Vortex-induced vibrations in a sheared flow: a new predictive method

M.S. Triantafyllou
Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

M.A. Grosenbaugh
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

R. Gopalkrishnan
Massachusetts Institute of Technology, Cambridge, Massachusetts, USA

ABSTRACT: It is shown that a linear hydrodynamic damping term is an intrinsic feature of the vortex-induced vibrations of slender cylinders in the lock-in regime. The damping coefficient can be directly evaluated from experimental measurements of the force acting on a section of the cylinder forced to move in a uniform flow.

1 INTRODUCTION

Elastically mounted cylinders and long, flexible cylinders undergo vortex-induced vibrations when placed normal to a flow. The amplitude of this process is self-limiting with a maximum value approximately equal to one to two cylinder diameters. Laboratory experiments have been conducted to measure forces on rigid cylinders that are oscillated at a specific amplitude and frequency transversely to a uniform flow. These tests confirm that there is power input into the cylinder vibrations at small amplitudes of motion, for frequencies close to the Strouhal frequency of natural vortex formation, whereas there is dissipation for larger amplitudes (King 1977, Staubli 1983, Bearman 1984). When a cylinder oscillates with a frequency that is within a narrow range about the Strouhal frequency, the vortex formation process synchronizes with the motion of the cylinder in what is called a condition of *lock-in*. Under lock-in conditions, a vibrating cylinder is subject to a significantly increased drag force, up to three or more times higher than that of a stationary cylinder.

Hartlen and Currie (1970) and several other authors (Bearman 1984) used the van der Pol oscillator to represent qualitatively the self-limiting nature of the excitation lift force. Alternatively, the concept of energy balance has been incorporated in models to predict the response of long, flexible cylinders (Vandiver 1988). In these models, the direction of energy transfer is dependent on whether or not the motion of the cylinder at a particular point is correlated with the vortex formation process. Energy is assumed to be transferred from the fluid to the cylinder at points where the motion is synchronized with vortex shedding (lock-in condition), while it is assumed that the cylinder loses energy to the fluid at points where the motion is not correlated with vortex shedding. At these points, the loss of energy is modelled by an "equivalent" hydrodynamic damping term, calculated by linearizing the quadratic drag force acting on the cylinder.

In this paper we show, on the basis of experimental results, that the vortex-induced lift force depends on the amplitude of the cylinder vibration in a manner which is characteristic of a process containing a purely linear damping term. This provides a direct way of evaluating the damping coefficient using laboratory measurements.

In §2, we derive a model of hydrodynamic damping for a simple harmonic response. The model is extended to the more general case of a narrow-band response in §3. In §4, we incorporate the model into a simple scheme that predicts the vortex-induced response of flexibly mounted, rigid cylinders and long, flexible cylinders. We compare the predictions to previously published experimental results.

2 HARMONIC RESPONSE

The force acting on a section of a slender circular cylinder of diameter d, vibrating harmonically in the transverse direction relative to an oncoming flow of velocity V, is a nonlinear function of the

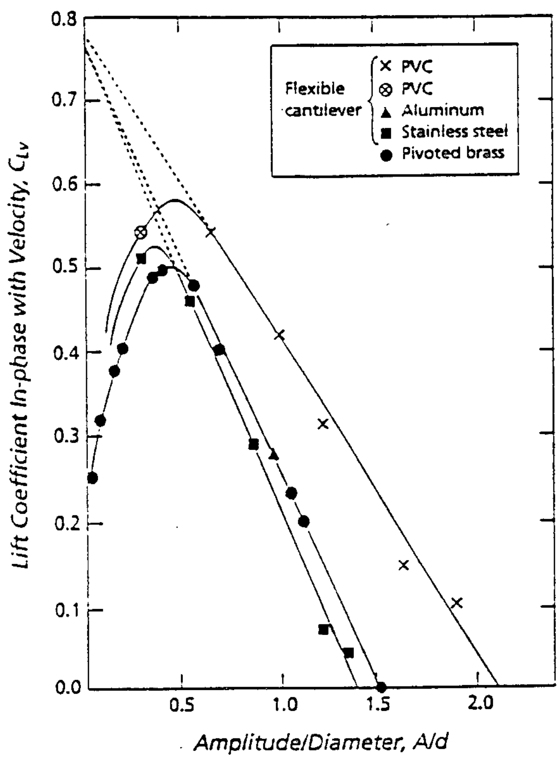

Figure 1. The lift coefficient in-phase with velocity as a function of the amplitude-to-diameter ratio (King 1977).

