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**13. ABSTRACT (Maximum 200 words)**

Work has continued on laser beam analysis, laser beam characterizarion, and laser beam quality. Unexpected analytical and numerical results have been developed, and experimental work on the development of quasi-monolithic diode-pumped unstable resonator lasers continues. A number of papers have been published or are in course of preparation, as detailed in the report.

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Progress Report
AFOSR Grant F49620-96-1-0006
1 August 1997 to 30 November 1997
Ginzton Laboratory, Stanford University, California

(with appendices covering December 1, 1995
through November 30, 1997)

This is an additional Technical Report on AFOSR Grant F49620-96-1-0006 to the Edward L. Ginzton Laboratory, Stanford University, Stanford CA, covering the period from 1 August 1997 to 30 November 1997. This grant provides for a "Program of Research in Laser Oscillator Physics and Laser Device Performance" under the direction of Prof. A. E. Siegman.

As a consequence of scheduling complexities associated with this contract, the period covered in this report is actually a brief extension of earlier report periods extending from 1 December 1995 to 31 July 1996 and 1 August 1996 to 31 July 1997, for which earlier initial and interim reports have already been supplied. In order to provide a unified record of accomplishments for the two-year period from 1 December 1995 through 30 November 1997, copies of these two earlier reports are appended to this report. (Note also that the dates on these two earlier reports were unfortunately garbled in preparation, but have been corrected on the attached copies.)

**Progress During the Reporting Period**

During this report period we continued to work on the general topics of novel laser resonators and modes, laser beam analysis, laser beam characterization and "laser beam quality", leading to two talks and two publications as listed below. Progress in understanding and improving the propagation and focusing properties of laser beams continues to be of particular importance to industrial as well as military applications of lasers. We also continued work on the development of miniaturized diode-pumped vanadate solid-state lasers using unstable resonators for increased power output as described in the earlier progress reports, although no additional publications on this topic were put out during the reporting period.

*1) Nonorthogonal Resonator Modes and Beams:*

Our theoretical work on laser beams and resonators, and on the unusual properties resulting from the nonorthogonal character of laser modes was reported during this period in:

A. E. Siegman, "Optical beams and resonators: an oral exam", invited talk presented at the Biannual Meeting of the Dutch Physical Society, Lunteren, The Netherlands (5 November 1997).

A. E. Siegman, "Nonorthogonal optical modes and resonators", invited lecture presented at the NATO Advanced Research Workshop on Optical Resonators, Slovak Republic, July 1-5, 1997, and since then published in *Optical Resonators: Science and Engineering*, edited by R. Kossowsky (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997); pp. 29-54.

## 2) Stadium Diode Lasers:

We have also been doing both experimental and theoretical work for some time on the novel oscillation modes of a "stadium diode laser". Researchers in chaos theory and nonlinear dynamics have known for many years that the trajectories of rays bouncing inside a stadium-shaped enclosure are totally chaotic in character. The eigenmode patterns of the resonant eigenmodes inside such a stadium structure may also display somewhat regular "scar" patterns which resemble closed bouncing ball orbits inside the stadium.

From another viewpoint, however, the closed ray paths inside such resonators represent unstable periodic optical lensguides or unstable resonators, such as are familiar to laser scientists. Although much attention has been given by chaos theorists to the complex properties of these stadium enclosures, there seems to have been no connection drawn in the past to the properties of corresponding optical resonators.

As a start in exploring this connection, we have used our diode-laser mirror-etching technology to construct diode lasers in the shape of a (half) stadium, with partially reflecting walls all around. In effect, we build a planar stadium resonator with internal gain, and examine the modes in which it oscillates. To provide a theoretical explanation of the interesting modal patters which we observe from these diodes, we have also done extended Fox-and-Li calculation, including "image sources" to take account of the reflections off the side walls of these structures. These calculations have been successful in explaining the experimental results, and some of our results from this work have now been published in:

T. Fukushima, S. A. Biellak, Y. Sun and A. E. Siegman, "Beam propagation behavior in a quasi-stadium laser diode," *Optics Express* **2**, 21–28 (19 January 1998).

It might be noted that the journal in which this is published, *Optics Express*, is a new multimedia on-line journal developed by the Optical Society of America. Articles in this journal can be accessed and read over the Internet at `http://epubs.osa.org/opticsexpress/`. Our article takes advantage of the multimedia capabilities of this journal in that two of the illustrations are actually short movies which can be viewed on screen, showing the propagation of the output beams from the stadium resonator.

# Progress Report
## AFOSR Grant F49620-96-1-0006
## Ginzton Laboratory, Stanford University, California
## 1 August 1996 to 31 July 1997

This is an interim progress report on AFOSR Grant F49620-96-1-0006 to the Edward L. Ginzton Laboratory, Stanford University, Stanford CA, covering the period from 1 August 1996 to 31 July 1997. This grant provides for a "Program of Research in Laser Oscillator Physics and Laser Device Performance" under the direction of Prof. A. E. Siegman.

## Progress During the Reporting Period

During this period we continued a large amount of work on the general topics of laser resonators and modes, laser beam analysis, laser beam characterization and "laser beam quality", since progress in understanding and improving the propagation and focusing properties of laser beams is of particular importance to industrial as well as military applications of lasers. We also continued the development of miniaturized diode-pumped vanadate solid-state lasers using unstable resonators for increased power output, and carried out some interesting secondary experiments and analysis on a novel chaotic "stadium resonator" diode laser.

