ARMY RESEARCH LABORATORY

JTCG/AS Interlaboratory Ballistic Test Program—Final Report

by Albert L. Chang
and Barry A. Bodt

ARL-TR-1577

December 1997

Army Research Laboratory
Aberdeen Proving Ground, MD 21005-5069

ARL-TR-1577 **December 1997**

JTCG/AS Interlaboratory Ballistic Test Program—Final Report

Albert L. Chang
Weapons and Materials Research Directorate, ARL

Barry A. Bodt
Information Science and Technology Directorate, ARL

Abstract

A two-phase study was conducted to determine the efficacy of MIL-STD-662E in ballistic protection limit determination for advanced lightweight armor technologies. The first-phase results demonstrated an unacceptable variability in limit estimates across sites. It was concluded that more stringent controls in implementing MIL-STD-662E, particularly as to test velocity selection, would be required to achieve adequate reproducibility. An unusual pattern in the data from the first phase suggested that there was a physical reason beyond operational concerns for the unacceptable variability. In the second phase, this new reason was investigated in an interlaboratory test involving six sites. It was determined that a physical phenomenon, termed shatter gap, was the principal cause of the irreproducible results from the first phase. For this setting, modifications of MIL-STD-662E to support reproducible results were suggested and successfully tested.

Acknowledgments

The authors would like to express their sincere thanks to Mr. Joseph P. Jolley, the U.S. Army representative to the Joint Technical Coordinating Group on Aircraft Survivability (JTCG/AS) Central Office and LTC John N. Lawless, Jr., Director, JTCG/AS Central Office, for their critical and patient support of this program. In spite of interruptions due to personnel and facility changes, their support for this program was directly responsible for its successful completion. The JTCG/AS project identification number is V-3-A1, *Aircraft Armor Materials Testing and Data Reporting Standards Project.* The authors gratefully acknowledge Mr. John H. Graves (formerly at U.S. Army Research Laboratory [ARL], Aberdeen Proving Ground [APG], MD) and MAJ Hermann Kolev (U.S. Military Academy [USMA]) for their contributions to Phase I of this project. We also acknowledge participation of the following people and organizations who performed the Phase II ballistic tests:

Mr. Thomas Carlson, U.S. Army Aberdeen Test Center (ATC), APG, MD;

Messrs. Harold Holland, Bryan Pilati, and Donald Campbell, Aviation Applied Technology Directorate (AATD), Fort Eustis, VA;

Mr. Alex Kurtz, U.S. Air Force Wright Laboratories (AFWL), Wright-Patterson Air Force Base (AFB), OH;

Messrs. John P. Bird Jon Miller, Ceradyne Inc., Costa Mesa, CA;

Mr. Stan Lyons, Simula, Inc., Phoenix, AZ; and

Messrs. John Riegel and Don Grosch, Southwest Research Institute (SRI), San Antonio, TX.

INTENTIONALLY LEFT BLANK.

Table of Contents

		Page
	Acknowledgments	iii
	List of Figures	vii
	List of Tables	ix
1.	**Introduction**	1
2.	**Review of Phase I Results**	4
2.1	Ballistic Data and Penetration Probability	5
2.2	Sources of Discrepancy	5
3.	**Phase II Results**	8
3.1	Confirmation of Nonmonotonic Penetration Probability Distribution	8
3.2	Observation of Shatter Gap	12
3.3	Statistical Models and Simulation	13
3.4	New Velocity Selection Rule	14
3.5	Modified Dixon's Maximum Likelihood Estimate	18
3.6	Phase II Interlaboratory V_{50} Test	18
4.	**Discussion**	20
4.1	Shatter Gap	22
4.2	V_{50} Data Scatter	22
5.	**Conclusions**	23
6.	**References**	25
	Appendix A: Proposed V_{50} Test Procedure	27
	Appendix B: FORTRAN Code Listing of MDMLE Program	31
	Distribution List	39
	Report Documentation Page	47

INTENTIONALLY LEFT BLANK.

List of Figures

Figure		Page
1.	A Schematic Illustration of Partial and Complete Penetrations	1
2.	Summary of Phase I Ballistic Data of VAR 4340 Steel Armor Plates From All Nine Laboratories	6
3.	Summary of Phase I Ballistic Data of Al_2O_3/Kevlar Armor Plates From All Nine Laboratories	6
4.	Penetration Probability of VAR 4340 Steel Armor Plates Obtained From Combined Phase I Data of All Nine Laboratories	7
5.	Penetration Probability of Al_2O_3/Kevlar Armor Plates Obtained From Combined Phase I Data of All Nine Laboratories	7
6.	Penetration Probability of Al_2O_3/Kevlar Armor Plates Obtained From 69 Shots in Phase II	9
7.	Shattering of 0.50-cal. AP M2 Bullets as a Function of Velocity After Impacting Al_2O_3/Kevlar Armor Plates	12
8.	Effect of Velocity Step Size on the Simulated Low-V_{50} Distribution in Armor Plates With a Nonmonotonic Pc(V)	16
9.	Effect of Velocity Step Size on the Simulated High-V_{50} Distribution in Armor Plates With a Nonmonotonic Pc(V)	17
10.	Overlapping of Low-V_{50} and High-V_{50} Distributions in Armor Plates With a Monotonic Pc(V)	17
11.	Summary of Phase II V_{50} Ballistic Test on Al_2O_3/Kevlar Armor Plates From All Six Laboratories	21
12.	Measured Low-V_{50} and High-V_{50} From Phase II Interlaboratory Test on Al_2O_3/Kevlar Armor Plates as Compared to Simulations	21

INTENTIONALLY LEFT BLANK.

List of Tables

Table | Page

1. Results of 69 Ballistic Shots on Phase II Al_2O_3/Kevlar Armor Plates 10

2. Data-Fitted Model Parameters for Observed Pc(V) 14

3. Results of V_{50} Ballistic Test on Phase II Al_2O_3/Kevlar Armor Plates From All Six Laboratories 19

INTENTIONALLY LEFT BLANK.

1. Introduction

The ballistic performance of lightweight armor materials has been evaluated based on the V_{50} ballistic limit, which is defined as the projectile striking velocity at which complete penetration and partial penetration of the armor materials are equally likely events. There are several types of V_{50} ballistic limits, which depending on the definition of the complete (or partial) penetration, included the Army, Navy, and protection criterion [1]. The protection ballistic limit (PBL) V_{50} is the most widely accepted criterion for assessing the performance of lightweight armor materials and serves as the focus of this study. The outcome of the PBL V_{50} ballistic test is determined by the final condition of a witness plate placed behind the armor panel. If the witness plate is perforated by the projectile or spall from the armor panel, the outcome is termed a complete penetration. If no perforation is observed through the witness plate, the result is termed a partial penetration. If the armor panel is perforated, but the witness plate remains intact, the result is still defined as a partial penetration. A schematic definition of the partial and complete penetration is shown in Figure 1.

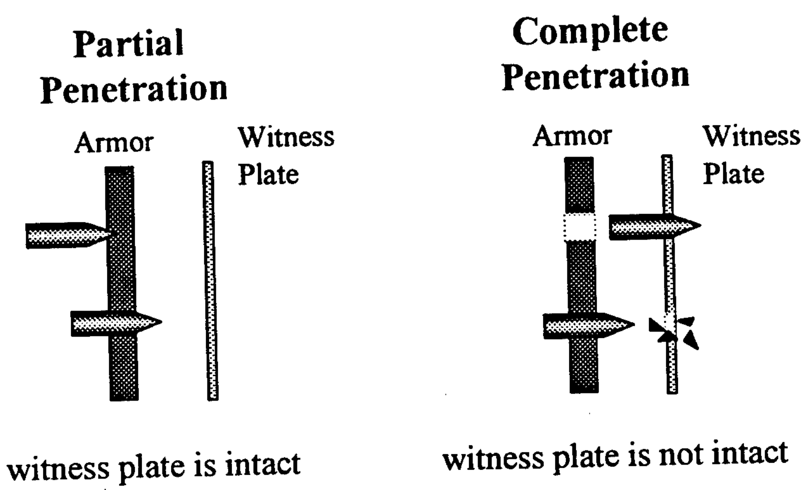

Figure 1. A Schematic Illustration of Partial and Complete Penetrations.

The outcome of a given ballistic event (partial or complete penetration) at a given velocity V is not always identical, due to the statistical nature of the various material failure processes involved in the ballistic event. Therefore, it is best described in terms of the probability of complete penetration, Pc(V). There is no a priori derivation of Pc(V) from first principles. It is generally accepted that Pc(V) is assumed to follow some unspecified statistical model. In general, Pc(V) = 0 at lower velocities, Pc(V) = 0.5 at V_{50}, where the partial and complete penetration are equally likely, and Pc(V) = 1 at higher velocities. Near V_{50}, some of the partial penetrations have velocities higher than some of the complete penetrations. The zone of mixed results (ZMR) is the velocity range over which such overlap occurs. Ideally, Pc(V) can be determined experimentally by conducting multiple ballistic tests at various velocities. However, this approach is not practical for obtaining the V_{50} ballistic limit of armor materials on a regular basis, due to the high cost associated with such large number of ballistic tests. Therefore, a more efficient and standardized test, namely MIL-STD-662E [2], using the up-and-down velocity-selection rule to sample the Pc(V) with fewer numbers of ballistic tests, was developed, and it has been widely used throughout the ballistic testing community. The up-and-down method is based on a bisection algorithm. If the nth shot at V_n yields a complete penetration, the velocity of the (n + 1)th shot should be at $V_{n+1} = V_n - \Delta V$ in order to obtain a partial penetration. Otherwise, if the nth shot yields a partial penetration, V_{n+1} should be at $V_n + \Delta V$ in order to obtain a complete penetration. The velocity step, ΔV, is typically 100 ft/s. After the occurrence of the first reversal, that is a complete penetration followed by a partial penetration or a partial penetration followed by a complete penetration, $\Delta V = 50$ ft/s and $\Delta V = 25$ ft/s for additional reversals. The up-and-down method assumes that Pc(V) is monotonic; (i.e., Pc(V) is always increasing with increasing striking velocity in the velocity range of interest).

MIL-STD-662E *V_{50} Ballistic Test for Armor* [2] defines the ballistic limit, protection criteria, V_{50} BL(P), as follows.

> The V_{50} BL(P) may be defined as the average of an equal number of highest partial penetration velocities and the lowest complete penetration velocities that occur within a specified velocity spread. The normal up-and-down firing procedure is used. A 0.20-in (0.051 mm)-thick 2024 T3 sheet of aluminum is placed 6 ±0.5 in (152 ±12.7 mm) behind and parallel to the target to witness complete penetrations. Normally, at least two partial and two complete penetration velocities are used to complete the BL(P). Four-, six-, and 10-round ballistic limits are frequently used.

The maximum allowable velocity span is dependent on the armor material and test conditions. Maximum velocity spans of 60, 90, 100, and 125 ft/s (18, 27, 30, and 38 m/s) are frequently used.

With the advancement of armor material technology, especially with the combinations of ceramic front tile with composite backing plate, there have been many instances where V_{50} data obtained from different laboratories differ from one another. Furthermore, in ballistic tests conducted by the same laboratory, there were instances where the velocity spreads between the highest partial penetration and the lowest complete penetration exceeded the allowable limits. In order to validate the current MIL-STD-662E V_{50} test procedure, an interlaboratory ballistic test program was initiated by the U.S. Army Research Laboratory (ARL), Aberdeen Proving Ground (APG), MD, in 1993.

The objectives of this project were to (1) evaluate the reproducibility of the V_{50} data using the current MIL-STD-662E by conducting interlaboratory ballistic tests on benchmark armor materials and (2) propose modifications to MIL-STD-662E. This project was sponsored by the Joint Technical Coordinating Group on Aircraft Survivability (JTCG/AS). The JTCG/AS project identification number is V-3-A1, *Aircraft Armor Materials Testing and Data Reporting Standards Project.* This project was divided into two phases as follows.