motion. We denote the lift force per unit span that is in-phase with the velocity by $L_v(t)$ and its amplitude by L_o and proceed to nondimensionalize it to obtain the lift coefficient that is in-phase with velocity, C_{L_v}:

$$C_{L_v} = \frac{L_o}{\frac{1}{2}\rho d V^2} \qquad (1)$$

where ρ denotes the fluid density.

Figure 1 shows a plot of the measured coefficient, C_{L_v}, of rigid pivoted cylinders versus the vibration amplitude for a nondimensional oscillation frequency close to the Strouhal number (King 1977). Power input occurs when C_{L_v} is positive and dissipation occurs when C_{L_v} is negative. Except for small amplitudes, when the vortex formation process is not well correlated along the span of the cylinder, there is clearly a linear relation between the lift coefficient and the amplitude of motion. Over a range of practical interest, typically for amplitude to diameter ratios higher than 0.4, the curve can be approximated by a straight line with negative slope. This is a distinct feature of a nonlinear process that contains a term that can be modelled through a linear damping coefficient. The damping coefficient can be directly obtained from the slope of the line. A simple representation of the lift force curve is

$$C_{L_v} = C_o - \lambda \frac{A}{d} \qquad (2)$$

where $\frac{A}{d}$ is the amplitude-to-diameter ratio and C_o and λ are curve-fitting constants. Equation (2) is accurate if the cylinder vibrates with an amplitude that is larger than the threshold amplitude.

It should be noted that the methodology to replace an amplitude-dependent excitation by equivalent motion-dependent terms has been applied before in other fields to analyze nonlinear phenomena, such as the value of wave-drift damping estimated from second order wave forces (Faltinsen 1990).

Experiments have been conducted in the MIT Testing Tank Facility on rigid circular cylinders of diameter 2.54 cm and span 30 cm, forced to move in a prescribed motion transversely to a flow with constant velocity V (Gopalkrishnan 1992). Figure 2 shows several plots of the coefficient, C_{L_v}, for harmonic motion versus the amplitude-to-diameter ratio for various imposed frequencies, which are near the frequency of the maximum in-phase lift coefficient. It is interesting to note that the slope of the various curves varies little over a range of nondimensional frequencies, $\frac{fd}{V}$, where f is the oscillation frequency in Hertz.

For a purely sinusoidal force at circular frequency $\omega = 2\pi f$, equation 2 provides the component of the lift force in phase with velocity:

$$L_v(t) = \frac{1}{2}\rho d V^2 \left(C_o - \lambda \frac{A}{d}\right) \sin \omega t \qquad (3)$$

If we define

$$L_e(t) = \left(\frac{1}{2}\rho d V^2\right) C_o \sin \omega t \qquad (4)$$

$$b_h = \left(\frac{1}{2}\rho d V^2\right) \frac{\lambda}{\omega d} \qquad (5)$$

$$v(t) = \omega A \sin \omega t \qquad (6)$$

where $v(t)$ is the cylinder velocity, we can write equation 3 more simply as

$$L_v(t) = L_e(t) - b_h v(t) \qquad (7)$$

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE 1994

3. REPORT TYPE AND DATES COVERED TECHNICAL

4. TITLE AND SUBTITLE
VORTEX-INDUCED VIBRATIONS IN A SHEARED FLOW: A NEW PREDICTIVE METHOD

5. FUNDING NUMBERS
ONR N00014-92-J-1269

6. AUTHOR(S)
M.S. TRIANTAFYLLOU, M.A.GROSENBAUGH, AND R. GOPALKRISHNAN

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
WOODS HOLE OCEANOGRAPHIC INSTITUTION
WOODS HOLE, MA 02543

8. PERFORMING ORGANIZATION REPORT NUMBER
WHOI CONTR. 8553

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
OFFICE OF NAVAL RESEARCH
ENVIRONMENTAL SCIENCES
DIRECTORATE
ARLINGTON, VA 22217-5660

