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A Twisted Turbine Blade Analysis for a Gas Turbine Engine

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Abstract

An analysis of a twisted turbine blade was performed to determine the regions of maximum stress and moment which occur on a typical gas turbine engine under variable revolutions per second. An extension of the Holzer and Myklestad methods was developed to simulate shear and moment effects occurring from the root of the blade to its outermost extremity. Under numerical simulations, shear force and moments of inertia were realized at changing revolutions per minute by solving the Den Hartog twisted blade equations.

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1. Introduction

The usual method for determining the natural frequencies or critical speeds of shafts or beams in bending is the iteration method of Stodola (Anderson 1967; Crandal 1956; Den Hartog 1956; Harris and Crede 1961; Kelly and Richman 1971; Meirovitch 1967; Timoshenko, Young, and Weaver 1967; Tong 1960; Vernon 1967; Andronow and Chaiken 1949; Cunningham 1958; Minorisky 1962; Stoker 1950) either in graph or numerical form. In addition, another extension was introduced called the Holzer method, which has been applied to flexural vibration. In the torsional problem, the angle and twisting moments are significant calculational quantities; in the flexural problem, there are usually the deflection, the slope, the bending moment, and the shear force, which are pertinent and relevant quantities. It is necessary to find the relations among these quantities from one section (station) to another along a turbine blade of constant cross section (nontwisted).

Field and point transfer matrices may also be used to obtain relations that govern the flexural motion and vibrations of turbine blades. The transfer matrices are suitable for solution by the Myklestad method (Tse 1978), which is essentially a Holzer procedure where the primary difference lies in the use of the equations for bending and the manipulation of these equations to solve readily in terms of the boundary conditions for the blade itself; in this particular case, a remainder slope is used as the test for the selection of the correct frequency.

However, the Myklestad, and/or Holzer method, can be extended to more complicated cases, for example, to a twisted turbine blade in which the principal axes of bending stiffness turn through an angle along the blade length. Then, a vibration in one plane is no longer possible (i.e., the motion in the axial direction is coupled to that in the tangential direction). Hence, the purpose of this report is to analyze the shear and moments of a typical twisted turbine blade as extracted from a gas turbine engine. The information yielded in this analysis will aid and assist the understanding of the mechanical vibrations anamolies associated with the Turbine Engine Diagnostics (TED) project (Helfman, Dumer, and Hanratty 1995) to develop acoustical signatures necessary for performance assessment and prediction.

2. Analysis

Let the axial direction be x, the tangential direction be y, and the radial direction along the blade be z. First calculate the mass per unit length, μ_1, the bending stiffnesses, I_z and I_y, in the axial and tangential directions, and the product of inertia, I_{xy}, or stiffness coupling. All of these are variable with the length z. Then cut the blade into a number of sections; take the average value of μ_1, I_z, I_y, and I_{xy} of each section and consider these values constant along each section. The vibration takes place in the x and y coordinates simultaneously so that there will be eight equations. There is cross coupling on account of the product of inertia I_{xy} term. The eight equations are given below without derivation (Den Hartog 1956).

$$S_{x,n+1} = S_{x,n} + m_n \omega^2 x_n, \tag{1}$$

$$S_{y,n+1} = S_{y,n} + m_n \omega^2 y_n, \tag{2}$$

$$M_{x,n+1} = M_{x,n} + S_{x,n+1} L, \tag{3}$$

$$M_{y,n+1} = M_{y,n} + S_{y,n+1} L, \tag{4}$$

$$x'_{n+1} = x'_n + \frac{L}{E\left(I_{xn} I_{yn} - I_{xyn}^2\right)} \left[I_{yn} \left(M_{x,n+1} - \frac{S_{x,n+1}}{2} L \right) + I_{xyn} \left(M_{y,n+1} - \frac{S_{y,n+1}}{2} L \right) \right], \tag{5}$$

$$y'_{n+1} = y'_n + \frac{L}{E\left(I_{xn} I_{yn} - I_{xyn}^2\right)} \left[I_{xn} \left(M_{y,n+1} - \frac{S_{y,n+1}}{2} L \right) + I_{xyn} \left(M_{y,n+1} - \frac{S_{x,n+1}}{2} L \right) \right], \tag{6}$$

