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SSC-398A

ASSESSMENT OF RELIABILITY OF SHIP STRUCTURES

Appendices

SHIP STRUCTURE COMMITTEE
1997

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ASSESSMENT OF RELIABILITY OF SHIP STRUCTURES

This work forms part of a series of Ship Structure Committee tasks in the structural reliability area. Previous work covered assessment of uncertainties associated with hull ultimate failure, uncertainties in stress analysis, uncertainties in strength models, probabilistic loads and load combinations. In addition, an introduction to structural reliability theory, a demonstration of probability based design procedures, and demonstration prototype design code have been funded.

This report presents a set of methodologies for assessing existing surface ship structural reliability. Areas included cover wave loads and load combinations, hull strength, the estimation of ship failure probabilities, fatigue reliability, and safety level selection. Methods for dealing with non-linearity associated with both loads and strength are presented. In addition to incorporating the results of previous work, the report presents additional information and developments in the various topic areas. In several cases results have been presented in the form of design charts and equations with worked out examples. Applications are made to four ships: two cruisers, a tanker, and an SL-7. For each of these ships loads, strength, reliability, and sensitivity to parameters have been estimated.

The report includes general guidelines for identifying significant parameters affecting reliability as well as recommendations. A set of 10 appendices provides more detail on selected topics.

JCCard
J. C. CARD
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee

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16. Abstract

A detailed approach has been developed for assessing structural safety and reliability of ships. The methodology provides a means for determining reliability levels associated with a hull girder, stiffened panel and unstiffened plate modes of failure. Procedures for esimating the non-linear extreme sea loads and structural strength which are required for the reliability analysis have been developed. Fatigue reliability of ship structural details was also addressed and further developed.

The methodology was demonstrated on four ships; two cruisers, a double hull tanker and an SL-7 containership. Reliability levels associated with each mode of failure of these ships were determined and compared. Sensitivity analysis has been conducted which provides sensitivity of a safety index to variations in design variables associated with extreme loading conditions as well as with fatigue loads.

Recommendations are made of target reliability levels for each ship type and failure mode. Design variables that have the highest impact on reliability have been identified and some guidelines are provided for improving design criteria.

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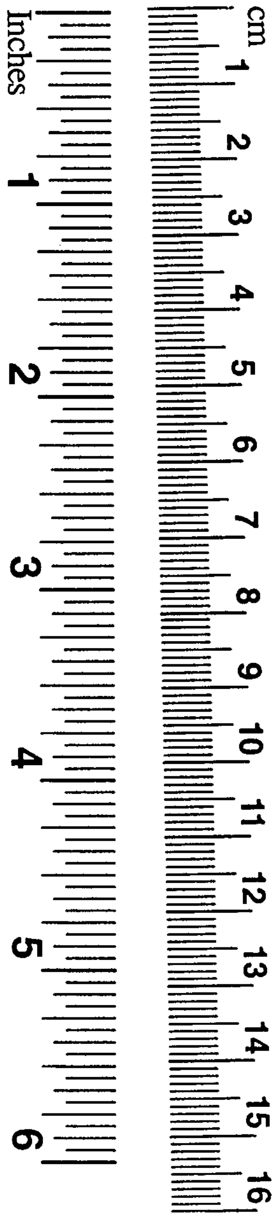
METRIC CONVERSION CARD

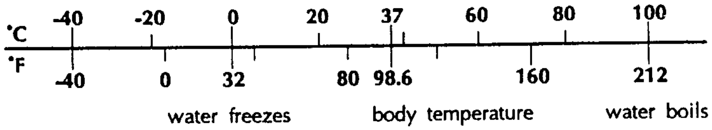
Approximate Conversions to Metric Measures

Symbol	*When You Know*	*Multiply by*	*To Find*	*Symbol*
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	metric ton	t
		VOLUME		
tsp	teaspoons	5	milliliters	mL
Tbsp	tablespoons	15	milliliters	mL
in^3	cubic inches	16	milliliters	mL
fl oz	fluid ounces	30	milliliters	mL
c	cups	0.24	liters	L
pt	pints	0.47	liters	L
qt	quarts	0.95	liters	L
gal	gallons	3.8	liters	L
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	degrees Fahrenheit	subtract 32, multiply by 5/9	degrees Celsius	°C



Approximate Conversions from Metric Measures

Symbol	*When You Know*	*Multiply by*	*To Find*	*Symbol*
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd.
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares ($10,000 m^2$)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	metric ton (1,000 kg)	1.1	short tons	
		VOLUME		
mL	milliliters	0.03	fluid ounces	fl oz
mL	milliliters	0.06	cubic inches	in^3
L	liters	2.1	pints	pt
L	liters	1.06	quarts	qt
L	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	degrees Celsius	multiply by 9/5, add 32	degrees Fahrenheit	°F



APPENDIX A

EXTREME LOADS AND LOAD COMBINATIONS

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APPENDIX A

Extreme Loads and Load Combinations

A. E. Mansour[1]

A simple model for combining extreme responses of correlated load components has been developed in this paper for use in design of marine structures. The combined response has the form $f_c = f_1 + Kf_2$ for a two correlated load case and $f_c = f_1 + K_2f_2 + K_3f_3$ for a three correlated load case. The load factors K_i are determined from probabilistic analysis of the combined response of a multiple component system subjected to common input (waves). Application examples are given and modeling errors are discussed. The model is suitable for use in the usual deterministic design analysis or probabilistic and reliability design procedures. This is the first of a three-paper series on this subject.

Introduction

THE OBJECTIVE of this paper is to provide a simple design procedure for determining the combined load or response due to several individual load components acting on a marine structure, taking into consideration the correlation between the load components. In the case of a ship, these load components may consist of global (hull girder) loads such as wave-induced vertical, horizontal, torsional and springing moments, and local loads such as the dynamic wave pressure or internal cargo inertia loads acting on hull stiffened panels. Each of these load components is usually calculated using a separate computer program or simplified analysis. In many cases a characteristic (design) value can be determined on the basis of extreme value theory and statistical data analysis. The purpose of this paper is to provide a simple procedure for combining these characteristic load components or their responses with appropriate attention given to their phasing and correlation. Although the developed procedure can be used for any design analyses, it is particularly useful for probability-based or reliability design analysis. Slamming and fatigue loads are not explicitly addressed in this paper.

A simple format of the combined response is sought, in the form:

$$f_c = f_1 + Kf_2 \qquad f_1 > f_2 \tag{1}$$

for the two-load case, or

$$f_c = f_1 + K_2f_2 + K_3f_3 \qquad f_1 > f_2 > f_3 \tag{2}$$

for the three-load case. f_1, f_2 or f_3 is the individual response to a characteristic value (extreme) of a load component and f_c is the combined extreme response (e.g., stress or deflection). The K-factors appearing in equations (1) and (2) must necessarily depend on the degree of correlation between the individual load components and, as will be seen later, on their relative magnitude and the frequency content of the underlying processes of each component.

In the simplest possible estimation of the extreme load effect, one assumes that the combined extreme load effect is the sum of the extreme values from individual processes that contribute additive effects. This method, referred to sometimes as the "peak coincidence method," leads to an oversized structure, since it is not typical that extreme values from individual processes occur at the same instant of time. There are also other simplified approaches, two worth mentioning being Turkstra's rule (1970) and the square root of the sum of the squares (SRSS) method (Mattu 1980). The approximations involved in these two methods will be discussed later.

The load coincidence technique due to Wen (1977) and Wen & Pearce (1982) is rather general one in that it accounts for load correlations. The method, however, requires the use of an average coincidence rate.

Another class of methods is those which calculate the outcrossing rate of a vector load process from a safe domain defined by load and strength variables. Until recently, the most general use of the method was based on outercrossing rate bounds, e.g., Larrabee & Cornell (1981), who developed an upper bound based on a "point crossing" formula. A lower bound is also obtainable, and the method can be extended to nonstationary load processes. For more than two load processes, see Ditlevsen & Madsen (1983).

Recently, Hagen & Tvedt (1992) proposed a method to calculate the mean outcrossing rate that is applicable to both stationary and nonstationary stochastic vector processes, provided that the random variables representing the process and its time derivative process can be mapped into a set of independent standard normal variates. This method has been used for outcrossing rate calculations when the threshold level is varying (Friis Hansen 1993).

One unique aspect of the loads acting on a marine structure is that most of them have a common source—ocean waves. Unlike many civil engineering structures, this commonality of input tends to increase the correlation between the loads. Aside from stillwater loads, all other important loads including low-frequency and high-frequency (slamming and springing) loads as well as external dynamic pressure are due to waves. Mansour (1981,1975) developed an approach for combining these loads, taking into consideration the commonality of the load source. The methodology for short-term load combinations assumes a Gaussian wave process as a common input and a linear vessel system. In effect, for a given sea state, the system transfer functions can be determined and the variance of the combined load effect is obtained. The approach will be extended and further developed in this paper. The combination of the effects of the vertical and horizontal moments and local pressure was considered in a recent report for the American Bureau of Shipping (ABS) by Mansour et al (1992).

In the first section of the paper, the basic approach for

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combining n-load components that have a common source is outlined. Later sections address two- and three-load combination cases; then, application examples are given covering combinations of hull girder (global) loads, e.g., combinations of vertical and horizontal moment effects and combinations of hull girder and local load effects. Next, modeling errors associated with hull girder loads and with the approach to load combinations are addressed. Finally, the paper summarizes the main results and provides some conclusions.

The basic approach

A ship traveling in oblique irregular seas can be considered as a multiple linear system with the ocean waves representing a common input to the system. Over a short period of time the waves can be represented as a stationary random process in the wide sense. In general, the output of the system can be a time variation of any measurable quantity, e.g., motion, velocities, accelerations, loads, and stresses. The sum of the outputs $y(t)$ of this multiple system represents the combined response, e.g., motion, acceleration, stress. Therefore, the probabilistic definition of the sum is of interest in design.

Figure 1 describes schematically the input/output procedure for n-linear systems. The analysis can be carried out in a frequency or time domain, both of which will be investigated here. For generality, the constants a_i are used to ensure uniformity of units and direction, e.g., to convert loads to stresses, all in the same direction. They can always be taken equal to one if not needed. The output is given by the convolution integral:

$$y(t) = \sum_{i=1}^{n} a_i \int_0^{\infty} h_i(\tau)x(t-\tau)d\tau \tag{3}$$

where $h_i(\cdot)$ are the impulse response functions of the individual components and $x(\cdot)$ is the common input, i.e., a time history of wave surface elevation.

Since $x(t)$ is a common input to all terms of equation (3) and since the summation and integration signs can be interchanged in this case, a composite impulse response function $h_c(t)$ can be defined as

$$h_c(t) = \sum_{i=1}^{n} a_i h_i(t) \tag{4}$$

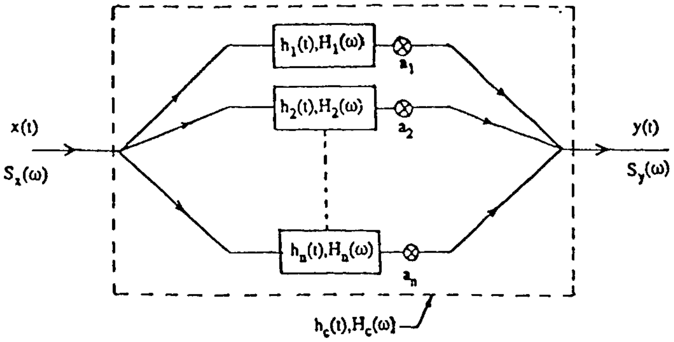

Fig. 1 Model for correlated wave loads acting on a marine structure

Therefore, all the usual auto and cross-correlation and spectral density relationships valid for a single linear system can be extended to the composite linear system using $h_c(t)$ as the system impulse response function.

In a frequency domain, the frequency response (transfer) function $H_i(\omega)$ for each component is obtained as the Fourier transform of $h_i(t)$, i.e.

$$H_i(\omega) = \int_0^{\infty} h_i(t)e^{-j\omega t}dt \tag{5}$$

Therefore, one can define a composite frequency response function $H_c(\omega)$ as

$$H_c(\omega) = \int_0^{\infty} h_c(t)e^{-j\omega t}dt \tag{6}$$

Substituting for $h_c(t)$ in (6) using equation (4) and noting equation (5), one can write:

$$H_c(\omega) = \sum_{i=1}^{n} a_i H_i(\omega) \tag{7}$$

The relation between the input (sea) spectrum $S_x(\omega)$ and the output (response) spectrum $S_y(\omega)$ for a single component is given by the usual equation

$$S_y(\omega) = H_i(\omega)H_i^*(\omega)S_x(\omega) = |H_i(\omega)|^2 S_x(\omega) \tag{8}$$

where $H_i^*(\omega)$ is the complex conjugate of $H_i(\omega)$. For the composite system, an equation similar to (8) can thus be written as

Nomenclature

a_i = conversion factor associated with load component i
f_i = characteristic value of response (stress or deflection) to load component i
f_c = combined response (stress or deflection)
$h_c(t)$ = composite impulse response function
$h_i(t)$ = impulse response function for load component i
$H_c(\omega)$ = composite frequency response function
$H_i(\omega)$ = frequency response function for load component i
$H_i^*(\omega)$ = conjugate complex of $H_i(\omega)$
K = load combination factor for two correlated load response
K_1,K_2,K_3 = load combination factors for three correlated load response
N_i = number of peaks associated with load component i
r_i = stress ratios
Re(·) = real part of a complex function
$S_x(\omega),S_y(\omega)$ = wave and response spectra, respectively
t = time
$x(t),y(t)$ = time histories of input and output, respectively
α = ship heading angle
α_i = a multiplier used to predict extreme response to load component i
ϵ = bandwidth parameter
ζ = peak amplitude
μ = wave spreading angle
ρ_{ij} = correlation coefficient between to response components i and j
σ_c^2 or m_0 = variance of the combined response
σ_i = standard deviation of response to load component i
ω = frequency

$$\begin{aligned} S_y(\omega) &= H_c(\omega)H_c^*(\omega)S_x(\omega) \\ &= S_x(\omega)\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j H_i(\omega)H_j^*(\omega) \\ &= S_x(\omega)\sum_{i=1}^{n} a_i^2 |H_i(\omega)|^2 \\ &\quad + S_x(\omega)\sum_{\substack{i=1}}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} a_i a_j H_i(\omega)H_j^*(\omega) \qquad (9) \end{aligned}$$

where $|H_i(\omega)|$ are the moduli of the individual frequency response functions and the double summation terms in equation (9) represent the cross spectra terms. The first term in equation (9) is simply the sum of the individual response spectra, each modified by the factor a_i^2. The second term, which can be either positive or negative, is a corrective term that reflects the correlation between the load components.

If the frequency response functions $H_i(\omega)$ do not overlap on a frequency axis, that is, if $H_i(\omega)H_j^*(\omega) = 0$, then the second term in equation (9) drops out and the load components are uncorrelated. Furthermore, if the wave input is considered a normal process with zero mean, then the respective outputs of the n-components are jointly normal, and if uncorrelated it follows that they are also independent.

In general, the variance σ_c^2 of the combined output response is given as the zero moment m_0 of the output spectrum, i.e.

$$\begin{aligned} \sigma_c^2 = m_0 &= \int_0^\infty S_y(\omega)d\omega \\ &= \sum_{i=1}^{n} a_i^2 \int_0^\infty |H_i(\omega)|^2 S_x(\omega)\,dw \\ &\quad + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} a_i a_j \int_0^\infty H_i(\omega)H_j^*(\omega)S_x(\omega)\,d\omega \qquad (10) \end{aligned}$$

Equation (10) can be written in a different form that makes it easier to define the correlation coefficient between the different response components.

$$\sigma_c^2 = \sum_{i=1}^{n} a_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} a_i a_j \rho_{ij}\sigma_i\sigma_j \qquad (11)$$

where σ_i^2 are variances of the individual load component

$$\sigma_i^2 = \int_0^\infty |H_i(\omega)|^2 S_x(\omega)\,d\omega \qquad (12)$$

and ρ_{ij} are correlation coefficients between individual load components

$$\rho_{ij} = \frac{1}{\sigma_i\sigma_j}\int_0^\infty \mathrm{Re}[H_i(\omega)H_j^*(\omega)]S_x(\omega)\,d\omega \qquad (13)$$

The above results can be generalized to the case of short-crested seas where the sea spectrum is defined in terms of frequency and a wave spreading angle μ. For a ship heading angle α, the combined response variance given by (11) is valid but with equations (12) and (13) replaced by

$$\sigma_i^2 = \int_{-\Pi/2}^{\Pi/2}\int_0^\infty |H_i(\omega,\alpha-\mu)|^2 S_x(\omega,\mu)\,d\omega d\mu \qquad (14)$$

and

$$\rho_{ij} = \frac{1}{\sigma_i\sigma_j}\int_{-\Pi/2}^{\Pi/2}\int_0^\infty \mathrm{Re}\{H_i(\omega,\alpha-\mu)H_j^*(\omega,\alpha-\mu)\} S_x(\omega,\mu)\,d\omega d\mu \qquad (15)$$

Re{ · } indicates the real part of the function and $H_j^*(\cdot)$ is the conjugate of the complex frequency response function.

Equation (11) with definitions (12) and (13) or (14) and (15) form the basis for combining the variances of a multiple system taking into consideration the correlation between the response components. If the response components are uncorrelated, i.e., if $\rho_{ij} = 0$, the second term in equation (11) drops out and the combined variance is simply the sum of the individual variances modified by the factors a_i^2. On the other hand, if the individual components are perfectly correlated, ρ_{ij} will approach plus or minus one, and the effect of the second term in equation (11) on the combined variance can be substantial.

Considering a normal (Gaussian) seaway as common input, the output of the multiple system is also normal. The probability density function of the output peaks for a general normal random process with bandwidth parameter ϵ is given by (Rice 1944):

$$f(\zeta) = \frac{1}{\sqrt{2\pi m_0}}\left\{\epsilon e^{-\zeta^2/2\epsilon^2 m_0} + \sqrt{\frac{2\pi(1-\epsilon^2)}{m_0}}\,\zeta e^{-\zeta^2/2m_0}\,\Phi\left(\frac{\sqrt{1-\epsilon^2}}{\epsilon}\cdot\frac{\zeta}{\sqrt{m_0}}\right)\right\} \qquad (16)$$

where

$$\epsilon^2 = 1 - \frac{m_2^2}{m_0 m_4};\ m_n = \int_0^\infty \omega^n S_y(\omega)\,d\omega\ ;\ n = 0, 2, 4$$

$$\Phi(u) = \int_{-\infty}^{u} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$$

When $\epsilon = 0$, that is, a narrow-band process, equation (16) reduces to the Rayleigh distribution usually used to characterize the peak probabilities. If $\epsilon = 1$, that is, a wide-band output spectrum, equation (16) of the peaks reduces to a normal distribution; that is, it reduces to the distribution of the process (elevation) itself.

Extreme values of the peaks of the combined response can be estimated from equation (16) (and the corresponding cumulative distribution function) using order statistics, outcrossing analysis, or Gumbel asymptotic distribution (see, for example, Mansour 1990).

Although the approach outlined above can be used to determine the extreme value of the combined response, equation (16) and the extreme value analysis are not suitable for direct use in design. For design purposes, simple formulations such as given by equations (1) and (2) are more suitable. A simplification of the described procedure is therefore necessary. In the next sections of the paper, a simplified procedure has been developed which reduces the above outlined analysis to equations (1) and (2) for the two- and three-load combination cases, respectively.

Two correlated load combinations

As mentioned earlier, a simple format of the combined response (stress) is sought, in the form

$$f_c = f_1 + Kf_2 \qquad f_1 > f_2 \qquad (1)$$

where K is a probabilistic load combination factor and f_1 and

f_2 are the individual extreme stresses (characteristic values) corresponding to two load components.

The characteristic design values f_1 and f_2 are usually determined from extreme value theory. For example, the expected extreme stress peak f_i in N_i peaks during a Gaussian design sea state is given by (Cartwright & Longuet-Higgins 1956):

$$f_i = E[f_{i\max}] = \alpha_i \sigma_i \tag{17}$$

where σ_i^2 is the variance of the stress process i and α_i is a multiplier that depends on the number of peaks N_i and the bandwidth parameter ϵ_i given by

$$\alpha_i = [2\ln(1-\epsilon_i^2)^{1/2}N_i]^{1/2} + 0.2886\,[2\ln(1-\epsilon_i^2)^{1/2}N_i]^{-1/2} \tag{18}$$

If the most probable extreme value (mode) instead of the expected extreme value (mean) is used as a characteristic (design) value of the stress f_i, then equation (17) still holds but with α_i given by the first term only of equation (18). Similarly, the average of the highest $1/m$th value, if used as a characteristic value, has the form of equation (17). Since for linear systems the individual responses to a Gaussian process is Gaussian, the combined response of the components is also Gaussian. This means that equations (17) and (18) are valid for the combined response f_c as well. Therefore, equation (1) can be solved for the probabilistic load combination factor K in terms of the variances, using equation (17):

$$K = \frac{f_c - f_1}{f_2} = \frac{\alpha_c\sigma_c - \alpha_1\sigma_1}{\alpha_2\sigma_2} \tag{19}$$

where σ_1, σ_2, and σ_c are the root-mean-square (rms) values of the two load effects and the combined effect, respectively, and α_1, α_2, and α_c are the corresponding multipliers as determined from equation (18).

If the most probable extreme values are used as the characteristic value, then the coefficients α_i in equation (19) are, using (18), given by:

$$\alpha_i = \sqrt{2\ln(1-\epsilon_i^2)^{1/2}N_i} \qquad i = 1, 2, c \tag{20}$$

Substituting for σ_c in equation (19) by its value given by equation (11) for the two-load case, and noting that all a_i are equal to unity [since equations (1) and (20) involve stresses rather than moments], one obtains

$$K = \frac{m_r}{r}[m_c(1 + r^2 + 2\rho r)^{1/2} - 1] \tag{21}$$

where

$$r = \frac{\sigma_2}{\sigma_1},\ m_r = \sqrt{\frac{\ln(1-\epsilon_1^2)^{1/2}N_1}{\ln(1-\epsilon_2^2)^{1/2}N_2}} \quad \text{and } m_c = \sqrt{\frac{\ln(1-\epsilon_c^2)^{1/2}N_c}{\ln(1-\epsilon_1^2)^{1/2}N_1}}$$

ρ is the correlation coefficient between the two rms stress components σ_1 and σ_2 given by equation (13) for long-crested seas or (15) for short-crested seas.

Typical values of ρ and the corresponding typical values of K will be given for specific load combinations in the next section of the paper. Equations (1) and (21) form the basis of the simplified approach to the two-load combination cases.

Figure 2 shows the trend of K as a function of the correlation coefficient and the ratio of the stresses for the special case when $m_r = m_c = 1$, e.g., all processes are narrow-band with approximately the same central frequency, as in the case of combining stresses due to vertical and horizontal bending moments. In this figure σ_1 was selected as the larger of the two stresses so that r always falls in the range zero to one. It is seen that for $\rho > 0.5$, K does not depend appreciably on r. From equation (21) or Fig. 2, the following extreme cases can be obtained:

(*a*) If $\rho = 1$, i.e., the two stresses are fully correlated, $K = 1$ independent of the stress ratio r.

(*b*) If $\rho = 0$, i.e., the two stresses are uncorrelated, $K = 0.05$ for $r = 0.1$ and $K = 0.41$ for $r = 1$.

The second extreme case indicates that even though the two loads or stresses are uncorrelated, the fact that a second load exists will contribute somewhat to the combined stress (5% of f_2 for $r = 0.1$ or 41% of f_2 for $r = 1$).

Comparison with Turkstra's rule

Turkstra (1970) proposed that structural safety can be checked using a set of design loads with each load at its maximum value and with the other loads at their accompanying "point in time" values. In the case of two-load effects, each a realization of a zero mean Gaussian random process x_1 or x_2, the accompanying load effect (stress) value can be estimated from (Thayamballi 1993):

$$E(x_2/x_1) = \rho\left(\frac{\sigma_2}{\sigma_1}\right)x_1^* \tag{22}$$

where ρ is the correlation coefficient and x_1^* is the value of x_1 for which x_2 needs to be predicted. The σ_i are the stress process rms values. Also, the variance of x_2, given x_1, is obtained from

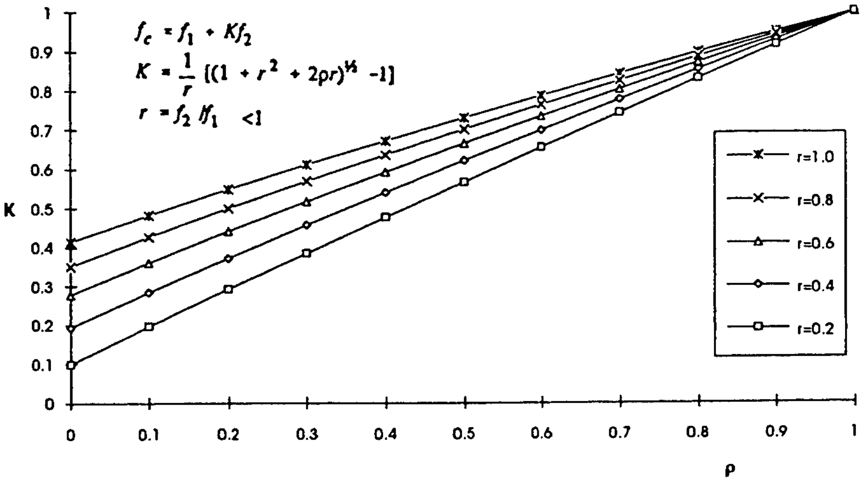

Fig. 2 Load factor for two correlated wave loads

$$\mathrm{Var}(x_2/x_1) = \sigma_2^2(1 - \rho^2) \tag{23}$$

Note that as ρ tends to ± 1, the variance of the predicted value of x_2 tends to zero; i.e., the prediction is more certain. If ρ tends to zero, the variance of the predicted value of x_2 is larger; i.e., the prediction is less certain.

Equations (22) and (23) can be used to provide an estimate of the coexisting x_2-value and the variability of that estimate when x_1 is an extreme value. Thus the combined stress according to Turkstra's rule, using equation (22), is

$$f_c = \max\left[f_1 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)f_1; f_2 + \rho\left(\frac{\sigma_1}{\sigma_2}\right)f_2\right] \tag{24}$$

If one uses the same approximation made in Fig. 2, i.e., $\alpha_1 \cong \alpha_2$, one can write

$$\frac{\sigma_2}{\sigma_1} \cong \frac{f_2}{f_1} \tag{25}$$

where f_1 and f_2 are the extreme (characteristic) values of the stresses and σ_1 and σ_2 are the rms values. Substituting (25) in (24) one gets:

$$f_c = \max[f_1 + \rho f_2; f_2 + \rho f_1] = f_1 + \rho f_2 \quad \text{for} \quad f_1 > f_2 \tag{26}$$

that is, in this case,

$$K = \rho$$

By comparison with Fig. 2, which shows how K derived in this paper varies with ρ and r, it can be seen that Turkstra's rule ($K = \rho$, i.e., a line connecting the origin and the point $\rho = 1, K = 1$) will underestimate the combined extreme stress. This conclusion was also reached by Nikolaidis & Kaplan (1991) when they compared Turkstra's rule with simulation results. Figure 2 shows also that the likely error in applying Turkstra's rule increases with increasing the stress ratio r.

Comparison with square root of sum of squares (SRSS) method

According to the SRSS method (Mattu 1980) the combined stress variance for the two-stress case is given by

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{27}$$

Comparing the above equation with equation (11), it is clear that the SRSS method neglects the effect of correlation between the stresses, i.e., the second term of equation (11). It is therefore expected to give accurate results only if $\rho = 0$.

For extreme characteristic stress values f_1 and f_2, the combined stress according to the SRSS method is

$$f_c = (f_1^2 + f_2^2)^{1/2}$$
$$= f_1 + Kf_2$$

where

$$K = \frac{m_r}{r}[m_c(1 + r^2)^{1/2} - 1] \tag{28}$$

The probabilistic load combination factor K derived in this paper [equation (21)] is again more accurate than that given by the SRSS method [equation (28)] because it reflects the effect of correlation between the stress components. The error in estimating the combined stress according to the SRSS method increases as the correlation coefficient increases (see Fig. 2). If $\rho = 0$, the SRSS method is expected to give accurate results.

Nikolaidis & Kaplan (1991) provided some results for combining wave-induced and slamming bending moments using simulation. The results were compared with Turksra's rule, the peak coincidence method, and the SRSS method. For a significant wave height of 6.14 m and ship speed of 25 knots the simulation result for the combined bending moment is 1.8 × 10^5 ton · m. Turkstra's rule gave 1.3 × 10^5 ton · m while the peak coincidence and the SRSS methods gave 2.1 × 10^5 and 1.5 × 10^5 ton · m, respectively. The average maximum wave and slamming bending moments are given as 0.94 × 10^5 and 1.17 × 10^5, respectively.

If equation (26) is used to determine approximately the correlation coefficient ρ according to Turkstra's combined moment result, one gets $\rho = 0.138$. Using this value in equation (21), and assuming that $m_r = m_c = 1$, one obtains $K \simeq 0.456$. This value gives a combined moment according to equation (1) of 1.60 × 10^5 ton · m., which is closer to the simulation results than the other methods.

Although the slamming bending moment is not linear with respect to the wave height, the proposed method results are closer to simulation results than the other methods. This example shows the potential of the presented method although, in this case, the correlation coefficient was estimated approximately.

Three correlated load combinations

The simplified procedure for the three-load case is similar to that for the two-load one. The sought combined stress has the form

$$\begin{aligned} f_c &= f_1 + K_2f_2 + K_3f_3 \\ f_c &= f_2 + K_1f_1 + K_3f_3 \\ f_c &= f_3 + K_1f_1 + K_2f_2 \end{aligned} \tag{29}$$

where K_1, K_2, and K_3 are the load combination factors corresponding to the individual characteristic stresses f_1, f_2, and f_3, respectively. By requiring that any of the above equations yield the same combined stress f_c, equations (29) can be solved simultaneously for the load factors, yielding

$$\begin{aligned} K_1 &= \frac{1}{2}\left(\frac{f_c}{f_1} + 1 - \frac{f_2}{f_1} - \frac{f_3}{f_1}\right) \\ K_2 &= \frac{1}{2}\left(\frac{f_c}{f_2} + 1 - \frac{f_1}{f_2} - \frac{f_3}{f_2}\right) \\ K_3 &= \frac{1}{2}\left(\frac{f_c}{f_3} + 1 - \frac{f_1}{f_3} - \frac{f_2}{f_3}\right) \end{aligned} \tag{30}$$

The characteristic stress (extreme) f_i in each instance is $\alpha_i(\mathrm{rms})_i$. Here we will consider only the case when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_c$, i.e., the case of narrow-band processes with approximately the same central frequency. This case is adequate for combining stresses due to vertical and horizontal bending moments together with stress due to torsional moment or to local lateral pressure. In this case, equation (11) can be written in terms of f_i instead of σ_i and used to eliminate f_c from equation (30). This yields, for K_1, K_2, and K_3:

$$K_1 = \frac{1}{2}(\rho^* - r_2 - r_3 + 1) \tag{31}$$

$$K_2 = \frac{1}{2r_2}(\rho^* + r_2 - r_3 - 1) \tag{32}$$

$$K_3 = \frac{1}{2r_3}(\rho^* + r_3 - r_2 - 1) \tag{33}$$

where

$$r_2 = \frac{f_2}{f_1} \text{ and } r_3 = \frac{f_3}{f_1}$$

and

$$\rho^* = [1 + r_2^2 + r_3^2 + 2\rho_{12}r_2 + 2\rho_{13}r_3 + 2\rho_{23}r_2r_3]^{1/2} \quad (34)$$

The correlation coefficients ρ_{12}, ρ_{13}, and ρ_{23} between the individual stress components f_1, f_2, and f_3 are to be determined from equation (13) for long-crested seas or (15) for short-crested seas. Experimental or simulation data, if available for these coefficients, may be used instead of equation (13) or (15). Typical values are given next in the "Application" section of the paper for specific load combinations.

Since any of equations (29) will give an identical result for the combined stress f_c, it is sufficient to use the first equation of (29) [or equation (2)] and equations (32) and (33) to determine K_2 and K_3 appearing in (29). The Appendix shows some design charts for K_2 and K_3 [equations (32) and (33)] as a function of r_2, r_3, ρ_{12}, ρ_{13}, and ρ_{23}. These are to be used with the first of equations (29). f_1 was selected to be the largest stress so that r_2 and r_3 will always have values between zero and one. Note that the K-factors can be negative, but they always yield the correct value for the combined stress.

Application examples

Equation (1) with K determined from equation (21) is applicable to many two-load combination cases in marine structures. For ships as an example, these two equations can be used to combine the effects of vertical and horizontal bending moments, vertical and torsional moments, vertical and springing moments, and horizontal and torsional moments. They can be used also to combine stresses due to primary vertical bending moment (or any of the other primary moments) with secondary stresses due to lateral pressure. In all cases the characteristic stresses f_1 and f_2 may be taken as the most probable extreme values (or the expected values) of the individual stress components as given by equation (17) in the considered design sea state (Mansour & Hovem 1993).

Note that the frequency response functions $H_i(\omega)$ are readily computed in many ship motion computer programs for individual loads or moments rather than stresses, e.g., primary vertical, horizontal and torsional moments as well as external hydrodynamic pressure. These individual load frequency response functions must be converted to stress frequency response functions by multiplying by an appropriate conversion factor, e.g., by one over a section modulus to convert a moment component to a stress component. These conversion factors are accounted for through the constants a_i appearing for example in equation (11). Therefore, in the case of a moment, a_i is equal to one over the section modulus. All stress components used in equations (1) and (2) must be at the same location and in the same direction. In case of a stress component due to external pressure, only the dynamic part of the pressure (i.e., excluding the stillwater pressure) is to be used in the calculation of the combined response; see Mansour & Thayamballi (1993). The stillwater stresses are to be added after obtaining the combined stress due to waves, in the usual manner.

The presented model for load combination can be also used in conjunction with the finite-element method. For example, in the case of vertical and horizontal moments, the K-factor determined from equation (21) provides the fraction of the horizontal bending moment to be applied simultaneously with the vertical bending moment on the hull.

The load combination factor K depends on the correlation coefficient ρ, which can be determined from equation (13) or (15), experimental data, or simulation. Some typical values of ρ are available for specific load combinations as follows:

- Stresses due to primary vertical and horizontal moments: For large tankers considered by Stiansen & Mansour (1975) the correlation coefficient was found to be dependent on sea state and heading with values close to 0.45. The International Ship Structures Congress (ISSC), in their 1973 session, recommended $\rho = 0.32$. For $r = 0.67$, the first value of ρ results in $K = 0.65$ and the second value gives $K = 0.55$ ($m_r = m_c = 1$). That is to say, only about 60% of the stresses due to the horizontal moment should be added to those due to vertical moment.
- Stresses due to vertical moment and external hydrodynamic pressure: For Mariner class ships, ρ was determined to be in the range 0.70 to 0.78 for panels near the midship section. If one assumes $\rho = 0.74$ and $r = 0.2$, equation (21) with $m_r = m_c = 1$ yields $K = 0.78$. Note the high correlation between the primary bending stress due to hull girder wave vertical moment and the secondary stress due to the hydrodynamic pressure near the midship section. Note also that ρ and the corresponding K-factor are associated with the time-dependent part of the individual stress components. The time-independent part of the stresses due to still water, for both hull girder bending and hydrostatic pressure, should be added to the resulting combined wave stress in the usual manner.

Equation (2) for the three-load combination case may be also used in many applications to ships and marine structures. The K-factors appearing in the equation can be determined from equations (32) and (33). To get an appreciation of the factors K_2 and K_3, consider the stress arising from vertical bending moment f_1, horizontal bending f_2, and local pressure f_3 near the midship section at a bottom plating. For $r_2 = 0.6$, $r_3 = 0.4$, $\rho_{12} = 0.4$, $\rho_{13} = 0.6$, and $\rho_{23} = 0.2$, equations (32) and (33) yield $K_2 = 0.67$ and $K_3 = 0.51$.

It is interesting to note that, for an extreme case when all three loads are fully correlated ($\rho_{12} = \rho_{13} = \rho_{23} = 1$), the values of the K-factors are always unity, i.e., $K_2 = K_3 = 1$, independent of the values of r_2 and r_3. If, on the other hand, all three loads are uncorrelated ($\rho_{12} = \rho_{13} = \rho_{23} = 0$), the K-factors will assume nonzero values and their magnitude will depend on r_2 and r_3.

It is also interesting to note that the two-load combination case when $m_r = m_c = 1$ can be retrieved from the three-load combination equations by inserting zero for one of the load components. For example, if one inserts $r_3 = 0$ in equations (2), (32), and (33) and noting that $f_3/r_3 = f_1$, one obtains an equation for K_2 identical to K for the two-load case, i.e., equation (21).

Modeling errors

The presented simple procedure for combining two correlated loads [equations (1) and (21)] and three correlated loads [equations (2), (32), and (33)] may be used in either the usual deterministic design analysis or in probabilistic and reliability analysis. It is noted, however, that the load factors are determined on a probabilistic basis. Therefore, it is more consistent to use these equations in connection with probabilistic analyses or reliability methods.

The developed load combination factors are based on linear systems and the associated spectral analysis. If used in connection with reliability analysis, it is suggested that the K-factors be taken as normally distributed and the associated modeling error to have a bias of 1.0 and a coefficient of variation of 15%. These values are based on experience. Monte-Carlo simulation can be used to estimate the statistics of the modeling error. The associated characteristic stresses f_1, f_2, and f_3 can be taken as the most probable extreme values given by equations (17) and (20) and should be calculated for a specified duration in a design sea state. In most cases the bandwidth parameter ϵ_i may be taken equal to zero in equation (20). If f_1, f_2, and f_3 are calculated on the basis of linear theory, which is likely the case, the modeling errors associated with them due to nonlinearities should be accounted for separately. For example, in the case of wave-induced vertical bending moment and the associated stress, the data provided by the Committee on "Applied Design" of the International Ship and Offshore Structures Congress (ISSC 1991) suggest a bias of 0.9 to be used to correct for overpredicting the moment or stresses due to the linearity assumption and short-crestedness of waves; i.e., the actual loads in high sea states are approximately 90% of the predicted values. In addition, since linear strip-theory spectral analysis programs give equal sagging and hogging moments, a bias of 1.15 is suggested to estimate sagging bending moments and a bias of 0.85 to estimate hogging bending moments. For more information on quadratic response, see Jensen et al (1992), and for a more complete discussion of modeling errors associated with extreme loads for use in reliability analysis, see Mansour & Hovem (1993).

Summary and conclusions

A simple model suitable for design analysis has been presented for combining extreme correlated loads and the associated stresses. The cases of two correlated loads [equations (1) and (21)] and three correlated loads [equations (2), (32), and (33)] have been modeled in a format suitable for use either in the usual deterministic design analysis or in probabilistic and reliability analysis.

The model is based on developing a composite frequency and impulse response functions of multiple linear systems subjected to common input (ocean waves). The requirement for the applicability of the model is the satisfaction of the stationarity condition of the common wave input and the linearity of the multiple system. The stationarity of the wave input implies short-term analysis, and the linearity assumption allows the use of the superposition principle but decreases the accuracy in high sea states. The model is consistent with the "standard" frequency-domain linear ship motion computer programs currently available in many design offices, classification societies, and government agencies. Modeling errors or correction factors can be incorporated in the model to account for the nonlinearities of the response in high sea states. Some typical values of the modeling errors are given in the paper, though additional work is necessary in this area.

The model is also suitable for use in the two level of analyses usually required in practice, i.e., the design-oriented and checking-oriented analyses. The design-oriented analysis usually requires preliminary estimates of load combinations, mostly to determine minimum scantlings and to develop the design further. Therefore, a design-oriented formulation must be simple and must be, to a large extent, independent of detailed information which is usually not available at the early stages of a design. The developed model can be used in this type of design-oriented analysis with load combination factors K_i estimated from typical values for the specific two- and three-load cases under consideration. Some typical values of the load factors are given in this paper, though a more thorough parametric study in this area is necessary.

Checking-oriented analysis, on the other hand, requires more accurate estimates of load combinations to be used to check the adequency of a completed preliminary design or an existing ship. This checking-oriented analysis will necessarily depend on more detailed information on the marine structure and the operation profile. The presented model for load combination can be used also for this type of checking analysis. A more accurate determination of the K-factors is possible for a completed design or an existing ship. A strip-theory ship motion program can be used to determine the frequency response functions (transfer functions) of each individual load or stress component. The correlation coefficients can be then determined from equation (13) for long-crested seas or (15) for short-crested seas. An accurate determination of the K-factors is then possible from equation (21) for the two correlated load combination or from equation (32) and (33) for the three correlated load combination.

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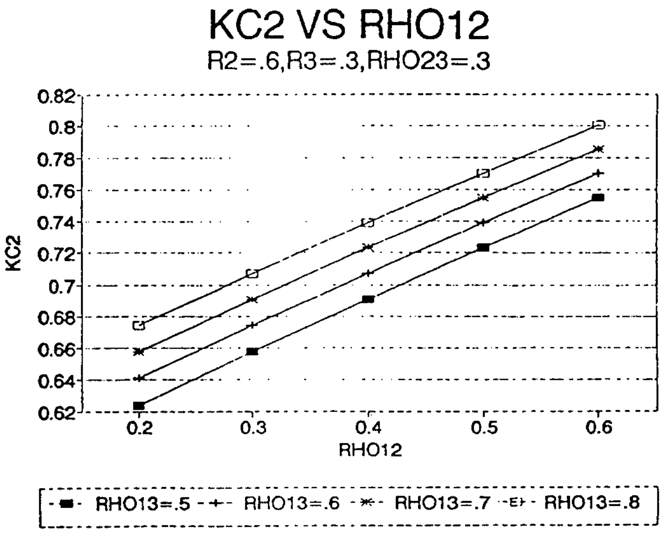
Appendix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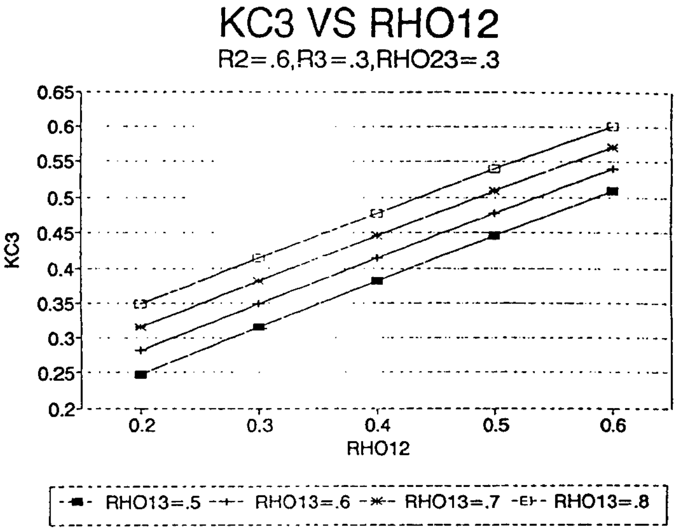
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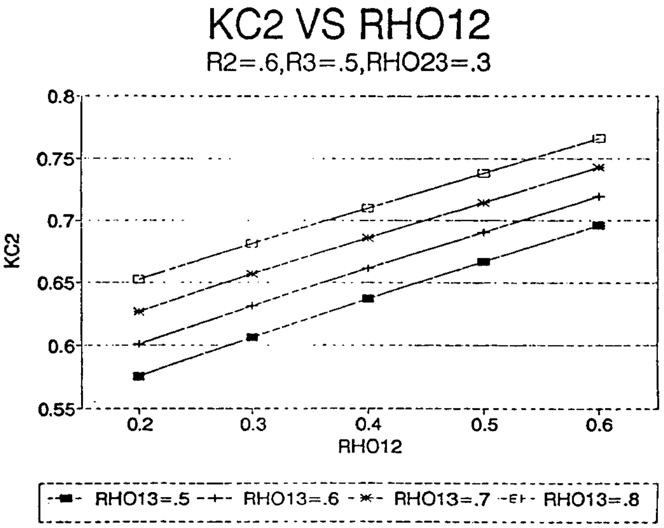
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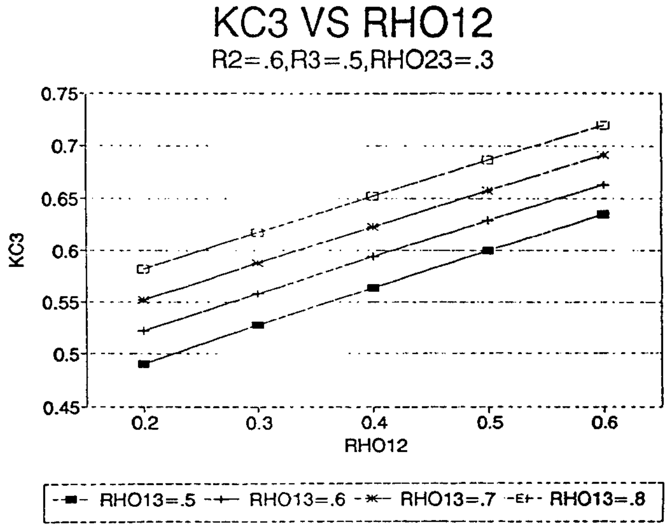
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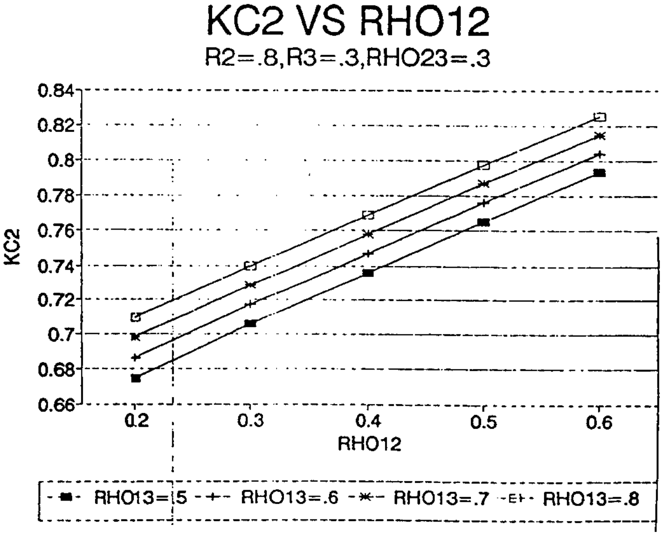
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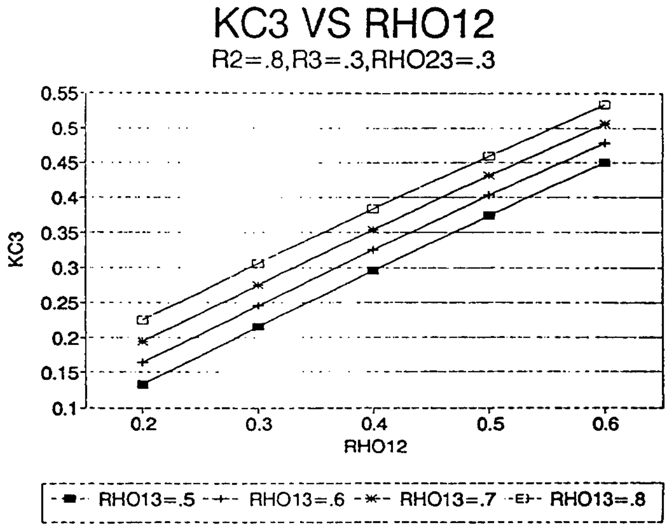
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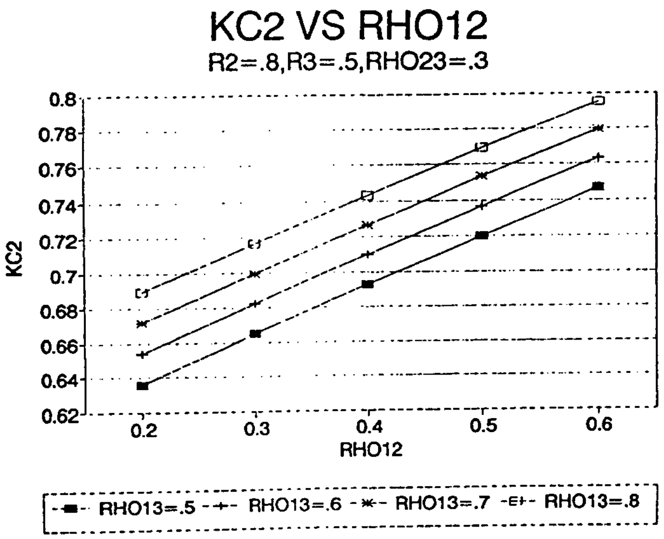
KC2 VS RHO12
R2=.8,R3=.5,RHO23=.3
KC2
0.8
0.78
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.2
0.3
0.4
0.5
0.6
RHO12
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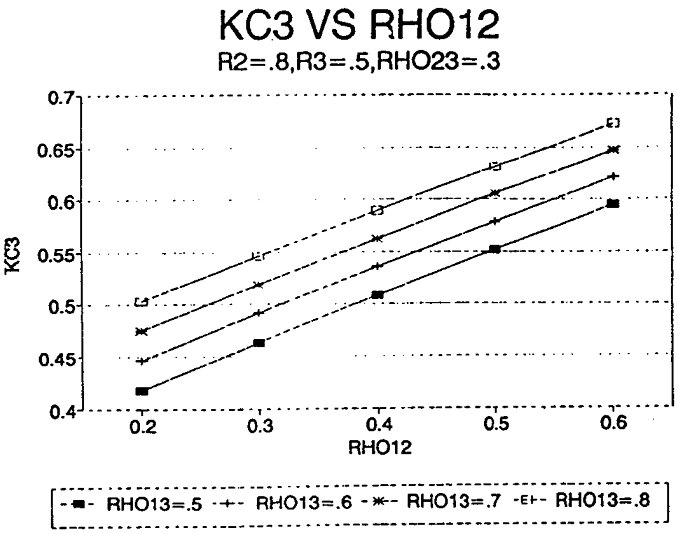
RHO13=.6
RHO13=.7
RHO13=.8


KC3 VS RHO12
R2=.8,R3=.5,RHO23=.3
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0.65
0.6
0.55
0.5
0.45
0.4
0.2
0.3
0.4
0.5
0.6
RHO12
RHO13=.5
RHO13=.6
RHO13=.7
RHO13=.8

APPENDIX B

SKEWNESS, KURTOSIS AND ZERO UPCROSSING RATE OF COMBINED RESPONSE

APPENDIX B

SKEWNESS, KURTOSIS AND ZERO UPCROSSING RATE OF COMBINED RESPONSE

The standard deviation, skewness and kurtosis of the combined response x_c can be approximately estimated from the corresponding values of the individual components x_i whose individual means are zero, as follows:

$$x_c = x_1 + x_2$$

and

$$\sigma_c^2 = E\left[x_c^2\right] = \sigma_1^2 + \sigma_2^2 + 2E\left[x_1 x_2\right]$$

$$= \sigma_1^2 + \sigma_2^2 + 2\rho_{12}\,\sigma_1\,\sigma_2 \tag{B.1}$$

where $E(*)$ denotes expected value, σ_i the standard deviation of x_i and

$$\rho_{12} = \frac{E\left[x_1 x_2\right]}{\sigma_1\,\sigma_2}$$

as zero individual means are assumed.

The combined response skewness α_c can be determined from:

$$\alpha_c\,\sigma_c^3 = E\left[x_c^3\right] = E\left[\left(x_1 + x_2\right)^3\right]$$

$$= \alpha_1\,\sigma_1^3 + \alpha_2\,\sigma_2^3 + 3E\left[x_1^2\,x_2\right] + 3E\left[x_1\,x_2^2\right]$$

If $x_1{}^2$ is assumed independent of x_2 and $x_2{}^2$ independent of x_1, then

$$\alpha_c = \frac{1}{\sigma_c^3}\left[\alpha_1\,\sigma_1^3 + \alpha_2\,\sigma_2^3\right] \tag{B.2}$$

The combined response kurtosis β_c can be determined from:

$$
\begin{aligned}
(\beta_c - 3)\sigma_c^4 &= E\left[x_c^4\right] = E\left[\left(x_1 + x_2\right)^4\right] \\
&= (\beta_1 - 3)\sigma_1^4 + (\beta_2 - 3)\sigma_2^4 + 4E\left[x_1^3\, x_2\right] + 4E\left[x_1\, x_2^3\right] + 6E\left[x_1^2\, x_2^2\right]
\end{aligned}
$$

If the x_i of higher powers are independent, then

$$\beta_c = \frac{1}{\sigma_c^4}\left[(\beta_1 - 3)\sigma_1^4 + (\beta_2 - 3)\sigma_2^4 + 6\sigma_1^2\,\sigma_2^2\right] + 3 \tag{B.3}$$

The combined response zero crossing can be approximately determined from the combined spectrum $S_c(\omega)$ as follows:

$$S_c(\omega) = S_x(\omega) \sum_i \left|H_i(\omega)\right|^2 + S_x(\omega) \sum_i \sum_{\substack{j \\ i \neq j}} H_i(\omega)\, H_j^*(\omega) \tag{B.4}$$

and

$$\nu_{0c}^2 = \left(\frac{1}{2\pi}\right)^2 \frac{m_{2,c}}{m_{0,c}} \tag{B.5}$$

where

$$\sigma_c^2 = m_{0,c} = \int_0^\infty S_c(\omega) d\omega \quad and \quad m_{2,c} = \int_0^\infty \omega^2 S_c(\omega) d\omega \tag{B.6}$$

In the above equations, $S_x(\omega)$ is the wave spectrum (common input spectrum to the two processes), $H_i(\omega)$ is the complex frequency response function and $H_j^*(\cdot)$ indicates the conjugate of $H_j(\cdot)$.