These activities led to three journal publications during the reporting period, along the presentation of number of invited talks and participation in several very useful international meetings and workshops. Our first publication reported some excellent results on high-power near-diffraction-limited diode lasers using reactive-ion-etched unstable resonator mirrors, as reported in:

> S. A. Biellak, G. Fanning, Y. Sun, S. S. Wong and A. E. Siegman, "Reaction-ion-etched diffraction-limited unstable-resonator semiconductor lasers," *IEEE J. Quantum Electronics* **QE–33**, 219–230 (February 1997).

These results may represent the highest-brightness single stripe diode laser results yet reported. As part of our vanadate laser development we also observed and explained a small but definite astigmatism in the output beam from an unstable-resonator vanadate laser, as reported in

> Y.-J. Cheng, G. Fanning and A. E. Siegman, "Transverse-mode astigmatism in a diode-pumped unstable resonator Nd:YVO4 laser," *Appl. Opt.* **36**, 1130–1134 (20 February 1997).

This effect had not been previously seen, but fortunately is also easily corrected or compensated for. Finally in work begun in the previous reporting period, we developed some very interesting and unexpected analytical and numerical results concerning eigenmode expansions for the fields in laser resonators and optical waveguides having nonhermitian or biorthogonal eigenmodes. This class of resonators includes the important classes of gain-guided resonators and lensguides, variable-reflectivity-mirror resonators, and unstable optical resonators. We have found that several of the conventional methods for modal

expansions no long apply (or at least do not converge properly) in these systems, and have developed new expansion methods which solve these problems in a very general way. An extensive manuscript reporting on these results has now been published in:

> A. Kostenbauder, Y. Sun and A. E. Siegman, "Eigenmode expansions using biorthogonal eigenfunctions: complex-valued Hermite gaussians," *J. Opt. Soc. Amer. A* **14**, 1780–1790 (August 1997).

Reprints of the above three papers are attached to this report. We also presented a large number of invited talks and lectures and participated in a number of very stimulating and useful meetings and workshops on the above topics, including:

> Y. Sun and C. G. Fanning, "Spatial solitary waves in the coupled wave-carrier systems of semiconductor gain media," presented at CLEO/Europe '96, Hamburg, Germany, Paper CTuJ5 (September 1996).

> A. E. Siegman, "Normal modes and not so normal modes (invited talk)," presented at Topical Meeting on Advanced Solid-State Lasers, Orlando, Florida, (January 1997).

> T. Fukushima, S. A. Biellak, Y. Sun, C. G. Fanning, Y. Cheng, S. S. Wong and A. E. Siegman, "Laser characteristics of a quasi-stadium laser diode," presented at CLEO/QELS '97, Baltimore, Md, CWF6 (18–23 May 1997).

> A. E. Siegman, "Eigenmode expansions in nonorthogonal systems (invited lecture)," presented at Workshop on Asymptotic and Approximate Methods in Optical Modelling, University of Rochester, New York, (12–13 June 1997).

> A. E. Siegman and C. G. Fanning, "Miniature high-performance end-pumped Nd:vanadate lasers," presented at LASERS 97: Novel Lasers, Devices and Applications, Munich, Germany, (18 June 1997).

> G. Nemes and J. Serna, "The ten physical parameters associated with a full general astigmatic beam: a Gauss-Schell model," presented at Laser Beam and Optics Characterization (LBOC 4), Munich, Germany (June 1997).

> G. Nemes and J. Serna, "Do not use spherical lenses and free spaces to characterize beams: a possible improvement of the ISO/DIS 11146 document," in Laser Beam and Optics Characterization (LBOC 4), Munich, Germany, (June 1997).

> A. E. Siegman, "Nonorthogonal optical modes and resonators (invited lecture)", presented at NATO Advanced Research Workshop on Optical Resonators, Slovak Republic, July 1–5, 1997, and to be published in *Optical Resonators: Theory and Design*, edited by R. Kossowsky (Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997).

Progress Report

AFOSR Grant F49620-96-1-0006

Edward L. Ginzton Laboratory, Stanford University, California

1 December 1995 to 31 July 1996

This is the initial progress report on AFOSR Grant F49620-96-1-0006 to the Edward
L. Ginzton Laboratory, Stanford University, Stanford CA, covering the period from 1
December 1995 to 31 July 1996. This grant provides for a "Program of Research in
Laser Oscillator Physics and Laser Device Performance" under the direction of Prof. A. E.
Siegman.

## 1. Progress During the Reporting Period

During this initial reporting period we continued work on the general topics of laser
beam analysis, laser beam characterization and "laser beam quality", since progress in
understanding and improving the propagation and focusing properties of laser beams is
of particular importance to industrial as well as military applications of lasers. Efforts
during this period included the preparation and presentation of two invited papers at
major conferences on this topic, namely:

> A. E. Siegman, "Laser beam quality: what's it good for?" (plenary talk), pre-
> sented at LASERS '95, Society for Optical and Quantum Electronics, Charleston,
> South Carolina, 4–8 December 1995 and to be published in *Proceedings of the
> International Conference on Lasers '95* (STS Press, 1995); pp. 1–12.

> A. E. Siegman, "Modes, Beams, Coherence and Orthogonality," (invited talk),
> presented at 3rd International Workshop on Laser Beam and Optics Characteri-
> zation, Quebec, Canada, (8-10 July 1996) and to be published in the Proceedings
> of this conference.

A paper on the modal analysis of waveguiding structures with laterally offset gain and
index guiding properties, a topic important in the design of unstable-cavity and thermally
steerable semiconductor diode lasers, was also published during this period:

> Y. Sun and A. E. Siegman, "Optical mode properties of laterally offset gain and
> index guiding structures," *IEEE J. Quantum Electron.* **QE–32**, 790–795 (May
> 1996).

