QUARTZ CRYSTAL RESONATORS AND OSCILLATORS
For Frequency Control and Timing Applications
A TUTORIAL

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Quartz Crystal Resonators and Oscillators for Frequency Control and Timing Applications - A Tutorial

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This report is a slight revision of Technical Report SLCET-TR-88-1 (Rev. 7.3, same title, dated November 1996, AD-A317164). This report is approved for public release. Distribution is unlimited.

Subjects covered include: applications of frequency standards; types of oscillators (quartz and atomic); quartz resonator properties; quartz growing, sweeping, and material characteristics; Q and its significance; resonator and oscillator stability, including aging, short-term instability, frequency vs. temperature characteristics, oscillator circuit caused instabilities, frequency vs. drive level effects, acceleration effects, the effect of shock, and radiation effects; emerging technologies; atomic frequency standards; comparison of the major oscillator types; oscillator specifications and selection guidelines; time and timekeeping; clock errors; relativistic time; time transfer; time and frequency subsystem; and other applications of quartz resonators.

The goal of this document is to assist in presenting to the nonspecialist the most frequently encountered concepts in frequency control and timing. The document originated as a set of "hard copies" of presentation visuals (i.e., vu-graphs).
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* Chapter-specific references are listed at the end of each chapter.
"Everything should be made as simple as possible - but not simpler," said Einstein. The main goal of this "tutorial" is to assist with presenting the most frequently encountered concepts in frequency control and timing, as simply as possible.

In my position as Chief, Frequency Control and Timing Branch, US Army Electronics Technology and Devices Laboratory, I am often called upon to brief visitors, management, and potential users of precision oscillators. I have also been invited to present seminars and review papers before universities, IEEE, and other professional groups. In the beginning, I spent a great deal of time preparing these presentations. Much of the time was spent on preparing the presentation visuals (i.e., the vu-graphs). As I acumulate.

John R. Vig
Applications and Requirements
### Applications of Quartz Crystals

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<th>Consumer</th>
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<td>Watches &amp; clocks</td>
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<td>Navigation</td>
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<td>Cellular &amp; cordless phones, pagers</td>
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<td>IFF</td>
<td>Mobile/cellular/portable radio, telephone &amp; pager</td>
<td>Radio &amp; hi-fi eq'pment</td>
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<td>Radar</td>
<td>Aviation</td>
<td>Color TV</td>
</tr>
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<td>Sensors</td>
<td>Marine</td>
<td>Home computers</td>
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<td>Guidance systems</td>
<td>Navigation</td>
<td>VCR &amp; video camera</td>
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<td>Fuzes</td>
<td>Instrumentation</td>
<td>CB &amp; amateur radio</td>
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<td>Electronic warfare</td>
<td>Computers</td>
<td>CATV</td>
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<tr>
<td>Sonobuoys</td>
<td>Digital systems</td>
<td>Toys &amp; games</td>
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<td><strong>Research &amp; Metrology</strong></td>
<td>CRT displays</td>
<td>Pacemakers</td>
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<td>Atomic clocks</td>
<td>Disk drives</td>
<td></td>
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<td>Instruments</td>
<td>Modems</td>
<td><strong>Automotive</strong></td>
</tr>
<tr>
<td>Astronomy &amp; geodesy</td>
<td>Tagging/identification</td>
<td>Engine control, stereo, clock</td>
</tr>
<tr>
<td>Space tracking</td>
<td>Utilities</td>
<td>Trip computer</td>
</tr>
<tr>
<td>Celestial navigation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Frequency Control Device Market

<table>
<thead>
<tr>
<th></th>
<th>Commercial</th>
<th>Military</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (no./yr.)</td>
<td>&gt;1 billion</td>
<td>&lt;1 million</td>
</tr>
<tr>
<td>Unit cost (typical)</td>
<td>~$1</td>
<td>~$100</td>
</tr>
<tr>
<td>Major markets</td>
<td>Watches, clocks, color TV, autos, cellular &amp; mobile radios, computers</td>
<td>C³, nav., radar, fuzes, sonobuoys, IFF</td>
</tr>
<tr>
<td>Major R&amp;D thrusts</td>
<td>Cheaper, smaller</td>
<td>Higher stability (aging, noise, temp., acceleration, radiation) lower power, smaller, more rugged, cheaper</td>
</tr>
</tbody>
</table>
# Commercial - Military Comparison

<table>
<thead>
<tr>
<th>Commercial</th>
<th>Parameters</th>
<th>Military &amp; Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical</td>
<td>State-Of-The-Art</td>
<td>Fielded Systems</td>
</tr>
<tr>
<td>~ 92%</td>
<td>~ 2%</td>
<td>~ 5%</td>
</tr>
<tr>
<td>CB radios, watches, color TVs, microcomputers</td>
<td>Instruments, Commercial Spacecraft</td>
<td>PRC-77, VRC-12</td>
</tr>
<tr>
<td>10&lt;sup&gt;-4&lt;/sup&gt;</td>
<td>10&lt;sup&gt;-7&lt;/sup&gt;</td>
<td>5 x 10&lt;sup&gt;-5&lt;/sup&gt;</td>
</tr>
<tr>
<td>0°C to 60°C</td>
<td>0°C to 71°C</td>
<td>-55°C to +100°C</td>
</tr>
<tr>
<td>No Requirement</td>
<td>No Requirement</td>
<td>No Requirement</td>
</tr>
<tr>
<td>100 to 1,000 g</td>
<td>100 g</td>
<td>100 g</td>
</tr>
<tr>
<td>100 cm&lt;sup&gt;3&lt;/sup&gt;</td>
<td>300 cm&lt;sup&gt;3&lt;/sup&gt;</td>
<td>100 cm&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>Not oven-controlled</td>
<td>4 W</td>
<td>Not oven-controlled</td>
</tr>
<tr>
<td>Not oven-controlled</td>
<td>10 min</td>
<td>Not oven-controlled</td>
</tr>
<tr>
<td>No Requirement</td>
<td>No Requirement</td>
<td>Radiation hardened</td>
</tr>
</tbody>
</table>

Number of U.S. Companies ~ 60
Navigation

Precise time is essential to precise navigation. Historically, navigation has been a principal motivator in man’s search for better clocks. Even in ancient times, one could measure latitude by observing the stars’ position. However, to determine longitude, the problem became one of timing. Since the earth makes one revolution (360°) in 24 hours, one can determine longitude from the time difference \( \Delta t \) between local time (which was determined from the sun’s position) and the time at the Greenwich meridian (which was determined by a clock). Longitude in degrees = \((360°/24 \text{ hours}) \times \Delta t \) in hours.

In 1714, the British government offered a reward of 20,000 pounds to the first person to produce a clock that allowed the determination of a ship’s longitude to 30 nautical miles at the end of a six week voyage (i.e., a clock accuracy of three seconds per day). The Englishman John Harrison won the competition in 1735 for his chronometer invention.

Today’s electronic navigation systems still require ever greater accuracies. Since light (radio waves) travels 300 meters per microsecond, e.g., if a vessel’s timing was in error by one millisecond, a navigational error of 300 kilometers would result. In the Global Positioning System (GPS), atomic clocks in the satellites and quartz oscillators in the receivers provide nanosecond-level accuracies. The resulting (worldwide) navigational accuracies are about ten meters (see chapter 9 for further details about GPS).
Commercial Two-way Radio

Historically, as the number of users of commercial two-way radios have grown, channel spacings have been narrowed, and higher-frequency spectra have had to be allocated to accommodate the demand. Narrower channel spacings and higher operating frequencies necessitate tighter frequency tolerances for both the transmitters and the receivers. In 1940, when only a few thousand commercial broadcast transmitters were in use, a 500 ppm tolerance was adequate. Today, the millions of cellular telephones (which operate at frequency bands above 800 MHz) must maintain a frequency tolerance of 2.5 ppm. TCXOs of 2 ppm frequency accuracy are used for frequency control. The 896-901 MHz and 935-940 MHz mobile radio bands require frequency tolerances of 0.1 ppm at the base station and 1.5 ppm at the mobile station.

The need to accommodate more users will continue to require higher and higher frequency accuracies. For example, a NASA concept for a personal satellite communication system would use walkie-talkie-like hand-held terminals, a 30 GHz uplink, a 20 GHz downlink, and a 10 kHz channel spacing. The terminals’ frequency accuracy requirement is a few parts in 10^8.
Digital Network Synchronization

- Synchronization plays a critical role in digital telecommunication systems. It ensures that information transfer is performed with minimal buffer overflow or underflow events, i.e., with an acceptable level of "slips." Slips cause problems, e.g., missing lines in FAX transmission, clicks in voice transmission, loss of encryption key in secure voice transmission, and data retransmission.

- In AT&T's network, timing is distributed down a hierarchy of nodes. A timing source-receiver relationship is established between pairs of nodes containing clocks. The clocks are of four types, in four "stratum levels."

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Accuracy (Free running) long term</th>
<th>Clock type</th>
<th>Number used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1×10^{-11}</td>
<td>GPS w. two Rb</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>1.6×10^{-8}</td>
<td>Rb or OCXO</td>
<td>~200</td>
</tr>
<tr>
<td>3</td>
<td>4.6×10^{-6}</td>
<td>OCXO or TCXO</td>
<td>1000's</td>
</tr>
<tr>
<td>4</td>
<td>3.2×10^{-5}</td>
<td>XO</td>
<td>~1 million</td>
</tr>
</tbody>
</table>
Phase Noise in PLL and PSK Systems

The phase noise of oscillators can lead to erroneous detection of phase transitions, i.e., to bit errors, when phase shift keyed (PSK) digital modulation is used. In digital communications, for example, where 8-phase PSK is used, the maximum phase tolerance is ±22.5°, of which ±7.5° is the typical allowable carrier noise contribution. Due to the statistical nature of phase deviations, if the RMS phase deviation is 1.5°, for example, the probability of exceeding the ±7.5° phase deviation is 6 X 10^{-7}, which can result in a bit error rate that is significant in some applications.

Shock and vibration can produce large phase deviations even in "low noise" oscillators. Moreover, when the frequency of an oscillator is multiplied by N, the phase deviations are also multiplied by N. For example, a phase deviation of 10^{-3} radian at 10 MHz becomes 1 radian at 10 GHz. Such large phase excursions can be catastrophic to the performance of systems, e.g., of those which rely on phase locked loops (PLL) or phase shift keying. Low noise, acceleration insensitive oscillators are essential in such applications.
Utility Fault Location

When a fault occurs, e.g., when a "sportsman" shoots out an insulator, a disturbance propagates down the line. The location of the fault can be determined from the time of arrival differences:

\[ x = \frac{1}{2} [ L - c(t_b - t_a) ] = \frac{1}{2} [ L - c\Delta t ] \]

where \( x \) = distance of the fault from substation A, \( L \) = A to B line length, \( c \) = speed of light, and \( t_a \) and \( t_b \) = time of arrival of disturbance at A and B, respectively.

Fault locator error = \( x_{error} = \frac{1}{2}(c\Delta t_{error}) \); if \( \Delta t_{error} = 1 \) microsecond, then

\[ x_{error} = 150 \text{ meters} = 1/2 \text{ of high voltage tower spacings.} \]
Space Exploration

How does NASA know where a spacecraft is in deep space? The spacecraft's precise range, velocity and angular position are determined with the aid of highly stable frequency standards. The range is determined from the propagation time of microwave radiation between an antenna on Earth and the spacecraft. The velocity is determined from the "doppler," i.e., by comparing the phase of the incoming carrier signal with that of a reference signal generated from the ground station frequency standard. The angular position is determined by Very Long Baseline Interferometry (VLBI) in which widely separated stations (in California, Spain and Australia) simultaneously receive signals from the spacecraft. Differences between times of arrival coupled with knowledge of the baseline vectors joining the station antennas provide direct geometric determination of the angles between the baseline vectors and the direction to the spacecraft. Hydrogen masers provide the best stability ($\sim 10^{-15}$) for the propagation times of interest, which typically range from minutes to hours. VLBI is also used for high resolution angular measurements in radioastronomy.
Military Requirements

Military needs are a prime driver of frequency control technology. Modern military systems require oscillators/clocks that are:

- Stable over a wide range of parameters (time, temperature, acceleration, radiation, etc.)
- Low noise
- Low power
- Small size
- Fast warm-up
- Low life-cycle cost
# Impacts of Oscillator Technology Improvements

- Higher jamming resistance & improved ability to hide signals
- Improved ability to deny use of systems to unauthorized users
- Longer autonomy period (radio silence interval)
- Faster signal acquisition (net entry)
- Lower power for reduced battery consumption
- Improved spectrum utilization
- Improved surveillance capability (e.g., slow-moving target detection, bistatic radar)
- Improved missile guidance (e.g., on-board radar vs. ground radar)
- Improved identification-friend-or-foe (IFF) capability
- Improved electronic warfare capability (e.g., emitter location via TOA)
- Lower error rates in digital communications
- Improved navigation capability
- Improved survivability and performance in radiation environment
- Improved survivability and performance in high shock applications
- Longer life, and smaller size, weight, and cost
- Longer recalibration interval (lower logistics costs)
Spread Spectrum Systems

- In a spread spectrum system, the transmitted signal (e.g., a voice channel of a few kHz bandwidth) is spread over a bandwidth that is much wider (e.g., many MHz) than the bandwidth required to transmit the information being sent. This is accomplished by modulating a carrier signal with the information being sent, and with a wideband pseudonoise (PN) encoding signal. A spread spectrum receiver with the appropriate PN code can demodulate and extract the information being sent. Those without the PN code may completely miss the signal, or if they detect the signal, it appears to them as noise.

- Two of the spread spectrum modulation types are: 1. direct sequence, in which the carrier is modulated by a digital code sequence, and 2. frequency hopping, in which the carrier frequency jumps from frequency to frequency, within some predetermined set, the order of frequencies being determined by a code sequence.

- Transmitter and receiver contain clocks which must be synchronized; e.g., in a frequency hopping system, the transmitter and receiver must hop to the same frequency at the same time. The faster the hopping rate, the higher the jamming resistance, and the more accurate the clocks must be.

- Advantages of spread spectrum systems include the following capabilities: 1. rejection of intentional and unintentional jamming, 2. low probability of intercept (LPI), 3. selective addressing, 4. multiple access, and 5. high accuracy navigation and ranging.
Clock for Very Fast Frequency Hopping Radio

To defeat a "perfect" follower jammer, need a hop-rate given by:

\[ t_m < (t_1 + t_2) - t_R \]

where \( t_m \approx \text{msg. duration/hop} \) 
\[ \approx \frac{1}{\text{hop-rate}} \]

Example

Let R1 to R2 = 1 km, R1 to J = 5 km, and J to R2 = 5 km. Then, since propagation delay = 3.3 \( \mu \)s/km, \( t_1 = t_2 = 16.5 \mu \)s, \( t_R = 3.3 \mu \)s, and \( t_m < 30 \mu \)s.

Allowed clock error \( \approx 0.2 \ t_m \)
\[ \approx 6 \ \mu \text{s} \]

For a 4 hour resynch interval, clock accuracy requirement is:
\[ 4 \times 10^{-10} \]
## Clocks and Frequency Hopping C³ Systems

<table>
<thead>
<tr>
<th>Slow hopping</th>
<th>Fast hopping</th>
<th>Extended radio silence</th>
<th>Extended calibration interval</th>
<th>Orthogonality</th>
<th>Interoperability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good clock</td>
<td>Better clock</td>
<td>Better clock</td>
<td>Better clock</td>
<td>Better clock</td>
<td>Better clock</td>
</tr>
</tbody>
</table>
Identification-Friend-Or-Foe (IFF)

Air Defense IFF Applications

AWACS

F-16

FAAD

PATRIOT

STINGER

FRIEND OR FOE?
Identification-Friend-Or-Foe (IFF)

In a modern battle, when the sky is filled with friendly and enemy aircraft, and a variety of advanced weapons are ready to fire from both ground and airborne platforms, positive identification of friend and foe is critically important. For example, fratricide due to identification errors was a major problem in the 1973 Arab-Israeli war.

Current IFF systems use an interrogation/response method which employs cryptographically encoded spread spectrum signals. The interrogation signal received by a friend is supposed to result in the "correct" code being automatically sent back via a transponder on the friendly platform. The "correct" code must change frequently to prevent a foe from recording and transmitting that code ("repeat jamming"), thereby appearing as a friend. The code is changed at the end of what is called the Code Validity Interval, or CVI.

The better the clock accuracy, the shorter can be the CVI, the more resistant the system can be to repeat jamming, and the longer can be the autonomy period for users who cannot resynchronize their clocks during a mission.
Effect of Noise in Doppler Radar System

- Echo = Doppler shifted echo from moving target + large "clutter" signal
- (Echo signal) - (reference signal) → Doppler shifted signal from target
- Phase noise of the local oscillator modulates (decorrelates) the clutter signal, generates higher frequency clutter components, and thereby degrades the radar’s ability to separate the target signal from the clutter signal.
Bistatic Radar

Conventional (i.e., "monostatic") radar, in which the illuminator and receiver are on the same platform, is vulnerable to a variety of countermeasures. Bistatic radar, in which the illuminator and receiver are widely separated, can greatly reduce the vulnerability to countermeasures such as jamming and antiradiation weapons, and can increase slow moving target detection and identification capability via "clutter tuning" (receiver maneuvers so that its motion compensates for the motion of the illuminator; creates zero Doppler shift for the area being searched). The transmitter can remain far from the battle area, in a "sanctuary". The receiver can remain "quiet."