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
In citing this report in a bibliography, the reference given should be:
In: Proceedings of the International Conference on Hydroelasticity in Marine Technology. May 1994. pgs. 31-38.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE:
DISTRIBUTION UNLIMITED

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

ABSTRACT: It is shown that a linear hydrodynamic damping term is an intrinsic feature of the vortex-induced vibrations of slender cylinders in the lock-in regime. The damping coefficient can be directly evaluated from experimental measurements of the force acting on a section of the cylinder forced to move in a uniform flow.

14. SUBJECT TERMS
1) hydrodynamic damping
2) vortex-induced vibrations
3) flow induced vibrations

15. NUMBER OF PAGES 8

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED

18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED

19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED

20. LIMITATION OF ABSTRACT

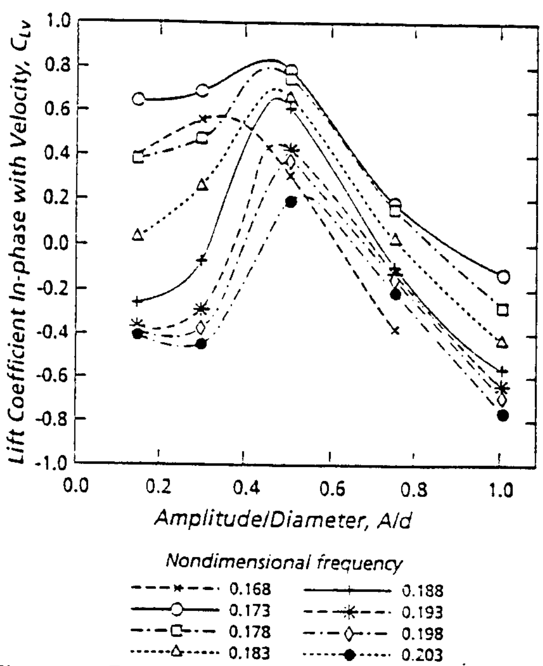

Figure 2. Experimental measurements (Gopalkrishnan 1992) of the lift coefficient in-phase with velocity as a function of the amplitude-to-diameter ratio for various values of the nondimensional frequency parameter $\frac{fd}{V}$.

In equation 7, the lift force that is in-phase with the cylinder velocity is decomposed into two parts, one of which is a pure excitation force $L_e(t)$ and a second term which is a linear damping force $b_h v(t)$. Equation 5 can be used to obtain a direct estimate of the hydrodynamic damping coefficient b_h, once the curve-fitting constant λ is determined from experimental data. We define the hydrodynamic damping ratio, ζ_h, as:

$$\zeta_h = \frac{b_h}{2m\omega} = \frac{\rho}{\rho_c}\frac{1}{4\pi^3}\frac{\lambda}{St^2} \qquad (8)$$

where m is the mass per unit length of the cylinder and ρ_c is the cylinder density. We have assumed that the harmonic motion is at the Strouhal frequency $\omega_v = 2\pi f_v$ and write the Strouhal number as $St = \frac{f_v d}{V}$. Then, taking a metallic cylinder with specific density equal to 5.0, specifying a Strouhal number of $St = 0.17$, and calculating $\lambda = 1.36$ from the experimental data in figure 2, we find that $\zeta_h = 0.076$ (7.6% of critical).

Both the damping coefficient, b_h, and the pure excitation force in the direction of the velocity, L_e, are independent of amplitude. The excitation force, however, is phase-correlated with the velocity. This is important in any numerical calculation, as shown in the sequel. For most applications in water, the structural damping is small in comparison to hydrodynamic damping and may be neglected. In air, the structural damping is significant and may be added directly to b_h.