Substituting (B.6) in (B.5) and using (B.4) one gets

$$\nu_{0c}^2 = \left(\frac{1}{2\pi}\right)^2 \frac{\sum_i m_{2,i} + \sum_i \sum_{\substack{j \\ i \neq j}} \rho_{ij,2} \sqrt{m_{2,i} \cdot m_{2,j}}}{\sum_i m_{0,i} + \sum_i \sum_{\substack{j \\ i \neq j}} \rho_{ij} \sigma_i \sigma_j} \tag{B.7}$$

where

$$\rho_{ij} = \frac{1}{\sigma_i \sigma_j} \int_0^{\infty} S_x(\omega)\, \mathrm{Re}\left\{ H_i(\omega)\, H_j^*(\omega) \right\} d\omega \tag{B.8}$$

$$\rho_{ij,2} = \frac{1}{\sqrt{m_{2,i}\, m_{2,j}}} \int_0^{\infty} \omega^2\, S_x(\omega)\, \mathrm{Re}\left\{ H_i(\omega)\, H_j^*(\omega) \right\} d\omega \tag{B.9}$$

If the two processes are uncorrelated or if the correlation terms are neglected, then (B.7) reduces to

$$\nu_{0c}^2 \cong \left(\frac{1}{2\pi} \right)^2 \frac{\sum_i m_{2,i}}{\sum_i m_{0,i}} \tag{B.10}$$

Otherwise the correlation terms must be calculated from (B.8) and (B.9). Using the relation

$$\nu_{0i}^2 \simeq \left(\frac{1}{2\pi} \right)^2 \frac{m_{2,i}}{m_{0,i}} \qquad i = 1,2$$

and for two processes only, i.e., $i = 1, 2$, (B.10) can be written as:

$$\nu_{0c}^2 = \frac{\sigma_1^2 \nu_{01}^2 + \sigma_2^2 \nu_{02}^2}{\sigma_1^2 + \sigma_2^2} \tag{B.11}$$

APPENDIX C

COMPUTER CODES FOR STRUCTURAL RELIABILITY ANALYSIS

CALREL

Program for Structural Reliability Analysis

CALREL (CAL-RELiability) is a general-purpose structural reliability analysis program designed to compute probability integrals of the form

$$p = \int_F f_{\mathbf{X}}(\mathbf{x})\,d\mathbf{x}$$

where $\mathbf{X}$ is a vector of random variables with the joint probability density function $f_{\mathbf{X}}(\mathbf{x})$, and F denotes the failure domain, which is defined as $F = \{g(\mathbf{x}) < 0\}$ for a component problem, as $F = \{\bigcup_i g_i(\mathbf{x}) \leq 0\}$ for a series system problem, and as $F = \{\bigcup_k \bigcap_{i \in C_k} g_i(\mathbf{x}) \leq 0\}$ for a general system problem, where $g_i(\mathbf{x})$ denote limit-state functions and C_k denote cut sets. The functions $g_i(\mathbf{x})$ are provided by the user through an user-defined subroutine, which itself may call other subroutines or an entire subprogram (e.g., a finite-element program) supplied by the user.

CALREL incorporates four general techniques for computing the above probability:

(1) First-order reliability method (FORM), where the limit-state surfaces are replaced by tangent hyperplanes at *design* points in a transformed standard normal space;

(2) Second-order reliability method (SORM), where the limit-state surfaces are replaced by hyperparaboloids by either curvature fitting or point fitting in the standard normal space;

(3) Directional simulation with exact or approximate surfaces; and

(4) Monte Carlo simulation.

In addition to the above, CALREL has routines for computing reliability sensitivity measures with respect to parameters defining probability distribution functions or limit-state functions.

CALREL has a large library of probability distributions for independent as well as dependent random variables. Additional distributions can be included through a user-defined subroutine.

CALREL is written in FORTRAN-77 and operates on IBM-PC or compatible personal computers, as well as on computers with the UNIX operating system. It has been developed by P-L. Liu, H-Z. Lin and A. Der Kiureghian at the University of California at Berkeley. Further information and price quotation can be obtained by contacting Ken Wong at NISEE, Department of Civil Engineering, University of California, Berkeley, CA 94720, or by calling (415) 642-5113.

VERITAS
SESAM
SYSTEMS

State-of-the-art computer program for probabilistic reliability and sensitivity analysis

PROBAN®

PROBAN helps to efficiently evaluate the impact of uncertainty on the reliability of a system - technical, financial, managerial or otherwise. Probabilistic reliability and sensitivity methods are used to quantify uncertainties and thereby to support achieving a required reliability and controlling risk. The methods provide a basis for decisions regarding optimal allocation of resources, and they complement and enhance experimental approaches and conventional deterministic analyses such as design-case evaluations or what-if sensitivity and parameter studies.

The benefits of a probabilistic approach are the clear treatment of uncertainty, the identification of key factors, the possibility of performing trade-off studies, and the fact that the only consistent way of reducing uncertainty when more information becomes available, e.g. through inspection, is through a probabilistic analysis.

Besides traditional reliability measures, the modern probabilistic methods in PROBAN provide a number of useful sensitivity results for reliability-based design and optimisation. Also, sensitivity measures may show the effect of changes in parameter values, and importance measures can be used in initial analyses to reduce the number of random variables and to focus attention on the important uncertain quantities for the problem at hand.

APPLICATIONS

PROBAN has been developed to be a general probabilistic analysis tool. Particularly efficient methods are available for computing small probabilities, as often arise in structural reliability problems. The program is used by engineers that run decision models already available in PROBAN, as well as by experienced analysts who formulate, implement, and apply new probabilistic models.

PROBAN is applied in many areas. The program has successfully been used in reliability studies for marine and offshore structures including:

- calibration of safety requirements in technical standards
- fatigue and fracture reliability
- cost-optimal inspection planning
- re-qualification of existing structures
- foundation analysis and consolidation updating
- pipeline reliability
- probability-based fire and explosion analysis
- ship hull reliability

PROBAN is also used in the mechanical industry and in aerospace companies and institutions. Applications here include:

- stochastic durability analysis
- probabilistic damage tolerance evaluation
- strength analysis of composites

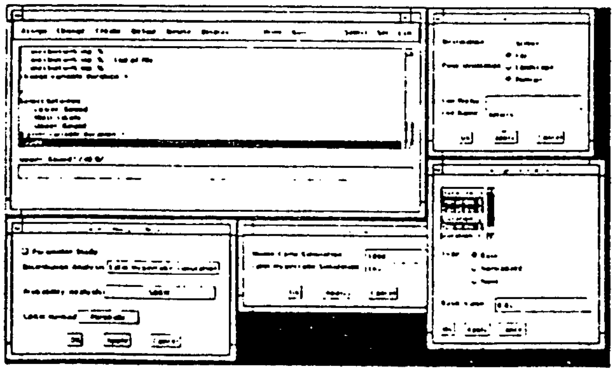
In civil engineering, PROBAN has been applied to large structures such as bridges, e.g. for:

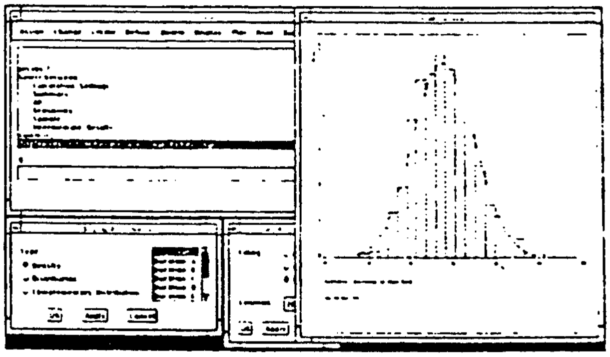
- design basis determination
- traffic load modelling
- ship collision evaluation

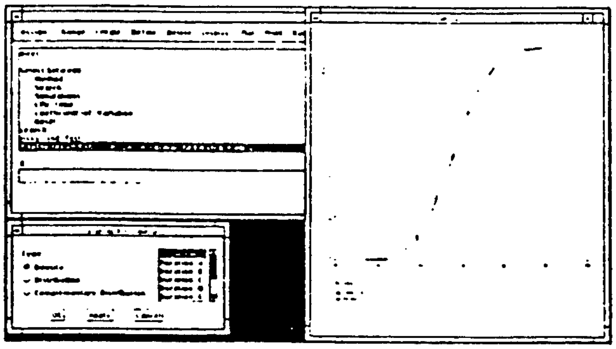
Other applications of PROBAN include:

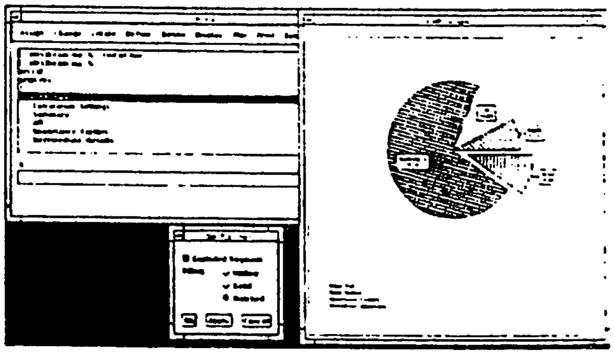
- stochastic finite element analysis
- economic risk analysis
- network scheduling under uncertainty

PROBAN® is a Registered Trademark of Veritas Sesam Systems









TECHNICAL CAPABILITIES

PROBAN provides state-of-the-art computation facilities for the analysis of random variable models. It features general methods to determine probability density and distributions, reliability measures and probabilistic sensitivity and importance measures. The numerical routines, the execution facilities, and the implementation are of a high quality. The program features include:

Extensive modelling capabilities with a library of one- and multi-dimensional probability distributions. User-defined distributions can easily be specified by providing the density and distribution functions. General multivariate distributions are established by a sequential modelling in which a function of random variables can be used as a parameter in the distribution of another random variable.

The functions used to model probability distributions and to define the events of interest (for example the failure of a system) can be provided by the user as subroutines. In addition, PROBAN comes with a library of standard functions that can be used directly.

Full-featured first and second order reliability methods (FORM and SORM) for probability computation of single events, unions, intersections and unions of intersections are available. FORM and SORM are particularly efficient for computing very small probabilities, e.g. $10^{-3} - 10^{-6}$. Exact parametric FORM sensitivity is provided for small and large intersections, as may be required in reliability updating, and the second order method includes exact SORM probability computation. Conditional reliability under inequality and equality events can be computed. PROBAN also contains a mean-based FORM, intended primarily for CPU-intensive models.

The approximate FORM/SORM results may be updated through importance sampling. The probability of general events can be computed by Monte Carlo simulation and directional sampling. Probability distribution computations can be performed by Monte Carlo simulation or Latin Hypercube sampling. Sensitivity analysis by simulation is also available.

DOCUMENTATION

The capabilities of PROBAN are documented in numerous scientific and technical papers and reports. The program comes well documented with User's Manual, Distribution Library Manual, Theory Manual and Example Manual.

USER-INTERFACE

PROBAN Version 3 (1991) is an interactive program, with a database for model and result data. The program is equipped with an efficient, graphical user-interface. The input is logged in a journal file from where it can be retrieved during a later (re)analysis. The program can also be executed in batch mode.

Many graphics and print options are available, for example, to display probability density and distribution functions of input and output random variables. Importance measures can be displayed in pie charts and automated parameter studies can be presented by graphs.

FURTHER DEVELOPMENT

PROBAN is the result of a major strategic research effort at Det norske Veritas, Norway, through A.S Veritas Research. The first version was made in the mid-seventies and it handled second-moment reliability computation for components. From 1984, PROBAN was developed at A.S Veritas Research, Høvik outside Oslo, Norway, in close cooperation with internationally leading researchers in the field. The first commercial version of PROBAN was made available in 1986.

Det norske Veritas intends to keep PROBAN at the leading edge. Further research and development are undertaken by the large group of specialists at A.S Veritas Research and Veritas Sesam Systems A.S, Norway. This ensures long-term maintenance and support of the program. The implementation of new features in PROBAN is prioritised according to user needs. A number of special-purpose probabilistic analysis modules based on PROBAN have been developed for Det norske Veritas and other organisations.

PROGRAM INFORMATION

PROBAN is designed and maintained to be a state-of-the-art, professional computer program. The program is supported worldwide by Veritas Sesam Systems. It is available for common computers from APOLLO/HP, DEC, IBM and SUN.

PROBAN is installed at an increasing number of companies in the petroleum industry, in engineering consultant and design firms, and in the aerospace industry. In addition, the program is installed at research centres and universities in Europe and the US.

VERITAS SESAM SYSTEMS

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Veritas Sesam Systems Ltd.
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1325 South Dairy Ashford
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DET NORSKE VERITAS

Det norske Veritas (DnV) is a corporation whose objective is to safeguard life, property and the environment through services for managing quality, safety and risk. DnV was established in 1864 as a ship classification society and has remained an independent foundation. DnV provides a wide variety of services in shipping, offshore development and production, land-based industry, and aerospace and information technology. DnV operates in 110 countries. The headquarter is at Høvik outside Oslo, Norway.

VERITAS SESAM SYSTEMS

Veritas Sesam Systems (VSS) is the company in the DnV-Group for marketing of engineering software. The company also develops, maintains, and operates software, and it serves as a market partner for R&D institutions. VSS' activities are based on the SESAM program system for structural engineering in the offshore and marine industry. VSS also offers the probabilistic analysis program PROBAN. VSS has subsidiary offices in London and Houston.

Published by Veritas Sesam Systems A.S, Høvik, Norway, July 15, 1991

SUMMARY OF NESSUS CAPABILITIES

NESSUS, under funding from NASA LeRC, is a general purpose probabilistic structural analysis program which can model uncertainties in loading, material properties, geometry, initial conditions, and any user-defined random variables. NESSUS employs advanced probabilistic methods which can efficiently compute structural system reliability and risk and identify critical uncertainty parameters. This information can be used for cost-effective reliability-based design and analysis.

Capabilities

Analysis Types

Static
Natural frequency
Buckling
Harmonic excitation
Random vibration
Transient dynamics

Nonlinearities

Plasticity
Large displacement

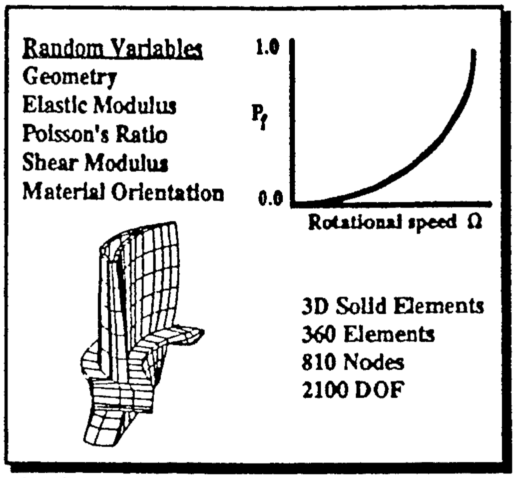
Element Library

Beam
Plate
Plane strain
Plane stress
Axisymmetric
3D solid
3D enhanced solid

Probabilistic Analysis

Fast probability integration
Efficient monte carlo
Adaptive importance sampling
Probabilistc fault tree
Probabilistic sensitivities

Southwest Research Institute
Dr. Y.-T. Wu
6220 Culebra Rd.
P.O. Drawer 28510
San Antonio, TX 78228-0510
(512) 522-3167



NESSUS computes the probability of failure of a turbine blade as a function of rotational speed

NESSUS can be used to Compute:

- CDF analysis
- Probability of failure
- Structural reliability
- System reliability
- Probability of exceedance of disp, stress, strain, freq, ...
- Optimized inspection schedules
- Fault tree analysis
- Probability of rotor instability

Performance Function:

- NESSUS finite element module
- User-defined subroutine
- Custom made interfaces to third party finite element programs and other programs

Random Variables

- Geometry
- Loads
    - Forces
    - Pressures
    - Temperatures
- Material properties
    - Elastic modulus
    - Poisson's ratio
    - Shear modulus
    - Material orientation
    - Yield stress
    - Hardening parameters
- Harmonic excitation
- Random vibration
- Initial conditions
- User-defied

Output Variables

Displacement
Stress
Strain
Plastic strain
Creep strain
Thermal strain
Natural frequency
Fatigue life
Fracture parameters
User-defined

Operating Systems

Cray/Unicos ™
Vax/VMS ™
Unix

Patran™ Interfaces

™ Cray is a registered trademark of Cray Research Inc., Vax/VMS is a registered trademark of the Digital Equipment Corp., Patran is a registered trademark of PDA Inc.

Structural Risk Assessment Code NESSUS

NESSUS integrates structural reliability methods with finite element and boundary element methods. The NESSUS code can simulate uncertainties in the loads, material properties, geometries, and other user-defined uncertainty inputs to quantitatively predict, in probability terms, the risk of component or system failure. NESSUS analysis can identify critical variables and failure modes for design optimization.

Random Variables

Loads
- Forces
- Pressures
- Temperatures
- Vibrations (PSD)

Material properties
- Moduli
- Poisson's ratio
- Yield stress
- Hardening parameters
- Material orientation

Geometry

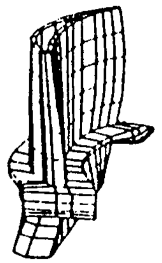
User-defined

Probabilistic Methods

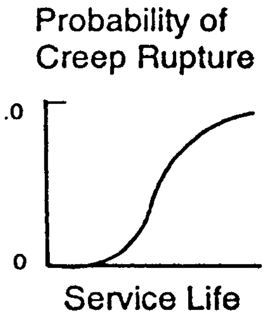
Fast Probability Analysis
- Advanced Mean-Value
- First and Second-Order
- Fast Convolution

Sampling
- Standard Monte Carlo
- Latin Hypercube
- Adaptive importance

Probabilistic Fault Tree



360 3D Solid Elements
810 Nodes, 2100 DOF



Probabilistic Results
- Full probability distribution
- Component/single-mode reliability
- System/multiple-modes reliability
- Probabilistic sensitivities
- Probability-based costs

Performance Functions
- Structural responses:
  stress, strain, disp., freq., etc.
- Fatigue and fracture life
- Creep rupture life
- User-defined subroutines
- External analysis programs
  (requires custom-made interface)

Analysis Types

Static
Transient dynamics
Buckling
Vibrations
Nonlinearities
- Plasticity
- Large displacement

Element Library

Beam
Plate
Plane strain
Plane stress
Axisymmetric
3D solid
Enhanced solids

Operating Systems

Mainframes
Workstations

Southwest Research Institute
Structural Engineering Dept.
P.O. Drawer 28510
San Antonio, TX 78228
(512) 522-3167

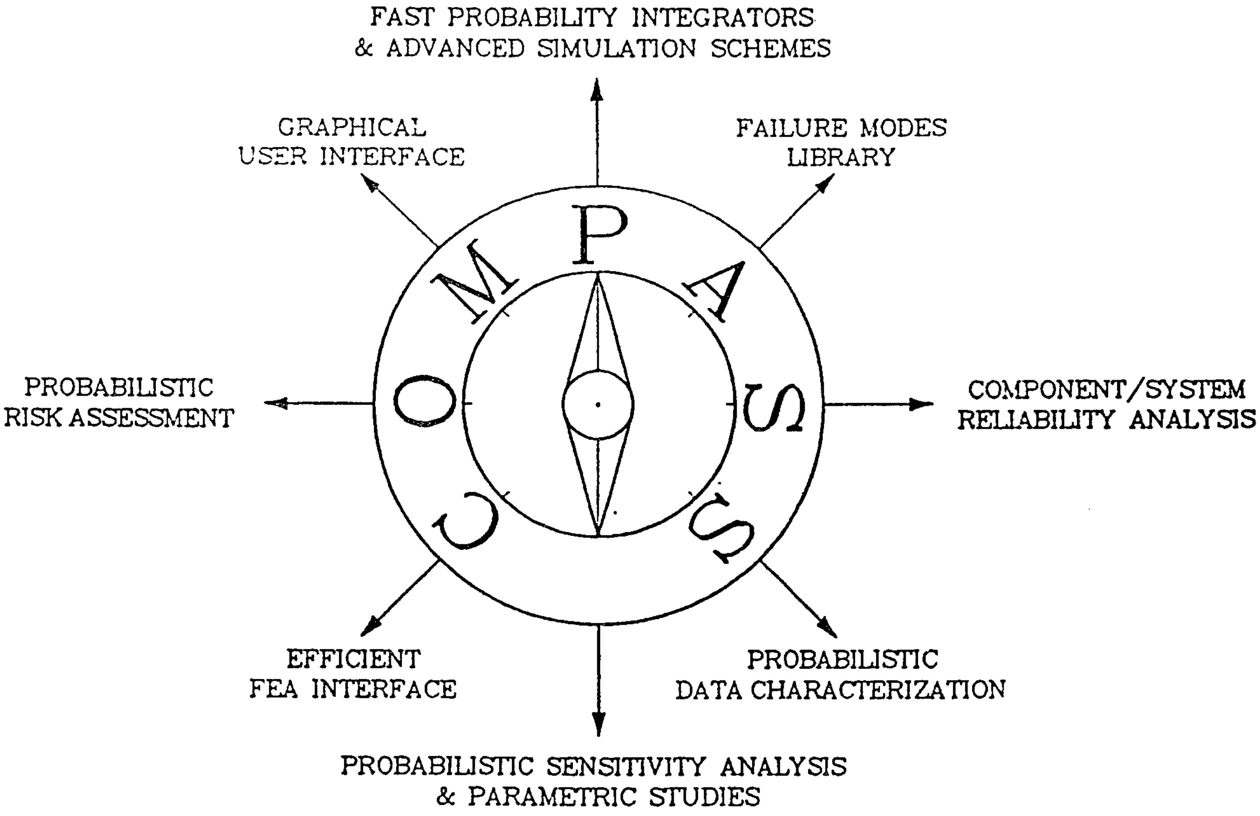
NESSUS is being developed at Southwest Research Institute since 1984 under NASA funding.

April 1992 NESSUS 6.0

COMPASS

COMPASS (acronym for Computer Methods for Probabilistic Analysis of Structures and Systems) is a general purpose software system for the reliability analysis of stochastic systems. The program is developed, maintained, marketed and supported by Martec Limited: an advanced engineering consultancy based in Halifax, Nova Scotia, Canada.

The main motivation for the development of COMPASS was the provision of a robust, efficient, user-friendly, and reasonably affordable computational tool for probabilistic reliability and risk assessment.

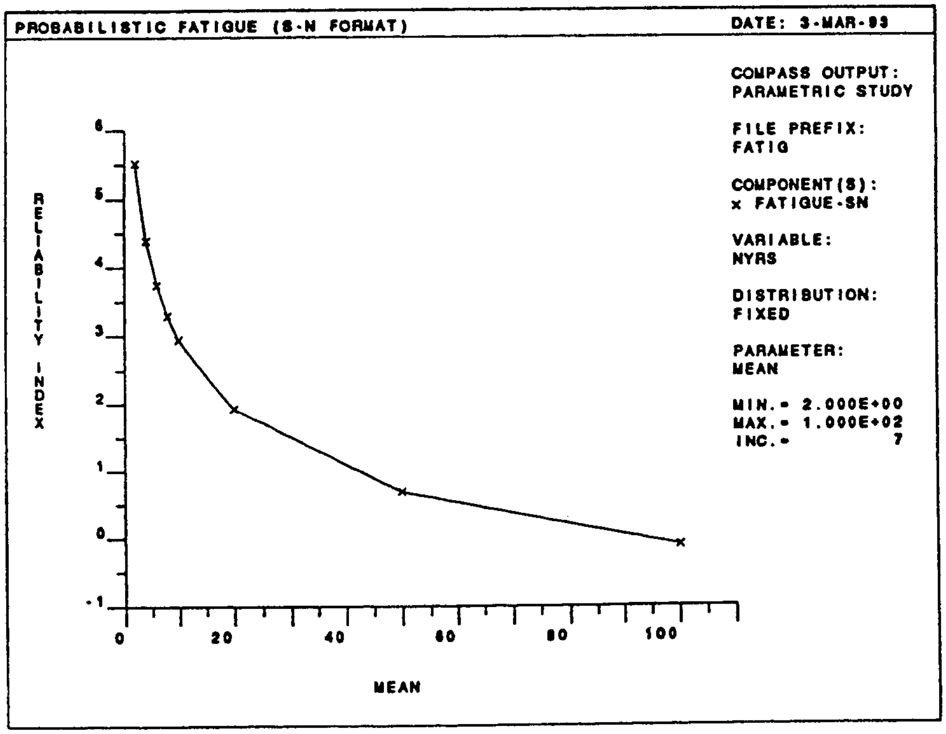


COMPASS has been demonstrated to produce accurate reliability and probabilistic sensitivity analysis results in several engineering applications.

General Features

COMPASS operates interactively or in a batch mode. The program currently has the following main features:

- ✓ Library of 16 probability distributions.
- ✓ Correlations between variables in U-space or X-space.
- ✓ Definition of limit state functions by user subroutine.
- ✓ Calculation of component reliability index (β) and failure probability (P_f) by:
    - First-order Reliability Methods (FORM)
    - Second-order Reliability Methods (SORM)
    - Direct Monte Carlo Simulation (DMCS)
    - Importance Sampling Scheme (ISS)
- ✓ Systems reliability analysis methods based on:
    - Unimodal and Bimodal Bounds
    - Probabilistic Network Evaluation Technique (PNET)
    - Direct Monte Carlo Simulation (DMCS)
    - Importance Sampling Scheme (ISS)
- ✓ Calculation of parametric sensitivity and importance factors.



Unique Features

In addition to these general purpose features, COMPASS has unique analysis capabilities that are directed at special engineering requirements. Some of these capabilities are:

✓ Built-in library of limit state functions:

  - Fatigue Damage Accumulation
  - Probabilistic Fracture Mechanics
  - Composite Failure Criteria

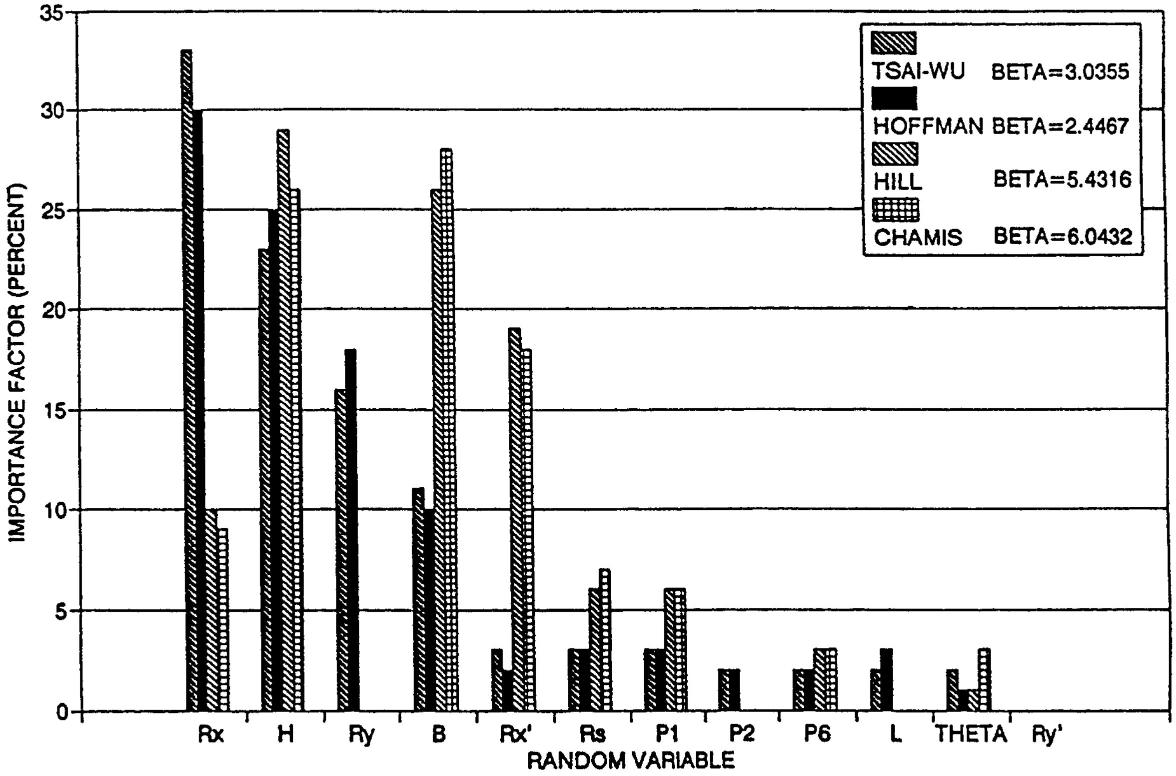
✓ Customized limit state functions provided on request.

✓ Graphics support capabilities.

✓ Probabilistic data characterization.

✓ Efficient interface with the commercial FEA program VAST (customized interfaces with other commercial FEA packages are available on request).

✓ Parametric studies



IMPORTANCE FACTORS FOR RANDOM VARIABLES OF A COMPOSITE LAMINATE

APPENDIX D

GENERAL INFORMATION
ON
THE FOUR SELECTED SHIPS

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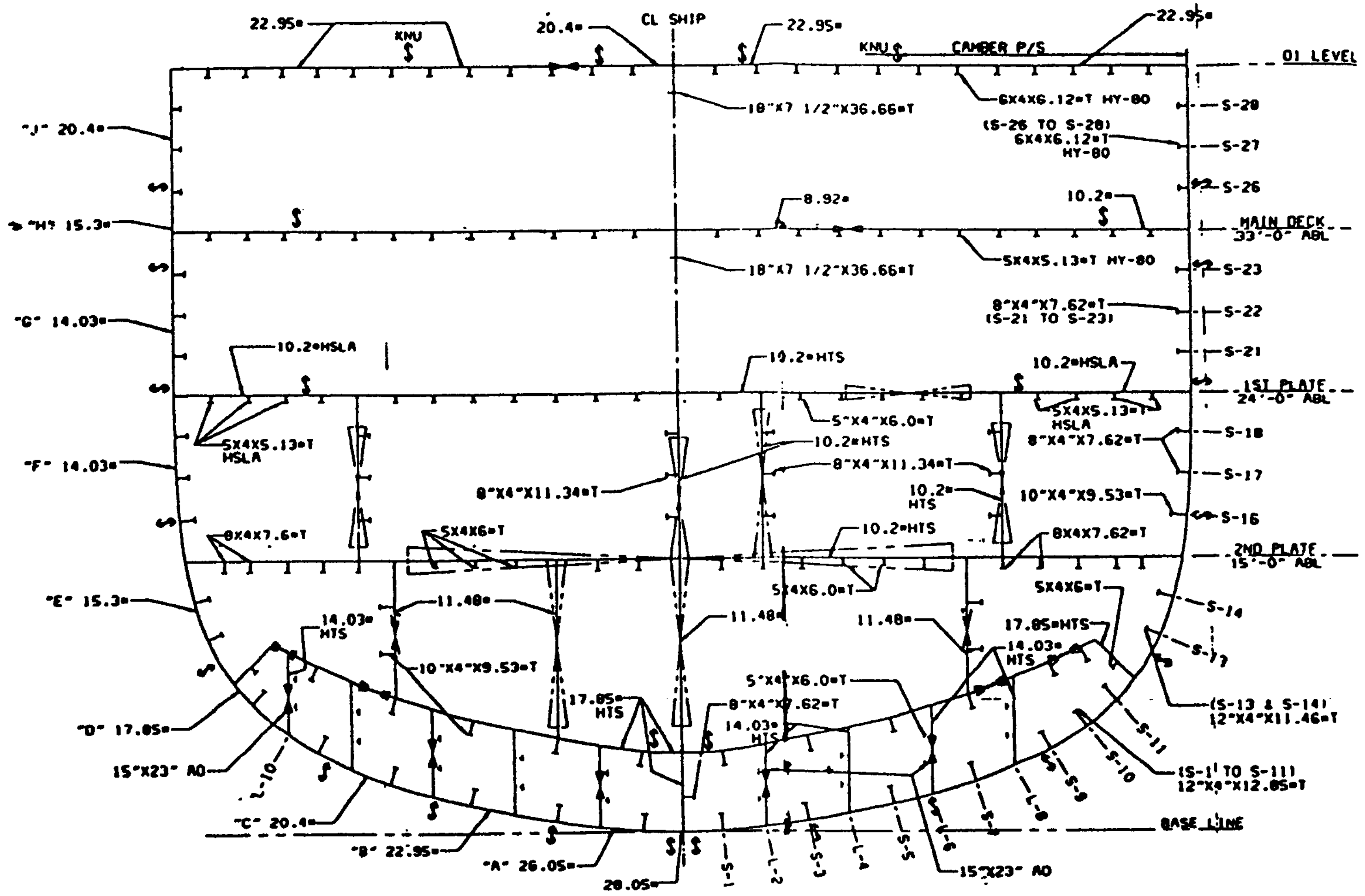
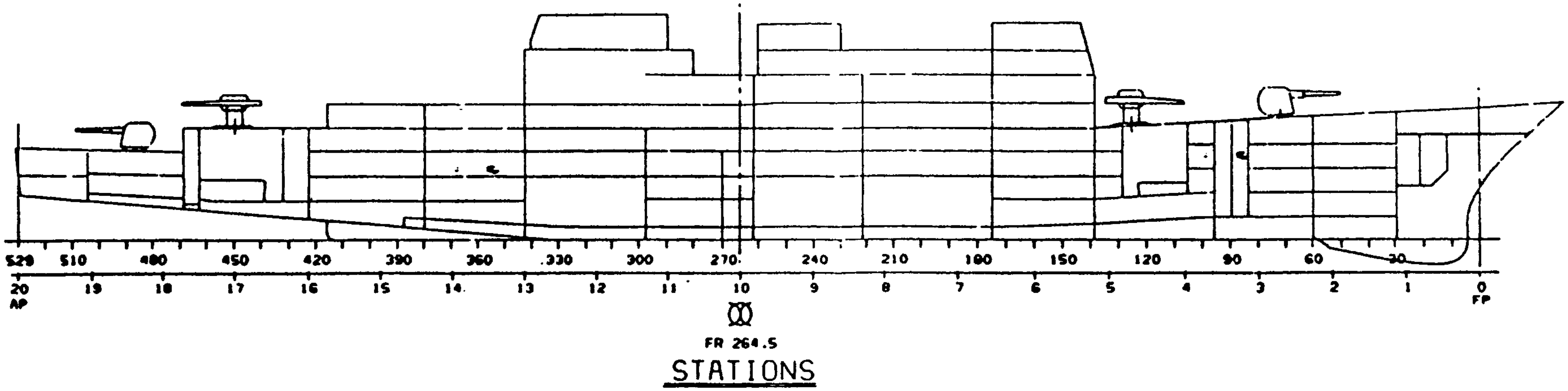


STA. 10 - 6" AFT FR 264

LOOKING FWD

CL SHIP

Cruiser No. 1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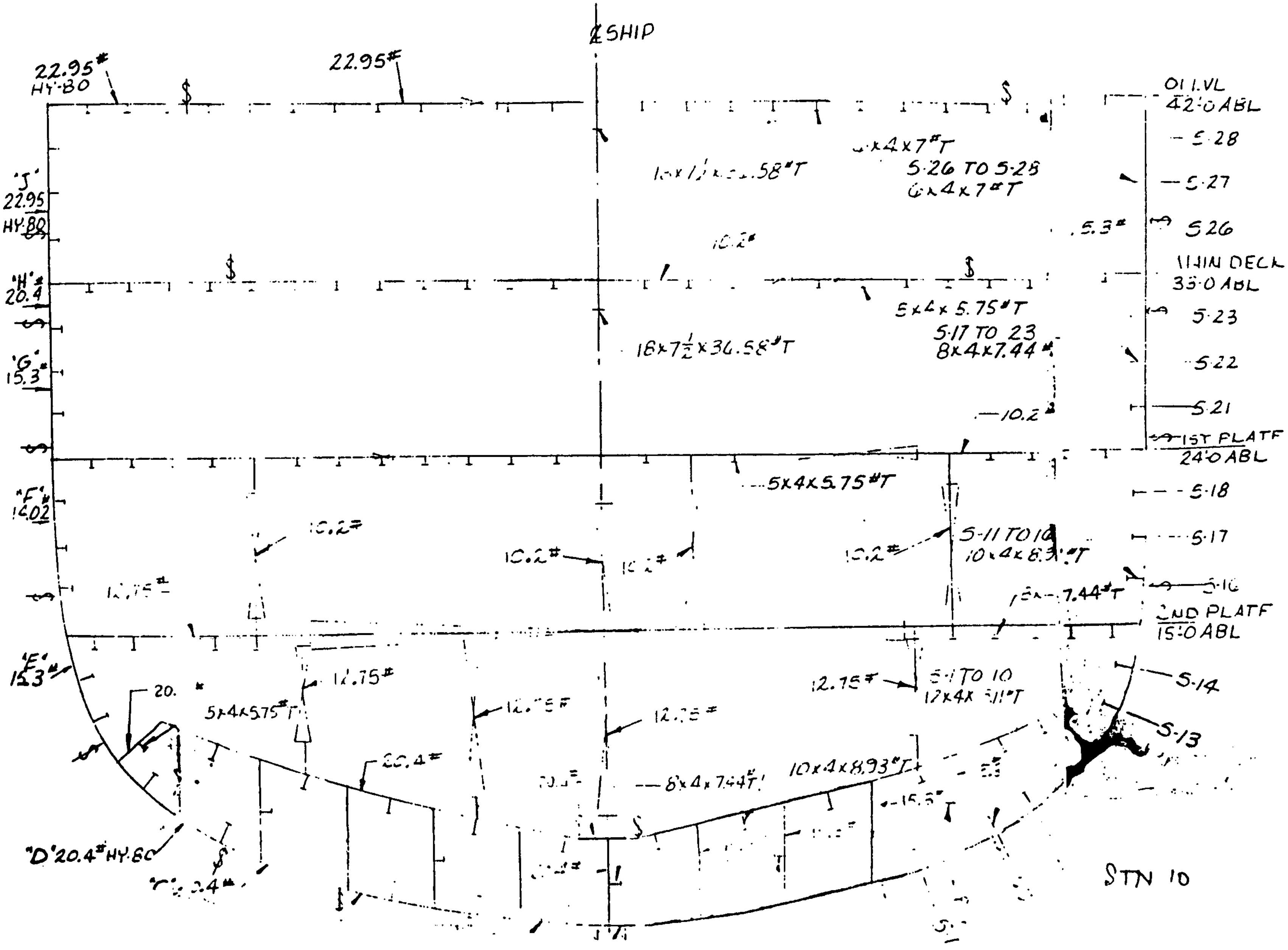
Cruiser No. 2


℄ SHIP
22.95# HY-80
22.95#
01 LVL 42'-0 ABL
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MAIN DECK 35'-0 ABL
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5-21
1ST PLATF 24'-0 ABL
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5-17
5-16
2ND PLATF 15'-0 ABL
5-14
5-13
STN 10
'J' 22.95 HY-80
'H' 20.4
'G' 15.3
'F' 14.02
'E' 15.3
"D" 20.4# HY-80
6x4x7#T
10x7½x...58#T
5-26 TO 5-28
6x4x7#T
10.2#
5.3#
5x4x5.75#T
5-17 TO 23
8x4x7.44#
16x7½x36.58#T
10.2#
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5-11 TO 16
10x4x8.93#T
12.75#
7.44#T
12.75#
5x4x5.75#T
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12.75#
12.75#
12.75#
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12x4x...#T
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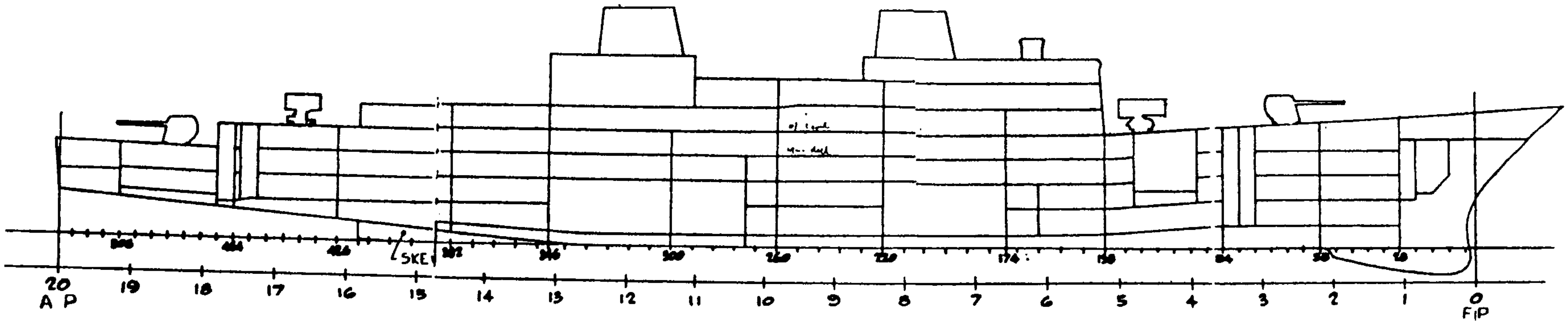
8x4x7.44#T
10x4x8.93#T
15.3#

Cruiser No. 2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300
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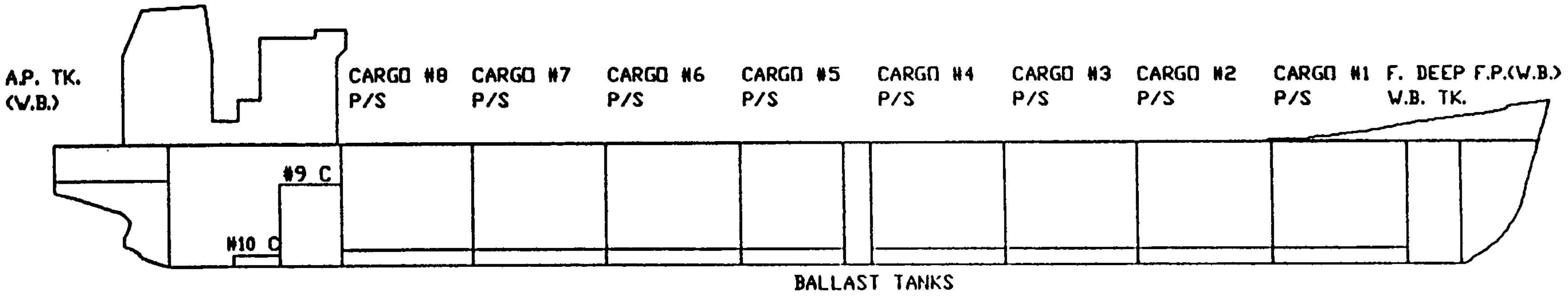
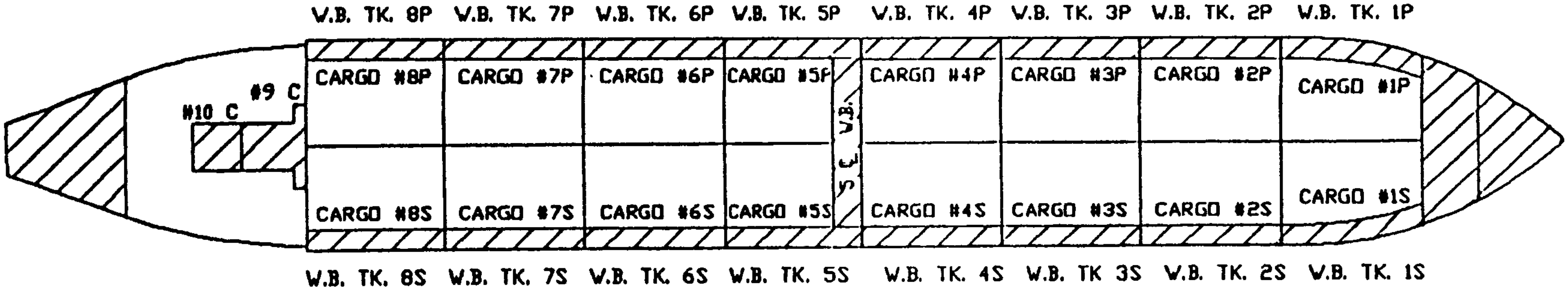
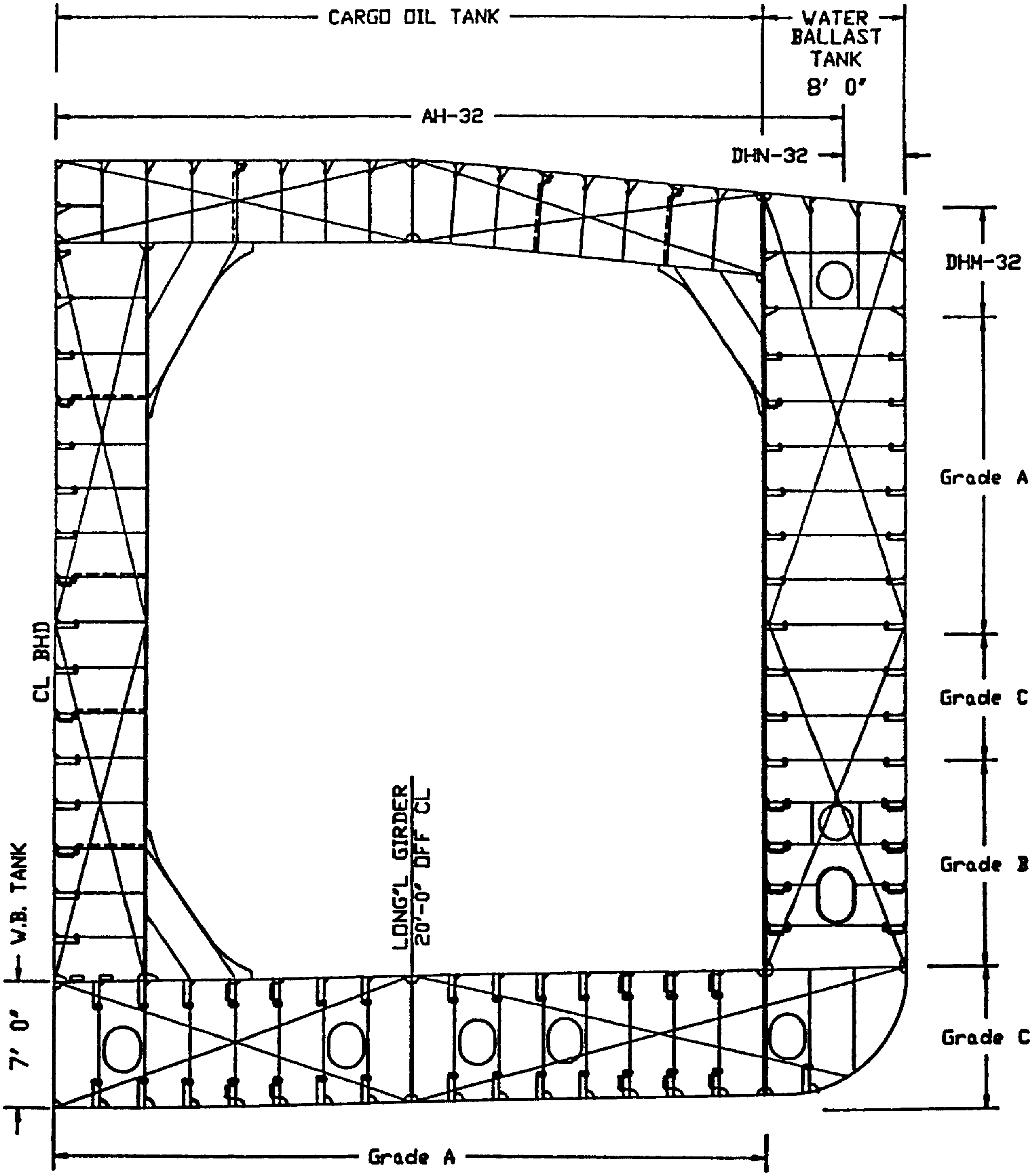
Double Hull Tanker



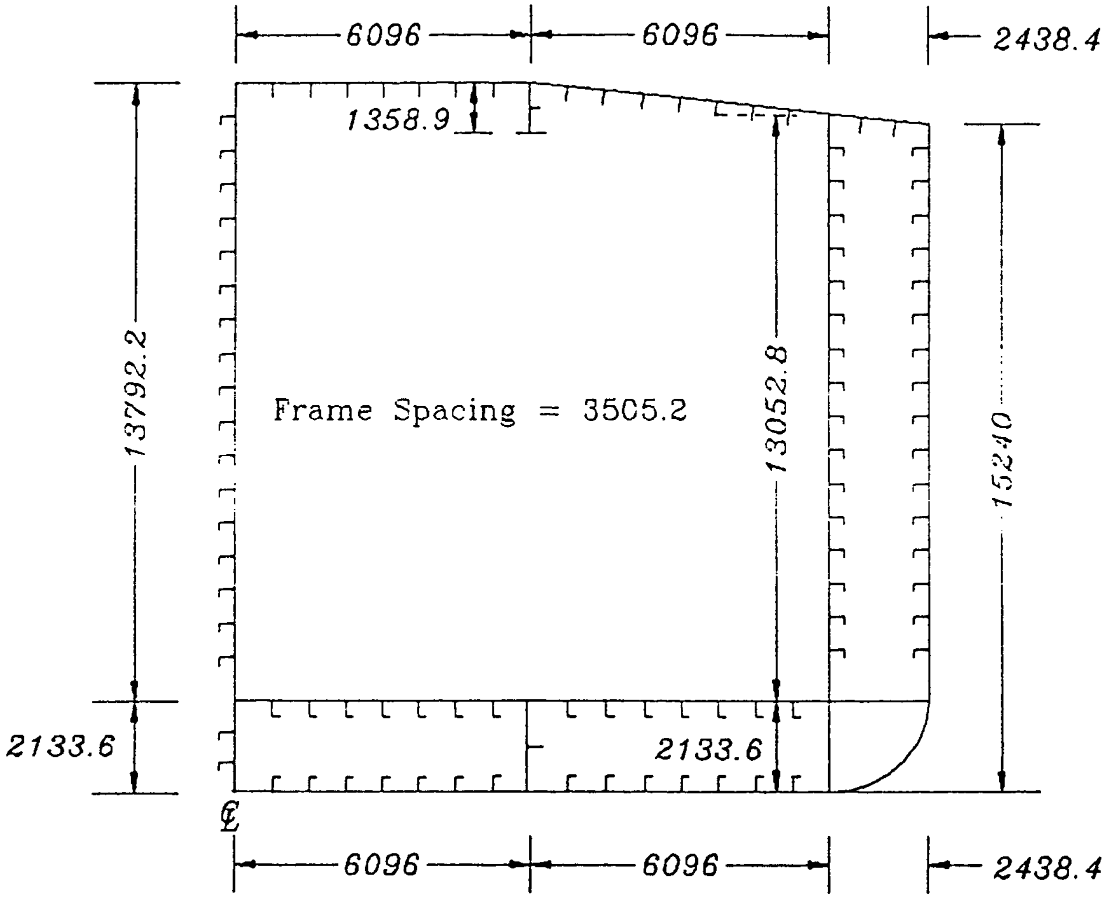


Tank Arrangement

Double Hull Tanker

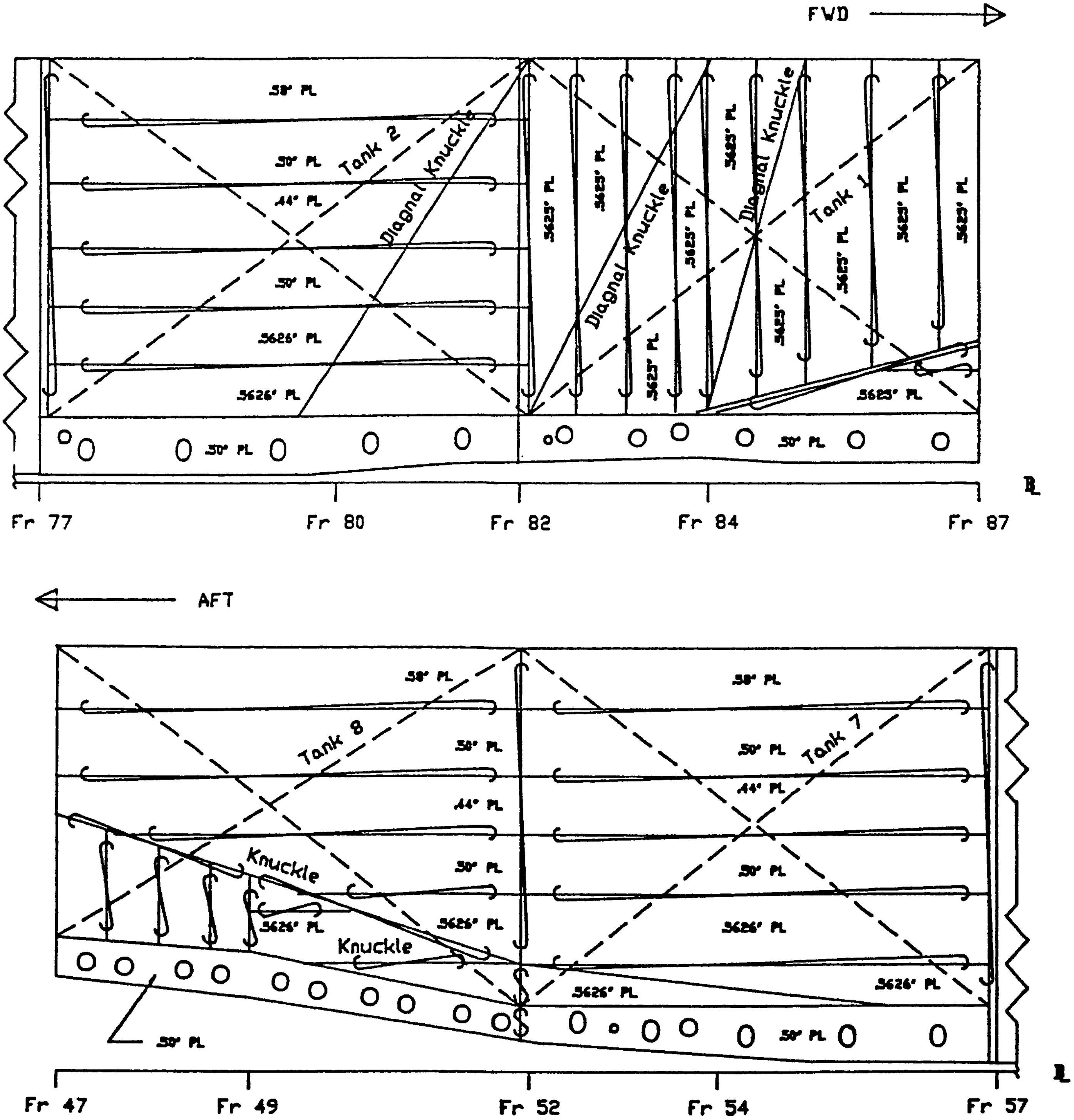


Midship Section



Midship section of a double hull tanker (unit mm)