The timing and phase coherence problems can be orders of magnitude more severe in bistatic than in monostatic radar, especially when the platforms are moving. The two reference oscillators must remain synchronized and syntonized during a mission so that the receiver knows when the transmitter emits each pulse, and so that the phase variations will be small enough to allow a satisfactory image to be formed. Low noise crystal oscillators are required for short term stability; atomic frequency standards are often required for long term stability.
Doppler Shifts

Doppler radars require low-phase-noise oscillators. The velocity of the target and the radar frequency are the primary factors that determine the oscillator noise requirements. For example, to detect slow-moving targets, the noise close to the carrier must be low.
Chapter 1 References*


* References with a ● are general references, those with a number refer to the page of the same number.


Quartz Crystal Oscillators
Crystal Oscillator

- Tuning Voltage
- Crystal Resonator
- Amplifier
- Output Frequency
At the frequency of oscillation, the closed loop phase shift = 2π.

When initially energized, the only signal in the circuit is noise. That component of noise, the frequency of which satisfies the phase condition for oscillation, is propagated around the loop with increasing amplitude. The rate of increase depends on the excess, i.e., small-signal, loop gain and on the BW of the crystal network.

The amplitude continues to increase until the amplifier gain is reduced either by nonlinearities of the active elements ("self limiting") or by some automatic level control.

At steady state, the closed-loop gain = 1.
Oscillation and Stability

- If a phase perturbation $\Delta \phi$ occurs, the frequency must shift $\Delta f$ to maintain the $2n\pi$ phase condition, where $\Delta f/f = -\Delta \phi/2Q_L$ for a series-resonance oscillator, and $Q_L$ is the loaded $Q$ of the crystal in the network. The "phase slope" $d\phi/df$ is proportional to $Q_L$ in the vicinity of the series resonance frequency (see "Equivalent Circuit" and "Frequency vs. Reactance" in Chapt. 3).

- Most oscillators operate at "parallel resonance," where the reactance vs. frequency slope, $dX/df$, i.e., the "stiffness," is inversely proportional to $C_1$, the motional capacitance of the crystal unit.

- For maximum frequency stability with respect to phase (or reactance) perturbations in the oscillator loop, the phase slope (or reactance slope) must be maximum, i.e., $C_1$ should be minimum and $Q_L$ should be maximum. A quartz crystal unit's high $Q$ and high stiffness makes it the primary frequency (and frequency stability) determining element in oscillators.
Tunability and Stability

Making an oscillator tunable over a wide frequency range degrades its stability because making an oscillator susceptible to intentional tuning also makes it susceptible to factors that result in unintentional tuning. The wider the tuning range, the more difficult it is to maintain a high stability. For example, if an OCXO is designed to have a short term stability of $1 \times 10^{-12}$ for some averaging time and a tunability of $1 \times 10^{-7}$, then the crystal's load reactance must be stable to $1 \times 10^{-5}$ for that averaging time. Achieving such stability is difficult because the load reactance is affected by stray capacitances and inductances, by the the stability of the varactor's capacitance vs. voltage characteristic, and by the stability of the voltage on the varactor. Moreover, the $1 \times 10^{-5}$ load reactance stability must be maintained not only under benign conditions, but also under changing environmental conditions (temperature, vibration, radiation, etc.). Whereas a high stability, ovenized 10 MHz voltage controlled oscillator may have a frequency adjustment range of $5 \times 10^{-7}$ and an aging rate of $2 \times 10^{-8}$ per year, a wide tuning range 10 MHz VCXO may have a tuning range of 50 ppm and an aging rate of 2 ppm per year.
Oscillator Acronyms

- **XO**.......... Crystal Oscillator
- **VCXO**....... Voltage Controlled Crystal Oscillator
- **OCXO**....... Oven Controlled Crystal Oscillator
- **TCXO**....... Temperature Compensated Crystal Oscillator
- **TCVCXO**.... Temperature Compensated/Voltage Controlled Crystal Oscillator
- **OCVCXO**.... Oven Controlled/Voltage Controlled Crystal Oscillator
- **MCXO**....... Microcomputer Compensated Crystal Oscillator
- **RbXO**....... Rubidium-Crystal Oscillator
Crystal Oscillator Categories

The three categories, based on the method of dealing with the crystal unit’s frequency vs. temperature characteristic, are:

- **XO, crystal oscillator**, which does not contain means for reducing the crystal’s f vs. T characteristic (also called PXO - packaged crystal oscillator).

- **TCXO, temperature compensated crystal oscillator**, in which the output signal from a temperature sensor (thermistor) is used to generate a correction voltage that is applied to a voltage-variable reactance (varactor) in the crystal network. The reactance variations compensate for the crystal’s f vs. T characteristic. Analog TCXO’s can provide about a 20X improvement over the crystal’s f vs. T variation.

- **OCXO, oven controlled crystal oscillator**, in which the crystal and other temperature sensitive components are in a stable oven which is adjusted to the temperature where the crystal’s f vs. T has zero slope. OCXO’s can provide a >1000X improvement over the crystal’s f vs. T variation.
Crystal Oscillator Categories

- **Crystal Oscillator (XO)**
  
  ![Diagram of Crystal Oscillator (XO)]

- **Temperature Compensated (TCXO)**
  
  ![Diagram of Temperature Compensated (TCXO)]

- **Oven Controlled (OCXO)**
  
  ![Diagram of Oven Controlled (OCXO)]
## Hierarchy of Oscillators

<table>
<thead>
<tr>
<th>Oscillator Type*</th>
<th>Accuracy**</th>
<th>Typical Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crystal oscillator (XO)</td>
<td>10^{-5} to 10^{-4}</td>
<td>Computer timing</td>
</tr>
<tr>
<td>Temperature compensated crystal oscillator (TCXO)</td>
<td>10^{-6}</td>
<td>Frequency control in tactical radios</td>
</tr>
<tr>
<td>Microcomputer compensated crystal oscillator (MCXO)</td>
<td>10^{-8} to 10^{-7}</td>
<td>Spread spectrum system clock</td>
</tr>
<tr>
<td>Oven controlled crystal oscillator (OCXO)</td>
<td>10^{-8}</td>
<td>Navigation system clock &amp; frequency standard, MTI radar</td>
</tr>
<tr>
<td>Small atomic frequency standard (Rb, RbXO)</td>
<td>10^{-9}</td>
<td>C³ satellite terminals, bistatic &amp; multistatic radar</td>
</tr>
<tr>
<td>High performance atomic standard (Cs)</td>
<td>10^{-12} to 10^{-11}</td>
<td>Strategic C³, EW</td>
</tr>
</tbody>
</table>

*Sizes range from < 5 cm³ for clock oscillators to > 30 liters for Cs standards. Costs range from < $5 for clock oscillators to > $40,000 for Cs standards.*

**Including the effects of military environments and one year of aging.
Oscillator Circuit Types

Of the numerous oscillator circuit types, three of the more common ones, the Pierce, the Colpitts and the Clapp, consist of the same circuit except that the rf ground points are at different locations. The Butler and modified Butler are also similar to each other; in each, the emitter current is the crystal current. The gate oscillator is a Pierce-type that uses a logic gate plus a resistor in place of the transistor in the Pierce oscillator. (Some gate oscillators use more than one gate.)
Oscillator Circuit Types - Comments

The choice of oscillator circuit type depends on factors such as the desired frequency, stability, input voltage and power, output power and waveform, tunability, design complexity, cost, and the crystal unit’s characteristics.

In the Pierce family, the ground point location has a profound effect on the performance. The Pierce configuration is generally superior to the others, e.g., with respect to the effects of stray reactances and biasing resistors, which appear mostly across the capacitors in the circuit rather than the crystal unit. It is one of the most widely used circuits for high stability oscillators. In the Colpitts configuration, a larger part of the strays appears across the crystal, and the biasing resistors are also across the crystal, which can degrade performance. The Clapp is seldom used because, since the collector is tied directly to the crystal, it is difficult to apply a dc voltage to the collector without introducing losses or spurious oscillations. (See the references for more details.)

The Pierce family usually operates at "parallel resonance" (see "Resonator Frequency vs. Reactance" in Chapt. 3), although it can be designed to operate at series resonance by connecting an inductor in series with the crystal. The Butler family usually operates at (or near) series resonance. The Pierce can be designed to operate with the crystal current above or below the emitter current.

Gate oscillators are common in digital systems when high stability is not a major consideration.
Each of the three main parts of an OCXO, i.e., the crystal, the sustaining circuit, and the oven, contribute to instabilities. The various instabilities are discussed in the rest of chapter 3 and in chapter 4.
Oscillator Instabilities - General Expression

\[
\frac{\Delta f}{f_{oscillator}} \approx \frac{\Delta f}{f_{resonator}} + \frac{1}{2Q_L} \left[ 1 + \left( \frac{2f_f Q_L}{f} \right)^2 \right]^{-1/2} d\phi(f_f)
\]

where \( Q_L \) = loaded Q of the resonator, and \( d\phi(f_f) \) is a small change in loop phase at offset frequency \( f_f \) away from carrier frequency \( f \). Systematic phase changes and phase noise within the loop can originate in either the resonator or the sustaining circuit. Maximizing \( Q_L \) helps to reduce the effects of noise and environmentally induced changes in the sustaining electronics.

In a properly designed oscillator, short-term instabilities are determined by the resonator at offset frequencies smaller than the resonator’s half-bandwidth, and by the sustaining circuit and the amount of power delivered from the loop for larger offsets.
Oscillator Instabilities - Sustaining Circuit

- **Load reactance change** - adding a load capacitance to a crystal changes the frequency by

\[
\delta f = \frac{\Delta f}{f} = \frac{C_1}{2(C_0 + C_L)}
\]

then,

\[
\frac{\Delta (\delta f)}{\Delta C_L} = -\frac{C_1}{2(C_0 + C_L)^2}
\]

**Example:** If \( C_0 = \text{pF}, \ C_1 = 14\text{fF} \) and \( C_L = 20 \text{ pF} \), then \( \Delta C_L = 10 \text{ fF} = 5 \times 10^{-4} \) causes \( \approx 1 \times 10^{-7} \) frequency change, and \( C_L \) aging of 10 ppm per day causes \( 2 \times 10^{-9} \) per day of oscillator aging.

- **Drive level changes:** Typically \( 10^{-8} \) per ma\(^2\) for a 10 MHz 3rd SC-cut.

- **DC bias** on the crystal contributes to oscillator aging.
Oscillator Instabilities - Tuned Circuits

Many oscillators contain tuned circuits - to suppress unwanted modes, as matching circuits, and as filters. The effects of small changes in the tuned circuit’s inductance and capacitance is given by

\[
\frac{\Delta f}{f_{oscillator}} \approx \frac{d\phi(f_f)}{2Q} \approx \left( \frac{1}{1 + \frac{2f_f}{BW}} \right) \left( \frac{Q_c}{Q} \right) \left( \frac{dC_c}{C_c} + \frac{dL_c}{L_c} \right)
\]

where BW is the bandwidth of the filter, \(f_f\) is the frequency offset of the center frequency of the filter from the carrier frequency, and \(Q_c\), \(L_c\) and \(C_c\) are the tuned circuit’s Q, inductance and capacitance, respectively.
Oscillator Instabilities - Circuit Noise

Flicker PM noise in the sustaining circuit causes flicker FM contribution to the oscillator output frequency given by:

$$L_{osc}(f_f) = L_{ckt}(1\text{Hz}) \frac{f^2}{4f_f^3Q_L^2}$$

and

$$\sigma_y(\tau) = \frac{1}{Q_L} \sqrt{\ln 2 L_{ckt}(1\text{Hz})}$$

where \(f_f\) is the frequency offset from the carrier frequency \(f\), \(Q_L\) is the loaded Q of the resonator in the circuit, \(L_{ckt}(1\text{Hz})\) is the flicker PM noise at \(f_f = 1\text{Hz}\), and \(\tau\) is any measurement time in the flicker floor range. For \(Q_L = 10^6\) and \(L_{ckt}(1\text{Hz}) = -140\text{dBc/Hz}\), \(\sigma_y(\tau) = 8.3 \times 10^{-14}\). \((L_{ckt}(1\text{Hz}) = -155\text{dBc/Hz} \text{ has been achieved.})\)
Oscillator Instabilities - External Load

If the external load changes, there is a change in the amplitude or phase of the signal reflected back into the oscillator. The portion of that signal which reaches the oscillating loop changes the oscillation phase, and hence the frequency by

\[
\frac{\Delta f}{f_{oscillator}} \approx \frac{d\Phi(f_f)}{2Q} \approx \left(\frac{1}{2Q}\right)\left(\frac{\Gamma - 1}{\Gamma + 1}\right)(\sin\theta)\sqrt{\text{isolation}}
\]

where \(\Gamma\) is the VSWR of the load, and \(\theta\) is the phase angle of the reflected wave; e.g., if \(Q \sim 10^6\), and isolation \(\sim 40\) dB (i.e., \(\sim 10^{-4}\)), then the worst case (100% reflection) pulling is \(\sim 5 \times 10^{-9}\). A VSWR of 2 reduces the maximum pulling by only a factor of 3. The problem of load pulling becomes worse at higher frequencies, because both the Q and the isolation are lower.
Most users require a sine wave, or a TTL-compatible, or a CMOS-compatible, or an ECL-compatible output. The latter three can be simply generated from a sine wave. The four output types are illustrated below, with the dashed lines representing the supply voltage inputs, and the bold solid lines, the outputs. (There is no "standard" input voltage for sine wave oscillators, and the input voltage for CMOS typically ranges from 5V to 15V.)

<table>
<thead>
<tr>
<th>+15V</th>
<th>+10V</th>
<th>+5V</th>
<th>0V</th>
<th>-5V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine</td>
<td>TTL</td>
<td>CMOS</td>
<td>ECL</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram showing sine, TTL, CMOS, and ECL outputs]
Chapter 2 References


Quartz Crystal Resonators
Why Quartz?

Quartz is the only material known that possesses the following combination of properties:

- Piezoelectric ("pressure-electric"; piezein = to press, in Greek)
- Zero temperature coefficient cuts exist
- Stress compensated cut exists
- Low loss (i.e., high Q)
- Easy to process; low solubility in everything, except fluoride etchants, under "normal" conditions; hard but not brittle
- Abundant in nature; easy to grow in large quantities, at low cost, and with relatively high purity and perfection. Of the man-grown single crystals, quartz, at ~3,000 tons per year, is second only to silicon in quantity grown (3 to 4 times as much Si is grown annually).
The Piezoelectric Effect

The direct piezoelectric effect was discovered by the Curie brothers in 1880. They showed that when a weight was placed on a quartz crystal, charges appeared on the crystal surface; the magnitude of the charge was proportional to the weight. In 1881, the converse piezoelectric effect was illustrated; when a voltage was applied to the crystal, the crystal deformed due to the lattice strains caused by the effect. The strains reversed when the voltage was reversed. The piezoelectric effect can, thereby, provide a coupling between an electrical circuit and the mechanical properties of a crystal. Under the proper conditions, a "good" piezoelectric resonator can stabilize the frequency of an oscillator circuit.

Of the 32 crystal classes, 20 exhibit the piezoelectric effect (but only a few of these are useful). Piezoelectric crystals lack a center of symmetry. When a force deforms the lattice, the centers of gravity of the positive and negative charges in the crystal can be separated so as to produce surface charges. The figures show one example (from Kelvin’s qualitative model) of the effect in quartz. Each silicon atom is represented by a plus, and each oxygen atom by a minus. When a strain is applied so as to elongate the crystal along the Y-axis, there are net movements of negative charges to the left and positive charges to the right (along the X-axis).
### The Piezoelectric Effect

<table>
<thead>
<tr>
<th>STRAIN</th>
<th>FIELD along:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>EXTENSIONAL</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>along:</td>
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<td></td>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>SHEAR</td>
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<td></td>
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<tr>
<td>about:</td>
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</tr>
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</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

In quartz, the five strain components shown may be generated by an electric field. The modes shown on the next page may be excited by suitably placed and shaped electrodes. The shear strain about the Z-axis produced by the Y-component of the field is used in the rotated Y-cut family which includes the AT, BT, and ST-cuts.
<table>
<thead>
<tr>
<th>Modes of Motion</th>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Face Shear Mode</strong></td>
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<tr>
<td><strong>Extensional Mode</strong></td>
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<td><strong>Flexure Mode</strong></td>
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<tr>
<td><strong>Third Overtone</strong></td>
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<tr>
<td><strong>Fundamental Mode</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Thickness Shear Mode</strong></td>
<td></td>
</tr>
</tbody>
</table>
Resonator Packaging

Two-point Mount Package

Three- and Four-point Mount Package

- Quartz Blank
- Cover
- Bonding Area
- Mounting Clips
- Seal
- Base
- Electrodes
- Pins

Top view of cover
Ceramic Flatpack and Metal-Enclosed Resonators
Resonator Vibration Amplitude Distribution

In an ideal resonator, the amplitude of vibration falls off exponentially outside the electrodes. In a properly designed resonator, a negligible amount of energy is lost to the mounting and bonding structure, i.e., the edges must be inactive in order for the resonator to be able to possess a high Q. The displacement of a point on the resonator surface is proportional to the drive current. At the typical drive currents used in (e.g., 10 MHz) thickness shear resonators, the peak displacement is on the order of a few atomic spacings.
Resonant Vibrations of a Quartz Plate

X-ray topographs (210 plane) of various modes excited during a frequency scan of a fundamental mode, circular, AT-cut resonator. The first peak, at 3.2 MHz, is the main mode; all others are unwanted modes. Dark areas correspond to high amplitudes of displacement.
Unwanted Modes vs. Temperature

(3 MHz rectangular AT-cut resonator, 22 x 27 x 0.552 mm)

Activity dips occur where the $f$ vs. $T$ curves of unwanted modes intersect the $f$ vs. $T$ curve of the wanted mode. Such activity dips are highly sensitive to drive level and load reactance.
Mathematical Description of a Quartz Resonator

- In piezoelectric materials, electrical current and voltage are coupled to elastic displacement and stress:

\[
\{T\} = [C] \{\varepsilon\} - [e] \{E\} \\
\{D\} = [e] \{\varepsilon\} + [e] \{E\}
\]

where \{T\} = stress tensor, [C] = elastic stiffness matrix, \{S\} = strain tensor, [e] = piezoelectric matrix, \{E\} = electric field vector, \{D\} = electric displacement vector, and \{e\} = is the dielectric matrix.