3 NARROW-BAND RESPONSE

Lock-in of a flexibly mounted cylinder, or a flexible structure is usually characterized by a narrow-band response with characteristic beating oscillations. We can extend heuristically the derivation of §2 to apply to these cases when the response is not harmonic. For example, a three-dimensional plot of the lift coefficient in phase with velocity as function of the amplitude-to-diameter ratio and the frequency of oscillation can be constructed from figure 2. Such plots have been provided by Staubli (1983) and Gopalkrishnan (1992). The lift force that is in-phase with the velocity can be then represented then by the following, more general equation

$$C_{L_v}(\omega) = H(\omega)C_o - \Lambda(\omega)\lambda\frac{A}{d} \qquad (9)$$

which is similar to equation 2, but includes the frequency dependence ω in the curve-fitting parameters H and Λ. The functional form of the curve-fitting parameters is determined from experimental data, and C_o and λ are as defined before. Because of the similarity in the the shapes of the curves in figure 2, we conclude that $\Lambda(\omega)$ is very nearly constant over a narrow frequency range and is equal to one. This results in considerable simplification for use in numerical calculations.

The accuracy of equation 9 is subject to the same amplitude-threshold considerations as those related to equation 2. In addition, we note that in order for equation 9 to apply to a multi-frequency response, linearity must be assumed. This is not correct for other parameters relevant to vortex-induced oscillations. For example, the excitation force for monochromatic excitation at nonlock-in conditions contains an additional component at the Strouhal frequency. Thus, in order to employ equation 9, we must assume that the dominant force component has a frequency content that is within a narrow band around a specific frequency ω, and that the response is still within the lock-in regime. Triantafyllou and Karniadakis (1989) have shown numerically and Gopalkrishnan (1992) and Gopalkrishnan *et al.* (1992) have shown experimentally that, in the case of a beating oscillation, i.e., an oscillation consisting of two (or three equidistant) sinusoidal components, the harmonic results can be used to predict the lift force in a multi-frequency response, provided that the frequencies are sufficiently close together and within the lock-in regime. However, the drag force in a multi-frequency response can not be calculated on the basis of harmonic results.

Hence, assuming that harmonic data can be used to calculate the lift force in a narrow-band response, we can write H and Λ as integro-differential operators in the time domain. The damping, as expressed by the term containing Λ, is still linear and resembles, in form, the well-known, frequency-dependent damping of floating bodies in the presence of a free surface (Faltinsen 1990).

If the cylinder motion has a narrow-band spectrum about $\omega = \omega_v$, then we can exploit the fact that the slope of the lift-force coefficient in-phase with the velocity for a given imposed amplitude appears to be nearly frequency-independent (figure 2), and we can write the time-dependent lift force approximately as

$$L_v(t) = \left[\frac{v(t)}{\hat{v}(t)}\right] L_{eo} - b_h v(t) \qquad (10)$$

where $\hat{v}(t)$ is the slowly varying envelope of $v(t)$. The damping coefficient, b_h, is given by equation 5 with $\omega = \omega_v$, and L_{eo} is approximately given as

$$L_{eo} = H(\omega_v) C_o \left(\frac{1}{2}\rho d V^2\right) \qquad (11)$$

As with the case of the purely sinusoidal response, the expression for b_h is simple and can be determined directly from experimental data (figure 2). The difficulty in this case consists of ensuring that the excitation is indeed properly correlated with the velocity. This is straightforward in time-domain simulations, since one must calculate the envelope of the velocity at each time step before using equation 10. Often, however, frequency domain techniques are employed, resulting in considerable savings in computational expense; an additional requirement must then be imposed, to ensure that the excitation is properly correlated, viz.

$$\left[\lim_{T\to\infty} \frac{1}{T}\int_0^T L_e(t) v(t) dt\right]^2 = \frac{1}{2} L_o^2 \left[\lim_{T\to\infty}\frac{1}{T}\int_0^T v(t) v(t) dt\right] \qquad (12)$$

where

$$L_o = \left(\frac{1}{2}\rho d V^2\right) C_o \qquad (13)$$

4 APPLICATIONS

Below, we provide the results of simple calculations of the vortex-induced response of cylinders based on the concepts and equations derived in §2 and §3.