Phase I (FY93–FY95): Nine laboratories, including four government laboratories (ARL/MD, Aviation Applied Technology Directorate [AATD], U.S. Air Force Wright Laboratory [AFWL], and Naval Surface Warfare Center [NSWC]), and five industrial laboratories, (H. P. White, Ceradyne, Simula, Southwest Research Institute [SRI], and University of Dayton Research Institute [UDRI]), participated in this project. These laboratories conducted V_{50} ballistic tests in accordance with MIL-STD-662E [2] on identical armor materials (material no. 1: VAR 4340 Steel, material no. 2: AD90 Al_2O_3/Kevlar) using identical 0.50 AP M2 bullets from the same lot. Results from Phase I [3] demonstrated the weakness of the MIL-STD-662E procedure for obtaining consistent V_{50} data from different laboratories. Statistical analysis of Phase I data suggested that Pc(V) for both 4340 steel and Al_2O_3/Kevlar armor plates were nonmonotonic. This nonmonotonic behavior implies that each laboratory might have sampled different parts of this Pc(V) by using slightly different starting velocities, thus obtaining significantly different V_{50} data. Other variations in experimental techniques from each laboratory, such as different fixtures for mechanically constraining the armor plates during impact, would increase the data scatter in the interlaboratory test even further.

Phase II (FY95–FY96): A new V_{50} test procedure was developed during this phase. This proposed V_{50} test procedure can sample both the monotonic and the nonmonotonic Pc(V) and was successfully verified in Phase II interlaboratory V_{50} ballistic tests that involved six laboratories, including three government laboratories (ARL/MD, AATD, and AFWL) and three industrial laboratories (Ceradyne, Simula, and SRI). The Phase II results are the main subject of this report.

In the following sections, we briefly summarize the Phase I results and identify the sources of discrepancy in Phase I data, describe efforts in confirming the nonmonotonic behavior of Pc(V) in Al_2O_3/Kevlar armor plates, and describe the development of the new velocity-selection rule for the V_{50} test by performing statistical model simulations. This new velocity-selection rule was field-tested by the six laboratories in the Phase II interlaboratory V_{50} ballistic tests, yielding consistent V_{50} data from the participants. We also discuss the physical processes responsible for the nonmonotonic behavior and evaluate the experimental techniques that contributed to the V_{50} data scatter. Finally, we provide a proposed V_{50} test procedure.

2. Review of Phase I Results

Two types of armor materials/systems were tested by nine laboratories in Phase I. The first type is the 0.175-in-thick VAR 4340 steel panels with a Rockwell C hardness of 48–52. The second type is the ceramic/composite panels with AD90 Al_2O_3 ceramic tile (5 in × 5 in × 0.535 in) backed by Kevlar-reinforced plastic (KRP) (15 in × 15 in × 18 ply). Each laboratory was given 3 VAR 4340 steel panels capable of sustaining 8 or 9 shots each, 16 Al_2O_3/Kevlar panels capable of sustaining a single shot, and enough 0.50-cal. AP M2 bullets that came from the same lot. The starting velocity of 1,600 ft/s was selected for both the steel panels and the ceramic/composite panels. The test engineer of each laboratory selected all subsequent velocities (by selecting the powder charges) according to the velocity-selection rule specified by MIL-STD-662E. All laboratories used variants of the up-and-down method to select the projectile velocities. Most of the participants focused their attention in the velocity range from 1,400 to 2,000 ft/s for both targets. After the V_{50} PBL had been satisfactorily determined, each laboratory used the remaining shots (usually two–four) to explore velocities outside the 1,400–2,000-ft/s range.

2.1 Ballistic Data and Penetration Probability. Figure 2 summarizes the ballistic data of VAR 4340 steel armor plates from all nine laboratories. It is clear that the velocity range where the partial and complete penetrations overlap, ZMR varies from laboratory to laboratory. For example, in Lab 6, the ZMR was as large as 600 ft/s, while in Lab 4, it was only about 150 ft/s. Therefore, the reported V_{50} data varied from laboratory to laboratory with little consistency. Figure 3 shows the ballistic data of Al_2O_3/Kevlar armor plates from all nine laboratories. Again, the ZMR varies from laboratory to laboratory and appears to be as large as 800 ft/s in Lab 9. By grouping the combined data from all nine laboratories into various velocity bins, the penetration probability for a given velocity bin, Pc(V), can be estimated by dividing the number of complete penetrations by the total number of shots in this velocity bin. Figure 4 shows the Pc(V) obtained from the combined data of VAR 4340 steel armor plates, and Figure 5 shows the Pc(V) obtained from the combined data of Al_2O_3/Kevlar armor plates. The solid squares in Figures 4 and 5 are observed penetration probability at various velocities. The solid curves are fitted to the data based on a proposed statistical model, which will be discussed in detail later. The observed penetration probability data in Figures 4 and 5 strongly suggest that the Pc(V) in these armor materials systems is nonmonotonic. In other words, the probability of complete penetration does not always increase with increasing velocity. In certain velocity ranges, Pc(V) actually decreases with increasing velocity.

2.2 Sources of Discrepancy. There are two kinds of discrepancy in this interlaboratory ballistic test, namely the extrinsic discrepancy vs. the intrinsic discrepancy. The extrinsic discrepancy could come from the difference in V_{50} ballistic test practice in each laboratory (such as difference in target fixture, velocity measurement and control, bullet pitch and yaw, and velocity selection). The intrinsic discrepancy, on the other hand, could come from the variation of material behaviors in bullets and armor panels as a function of bullet-striking velocity. This intrinsic discrepancy, if exists, should be observable even within each laboratory's own data. Careful examination of Figures 2 and 3 indicates that even within a single laboratory's data, the indication of a nonmonotonic Pc(V)

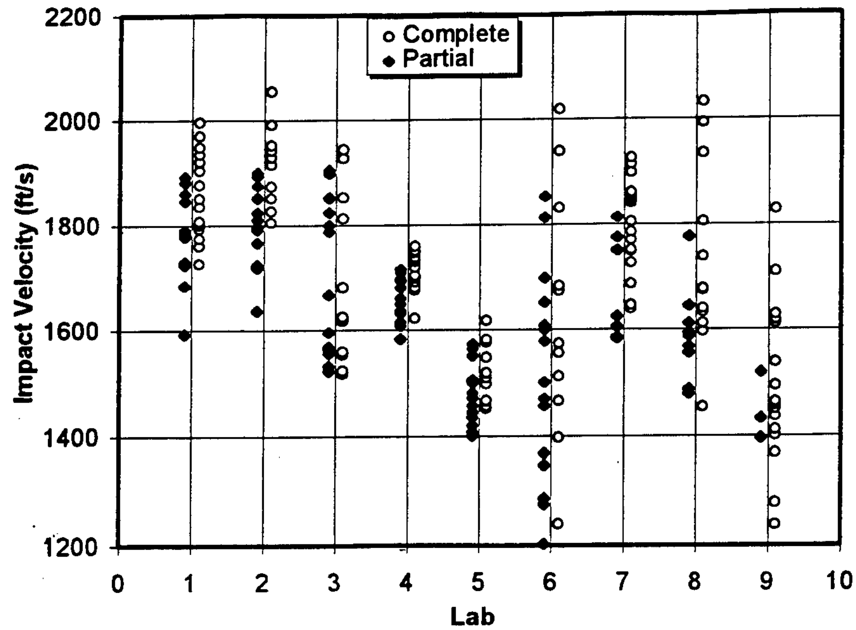

Figure 2. Summary of Phase I Ballistic Data of VAR 4340 Steel Armor Plates From All Nine Laboratories.

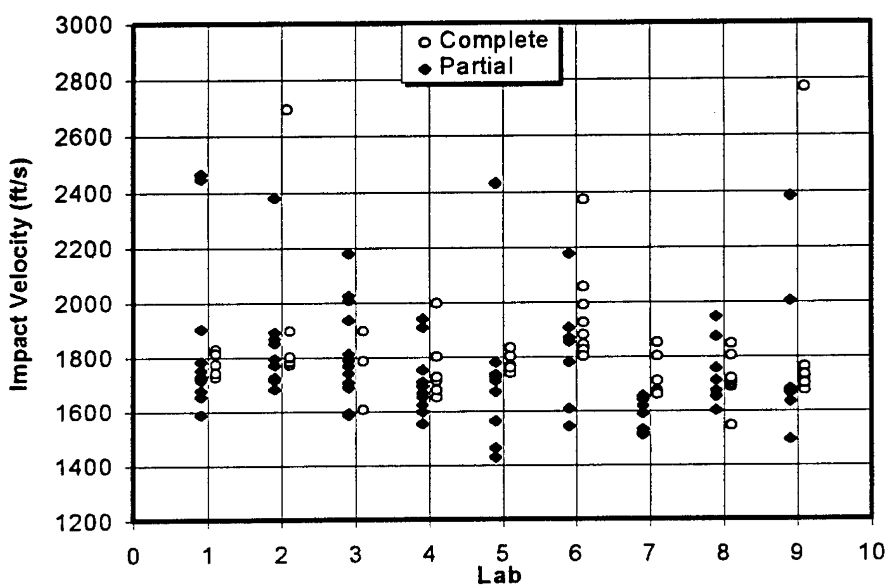

Figure 3. Summary of Phase I Ballistic Data of Al_2O_3 /Kevlar Armor Plates From All Nine Laboratories.

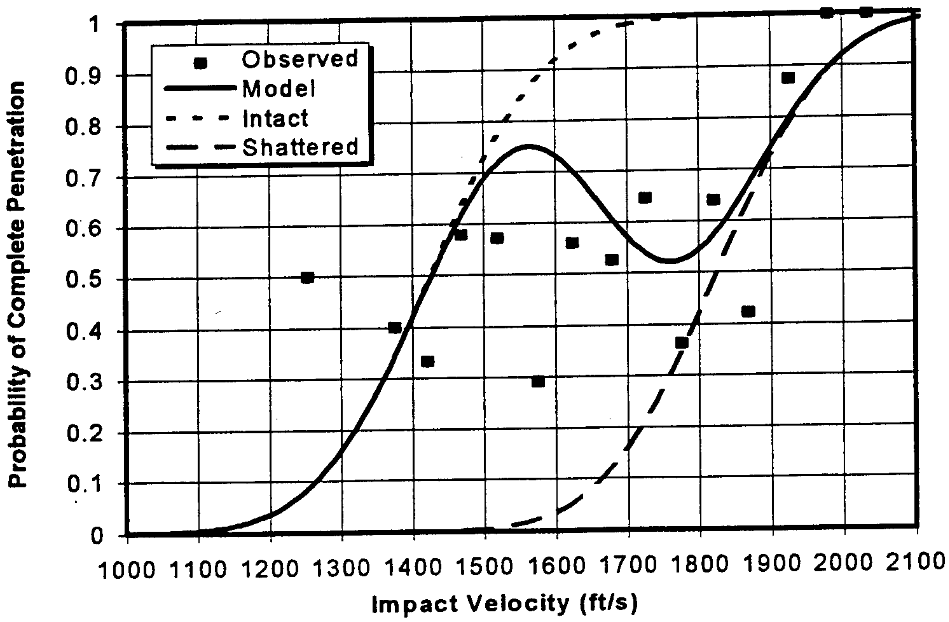

Figure 4. Penetration Probability of VAR 4340 Steel Armor Plates Obtained From Combined Phase I Data of All Nine Laboratories.