- For a linear piezoelectric material:

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6 \\
D_1 \\
D_2 \\
D_3 \\
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & -e_{11} & -e_{21} & -e_{31} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & -e_{12} & -e_{22} & -e_{32} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & -e_{13} & -e_{23} & -e_{33} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & -e_{14} & -e_{24} & -e_{34} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & -e_{15} & -e_{25} & -e_{35} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & -e_{16} & -e_{26} & -e_{36} \\
e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{11} & e_{12} & e_{13} \\
e_{21} & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & e_{21} & e_{22} & e_{23} \\
e_{31} & e_{32} & e_{33} & e_{34} & e_{35} & e_{36} & e_{31} & e_{32} & e_{33}
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6 \\
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]

where

\[
T_1 = T_{11}, \quad S_1 = S_{11}, \\
T_2 = T_{22}, \quad S_2 = S_{22}, \\
T_3 = T_{33}, \quad S_3 = S_{33}, \\
T_4 = T_{23}, \quad S_4 = 2S_{23}, \\
T_5 = T_{13}, \quad S_5 = 2S_{13}, \\
T_6 = T_{12}, \quad S_6 = 2S_{12}.
\]

- Elasto-electric matrix for quartz:

LINES JOIN NUMERICAL EQUALITIES EXCEPT FOR COMPLETE RECIPROCITY ACROSS PRINCIPAL DIAGONAL.
○ INDICATES NEGATIVE OF ●
● ● INDICATES TWICE THE NUMERICAL EQUALITIES
x INDICATES \(1/2 (e_{11} - c_{11})\)
Mathematical Description - Continued

- Number of independent non-zero constants depend on crystal symmetry. For quartz (trigonal, class 32), there are 10 independent linear constants - 6 elastic, 2 piezoelectric and 2 dielectric. "Constants" depend on temperature, stress, coordinate system, etc.

- To describe the behavior of a resonator, the differential equations for Newton's law of motion for a continuum, and for Maxwell's equation* must be solved, with the proper electrical and mechanical boundary conditions at the plate surfaces. ( \( F = ma \Rightarrow \frac{\partial T_{ij}}{\partial x_j} = \rho \ddot{u}_i \); \( \mathbf{\nabla} \cdot \mathbf{D} = 0 \Rightarrow \frac{\partial D_i}{\partial x_j} = 0 \),

\[
E_i = - \frac{\partial \phi}{\partial x_i} \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad ; \text{etc.}
\]

- Equations are very "messy" - have never been solved in closed form for physically realizable three-dimensional resonators. Nearly all theoretical work has used approximations.

- Some of the most important resonator phenomena (e.g., acceleration sensitivity) are due to nonlinear effects. Quartz has numerous higher order constants, e.g., 14 third-order and 23 fourth-order elastic constants, as well as 16 third-order piezoelectric coefficients are known; nonlinear equations are extremely messy.

\(*\) Magnetic field effects are generally negligible; quartz is diamagnetic.
Infinite Plate Thickness Shear Resonator

\[ f_n = \frac{n}{2h} \sqrt{\frac{c_{ij}}{\rho}} \quad n = 1, 3, 5 \]

Where \( f_n \) = resonant frequency of \( n \)-th harmonic
\( h \) = plate thickness
\( \rho \) = density
\( c_{ij} \) = elastic modulus associated with the elastic wave being propagated

\[ T_f = \frac{d(\log f_n)}{dT} = \frac{1}{f_n} \frac{df_n}{dT} = \frac{-1}{h} \frac{dh}{dT} - \frac{1}{2\rho} \frac{d\rho}{dT} + \frac{1}{2c_{ij}} \frac{dc_{ij}}{dT} \]

\( T_f \), the linear temperature coefficient of frequency, is negative for most materials (i.e., "springs" become "softer" as \( T \) increases). The coefficient for quartz can be +, - or zero (see next page).
Quartz Is Highly Anisotropic

- The properties of quartz vary greatly with crystallographic direction. For example, when a quartz sphere is etched deeply in HF, the sphere takes on a triangular shape when viewed along the Z-axis, and a lenticular shape when viewed along the Y-axis. The etching rate is more than 100 times faster along the fastest etching rate direction (the Z-direction) than along the slowest direction (the slow-X-direction).

- The thermal expansion coefficient is $7.8 \times 10^{-6}/^{\circ}C$ along the Z-direction, and $14.3 \times 10^{-6}/^{\circ}C$ perpendicular to the Z-direction; the temperature coefficient of density is, therefore, $-36.4 \times 10^{-6}/^{\circ}C$.

- The temperature coefficients of the elastic constants range from $-3300 \times 10^{-6}/^{\circ}C$ (for $C_{12}$) to $+164 \times 10^{-6}/^{\circ}C$ (for $C_{66}$).

- For the proper angles of cut, the sum of the first two terms in $T_f$ on the previous page is cancelled by the third term, i.e., temperature compensated cuts exist in quartz. (See next page.)
Zero Temperature Coefficient Quartz Cuts

The AT, FC, IT, SC, BT, and RT-cuts are on the loci of zero temperature coefficient cuts. The LC is a "linear coefficient" cut that is used in a thermometer.

Y-cut: \( \approx +90 \text{ ppm/}^\circ\text{C} \) (thickness-shear mode)

X-cut: \( \approx -20 \text{ ppm/}^\circ\text{C} \) (extensional mode)
Equivalent Circuits

The mechanically vibrating system and the circuit shown in the figure are "equivalent," because each can be described by the same differential equation. The mass, spring and damping element (i.e., the dashpot) correspond to the inductor, capacitor and resistor. The driving force corresponds to the voltage, the displacement of the mass to the charge on the capacitor, and the velocity to the current.

A crystal resonator is a mechanically vibrating system that is linked, via the piezoelectric effect, to the electrical world. In the (simplified) equivalent circuit (of one mode of vibration) of a resonator, on the next page, $C_0$ is called the "shunt" capacitance. It is the capacitance due to the electrodes on the crystal plate (plus the stray capacitances due to the crystal enclosure). The $R_1$, $L_1$, $C_1$ portion of the circuit is the "motional arm" which arises from the mechanical vibrations of the crystal. The $C_0$ to $C_1$ ratio is a measure of the interconversion between electrical and mechanical energy stored in the crystal, i.e., of the piezoelectric coupling factor, $k$, and $C_1$ is a measure of the crystal's "stiffness," i.e., its tunability - see the equation under the equivalent circuit on the next page. When a dc voltage is applied to the electrodes of a resonator, the $C_0/C_1$ is also a measure of the ratio of electrical energy stored in the capacitor formed by the electrodes to the energy stored elastically in the crystal due to the lattice strains produced by the piezoelectric effect. The $C_0/C_1$ is also a measure of the antiresonance-resonance frequency separation. (Let $r = C_0/C_1$, then $f_A - f_R = f_R/2r$, and $2r = (\pi N/2k)^2$, where $N = 1, 3, 5, \ldots$ is the overtone number.)

Some of the numerous advantages of quartz crystal resonator over a tank circuit built from discrete R's, C's and L's are that the crystal is far stiffer and has a far higher Q than what could be built from normal discrete components. For example, a 5 MHz fundamental mode AT-cut crystal may have $C_1 = 0.01$ pF, $L_1 = 0.1$ H, $R_1 = 5$ $\Omega$, and $Q = 10^6$. A 0.01pF capacitor is not available, since the leads attached to such a capacitor would alone probably contribute more than 0.01 pF. Similarly, a 0.1 H inductor would be physically large, it would need to include a large number of turns, and would need to be superconducting in order to have a $\leq 5$ $\Omega$ resistance.
Equivalent Circuit of a Resonator

Symbol for crystal unit

\[ \frac{\Delta f}{f_s} \approx \frac{C_1}{2(C_0 + C_L)} \]

1. Voltage control (VCXO)
2. Temperature compensation (TCXO)
Crystal Oscillator f vs. T Compensation

Uncompensated Frequency

Compensating Voltage on Varactor $C_L$

Compensated Frequency of TCXO
Resonator Frequency vs. Reactance

- Reactance vs. Frequency plot
- Area of Usual "Parallel Resonance"
- Series Resonance
- Antiresonance
- $\frac{1}{2\pi fC_0}$
- $f_S$
- $f_A$
Equivalent Circuit Parameter Relationships

\[
C_0 \equiv \varepsilon \frac{A}{t} \quad r \equiv \frac{C_0}{C_1}
\]

\[
f_s = \frac{1}{2\pi} \sqrt{\frac{1}{L_1 C_1}} \quad f_a - f_s \equiv \frac{f_s}{2r}
\]

\[
Q = \frac{1}{2\pi f_s R_1 C_1} \quad \omega L_1 - \frac{1}{\omega C_1} = \frac{\omega L_1}{R_1}
\]

\[
\tau_i = R_1 C_1 \approx 10^{-14} \text{s} \quad \frac{d\varphi}{df} \approx \frac{360 Q}{\pi f_s}
\]

\[
C_{1n} \approx \frac{f C_{11}}{n^3} \quad L_{1n} \approx \frac{n^3 L_{11}}{f^3} \quad R_{1n} \approx \frac{n^3 R_{11}}{f} \quad 2r = \left(\frac{\pi n}{2k}\right)^2
\]

- \(n\): Overtone number
- \(C_0\): Static capacitance
- \(C_1\): Motional capacitance
- \(C_{1n}\): \(C_1\) of \(n\)-th overtone
- \(L_1\): Motional inductance
- \(L_{1n}\): \(L_1\) of \(n\)-th overtone
- \(R_1\): Motional resistance
- \(R_{1n}\): \(R_1\) of \(n\)-th overtone
- \(\varepsilon\): Dielectric permittivity of quartz
  \(\approx 40 \times 10^{-13} \text{ pF/mm (average)}\)
- \(A\): Electrode area
- \(t\): Plate thickness
- \(r\): Capacitance ratio
- \(f_s\): Series resonance frequency \(\approx f_R\)
- \(f_a\): Antiresonance frequency
- \(Q\): Quality factor
- \(\tau_i\): Motional time constant
- \(\omega\): Angular frequency \(= 2\pi f\)
- \(\varphi\): Phase angle of the impedance
- \(k\): Piezoelectric coupling factor
  \(= 8.8\% \text{ for AT-cut, } 4.99\% \text{ for SC}\)
What is \( Q \) and Why Is It Important?

\[
Q \equiv 2\pi \frac{\text{Energy stored during a cycle}}{\text{Energy lost during the cycle}}
\]

- \( Q \) is proportional to the decay-time, and is inversely proportional to the linewidth of resonance (see next page).

- The higher the \( Q \), the higher the frequency stability and accuracy \textbf{capability} of a resonator (i.e., high \( Q \) is a necessary but not a sufficient condition). If, e.g., \( Q = 10^6 \), then \( 10^{-10} \) accuracy requires ability to determine center of resonance curve to 0.01% of the linewidth, and stability (for some averaging time) of \( 10^{-12} \) requires ability to stay near peak of resonance curve to \( 10^{-6} \) of linewidth.

- Phase noise close to the carrier has an especially strong dependence on \( Q \) \((S_\phi \propto 1/Q^4)\).
Decay Time, Linewidth, and Q

\[ W \approx \frac{1}{\pi t_d} \]

\[ Q = \frac{v_0}{W} \equiv v_0 \pi t_d \]
Factors That Determine Resonator $Q$

The maximum $Q$ of a quartz crystal resonator is given by:

$$Q_{\text{max}} = \frac{1}{2\pi f \tau},$$

where $f$ is the frequency in Hz, and $\tau$ is an empirically determined time constant in seconds, which varies with the angles of cut and the mode of vibration. For example, $\tau = 1 \times 10^{-14}$s for the AT-cut’s c-mode ($Q_{\text{max}} = 3.2$ million at 5 MHz), $\tau = 9.9 \times 10^{-15}$s for the SC-cut’s c-mode, and $\tau = 4.9 \times 10^{-15}$s for the BT-cut’s b-mode.

**Other factors** which affect the $Q$ of a resonator include:

- Overtone
- Surface finish
- Material impurities and defects
- Mounting stresses
- Bonding stresses
- Temperature
- Electrode geometry and type
- Blank geometry (contour, dimensional ratios)
- Drive level
- Gases inside the enclosure (pressure, type of gas)
- Interfering modes
- Ionizing radiation
Resonator Fabrication Steps

1. Design Resonators
2. Grow Quartz
3. Sweep
4. Cut
5. Lap
6. Round
7. Orient in Mask
8. Clean (Chemical Polish)
9. Contour
10. Angle Correct
11. X-Ray Orient
12. Deposit Contacts
13. Prepare Enclosure
14. Mount
15. Bond
16. Inspect
17. Clean
18. Test
19. Seal
20. Frequency Adjust
21. Plate
22. Bake
23. Final Clean

Oscillator
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880</td>
<td>Piezoelectric effect discovered by Jacques and Pierre Curie</td>
</tr>
<tr>
<td>1905</td>
<td>First hydrothermal growth of quartz in a laboratory - by G. Spezia</td>
</tr>
<tr>
<td>1917</td>
<td>First application of piezoelectric effect, in sonar</td>
</tr>
<tr>
<td>1918</td>
<td>First use of piezoelectric crystal in an oscillator</td>
</tr>
<tr>
<td>1926</td>
<td>First quartz crystal controlled broadcast station</td>
</tr>
<tr>
<td>1927</td>
<td>First temperature compensated quartz cut discovered</td>
</tr>
<tr>
<td>1927</td>
<td>First quartz crystal clock built</td>
</tr>
<tr>
<td>1934</td>
<td>First practical temp. compensated cut, the AT-cut, developed</td>
</tr>
<tr>
<td>1949</td>
<td>Contoured, high-Q, high stability AT-cuts developed</td>
</tr>
<tr>
<td>1956</td>
<td>First commercially grown cultured quartz available</td>
</tr>
<tr>
<td>1956</td>
<td>First TCXO described</td>
</tr>
<tr>
<td>1972</td>
<td>Miniature quartz tuning fork developed; quartz watches available</td>
</tr>
<tr>
<td>1974</td>
<td>The SC-cut (and TS/TTC-cut) predicted; verified in 1976</td>
</tr>
<tr>
<td>1982</td>
<td>First MCXO with dual c-mode self-temperature sensing</td>
</tr>
</tbody>
</table>
Quartz Resonators for Wristwatches

- Requirements:
  - Small size
  - Low power dissipation (of oscillator)
  - Low cost
  - High stability (temperature, aging, shock, attitude)

- These requirements can be met with 32,768 Hz quartz tuning forks.
Why 32,768 Hz?

\[ 32,768 = 2^{15} \]

- In an analog watch, a stepping motor receives one impulse per second which advances the second hand by 6°, i.e., 1/60th of a circle, every second.

- Dividing 32,768 Hz by two 15 times results in 1 Hz.

- The 32,768 Hz is a compromise among size, power requirement (i.e., battery life) and stability.
Quartz Tuning Fork

a) natural faces and crystallographic axes of quartz

b) crystallographic orientation of tuning fork

c) vibration mode of tuning fork
Watch Crystal

2 mm diameter x 6 mm tall cylinder
(0.08" diameter x 0.24" tall)
Chapter 3 References


Oscillator Stability
The Units of Stability in Perspective

- What is one part in $10^{10}$? (As in $1 \times 10^{-10}$/day aging.)
  - $\sim 1/2$ cm out of the circumference of the earth.
  - $\sim 1/4$ second per human lifetime ($\sim 80$ years).

- What is -170 dB? (As in -170 dBC/Hz phase noise.)
  - -170 dB = 1 part in $10^{17} \approx$ thickness of a sheet of paper out of total distance traveled by all the cars in the world in a day.
Accuracy, Precision and Stability

- Precise but not accurate
- Not accurate and not precise
- Accurate but not precise
- Accurate and precise

- Stable but not accurate
- Not stable and not accurate
- Accurate but not stable
- Stable and accurate
# Influences on Oscillator Frequency

- **Time**
  - Short term (noise)
  - Intermediate term (e.g., due to oven fluctuations)
  - Long term (aging)

- **Temperature**
  - Static frequency vs. temperature
  - Dynamic frequency vs. temperature (warmup, thermal shock)
  - Thermal history ("hysteresis," "retrace")

- **Acceleration**
  - Gravity (2g tipover)
  - Vibration
  - Acoustic noise
  - Shock

- **Ionizing radiation**
  - Steady state
  - Pulsed
  - Photons (X-rays, γ-rays)
  - Particles (neutrons, protons, electrons)

- **Other**
  - Power supply voltage
  - Atmospheric pressure (altitude)
  - Humidity
  - Magnetic field
  - Load impedance
Idealized Frequency-Time-Influence Behavior

- Temperature Step
- Vibration
- Shock
- Oscillator Turn Off & Turn On
- 2-g Tipover
- Radiation
- Aging

Short-Term Instability

\[ \Delta f \frac{x}{f} \times 10^8 \]

- \( t_0 \)
- \( t_1 \)
- \( t_2 \)
- \( t_3 \)
- \( t_4 \)
- \( t_5 \)
- \( t_6 \)
- \( t_7 \)
- \( t_8 \)

Time
Aging and Short-Term Stability

Short-term instability (Noise)

\[ \Delta f/f \text{ (ppm)} \]

Time (days)
Aging Mechanisms

- **Mass transfer due to contamination**
  Since \( f \propto 1/t \), \( \Delta f/f = -\Delta t/t \); e.g., \( f_{5\text{MHz}} \approx 10^6 \) molecular layers, therefore, 1 quartz-equivalent monolayer \( \Rightarrow \Delta f/f \approx 1 \) ppm

- **Stress relief** in the resonator’s: mounting and bonding structure, electrodes, and in the quartz (?)

