V|$ -dimensional vector such that

$$d_i^{(s)} = \begin{cases} +1 & \text{if node } i \text{ is the source of session } s \\ -1 & \text{if node } i \text{ is the sink of session } s \\ 0 & \text{otherwise} \end{cases}$$

Also, the link capacity imposes the following constraint:

$$\sum_{s=1}^S x_j^{(s)} \leq c_j, \quad j = 1, 2, \dots, |E|$$

Finding feasible flows $x^{(s)}$ satisfying these constraints or proving the nonexistence of such flows can be done through Linear Programming [2]. More specifically, we can add augmented variables $y^{(s)}, s = 1, 2, \dots, S$ and solve the following Linear Programming problem:

$$\begin{aligned} & \text{minimize} && \sum_{s=1}^S \sum_{i=1}^{|V|} y_i^{(s)} \\ & \text{subject to} && Ax^{(s)} + Iy^{(s)} = f_s d^{(s)}, \quad s = 1, 2, \dots, S \\ & && \sum_{s=1}^S x_j^{(s)} \leq c_j, \quad j = 1, 2, \dots, |E| \\ & && x^{(s)}, y^{(s)} \geq 0 \end{aligned}$$

If the minimal cost of this problem is 0, the values of the minimizing vectors $x^{(s)}$ are feasible flows. If the minimal value is positive, the network is not accommodating the traffic. The Linear Programming can be efficiently solved by the Simplex Algorithm [2] for the most part. If computational efficiency is desired for the worst instance, one can choose an interior point method [3].

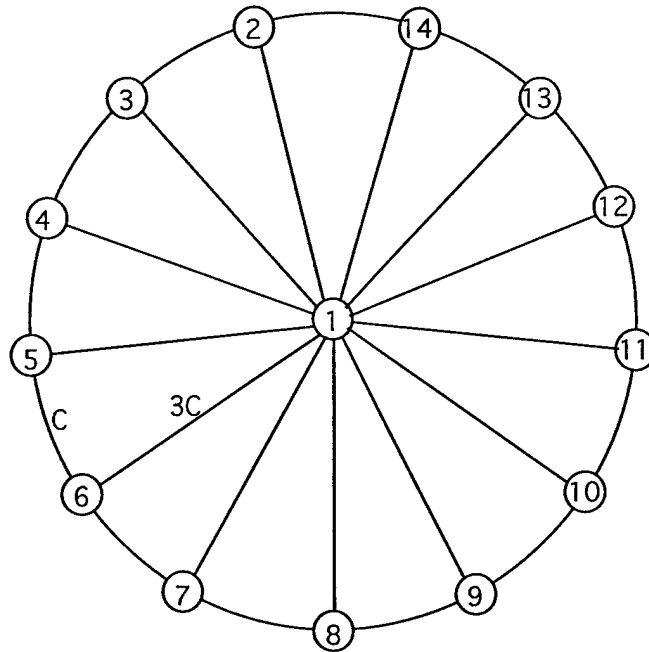


Figure 1: 14-node Ring-Star Network

2.2 Reliability Analysis Based on Traffic Accommodation

With this formulation we can perform a wide range of reliability analysis for a given network from the standpoint of the traffic accommodation. For example, we can analyze the vulnerability of the network operation to a certain link failure, a set of link failures, a certain node failure, etc.

Example 1: link failure in 14-node ring-star network

Consider the ring-star network in Figure 1. This example is motivated by 14 regional hubs of a Wide Area Network. All the links are full-duplex. Each link of the ring has an identical capacity C , and each radial link has capacity $3C$. Consider uniform traffic; all 14×13 sessions originating from and destined to distinct nodes have an identical traffic demand (intensity, rate) $f = C/3$. We first discuss that this network can accommodate the specified traffic demand. Consider a routing scheme that routes the traffic according to the following rule, which we refer to as “two-hops-in-ring”:

- For a session whose source or destination is the center node (node 1), the traffic is made to flow through the radial link from the source to the destination. (It only takes one hop.)
- For a session whose source and destination are both ring nodes and no more than two hops away from each other on the ring, the traffic is routed through the shortest path on the ring.

For example, for a session whose source is node 2 and whose destination is node 13, the traffic goes through nodes 2, 14, 13, successively. As another example, for a session whose source is node 2 and whose destination is node 3, the traffic goes from source node 2 to destination node 3 through one link.