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In work which is continuing beyond this reporting period, we have been developing some very interesting and unexpected analytical and numerical results concerning eigenmode expansions for the fields in laser resonators and optical waveguides having nonhermitian or biorthogonal eigenmodes. This class of resonators includes in particular the important classes of gain-guided resonators and lensguides, variable-reflectivity-mirror resonators, and unstable optical resonators. An extensive manuscript reporting on these results is currently in preparation.

On the experimental side, we continued work on the development of small, quasi-monolithic diode-pumped unstable-resonator lasers, primarily using neodymium vanadate as the laser medium, and on the measurement of quantum noise fluctuations in these lasers. We first developed these lasers in order to measure an unusual excess quantum noise process in unstable-resonator laser oscillators which we have been attempting to confirm experimentally for several years. This work has now come to a successful conclusion after several years of effort with the very clear and definitive observation of a large excess Schawlow-Townes noise in such oscillators, as predicted by our theory, and shown for a typical case in the following figure.
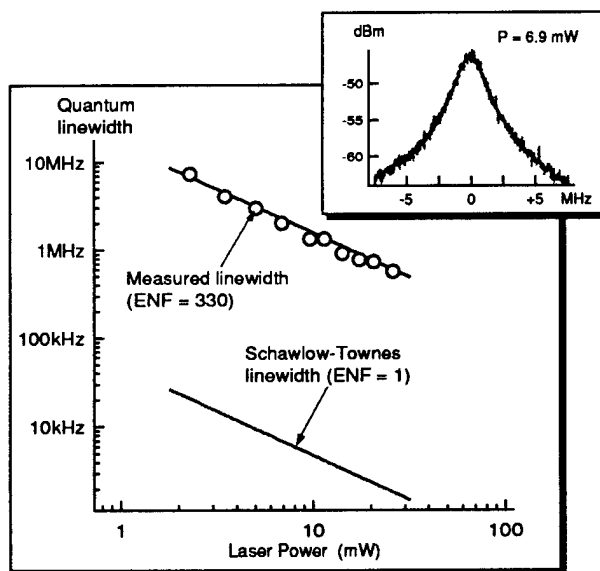
Figure 1. Experimental results showing a large excess noise figure in a quasi-monolithic diode-pumped neodymium vanadate laser using an unstable optical resonator. The insert shows the very accurately lorentzian spectrum of the laser, as expected for a quantum-noise-broadened oscillator.

These results were presented as a post-deadline at the 1996 QELS conference and have also been published in Phys. Rev. Letters, as follows:

Y.-J. Cheng, G. Fanning and A. E. Siegman, "Experimental observation of a large excess quantum noise factor in the linewidth of a laser oscillator using nonorthogonal modes," QELS '96, Anaheim, (post-deadline paper) (June 1996).

Y.-J. Cheng, G. Fanning and A. E. Siegman, "Experimental observation of a large excess quantum noise factor in the linewidth of a laser oscillator using nonorthogonal modes," *Phys. Rev. Lett.* **77**, 627–630 (22 July 1996).

In the course of this work we also discovered that the optical anisotropy of the vanadate crystal leads to a small but significant difference between the ABCD matrices in the $x$ and $y$ transverse directions, and this in turn leads to a small but readily observable astigmatism, that is, a small difference in the wavefront radius of curvature in these two directions, for the output beam from the unstable resonator. (For the case of a stable resonator this same astigmatism would cause a small difference in the spot sizes rather than the wavefront curvatures in the $x$ and $y$ directions; both results seem to have gone unnoticed in previous work.) This result was published in:

Y.-J. Cheng, G. Fanning and A. E. Siegman, "Transverse mode astigmatism in a diode-pumped unstable-resonator Nd:YVO4 laser," *Appl. Opt.* **35**, accepted for publication (submitted April 1996).

Although these miniature vanadate lasers were initially developed by us in order to carry out the noise observations described above, we have also realized that they provide small, rugged, quasi-monolithic diode-pumped laser structures which operate in a single transverse and longitudinal mode, but which have substantially larger mode volume and therefore potentially larger power outputs than the NPRO or microchip lasers developed by others to date. We are therefore continuing work on the development of these lasers, and attempting to optimize their power output and efficiency for use as small but still moderate power laser sources for various practical applications.

## 2. Additional ASFOSR-Related Activities, April 1995–August 1995

During the period from April through August of 1995, prior to the beginning of this grant, our group was operating with primary support from various Stanford University funding sources, but continuing to work on activities that had been supported by earlier AFOSR grants and that continued into the current grant period. For completeness, therefore, we also list the following publications, submissions and PhD dissertations which were completed during this interim period and which represent accomplishments supported at least in part by earlier AFOSR support:

D. Francis, Y. Sun, S. Biellak, A. E. Siegman and C. J. Chang-Hasnain, "Optical characteristics of a beam-steerable semiconductor-fan laser array," presented at *Conference on Lasers and Electro-Optics (CLEO '95)*, May 1995.

A. E. Siegman, "Lasers Without Photons," invited talk presented at *CQO7*, Rochester NY, June 1995 and published in *Coherence and Quantum Optics VII*, ed. by L. Eberly, L. Mandel and L. Wolf (Plenum Press, New York, 1995).

Y. Sun, *Lateral Mode Control of Semiconductor Lasers*, Ph.D. dissertation, submitted to Department of Electrical Engineering, Stanford University, August 1995.