- **Other effects**
  - Quartz outgassing
  - Diffusion effects
  - Chemical reaction effects
  - Pressure changes in resonator enclosure (leaks and outgassing)
  - Oscillator circuit aging (load reactance and drive level changes)
  - Electric field changes (doubly rotated crystals only)
  - Oven-control circuitry aging
Aging can be positive or negative. Occasionally, a reversal of aging direction is observed. The above (computer generated) curves illustrate the three types of aging behaviors. The curve showing the reversal is the sum of the other two curves. Reversal indicates the presence of at least two aging mechanisms.
Stresses on a Quartz Plate

**Causes:**
- Thermal expansion coefficient differences
- Bonding materials changing dimensions upon solidifying/curing
- Residual stresses due to clip forming and welding operations, sealing
- Intrinsic stresses in electrodes
- Nonuniform growth, impurities & other defects during quartz growing
- Surface damage due to cutting, lapping and (mechanical) polishing

**Effects:**
- In-plane diametric forces
- Tangential (torsional) forces, especially in 3 and 4-point mounts
- Bending (flexural) forces, e.g., due to clip misalignment and electrode stresses
- Localized stresses in the quartz lattice due to dislocations, inclusions, other impurities, and surface damage
Thermal Expansion Coefficient of Quartz

THERMAL EXPANSION COEFFICIENT, $\alpha$, OF AT-CUT QUARTZ, $10^{-6}/\text{K}$

RADIAL

TANGENTIAL

$\alpha$ (THICKNESS) = 11.64
Force-Frequency Coefficient

\[ \Delta f = K_F \left( \frac{\text{Force}}{\text{Diameter}} \right) \left( \frac{\text{Frequency-constant}}{\text{Thickness}} \right) \]

* \( 10^{-15} \text{ m} \cdot \text{s} / \text{N} \)

AT-cut quartz

\[ K_F (\psi) \]

\[ 0^\circ \ 10^\circ \ 20^\circ \ 30^\circ \ 40^\circ \ 50^\circ \ 60^\circ \ 70^\circ \ 80^\circ \ 90^\circ \]
Strains Due To Mounting Clips

Photograph of a 1 cm diameter AT-cut resonator and its X-ray topograph. The topograph shows the lattice distortion due to the mounting stresses.
Strains Due To Bonding Cements

X-ray topographs showing lattice distortions caused by bonding cements; (a) Bakelite cement - expanded upon curing, (b) DuPont 5504 cement - shrank upon curing.
Mounting Force Induced Frequency Changes

The force-frequency coefficient, $K_F (\Psi)$, is defined by

$$\frac{\Delta f}{f} = K_F \frac{(\text{Force})(\text{Frequency-constant})}{(\text{Diameter})(\text{Thickness})}$$

Maximum $K_F (\text{AT-cut}) = 24.5 \times 10^{-15} \text{ m-s/N at } \Psi = 0^\circ$

Maximum $K_F (\text{SC-cut}) = 14.7 \times 10^{-15} \text{ m-s/N at } \Psi = 44^\circ$.

As an example, consider 5 MHz 3rd overtone, 14 mm diameter resonators. Then, since 1 gram = $9.81 \times 10^{-3}$ newtons, and assuming the presence of diametrical forces only,

$$\left(\frac{\Delta f}{f}\right)_{\text{Max}} = \begin{cases} 2.9 \times 10^{-8} \text{ per gram for an AT-cut resonator} \\ 1.7 \times 10^{-8} \text{ per gram for an SC-cut resonator} \end{cases}$$

$$\left(\frac{\Delta f}{f}\right)_{\text{Min}} = 0 \text{ at } \Psi = 61^\circ \text{ for an AT-cut resonator, and at } \Psi = 82^\circ \text{ for an SC-cut.}$$
When 22 MHz fundamental mode AT-cut resonators were reprocessed so as to vary the bonding orientations, the $f$ vs. $T$ characteristics of the resonators changed as if the angles of cut had been changed. The resonator blanks were 6.4 mm in diameter, plano-plano, and were bonded to low-stress mounting clips by nickel electrobonding.
Short Term Instability (Noise)

### Stable Frequency (Ideal Oscillator)

\[ V(t) = V_0 \sin(2\pi v_0 t) \]

\[ \Phi(t) = 2\pi v_0 t \]

### Unstable Frequency (Real Oscillator)

\[ V(t) = [V_0 + \varepsilon(t)] \sin[2\pi v_0 t + \phi(t)] \]

\[ \Phi(t) = 2\pi v_0 t + \phi(t) \]

Instantaneous frequency, \( v(t) = \frac{1}{2\pi} \frac{d\Phi(t)}{dt} = v_0 + \frac{1}{2\pi} \frac{d\phi(t)}{dt} \)

- \( V(t) = \) Oscillator output voltage, \( V_0 = \) Nominal peak voltage amplitude
- \( \varepsilon(t) = \) Amplitude noise, \( v_0 = \) Nominal (or "carrier") frequency
- \( \Phi(t) = \) Instantaneous phase, and \( \phi(t) = \) Deviation of phase from nominal (i.e., the ideal)
Impacts of Oscillator Noise

- Limits the ability to determine the current state and the predictability of precision oscillators
- Limits syntonization and synchronization accuracy
- Limits receivers’ useful dynamic range, channel spacing, and selectivity; can limit jamming resistance
- Limits radar performance (especially Doppler radar’s)
- Causes timing errors [$\sim \tau \sigma_y(\tau)$]
- Causes bit errors in digital communication systems
- Limits number of communication system users, as noise from transmitters interfere with receivers in nearby channels
- Limits navigation accuracy
- Limits ability to lock to narrow-linewidth (atomic) resonances
- Can cause loss of lock; can limit acquisition/reacquisition capability in phase-locked-loop systems
## Causes of Short Term Instabilities

- Johnson noise (thermally induced charge fluctuations, i.e., "thermal emf" in resistive elements)
- Phonon scattering by defects & quantum fluctuations (related to Q)
- Noise due to oscillator circuitry (active and passive components)
- Temperature fluctuations  - thermal transient effects  
  - activity dips at oven set-point
- Random vibration
- Fluctuations in the number of adsorbed molecules
- Stress relief, fluctuations at interfaces (quartz, electrode, mount, bond)
- Shot noise in atomic frequency standards
- ???
Example (a) shows a sine wave and its second harmonic. A signal consisting of the sum of the two waves is shown in the time domain (b), and in the frequency domain (c). In the time domain, all frequency components are summed together. In the frequency domain, signals are separated into their frequency components and the power level at each frequency is displayed.
# Short-Term Stability Measures

<table>
<thead>
<tr>
<th>Measure</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-sample deviation (square-root of Allan variance)</td>
<td>$\sigma_y(\tau)^*$</td>
</tr>
<tr>
<td>Spectral density of phase deviations</td>
<td>$S_\phi(f)$</td>
</tr>
<tr>
<td>Spectral density of fractional frequency deviations</td>
<td>$S_y(f)$</td>
</tr>
<tr>
<td>Phase noise</td>
<td>$\mathcal{L}(f)^*$</td>
</tr>
</tbody>
</table>

* Most frequently found on oscillator specification sheets

\[
f^2 S_\phi(f) = \nu^2 S_y(f); \quad \mathcal{L}(f) \equiv \frac{1}{2}[S_\phi(f)] \text{ (per IEEE Std.1139-1988)},
\]

\[
\sigma_y^2(\tau) = \frac{2}{(\pi \nu \tau)^2} \int_0^\infty S_\phi(f) \sin^4(\pi f \tau) df
\]

where $\tau =$ averaging time, $f =$ Fourier frequency, or "frequency from the carrier", and $\nu =$ carrier frequency.
Allan Variance

The two-sample deviation, or square-root of the "Allan variance," is the standard method of describing the short-term stability of oscillators in the time domain. It is usually denoted by $\sigma_y(\tau)$, where

$$\sigma_y^2(\tau) = \frac{1}{2} \langle (y_{k+1} - y_k)^2 \rangle .$$

The fractional frequencies, $y = \frac{\Delta f}{f}$, are measured over a time interval, $\tau$; $(y_{k+1} - y_k)$ are the differences between pairs of successive measurements of $y$, and, ideally, $\langle \rangle$ denotes a time average of an infinite number of $(y_{k+1} - y_k)^2$. A good estimate can be obtained by a limited number, $m$, of measurements $(m \geq 100)$. $\sigma_y(\tau)$ generally denotes $\sqrt{\sigma_y^2(\tau, m)}$, i.e.,

$$\sigma_y^2(\tau) = \sigma_y^2(\tau, m) = \frac{1}{m} \sum_{j=1}^{m} \frac{1}{2} (y_{k+1} - y_k)^2 .$$
Why Allan Variance?

- Classical variance: \( \sigma^2 = \frac{1}{m-1} \sum (y_i - \bar{y})^2 \),

  diverges for commonly observed noise processes, such as random walk, i.e., the variance increases with increasing number of data points.

- Allan variance:
  - Converges for all noise processes observed in precision oscillators.
  - Has straightforward relationship to power law spectral density types.
  - Is easy to compute.
  - Is faster and more accurate in estimating noise processes than the Fast Fourier Transform.
Frequency Noise and $\sigma_y(\tau)$

\[ \frac{\Delta f}{f} \]

$0.1 \text{ s averaging time}$

\[ \frac{\Delta f}{f} \]

$1.0 \text{ s averaging time}$

\[ \sigma_y(\tau) \]

$10^{-10}$

$10^{-11}$

$10^{-12}$

.01 .1 1 10 100

Averaging time, $\tau$, s
**Time Domain Stability**

For $\sigma_y(\tau)$ to be a proper measure of random frequency fluctuations, aging must be properly subtracted from the data at long $\tau$'s.
Below the flicker of frequency noise (i.e., the "flicker floor") region, crystal oscillators typically show $\tau^{-1}$ (white phase noise) dependence. Atomic standards show $\tau^{-1/2}$ (white frequency noise) dependence down to about the servo-loop time constant, and $\tau^{-1}$ dependence at less than that time constant. Typical $\tau$'s at the start of flicker floors are: 1 second for a crystal oscillator, $10^3$ s for a Rb standard and $10^5$ s for a Cs standard.
Spectral Densities

\[ V(t) = [V_0 + \varepsilon(t)] \sin [2\pi v_0 t + \phi(t)] \]

In the frequency domain, due to the "phase noise", \( \phi(t) \), some of the power is at frequencies other than \( v_0 \). The stabilities are characterized by "spectral densities." The spectral density \( S_\nu(f) \), the mean-square voltage \(<V^2(t)\>\) in a unit bandwidth centered at \( f \), is not a good measure of frequency stability because both \( \varepsilon(t) \) and \( \phi(t) \) contribute to it, and because it is not uniquely related to frequency fluctuations (although \( \varepsilon(t) \) is usually negligible in precision frequency sources.)

The spectral densities of phase and fractional-frequency fluctuations, \( S_\phi(f) \) and \( S_y(f) \), respectively, are used to characterize stabilities in the frequency domain. The spectral density \( S_g(f) \) of a quantity \( g(t) \) is the mean square value of \( g(t) \) in a unit bandwidth centered at \( f \). Moreover, the RMS value of \( g^2 \) in bandwidth \( BW \) is given by \( g_{\text{RMS}}^2(t) = \int_{\text{BW}} S_g(f) \, df \).
## Pictures of Noise

<table>
<thead>
<tr>
<th>Plot of $z(t)$ vs. $t$</th>
<th>$S_z(f) = h_\alpha f^\alpha$</th>
<th>Noise name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = 0$</td>
<td>White</td>
</tr>
<tr>
<td></td>
<td>$\alpha = -1$</td>
<td>Flicker</td>
</tr>
<tr>
<td></td>
<td>$\alpha = -2$</td>
<td>Random walk</td>
</tr>
<tr>
<td></td>
<td>$\alpha = -3$</td>
<td></td>
</tr>
</tbody>
</table>

Plots show fluctuations of a quantity $z(t)$, which can be, e.g., the output of a counter ($\Delta f$ vs. $t$) or of a phase detector ($\phi[t]$ vs. $t$). The plots show simulated time-domain behaviors corresponding to the most common (power-law) spectral densities; $h_\alpha$ is an amplitude coefficient. Note: since $S_{\Delta f} = f^2 S_\phi$, e.g. white frequency and random walk of phase are equivalent.
Mixer Functions

\[ V_1 = A_1 \sin (\omega_1 t + \phi_1) \]
\[ V_2 = A_2 \sin (\omega_2 t + \phi_2) \]

Trigonometric identities:  
\[ \sin(x)\sin(y) = \frac{1}{2} \cos(x-y) - \frac{1}{2} \cos(x+y) \]
\[ \cos(x \pm \pi/2) = \sin(x) \]

Let \( \omega_1 = \omega_2; \Phi_1 \equiv \omega_1 t + \phi_1, \) and \( \Phi_2 \equiv \omega_2 t + \phi_2. \) Then the mixer can become:

- **Phase detector:** When \( \Phi_1 = \Phi_2 + \pi/2 \) and \( A_1 = A_2 = 1, \) then
  \[ V_0 = \frac{1}{2} \sin(\phi_1 - \phi_2) = \frac{1}{2} (\phi_1 - \phi_2) \] for small \( \phi'\)s

- **AM detector:** When \( A_2 = 1 \) and the filter is a low-pass filter, then
  \[ V_0 = \frac{1}{2} A_1 \cos (\phi_1 - \phi_2); \] if \( \phi_1 \approx \phi_2, \) then \( V_0 \approx \frac{1}{2} A_1 \)

- **Frequency multiplier:** When \( V_1 = V_2 \) and the filter is band-pass at \( 2\omega_1, \) then
  \[ V_0 = \frac{1}{2} A_1^2 \cos(2\omega_1 t + 2\phi_1) \] \( \Rightarrow \) Doubles the frequency and phase error.
i.e., the phase detector converts phase fluctuations to voltage fluctuations.

\[ (1) \phi K = (1) \phi \Lambda / (1) \phi >> (1) \phi / (1) \phi \]

For the low-pass filter (LPF) eliminating the second cosine term, then,

\[ [\cdots + (1) \phi V / 2 + \cos(2\pi (1) \phi) - (1) \phi] \Lambda \cos K \cos(2\pi (1) \phi) - (1) \phi] \Lambda = (1) \Lambda \Lambda = (1) \Lambda ^{0} \Lambda \]

Then, double-balanced mixer. Then, frequency and in phase quadrature (i.e., 90° out of phase), are input to a device under test (DUT) and a reference source, at the same time.

**Phase Detector**
Phase Noise Measurements

\[ V(t) = V_o \sin [2\pi v_o t + \Phi(t)] \]

\[ V_\phi(t) = k\Phi(t) \]

Oscilloscope

\[ \Phi(t) \]

RF Voltmeter

\[ \Phi_{RMS}(t) \text{ in BW of meter} \]

Spectrum Analyzer

\[ S_\phi(f) \text{ vs. } f \]
Frequency - Phase - Time Relationships

\[ \nu(t) = \nu_0 + \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \text{"instantaneous" frequency;} \quad \phi(t) = \phi_0 + \int_0^t 2\pi[\nu(t') - \nu_0]dt' \]

\[ y(t) = \frac{v(t) - \nu_0}{\nu_0} = \frac{\dot{\phi}(t)}{2\pi\nu_0} = \text{normalized frequency;} \quad \phi^2 = \int_{\text{RMS}} S_\phi(f)df \]

\[ S_\phi(f) = \frac{\phi^2_{\text{RMS}}}{\text{BW}} = \left(\frac{\nu_0}{f}\right)^2 S_y(f) ; \quad \mathcal{L}(f) = 1/2 S_\phi(f), \text{ per IEEE Standard 1139-1988} \]

\[ \sigma_y^2(\tau) = 1/2 \langle (\tilde{y}_{k+1} - \tilde{y}_k)^2 \rangle = \frac{2}{(\pi \nu_0 \tau)^2} \int_0^\infty S_\phi(f) \sin^4(\pi f \tau) df \]

The five common power-law noise processes in precision oscillators are:

\[ S_y(f) = h_2 f^2 + h_1 f + h_0 + h_{-1} f^{-1} + h_{-2} f^{-2} \]

(White PM) (Flicker PM) (White FM) (Flicker FM) (Random-walk FM)

Time deviation \[ x(t) = \int_0^t y(t') dt' = \frac{\phi(t)}{2\pi \nu_0} \]

* MIL-O-55310B's definition of phase noise is \[ \mathcal{L}(f) = 10 \log [S_\phi(f)/2], \] where the unit of \[ \mathcal{L}(f) \] is dBc.
**$S_\phi(f)$ to SSB Power Ratio Relationship**

Consider the "simple" case of sinusoidal phase modulation at frequency $f_m$. Then, $\phi(t) = \phi_0(t)\sin(2\pi f_m t)$, and $V(t) = V_0\cos[2\pi f_c t + \phi(t)] = V_0\cos[2\pi f_c t + \phi_0(t)\sin(2\pi f_m t)]$, where $\phi_0(t) = \text{peak phase excursion}$, and $f_c = \text{carrier frequency}$. Cosine of a sine function suggests a Bessel function expansion of $V(t)$ into its components at various frequencies via the identities:

\[
\begin{align*}
\cos(X + Y) &= \cos X \cos Y - \sin X \sin Y \\
\cos X \cos Y &= \frac{1}{2}[\cos(X + Y) + \cos(X - Y)] \\
-\sin X \sin Y &= [\cos(X + Y) - \cos(X - Y)] \\
\cos(B \sin X) &= J_0(B) + 2\sum_{n=1}^{\infty} J_{2n}(B) \cos(2nX) \\
\sin(B \sin X) &= 2\sum_{n=0}^{\infty} J_{2n+1}(B) \sin((2n + 1)X)
\end{align*}
\]