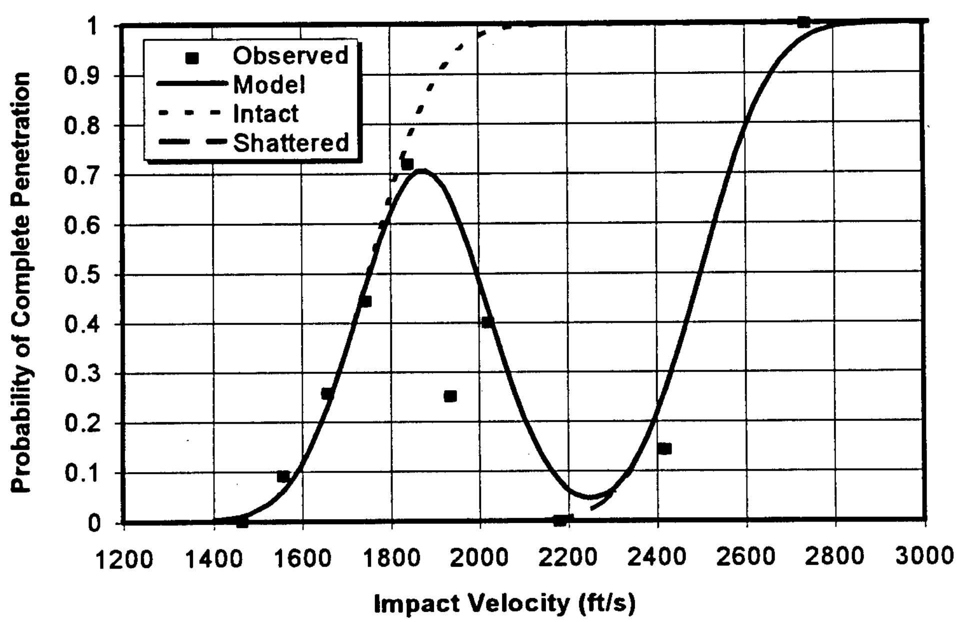

Figure 5. Penetration Probability of Al_2O_3 /Kevlar Armor Plates Obtained From Combined Phase I Data of All Nine Laboratories.

distribution is evident. Assuming the existence of a nonmonotonic Pc(V) distribution, it is postulated that the current velocity-selection rule could make each laboratory sample different parts of a nonmonotonic Pc(V) distribution. This intrinsic discrepancy, if not corrected, could lead to drastically different V_{50} results, overwhelming the effect of the extrinsic discrepancy.

Therefore, the strategies of the Phase II study were (1) to confirm the existence of a nonmonotonic penetration probability distribution, Pc(V) in Al_2O_3/Kevlar armor plates by conducting systematic ballistic tests at various velocities by a single laboratory, thus eliminating the possible effects of extrinsic discrepancy due to interlaboratory inconsistencies and bypassing the effect of the current velocity-selection rule; (2) to develop a new velocity-selection rule, suitable for both the nonmonotonic and the monotonic Pc(V) distributions, from statistical model simulations; and (3) to confirm the benefit of the new velocity-selection rule by conducting the Phase II interlaboratory V_{50} test.

3. Phase II Results

In the following sections, we present the experimental and analytical details of the confirmation of the nonmonotonic behavior of Pc(V), the direct observation of bullet shattering as a function of velocity, physical evidence of the shatter-gap behavior, the development of statistical models to describe this nonmonotonic Pc(V), the development of a new velocity-selection rule by Monte-Carlo simulation of the V_{50} test procedure, and the results of the Phase II interlaboratory V_{50} test (involving six laboratories) using this new velocity-selection rule for nonmonotonic Pc(V).

3.1 Confirmation of Nonmonotonic Penetration Probability Distribution. A new batch of 144 Al_2O_3/Kevlar armor plates, identical to the Phase I specifications, was procured. In collaboration with the U.S. Army Aberdeen Test Center (ATC), ARL/MD, conducted 69 ballistic shots to identify the Pc(V). The remaining 75 armor plates were used by five additional laboratories to confirm the new V_{50} test procedure. Using Phase I results as the guide, the velocity window of interest was identified to be from 1,400 to 2,800 ft/s. The velocity window was divided into 11 bins. The number of shots per bin was selected depending on the velocity. For velocity bins at which nonmonotonic behavior was anticipated, a greater number of shots were allocated to increase the confidence of the observations. Flash x-ray radiographs and yaw cards were used to confirm that the 0.50-cal. AP M2 bullet had acceptable pitch and yaw (less than 3°).

Figure 6 shows the penetration probability derived from the 69 shots. The solid squares in Figure 6 are observed penetration probabilities at various velocities. The upper and lower 80% confidence intervals associated with each observed probability are also shown as hollowed triangles and hollowed circles, respectively. The solid curve is based on a data-fitted statistical model that will be discussed later.

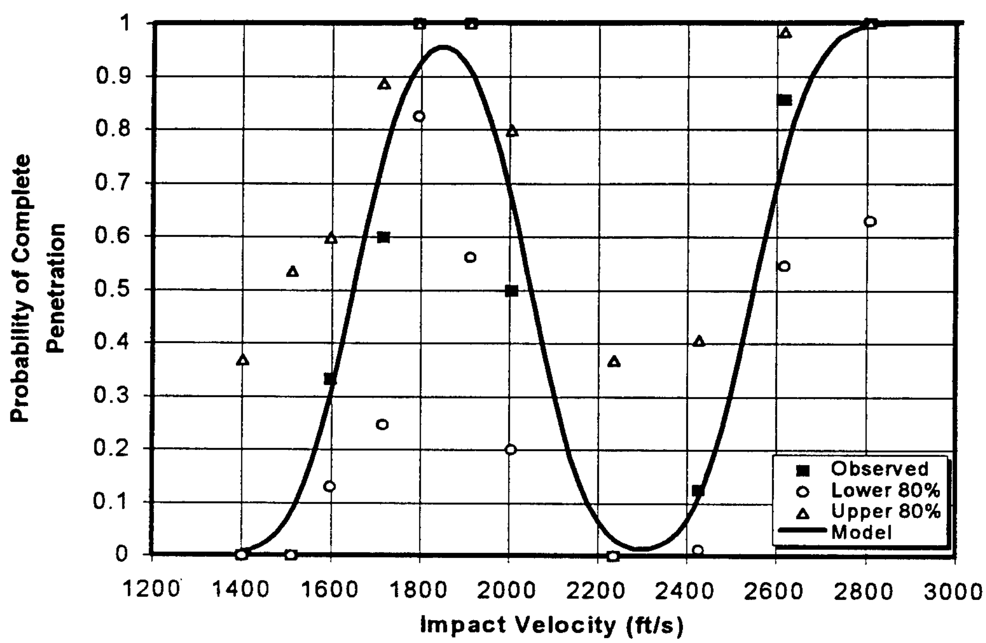

Figure 6. Penetration Probability of Al_2O_3 /Kevlar Armor Plates Obtained From 69 Shots in Phase II.

Table 1 lists the results of the 69 shots along with the calculated Pc(V) and 80% and 90% confidence intervals. The Pc(V) of a velocity bin at Vmean is simply the number of complete penetrations (C), divided by the total number of shots in this velocity bin.

Table 1. Results of 69 Ballistic Shots on Phase II $A1_2O_3$ /Kevlar Armor Plates

Velocity (ft/s)	P/C	Vmean (ft/s)	Pc(V)	Lower 80%	Upper 80%	Lower 90%	Upper 90%
1,371 1,391 1,411 1,422	P P P P	1,401.2	0/5	0	0.369	0	0.451
1,492 1,518 1,526	P P P	1,512	0/3	0	0.536	0	0.632
1,561 1,585 1,589 1,597 1,598 1,601 1,609 1,616 1,630	P P P P C C P C P	1,598.44	3/9	0.13	0.599	0.098	0.655
1,679 1,705 1,724 1,724 1,746	C C P C P	1,715.6	3/5	0.247	0.888	0.189	0.924
1,758 1,767 1,769 1,774 1,787 1,794 1,799 1,807 1,813 1,817 1,817 1,840	C C C C C C C C C C C C	1,795.17	12/12	0.825	1	0.779	1

Note: P - Partial
C - Complete

Table 1. Results of 69 Ballistic Shots on Phase II $A1_2O_3$ /Kevlar Armor Plates (continued)

Velocity (ft/s)	P/C	Vmean (ft/s)	Pc(V)	Lower 80%	Upper 80%	Lower 90%	Upper 90%
1,897	C	1,912	4/4	0.562	1	0.473	1
1,902	C						
1,920	C						
1,929	C						
1,954	P	2,004	3/6	0.201	0.799	0.153	0.847
1,966	C						
2,005	P						
2,019	P						
2,023	C						
2,057	C						
2,197	P	2,235.4	0/5	0	0.369	0	0.451
2,224	P						
2,240	P						
2,253	P						
2,263	P						
2,383	P	2,426.5	1/8	0.013	0.406	0.006	0.471
2,398	P						
2,403	P						
2,425	P						
2,426	P						
2,437	P						
2,454	P						
2,486	C						
2,597	C	2,618.14	6/7	0.547	0.985	0.479	0.993
2,605	C						
2,618	P						
2,621	C						
2,626	C						
2,626	C						
2,634	C						
2,747	C	2,807.2	5/5	0.631	1	0.549	1
2,816	C						
2,823	C						
2,824	C						
2,826	C						

Note: P - Partial
C - Complete

3.2 Observation of Shatter Gap. Flash x-ray was also used to record and measure residual bullet length from complete penetrations. For partial penetrations, the residual bullet was collected and the residual bullet length measured, whenever possible. Based on the residual bullet length measurement from both the partial and the complete penetrations, the bullet-shattering phenomenon, the shatter gap, was directly confirmed by this test. Figure 7 shows the shattering of 0.50-cal. AP M2 bullet after impacting Al_2O_3/Kevlar armor plates as a function of impact velocity. The dotted curve is the same data-fitted model Pc(V) as shown in Figure 6. Residual core lengths obtained from the partial and the complete penetrations are plotted separately as hollow squares and solid triangles, respectively.

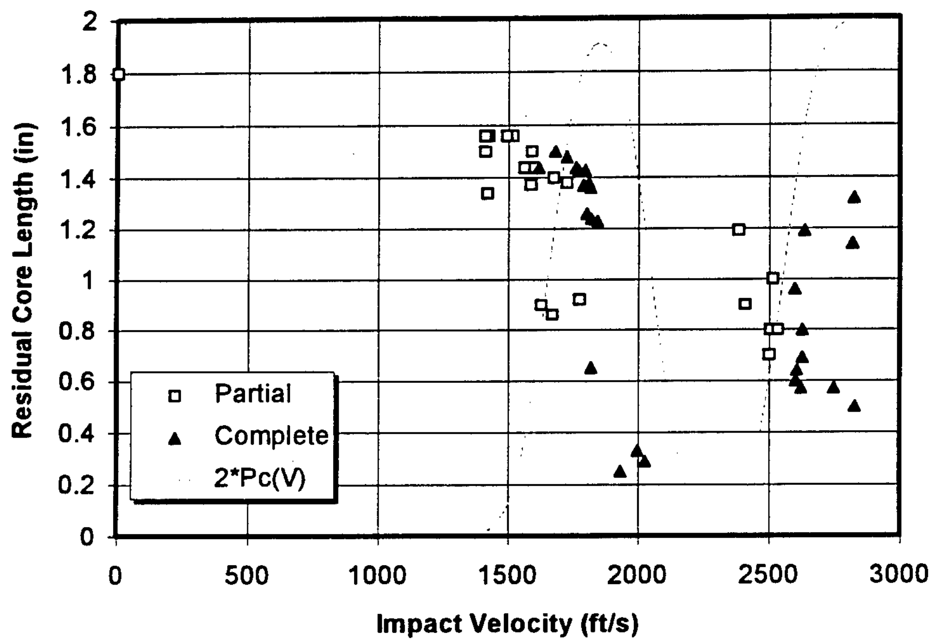

Figure 7. Shattering of 0.50-cal. AP M2 Bullets as a Function of Velocity After Impacting Al_2O_3 /Kevlar Armor Plates.