S. Biellak, *Reactive Ion Etched Semiconductor Diode Laser Resonators*, Ph.D. dissertation, Department of Applied Physics, Stanford University (August 1995).

S. A. Biellak, Y. Sun, S. S. Wong and A. E. Siegman, "Lateral mode behavior of reactive-ion-etched stable-resonator semiconductor lasers," *J. Appl. Phys.* **78**, 4294–4296 (15 September 1995).

S. A. Biellak, G. Fanning, Y. Sun, S. S. Wong and A. E. Siegman, "Reaction-ion-etched diffraction-limited unstable-resonator semiconductor lasers," *IEEE J. Quantum Electron.* **QE–33**, submitted for publication, August 1995.

S. A. Biellak, Y. Sun, S. Wong and A. E. Siegman, "Monolithic stable-resonator semiconductor laser," presented at *Semiconductor Lasers: Advanced Devices and Applications (1995 Topical Meeting)*, Keystone, Colorado, 21–23 August 1995.

Y. Sun, "Analytic spatial solitary wave modes of the coupled wave-carrier equations in a semiconductor waveguide," *Opt. Lett.* **20**, 1–3 (1 September 1995).

# Beam propagation behavior in a quasi-stadium laser diode

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**Abstract:** The beam propagation behavior of a quasi-stadium laser diode is theoretically investigated. The resonator that we analyzed consists of one flat end-mirror, one convex curved end-mirror and two straight side wall mirrors. The cavity dimension is much larger than the oscillation wavelength. We derived one-dimensional Huygen's integral equations for this laser cavity and carried out eigenmode calculations using the Fox and Li mode calculation method taking into account the effect of the side wall reflections and visualized the propagation beams. Unique beam propagation behaviors were obtained. These results well agree with our previous experimental results.

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## References and links

1. M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York Inc., 1990)

2. E. J. Heller, "Bound-State Eigenfunctions of Classically Chaotic Hamiltonian Systems: Scars of Periodic Orbits," *Phys. Rev. Lett.* **53**, 1515 (1984)

3. E. J. Heller and S. Tomsovic, "Postmodern Quantum Mechanics," *Phys. Today* **46**, 38 (1993)

4. S. W. McDonald and A. N. Kaufman, "Wave Chaos in the Stadium: Statistical Properties of Short-Wave Solutions of the Helmholts Equation," *Phys. Rev. A* **37**, 3067 (1988)

5. S. Tomsovic and E. J. Heller, "Semiclassical Dynamics of Chaotic Motion: Unexpected Long-Time Accuracy," *Phys. Rev. Lett.* **67**, 664 (1991)

6. S. A. Biellak, "Reactive Ion Etched Unstable and Stable Semiconductor Diode Lasers," Ph.D. thesis, Stanford University (1995)

7. S. A. Biellak, C. G. Fanning, Y. Sun, S. S. Wong, and A. E. Siegman, High Power Diffraction Limited Reactive-Ion-Etched Unstable Resonator Diode Lasers, in *Conference on Lasers and Electro-Optics*, Vol. **8** of 1994 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1994), p. 4.

8. Y. Sun, "Lateral Mode Control of Semiconductor Lasers," Ph.D. thesis, Stanford University (1995)

9. T. Fukushima, S. A. Biellak, Y. Sun, C. G. Fanning, Y. Cheng, S. S. Wong, and A. E. Siegman, Lasing Characteristics of a Quasi-Stadium Laser Diode, in *Conference on Lasers and Electro-Optics*, Vol. **11** of 1997 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1997), p. 227.

10. A. E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986)

# 1. Introduction

The stadium resonator, which consists of two half-circles connected by two straight side walls, and the half-stadium resonator, one side of the stadium resonator, have been extensively investigated as examples of classical and quantum chaotic systems.[1-5]

A classical particle moving freely and bouncing elastically inside these structures shows two kinds of orbits. One is closed and periodic orbits that depend on the bouncing number at the straight side walls. These closed orbits are not perturbation stable. If a particle moves away from the closed orbit a little bit, the initial perturbation is rapidly enhanced as the particle repeats the bouncing at the half-circles. The other is a fully chaotic orbit which covers entire interior of the structure. In a quantum model, the wave equation is solved with appropriate boundary conditions.[2,3] In this case, "scars" patterns, which are narrow linear regions with an enhanced intensity of eigenfunction, are formed around the classical closed orbits.[1,2]

It is physically interesting to study the propagation of an optical beam inside these resonators, because the propagation can be described by a classical ray trace picture that is equivalent to the orbits of the classical particle, as long as the wavelength of the optical beam is much smaller than the cavity dimension and the beam width and divergence are negligible. To examine the beam propagation behavior, some of the authors fabricated quasi-stadium semiconductor lasers by using reactive ion etching technique[6,7] and observed the lasing characteristics, such as light output injection current characteristics, lasing spectra and output beam patterns from both end-mirrors.[8,9] Unique fringe patterns on both end-mirrors and a focused image outside the curved end-mirror were observed. Some of these characteristics are well explained by using ray trace picture based on geometrical optics.[8] However, the fringe patterns and beam divergence can not be explained by using such geometrical optics. Moreover, it is speculated that the optical beams propagating inside the laser cavity have sizable divergence and width. To better understand the beam propagation behavior, it is necessary to analyze the eigenmode of the laser cavity.