After some messy algebra, $S_V(f)$ and $S_\phi(f)$ are as shown on the next page. Then,

SSB Power Ratio at $f_m = \frac{V_0^2 J_1^2[\Phi(f_m)]}{V_0^2 J_0^2[\Phi(f_m)] + 2\sum_{i=1}^{\infty} J_i^2[\Phi(f_m)]}$

if $\Phi(f_m) << 1$, then $J_0 = 1$, $J_1 = 1/2\Phi(f_m)$, $J_n = 0$ for $n > 1$, and

SSB Power Ratio $= \mathcal{L}(f_m) = \frac{\Phi^2(f_m)}{4} = \frac{S_\phi(f_m)}{2}$
\[ S_\phi(f), S_v(f) \text{ and } \mathcal{L}(f) \]

\[ \Phi(t) = \Phi(f_m) \cos(2\pi f_m t) \]

\[ V(t) = V_0 \cos[2\pi f_c t + \Phi(f_m)] \]

SSB Power Ratio

\[ \frac{V_0^2 J_1^2 [\Phi(f_m)]}{V_0^2 J_0^2 [\Phi(f_m)] + 2 \sum_{i=1} J_i^2 [\Phi(f_m)]} \approx \mathcal{L}(f_m) \approx \frac{S_\phi(f_m)}{2} \]
Types of Phase Noise

40 dB/Decade ($f_f^{-4}$)
Random Walk of Frequency

30 dB/Decade ($f_f^{-3}$)
Flicker of Frequency

20 dB/Decade ($f_f^{-2}$)
White Frequency; Random Walk of Phase

10 dB/Decade ($f_f^{-1}$)
Flicker of Phase

0 dB/Decade ($f_f^0$)
White Phase

$L(f_f)$

$\sim$ BW of Resonator

$F_f$

Fourier Frequency
(Sideband Frequency)
(Offset Frequency)
(Modulation Frequency)
Noise in Crystal Oscillators

- The resonator is the primary noise source close to the carrier; the oscillator circuitry is the primary source far from the carrier.
- Frequency multiplication by N increases the phase noise as $N^2$ (i.e., by 20 log N, in dB's).
- Vibration-induced "noise" can dominate all other sources of noise in many applications (see acceleration effects section, later).
- Close to the carrier (within BW of resonator), $S_y(f)$ varies as $1/f$, $S_\phi(f)$ as $1/f^3$, where $f$ = offset from carrier frequency, $v$; $S_\phi(f)$ also varies as $1/Q^4$, where $Q$ = unloaded $Q$. Since $Q_{\text{max}}v = \text{const.}$, $S_\phi(f) \propto v^4$. $(Q_{\text{max}}v)_{\text{BAW}} = 1.6 \times 10^{13}$ Hz; $(Q_{\text{max}}v)_{\text{SAW}} = 1.05 \times 10^{13}$ Hz.
- In the time domain, noise floor is $\sigma_y(\tau) \geq (2.0 \times 10^{-7})Q^{-1} \approx 1.2 \times 10^{-20} v$, $v$ in Hz. In the regions where $\sigma_y(\tau) \text{ varies as } \tau^{-1}$ and $\tau^{-1/2}$ ($\tau^{-1/2}$ occurs in atomic frequency standards), $\sigma_y(\tau) \propto (Q_S R)^{-1}$, where $S_R$ is the signal-to-noise ratio; i.e., the higher the $Q$ and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).
- Loaded $Q$ of oscillator affects noise when the oscillator circuitry is a significant noise source.
- Noise floor is limited by Johnson noise; noise power, $kT = -174$ dBm/Hz at 290°K.
- Higher signal level will improve the noise floor but not the close-in noise. (In fact, high drive levels generally degrade the close-in noise.)
- Low noise SAW vs. low noise BAW multiplied up: BAW is lower noise at $f < ~1$ kHz, SAW is lower noise at $f > ~1$ kHz; can phase lock the two to get the best of both.
TCXO Noise

The short term stability of TCXOs is temperature (T) dependent, and is generally significantly worse than that of OCXOs, for the following reasons:

- The slope of the TCXO crystal’s frequency (f) vs. T varies with T. For example, the f vs. T slope may be near zero at ~20°C, but it will be ~1ppm/°C at the T extremes. T fluctuations will cause small f fluctuations at laboratory ambient T’s, so the stability can be good there, but millidegree fluctuations will cause ~10⁻⁹ f fluctuations at the T extremes. The TCXO’s f vs. T slopes also vary with T; the zeros and maxima can be at any T, and the maximum slopes can be on the order of 1 ppm/°C.

- AT-cut crystals’ thermal transient sensitivity makes the effects of T fluctuations depend not only on the T but also on the rate of change of T (whereas the SC-cut crystals typically used in precision OCXOs are insensitive to thermal transients). Under changing T conditions, the T gradient between the T sensor (thermistor) and the crystal will aggravate the problems.

- TCXOs typically use fundamental mode AT-cut crystals which have lower Q and larger C₁ than the crystals typically used in OCXOs. The lower Q makes the crystals inherently noisier, and the larger C₁ makes the oscillators more susceptible to circuitry noise.

- AT-cut crystals’ f vs. T often exhibit activity dips. At the T’s where the dips occur, the f vs. T slope can be very high, so the noise due to T fluctuations will also be very high, e.g., 100x degradation of σ_y(τ) and 30 dB degradation of phase noise are possible. Activity dips can occur at any T.
Low-Noise SAW and BAW Multiplied to 10 GHz
(in a nonvibrating environment)

BAW = bulk-acoustic wave oscillator
SAW = surface acoustic wave oscillator

Offset frequency in Hz

10^{-1}  10^{0}  10^{1}  10^{2}  10^{3}  10^{4}  10^{5}  10^{6}

g(f) in dBC / Hz

-160  -140  -120  -100  -80  -60  -40  -20  0

BAW
5 MHz x 2000

BAW
100 MHz x 100

SAW
500 MHz x 20
Low-Noise SAW and BAW Multiplied to 10 GHz
(in a vibrating environment)

Vibration induced phase noise dominates the phase noise of both (whichever has lower acceleration sensitivity will have lower phase noise; currently, BAW can provide lower sensitivity than SAW.) Illustration assumes $1 \times 10^{-9}/g$ acceleration sensitivity for both BAW and SAW, and 0.01 $g^2/Hz$ random vibration power spectral density at all vibration frequencies.

Offset frequency in Hz

$\varphi(f)$ in dBc / Hz

- 5 MHz x 2000
- BAW

- 100 MHz x 100
- BAW and SAW

- 500 MHz x 20
Effects of Frequency Multiplication

\[ f_i \equiv f_{\text{in}} \]
\[ \Delta f_i \]
\[ \frac{\Delta f_i}{f_i} \equiv y \]
\[ \Delta \phi_i \]
\[ L(f)_i \]
\[ S_{\phi}(f)_i \]
\[ S_y(f)_i \]
\[ \sigma_y(\tau)_i \]

\[ f_i \times M = f_o \]

Noiseless Multiplier

\[ f_o \equiv f_{\text{out}} = Mf_i \]
\[ \Delta f_o = M\Delta f_i \]
\[ \frac{\Delta f_o}{f_o} = \frac{\Delta f_i}{f_i} \]
\[ \Delta \phi_o = M\Delta \phi_i \]
\[ L(f)_o = L(f)_i + 20 \log M \]
\[ S_{\phi}(f)_o = M^2 S_{\phi}(f)_i \]
\[ S_y(f)_o = S_y(f)_i \]
\[ \sigma_y(\tau)_o = \sigma_y(\tau)_i \]

Note that \( y = \frac{\Delta f}{f}, S_y(f), \) and \( \sigma_y(\tau) \) are uneffected by frequency multiplication.
Quartz Wristwatch Accuracy vs. Temperature

Temperature coefficient of frequency = \(-0.035 \text{ ppm/°C}^2\)

Time Error per Day (seconds)

-55°C → 20°C → 85°C

-10°C

Winter

Military

"Cold"

"Hot"

The crystals are cut so that the zero temperature coefficient is at 25°C. This has been found to provide the highest probability of accuracy, based on the typical durations and temperatures while the watch is on the wrist and while it is off the wrist.
This frequency vs. temperature characteristic is typical of AT-cut and SC-cut resonators. The upper and lower turnover points (UTP and LTP) are the points where $\frac{df(T)}{dT} = 0$. The inflection point is where $\frac{d^2f(T)}{dT^2} = 0$.

The inflection temperatures are $\approx 26^\circ$C for AT-cuts, and $\approx 96^\circ$ to $105^\circ$C for SC-cuts.
Resonator f vs. T Determining Factors

- **Primary:** Angles of cut

- **Secondary:**
  - Overtone
  - Blank geometry (contour, dimensional ratios)
  - Material impurities and strains
  - Mounting & bonding stresses (magnitude and direction)
  - Electrodes (size, shape, thickness, density, stress)
  - Drive level
  - Interfering modes
  - Load reactance (value & temperature coefficient)
  - Temperature rate of change
  - Thermal history
  - Ionizing radiation
Frequency-Temperature vs. Angle-of-Cut, AT-cut

\[ \theta = 350^\circ 20' + \Delta \theta, \phi = 0 \]

for 5th overtone AT-cut

\[ \theta = 350^\circ 12.5' + \Delta \theta, \phi = 0 \]

for fundamental mode plano-plano AT-cut

Temperature (OC)
Desired $f$ vs. $T$ for SC-cut Resonator

Frequency Offset (ppm)

Temperature (°C)

frequency remains within ± 1 ppm over a ± 25°C range about $T_i$
OCXO Oven’s Effect on Stability

Typical f vs. T characteristic for AT and SC-cut resonators

Oven Parameters vs. Stability for SC-cut Oscillator
Assuming $T_i - T_{LTP} = 10^\circ C$

<table>
<thead>
<tr>
<th>$T_i - T_{LTP} = 10^\circ C$</th>
<th>Oven Cycling Range (millidegrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Oven Offset (millidegrees)</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>$4 \times 10^{-12}$</td>
</tr>
<tr>
<td>10</td>
<td>$6 \times 10^{-13}$</td>
</tr>
<tr>
<td>1</td>
<td>$2 \times 10^{-13}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$2 \times 10^{-13}$</td>
</tr>
<tr>
<td>0</td>
<td>$2 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

A comparable table for AT and other non-thermal-transient compensated cuts of oscillators would not be meaningful because the dynamic f vs. T effects would generally dominate the static f vs. T effects.
Oven Stability Limits

- Thermal gain of $10^5$ has been achieved with a feed-forward compensation technique (i.e., measure outside $T$ of case & adjust setpoint of the thermistor to compensate). With a $10^5$ gain, e.g., if outside $\Delta T = 100^\circ C$, inside $\Delta T = 1$ mK.

- Stability of a good amplifier $\sim 1 \mu K/K$

- Stability of thermistors $\sim 1 mK/year$ to $100 mK/year$

- Noise $< 1 \mu K$ (Johnson noise in thermistor + amplifier noise + shot noise in the bridge current)

- Quantum limit of temperature fluctuations $\sim 1 nK$

- Optimum oven design can provide very high $f$ vs. $T$ stability
Effects of Harmonics on $f$ vs. $T$

$\Delta f/f \times 10^{-6}$

$\Delta T, ^\circ C$

**AT-cut**
Reference angle-of-cut ($\theta$) is about 8 minutes higher for the overtone modes. (For the overtone modes of the SC-cut, the reference $\theta$-angle-of-cut is about 30 minutes higher.)
Warmup of AT- and SC-cut Resonators

Deviation from static $f$ vs. $T = \ddot{a} \frac{dT}{dt}$, where, for example, $\ddot{a} \approx -2 \times 10^{-7} \text{ s/K}^2$ for a typical AT-cut resonator.
TCXO Thermal Hysteresis

TCXO = Temperature Compensated Crystal Oscillator
In (a), the oscillator was kept on continuously while the oven was cycled off and on. In (b), the oven was kept on continuously while the oscillator was cycled off and on.
TCXO Trim Effect

In TCXO's, temperature sensitive reactances are used to compensate for f vs. T variations. A variable reactance is also used to compensate for TCXO aging. The effect of the adjustment for aging on f vs. T stability is the "trim effect." Curves show f vs. T stability of a "0.5 ppm TCXO," at zero trim and at ±6 ppm trim. (Curves have been vertically displaced for clarity.)
Why the Trim Effect?

\[ \frac{\Delta f}{f_s} \approx \frac{C_1}{2(C_0 + C_L)} \]

Compensating \( f \) vs. \( T \)

Compensating \( C_L \) vs. \( T \)
Activity dips in the $f$ vs. $T$ when operated with an without load capacitors. (Curves have been vertically displaced for clarity.) Dip temperatures are a function of $C_L$, which indicates that the dip is caused by a mode (probably flexure) with a large negative temperature coefficient. See also "Unwanted Modes vs. Temperature" in Chapter 3.
Frequency Jumps

When the frequencies of oscillators are observed for long periods, occasional frequency jumps can be observed. In precision oscillators, the magnitudes of the jumps are typically in the range of $10^{-11}$ to $10^{-9}$. The jumps can be larger in general purpose units. The jumps occur many times a day in some oscillators, and much less than once a day in others. The frequency excursions can be positive or negative. The causes (and cures) are not well understood.

The causes are believed to include nearby spurious resonances, stress relief, changes in surface and electrode irregularities, and noisy active and passive circuit components. The effect can depend on resonator drive level; in some units, frequency jumps can be produced at certain drive levels (but not below or above). Aging affects the incidence. Well aged units show a lower incidence of jumps than new units.

Environmental effects can also produce jumps. Magnetic field, pressure, temperature, and power transients can produce sudden frequency excursions, as can shock and vibration. It is not unusual for example, to experience shock and vibration levels of $>0.01\text{g}$ in buildings as trucks pass by, heavy equipment is moved, boxes are dropped, etc. $[0.02\times 10^{-9}/\text{g} = 2 \times 10^{-11}]$
Effect of Load Capacitance on $f$ vs. $T$

The $f$ vs. $T$ characteristics with and without a load capacitor: 1. $C_L$ raises the frequency at all $T$'s (curve with $f_L$ has been vertically displaced for clarity), 2. $C_L$ rotates the $f$ vs. $T$ to lower the apparent angle of cut, i.e., it reduces peak-to-peak $f$ and turning-point-to-turning-point $T$, and 3. $T$-coefficient of $C_L$ can greatly amplify the $f$ vs. $T$ rotation.
Frequency vs. Drive Level

Crystal Current (µ amp)

Frequency Change (parts in 10⁻⁹)
At high drive levels, resonance curves become asymmetric due to the nonlinearities of quartz.
Drive Level vs. Resistance

- Normal operating range
- Anomalous starting resistance
- Drive level effects

Resistance $R_\perp$

$10^{-3}$ $10^{-2}$ $10^{-1}$ $1$ $10$ $100$ $I_x$ (mA)
A 'good' crystal will follow the path OABCBAO without hysteresis. A 'bad' crystal will follow a path OADBCBAO: hence the term 'second level of drive'. On again increasing the drive, there is a tendency for the magnitude of the effect to decrease, but in a very irregular and irreproducible manner. The effect is usually due to particulate contamination, loose electrodes, or other surface defects.
Acceleration vs. Frequency Change

Frequency shift is a function of the magnitude and direction of the acceleration, and is linear with magnitude up to at least 50 g’s.
2-g Tipover Test

($\Delta f$ vs. attitude about three axes)

- 10.000 MHz
tipover test - ppb

(F(max) - F(min))/2 = 1.889 -09 (ccw)
(F(max) - F(min))/2 = 1.863 -09 (cw)
delta THETA = 106.0 deg.

(F(max) - F(min))/2 = 6.841 -10 (ccw)
(F(max) - F(min))/2 = 6.896 -10 (cw)
delta THETA = 150.0 deg.

(F(max) - F(min))/2 = 1.882 -09 (ccw)
(F(max) - F(min))/2 = 1.859 -09 (cw)
delta THETA = 16.0 deg.
Sinusoidal Vibration Modulated Frequency

\[ t = 0 \]

\[ t = \frac{\pi}{2f_v} \]

\[ t = \frac{3\pi}{2f_v} \]

\[ t = \frac{2\pi}{f_v} \]
## Acceleration Levels and Effects

<table>
<thead>
<tr>
<th>Environment</th>
<th>Acceleration typical levels*, in g's</th>
<th>Frequency change $\times 10^{-11}$, for $1\times 10^{-9}/g$ oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings**, quiescent</td>
<td>0.02 rms</td>
<td>2</td>
</tr>
<tr>
<td>Tractor-trailer (3-80 Hz)</td>
<td>0.2 peak</td>
<td>20</td>
</tr>
<tr>
<td>Armored personnel carrier</td>
<td>0.5 to 3 rms</td>
<td>50 to 300</td>
</tr>
<tr>
<td>Ship - calm seas</td>
<td>0.02 to 0.1 peak</td>
<td>2 to 10</td>
</tr>
<tr>
<td>Ship - rough seas</td>
<td>0.8 peak</td>
<td>80</td>
</tr>
<tr>
<td>Propeller aircraft</td>
<td>0.3 to 5 rms</td>
<td>30 to 500</td>
</tr>
<tr>
<td>Helicopter</td>
<td>0.1 to 7 rms</td>
<td>10 to 700</td>
</tr>
<tr>
<td>Jet aircraft</td>
<td>0.02 to 2 rms</td>
<td>2 to 200</td>
</tr>
<tr>
<td>Missiles - boost phase</td>
<td>15 peak</td>
<td>1,500</td>
</tr>
<tr>
<td>Railroads</td>
<td>0.1 to 1 peak</td>
<td>10 to 100</td>
</tr>
</tbody>
</table>

* Levels at the oscillator depend on how and where the oscillator is mounted. Platform resonances can greatly amplify the acceleration levels.