Double Hull Tanker



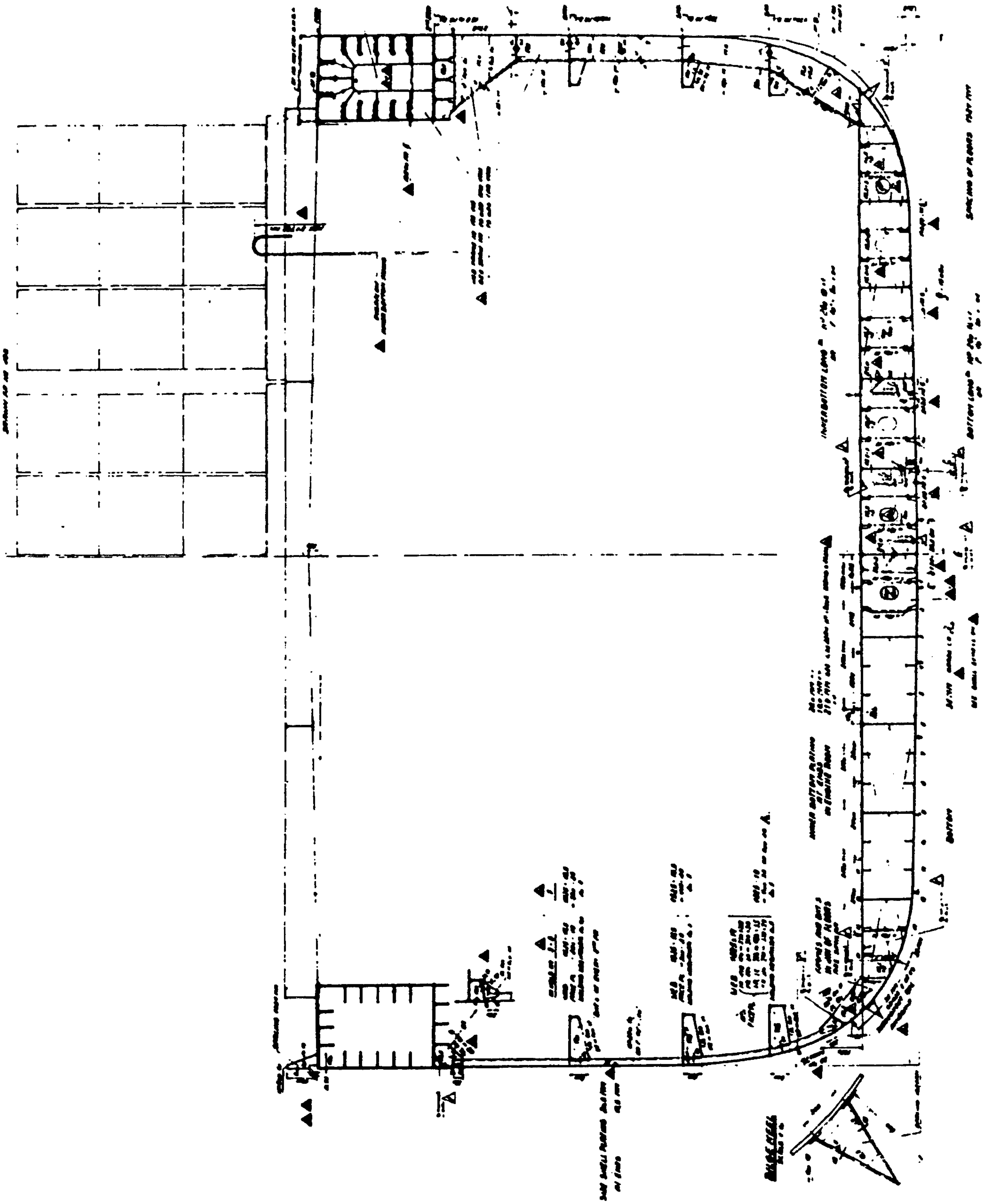
Longitudinal Bulkhead at 40 ft from CL

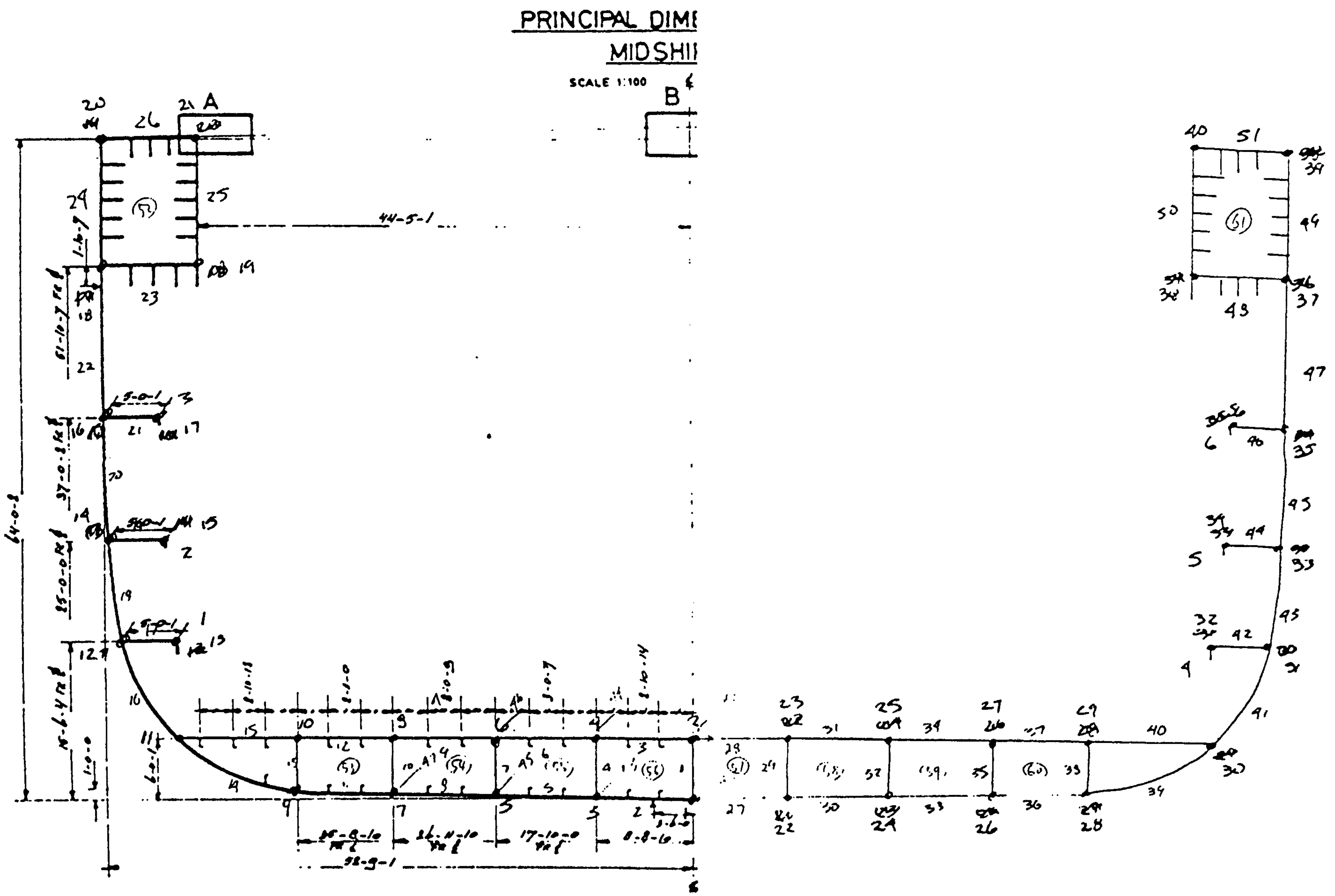
CHARACTERISTICS OF *S. S. SEA-LAND McLEAN*

Name·	**SEA-LAND McLEAN**
Builder:	**Rotterdam Dry Dock (Hull 330)**
Class:	**SL-7 Containership**
Length, overall	**946' 1-1/2"**
Length, between perpendiculars	**830' 6"**
Beam, molded	**105' 6"**
Depth to main deck, forward	**64' 0"**
Depth to main deck, aft	**68' 6"**
Draft, design	**30' 0"**
Draft, scantling	**34' 0"**
Dead weight - long tons	**27,315**
Displacement (34' 0" draft) - long tons	**50,315**
Machinery	**Two separate cross-compound steam turbines driving two propeller shafts**
Shaft horsepower-maximum continuous, both shafts	**120,000**
Propeller RPM	**135**
Speed, maximum, knots	**33**
Center of gravity - full load	**399.32' forward of aft perpendicular 42.65' above base line**

Container Capacity

	8' x 8.5' x 35'	8' x 8.5' x 40'	Total
Below deck	554	140	694
Above deck	342	60	402
TOTAL	896	200	1,096


TYP WEB FRAME
INNER BOTTOM LONG'L
BOTTOM LONG'L
BOTTOM
INNER BOTTOM PLATING
SIDE SHELL PLATING
BILGE KEEL



SL-7 Container

APPENDIX E

COLLECTED LOAD DATA (SAMPLE)
ON THE FOUR SELECTED SHIPS

Cruiser No. 1

CHARACTERISTICS

LBP	529 ft
B midship	55 ft
T midship	2 4 ft

Station of max area	290.95 ft aft of FP
Station spacing	26.45 ft
Total Displacement	9680 tons

TROCHOIDAL WAVE CALCULATION RESULTS

Displacement	9335 L.Tons
LCG	10.8 ft aft
Wave length L	529 ft
Wave type	Trochoidal
Wave height	1.1 * sqrt (L)

Max BM and Min Shear force occur close to midship
(About station 11, aft of midship station 10, 0 FP 20 AP)
Max Shear force occurs about Station 6 fwd, 15 aft

ALLOWABLE STRESSES (per specs)

8.5 TSI at keel
9.5 TSI at 01 Level

SECTION AND MOMENT DATA

Neutral Axis Location

Stations 9,10,11	20.07, 19.76, 19.1 ft ABL 21.93, 22.24, 22.9 ft from 01 Level 20.17, 19.76, 19.1 ft above keel

Station	SM (top) 01 Level (in**2-ft)	SM (keel) (in**2-ft)	BM hogging (ft-tons)	BM sagging (ft-tons)
9	21388	23371	194236	105358
10	22805	25667	210234	111253
11	23168	27777	214972	108553

TROCHOIDAL STRESSES (TSI)

Station	01 Level Tension	Compression	Keel Tension	Compression
9	9.08	4.93	4.51	8.31
10	9.22	4.88	4.33	8.19
11	9.28	4.69	3.91	7.74

STILL WATER BENDING MOMENTS AND STRESSES

M_{sw} = 71,926 ft-ton Hogging @ midship section

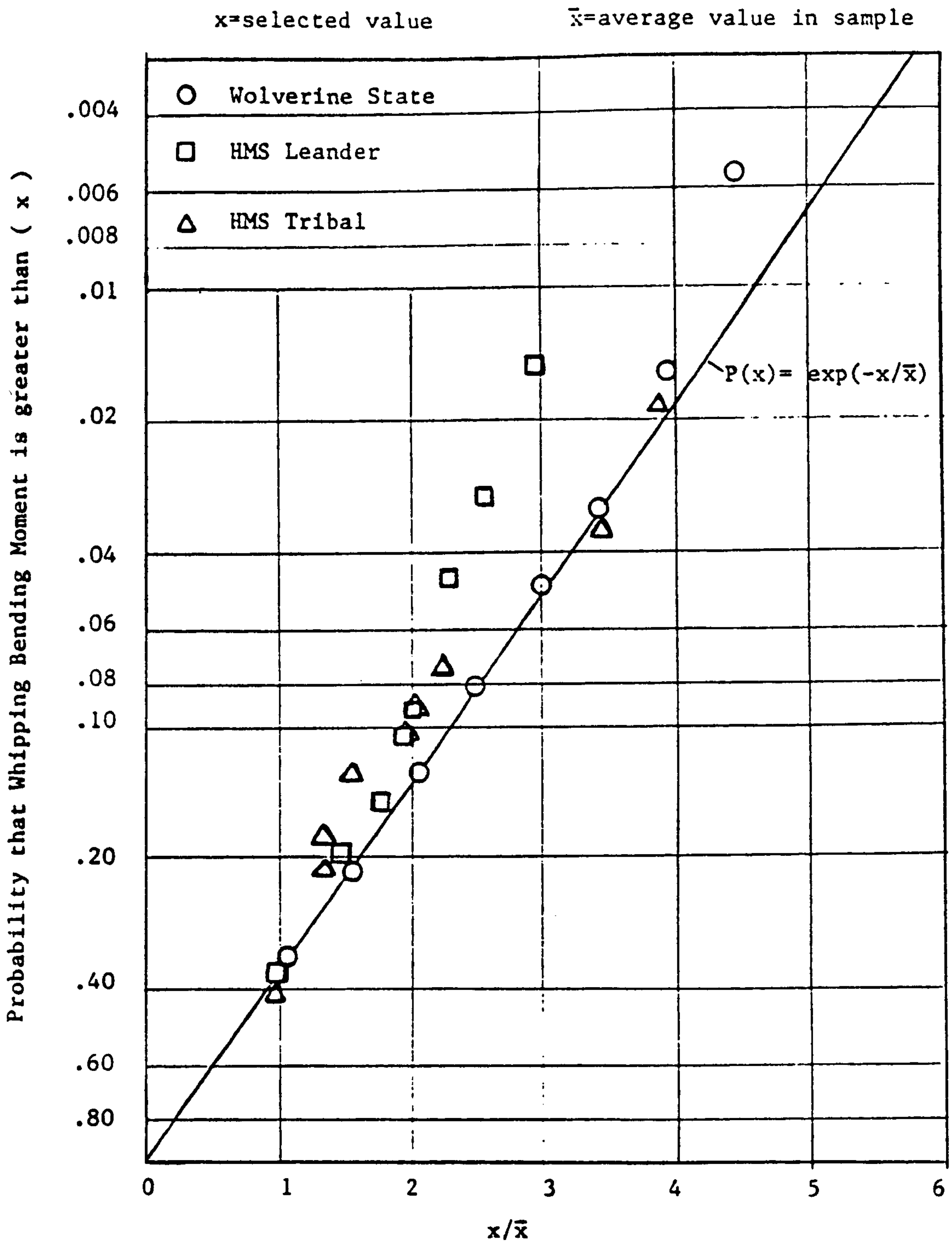
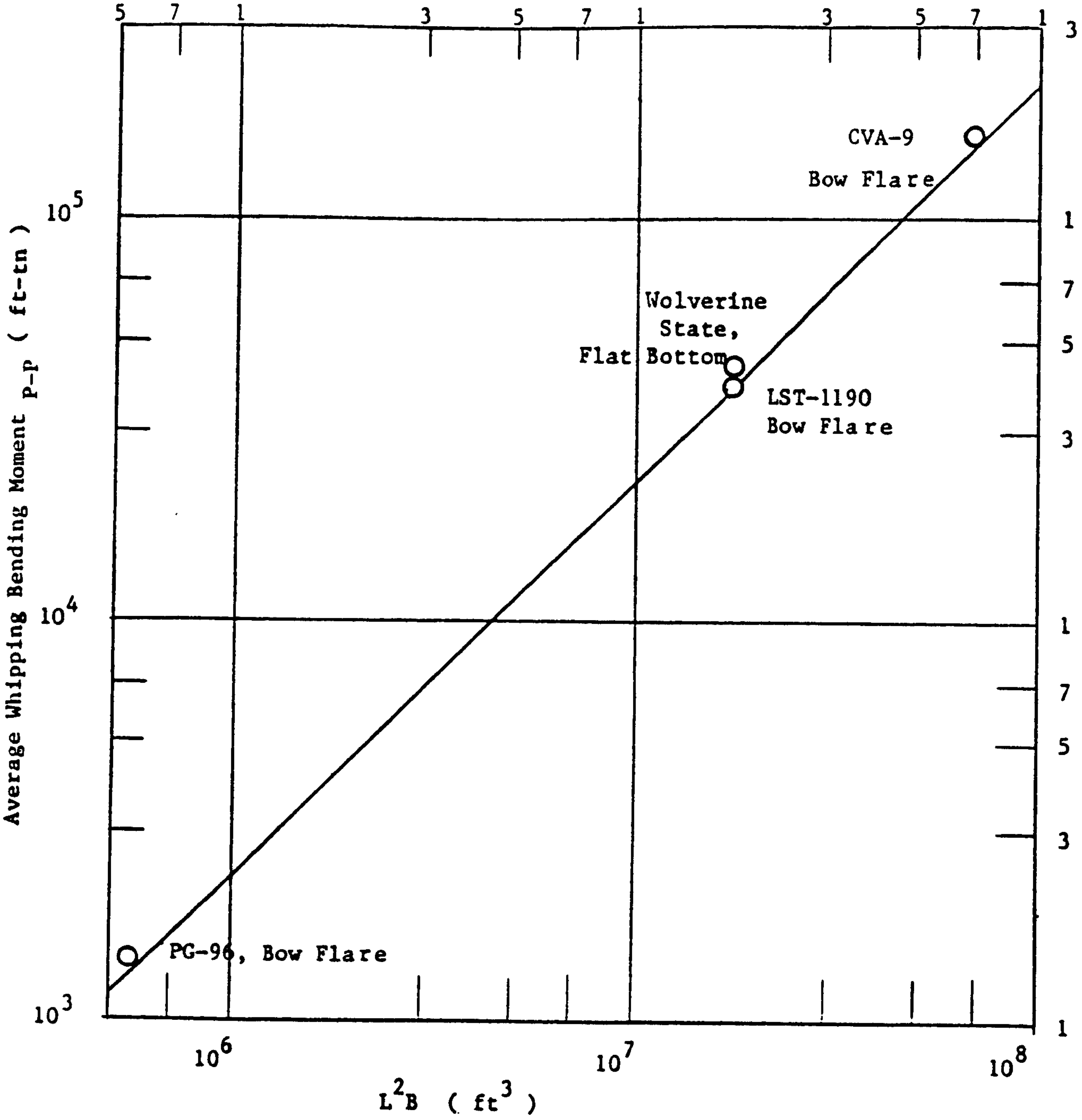
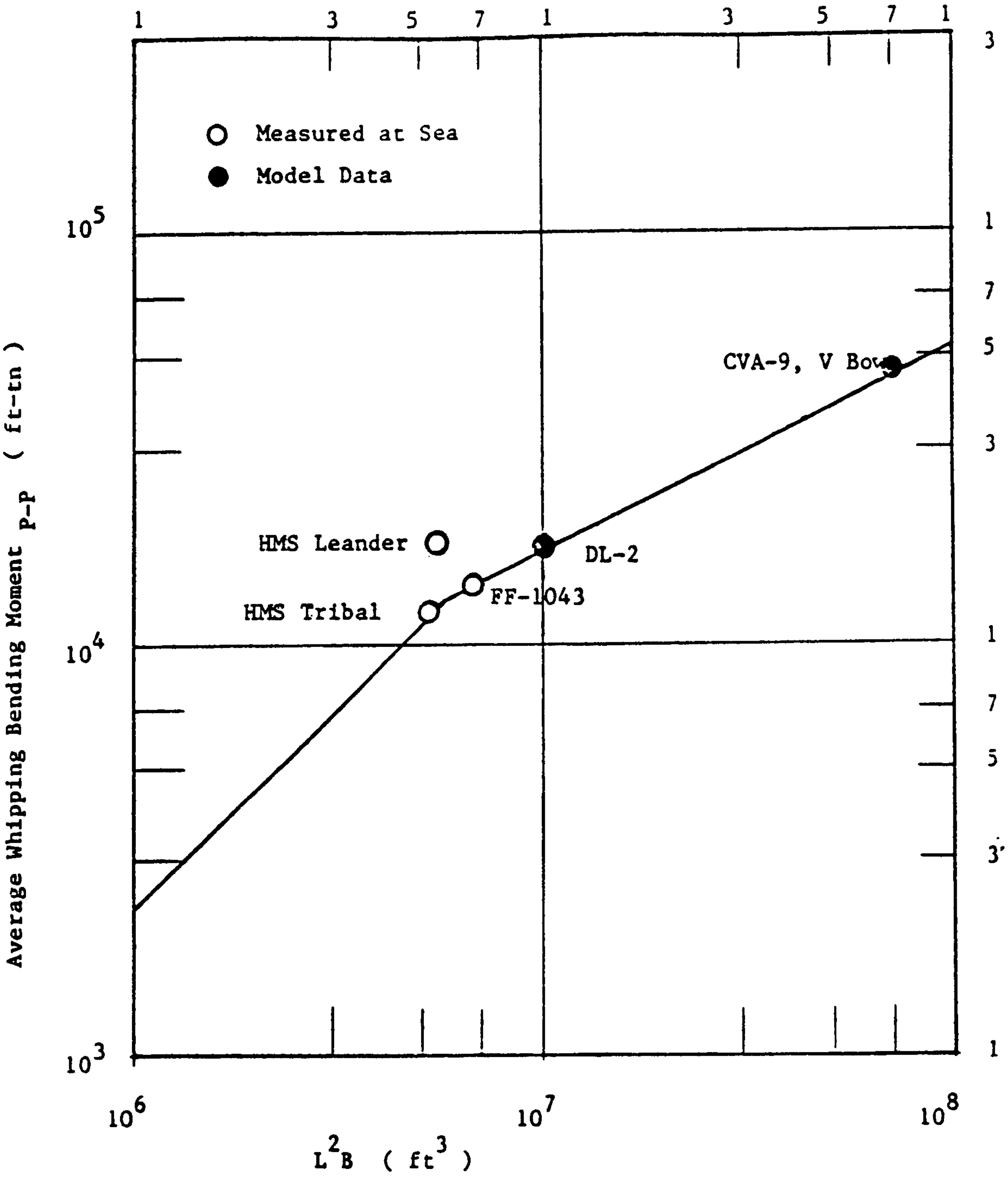


Figure 3 - Probability Distribution for Whipping Bending Moments



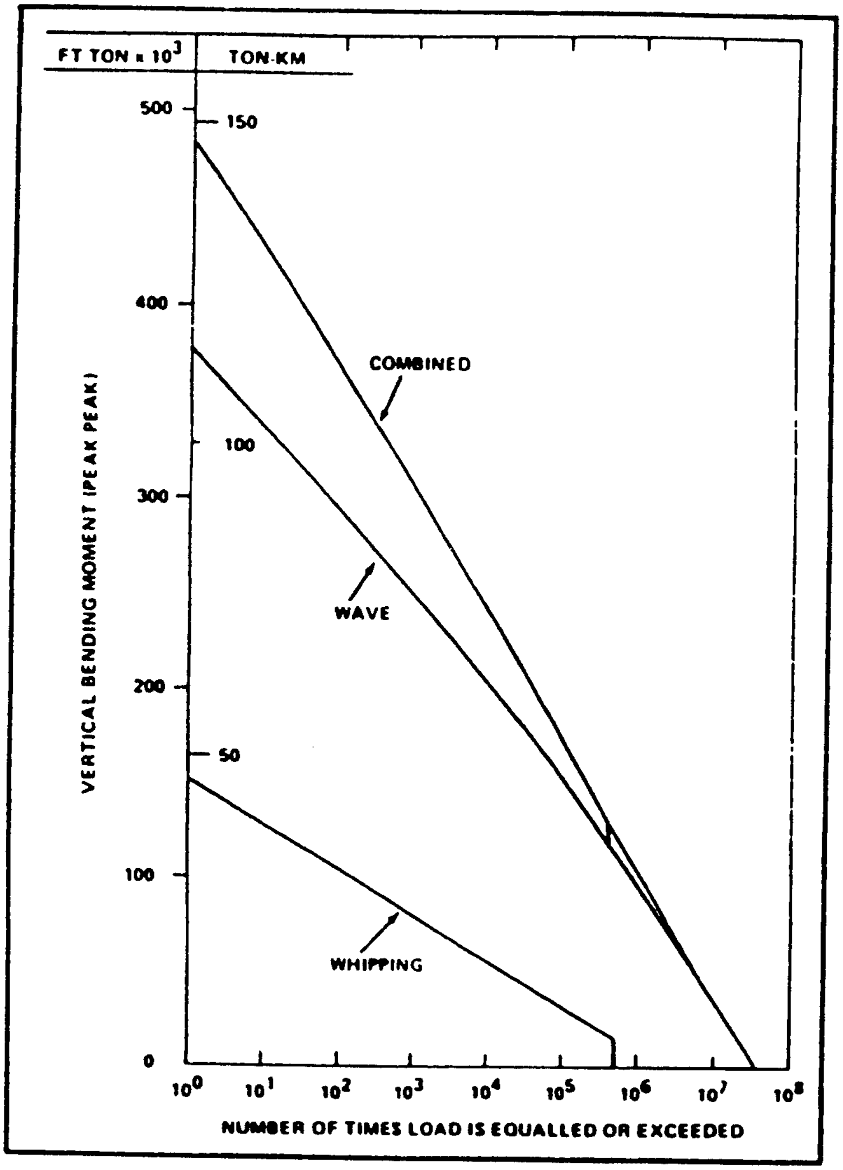
6 (a) Sea Trial Data for Hulls Disposed Toward Whipping

Figure 6 - Whipping Bending Moments

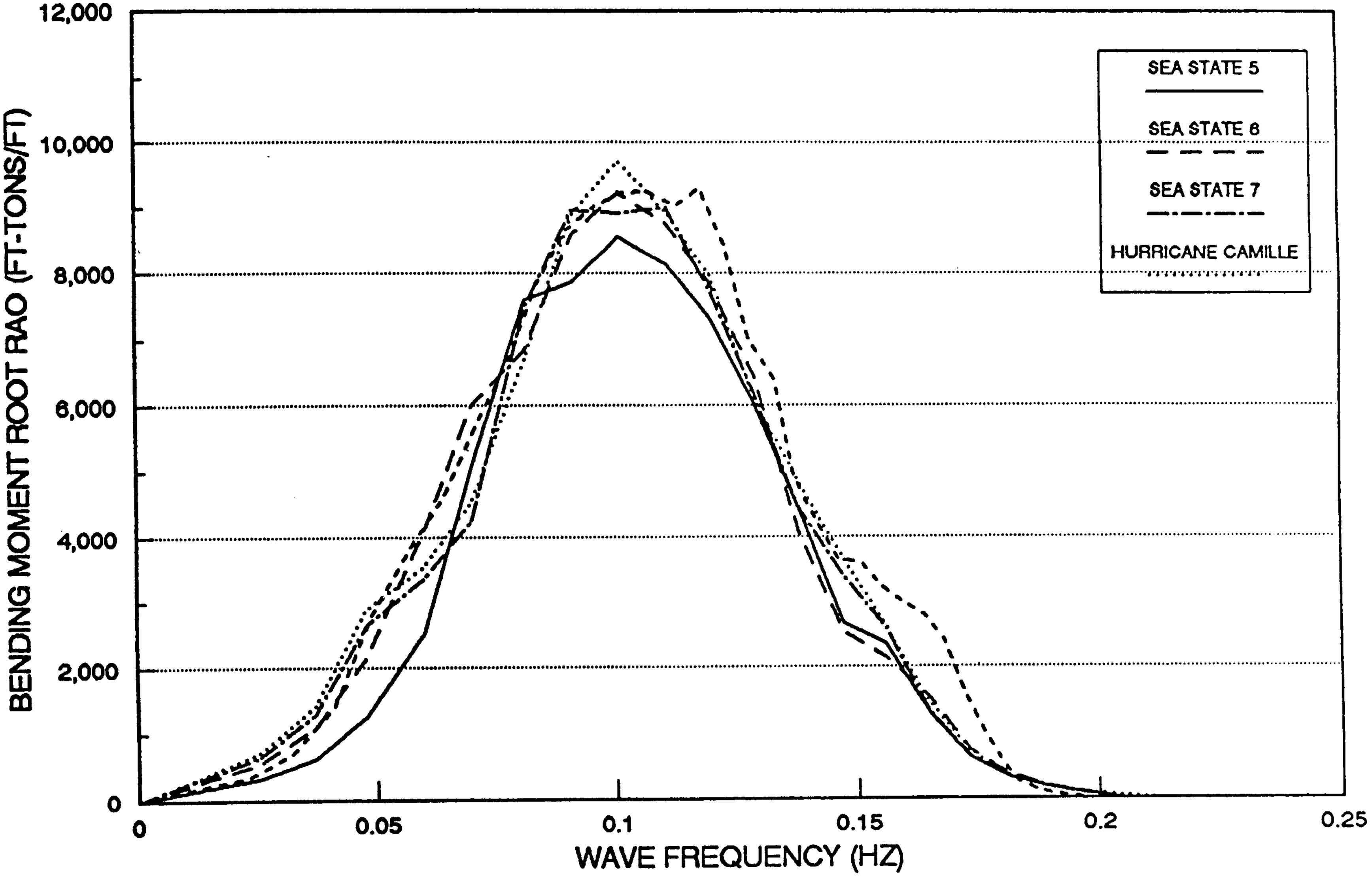


6 (b) Data from Fine Bow Hulls

Figure 6 continued

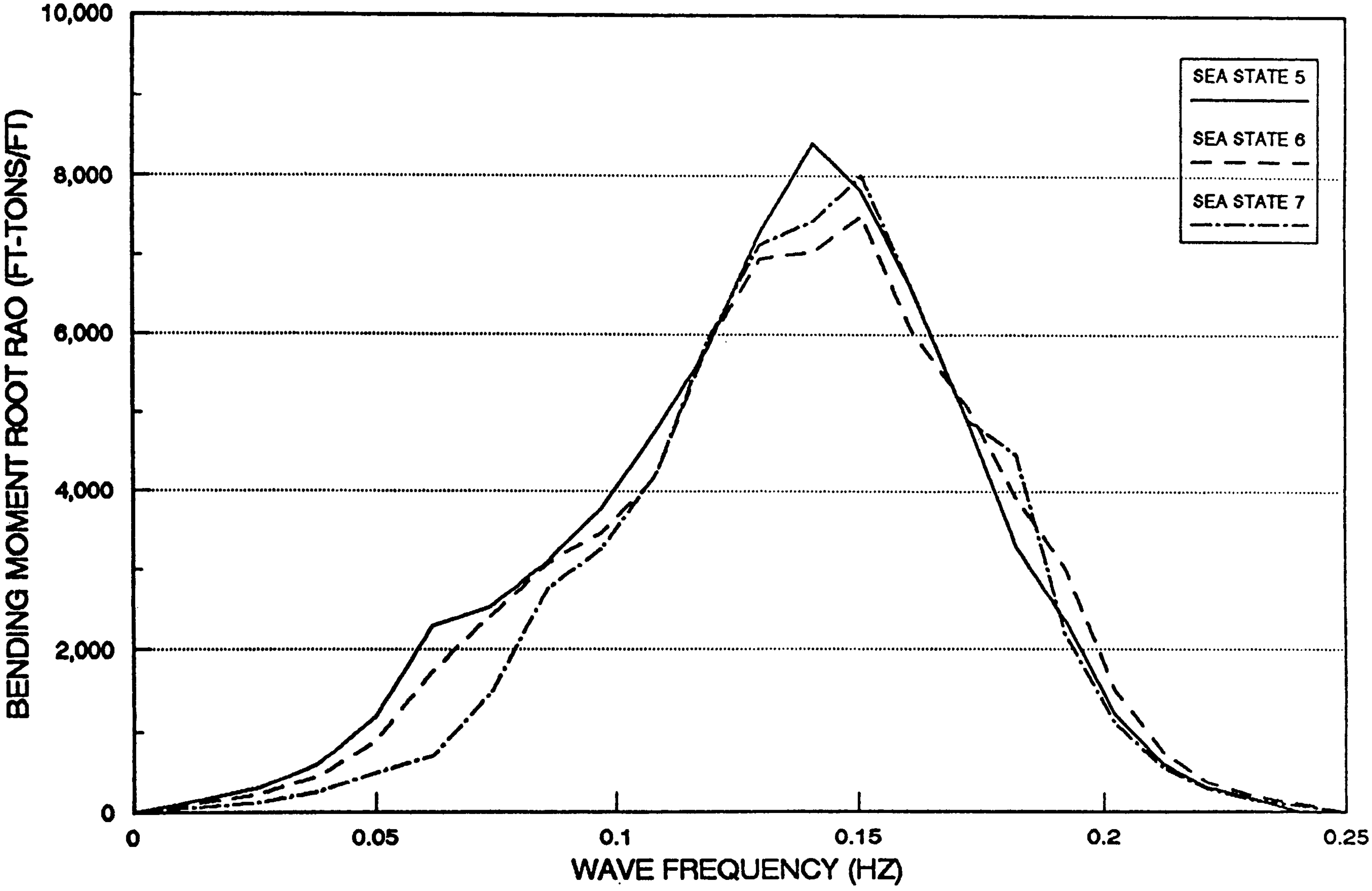


Lifetime Bending Moments for Ship 2.

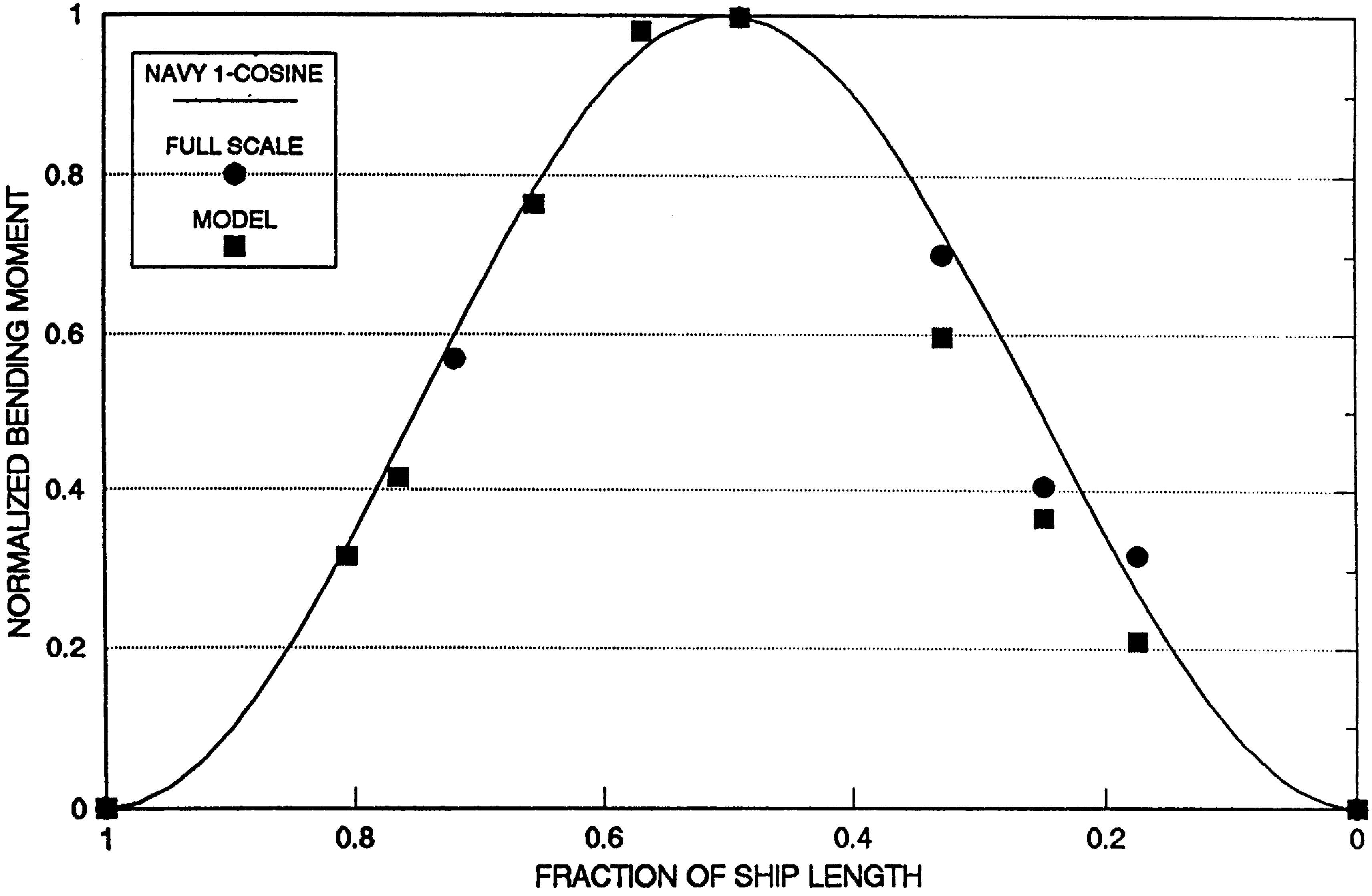


HEAD SEA VERTICAL BENDING MOMENT RAO'S

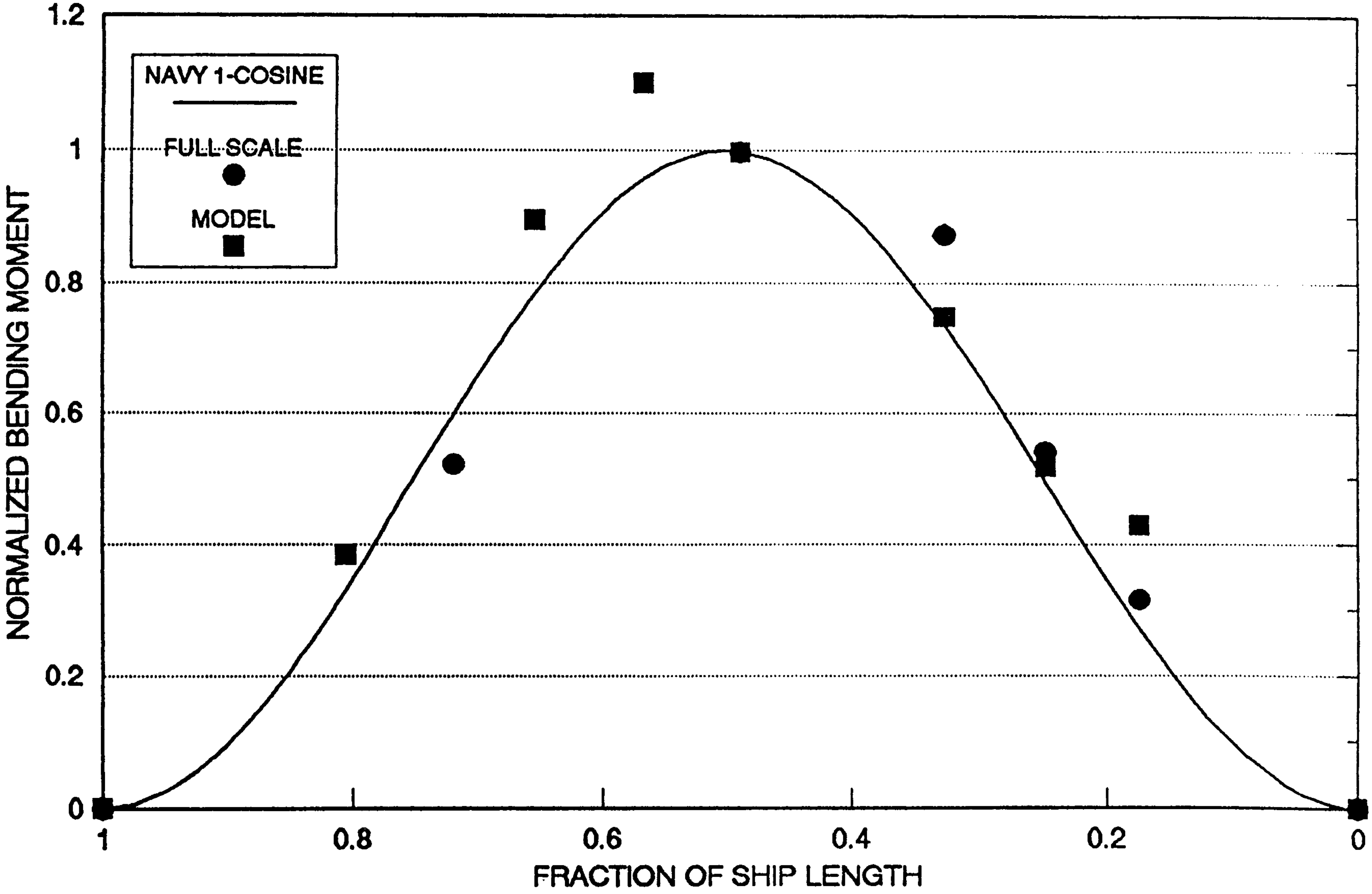
60 DEGREE LATERAL BENDING MOMENT RAO'S



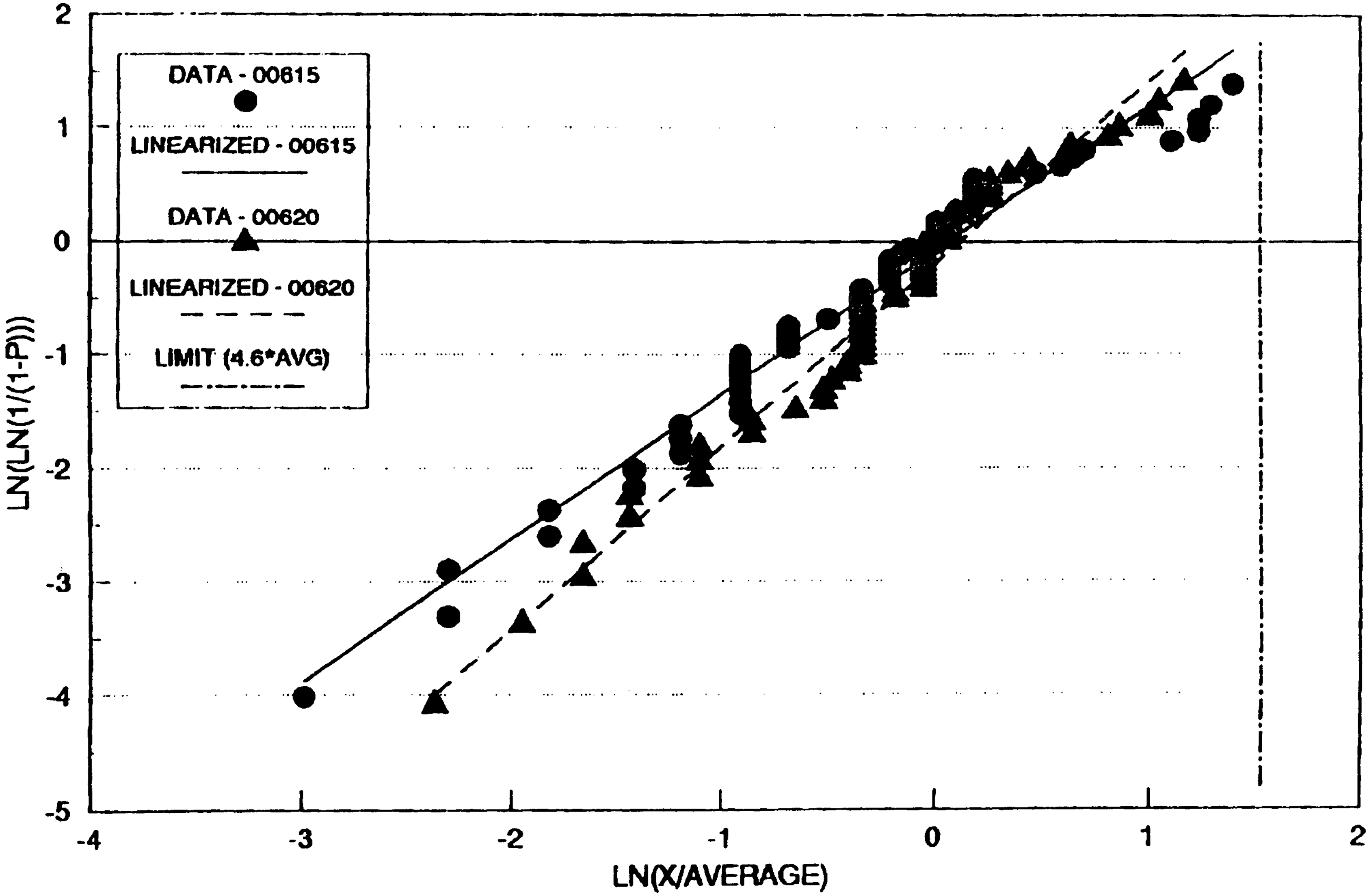
ORDINARY WAVE
VERTICAL BENDING MOMENT DISTRIBUTION



ORDINARY WAVE
LATERAL BENDING MOMENT DISTRIBUTION



WEIBULL PLOTS
MIDSHIP VERTICAL WHIPPING MOMENTS



Cruiser No. 2

CHARACTERISTICS

LBP	529 ft
Draft	21.48 ft
LCG from amidships	-7.109 ft (+ fwd)
Displacement	9019 Tons
Station of max area	288.95 ft from FP
Beam at that station	55.92 ft
Section area coefft	0.841
Prismatic coefft	0.5754
Block coefft	0.4839

(Above data from SHCP output, baseline ship)

TROCHOIDAL WAVE CALCULATION RESULTS

Displacement	7818 Tons
Wave length L	529 ft
Wave type	Trochoidal
Wave height	1.1 * sqrt (L), 25.3 ft

Max BM and Min Shear force occur close to midship
(Between station 11 aft, and midship station 10, 0 FP 20 AP)
Max Shear force occurs between Station 6,7 fwd, 14 aft

SECTION AND MOMENT DATA

Location of Neutral Axis

Station 9,10,11	19.84, 20.11, 19 ft ABL
	22.16, 21.89, 23 ft from top
	19.94, 20.11, 19 ft from keel

Station	SM (01 Levl) (in**2-ft)	SM (keel) (in**2-ft)	BM hogging (ft-tons)	BM sagging (ft-tons)
9	22307	24906	179916	103400
10	24419	26581	190670	110224
11	22644	27411	190034	111258

TROCHOIDAL STRESSES (TSI)

Station	01 Level Tension	Compression	Keel Tension	Compression
9	8.07	4.64	4.16	7.25
10	7.80	4.51	4.16	7.18
11	8.39	4.91	4.06	6.93

STILL WATER BENDING MOMENTS AND STRESSES
(Baseline ship, SHCP output)

Station	Bending Moment (ft-tons)	Stresses (TSI) 01 Level Tension	Keel Compression
9	51771 hogging	2.32	2.08
10	57875	2.37	2.18
11	61369	2.71	2.24

“SCORES” RESULTS — SAMPLE

Cruiser No. 2

```
RUN IDENTIFICATION:  SEA STATE 6

INPUT VERIFICATION:

   1- WATERLINE LENGTH LWL (M)                =    161.28

   2- VESSEL DISPLACEMENT (TONNES)            =    8835.5

   3- VERTICAL CENTER OF GRAVITY (M)          =      7.10

   4- ROLL RADIUS OF GYRATION (M)             =      6.95

   5- FRACTION OF CRITICAL ROLL DAMPING       =     .1000

   6- SHIP SPEED (KNOTS)                      =     15.00

   7- SHIP HEADING RELATIVE TO WAVES (DEG)    =    135.00

   8- WATER TYPE                              =   SALT@15C

   9- ISSC TWO PARAMETER SPECTRUM EXCITATION

  10- SIGNIFICANT WAVE HEIGHT (M)             =      5.09

  11- CHARACTERISTIC WAVE PERIOD (S)          =     10.24

  12- LOWER FREQ.INTEGRATION LIMIT (R/S)      =       .26

  13- UPPER FREQ.INTEGRATION LIMIT (R/S)      =      1.70


     STA.     BEAM[M]  AREA[M*M]   DRAFT[M]  WEIGHT[T]

      0          .00        .00        .00      300.0
      1         4.02      20.26       7.13      375.6
      2         7.98      42.53       7.02      775.0
      3        11.74      61.82       6.89      926.1
      4        15.32      80.08       6.78     1324.4
      5        16.73      92.16       6.55     1295.7
      6        16.79      89.37       6.42     1320.0
      7        16.26      69.52       6.31      820.1
      8        15.23      46.79       6.20      710.0
      9        13.96      24.14       6.08      688.6
     10        12.46       7.45       5.96      300.0
```

Cruiser No. 2 (SCORES)

```
RUN IDENTIFICATION:  SEA STATE 6

MOTION NATURAL FREQUENCIES AND PERIODS:

    HEAVE NATURAL FREQUENCY = 1.120 RAD/S    HEAVE NATURAL PERIOD =  5.61 SEC.

    PITCH NATURAL FREQUENCY = 1.152 RAD/S    PITCH NATURAL PERIOD =  5.45 SEC.

    ROLL NATURAL FREQUENCY  =  .412 RAD/S    ROLL NATURAL PERIOD  = 15.25 SEC.

      ROLL WAVE DAMPING           = 0.103E+03

      ADDED VISCOUS ROLL DAMPING = 0.397E+04


SEAKEEPING RESPONSE RESULTS:

  SHIP SPEED                                 = 15.0 KNOTS = 7.72 M/S
  WAVE ANGLE [WITH HEAD SEAS 180 DEG.]       =135.0 DEG.
  ISSC TWO PARAMETER SPECTRUM - SIGN.HEIGHT = 5.09 M     CHAR. PERIOD = 10.24 S

 NONDIMENSIONAL MOTION RESPONSE:

    WAVE  ENCOUNT.  WAVE         HEAVE               PITCH                ROLL
    FREQ.   FREQ.  LENGTH    AMPL.     PHASE     AMPL.     PHASE     AMPL.     PHASE
     R/S     R/S      M        ND       DEG.       ND       DEG.       ND       DEG.
    .260    .298   911.5  0.998E+00  179.9  0.719E+00   84.4  0.148E+01 -136.0
    .340    .404   533.0  0.986E+00  179.8  0.728E+00   80.2  0.420E+01  155.8
    .420    .518   349.3  0.963E+00 -180.0  0.731E+00   74.0  0.180E+01   61.0
    .500    .639   246.5  0.932E+00 -179.1  0.721E+00   65.3  0.918E+00   40.6
    .580    .767   183.2  0.923E+00 -178.2  0.688E+00   52.7  0.585E+00   31.0
    .660    .902   141.5  0.986E+00  176.5  0.614E+00   33.0  0.381E+00   25.0
    .740   1.045   112.5  0.949E+00  147.0  0.441E+00     .4  0.225E+00   21.1
    .820   1.194    91.6  0.358E+00   97.1  0.189E+00  -33.6  0.105E+00   21.4
    .900   1.351    76.1  0.553E-01 -115.3  0.554E-01  -70.2  0.304E-01   44.7
    .980   1.514    64.2  0.125E+00 -145.7  0.170E-01 -176.7  0.220E-01  134.5
   1.060   1.685    54.8  0.732E-01 -168.4  0.169E-01  122.4  0.238E-01  151.5
   1.140   1.863    47.4  0.227E-01  162.8  0.811E-02   94.3  0.130E-01  156.5
   1.220   2.048    41.4  0.697E-02   85.6  0.151E-02   56.2  0.292E-02  174.9
   1.300   2.240    36.5  0.495E-02   49.8  0.648E-03  -61.7  0.947E-03  -76.6
   1.380   2.440    32.4  0.151E-02   55.7  0.441E-03  -22.0  0.173E-03  128.2
   1.460   2.646    28.9  0.116E-02  -82.0  0.641E-03   -5.2  0.411E-03  142.5
   1.540   2.860    26.0  0.255E-02  -96.9  0.318E-03  -80.5  0.745E-03 -110.8
   1.620   3.080    23.5  0.109E-02 -135.3  0.503E-03 -163.0  0.744E-03 -139.2
   1.700   3.308    21.3  0.156E-02  106.9  0.277E-03  144.9  0.698E-03  108.6
```

Cruiser No. 2 (SCORES)

NONDIMENSIONAL MOMENT RESPONSE FOR RUN: SEA STATE 6

WAVE FREQ. R/S	ENCOUNT. FREQ. R/S	WAVE LENGTH M	VERTICAL MOMENT AMPL. ND	VERTICAL MOMENT PHASE DEG.	TRANS. MOMENT AMPL. ND	TRANS. MOMENT PHASE DEG.	TORS. MOMENT AMPL. ND	TORS. MOMENT PHASE DEG.
.260	.298	911.5	0.336E-03	.7	0.183E-03	79.9	0.119E-03	-164.8
.340	.404	533.0	0.139E-02	-.3	0.246E-03	53.1	0.850E-03	146.8
.420	.518	349.3	0.349E-02	-1.5	0.809E-03	91.3	0.684E-03	64.9
.500	.639	246.5	0.674E-02	-3.7	0.208E-02	79.9	0.516E-03	48.7
.580	.767	183.2	0.106E-01	-6.9	0.445E-02	72.6	0.385E-03	30.1
.660	.902	141.5	0.132E-01	-9.6	0.762E-02	65.8	0.274E-03	-14.9
.740	1.045	112.5	0.136E-01	-3.1	0.105E-01	61.1	0.395E-03	-68.6
.820	1.194	91.6	0.149E-01	5.6	0.119E-01	58.7	0.604E-03	-89.3
.900	1.351	76.1	0.138E-01	8.1	0.110E-01	58.8	0.685E-03	-96.5
.980	1.514	64.2	0.956E-02	2.8	0.804E-02	61.8	0.547E-03	-95.4
1.060	1.685	54.8	0.266E-02	-16.3	0.381E-02	70.3	0.271E-03	-73.4
1.140	1.863	47.4	0.264E-02	179.7	0.709E-03	149.4	0.208E-03	3.0
1.220	2.048	41.4	0.334E-02	163.4	0.147E-02	-140.9	0.245E-03	25.0
1.300	2.240	36.5	0.125E-02	164.5	0.773E-03	-168.7	0.131E-03	22.4
1.380	2.440	32.4	0.606E-03	-101.2	0.140E-02	97.9	0.503E-04	-10.5
1.460	2.646	28.9	0.719E-03	163.5	0.176E-02	97.6	0.807E-04	-8.6
1.540	2.860	26.0	0.180E-02	107.7	0.984E-03	92.8	0.581E-04	-17.5
1.620	3.080	23.5	0.892E-03	58.4	0.525E-03	-2.9	0.919E-04	169.6
1.700	3.308	21.3	0.170E-02	-74.5	0.632E-03	-76.9	0.155E-03	122.2

AMPLITUDE RESPONSE SPECTRA:

FREQ R/S	WAVE AMP. M	HEAVE M	PITCH DEG.	ROLL DEG.	VERT. MOM. T-M	LAT. MOM. T-M	TORS. MOM. T-M
.260	0.000	0.000	0.000	0.000	0.769E+01	0.229E+01	0.974E+00
.340	.798	.776	.193	6.414	0.306E+06	0.966E+04	0.115E+06
.420	4.112	3.811	2.335	14.181	0.997E+07	0.537E+06	0.384E+06
.500	4.765	4.137	5.282	8.560	0.432E+08	0.412E+07	0.252E+06
.580	3.567	3.039	6.514	4.713	0.795E+08	0.141E+08	0.105E+06
.660	2.342	2.276	5.725	2.204	0.819E+08	0.271E+08	0.351E+05
.740	1.493	1.346	2.973	.770	0.553E+08	0.328E+08	0.464E+05
.820	.960	.123	.527	.164	0.423E+08	0.269E+08	0.697E+05
.900	.629	.002	.043	.013	0.240E+08	0.153E+08	0.588E+05
.980	.423	.007	.004	.006	0.771E+07	0.544E+07	0.252E+05
1.060	.291	.002	.004	.007	0.411E+06	0.840E+06	0.424E+04
1.140	.205	0.000	.001	.002	0.285E+06	0.205E+05	0.176E+04
1.220	.147	0.000	0.000	0.000	0.326E+06	0.630E+05	0.176E+04
1.300	.108	0.000	0.000	0.000	0.336E+05	0.128E+05	0.369E+03
1.380	.080	0.000	0.000	0.000	0.587E+04	0.313E+05	0.406E+02
1.460	.061	0.000	0.000	0.000	0.626E+04	0.377E+05	0.790E+02
1.540	.047	0.000	0.000	0.000	0.303E+05	0.901E+04	0.314E+02
1.620	.036	0.000	0.000	0.000	0.576E+04	0.200E+04	0.611E+02
1.700	.029	0.000	0.000	0.000	0.166E+05	0.228E+04	0.136E+03

RESPONSE AMPLITUDE STATISTICS:

	M	M	DEG.	DEG.	T-M	T-M	T-M
R.M.S.	1.267	1.114	1.374	1.721	0.526E+04	0.319E+04	0.297E+03
AVE.	1.584	1.393	1.718	2.152	0.657E+04	0.399E+04	0.371E+03
SIGNIF.	2.535	2.228	2.748	3.443	0.105E+05	0.638E+04	0.593E+03
AVE1/10	3.232	2.841	3.504	4.389	0.134E+05	0.814E+04	0.756E+03
DESIGN VALUE WITH N=1000 AND ALPHA=0.01					0.252E+05	0.153E+05	0.142E+04

Cruiser No. 2 (SCORES)

```
RUN IDENTIFICATION:  SEA STATE 7

INPUT VERIFICATION:

   1- WATERLINE LENGTH LWL (M)                 =    161.28

   2- VESSEL DISPLACEMENT (TONNES)             =    8835.5

   3- VERTICAL CENTER OF GRAVITY (M)           =      7.10

   4- ROLL RADIUS OF GYRATION (M)              =      6.95

   5- FRACTION OF CRITICAL ROLL DAMPING        =     .1000

   6- SHIP SPEED (KNOTS)                       =     10.00

   7- SHIP HEADING RELATIVE TO WAVES (DEG)     =    135.00

   8- WATER TYPE                               =  SALT@15C

   9- ISSC TWO PARAMETER SPECTRUM EXCITATION

  10- SIGNIFICANT WAVE HEIGHT (M)              =      7.32

  11- CHARACTERISTIC WAVE PERIOD (S)           =     10.90

  12- LOWER FREQ.INTEGRATION LIMIT (R/S)       =       .26

  13- UPPER FREQ.INTEGRATION LIMIT (R/S)       =      1.70

     STA.     BEAM[M]   AREA[M*M]    DRAFT[M]  WEIGHT[T]

      0         .00        .00          .00      300.0
      1        4.02      20.26         7.13      375.6
      2        7.98      42.53         7.02      775.0
      3       11.74      61.82         6.89      926.1
      4       15.32      80.08         6.78     1324.4
      5       16.73      92.16         6.55     1295.7
      6       16.79      89.37         6.42     1320.0
      7       16.26      69.52         6.31      820.1
      8       15.23      46.79         6.20      710.0
      9       13.96      24.14         6.08      688.6
     10       12.46       7.45         5.96      300.0
```

Cruiser No. 2 (SCORES)

```
RUN IDENTIFICATION:  SEA STATE 7

MOTION NATURAL FREQUENCIES AND PERIODS:

    HEAVE NATURAL FREQUENCY = 1.120 RAD/S    HEAVE NATURAL PERIOD =  5.61 SEC.

    PITCH NATURAL FREQUENCY = 1.152 RAD/S    PITCH NATURAL PERIOD =  5.45 SEC.

    ROLL NATURAL FREQUENCY  =  .412 RAD/S    ROLL NATURAL PERIOD  = 15.25 SEC.

      ROLL WAVE DAMPING          = 0.103E+03

      ADDED VISCOUS ROLL DAMPING = 0.397E+04


SEAKEEPING RESPONSE RESULTS:

  SHIP SPEED                                = 10.0 KNOTS = 5.14 M/S
  WAVE ANGLE [WITH HEAD SEAS 180 DEG.]      =135.0 DEG.
  ISSC TWO PARAMETER SPECTRUM - SIGN.HEIGHT = 7.32 M     CHAR. PERIOD = 10.90 S

 NONDIMENSIONAL MOTION RESPONSE:

   WAVE  ENCOUNT.  WAVE        HEAVE               PITCH                ROLL
   FREQ.  FREQ.   LENGTH   AMPL.     PHASE     AMPL.     PHASE     AMPL.     PHASE
    R/S    R/S      M        ND       DEG.       ND       DEG.       ND       DEG.
   .260   .285   911.5  0.994E+00  179.9  0.716E+00   85.2  0.135E+01 -132.3
   .340   .383   533.0  0.976E+00  179.8  0.719E+00   81.7  0.344E+01 -179.9
   .420   .485   349.3  0.938E+00  179.9  0.714E+00   76.7  0.245E+01   71.5
   .500   .593   246.5  0.876E+00 -179.2  0.693E+00   69.7  0.114E+01   44.8
   .580   .705   183.2  0.799E+00 -177.1  0.647E+00   60.0  0.705E+00   34.2
   .660   .822   141.5  0.740E+00 -174.8  0.567E+00   46.3  0.455E+00   28.9
   .740   .943   112.5  0.696E+00  179.2  0.442E+00   25.2  0.278E+00   25.8
   .820  1.069    91.6  0.452E+00  151.2  0.250E+00   -7.5  0.133E+00   27.0
   .900  1.200    76.1  0.593E-01 -154.2  0.753E-01  -43.9  0.418E-01   49.6
   .980  1.336    64.2  0.174E+00 -128.9  0.155E-01 -151.7  0.286E-01  134.4
  1.060  1.477    54.8  0.114E+00 -152.9  0.215E-01  133.8  0.321E-01  154.3
  1.140  1.622    47.4  0.371E-01  177.9  0.113E-01  104.7  0.185E-01  159.3
  1.220  1.772    41.4  0.921E-02   98.0  0.197E-02   67.6  0.506E-02  175.2
  1.300  1.927    36.5  0.666E-02   61.5  0.798E-03  -69.9  0.110E-02  -80.8
  1.380  2.086    32.4  0.286E-02   78.3  0.620E-03    7.0  0.304E-03   54.5
  1.460  2.251    28.9  0.129E-02  -82.3  0.983E-03    5.4  0.578E-03  110.0
  1.540  2.420    26.0  0.362E-02  -95.4  0.510E-03  -76.9  0.969E-03 -115.4
  1.620  2.593    23.5  0.221E-02 -143.8  0.905E-03 -159.4  0.121E-02 -138.8
  1.700  2.772    21.3  0.315E-02  112.9  0.631E-03  136.9  0.931E-03  109.4
```

Cruiser No. 2 (SCORES)

NONDIMENSIONAL MOMENT RESPONSE FOR RUN: SEA STATE 7

WAVE	ENCOUNT.	WAVE	VERTICAL MOMENT		TRANS. MOMENT		TORS. MOMENT	
FREQ. R/S	FREQ. R/S	LENGTH M	AMPL. ND	PHASE DEG.	AMPL. ND	PHASE DEG.	AMPL. ND	PHASE DEG.
.260	.285	911.5	0.379E-03	-3.4	0.237E-03	84.9	0.101E-03	-162.3
.340	.383	533.0	0.148E-02	-2.3	0.437E-03	69.2	0.649E-03	168.5
.420	.485	349.3	0.364E-02	-2.4	0.843E-03	94.9	0.870E-03	72.1
.500	.593	246.5	0.701E-02	-3.4	0.211E-02	86.8	0.611E-03	49.2
.580	.705	183.2	0.112E-01	-5.4	0.433E-02	79.9	0.468E-03	30.2
.660	.822	141.5	0.149E-01	-8.0	0.757E-02	73.3	0.351E-03	-7.6
.740	.943	112.5	0.161E-01	-9.1	0.107E-01	67.3	0.440E-03	-59.9
.820	1.069	91.6	0.148E-01	-2.0	0.125E-01	63.0	0.646E-03	-84.3
.900	1.200	76.1	0.133E-01	5.4	0.118E-01	61.0	0.733E-03	-94.2
.980	1.336	64.2	0.943E-02	6.9	0.857E-02	62.3	0.594E-03	-95.5
1.060	1.477	54.8	0.333E-02	2.1	0.402E-02	70.9	0.299E-03	-78.4
1.140	1.622	47.4	0.193E-02	171.4	0.931E-03	152.6	0.194E-03	-3.0
1.220	1.772	41.4	0.306E-02	162.3	0.174E-02	-142.2	0.250E-03	25.6
1.300	1.927	36.5	0.107E-02	165.6	0.757E-03	-150.7	0.147E-03	28.5
1.380	2.086	32.4	0.784E-03	-96.5	0.115E-02	87.7	0.446E-04	6.8
1.460	2.251	28.9	0.785E-03	174.8	0.181E-02	85.9	0.748E-04	-13.6
1.540	2.420	26.0	0.187E-02	111.0	0.122E-02	91.2	0.702E-04	-26.4
1.620	2.593	23.5	0.129E-02	46.7	0.418E-03	-13.1	0.824E-04	168.2
1.700	2.772	21.3	0.230E-02	-65.9	0.827E-03	-83.2	0.167E-03	122.7

AMPLITUDE RESPONSE SPECTRA:

FREQ R/S	WAVE AMP. M	HEAVE M	PITCH DEG.	ROLL DEG.	VERT. MOM. T-M	LAT. MOM. T-M	TORS. MOM. T-M
.260	.012	.012	.001	.003	0.336E+03	0.131E+03	0.238E+02
.340	3.654	3.478	.861	19.668	0.159E+07	0.139E+06	0.307E+06
.420	10.376	9.122	5.616	66.015	0.274E+08	0.147E+07	0.157E+07
.500	9.598	7.359	9.830	26.693	0.941E+08	0.853E+07	0.714E+06
.580	6.500	4.155	10.511	12.482	0.162E+09	0.243E+08	0.283E+06
.660	4.060	2.221	8.466	5.449	0.179E+09	0.464E+08	0.997E+05
.740	2.520	1.222	5.036	1.987	0.130E+09	0.577E+08	0.973E+05
.820	1.594	.326	1.543	.436	0.693E+08	0.497E+08	0.133E+06
.900	1.035	.004	.131	.041	0.368E+08	0.288E+08	0.111E+06
.980	.691	.021	.005	.018	0.123E+08	0.101E+08	0.486E+05
1.060	.474	.006	.009	.021	0.105E+07	0.152E+07	0.845E+04
1.140	.332	0.000	.002	.007	0.248E+06	0.575E+05	0.251E+04
1.220	.238	0.000	0.000	0.000	0.446E+06	0.144E+06	0.298E+04
1.300	.174	0.000	0.000	0.000	0.397E+05	0.199E+05	0.750E+03
1.380	.130	0.000	0.000	0.000	0.159E+05	0.340E+05	0.515E+02
1.460	.098	0.000	0.000	0.000	0.121E+05	0.643E+05	0.110E+03
1.540	.075	0.000	0.000	0.000	0.528E+05	0.224E+05	0.742E+02
1.620	.059	0.000	0.000	0.000	0.194E+05	0.204E+04	0.793E+02
1.700	.046	0.000	0.000	0.000	0.489E+05	0.630E+04	0.257E+03

RESPONSE AMPLITUDE STATISTICS:

	M	M	DEG.	DEG.	T-M	T-M	T-M
R.M.S.	1.825	1.494	1.833	3.260	0.756E+04	0.428E+04	0.520E+03
AVE.	2.281	1.868	2.292	4.075	0.945E+04	0.535E+04	0.649E+03
SIGNIF.	3.650	2.989	3.667	6.519	0.151E+05	0.856E+04	0.104E+04
AVE1/10	4.654	3.811	4.675	8.312	0.193E+05	0.109E+05	0.132E+04
DESIGN VALUE WITH N=1000 AND ALPHA=0.01					0.363E+05	0.205E+05	0.249E+04

DOUBLE HULL TANKER

CHARACTERISTICS

LBP	625 ft
B molded	96 ft
Depth	50 ft
T design load	34 ft
Displacement	44513 L.Tons
Deadweight	34700 L.Tons
Web frame spacing	11.5 ft
Tank Length, typical	57.5 ft

VESSEL LOADING IS LOADMASTER CONTROLLED

ALLOWABLE STILL WATER BENDING MT AND SHEAR FORCE

Frame No.	BM (L.Ton-ft)	SF (L.Tons)
57 Aft	421665	5810
67 Midship	421665	6323
77 Fwd	380060	5305

TYPICAL STILL WATER BM AND SF

Condition	BM as % of Allowable /(Location)	SF as % of Allowable (Location)
Light ship	52(Bhd 62)	32(Bhd 47)
Fairweather ballast	80(Bhd 57)	51(Bhd 47)
Max ballast	98(Bhd 62)	60(Bhd 47)
Homogenous cargo	20(Bhd 67)	10(Bhd 67)
Half cargo	70(Bhd 72)	60(Bhd 77)
3/4 cargo	65(Bhd 63)	60(Bhd 77)

SL-7 SHIP

Weights, Centers and Gyradii for "Light" Load Condition

SEGMENT	WEIGHT[1]	LCG[2]	VCG[3]	K_{xx}[4]	K_{yy}[5]	K_{zz}[6]
1	777.4	421.25	43.40	24.9	31.4	21.8
2	1859.9	355.93	32.88	25.3	30.3	22.8
3	1217.9	297.07	58.52	36.7	32.6	31.0
4	1151.8	254.73	47.36	30.0	25.9	21.7
5	1379.2	214.75	48.67	33.2	27.2	25.1
6	1844.3	174.71	44.99	33.6	26.7	25.7
7	1990.6	134.72	33.36	32.7	25.6	25.9
8	2429.0	94.72	35.89	35.6	26.6	28.5
9	2547.5	54.73	34.42	36.1	26.3	29.4
10	2707.6	14.74	33.81	36.6	26.2	30.4
11	2714.9	-27.74	31.54	37.0	25.6	31.7
12	2697.9	-72.74	31.49	37.0	25.7	31.6
13	3284.9	-109.75	42.97	42.2	30.0	31.9
14	3031.4	-147.25	45.39	46.2	34.2	36.7
15	2726.3	-194.75	41.65	37.9	24.8	32.3
16	2757.4	-234.10	42.03	37.3	26.2	31.8
17	1631.3	-275.85	46.21	36.8	26.1	31.0
18	1217.7	-316.15	47.13	35.1	26.4	29.4
19	982.5	-355.30	41.47	32.7	24.1	28.4
20	901.2	-395.25	40.77	31.2	25.1	27.0
21	889.3	-429.25	44.36	24.3	21.1	18.6
22	682.9	-460.25	52.05	22.5	18.1	18.9
TOTAL	41422.8	-37.43	40.26	36.7	214.8	215.0

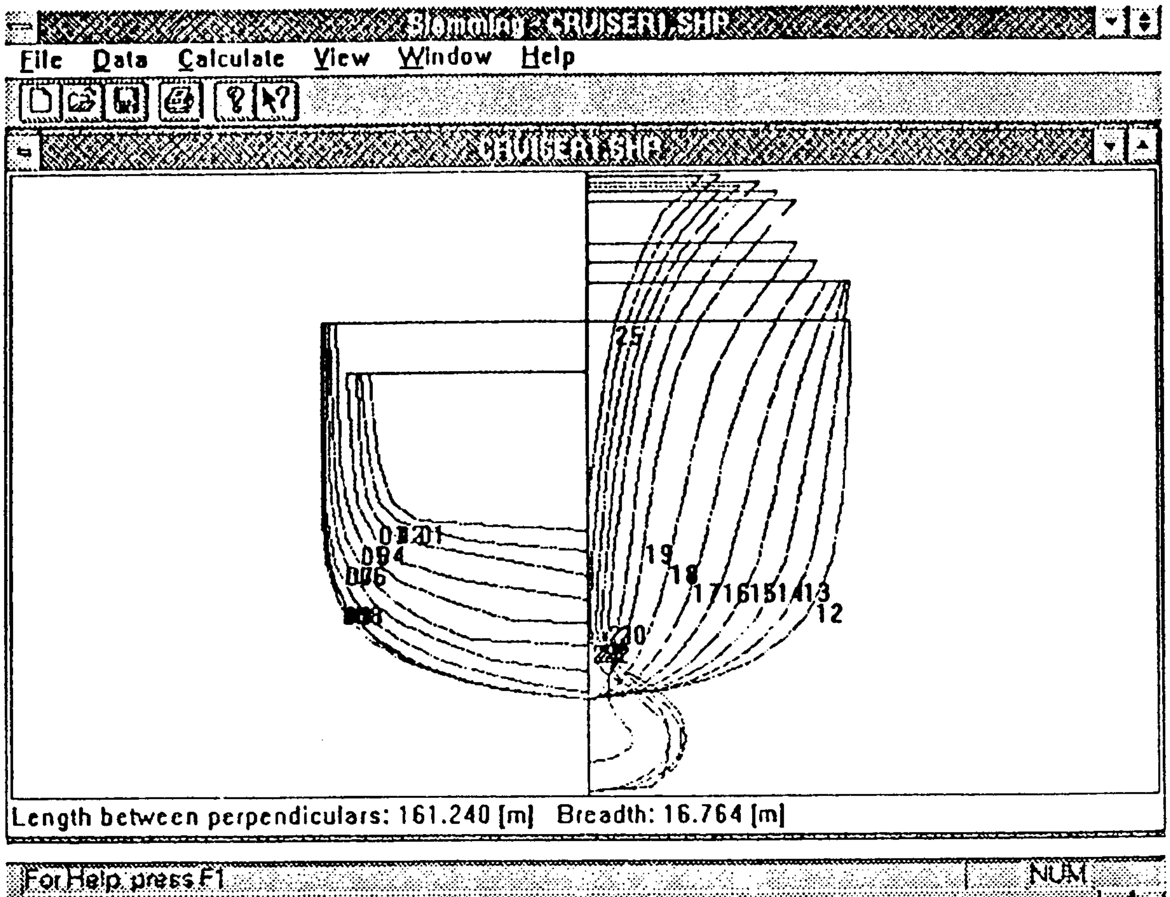
[1] Long Tons (2240 lb)
[2] Feet Forward of Midship
[3] Feet Above Baseline
[4] Roll Gyradius, Feet
[5] Pitch Gyradius, Feet
[6] Yaw Gyradius, Feet

APPENDIX F

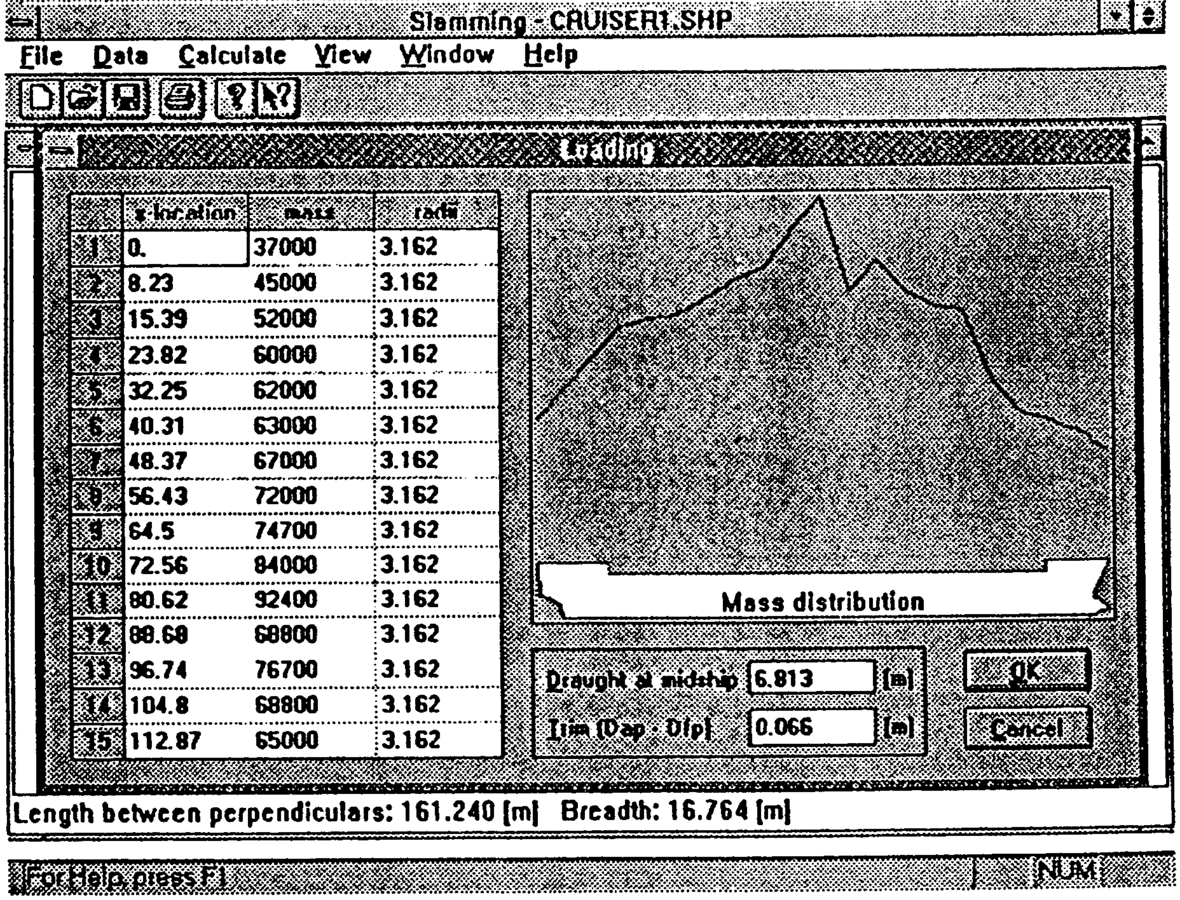
SAMPLE OF INPUT/OUTPUT FILES OF:

SLAM
SOST
ALPS/ISUM
CALREL

SLAM



Length between perpendiculars: 161.240 [m] Breadth: 16.764 [m]



	x-location	mass	radii
1	0.	37000	3.162
2	8.23	45000	3.162
3	15.39	52000	3.162
4	23.82	60000	3.162
5	32.25	62000	3.162
6	40.31	63000	3.162
7	48.37	67000	3.162
8	56.43	72000	3.162
9	64.5	74700	3.162
10	72.56	84000	3.162
11	80.62	92400	3.162
12	88.68	68800	3.162
13	96.74	76700	3.162
14	104.8	68800	3.162
15	112.87	65000	3.162

Draught at midship 6.813 [m]

Trim (Dap - Dfp) 0.066 [m]

Length between perpendiculars: 161.240 [m] Breadth: 16.764 [m]

3. Data Entry

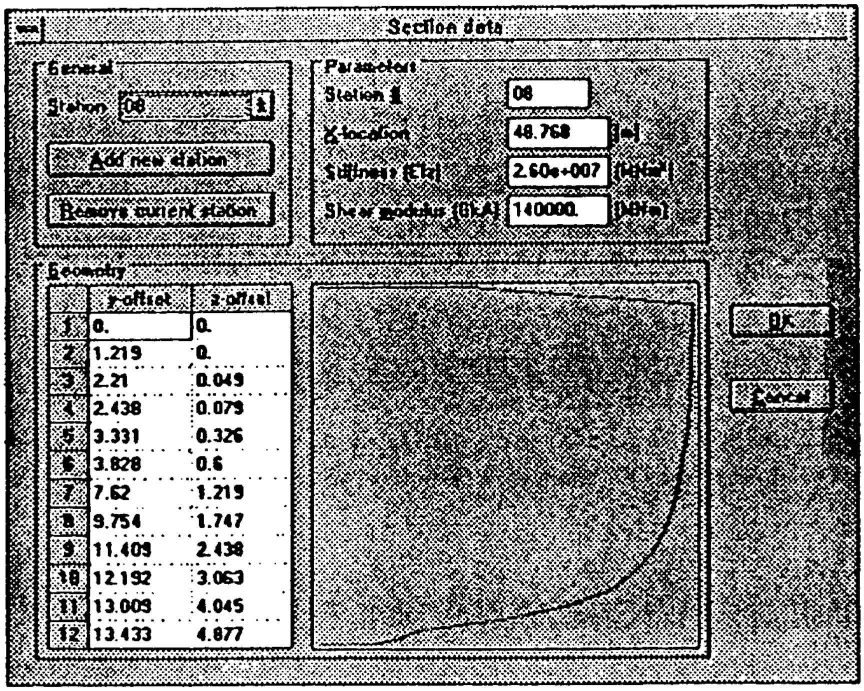
The program provides several data input screens which ask for all necessary information: ship sections, loading, transfer function, sea state, and analysis. All of the required data is able to be input into the program manually. The ship sections and mass distribution, however, can also be imported from an outside source.

3.1 Ship Sections

The ship sections screen asks for the offsets, stiffness, shear modulus, station number, and location of the station from the forward perpendicular. The units and a brief description are as follows:

Input	Units
offsets, *y* and *z* coordinates	m
stiffness, *EIz*	MNm^2
E is Young's Modulus	MN/m
Iz is the moment of inertia around the z-axis	m^3
shear modulus, *GkA*	MN
G is the shear modulus	MN/m
k is the effective shear area factor	dimensionless
A is the area of the cross section	m^2
x-location, distance from the forward perpendicular	m

The ship geometry can be input manually by opening the "Ship Sections" sheet and typing in the y and z coordinates, stiffnesses, and location for the given station. This process may then be repeated for as many stations as desired. See Figure 1 for a sample sheet.



Station 08

Station # 08

X-location 48.768 [m]

Stiffness (EIz) 2.60e+007 [MNm²]

Shear modulus (GkA) 140000. [MN/m]

	y-offset	z-offset
1	0.	0.
2	1.219	0.
3	2.21	0.049
4	2.438	0.079
5	3.331	0.326
6	3.828	0.6
7	7.62	1.219
8	9.754	1.747
9	11.409	2.438
10	12.192	3.063
11	13.009	4.045
12	13.433	4.877

Figure 1 – Ship Section Sheet

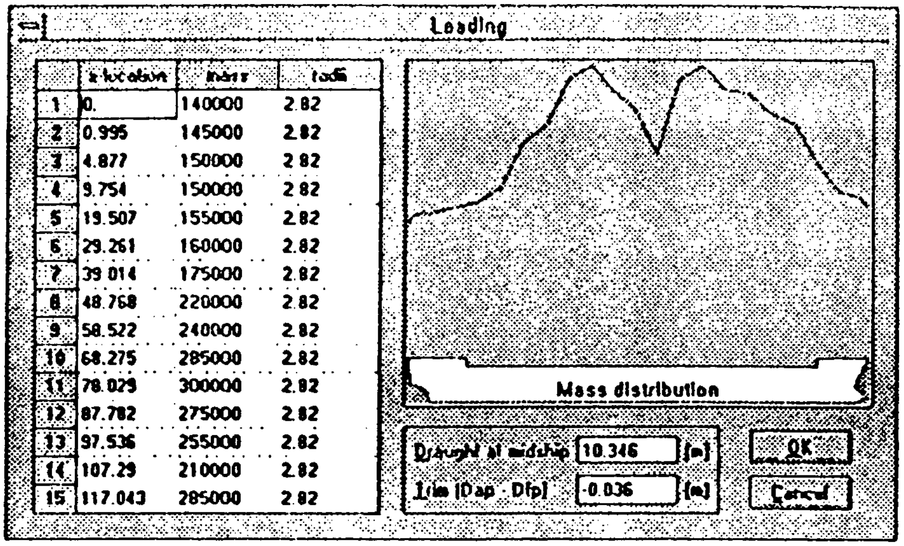
It is also possible to inport data from ASCII files. The file must be named "ship.sec" and the format of the data must be as follows:

File Format	Example
[number of stations on file]	25
[station number x-coordinate stiffness shear modulus]	1 0 1E6 1E4
[number of offset points for the station]	3
[y offsets z offsets]	0 9.754
	0.003 16.002
	0 16.002

The last three steps are then repeated for the appropriate number of stations.

3.2 Ship Loading

The mass distribution can be either input manually or imported. For manual input, the loading screen will require input of the position of the stations, the mass corresponding to that station, and the radius of gyration. See Figure 2 for a sample sheet.



	x-location	mass	radii
1	0.	140000	2.82
2	0.995	145000	2.82
3	4.877	150000	2.82
4	9.754	150000	2.82
5	19.507	155000	2.82
6	29.261	160000	2.82
7	39.014	175000	2.82
8	48.768	220000	2.82
9	58.522	240000	2.82
10	68.275	285000	2.82
11	78.029	300000	2.82
12	87.782	275000	2.82
13	97.536	255000	2.82
14	107.29	210000	2.82
15	117.043	285000	2.82

Draught at midship 10.346 [m]
Trim (Dap - Dfp) -0.036 [m]

Figure 2 – Ship Loading Sheet

The position of the stations must be input starting from the bow. The corresponding mass at that station may be in any loading condition desired, if applicable. The radius of gyration of the mass is defined as

$$r = \sqrt{\frac{h}{2}}$$ where h = the height of the side of the vessel at that particular station.

The units of these inputs are as follows:

Input	Units
position (*x* coordinate) of station	m
mass	kg
radius of gyration	$m^{1/2}$

The mass distribution can be imported from an ASCII file named "ship.loa" and has the following format:

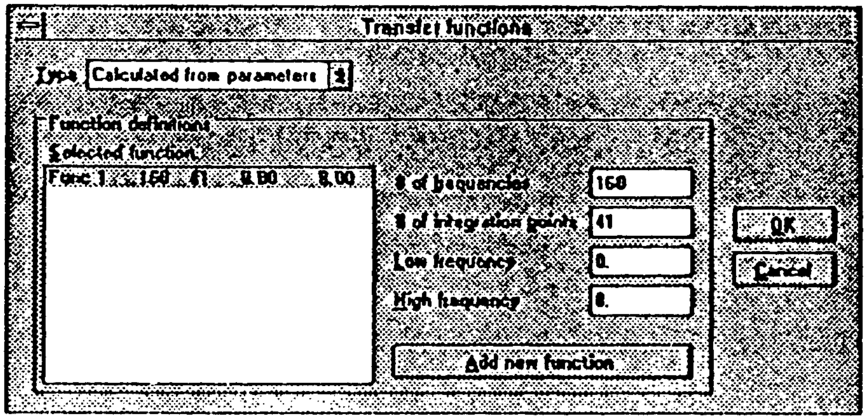
File Format	Example
[number of loading points in file]	25
[*x* coordinate mass radius of gyration]	0 140000 2.82

The last step is then repeated for the appropriate number of stations.

3.3 Transfer Functions

The transfer function sheet consists of the following fields as shown in Figure 3.

- number of frequencies
  This tells the program how many frequencies should be run in the range specified in the following fields.
- low frequency (radians/second)
  This tells the program the frequency at which to begin calculation.
- high frequency (radians/second)
  This tells the program the frequency at which to stop calculation.
- integration points
  This tells the program how many longitudinal points along the vessel are to be used for the numberical methods calculations.



Type: Calculated from parameters

Selected function: Func 1 160 41 0.00 8.00

# of frequencies: 160

# of integration points: 41

Low frequency: 0.

High frequency: 8.

Figure 3 – Transfer Function Sheet

3.4 Sea State

The sea state sheet consists of the following fields as shown in Figure 4.

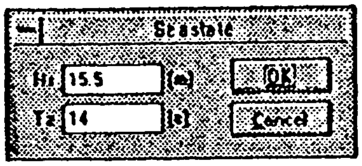
- significant wave height, H_s (meters)

  Significant wave height is defined as the average of the highest 1/3 waves to be encountered.

- zero crossing period, T_Z (seconds)

  Zero crossing period is the period of the wave and can be calculated by

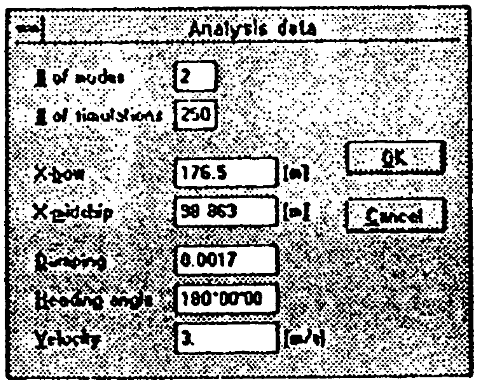
$$T_z = 11.12\sqrt{\frac{H_s}{g}} \qquad \text{where } g \text{ is the acceleration of gravity}$$



Hs 15.5 [m]

Tz 14 [s]

Figure 4 – Sea State Sheet



# of modes 2

# of simulations 250

X-bow 176.5 [m]

X-midship 98 863 [m]

Damping 0.0017

Heading angle 180°00'00

Velocity 3. [m/s]

Figure 5 – Analysis Sheet

3.5 Analysis

The analysis sheet consists of the following fields as shown in Figure 5.

- number of modes

  Defines the number of modes used when the dynamic response due to the slamming impact is calculated. Two modes were used in all calculations for these vessels as higher modes produced insignificant changes in the results.
- number of simulations

  The statistics of the response moments are calculated by simulations.
- *x*-bow (meters)

  This is the longitudinal position at which slamming impact takes place. For this analysis, the position of slamming impact was taken as the location of damage which was determined using Figure 6. The percent of total length read from the chart was the mean value for a given block coefficient and in some cases had to be extrapolated. As will be shown later, the position of slamming impact will greatly influence the calculated slamming induced bending moments.

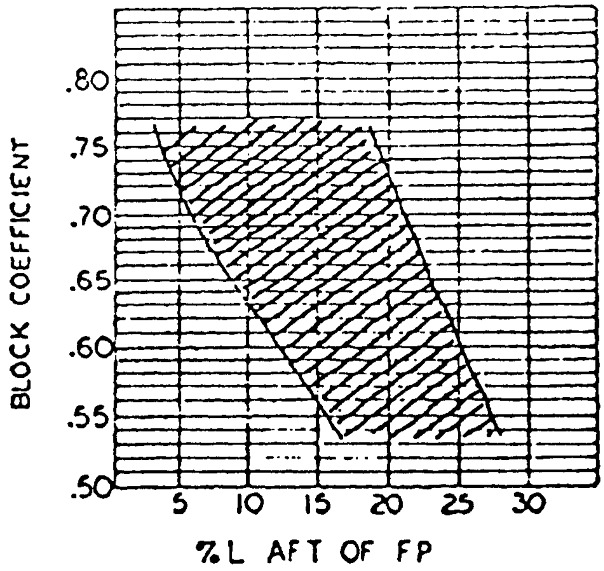


Figure 6 – Longitudinal Location of Damage [6]

- *x*-midship (meters)

  This is the longitudinal position along the vessel at which the response is to be calculated. In the analysis, this position was taken to be the midship of the vessel.
- damping ratio

  This is a structural coefficient in the dynamic equations of motion. This analysis used a damping ratio of 0.0017.
- heading angle (degrees)

  This is the angle of the vessel relative to the encountered waves where 0° signifies following seas and 180° signifies head seas.
- velocity (meters/second)