At velocities below 1,700 ft/s where partial penetrations were most probable, the residual core length reduced only slightly from the original 1.8 in by breaking the nose tip. At velocities between 1,700 and 2,000 ft/s, more bullet shattering (core length reduction) was observed with increasing velocity, and the probability of complete penetration began to decrease with increasing velocity. Between 2,000 and 2,400 ft/s, where no complete penetrations were observed, we did not recover any residual bullet in partial penetrations. At this velocity range, it was believed that the bullet was so severely shattered that it was extremely difficult to recover the residual bullet from partial penetrations. At velocities between 2,400 and 2,800 ft/s, the residual core length showed an increase, probably due to the large deformation of the Kevlar backing plate reducing the impact stress on the bullet. The probability of complete penetration increased with increasing velocity. This observed shatter-gap behavior is believed to be the physical processes responsible for the observed nonmonotonic behavior of Pc(V).

3.3 Statistical Models and Simulation. Adams and Zwissler [4] suggested that the nonmonotonic Pc(V) can be expressed in terms of a combination of three probability distributions such that

$$Pc(V) = (1 - P(Tm, Ts, V)) * P(Im, Is, V) + P(Tm, Ts, V) * P(Sm, Ss, V), \quad (1)$$

where

P(Im, Is, V) = probability of complete penetration by an intact bullet at V;
P(Sm, Ss, V) = probability of complete penetration by a shattered bullet at V;
P(Tm, Ts, V) = probability of transition between intact (0.0) and shatter (1.0) at V;
Im, Sm, Tm = mean of intact, shatter, and transition, respectively; and
Is, Ss, Ts = standard deviation of intact, shatter, and transition, respectively.

Equation (1) defines Pc(V) as a sum of two probability contributions. The first one is from having an intact bullet at V that completely penetrates the target, the (1 - P(Tm, Ts, V)) * P(Im, Is, V) term. The second one is from having a shattered bullet at V that completely penetrates the target, the P(Tm, Ts, V) * P(Sm, Ss, V) term. It was found that this model can adequately describe the observed nonmonotonic Pc(V) with appropriately chosen parameters. Table 2 shows the data-fitted parameters used to describe the observed Pc(V) of VAR 4340 Steel, Phase I Al_2O_3/Kevlar, and Phase II Al_2O_3/Kevlar armor plates.

Table 2. Data-Fitted Model Parameters From Observed Pc(V)

Armor Plates	Im (ft/s)	Is (ft/s)	Tm (ft/s)	Ts (ft/s)	Sm (ft/s)	Ss (ft/s)
VAR 4340 Steel	1,425	125	1,700	125	1,825	125
Phase I Al_2O_3/Kevlar	1,750	125	2,000	125	2,500	125
Phase II Al_2O_3/Kevlar	1,650	100	2,050	100	2,550	100

3.4 New Velocity Selection Rule. The current program offered a unique opportunity to re-examine various assumptions of the MIL-STD-662E V_{50} test procedure. The MIL-STD-662E selection of initial velocity at the "expected V_{50}" and the choice of reducing the step size after an observed reversal were based on the assumption of a monotonic Pc(V), such that the subsequent velocities would converge rapidly toward the "actual V_{50}." The MIL-STD-662E method of V_{50} calculation by averaging equal numbers of partial and complete penetrations around the ZMR was also based on the assumption that Pc(V) is always monotonic and Pc(V) is symmetrical around V_{50}. Since the assumption of a monotonic Pc(V) was nullified by the direct measurement of a nonmonotonic Pc(V) in Al_2O_3/Kevlar systems, modifications to various aspects of the V_{50} test procedure were studied in detail with Monte-Carlo simulations.

It was immediately evident that in a system with nonmonotonic Pc(V), the value of V_{50} was dictated by the initial velocity selection. Therefore, the only sensible alternative was to have two starting velocities, one from the low end of the velocity window of interest and going up and the other from the high end and coming down. In other words, one should perform two independent V_{50} tests and report both values of low-V_{50} and high-V_{50}. If the two values agreed within the uncertainty limits, the system had a monotonic Pc(V) and the V_{50} = (low-V_{50} + high-V_{50})/2. Otherwise, the system was nonmonotonic, and one should report both values. From the personnel protection point of view, the low-V_{50} value should be more significant; therefore, a greater number of shots should be allocated toward the low-V_{50} test. Compromise between statistical accuracy and conservation of resources led to the allocation of nine shots for the low-V_{50} test and six shots for the high-V_{50} test.

Important to both design and estimation is the selection of velocity step size. In the design, it governs what data are collected. In estimation, it impacts the V_{50} estimate. One danger of selecting a step size too small is that if Pc(V) is truly monotonic, the modest sample size (nine or six) might not permit the design to travel quickly enough to the lone V_{50}. When the step size is too large, the sequential design "walks" past the low-V_{50}. This causes the estimates for V_{50} to be more variable and biased toward the right.

Although by no means rigorous, we would suggest discussions along the following lines to set the step size, we suggest that the tester first guesses the lowest velocity, V_{low}, at which a complete penetration would be observed and then the highest practicable velocity, V_{high}, based on physical considerations of the gun, bullet, etc., at which a partial penetration would be observed. V_{low} and V_{high} would define the velocity window of interest. The velocity window of interest for the current system was determined from Phase I data to be between $V_{low} = 1{,}400$ and $V_{high} = 2{,}800$ ft/s. We also suggest that more shots be taken for the low-V_{50} than for high-V_{50}. In this project, we used nine shots for the low-V_{50} ($N_{low} = 9$) and six shots for the high-V_{50} ($N_{high} = 6$). In order to make sure that both V tests can sample a monotonic Pc(V) situated somewhere in the middle of the velocity window, we suggest that the selected step size would provide an overlap, $\Delta V_{overlap}$, in the two sampling ranges. Therefore, the step size ΔV can be roughly estimated to be

$$\Delta V = (V_{high} - V_{low} + \Delta V_{overlap})/(N_{low} + N_{high}), \qquad (2)$$

In the current test, we have $\Delta V = (2{,}800 - 1{,}400 + 100)/(9 + 6) = 100$ ft/s.

The intended velocity for the first shot of the low-V_{50} test was set at $V_1 = V_{low} + \Delta V$, and the subsequent eight intended velocities were set at $V_{i+1} = V_i + \Delta V$, if the previous shot showed a partial penetration and $V_{i+1} = V_i - \Delta V$ for complete penetrations. Similarly, the intended velocity for the first shot of the high-V_{50} test was set at $V_{10} = V_{high} - \Delta V$, and the subsequent five intended velocities were set at $V_{i+1} = V_i + \Delta V$ for partial penetrations and $V_{i+1} = V_i - \Delta V$ for complete penetrations. This is a modified up-and-down method without reducing the step size after an observed reversal. The reason for using the intended velocities (instead of the actual velocities) for selecting the next velocity was to average out the fluctuations in velocity control (the gun noise) from the actual velocity sequence. In simulations, the gun noise was assumed to have a maximum range of ±25 ft/s, and the random velocity fluctuations were assumed to be centered around each of the intended velocities.

The effect of velocity step size and of up-and-down velocity selection on the simulated V_{50} distribution was studied in detail. Figure 8 shows the effect of the velocity step size ($\Delta V = 50$, 100, and 200 ft/s) on the simulated low-V_{50} distribution, with the starting velocity $V_1 = 1{,}400 + 50$; $1{,}400 + 100$; and $1{,}400 + 200$ ft/s. Figure 9 shows the same for the simulated high-V_{50} distribution, with the starting velocity $V_{10} = 2{,}800 - 50$; $2{,}800 - 100$; and $2{,}800 - 200$ ft/s. These results indicated that the choice of step size between 50 and (through) 200 ft/s did not change the V_{50} distribution significantly.

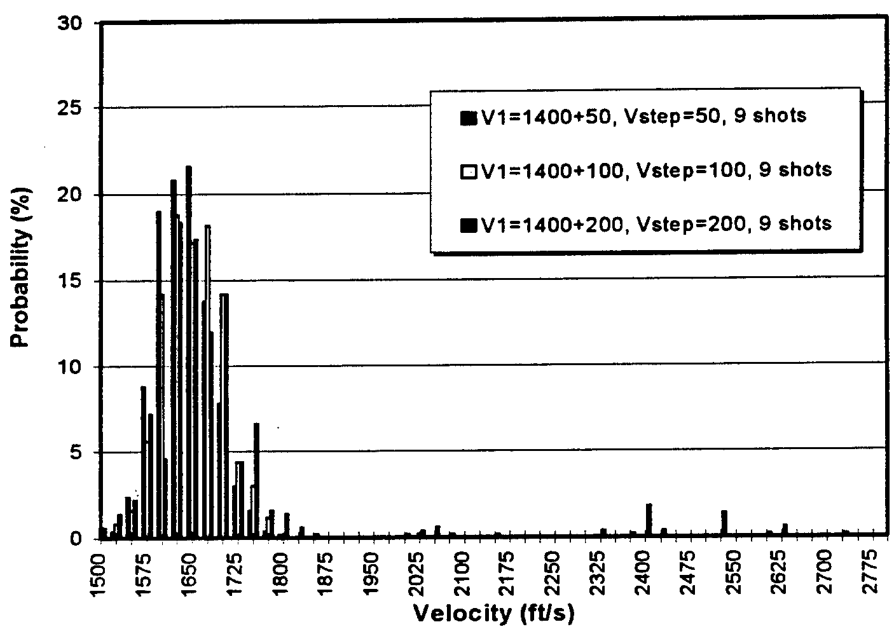

Figure 8. Effect of Velocity Step Size on the Simulated Low-V_{50} Distribution in Armor Plates With a Nonmonotonic Pc(V).

Figure 10 shows that the proposed V_{50} test procedure is also applicable to armor plates with an assumed monotonic Pc(V). The overlapping low-V_{50} and high-V_{50} distributions (with a step size of 100 ft/s) suggested that the actual $V_{50} = (\text{low-}V_{50} + \text{high-}V_{50})/2$.

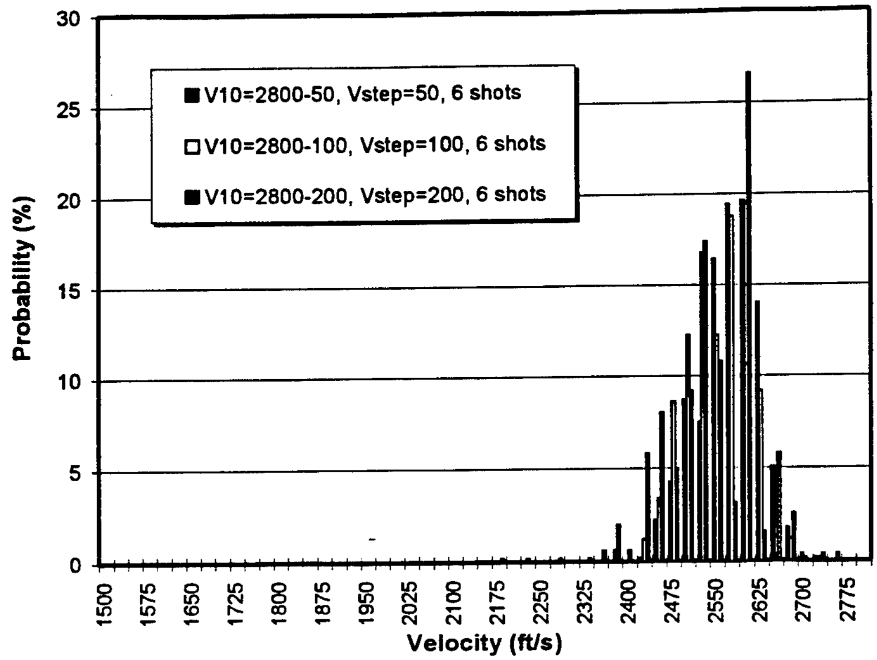

Figure 9. Effect of Velocity Step Size on the Simulated High-V_{50} Distribution in Armor Plates With a Nonmonotonic Pc(V).