** Building vibrations can have significant effects on noise measurements.
Acceleration-sensitivity is a vector, i.e., the acceleration-induced frequency shift is maximum when the acceleration is along the acceleration-sensitivity vector; $\Delta f = \vec{\Gamma} \cdot \vec{A}$. 

\[
\vec{\Gamma} = \gamma_1 \hat{i} + \gamma_2 \hat{j} + \gamma_3 \hat{k} \\
|\vec{\Gamma}| = \sqrt{\gamma_1^2 + \gamma_2^2 + \gamma_3^2}
\]
Vibration-Induced Allan Variance Degradation

Vibration modulates the frequency and, thereby, degrades the short-term stability. The typical degradation due to sinusoidal vibration varies with averaging time, as shown. Since a full sine wave averages to zero, the degradation is zero for averaging times that are integer multiples of the period of vibration. The peaks occur at averaging times that are odd multiples of half the period of vibration. The $\sigma_y(\tau)$ due to a single-frequency vibration is:

$$\sigma_y(\tau) = \frac{\Gamma \cdot a}{\pi} \frac{\tau_v}{\tau} \sin^2\left(\pi \frac{\tau}{\tau_v}\right),$$

where $\tau_v$ is the period of vibration, $\tau$ is the measurement averaging time, $\Gamma$ is the acceleration sensitivity vector, and $a$ is the acceleration.

**Example:**

$f_v = 20$ Hz
$a = 1.0$ g along $\Gamma$
$|\Gamma| = 1 \times 10^{-9}$/g
Vibration-Induced Phase Excursion

The phase of a vibration modulated signal is

\[ \phi(t) = 2\pi f_0 t + \left( \frac{\Delta f}{f_v} \right) \sin (2\pi f_v t) . \]

When the oscillator is subjected to a simple sinusoidal vibration, the peak phase excursion is

\[ \Delta \phi_{peak} = \frac{\Delta f}{f_v} = \frac{(\bar{\Gamma} \cdot \bar{a}) f_0}{f_v} \]

**Example:** if a 10 MHz, 1 x 10^{-9}/g oscillator is subjected to a 10 Hz sinusoidal vibration of amplitude 1 g, the peak vibration-induced phase excursion is 1 x 10^{-3} radians. If this oscillator is used as the reference oscillator in a 10 GHz radar system, the peak phase excursion at 10 GHz will be 1 radian. Such a large phase excursion can be catastrophic to the performance of many systems, such as those which employ phase locked loops (PLL) or phase shift keying (PSK).
Vibration-Induced Sidebands

NOTE: The "sidebands" are spectral lines at \( \pm f_v \) from the carrier frequency (where \( f_v \) = vibration frequency). The lines are broadened because of the finite bandwidth of the spectrum analyzer.

\[ 10g \text{ amplitude @ 100 Hz} \]

\[ |\Gamma| = 1.4 \times 10^{-9} \text{ per g} \]
Vibration-Induced Sidebands

Each frequency multiplication by 10 increases the sidebands by 20 dB.
Sine Vibration-Induced Phase Noise

Sinusoidal vibration produces spectral lines at ±f_v from the carrier, where f_v is the vibration frequency.

\[ \mathcal{L}'(f_v) = 20 \log \left( \frac{\Gamma \cdot Af_0}{2f_v} \right) \]

e.g., if \( |\Gamma| = 1 \times 10^{-9}/g \) and \( f_0 = 10 \text{ MHz} \), then even if the oscillator is completely noise free at rest, the phase "noise," i.e., the spectral lines, due solely to a sine vibration level of 1g will be:

<table>
<thead>
<tr>
<th>Vibr. freq., f_v, in Hz</th>
<th>( \mathcal{L}'(f_v) ), in dBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-46</td>
</tr>
<tr>
<td>10</td>
<td>-66</td>
</tr>
<tr>
<td>100</td>
<td>-86</td>
</tr>
<tr>
<td>1,000</td>
<td>-106</td>
</tr>
<tr>
<td>10,000</td>
<td>-126</td>
</tr>
</tbody>
</table>
Random Vibration-Induced Phase Noise

Random vibration's contribution to phase noise is given by:

\[ L(f) = 20 \log \left( \frac{\Gamma \cdot Af_0}{2f} \right), \quad \text{where} \quad |A|=\left[2\text{(PSD)}\right]^{1/2} \]

e.g., if \( |\Gamma| = 1 \times 10^{-9}/g \) and \( f_0 = 10 \text{ MHz} \), then even if the oscillator is completely noise free at rest, the phase noise due solely to a vibration PSD = 0.1 g²/Hz will be:

<table>
<thead>
<tr>
<th>Offset freq., f, in Hz</th>
<th>( L(f) ), in dBc/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-53</td>
</tr>
<tr>
<td>10</td>
<td>-73</td>
</tr>
<tr>
<td>100</td>
<td>-93</td>
</tr>
<tr>
<td>1,000</td>
<td>-113</td>
</tr>
<tr>
<td>10,000</td>
<td>-133</td>
</tr>
</tbody>
</table>
Random-Vibration-Induced Phase Noise

Phase noise under vibration is for $\Gamma = 1 \times 10^{-9}$ per g and $f = 10 \text{ MHz}$

Offset Frequency (Hz)

Typical Aircraft Random Vibration Envelope
The acceleration sensitivity, $\Gamma$, can be calculated from the vibration induced sidebands. In an ideal oscillator, $\Gamma$ vs. $f_v$ would be a constant, but real oscillators exhibit resonances. In the above example, the resonance at 424 Hz resulted in a 17-fold increase in $\Gamma$. The preferred test method includes measurement of $\Gamma$ at a number of vibration frequencies in order to reveal resonances.
Acceleration Sensitivity of Quartz Resonators

Resonator acceleration sensitivities range from the low parts in $10^{10}$ per g for the best commercially available SC-cuts, to parts in $10^7$ per g for tuning-fork-type watch crystals. When a wide range of resonators were examined: AT, BT, FC, IT, SC, AK, and GT-cuts; 5 MHz 5th overtones to 500 MHz fundamental mode inverted mesa resonators; resonators made of natural quartz, cultured quartz, and swept cultured quartz; numerous geometries and mounting configurations (including rectangular AT-cuts); nearly all of the results were within a factor of three of $1x10^-9$ per g. On the other hand, the fact that a few resonators have been found to have sensitivities of less than $1x10^{-10}$ per g indicates that the observed acceleration sensitivities are not due to any inherent natural limitations.

Recent theoretical and experimental evidence indicates that the major variables yet to be controlled properly are the mode shape and location (i.e., the amplitude of vibration distribution), and the strain distribution associated with the mode of vibration. Theoretically, when the mounting is completely symmetrical with respect to the mode shape, the acceleration sensitivity can be zero, but tiny changes from this ideal condition can cause a significant sensitivity. Until the acceleration sensitivity problem is solved, acceleration compensation and vibration isolation can provide lower than $1x10^{-10}$ per g, for a limited range of vibration frequencies, and at a cost.
Phase Noise Degradation Due to Vibration

- Data shown is for a 10 MHz, $2 \times 10^{-9}$ per g oscillator
- Radar spec. shown is for a coherent radar (e.g., SOTAS)

Impacts on Radar Performance
- Lower probability of detection
- Lower probability of identification
- Shorter range
- False targets

Radar Oscillator Specification

Required to 'see' 4Km/hr target

'GOOD' OSCILLATOR ON VIBRATING PLATFORM (1g)

'GOOD' OSCILLATOR AT REST

OFFSET FROM CARRIER (Hz)
Coherent Radar Probability of Detection

To "see" 4 km/h targets, low phase noise 70 Hz from the carrier is required. Shown is the probability of detection of 4 km/h targets vs. the phase noise 70 Hz from the carrier of a 10 MHz reference oscillator. (After multiplication to 10 GHz, the phase noise will be at least 60 dB higher.) The phase noise due to platform vibration, e.g., on an aircraft, reduces the probability of detection of slow-moving targets to zero.

![Graph showing the relationship between phase noise and probability of detection for coherent radar.]
Vibration Isolation

Limitations

- Poor at low frequencies
- Adds size, weight and cost
- Ineffective for acoustic noise
Vibration Compensation

**Stimulus**
- DC Voltage on Crystal
  - Circuit diagram: DC voltage source connected to the crystal
  - Label: OSC.

**Response**
- 5 MHz fund. SC
  - Frequency response graph
  - Labels: $f$, $f_0$, $f_0 + f_v$, $f_0 - f_v$

**Compensated Oscillator**
- Vibration Compensated Oscillator
  - Circuit diagram: oscillator, amplifier, accelerometer
  - Label: ACC = accelerometer

**Response to Vibration**
- Circuit diagram: crystal being vibrated
  - Label: OSC.
Vibration Sensitivity Measurement System

- Controller
- Plotter or printer
- Spectrum analyzer
- Frequency multiplier (x10)
- Synthesizer (local oscillator)
- Signal generator \( F_V \)
- Accelerometer
- Test oscillator
- Shake table
- Power amplifier
- Vibration level controller
The frequency excursion during a shock is due to the resonator's stress sensitivity. The magnitude of the excursion is a function of resonator design, and of the shock induced stresses on the resonator. (Resonances in the mounting structure will amplify the stresses.) The permanent frequency offset can be due to: shock induced stress changes, the removal of (particulate) contamination from the resonator surfaces, and changes in the oscillator circuitry. Survival under shock is primarily a function of resonator surface imperfections. Chemical-polishing-produced scratch-free resonators have survived shocks of up to 36,000 g in air gun tests, and have survived the shocks due to being fired from a 155 mm howitzer (16,000 g, 12 ms duration).
Radiation-Induced Frequency Shifts

\[ f_0 = \text{original, preirradiation frequency} \]
\[ f_{ss} = \text{steady-state frequency} \] (0.2 to 24 hours after exposure)
\[ \Delta f_{ss} = \text{steady-state frequency offset} \]
\[ f_t = \text{frequency at time } t \]

\[ \Delta f_{ss}/\text{rad}^* = \begin{cases} 10^{-11} & \text{for natural quartz (and R increase can stop the oscillation)} \\ 10^{-12} & \text{for cultured quartz} \\ 10^{-13} & \text{for swept cultured quartz} \end{cases} \]

* for a 1 megarad dose (coefficients are dose dependent)

Idealized frequency vs. time behavior for a quartz resonator following a pulse of ionizing radiation.
Effects of Repeated Irradiations

Five irradiations; responses during 4th and 5th repeated 3rd; at least 2 days elapsed between successive irradiations

1. Initial irradiation
2. Preirradiation (2.5 x 10^4 rad)
3. Preirradiation (> 10^6 rad)

Initial slopes:
1st: -1 x 10^-9/rad
2nd: +1 x 10^-11/rad
3rd: +3 x 10^-12/rad
4th: +3 x 10^-12/rad
5th: +5 x 10^-12/rad

Fractional Frequency, ppb

Dose, rad(SiO_2)
Radiation Induced $\Delta f$ vs. Dose and Quartz-Type

10 MeV Electrons,
5 MHz 5th overtone
AT-cut resonators

Z-growth
Cultured

Swept Z-growth
Cultured

Natural

Frequency Change (Hz)

10 MeV Electrons, 5 MHz 5th overtone AT-cut resonators

Z-growth
Cultured

Swept Z-growth
Cultured

Natural
Annealing of Radiation Induced f Changes

- For 4 MHz AT-cut resonator, X-ray dose of 6 x 10^6 rads produced Δf = 41 Hz.

- Activation energies were calculated from the temperature dependence of the annealing curves. The experimental results can be reproduced by two processes, with activation energies $E_1 = 0.3 \pm 0.1$ eV and $E_2 = 1.3 \pm 0.3$ eV.

- Annealing was complete in less than 3 hours at > 240°C.
Transient $\Delta f$ After a Pulse of $\gamma$ Radiation

- Experimental data, dose = $1.3 \times 10^4$ rads, SC-cut
- Experimental data, dose = $2.3 \times 10^4$ rads, AT-cut
- Model calculation: AT-cut
Effects of Flash X-rays on $R_s$

The curves show the series resonance resistance, $R_s$, vs. time following a $4 \times 10^4$ rad pulse. Resonators made of swept quartz show no change in $R_s$ from the earliest measurement time (1 ms) after exposure, at room temperature. Large increase in $R_s$ (i.e., large decrease in the Q) will stop the oscillation.
Frequency Change due to Neutrons

Curve shows the nearly linear increase in resonant frequency of a crystal unit as a function of reactor irradiation. At other fluences, the slopes are, for example, $8 \times 10^{-21}/\text{n/cm}^2$ at $10^{10}$ to $10^{12}\text{n/cm}^2$, and $5 \times 10^{-21}/\text{n/cm}^2$ at $10^{12}$ to $10^{13}\text{n/cm}^2$. 
A fast neutron can displace about 50 to 100 atoms before it comes to rest. Most of the damage is done by the recoiling atoms. The net result is that each neutron can cause numerous vacancies and interstitials.
Summary - Steady-State Radiation Results

- Dose vs. frequency change is nonlinear; f change per rad is larger at low doses.

- At doses > 1 KRad, f change is quartz impurity dependent. The ionizing radiation produces electron-hole pairs; the holes are trapped by the impurity Al sites while the compensating cation (e.g., Li or Na) is released. The freed cations are loosely trapped along the optic axis. The lattice near the Al is altered, the elastic constant is changed; therefore, the f shifts. Ge impurity is also troublesome.

- At a 1 MRad dose, f change ranges from pp $10^{11}$ per rad for natural quartz to pp $10^{14}$ per rad for high quality swept quartz.

- Frequency change is negative for natural quartz; it can be positive or negative for cultured and swept cultured quartz.

- Frequency change saturates at doses > $10^6$ rads.

- Q degrades if quartz contains high concentration of alkali impurities; Q of resonators made of properly swept cultured quartz is unaffected.

- High dose radiation can also rotate f vs. T characteristic.

- Frequency change anneals at $T > 240^\circ C$ in less than 3 hours.

- Preconditioning (e.g., with doses > $10^5$ rads) reduces the high dose radiation sensitivities upon subsequent irradiations.

- At doses < 100 rad, f change is not well understood. Radiation induced stress relief and surface effects (adsorption, desorption, dissociation, polymerization and charging) may be significant.
Summary - Pulse Irradiation Results

- For applications requiring circuits hardened to pulse irradiation, quartz resonators are the least tolerant element in properly designed oscillator circuits.

- Resonators made of unswept quartz or natural quartz can experience a large increase in $R_s$ following a pulse of radiation; the radiation pulse can stop the oscillation.

- Natural, cultured, and swept cultured AT-cut quartz resonators experience an initial negative frequency shift immediately after exposure to a pulse of X-rays (e.g., $10^4$ to $10^5$ Rad of flash X-rays), $\Delta f/f$ is as large as -3 ppm at 0.02 sec after burst of $10^{12}$ Rad/sec.

- Transient $f$ offset anneals as $t^{-1/2}$; the nonthermal-transient part of the $f$ offset is probably due to the diffusion and retrapping of hydrogen at the Al$^{3+}$ trap.

- Resonators made of properly swept quartz experience a negligibly small change in $R_s$ when subjected to pulsed ionizing radiation (the oscillator circuit does not require a large reserve of gain margin).

- SC-cut quartz resonators made of properly swept high Q quartz do not exhibit transient frequency offsets following a pulse of ionizing radiation.

- Crystal oscillators will stop oscillating during an intense pulse of ionizing radiation because of the large prompt photoconductivity in quartz and in the transistors comprising the oscillator circuit. Oscillation will start up within 15 $\mu$s after burst if swept quartz is used in the resonator and the oscillator circuit is properly designed for the radiation environment.
Summary - Neutron Irradiation Results

- When a fast neutron (~MeV energy) hurtles into a crystal lattice and collides with an atom, it is scattered like a billiard ball. The recoiling atom, having an energy (~10^4 to 10^6 eV) that is much greater than its binding energy in the lattice, leaves behind a vacancy and, as it travels through the lattice, it displaces and ionizes other atoms. A single fast neutron can thereby produce numerous vacancies, interstitials, and broken interatomic bonds. Neutron damage thus changes both the elastic constants and the density of quartz. Of the fast neutrons that impinge on a resonator, most pass through without any collisions, i.e., without any effects on the resonator. The small fraction of neutrons that collide with atoms in the lattice cause the damage.

- Frequency increases approximately linearly with fluence. For AT- and SC-cut resonators, the slopes range from +0.7 x 10^-21/n/cm^2 at very high fluences (10^{17} to 10^{18} n/cm^2) to 5 x 10^-21/n/cm^2 at 10^{12} to 10^{13} n/cm^2, and 8 x 10^-21/n/cm^2 at 10^{10} to 10^{12} n/cm^2. Sensitivity probably depends somewhat on the quartz defect density and on the neutron energy distribution. (Thermonuclear neutrons cause more damage than reactor neutrons.)

- Neutron irradiation also rotates the frequency vs. temperature characteristic.

- When a heavily neutron irradiated sample was baked at 500° C for six days, 90% of the neutron-induced frequency shift was removed (but the 10% remaining was still 93 ppm).
Other Effects on Stability

- **Electric field** - affects doubly-rotated resonators; e.g., a voltage on the electrodes of a 5 MHz fundamental mode SC-cut resonator results in a $\Delta f/f = 7 \times 10^{-9}$ per volt. The voltage can also cause sweeping, which can affect the frequency (of all cuts).