4.1 *Cylinder in a Uniform Current*

We begin by considering the narrow-band, lock-in response of a flexibly-mounted, rigid cylinder. The natural frequency of the system is equal to the frequency of maximum lift coefficient in-phase with the velocity. A compilation of data for vibrating cylinders as a function of the reduced damping from Griffin (1981) is shown, for comparison, in figure 3. The reduced damping is defined as the ratio of the structural damping ratio ζ_s and the quantity μ, where

$$\zeta_s = \frac{b_s}{2m\omega} \qquad (14)$$

$$\mu = \frac{\frac{1}{2}\rho d V^2}{m\omega^2 d} \qquad (15)$$

The term b_s is the structural damping coefficient per unit span.

Superimposed on the figure are calculations by the present method. Here, we have modelled the transverse motion, $y(t)$, of a rigid cylinder of unit span, having a mass m, mounted on a spring of constant k and a linear dashpot of constant b_s, and placed transversely to a constant flow of velocity V. The following is the equation of motion that is used for the calculations

$$m\frac{d^2y(t)}{dt^2} + b_s\frac{dy(t)}{dt} + ky(t) = f(t) \qquad (16)$$

The right hand side of the equation, $f(t)$, is the fluid force, which is written as the sum of an added mass term and the lift force in-phase with the velocity, which is further decomposed in accordance with equation 9. The method of harmonic balance together with equation 12 provides the solution plotted in figure 3. The calculations show good agreement with experimental data, even for large values of structural damping when the response is smaller than the threshold value.