  The speed of the vessel corresponding to the particular sea state.

```
Results for CRUISER1.SHP --- page 1

RESULT IN AIR
Total mass of ship 9694063.3
Natural frequency 1 =     8.39 rad/sec    w(L/g)^½ =    34.02
Natural frequency 2 =    17.67 rad/sec    w(L/g)^½ =    71.64

RESULT IN WATER
Total mass of ship 9694063.3
Natural frequency 1 =     6.23 rad/sec    w(L/g)^½ =    25.26
Natural frequency 2 =    12.67 rad/sec    w(L/g)^½ =    51.36


TRANSFER FUNCTIONS

# of wave frequencies 160
Velocity  4.00   Heading 180.0   XcG  76.90

 Omega      H_a       H_p      P_a       P_p      M_a       M_p
-------   -------   -------  -------   -------  -------   -------
 0.050    0.99931   6.2825   0.00029   1.5488   0.00008   1.9097
 0.101    0.99803   6.2823   0.00107   1.5662   0.00011   1.8899
 0.151    0.99447   6.2825   0.00237   1.5791   0.00003   5.3136
 0.201    0.98617   0.0002   0.00418   1.5963   0.00049   5.1542
 0.252    0.96948   0.0020   0.00648   1.6205   0.00143   5.1657
 0.302    0.93976   0.0041   0.00922   1.6535   0.00302   5.1798
 0.352    0.89211   0.0051   0.01226   1.6976   0.00541   5.1958
 0.403    0.82266   0.0015   0.01539   1.7547   0.00864   5.2187
 0.453    0.73132   6.2696   0.01818   1.8284   0.01218   5.2397
 0.503    0.61971   6.2308   0.02023   1.9098   0.01642   5.3327
 0.553    0.49455   6.1447   0.02135   2.0192   0.02052   5.3907
 0.604    0.37951   5.9728   0.02092   2.1597   0.02366   5.4737
 0.654    0.29959   5.7030   0.01865   2.3481   0.02503   5.5917
 0.704    0.25131   5.3929   0.01437   2.6121   0.02434   5.7586
 0.755    0.20324   5.1034   0.00866   3.0940   0.02137   5.9870
 0.805    0.21107   4.5605   0.00391   4.4329   0.01676   6.2624
 0.855    0.29407   4.7795   0.00567   5.9653   0.01044   0.2933
 0.906    0.21030   5.4482   0.00723   0.3827   0.00381   1.1502
 0.956    0.08620   0.2660   0.00585   1.1722   0.00448   2.9396
 1.006    0.04799   2.0559   0.00325   2.2386   0.00525   3.4844
 1.057    0.03917   3.1075   0.00200   3.6013   0.00332   3.9486
 1.107    0.01825   4.0111   0.00155   4.5826   0.00175   5.0521
 1.157    0.01079   5.4708   0.00085   5.4181   0.00194   6.1253
 1.208    0.00934   0.1259   0.00040   0.3101   0.00166   0.5347
 1.258    0.00495   0.6203   0.00032   1.6138   0.00079   1.2757
 1.308    0.00060   2.0676   0.00021   2.3113   0.00060   3.1552
 1.358    0.00152   3.7744   0.00005   3.3060   0.00075   3.7080
 1.409    0.00063   3.5464   0.00005   5.4432   0.00032   3.5027
 1.459    0.00090   1.7619   0.00003   5.3216   0.00040   1.8484
 1.509    0.00060   1.1539   0.00004   3.5958   0.00026   1.0668
 1.560    0.00057   5.3204   0.00002   3.4898   0.00035   5.4362
 1.610    0.00059   4.9183   0.00003   0.4052   0.00026   5.1481
 1.660    0.00033   2.2520   0.00004   0.7726   0.00042   2.0707
 1.711    0.00066   2.1527   0.00003   2.8978   0.00063   2.2072
 1.761    0.00021   4.2314   0.00005   3.7981   0.00023   4.3057
 1.811    0.00067   5.1645   0.00003   5.4406   0.00063   5.2744
 1.862    0.00026   0.3279   0.00004   0.4601   0.00033   0.8164
 1.912    0.00057   1.8596   0.00003   2.0473   0.00073   1.9993
 1.962    0.00024   3.3890   0.00004   3.4962   0.00034   3.6621
 2.013    0.00044   4.9192   0.00002   5.1826   0.00064   5.0592
 2.063    0.00022   0.5074   0.00003   0.3490   0.00040   0.7745
 2.113    0.00034   1.7913   0.00002   2.2483   0.00058   1.9079
 2.164    0.00023   3.9971   0.00002   3.6242   0.00044   4.1235
 2.214    0.00022   5.0759   0.00002   5.6723   0.00039   5.2569
 2.264    0.00023   1.0696   0.00001   0.8832   0.00051   1.2188
 2.314    0.00010   2.6152   0.00002   2.8411   0.00024   2.9183
 2.365    0.00017   4.4392   0.00001   4.9168   0.00043   4.5606
 2.415    0.00012   0.7428   0.00001   0.1745   0.00035   0.7690
 2.465    0.00007   1.9754   0.00001   2.4551   0.00019   2.0508
 2.516    0.00013   4.1593   0.00001   4.5028   0.00036   4.2139
 2.566    0.00008   0.4689   0.00001   6.0962   0.00026   0.5764
 2.616    0.00004   1.5401   0.00001   2.2220   0.00010   1.7060
 2.667    0.00008   3.8054   0.00000   4.5625   0.00024   3.8966
 2.717    0.00005   0.3718   0.00000   5.6791   0.00019   0.6297
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Marsden area No.  8: Fraction of time .........:      .059
Marsden area No.  9: Fraction of time .........:      .059
Marsden area No. 10: Fraction of time .........:      .059
Marsden area No. 11: Fraction of time .........:      .059
Marsden area No. 15: Fraction of time .........:      .118
Marsden area No. 16: Fraction of time .........:      .118
Marsden area No. 17: Fraction of time .........:      .059
Marsden area No. 23: Fraction of time .........:      .118
Marsden area No. 24: Fraction of time .........:      .118
Marsden area No. 25: Fraction of time .........:      .118
Marsden area No. 26: Fraction of time .........:      .059
Marsden area No. 27: Fraction of time .........:      .059

Total period (years) .........................:    15.000

Hs1 = practical Hs limit for service speed ....:    16.500
Fraction of time with service speed when Hs<Hs1:      .800
Fraction of time with minimum speed when Hs>Hs1:     1.000
Service speed >=..............................:    30.500

Fraction of time with heading 0 deg (following):      .111
Fraction of time with heading 45 deg ..........:      .222
Fraction of time with heading 90 deg ..........:      .222
Fraction of time with heading 135 deg .........:      .333
Fraction of time with heading 180 deg (head) ..:      .111

 Main dimensions: L,B,T = 640.000, 96.000, 34.100
 Rigid hull,    bending moments,      sagging statistics

Response at Station No. ......................:     11

Fraction of time with non-zero response spectra:  .899D+00

Stress conversion factor for fatigue analysis..:  .500D+08
Scale factor for S-N curve ...................:  .329D+13
Slope of S-N curve ...........................:   -3.000
Resulting fatigue damage .....................:  .273D-02
Resulting fatigue damage (linear) ............:  .258D-02


Long term probability of exceedance of + peaks
during  15.0 years, corr. to   59944559 peaks (  59588777 if Gaussian)
 |-----------|-----------|-----------|-----------|-----------|-----------|
 |   Level   |  Poisson  |   Order   |  Poisson  |  Poisson  |Individual |
 | from zero |upcrossing |statistics |(k3=0,k4=3)|Linear resp|peak exced.|
 |-----------|-----------|-----------|-----------|-----------|-----------|
 | 0.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 8.993D-01 |
 | 2.200D+07 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 7.209D-01 |
 | 4.400D+07 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 5.154D-01 |
 | 6.600D+07 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 3.738D-01 |
 | 8.800D+07 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 2.767D-01 |
 | 1.100D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 2.076D-01 |
 | 1.320D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.576D-01 |
 | 1.540D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.211D-01 |
 | 1.760D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 9.406D-02 |
 | 1.980D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 7.366D-02 |
 | 2.200D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 5.809D-02 |
 | 2.420D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 4.608D-02 |
 | 2.640D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 3.673D-02 |
 | 2.860D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 2.941D-02 |
 | 3.080D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 2.363D-02 |
 | 3.300D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.905D-02 |
 | 3.520D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.540D-02 |
 | 3.740D+08 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.000D+00 | 1.247D-02 |
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3.960D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.013D-02
4.180D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.234D-03
4.400D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.707D-03
4.620D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.472D-03
4.840D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.471D-03
5.060D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.658D-03
5.280D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.997D-03
5.500D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.459D-03
5.720D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.021D-03
5.940D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.663D-03
6.160D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.370D-03
6.380D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.131D-03
6.600D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	9.342D-04
6.820D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	7.729D-04
7.040D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.403D-04
7.260D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.311D-04
7.480D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.411D-04
7.700D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.668D-04
7.920D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.054D-04
8.140D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.545D-04
8.360D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.124D-04
8.580D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.774D-04
8.800D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.484D-04
9.020D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.242D-04
9.240D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.041D-04
9.460D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.734D-05
9.680D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	7.335D-05
9.900D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.166D-05
1.012D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.188D-05
1.034D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.370D-05
1.056D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.684D-05
1.078D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.108D-05
1.100D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.625D-05
1.122D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.218D-05
1.144D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.876D-05
1.166D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.589D-05
1.188D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.346D-05
1.210D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.141D-05
1.232D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	9.681D-06
1.254D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.219D-06
1.276D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.982D-06
1.298D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.936D-06
1.320D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.049D-06
1.342D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.297D-06
1.364D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.659D-06
1.386D+09	1.000D+00	1.000D+00	1.000D+00	9.999D-01	3.117D-06
1.408D+09	1.000D+00	1.000D+00	1.000D+00	9.995D-01	2.656D-06
1.430D+09	1.000D+00	1.000D+00	9.999D-01	9.973D-01	2.265D-06
1.452D+09	1.000D+00	1.000D+00	9.991D-01	9.898D-01	1.932D-06
1.474D+09	1.000D+00	1.000D+00	9.959D-01	9.711D-01	1.649D-06
1.496D+09	1.000D+00	1.000D+00	9.863D-01	9.352D-01	1.408D-06
1.518D+09	1.000D+00	1.000D+00	9.643D-01	8.786D-01	1.202D-06
1.540D+09	1.000D+00	1.000D+00	9.248D-01	8.027D-01	1.027D-06
1.562D+09	1.000D+00	1.000D+00	8.653D-01	7.125D-01	8.773D-07
1.584D+09	1.000D+00	1.000D+00	7.879D-01	6.155D-01	7.498D-07
1.606D+09	1.000D+00	1.000D+00	6.981D-01	5.189D-01	6.410D-07
1.628D+09	1.000D+00	1.000D+00	6.027D-01	4.282D-01	5.481D-07
1.650D+09	1.000D+00	1.000D+00	5.086D-01	3.471D-01	4.687D-07
1.672D+09	1.000D+00	1.000D+00	4.206D-01	2.770D-01	4.009D-07
1.694D+09	1.000D+00	1.000D+00	3.419D-01	2.184D-01	3.430D-07
1.716D+09	1.000D+00	1.000D+00	2.740D-01	1.704D-01	2.935D-07
1.738D+09	1.000D+00	1.000D+00	2.169D-01	1.318D-01	2.511D-07
1.760D+09	9.998D-01	1.000D+00	1.701D-01	1.012D-01	2.149D-07
1.782D+09	9.993D-01	1.000D+00	1.322D-01	7.725D-02	1.839D-07

1.804D+09	9.980D-01	9.999D-01	1.021D-01	5.866D-02	1.574D-07
1.826D+09	9.951D-01	9.997D-01	7.838D-02	4.434D-02	1.348D-07
1.848D+09	9.895D-01	9.990D-01	5.987D-02	3.339D-02	1.154D-07
1.870D+09	9.797D-01	9.973D-01	4.552D-02	2.505D-02	9.877D-08
1.892D+09	9.643D-01	9.937D-01	3.448D-02	1.873D-02	8.456D-08
1.914D+09	9.423D-01	9.870D-01	2.603D-02	1.397D-02	7.239D-08
1.936D+09	9.130D-01	9.757D-01	1.959D-02	1.039D-02	6.198D-08
1.958D+09	8.762D-01	9.585D-01	1.470D-02	7.701D-03	5.307D-08
1.980D+09	8.327D-01	9.344D-01	1.099D-02	5.696D-03	4.543D-08
2.002D+09	7.836D-01	9.029D-01	8.205D-03	4.202D-03	3.890D-08
2.024D+09	7.302D-01	8.642D-01	6.107D-03	3.093D-03	3.330D-08
2.046D+09	6.741D-01	8.190D-01	4.535D-03	2.271D-03	2.851D-08
2.068D+09	6.169D-01	7.685D-01	3.359D-03	1.663D-03	2.441D-08
2.090D+09	5.601D-01	7.142D-01	2.483D-03	1.215D-03	2.090D-08
2.112D+09	5.048D-01	6.578D-01	1.830D-03	8.858D-04	1.789D-08
2.134D+09	4.520D-01	6.006D-01	1.346D-03	6.442D-04	1.531D-08
2.156D+09	4.024D-01	5.442D-01	9.880D-04	4.674D-04	1.311D-08
2.178D+09	3.563D-01	4.896D-01	7.234D-04	3.383D-04	1.122D-08
2.200D+09	3.141D-01	4.376D-01	5.284D-04	2.443D-04	9.602D-09

Long term probability of exceedance of - peaks

Level from zero	Poisson upcrossing	Order statistics	Poisson (k3=0,k4=3)	Poisson Linear resp	Individual peak exced.
0.000D+00	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.993D-01
2.200D+07	1.000D+00	1.000D+00	1.000D+00	1.000D+00	7.183D-01
4.400D+07	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.075D-01
6.600D+07	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.614D-01
8.800D+07	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.617D-01
1.100D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.918D-01
1.320D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.424D-01
1.540D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.071D-01
1.760D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.142D-02
1.980D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.238D-02
2.200D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.806D-02
2.420D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.719D-02
2.640D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.887D-02
2.860D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.247D-02
3.080D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.752D-02
3.300D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.367D-02
3.520D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.068D-02
3.740D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.347D-03
3.960D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.526D-03
4.180D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.104D-03
4.400D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.993D-03
4.620D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.125D-03
4.840D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.447D-03
5.060D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.917D-03
5.280D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.503D-03
5.500D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.179D-03
5.720D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	9.257D-04
5.940D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	7.274D-04
6.160D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.720D-04
6.380D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.502D-04
6.600D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.546D-04
6.820D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.796D-04
7.040D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.206D-04
7.260D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.742D-04
7.480D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.378D-04
7.700D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.090D-04
7.920D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.634D-05

TANKER

8.140D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.845D-05
8.360D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.433D-05
8.580D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.316D-05
8.800D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.432D-05
9.020D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.733D-05
9.240D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.178D-05
9.460D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.738D-05
9.680D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.388D-05
9.900D+08	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.110D-05
1.012D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.881D-06
1.034D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	7.115D-06
1.056D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.705D-06
1.078D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.578D-06
1.100D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.676D-06
1.122D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.954D-06
1.144D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.375D-06
1.166D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.910D-06
1.188D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.536D-06
1.210D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	1.236D-06
1.232D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	9.944D-07
1.254D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	8.000D-07
1.276D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	6.435D-07
1.298D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	5.175D-07
1.320D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	4.161D-07
1.342D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	3.344D-07
1.364D+09	1.000D+00	1.000D+00	1.000D+00	1.000D+00	2.687D-07
1.386D+09	1.000D+00	1.000D+00	9.999D-01	9.999D-01	2.157D-07
1.408D+09	9.998D-01	1.000D+00	9.992D-01	9.995D-01	1.732D-07
1.430D+09	9.990D-01	9.998D-01	9.961D-01	9.973D-01	1.389D-07
1.452D+09	9.959D-01	9.987D-01	9.863D-01	9.898D-01	1.114D-07
1.474D+09	9.878D-01	9.952D-01	9.639D-01	9.711D-01	8.922D-08
1.496D+09	9.707D-01	9.862D-01	9.234D-01	9.352D-01	7.144D-08
1.518D+09	9.406D-01	9.675D-01	8.623D-01	8.786D-01	5.717D-08
1.540D+09	8.953D-01	9.354D-01	7.829D-01	8.027D-01	4.571D-08
1.562D+09	8.352D-01	8.880D-01	6.912D-01	7.125D-01	3.653D-08
1.584D+09	7.629D-01	8.259D-01	5.944D-01	6.155D-01	2.917D-08
1.606D+09	6.828D-01	7.522D-01	4.994D-01	5.189D-01	2.327D-08
1.628D+09	5.996D-01	6.712D-01	4.111D-01	4.282D-01	1.855D-08
1.650D+09	5.176D-01	5.877D-01	3.327D-01	3.471D-01	1.478D-08
1.672D+09	4.402D-01	5.060D-01	2.654D-01	2.770D-01	1.177D-08
1.694D+09	3.696D-01	4.293D-01	2.092D-01	2.184D-01	9.358D-09
1.716D+09	3.069D-01	3.597D-01	1.633D-01	1.704D-01	7.437D-09
1.738D+09	2.526D-01	2.981D-01	1.265D-01	1.318D-01	5.905D-09
1.760D+09	2.062D-01	2.449D-01	9.726D-02	1.012D-01	4.685D-09
1.782D+09	1.673D-01	1.996D-01	7.437D-02	7.725D-02	3.714D-09
1.804D+09	1.350D-01	1.617D-01	5.658D-02	5.866D-02	2.942D-09
1.826D+09	1.084D-01	1.303D-01	4.287D-02	4.434D-02	2.328D-09
1.848D+09	8.672D-02	1.045D-01	3.235D-02	3.339D-02	1.841D-09
1.870D+09	6.916D-02	8.348D-02	2.434D-02	2.505D-02	1.454D-09
1.892D+09	5.500D-02	6.649D-02	1.825D-02	1.873D-02	1.148D-09
1.914D+09	4.363D-02	5.282D-02	1.364D-02	1.397D-02	9.052D-10
1.936D+09	3.455D-02	4.185D-02	1.017D-02	1.039D-02	7.132D-10
1.958D+09	2.730D-02	3.310D-02	7.567D-03	7.701D-03	5.615D-10
1.980D+09	2.154D-02	2.613D-02	5.614D-03	5.696D-03	4.417D-10
2.002D+09	1.697D-02	2.059D-02	4.155D-03	4.202D-03	3.471D-10
2.024D+09	1.335D-02	1.621D-02	3.068D-03	3.093D-03	2.726D-10
2.046D+09	1.049D-02	1.274D-02	2.260D-03	2.271D-03	2.138D-10
2.068D+09	8.235D-03	9.998D-03	1.661D-03	1.663D-03	1.676D-10
2.090D+09	6.457D-03	7.839D-03	1.218D-03	1.215D-03	1.313D-10
2.112D+09	5.058D-03	6.139D-03	8.911D-04	8.858D-04	1.027D-10
2.134D+09	3.958D-03	4.804D-03	6.504D-04	6.442D-04	8.033D-11
2.156D+09	3.094D-03	3.755D-03	4.736D-04	4.674D-04	6.275D-11
2.178D+09	2.417D-03	2.932D-03	3.441D-04	3.383D-04	4.898D-11
2.200D+09	1.886D-03	2.288D-03	2.495D-04	2.443D-04	3.821D-11

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Marsden area No.  8: Fraction of time .........:      .059
Marsden area No.  9: Fraction of time .........:      .059
Marsden area No. 10: Fraction of time .........:      .059
Marsden area No. 11: Fraction of time .........:      .059
Marsden area No. 15: Fraction of time .........:      .118
Marsden area No. 16: Fraction of time .........:      .118
Marsden area No. 17: Fraction of time .........:      .059
Marsden area No. 23: Fraction of time .........:      .118
Marsden area No. 24: Fraction of time .........:      .118
Marsden area No. 25: Fraction of time .........:      .118
Marsden area No. 26: Fraction of time .........:      .059
Marsden area No. 27: Fraction of time .........:      .059

Total period (years) ..........................:    15.000

Hsl = practical Hs limit for service speed ....:    16.500
Fraction of time with service speed when Hs<Hsl:      .800
Fraction of time with minimum speed when Hs>Hsl:     1.000
Service speed >=...............................:    30.500

Fraction of time with heading 0 deg (following):      .111
Fraction of time with heading 45 deg ..........:      .222
Fraction of time with heading 90 deg ..........:      .222
Fraction of time with heading 135 deg .........:      .333
Fraction of time with heading 180 deg (head) ..:      .111

 Main dimensions: L,B,T = 880.000,105.510, 32.429
 Rigid hull,    bending moments,      sagging statistics

Response at Station No. .......................:    11

Fraction of time with non-zero response spectra:  .894D+00

Stress conversion factor for fatigue analysis..:  .500D+08
Scale factor for S-N curve ....................:  .329D+13
Slope of S-N curve ............................:    -3.000
Resulting fatigue damage ......................:  .743D-02
Resulting fatigue damage (linear) .............:  .699D-02


Long term probability of exceedance of + peaks
during  15.0 years, corr. to   55596001 peaks (  54925809 if Gaussian)
| ----------- | ----------- | ----------- | ----------- | ----------- | ----------- |
|   Level     |   Poisson   |    Order    |   Poisson   |   Poisson   | Individual  |
|  from zero  | upcrossing  | statistics  |(k3=0,k4=3)  |Linear resp  |peak exced.  |
| ----------- | ----------- | ----------- | ----------- | ----------- | ----------- |
|  0.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  8.938D-01  |
|  2.200D+07  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  7.448D-01  |
|  4.400D+07  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  5.668D-01  |
|  6.600D+07  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  4.372D-01  |
|  8.800D+07  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  3.445D-01  |
|  1.100D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  2.768D-01  |
|  1.320D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  2.257D-01  |
|  1.540D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.859D-01  |
|  1.760D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.543D-01  |
|  1.980D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.291D-01  |
|  2.200D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.087D-01  |
|  2.420D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  9.211D-02  |
|  2.640D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  7.847D-02  |
|  2.860D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  6.716D-02  |
|  3.080D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  5.772D-02  |
|  3.300D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  4.980D-02  |
|  3.520D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  4.311D-02  |
|  3.740D+08  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  1.000D+00  |  3.744D-02  |
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*                                                                              *
*                                                                              *
*                                                                              *
*                          A L P S / I S U M                                   *
*                                                                              *
*                                                                              *
*                                                                              *
*                  A   C O M P U T E R   P R O G R A M   O F                   *
*                  N O N L I N E A R   A N A L Y S I S   O F                   *
*           L A R G E   P L A T E D   S T R U C T U R E S   U S I N G          *
*        I D E A L I Z E D   S T R U C T U R A L   U N I T   M E T H O D       *
*                                                                              *
*                                                                              *
*                         D E V E L O P E D   B Y                              *
*                                                                              *
*                         J E O M   K .   P A I K                              *
*                                                                              *
*      D E P A R T M E N T   O F   N A V A L   A R C H I T E C T U R E         *
*                  A N D   O C E A N   E N G I N E E R I N G                   *
*              P U S A N   N A T I O N A L   U N I V E R S I T Y               *
*                          P U S A N, K O R E A                                *
*                                                                              *
*                                                                              *
*                                                                              *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *


              <<<<< I N P U T   D A T A   E C H O >>>>>

      NUMBER OF PLATE ELEMENT(NEP)..........=        140
      NUMBER OF STIFFENED PLATE ELEMENT(NES)=         51
      NUMBER OF BEAM ELEMENT(NEF)...........=         25
      NUMBER OF NODAL POINT(NP).............=        118
      NUMBER OF LOADING STEP(NSTEP).........=        500
      SKIP NUMBER OF OUTPUT PRINT(NSKIP)....=        200
      NUMBER OF LOADING POINT(NR)...........=         59
      TYPE OF LOADING CONDITION(KTYPE)......=          2

      ARRANGEMENT NUMBER OF PLATE (NPNOP)
      (    1)    1    2   61   60  (    2)   11   38   97   70
      (    3)   21   59  118   80  (    4)   24   58  117   83
      (    5)   29   57  116   88  (    6)    1    3   62   60
      (    7)    4    2   61   63  (    8)    3    4   63   62
      (    9)    3    5   64   62  (   10)    5    6   65   64
      (   11)    7    4   63   66  (   12)    6    7   66   65
      (   13)    6    8   67   65  (   14)    9    7   66   68
      (   15)    8    9   68   67  (   16)    8   12   71   67
      (   17)   10    9   68   69  (   18)   11   10   69   70
      (   19)   13   10   69   72  (   20)   12   13   72   71
      (   21)   12   14   73   71  (   22)   14   15   74   73
      (   23)   16   13   72   75  (   24)   15   16   75   74
      (   25)   15   17   76   74  (   26)   17   18   77   76
      (   27)   18   11   70   77  (   28)   18   19   78   77
      (   29)   19   20   79   78  (   30)   20   21   80   79
      (   31)   20   22   81   79  (   32)   22   23   82   81
      (   33)   23   24   83   82  (   34)   23   25   84   82
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(    35)   25   26   85   84  (    36)   26   27   86   85
(    37)   27   28   87   86  (    38)   28   29   88   87
(    39)   30    1   60   89  (    40)    2   31   90   61
(    41)   31   30   89   90  (    42)   32   30   89   91
(    43)   33   32   91   92  (    44)   31   34   93   90
(    45)   34   33   92   93  (    46)   35   33   92   94
(    47)   34   36   95   93  (    48)   36   35   94   95
(    49)   39   35   94   98  (    50)   36   37   96   95
(    51)   37   38   97   96  (    52)   37   40   99   96
(    53)   40   39   98   99  (    54)   41   39   98  100
(    55)   42   41  100  101  (    56)   40   43  102   99
(    57)   43   42  101  102  (    58)   44   42  101  103
(    59)   45   44  103  104  (    60)   38   45  104   97
(    61)   46   45  104  105  (    62)   47   46  105  106
(    63)   48   47  106  107  (    64)   49   47  106  108
(    65)   50   49  108  109  (    66)   51   50  109  110
(    67)   52   50  109  111  (    68)   53   52  111  112
(    69)   54   53  112  113  (    70)   55   54  113  114
(    71)   56   55  114  115  (    72)   59   48  107  118
(    73)   58   51  110  117  (    74)   57   56  115  116
(    75)   25   26   28   24  (    76)   23   27   28   24
(    77)   24   28   29   58  (    78)   24   29   57   58
(    79)   58   56   55   51  (    80)   51   55   54   50
(    81)   51   55   53   52  (    82)   84   85   87   83
(    83)   82   86   87   83  (    84)   83   87   88  117
(    85)   83   88  116  117  (    86)  117  115  114  110
(    87)  110  114  113  109  (    88)  110  114  112  111
(    89)   20   23   24   21  (    90)   21   24   58   59
(    91)   59   58   51   48  (    92)   48   51   50   47
(    93)   79   82   83   80  (    94)   80   83  117  118
(    95)  118  117  110  107  (    96)  107  110  109  106
(    97)   18   20   21   11  (    98)   11   21   59   38
(    99)   11   59   48   38  (   100)   38   48   47   45
(   101)   77   79   80   70  (   102)   70   80  118   97
(   103)   70  118  107   97  (   104)   97  107  106  104
(   105)   15   16   13   12  (   106)   12   13    9    8
(   107)    8    9    7    6  (   108)    6    7    4    3
(   109)    3    4    2    1  (   110)    1    2   31   30
(   111)   30   31   34   33  (   112)   33   34   36   35
(   113)   35   36   40   39  (   114)   39   40   43   42
(   115)   74   75   72   71  (   116)   71   72   68   67
(   117)   67   68   66   65  (   118)   65   66   63   62
(   119)   62   63   61   60  (   120)   60   61   90   89
(   121)   89   90   93   92  (   122)   92   93   95   94
(   123)   94   95   99   98  (   124)   98   99  102  101
(   125)   10   11   38   37  (   126)   69   70   97   96
(   127)   20   22   24   21  (   128)   79   81   83   80
(   129)   48   51   49   47  (   130)  107  110  108  106
(   131)   19   20   21   11  (   132)   78   79   80   70
(   133)   38   48   47   46  (   134)   97  107  106  105
(   135)   17   18   11   16  (   136)   76   77   70   75
(   137)   43   38   45   44  (   138)  102   97  104  103
(   139)   58   57   56   51  (   140)  117  116  115  110

MATERIAL PROPERTY OF PLATE (E,SY,SYT,TC,EFCR)
(     1)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(     2)    .211E+05  .330E+02  .462E+02  .100E-01  .500E-01
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(     3)    .211E+05  .330E+02  .462E+02  .635E+01  .500E-01
(     4)    .211E+05  .562E+02  .787E+02  .556E+01  .500E-01
(     5)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(     6)    .211E+05  .547E+02  .765E+02  .175E+02  .500E-01
(     7)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(     8)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(     9)    .211E+05  .562E+02  .787E+02  .175E+02  .500E-01
(    10)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    11)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    12)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    13)    .211E+05  .544E+02  .762E+02  .143E+02  .500E-01
(    14)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    15)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    16)    .211E+05  .542E+02  .759E+02  .127E+02  .500E-01
(    17)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    18)    .211E+05  .536E+02  .751E+02  .714E+01  .500E-01
(    19)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    20)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    21)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    22)    .211E+05  .531E+02  .743E+02  .111E+02  .500E-01
(    23)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    24)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    25)    .211E+05  .562E+02  .787E+02  .111E+02  .500E-01
(    26)    .211E+05  .543E+02  .760E+02  .953E+01  .500E-01
(    27)    .211E+05  .330E+02  .462E+02  .635E+01  .500E-01
(    28)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    29)    .211E+05  .540E+02  .756E+02  .873E+01  .500E-01
(    30)    .211E+05  .562E+02  .787E+02  .635E+01  .500E-01
(    31)    .211E+05  .540E+02  .756E+02  .873E+01  .500E-01
(    32)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    33)    .211E+05  .562E+02  .787E+02  .635E+01  .500E-01
(    34)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    35)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    36)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    37)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    38)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    39)    .211E+05  .547E+02  .765E+02  .175E+02  .500E-01
(    40)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    41)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    42)    .211E+05  .562E+02  .787E+02  .175E+02  .500E-01
(    43)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    44)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    45)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    46)    .211E+05  .544E+02  .762E+02  .143E+02  .500E-01
(    47)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    48)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    49)    .211E+05  .542E+02  .759E+02  .127E+02  .500E-01
(    50)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    51)    .211E+05  .536E+02  .751E+02  .714E+01  .500E-01
(    52)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    53)    .211E+05  .330E+02  .462E+02  .873E+01  .500E-01
(    54)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    55)    .211E+05  .531E+02  .743E+02  .111E+02  .500E-01
(    56)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    57)    .211E+05  .330E+02  .462E+02  .111E+02  .500E-01
(    58)    .211E+05  .562E+02  .787E+02  .111E+02  .500E-01
(    59)    .211E+05  .543E+02  .760E+02  .953E+01  .500E-01
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(    60)    .211E+05  .330E+02  .462E+02  .635E+01  .500E-01
(    61)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    62)    .211E+05  .540E+02  .756E+02  .873E+01  .500E-01
(    63)    .211E+05  .562E+02  .787E+02  .635E+01  .500E-01
(    64)    .211E+05  .540E+02  .756E+02  .873E+01  .500E-01
(    65)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    66)    .211E+05  .562E+02  .787E+02  .635E+01  .500E-01
(    67)    .211E+05  .562E+02  .787E+02  .953E+01  .500E-01
(    68)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    69)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    70)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    71)    .211E+05  .562E+02  .787E+02  .143E+02  .500E-01
(    72)    .211E+05  .330E+02  .462E+02  .635E+01  .500E-01
(    73)    .211E+05  .562E+02  .787E+02  .556E+01  .500E-01
(    74)    .211E+05  .562E+02  .787E+02  .127E+02  .500E-01
(    75)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    76)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    77)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    78)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    79)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    80)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    81)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    82)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    83)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    84)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    85)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    86)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    87)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    88)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    89)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    90)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    91)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    92)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    93)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    94)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    95)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    96)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    97)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    98)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(    99)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   100)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   101)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   102)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   103)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   104)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   105)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   106)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   107)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   108)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   109)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   110)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   111)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   112)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   113)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   114)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   115)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(   116)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
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(  117)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  118)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  119)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  120)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  121)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  122)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  123)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  124)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  125)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  126)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  127)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  128)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  129)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  130)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  131)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  132)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  133)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  134)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  135)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  136)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  137)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  138)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  139)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01
(  140)    .211E+05  .330E+02  .462E+02  .500E+00  .500E-01

PROPERTY OF STIFFENER IN STIFFENED PLATE
      (NTYPE,STIFFENER NO.,H x B x T1/T2,A,I)
(     1)    3  1   151.380 109.220   3.560    5.840  .118E+04  .209E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     3)    3  8   125.480 100.330   4.570    5.180  .875E+04  .976E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     4)    3  9   125.480 106.680   3.180    5.590  .896E+04  .112E+09
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     5)    3  1   151.380 109.220   3.580    5.840  .118E+04  .211E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     6)    3  1   152.400 101.600   5.840    6.830  .158E+04  .269E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     7)    3  1   125.480 100.330   4.570    5.180  .109E+04  .127E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(     8)    3  2   125.480 100.330   4.570    5.180  .219E+04  .249E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    11)    3  1   125.480 100.330   4.570    5.180  .109E+04  .127E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    12)    3  2   125.480 100.330   4.570    5.180  .219E+04  .249E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    13)    3  1   152.400 101.600   5.840    6.830  .158E+04  .263E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    14)    3  1   125.480 100.330   4.570    5.180  .109E+04  .127E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    15)    3  2   125.480 100.330   4.570    5.180  .219E+04  .249E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    16)    3  1   152.400 101.600   5.840    6.830  .158E+04  .260E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    18)    3  2   125.480 100.330   4.570    5.180  .219E+04  .246E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
(    20)    3  2   125.480 100.330   4.570    5.180  .219E+04  .249E+08
            0  0      .000    .000    .000     .000  .000E+00  .000E+00
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(    22)    3  2    152.400 101.600    5.840    6.830   .317E+04   .515E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    23)    3  2    125.480 100.330    4.570    5.180   .219E+04   .254E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    24)    3  1    125.480 100.330    4.570    5.180   .109E+04   .127E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    26)    3  1    151.260 100.790    5.030    5.690   .133E+04   .208E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    27)    3  5    100.330 100.080    4.320    5.180   .476E+04   .372E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    29)    3  2    100.330 100.080    4.320    5.180   .190E+04   .152E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    30)    3  3    125.480 106.680    3.180    5.590   .299E+04   .377E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    31)    3  2    100.330 100.080    4.320    5.180   .190E+04   .152E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    33)    3  2    125.480 106.680    3.180    5.590   .199E+04   .251E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    35)    3  1    151.380 109.220    3.580    5.840   .118E+04   .211E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    38)    3  9    151.380 109.220    3.580    5.840   .106E+05   .192E+09
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    39)    3  1    152.400 101.600    5.840    6.830   .158E+04   .269E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    40)    3  1    125.480 100.330    4.570    5.180   .109E+04   .127E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    41)    3  2    125.480 100.330    4.570    5.180   .219E+04   .249E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    44)    3  1    125.480 100.330    4.570    5.180   .109E+04   .127E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    45)    3  2    125.480 100.330    4.570    5.180   .219E+04   .249E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    46)    3  1    152.400 101.600    5.840    6.830   .158E+04   .263E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    47)    3  1    125.480 100.330    4.570    5.180   .109E+04   .127E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    48)    3  2    125.480 100.330    4.570    5.180   .219E+04   .249E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    49)    3  1    152.400 101.600    5.840    6.830   .158E+04   .260E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    51)    3  2    125.480 100.330    4.570    5.180   .219E+04   .246E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    53)    3  2    125.480 100.330    4.570    5.180   .219E+04   .249E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    55)    3  2    152.400 101.600    5.840    6.830   .317E+04   .515E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    56)    3  2    125.480 100.330    4.570    5.180   .219E+04   .254E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    57)    3  1    125.480 100.330    4.570    5.180   .109E+04   .127E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    59)    3  1    151.260 100.790    5.030    5.690   .133E+04   .208E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    60)    3  5    100.330 100.080    4.320    5.180   .476E+04   .372E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    62)    3  2    100.330 100.080    4.320    5.180   .190E+04   .152E+08
            0  0       .000    .000     .000     .000   .000E+00   .000E+00
(    63)    3  3    125.480 106.680    3.180    5.590   .299E+04   .377E+08
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              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    64)      3  2    100.330  100.080    4.320    5.180   .190E+04   .152E+08
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    66)      3  2    125.480  106.680    3.180    5.590   .199E+04   .251E+08
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    68)      3  1    151.380  109.220    3.580    5.840   .118E+04   .211E+08
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    71)      3  9    151.380  109.220    3.580    5.840   .106E+05   .192E+09
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    72)      3  8    125.480  100.330    4.570    5.180   .875E+04   .976E+08
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    73)      3  9    125.480  106.680    3.180    5.590   .896E+04   .112E+09
              0  0       .000     .000     .000     .000   .000E+00   .000E+00
(    74)      3  1    151.380  109.220    3.580    5.840   .118E+04   .211E+08
              0  0       .000     .000     .000     .000   .000E+00   .000E+00

ARRANGEMENT NUMBER OF BEAM-COLUMN (NPNOF)
(     1)     5    64  (     2)    14    73  (     3)    17    76
(     4)    19    78  (     5)    21    80  (     6)    22    81
(     7)    25    84  (     8)    24    83  (     9)    26    85
(    10)    28    87  (    11)    29    88  (    12)    32    91
(    13)    41   100  (    14)    44   103  (    15)    46   105
(    16)    48   107  (    17)    49   108  (    18)    52   111
(    19)    51   110  (    20)    53   112  (    21)    55   114
(    22)    56   115  (    23)    57   116  (    24)    58   117
(    25)    59   118  (

MATERIAL PROPERTY OF BEAM-COLUMN (NTYP,H1,B,T1,T2,ARF,ZIF)
(     1) 3      33.00   152.40   101.60     5.84      6.83   .158E+04   .257E+08
(     2) 3      33.00   152.40   101.60     5.84      6.83   .158E+04   .237E+08
(     3) 3      33.00   151.26   100.79     5.03      5.69   .133E+04   .204E+08
(     4) 3      33.00   125.48   100.33     4.57      5.18   .109E+04   .121E+08
(     5) 3      33.00   125.48   100.33     4.57      5.18   .109E+04   .129E+08
(     6) 3      33.00   100.33   100.08     4.32      5.18   .952E+03   .833E+07
(     7) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .209E+08
(     8) 3      56.20   125.48   106.68     3.18      5.59   .995E+03   .128E+08
(     9) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .219E+08
(    10) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .214E+08
(    11) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .209E+08
(    12) 3      33.00   152.40   101.60     5.84      6.83   .158E+04   .253E+08
(    13) 3      33.00   152.40   101.60     5.84      6.83   .158E+04   .263E+08
(    14) 3      33.00   151.26   100.79     5.03      5.69   .133E+04   .211E+08
(    15) 3      33.00   125.48   100.33     4.57      5.18   .109E+04   .125E+08
(    16) 3      33.00   125.48   100.33     4.57      5.18   .109E+04   .129E+08
(    17) 3      33.00   100.33   100.08     4.32      5.18   .952E+03   .781E+07
(    18) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .203E+08
(    19) 3      56.20   125.48   106.68     3.18      5.59   .995E+03   .131E+08
(    20) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .206E+08
(    21) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .211E+08
(    22) 3      56.20   151.38   109.22     3.58      5.84   .118E+04   .209E+08
(    23) 3      56.20   228.60   190.50     9.09     14.48   .484E+04   .200E+09
(    24) 3      56.20   228.60   190.50     9.09     14.48   .484E+04   .200E+09
(    25) 3      33.00   228.60   190.50     9.09     14.48   .484E+04   .200E+09

C O O R D I N A T E (XXG)
(  1)     .000E+00  .000E+00  .000E+00
(  2)     .000E+00  .000E+00  .137E+04
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(  3)    .000E+00  .129E+04  .762E+02
(  4)    .000E+00  .129E+04  .145E+04
(  5)    .000E+00  .193E+04  .172E+03
(  6)    .000E+00  .258E+04  .293E+03
(  7)    .000E+00  .258E+04  .166E+04
(  8)    .000E+00  .387E+04  .566E+03
(  9)    .000E+00  .387E+04  .194E+04
( 10)    .000E+00  .451E+04  .211E+04
( 11)    .000E+00  .451E+04  .457E+04
( 12)    .000E+00  .516E+04  .988E+03
( 13)    .000E+00  .516E+04  .236E+04
( 14)    .000E+00  .580E+04  .127E+04
( 15)    .000E+00  .741E+04  .216E+04
( 16)    .000E+00  .677E+04  .282E+04
( 17)    .000E+00  .774E+04  .308E+04
( 18)    .000E+00  .821E+04  .457E+04
( 19)    .000E+00  .825E+04  .526E+04
( 20)    .000E+00  .838E+04  .732E+04
( 21)    .000E+00  .580E+04  .732E+04
( 22)    .000E+00  .838E+04  .937E+04
( 23)    .000E+00  .838E+04  .101E+05
( 24)    .000E+00  .645E+04  .101E+05
( 25)    .000E+00  .838E+04  .107E+05
( 26)    .000E+00  .838E+04  .121E+05
( 27)    .000E+00  .838E+04  .128E+05
( 28)    .000E+00  .774E+04  .128E+05
( 29)    .000E+00  .129E+04  .128E+05
( 30)    .000E+00 -.129E+04  .762E+02
( 31)    .000E+00 -.129E+04  .145E+04
( 32)    .000E+00 -.193E+04  .172E+03
( 33)    .000E+00 -.258E+04  .293E+03
( 34)    .000E+00 -.258E+04  .166E+04
( 35)    .000E+00 -.387E+04  .566E+03
( 36)    .000E+00 -.387E+04  .194E+04
( 37)    .000E+00 -.451E+04  .211E+04
( 38)    .000E+00 -.451E+04  .457E+04
( 39)    .000E+00 -.516E+04  .988E+03
( 40)    .000E+00 -.516E+04  .236E+04
( 41)    .000E+00 -.580E+04  .127E+04
( 42)    .000E+00 -.741E+04  .216E+04
( 43)    .000E+00 -.677E+04  .282E+04
( 44)    .000E+00 -.774E+04  .308E+04
( 45)    .000E+00 -.821E+04  .457E+04
( 46)    .000E+00 -.825E+04  .526E+04
( 47)    .000E+00 -.838E+04  .732E+04
( 48)    .000E+00 -.580E+04  .732E+04
( 49)    .000E+00 -.838E+04  .937E+04
( 50)    .000E+00 -.838E+04  .101E+05
( 51)    .000E+00 -.645E+04  .101E+05
( 52)    .000E+00 -.838E+04  .107E+05
( 53)    .000E+00 -.838E+04  .121E+05
( 54)    .000E+00 -.838E+04  .128E+05
( 55)    .000E+00 -.774E+04  .128E+05
( 56)    .000E+00 -.129E+04  .128E+05
( 57)    .000E+00  .000E+00  .128E+05
( 58)    .000E+00  .000E+00  .101E+05
( 59)    .000E+00  .000E+00  .732E+04
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( 60)    .244E+04  .000E+00  .000E+00
( 61)    .244E+04  .000E+00  .137E+04
( 62)    .244E+04  .129E+04  .762E+02
( 63)    .244E+04  .129E+04  .145E+04
( 64)    .244E+04  .193E+04  .172E+03
( 65)    .244E+04  .258E+04  .293E+03
( 66)    .244E+04  .258E+04  .166E+04
( 67)    .244E+04  .387E+04  .566E+03
( 68)    .244E+04  .387E+04  .194E+04
( 69)    .244E+04  .451E+04  .211E+04
( 70)    .244E+04  .451E+04  .457E+04
( 71)    .244E+04  .516E+04  .988E+03
( 72)    .244E+04  .516E+04  .236E+04
( 73)    .244E+04  .580E+04  .127E+04
( 74)    .244E+04  .741E+04  .216E+04
( 75)    .244E+04  .677E+04  .282E+04
( 76)    .244E+04  .774E+04  .308E+04
( 77)    .244E+04  .821E+04  .457E+04
( 78)    .244E+04  .825E+04  .526E+04
( 79)    .244E+04  .838E+04  .732E+04
( 80)    .244E+04  .580E+04  .732E+04
( 81)    .244E+04  .838E+04  .937E+04
( 82)    .244E+04  .838E+04  .101E+05
( 83)    .244E+04  .645E+04  .101E+05
( 84)    .244E+04  .838E+04  .107E+05
( 85)    .244E+04  .838E+04  .121E+05
( 86)    .244E+04  .838E+04  .128E+05
( 87)    .244E+04  .774E+04  .128E+05
( 88)    .244E+04  .129E+04  .128E+05
( 89)    .244E+04 -.129E+04  .762E+02
( 90)    .244E+04 -.129E+04  .145E+04
( 91)    .244E+04 -.193E+04  .172E+03
( 92)    .244E+04 -.258E+04  .293E+03
( 93)    .244E+04 -.258E+04  .166E+04
( 94)    .244E+04 -.387E+04  .566E+03
( 95)    .244E+04 -.387E+04  .194E+04
( 96)    .244E+04 -.451E+04  .211E+04
( 97)    .244E+04 -.451E+04  .457E+04
( 98)    .244E+04 -.516E+04  .988E+03
( 99)    .244E+04 -.516E+04  .236E+04
(100)    .244E+04 -.580E+04  .127E+04
(101)    .244E+04 -.741E+04  .216E+04
(102)    .244E+04 -.677E+04  .282E+04
(103)    .244E+04 -.774E+04  .308E+04
(104)    .244E+04 -.821E+04  .457E+04
(105)    .244E+04 -.825E+04  .526E+04
(106)    .244E+04 -.838E+04  .732E+04
(107)    .244E+04 -.580E+04  .732E+04
(108)    .244E+04 -.838E+04  .937E+04
(109)    .244E+04 -.838E+04  .101E+05
(110)    .244E+04 -.645E+04  .101E+05
(111)    .244E+04 -.838E+04  .107E+05
(112)    .244E+04 -.838E+04  .121E+05
(113)    .244E+04 -.838E+04  .128E+05
(114)    .244E+04 -.774E+04  .128E+05
(115)    .244E+04 -.129E+04  .128E+05
(116)    .244E+04  .000E+00  .128E+05
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(117)    .244E+04  .000E+00  .101E+05
(118)    .244E+04  .000E+00  .732E+04

L O A D I N G   P O I N T
(   60)    1  (   61)    1  (   62)    1  (   63)    1
(   64)    1  (   65)    1  (   66)    1  (   67)    1
(   68)    1  (   69)    1  (   70)    1  (   71)    1
(   72)    1  (   73)    1  (   74)    1  (   75)    1
(   76)    1  (   77)    1  (   78)    1  (   79)    1
(   80)    1  (   81)    1  (   82)    1  (   83)    1
(   84)    1  (   85)    1  (   86)    1  (   87)    1
(   88)    1  (   89)    1  (   90)    1  (   91)    1
(   92)    1  (   93)    1  (   94)    1  (   95)    1
(   96)    1  (   97)    1  (   98)    1  (   99)    1
(  100)    1  (  101)    1  (  102)    1  (  103)    1
(  104)    1  (  105)    1  (  106)    1  (  107)    1
(  108)    1  (  109)    1  (  110)    1  (  111)    1
(  112)    1  (  113)    1  (  114)    1  (  115)    1
(  116)    1  (  117)    1  (  118)    1  (

BOUNDARY COND. : 0=FIX, 1=FREE, -1=PRESCRIBED DISPLACEMENT
(    1)    0    0    0   (    2)    0    0    1
(    3)    0    1    1   (    4)    0    1    1
(    5)    0    1    1   (    6)    0    1    1
(    7)    0    1    1   (    8)    0    1    1
(    9)    0    1    1   (   10)    0    1    1
(   11)    0    1    1   (   12)    0    1    1
(   13)    0    1    1   (   14)    0    1    1
(   15)    0    1    1   (   16)    0    1    1
(   17)    0    1    1   (   18)    0    1    1
(   19)    0    1    1   (   20)    0    1    1
(   21)    0    1    1   (   22)    0    1    1
(   23)    0    1    1   (   24)    0    1    1
(   25)    0    1    1   (   26)    0    1    1
(   27)    0    1    1   (   28)    0    1    1
(   29)    0    1    1   (   30)    0    1    1
(   31)    0    1    1   (   32)    0    1    1
(   33)    0    1    1   (   34)    0    1    1
(   35)    0    1    1   (   36)    0    1    1
(   37)    0    1    1   (   38)    0    1    1
(   39)    0    1    1   (   40)    0    1    1
(   41)    0    1    1   (   42)    0    1    1
(   43)    0    1    1   (   44)    0    1    1
(   45)    0    1    1   (   46)    0    1    1
(   47)    0    1    1   (   48)    0    1    1
(   49)    0    1    1   (   50)    0    1    1
(   51)    0    1    1   (   52)    0    1    1
(   53)    0    1    1   (   54)    0    1    1
(   55)    0    1    1   (   56)    0    1    1
(   57)    0    1    1   (   58)    0    1    1
(   59)    0    1    1   (   60)   -1    0    0
(   61)   -1    0    1   (   62)   -1    1    1
(   63)   -1    1    1   (   64)   -1    1    1
(   65)   -1    1    1   (   66)   -1    1    1
(   67)   -1    1    1   (   68)   -1    1    1
(   69)   -1    1    1   (   70)   -1    1    1
(   71)   -1    1    1   (   72)   -1    1    1
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(   73)   -1     1     1   (   74)   -1     1     1
(   75)   -1     1     1   (   76)   -1     1     1
(   77)   -1     1     1   (   78)   -1     1     1
(   79)   -1     1     1   (   80)   -1     1     1
(   81)   -1     1     1   (   82)   -1     1     1
(   83)   -1     1     1   (   84)   -1     1     1
(   85)   -1     1     1   (   86)   -1     1     1
(   87)   -1     1     1   (   88)   -1     1     1
(   89)   -1     1     1   (   90)   -1     1     1
(   91)   -1     1     1   (   92)   -1     1     1
(   93)   -1     1     1   (   94)   -1     1     1
(   95)   -1     1     1   (   96)   -1     1     1
(   97)   -1     1     1   (   98)   -1     1     1
(   99)   -1     1     1   (  100)   -1     1     1
(  101)   -1     1     1   (  102)   -1     1     1
(  103)   -1     1     1   (  104)   -1     1     1
(  105)   -1     1     1   (  106)   -1     1     1
(  107)   -1     1     1   (  108)   -1     1     1
(  109)   -1     1     1   (  110)   -1     1     1
(  111)   -1     1     1   (  112)   -1     1     1
(  113)   -1     1     1   (  114)   -1     1     1
(  115)   -1     1     1   (  116)   -1     1     1
(  117)   -1     1     1   (  118)   -1     1     1

HARD PLATE UNITS
   36   37   69   70

LENGTH(AA) AND BREADTH(BB)
(     1)     2438.400     1371.600   (     2)     2438.400     9026.800
(     3)     2438.400     5802.900   (     4)     2438.400     6447.700
(     5)     2438.400     1289.500   (     6)     2438.400     1291.749
(     7)     2438.400     1291.749   (     8)     2438.400     1371.600
(     9)     2438.400      651.819   (    10)     2438.400      656.073
(    11)     2438.400     1307.647   (    12)     2438.400     1371.600
(    13)     2438.400     1318.227   (    14)     2438.400     1318.227
(    15)     2438.400     1371.600   (    16)     2438.400     1356.735
(    17)     2438.400      666.630   (    18)     2438.400     2464.800
(    19)     2438.400      692.403   (    20)     2438.400     1371.600
(    21)     2438.400      703.798   (    22)     2438.400     1841.032
(    23)     2438.400     1675.486   (    24)     2438.400      920.695
(    25)     2438.400      973.972   (    26)     2438.400     1566.849
(    27)     2438.400     3697.900   (    28)     2438.400      687.103
(    29)     2438.400     2061.310   (    30)     2438.400     2577.600
(    31)     2438.400     2057.401   (    32)     2438.400      685.800
(    33)     2438.400     1934.300   (    34)     2438.400      673.300
(    35)     2438.400     1346.000   (    36)     2438.400      673.100
(    37)     2438.400      644.816   (    38)     2438.400     6447.866
(    39)     2438.400     1291.749   (    40)     2438.400     1291.749
(    41)     2438.400     1371.600   (    42)     2438.400      651.819
(    43)     2438.400      656.073   (    44)     2438.400     1307.647
(    45)     2438.400     1371.600   (    46)     2438.400     1318.227
(    47)     2438.400     1318.227   (    48)     2438.400     1371.600
(    49)     2438.400     1356.735   (    50)     2438.400      666.630
(    51)     2438.400     2464.800   (    52)     2438.400      692.403
(    53)     2438.400     1371.600   (    54)     2438.400      703.798
(    55)     2438.400     1841.032   (    56)     2438.400     1675.486
(    57)     2438.400      920.695   (    58)     2438.400      973.972
```

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(   59)   2438.400   1566.849  (   60)   2438.400   3697.900
(   61)   2438.400    687.103  (   62)   2438.400   2061.310
(   63)   2438.400   2577.600  (   64)   2438.400   2057.401
(   65)   2438.400    685.800  (   66)   2438.400   1934.300
(   67)   2438.400    673.300  (   68)   2438.400   1346.000
(   69)   2438.400    673.100  (   70)   2438.400    644.816
(   71)   2438.400   6447.866  (   72)   2438.400   5802.900
(   73)   2438.400   6447.700  (   74)   2438.400   1289.500
(   75)   1491.786   2167.709  (   76)   1289.558   2840.909
(   77)   6447.783   3010.291  (   78)   3868.600   4292.737
(   79)   6447.783   3010.291  (   80)   1289.558   2840.909
(   81)   1491.786   2167.709  (   82)   1491.786   2167.709
(   83)   1289.558   2840.909  (   84)   6447.783   3010.291
(   85)   3868.600   4292.737  (   86)   6447.783   3010.291
(   87)   1289.558   2840.909  (   88)   1491.786   2167.709
(   89)   2255.950   2780.582  (   90)   6125.300   2780.581
(   91)   6125.300   2780.581  (   92)   2255.950   2780.582
(   93)   2255.950   2780.582  (   94)   6125.300   2780.581
(   95)   6125.300   2780.581  (   96)   2255.950   2780.582
(   97)   3137.750   2889.789  (   98)   7414.850   4156.412
(   99)   7414.850   4156.412  (  100)   3137.750   2889.789
(  101)   3137.750   2889.789  (  102)   7414.850   4156.412
(  103)   7414.850   4156.412  (  104)   3137.750   2889.789
(  105)   2109.095   1146.147  (  106)   1356.735   1371.600
(  107)   1318.227   1371.600  (  108)   1307.647   1371.600
(  109)   1291.749   1371.600  (  110)   1291.749   1371.600
(  111)   1307.647   1371.600  (  112)   1318.227   1371.600
(  113)   1356.735   1371.600  (  114)   2109.095   1146.147
(  115)   2109.095   1146.147  (  116)   1356.735   1371.600
(  117)   1318.227   1371.600  (  118)   1307.647   1371.600
(  119)   1291.749   1371.600  (  120)   1291.749   1371.600
(  121)   1307.647   1371.600  (  122)   1318.227   1371.600
(  123)   1356.735   1371.600  (  124)   2109.095   1146.147
(  125)   9026.800   2464.800  (  126)   9026.800   2464.800
(  127)   2314.938   2437.682  (  128)   2314.938   2437.682
(  129)   2314.938   2437.682  (  130)   2314.938   2437.682
(  131)   3190.077   2546.237  (  132)   3190.077   2546.237
(  133)   3190.077   2546.237  (  134)   3190.077   2546.237
(  135)   2349.918   2212.915  (  136)   2349.918   2212.915
(  137)   2349.918   2212.915  (  138)   2349.918   2212.915
(  139)   3868.600   4292.737  (  140)   3868.600   4292.737

BUCKLING TERM OF INITIAL DEFLECTION OF PLATE
(    1)   5.555  (    2)    .005  (    3)   3.175  (    4)   2.780
(    5)   6.350  (    6)   8.730  (    7)   5.555  (    8)   4.365
(    9)   8.730  (   10)   7.145  (   11)   5.555  (   12)   4.365
(   13)   7.145  (   14)   5.555  (   15)   4.365  (   16)   6.350
(   17)   5.555  (   18)   3.570  (   19)   5.555  (   20)   4.365
(   21)   6.350  (   22)   5.555  (   23)   5.555  (   24)   5.555
(   25)   5.555  (   26)   4.765  (   27)   3.175  (   28)   4.765
(   29)   4.365  (   30)   3.175  (   31)   4.365  (   32)   4.765
(   33)   3.175  (   34)   4.765  (   35)   6.350  (   36)   6.350
(   37)   7.145  (   38)   7.145  (   39)   8.730  (   40)   5.555
(   41)   4.365  (   42)   8.730  (   43)   7.145  (   44)   5.555
(   45)   4.365  (   46)   7.145  (   47)   5.555  (   48)   4.365
(   49)   6.350  (   50)   5.555  (   51)   3.570  (   52)   5.555
(   53)   4.365  (   54)   6.350  (   55)   5.555  (   56)   5.555
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```
(   57)   5.555  (   58)   5.555  (   59)   4.765  (   60)   3.175
(   61)   4.765  (   62)   4.365  (   63)   3.175  (   64)   4.365
(   65)   4.765  (   66)   3.175  (   67)   4.765  (   68)   6.350
(   69)   6.350  (   70)   7.145  (   71)   7.145  (   72)   3.175
(   73)   2.780  (   74)   6.350  (

INITIAL DEFLECTION OF BEAM-COLUMN
(    1)  1219.200  (    2)  1219.200  (    3)  1219.200  (    4)  1219.200
(    5)  1219.200  (    6)  1219.200  (    7)  1219.200  (    8)  1219.200
(    9)  1219.200  (   10)  1219.200  (   11)  1219.200  (   12)  1219.200
(   13)  1219.200  (   14)  1219.200  (   15)  1219.200  (   16)  1219.200
(   17)  1219.200  (   18)  1219.200  (   19)  1219.200  (   20)  1219.200
(   21)  1219.200  (   22)  1219.200  (   23)  1219.200  (   24)  1219.200
(   25)  1219.200  (

ACTUAL COMPRESSIVE RESIDUAL STRESS OF PLATE
(    1)   -3.30     -.93  (    2)   -3.30   -12.22  (    3)   -3.30     -.87
(    4)   -5.62    -1.49  (    5)   -5.62    -1.49  (    6)   -5.47    -1.45
(    7)   -3.30     -.87  (    8)   -3.30     -.62  (    9)   -5.62    -1.50
(   10)   -5.62    -1.51  (   11)   -3.30     -.88  (   12)   -3.30     -.62
(   13)   -5.44    -1.47  (   14)   -3.30     -.89  (   15)   -3.30     -.62
(   16)   -5.42    -1.51  (   17)   -3.30     -.90  (   18)   -5.36    -1.81
(   19)   -3.30     -.94  (   20)   -3.30     -.62  (   21)   -5.62    -1.62
(   22)   -5.31    -1.34  (   23)   -3.30     -.76  (   24)   -3.30     -.62
(   25)   -5.62    -2.24  (   26)   -5.43    -1.74  (   27)   -3.30     -.83
(   28)   -5.62    -1.58  (   29)   -5.40    -1.52  (   30)   -5.62    -1.49
(   31)   -5.40    -1.52  (   32)   -5.62    -1.58  (   33)   -5.62    -1.49
(   34)   -5.62    -1.55  (   35)   -5.62    -1.55  (   36)   -5.62    -1.55
(   37)   -5.62    -1.49  (   38)   -5.62    -1.49  (   39)   -5.47    -1.45
(   40)   -3.30     -.87  (   41)   -3.30     -.62  (   42)   -5.62    -1.50
(   43)   -5.62    -1.51  (   44)   -3.30     -.88  (   45)   -3.30     -.62
(   46)   -5.44    -1.47  (   47)   -3.30     -.89  (   48)   -3.30     -.62
(   49)   -5.42    -1.51  (   50)   -3.30     -.90  (   51)   -5.36    -1.81
(   52)   -3.30     -.94  (   53)   -3.30     -.62  (   54)   -5.62    -1.62
(   55)   -5.31    -1.34  (   56)   -3.30     -.76  (   57)   -3.30     -.62
(   58)   -5.62    -2.24  (   59)   -5.43    -1.74  (   60)   -3.30     -.83
(   61)   -5.62    -1.58  (   62)   -5.40    -1.52  (   63)   -5.62    -1.49
(   64)   -5.40    -1.52  (   65)   -5.62    -1.58  (   66)   -5.62    -1.49
(   67)   -5.62    -1.55  (   68)   -5.62    -1.55  (   69)   -5.62    -1.55
(   70)   -5.62    -1.49  (   71)   -5.62    -1.49  (   72)   -3.30     -.87
(   73)   -5.62    -1.49  (   74)   -5.62    -1.49  (

COMPRESSIVE RESIDUAL STRESS OF BEAM-COLUMN
(    1)        .00  (    2)        .00  (    3)        .00  (    4)        .00
(    5)        .00  (    6)        .00  (    7)        .00  (    8)        .00
(    9)        .00  (   10)        .00  (   11)        .00  (   12)        .00
(   13)        .00  (   14)        .00  (   15)        .00  (   16)        .00
(   17)        .00  (   18)        .00  (   19)        .00  (   20)        .00
(   21)        .00  (   22)        .00  (   23)        .00  (   24)        .00
(   25)        .00  (

TOTAL NUMBER OF UNKNOWNS =       231
REAL VALUE OF NWKPA      =     16411
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                  HULL MODULE DATA
                 =================

     LENGTH OF HULL MODULE       (mm) :   .24384E+04

     DEPTH OF HULL MODULE        (mm) :   .12802E+05

     BREADTH OF HULL MODULE      (mm) :   .16764E+05


     CROSS-SECTIONAL AREA       (mm2) :   .14220E+07

     HEIGHT TO NEUTRAL AXIS      (mm) :   .60288E+04

     MOMENT OF INERTIA, VERT.    (m4) :   .31113E+02

     SECTION MODULUS, BOTTOM     (m3) :   .51607E+01

     SECTION MODULUS, DECK       (m3) :   .45938E+01

     MOMENT OF INERTIA, HORI.    (m4) :   .39306E+02

     PLASTIC BENDING MOMENT, VERT.(ton-m)x10**5 :   .29756E+01

     PLASTIC BENDING MOMENT, HORI.(ton-m)x10**5 :   .32601E+01

     WEIGHT OF FULL-HULL MODULE (ton) :   .27218E+02


     L O A D I N G   S T E P   =      200
    =======================================

VERTICAL CURVATURE      x 10**-7(1/mm) = -.33014E+01
VERTICAL BENDING MOMENT x 10**5(ton-m) = -.17142E+01
HEIGHT TO NEUTRAL AXIS            (mm) =  .54780E+04

HORIZONTAL CURVATURE      x 10**-7(1/mm) =  .00000E+00
HORIZONTAL BENDING MOMENT x 10**5(ton-m) =  .00000E+00
WIDTH TO NEUTRAL AXIS               (mm) =  .00000E+00

E X T E R N A L   L O A D / D I S P L  C E M E N T
(   60,     1)    .47373E+01     (   61,     1)    .36332E+01
(   62,     1)    .46760E+01     (   63,     1)    .35718E+01
(   64,     1)    .45992E+01     (   65,     1)    .45017E+01
(   66,     1)    .33975E+01     (   67,     1)    .42813E+01
(   68,     1)    .31772E+01     (   69,     1)    .30410E+01
(   70,     1)    .10568E+01     (   71,     1)    .39420E+01
(   72,     1)    .28379E+01     (   73,     1)    .37148E+01
(   74,     1)    .29989E+01     (   75,     1)    .24698E+01
(   76,     1)    .22590E+01     (   77,     1)    .10568E+01
(   78,     1)    .50474E+00     (   79,     1)   -.11515E+01
(   80,     1)   -.11515E+01     (   81,     1)   -.28077E+01
(   82,     1)   -.33598E+01     (   83,     1)   -.33598E+01
(   84,     1)   -.39018E+01     (   85,     1)   -.49853E+01
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(   86,     1)  -.55272E+01     (   87,     1)  -.55309E+01
(   88,     1)  -.55681E+01     (   89,     1)   .46760E+01
(   90,     1)   .35718E+01     (   91,     1)   .45992E+01
(   92,     1)   .45017E+01     (   93,     1)   .33975E+01
(   94,     1)   .42813E+01     (   95,     1)   .31772E+01
(   96,     1)   .30410E+01     (   97,     1)   .10568E+01
(   98,     1)   .39420E+01     (   99,     1)   .28379E+01
(  100,     1)   .37148E+01     (  101,     1)   .29989E+01
(  102,     1)   .24698E+01     (  103,     1)   .22590E+01
(  104,     1)   .10568E+01     (  105,     1)   .50474E+00
(  106,     1)  -.11515E+01     (  107,     1)  -.11515E+01
(  108,     1)  -.28077E+01     (  109,     1)  -.33598E+01
(  110,     1)  -.33598E+01     (  111,     1)  -.39018E+01
(  112,     1)  -.49853E+01     (  113,     1)  -.55272E+01
(  114,     1)  -.55309E+01     (  115,     1)  -.55681E+01
(  116,     1)  -.55681E+01     (  117,     1)  -.33598E+01
(  118,     1)  -.11515E+01     (

C O L L A P S E   M O D E   O F   P L A T E
(    1)  5      .000     .000     .000     .992     .833     .510
(    2)  0      .000     .000     .000     .049     .000     .000
(    3)  0      .354     .490     .000     .000     .000     .000
(    4)  1      .996     .840     .000     .000     .000     .000
(    5)  2      .922     .993     .000     .000     .000     .000
(    6)  0      .000     .000     .000     .566     .000     .000
(    7)  0      .003     .000     .000     .916     .000     .000
(    8)  5      .000     .000     .000     .993     .817     .492
(    9)  0      .000     .000     .000     .449     .000     .000
(   10)  0      .000     .000     .000     .432     .000     .000
(   11)  0      .002     .000     .000     .855     .000     .000
(   12)  5      .000     .000     .000     .994     .796     .471
(   13)  0      .000     .000     .000     .511     .000     .000
(   14)  0      .002     .000     .000     .760     .000     .000
(   15)  5      .000     .000     .000     .994     .771     .447
(   16)  0      .000     .000     .000     .457     .000     .000
(   17)  0      .002     .000     .000     .582     .000     .000
(   18)  0      .000     .000     .000     .120     .000     .000
(   19)  0      .002     .000     .000     .524     .000     .000
(   20)  5      .000     .000     .000     .992     .724     .418
(   21)  0      .000     .000     .000     .297     .000     .000
(   22)  0      .000     .000     .000     .358     .000     .000
(   23)  0      .001     .000     .000     .540     .000     .000
(   24)  0      .000     .000     .000     .564     .000     .000
(   25)  0      .183     .000     .000     .000     .000     .000
(   26)  0      .091     .000     .000     .000     .000     .000
(   27)  0      .000     .000     .000     .100     .000     .000
(   28)  0      .002     .000     .000     .013     .000     .000
(   29)  0      .016     .090     .000     .000     .000     .000
(   30)  0      .145     .278     .000     .000     .000     .000
(   31)  0      .268     .570     .000     .000     .000     .000
(   32)  0      .394     .000     .000     .000     .000     .000
(   33)  0      .924     .786     .000     .000     .000     .000
(   34)  0      .539     .000     .000     .000     .000     .000
(   35)  2      .866     .992     .000     .000     .000     .000
(   36)  0      .000     .000     .000     .654     .000     .000
(   37)  0      .000     .000     .000     .723     .000     .000
(   38)  2      .865     .994     .000     .000     .000     .000
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(   39)  0       .651    .000    .000    .000    .000    .000
(   40)  0       .000    .000    .000    .905    .000    .000
(   41)  5       .000    .000    .000    .992    .815    .492
(   42)  0       .523    .000    .000    .000    .000    .000
(   43)  0       .504    .000    .000    .000    .000    .000
(   44)  0       .000    .000    .000    .846    .000    .000
(   45)  5       .000    .000    .000    .994    .796    .471
(   46)  0       .587    .000    .000    .000    .000    .000
(   47)  0       .000    .000    .000    .752    .000    .000
(   48)  5       .000    .000    .000    .993    .770    .447
(   49)  0       .526    .000    .000    .000    .000    .000
(   50)  0       .000    .000    .000    .577    .000    .000
(   51)  0       .000    .000    .000    .120    .000    .000
(   52)  0       .000    .000    .000    .517    .000    .000
(   53)  5       .000    .000    .000    .995    .730    .418
(   54)  0       .000    .000    .000    .295    .000    .000
(   55)  0       .063    .000    .000    .373    .000    .000
(   56)  0       .000    .000    .000    .539    .000    .000
(   57)  0       .000    .000    .000    .566    .000    .000
(   58)  0       .055    .000    .000    .154    .000    .000
(   59)  0       .012    .000    .000    .076    .000    .000
(   60)  0       .000    .000    .000    .100    .000    .000
(   61)  0       .002    .000    .000    .013    .000    .000
(   62)  0       .016    .094    .000    .000    .000    .000
(   63)  0       .145    .276    .000    .000    .000    .000
(   64)  0       .268    .569    .000    .000    .000    .000
(   65)  0       .395    .000    .000    .000    .000    .000
(   66)  0       .923    .786    .000    .000    .000    .000
(   67)  0       .537    .000    .000    .000    .000    .000
(   68)  2       .853    .992    .000    .000    .000    .000
(   69)  0       .000    .000    .000    .653    .000    .000
(   70)  0       .000    .000    .000    .722    .000    .000
(   71)  2       .865    .995    .000    .000    .000    .000
(   72)  0       .354    .486    .000    .000    .000    .000
(   73)  1       .995    .838    .000    .000    .000    .000
(   74)  2       .915    .990    .000    .000    .000    .000

COLLAPSE  MODE  OF  BEAM-COLUMN
(    1)      5  (    2)      0  (    3)      0  (    4)      0
(    5)      0  (    6)      0  (    7)      0  (    8)      0
(    9)      0  (   10)      0  (   11)      0  (   12)      5
(   13)      0  (   14)      0  (   15)      0  (   16)      0
(   17)      0  (   18)      0  (   19)      0  (   20)      0
(   21)      0  (   22)      0  (   23)      0  (   24)      0
(   25)      0  (

NODAL  DEFORMATIONS
(    1)     .000E+00  .000E+00  .000E+00
(    2)     .000E+00  .000E+00 -.912E+00
(    3)     .000E+00 -.733E+00 -.528E+00
(    4)     .000E+00 -.493E+00 -.123E+01
(    5)     .000E+00 -.697E+00 -.335E+01
(    6)     .000E+00 -.150E+01 -.108E+01
(    7)     .000E+00 -.913E+00 -.175E+01
(    8)     .000E+00 -.202E+01 -.198E+01
(    9)     .000E+00 -.122E+01 -.259E+01
(   10)     .000E+00 -.996E+00 -.433E+01
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(    11)      .000E+00  .202E-01 -.479E+01
(    12)      .000E+00 -.237E+01 -.305E+01
(    13)      .000E+00 -.155E+01 -.355E+01
(    14)      .000E+00 -.745E+01  .749E+01
(    15)      .000E+00 -.207E+01 -.397E+01
(    16)      .000E+00 -.189E+01 -.419E+01
(    17)      .000E+00 -.102E+01 -.449E+01
(    18)      .000E+00 -.446E+00 -.491E+01
(    19)      .000E+00 -.103E-01 -.499E+01
(    20)      .000E+00  .276E-01 -.495E+01
(    21)      .000E+00  .851E-01 -.475E+01
(    22)      .000E+00  .447E-01 -.502E+01
(    23)      .000E+00 -.162E+00 -.503E+01
(    24)      .000E+00 -.156E+00 -.468E+01
(    25)      .000E+00  .166E+00 -.520E+01
(    26)      .000E+00  .119E+01 -.547E+01
(    27)      .000E+00  .146E+01 -.504E+01
(    28)      .000E+00  .103E+01 -.494E+01
(    29)      .000E+00  .167E+00 -.445E+01
(    30)      .000E+00  .659E+00 -.794E+00
(    31)      .000E+00  .441E+00 -.149E+01
(    32)      .000E+00  .130E+01  .118E+01
(    33)      .000E+00  .114E+01 -.149E+01
(    34)      .000E+00  .804E+00 -.216E+01
(    35)      .000E+00  .159E+01 -.241E+01
(    36)      .000E+00  .108E+01 -.302E+01
(    37)      .000E+00  .109E+01 -.380E+01
(    38)      .000E+00  .585E+00 -.429E+01
(    39)      .000E+00  .186E+01 -.344E+01
(    40)      .000E+00  .127E+01 -.395E+01
(    41)      .000E+00  .750E+01  .830E+01
(    42)      .000E+00  .171E+01 -.385E+01
(    43)      .000E+00  .169E+01 -.425E+01
(    44)      .000E+00  .820E+00 -.438E+01
(    45)      .000E+00  .103E+01 -.459E+01
(    46)      .000E+00  .556E+00 -.468E+01
(    47)      .000E+00  .521E+00 -.463E+01
(    48)      .000E+00  .492E+00 -.448E+01
(    49)      .000E+00  .585E+00 -.469E+01
(    50)      .000E+00  .857E+00 -.469E+01
(    51)      .000E+00  .840E+00 -.442E+01
(    52)      .000E+00  .615E+00 -.490E+01
(    53)      .000E+00 -.138E+00 -.524E+01
(    54)      .000E+00 -.545E+00 -.481E+01
(    55)      .000E+00 -.117E+00 -.484E+01
(    56)      .000E+00  .340E+00 -.442E+01
(    57)      .000E+00  .252E+00 -.418E+01
(    58)      .000E+00  .385E+00 -.434E+01
(    59)      .000E+00  .335E+00 -.439E+01
(    60)      .474E+01  .000E+00  .000E+00
(    61)      .363E+01  .000E+00 -.494E+00
(    62)      .468E+01 -.734E+00  .410E+00
(    63)      .357E+01 -.508E+00 -.304E+00
(    64)      .460E+01 -.562E+00 -.325E+01
(    65)      .450E+01 -.151E+01 -.113E+00
(    66)      .340E+01 -.918E+00 -.780E+00
(    67)      .428E+01 -.202E+01 -.101E+01
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(    68)      .318E+01 -.122E+01 -.162E+01
(    69)      .304E+01 -.100E+01 -.335E+01
(    70)      .106E+01  .761E-02 -.379E+01
(    71)      .394E+01 -.236E+01 -.208E+01
(    72)      .284E+01 -.155E+01 -.258E+01
(    73)      .371E+01 -.722E+01  .799E+01
(    74)      .300E+01 -.207E+01 -.299E+01
(    75)      .247E+01 -.189E+01 -.321E+01
(    76)      .226E+01 -.102E+01 -.351E+01
(    77)      .106E+01 -.448E+00 -.392E+01
(    78)      .505E+00 -.423E-01 -.401E+01
(    79)     -.115E+01  .193E-01 -.396E+01
(    80)     -.115E+01  .780E-01 -.369E+01
(    81)     -.281E+01  .646E-01 -.403E+01
(    82)     -.336E+01 -.168E+00 -.404E+01
(    83)     -.336E+01 -.163E+00 -.357E+01
(    84)     -.390E+01  .135E+00 -.421E+01
(    85)     -.499E+01  .131E+01 -.448E+01
(    86)     -.553E+01  .147E+01 -.404E+01
(    87)     -.553E+01  .104E+01 -.377E+01
(    88)     -.557E+01  .177E+00 -.282E+01
(    89)      .468E+01  .656E+00  .141E+00
(    90)      .357E+01  .455E+00 -.568E+00
(    91)      .460E+01  .110E+01  .918E+00
(    92)      .450E+01  .115E+01 -.525E+00
(    93)      .340E+01  .809E+00 -.119E+01
(    94)      .428E+01  .159E+01 -.145E+01
(    95)      .318E+01  .107E+01 -.206E+01
(    96)      .304E+01  .110E+01 -.281E+01
(    97)      .106E+01  .587E+00 -.327E+01
(    98)      .394E+01  .185E+01 -.246E+01
(    99)      .284E+01  .127E+01 -.298E+01
(   100)      .371E+01  .721E+01  .871E+01
(   101)      .300E+01  .171E+01 -.287E+01
(   102)      .247E+01  .169E+01 -.328E+01
(   103)      .226E+01  .824E+00 -.340E+01
(   104)      .106E+01  .103E+01 -.361E+01
(   105)      .505E+00  .577E+00 -.369E+01
(   106)     -.115E+01  .531E+00 -.364E+01
(   107)     -.115E+01  .503E+00 -.334E+01
(   108)     -.281E+01  .536E+00 -.370E+01
(   109)     -.336E+01  .855E+00 -.369E+01
(   110)     -.336E+01  .844E+00 -.321E+01
(   111)     -.390E+01  .580E+00 -.390E+01
(   112)     -.499E+01 -.302E+00 -.424E+01
(   113)     -.553E+01 -.555E+00 -.381E+01
(   114)     -.553E+01 -.127E+00 -.358E+01
(   115)     -.557E+01  .349E+00 -.288E+01
(   116)     -.557E+01  .262E+00 -.274E+01
(   117)     -.336E+01  .386E+00 -.289E+01
(   118)     -.115E+01  .337E+00 -.307E+01