- **Magnetic field** - quartz is diamagnetic, however, magnetic fields can induce Eddy currents, and will affect magnetic materials in the resonator package and the oscillator circuitry. Induced ac voltages can affect varactors, AGC circuits and power supplies. Typical frequency change of a "good" quartz oscillator is $<<10^{-10}$ per gauss.

- **Ambient pressure (altitude)** - deformation of resonator and oscillator packages, and change in heat transfer conditions affect the frequency.

- **Humidity** - can affect the oscillator circuitry, and the oscillator’s thermal properties.

- **Power supply voltage, and load impedance** - affect the oscillator circuitry, and indirectly, the resonator’s drive level and load reactance. A change in load impedance changes the amplitude or phase of the signal reflected into the oscillator loop, which changes the phase (and frequency) of the oscillation. The effects can be minimized through voltage regulation and buffering.

- **Gas permeation** - stability can be affected by excessive levels of atmospheric hydrogen and helium diffusing into "hermetically sealed" metal and glass enclosures (e.g., hydrogen diffusion through nickel resonator enclosures, and helium diffusion through glass Rb standard bulbs).
## Interactions Among Influences

In attempting to measure the effect of a single influence, one often encounters interfering influences, the presence of which may or may not be obvious.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Interfering Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonator aging</td>
<td>$\Delta T$ due to oven $T$ (i.e., thermistor) aging</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ drive level due to osc. circuit aging</td>
</tr>
<tr>
<td>Short term stability</td>
<td>Vibration</td>
</tr>
<tr>
<td>Vibration sensitivity</td>
<td>Induced voltages due to magnetic fields</td>
</tr>
<tr>
<td>2-g tipover sensitivity</td>
<td>$\Delta T$ due to convection inside oven</td>
</tr>
<tr>
<td>Resonator f vs. $T$</td>
<td>Thermal transient effect, humidity</td>
</tr>
<tr>
<td>(static)</td>
<td>$T$-coefficient of load reactances</td>
</tr>
<tr>
<td>Radiation sensitivity</td>
<td>$\Delta T$, thermal transient effect, aging</td>
</tr>
</tbody>
</table>
Chapter 4 References


4-8 Provided by Arthur Ballato, U.S. Army LABCOM, private communication, circa 1978.


4-30 The analysis and graphs on pp. 4-30 and 4-31 were provided by Raymond L. Filler, U.S. Army LABCOM, 1989.


Quartz Material Properties
Hydrothermal Growth of Quartz

- The autoclave is filled to some predetermined factor with water plus mineralizer (NaOH or Na₂CO₃).
- The baffle localizes the temperature gradient so that each zone is nearly isothermal.
- The seeds are thin slices of (usually) Z-cut single-crystals.
- The nutrient consists of small (~1/2") pieces of single-crystal quartz ("lascas").
- The temperatures and pressures are typically about 350°C and 800 to 2,000 atmospheres; T₂ - T₁ is typically 4°C to 10°C.
- The nutrient dissolves slowly (30 to 60 days per run), diffuses to the growth zone, and deposits onto the seeds.
Left-Handed and Right-Handed Quartz

Right-Handed

Left-Handed
The Quartz Lattice

- Si
- O
- Z
- Y

Angles:
- 144.1°
- 109°
# Quartz Properties' Effects on Device Properties

<table>
<thead>
<tr>
<th>Quartz Property</th>
<th>Device and Device-Fabrication Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>Oscillator short-term stability, phase noise close to carrier, long-term stability, filter loss</td>
</tr>
<tr>
<td>Purity (Al, Fe, Li, Na, K, -OH, H₂O)</td>
<td>Radiation hardness, susceptibility to twinning, optical characteristics</td>
</tr>
<tr>
<td>Crystalline Perfection, Strains</td>
<td>Sweepability, etchability for chem. polishing and photolithographic processing, optical properties, strength, aging(?), hysteresis (?)</td>
</tr>
<tr>
<td>Inclusions</td>
<td>High-temperature processing and applications, optical characteristics, etchability</td>
</tr>
</tbody>
</table>
Ions in Quartz - Simplified Model

Model shows the positions of $\text{H}^+$ and alkali ions in the channels of the quartz lattice, and the corresponding trends in the potential energy curves. Radiation can move ions from one potential well to another, thereby changing the elastic constants, and consequently, resulting in a frequency change.
Aluminum Associated Defects

Al-OH\(^-\) center

Al\(^{3+}\) center

OH\(^-\) molecule

Al-M\(^+\) center

Interstitial Alkali

\([\text{Al}_{E^+}]^0\) center
(aluminum-hole center)

Hole trapped in nonbonding oxygen p orbital
Sweeping

Sweeping is a purification process which removes certain impurities from the quartz and thereby improves the radiation hardness and etching properties of quartz crystals. It is an electric-field driven, solid-state diffusion process that is performed at an elevated temperature. The major steps of a typical sweeping process consist of applying electrodes to the Z-surfaces of a lumbered quartz bar, heating the bar slowly to 500°C, applying a voltage to the electrodes such that the electric field along the Z-direction is about 1 kV/cm, monitoring the current through the bar (as the sweeping progresses, the current decreases), and after the current decays to some constant value, cooling the bar slowly to room temperature, then removing the voltage.

Under the influences of the high electric field and the high temperature, the positive impurity ions, such as Li⁺ and Na⁺, diffuse to the cathode and are removed when the electrodes are removed in subsequent processing. In addition to improving radiation hardness, sweeping also greatly reduces the number of etch channels that are produced when quartz is etched.
Typical Sweeping Method

- Oven
  - $T = 500^\circ C$

- Cr-Au Quartz Bar
  - $E = 1000 \text{ V/cm}$

- Thermometer
- Ammeter
- High V Power Supply

$I$
- $0.5 \mu\text{a/cm}^2$
- Time
Quartz Quality Indicators

- Infrared absorption coefficient *
- Etch-channel density *
- Etch-pit density
- Inclusion density *
- Acoustic attenuation
- Impurity analysis
- X-ray topography
- UV absorption
- Birefringence along the optic axis
- Thermal shock induced fracture
- Electron spin resonance
- ???

* EIA Standard 477-1 contains standard test method for this quantity
Infrared Absorption

![Graph showing infrared absorption spectrum with labeled peaks at 3500, 3585, 3410, 3200, and 3300 cm⁻¹, with wavelength in μm on the x-axis and transmission (%) on the y-axis.]
Infrared Absorption

One of the factors that determine the maximum achievable resonator Q is the OH content of the quartz. Infrared absorption measurements are routinely used to measure the intensities of the temperature-broadened OH defect bands. The \textit{infrared absorption coefficient} $\alpha$ is defined by EIA Standard 477-1 as

$$\alpha = \frac{A (3500 \text{ cm}^{-1}) - A (3800 \text{ cm}^{-1})}{\text{Y-cut thickness in cm}}$$

where the A's are the logarithm (base 10) of the fraction of the incident beam absorbed at the wave numbers in the parentheses.

<table>
<thead>
<tr>
<th>Grade</th>
<th>$\alpha$, in cm$^{-1}$</th>
<th>Approx. max. Q$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.03</td>
<td>3.0</td>
</tr>
<tr>
<td>B</td>
<td>0.045</td>
<td>2.2</td>
</tr>
<tr>
<td>C</td>
<td>0.060</td>
<td>1.8</td>
</tr>
<tr>
<td>D</td>
<td>0.12</td>
<td>1.0</td>
</tr>
<tr>
<td>E</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

* In millions, at 5 MHz ($\alpha$ is a quality indicator for unswept quartz only).
Quartz Twinning

- The X-axes of quartz, the electrical axes, are parallel to the line bisecting adjacent prism faces; the +X-direction is positive upon extension due to tension.

- Electric twinning (also called Dauphiné twinning) consists of localized reversal of the X-axes. It usually consists of irregular patches, with irregular boundaries. It can be produced artificially by inversion from high-quartz, thermal shock, high local pressure (even at room temperature), and by an intense electric field.

- In right-handed quartz, the plane of polarization is rotated clockwise as seen by looking toward the light source; in left handed, it is CCW. Optically twinned (also called Brazil twinned) quartz contains both left and right-handed quartz. Boundaries between optical twins are usually straight.

- Etching can reveal both kinds of twinning.
The diagrams illustrate the relationship between the axial system and hand of twinned crystals. The arrows indicate the hand.
Quartz Lattice and Twinning

Z-axis projection showing electric (Dauphinè) twins separated by a twin wall of one unit cell thickness. The numbers in the atoms are atom heights expressed in units of percent of a unit cell height. The atom shifts during twinning involve motions of < 0.03 nm.
Quartz Inversion

- Quartz undergoes a high-low inversion (α – β transformation) at 573°C. (It is 573°C at 1 atm on rising temperature; it can be 1°C to 2°C lower on falling temperature.)

- Bond angles between adjoining (SiO₄) tetrahedra change at the inversion. Whereas low-quartz (α - quartz) is trigonal, high quartz (β - quartz) is hexagonal. Both forms are piezoelectric.

- An abrupt change in nearly all physical properties takes place at the inversion point; volume increases by 0.86% during inversion from low to high quartz. The changes are reversible, although Dauphiné twinning is usually acquired upon cooling through the inversion point.

- Inversion temperature decreases with increasing Al and alkali content, increases with Ge content, and increases 1°C for each 40 atm increase in hydrostatic pressure.
Phase Diagram of Silica (SiO$_2$)
Internal Friction of Quartz

Empirically determined Q vs. frequency curves indicate that the maximum achievable Q times the frequency is a constant, $16 \times 10^6$ for AT-cut resonators, when $f$ is in MHz.
Chapter 5 References


Emerging Oscillator Technologies
Emerging/Improving Technologies

• SC-cut resonators
• Resonator theory and finite element modeling
• Advanced fabrication techniques
  • Surface cleaning (UV-ozone, plasma; ice scrubber for particle removal)
  • Chemical polishing & chemical milling
  • Plate, mount and electrode geometries (lateral field, BVA, polygonal)
  • Bonding (parallel gap, thermocompression)
  • Packaging (ceramic flatpack, ceramic-metal, all-quartz)
  • Ultrahigh vacuum, high temperature & automated processing
• High purity, low defect density quartz
• UHF and miniature (photolithography/etching produced) resonators
• Acceleration sensitivity reduction and compensation techniques
• Microcomputer compensation (temperature, acceleration, radiation)
• Miniature fast warmup OCXO & directly heated crystal plate
• Rubidium-crystal oscillator (RbXO)
• Optically pumped & trapped ion/atom atomic frequency standards
Comparison of SC and AT-cuts

- **Advantages of the SC-cut**
  - Thermal transient compensated (allows faster warmup OCXO)
  - Static and dynamic f vs. T allow higher stability OCXO and MCXO
  - Better f vs. T repeatability allows higher stability OCXO and MCXO
  - Far fewer activity dips
  - Lower drive level sensitivity
  - Planar stress compensated; lower Δf due to edge forces and bending
  - Lower sensitivity to radiation
  - Higher capacitance ratio (less Δf for oscillator reactance changes)
  - Higher Q for fundamental mode resonators of similar geometry
  - Less sensitive to plate geometry - can use wide range of contours

- **Disadvantage of the SC-cut**: More difficult to manufacture for OCXO (but is easier to manufacture for MCXO than is an AT-cut for precision TCXO)

- **Other Significant Differences**
  - B-mode is excited in the SC-cut, although not necessarily in LFR's
  - The SC-cut is sensitive to electric fields (can be used for compensation)
In lateral field resonators (LFR): 1. the electrodes are absent from the regions of greatest motion, and 2. varying the orientation of the gap between the electrodes varies certain important resonator properties. Advantages of LFR are:

- Ability to eliminate undesired modes, e.g., the b-mode in SC-cuts
- Potentially higher Q (less damping due to electrodes and mode traps)
- Potentially higher stability (less electrode and mode trap effects, smaller $C_1$)
Microcomputer Compensated Crystal Oscillator (MCXO)

- Accuracy: 5 msec per day ($5 \times 10^{-8}$), with $< 50$ mW power

- Major barriers: thermal hysteresis, thermometry, circuit instabilities

- Solutions: high stability overtone SC-cut and lateral field resonators, dual-mode oscillator and digital compensation techniques

- Advantages over analog TCXO: much higher accuracy possible, rapid and easy compensation and recalibration
MCXO - Description of Operation

The following analogy illustrates the difference between an MCXO-based clock and a conventional TCXO-based clock. Suppose one has a clock that gains 24 seconds per day. The conventional way to maintain accurate time with such a clock is to adjust the frequency of the internal oscillator to the proper frequency, and then to maintain that frequency, e.g., with a TCXO. Another way to maintain accurate time is to set the clock, and then to stop the clock for 1 second every hour, or for 1/60 sec every minute, or for 1/3600 sec every second, etc. For a conventional clock, the second method would be very inconvenient and difficult to use for accurate timekeeping. The MCXO, however, operates somewhat as the second method, i.e., the MCXO provides accurate time from an inaccurate (but highly reproducible) frequency source, by correcting for the known frequency inaccuracies.

Simplified block diagrams of two implementations of the MCXO are shown on the two pages following this description. In the pulse deletion method, the dual-mode oscillator provides output signals at two frequencies, one of which, \( f_\beta \), is the resonator temperature indicator. The signals are processed by the microcomputer which, from \( f_\beta \), determines the necessary correction to \( f_c \) and then subtracts the required number of pulses from \( f_c \) to obtain the corrected output \( f_o \). Fractions of pulses that cannot be subtracted within the update interval (~ 1 s) are used as a carry, so that the long-term average is within the \( \pm 2 \times 10^{-8} \) design accuracy. Correction data in the PROM are unique to each crystal and are obtained from a precise thermal characterization of the \( f_c \) and \( f_\beta \) output signals. The corrected output signal \( f_o \) can be divided down to produce a 1 pps time reference or can be used directly to drive a clock. Due to the objectionable noise characteristics
MCXO - Description of Operation, Cont’d

created by the pulse deletion process, additional signal processing is necessary to provide a useful RF output for frequency control applications. This can be accomplished by, for example, imparting the MCXO frequency accuracy to a second, low-noise, low-cost, voltage controlled crystal oscillator.

A sinewave RF output may be obtained directly by using an alternate MCXO approach that is based on phase-locked-loop frequency summing instead of pulse deletion. As in the pulse deletion method, a dual-mode oscillator generates the two output frequencies, \( f_c \) and \( f_B \). The microcomputer computes a number \( N \) which is used to control a direct digital synthesizer (DDS). The DDS generates a correction frequency \( df_c \) which, when added to \( f_c \), results in the compensated output frequency \( f_o \). The phase-locked-loop frequency summer incorporates a VCXO that is adjusted, in frequency and phase, to the desired sum frequency \( f_o \).

The MCXO has major advantages over conventional temperature compensated crystal oscillators (TCXO’s) for the following reasons: 1. the MCXO circumvents the need for pulling the crystal frequency and, therefore, permits the use of "stiff", high stability SC-cut crystal units, 2. the MCXO allows resonator self-temperature sensing, using a dual-mode oscillator; thermometry-caused errors are thus eliminated, 3. the trim effect is eliminated, 4. automatic recalibration features can be designed into the MCXO algorithm; an offset can be stored in memory following simple injection of an external, higher-accuracy reference signal, and 5. an accurate but very low-power clock is possible through duty-cycling the MCXO to periodically update a low-power, wristwatch-type clock (e.g., six seconds on, one minute off).
MCXO - Pulse Deletion Method

Microcomputer compensated crystal oscillator (MCXO) block diagram - pulse deletion method.
MCXO - PLL Frequency Summing Method

Microcomputer compensated crystal oscillator (MCXO) block diagram - phase-locked-loop frequency summing method.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>MCXO</th>
<th>TCXO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut, overtone</td>
<td>AT-cut, fund.</td>
<td>Tight</td>
</tr>
<tr>
<td>Angle-of-cut tolerance</td>
<td>SC-cut, 3rd</td>
<td>Tight</td>
</tr>
<tr>
<td>Blank f and plating tolerance</td>
<td></td>
<td>Significant</td>
</tr>
<tr>
<td>Activity dip incidence</td>
<td></td>
<td>10^{-7} to 10^{-6}</td>
</tr>
<tr>
<td>Hysteresis (-55°C to +85°C)</td>
<td></td>
<td>10^{-7} to 10^{-6}</td>
</tr>
<tr>
<td>Aging per year</td>
<td></td>
<td>10^{-9} to 10^{-8}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10^{-8} to 10^{-7}</td>
</tr>
</tbody>
</table>
Rubidium - Crystal Oscillator (RbXO)

<table>
<thead>
<tr>
<th>Rubidium Frequency Standard</th>
<th>RbXO Interface</th>
<th>Low-power Crystal Oscillator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(∼ 25W @ -55°C)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The RbXO provides the best of both worlds - the long term stability of a Rb standard and the low power requirement of a crystal oscillator. Occasionally, power is applied to the Rb standard for a few minutes. Upon warmup of the Rb standard, the RbXO interface syntonizes the crystal oscillator and cuts off power to the Rb standard. When the crystal oscillator is an MCXO, the MCXO digital circuit can include the RbXO interface, and the average RbXO power consumption can be less than 100 mW.
The Rb reference is a miniature Rb frequency standard (RFS) that has been modified to control an external crystal oscillator. The OCXO includes a digital tuning memory to hold the frequency control voltage while the Rb reference is off. The OCXO is ON continually. Periodically, the system applies power to the RFS. After the warmup of the RFS (a few minutes), the interface circuits adjust the frequency of the OCXO to the RFS reference, then shut off the RFS. For manpack applications, the OCXO will be separable from the rest of the RbXO so that the manpack can operate with minimum size, weight, and power, and with nearly the accuracy of the RFS for the duration of a mission. An MCXO can replace the OCXO for even lower power consumption.
Rubidium Crystal Oscillator