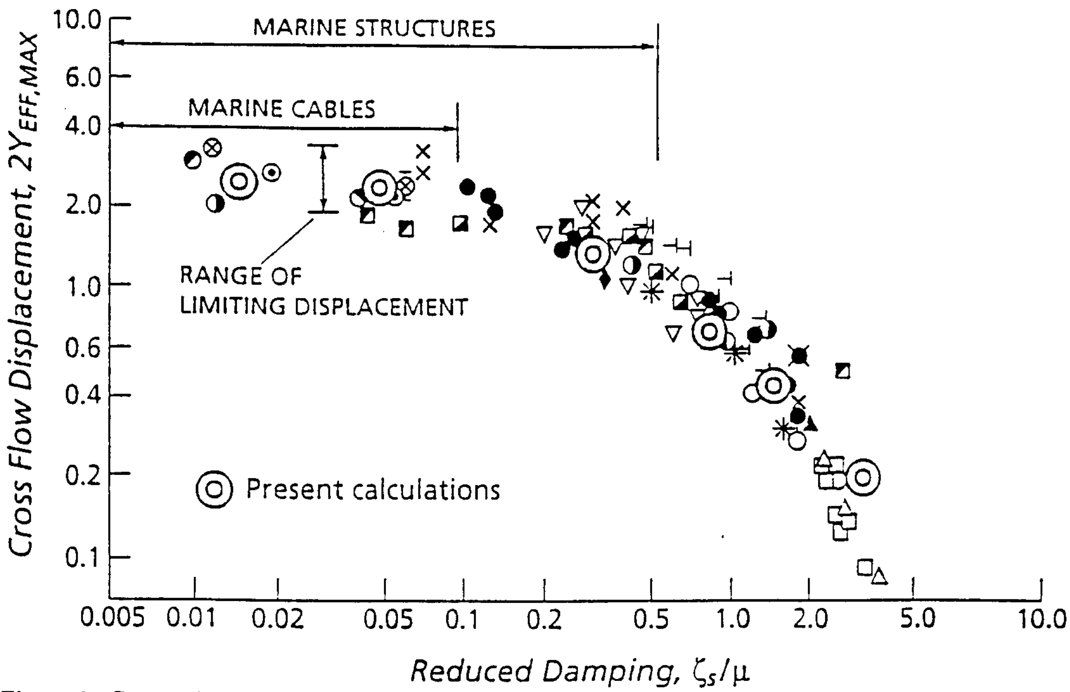

Figure 3. Comparison between the measurements of the maximum double-amplitude motion of a circular cylinder as a function of the reduced damping $\frac{\zeta}{\mu}$ (Griffin 1981) and calculations using the method outlined in this paper.

4.2 *Taut String in a Shear Current*

For the next application, we consider a taut string of length L placed normal to a spatially-varying current with nominal velocity V. The transverse response of the string, $y(t,s)$, is assumed to be described accurately by the following linear structural model and a hydrodynamic force, $f(s,t)$,

$$m\frac{\partial^2 y}{\partial t^2} + b_s\frac{\partial y}{\partial t} = \frac{\partial}{\partial s}\left[T(s)\frac{\partial y}{\partial s}\right] + f(s,t) \qquad (17)$$

where s is the Lagrangian coordinate along the string, $T(s)$ is the static tension, m is the mass per-unit-length and b_s is the structural damping per-unit-length. The force, $f(s,t)$, can be decomposed into the approximate form of equation 10 with a hydrodynamic damping force and an excitation force that is in-phase with the velocity. The decomposition also yields a term that represents the added mass force. It is further assumed that the characteristic wavelength of the string oscillations is much smaller than the length of the string, hence the response is effectively that of an infinitely long string.

An important consideration in studying the response of long structures is the length over which the vortex formation process can be assumed to be correlated. It is assumed herein that vortex shedding is fully correlated over half of a wavelength of a travelling wave. This is based on experimental measurements by Ramberg and Griffin (1976), who evaluated the cross-correlation between velocities measured at two locations in the wake of a vibrating cable, separated by a distance s along the axis of the cable: They found nearly perfect correlation for all points between two successive nodes of the vibrating cable for vibrational amplitudes above a threshold value. Also, Gharib (1989) showed through visualization of the response of a flexible cylinder that there is full correlation in the vortex formation process between two successive nodes, while, at the nodes, longitudinal vortical structures destroy any vortex interconnection. The frequency of excitation within a half wavelength is assumed to be equal to the frequency at the anti-node, where the maximum amplitude occurs.

Equation 17 can be solved together with equation 10 to provide the time-domain response of a cable, even when the response is not monochromatic. The present analysis is applicable provided that the response is narrow-banded and the maximum response amplitude is larger than about 0.4

diameters.

For this paper, we used standard frequency domain techniques to solve equations 10 and 17 and obtain the vibration amplitude of a tow cable in a shear current. The presence of shear current causes the vortex-induced vibrations to be amplitude modulated. The excitation force depends on the slowly varying envelope of the velocity of vibration, hence the solution is obtained by iteration. Once we calculate the vibration amplitude, we use the laboratory measurements of Gopalkrishnan (1992) to estimate the drag coefficient.

We compared our predictions against the following data from full-scale experiments of towed cables in shear currents:

1. Data from Yoerger *et al.* (1991) for run 1A in the authors' notation, involving a 1,200-meter cable towed nearly vertical at 0.5 m/s in the presence of a measured shear current. The configuration of the cable was recorded using acoustic transponders, and from these measurements the drag coefficient was estimated to be equal to 2.47 ±0.24. By using the measured shear current and the procedure outlined above, we obtained a spatially varying drag coefficient along the cable length, between the values of 2.0 and 2.7, with an average value of 2.21.

2. Data from Yoerger *et al.* (1991) for run 2A involving an 800-meter cable towed at 0.5 m/s in the presence of a shear current. The calculations provided an average drag coefficient of 2.05. The measured full-scale drag coefficient was 2.24 ±0.24.