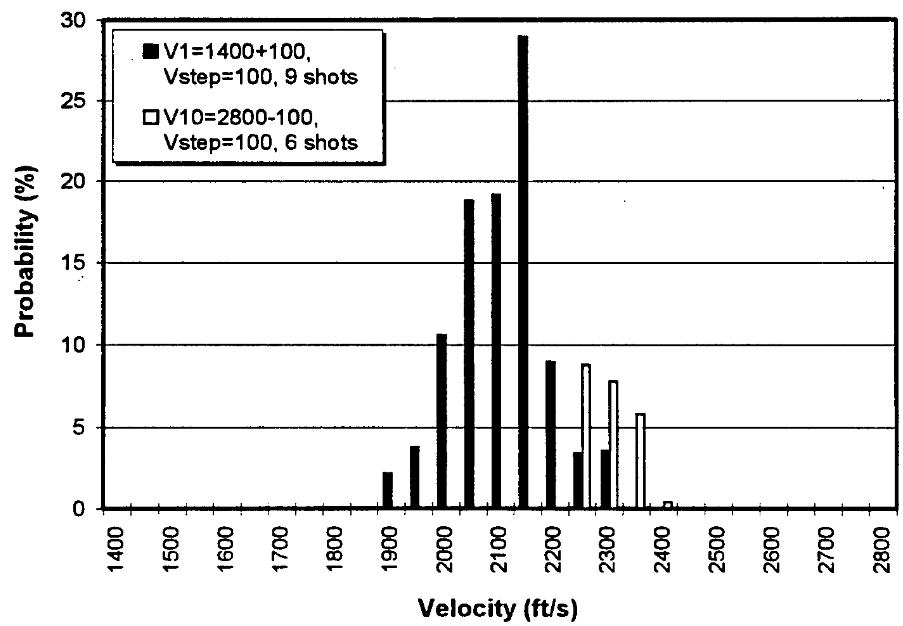

Figure 10. Overlapping of Low-V_{50} and High-V_{50} Distributions in Armor Plates With a Monotonic Pc(V).

The proposed V_{50} test procedure is summarized in Appendix A. This procedure was sent out to the participating laboratories for the Phase II interlaboratory V_{50} test program.

3.5 Modified Dixon's Maximum Likelihood Estimate. Since there is no guarantee that Pc(V) is symmetrical around ZMR in a nonmonotonic Pc(V), averaging equal numbers of partial and complete penetrations, as suggested by MIL-STD-662E, would not always give the best estimate of the V_{50} value. Instead, we have developed a modified Dixon's method for calculating the maximum likelihood estimate (MLE) of V_{50} (known as MDMLE).

The MLE method selects a V_{50} from a data set of complete and partial penetrations with their respective velocities that would make V_{50} most likely to have occurred. Estimation of low-V_{50} and high-V_{50} from two separate sets of data was accomplished through a modification to the usual MLE approach. Following Dixon's [5] approach, the standard deviation of the response curve was assumed to be known and equal to the fixed step size taken in the design. Under this modification, the conditions for estimate existence are only that at one complete penetration and one partial penetration are observed for each of the low-V_{50} and high-V_{50} test. A logistic response function was selected to locally describe the response curve about each V_{50}. The Newton-Raphson approximation method was used to carry out this estimation procedure. A FORTRAN code listing of program MDMLE is included in Appendix B.

3.6 Phase II Interlaboratory V_{50} Test. In the Phase II interlaboratory V_{50} test program, each laboratory followed the proposed V_{50} test procedure (Appendix A), instead of the MIL-STD-662E procedure. In addition to identical armor plates and bullets, the same identical target fixture was used by all six laboratories. The reported low-V_{50} and high-V_{50} values from each laboratory were calculated by ARL/MD using the identical MDMLE program. Table 3 lists the results of the Phase II interlaboratory V_{50} test from six laboratories. Figure 11 is the graphical representation of Table 3. The measured low-V_{50} and high-V_{50} values from each laboratory are shown as solid squares with an 'x.' The dotted lines represent the low-V_{50} and high-V_{50} values obtained from the model V_{50} values fitted to the observed Pc(V). The consistency among the interlaboratory V_{50} data is excellent, far superior than that obtained in Phase I using MIL-STD-662E. Figure 12 displays the observed V_{50} values with simulated distributions for these values under the model [1].

Table 3. Results of V_{50} Ballistic Test on Phase II $A1_2O_3$ /Kevlar Armor Plates From all Six Laboratories

Lab	Velocity	P/C	Low-V_{50}	Lab	Velocity	P/C	High-V_{50}
1	1,526	P	1,609	1	2,747	C	2,475
1	1,598	C		1	2,597	C	
1	1,518	P		1	2,454	P	
1	1,597	P		1	2,621	C	
1	1,679	C		1	2,486	C	
1	1,609	P		1	2,403	P	
1	1,705	C					
1	1,616	C					
1	1,492	P					
2	1,419	P	1,727	2	2,692	C	2,524
2	1,667	P		2	2,504	P	
2	1,784	C		2	2,596	C	
2	1,678	C		2	2,492	C	
2	1,625	P		2	2,406	P	
2	1,673	P		2	2,500	P	
2	1,773	P					
2	1,843	C					
2	1,778	C					
3	1,491	P	1,647	3	2,712	C	2,543
3	1,603	C		3	2,600	C	
3	1,502	P		3	2,511	P	
3	1,604	P		3	2,608	C	
3	1,730	C		3	2,532	P	
3	1,604	C		3	2,601	C	
3	1,474	P					
3	1,609	P					
3	1,703	P					
4	1,505	P	1,632	4	2,732	C	2,570
4	1,657	C		4	2,623	P	
4	1,523	P		4	2,680	C	
4	1,646	C		4	2,597	C	
4	1,447	P		4	2,495	P	
4	1,648	P		4	2,597	C	
4	1,738	C					
4	1,637	P					
4	1,656	C					

Note: P - Partial
C - Complete

Table 3. Results of V_{50} Ballistic Test on Phase II Al_2O_3 /Kevlar Armor Plates From All Six Laboratories (continued)

Lab	Velocity	P/C	Low-V_{50}	Lab	Velocity	P/C	High-V_{50}
5	1,396	P	1,682	5	2,663	C	2,428
5	1,625	P		5	2,519	P	
5	1,696	C		5	2,747	C	
5	1,613	P		5	2,642	C	
5	1,711	P		5	2,491	C	
5	1,810	C		5	2,415	C	
5	1,667	C					
5	1,621	P					
5	1,733	C					
6	1,513	P	1,605	6	2,700	C	2,277
6	1,596	P		6	2,593	C	
6	1,711	C		6	2,566	C	
6	1,607	C		6	2,407	C	
6	1,513	P		6	2,225	P	
6	1,529	P		6	2,358	C	
6	1,704	C					
6	1,630	C					
6	1,518	P					

Note: P - Partial
C - Complete

Results of this test confirmed the usefulness of the proposed V_{50} test procedure and provided the valuable operational experience for future implementation of this procedure at large.

4. Discussion

The current program had the benefit of a well-characterized Pc(V), which directly confirmed the shatter-gap behavior postulated by many previous investigators. Armed with a statistical model of this nonmonotonic Pc(V) and a Monte-Carlo simulation program, we can verify various strategies for V_{50} test and quantify the build-in uncertainties in V_{50} test.

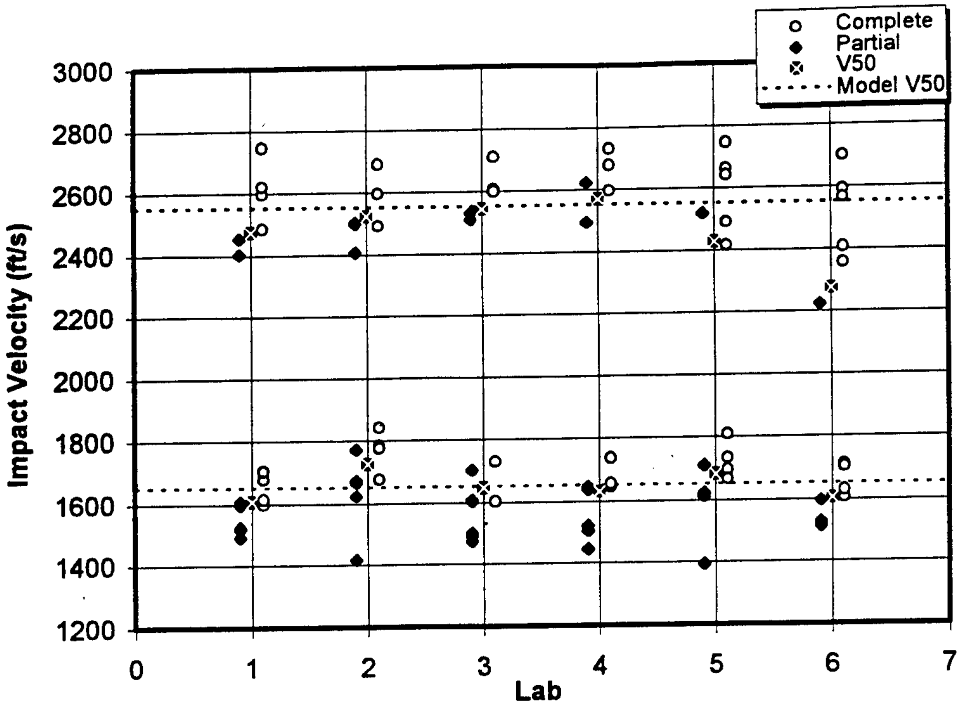

Figure 11. Summary of Phase II V_{50} Ballistic Test on Al_2O_3/Kevlar Armor Plates From All Six Laboratories.

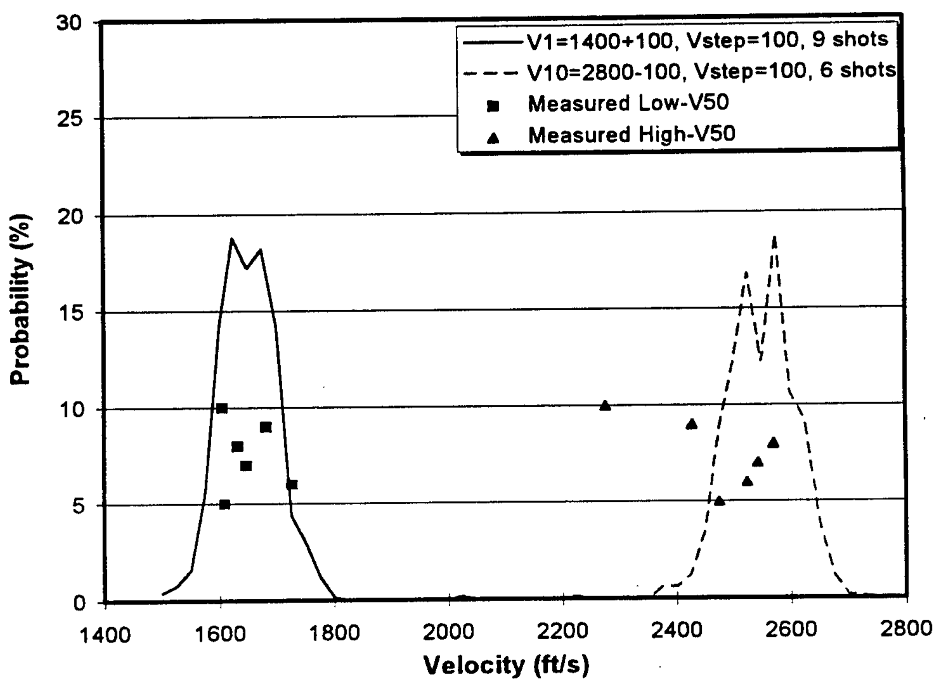

Figure 12. Measured Low-V_{50} and High-V_{50} From Phase II Interlaboratory Test on Al_2O_3/Kevlar Armor Plates as Compared to Simulations.