In this work, we carried out eigenmode calculations and visualized the beam propagation behavior inside the quasi-stadium laser cavity. From the view point of classification of laser cavities, the quasi-stadium resonator can be assumed to be a geometrical unstable resonator consisting of flat and curved end-mirrors with side wall reflection mirrors. We adopted the Fox and Li mode calculation method[10] with taking into account the effect of side wall reflections. In our experiment, we observed the output beam patterns just above the threshold current.[9] In this condition, the intensity of the propagating beams inside the laser cavity is relatively weak and the spatial hole burning effects are negligible. In our calculations, therefore, we do not take into account the spatial hole burning effects. We calculated the spectrum of round-trip eigenvalue, output beam patterns from both end-mirrors and beam propagation behavior inside the laser cavity for both directions (forward and backward) and visualized them. The calculated output beam patterns show excellent agreement with our experimental results.[9] This calculation method is adequate for the analysis of this kind of laser cavity, as long as the spatial hole burning effects are negligible. It is also found that the interference patterns are formed along the classical ray trajectories inside the laser cavity. These intensity patterns are similar to the "scars" patterns that are calculated in the quantum model.

## 2. Device structure and theoretical model

Figure 1 shows the structure of the quasi-stadium laser diode. This structure is same as the device that we measured the lasing characteristics in [9]. The flat and curved end-mirrors are connected by straight side wall mirrors. The cavity length $L$ and width $W$ are 660 and 60$\mu$m, respectively. The radius of curved end-mirror $R$ is 60$\mu$m. The width of flat end-mirror $W_r$ is 20$\mu$m. The side walls are separated from both cavity ends by distance $W_s$, which is 90$\mu$m. The purpose of the open unpumped corner regions is to suppress the higher order ray trajectories.

Figure 2 shows the top view of the laser cavity. The quasi-stadium resonator can be assumed to be a geometrical unstable resonator consisting of a flat end-mirror and a curved end-mirror with straight side wall mirrors. We analyzed the resonator by using the Fox and Li mode calculation method taking into account the effect of the side wall reflections. The resonator is an unstable resonator, therefore the forward propagation is different from the backward propagation.
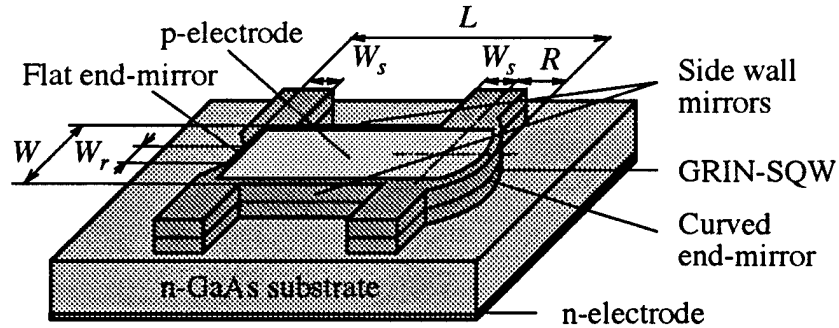


Fig. 1. Structure of the quasi-stadium laser diode.
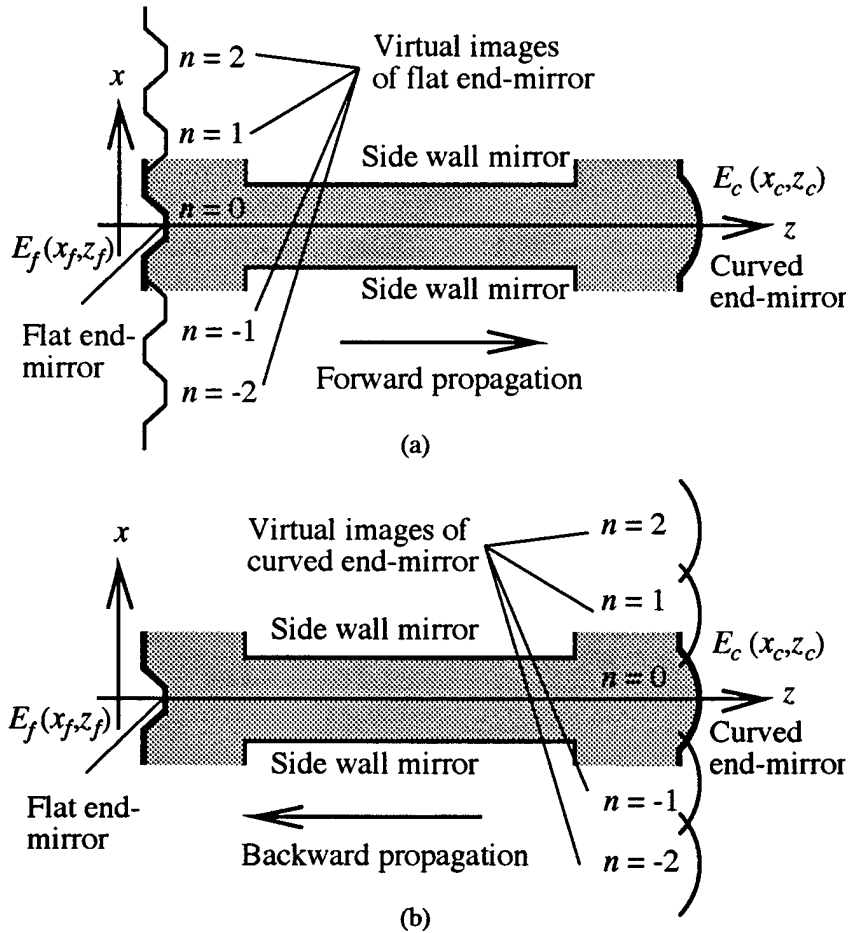


Fig. 2 . Theoretical model for the analysis of eigenmode. (a) Virtual images of flat end-mirror, (b) virtual images of curved end-mirror.

