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RAND

Weapon Mix and Exploratory Analysis

A Case Study

Arthur Brooks, Steve Bankes, Bart Bennett

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Project AIR FORCE

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PREFACE

Over the last several years, a new approach to model-based analysis has been developed at RAND. This approach—exploratory analysis greatly expands on traditional analytic approaches, particularly sensitivity analysis, to enhance understanding of complex problems, provide a wider range of information for decisionmakers, improve comparisons among alternative modeling venues, and thereby enable greater comprehension of, and differences among, policy options.

This documented briefing reviews the methodology of exploratory analysis and its advantages over traditional analysis in the context of a search for the preferred weapon mix. We find that exploratory analysis can enhance decisionmaking flexibility, indicate robustness of options, neutralize risk, and provide greater understanding of the policy problem and model.

This work was conducted as part of the Theater Modeling Improvement Project sponsored by the Air Force Director of Modeling, Simulation, and Analysis (AF/XOM). The Air Force point of contact for the study is Col Ed Crowder of the Air Force Studies and Analysis Agency (AFSAA). A companion document, *Modeling and Simulation Infrastructure to Support Adaptive Planning in the Air Force*, DB-210-AF, describes exploratory analysis more generally and provides brief examples from other RAND work.¹

Project AIR FORCE

Project AIR FORCE, a division of RAND, is the Air Force federally funded research and development center (FFRDC) for studies and analyses. It provides the Air Force with independent analyses of policy alternatives affecting the development, employment, combat readiness, and support of current and future aerospace forces.

¹This methodology has had a number of other applications to date; for example, see Steve Bankes, "Computational Experiments and Exploratory Modeling," *CHANCE*, Vol. 7, No. 1, 1994, pp. 50–57.

Research is performed in three programs: Strategy and Doctrine, Force Modernization and Employment, and Resource Management and System Acquisition.

SUMMARY

This documented briefing discusses the advantages of exploratory analysis over more traditional model-based analysis in the context of the weapon mix problem. We illustrate this difference first by example and then by definition. In the example, we walk through a traditional analytic approach and show the kinds of results that are often observed. We then perform a different kind of analysis—based on a large number of computational experiments—on the same problem, and show that this kind of analysis provides more information and keener insights than we originally obtained. We continue by describing more generally this methodology, which we define as "exploratory analysis," and demonstrate its benefits to the decisionmaker and the analyst. We also discuss what is required to perform this type of analysis.

TRADITIONAL ANALYSIS VS. EXPLORATORY ANALYSIS: AN APPLICATION

We illustrate the value of exploratory analysis by applying it to a problem that is of particular relevance to the Department of Defense and the services: the weapon mix problem from the Deep Attack/ Weapon Mix Study (DAWMS). A best-estimate scenario is developed and a similar model to that in DAWMS is used to calculate the preferred weapon mix. We then perform sensitivity analysis to measure the impact of uncertainty in the weapon reliability, the sortie rate, and the deployment schedule. We find that the prescribed weapon mixes change in non-intuitive, seemingly erratic ways as we marginally change these inputs, which would appear to imply that either the model is faulty or that there are errors in the data. Either way, we are left with little confidence in the outcomes of the study.

When we extend the conventional approach by applying much broader computational experiments than those used originally, we discover the explanation for the seemingly erroneous results obtained earlier: The model and data are not causing the error; rather, the culprit is the way in which the model has been used. Changing and expanding the use of the model in ways carefully explained in this briefing provide a clear explanation of the previously troubling sensitivity analysis results. We call the method for using the model in this different way "exploratory analysis." Broadly stated, exploratory analysis is a methodology designed to help us comprehend complex systems, such as (in the case at hand) those generally represented in theater-level combat models with many imperfectly known parameters, decisions, and measures of effectiveness. For example, we begin by determining a range of desirable (or tolerable) outcomes in terms of such measures as time to complete a campaign or total casualties. We then explore the spaces of scenario conditions, decision options, or combinations that correspond to this range of outcome values. A much more comprehensive set of trades among options (than that gotten from the traditional analysis) can then be presented to illuminate the decisionmaking process.

In the case at hand, exploratory analysis provides the means for collecting greater information about the weapon mix problem and the tool we are using to evaluate alternative options. Further, we are able to determine the full range of weapon mix trades that accomplish campaign objectives; demonstrate how a decisionmaker can see the impact of imposing additional constraints (such as on cost or risk); suggest how a weapon mix could be selected that is robust (or as robust as possible) across contingencies; and reduce or neutralize the risk of uncertainties in the scenario. As this document shows, without exploratory analysis in the case examined here, the decisionmaker would have been left with unacceptable results, unnecessary expenses to fix a model that was not broken, and a weapon mix study that still needed to be done.

EXPLORATORY ANALYSIS EXTENDS TRADITIONAL ANALYSIS

To understand how exploratory analysis extends traditional scenariobased analysis, we now step back and more generally describe how both traditional analysis and exploratory analysis are performed.

Traditional Analysis

Traditional analysis determines the value or impact of a system or policy within the context of plausible, best-estimate current or future scenarios. To perform traditional analysis (as stated in the description of the present case study), we first define these scenarios. Concerns about uncertain or unknown scenario conditions and specific data items often stimulate large-scale efforts to determine precisely the best (or most acceptable) estimates of the data for the analysis as a whole and, in particular, the model(s) used to calculate performance measures. Sensitivity analysis is used to augment the best-estimate analysis to determine the impact of variations or errors in these estimates.

Frequently, the sensitivity analysis produces troubling results: changing the input conditions causes some output values to change erratically—and not infrequently in non-intuitive ways. Analysts and decisionmakers often interpret these outcomes as errors in the model, which sometimes leads to the model (and the analysis) being discredited. One recourse is to instigate a major (and most likely very costly) review of the model and/or the associated databases in an attempt to fix the errors or to increase "realism" by including a broader scope or more details. These efforts may make the model only more cumbersome and difficult to validate. It is often not possible to know what value has actually been added to the decisionmaking process.

Exploratory Analysis

Exploratory analysis differs from traditional analysis. Operationally, it changes the way the model is used: the model is run many times with many different input levels as opposed to one best-estimate case run followed by a (probably) fairly limited sensitivity analysis. Methodologically, the difference is starker: exploratory analysis represents a fundamentally different way of looking at the problem. While traditional analysis works in a sense from the inside out—the solution is found and then the area around it examined—exploratory analysis features an "outside in" approach: A large set of plausible scenario conditions, decision options, and desirable outcomes is examined, and then a preferred solution is selected.

As this document will make explicit, this methodology has a number of advantages over the traditional approach.

- Exploratory analysis can provide greater insights than can traditional methods, including the information that explains and sometimes resolves troubling sensitivity analysis results—without resorting to changes in the model or data.
- Exploratory analysis can help decisionmakers choose options that are robust across different scenario conditions, operational or technical preferences, and costs.
- Exploratory analysis can be an effective tool in the process of the verification and validation of the models used.

As systemic complexity is a feature central to many problems, exploratory analysis can be and has been useful in many contexts. Our case study makes its use clear for military modeling and simulation; it has also been used to address problems in areas as divergent as climate change, information infrastructure, and investment strategy. As we will show, however, there are limits on the scale of explorations we can undertake in different problems. To perform the computational experiments that are essential for undertaking this analysis (often consisting of tens or hundreds of thousands of computer runs), considerable computational horsepower must be made available to a study team. In addition, the means for defining, implementing, and managing the experiments as well as the vehicle for characterizing and analyzing the outputs must be provided. In other words, the use of exploratory analysis is often bound by infrastructural constraints.

In this documented briefing, we suggest ways to enhance the infrastructure for exploratory analysis and thus make the approach as practical as possible. We believe that the applicability of exploratory analysis is very wide and will provide marked improvements in how analysis can aid the decisionmaking process.

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This briefing presents an innovative approach to decisionmaking called exploratory analysis; we explain it in the context of a problem that is of particular importance to the Department of Defense and services.

1



The problem is taken from the Deep Attack/Weapon Mix Study (DAWMS), which entails a determination of the appropriate mix of weapons and forces for performing the deep attack mission. Specifically, we will look at two questions that are part of this study. First, what weapons are needed, given the set of scenarios judged most probable? Second, what would be the impact of changing the availability of these weapons?

While we look at an actual problem here, the purpose is not to recommend specific acquisition decisions. Rather, we are interested in comparing exploratory analysis to traditional analysis and demonstrating how this methodology could be applied in reaching such recommendations. Therefore, we have taken the liberty of using a model and a database that were readily available to us at the beginning of this study and, although similar, were not specifically used in DAWMS.



We will reach three basic conclusions here. First, traditional analysis of the weapon mix problem is useful but incomplete in providing information to the decisionmaker. We are not saying that traditional analysis is bad, but rather that it needs to be broadened.

Second, we will see that, on encountering counterintuitive results in this analysis, the standard remedies of seeking resolution within the model or via data modifications is simply wrong. The real problem is how the model has been used or, more properly, how it has not been used to its fullest extent. Further, if fixing the model or data has any ameliorating effect at all, it may not be the most efficient way to achieve better results.

Finally, we will find that exploratory analysis has four advantages over traditional analysis. First, this methodology increases flexibility for decisionmakers, including planners, operators, and budgeters (as we will demonstrate). Second, such modeling allows the assessment of the robustness of a particular weapon mix across different contingencies, including major regional conflicts, lesser regional conflicts, and missions other than war. Third, it allows us effectively to neutralize risk that comes from imperfect information, as well as an uncertain future. And fourth, we will see that this methodology affords us greater insights into the policy problem as well as a better understanding of the model.



We will compare exploratory analysis to traditional analysis by first demonstrating a traditional analysis of the weapon mix problem. This will lead to some troubling results. We will then explain what is really happening by applying exploratory analysis. In the third section, the methodology is defined and described in greater generality. In the fourth section, a variety of examples are given to help decisionmakers see what benefits exploratory analysis may provide. Finally, we discuss the potential and actual uses of exploratory analysis, as well as what is required for its routine application.

We now proceed with the first part, a traditional scenario-based analysis of the weapon mix problem, followed by some general conclusions from this analysis.



The first step in a traditional analysis is the selection of a scenario. Much of the work to determine the best weapon mix occurs in defining the scenarios. This involves the location and time of the operation, the systems under consideration (including the targets, threats, platforms, and weapons), the necessary constraints on the problem (such as attrition, concepts of operation (CONOPs), or budget), and measures of effectiveness (MOEs), such as the value of the targets destroyed or the time spent in completing the phases of the campaign. An analysis of the scenario ensues, which requires an estimation of the most likely conditions for the scenario—the best estimates of the values of the parameters in the model. As a second step in this process, the best weapon mix is determined, often calculated using a model, from the best-estimate scenario. As a result of examining the model output, changes to the base case may be made to correct for unforeseen outcomes.

The last step in the traditional analysis is to conduct sensitivity analysis. Sensitivity analysis examines variations to input values from the bestestimate (base) case. Its purpose is to gauge how sensitive outcomes are to small changes in the parameters, and in this way address the potential effects of uncertainty.



The analytic process described in the preceding chart is often undertaken with the aid of computer models. In our analysis, we, too, rely on a model to determine the best weapon mix. Our model process employs an optimizing model and is diagnosed above.¹

First, the measure of effectiveness is designated to drive the optimization. We may select from a variety of measures to drive the optimization, such as time to complete campaign phases, while we still capture and calculate other measures, such as targets destroyed or cost. Then, we input the systems (the quantity, quality, and characteristics of the aircraft, weapons, and targets) and constrain the model appropriately (limiting attrition or budget, or the manner in which the systems must perform). Next, we run the optimizer,² which produces three pieces of information. The first is the best possible value of our objective; the value of other measures of effectiveness can also be

¹The reader might question whether optimization is in fact the right tactic, given the uncertainty inherent in warfighting. The point is well-taken, but we proceed for two reasons. First, we are demonstrating the exploratory analysis approach in a given context; that given context assumes that requirements have been specified and an optimization will be performed. Second, the approach we discuss will work just as well (and in some respects, even better) with other types of models, such as simulations.

² The actual model used here was the Conventional Targeting Effectiveness Model (CTEM). We note again that DAWMS used a different optimization model; but any model, either prescriptive or descriptive, could be used.

calculated. Second, an allocation is given; this is a schedule indicating what aircraft/weapon combination attacks each target, how it is employed, and in what sequence. Third, we calculate the mix of weapons to accomplish the task. The weapon mix is probably the most important piece of information to us, as it represents the recommendation to the decisionmaker: it is the combination of weapons we need to attain the optimal value of our measure of effectiveness.



We now proceed to demonstrate the analytic and modeling process we have just described. In phase one, we determine the best estimate for all the inputs that define the scenario.

Our results are based on a sample: SWA 2001 scenario.³ The measure of effectiveness selected as the objective is the time to complete the entire campaign (which we will minimize). Data are taken from a variety of authoritative sources, as noted on the next slides.

To simplify our examination of this scenario, we have assumed some special features regarding attrition and weather that are particularly optimistic. However, this should not pose a problem, since our purpose, as we have already mentioned, is simply to demonstrate the difference between traditional and exploratory analysis.

³ This scenario was assembled by Keith Henry at RAND. We note that this is not one of the DAWMS scenarios.

Best-Estima	te Results			
OBJECTIVE CAMPAIGN IS COMPLETED	N 22 DAYS			
DECISIONS				
TOTAL Weapon 1 USED:	15,502			
TOTAL Weapon 2 USED:	6,960			
TOTAL Weapon 3 USED:	5,003			
		9	10/96	RAND

Once the scenario is completely defined, we move on to step two in the analytic process. Running the model with all the input information described produces the results shown here: The minimal campaign completion time is 22 days. To achieve this best-estimate result, the model tells us to inform the decisionmaker that 15,502 of Weapon 1 should be in inventory, along with 6,960 of Weapon 2 and 5,003 of Weapon 3 (among other munitions).⁴

A striking observation is the recommendation to acquire so many of Weapon 1. Given this, one wonders what would happen if Weapon 1 were not to work as advertised—suppose jamming or some other impediment to Weapon 1's effectiveness were to occur. Would this have a detrimental impact on the ability to complete the campaign in a timely fashion? These and other questions lead us into step three of the traditional analysis: sensitivity analysis.

⁴ The model determines the quantities for each weapon type. We have aggregated these into the three weapon classes (Weapon 1, Weapon 2, and Weapon 3) as a matter of convenience.



To undertake sensitivity analysis, we need first to decide on the parameters to vary. The criterion for choosing a particular parameter generally is an *a priori* belief that a change in it will result in some change in our measure of effectiveness.

In the previous chart, we saw that the prescribed level of Weapon 1 was quite high relative to Weapon 2 and Weapon 3. Therefore, we would likely expect a change in the effectiveness of these weapons to have a disproportionately great effect on our ability to complete the campaign in 22 days; hence, the first parameter we choose to vary is Weapon 1 reliability. We scale back this reliability to see how it affects the number of days to complete the campaign.

The other parameters we examine affect the number of weapons that can be delivered on targets within a set amount of time: the sortie rate and the time-phased force deployment (TPFD) schedule. As a first attempt to understand the effect of sortie rate changes, we will simply change the daily rate across the board without any preferential treatment of any particular aircraft. Changes in the deployment schedule are also made without preference for any platform type. Here, we degrade the entire schedule: The graph above on the left shows the baseline TPFD schedule; the graph on the right reflects platforms arriving at the theater of operations on the same schedule, but with 20 percent fewer arriving on each day. Note that the schedule is cumulative—by the 24th day, all the platforms that are coming have arrived. It is also monotonic, because the model has no attrition. In sum, in this initial sensitivity analysis we measure the variance of the parameters as a simple degradation from their baseline (best-estimate) levels.



The results of this sensitivity analysis can be viewed graphically. It is easiest to visualize the results by holding one of the parameters constant and by representing changes in it with a series of graphs. Thus, we hold sorties constant at 100 percent (we have left sorties at their baseline level). The above chart shows the effect on the objective (days to complete the campaign) from varying the Weapon 1 reliability and the deployment schedule. The original, best-estimate solution value—22 days to complete—can be found in the front corner, where the two parameters are at their best-estimate (100 percent) values.

The graph is monotonically increasing over its entire range—as either parameter is degraded independently, the value of the objective always increases, and takes longer to complete the campaign. This makes intuitive sense, as might the fact that degrading the deployment schedule seems to have relatively greater impact than does poor Weapon 1 reliability (since we can substitute between weapons when Weapon 1 weapons are less effective). It can be shown that degrading the sortie rate has a similarly monotonically increasing relationship with the objective.

The slope of the graph is also quite gradual. All of this is good news from the point of view of traditional sensitivity analysis: The gentle grade means that a small error in measuring the parameters will result in a small error in the value of the objective; the monotonicity means that this error will also be predictable (we know that estimating a parameter level too optimistically will always result in an increase in the days to complete, for example).



The results on the previous chart would represent a rather thorough sensitivity analysis by the standards of many studies. We might go even further, though, and question whether the apparent smoothness of the response surface is an artifact of the coarseness with which we have sampled from the parameter space. That is, some roughness in the graph might be masked if it happens to fall between the tick marks designated. For this reason, we undertake a more detailed sensitivity analysis.

In this chart, sorties are degraded to 60 percent, and we look at deployment between 60 and 90 percent (drawn down in 5-percent increments) and Weapon 1 reliability between 100 and 50 percent (also in 5-percent increments). Note that the 60 percent sortie level has raised the surface (it takes longer to complete the campaign). This graph reinforces the good news of the last: the results are monotonic and gently sloping.⁵ A small parameter misestimate will lead to a small, predictable error in the measure of effectiveness.

Unfortunately, these intuitive observations are not the end of the story. Recall that the recommendation we wish to give the decisionmaker is not the number of days to complete the campaign, but rather how

⁵Although the chart may appear to be a little "bumpy," we can assure the reader that it is really monotonic. The apparent roughness occurs because of the perspective-drawing limitations of the drawing software.

many of each type of munition to have in inventory. Hence, we should be concerned with not only the sensitivity of the objective, but also of the decisions: how many weapons do we need if conditions change?



This chart looks at the sensitivity of the Weapon 1 decision, within the same parameter values as those in the previous chart. The axes are reversed for reasons of perspective.

Clearly, what we observe is quite different from what we saw in the last chart: There is considerable nonmonotonicity in this graph. These features are apparent with respect to each of the parameters individually (including sorties, as can be shown).

What does this mean? Consider the case in which the deployment schedule is held at 80 percent and Weapon 1 reliability is decreased. Initially, the number of Weapon 1 used by the model decreases, as makes intuitive sense. However, we can see that a continued degradation in reliability leads to *more* of Weapon 1 being used. This is an odd result: to have less, then more, then less, then more of Weapon 1 used as Weapon 1 reliability degrades. What is more troubling is that these same sorts of nonmonotonic changes in Weapon 1 occur as the deployment is degraded.

One explanation for this behavior might be that the number of Weapon 1 increases to compensate for decreases in their reliability *up to a certain point*. Once Weapon 1 reliability falls beyond a certain point, our preferred solution is to use more of some other kind of weapon and, hence, fewer of Weapon 1. However, we would not expect this behavior to repeat itself as Weapon 1 reliability declines or to see it across different levels of deployment, which is the case. Thus, we are left to question what is really going on in this analysis. Not only are the model results troubling, but it is impossible to predict whether more or fewer of Weapon 1 are really required. A small change or misestimate in these uncertain parameter values (Weapon 1 reliability and deployment schedule) might cause a weapon usage that is substantially different from that found to solve the best-estimate scenario.



What conclusions might we reach based on this sensitivity analysis? First, we might conclude that there is a grave problem with the model; perhaps the algorithms are unstable, or maybe we have some numerical problem. A linear programming expert might conclude that we are simply alternating between corner-point solutions. To address this problem, we would likely seek out expertise in the model's code. Perhaps we would even recommend putting greater detail or better representations in the model.

Second, we might look at the measurement of the inputs: If we can ensure that the parameters are all accurate and precise going into the calculation, we have no need for the sensitivity analysis because the best estimates are absolutely correct and we assert there will be no deviation from them. Then, the ruggedness that occurs around the best estimate weapon combination is no longer of consequence. To improve these measurements, we might (for instance) assemble committees or battle staffs to derive highly precise inputs. But, in making this assertion, we surely are only denying the enormous uncertainties of predicting future scenarios, battlefield outcomes, and system operations.

Either of these remedies would take some time to complete and may lead to a large capital expense. Thus, a third conclusion (which is more near term) might be that we should "buy the problem out": acquire enough munitions to cover all the uncertainty. After all, perhaps this is just the "noise" caused by the large uncertainties in this problem, and the only thing we can do is either suffer the risk or buy it out. The problem with this remedy, naturally, is simply one of feasibility. There are likely many situations in which buying out the largest deviation is fiscally infeasible.

At this juncture, we have completed the traditional analysis, although it has left us with some generally unsatisfying conclusions. The alternatives all seem costly. Analysts and decisionmakers look at our troubling conclusions suspiciously. Not only is it likely that the model and the analysis have been discredited, but we now face the dilemma of spending additional time and money to fix the model or data, or be left with a weapon mix study that still needs to be done.



We now move on to get to the bottom of the troubling results that plagued our traditional analysis.



As familiar as the preceding conclusions might seem, they are, at least in this case, not correct: Neither the model nor the data are the problem, and acquiring additional munitions is not necessary. As we shall show, the real problem is *how the model has been used*, not the model itself.

Optimal Solutions					
Weapon M	<u>Weapon Mix</u>		weapon 1 reliability		
		90%	85%	80%	
Weapon 1	12,781				
Weapon 2	9,570	22 Days			
Weapon 3	5,794				
Weapon 1	12,927				
Weapon 2	9,617		22 Days		
Weapon 3	5,206				
Weapon 1	12,446				
Weapon 2	10,291			23 Days	
Weapon 3	6,101	Service and the service of the servi			

To justify our assertion, consider the following example. This chart shows three levels of Weapon 1 reliability across the upper right. On the left are the optimal weapon mixes for each of these cases, with the corresponding number of days to complete the campaign shown in the box.

When Weapon 1 reliability is 90 percent, the campaign finishes in 22 days; and we are told to use 12,781 of Weapon 1, 9,570 of Weapon 2, and 5,794 of Weapon 3 (the top weapon mix). Similarly, at 85-percent Weapon 1 reliability, we finish in 22 days and are told to use the middle weapon mix; at 80 percent, we finish in 23 days and use the bottom mix.

These different weapon mixes exhibit precisely the fluctuations seen before. As Weapon 1 reliability falls from 90 to 80 percent, for example, the number of Weapon 1 required first increases, and then decreases, while the number of Weapon 3 first decreases and then increases.

Although these weapon mixes differ from one another, what would happen to the value of our objective (the number of days to complete the campaign) in the 90-percent case if we were to use the weapon mixes calculated for the 85-percent or 80-percent cases? Since we know that 22 days is an optimal value, we won't do any better than this, but how much worse will we do with the alternative mixes?

Weapon Mix	<u> </u>	W	leapon 1 relia	bility
		90%	85%	80%
Weapon 1	12,781			
Weapon 2	9,570	22 Days		
Weapon 3	5,794			
Weapon 1	12,927			
Weapon 2	9,617	22 Days	22 Days	
Weapon 3	5,206			
Weapon 1	12,446			
Weapon 2	10,291	22 Days		23 Days
Weapon 3	6.101	L	****	

As it happens, we will do no worse at all, technically: in both cases, we can *still* finish the campaign in about 22 days. While in truth the first weapon mix performs fractionally better than the others (the linear program does find the best fractional solution), when rounded to the nearest day there is no difference. These fractional savings are well within the overall calculation and likely to be irrelevant to the decision-maker when compared with accompanying differences in other measures of interest, such as cost or flexibility. Hence, we have found, at least in terms of days to complete the campaign, equally good, near-optimal alternative weapon mixes.

Similarly, what would happen to our objective value if we were to take the weapon mixes calculated for 90- and 80-percent Weapon 1 reliability and use them in the model, while the Weapon 1 reliability is set to 85 percent?

More Solutions				
Weapon Mix	lix	W	eapon 1 reliab	oility
		90%	85%	80%
Weapon 1	12,781	1		
Weapon 2	9,570	22 Days	22 Days	
Weapon 3	5,794			
Weapon 1	12,927			
Weapon 2	9,617	22 Days	22 Days	
Weapon 3	5,206			
Weapon 1	12,446			
Weapon 2	10,291	22 Days	22 Days	23 Days
Weapon 3	6,101			

Again, the value of our objective (22 days) remains unchanged for these alternative weapon mixes. Completing the example, we plug in the mixes from 90- and 85-percent reliability and run our model at 80 percent.

Weapon M	lix	w	eapon 1 relia	oility
<u>ITOUPOIT II</u>		<u>90%</u>	85%	80%
Weapon 1	12,781	Batternange,		tertent de tratte occasionen "server alemanis
Weapon 2	9,570	22 Days	22 Days	23 Days
Weapon 3	5,794			
Weapon 1	12,927			
Weapon 2	9,617	22 Days	22 Days	23 Days
Weapon 3	5,206			
Weapon 1	12,446			
Weapon 2	10,291	22 Days	22 Days	23 Days
Weapon 3	6,101	 But pressure to the set of the	-	

As by now we would expect, the objective value is unchanged at 23 days to complete the campaign. So what is going on here?

What this example tells us is that the solutions calculated by the optimization model are not unique; different combinations of weapons can be used with essentially the same impact on the number of days to complete the mission. And the few solutions found here are probably not coincidental: We are led to suspect that there are likely *many* combinations that will do as good a job. This supports our intuition that there are a variety of ways to successfully fight a war. This is encouraging news, but can we find all of them? Yes, by systematically searching across the space of weapon mixes. To do this, we no longer use the model in the typical way to find an optimal solution. Instead, we are changing it into a search engine to uncover all the optimal, or near-optimal, solutions.



This chart presents the findings of such a search. Discrete sampling found 196 different combinations of Weapons 1, 2, and 3 that all produced a value of 22–23 days to complete the campaign. Each combination of weapons (rounded up to the nearest hundred) is represented by the position and height of each column (the point corresponding to a particular combination is at the top of each column). Note that the original solution is found among these solutions (it would be troubling if it were not, of course).

In actuality, any weapon mix that lies within the boundary formed by the top of these bars is a solution. Instead of knowing just the one solution picked up by traditional analysis, we can calculate a wide range of alternative weapon mixes, all of which are clustered in a particular region—a strip stretching from the area of low Weapon 2 levels and high Weapon 1 levels to the area of low Weapon 1 and high Weapon 2.

This chart has two salient features worth mentioning, both of which contain important information for the decisionmaker and neither of which is apparent via traditional analysis. First, this graph indicates the way to trade off weapons to accomplish the mission in 22–23 days. Specifically, note that, at the corner in which Weapon 1 levels are low but Weapon 2 levels are high, we need to use more of Weapon 3 than in the opposite case. Why might this be? The answer involves the nature of the platforms being used. While fighters carry all three weapon types, bombers carry only Weapon 1 and Weapon 3. Therefore, at low numbers of Weapon 1, the bombers use an increasingly large number of Weapon 3 as their only alternative weapon type. This will cause a disproportionate substitution into Weapon 3 at this corner and create the observed skewness.

Second, we may ask why there are no weapon mixes in the front or back portions of the box. After thinking about the region in front, we conclude that the small numbers of Weapons 1 and 2 do not allow us successfully to attack some targets in the time prescribed (22–23 days). Certain targets are most efficiently serviced by Weapon 1 or Weapon 2, which are very accurate; at low levels of each (the front portion of the box), the less-accurate substitute—Weapon 3—cannot be utilized effectively against these targets *within the time limit*.

What about the region in the back of the box? Here, both Weapon 1 and Weapon 2 are at relatively high levels, so why aren't there any solutions? The reason has to do with the availability of platforms to deliver these numbers of munitions to their targets. Simply put, in this region we do not have sufficient aircraft or sorties to use all these weapons. This suggests that, were we to impose a more favorable TPFD schedule (in which more platforms arrived earlier in the campaign), enable a higher sortie rate, or change the platform mix in theater, we would begin to find solutions in the back region. Here is a connection between simply determining the weapon mix for a given force (similar to DAWMS, phase one) and examining alternative forces along with the weapon mix (phase two).

Another kind of tradeoff can be best seen by looking at this same set of tradeoffs from above, as we do in the next slide.



This chart shows the tradeoff between Weapon 1 and Weapon 2, with Weapon 3 (the third dimension) depicted with shading. We can see the original solution, as well as the features described in the last slide—the skewness in Weapon 3 in one corner, as well as the areas where the weapon mix is not feasible (the light boxes).

This graph gives us a look into the nature of the tradeoff between Weapons 1 and 2. Notice the distinct "dogleg" in the set of potential weapon mixes. What this indicates is a relatively low level of substitutability between these weapons in the more extreme regions. For instance, a relatively small decrease in Weapon 2, when the quantity is already low (say 4,000), will require a disproportionately large increase in the number of Weapon 1 to make up the difference. The converse is true at low Weapon 1 and high Weapon 2 levels.

So how "good" was the original solution? Given this picture of the full set of possible weapon mixes, the solution found using traditional analysis is potentially not a preferred one. For example, we may reduce the sheer quantity of weapons needed by taking a large decrease in Weapon 1 and only a small increase in Weapon 2. Thus, this chart tells us something about the relative efficiency of different solutions.

None of the preceding information—about the whole set of solutions, the regions of feasibility and infeasibility, the nature of the weapon tradeoffs, and the relative efficiency of alternative solutions—can be derived from the traditional analysis, which provides the single, optimum solution.

Before turning to a more general discussion of the type of analysis we have performed here, we return to the troubling result we observed as Weapon 1 reliability was reduced and the Weapon 1 component of the mix increased and decreased repeatedly.



In the preceding two charts, we have looked at the set of weapon mixes that achieve a fixed objective value and a fixed Weapon 1 reliability, sortie rate, and deployment schedule (all at their nominal values). What happens as we degrade one of these, say, Weapon 1 weapon reliability?

First, we now know that we should look at a whole set of solutions for each Weapon 1 reliability value; the point-to-point comparison from our original sensitivity analysis was incomplete. The previous charts showed the trade space for 100 percent reliability. What is the trade space for lower Weapon 1 reliability? Should we still worry about the troubling, nonmonotonic results that we originally observed as we change Weapon 1 reliability?

To address the first question, the chart superimposes the set of weapon mix solutions for the baseline (100 percent) case on the cases in which Weapon 1 reliability is 90 percent and 85 percent (sorties and deployment are maintained constant at 100 percent in all cases), all of which correspond to an objective value of 22–23 days to complete the campaign.

For visual ease, we look at only two dimensions (Weapons 1 and 3). The regions are inclusive; that is, all points within the darkest region's perimeter are solutions for Weapon 1 reliability of 100 percent, all points within the next lightest region's perimeter are solutions at 90 percent, and so on. In addition, the site of the solutions calculated by the traditional use of the model for each of these parameter values is indicated. Note the counterintuitive change among these "optima" from using more, to fewer, to more of Weapon 1, as seen earlier.

This graph explains the troubling results we saw before. It is clear that many possible solutions to our problem yield the same objective value across these values of Weapon 1 reliability. Many of them are coincident (those near-optimal mixes in the lightest region are also near-optimal solutions for the 90 percent and 100 percent case). When we ran the model in the traditional way, however, this was not indicated. The model (arbitrarily) calculated only one of the many possible weapon mixes in each case, and the resulting three mixes happened to first require more of Weapon 1, then less, then more as reliability dropped off. Because traditional sensitivity analysis provided limited results, we incorrectly interpreted the nonmonotonicity of the results as being troubling, and, perhaps, as even indicative of errors in the model or data.

These more comprehensive results explain what is really happening. The reduction in Weapon 1 reliability is actually changing the set of weapon mixes that can accomplish the campaign objectives. Furthermore, we are also provided with the answer to what we should do at fairly modest amounts of uncertainty in Weapon 1 reliability: Just pick a point somewhere in the middle of the lightest region. We know that this mix is an optimal solution across all three values of the parameter. In other words, we don't need to agonize over the accuracy or precision in Weapon 1 reliability (which is highly uncertain in future campaigns). Instead, we can select a weapon mix that is robust across a range of Weapon 1 reliability conditions. In other words, we can use our decision to select a weapon mix to not only fulfill campaign objectives, but also to resolve uncertainties in the scenario conditions.

This example sums up the intrinsic difference between traditional analysis and our nontraditional approach. Traditional analysis found a single point (somewhat arbitrarily, as it turns out), which we regarded as *the* solution and examined the variation around it; with our approach, we systematically search for *all* the possible optimal and near-optimal solutions, then select a preferred one. The traditional approach works from the inside out. We are suggesting that working from the outside—the big picture—and selecting a solution within provides important additional information for both analysts and decisionmakers. An additional significant difference with this approach is the ability to *choose the most advantageous weapon mix* from the set of good solutions based on a variety of preferences (e.g., costs, risk, operational considerations) that may be difficult to capture in a single model.



We call the methodology we have just employed "exploratory analysis." In this next section we will explain the generality of the approach and the details of how it can be applied.



Exploratory analysis searches across the space of scenarios, decisions, and measures of effectiveness in search of robustness. It allows us to look at robustness in a variety of ways. So far in our examples, we have determined weapon mix (decision recommendations) solutions that are robust across the variability or uncertainty in Weapon 1 reliability (an input value). We might also look for decision recommendations that are common to different scenarios—for instance, across two or more theaters of operations. We could consider looking for decision recommendations that are robust across models. For example, suppose we are not sure which of three different models' solutions are "best" (however that might be defined). By sampling across the decision space produced by all three and choosing a solution common to all, we no longer have to argue the highly subjective question of whose model is best.

How is exploratory modeling different from traditional analysis? First, it changes the way the model is used. The model is run many times with many different input levels, which requires fairly large numbers of computational experiments (perhaps tens or hundreds of thousands). These experiments are made possible in our example by today's computing environment—high-powered, networked workstations.

Second, as alluded to before, exploratory modeling represents a fundamentally different way of looking at the problem. While traditional analysis works in a sense from the inside out—the solution is found and the area around it is examined—exploratory modeling allows an "outside in" approach: A large space in the domain of interest (and all the solutions in it) is examined, then a solution is selected. The next few pages address this difference in more detail.

This more inductive method to solving the problem has a number of advantages over traditional analysis. Using the weapon mix problem, we will demonstrate in the last section of this documented briefing that the advantages include greater decisionmaking flexibility, ability to assess robustness, and risk neutralization.



This diagram provides one view of how exploratory analysis differs from traditional sensitivity analysis. Simply put, sensitivity analysis tries to measure the impact of changing inputs on outputs. More carefully and broadly, sensitivity analysis tries to capture how measures of effectiveness change as either the scenario or the decisions change.⁶

Exploratory analysis takes a broader perspective and not only considers much more comprehensive sensitivity analysis, but a greater set of effects. We can determine the impact of a particular measure (such as days to complete a campaign) on the inputs or the decision variables (such as the mix of weapons). This effect (how changes to the output affect inputs) could be thought of as sensitivity analysis in reverse. But this is not all. We may also desire to capture the interaction between or within the set of values that make up the scenario, the decision variables, or the measures of effectiveness.

⁶We recognize that in a prescriptive model (such as an optimizing model) the decision variables are outputs whereas in a descriptive model (such as a simulation) the decision variables are part of the inputs. For sensitivity analysis, we typically desire to know what impact the decision variables have on measures of performance, regardless of whether they are inputs or outputs.



On a slightly more mathematical level, we see that exploratory analysis can be distinguished from sensitivity analysis in terms of the implied *directions* in which we can capture changes in values. Say that the model we are using can be represented by the function f(X). While traditional sensitivity analysis and exploratory analysis concern themselves with changing the inputs X and seeing the effect on the output(s) Y via f(X), exploratory analysis goes further and allows us to consider the range of X (the combinations of inputs) that corresponds to a particular range of Y via $f^1(Y)$. We can also look at interactions among the set of inputs X and the set of outputs Y. Note that exploratory analysis goes beyond sensitivity analysis to the extent that it allows us to examine the comparative statics of our problem as opposed to just the partial derivatives of the objective function.



This section gives a few concrete examples of how exploratory analysis can help the decisionmaker.



This chart represents in numerical form the data presented in prior graphs. Levels of Weapon 1 are on the horizontal axis; levels of Weapon 2 are on the vertical axis; and the numbers in the inside cells correspond to Weapon 3 (all weapons are rounded up to the nearest 500). Blank cells are the infeasible combinations. Under traditional analysis, we would have gotten the one solution pointed to with the arrow.

How are additional weapon mixes found via exploratory analysis superior to the single-point solution found by conventional means? Consider first an operational commander. While a commander does not buy weapons, his staff *does* perform analysis to determine needs. Suppose the commander's staff's traditional analysis has come up with the single point, and that for whatever reason (for example, jamming or technological risk), the commander is uncomfortable with using so many of Weapon 1 and would prefer to substitute Weapon 2 for them, if possible. With the single-point solution, the commander has no idea if by using more Weapon 2 there are in fact combinations of Weapons 1 and 3 that allow him to complete his mission in approximately the same amount of time. If, however, the commander's staff had used exploratory analysis to generate the table above, the commander could readily choose among a large number of solutions in the upper righthand corner that require relatively few of Weapon 1. Additionally, from an operational perspective, this table of tradeoffs would also prove useful in the event that in-theater inventories of a particular munition were below the single-point, "optimal" level. Could other munitions be used to substitute for this deficiency? Will the munition deficiency prohibit the commander from accomplishing mission objectives? A table like this readily provides the alternatives that permit the same level of mission success.



Exploratory analysis can also increase the force programmer's flexibility. This chart exhibits the same solutions as on the last, but shades about half of them. The shaded cells represent solutions that fall within a notional budget constraint (the less costly weapon mixes). If this were the actual budget constraint, flexibility would be enhanced by allowing selection from the variety of mixes (including the least expensive one) feasible for the budget, as opposed to just hoping that the one solution found with traditional analysis might fit.

Each of these examples demonstrates how exploratory analysis allows additional preferences to be included and their impact displayed, thus improving decisionmaking flexibility.



Exploring the space of solutions provides the ability to assess robustness better across two contingencies and to allow the decisionmaker to choose a weapon mix that is robust for either case. This chart considers two distinct contingencies: SWA and NEA.⁷ If we are uncertain as to which might occur, how do we select the weapon mix to have in inventory? Buying enough for each one separately leads to an over costly solution. Traditional analysis can determine the requisite inventory for each, without buying overlapping types or quantities. Although better, the traditional approach may also lead to excessive inventory.

Using exploratory analysis, we can display the full trade space for each contingency, as shown above. The darker cells are the solutions we found for SWA. The checkered cells correspond to notional solutions for another theater, NEA. Clearly, given these two solution sets, we can select a weapon mix that provides success in either SWA or NEA (the medium-shaded cells); we can *see* the robustness of weapon mixes across contingencies.

It is possible, of course, that the solution set for the second theater is disjoint from the first, so no common weapon mixes exist. In such a case, exploring the space of solutions is still extremely valuable. With

⁷ The definition of the two contingencies in this example are completely arbitrary. One could have been a combined SWA and NEA, the second an alternative major regional contingency (MRC) and a mission other than war.

traditional analysis, we calculate only single-point solutions for each theater, which are arbitrarily far apart. With exploratory analysis, the full set of possibilities is seen, so that a weapon mix closest to both can be selected. Implications of the risk are more clearly understood.



An example of how exploratory analysis can neutralize risk has already been given, but we do so more explicitly here. This chart contains just a few cells from the weapon mix table shown previously. These weapon mixes all correspond to the nominal 100 percent Weapon 1 reliability. But what if Weapon 1 reliability is 95 percent or, perhaps, 90 percent?



This chart shows the weapon mixes that are solutions for the 90 percent Weapon 1 reliability case. The neutralization of the risk or uncertainty about Weapon 1 can be accomplished by selecting a solution that is the same in this and the previous chart. The next chart compares the weapon mixes above to those at 100 percent reliability to find a few of these common mixes.



The common solutions are the darker cells: Any of these weapon mixes assures us of achieving the same outcome of 22–23 days to complete the campaign regardless of whether Weapon 1 is completely or somewhat less effective. It might be noted that the common cells here have a small variance in the numbers of Weapon 3. For ease of exposition, the solutions are made common by rounding up to the higher Weapon 3 level of the two. Given more exhaustive sampling for solutions, we are likely to find weapon mixes for all three weapon types that are truly common across levels of Weapon 1 reliability.



The final section discusses uses and requirements of exploratory analysis.



In this briefing we have used exploratory analysis only in a limited way. The full extent of this approach goes beyond our examples both in scale and in scope. This methodology can and has been used to address a wide variety of problems. For example, RAND projects have used exploratory analysis in analyses of military strategy, force structure, logistics, and adaptive defense planning.⁸ In addition, it has been applied to such diverse nonmilitary topics as science and technology investment strategy, drug control policy, global warming,⁹ and the future of higher education.

We note again that beyond providing more effective *uses* of models built for analysis, exploratory analysis also provides an effective vehicle for improvement of the models themselves. After all, the best way to uncover anomalous model behavior is via large-scale systematic experimentation under a wide variety of conditions and input values. Exploratory analysis can be used as an effective tool in the verification and validation process.

⁸ See James A. Dewar, Steven C. Bankes, James S. Hodges, Thomas Lucas, Desmond K. Saunders-Newton, and Patrick Vye, *Credible Uses of the Distributed Interactive Simulation (DIS) Environment*, Santa Monica, Calif.: RAND, MR-607-A, 1996, or Steve Bankes, *Modeling and Simulation Infrastructure to Support Adaptive Planning in the Air Force*, Santa Monica, Calif.: RAND, DB-210-AF, 1997.

⁹ See Robert J. Lempert, Michael E. Schlesinger, and Steve Bankes, "When We Don't Know the Costs or the Benefits: Adaptive Strategies for Abating Climate Change," *Climatic Change*, Vol. 33, 1996, pp. 235–274.



We have mentioned throughout that exploratory analysis requires large-scale computational experiments. To make this more concrete, this chart summarizes what exactly went into the explorations performed over the course of the analysis described in this briefing. Given the model run time, number of work stations, and scope of this project, the number of runs, while substantial, was not prohibitive. In other analyses and with other models, the time required to perform comprehensive exploratory analysis may or may not be affordable.

In a typical analysis, it is not feasible to significantly alter run time; additionally, time for completion of the analysis is often fixed. Given this information and the number of work stations available, the maximum number of model runs can be determined. We seek to set up the maximum number of runs so that as much useful information as possible is provided for the decisionmaking process.

The run time in our example could have been better spent if we had used more clever sampling and searching techniques. Referring back to the weapon mix trade spaces shown on pages 25 and 27, we note that the critical information on these charts is the boundary *and* the area inside the boundary. With our limited set of tools to support exploratory analysis, we were required to calculate not only the cells where there was an alternative weapon mix, but also the cells where there were none to find. This brute force approach was necessary but inefficient. Given a better infrastructure to perform exploratory analysis, either the same information could have been obtained from far fewer runs, or the same number of runs could have been used to produce even more information.



The case study presented in this documented briefing, while effective in demonstrating some of the advantages of exploratory analysis, was not a taxing exercise of the methodology. It is not difficult to imagine studies in which a far greater variety of uncertainties, decision options, or measures of effectiveness must be considered. For such studies, it might be difficult to define precisely what is the space to explore vis-à-vis the computational experiments. Other problems can arise in manipulating large volumes of data going into the experiments as well as the consumption and interpretation of potentially huge quantities of output generated.

In our case we observed the interconnected effects of changes in just three scenario parameters (weapon reliability, sortie rates, and deployment rates), three decision options (the numbers of three different weapon types), and one output measure (days to complete the campaign).¹⁰ Certainly, these are not the only interactions we could have looked at; we limited ourselves in this way precisely because it kept us within a manageable level of dimensionality (of the trade space) to conduct an example analysis given the present state of the methodology's infrastructure.

What practical steps could be taken to improve our ability to tackle more highly dimensional problems with exploratory analysis? Several

¹⁰We also included measures of cost and risk in the analysis, but this was done externally to the principal exploration.

are immediately clear. First, we could provide operation system interface for multiple-machine processing of model cases. Exploratory analysis relies on large computational experiments that can be most efficiently conducted across a large number of networked computers. This step would provide a standard means for accessing and controlling UNIX-based machines on the network by using a generalized, transportable interface to the operation system. This controller would provide coordinated access to all the workstations on a network whether there were tens or hundreds available.

Second, we could devise and implement software to facilitate case generation and model specification. This would provide automation for the generation of the computational experiments.

Third, we could adopt and implement software to support result databases. Exploratory analyses generate large quantities of data that must be stored and manipulated. These software tools would provide standardized facilities for handling these databases, including improved capabilities for output database maintenance and the ability to use multiple hardware and database systems.

Fourth, we could find or develop improved visualization tools for multidimensional model results. As noted above, exploratory analysis calculates the effect of interactions among many important problem parameters such as force levels, system effectiveness, timing, and tactics. Being able to understand and communicate these multidimensional effects can be difficult; this step would ideally provide a portfolio of visualization tools that were readily available as part of the basic exploratory analytic environment.

Fifth, we could implement a portfolio of search and sampling methods. The exploratory analysis conducted here used brute force methods for discovering the critical points at which decisions change. This task would research algorithmic options for exploring high-dimensional model spaces automatically and finding these critical points much more efficiently. As a result of these tools, study results could be derived more rapidly and more extensive analysis could be performed.

After these steps were taken, we would then likely turn our attention to methods for transferring the tools of exploratory analysis to those outside our immediate analytic environment. In this way, the advantages of this methodology could be exploited widely.



The conclusions described at the outset can now be put into better focus. First, we have seen that exploratory analysis is not at odds with traditional analysis, but, rather, that it expands traditional analysis to provide greater information to the decisionmaker. Essentially, what we have shown is that, to assemble an adequate body of information about the weapon mix problem, we need to perform traditional analysis many times in a schematic way across the entire space of potential solutions. Second, we have demonstrated how exploratory analysis can deal with problems, such as troubling results commonly encountered in traditional analysis. Our demonstration has also pointed out that the responses often made to such problems (absent exploration) are not likely to be effective, since the true nature of the problem is not inadequacies of the model or data, but inadequacies of the analysis (how the model has been used). Finally, we have demonstrated that exploratory analysis provides some useful advantages, including allowing greater flexibility and robustness, lowering risk, and improving understanding.

The central concept behind exploratory analysis is relatively simple. For the test case we have described here, the benefits are easy to see, inexpensive to obtain, yet potentially quite costly to neglect.

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