4.1 Shatter Gap. The exact physical-loading conditions and mechanisms for initiating extensive bullet shattering are not known at this point. More instrumented ballistic tests are needed. It was noted that as the bullet shattering progressed with increasing velocity, the Kevlar backing plate began to deform significantly, forming a big bulge without perforation for partial penetrations. The large deformation of the backing plate seemed to decrease the loading on the bullet and reduce the bullet shattering. At even higher velocities, more complete penetrations began to appear and the severely deformed backing plate began to form a hole by the residual bullet.

The existence of a nonmonotonic Pc(V) in armor systems has profound implications in regard to the armor system acceptance tests. Lack of complete penetrations at muzzle velocities does not guarantee that the system will not be penetrated at lower velocities. It is hoped that the proposed V_{50} test procedure will serve as the basis for a industry-adopted test standard.

4.2 V_{50} Data Scatter. Due to the small number of shots allocated and the build-in ±25 ft/s gun noise, the simulated low-V_{50} distribution (nine shots, $\Delta V = 100$ ft/s) was centered at 1,650 ft/s with a spread width of ~120 ft/s (measured between the 10th and the 90th quantile) while the simulated high-V_{50} distribution (six shots, $\Delta V = 100$ ft/s) was centered at 2,550 ft/s and had a spread width of ~140 ft/s. These spread widths represented the best accuracy in V_{50} values obtainable under ideal conditions. In reality, the velocity measurement apparatus and technique of various laboratories were seldom calibrated with each other. Some laboratories applied air-drag-correction procedures to the measured velocity, while others did not. Few laboratories used flash x-ray to monitor bullet pitch and yaw. Most used yaw cards instead to cut down cost. It was found that both ends of the gun barrel must be rigidly restrained to obtain consistently low pitch and yaw. In the current program, all six laboratories used the same target fixture that was used to generate the Pc(V). In future implementations, difference in target fixture geometry and attachment technique from each laboratory will undoubtedly contribute to additional data scatter. Perhaps the most significant source of interlaboratory data scatter came from the large-gun noise, which well exceeded the ±25 ft/s value used in simulations. Laboratories situated in dry-climate regions tend to get lower gun noise than those located in humid-climate regions.

5. Conclusions

Specifically, we conclude the following.

(1) The current MIL-STD-662E V_{50} test procedure was based on the assumption that the Pc(V) of armor/bullet systems was always monotonic.

(2) Direct measurement of Pc(V) in Al_2O_3/Kevlar systems confirmed that the Pc(V) was nonmonotonic.

(3) Observation of the bullet-shattering behavior as a function of velocity strongly suggested that the nonmonotonic Pc(V) in Al_2O_3/Kevlar systems was caused by the shatter-gap behavior.

(4) A proposed V_{50} test procedure was developed applicable for testing systems with either monotonic or nonmonotonic Pc(V).

(5) Results of Phase II interlaboratory V_{50} tests program using the proposed V_{50} test procedure showed a significantly reduced V_{50} data scatter as compared to Phase I interlaboratory data.

(6) To further reduce other sources of interlaboratory V_{50} data scatter, we recommend frequent velocity measurement calibration and certification, gun-noise reduction, bullet pitch and yaw measurement and reduction, and standardization of the target fixture, especially for ceramic/composite armor plates.

INTENTIONALLY LEFT BLANK.

6. References

1. Mascianica, F. "Ballistic Technology of Lightweight Armor." AMMRC TR 81-20, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, March 1981.

2. U.S. Army Research Laboratory. *V_{50} Ballistic Test for Armor.* MIL-STD-662E, Aberdeen Proving Ground, MD, January 1987.

3. Graves, J. H., and H. Kolev. "JTCG/AS Interlaboratory Ballistic Test Program." ARL-TR-755, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, June 1995.

4. Adams, M. A., and J. D. Zwissler. Private communication. Jet Propulsion Laboratory, Pasedena CA, March 1995.

5. Dixon, W. J. "The Up-and-Down Method for Small Samples." *American Statistical Association Journal*, pp. 967–978, December 1965.

INTENTIONALLY LEFT BLANK.

Appendix A:

Proposed V_{50} Test Procedure

INTENTIONALLY LEFT BLANK.

1. Total number of shots = 15.

2. Target fixture:
For ceramic/composite targets, it should be sandwiched tightly between 2 steel plates having the same outside dimensions as the target, a central widow at least 2 in larger than the ceramic tile dimensions and a contact area between the steel plates and the target at least 1.5 in wide. The target should be mounted to a rigid holder so that the bullet impacts at 0° obliquity.

3. Use a 0.020-in-thick 2024 aluminum sheet, ~6 in behind the armor, as a witness plate to record partial or complete penetration.

4. Test engineer should develop and use his/her own powder curve for bullets to control projectile velocity within ± 25 fps.

5. Test engineer should ensure low projectile pitch-and-yaw angle (below 3°) for the entire velocity window of interest and use yaw-card or flash x-ray to record pitch-and-yaw for each shot. Discard data from shots with excessive pitch-and- yaw (more than 10°).

6. Test engineer should determine the velocity window of interest: between V_{low} to V_{high}.

7. For low-V_{50} test:
number of shots = 9, $V_{i+1} = V_i + \Delta V$ (V_i = intended velocities, i = 1–9)
$V_1 = V_{low}$ + 100 fps (starting velocity)
ΔV = +100 fps (if previous shot is partial)
ΔV = −100 fps (if previous shot is complete)

8. For high-V_{50} test:
number of shots = 6, $V_{i+1} = V_i + \Delta V$ (V_i = intended velocities, i = 10–15)
$V_{10} = V_{high}$ - 100 fps (starting velocity)
ΔV = +100 fps (if previous shot is partial)
ΔV = −100 fps (if previous shot is complete)

9. Use a statistical computer program, MDMLE, to evaluate the values of Low-V_{50} and High-V_{50} from the firing data.

10. Testing engineer should prepare firing data sheets which should include firing sequence, impact velocities, partial/complete, yaw, low-V_{50} and high-V_{50} obtained.

INTENTIONALLY LEFT BLANK.

Appendix B:

FORTRAN Code Listing of MDMLE Program

INTENTIONALLY LEFT BLANK.

```
c
c This program provides a Modified Dixon Maximum Likelihood Estimate
c for V50.
c
      program mdmle
      character f1name*64
      dimension stress(100,2)
c
c Establish necessary inputs.
c
      call set(delta,nsample,stress,numit,epsilon,nonresp,f1name)
c
c Order the nonresponse velocities followed by the response velocities
c
      call sort(stress,nsample,nonresp)
c
c Compute the Modified Dixon Method MLE
c
      call mle(stress,v50,numit,epsilon,nsample,delta,nonresp)
c
c Display results
c
      call out(delta,nsample,stress,v50)
      stop
      end
c
c Subroutine Set
c
c This subroutine allows for data to be read in from a file
c or from the screen.
c
      subroutine set(delta,nsample,stress,numit,epsilon,nonresp,f1name)
      dimension stress(100,2)
      character f1name*64,dansa*64,dansb*64
      nonresp=0
      write(*,*) 'Name the output file'
      read(*,'(a)')f1name
      write(*,*) 'Name the output file'
      read(*,'(a)')f1name
      open(8,file=f1name,status='new')
      write(*,*)'Would you like to read the program information? y/n'
      read(*,'(a)')dansa
      if(dansa.eq.'y')then
      write(*,*)'This FORTRAN routine delivers a maximum likelihood'
      write(*,*)'estimate for the mean of the logistic response distr-'
```

```
  write(*,*)'ibution. This is a suitable substitute for the normal'
  write(*,*)'distribution over the .2 to .8 quantiles. Assumed is'
  write(*,*)'that the standard deviation of the response distr-'
  write(*,*)'ibution is known to be the "delta velocity change".'
  write(*,*)
  write(*,*)'Because of this assumption, the estimate will exist'
  write(*,*)'if at least one nonresponse and one response are'
  write(*,*)'present in the data. If not, a code of -9999 will be'
  write(*,*)'printed. Occasionally, the Newton-Raphson procedure'
  write(*,*)'will diverge. A code of -7777 is given when this'
  write(*,*)'occurs. If convergence has not been achieved in the'
  write(*,*)'number of iterations allowed by the user, a code of'
  write(*,*)'-8888 is given. Increase the number of iterations'
  write(*,*)'allowed or relax the epsilon for convergence to '
  write(*,*)'correct this.'
  write(*,*)
  write(*,*)'Hit enter to continue.'
  read(*,'(a)')dansb
  write(*,*)
  write(*,*)'There is no attempt on the part of the author to'
  write(*,*)'pass this as polished software. It is not. It is,'
  write(*,*)'however, functional and will allow the user to'
  write(*,*)'quickly and easily get a maximum likelihood estimate'
  write(*,*)'for the V50 instead of using the rough approximation'
  write(*,*)'that involves averaging stimulus levels.'
  write(*,*)
  write(*,*)'Dr. Barry A. Bodt'
  write(*,*)'Army Research Lab'
  write(*,*)'babodt@arl.army.mil'
  write(*,*)'4 June, 1996'
  write(*,*)
  else
  endif
  write(*,*)
  write(*,*)'Enter the delta velocity change.'
  read(*,*)delta
  write(*,*)'Enter the number of number of shots taken.'
  read(*,*)nsample
  write(*,*)'Enter each velocity, followed by a response (1)'
  write(*,*)'or a nonresponse (0).'
  do 10 j=1,nsample
  write(*,*)'test ',j
  read(*,*)stress(j,1),stress(j,2)
  if(stress(j,2).eq.0)nonresp=nonresp+1
10 continue
```

```
      write(*,*)
      write(*,*)'How many iterations are to be allowed for the'
      write(*,*)'Newton-Raphson algorithm and what is the epsilon'
      write(*,*)'for convergence? Enter the number of iterations'
      write(*,*)'followed by the convergence criterion.'
      read(*,*)numit,epsilon
      return
      end
c
c Subroutine Sort
c
c This routine is just a simple bubble sort to order the data in
c the stress array according to increasing velocity within response
c
      subroutine sort(stress,nsample,nonresp)
      dimension stress(100,2)
c
c     First order the data according to response type with
c     nonresponse (0) and response (1).
c
      do 30 i=1,nsample-1
      do 20 j=i+1,nsample
      if(stress(j,2).lt.stress(i,2))then
      do 10 k=1,2
      temp=stress(i,k)
      stress(i,k)=stress(j,k)
      stress(j,k)=temp
   10 continue
      endif
   20 continue
   30 continue
c
c     Next order the nonresponse data according to velocity.
c
      do 60 ii=1,nonresp-1
      do 50 jj=ii+1,nonresp
      if(stress(jj,1).lt.stress(ii,1))then
      do 40 kk=1,2
      temp=stress(ii,kk)
      stress(ii,kk)=stress(jj,kk)
      stress(jj,kk)=temp
   40 continue
      endif
   50 continue
   60 continue
```

```
c
c   Finally order the response data according to velocity.
c
      do 90 iii=nonresp+1,nsample-1
      do 80 jjj=iii+1,nsample
      if(stress(jjj,1).lt.stress(iii,1))then
      do 70 kkk=1,2
      temp=stress(iii,kkk)
      stress(iii,kkk)=stress(jjj,kkk)
      stress(jjj,kkk)=temp
 70   continue
      endif
 80   continue
 90   continue
      return
      end
c
c Output of initial conditions
c
      subroutine out(delta,nsample,stress,v50)
      dimension stress(100,2)
      write(8,*)
      write(*,*)
      write(8,100)delta,nsample
      write(*,100)delta,nsample
      write(8,*)
      write(*,*)
      write(8,200)
      write(*,200)
      do 10 j=1,nsample
      write(8,300)(stress(j,k),k=1,2)
      write(*,300)(stress(j,k),k=1,2)
 10   continue
      write(8,*)
      write(*,*)
      write(8,400)v50
      write(*,400)v50
      return
      100 format(1x,'delta = ',f8.3,5x,'shots = ',i8)
      200 format(1x,'velocity response')
      300 format(1x,f8.3,6x,f4.2)
      400 format(1x,'mle v50 estimate = ',f8.3)
      end
c
      subroutine mle(stress,v50,numit,epsilon,nsample,delta,nonresp)
```

```
      dimension stress(100,2)
c
c check to see that the estimate exists (at least one
c one nonresponse and one response
c
      nresp=nsample-nonresp
      if(nonresp.eq.nsample.or.nresp.eq.nsample)then
      v50=-9999
      return
      endif
c
c With the logistic model f(alpha+beta*x), the standard
c deviation is related to beta through the equation
c beta=pi/(sigma*sqrt(3)). In our model, the standard
c deviation is being taken to be the step value for the
c data collection, hence the next line of code.
c
      beta=3.141592/(delta*sqrt(3))
c
c establish a reasonable starting value for the Newton-
c Raphson procedure using the average of the lowest complete
c and the highest partial. Then convert that v50 start to
c the parameters for the f(alpha + beta*X) formulation to
c form alpha0 for the Newton-Raphson.
c
      xmu0=(stress(nonresp,1)+stress(nonresp+1,1))/2
      alpha0=-beta*xmu0
      alphnm1=alpha0
      do 20 k=1,numit
      xnumer=0
      xdenom=0
c
c Compute the update for the Newton Raphson approximation.  It
c involves the computation of probabilities from the logistic
c distribution.  The update will be called xmargin.
c
      do 10 j=1,nsample
      dist=1/(1+exp(-alphnm1-beta*stress(j,1)))
      xnumer=xnumer+stress(j,2)-dist
      xdenom=xdenom+dist*(1-dist)
   10 continue
c
c Because of the instability of the Newton-Raphson in this
c situation, it is possible for the xdenom above to be very
c close to zero.  This will occur when the "alpha n minus 1"
```

```
c value, the current parameter estimate, is very far from
c the actual root. Then the probabilites will be computed
c for velocity levels out in the tail of the distribution.
c Either dist or 1-dist is likely to produce zeros in that
c instance. A good starting value is a hedge against this.
c The shape of the function for which the root is desired
c is such that if you are out a little too far, the algorithm
c will diverge, not converge. This is likely to be first
c manifested in a zero denominator.
c
      if(xdenom.eq.0)then
      v50=-7777
      return
      endif
c
c Update the parameter alpha. Check to see if it satisfies the
c convergence criterion. If it does, compute the mean based on
c the two parameters of f(alpha + beta*X)
c
      xmargin=xnumer/xdenom
      alphnm1=alphnm1+xmargin
      xtest=abs(xmargin)
      if(xtest.lt.epsilon)then
      alpha=alphnm1
      v50=-alpha/beta
      return
      endif
 20   continue
c
c If convergence is so slow that it cannot occur in the specified
c number of iterations, assign it a code of -8888. This is unlikely
c to occur, but is possible.
c
      v50=-8888
      return
      end
```