During the beam propagation from the flat end-mirror to the curved end-mirror (forward propagation), the side wall mirrors form some virtual images of the flat end-mirror as a result of the side wall reflections as shown in Fig. 2a. During the beam propagation from the curved end-mirror to the flat end-mirror (backward propagation), some virtual images of the curved end-mirror are formed as shown in Fig. 2b in the same manner.

To begin our calculation, we assumed a field distribution on the flat end-mirror $E_f(x_f, z_f)$ with uniform intensity and phase as an initial condition. The field distribution on the curved end-mirror $E_c(x_c, z_c)$ is calculated by using following one-dimensional Huygen's integral equation from the images of the flat end-mirror including the virtual images,

$$E_c(x_c, z_c) = \sqrt{\frac{j}{\lambda}} \exp[-jk(z_c - z_f)] \sum_n \int_{flat} E_f(x_f, z_f)$$

$$\times \sqrt{\frac{z_c - z_f}{(x_c - x_f)^2 + (z_c - z_f)^2}} \exp\left[-jk \frac{(x_c - x_f)^2}{2(z_c - z_f)}\right] \cos\theta dx_f. \tag{1}$$

Where, $\lambda$ and $k = 2\pi / \lambda = 2\pi n_{eff} / \lambda_0$ are oscillation wavelength and wavenumber inside the laser cavity. $\lambda_0$ is wavelength in a vacuum and $n_{eff}$ is effective index of the laser diode. $n$ indicates the number of the flat end-mirror images as shown in Fig. 2a. $\cos\theta$ is the obliquity factor which depends on the angle $\theta$ between the line element $(x_f, z_f)$-$(x_c, z_c)$ and the normal to the surface element $dx_f$.

Following that, the field distribution on the flat end-mirror is calculated by using following one-dimensional Huygen's integral equation from the images of curved end-mirror including virtual images in the same manner,

$$E_f(x_f, z_f) = \sqrt{\frac{j}{\lambda}} \sum_n \int_{curve} E_c(x_c, z_c) \sqrt{\frac{z_c - z_f}{(x_c - x_f)^2 + (z_c - z_f)^2}}$$

$$\times \exp\left[-jk(z_c - z_f) - jk \frac{(x_c - x_f)^2}{2(z_c - z_f)}\right] \cos\theta dr. \tag{2}$$

Where $n$ is the number of the curved end-mirror images. $\theta$ is the angle between the line element $(x_f, z_f)$-$(x_c, z_c)$ and the normal to the surface element $dr$.

Equations (1) and (2) are obtained from the conventional two-dimensional Huygen's integral[10] by using Fresnel approximation and separating in rectangular coordinates $x$ and $y$. During these Huygen's integrals, we took into account the effect of the open unpumped corner regions by removing the field propagating from the surface element $dx_f$ or $dr$ whose line element $(x_f, z_f)$-$(x_c, z_c)$ crosses the open regions of the side walls. These round-trip calculations are repeated until the round-trip eigenvalue $\gamma$ converges. We defined $\gamma$ as,

$$\gamma = \lim_{m \to \infty} \frac{|E_f^{m+1}(x_f, z_f)|^2}{|E_f^m(x_f, z_f)|^2}. \tag{3}$$

Where $m$ is the number of the round-trip.

Through the calculations, we obtain the field distributions on both flat and curved end-mirrors. Therefore we can calculate the beam propagation behavior inside the laser cavity

in both directions (forward and backward) by using one-dimensional Huygen's integrals. The output beam patterns from both end-mirrors are calculated by using two dimensional Huygen's integral. Here we assumed the field distribution perpendicular to the active layer was Gaussian. In the actual device, the output beams propagating from both end-mirrors are affected by Lloyd's mirror reflection at the surface of the substrate. To obtain good agreement with our experimental results, we also took into account the effect of the Lloyd's mirror reflection in this calculation.

## 3. Calculation results

### 3.1 Spectrum of round-trip eigenvalue

It is speculated that the laser diode oscillates at the wavelength where the round-trip loss becomes minimum (the round-trip eigenvalue becomes maximum) near the wavelength of the gain peak. Figure 3 shows the spectrum of the round-trip eigenvalue. In this calculation, we determine the effective index $n_{eff}$ as 3.3, so that the calculation results agree with our experimental results.[9] $n_v$ is the number of the virtual images taken into account on each side of the laser cavity during the eigenmode calculation. When $n_v = 0$, the resonator becomes simple unstable resonator without side wall reflections. In this case, the round-trip eigenvalue is relatively small. When we take one virtual image on each side of the laser cavity ($n_v = 1$), the round-trip eigenvalue spectrum shows some ripples. It is speculated that these ripples are caused by the interference among the beam propagating directly and the beams reflected at the side walls. As the number of the virtual images increases, the round-trip eigenvalue increases and the spectrum becomes more complex shape, namely, some narrow and sharp peaks appear in the spectrum. However, there is no difference between the spectrums calculated for $n_v = 2$ and $n_v = 4$. The reason is that the open unpumped corner regions in the laser cavity restrict the higher order ray trajectories. It is found that $n_v = 2$ is enough number to analyze this laser cavity.

