AFOSR-TR-97-0481

REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	1 Oct 95	Final Technical Report 1 Aug 94 to 30 Sep 95

4. TITLE AND SUBTITLE
Coupled Fields and Mechanics in Material Space

5. FUNDING NUMBERS
F49620-92-J-0421

6. AUTHOR(S)
George Hermann

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Division of Applied Machanics
Department of Mechanical Engineering
Stanford University
Stanford, CA 94305-4040

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NA
110 Duncan Avenue, Suite B 115
Bolling AFB, DC 20332-8050

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

19971003 066

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT

Approval for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
During the period covered by this report, significient advances in several diverse areas have been achieved as outlined below under separate headings: 1. Damage Mechanics, 2. Functionally gradient Materials with Defects, 3. Problems in Heterogenization and 4. Eshelby's Energy Momentum Tensor. It is experimentally observed that brittle materials and structural elements fabricated with them, undergo development of damage (growing microcracks, microvoids, grain boundary sliding , etc.) as the severity of environment (e.g. loads) increases, and before gross fracture sets in. The heterogenization technique is a procedure, recently developed with partial support of AFOSR, of expressing the solution to a heterogeneous problem in terms of the solution to the corresponding homogeneous problem.

14. SUBJECT TERMS	15. NUMBER OF PAGES
	6
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

Final Technical Report

submitted to

Air Force Office of Scientific Research
Bolling Air Force Base
Washington, DC 20332-6448

of Research in the Field of

Coupled Fields and
Mechanics in Material Space

for the period

August 1, 1994 to September 30, 1995

submitted by

George Herrmann

Division of Applied Mechanics
Department of Mechanical Engineering
Stanford University
Stanford, CA 94305-4040

October 1995

During the period covered by this report, significant advances in several diverse areas have been achieved as outlined below under separate headings:

1. Damage Mechanics
2. Functionally Gradient Materials with Defects
3. Problems in Heterogenization
4. Eshelby's Energy Momentum Tensor

Damage Mechanics

It is experimentally observed that brittle materials and structural elements fabricated with them, undergo development of damage (growing microcracks, microvoids, grain boundary sliding, etc.) as the severity of environment (e.g. loads) increases, and before gross fracture sets in.

In collaboration with the late Professor J. Kestin of Brown University we succeeded during the last few years and with the sponsorship, in part, of the Air Force Office of Scientific Research, to develop a rudimentary model of damage based on thermodynamic considerations. This initial theory leads to results which are in encouraging agreement with the few experimental data which have been published so far.

In developing the initial elements of this theory we obviously based ourselves on the "classical" or "conventional" thermodynamics, as expounded by J. Kestin in a series of recent papers and who was one of the very few thermodynamicists interested in applications to solid mechanics.

It is the study of irreversible processes which leads thermodynamicists to rather divergent views. We adopted the following position: A distinction has to be made between intensive parameters which appear in physical space and those which describe states of constrained equilibrium in the Gibbsian phase (or state) space. The latter consists of a set of extensive variables, namely the internal energy, the external deformation variables and the internal deformation variables, which, by contrast to the intensive ones, can be measured (in principle) in equilibrium as well as in nonequilibrium. For a given system under study, the introduction of these different variables is based on physical insight, experimental findings, intuition, etc.

Since temperature and entropy can be defined (or introduced) only for reversible processes it is imperative to introduce the so-called principle of local state (or the method of local equilibrium). This principle is applied by associating with every nonequilibrium state an accompanying equilibrium state of equal values of internal energy, as well as external and internal deformation variables. It is then asserted that the temperature and entropy in physical space (irreversible processes) can be approximated by their values in the Gibbsian phase space (reversible processes) by standard, classical methods. A continuous sequence of accompanying equilibrium states may be called an accompanying reversible process and it is conceived as an adiabatic projection of the continuous sequence of nonequilibrium states which constitute the irreversible process. This allows to express the classical Gibbs equation in rate form and to derive explicit expressions for the rate of entropy production by eliminating the rate of internal energy between it and the energy balance equation.

The essential part of this methodology consists in the formulation of the Gibbs equation for the accompanying process in phase space. This is obtained from the knowledge of the physics of the situation and leads to the identification of the internal deformation variables (which can be observed and measured, but not controlled) and the imagined virtual (i.e. reversible) work done against them by the associated affinities.

Based on the methodology briefly summarized above we were successful in obtaining, for one-dimensional systems and for isothermal processes, a basic theory of damage in brittle solids. Starting with this experience we propose to derive a more complete theory, including non-isothermal processes and by extending it to two and three dimensions. To study relevant boundary and initial value problems we would like to establish the governing partial differential equations for a geometrically given region, and thus we should be able to study the evolution of damage not only in time, but also in the space of the material, for given loads and given geometry.

Functionally Gradient Materials with Defects

Functionally gradient materials (FGM) were first produced in Japan for the primary purpose of developing thermal protection for a future space plane which would have to withstand severe aerdynamic heating. Since that time it was found that FGM, whose material properties vary continuously in a particular direction, would have useful applications in a variety of industries. This writer, together with his associates, has already been involved in some aspects concerning stress analysis of FGM (e.g., crack mechanics in smoothly nonhomogeneous materials as well as composites with nonhomogeneous fibers) and has been invited to present a paper [1] at the 3rd International Symposium on Structural and Functional Gradient Materials held at the Swiss Federal Institute of technology in Lausanne, Switzerland, in October 1994.