AVERAGE  STRESS  OF  PLATE
(     1)    33.041     .812  -2.218     (     2)     7.874   1.267     .001
(     3)    -8.061    -.183    .010     (     4)   -20.522   2.255     .012
(     5)   -26.001    -.292   -.008     (     6)    41.258    .207    -.103
(     7)    32.122    1.117    .085     (     8)    33.362    .992    -.134
```

```
(    9)    37.928     .527    -.025     (    10)    37.212     .518    -.064
(   11)    31.046    1.098     .052     (    12)    33.344     .919    -.039
(   13)    39.055     .325    -.035     (    14)    29.253     .985     .020
(   15)    33.309     .845    -.037     (    16)    36.879     .460    -.001
(   17)    25.647     .973     .025     (    18)    19.284    1.506    -.049
(   19)    24.123     .477    -.003     (    20)    33.408    1.105    -.029
(   21)    32.188    3.422     .113     (    22)    32.011     .469    -.075
(   23)    24.571     .665     .004     (    24)    25.070     .606    -.012
(   25)    22.666    -.233    -.014     (    26)    15.570    -.049    -.002
(   27)    10.505     .118     .023     (    28)     6.478     .122     .006
(   29)    -2.108     .082     .013     (    30)    -7.147    -.068     .026
(   31)   -11.365    -.114     .023     (    32)   -14.868    -.182     .033
(   33)   -19.486     .040     .023     (    34)   -17.503    -.207     .037
(   35)   -25.568    -.075     .036     (    36)   -45.500    -.120     .049
(   37)   -47.893    -.238    -.025     (    38)   -26.716    -.367    -.004
(   39)    43.506    -.253     .095     (    40)    32.256    1.795    -.085
(   41)    33.381    1.049     .140     (    42)    40.073    -.162    -.025
(   43)    39.317    -.175     .083     (    44)    31.154    1.680    -.053
(   45)    33.343     .917     .042     (    46)    41.153    -.121     .025
(   47)    29.348    1.513    -.020     (    48)    33.318     .871     .040
(   49)    38.837    -.007    -.009     (    50)    25.717    1.336    -.015
(   51)    19.227    1.315     .068     (    52)    24.230    1.027    -.005
(   53)    33.334     .861     .029     (    54)    32.399    4.125    -.127
(   55)    32.631     .392     .080     (    56)    24.574     .728    -.004
(   57)    24.967     .264     .012     (    58)    22.102     .124     .013
(   59)    15.118     .242    -.001     (    60)    10.533     .210    -.007
(   61)     6.497     .135    -.010     (    62)    -2.115     .112    -.021
(   63)    -7.145    -.035    -.036     (    64)   -11.363    -.080    -.032
(   65)   -14.893    -.154    -.042     (    66)   -19.476     .055    -.003
(   67)   -17.449    -.297    -.047     (    68)   -25.317    -.221    -.042
(   69)   -45.525    -.200    -.054     (    70)   -47.931    -.368     .029
(   71)   -26.736    -.350     .002     (    72)    -8.045    -.213    -.022
(   73)   -20.519    2.071    -.003     (    74)   -25.869    -.351     .000

N E C K I N G   S T R E S S   O F   P L A T E
(    1)      34.810      32.460      32.902     -11.732
(    2)      33.457      32.933       5.845     -13.553
(    3)      33.000      33.000      -3.300       -.873
(    4)      56.200      56.200      -5.620      -1.486
(    5)      56.200      56.200      -5.620      -1.486
(    6)      56.736      54.099      35.243     -13.463
(    7)      34.498      32.607      26.657      -8.727
(    8)      34.784      32.455      32.373     -11.510
(    9)      58.206      55.624      34.496     -13.028
(   10)      58.168      55.635      33.742     -12.820
(   11)      34.449      32.621      25.677      -8.472
(   12)      34.708      32.488      30.866     -10.862
(   13)      56.299      53.846      32.550     -12.553
(   14)      34.367      32.640      24.034      -8.102
(   15)      34.613      32.530      28.960     -10.021
(   16)      55.978      53.689      30.149     -11.734
(   17)      34.311      32.657      22.921      -7.755
(   18)      54.486      53.407      12.365      -5.663
(   19)      34.222      32.658      21.137      -7.771
(   20)      34.466      32.613      26.025      -8.353
(   21)      57.856      55.870      27.497      -8.227
(   22)      54.552      52.688      23.728      -9.581
```

```
(    23)      34.133     32.694     19.360     -6.884
(    24)      34.183     32.694     20.360     -6.886
(    25)      56.283     58.695     20.856     -2.186
(    26)      56.234     56.295     -4.856     -2.929
(    28)      53.854     52.869     -5.855     -1.989
(    28)      58.468     58.888      5.859     -3.066
(    29)      56.068     56.006     -5.289     -1.856
(    30)      56.000     56.000     -5.680     -1.586
(    30)      56.000     56.000     -5.600     -1.589
(    32)      56.000     56.000     -5.600     -1.589
(    32)      56.200     56.200     -5.620     -1.586
(    34)      56.200     56.200     -5.620     -1.588
(    35)      56.200     56.200     -5.620     -1.552
(    36)      56.200     56.200     -5.620     -1.551
(    38)      56.200     56.200     -5.620     -1.586
(    38)      56.200     56.200     -5.620     -1.486
(    39)      56.200     56.200     -5.620     -1.488
(    40)      54.588     52.609     25.860     -8.488
(    40)      34.588     32.659     32.863    -18.488
(    42)      56.280     58.289     38.620    -11.508
(    43)      56.200     56.200     -5.620     -1.502
(    44)      56.800     58.809     26.620     -8.502
(    45)      34.508     32.688     26.866    -10.864
(    46)      54.708     52.488     38.866    -10.860
(    48)      54.400     52.600     25.480     -7.960
(    48)      34.622     32.631     28.969     -9.964
(    49)      54.800     52.800     28.960     -9.998
(    50)      54.208     52.860     25.590     -7.608
(    50)      54.886     53.688     23.385     -5.688
(    52)      54.286     53.668     22.389     -5.878
(    53)      34.266     32.668     28.029     -8.588
(    54)      54.856     58.602     26.095     -8.588
(    55)      53.896     58.804     22.582     -6.958
(    56)      58.998     52.691     19.689     -6.945
(    58)      34.188     32.698     20.659     -6.966
(    58)      56.182     58.658     28.023     -5.266
(    59)      56.780     56.068      5.088     -5.298
(    60)      58.480     52.868      5.868     -4.378
(    60)      58.455     58.830      5.845     -3.984
(    62)      56.088     56.000     -5.500     -2.982
(    63)      56.000     56.000     -5.600     -1.588
(    64)      56.000     56.000     -5.600     -1.589
(    65)      56.000     56.000     -5.600     -1.589
(    66)      56.200     56.200     -5.620     -1.586
(    68)      56.200     56.200     -5.620     -1.588
(    68)      56.200     56.200     -5.620     -1.552
(    69)      56.200     56.200     -5.620     -1.551
(    69)      56.200     56.200     -5.620     -1.586
(    70)      56.200     56.200     -5.620     -1.486
(    72)      58.000     58.000     -5.600     -1.888
(    72)      33.000     33.000     -3.300      -.873
(    73)      56.200     56.200     -5.620     -1.486
(    74)      56.200     56.200     -5.620     -1.486

AXIAL   STRESS   OF   BEAM-COLUMN
(    1)    33.185     (     2)    32.158  (     3)    19.551
(    4)     4.368     (     5)    -9.450  (     6)   -22.512
```

```
(    7)   -32.510      (    8)   -27.603  (     9)   -41.566
(   10)   -46.051      (   11)   -46.315  (    12)    33.185
(   13)    32.158      (   14)    19.551  (    15)     4.368
(   16)    -9.450      (   17)   -22.392  (    18)   -32.474
(   19)   -27.631      (   20)   -41.466  (    21)   -46.030
(   22)   -46.315      (   23)   -47.370  (    24)   -28.603
(   25)    -9.809      (

A V E R A G E   S T R A I N   O F   P L A T E
(    1)    .172E-02 -.512E-03 -.317E-03
(    2)    .434E-03 -.634E-04  .215E-05
(    3)   -.472E-03 -.439E-04  .120E-05
(    4)   -.138E-02 -.845E-04  .139E-05
(    5)   -.228E-02 -.658E-04 -.383E-05
(    6)    .193E-02 -.570E-03 -.128E-04
(    7)    .148E-02 -.390E-03  .105E-04
(    8)    .169E-02 -.516E-03 -.202E-04
(    9)    .190E-02 -.546E-03 -.306E-05
(   10)    .187E-02 -.536E-03 -.788E-05
(   11)    .143E-02 -.376E-03  .649E-05
(   12)    .162E-02 -.486E-03 -.562E-05
(   13)    .180E-02 -.525E-03 -.430E-05
(   14)    .135E-02 -.357E-03  .247E-05
(   15)    .153E-02 -.446E-03 -.505E-05
(   16)    .169E-02 -.485E-03 -.176E-06
(   17)    .128E-02 -.335E-03  .311E-05
(   18)    .840E-03 -.183E-03 -.607E-05
(   19)    .121E-02 -.337E-03 -.379E-06
(   20)    .139E-02 -.367E-03 -.365E-05
(   21)    .157E-02 -.313E-03  .141E-04
(   22)    .138E-02 -.391E-03 -.930E-05
(   23)    .109E-02 -.295E-03  .529E-06
(   24)    .112E-02 -.309E-03 -.149E-05
(   25)    .108E-02 -.153E-03 -.179E-05
(   26)    .680E-03 -.140E-03 -.275E-06
(   27)    .434E-03 -.125E-03  .289E-05
(   28)    .320E-03 -.849E-04  .690E-06
(   29)   -.133E-03  .248E-04  .165E-05
(   30)   -.472E-03 -.226E-04  .323E-05
(   31)   -.812E-03 -.366E-04  .291E-05
(   32)   -.126E-02 -.119E-04  .406E-05
(   33)   -.138E-02 -.270E-05  .287E-05
(   34)   -.149E-02 -.249E-03  .458E-05
(   35)   -.182E-02 -.200E-03  .473E-05
(   36)   -.216E-02  .641E-03  .599E-05
(   37)   -.227E-02  .670E-03 -.306E-05
(   38)   -.228E-02  .134E-03 -.261E-05
(   39)    .193E-02 -.523E-03  .118E-04
(   40)    .148E-02 -.361E-03 -.106E-04
(   41)    .169E-02 -.513E-03  .212E-04
(   42)    .190E-02 -.508E-03 -.313E-05
(   43)    .187E-02 -.492E-03  .102E-04
(   44)    .143E-02 -.352E-03 -.656E-05
(   45)    .162E-02 -.486E-03  .597E-05
(   46)    .180E-02 -.471E-03  .304E-05
(   47)    .135E-02 -.335E-03 -.245E-05
(   48)    .153E-02 -.444E-03  .542E-05
```

```
(   49)    .169E-02 -.423E-03 -.107E-05
(   50)    .128E-02 -.321E-03 -.184E-05
(   51)    .840E-03 -.192E-03  .840E-05
(   52)    .121E-02 -.315E-03 -.580E-06
(   53)    .139E-02 -.378E-03  .362E-05
(   54)    .157E-02 -.281E-03 -.157E-04
(   55)    .138E-02 -.373E-03  .990E-05
(   56)    .109E-02 -.293E-03 -.536E-06
(   57)    .112E-02 -.324E-03  .151E-05
(   58)    .108E-02 -.212E-03  .158E-05
(   59)    .680E-03 -.169E-03 -.899E-07
(   60)    .434E-03 -.120E-03 -.919E-06
(   61)    .320E-03 -.841E-04 -.129E-05
(   62)   -.133E-03  .248E-04 -.255E-05
(   63)   -.472E-03 -.109E-04 -.442E-05
(   64)   -.812E-03 -.302E-04 -.394E-05
(   65)   -.126E-02  .111E-04 -.522E-05
(   66)   -.138E-02 -.715E-05 -.368E-06
(   67)   -.149E-02 -.312E-03 -.579E-05
(   68)   -.182E-02 -.253E-03 -.551E-05
(   69)   -.216E-02  .638E-03 -.671E-05
(   70)   -.227E-02  .664E-03  .360E-05
(   71)   -.228E-02  .730E-04 -.111E-05
(   72)   -.472E-03 -.278E-04 -.274E-05
(   73)   -.138E-02 -.708E-04 -.972E-06
(   74)   -.228E-02 -.676E-04 -.403E-05

A X I A L   S T R A I N   O F   B E A M - C O L U M N
(    1)    .189E-02  (    2)    .152E-02  (    3)    .927E-03
(    4)    .207E-03  (    5)   -.472E-03  (    6)   -.115E-02
(    7)   -.160E-02  (    8)   -.138E-02  (    9)   -.204E-02
(   10)   -.227E-02  (   11)   -.228E-02  (   12)    .189E-02
(   13)    .152E-02  (   14)    .927E-03  (   15)    .207E-03
(   16)   -.472E-03  (   17)   -.115E-02  (   18)   -.160E-02
(   19)   -.138E-02  (   20)   -.204E-02  (   21)   -.227E-02
(   22)   -.228E-02  (   23)   -.228E-02  (   24)   -.138E-02
(   25)   -.472E-03  (
```

A1

CALREL

```
************************************************************
*      U n i v e r s i t y   o f   C a l i f o r n i a     *
*            Department  of  Civil  Engineering            *
*                                                          *
*                       C  A  L  R  E  L                   *
*                   CAL-RELiability program                *
*                         Developed by                     *
*          P.-L. Liu, H.-Z. Lin and A. Der Kiureghian      *
*                                                          *
*               Last Revision:   January 1990              *
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************************************************************

>>>> NEW PROBLEM <<<<

number of limit-state functions..........ngf=       1
number of independent variable groups ...nig=       1
total number of random variables ........nrx=       6
number of limit-state parameters ........ntp=       0

>>>> INPUT DATA <<<<

Ship Reliability Project
Cruiser 1
Primary Mode -- Ultimate Strength
Sagging Condition, Short-Term (CR1_PYSS)
type of system .........................icl=       1
  icl=1 ..................................component
  icl=2 ..............................series system
  icl=3 .............................general system
flag for gradient computation ..........igr=       0
  igr=0 ..........................finite difference
  igr=1 ...................formulas provided by user

optimization scheme used ...............iop=       1
  iop=1 ..............................HL-RF method
  iop=2 .....................modified HL-RF method
  iop=3 ..................gradient projection method
  iop=4 .................sequential quadratic method
maximum number of iteration cycles ......ni1=     100
maximum steps in line search ...........ni2=       4
convergence tolerance ................tol= 1.000E-03
optimization parameter 1 ............op1= 1.000E+00
optimization parameter 2 ............op2= 0.000E+00
optimization parameter 3 ............op3= 0.000E+00

statistical data of basic varibles:
available probability distributions:
  determinitic .............ids=0
  normal ...................ids=1
  lognormal ................ids=2
  gamma ....................ids=3
  shifted exponential ......ids=4
  shifted rayleigh .........ids=5
  uniform ..................ids=6
  beta .....................ids=7
  type i largest value .....ids=11
  type i smallest value ....ids=12
  type ii largest value ....ids=13
  weibull ..................ids=14
  user defined .............ids>50
```

```
group no.:     1          group type:     1
var    ids    mean    st. dev.   param1    param2    param3    param4    init. pt
Mu      2  5.96E+01  5.96E+00  4.08E+00  9.98E-02                        5.96E+01
Ms      1  6.14E+00  9.22E-01  6.14E+00  9.22E-01                        6.14E+00
Mw     51            1.99E+00  1.99E+01  1.99E+00  0.00E+00  0.00E+00  1.99E+01
Md     52            2.39E+00  7.96E+00  2.39E+00  0.00E+00  0.00E+00  7.96E+00
Kw      1  1.00E+00  5.00E-02  1.00E+00  5.00E-02                        1.00E+00
Kd      1  7.00E-01  1.05E-01  7.00E-01  1.05E-01                        7.00E-01

>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval ..........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      0
  ini=0 ......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 ......................analyze a new problem
  ist=1 ............continue an unconverged problem

limit-state function     1
-----------------------------------------------------------------------------
iteration number ..............iter=       10
value of limit-state function..g(x)=-5.507E-07
reliability index ............beta=    6.4746
probability ...................Pf1= 4.752E-11
var          design point                        sensitivity vectors
           x*             u*              alpha      gamma      delta       eta
Mu      4.077E+01   -3.750E+00           -.5791     -.5791      .8024   -2.2182
Ms      5.360E+00   -8.499E-01           -.1313     -.1313      .1313    -.1116
Mw      3.142E+01    4.064E+00            .6277      .6277
Md      1.265E+01    2.250E+00            .3474      .3474
Kw      1.097E+00    1.939E+00            .2994      .2994     -.2994    -.5805
Kd      8.411E-01    1.343E+00            .2075      .2075     -.2075    -.2787
-----------------------------------------------------------------------------

>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used ..................itg=    2
  itg=1 ..........................improved Breitung formula
  itg=2 ..........................improved Breitung formula
        ..........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function     1
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)         u'i    u'n     G(u)           a'i
   1 2.941  6.502 -4.035E-05   -2.935  6.504 -1.018E-05     3.2933E-03
   2 2.979  6.484 -2.421E-06   -2.979  6.485 -3.334E-06     1.1143E-03
   3 2.981  6.483 -7.339E-06   -3.000  6.394  6.982E-09    -4.2385E-03
   4 3.000  6.389  8.723E-10   -3.000  6.329  3.568E-10    -1.3009E-02
   5 2.884  6.527 -9.162E-05   -2.863  6.536 -5.451E-05     6.8979E-03

                                      improved Breitung        Tvedt's EI
generalized reliability index betag =       6.4669               6.4670
probability                     Pf2 =     5.001E-11            4.999E-11
-----------------------------------------------------------------------------
```

A3

```
>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
..........................................isv=      0
  isv=1 ......................distribution parameters
  isv=2 ...................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    1
----------------------------------------------------------------------
d(beta)/d(parameter) :
var      mean      std dev     par 1       par 2       par 3      par 4
Mu    1.347E-01 -3.724E-01  5.806E+00 -2.177E+01
Ms    1.424E-01 -1.210E-01  1.424E-01 -1.210E-01
Mw                         -1.764E-01 -7.033E-01  0.000E+00  0.000E+00
Md                         -1.565E-01 -1.747E-01  0.000E+00  0.000E+00
Kw   -5.988E+00 -1.161E+01 -5.988E+00 -1.161E+01
Kd   -1.976E+00 -2.654E+00 -1.976E+00 -2.654E+00

d(Pf1)/d(parameter) :
var      mean      std dev     par 1       par 2       par 3      par 4
Mu   -4.239E-11  1.172E-10 -1.827E-09  6.851E-09
Ms   -4.481E-11  3.808E-11 -4.481E-11  3.808E-11
Mw                          5.551E-11  2.213E-10  0.000E+00  0.000E+00
Md                          4.924E-11  5.497E-11  0.000E+00  0.000E+00
Kw    1.885E-09  3.653E-09  1.885E-09  3.653E-09
Kd    6.218E-10  8.353E-10  6.218E-10  8.353E-10
----------------------------------------------------------------------
Stop - Program terminated.
```

```
************************************************************
*     U n i v e r s i t y   o f   C a l i f o r n i a     *
*          Department  of  Civil  Engineering               *
*                                                           *
*                    C  A  L  R  E  L                       *
*               CAL-RELiability program                     *
*                    Developed by                           *
*       P.-L. Liu, H.-Z. Lin and A. Der Kiureghian          *
*                                                           *
*          Last Revision:   January 1990                    *
*                  Copyright @ 1990                         *
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*   prohibited by law.                                      *
************************************************************


>>>> NEW PROBLEM <<<<

number of limit-state functions.........ngf=      1
number of independent variable groups ...nig=      1
total number of random variables ........nrx=      5
number of limit-state parameters ........ntp=      1


>>>> INPUT DATA <<<<

***  Cruiser 1 --> secondary mode, short term loading, hogging ***
type of system ..........................icl=      1
  icl=1 ..................................component
  icl=2 ..............................series system
  icl=3 .............................general system
flag for gradient computation ...........igr=      0
  igr=0 ..........................finite difference
  igr=1 ..................formulas provided by user

optimization scheme used ...............iop=      1
  iop=1 ................................HL-RF method
  iop=2 .......................modified HL-RF method
  iop=3 ..................gradient projection method
  iop=4 .................sequential quadratic method
maximum number of iteration cycles ......nil=    100
maximum steps in line search ...........ni2=      4
convergence tolerance ..................tol= 1.000E-03
optimization parameter 1 ...............op1= 1.000E+00
optimization parameter 2 ...............op2= 0.000E+00
optimization parameter 3 ...............op3= 0.000E+00

statistical data of basic varibles:
available probability distributions:
  determinitic .............ids=0
  normal ...................ids=1
  lognormal ................ids=2
  gamma ....................ids=3
  shifted exponential ......ids=4
  shifted rayleigh .........ids=5
  uniform ..................ids=6
```

```
  beta .....................ids=7
  type i largest value .....ids=11
  type i smallest value ....ids=12
  type ii largest value ....ids=13
  weibull ..................ids=14
  user defined .............ids>50

group no.:     1          group type:    1
var   ids    mean     st. dev.   param1     param2     param3     param4    init. pt
su     2  2.33E+01  2.33E+00  3.14E+00  9.98E-02                           0.00E+00
smb    2  2.67E+01  1.07E+00  3.28E+00  4.00E-02                           0.00E+00
ms     1  6.14E+01  9.22E+00  6.14E+01  9.22E+00                           0.00E+00
kw     1  1.00E+00  5.00E-02  1.00E+00  5.00E-02                           0.00E+00
mw    51            1.69E+01  1.69E+02  1.69E+01  0.00E+00  0.00E+00  1.69E+02


>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval ..........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      0
  ini=0 ........................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 ........................analyze a new problem
  ist=1 .............continue an unconverged problem

limit-state function     1
--------------------------------------------------------------------------------
iteration number ..............iter=         7
value of limit-state function..g(x)= 1.699E-07
reliability index .............beta=    6.7292
probability .....................Pf1= 8.532E-12
var           design point                        sensitivity vectors
              x*           u*            alpha      gamma      delta       eta
su       1.501E+01    -4.337E+00        -.6444     -.6444      .9306   -2.8447
smb      2.492E+01    -1.738E+00        -.2582     -.2582      .2767    -.4586
ms       7.131E+01     1.071E+00         .1591      .1591     -.1591    -.1704
kw       1.081E+00     1.627E+00         .2417      .2417     -.2417    -.3932
mw       2.800E+02     4.434E+00         .6590      .6590
--------------------------------------------------------------------------------


>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.........................................isv=      0
  isv=1 .......................distribution parameters
  isv=2 ....................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    1
------------------------------------------------------------------------
d(beta)/d(parameter) :
var       mean      std dev      par 1       par 2       par 3       par 4
su    4.001E-01 -1.223E+00  6.459E+00 -2.801E+01
smb   2.588E-01 -4.290E-01  6.459E+00 -1.122E+01
ms   -1.727E-02 -1.849E-02 -1.727E-02 -1.849E-02
kw   -4.835E+00 -7.865E+00 -4.835E+00 -7.865E+00
```

```
mw                         -2.157E-02 -9.345E-02  0.000E+00  0.000E+00

d(Pf1)/d(parameter) :
var     mean      std dev     par 1      par 2      par 3      par 4
su   -2.346E-11  7.171E-11 -3.787E-10  1.642E-09
smb  -1.517E-11  2.516E-11 -3.787E-10  6.581E-10
ms     1.012E-12  1.084E-12  1.012E-12  1.084E-12
kw     2.835E-10  4.611E-10  2.835E-10  4.611E-10
mw                          1.265E-12  5.479E-12  0.000E+00  0.000E+00
----------------------------------------------------------------------

sensitivity with respect to limit-state function parameters

limit-state function    1
----------------------------------------------------------------------
par   d(beta)/d(parameter)      d(Pf1)/d(parameter)
 1          0.000E+00                0.000E+00
----------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

type of integration scheme used ...................itg=    2
  itg=1 ...........................improved Breitung formula
  itg=2 ...........................improved Breitung formula
        ...........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    1
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)        u'i    u'n    G(u)           a'i
   1 2.885  6.779 -1.078E-04    -3.000  6.714  4.028E-11     2.0369E-03
   2 2.993  6.732 -5.892E-08    -2.996  6.731 -5.143E-08     2.5304E-04
   3 2.996  6.731 -3.290E-08    -2.996  6.731 -3.844E-08     1.8807E-04
   4 2.906  6.770 -2.677E-05    -2.890  6.777 -1.399E-05     5.2853E-03

                                      improved Breitung      Tvedt's EI
generalized reliability index betag =        6.7367             6.7367
probability                    Pf2 =     8.101E-12          8.100E-12
-----------------------------------------------------------------------------
Stop - Program terminated.
```

```
*************************************************************
*      U n i v e r s i t y   o f   C a l i f o r n i a      *
*           Department  of  Civil  Engineering              *
*                                                           *
*                   C  A  L  R  E  L                        *
*                CAL-RELiability program                    *
*                      Developed by                         *
*        P.-L. Liu, H.-Z. Lin and A. Der Kiureghian         *
*                                                           *
*          Last Revision:   January 1990                    *
*                 Copyright @ 1990                          *
*************************************************************


*************************************************************
*    This version of CALREL is for the exclusive use of     *
*    students and faculty at the University of California   *
*    at Berkeley, California, USA. Unauthorized use is      *
*    prohibited by law.                                     *
*************************************************************


>>>> NEW PROBLEM <<<<

number of limit-state functions..........ngf=      1
number of independent variable groups ...nig=      1
total number of random variables ........nrx=      7
number of limit-state parameters ........ntp=      1

>>>> INPUT DATA <<<<

***  Cruiser 1 --> secondary mode, short term loading, sagging ***
type of system ..........................icl=      1
   icl=1 .................................component
   icl=2 ...............................series system
   icl=3 ..............................general system
flag for gradient computation ...........igr=      0
   igr=0 ...........................finite difference
   igr=1 ...................formulas provided by user

optimization scheme used ................iop=      1
   iop=1 ................................HL-RF method
   iop=2 .......................modified HL-RF method
   iop=3 ..................gradient projection method
   iop=4 ..................sequential quadratic method
maximum number of iteration cycles ......ni1=    100
maximum steps in line search ............ni2=      4
convergence tolerance ................tol= 1.000E-03
optimization parameter 1 .............op1= 1.000E+00
optimization parameter 2 .............op2= 0.000E+00
optimization parameter 3 .............op3= 0.000E+00

statistical data of basic varibles:
available probability distributions:
   determinitic .............ids=0
   normal ...................ids=1
   lognormal ................ids=2
   gamma ....................ids=3
   shifted exponential ......ids=4
   shifted rayleigh .........ids=5
   uniform ..................ids=6
```

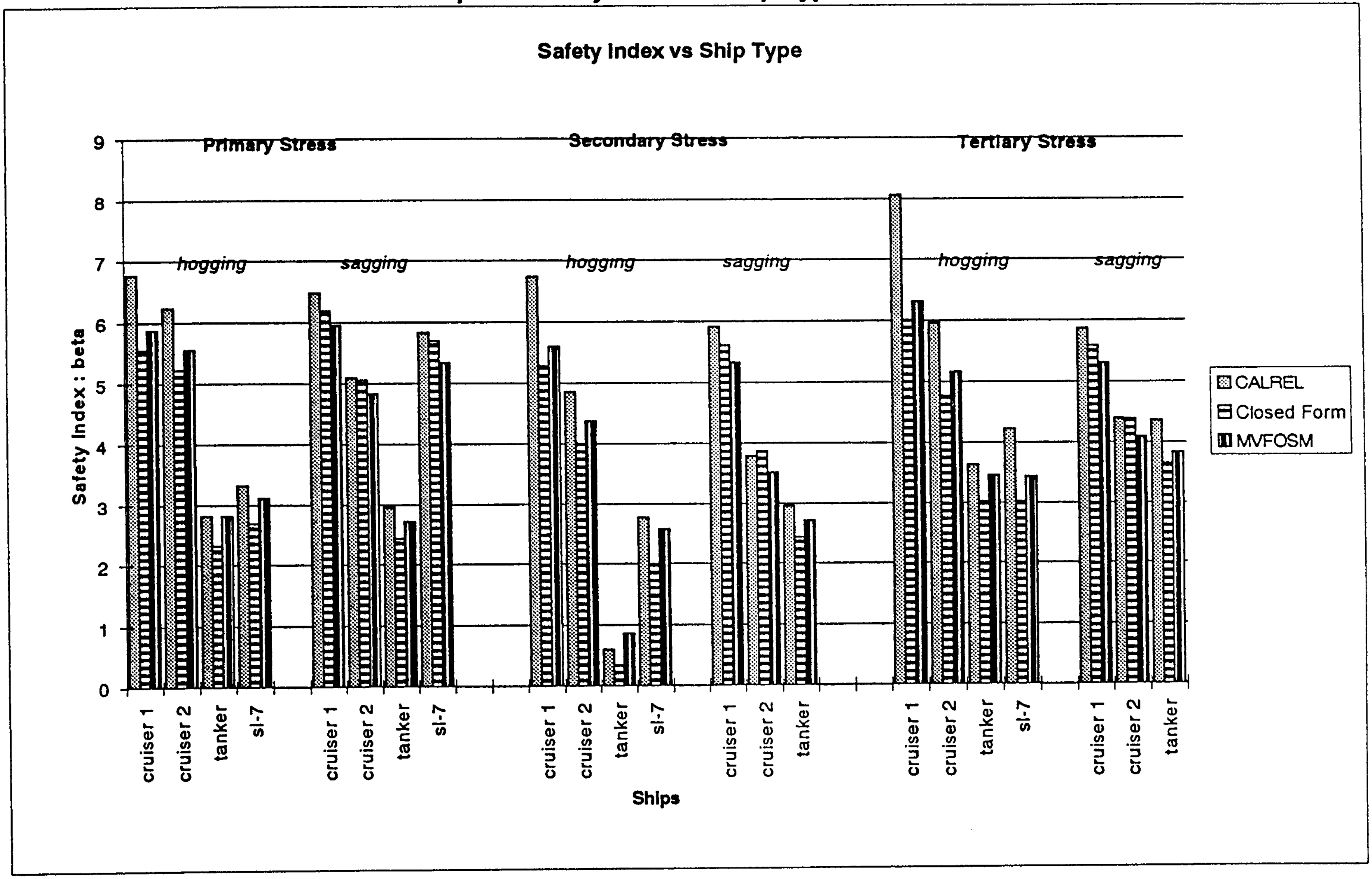
```
   beta .....................ids=7
   type i largest value .....ids=11
   type i smallest value ....ids=12
   type ii largest value ....ids=13
   weibull ..................ids=14
   user defined .............ids>50

group no.:     1         group type:    1
var    ids    mean     st. dev.    param1     param2     param3     param4    init. pt
su      2  2.37E+01  2.37E+00  3.16E+00  9.98E-02                          0.00E+00
smd     2  2.34E+01  9.35E-01  3.15E+00  4.00E-02                          0.00E+00
ms      1  6.14E+01  9.22E+00  6.14E+01  9.22E+00                          0.00E+00
kw      1  1.00E+00  5.00E-02  1.00E+00  5.00E-02                          0.00E+00
mw     51            1.99E+01  1.99E+02  1.99E+01  0.00E+00  0.00E+00  1.99E+02
kd      1  7.00E-01  1.05E-01  7.00E-01  1.05E-01                          0.00E+00
md     52            2.39E+01  7.96E+01  2.39E+01  0.00E+00  0.00E+00  7.96E+01


>>>> FIRST-ORDER RELIABILITY ANALYSIS <<<<

print interval ..........................npr=      0
  npr<0 ..........no first order results are printed
  npr=0 ........print the final step of FORM results
  npr>0 ........print the results of every npr steps
initialization flag .....................ini=      0
  ini=0 .......................start from mean point
  ini=1 ..........start from point specified by user
  ini=-1 ....start from previous linearization point
restart flag ............................ist=      0
  ist=0 .......................analyze a new problem
  ist=1 .............continue an unconverged problem

limit-state function     1
---------------------------------------------------------------------------------
iteration number ..............iter=          9
value of limit-state function..g(x)=-9.490E-07
reliability index .............beta=     5.9050
probability ....................Pf1= 1.763E-09
var          design point                         sensitivity vectors
             x*             u*            alpha     gamma     delta       eta
su       1.701E+01     -3.281E+00        -.5556    -.5556     .7439    -1.8689
smd      2.217E+01     -1.315E+00        -.2226    -.2226     .2347     -.3013
ms       5.403E+01     -8.036E-01        -.1361    -.1361     .1361     -.1094
kw       1.087E+00      1.730E+00         .2930     .2930    -.2930     -.5070
mw       2.977E+02      3.644E+00         .6173     .6173    -.3520    -1.2579
kd       8.256E-01      1.196E+00         .2024     .2024    -.2024     -.2421
md       1.202E+02      2.007E+00         .3397     .3397    -.3671     -.3702
---------------------------------------------------------------------------------


>>>> SENSITIVITY ANALYSIS AT COMPONENT LEVEL <<<<

type of parameters for sensitivity analysis
.........................................isv=      0
  isv=1 .......................distribution parameters
  isv=2 ...................limit-state fcn parameters
  isv=0 ..distribution and limit-state fcn parameters

sensitivity with respect to distribution parameters

limit-state function    1
-------------------------------------------------------------------------------
d(beta)/d(parameter) :
var      mean       std dev      par 1       par 2       par 3       par 4
```

```
su     3.136E-01 -7.879E-01  5.570E+00 -1.827E+01
smd    2.511E-01 -3.222E-01  5.570E+00 -7.321E+00
ms     1.477E-02 -1.187E-02  1.477E-02 -1.187E-02
kw    -5.860E+00 -1.014E+01 -5.860E+00 -1.014E+01
mw                          -1.769E-02 -6.321E-02  0.000E+00  0.000E+00
kd    -1.928E+00 -2.305E+00 -1.928E+00 -2.305E+00
md                          -1.536E-02 -1.549E-02  0.000E+00  0.000E+00

d(Pf1)/d(parameter) :
var      mean       std dev     par 1      par 2       par 3      par 4
su    -3.354E-09  8.426E-09 -5.956E-08  1.954E-07
smd   -2.685E-09  3.446E-09 -5.956E-08  7.830E-08
ms    -1.579E-10  1.269E-10 -1.579E-10  1.269E-10
kw     6.268E-08  1.084E-07  6.268E-08  1.084E-07
mw                           1.892E-10  6.760E-10  0.000E+00  0.000E+00
kd     2.062E-08  2.466E-08  2.062E-08  2.466E-08
md                           1.642E-10  1.657E-10  0.000E+00  0.000E+00
----------------------------------------------------------------------

sensitivity with respect to limit-state function parameters

limit-state function    1
----------------------------------------------------------------------
par   d(beta)/d(parameter)      d(Pf1)/d(parameter)
 1          0.000E+00                0.000E+00
----------------------------------------------------------------------


>>>> SECOND-ORDER RELIABILITY ANALYSIS -- POINT FITTING <<<<

:ype of integration scheme used ...................itg=    2
  itg=1 ...........................improved Breitung formula
  itg=2 ...........................improved Breitung formula
        ...........................& Tvedt's exact integral
max. number of iterations for each fitting point ..inp=    4

limit-state function    1
-----------------------------------------------------------------------------
coordinates and  ave. main curvatures of fitting points in rotated space
axis  u'i    u'n     G(u)        u'i    u'n     G(u)          a'i
   1 3.000  5.867  3.126E-10   -3.000  5.825  1.764E-09    -6.5869E-03
   2 3.000  5.902  3.061E-12   -3.000  5.902  5.765E-12    -3.3805E-04
   3 2.988  5.911 -2.882E-07   -2.987  5.911 -3.200E-07     7.1046E-04
   4 2.916  5.947 -2.115E-05   -2.900  5.955 -7.200E-06     5.4309E-03
   5 3.000  5.863 -1.980E-08   -3.000  5.717  2.424E-08    -1.3451E-02
   6 2.950  5.930 -5.424E-06   -2.944  5.933 -3.997E-06     3.0581E-03