3. Data from figure 14 in Grosenbaugh (1991) for a cable 1,200-meters long towed nearly vertically in a transient condition. Our calculations gave a spatially varying drag coefficient in the range of 1.7 to 2.7, with an average value of 2.08. The average drag coefficient from the full-scale measurements was 1.95 ±0.20.

4. Data from figure 3 in Grosenbaugh (1991) corresponding to a 1,200-meter tow cable that had reached steady-state conditions. The calculation provided a spatially varying drag coefficient in the range of 1.6 to 2.4, with an average value of 1.95. The estimated average drag coefficient from the full-scale measurements was 2.15 ±0.20.

5 SUMMARY

The basic result of the present paper is that a linear hydrodynamic damping term is an intrinsic feature of vortex-induced vibrations in the lock-in regime, as experimental results demonstrate. The value of the linear damping term can be obtained directly from forced-motion tests on rigid cylinders. This allows simple and efficient calculations of the vortex-induced response under lock-in conditions of flexibly-mounted, rigid cylinders and long flexible cylinders.

ACKNOWLEDGEMENTS

The authors wish to acknowledge support from the Office of Naval Research (Ocean Engineering Division) under grant numbers N00014-89-J-3061 and N00014-92-J-1269.

REFERENCES

Bearman, P. 1984. Vortex shedding from oscillating bluff bodies. *Annual Review of Fluid Mechanics* 16, 195-222.

Gharib, Morteza 1991. Ordered and chaotic vortex streets behind vibrating circular cylinders. *Marine Industry Collegium Symposium*, April 23-24, MIT, Cambridge, Massachusetts.

Gopalkrishnan, R. 1992. Vortex-induced forces on oscillating bluff cylinders. *Ph.D. Dissertation, Joint Program of Massachusetts Institute of Technology and Woods Hole Oceanographic Institution.*

Gopalkrishnan, R., Grosenbaugh, M.A., and Triantafyllou, M.S. 1992. Amplitude modulated cylinders in constant flow: Fundamental experiments to predict response in shear flow. *Proc. Third International Symposium on Flow Induced Vibrations and Noise* ASME, Anaheim, California.

Griffin, O.E. 1981. OTEC cold water pipe design for problems caused by vortex-excited oscillations. *Ocean Engng.* 8, 129-209.

Grosenbaugh, M.A. 1991. The effect of unsteady motion on the drag forces and flow-induced vibrations of a long vertical tow cable. *Int. J. Offshore and Polar Engng.* 1, 18-26.

Hartlen, R.T. and Currie, I.G. 1970. Lift oscillator model of vortex-induced vibrations, *J. Eng. Mech. Div. ASCE* 96, 577-591.

Faltinsen, O. 1990. *Sea Loads on Ships and Offshore Structures.* Cambridge University Press.

King, R. 1977. A review of vortex shedding research and its applications. *Ocean Engng* 4, 141-171.

Ramberg S.E. and Griffin O.M. 1976. Velocity correlation and vortex spacing in the wake of a vibrating cable. *Journal of Fluids Engineering* 98, 10-18.

Staubli, T. 1983. Calculation of the vibration of an elastically mounted cylinder using experimental data from forced oscillation. *Journal of Fluids Engineering* 105, 225-229.

Triantafyllou, G.S. and Karniadakis, G.E. 1989. Forces on a vibrating cylinder in steady crossflow. *Proc. Eighth International Conference on Offshore Mechanics and Arctic Engineering, The Hague, The Netherlands.* 2, 247-252.

Vandiver, J.K. 1988. Predicting the response characteristics of long, flexible cylinders in ocean currents. *Ocean Structural Dynamics Symposium '88, Corvallis, Oregon.*

Yoerger, D.R., Grosenbaugh, M.A., Triantafyllou, M.S. and Burgess, J. J. 1988. Drag forces and flow-induced vibrations of a long vertical tow cable. Part I: Steady-state towing conditions. *J. Offshore Mech. Arctic Engng. ASME* 113, 117-127.