$$x_{n+1} = x_n + x'_n L + \frac{L^2}{E(I_{xn} I_{yn} - I_{xyn}^2)} \left[I_{yn} \left(M_{x,n+1} - \frac{2}{3} S_{x,n+1} L \right) + I_{xyn} \left(M_{y,n+1} - \frac{2}{3} S_{y,n+1} L \right) \right], \tag{7}$$

$$y_{n+1} = y_n + y'_n L + \frac{L^2}{E(I_{xn} I_{yn} - I_{xyn}^2)} \left[I_{xn} \left(M_{y,n+1} - \frac{2}{3} S_{y,n+1} L \right) + I_{xyn} \left(M_{x,n+1} - \frac{2}{3} S_{x,n+1} L \right) \right]. \quad (8)$$

Here,

S_x, S_y represent the shear forces in the x and y directions, respectively;

M_x, M_y represent the moments in the x and y directions, respectively;

m_n is the mass of nth section or station of the twisted blade;

L is the total length of the twisted blade;

I_x, I_y represent the moment of inertia in the x and y directions, respectively;

and I_{xy} is the product of inertia.

The boundary conditions are as follows:

(1) At station zero (root of blade):

$$x = y = x' = y' = O. \quad (9)$$

The bending moments and shear forces at the root are unknown. Take $M_x = 1$, $M_y = M_{yo}$, $S_x = S_{xo}$, and $S_y = S_{yo}$ at the root.

(2) At the free end, we have

$$M_x = M_y = S_x = S_y = O. \tag{10}$$

The solution methodology will involve casting the set of eight equations in matrix notation to effectuate a solution for given values of the frequency, w. The matrix system to be solved is shown in Table 1 in which the coefficients C1, ... , C16 can be defined as:

$$C_1 = -L^2 I_y / [\, E(\, I_x I_y - I^2_{xy})\,] \tag{11}$$

$$C_2 = -2C_1 L / 3 \tag{12}$$

$$C_3 = -L^2 I_{xy} / [\, E(\, I_x I_y - I^2_{xy})\,] \tag{13}$$

$$C_4 = -2C_3 L / 3 \tag{14}$$

$$C_5 = -L^2 I_x / [\, E(I_x I_y - I^2_{xy})\,] \tag{15}$$

$$C_6 = -2C_5 L / 3 \tag{16}$$

$$C_7 = C_3 \tag{17}$$

$$C_8 = C_4 \tag{18}$$

$$C_9 = C_1 / L \tag{19}$$

$$C_{10} = C_9 L / 2 \tag{20}$$

$$C_{11} = C_3 / L \tag{21}$$

$$C_{12} = -C_{11} L / 2 \tag{22}$$

$$C_{13} = C_5 / L \quad (23)$$

$$C_{14} = -C_{13}L / 2 \quad (24)$$

$$C_{15} = C_3 / L \quad (25)$$

$$C_{16} = -C_{15}L / 2 \quad (26)$$

such that the index, n, has been omitted for convenience.

The solution was achieved through a matrix inversion technique in the IMSL suite of software residing on the Cray C90 in the Waterways Experimental Station. Several values of the angular velocity, ω were varied such as 1,000–9,000 rpm to obtain a realistic range of operating conditions in a gas turbine engine.

3. Results

The distribution of the x-component of the shear stress along various stations on the blade itself at numerous revolutions per minute is shown in Figure 1. Here, the maximum shear takes place next to the root of the blade and then diminishes toward the end or tip of the blade itself, which is very similar to a typical cantelever beam under a uniformly distributed load. Figure 2 depicts the distribution of the y-component of the shear stress at stations along the blade for variable revolutions per minute where again the maximum shear occurs immediately adjacent to the root of the blade as intuitively expected.

Figure 3 illustrates the axial distribution of the x-component moment of inertia for the blade at different stations and at variable revolutions per minute; clearly, it is seen that the moment becomes a maximum at around one third the distance from the root of the blade for the largest revolutions per minute (or for increasing revolutions per minute). Similarly, Figure 4 shows the maximum moment of inertia (y-component) occurring near the immediate vicinity of the root of the blade for large revolutions per minute.

Table 1.

$$
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -m_n\omega^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -m_n\omega^2 & 0 & 0 & 0 & 0 & 0 \\
0 & L & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -L & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & C_2 & 0 & C_4 & 0 & C_1 & 0 & C_3 & -1 & 1 & 0 & 0 & -L & 0 & 0 & 0 \\
0 & C_8 & 0 & C_6 & 0 & C_7 & 0 & C_5 & 0 & 0 & -1 & 1 & 0 & 0 & -L & 0 \\
0 & C_{10} & 0 & C_{12} & 0 & C_9 & 0 & C_{11} & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & C_{16} & 0 & C_{14} & 0 & C_{15} & 0 & C_{13} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
S_{x,n} \\ S_{x,n+1} \\ S_{y,n} \\ S_{y,n+1} \\ M_{x,n} \\ M_{x,n+1} \\ M_{y,n} \\ M_{y,n+1} \\ X_n \\ X_{n+1} \\ y_n \\ y_{n+1} \\ x'_n \\ x'_{n+1} \\ y'_n \\ y'_{n+1}
\end{bmatrix}
= 0
$$

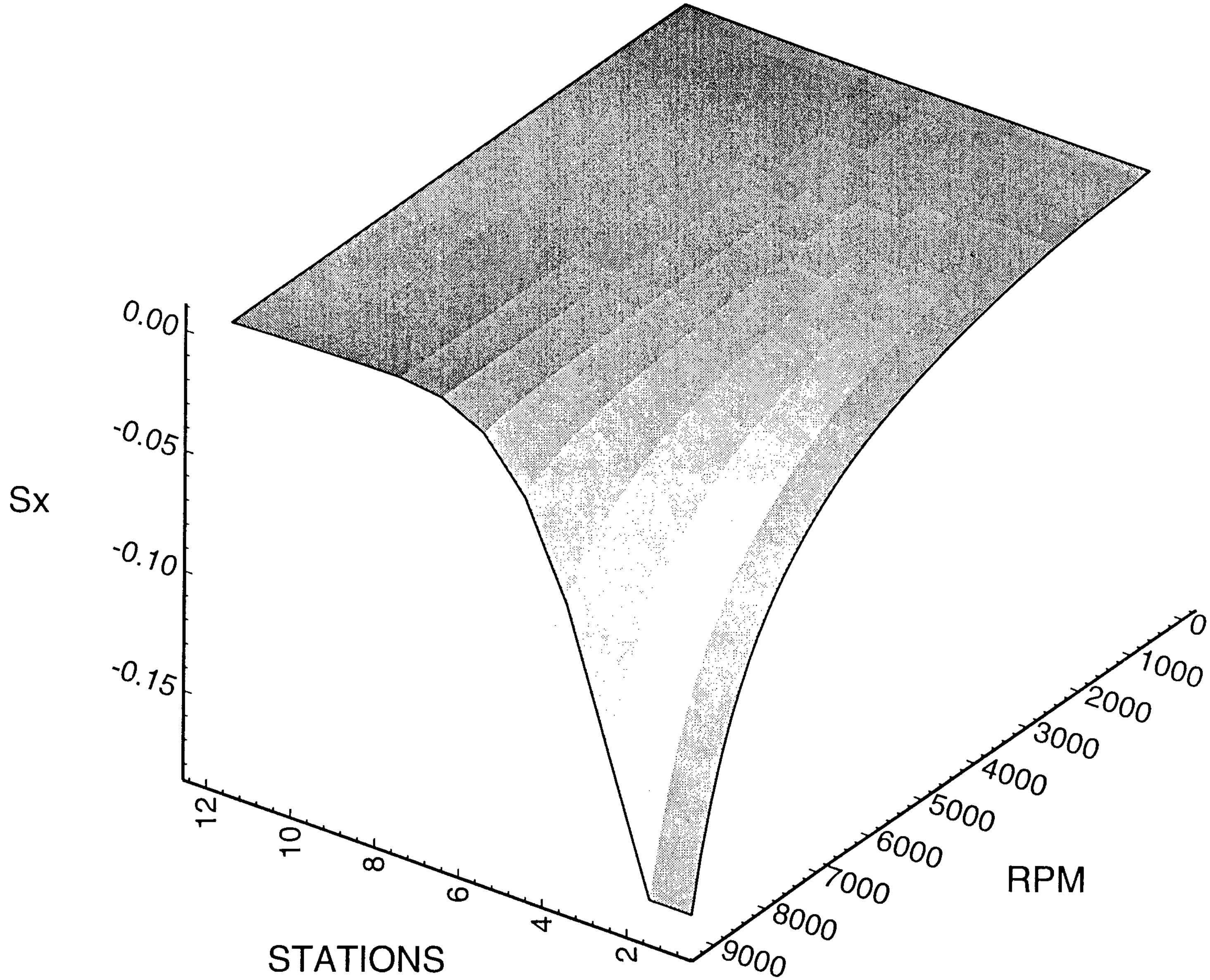

Figure 1. Distribution of Axial Shear Force Along Blade Stations for Variable Revolutions Per Minute.

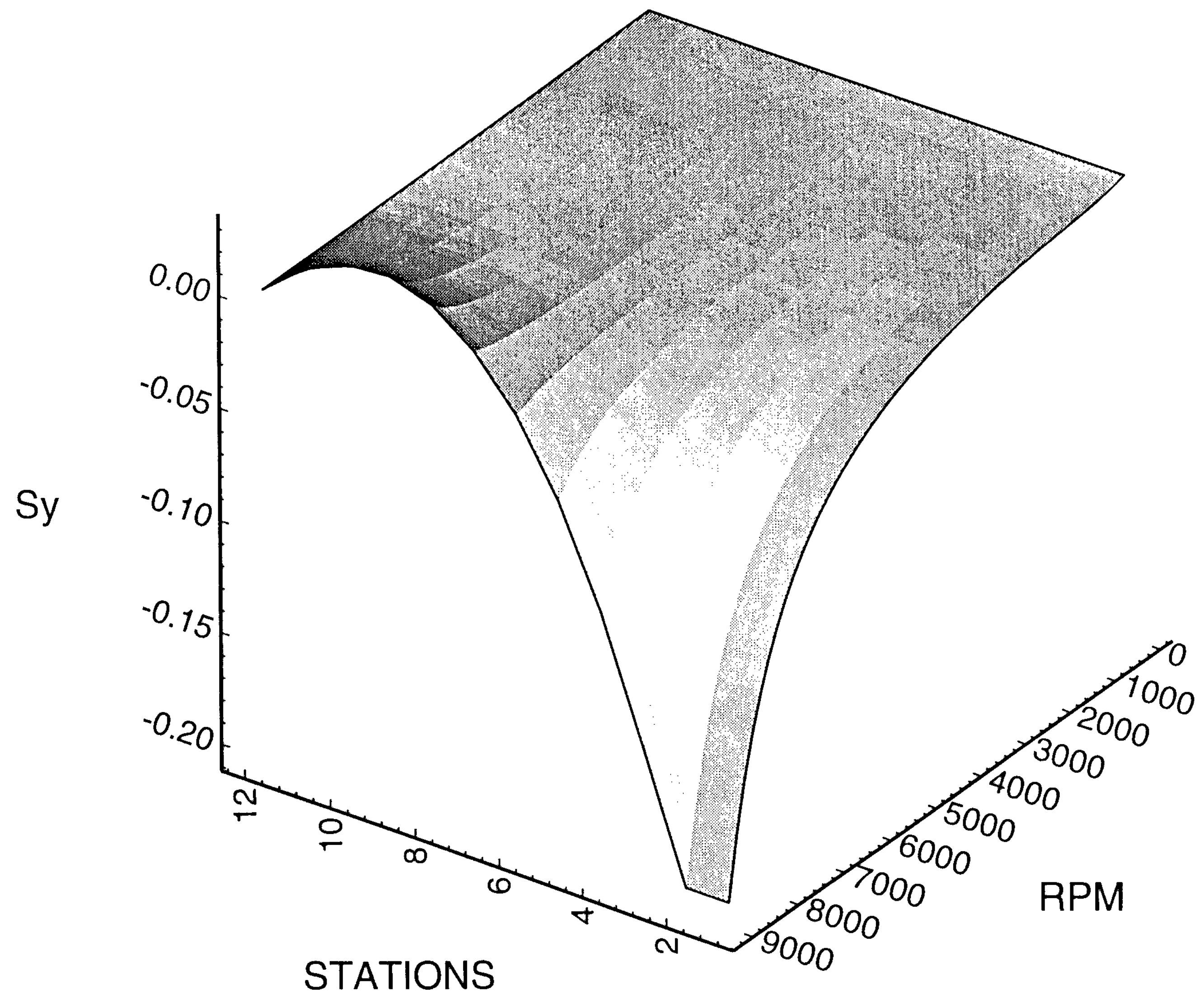

Figure 2. Distribution of Transverse Shear Force Along Blade Stations for Variable Revolutions Per Minute.

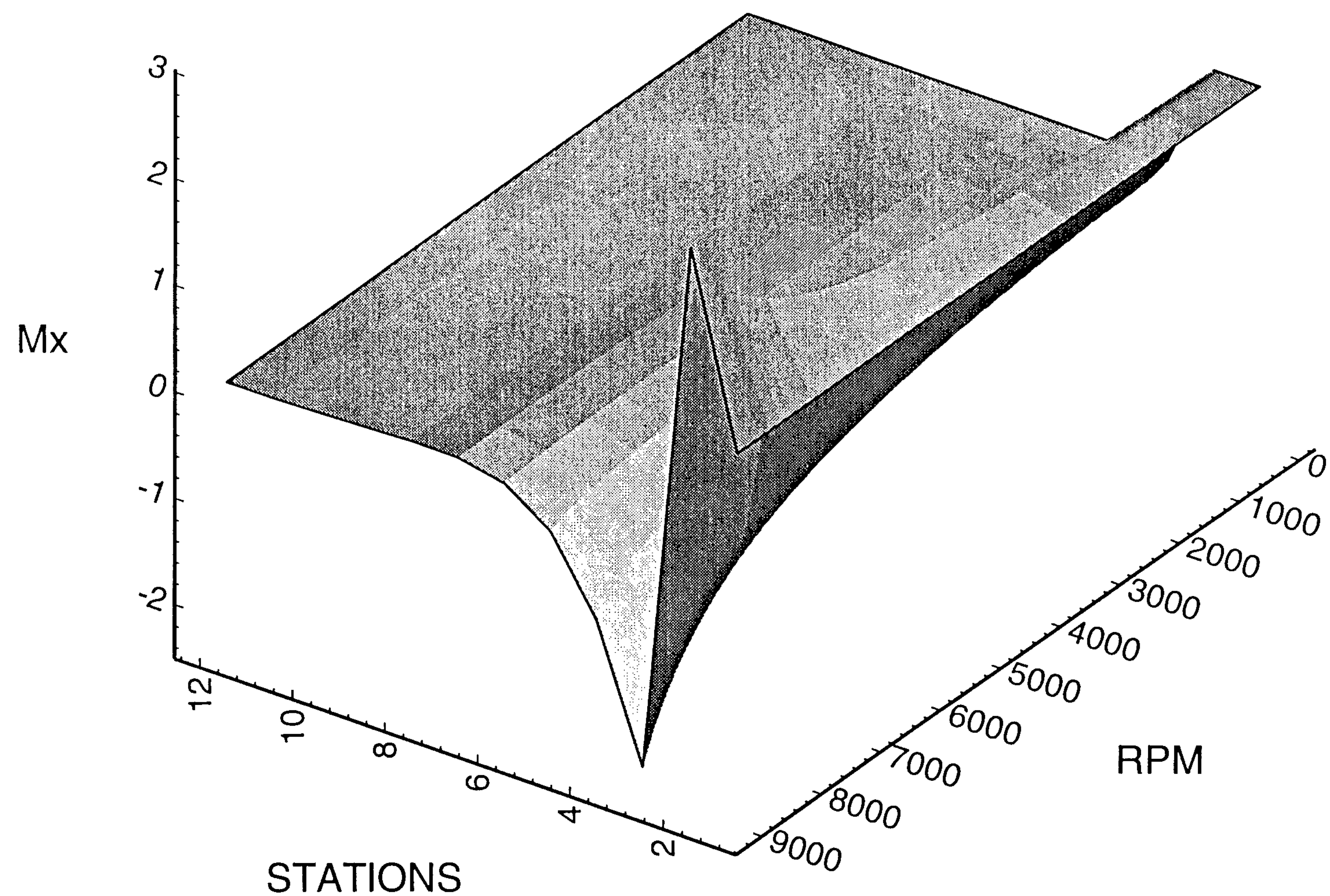

Figure 3. Distribution of Axial Moment Along Blade Stations for Variable Revolutions Per Minute.