                                          improved Breitung      Tvedt's EI
generalized reliability index betag =         5.8921              5.8922
probability                     Pf2 =      1.907E-09           1.906E-09
-----------------------------------------------------------------------------
Stop - Program terminated.
```

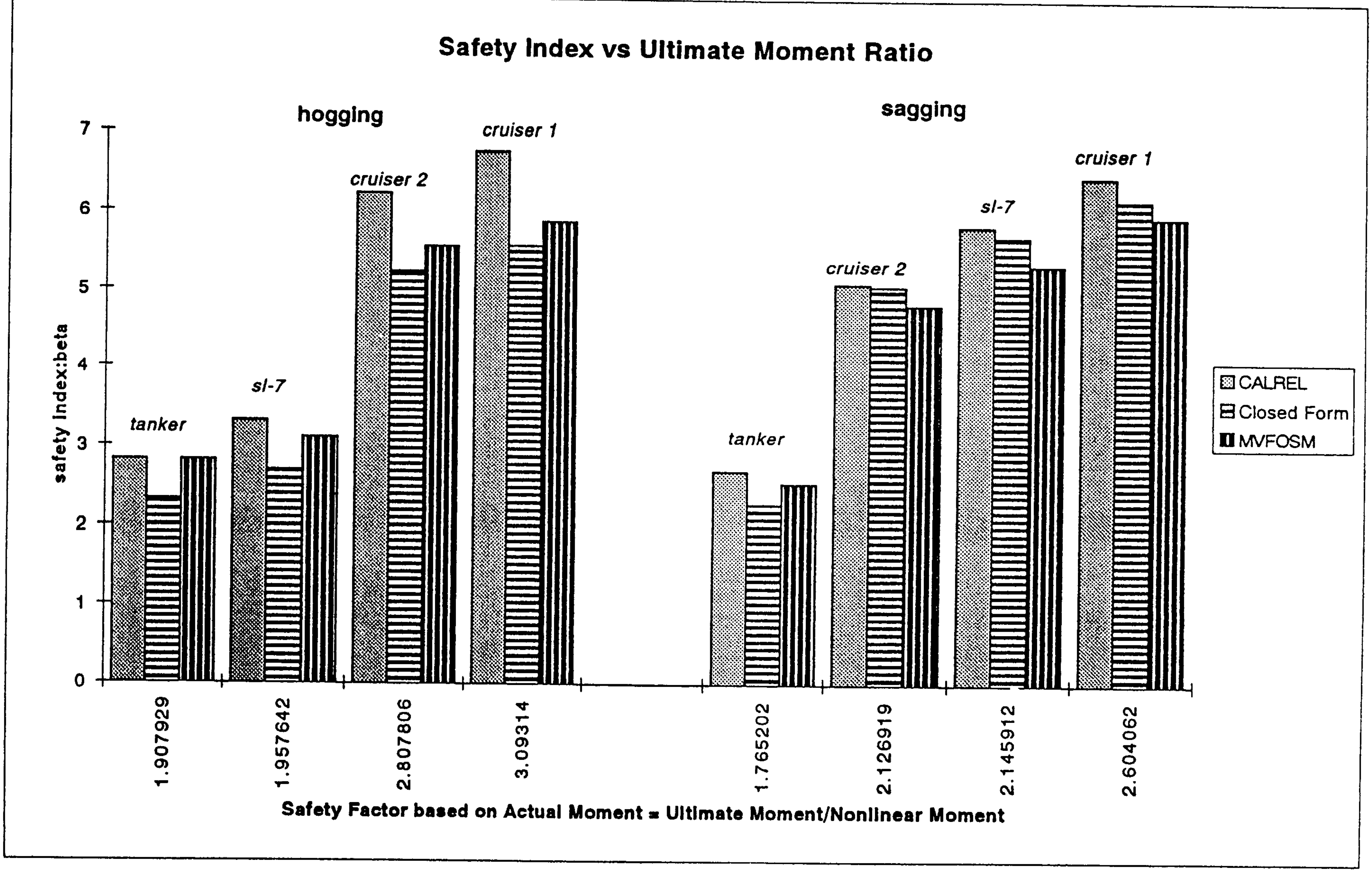
APPENDIX G

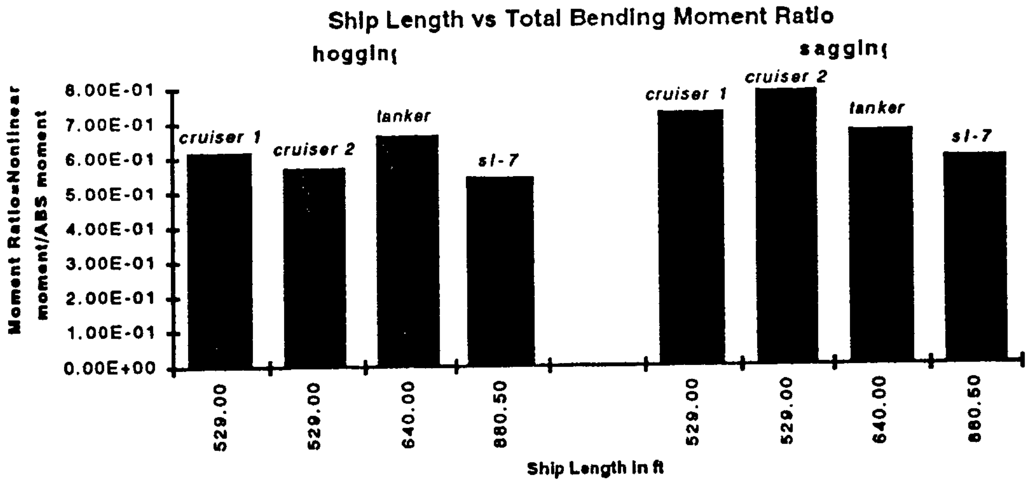
PARAMETRIC STUDY AND COMPARISON OF RELIABILITY INDICES

Graph 1 : Safety Index vs Ship Type



Graph 5 : Safety Index vs Ultimate Moment Ratio

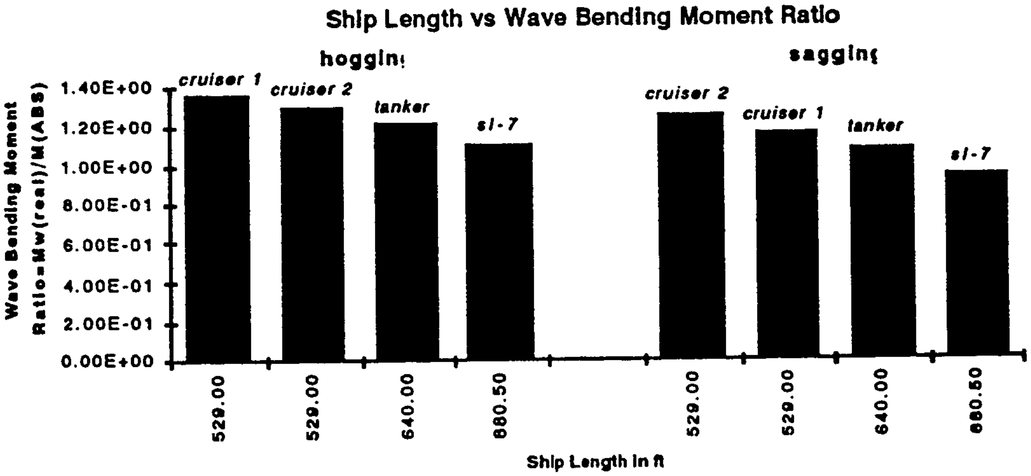




2. Ship Length(ft) vs Wave Bending Moment Ratio

Table 6.1.5II Ship Length(ft) vs Wave Bending Moment Ratio

Ship Length vs Wave Bending Moment Ratio				
		Ships	**Length**	**Ratio(wave)**
Primary	**hogging**	*cruiser 1*	529.00	1.37E+00
		cruiser 2	529.00	1.30E+00
		tanker	640.00	1.21E+00
		sl-7	880.50	1.10E+00
	sagging	*cruiser 2*	529.00	1.26E+00
		cruiser 1	529.00	1.15E+00
		tanker	640.00	1.08E+00
		sl-7	880.50	9.42E-01
ps : Ratio(wave)=Wave Bending Moment(real)/Wave Bending Moment(ABS)				



(IV) Factor of Safety vs Safety Index (see Graph 4,5)

Table 6.1.4 Factor of Safety vs Safety Index

	Factor of Safety vs Safety Index					
Primary Stress						
deck						
Ships	**Initial yield moment**	**ABS moment**	**SFI**	**beta 1**	**beta 2**	**beta 3**
sl-7	3.00E+06	1.80E+06	1.67E+00	3.32E+00	2.69E+00	3.11E+00
tanker	1.51E+06	8.80E+05	1.71E+00	2.82E+00	2.32E+00	2.82E+00
cruiser 2	5.24E+05	2.71E+05	1.93E+00	6.23E+00	5.23E+00	5.56E+00
cruiser 1	8.34E+05	2.74E+05	3.04E+00	6.76E+00	5.56E+00	5.87E+00
bottom						
Ships	**Initial yield moment**	**ABS moment**	**SFI**	**beta 1**	**beta 2**	**beta 3**
sl-7	2.38E+06	1.80E+06	1.33E+00	5.83E+00	5.70E+00	5.34E+00
tanker	1.58E+06	8.79E+05	1.80E+00	2.70E+00	2.29E+00	2.55E+00
cruiser 2	5.78E+05	2.71E+05	2.13E+00	5.10E+00	5.07E+00	4.83E+00
cruiser 1	9.12E+05	2.74E+05	3.32E+00	6.47E+00	6.18E+00	5.95E+00
method1 : CALREL structural program						
method2 : Closed Form (by approximation)						
method3 : Mean Value First Order Second Moment						
hogging						
Ships	**Ultimate moment**	**Nonlinear moment**	**SFu**	**beta 1**	**beta 2**	**beta 3**
tanker	1.12E+06	5.86E+05	1.91E+00	2.82E+00	2.32E+00	2.82E+00
sl-7	1.90E+06	9.70E+05	1.96E+00	3.32E+00	2.69E+00	3.11E+00
cruiser 2	4.38E+05	1.56E+05	2.81E+00	6.23E+00	5.23E+00	5.56E+00
cruiser 1	5.23E+05	1.69E+05	3.09E+00	6.76E+00	5.56E+00	5.87E+00
sagging						
Ships	**Ultimate moment**	**Nonlinear moment**	**SFu**	**beta 1**	**beta 2**	**beta 3**
tanker	1.05E+06	5.95E+05	1.77E+00	2.70E+00	2.29E+00	2.55E+00
cruiser 2	4.55E+05	2.14E+05	2.13E+00	5.10E+00	5.07E+00	4.83E+00
sl-7	2.29E+06	1.07E+06	2.15E+00	5.83E+00	5.70E+00	5.34E+00
cruiser 1	5.18E+05	1.99E+05	2.60E+00	6.47E+00	6.18E+00	5.95E+00
method1 : CALREL structural program						
method2 : Closed Form (by approximation)						
method3 : Mean Value First Order Second Moment						

(V) Ship Length vs Moment Ratio

1. Ship Length(ft) vs Total Bending Moment Ratio

Table 6.1.5I Ship Length(ft) vs Total Bending Moment Ratio

Ship Length vs Total Bending Moment Ratio						
		Ships	**Length**	**Nonlinear Moment**	**ABS Moment**	**Ratio(total)**
Primary	**hogging**	*cruiser 1*	529.00	1.69E+05	2.74E+05	6.16E-01
		cruiser 2	529.00	1.56E+05	2.71E+05	5.75E-01
		tanker	640.00	5.86E+05	8.80E+05	6.66E-01
		sl-7	880.50	9.70E+05	1.80E+06	5.39E-01
	sagging	*cruiser 1*	529.00	1.99E+05	2.74E+05	7.25E-01
		cruiser 2	529.00	2.14E+05	2.71E+05	7.89E-01
		tanker	640.00	5.95E+05	8.79E+05	6.77E-01
		sl-7	880.50	1.07E+06	1.80E+06	5.92E-01
ps : Ratio(total)=Nonlinear Moment/ABS Moment						

Appendix 5. Results of MVFOSM

Primary Stress

Ship	**: Cruiser1**		
Condition	**: short term , hogging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e-ew-ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw-n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	1.69100E+05	*mean of wave bending moment*	*
fd(mean)	8.76400E+04	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	6.01508E+04	*standard deviation of response to load component u*	*
sigma s	9.21600E+03	*standard deviation of response to load component s*	*
sigma w	1.69100E+04	*standard deviation of response to load component w*	*
sigma d	2.02920E+04	*standard deviation of response to load component d*	*
K-Kd	0	*load combination factor for two correlated load response*	*
r	0.833333333	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0	*correlation coefficient between w and d*	
sigma g	6.31585E+04	*s.t.d. of limit-state function*	**
mean			
mu u	6.01508E+05	*mean of load component u*	*
mu s	8.14400E+04	*mean of load component s*	*
mu w	1.69100E+05	*mean of load component w*	*
mu d	8.76400E+04	*mean of load component d*	*
mu g	3.70968E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*5.87360E+00*	*safety index*	
Pf	*2.13920E-09*	*probability of failure*	
*ps : " * " means the value is from the input varibles table " inputvars "*			
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*			

Ship	**: Cruiser1**		
Condition	**: short term , sagging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	1.98900E+05	*mean of wave bending moment*	*
fd(mean)	7.95600E+04	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	5.95840E+04	*standard deviation of response to load component u*	*
sigma s	9.21600E+03	*standard deviation of response to load component s*	*
sigma w	1.98900E+04	*standard deviation of response to load component w*	*
sigma d	2.38680E+04	*standard deviation of response to load component d*	*
K=Kd	0.7	*load combination factor for two correlated load response*	
r	1.2	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0.394	*correlation coefficient between w and d*	
sigma g	8.75975E+04	*s.t.d. of limit-state function*	**
mean			
mu u	5.95840E+05	*mean of load component u*	*
mu s	-8.14400E+04	*mean of load component s*	*
mu w	1.98900E+05	*mean of load component w*	*
mu d	7.95600E+04	*mean of load component d*	*
mu g	4.02488E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	***5.95418E+00***	*safety index*	
Pf	***1.31139E-09***	*probability of failure*	
*ps : " * " means the value is from the input varibles table " inputvars "*			
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*			

Ship	**: Cruiser2**		
Condition	**: short term , hogging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	1.55900E+05	*mean of wave bending moment*	*
fd(mean)	6.23600E+04	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	5.03398E+04	*standard deviation of response to load component u*	*
sigma s	7.74000E+03	*standard deviation of response to load component s*	*
sigma w	1.55900E+04	*standard deviation of response to load component w*	*
sigma d	1.87080E+04	*standard deviation of response to load component d*	*
K=Kd	0	*load combination factor for two correlated load response*	*
r	0.833333333	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0	*correlation coefficient between w and d*	
sigma g	5.32839E+04	*s.t.d. of limit-state function*	**
mean			
mu u	5.03398E+05	*mean of load component u*	*
mu s	5.16000E+04	*mean of load component s*	*
mu w	1.55900E+05	*mean of load component w*	*
mu d	6.23600E+04	*mean of load component d*	*
mu g	2.95898E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*5.55531E+00*	*safety index*	
Pf	*1.38919E-08*	*probability of failure*	

*ps : " * " means the value is from the input varibles table " inputvars "*
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*

Ship	**: Cruiser2**		
Condition	**: short term , sagging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	2.13900E+05	*mean of wave bending moment*	*
fd(mean)	8.55600E+04	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	5.23190E+04	*standard deviation of response to load component u*	*
sigma s	7.74000E+03	*standard deviation of response to load component s*	*
sigma w	2.13900E+04	*standard deviation of response to load component w*	*
sigma d	2.56680E+04	*standard deviation of response to load component d*	*
K=Kd	0.7	*load combination factor for two correlated load response*	
r	1.2	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0.394	*correlation coefficient between w and d*	
sigma g	6.22929E+04	*s.t.d. of limit-state function*	**
mean			
mu u	5.23190E+05	*mean of load component u*	*
mu s	-5.16000E+04	*mean of load component s*	*
mu w	2.13900E+05	*mean of load component w*	*
mu d	8.55600E+04	*mean of load component d*	*
mu g	3.00998E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*4.83198E+00*	*safety index*	
Pf	*6.76800E-07*	*probability of failure*	

*ps : " * " means the value is from the input varibles table " inputvars "*
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*

Ship	: sl-7		
Condition	: short term , hogging , primary stress		
Method	: mean value first order second moment		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	9.69600E+05	*mean of wave bending moment*	*
fd(mean)	1.93920E+05	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	2.40113E+05	*standard deviation of response to load component u*	*
sigma s	8.98650E+04	*standard deviation of response to load component s*	*
sigma w	9.69600E+04	*standard deviation of response to load component w*	*
sigma d	5.81760E+04	*standard deviation of response to load component d*	*
K=Kd	0	*load combination factor for two correlated load response*	
r	0.6	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0	*correlation coefficient between w and d*	
sigma g	2.74101E+05	*s.t.d. of limit-state function*	**
mean			
mu u	2.18285E+06	*mean of load component u*	*
mu s	3.59460E+05	*mean of load component s*	*
mu w	9.69600E+05	*mean of load component w*	*
mu d	1.93920E+05	*mean of load component d*	*
mu g	8.53790E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*3.11487E+00*	*safety index*	
Pf	*9.20196E-04*	*probability of failure*	
*ps : " * " means the value is from the input varibles table " inputvars "*			
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*			

Ship	: sl-7		
Condition	: short term , sagging , primary stress		
Method	: mean value first order second moment		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	1.06500E+06	*mean of wave bending moment*	*
fd(mean)	2.13000E+05	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	2.89103E+05	*standard deviation of response to load component u*	*
sigma s	8.98650E+04	*standard deviation of response to load component s*	*
sigma w	1.06500E+05	*standard deviation of response to load component w*	*
sigma d	8.39000E+04	*standard deviation of response to load component d*	*
K=Kd	0.7	*load combination factor for two correlated load response*	
r	0.6	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0.547	*correlation coefficient between w and d*	
sigma g	3.31980E+05	*s.t.d. of limit-state function*	**
mean			
mu u	2.62821E+06	*mean of load component u*	*
mu s	-3.59460E+05	*mean of load component s*	*
mu w	1.06500E+06	*mean of load component w*	*
mu d	2.13000E+05	*mean of load component d*	*
mu g	1.77357E+06	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*5.34239E+00*	*safety index*	
Pf	*4.59638E-08*	*probability of failure*	
*ps : " * " means the value is from the input varibles table " inputvars "*			
*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*			

Ship	**: Tanker**		
Condition	**: short term , hogging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	5.86100E+05	*mean of wave bending moment*	*
fd(mean)	1.17220E+05	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	1.41457E+05	*standard deviation of response to load component u*	*
sigma s	5.92350E+04	*standard deviation of response to load component s*	*
sigma w	5.86100E+04	*standard deviation of response to load component w*	*
sigma d	3.51660E+04	*standard deviation of response to load component d*	*
K=Kd	0	*load combination factor for two correlated load response*	*
r	0.6	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0	*correlation coefficient between w and d*	
sigma g	1.64177E+05	*s.t.d. of limit-state function*	**
mean			
mu u	1.28597E+06	*mean of load component u*	*
mu s	2.36940E+05	*mean of load component s*	*
mu w	5.86100E+05	*mean of load component w*	*
mu d	1.17220E+05	*mean of load component d*	*
mu g	4.62933E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*2.81972E+00*	*safety index*	
Pf	*2.40333E-03*	*probability of failure*	

*ps : " * " means the value is from the input varibles table " inputvars "*

*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*

Ship	**: Tanker**		
Condition	**: short term , sagging , primary stress**		
Method	**: mean value first order second moment**		
Part 1: Calculation of the s.t.d. and mean of limit-state function			
s.t.d.			
e=ew=ed	0	*bandwidth parameter*	
t	3	*term period (hours)*	
Nw=n	7.71429E+02	*number of peaks associated with load component w*	
Nd	3.85714E+03	*number of peaks associated with load component d*	
fw(mean)	5.94800E+05	*mean of wave bending moment*	*
fd(mean)	1.18960E+05	*mean of dynamic bending moment*	*
alafa w	3.72558E+00	*conversion factor associated with load component w*	
alafa d	4.13493E+00	*conversion factor associated with load component d*	
sigma u	1.32818E+05	*standard deviation of response to load component u*	*
sigma s	3.25500E+04	*standard deviation of response to load component s*	*
sigma w	5.94800E+04	*standard deviation of response to load component w*	*
sigma d	3.56880E+04	*standard deviation of response to load component d*	*
K=Kd	0.7	*load combination factor for two correlated load response*	*
r	0.6	*stress ratio*	
mr=mc	1	*coefficients associated with loading factor*	
ro	0.547	*correlation coefficient between w and d*	
sigma g	1.56485E+05	*s.t.d. of limit-state function*	**
mean			
mu u	1.20743E+06	*mean of load component u*	*
mu s	1.30200E+05	*mean of load component s*	*
mu w	5.94800E+05	*mean of load component w*	*
mu d	1.18960E+05	*mean of load component d*	*
mu g	3.99161E+05	*mean of limit-state function*	**
Part 2 : Probability of Failure			
beta g	*2.55080E+00*	*safety index*	
Pf	*5.37389E-03*	*probability of failure*	

*ps : " * " means the value is from the input varibles table " inputvars "*

*ps : " ** " means the value is after some calculations of the input variables from " inputvars "*

Appendix 6. Comparison of the Short-Term Primary Stress

Comparison of " Pf " and " beta " among Three Methods					
Condition : short term , primary stress					
	ship	**cruiser 1**		**cruiser 2**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	6.940E-12	6.760E+00	2.340E-10	6.230E+00
	method2	1.376E-08	5.557E+00	8.542E-08	5.229E+00
	method3	2.139E-09	5.874E+00	1.389E-08	5.555E+00
sagging					
	method1	4.920E-11	6.470E+00	1.700E-07	5.100E+00
	method2	3.265E-10	6.178E+00	2.040E-07	5.066E+00
	method3	1.311E-09	5.954E+00	6.768E-07	4.832E+00
	ship	**tanker**		**sl-7**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	2.400E-03	2.820E+00	4.500E-04	3.320E+00
	method2	1.008E-02	2.323E+00	3.536E-03	2.693E+00
	method3	2.403E-03	2.820E+00	9.202E-04	3.115E+00
sagging					
	method1	3.470E-03	2.700E+00	2.780E-09	5.830E+00
	method2	1.115E-02	2.285E+00	5.933E-09	5.702E+00
	method3	5.374E-03	2.551E+00	4.596E-08	5.342E+00
Limit-State-Function :		$g = M_u - [M_s + k_w(M_w + k_d M_d)]$; hogging: $k_d = 0$			
method1 : CALREL structural program					
method2 : Closed Form (by approximation)					
method3 : Mean Value First Order Second Moment					

Appendix 7. Comparison of the Short-Term Secondary Stress

Comparison of " Pf " and " beta " among Three Methods					
Condition : short term , secondary stress					
	ship	**cruiser 1**		**cruiser 2**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	8.530E-12	6.730E+00	5.880E-07	4.860E+00
	method2	6.244E-08	5.287E+00	3.067E-05	4.009E+00
	method3	1.018E-08	5.609E+00	5.748E-06	4.387E+00
sagging					
	method1	1.720E-09	5.910E+00	7.840E-05	3.780E+00
	method2	9.899E-09	5.614E+00	5.333E-05	3.874E+00
	method3	4.850E-08	5.333E+00	2.180E-04	3.517E+00
	ship	**tanker**		**sl-7**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	2.740E-01	6.000E-01	2.800E-03	2.770E+00
	method2	3.707E-01	3.299E-01	2.464E-02	1.966E+00
	method3	1.981E-01	8.483E-01	5.070E-03	2.571E+00
sagging					
	method1	1.540E-03	2.960E+00	N	N
	method2	7.544E-03	2.430E+00	N	N
	method3	3.297E-03	2.717E+00	N	N
Limit-State Function :		$g = \sigma_u SM_b - [M_s + k_w(M_w + k_d M_d)]$; hogging: $k_d = 0$			
method1 : CALREL structural program					
method2 : Closed Form (by approximation)					
method3 : Mean Value First Order Second Moment					

Appendix 8. Comparison of the Short-Term Tertiary Stress

Comparison of " Pf " and " beta " among Three Methods					
Condition : short term , tertiary stress					
	ship	**cruiser 1**		**cruiser 2**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	3.330E-16	8.060E+00	1.270E-09	5.960E+00
	method2	9.555E-10	6.006E+00	6.763E-07	4.768E+00
	method3	1.447E-10	6.305E+00	1.210E-07	5.164E+00
sagging					
	method1	2.190E-09	5.870E+00	5.420E-06	4.400E+00
	method2	1.137E-08	5.590E+00	5.648E-06	4.391E+00
	method3	5.578E-08	5.307E+00	2.212E-05	4.084E+00
	ship	**tanker**		**sl-7**	
		Pf	*beta*	*Pf*	*beta*
hogging					
	method1	1.420E-04	3.630E+00	1.170E-05	4.230E+00
	method2	1.238E-03	3.026E+00	1.246E-03	3.024E+00
	method3	2.727E-04	3.457E+00	3.013E-04	3.430E+00
sagging					
	method1	6.220E-06	4.370E+00	N	N
	method2	1.331E-04	3.646E+00	N	N
	method3	6.204E-05	3.838E+00	N	N
Limit-State Function :		$g = \sigma_u SM_b - [M_s + k_w(M_w + k_d M_d)]$; hogging: $k_d = 0$			
method1 : CALREL structural program					
method2 : Closed Form (by approximation)					
method3 : Mean Value First Order Second Moment					

Nomenclature

length between perpendiculars	LBP
beam	B
waterplane coefficient	C_{WP}
block coefficient	C_B
heading (0° = head seas)	θ
speed	V_S
stillwater bending moment	M_{SW}
ultimate failure bending moment	M_{ult}
mean	μ
standard deviation	σ
wave frequency	ω
encounter frequency	ω_e
significant wave height	$H_{1/3}$
mean wave period	T_m
moments of the response spectrum	
zeroth	m_0
second	m_2
fourth	m_4
probability density function	$f_x(x)$
cumulative distribution function	$F_x(x)$
standard normal cumulative distribution function	$\Phi_x(x)$
average response period	T_{avg}
bandwidth parameter	ε
number of encounters	N
expected maximum in N encounters	Q_N
value with a probability of exceedance of α	$q(\alpha)$
probability of failure	P_f

Cruiser I Particulars

Length (BP)	529.00	feet
Beam	55.00	feet
Draft	22.07	feet
Displacement	9403.40	LT
Speed	30+	knots
Trim	1.83	feet by stern
GM_T	2.56	feet
LCG	7.37	feet aft amidships
KG	23.28	feet
C_B	0.61	
C_{WP}	0.753	

Assumptions

The following assumptions are made concerning the ship and the environment for this analysis:

- M_{SW} is deterministic and known.
- Sea conditions are statistically stationary and the spectral content of the waves can be represented by a two-dimensional spectrum.
- The seas are long-crested and fully-developed.
- The ship's response to the waves is linear and can be represented by an RAO.
- M_{ult} is normally distributed with mean μ and a known coefficient of variation. (applies to both hogging and sagging strength)
- The bending moment response is a narrowband process and its peaks follow a Rayleigh distribution.
- Order statistics can be used to determine the extreme characteristics of the bending moment response.
- The ship's strength is statistically independent of the wave-induced bending moment.

Development of the Model

Determining the Response Spectrum

The wave forces that the ship encounters are modeled by a two-dimensional sea spectrum. This procedure utilizes the ISSC-63 wave spectrum. This is a two-parameter spectrum, with significant wave height and mean wave period as its parameters. A spectrum is generated for each case by looking up the given sea state in Table 1 and reading the corresponding $H_{1/3}$ and T_m. The ISSC-63 spectrum is given by

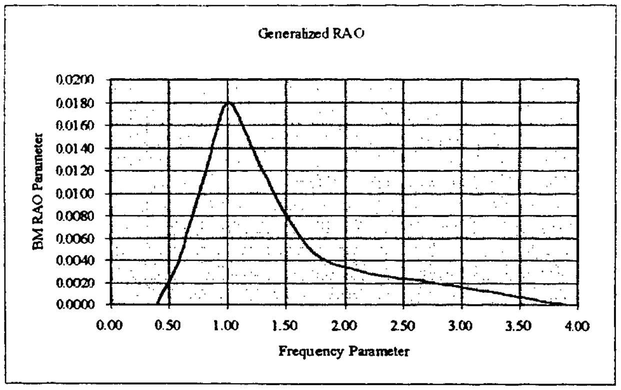
$$S_W(\omega) = AB\omega^{-5}e^{-B\omega^{-4}} \quad \text{where} \quad A = (0.25)\left(H_{1/3}\right)^2 \quad \text{and} \quad B = \left(0.817 \times \frac{2\pi}{T_m}\right)^4$$

Table 1: NATO Sea States

Sea State	$H_{1/3}$ (m)	$H_{1/3}$ (ft)	T_m (sec)	% Occurance	
2	0.30	0.98	7.5	7.2	
3	0.88	2.89	7.5	22.4	
4	1.88	6.17	8.8	28.7	
5	3.25	10.66	9.7	15.5	
6	5.00	16.40	12.4	18.7	
7	7.50	24.61	15.0	6.1	
8	11.50	37.73	16.4	1.2	
9	~~20.00~~	~~65.62~~	20.0	0.05	
	14.00	45.9			
NOTES:	open-ocean, North Atlantic, fully developed seas,				
	most probable wave heights and modal periods				

In order to get the response spectrum, the bending moment RAO for the ship must be known at the given speed and heading. In this case, the RAO's are determined from a plot of non-dimensional RAO's. (Figure 1) The information in this plot is valid for ships with cruiser/destroyer-type hullforms ($0.44 < C_B < 0.62$ and $0.72 < C_{WP} < 0.84$). The plot is entered with the frequency parameter and the returned value, the bending moment parameter is converted into the bending moment RAO value for the input frequency. Note that the RAO is a function of the length and beam of the ship and the given heading and speed.

Figure 1: RAO Plot



$$\text{Frequency Parameter} = \frac{\omega\sqrt{|\cos\theta|}}{\sqrt{2\pi g/LBP}}$$

$$RAO(\omega) = \left\{\rho g B(LBP)^2 F_1 F_2 \,\text{BM Parameter}(\omega)\right\}^2$$

$$\text{where} \quad F_1 = \sqrt[3]{|\cos\theta|} \quad \text{and} \quad F_2 = 1.1\tanh(1.5 + V_s/g) + 0.03(V_s/g)^2$$

Now, the response spectrum is simply $S_{BM}(\omega) = S_W(\omega) \times RAO(\omega)$. Converting the response in wave frequency into the response in encounter frequency is the next step. First, the response is divided into discreet, evenly spaced blocks. The total area and center frequency (ω_c) are calculated for each block. Next, each center frequency is converted to the corresponding encounter frequency by the formula

$$\omega_{c,e} = \omega_c + \frac{V_S\,\omega_c^{\,2}}{g}\cos\theta$$

The n^{th} moment of the response spectrum, now in terms of encounter frequencies, is given by

$$m_n = \sum \omega_{c,e}^n \times Area@\omega_{c,e}$$

Statistics of the Extreme Responses

It is now necessary to determine some of the characteristics of the extreme responses. First, the bandwidth parameter is calculated $\varepsilon = \sqrt{1 - m_2^2/m_0 m_4}$ So long as this is less than 0.6, we are well justified in assuming a narrowband process. Next, the average period of the response is determined by $T_{avg} \approx 2\pi\sqrt{m_0/m_2}$ This value is combined with the duration over which the analysis is being conducted (this must be ≤ 3 hours) to calculate the number of cycles (encounters) to be expected in the analysis period

$$N \approx \frac{\text{duration in seconds}}{T_{avg}}$$

At this point, it is useful to calculate, as a check, the expected maximum value of the bending moment in N cycles. This value, derived from the use of order statistics, is given by

$$Q_N \approx \sqrt{m_0}\left\{\sqrt{2\ln N} + \frac{0.5772}{\sqrt{2\ln N}}\right\}$$ assuming a Rayleigh distribution for the peaks

or by $$Q_N \approx \sqrt{m_0}\left\{\sqrt{2\ln\left[N\left(1-\varepsilon^2\right)\right]} + \frac{0.5772}{\sqrt{2\ln\left[N\left(1-\varepsilon^2\right)\right]}}\right\}$$ assuming a Rice distribution.

These two values can then be compared with each other to get a qualitative measure of how much error is incurred by assuming that the process is perfectly narrowbanded for the remainder of the analysis. In practice, this error is small. For example, at 30 knots, sea state 9, head seas, we have $\varepsilon = 0.661$. This difference in the Q_N values is only 3.74 %, and the value of Q_N derived from the Rayleigh distribution is larger. Thus, it seems that narrowbandness is a well-justified assumption even for values of ε slightly greater than 0.6 -- if anything, we are being more conservative.

Other values that are of use in obtaining a qualitative feel for the extreme bending moment are the value with a probability of exceedance of 0.1% and the value with a probability of exceedance of 50%. These are given by

$$q(\alpha) = \sqrt{2m_0\left\{\ln N + \ln\left[1/\ln\left(\tfrac{1}{1-\alpha}\right)\right]\right\}}$$

Calculating the Probability of Failure

The first step in the actual calculation of the probability of failure is determining the probability distributions of both the strength and the load. In this case, strength is represented by the ultimate bending moment in both the hogging and sagging modes. We assume that the ultimate bending moments are normally distributed with the given mean and an assumed coefficient of variation. Note that the results are strongly dependent on the coefficient of variation that is assumed. For example, in the sample condition above, for a coefficient of variation of 15% the probability of failure (hogging) is 5.49×10^{-6} The corresponding probability of failure for a coefficient of variation of 12% is 2.22×10^{-8}

For the loading, we assume that the peaks of the bending moment can be closely approximated by a Rayleigh distribution. While it would be more exact to represent the peaks of the bending moment by Rice's distribution, we have shown above that assuming the response in narrowbanded incurs only a small error.

Since we are also assuming that the strength and the load are statistically independent, we can represent the joint probability density function as the product of the probability density function of the load and the probability density function of the strength. By the application of order statistics, the probability of failure is given by

$$P_f = 1 - \int_{M_{SW}}^{\infty} f_{Str}(z)\left[F_{BM}(z)\right]^N dz$$

where $$F_{BM}(z) = 1 - e^{-\left(\frac{z-M_{SW}}{\sqrt{2m_0}}\right)^2}$$

and $$f_{Str}(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}$$

These equations are integrated numerically for both the hogging and sagging modes to determine the probability of failure for each mode.

Limitations

Any answers arrived at through a modeling procedure are only as accurate as the model is realistic. There are several simplifications made in modeling the ship's response and calculating the probabilities of failure with this procedure that anyone who is using it must understand. First, the probabilities generated by the model are conditional on the ship actually encountering the specified sea condition for the specified duration at the specified course and speed. Second, the sea spectrum is only two-dimensional and assumes long-crested, fully-developed seas. However, the model can be made more realistic by modifying the spectrum to include directional and transient effects.

A more pressing consideration is the use of a regression fit for the ship's bending moment RAO's. A more accurate procedure could involve using the ship's actual RAO's (from full-scale or model testing) instead of the regression fitted ones. A significant obstacle to this enhancement is the difficulty in obtaining RAO's for each speed and heading condition to be investigated. It is obvious the use of the regression RAO's is much simpler, but how much accuracy is lost? Table 2 shows a comparison between the method used here and those derived from the second order strip theory program SOST. One can see that the values obtained from this procedure are very close to the linear SOST analysis. Therefore, so long as a second-order analysis is not necessary, using the regression RAO's is a valid simplification, at least for this ship.

Table 2: Comparison with SOST

Cruiser I -- 6 knots, $H_{1/3} = 45'$, $T_m = 14$ s					
duration: 2.78 hours			q(50%)		
			SOST	Lvl 3 Short-Term	
Condition A: $\theta = 0°$		sagging	2.032E+05		ft-LT
		hogging	1.691E+05		ft-LT
		linear	1.811E+05	1.783E+05	ft-LT
Condition A: $\theta = 45°$		sagging	1.408E+05		ft-LT
		hogging	1.134E+05		ft-LT
		linear	1.217E+05	1.227E+05	ft-LT

There is one other factor of importance that is not covered in this approach: loads on the ship due to slamming. This cannot be simply implemented in this framework. Its effects would be greatest in the higher sea states, adding approximately 1/3 of the expected wave bending moment to the total load.

Application of the Model

Figure 2: Probability of Failure at $\theta = 0°$

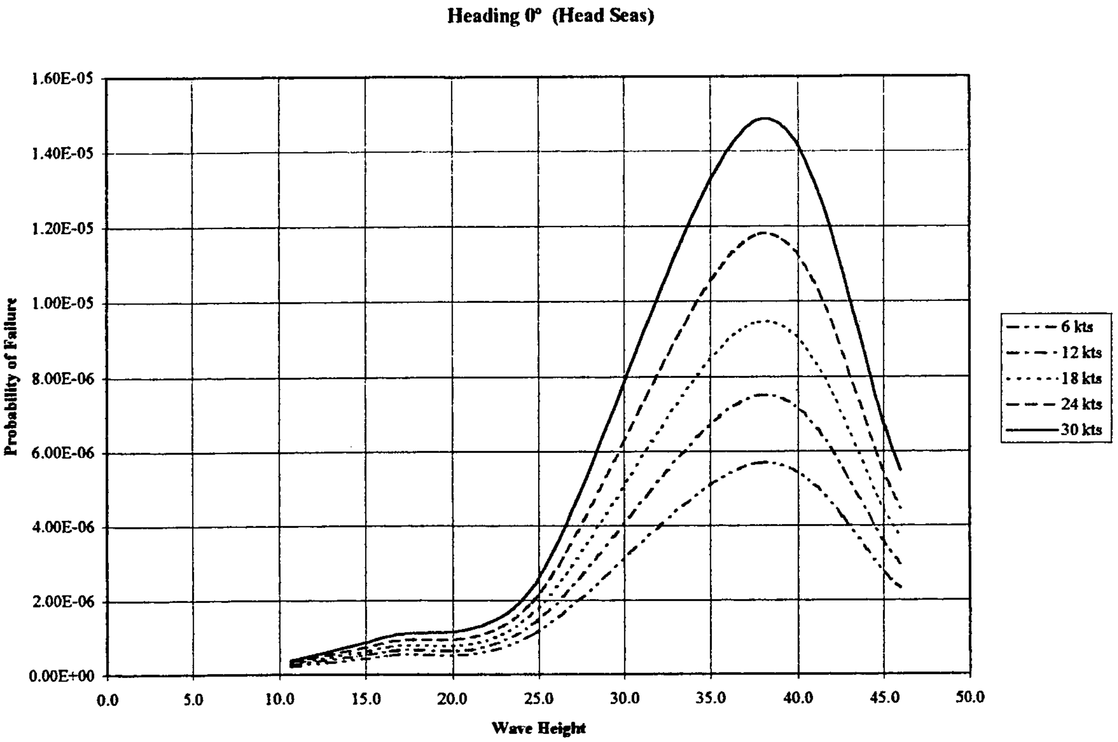


Figure 3: Probability of Failure at $\theta = 30°$

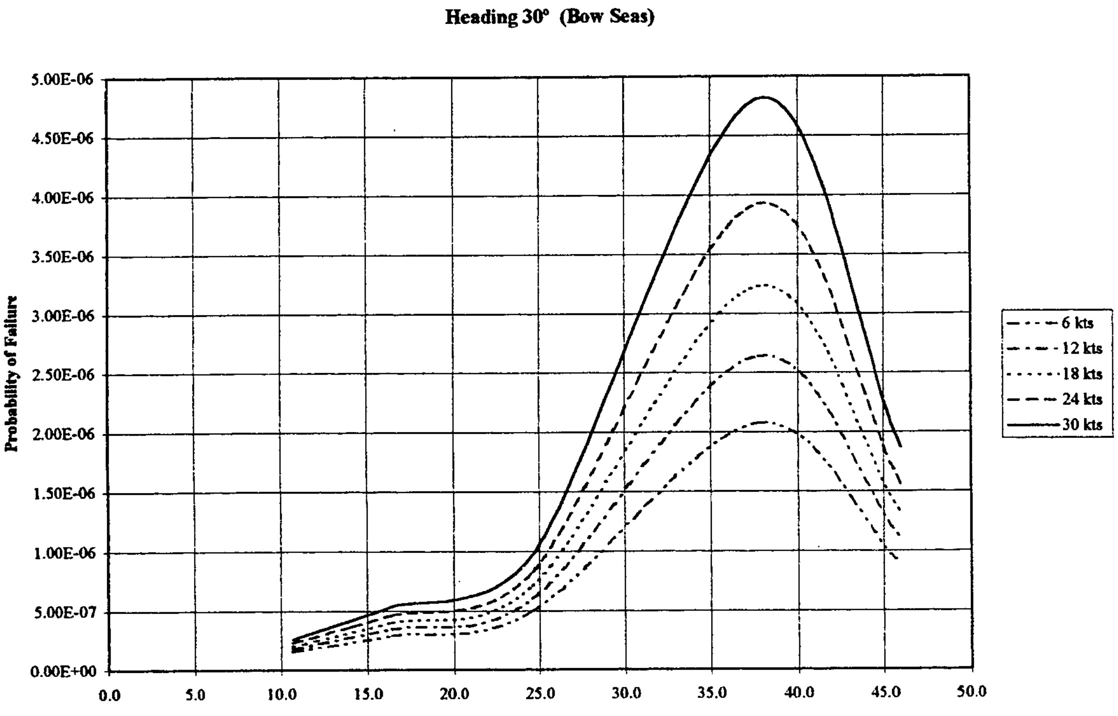


Figure 4: Probability of Failure at $\theta = 45°$

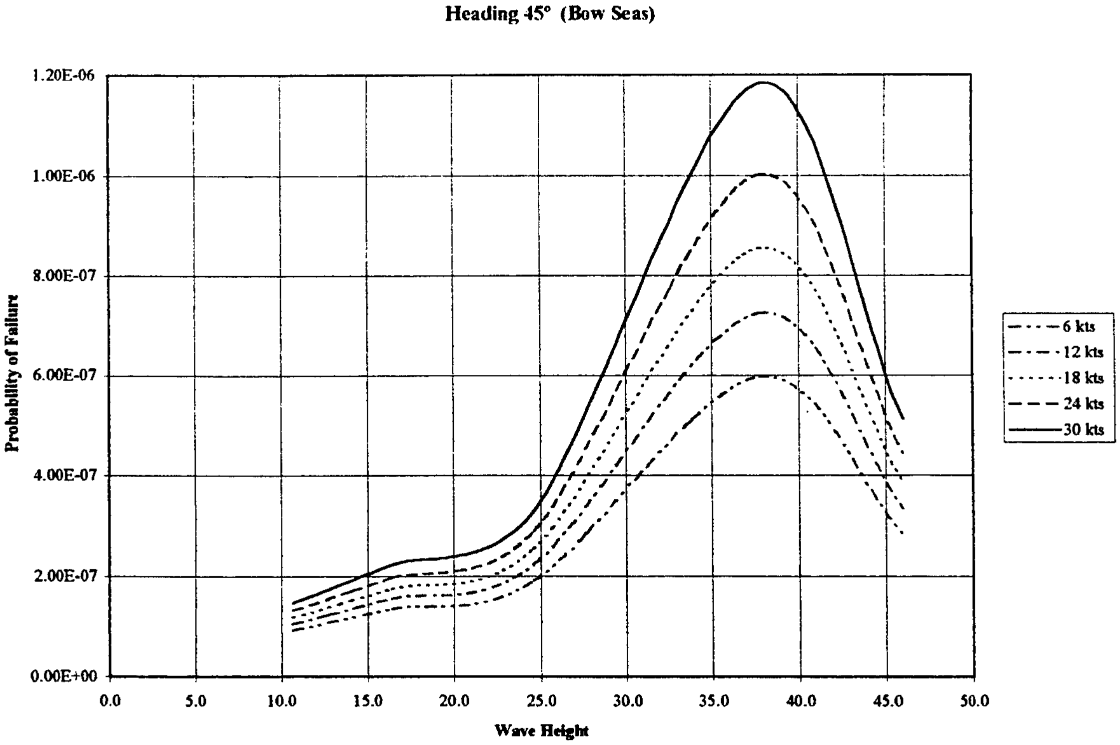
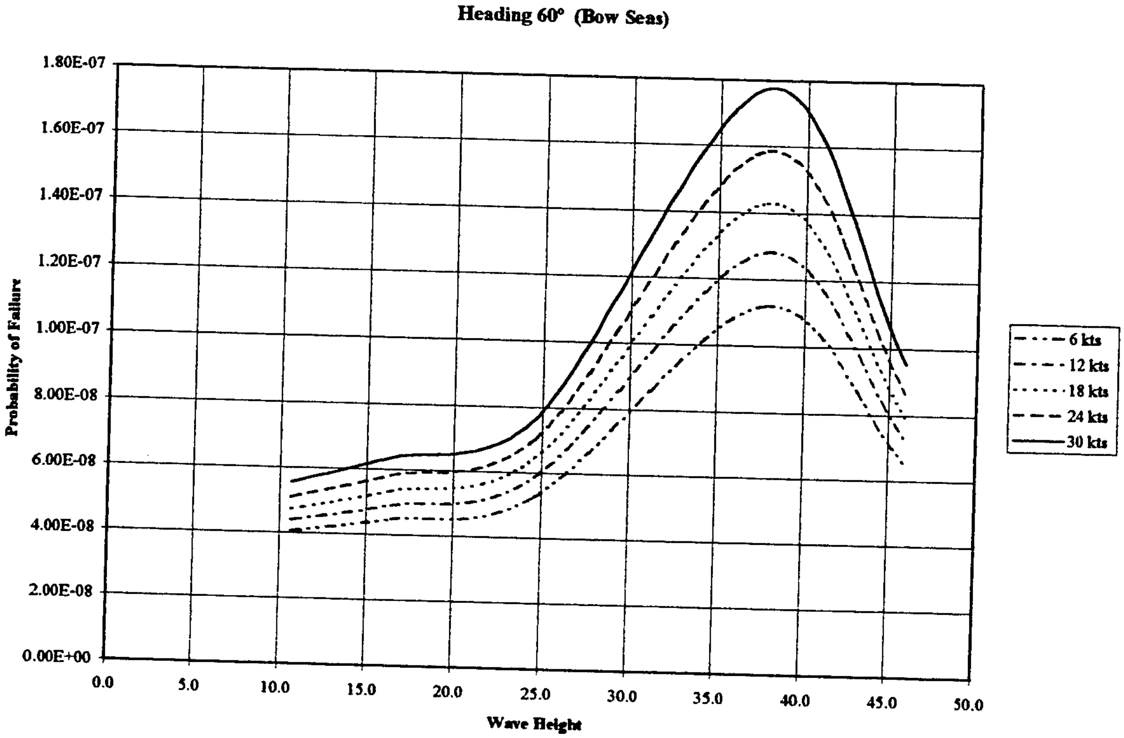


Figure 5: Probability of Failure at $\theta = 60°$



Conclusions

The fully-probabilistic (Level III) reliability analysis is a viable option for the analysis of wave-induced longitudinal bending loads over a short period of time. Various simplifying assumptions can make this process tractable, even if the user has only limited amounts of computer resources and ship data available. The method can also be easily modified to increase its accuracy, at the price of needing more information. The only significant failing of the model is in its failure to account for the slamming loads at high sea states. So long as one is mindful of the limitations of the procedure, the data derived from it can be of great use to both designers and operators.

Sample Run of Model

Input Data

Ship: *Cruiser I*

Hydrostatic Data

LBP:	529	ft
Beam:	55	ft
Draft:	22.07	ft
Δ:	9403.4	LT
C_B:	0.61	
C_{WP}:	0.753	

Condition Data

Speed:	30	knots
Heading:	0	° (relative)
Sea State	9	
Duration:	3	hour(s)

Strength Data

M_{SW} 76,821 ft-LT

Ultimate Failure Bending Moment

	Sagging	Hogging	
μ	-616,241	574,489	ft-LT
COV	15%	15%	
σ	92436	86173	ft-LT

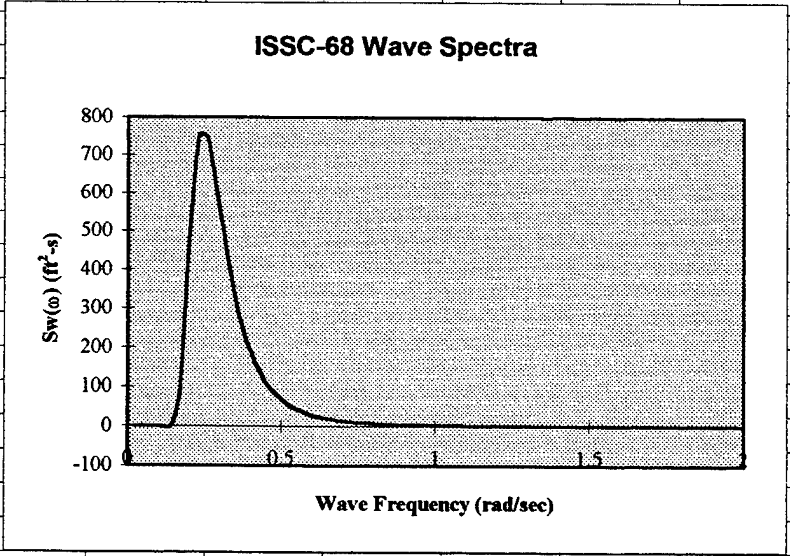
Wave Response Calculations

Density:	1.9905	lb-ft/s^2
Accel of Gravity:	32.174	ft/s^2
Sig Wave Height:	45.9	ft
Mean Wave Period:	20.0	sec

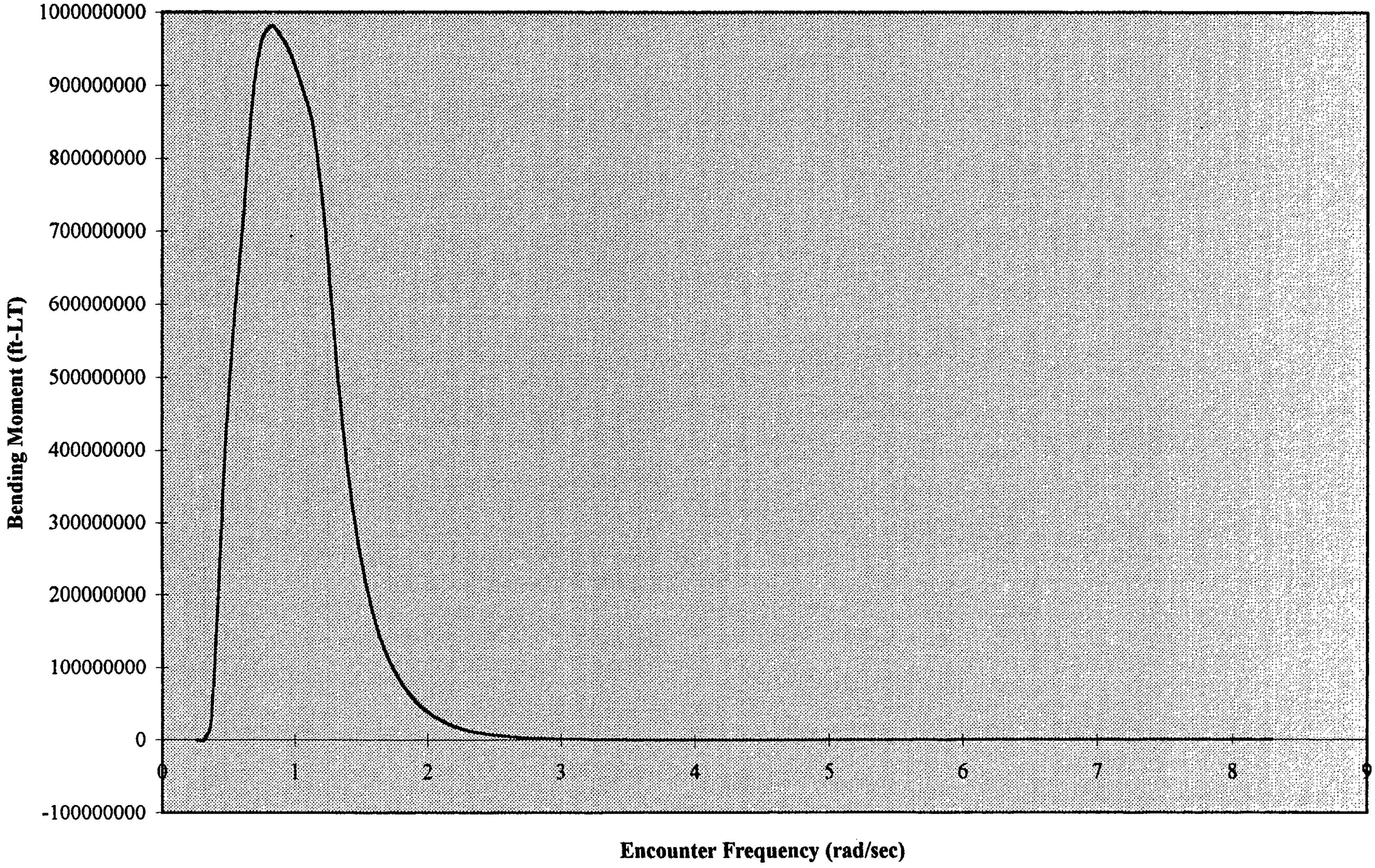
m_0	842.7E+6
m_2	958.7E+6
m_4	1.9E+9

ω	$S_w(\omega)$	RAO	$S_{BM}(\omega)$	ω_c	ω_e	Area Blks	δm_2	δm_4	ω_e	$S_{BM}(\omega_e)$
s^{-1}	ft^2/s	(ft-LT/ft)^2	(ft-LT)^2/s	s^{-1}	s^{-1}	(ft-LT)^2	(ft-LT)^2/s^2	(ft-LT)^2/s^4	s^{-1}	(ft-LT)^2/s
0.02	0	0	0	0.035	0.036928	0	0	0	0.02063	0
0.05	2E-295	0	0	0.065	0.071649	0	0	0	0.053934	0
0.08	6.73E-41	0	0	0.095	0.109203	0	0	0	0.090072	0
0.11	1.9E-08	0	0	0.125	0.14959	0	0	0	0.129043	0
0.14	0.528018	0	0	0.155	0.19281	0	0	0	0.170846	0
0.17	89.26811	0	0	0.185	0.238862	0	0	0	0.215482	0
0.2	474.7726	0	0	0.215	0.287747	0	0	0	0.262951	0
0.23	754.1868	0	0	0.245	0.339465	635559.8	73239.76	8439.9032	0.313252	0
0.26	745.2984	56850.59	42370650	0.275	0.394016	6441821	1000083	155261.51	0.366386	23301606
0.29	604.1912	640664.9	3.87E+08	0.305	0.451399	18362265	3741522	762377.9	0.422354	2.02E+08
0.32	450.9752	1856126	8.37E+08	0.335	0.511616	30685395	8031922	2102360.6	0.481153	4.17E+08
0.35	326.3703	3703235	1.21E+09	0.365	0.574665	41503215	13706005	4526265.4	0.542786	5.75E+08
0.38	234.6077	6641960	1.56E+09	0.395	0.640547	53432141	21923204	8995089.1	0.607252	7.1E+08
0.41	169.4466	11826071	2E+09	0.425	0.709261	64355179	32373965	16285770	0.67455	8.75E+08
0.44	123.624	18495255	2.29E+09	0.455	0.780809	70799058	43163498	26315146	0.744681	9.59E+08
0.47	91.31423	26649511	2.43E+09	0.485	0.855189	73839839	54002601	39494681	0.817645	9.82E+08
0.5	68.33553	36425799	2.49E+09	0.515	0.932402	74976619	65182638	56668016	0.893441	9.67E+08
0.53	51.80684	48435014	2.51E+09	0.545	1.012447	74743816	76616125	78535334	0.97207	9.4E+08
0.56	39.76932	62200100	2.47E+09	0.575	1.095326	72947030	87517377	104997987	1.053533	8.95E+08
0.59	30.8915	77350707	2.39E+09	0.605	1.181037	67015949	93477107	130386418	1.137827	8.36E+08
0.62	24.26273	85656089	2.08E+09	0.635	1.269581	55061985	88750905	143051931	1.224955	7.04E+08
0.65	19.25432	82711231	1.59E+09	0.665	1.360958	40983632	75910152	140601283	1.314915	5.23E+08
0.68	15.42744	73874433	1.14E+09	0.695	1.455168	29055531	61525453	130280922	1.407709	3.63E+08
0.71	12.47225	63929242	7.97E+08	0.725	1.55221	20302888	48916878	117858154	1.503335	2.46E+08
0.74	10.16739	54702726	5.56E+08	0.755	1.652085	14231042	38841993	106014753	1.601793	1.67E+08
0.77	8.352922	46995763	3.93E+08	0.785	1.754793	10027319	30877107	95079825	1.703085	1.15E+08
0.8	6.911977	39921429	2.76E+08	0.815	1.860334	7025991	24315841	84153268	1.807209	78434959
0.83	5.758267	33423844	1.92E+08	0.845	1.968707	4878484	18908065	73284023	1.914166	53277828
0.86	4.827429	27503007	1.33E+08	0.875	2.079913	3386100	14648403	63369571	2.023956	35816912
0.89	4.070987	22837506	92971192	0.905	2.193952	2365090	11384187	54796952	2.136579	24457732
0.92	3.452103	18742621	64701451	0.935	2.310824	1634887	8730147	46618185	2.252034	16608315
0.95	2.942536	15051990	44291017	0.965	2.430529	1109187	6552488	38708624	2.370322	11100082
0.98	2.520461	11765613	29654769	0.995	2.553066	757949.4	4940425	32202415	2.491443	7260179
1.01	2.168881	9624867	20875190	1.025	2.678436	537939.4	3859188	27685897	2.615397	4995255
1.04	1.874459	7995606	14987438	1.055	2.806639	383833.5	3023542	23817118	2.742184	3507121
1.07	1.626661	6517317	10601465	1.085	2.937675	269343.8	2324419	20059583	2.871803	2427155
1.1	1.417108	5189999	7354791	1.115	3.071543	192608.5	1817141	17143597	3.004255	1648213
1.13	1.239098	4427232	5485774	1.145	3.208244	147794.3	1521222	15657686	3.13954	1203890
1.16	1.087234	4016782	4367181	1.175	3.347778	117571.2	1317694	14768211	3.277657	938949.8
1.19	0.957148	3626294	3470901	1.205	3.490145	93344.39	1137039	13850396	3.418608	731399.7

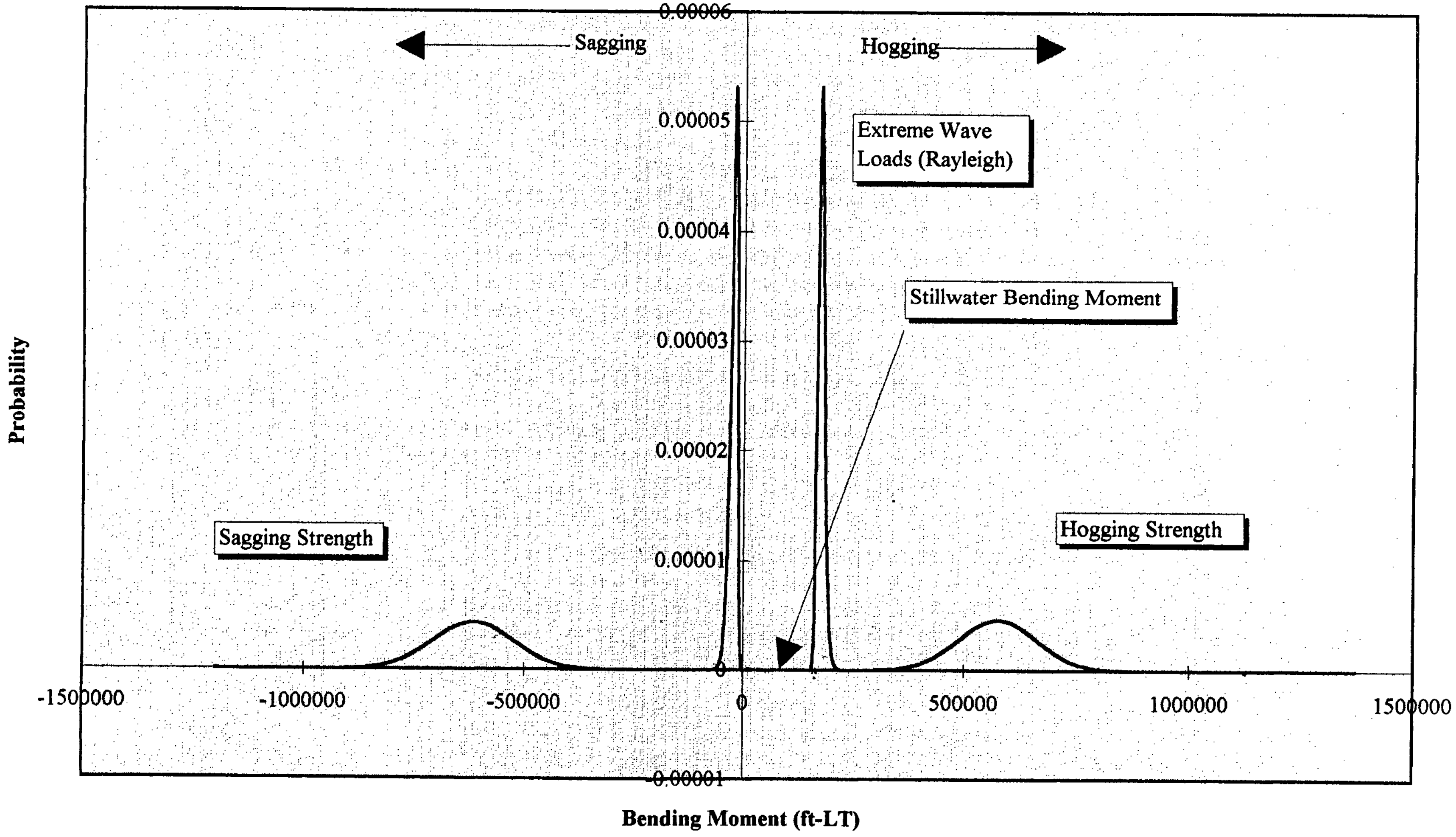
1.22	0.845286	3255770	2752058	1.235	3.635345	74345.89	982535.3	12984923	3.562391	568608.8
1.25	0.748741	2944053	2204334	1.265	3.783377	59927.97	857805.5	12278579	3.709007	446726.9
1.28	0.665123	2692531	1790864	1.295	3.934242	48655.68	753105.3	11656761	3.858456	356118.9
1.31	0.592458	2452239	1452848	1.325	4.08794	39437.25	659045.9	11013485	4.010737	283578.6
1.34	0.529109	2223175	1176302	1.355	4.244471	32062.03	577614.5	10406033	4.165851	225445.1
1.37	0.473714	2029002	961166.3	1.385	4.403834	26464.77	513251.4	9953871.6	4.323798	180938.5
1.4	0.425131	1889186	803152	1.415	4.56603	22110.41	460971.8	9610631.9	4.484578	148551.9
1.43	0.382404	1754360	670875.2	1.445	4.731059	18463.38	413264.5	9250067.3	4.648191	121955.9
1.46	0.344727	1624525	560017.1	1.475	4.898921	15466.41	371185.1	8908229.9	4.814636	100085.4
1.49	0.311415	1512697	471077.2	1.505	5.069616	13205.27	339388.6	8722626.8	4.983914	82793.01
1.52	0.281892	1451882	409273.9	1.535	5.243143	11478.57	315552.1	8674700.5	5.156025	70756.68
1.55	0.255664	1392314	355963.9	1.565	5.419503	9987.94	293355.9	8616161.2	5.330969	60551.78
1.58	0.232309	1333994	309898.8	1.595	5.598696	8687.61	272316.6	8535871.3	5.508745	51882.44
1.61	0.211468	1273364	269275.2	1.625	5.780722	7451.508	249005.1	8320940.1	5.689355	44379.78
1.64	0.19283	1179755	227492	1.655	5.96558	6291.339	223897.1	7968081.6	5.872797	36918.84
1.67	0.176128	1089720	191930.6	1.685	6.153271	5303.826	200817.4	7603498.5	6.059072	30677.62
1.7	0.161133	1003259	161657.7	1.715	6.343795	4463.16	179614.2	7228348.3	6.248179	25454.71
1.73	0.147643	920370.3	135886.3	1.745	6.537152	3747.56	160149.6	6843889.1	6.440119	21083.24
1.76	0.135486	841055.2	113951.1	1.775	6.733341	3138.607	142297.8	6451482	6.634893	17424.65
1.79	0.12451	765313.3	95289.35	1.805	6.932364	2620.693	125944.4	6052593.6	6.832499	14363.62
1.82	0.114584	693144.8	79423.51	1.835	7.134219	2180.572	110984.7	5648798.2	7.032937	11804.04
1.85	0.105593	624549.5	65947.93	1.865	7.338907	1806.98	97323.13	5241780.3	7.236209	9665.634
1.88	0.097435	559527.5	54517.41	1.895	7.546427	1490.327	84871.97	4833337.1	7.442313	7881.251
1.91	0.090021	498078.9	44837.72	1.925	7.756781	1222.432	73550.84	4425380.8	7.65125	6394.627
1.94	0.083275	440203.4	36657.73	1.955	7.969967	996.308	63285.86	4019941.1	7.86302	5158.552
1.97	0.077125	385901.3	29762.8	1.985	8.185986	805.9808	54009.07	3619167.3	8.077622	4133.36
2	0.071513	335172.5	23969.25						8.295058	3285.684



Bending Moment Spectrum



PDF's of Load And Strength



Probability of Failure Calculation

M_{SW}	76,821	ft-LT		BM_{RMS}	29,030	ft-LT			Rice	Rayleigh	
Step Size	4,465	ft-LT		ε	0.661	Wideband	Expected max				
m_0	842.7E+6	$(\text{ft-LT})^2$		T_{avg}	5.89	s		in N peaks	1.13E+05	1.17E+05	ft-LT
m_2	958.7E+6	$(\text{ft-LT})^2/s^2$		N	1833.3	peaks		semi-prob FS	3.03	2.97	
m_4	1.9E+9	$(\text{ft-LT})^2/s^4$		1/1000 value	1.56E+05	ft-LT					
				50% value	1.15E+05	ft-LT		% difference	3.74%		
					$P_{f,sag}$	3.48E-10					
					$P_{f,hog}$	5.49E-06					

Distribution Calculations

BM	$f_{str,sag}(str)$	$f_{str,hog}(str)$	$[f_{BM}(BM)]^N$	$f_{BM}(BM)$	$F_{BM}(BM)$	S(BM)	H(BM)
(ft-LT)	Normal	Normal	Rayleigh	Rayleigh	Rayleigh		
-1209168	5.02E-15	4.3E-99	0	0	1	5.02E-15	
-1204702	6.83E-15	1.26E-98	0	0	1	2.73E-14	
-1200237	9.28E-15	3.66E-98	0	0	1	1.86E-14	
-1195772	1.26E-14	1.06E-97	0	0	1	5.03E-14	
-1191307	1.7E-14	3.07E-97	0	0	1	3.4E-14	
-1186841	2.29E-14	8.88E-97	0	0	1	9.18E-14	
-1182376	3.09E-14	2.56E-96	0	0	1	6.18E-14	
-1177911	4.15E-14	7.34E-96	0	0	1	1.66E-13	