- For a session whose source and destination are both ring nodes and more than two hops away from each other on the ring, the traffic is routed through the radial links. For example, for a session whose source is node 2 and whose destination is node 12, the traffic goes through nodes 2, 1, 12, successively.

In this example, the link from node 2 to 3 carries traffic belonging to Origin-Destination pairs (sessions) (2, 3), (2, 4), and (14, 3), each with load $f = C/3$. Therefore, the load on this link is $3f = C$. The counting of routes going through each link shows that the load of each ring link is $3f = C$, and the load of each radial link is $9f = 3C$. Thus, the routing scheme saturates all the links. However, traffic through each link does not exceed the capacity, so the “two-hops-in-ring” routing results in a feasible flow. Thus, the network is accommodating the traffic demand. In relation to reliability, the following question is to be asked: if one radial link or a ring link is down, can the network accommodate the traffic? It turns out that a link failure in this case makes the network unable to accommodate the traffic. Suppose a radial link incident on a ring node i fails. Then, the total capacity of the incident directed links incoming to node i becomes $2C$. The total load destined for node i is $13 \times C/3 > 2C$. Therefore, network cannot accommodate the traffic. Suppose a ring link incident on node i fails. Then the total capacity of the incoming links incident on node i is $4C$, and it is less than $13 \times C/3$. Therefore, again the network cannot accommodate the traffic. \square

Example 2: reduced traffic demand

For the same network (illustrated in Figure 1), we now consider uniform traffic with $f = C/8$. The network certainly accommodates this traffic. The “two-hops-in-ring” routing scheme specified in the previous example does not load links above their capacities. The load is $3(C/8)$ for each ring link, and $9(C/8)$ for each radial link. Now we consider link failures. If one ring link fails, the network still accommodates the traffic. For each path that includes the failed link, we can find an alternate route through the two radial links. For example, we consider the failure of the link between node 2 and node 14 in Figure 2. This link failure disables traffic flow of the Origin-Destination pairs (2, 14), (14, 2), (2, 13), (13, 2), (3, 14), (14, 3) in the “two-hops-in-ring” routing. Consider the restoration routes for these pairs that go through the central node (node 1) via two radial links. For example, Origin-Destination pair (2, 14) goes through nodes 2,1,14; Origin-Destination pair (2, 13) goes through nodes 2,1,13; etc. These restoration routes increase the load of the radial link between node 1 and node 2 by $2f = 2C/8$ in each direction. The restoration routes affect the radial link between node 1 and node 14 in the same way. Therefore, the new load for those two radial links

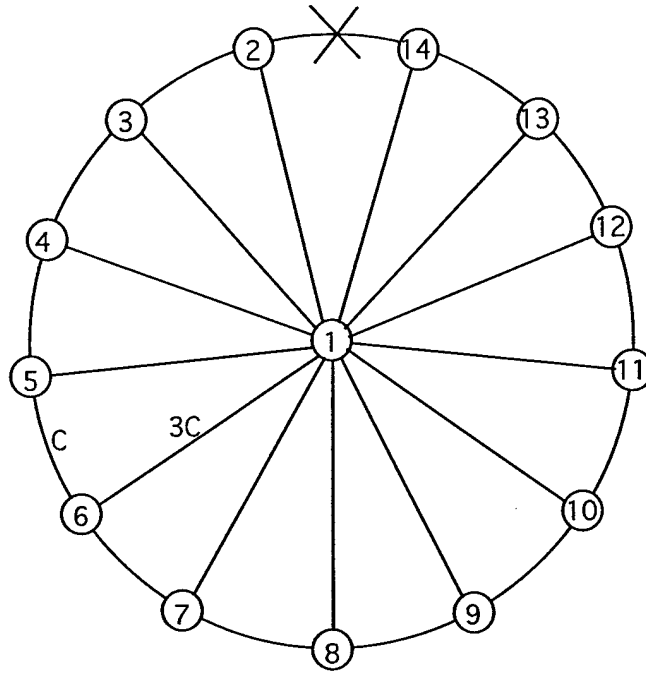


Figure 2: Failure of a Ring Link

is $11C/8$, which is less than the capacity $3C$. The restoration routes also increase the load of radial links between node 1 and node 3 and between node 1 and node 13 by f in each direction. The new load of these links is $10C/8$. On the other hand, the links between node 2 and node 3 and between node 13 and node 14 have reduction of load by $f = C/8$ in each direction. Therefore, the network still accommodates the traffic demand after the failure of one ring link.