The round-trip eigenvalue becomes maximum at 857nm. We believe that the laser diode oscillates at this wavelength when the gain peak is located near the wavelength. We carried out following calculations at the wavelength of 857nm.
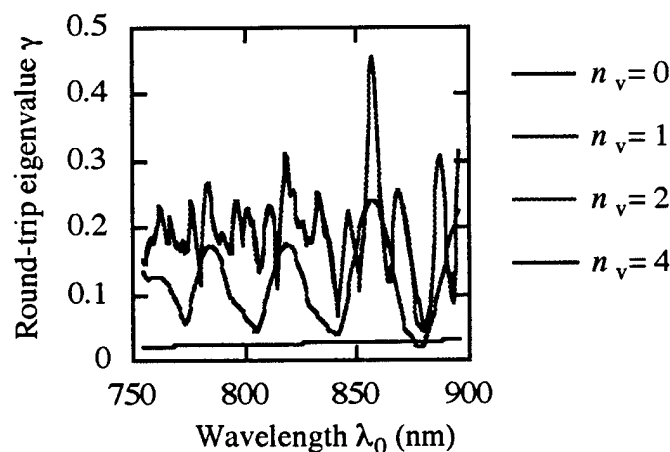


Fig. 3. Spectrum of round-trip eigenvalue.

### 3.2 Output beam patterns propagating from both end-mirrors

Figures 4a and 4b that are short animations show the variation of the output beam patterns against the distance from the flat and curved end-mirrors, respectively. A unique fringe pattern is formed on the flat end-mirror (Fig. 4a). Upon leaving the flat end-mirror, the fringe pattern diverges and eventually split into three main lobes. On the curved end-mirror (Fig. 4b), a

fringe pattern is also formed. Upon leaving the curved end-mirror, however, the fringe pattern focuses to three spots at the distance of 24μm. Beyond this distance, the spots again diverge and reform a fringe pattern. The variation of the output beam patterns well agrees with our experimental results.[9]

### 3.3 Beam propagation behavior inside the laser cavity

Finally we calculated the beam propagation behavior inside the laser cavity. Figure 5 shows the intensity patterns of the propagation beams inside the laser cavity for both directions. In this calculation, the field distribution on the line perpendicular to the laser cavity was calculated by using one dimensional Huygen's integral from the end-mirrors.
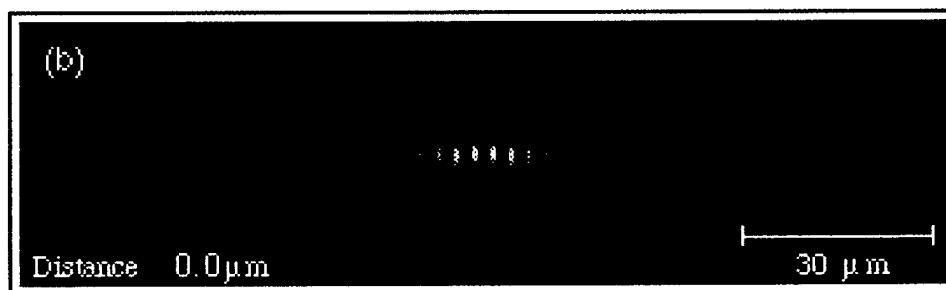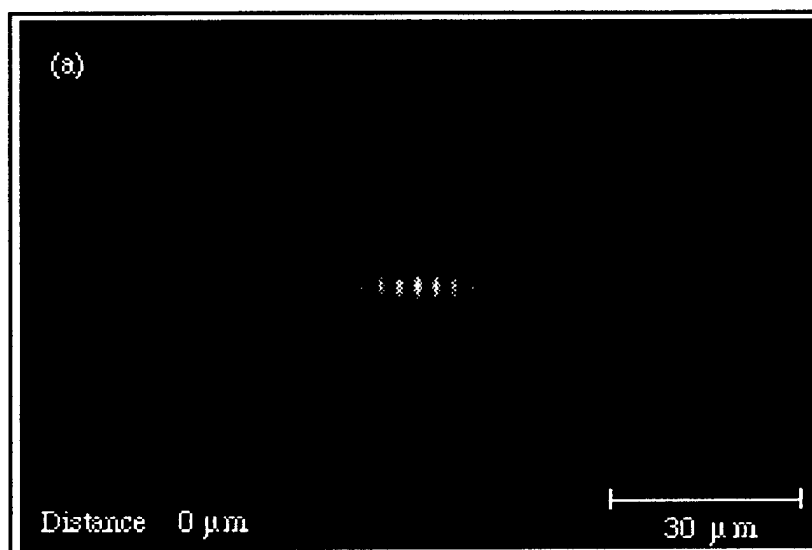




Fig. 4. Click in the space to start the animations that show the variations of output beam patterns versus the distance (a) from the flat end-mirror and (b) from the curved end-mirror, respectively.

Then the intensity distribution is calculated and normalized on the line so that the maximum intensity becomes unity. These calculations are then repeated, changing the location of the line from one end-mirror to the other end-mirror. After leaving the end-mirror, the intensity of the propagating beam decreases because the round-trip eigenvalue is less than unity and the beam divergence is relatively large. The purpose of the normalization of the intensity is to show the intensity patterns clearly through the entire laser cavity.