-1173446	5.55E-14	2.1E-95	0	0	1	1.11E-13	
-1168980	7.42E-14	6.01E-95	0	0	1	2.97E-13	
-1164515	9.9E-14	1.71E-94	0	0	1	1.98E-13	
-1160050	1.32E-13	4.87E-94	0	0	1	5.27E-13	
-1155585	1.75E-13	1.38E-93	0	0	1	3.49E-13	
-1151120	2.31E-13	3.9E-93	0	0	1	9.25E-13	
-1146654	3.06E-13	1.1E-92	0	0	1	6.11E-13	
-1142189	4.03E-13	3.09E-92	0	0	1	1.61E-12	
-1137724	5.3E-13	8.66E-92	0	0	1	1.06E-12	
-1133259	6.95E-13	2.42E-91	0	0	1	2.78E-12	
-1128793	9.09E-13	6.75E-91	0	0	1	1.82E-12	
-1124328	1.19E-12	1.88E-90	0	0	1	4.75E-12	
-1119863	1.55E-12	5.21E-90	0	0	1	3.09E-12	
-1115398	2.01E-12	1.44E-89	0	0	1	8.04E-12	
-1110932	2.61E-12	3.97E-89	0	0	1	5.21E-12	
-1106467	3.37E-12	1.09E-88	0	0	1	1.35E-11	
-1102002	4.35E-12	3E-88	0	0	1	8.7E-12	
-1097537	5.6E-12	8.21E-88	0	0	1	2.24E-11	
-1093071	7.19E-12	2.24E-87	0	0	1	1.44E-11	
-1088606	9.22E-12	6.1E-87	0	0	1	3.69E-11	
-1084141	1.18E-11	1.66E-86	0	0	1	2.36E-11	
-1079676	1.5E-11	4.48E-86	0	0	1	6.01E-11	
-1075210	1.91E-11	1.21E-85	0	0	1	3.82E-11	
-1070745	2.43E-11	3.26E-85	0	0	1	9.71E-11	
-1066280	3.08E-11	8.76E-85	0	0	1	6.15E-11	
-1061815	3.89E-11	2.35E-84	0	0	1	1.55E-10	

-1057349	4.9E-11	6.26E-84	0	0	1	9.8E-11					
-1052884	6.16E-11	1.67E-83	0	0	1	2.46E-10					
-1048419	7.73E-11	4.43E-83	0	0	1	1.55E-10					
-1043954	9.68E-11	1.18E-82	0	0	1	3.87E-10					
-1039489	1.21E-10	3.11E-82	0	0	1	2.42E-10					
-1035023	1.51E-10	8.19E-82	0	0	1	6.03E-10					
-1030558	1.87E-10	2.15E-81	0	0	1	3.75E-10					
-1026093	2.32E-10	5.64E-81	0	0	1	9.29E-10					
-1021628	2.87E-10	1.47E-80	0	0	1	5.75E-10					
-1017162	3.55E-10	3.85E-80	0	0	1	1.42E-09					
-1012697	4.37E-10	1E-79	0	0	1	8.74E-10					
-1008232	5.37E-10	2.59E-79	1E-303	5.5E-307	1	2.15E-09					
-1003767	6.58E-10	6.71E-79	3.1E-301	1.7E-304	1	1.32E-09					
-999301	8.05E-10	1.73E-78	9.4E-299	5.1E-302	1	3.22E-09					
-994836	9.83E-10	4.45E-78	2.8E-296	1.5E-299	1	1.97E-09					
-990371	1.2E-09	1.14E-77	8E-294	4.3E-297	1	4.78E-09					
-985906	1.45E-09	2.92E-77	2.2E-291	1.2E-294	1	2.91E-09					
-981440	1.76E-09	7.46E-77	6.1E-289	3.4E-292	1	7.04E-09					
-976975	2.13E-09	1.9E-76	1.6E-286	9E-290	1	4.26E-09					
-972510	2.57E-09	4.82E-76	4.3E-284	2.4E-287	1	1.03E-08					
-968045	3.09E-09	1.22E-75	1.1E-281	6E-285	1	6.18E-09					
-963579	3.71E-09	3.08E-75	2.8E-279	1.5E-282	1	1.48E-08					
-959114	4.44E-09	7.76E-75	6.7E-277	3.7E-280	1	8.88E-09					
-954649	5.3E-09	1.95E-74	1.6E-274	8.7E-278	1	2.12E-08					
-950184	6.32E-09	4.88E-74	3.7E-272	2E-275	1	1.26E-08					
-945719	7.52E-09	1.22E-73	8.5E-270	4.6E-273	1	3.01E-08					
-941253	8.92E-09	3.04E-73	1.9E-267	1E-270	1	1.78E-08					
-936788	1.06E-08	7.55E-73	4.1E-265	2.2E-268	1	4.23E-08					
-932323	1.25E-08	1.87E-72	8.6E-263	4.7E-266	1	2.49E-08					
-927858	1.47E-08	4.62E-72	1.8E-260	9.7E-264	1	5.88E-08					
-923392	1.73E-08	1.14E-71	3.6E-258	2E-261	1	3.46E-08					
-918927	2.03E-08	2.8E-71	7.1E-256	3.8E-259	1	8.1E-08					
-914462	2.37E-08	6.86E-71	1.4E-253	7.4E-257	1	4.74E-08					
-909997	2.77E-08	1.68E-70	2.6E-251	1.4E-254	1	1.11E-07					
-905531	3.22E-08	4.09E-70	4.7E-249	2.6E-252	1	6.45E-08					
-901066	3.74E-08	9.95E-70	8.4E-247	4.6E-250	1	1.5E-07					
-896601	4.34E-08	2.41E-69	1.5E-244	8E-248	1	8.68E-08					
-892136	5.02E-08	5.84E-69	2.5E-242	1.4E-245	1	2.01E-07					
-887670	5.79E-08	1.41E-68	4.2E-240	2.3E-243	1	1.16E-07					
-883205	6.67E-08	3.39E-68	6.8E-238	3.7E-241	1	2.67E-07					
-878740	7.65E-08	8.13E-68	1.1E-235	5.9E-239	1	1.53E-07					
-874275	8.77E-08	1.94E-67	1.7E-233	9.2E-237	1	3.51E-07					
-869809	1E-07	4.64E-67	2.6E-231	1.4E-234	1	2E-07					
-865344	1.14E-07	1.1E-66	3.8E-229	2.1E-232	1	4.57E-07					
-860879	1.3E-07	2.62E-66	5.5E-227	3E-230	1	2.6E-07					
-856414	1.48E-07	6.21E-66	7.8E-225	4.3E-228	1	5.9E-07					
-851948	1.67E-07	1.47E-65	1.1E-222	5.9E-226	1	3.34E-07					
-847483	1.89E-07	3.45E-65	1.5E-220	7.9E-224	1	7.55E-07					
-843018	2.13E-07	8.1E-65	1.9E-218	1E-221	1	4.26E-07					
-838553	2.39E-07	1.9E-64	2.5E-216	1.3E-219	1	9.57E-07					

-834088	2.69E-07	4.43E-64	3.1E-214	1.7E-217	1	5.37E-07					
-829622	3.01E-07	1.03E-63	3.8E-212	2.1E-215	1	1.2E-06					
-825157	3.36E-07	2.4E-63	4.6E-210	2.5E-213	1	6.71E-07					
-820692	3.74E-07	5.56E-63	5.3E-208	2.9E-211	1	1.5E-06					
-816227	4.16E-07	1.28E-62	6.1E-206	3.3E-209	1	8.31E-07					
-811761	4.61E-07	2.96E-62	6.8E-204	3.7E-207	1	1.84E-06					
-807296	5.1E-07	6.8E-62	7.4E-202	4E-205	1	1.02E-06					
-802831	5.63E-07	1.56E-61	7.9E-200	4.3E-203	1	2.25E-06					
-798366	6.2E-07	3.57E-61	8.2E-198	4.5E-201	1	1.24E-06					
-793900	6.81E-07	8.13E-61	8.3E-196	4.5E-199	1	2.72E-06					
-789435	7.46E-07	1.85E-60	8.3E-194	4.5E-197	1	1.49E-06					
-784970	8.16E-07	4.19E-60	8E-192	4.4E-195	1	3.26E-06					
-780505	8.9E-07	9.48E-60	7.6E-190	4.1E-193	1	1.78E-06					
-776039	9.69E-07	2.14E-59	7E-188	3.8E-191	1	3.87E-06					
-771574	1.05E-06	4.81E-59	6.3E-186	3.4E-189	1	2.1E-06					
-767109	1.14E-06	1.08E-58	5.5E-184	3E-187	1	4.56E-06					
-762644	1.23E-06	2.42E-58	4.8E-182	2.6E-185	1	2.46E-06					
-758178	1.33E-06	5.39E-58	4E-180	2.2E-183	1	5.31E-06					
-753713	1.43E-06	1.2E-57	3.3E-178	1.8E-181	1	2.86E-06					
-749248	1.53E-06	2.66E-57	2.6E-176	1.4E-179	1	6.13E-06					
-744783	1.64E-06	5.89E-57	2.1E-174	1.1E-177	1	3.28E-06					
-740317	1.75E-06	1.3E-56	1.6E-172	8.6E-176	1	7.01E-06					
-735852	1.87E-06	2.86E-56	1.2E-170	6.4E-174	1	3.74E-06					
-731387	1.99E-06	6.29E-56	8.6E-169	4.7E-172	1	7.95E-06					
-726922	2.11E-06	1.38E-55	6.1E-167	3.3E-170	1	4.21E-06					
-722457	2.23E-06	3.01E-55	4.2E-165	2.3E-168	1	8.92E-06					
-717991	2.35E-06	6.55E-55	2.9E-163	1.6E-166	1	4.71E-06					
-713526	2.48E-06	1.42E-54	1.9E-161	1E-164	1	9.92E-06					
-709061	2.61E-06	3.08E-54	1.2E-159	6.7E-163	1	5.21E-06					
-704596	2.73E-06	6.67E-54	7.8E-158	4.3E-161	1	1.09E-05					
-700130	2.86E-06	1.44E-53	4.8E-156	2.6E-159	1	5.72E-06					
-695665	2.98E-06	3.09E-53	2.9E-154	1.6E-157	1	1.19E-05					
-691200	3.11E-06	6.62E-53	1.7E-152	9.3E-156	1	6.21E-06					
-686735	3.23E-06	1.41E-52	9.8E-151	5.4E-154	1	1.29E-05					
-682269	3.34E-06	3.02E-52	5.5E-149	3E-152	1	6.69E-06					
-677804	3.46E-06	6.41E-52	3E-147	1.6E-150	1	1.38E-05					
-673339	3.57E-06	1.36E-51	1.6E-145	8.8E-149	1	7.13E-06					
-668874	3.67E-06	2.88E-51	8.5E-144	4.6E-147	1	1.47E-05					
-664408	3.77E-06	6.07E-51	4.3E-142	2.4E-145	1	7.54E-06					
-659943	3.86E-06	1.28E-50	2.2E-140	1.2E-143	1	1.54E-05					
-655478	3.94E-06	2.68E-50	1.1E-138	5.7E-142	1	7.89E-06					
-651013	4.02E-06	5.6E-50	5E-137	2.7E-140	1	1.61E-05					
-646547	4.09E-06	1.17E-49	2.3E-135	1.3E-138	1	8.18E-06					
-642082	4.15E-06	2.43E-49	1.1E-133	5.7E-137	1	1.66E-05					
-637617	4.2E-06	5.05E-49	4.7E-132	2.5E-135	1	8.4E-06					
-633152	4.24E-06	1.04E-48	2E-130	1.1E-133	1	1.7E-05					
-628686	4.28E-06	2.16E-48	8.5E-129	4.7E-132	1	8.55E-06					
-624221	4.3E-06	4.44E-48	3.5E-127	1.9E-130	1	1.72E-05					
-619756	4.31E-06	9.12E-48	1.4E-125	7.7E-129	1	8.63E-06					
-615291	4.32E-06	1.87E-47	5.6E-124	3E-127	1	1.73E-05					

APPENDIX H

SENSITIVITY ANALYSIS RESULTS

Primary (IY)			β =	10.29		
	x*	u*	α	γ	δ	η
Mi	6.01E+01	-6.16E+00	-0.5978	-0.5978	0.9337	-3.7184
Ms	5.18E+00	-1.05E+00	-0.1022	-0.1022	0.1022	-0.1074
Mw	4.00E+01	6.05E+00	0.5876	0.5876	-0.33034	-1.84433
Md	1.71E+01	3.80E+00	0.3696	0.3696	-0.42088	-0.68545
Kw	1.16E+00	3.21E+00	0.3115	0.3115	-0.3115	-0.9991
Kd	9.49E-01	2.38E+00	0.2307	0.2307	-0.2307	-0.548

Primary (ULT)			β =	6.47		
	x*	u*	α	γ	δ	η
Mu	40.77	-3.75	-0.5791	-0.5791	0.8024	-2.2182
Ms	5.36	-0.85	-0.1313	-0.1313	0.1313	-0.1116
Mw	31.42	4.06	0.6277	0.6277	-0.35104	-1.39957
Md	12.65	2.25	0.3474	0.3474	-0.37404	-0.41753
Kw	1.10	1.94	0.2994	0.2994	-0.2994	-0.5805
Kd	0.84	1.34	0.2075	0.2075	-0.2075	-0.2787

Secondary			β =	5.89		
	x*	u*	α	γ	δ	η
Su	1.70E+01	-3.28E+00	-0.5556	-0.5556	0.7439	-1.8689
SMd	2.22E+01	-1.32E+00	-0.2226	-0.2226	0.2347	-0.3013
Ms	5.40E+01	-8.04E-01	-0.1361	-0.1361	0.1361	-0.1094
Kw	1.09E+00	1.73E+00	0.293	0.293	-0.293	-0.507
Mw	2.98E+02	3.64E+00	0.6173	0.6173	-0.35203	-1.25788
Kd	8.26E-01	1.20E+00	0.2024	0.2024	-0.2024	-0.2421
Md	1.20E+02	2.01E+00	0.3397	0.3397	-0.3671	-0.37021

Tertiary			β =	5.86		
	x*	u*	α	γ	δ	η
Su	1.69E+01	-3.26E+00	-0.5551	-0.5551	0.7419	-1.8548
SMd	2.22E+01	-1.31E+00	-0.2225	-0.2225	0.2345	-0.2991
Ms	5.41E+01	-8.01E-01	-0.1365	-0.1365	0.1365	-0.1094
Kw	1.09E+00	1.72E+00	0.2931	0.2931	-0.2931	-0.5043
Mw	2.97E+02	3.62E+00	0.6175	0.6175	-0.35263	-1.25251
Kd	8.25E-01	1.19E+00	0.2025	0.2025	-0.2025	-0.2408
Md	1.20E+02	2.00E+00	0.3398	0.3398	-0.36734	-0.36854

Primary (IY)			β =	10.45		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mi	5.10E+01	-7.00E+00	-0.6698	-0.6698	1.0969	-4.731
Ms	7.44E+00	1.41E+00	0.1349	0.1349	-0.1349	-0.1902
Mw	3.80E+01	7.06E+00	0.6752	0.6752	-0.39377	-2.43191
Kw	1.15E+00	2.91E+00	0.278	0.278	-0.278	-0.808

Primary (ULT)			β =	6.75		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mu	3.82E+01	-4.52E+00	-0.6686	-0.6686	0.9772	-3.0698
Ms	7.15E+00	1.09E+00	0.162	0.162	-0.162	-0.1771
Mw	2.86E+01	4.60E+00	0.681	0.681	-0.37603	-1.68409
Kw	1.09E+00	1.70E+00	0.251	0.251	-0.251	-0.4255

Secondary			β =	6.74		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	1.50E+01	-4.34E+00	-0.6444	-0.6444	0.9306	-2.8447
SMb	2.49E+01	-1.74E+00	-0.2582	-0.2582	0.2767	-0.4586
Ms	7.13E+01	1.07E+00	0.1591	0.1591	-0.1591	-0.1704
Kw	1.08E+00	1.63E+00	0.2417	0.2417	-0.2417	-0.3932
Mw	2.80E+02	4.43E+00	0.659	0.659	-0.36453	-1.57931

Tertiary			β =	8.06		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	1.68E+01	-5.26E+00	-0.6528	-0.6528	1.0028	-3.4835
SMb	2.46E+01	-2.11E+00	-0.2617	-0.2617	0.2843	-0.562
Ms	7.23E+01	1.18E+00	0.1462	0.1462	-0.1462	-0.1724
Kw	1.10E+00	1.98E+00	0.2456	0.2456	-0.2456	-0.4862
Mw	3.10E+02	5.25E+00	0.6509	0.6509	-0.35946	-1.80154

Primary (IY)			β =	7.92		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mi	6.93E+01	-4.56E+00	-0.5751	-0.5751	0.8159	-2.6623
Ms	5.52E+00	-6.75E-01	-0.0851	-0.0851	0.0851	-0.0575
Mw	4.91E+01	4.95E+00	0.6242	0.6242	-0.34416	-1.64326
Md	2.00E+01	2.86E+00	0.3605	0.3605	-0.39078	-0.52705
Kw	1.12E+00	2.44E+00	0.3081	0.3081	-0.3081	-0.7521
Kd	8.81E-01	1.73E+00	0.2178	0.2178	-0.2178	-0.3759

Primary (ULT)			β =	4.27		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mu	4.63E+01	-2.48E+00	-0.5771	-0.5771	0.7265	-1.4801
Ms	5.69E+00	-4.95E-01	-0.1152	-0.1152	0.1152	-0.057
Mw	3.68E+01	2.68E+00	0.6243	0.6243	-0.38976	-0.99302
Md	1.51E+01	1.53E+00	0.3553	0.3553	-0.39078	-0.3019
Kw	1.07E+00	1.31E+00	0.3049	0.3049	-0.3049	-0.399
Kd	7.95E-01	9.09E-01	0.2116	0.2116	-0.2116	-0.1923

Secondary			β =	3.75		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	1.91E+01	-2.12E+00	-0.5625	-0.5625	0.6884	-1.2448
SMd	2.26E+01	-8.51E-01	-0.2254	-0.2254	0.2335	-0.2007
Ms	5.73E+01	-4.55E-01	-0.1205	-0.1205	0.1205	-0.0548
Kw	1.06E+00	1.14E+00	0.3021	0.3021	-0.3021	-0.3447
Mw	3.49E+02	2.26E+00	0.5985	0.5985	-0.39865	-0.83039
Kd	7.83E-01	7.95E-01	0.2104	0.2104	-0.2104	-0.1672
Md	1.45E+02	1.33E+00	0.3529	0.3529	-0.39245	-0.26272

Tertiary			β =	3.71		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	1.90E+01	-2.10E+00	-0.5625	-0.5625	0.6872	-1.2333
SMd	2.26E+01	-8.43E-01	-0.2254	-0.2254	0.2335	-0.1989
Ms	5.73E+01	-4.53E-01	-0.121	-0.121	0.121	-0.0548
Kw	1.06E+00	1.13E+00	0.3025	0.3025	-0.3025	-0.3422
Mw	3.48E+02	2.24E+00	0.5978	0.5978	-0.39976	-0.82232
Kd	7.83E-01	7.88E-01	0.2107	0.2107	-0.2107	-0.166
Md	1.45E+02	1.32E+00	0.3534	0.3534	-0.39312	-0.26085

Primary (IY)			β =	7.4		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mi	6.35E+01	-4.56E+00	-0.6161	-0.6161	0.8742	-2.8542
Ms	6.82E+00	7.38E-01	0.0997	0.0997	-0.0997	-0.0735
Mw	5.14E+01	5.41E+00	0.7304	0.7304	-0.40398	-2.07653
Kw	1.10E+00	2.06E+00	0.2775	0.2775	-0.2775	-0.5705

Primary (ULT)			β =	4.09		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Mu	4.61E+01	-2.63E+00	-0.6422	-0.6422	0.8182	-1.7434
Ms	6.63E+00	5.28E-01	0.1289	0.1289	-0.1289	-0.068
Mw	3.74E+01	2.90E+00	0.7089	0.7089	-0.43065	-1.2012
Kw	1.05E+00	1.07E+00	0.2616	0.2616	-0.2616	-0.28

Secondary			β =	4.16		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	1.79E+01	-2.59E+00	-0.6225	-0.6225	0.7907	-1.6666
SMb	2.56E+01	-1.04E+00	-0.2494	-0.2494	0.2603	-0.2687
Ms	6.63E+01	5.23E-01	0.1256	0.1256	-0.1256	-0.0657
Kw	1.05E+00	1.06E+00	0.2535	0.2535	-0.2535	-0.2674
Mw	3.72E+02	2.85E+00	0.6857	0.6857	-0.41855	-1.14565

Tertiary			β =	5.43		
	x*	u*	αλπηα	γαμμα	δελτα	ετα
Su	2.03E+01	-3.35E+00	-0.6173	-0.6173	0.831	-2.1215
SMb	2.53E+01	-1.34E+00	-0.2474	-0.2474	0.2612	-0.3422
Ms	6.70E+01	6.03E-01	0.1109	0.1109	-0.1109	-0.0668
Kw	1.07E+00	1.37E+00	0.2519	0.2519	-0.2519	-0.3448
Mw	4.19E+02	3.77E+00	0.6942	0.6942	-0.39243	-1.45283

Primary (IY)			β =	6.75		
	x*	u*	α	γ	δ	η
Mi	4.72E+01	-3.76E+00	-0.5563	-0.5563	0.7493	-2.132
Ms	4.64E+00	-6.82E-01	-0.101	-0.101	0.101	-0.0689
Mw	3.50E+01	4.33E+00	0.6411	0.6411	-0.35567	-1.50656
Md	1.41E+01	2.42E+00	0.3584	0.3584	-0.38524	-0.45695
Kw	1.10E+00	2.08E+00	0.308	0.308	-0.308	-0.6409
Kd	8.52E-01	1.45E+00	0.2144	0.2144	-0.2144	-0.3107
Primary (ULT)			β =	5.1		
	x*	u*	α	γ	δ	η
Mu	3.88E+01	-2.95E+00	-0.5766	-0.5766	0.7532	-1.7516
Ms	4.71E+00	-5.88E-01	-0.1148	-0.1148	0.1148	-0.0675
Mw	3.04E+01	3.23E+00	0.6311	0.6311	-0.37086	-1.16609
Md	1.23E+01	1.79E+00	0.3495	0.3495	-0.37985	-0.34412
Kw	1.08E+00	1.54E+00	0.3009	0.3009	-0.3009	-0.4638
Kd	8.12E-01	1.07E+00	0.2081	0.2081	-0.2081	-0.2218
Secondary			β =	3.74		
	x*	u*	α	γ	δ	η
Su	1.36E+01	-2.11E+00	-0.5577	-0.5577	0.6815	-1.2237
SMd	2.42E+01	-8.43E-01	-0.2233	-0.2233	0.2313	-0.1971
Ms	4.78E+01	-4.98E-01	-0.1318	-0.1318	0.1318	-0.0656
Kw	1.06E+00	1.14E+00	0.3029	0.3029	-0.3029	-0.3463
Mw	2.68E+02	2.27E+00	0.6003	0.6003	-0.39975	-0.83546
Kd	7.84E-01	7.96E-01	0.2109	0.2109	-0.2109	-0.1679
Md	1.12E+02	1.34E+00	0.3538	0.3538	-0.39347	-0.26394
Tertiary			β =	4.38		
	x*	u*	α	γ	δ	η
Su	1.47E+01	-2.46E+00	-0.5579	-0.5579	0.7011	-1.4191
SMd	2.40E+01	-9.84E-01	-0.2234	-0.2234	0.2327	-0.2286
Ms	4.74E+01	-5.41E-01	-0.1227	-0.1227	0.1227	-0.0663
Kw	1.07E+00	1.31E+00	0.2977	0.2977	-0.2977	-0.3904
Mw	2.83E+02	2.69E+00	0.6101	0.6101	-0.38092	-0.97284
Kd	7.96E-01	9.10E-01	0.2066	0.2066	-0.2066	-0.1881
Md	1.16E+02	1.53E+00	0.3468	0.3468	-0.38165	-0.29529

Primary (IY)			β =	7.77		
	x*	u*	α	γ	δ	η
Mi	3.85E+01	-4.94E+00	-0.6359	-0.6359	0.9238	-3.1848
Ms	6.01E+00	1.10E+00	0.1415	0.1415	-0.1415	-0.1554
Mw	2.94E+01	5.51E+00	0.7089	0.7089	0.393276	-2.04672
Kw	1.11E+00	2.10E+00	0.2704	0.2704	-0.2704	-0.5679

Primary (ULT)			β =	6.22		
	x*	u*	α	γ	δ	η
Mu	3.33E+01	-4.11E+00	-0.6602	-0.6602	0.9381	-2.7628
Ms	5.89E+00	9.53E-01	0.1532	0.1532	-0.1532	-0.146
Mw	2.54E+01	4.30E+00	0.6906	0.6906	-0.3836	-1.61304
Kw	1.08E+00	1.57E+00	0.2524	0.2524	-0.2524	-0.3964

Secondary			β =	4.86		
	x*	u*	α	γ	δ	η
Su	1.11E+01	-3.10E+00	-0.638	-0.638	0.8428	-2.0319
SMb	2.62E+01	-1.24E+00	-0.2557	-0.2557	0.2689	-0.3277
Ms	5.80E+01	8.24E-01	0.1695	0.1695	-0.1695	-0.1397
Kw	1.06E+00	1.18E+00	0.242	0.242	-0.242	-0.2847
Mw	2.21E+02	3.23E+00	0.6635	0.6635	-0.39047	-1.2246

Tertiary			β =	5.96		
	x*	u*	α	γ	δ	η
Su	1.24E+01	-3.80E+00	-0.6366	-0.6366	0.8849	-2.4678
SMb	2.59E+01	-1.52E+00	-0.2552	-0.2552	0.2712	-0.3982
Ms	5.87E+01	9.21E-01	0.1543	0.1543	-0.1543	-0.1421
Kw	1.07E+00	1.45E+00	0.243	0.243	-0.243	-0.3521
Mw	2.44E+02	3.99E+00	0.6684	0.6684	-0.37487	-1.46687

Primary (IY)			β =	4.67		
	x*	u*	α	γ	δ	η
Mu	5.25E+01	-2.57E+00	-0.5478	-0.5478	0.6796	-1.4523
Ms	4.84E+00	-4.20E-01	-0.0894	-0.0894	0.0894	-0.0375
Mw	4.03E+01	3.04E+00	0.6477	0.6477	-0.3879	-1.13956
Md	1.64E+01	1.69E+00	0.3605	0.3605	-0.39297	-0.33748
Kw	1.07E+00	1.46E+00	0.3104	0.3104	-0.3104	-0.4524
Kd	8.06E-01	1.01E+00	0.2147	0.2147	-0.2147	-0.2164

Primary (ULT)			β =	3.09		
	x*	u*	α	γ	δ	η
Mu	4.33E+01	-1.85E+00	-0.5917	-0.5917	0.7078	-1.1467
Ms	4.91E+00	-3.30E-01	-0.1055	-0.1055	0.1055	-0.0348
Mw	3.47E+01	1.86E+00	0.5968	0.5968	-0.42864	-0.70684
Md	1.45E+01	1.15E+00	0.368	0.368	-0.41391	-0.23428
Kw	1.05E+00	9.83E-01	0.3147	0.3147	-0.3147	-0.3094
Kd	7.72E-01	6.86E-01	0.2194	0.2194	-0.2194	-0.1505

Secondary			β =	1.73		
	x*	u*	α	γ	δ	η
Su	1.51E+01	-1.04E+00	-0.5884	-0.5884	0.6567	-0.6684
SMd	2.46E+01	-4.17E-01	-0.2356	-0.2356	0.24	-0.1076
Ms	4.99E+01	-2.18E-01	-0.123	-0.123	0.123	-0.0268
Kw	1.03E+00	5.76E-01	0.3252	0.3252	-0.3252	-0.1872
Mw	3.13E+02	9.49E-01	0.5361	0.5361	-0.47899	-0.33378
Kd	7.41E-01	3.95E-01	0.2227	0.2227	-0.2227	-0.0879
Md	1.30E+02	6.64E-01	0.3746	0.3746	-0.4366	-0.11772

Tertiary			β =	2.39		
	x*	u*	α	γ	δ	η
Su	1.63E+01	-1.42E+00	-0.5824	-0.5824	0.6716	-0.878
SMd	2.44E+01	-5.67E-01	-0.2333	-0.2333	0.239	-0.1414
Ms	4.95E+01	-2.76E-01	-0.1136	-0.1136	0.1136	-0.0314
Kw	1.04E+00	7.68E-01	0.3161	0.3161	-0.3161	-0.2428
Mw	3.27E+02	1.35E+00	0.5569	0.5569	-0.4493	-0.49703
Kd	7.56E-01	5.33E-01	0.2193	0.2193	-0.2193	-0.1169
Md	1.37E+02	8.94E-01	0.3679	0.3679	-0.42159	-0.17467

Primary (IY)			β =	4.54		
	x*	u*	α	γ	δ	η
Mi	4.72E+01	-2.68E+00	-0.5907	-0.5907	0.7386	-1.6298
Ms	5.53E+00	4.86E-01	0.1073	0.1073	-0.1073	-0.0521
Mw	3.92E+01	3.41E+00	0.7515	0.7515	-0.43497	-1.44963
Kw	1.06E+00	1.24E+00	0.2734	0.2734	-0.2734	-0.3389

Primary (ULT)			β =	3.18		
	x*	u*	α	γ	δ	η
Mu	4.07E+01	-2.08E+00	-0.6503	-0.6503	0.7929	-1.4102
Ms	5.46E+00	3.94E-01	0.1233	0.1233	-0.1233	-0.0486
Mw	3.38E+01	2.24E+00	0.6991	0.6991	-0.46737	-0.96228
Kw	1.04E+00	8.65E-01	0.2705	0.2705	-0.2705	-0.234

Secondary			β =	1.89		
	x*	u*	α	γ	δ	η
Su	1.34E+01	-1.28E+00	-0.6678	-0.6678	0.7611	-0.9163
SMb	2.70E+01	-5.13E-01	-0.2676	-0.2676	0.2736	-0.1478
Ms	5.37E+01	2.75E-01	0.1437	0.1437	-0.1437	-0.0396
Kw	1.03E+00	5.31E-01	0.2775	0.2775	-0.2775	-0.1474
Mw	2.99E+02	1.19E+00	0.6203	0.6203	-0.52002	-0.48654

Tertiary			β =	3.03		
	x*	u*	α	γ	δ	η
Su	1.49E+01	-1.93E+00	-0.635	-0.635	0.7649	-1.2834
SMb	2.67E+01	-7.74E-01	-0.2545	-0.2545	0.2629	-0.2071
Ms	5.45E+01	3.77E-01	0.124	0.124	-0.124	-0.0468
Kw	1.04E+00	8.03E-01	0.264	0.264	-0.264	-0.2121
Mw	3.30E+02	2.03E+00	0.6685	0.6685	-0.46332	-0.8505

Primary (IY)			β =	6.26		
	x*	u*	α	γ	δ	η
Mi	1.92E+02	-3.52E+00	-0.5618	-0.5618	0.7653	-2.0216
Ms	2.11E+01	-1.65E+00	-0.2636	-0.2636	0.2636	-0.4349
Mw	1.77E+02	4.45E+00	0.7112	0.7112	-0.39547	-1.71949
Md	2.50E+01	8.83E-01	0.1409	0.1409	-0.16148	-0.06575
Kw	1.09E+00	1.80E+00	0.2868	0.2868	-0.2868	-0.5149
Kd	7.55E-01	5.26E-01	0.084	0.084	-0.084	-0.0442
Primary (ULT)			β =	5.83		
	x*	u*	α	γ	δ	η
Mu	1.79E+02	-3.43E+00	-0.5871	-0.5871	0.8159	-2.0632
Ms	2.19E+01	-1.57E+00	-0.2682	-0.2682	0.2682	-0.4197
Mw	1.68E+02	4.04E+00	0.6925	0.6925	-0.38927	-1.54294
Md	2.47E+01	8.18E-01	0.1402	0.1402	-0.16148	-0.05934
Kw	1.08E+00	1.62E+00	0.2778	0.2778	-0.2778	-0.4503
Kd	7.51E-01	4.88E-01	0.0835	0.0835	-0.0835	-0.0407
Secondary			β =			
	undefined					
Tertiary			β =			
	undefined					

Primary (IY)			β =	6.58		
	x*	u*	α	γ	δ	η
Mi	2.22E+02	-4.39E+00	-0.6681	-0.6681	0.9685	-2.987
Ms	5.20E+01	1.79E+00	0.2715	0.2715	-0.2715	-0.4848
Mw	1.57E+02	4.28E+00	0.6507	0.6507	-0.36142	-1.51417
Kw	1.08E+00	1.56E+00	0.2378	0.2378	-0.2378	-0.3717
Primary (ULT)			β =	3.32		
	x*	u*	α	γ	δ	η
Mu	1.67E+02	-2.39E+00	-0.7138	-0.7138	0.911	-1.7732
Ms	4.65E+01	1.17E+00	0.3505	0.3505	-0.3505	-0.4112
Mw	1.16E+02	1.88E+00	0.5626	0.5626	-0.40255	-0.67231
Kw	1.04E+00	7.57E-01	0.2262	0.2262	-0.2262	-0.1713
Secondary			β =	2.74		
	x*	u*	α	γ	δ	η
Su	9.16E+00	-1.94E+00	-0.6994	-0.6994	0.858	-1.4222
SMb	1.73E+02	-7.06E-01	-0.2549	-0.2549	0.2626	-0.1899
Ms	4.50E+02	1.00E+00	0.3618	0.3618	-0.3618	-0.3625
Kw	1.03E+00	6.13E-01	0.2215	0.2215	-0.2215	-0.1358
Mw	1.10E+03	1.43E+00	0.5156	0.5156	-0.40856	-0.48296
Tertiary			β =	4.21		
	x*	u*	α	γ	δ	η
Su	1.04E+01	-2.89E+00	-0.6824	-0.6824	0.908	-2.0322
SMb	1.71E+02	-1.05E+00	-0.2488	-0.2488	0.2598	-0.2716
Ms	4.79E+02	1.33E+00	0.3151	0.3151	-0.3151	-0.42
Kw	1.05E+00	9.18E-01	0.2172	0.2172	-0.2172	-0.1995
Mw	1.24E+03	2.41E+00	0.5709	0.5709	-0.37093	-0.83595

Primary (IY)			β =	5.88		
	x*	u*	α	γ	δ	η
Mi	2.34E+02	-3.86E+00	-0.6569	-0.6569	0.9175	-2.5894
Ms	4.93E+01	1.49E+00	0.2532	0.2532	-0.2532	-0.3769
Mw	1.72E+02	3.92E+00	0.6675	0.6675	-0.37518	-1.44522
Kw	1.07E+00	1.43E+00	0.2425	0.2425	-0.2425	-0.3457

Primary (ULT)			β =	2.67		
	x*	u*	α	γ	δ	η
Mu	1.75E+02	-1.94E+00	-0.7186	-0.7186	0.8818	-1.464
Ms	4.41E+01	9.07E-01	0.3359	0.3359	-0.3359	-0.3046
Mw	1.27E+02	1.51E+00	0.5606	0.5606	-0.43512	-0.55378
Kw	1.03E+00	6.41E-01	0.2376	0.2376	-0.2376	-0.1524

Secondary			β =	2.11		
	x*	u*	α	γ	δ	η
Su	9.61E+00	-1.50E+00	-0.7045	-0.7045	0.831	-1.1303
SMb	1.74E+02	-5.48E-01	-0.2568	-0.2568	0.2629	-0.151
Ms	4.26E+02	7.38E-01	0.3454	0.3454	-0.3454	-0.2548
Kw	1.03E+00	4.99E-01	0.2335	0.2335	-0.2335	-0.1164
Mw	1.22E+03	1.10E+00	0.5137	0.5137	-0.44145	-0.37241

Tertiary			β =	3.58		
	x*	u*	α	γ	δ	η
Su	1.09E+01	-2.46E+00	-0.6829	-0.6829	0.8769	-1.745
SMb	1.72E+02	-8.97E-01	-0.249	-0.249	0.2584	-0.2332
Ms	4.56E+02	1.08E+00	0.2991	0.2991	-0.2991	-0.3224
Kw	1.04E+00	8.15E-01	0.2262	0.2262	-0.2262	-0.1844
Mw	1.36E+03	2.07E+00	0.5754	0.5754	-0.39649	-0.74459

Primary (IY)			β =	5.87		
	x*	u*	α	γ	δ	η
Mi	1.24E+02	-3.81E+00	-0.6494	-0.6494	0.9038	-2.5279
Ms	1.19E+01	7.73E-01	0.1317	0.1317	-0.1317	-0.1019
Mw	9.30E+01	3.99E+00	0.6789	0.6789	-0.38068	-1.48869
Md	1.38E+01	8.10E-01	0.138	0.138	-0.15897	-0.05762
Kw	1.08E+00	1.60E+00	0.2723	0.2723	-0.2723	-0.4353
Kd	7.51E-01	4.83E-01	0.0822	0.0822	-0.0822	-0.0397

Primary (ULT)			β =	3.02		
	x*	u*	α	γ	δ	η
Mu	9.47E+01	-2.16E+00	-0.709	-0.709	0.8873	-1.6018
Ms	1.13E+01	5.21E-01	0.1707	0.1707	-0.1707	-0.0889
Mw	7.07E+01	1.83E+00	0.5989	0.5989	-0.43346	-0.69794
Md	1.27E+01	4.90E-01	0.1607	0.1607	-0.18975	-0.02958
Kw	1.04E+00	8.34E-01	0.2733	0.2733	-0.2733	-0.2278
Kd	7.31E-01	2.90E-01	0.0952	0.0952	-0.0952	-0.0276

Secondary			β =	3.24		
	x*	u*	α	γ	δ	η
Su	1.22E+01	-2.24E+00	-0.6842	-0.6842	0.8619	-1.5964
SMd	7.82E+01	-8.16E-01	-0.2495	-0.2495	0.2581	-0.2133
Ms	1.13E+02	5.34E-01	0.1632	0.1632	-0.1632	-0.0871
Kw	1.04E+00	8.62E-01	0.2636	0.2636	-0.2636	-0.2273
Mw	7.14E+02	1.91E+00	0.5843	0.5843	-0.41579	-0.70686
Kd	7.31E-01	2.99E-01	0.0914	0.0914	-0.0914	-0.0273
Md	1.28E+02	5.05E-01	0.1543	0.1543	-0.18196	-0.03009

Tertiary			β =	4.63		
	x*	u*	α	γ	δ	η
Su	1.39E+01	-3.10E+00	-0.666	-0.666	0.9014	-2.1222
SMd	7.73E+01	-1.13E+00	-0.2428	-0.2428	0.2542	-0.2835
Ms	1.16E+02	6.56E-01	0.1412	0.1412	-0.1412	-0.0927
Kw	1.06E+00	1.19E+00	0.2556	0.2556	-0.2556	-0.3037
Mw	8.07E+02	2.89E+00	0.6211	0.6211	-0.37812	-1.04899
Kd	7.41E-01	3.87E-01	0.0831	0.0831	-0.0831	-0.0322
Md	1.32E+02	6.50E-01	0.1398	0.1398	-0.16318	-0.04263

Primary (IY)				β =	5.01	
	x*	u*	α	γ	δ	η
Mi	1.23E+02	-3.38E+00	-0.6738	-0.6738	0.909	-2.3353
Ms	3.70E+01	1.77E+00	0.3528	0.3528	-0.3528	-0.625
Mw	8.13E+01	3.06E+00	0.6096	0.6096	-0.36408	-1.07824
Kw	1.06E+00	1.12E+00	0.2234	0.2234	-0.2234	-0.2506

Primary (ULT)				β =	2.82	
	x*	u*	α	γ	δ	η
Mu	1.02E+02	-2.07E+00	-0.7255	-0.7255	0.9002	-1.569
Ms	3.33E+01	1.19E+00	0.4167	0.4167	-0.4167	-0.4946
Mw	6.66E+01	1.43E+00	0.5034	0.5034	-0.39801	-0.47355
Kw	1.03E+00	6.15E-01	0.2161	0.2161	-0.2161	-0.1329

Secondary				β =	0.57	
	x*	u*	α	γ	δ	η
Su	8.63E+00	-4.21E-01	-0.7045	-0.7045	0.7475	-0.3717
SMb	1.01E+02	-1.54E-01	-0.2567	-0.2567	0.2588	-0.0496
Ms	2.75E+02	2.84E-01	0.4748	0.4748	-0.4748	-0.1348
Kw	1.01E+00	1.30E-01	0.2179	0.2179	-0.2179	-0.0284
Mw	5.89E+02	2.43E-01	0.4061	0.4061	-0.43698	-0.0162

Tertiary				β =	3.61	
	x*	u*	α	γ	δ	η
Su	1.10E+01	-2.53E+00	-0.6958	-0.6958	0.8985	-1.8241
SMb	9.76E+01	-9.22E-01	-0.2537	-0.2537	0.2635	-0.2437
Ms	3.45E+02	1.38E+00	0.3798	0.3798	-0.3798	-0.5243
Kw	1.04E+00	7.53E-01	0.2072	0.2072	-0.2072	-0.156
Mw	7.00E+02	1.87E+00	0.5141	0.5141	-0.369	-0.61003

Primary (IY)			β =	4.5	
	x*	u*	α	γ	δ	η
Mi	1.41E+02	-2.47E+00	-0.5486	-0.5486	0.6903	-1.4035
Ms	1.84E+01	-1.13E+00	-0.25	-0.25	0.25	-0.2816
Mw	1.34E+02	3.24E+00	0.7202	0.7202	-0.42275	-1.33529
Md	2.12E+01	7.01E-01	0.1556	0.1556	-0.18084	-0.05312
Kw	1.07E+00	1.31E+00	0.2917	0.2917	-0.2917	-0.3834
Kd	7.44E-01	4.17E-01	0.0926	0.0926	-0.0926	-0.0386
Primary (ULT)			β =	2.17		
	x*	u*	α	γ	δ	η
Mu	1.04E+02	-1.29E+00	-0.5859	-0.5859	0.6773	-0.8147
Ms	2.10E+01	-7.24E-01	-0.3292	-0.3292	0.3292	-0.2384
Mw	1.07E+02	1.40E+00	0.6364	0.6364	-0.50724	-0.58589
Md	1.98E+01	4.11E-01	0.1867	0.1867	-0.22182	-0.02268
Kw	1.03E+00	6.83E-01	0.3104	0.3104	-0.3104	-0.212
Kd	7.26E-01	2.43E-01	0.1103	0.1103	-0.1103	-0.0268
Secondary			β =	2.39		
	x*	u*	α	γ	δ	η
Su	1.34E+01	-1.38E+00	-0.572	-0.572	0.667	-0.848
SMd	7.92E+01	-5.04E-01	-0.2086	-0.2086	0.2132	-0.1133
Ms	2.08E+02	-7.61E-01	-0.315	-0.315	0.315	-0.2397
Kw	1.04E+00	7.27E-01	0.3009	0.3009	-0.3009	-0.2186
Mw	1.08E+03	1.52E+00	0.6292	0.6292	-0.48753	-0.62427
Kd	7.27E-01	2.57E-01	0.1064	0.1064	-0.1064	-0.0273
Md	1.99E+02	4.35E-01	0.1799	0.1799	-0.21338	-0.02528
Tertiary			β =	3.56		
	x*	u*	α	γ	δ	η
Su	1.56E+01	-2.03E+00	-0.5673	-0.5673	0.7019	-1.2073
SMd	7.85E+01	-7.41E-01	-0.2068	-0.2068	0.2134	-0.1613
Ms	1.95E+02	-9.69E-01	-0.2705	-0.2705	0.2705	-0.2621
Kw	1.05E+00	1.02E+00	0.2851	0.2851	-0.2851	-0.291
Mw	1.20E+03	2.39E+00	0.6675	0.6675	-0.4351	-0.9694
Kd	7.36E-01	3.44E-01	0.0959	0.0959	-0.0959	-0.033
Md	2.06E+02	5.79E-01	0.1617	0.1617	-0.18961	-0.04068

Primary (IY)			β =	5.77		
	x*	u*	α	γ	δ	η
Mi	1.24E+02	-3.28E+00	-0.5674	-0.5674	0.7594	-1.9061
Ms	8.35E+00	-6.61E-01	-0.1144	-0.1144	0.1144	-0.0756
Mw	1.23E+02	4.42E+00	0.7656	0.7656	-0.4239	-1.83126
Kw	1.08E+00	1.62E+00	0.2807	0.2807	-0.2807	-0.4552

Primary (ULT)			β =	3.98		
	x*	u*	α	γ	δ	η
Mu	9.89E+01	-2.34E+00	-0.5862	-0.5862	0.7448	-1.4253
Ms	8.66E+00	-5.37E-01	-0.1351	-0.1351	0.1351	-0.0726
Mw	1.02E+02	2.99E+00	0.7498	0.7498	-0.45128	-1.30073
Kw	1.06E+00	1.10E+00	0.2755	0.2755	-0.2755	-0.3023

Secondary			β =	0.61		
	x*	u*	α	γ	δ	η
Su	8.59E+00	-4.63E-01	-0.719	-0.719	0.7662	-0.4094
SMb	1.01E+02	-1.69E-01	-0.2621	-0.2621	0.2643	-0.0547
Ms	1.03E+02	1.22E-01	0.1898	0.1898	-0.1898	-0.0232
Kw	1.01E+00	1.84E-01	0.2859	0.2859	-0.2859	-0.0526
Mw	7.53E+02	3.51E-01	0.5446	0.5446	-0.56956	-0.07576

Tertiary			β =	3.57		
	x*	u*	α	γ	δ	η
Su	1.12E+01	-2.32E+00	-0.6477	-0.6477	0.8218	-1.5655
SMb	9.79E+01	-8.47E-01	-0.2361	-0.2361	0.2446	-0.2092
Ms	1.12E+02	4.82E-01	0.1343	0.1343	-0.1343	-0.0647
Kw	1.05E+00	9.09E-01	0.2536	0.2536	-0.2536	-0.2306
Mw	9.45E+02	2.38E+00	0.6652	0.6652	-0.434	-0.96386

Primary (IY)			$\beta =$	3.86		
	x*	u*	α	γ	δ	η
Mi	1.33E+02	-2.58E+00	-0.6667	-0.6667	0.8464	-1.7803
Ms	3.37E+01	1.25E+00	0.3223	0.3223	-0.3223	-0.4026
Mw	9.51E+01	2.43E+00	0.6283	0.6283	-0.40669	-0.92602
Kw	1.05E+00	9.25E-01	0.2386	0.2386	-0.2386	-0.2206
Primary (ULT)			$\beta =$	1.72		
	x*	u*	α	γ	δ	η
Mu	1.11E+02	-1.28E+00	-0.7334	-0.7334	0.8472	-1.0143
Ms	3.00E+01	6.76E-01	0.3866	0.3866	-0.3866	-0.2612
Mw	7.94E+01	8.83E-01	0.5055	0.5055	-0.45908	-0.28997
Kw	1.02E+00	4.18E-01	0.239	0.239	-0.239	-0.0998
Secondary			$\beta =$	-0.51		
	x*	u*	α	γ	δ	η
Su	9.39E+00	3.51E-01	-0.7137	-0.7137	0.697	0.1714
SMb	1.02E+02	1.28E-01	-0.2601	-0.2601	0.2593	0.0229
Ms	2.43E+02	-2.15E-01	0.4372	0.4372	-0.4372	0.0941
Kw	9.94E-01	-1.20E-01	0.244	0.244	-0.244	0.0293
Mw	7.17E+02	-2.04E-01	0.4151	0.4151	-0.50226	0.171996
Tertiary			$\beta =$	2.55		
	x*	u*	α	γ	δ	η
Su	1.19E+01	-1.80E+00	-0.7	-0.7	0.8485	-1.3306
SMb	9.86E+01	-6.57E-01	-0.2552	-0.2552	0.2624	-0.1778
Ms	3.15E+02	9.00E-01	0.3494	0.3494	-0.3494	-0.3144
Kw	1.03E+00	5.84E-01	0.2269	0.2269	-0.2269	-0.1326
Mw	8.34E+02	1.34E+00	0.5208	0.5208	-0.42116	-0.46011

APPENDIX I

THE LOGNORMAL FORMAT

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THE LOGNORMAL FORMAT

The lognormal distribution and the lognormal format are summarized in Sections D.1 and D.2. Section D.3 provides the derivations of the properties of the lognormal. These are provided because they are not well documented in the literature.

I.1 The Lognormal Distribution

Consider the random variable, X. If $Y = \ell n\, X$ has a normal distribution with mean and standard deviation (μ_Y, σ_Y), then X has a lognormal distribution with mean and standard deviation (μ_X, σ_X).

The probability density function of X is

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_Y\, x} \exp\left[-\frac{(\ell n\, x - \mu_Y)^2}{2\sigma_Y^2} \right] \tag{I.1}$$

The moments of Y in terms of the moments of X are,

$$\mu_Y = \ell n\, \tilde{X} \tag{I.2}$$

$$\sigma_Y^2 = \ell n\left(1 + C_X^2\right) \tag{I.3}$$

where C_X is the coefficient of variation (COV)

$$C_X = \frac{\sigma_X}{\mu_X} \tag{I.4}$$

and where the median of X, denoted as $\tilde{X}$, in terms of the mean value is

$$\tilde{X} = \frac{\mu_X}{\sqrt{1 + C_X^2}} \tag{I.5}$$

I.2 The Lognormal Format

Let $g(\underset{\sim}{X})$ be a function of the design factors, $\underset{\sim}{X}$. Define the failure function $g(\underset{\sim}{X})$ so that the failure condition is $g \leq 1$. Assume that $g(\underset{\sim}{X})$ is a multiplicative function of K random design factors

$$g(\underset{\sim}{X}) = B \prod_{i=1}^{K} X_i^{a_i} \tag{I.6}$$

where B and all a_i are constants. Let $Z = \ell n\ g$.

$$Z = \ell n\ B + \sum_{i=1}^{K} a_i\ \ell n\ X_i \tag{I.7}$$

Now assume that all X_i have lognormal distributions. Because X_i is lognormal, it follows that all $\ell n\ X_i$ are normal.

The sum of normally distributed random variables is also normal. thus the probability of failure can be written as

$$\begin{aligned} p_f &= P(g \leq 1) = P(Z \leq 0) \\ &= P\left(\frac{Z - \mu_Z}{\sigma_Z} \leq -\frac{\mu_Z}{\sigma_Z}\right) \end{aligned} \tag{I.8}$$

The term on the left hand side is the standard normal variate. Define the safety index as

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{I.9}$$

Then,

$$p_f = \Phi(-\beta) \tag{I.10}$$

where Φ is the standard normal distribution function. μ_Z and σ_Z are,

$$\mu_Z = \tilde{Z} = \ell n\ \tilde{g} = \ell n\left[B \prod_{i=1}^{K} \tilde{X}_i^{a_i}\right] \tag{I.11}$$

$$\sigma_Z^2 = \ell n\left[\prod_{i=1}^{K}\left(1+C_i^2\right)^{a_i^2}\right] \tag{I.12}$$

The tildes indicate median values and the C's are the COV's.

I.3 Derivations of the Properties of Lognormal Variables

Given X is lognormal with mean and standard deviation (μ_X, σ_X), and median and coefficient of variation, ($\tilde{X}$, C_X)

$Y = \ell n\, X$. Thus Y is normal with mean and standard deviation (μ_Y, σ_Y).

(1) Derive the expression for $f_X(x)$. The pdf of Y is,

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_Y}\exp\left[-\frac{\left(y-\mu_Y\right)^2}{\sigma_Y{}^2}\right]$$

In general, for a monotonically increasing function,

$$f_X(x) = f_Y(y)\left|\frac{dy}{dx}\right|$$

Here,

$$\frac{dy}{dx} = \frac{1}{x}$$

So that,

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_Y\, x}\cdot\exp\left[-\frac{\left(\ell n\, x-\mu_Y\right)^2}{2\sigma_Y{}^2}\right] \tag{I.13}$$

(2) Show that $\mu_Y = \ell n\,\tilde{X}$. The 50% point is the same for both X and Y.

$$\tilde{Y} = \ell n\,\tilde{X}$$

But, $\mu_Y = \tilde{Y}$ (because Y is normal; f_Y is symmetrical). So that,

$$\mu_Y = \ell n\,\tilde{X} \tag{I.14}$$

(3) Find the kth moment of X about the origin

$$E(X^k) = \frac{1}{\sqrt{2\pi}\,\sigma_Y x} \int_{-\infty}^{\infty} x^k \exp\left[-\frac{(\ell n\, x - \mu_Y)^2}{2\sigma_Y{}^2}\right] dx$$

After considerable manipulation, it can be shown that,

$$E(X^k) = \exp\left[k\mu_Y + \frac{k^2\,\sigma_Y{}^2}{2}\right] \qquad (I.15)$$

(4) Show that $\sigma_Y{}^2 = \ell n\left(1 + C_X{}^2\right)$

From the expression for $E(X^k)$,

$$\mu_X = E(X) = \exp\left[\mu_Y + \frac{\sigma_Y{}^2}{2}\right] \qquad (I.16)$$

$$E(X^2) = \exp\left[2\mu_Y + 2\sigma_Y{}^2\right]$$

Thus,

$$\sigma_X{}^2 = E(X^2) - \mu_X{}^2 = \exp\left[2\mu_Y + \sigma_Y{}^2\right]\left[\exp\sigma_Y{}^2 - 1\right]$$
$$= \mu_X^2\left[\exp\sigma_Y{}^2 - 1\right]$$

or $$\exp\sigma_Y{}^2 = 1 + \frac{\sigma_X{}^2}{\mu_X{}^2}$$

and thus $$\sigma_Y{}^2 = \ell n\left(1 + C_X^2\right) \qquad (I.17)$$

(5) Derive an expression for μ_Y in terms of the moments of X.

From Eq. I.16

$$\mu_Y = \ell n\, \mu_X - \frac{1}{2}\sigma_Y^2$$

From Eq. I.17

$$\mu_Y = \ell n\, \mu_X - \frac{1}{2}\, \ell n \left(1 + C_X^2\right)$$

(6) Show that $$\tilde{X} = \frac{\mu_X}{\sqrt{1 + C_X^2}}$$

From Eq. I.14

$$X = e^{\mu_Y} = \frac{\mu_X}{\exp\left[\frac{1}{2}\, \ell n \left(1 + C_X^2\right)\right]}$$

Thus,

$$\tilde{X} = \frac{\mu_X}{\sqrt{1 + C_X^2}} \qquad (I.18)$$

(7) Multiplicative Functions of Lognormal Variates are Lognormal. Consider

$$g = B \prod_{i=1}^{k} X_i^{a_i} \qquad (I.19)$$

B, a_i are constants. All X_i have lognormal distributions. Take the log of both sides of Eq. I.19,

$$\ell n\, g = \ell n\, B + \sum_{i=1}^{k} a_i\; \ell n\, X_i$$

Note that $Y_i = \ell n\, X_i$ has a normal distribution. Let $Z = \ell n\, g$. Then, Z also has a normal distribution. The mean of $\hat{Z}$ is,

$$\mu_Z = \ell n\, B + \sum a_i\; E(\ell n\, X_i)$$

But $E(\ell n\, X_i) = \mu_{Y_i} = \ell n\, \tilde{X}_i$

Thus

$$\mu_Z = \ell n \left[B \prod_{i=1}^{k} \tilde{X}_i^{a_i} \right] \qquad (I.20)$$

The variance of Z is,

$$\sigma_Z^2 = \sum a_i^2 \underbrace{V(\ell n\, X_i)}$$

where the variance of $\ell n\, X_i = V(\ell n\, X_i) \equiv \sigma_{Y_i}^2$. But from Eq. I.17, $\sigma_Y^2 = \ell n\left(1 + C_i^2\right)$, where C_i is the COV of X_i and

$$\sigma_Z^2 = \ell n\left[\prod_{i=1}^{k}\left(1 + C_i^2\right)^{a_i^2}\right] \tag{I.21}$$

I.4 Base 10 Logs

All of the previous discussions related to base C logs. A summary of the base 10 log relationships is given below.

$$\mu_Y = \log_{10} \tilde{X},\ \text{or,}$$

$$= \log_{10} \mu_X - \frac{1}{2}\log_{10}\left(1 + C_X^2\right)$$

$$\sigma_Y^2 = 0.434\ \log_{10}\left(1 + C_X^2\right)$$

$$\mu_X = 10^{\left\{\mu_Y + \frac{1}{2}\left(\sigma_Y^2/0.434\right)\right\}}$$

$$C_X = \sqrt{10^{\left(\sigma_Y^2/.434\right)} - 1}$$

APPENDIX J

CRITICAL DESIGN VARIABLES BASED ON SENSITIVITY ANALYSIS

Critical Variables

- Which variables have the most impact on reliability?
- Determined by ranking importance factors for critical cases

	Cruiser 1 **and** *Cruiser 2*	*SL-7* **and** *Tanker*
First	wave bending moment (M_w)	strength (M_U, $S_{u,2}$, or $S_{u,3}$)
Second	strength (M_U, $S_{u,2}$, or $S_{u,3}$)	wave bending moment (M_w)
Third	dynamic bending moment (M_d)	stillwater bending moment (M_{sw})

Top three most important variables

Task VIII -- Recommendations for Improvements

- *Goal:* Given the variables shown to be most important in the sensitivity analysis, determine
  - what actual design parameters go into each of these variables
  - how much control the naval architect has over these design parameters
- Examined four variables...
  - wave bending moment
  - structural strength (primary, secondary, and tertiary)
  - dynamic bending moment (slamming)
  - stillwater bending moment

Variable: Wave Moment

- Contributing Factors
  1. environmental condition (waves)
  2. operating conditions (speed, heading, operating area)
  3. hull form
  4. weight distribution (specifically, radii of gyration)
- Controllable?
  1. no, natural forces
  2. marginal, requires restricting operation of ship
  3. marginal, cause/effect relationship not well understood, restricted by mission-driven limitations (e.g.cargo requirements and shape of holds)
  4. marginal, very difficult to reduce radii of gyration

Variables: Strengths (M_u, $S_{u,2}$, $S_{u,3}$)

- Contributing Factors
  1. section modulus
  2. material yield strength
  3. stiffening system design
  4. quality control in construction
- Controllable?
  1. yes, alter scantlings
  2. yes, change material (caution: fatigue and buckling)
  3. yes, add more and/or stronger stiffeners (cost!)
  4. somewhat, high precision construction is very expensive

Variable: Dynamic Moment

- Contributing Factors
  1. environmental conditions
  2. operating conditions
  3. weight distribution (gyradius)
  4. shape of hull near bow (bow flare and flat of bottom forward)
- Controllable?
  1. no, natural forces
  2. marginal, requires restricting operation of ship
  3. marginal, very difficult to reduce radii of gyration
  4. yes, interactions well understood, changes are localized

Variable: Stillwater Bending Moment

- Contributing Factors
  1. weight distribution
  2. hull form (buoyancy distribution)
- Controllable?
  1. yes, modifying weights to match buoyancy distribution is much easier than trying to change the gyradius
  2. yes -- mostly, procedures for obtaining a desired sectional area curve by changing hull shape are well defined and widely understood, only limitation is <u>mission-driven constraints</u> on required volumes at different locations

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The following persons were members of the committee that represented the Ship Structure Committee to the Contractor as resident subject matter experts. As such they performed technical review of the initial proposals to select the contractor, advised the contractor in cognizant matters pertaining to the contract of which the agencies were aware, performed technical review of the work in progress and edited the final report.

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Ship Structure Committee Publications - A Special Bibliography This bibliography of SSC reports may be downloaded from the internet at: "http://www.dot.gov/dotinfo/uscg/hq/nmc/nmc/ssc1/index.htm".

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