NO. OF COPIES	ORGANIZATION
2	DEFENSE TECHNICAL INFORMATION CENTER DTIC DDA 8725 JOHN J KINGMAN RD STE 0944 FT BELVOIR VA 22060-6218
1	HQDA DAMO FDQ DENNIS SCHMIDT 400 ARMY PENTAGON WASHINGTON DC 20310-0460
1	CECOM SP & TRRSTRL COMMCTN DIV AMSEL RD ST MC M H SOICHER FT MONMOUTH NJ 07703-5203
1	PRIN DPTY FOR TCHNLGY HQ US ARMY MATCOM AMCDCG T M FISETTE 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	PRIN DPTY FOR ACQUSTN HQS US ARMY MATCOM AMCDCG A D ADAMS 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	DPTY CG FOR RDE HQS US ARMY MATCOM AMCRD BG BEAUCHAMP 5001 EISENHOWER AVE ALEXANDRIA VA 22333-0001
1	DPTY ASSIST SCY FOR R&T SARD TT T KILLION THE PENTAGON WASHINGTON DC 20310-0103
1	OSD OUSD(A&T)/ODDDR&E(R) J LUPO THE PENTAGON WASHINGTON DC 20301-7100

NO. OF COPIES	ORGANIZATION
1	INST FOR ADVNCD TCHNLGY THE UNIV OF TEXAS AT AUSTIN PO BOX 202797 AUSTIN TX 78720-2797
1	USAASA MOAS AI W PARRON 9325 GUNSTON RD STE N319 FT BELVOIR VA 22060-5582
1	CECOM PM GPS COL S YOUNG FT MONMOUTH NJ 07703
1	GPS JOINT PROG OFC DIR COL J CLAY 2435 VELA WAY STE 1613 LOS ANGELES AFB CA 90245-5500
1	ELECTRONIC SYS DIV DIR CECOM RDEC J NIEMELA FT MONMOUTH NJ 07703
3	DARPA L STOTTS J PENNELLA B KASPAR 3701 N FAIRFAX DR ARLINGTON VA 22203-1714
1	USAF SMC/CED DMA/JPO M ISON 2435 VELA WAY STE 1613 LOS ANGELES AFB CA 90245-5500
1	US MILITARY ACADEMY MATH SCI CTR OF EXCELLENCE DEPT OF MATHEMATICAL SCI MDN A MAJ DON ENGEN THAYER HALL WEST POINT NY 10996-1786
1	DIRECTOR US ARMY RESEARCH LAB AMSRL CS AL TP 2800 POWDER MILL RD ADELPHI MD 20783-1145

NO. OF COPIES	ORGANIZATION
1	DIRECTOR US ARMY RESEARCH LAB AMSRL CS AL TA 2800 POWDER MILL RD ADELPHI MD 20783-1145
3	DIRECTOR US ARMY RESEARCH LAB AMSRL CI LL 2800 POWDER MILL RD ADELPHI MD 20783-1145
	ABERDEEN PROVING GROUND
4	DIR USARL AMSRL CI LP (305)

NO. OF COPIES	ORGANIZATION
1	OUSD DR&E THE PENTAGON WASHINGTON DC 20301
2	OSD DIR OF LIVE FIRING TESTING O'BRYON JULIAN ROOM 3E1060 THE PENTAGON WASHINGTON DC 20301-3110
1	DIR DEFNS INTLLGNC AGCY PAQ 4B K CRELLING WASHINGTON DC 20340-6053
3	US SECRET SERVICE TECH DEV & PLANNING DIV T THOMAS 1800 G ST NW WASHINGTON DC 20223
3	CDR HQ NASC AIR 418 LTC J LAWLESS JR J JOLLEY J ALDRIDGE 1421 JEFFERSON DAVIS HWY ARLINGTON VA 22243-5160
2	CDR DEFNS TECHL INFO CTR CAMERON STATION BLDG 5 DTIC FDAC 5010 DUKE ST ALEXANDRIA VA 23304-6145
1	CDR ARO INFO PROCESSING OFC PO BOX 12211 RESEARCH TRIANGLE PARK NC 27709-2211
1	CDR US AMC 5001 EISENHOWER AVE ALEXANDRIA VA 22333
1	CDR US ARMY MIS CMND AMSMI RD CS R/DOC REDSTONE ARSENAL AL 35809

NO. OF COPIES	ORGANIZATION
5	CDR USARMY ARDEC SATNC MI TECH LIB SATNC ITF J WARD M MAFFEO P CUNNIFF NATICK MA 01760-5010
1	CDR US ARMY SAT COMMCNT AGCY TECH DOCUMENT CTR FORT MONMOUTH NJ 07703
3	CDR USA TACOM AMSTA ZSK AMSTA TSL TECH LIB AMSTA TSK S GOODMAN WARREN MI 48397-5000
1	PRESIDENT AIRBORNE ELECTRONICS & SPECIAL WARFARE BOARD LIBRARY FORT BRAGG NC 28307
1	CDR DUGWAY PRVNG GRND TECH LBRY DUGWAY PROVING GROUND UT 84022
4	DIR USA AMCCOM BENET WEAPONS LAB AMSMC LCB TL AMSMC LCB R AMSMC LCB RM AMSMC LCB RP WATERVLIET NY 12189
2	CDR NGIC ATTN W MARLEY CPT T POWERS 220 7TH AVE CHARLOTTESVILLE VA 22901-5396
1	CDR US ARMY AEROMEDICAL RSRCH UNIT TECH LBRY PO BOX 577 FORT RUCKER AL 36360

NO. OF COPIES	ORGANIZATION
1	US ARMY AVN TRAIN LBRY BLDG 5906 5907 FORT RUCKER AL 36360
1	CDR US ARMY AGCY FOR AVN SFTY TECH LBRY FORT RUCKER AL 36362
1	DIR COMBAT DEV US ARMY AVN CTR ATZQ CD R MCCABE FORT RUCKER AL 36362-5000
1	CDR CLARKE ENGRG SCHL LBRY 3202 NEBRASKA AVE NORTH FORT LEONARD WOOD MO 65473-5000
1	CDR US ARMY ENGRG WATERWAYS EXPERIMENT STATION RSRCH CTR LBRY PO BOX 631 VICKSBURG MS 39180
1	COMDT US ARMY QTRMSTR SCHL QTRMSTR SCHL LBRY FORT LEE VA 23801
1	DEPT OF ARMY AEROSTRUCTURES MS 266 SU ARMY AVIATION R&T ACTIVITY AVSCOM LANGLEY RSRCH CTR HAMPTON VA 23665-5225
1	CDR NASA LANGLEY RSRCH CTR HAMPTON VA 23665-5225
1	CDR US ARMY VEHIC PROPULSION NASA LEWIS RSRCH CTR AMSRL VP 2100 BROOK PARK RD CLEVELAND OH 44135-3191
1	CDR US ARMY COMMCT & ELECT CMND TECH LBRY FORT MONMOUTH NJ 07703

NO. OF COPIES	ORGANIZATION
4	CDR US ARMY AVN & TROOP CMND AVN RSRCH & TECH ACTVTY APPLIED TECH DIR AMSAT R TV G MCALLISTER L BURROWS H HOLLAND F KEESEE FORT EUSTIS VA 23604-5577
11	CDR US ARMY AVN & TROOP CMND SFAE AV ASH COL J HUEY SFAE AV MG D IRBY JR SFAE AV SOA LTC M ROGERS SFAE AV AAH COL S DELOACH SFAE AV BH LTC J LANIER SFAE AV AEC COL T REINKOBER SFAE AV RAH BG O MULLEN SFAE AV CH COL R WILLIAMS SFAE AV RAH TV M SMITH AMSAT R ESC G KOVACS AMSAV EFM DR K BHANSALI 4300 GOODFELLOW BLVD ST LOUIS MO 63120-1798
1	CDR NAVAL RSRCH LAB CODE 6384 WASHINGTON DC 20375
1	CHIEF OF NAVAL RSRCH CODE 471 ARLINGTON VA 22217
5	CDR NSWC DR B SMITH CODE G22 2 CP E ROWE CODE G22 2 CP R WALTERS CODE G22 DAHLGREN VA 22448
4	CDR NWC R HORTON C218 J BATES C31801 J DUZAN C2183 L BUDD C2183 CHINA LAKE CA 93555-6001
1	CDR NVL POSTGRADUATE SCHL PROF R BALL CODE 67BP MONTEREY CA 93943