Now we consider the failure of a single radial link. Figure 3 illustrates the failure of the radial link between node 1 and node 8. It is suggested that the restoration route be through the adjacent radial links; routes $[1, 7, 8]$, $[1, 9, 8]$, $[8, 7, 1]$, and $[8, 9, 1]$. Total $9f$ load is carried through each radial link in each direction in the original two-hops-in-ring strategy. We split the load equally for two-hop restoration paths in both sides. Therefore, for the two radial links (the link between node 1 and node 7 and the link between node 1 and node 9) and the two ring links (the link between node 7 and node 8 and the link between node 9 and node 8) have increase of load by $4.5f$ each. The new load of those two radial links is $9f + 4.5f = 13.5f = 13.5C/8$ each, and it is less than the capacity $3C$. The new load of the two ring links is $3f + 4.5f = 7.5f = 7.5C/8$ each, and it is less than the capacity C . Thus, again every link has traffic under its capacity, and the network accommodates the traffic demand after the failure of one ring link. \square

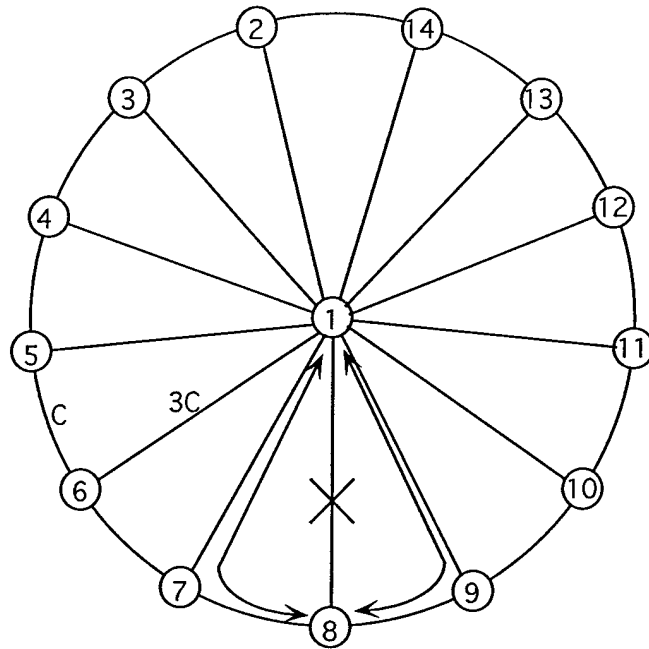


Figure 3: Failure of a Radial Link

In these examples, traffic accommodation in the case of various link failures was determined through simple observations. In more general (complex) network topologies and traffic requirements, the network accommodation can be determined through the algorithm presented in Subsection 2.1.

As for a node failure, if the failed node is either a source or a destination of any traffic session, the traffic cannot be accommodated. If the node is neither a source nor a destination, one can define a new directed graph G' that is obtained by removing all the edges incident on the failed node and the corresponding node-arc incidence matrix A' . Then, the network accommodation can be determined by using the algorithm in Subsection 2.1 with matrix A' .

3 Reliability Measures

In addition to studying the vulnerability of critical mission to the failure of particular links or nodes, from the standpoint of traffic accommodation one can construct a wide range of reliability measures. As examples, this report defines *edge accommodativeness*, *disaccommodation frequency vector*, and *probability of accommodation*. *Edge accommodativeness* is similar in spirit to *edge connectivity* [1]. *Edge accommodativeness* is defined as the minimum cardinality of a set of edges (links) whose

removal makes the network not accommodating the traffic. *Disaccommodation frequency vector* is a modification of the *cut frequency vector* [1] [4] for traffic accommodation criteria. Denoting the *disaccommodation frequency vector* by $u \equiv (u_1, u_2, \dots, u_{|E|})$, we define u_k as the number of edge subsets of size k whose removal makes the network not accommodating the traffic. When the probabilities of node and link failures are available, one can define the probability that the network can accommodate the traffic.

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