The writer has also been invited to be a member of the Scientific Committee which organized another Conference on FGM, sponsored by the Engineering Foundation, held at the Congress Center in Davos, Switzerland, in August 1995. At this conference the writer presented an unusually well-received lecture on fracture analysis of Functionally Gradient Materials, emphasizing in particular the J-integral for such materials.

The micromechanical analysis is effected by employing the newly proposed methodology of heterogenization [2-4], whereby the solution to a heterogeneous problem is obtained by performing a transformation on the solution to a corresponding homogeneous problem.

We proceeded to write down the solution for a multilayered fiber perfectly bonded to an infinite matrix, which is subjected to arbitrary loading. The solution is then expressed in terms of the solution of the corresponding homogeneous problem i.e., when the fiber is absent and the matrix material still subjected to the same loading (singularities) occupies the whole space. We achieve this by exploiting a connection between the solution to the heterogeneous problem and a group structure on the set $(-1,1)$ of real numbers x such that $-1 < x < 1$.

We then considered the case where the shear modulus is a continuous function of r throughout an annular region. The problem is formulated in a manner that leads to a Riccati differential equation, the solution of which can be obtained by considering the limit of our multilayered fiber solution.

Finally, the results are illustrated by considering the problem of reducing stress concentration around holes.

Problems in Heterogenization

The heterogenization technique is a procedure, recently developed with partial support of AFOSR, of expressing the solution to a heterogeneous problem in terms of the solution to the corresponding homogeneous problem. In one study this methodology was applied to the problem, in plane elastostatics, of two circular inclusions of arbitrary radii and of different elastic moduli, and perfectly bonded to a matrix, of infinite extent, which is subjected to arbitrary loading. The solution was formulated in a manner which leads to governing functional differential equations, i.e., equations were then solved by employing novel techniques. Several illustrative examples are being worked out. Particular attention is devoted to the limiting, but important, cases of two rigid inclusions or circular holes. Moreover, the asymptotic behavior of the stress field at the closest points of these two defects as they approach each other is being investigated.

Some of the results were presented by invitation at the recent National Congress of Applied Mechanics and a paper is being prepared for journal publication.

In another study the effect of a dislocation in a fiber reinforced composite was investigated and presented by invitation at the ASME Annual Meeting in November 1994. A paper summarizing these findings is being completed and will be also submitted for journal publication.

Eshelby's Energy Momentum Tensor

Eshelby's static energy momentum tensor $b_{ij} = W\delta_{ij} - \sigma_{ik}u_{k,j}$ (where W is the strain energy density, σ_{ik} is the stress tensor and $u_{k,j}$ is the displacement gradient) or the material momentum tensor, as it is also called sometimes, plays a preponderant role in fracture and defect mechanics. Its most prominent appearence occurs in the integrand of the popular J-integral of fracture mechanics. Yet, in spite of its frequent use and as a great surprise, its properties have not been investigated heretofore. All what is usually known about a tensor, such as invariants, principal values, principal directions, physical significance of components, remained completely in the dark. We have succeeded now to carefully and rather completely investigate the various features of this tensor and are now in the process of preparing a manuscript to be submitted for publication.

References

[1] T. Honein, E. Honein, and G. Herrmann, "Stress Analysis of Functionally Gradient Materials Based on a Heterogenization Procedure," *3rd International Symposium on Structural and Functional Gradient Materials,* October 1994, Lausanne, Switzerland.

[2] E. Honein, T. Honein, and G. Herrmann, "On Two Circular Inclusions in Harmonic Problems," *Quarterly of Applied Mathematics,* Vol. L, No. 3, pp. 479-499, 1992.

[3] E. Honein, T. Honein, and G. Herrmann, "Further Aspects of the Elastic Field for Two Circular Inclusions in Antiplane Elastostatics," *Journal of Applied Mechanics*, **59**, pp. 774-779, 1992.

[4] T. Honein, E. Honein, and G. Herrmann, "Circularly Cylindrical and Plane Layered Media in Antiplane Elastostatics," *Journal Applied Mechanics,* **61**, pp. 243-249, 1994.

Publications

[1] E. Honein, T. Honein, and G. Herrmann, "Stress Analysis of Functionally Gradient Materials Based on a Heterogenization Procedure," *3rd International Symposium on Structural and Functional Gradient Materials,* October 1994, Lausanne, Switzerland.

[2] T. Honein, E. Honein, and G. Herrmann, "Circularly Cylindrical and Plane Layered Media in Antiplane Elastostatics," *Journal Applied Mechanics,* **61**, pp. 243-249, 1994.

[3] E. Honein, T. Honein, and G. Herrmann, "The Elastic Field of Two Circular Inclusions in Plane Elastostatics." To be published.

[4] E. Honein, T. Honein, and G. Herrmann, "The Effect of a Dislocation in a Fiber-Reinforced Composite." To be published.

[5] E. Honein, T. Honein, and G. Herrmann, "On a Thermodynamic Formulation of Damage in Solids with Application to Ceramics." Under preparation.

[6] T. Honein, and G. Herrmann, "Conservation Laws and Path-Independent Integral." To be published.

Number of Researchers Working with the Pricipal Investigator

Faculty (include the PI): 1
Engineering Research Associate: 1

Professional Honors

During the reporting period, Prof. George Herrmann was honored by being awarded a fellowship in a new program at the Mathematical Research Institute in Oberwolfach, Germany. This special fellowship, sponsered by the Volkswagen Foundation, is reserved for the new, so-called "Research-in-Pairs"-(RiP) program. Together with one of his former postdoctoral researchers, Dr. R. Kienzler, who is now on the faculty of the University of Breenen, Prof. George Herrmann spent three weeks in August/September at the above-named Institute, concentrating on research in the area of defect mechanics.