NO. OF COPIES	ORGANIZATION
1	OFC OF NAVAL RSRCH MATERIALS ENGRG DIV G YODER CODE 1311 800 N QUINCY ST ARLINGTON VA 22217-5000
2	CRD NAVAL AIR ENGRG CTR K MEGERLE SESD CODE 5314 KM G FISHER SESD CODE MT14 LAKEHURST NJ 08733
5	CRD NAVAL AIR SYSTEMS CMND AIR 434 S BETTADUPUR J COLLINS H VARMALL J THOMPSON W KOEGEL JEFFERSON PLAZA ONE WASHINGTON DC 20361-5300
1	CDR NAVAL AIR SYSTEMS CMND AIR 447 L SLOTER JEFFERSON PLAZA ONE WASHINGTON DC 20361-5300
2	CDR NAWC AIRCRAFT DIV J KOZEL CODE 6063 TECH LIB WARMINSTER PA 18974-5000
1	CDR US AIR FORCE WRIGHT LAB A KURTZ BLD 63 1901 10TH ST WRIGHT PATERSON AFB OH 45433-7605
1	CDR US AIR FORCE WRIGHT LAB LIZZA BLD 45 2130 8TH ST STE 11 WRIGHT PATTERSON AFB OH 45433-7552

NO. OF COPIES	ORGANIZATION
2	CDR US AFW RSRCH & DEV CTR WRDC MLLP M FOMEY JR WRDC MLBC S SCHULMAN WRIGHT PATTERSON AFB OH 45433-6523
3	CDR US AIR FORCE WRIGHT LAB WL MLSE C HARMSWORTH WL MLSE J COATE WL MLLM J PETRAK WRIGHT PATTERSON AFB OH 45433-6533
1	46TH OG OGM W DYESS 205 WEST D AVE STE 241 EGLIN AIR FORCE BASE FL 32542-5000
1	ASC YHR LTC SULLIVAN EGLIN AIR FORCE BASE FL 32542-5000
2	US DEPT OF COMMERCE NATL INST OF STAND & TECH S HSU CHIEF CERAMICS DIV DR M VANGEL BLDG 101 RM A337 GAITHERSBURG MD 20899-0001
7	CDR US MILITARY ACADEMY DEPT OF MATH SCIENCES CPT H KOLEV 5 CP DEPT HEAD 2 CP WEST POINT NY 10996-1792
1	CRD US MILITARY ACADEMY DEPT OF CIVIL & MECH ENGRG PROF E LENOE WEST POINT NY 10996-1792
1	MARTIN MARIETTA LAB J GREEN 1450 S TOLLING RD BALTIMORE MD 21227-3898

NO. OF COPIES	ORGANIZATION
2	BOEING DEFENSE & SPACE GROUP HELICOPTER DIV P MCINTYRE MS P38 21 B THOMPSON MS P38 50 PO BOX 16856 PHILADELPHIA PA 19142-0858
2	BOEING HELICOPTERS N CARAVASOS J LYONS PO BOX 16858 MS P30 27 PHILADELPHIA PA 19142-0858
1	BOEING DEFNS & SPACE GROUP E WILHELM MS 4C 67 PO BOX 3707 SEATTLE WA 98124-2207
1	BOEING AEROSPACE CO BOEING MLTRY AIRPLANE DEV CHIEF OF MATERIALS & PROCESSES PO BOX 3999 SEATTLE WA 98124
1	TEXTRON LYCOMING ADVANCED MATERIALS TECH LAB V NANGIA 550 S MAIN ST STRATFORD CT 06497-2452
1	MCDONNELL DOUGLAS MISSILE SYST M/C 1063246 D CHONG PO BOX 516 ST LOUIS MO 63166-0516
2	VOUGHT CORP CHIEF OF MATERIALS & PROCESSES D PETERSON ADVNCD TECH CTR PO BOX 226144 DALLAS TX 75222
7	GRUMMAN AEROSPACE CORP CHIEF OF MATERIALS & PROCESSES P ADLER MS A02 26 S DEMAY PL12 DEPT 441 J KENNEDY MS A02 26 P SHAW MS A04 12 J GREENSPAN MS A04 12 L BOHANIC BETHPAGE NY 11714

NO. OF COPIES	ORGANIZATION
4	LOCKHEED ARNTCL SYST CO D CHELLMAN CHIEF OF MATERIALS & PROCESSES J WHITEHEAD 86 S COBB DR ZONE 150 MARIETTA GA 30063-0199
1	LOCKHEED ARNTCL SYST CO D RICHARDSON DEPT 73 C2 86 S COBB DR ZONE 0199 MARIETTA GA 30063-0199
1	FOSTER MILLER INC M SMIRLOCK 350 SECOND AVE WALTHAM MA 02154-1196
1	DIR LLNL UNIV OF CA CHEM & MTRL SCIENCE DEPT T SHELL PO BOX 808 L 369 LIVERMORE CA 94551
1	CREUSOT MARREL INC B HOLCOMBE 724 W LANCASTER AVE STE 213 WAYNE PA 19087
3	THE SURVICE ENGRG CO R LEVY G THOMPSON D VAN DUSEN 1003 OLD PHILADELPHIA RD STE 103 ABERDEEN MD 21001
1	CERCOM INC R PALICKA 1960 WATSON WAY PO BOX 70 VISTA CA 92083
2	CARBORUNDUM J RUPPEL R PALIA PO BOX 1054 NIAGARA FALLS NY 14302-1054

NO. OF COPIES	ORGANIZATION
1	D ATKINSON 7703 CARRLEIGH PARKWAY SPRINGFIELD VA 22152
1	P ZABEL ROUTE 1 BOX 397D DEVINE TX 78016
1	ALLIED SIGNAL CORP ARMOR BUSINESS GROUP D LANG PO BOX 31 PETERSBURG VA 23804
1	ALLIED SIGNAL CORP TECH CONTRACTING M CHOBRDA PO BOX 1021 MORRISTOWN NJ 07962-1021
1	THE ANALYTIC SCIENCES CORP FLORIDA OFFICE COMBS 1992 LEWIS TURNER BLVD FORT WALTON BEACH FL 32547
1	TITAN RSRCH & TECH D ORPHAL 5117 JOHNSON DR PLEASANTON CA 94588
1	W VIKESTAD 3007 ROLLING GREEN DR CHURCHVILLE MD 21028
1	BALLISTIC TECH INC J ROBINSON 23922 BROADHORN DR LAGUNA NIGUEL CA 92677
1	BLAZE TECH CORP N MOUSSA 21 ERIE ST CAMBRIDGE MA 02139
1	UNIV OF TEXAS INST FOR ADVANCED TECH BLESS 4030 2 WEST BRAKER LN AUSTIN TX 78759-5329

NO. OF COPIES	ORGANIZATION
1	UNIV OF DENVER COLORADO SMR DENVER RSRCH INST ULLYATT BW 227 PO BOX 100758 DENVER CO 80250
1	BOOZ ALLEN & HAMILTON INC J VICE 4141 COL GLENN HWY STE 131 WRIGHT PATTERSON AFB OH 45433-7542
1	DIR SURVIVABILITY VULNERABILITY INFO ACTVTY CTR 2130 EIGHTH ST STE 1 WRIGHT PATTERSON AFB OH 45433-7542
1	DR J COWIE 70 ISLETA DR WHITE ROCK NM 87544
1	CTR FOR NAVAL ANALYSIS 4401 FORT AVE PO BOX 16268 ALEXANDRIA VA 22302-0268
2	JET PROPULSION LAB CA INST OF TECH DR M ADAMS DR J ZWISSLER 4800 OAK GROVE DR PASADENA CA 91109
3	SIMULA GOVT PRODUCTS INC F LYONS C BRADNEY W PERCIBALLI 10016 S 51ST ST PHOENIX AZ 85044
2	HP WHITE LAB INC C DUNN E DUNN 3114 SCARBORO RD STREET MD 21154-1822

NO. OF COPIES	ORGANIZATION
5	SOUTHWEST RSRCH INST JAMES WALKER DIV 06 S ROYAL B MORRIS J RIEGEL DR C ANDERSON JR 6220 CULEBRA RD SAN ANTONIO TX 78228-0510
4	CERADYNE INC J MOSCOWITZ D REED J BIRD J WALSH 3169 REDHILL AVE COSTA MESA CA 92626
1	CERADYNE INC E CONABEE PO BOX 925 3449 CHURCH ST SCOTTDALE GA 30079
1	A ANCTIL 9 HARRIS DR BURLINGTON MA 01803
1	DR M AZRIN 107 MANNING ST NEDHAM MA 02194
1	J GRAVES 10 MIDDLEBURY LN BEVERLY MA 01915-1358
	ABERDEEN PROVING GROUND
38	DIR USARL AMSRL WM ME M WELLS J MONTGOMERY E CHIN AMSRL WM MD R DOWDING AMSRL WM MF A CHANG (5 CP) S CHOU (10 CP) R SQUILLACIOTI J MACKIEWICZ

NO. OF COPIES	ORGANIZATION
	AMSRL WM TA W BRUCHEY W GOOCH E RAPACKI N RUPERT F GRACE TECH LIB AMSRL SL BA R GRAYSON AMSRL IS CI B BODT (10 CP)
2	DIR AMSAA AMXSY J A LAGRANGE AMXSY MP H COHEN
7	DIR USA ATC STEAC LI V T WAGNER E SANDERSON T CARLSON M SIMON STEAC LI B SOKOLIS

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1997	Final, Jan 95 - Sep 96

4. TITLE AND SUBTITLE

JTCG/AS Interlaboratory Ballistic Test Program—Final Report

5. FUNDING NUMBERS

6MKCER

6. AUTHOR(S)

Albert L. Chang and Barry A. Bodt

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

U.S. Army Research Laboratory
ATTN: AMSRL-WM-MF
Aberdeen Proving Ground, MD 21005-5066

8. PERFORMING ORGANIZATION REPORT NUMBER

ARL-TR-1577

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A two-phase study was conducted to determine the efficacy of MIL-STD-662E in ballistic protection limit determination for advanced lightweight armor technologies. The first-phase results demonstrated an unacceptable variability in limit estimates across sites. It was concluded that more stringent controls in implementing MIL-STD-662E, particularly as to test velocity selection, would be required to achieve adequate reproducibility. An unusual pattern in the data from the first phase suggested that there was a physical reason beyond operational concerns for the unacceptable variability. In the second phase, this new reason was investigated in an interlaboratory test involving six sites. It was determined that a physical phenomenon, termed shatter gap, was the principal cause of the irreproducible results from the first phase. For this setting, modifications of MIL-STD-662E to support reproducible results were suggested and successfully tested.

14. SUBJECT TERMS

V_{50} test procedure, shatter gap, ballistic test

15. NUMBER OF PAGES

48

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18 298-102

INTENTIONALLY LEFT BLANK.

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. ARL Report Number/Author ARL-TR-1577 (Chang) ____________ Date of Report December 1997

2. Date Report Received ____________

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ____________

4. Specifically, how is the report being used? (Information source, design data, procedure, source of ideas, etc.) ____________

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided, or efficiencies achieved, etc? If so, please elaborate. ____________

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.) ____________

CURRENT ADDRESS

Organization

Name — E-mail Name

Street or P.O. Box No.

City, State, Zip Code

7. If indicating a Change of Address or Address Correction, please provide the Current or Correct address above and the Old or Incorrect address below.

OLD ADDRESS

Organization

Name

Street or P.O. Box No.

City, State, Zip Code

(Remove this sheet, fold as indicated, tape closed, and mail.)

(DO NOT STAPLE)

DEPARTMENT OF THE ARMY

OFFICIAL BUSINESS

BUSINESS REPLY MAIL
FIRST CLASS PERMIT NO 0001,APG,MD

POSTAGE WILL BE PAID BY ADDRESSEE

DIRECTOR
US ARMY RESEARCH LABORATORY
ATTN AMSRL WM MF
ABERDEEN PROVING GROUND MD 21005-5069

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES