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BACKGROUND

The Toplosky solution (Toplosky and Vogelsong, 1993) for scattering from a rib (or ribs), based on the approach by Stepanishen (1978), is expressed in terms of several response Green's functions. The response Green's functions (or plate cross-admittance functions) are

$$\Gamma_0 \equiv G_F(\varkappa_m|\varkappa_n) = \frac{k}{2\pi}\int_{-\infty}^{+\infty} Y(\xi)e^{ik\xi(x_m - x_n)}d\xi ,$$

$$\Gamma_1 \equiv G_M(\varkappa_m|\varkappa_n) = \frac{k}{2\pi}\int_{-\infty}^{+\infty} ik\xi Y(\xi)e^{ik\xi(x_m - x_n)}d\xi ,$$

$$\Gamma_2 \equiv G'_M(\varkappa_m|\varkappa_n) = \frac{k}{2\pi}\int_{-\infty}^{+\infty} (ik\xi)^2\, Y(\xi)e^{ik\xi(x_m - x_n)}d\xi ,$$

where k is the wavenumber and $Y(\xi)$ is the admittance of a fluid loaded plate.

$$Y(\xi) = \frac{1}{Z_p(\xi) + Z_a(\xi)} ,$$

where

$$Z_a(\xi) = \frac{\rho c}{\sqrt{1 - \xi^2}}$$

is the acoustic radiation impedance, with ρ and c the fluid density and sound speed.

$$Z_p(\xi) = i\omega\rho_p h\left[\Omega^2\xi^4 - 1\right]$$

is the impedance of an unstiffened Bernoulli–Euler (thin) plate, where ρ_p and h are the density and thickness of the plate and $\Omega = \frac{\omega}{\omega_c}$. The coincidence frequency

$$\omega_c = \frac{\sqrt{12}c^2}{hc_p} ,$$

with $c_p = \sqrt{\frac{E}{\rho_p(1 - \nu^2)}}$. E and ν are the Young's modulus and Poisson's ratio of the plate.

Note that the use of a Bernoulli–Euler plate impedance is not justified in the case of a thick plate. When $\Omega > 1$, the results are not valid. Toplosky used it to avoid divergences in his formulation encountered with a Timoshenko–Mindlin plate impedance. The root of the problem was his use of a force dipole line moment, which cannot be supported by a Timoshenko–Mindlin plate (Rumerman, 1979).

INTEGRALS

The Green's functions are evaluated using contour integration. There are branch points at $\xi = \pm 1$ as well as poles in the complex ξ-plane, some on the real axis. The L-shaped Sommerfeld branch cuts are used. The results may be expressed as

$$\Gamma_J = 2\pi i \sum \text{residues}_J - I_{35,J} - I_{26,J},$$

where

$$I_{35,J}(d) = -\frac{k}{\pi}\int_0^1 (ik\xi)^J \frac{\rho c\sqrt{1-\xi^2}}{(\omega\rho_p h)^2(\Omega^2\xi^4-1)^2(1-\xi^2)+(\rho c)^2} e^{ikd\xi}\, d\xi,$$

$$I_{26,J}(d) = i\frac{k}{\pi}\int_0^\infty (-k\xi)^J \frac{\rho c\sqrt{1+\xi^2}}{(\omega\rho_p h)^2(\Omega^2\xi^4-1)^2(1+\xi^2)+(\rho c)^2} e^{-kd\xi}\, d\xi.$$

The advantages of these expressions for numerical integration may be seen. In the case of an oscillatory integrand, the interval is simply [0,1]. In the case of an infinite interval, the integrand has exponential damping. The upper limit may therefore be set to a reasonable value (Toplosky uses 8).

NUMERICAL CONSIDERATIONS

Toplosky calculates the Green's functions using simple Romberg integration. This is sufficient at low frequencies. However, at high frequencies, the rational factor has a sharp peak near $\xi = \Omega^{-1/2}$ (width $\Delta\xi \propto \Omega^{-3/2}$), which may cause convergence problems, especially in combination with a rapidly oscillating exponential factor. Variable step size integration, such as in ordinary differential equation solvers, will provide superior performance. For $I_{26,J}$, when $d \neq 0$, the decaying exponential factor negates the significance of the peak.

HIGH-FREQUENCY APPROXIMATIONS

Based on the previous considerations, the integrals may be approximated when $\Omega >> 1$. Again, note that the Toplosky formulation is not valid when $\Omega > 1$. However, despite this, we will find some useful results, as will be seen later.

We will begin with $I_{35,\,J}$ first. Change the integration variable to $z = \sqrt{\Omega\xi}$, so

$$I_{35,J}(d) = -\frac{k}{\pi}\frac{\rho c}{\sqrt{\Omega}}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_0^{\sqrt{\Omega}} z^J \frac{\sqrt{1-\frac{z^2}{\Omega}}}{\left(\omega\rho_p h\right)^2\left(z^4-1\right)^2\left(1-\frac{z^2}{\Omega}\right)+\left(\rho c\right)^2}\exp\left(i\frac{kd}{\sqrt{\Omega}}z\right)dz\,,$$

and the peak is now at $z = 1$. Note that $z^4 - 1 = \left(z^2+1\right)\left(z+1\right)\left(z-1\right) \approx 4\left(z-1\right)$ in the vicinity of the peak, and that z^2/Ω is negligible there as well. Therefore,

$$I_{35,J}(d) \approx -\frac{\rho ck}{\pi}\frac{\sqrt{\Omega-1}}{\Omega}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_0^{\sqrt{\Omega}} \frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}z\right)dz}{\left[4\omega\rho_p h\sqrt{1-\frac{1}{\Omega}}\right]^2\left(z-1\right)^2+\left(\rho c\right)^2}\,,$$

$$\approx -\frac{k}{\pi}\frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}\right)}{4\omega\rho_p h\sqrt{\Omega}}\left(\frac{ik}{\sqrt{\Omega}}\right)^J \int_{-y_0}^{y_0\sqrt{\Omega}} \frac{\exp\left(i\frac{kd}{\sqrt{\Omega}}\frac{y}{y_0}\right)dy}{y^2+1}\,,$$

where $y_0 = 4kh\frac{\rho_p}{\rho}\sqrt{1-1/\Omega} = 8\sqrt{3}\,\frac{\rho_p c}{\rho c_p}\Omega\sqrt{1-1/\Omega} \gg 1$ for all practical problems. Thus, taking the integration limits as $\pm\infty$, we finally find

$$I_{35,J}(d) \approx -\frac{\exp\left(-\frac{\rho d}{4\rho_p h\sqrt{\Omega}}\right)}{4\rho_p ch\sqrt{\Omega}}\exp\left(i\frac{\omega_c d\sqrt{\Omega}}{c}\right)\left(i\frac{\omega_c\sqrt{\Omega}}{c}\right)^J.$$

In considering approximations to $I_{26,\,J}$, we must examine the case $d = 0$ separately.

$$I_{26,J}(0) = i\frac{k}{\pi}\frac{\rho c}{\sqrt{\Omega}}\left(-\frac{k}{\sqrt{\Omega}}\right)^J \int_0^{\infty} z^J \frac{\sqrt{1+\frac{z^2}{\Omega}}}{\left(\omega\rho_p h\right)^2\left(z^4-1\right)^2\left(1+\frac{z^2}{\Omega}\right)+\left(\rho c\right)^2}\,dz\,.$$

The development follows that for $I_{35,\,J}$,

$$I_{26,J}(0) \approx i\frac{k}{\pi}\frac{1}{4\,\omega\rho_p h\sqrt{\Omega}}\left(-\frac{k}{\sqrt{\Omega}}\right)^J \int_{-y_0}^{\infty}\frac{dy}{y^2+1}\,,$$

where $y_0 = 4kh\frac{\rho_p}{\rho}\sqrt{1+1/\Omega} = 8\sqrt{3}\,\frac{\rho_p c}{\rho c_p}\Omega\sqrt{1+1/\Omega} \gg 1$. Thus,

$$I_{26,J}(0) \approx i\frac{1}{4\rho_p ch\sqrt{\Omega}}\left(-\frac{\omega_c\sqrt{\Omega}}{c}\right)^J.$$

When $d \neq 0$, the main contribution to $I_{26,J}$ comes from a region where $\xi \ll 1$. Therefore,

$$I_{26,J}(d) \approx i\frac{k}{\pi}\frac{\rho c}{\left(\omega\rho_p h\right)^2 + (\rho c)^2}(-k)^J \int_0^\infty \xi^J e^{-kd\xi} d\xi \,,$$

$$\approx i\frac{1}{\pi d}\frac{\rho c}{\left(\omega_c \rho_p h\Omega\right)^2 + (\rho c)^2}\left(-\frac{1}{d}\right)^J (J!) \;.$$

EXAMPLE

Let us consider a 5-cm-thick steel plate (coincidence frequency of 4.7 kHz). The "exact" and approximate results are compared in figures 1 through 9. We find that the approximation of $I_{35,J}$ is accurate (within 10%) for $\Omega > 2$. The approximation of $I_{26,J}$ is accurate for $\Omega > 0.2$. It is this result that makes the analysis worthwhile. Similar results are found for a 2.5-cm-thick steel plate (coincidence frequency of 9.4 kHz).

SUMMARY

Approximations to integrals in the Toplosky rib formulation have been found. Though derived for high frequencies, many are useful over a much broader range. These approximations save a great deal of computation time.

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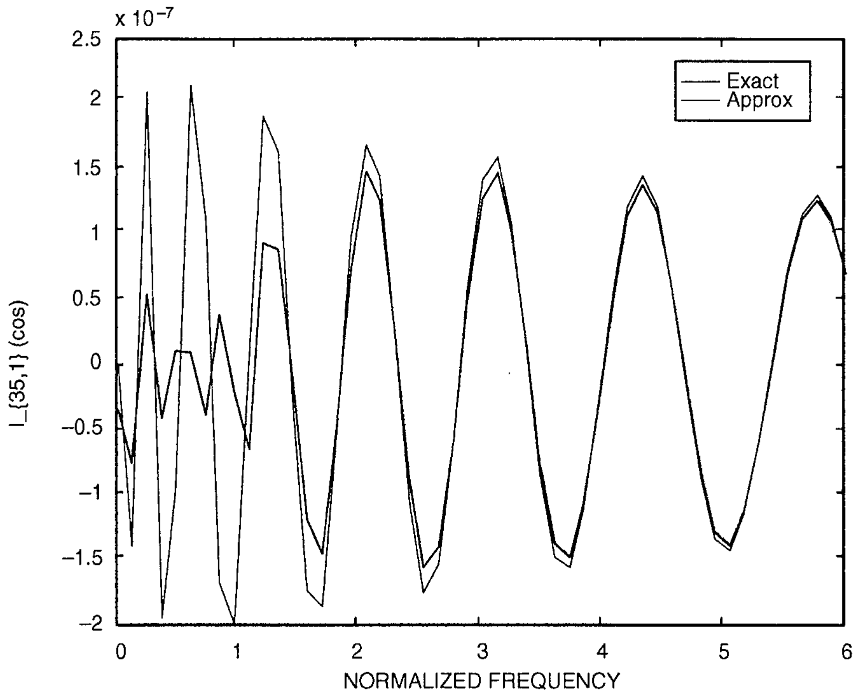

Figure 1. Comparison for $I_{35,1}$ (cosine part).

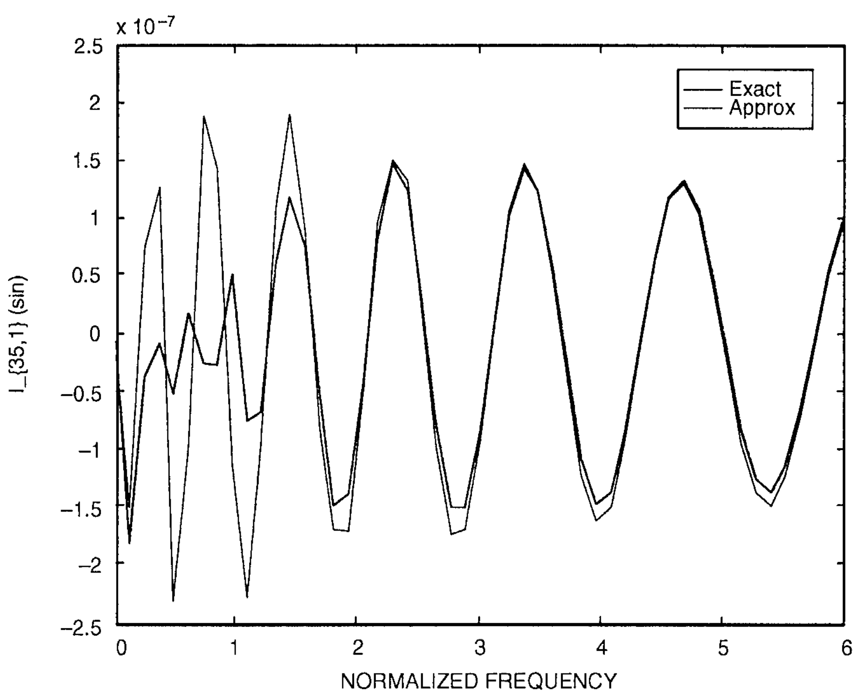

Figure 2. Comparison for $I_{35,1}$ (sine part).

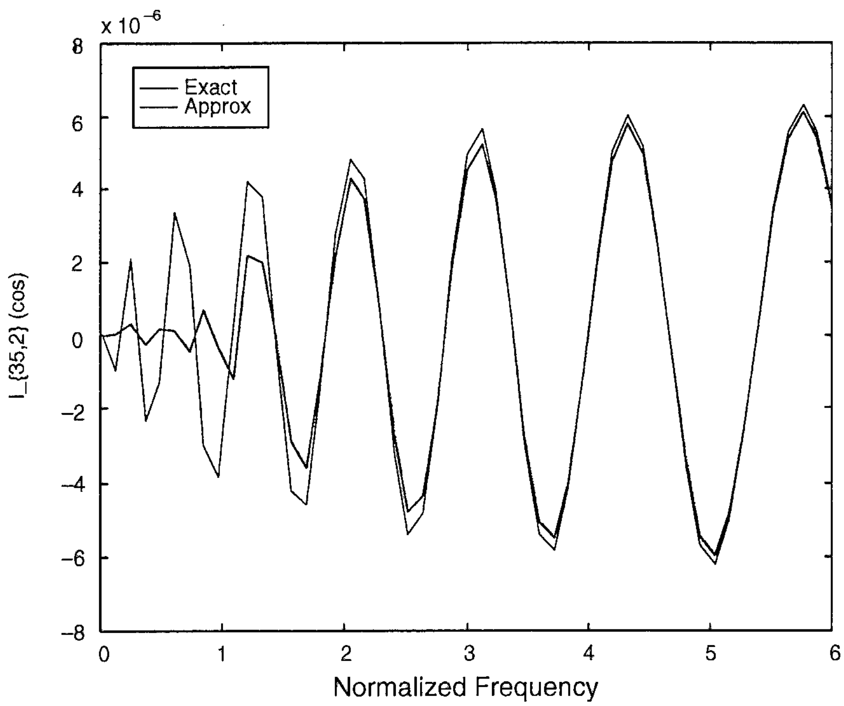

Figure 3. Comparison for $I_{35,2}$ (cosine part).

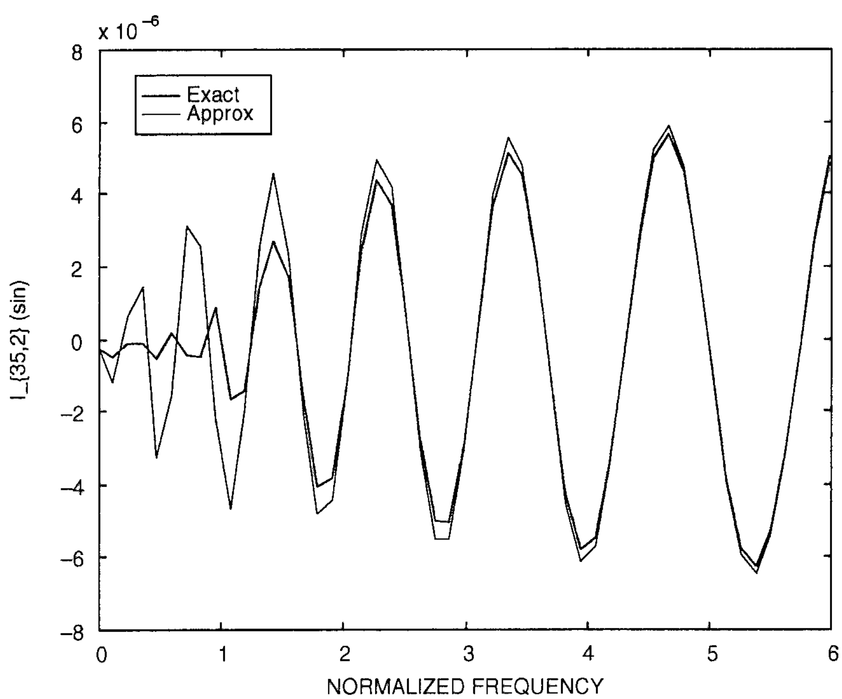

Figure 4. Comparison for $I_{35,2}$ (sine part).

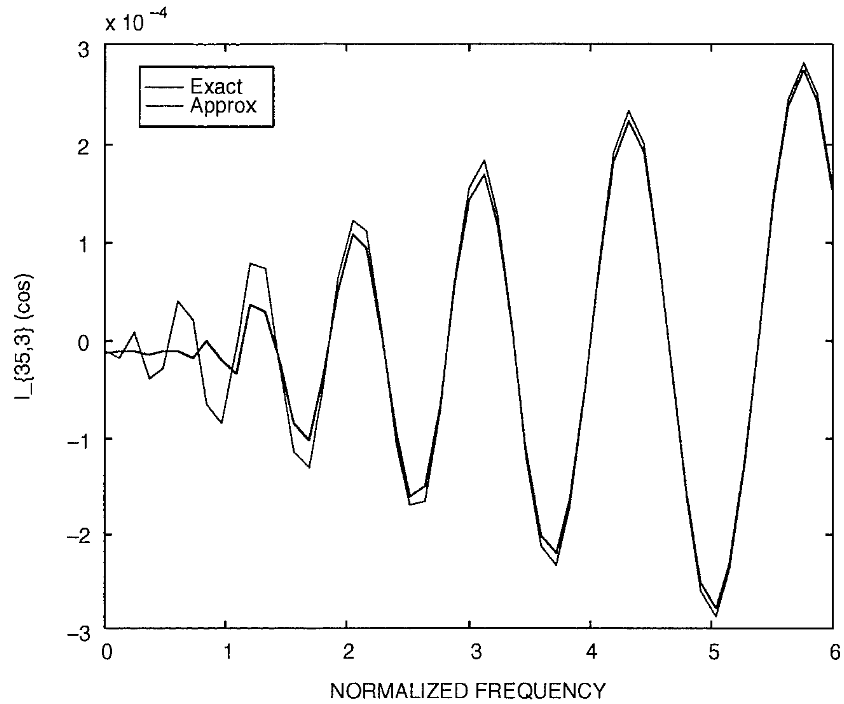

Figure 5. Comparison for $I_{35,3}$ (cosine part).

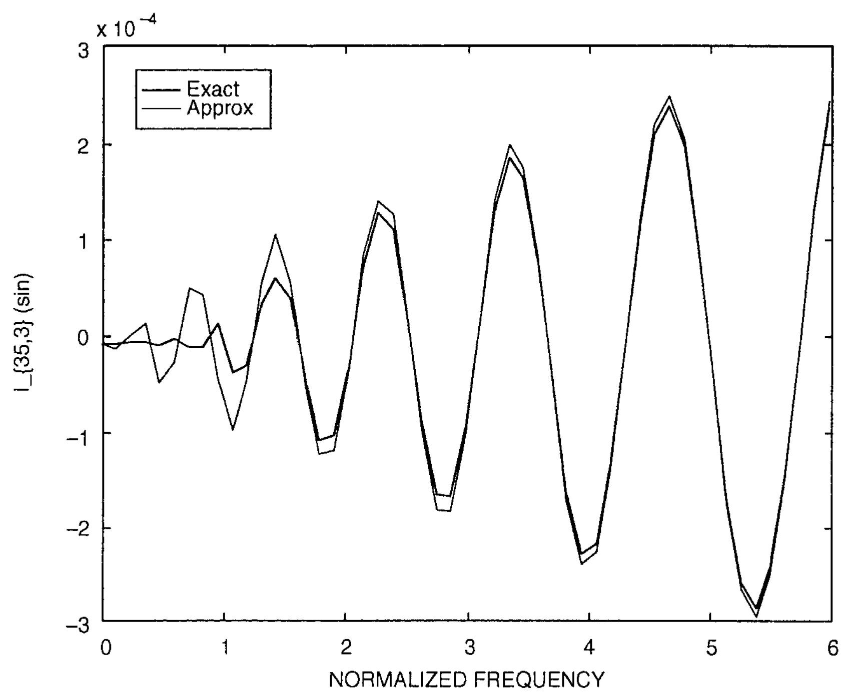

Figure 6. Comparison for $I_{35,3}$ (sine part).

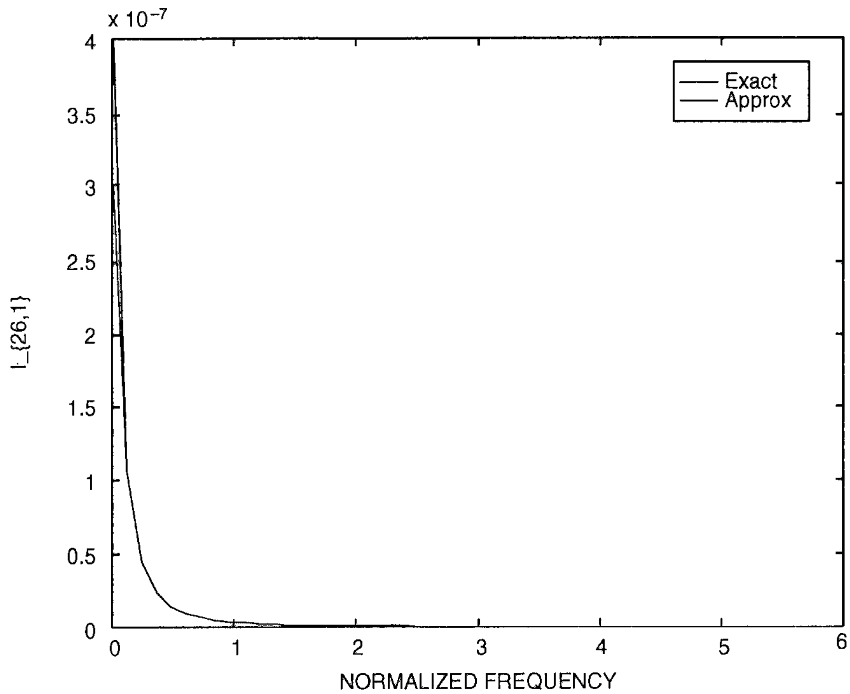

Figure 7. Comparison for $I_{26,1}$.

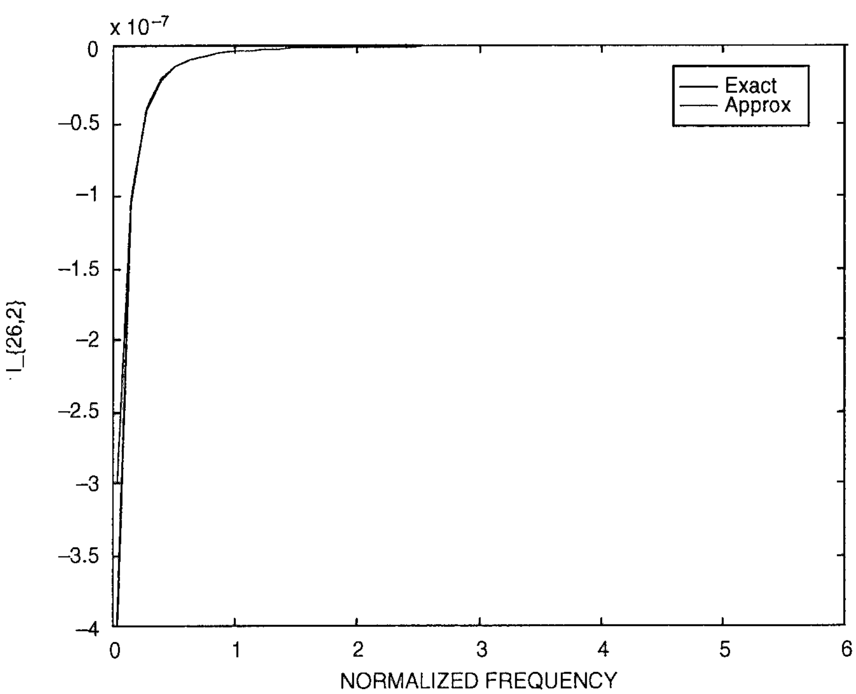

Figure 8. Comparison for $I_{26,2}$.

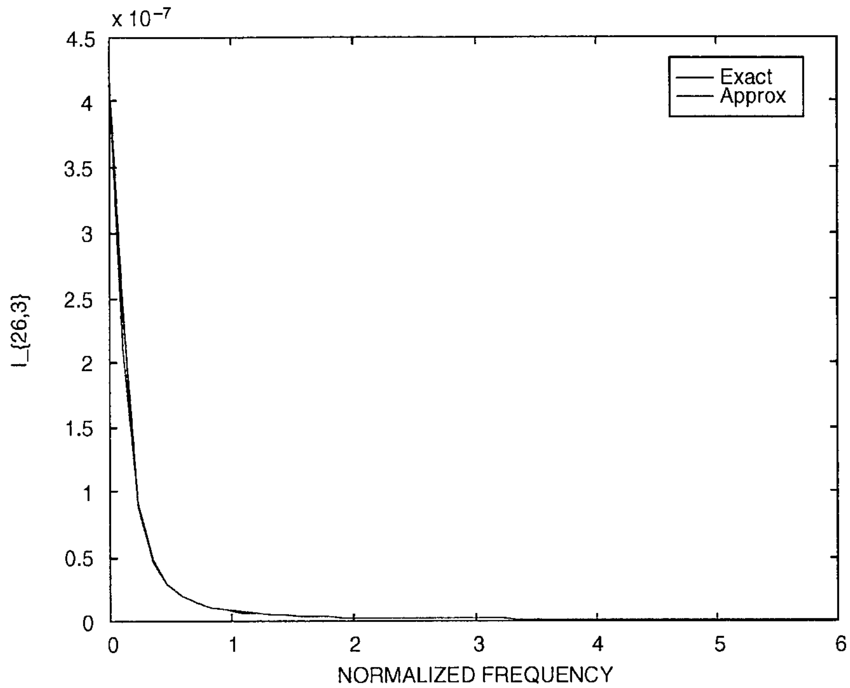

Figure 9. Comparison for $I_{26,3}$.

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