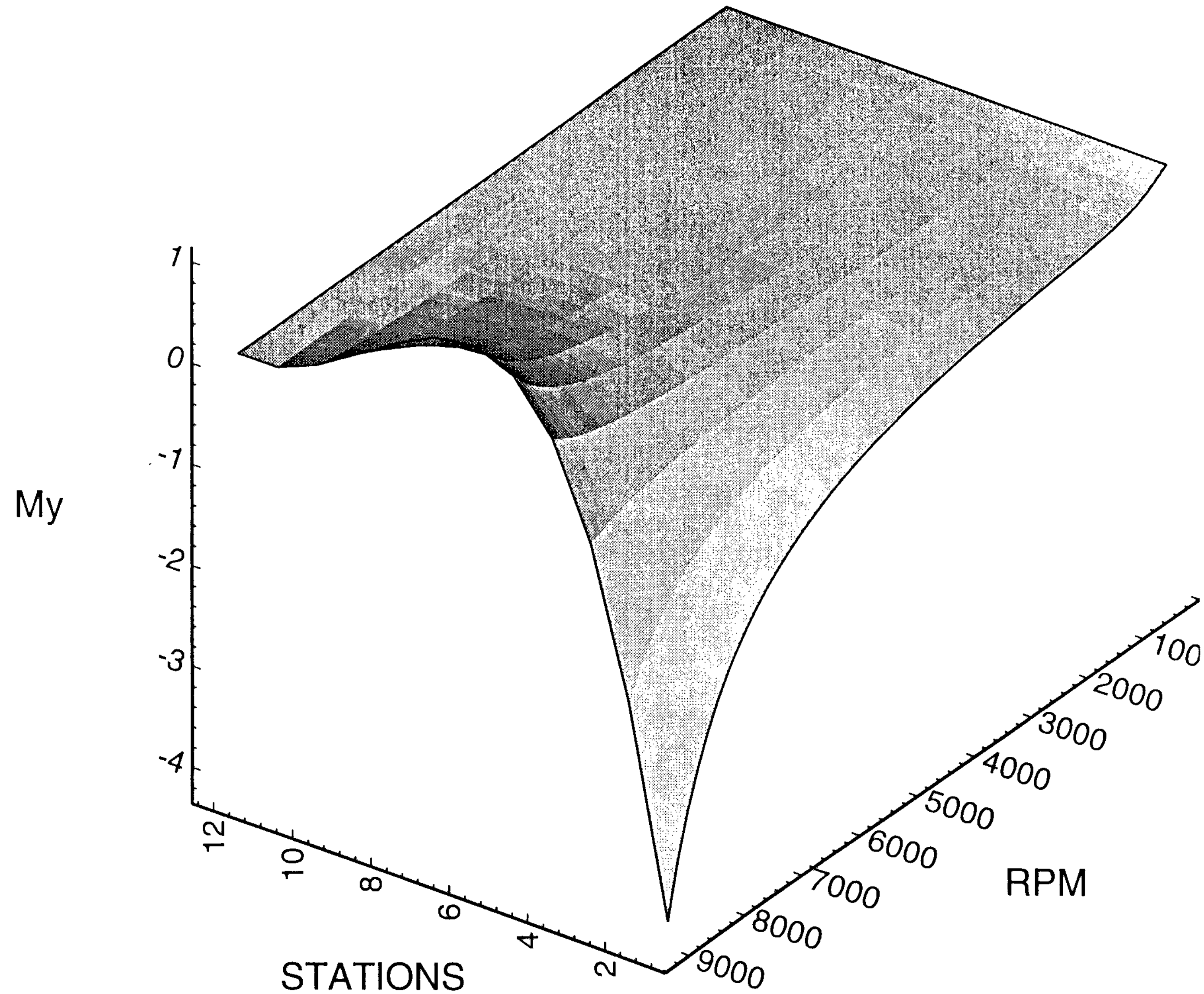

Figure 4. Distribution of Transverse Moment Along Blade Stations for Variable Revolutions Per Minute.

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