It is found that the interference patterns are formed along the closed and repetitive trajectories that is expected from the classical ray trace picture.[8] These intensity patterns

look like the "scars" patterns that are obtained from the quantum model.[1,2] It is speculated that these intensity patterns are formed by the interference between the beams propagating along the classical ray trajectories.
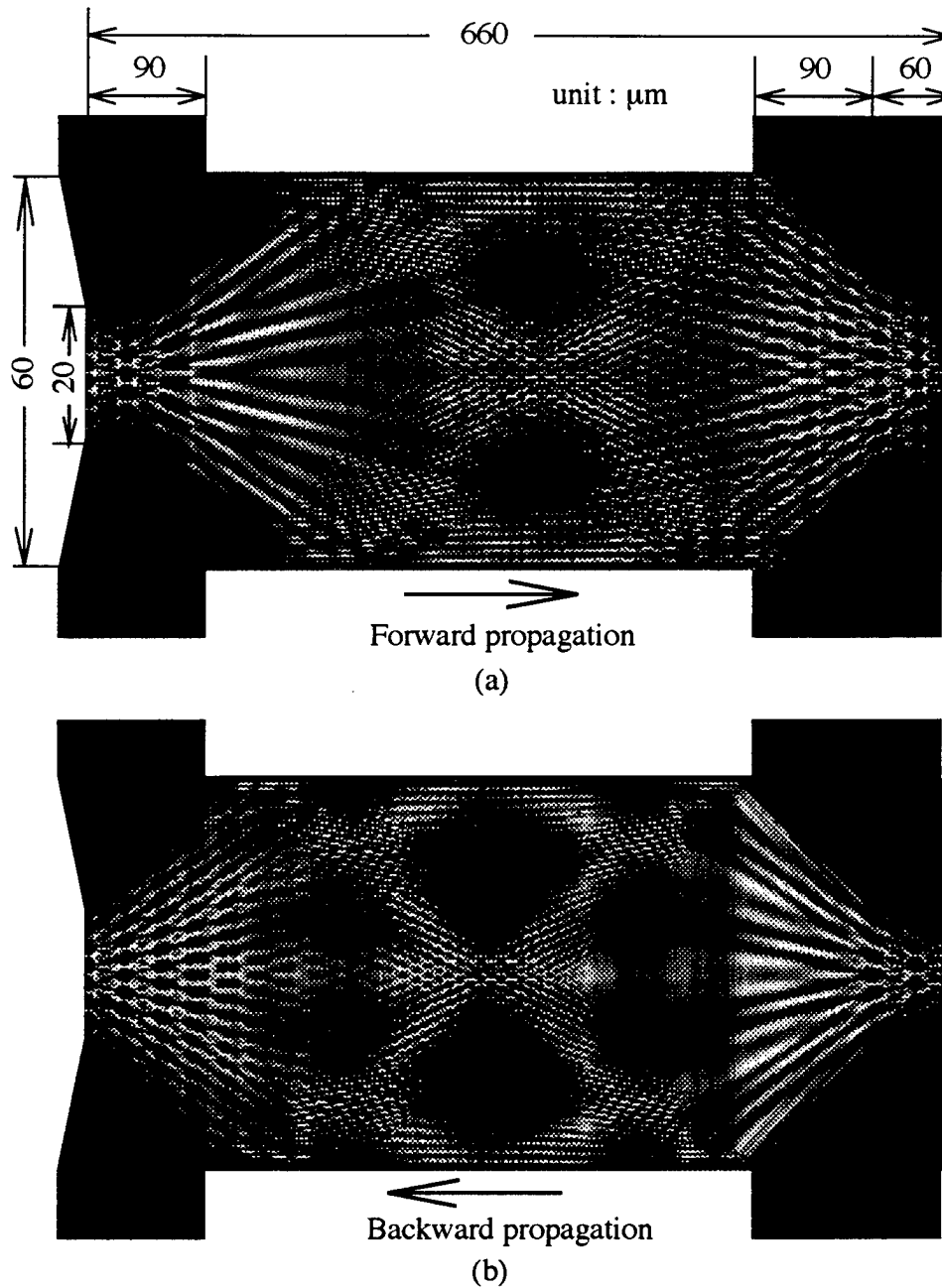


Fig. 5. Beam propagation behavior inside the laser cavity, (a) forward propagation and (b) backward propagation.

The round-trip beam propagation can be explained as follows. The fringe pattern on the flat end-mirror diverges and eventually splits into three directions, each beam propagating toward the curved end-mirror. After bouncing on the side wall mirrors a fringe pattern is formed on

the curved end-mirror. The reflected beams at the curved end-mirror then focus to three spots at the point $R/2$ away from the top of the curved end-mirror. These spots again diverge and propagate toward the flat end-mirror with large beam divergence. Eventually the same fringe pattern is formed on the flat end-mirror after one round-trip.

In the classical ray trace picture, the closed repetitive trajectories are independent of each other. In the actual quasi-stadium laser cavity, however, it is speculated that the beams propagating along the classical ray trajectories have sizable divergence and they are coupled to each other.

## 4. Conclusions

We analyzed the beam propagation behavior in a quasi-stadium laser diode by using the Fox and Li mode calculation method taking into account the effect of the side wall reflections. Unique fringe patterns from both end-mirrors, and a focused image outside the curved end-mirror are obtained. These results show the excellent agreement with our experimental results. We also visualized the beam propagation behavior inside the laser cavity. It is found that the interference patterns are formed along the classical closed ray trajectories. These patterns are formed by the interference between the beams propagating along different ray trajectories. This calculation method is useful for the analysis of this kind of laser cavity.

The laser structure analyzed in this work is specialized compared to a conventional stadium resonator. For example, the laser cavity has open unpumped corner regions to suppress the higher order ray trajectories, and the radius of the curved end-mirror is larger than the half of the cavity width. Further research will be conducted to analyze dependency of the beam propagation behavior on the structural parameters of the quasi-stadium laser cavity.

## Acknowledgments