13cm
Accuracy vs. Power-Requirement*

(Goal is to move the technologies toward the upper left)

* Accuracy vs. size, and accuracy vs. cost have similar relationships.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Causes</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aging</td>
<td>Contamination transfer, stress relief, material defects</td>
<td>Ultraclean processing; SC-cut; &quot;good&quot; quartz, mounting, bonding, electrodes &amp; packaging</td>
</tr>
<tr>
<td>Thermal Hysteresis &amp; Retrace</td>
<td>Stress relief, contamination transfer, material defects</td>
<td>&quot;good&quot; quartz, mounting, bonding, electrodes &amp; packaging, precision X-ray system, angle correction, SC-cut, &quot;good&quot; oven</td>
</tr>
<tr>
<td>Frequency vs. Temperature (static or dynamic)</td>
<td>Incorrect angles of cut, interfering modes, oven fluctuations</td>
<td>SC-cut, ceramic flatpacks</td>
</tr>
<tr>
<td>Thermal Shock (Warmup)</td>
<td>Stress-sensitivity of quartz, thermal time constant of package fluctuations</td>
<td>Mounting stress &amp; mode shape compensation, SC-cut, symmetry</td>
</tr>
<tr>
<td>Acceleration Sensitivity</td>
<td>Stress-sensitivity of quartz, lack of mounting &amp; mode-shape symmetry</td>
<td>Chemical polishing; blank inspection; &quot;good&quot; quartz, mounting &amp; bonding</td>
</tr>
<tr>
<td>Mechanical Shock</td>
<td>Surface &amp; bulk imperfections, deformation in mount/bond</td>
<td>High purity (sweep) quartz, compensation</td>
</tr>
<tr>
<td>Radiation Sensitivity (photons &amp; particles)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6 References


Atomic Frequency Standards*

* There are two important reasons for including this section: 1. atomic frequency standards are one of the most important applications of precision quartz oscillators, and 2. those who study or use crystal oscillators ought to be aware of what is available in case they need an oscillator with better long-term stability than what crystal oscillators can provide.
Precision Frequency Standards

- Quartz crystal resonator-based \( (f \sim 5\, \text{MHz},\, Q \sim 10^6) \)

- Atomic resonator-based
  - Rubidium\(^{87} \) \( (f_0 = 6.8\, \text{GHz},\, Q \sim 10^7) \)
  - Cesium\(^{133} \) \( (f_0 = 9.2\, \text{GHz},\, Q \sim 10^8) \)
  - Hydrogen \( (f_0 = 1.4\, \text{GHz},\, Q \sim 10^9) \)
  - Trapped ions \( (f_0 > 10\, \text{GHz},\, Q > 10^{11}) \)
Atomic Frequency Standard Basic Concepts

When an atomic system changes energy from an excited state to a lower energy state, a photon is emitted. The photon frequency $\nu$ is given by Planck's law

$$\nu = \frac{E_2 - E_1}{h}$$

where $E_2$ and $E_1$ are the energies of the upper and lower states, respectively, and $h$ is Planck's constant. An atomic frequency standard produces an output signal the frequency of which is determined by this intrinsic frequency rather than some property of a bulk material (as it is in quartz oscillators).

The properties of isolated atoms at rest, and in free space, would not change with space and time. Therefore, the frequency of an ideal atomic frequency standard would not change with time or with changes in the environment. Unfortunately, in real atomic frequency standards: 1) the atoms are moving at thermal velocities, 2) the atoms are not isolated but experience collisions and electric and magnetic fields, and 3) some of the components needed for producing and observing the atomic transitions contribute to instabilities.
Hydrogen-Like Atoms

Hydrogen-Like (or Alkali) Atoms

Hyperfine structure of $^{87}$Rb, with nuclear spin $I = 3/2$, $v_o = \Delta W/h = 6,834,682,605$ Hz and $X = [(-\mu J/J) + (\mu I/I)]H_0/\Delta W$ calibrated in units of $2.44 \times 10^3$ Oe.
A voltage controlled crystal oscillator (VCXO) is locked to the atomic resonator, which is a highly stable frequency reference generated from an atomic transition. Of the many atomic transitions available, the ones selected are from those which are least sensitive to environmental effects and which can be conveniently locked to the VCXO. The long term stability is determined by the atomic resonator, the short term stability, by the crystal oscillator.
Generalized Atomic Resonator

- Let A and B be two possible energy states of an atom, separated by energy \( h\nu_o \); then \( \nu_o \) is the frequency of the electromagnetic radiation required to convert the atoms from A to B, or from B to A; \( \nu_o \) is in the microwave range for all currently manufactured atomic standards.

- Population difference between energy states, when \( h\nu_o << kT \), is near zero. Therefore, in a natural ensemble of atoms, when \( \nu_o \) is applied, about half the atoms absorb \( h\nu_o \) and half emit \( h\nu_o \); the net effect is zero.

- A nonthermal distribution is prepared, i.e., one of the states is "selected," by optical excitation from one of the levels to a third level or by magnetic deflection of an atomic beam.

- Microwave energy is absorbed in the process of converting the selected atoms to the other energy state, e.g., from A to B. Thus, the applied microwave frequency can be "locked" to the frequency corresponding to the atomic transition.
Atomic Resonator Concepts

- The energy levels used are due to the spin-spin interaction between the atomic nucleus and the outer electron in the ground state \( ^2S_{1/2} \) of the atom; i.e., the ground state hyperfine transitions.

- Nearly all atomic standards use Rb or Cs atoms; nuclear spins \( I = 3/2 \) and \( 7/2 \), respectively.

- Energy levels split into \( 2(I \pm 1/2) + 1 \) sublevels in a magnetic field; the "clock transition" is the transition between the least magnetic-field-sensitive sublevels. A constant magnetic field, the "C-field," is applied to minimize the probability of the more magnetic-field-sensitive transitions.

- Magnetic shielding is used to reduce external magnetic fields at least 100-fold (e.g., the earth's).

- The Heisenberg uncertainty principle limits the achievable accuracy: \( \Delta E \Delta t \geq \hbar / 2\pi \), \( E = h\nu \), therefore, \( \Delta \nu \Delta t \geq 1 \), therefore, long observation time \( \rightarrow \) small frequency uncertainty.

- Resonance linewidth (i.e., \( 1/Q \)) is inversely proportional to coherent observation time \( \Delta t \); \( \Delta t \) is limited by: 1.) when atom enters and leaves the apparatus, and 2.) when the atom stops oscillating due to collisions with other atoms or with container walls (collisions disturb atom's electronic structure).

- Since atoms move with respect to the microwave source, resonance frequency is shifted due to the Doppler effect \( (k \cdot v) \); velocity distribution results in "Doppler broadening"; the second-order Doppler shift \( (1/2 \nu^2/c^2) \) is due to relativistic time dilation.
Cesium-Beam Frequency Standard

Cs atomic resonator schematic diagram

Atomic state selection

Cs atom detection
Cesium-Beam Frequency Standard

- The atomic resonance used is at 9,192,631,770 Hz - by definition (of the second).

- Oven is at ~100°C, Cs pressure in the oven ~ 10^{-3} torr, cavity is at ~ 10^{-9} torr; typical atom speed is 100 m/s; typical cavity length in commercial standards is 10 to 20 cm; interaction time ~ 1 to 2 x 10^{-3} s; linewidth ~ 0.5 to 1 kHz; Q ~ 10^7; in standard lab’s, length ~ 4 meters, Q ~ 10^8.

- It would be desirable to operate at zero magnetic field - all transitions would behave as a single transition, signal would be 7X larger, but that would require < 10^{-8} gauss for errors < 1 x 10^{-12}; not feasible; C-field must be applied; a 0.06 gauss C-field separates the sublevels by 40 kHz.

- The (3,0) to (4,0) clock transition has a small quadratic dependence on magnetic field; C-field must be stable and uniform; high degree of shielding is required for ±1x10^{-13}/gauss (e.g., the HP 004 uses a triple shield).

- State selecting magnet A "selects" one of the two atomic levels; the applied microwave causes a state change; the second magnet deflects to the detector the atoms which have undergone the state change; A and B magnets’ peak field ~ 10 kgauss.

- Atom detector is a ribbon or wire (e.g., W or Pt) at ~ 900°C; Cs atoms are ionized, ions are collected, current is amplified and fed back into feedback network; microwave frequency is locked to the frequency of maximum ion current, thus the atomic transition frequency controls the microwave frequency, i.e., the frequency of the crystal oscillator. Much less than 1% of the Cs atoms reach the detector in conventional Cs standards (hence optical pumping’s advantage.)
Magnetic field dependence of the hyperfine energy levels in the ground state of the cesium atom (nine in the upper state, seven in the lower). The magnetic field is plotted up to the value $H_0$. The solid arrow represents the "clock" transition; the dashed arrows depict the magnetic-field-sensitive (Zeeman) transitions. $F$ is the hyperfine quantum number, and $m_F$ is the magnetic quantum number of the atom.
Rubidium Frequency Standard

Energy level diagram for a rubidium atomic standard

Rubidium atomic resonator schematic diagram
Rubidium Gas Cell Resonator

- The atomic resonance used is at 6,834,682,608 Hz.

- Cell contains Rb gas at ~10^-6 torr and an inert buffer gas at ~1 torr; Rb atom oscillation lifetime is limited by collisions to ~10^-2 s; linewidth ~100 Hz; Q ~5 x 10^7. Buffer gas, a mixture of positive (e.g., N_2) and negative (e.g., Ar) pressure-shift gases, provides zero temperature coefficient at some T, confines Rb atoms to small region to reduce wall-collision and 1st order Doppler effects.

- Optical pumping relies on the natural coincidence of optical resonance frequencies between ^85Rb and ^87Rb, both at 795 nm.

- Rf excited ^87Rb lamp emits wavelengths corresponding to both the F=1 and F=2 transitions; ^85Rb filter cell absorbs more of the F=2 transition light; light which passes through filter is absorbed by the ^87Rb F=1 state; excited atoms relax to both the F=1 and F=2 states, but the F=1 states are excited again; F=2 state is overpopulated; 6.8 GHz converts F=2 back to F=1, which provides more atoms to absorb light. Microwave resonance causes increased light absorption, i.e., a (< 1%) dip in the light detected by the photocell; microwave frequency is locked to photocell detection dip, thus the atomic transition frequency controls the microwave frequency, i.e., the frequency of the crystal oscillator.
Atomic Resonator Instabilities

- **Noise** - due to the circuitry, crystal resonator, and atomic resonator. (See next page.)

- **Cavity pulling** - microwave cavity is also a resonator; atoms and cavity behave as two coupled oscillators; effect can be minimized by tuning the cavity to the atomic resonance frequency, and by maximizing the atomic resonance $Q$ to cavity $Q$ ratio.

- **Collisions** - cause frequency shifts and shortening of oscillation duration.

- **Doppler effects** - 1st order is classical, can be minimized by design; 2nd order is relativistic.

- **Magnetic field** - this is the only influence that directly affects the atomic resonance frequency.

- **Microwave spectrum** - asymmetric frequency distribution causes frequency pulling; can be made negligible through proper design.

- **Environmental effects** - magnetic field changes, temperature changes, vibration, shock, radiation, atmospheric pressure changes, and He permeation into Rb bulbs.
Noise in Atomic Frequency Standards

If the time constant for the atomic-to-crystal servo-loop is \( t_0 \), then at \( \tau < t_0 \), the crystal oscillator determines \( \sigma_y(\tau) \), i.e., \( \sigma_y(\tau) \sim \tau^{-1} \). From \( \tau > t_0 \) to the \( \tau \) where the "flicker floor" begins, variations in the atomic beam intensity (shot-noise) determine \( \sigma_y(\tau) \), and \( \sigma_y(\tau) \sim (i\tau)^{-1/2} \), where \( i = \) number of signal events per second. Shot noise within the feedback loop shows up as white frequency noise (random walk of phase). Shot noise is generally present in any electronic device (vacuum tube, transistor, photodetector, etc.) where discrete particles (electrons, atoms) move across a potential barrier in a random way.

In commercial standards, \( t_0 \) ranges from 0.01 s for a small Rb standard to 60 s for a high-performance Cs standard. In the regions where \( \sigma_y(\tau) \) varies as \( \tau^{-1} \) and \( \tau^{-1/2} \), \( \sigma_y(\tau) \propto (QS_R)^{-1} \), where \( S_R \) is the signal-to-noise ratio, i.e., the higher the Q and the signal-to-noise ratio, the better the short term stability (and the phase noise far from the carrier, in the frequency domain).
Short-Term Stability of a Cs Standard

*The 60 s time constant provides better short-term stability, but it is usable only in benign environments.
Acceleration Sensitivity of Atomic Standards

Let the servo loop time constant = $t_0$, let the atomic standard’s $\Gamma = \Gamma_A$, and the VCXO’s $\Gamma = \Gamma_O$. Then,

- For fast acceleration changes ($f_{\text{vib}} >> 1/2\pi t_0$), $\Gamma_A = \Gamma_O$
- For slow acceleration changes, ($f_{\text{vib}} >> 1/2\pi t_0$), $\Gamma_A << \Gamma_O$
- For $f_{\text{vib}} \approx f_{\text{mod}}, 2f_{\text{mod}}$, servo confused, $\Gamma_A \approx \Gamma_O$, plus f offset
- For small $f_{\text{vib}}$, (at Bessel function null), loss of lock, $\Gamma_A \approx \Gamma_O$
**Atomic Standard Acceleration Effects**

**In Rb cell standards**, high acceleration can cause $\Delta f$ due to light shift, power shift, and servo effects:
- Location of molten Rb in the Rb lamp can shift
- Mechanical changes can deflect light beam
- Mechanical changes can cause rf power changes

**In Cs beam standards**, high acceleration can cause $\Delta f$ due to changes in the atomic trajectory with respect to the tube & microwave cavity structures:
- Vibration modulates the amplitude of the detected signal. Worst when $f_{\text{vib}} = f_{\text{mod}}$.
- Beam to cavity position change causes cavity phase shift effects
- Velocity distribution of Cs atoms can change
- Rocking effect can cause $\Delta f$ even when $f_{\text{vib}} < f_{\text{mod}}$

**In H-masers**, cavity deformation causes $\Delta f$ due to cavity pulling effect
# Magnetic Field Sensitivities of Atomic Clocks

Clock transition frequency $v = v_o + C_H H_o^2$, where $C_H$ is the quadratic Zeeman effect coefficient (which varies as $1/v_o$).

<table>
<thead>
<tr>
<th>Atom</th>
<th>Transition Frequency</th>
<th>C-field* (milligauss)**</th>
<th>Shielding Factor*</th>
<th>Sensitivity* per gauss**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb</td>
<td>$v = 6.8$ GHz + (574 Hz/G$^2$) $B_o^2$</td>
<td>250</td>
<td>5,000</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>Cs</td>
<td>$v = 9.2$ GHz + (427 Hz/G$^2$) $B_o^2$</td>
<td>60</td>
<td>50,000</td>
<td>$10^{-13}$</td>
</tr>
<tr>
<td>H</td>
<td>$v = 1.4$ GHz + (2750 Hz/G$^2$) $B_o^2$</td>
<td>0.5</td>
<td>50,000</td>
<td>$10^{-13}$</td>
</tr>
</tbody>
</table>

* Typical values.

** 1 gauss = $10^{-4}$ Tesla; Tesla is the SI unit of magnetic flux density.
Crystal’s Influences on Atomic Standard

- **Short term stability** - for averaging times less than the atomic-to-crystal servo loop time constant, $\tau_L$, the crystal oscillator determines $\sigma_y(\tau)$.

- **Loss of lock** - caused by large phase excursions in $t < \tau_L$ (due to shock, attitude change, vibration, thermal transient, radiation pulse). At a Rb standard’s 6.8 GHz, for a $\Delta f = 1 \times 10^{-9}$ in 1s, as in a 2g tipover in 1s, $\Delta \phi \sim 7\pi$. Control voltage sweeping during reacquisition attempt can cause the phase and frequency to change wildly.

- **Maintenance or end of life** - when crystal oscillator frequency offset due to aging approaches EFC range (typically $\sim 1$ to $2 \times 10^{-7}$).

- **Long term stability** - noise at second harmonic of modulation $f$ causes time varying $\Delta f$’s; this effect is significant only in the highest stability (e.g., H and Hg) standards.
Miniature Optically Pumped Cs Standard

Essential Elements of An Optically Pumped Cesium Beam Standard

The proper atomic energy levels are populated by optical pumping with a laser diode. This method provides superior utilization of Cs atoms, and provides the potential advantages of: higher S/N, longer life, lower weight, and the possibility of trading off size for accuracy. A miniature Cs standard of $1 \times 10^{-11}$ accuracy, and <1 liter volume, i.e., about 100x higher accuracy than a Rb standard, in about the same volume (but not necessarily the same shape factor) seems possible.
Chapter 7 References


- Several review papers, including three on the environmental sensitivities of atomic frequency standards, are contained in the Proc. 22nd Ann. Precise Time and Time Interval (PTTI) Applications and Planning Meeting, NASA Conference Publ. 3116, 1990; AD-A239372.


7-14 Hewlett-Packard 5061B Data Sheet (Pub. 5952-7912D), Hewlett-Packard, Attn: Inquiry Manager, 1820 Embarcadero Road, Palo Alto, CA 94303.
Oscillator Comparison and Specification
## Oscillator Comparison

<table>
<thead>
<tr>
<th>Atomic Oscillators</th>
<th>Quartz Oscillators</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCXO</td>
<td>RbXO</td>
</tr>
<tr>
<td></td>
<td>CeUim</td>
</tr>
<tr>
<td><strong>Accuracy</strong> *</td>
<td><strong>Aging/Year (per year)</strong></td>
</tr>
<tr>
<td><strong>Temp. Stab.</strong></td>
<td><strong>Temp. Stab.</strong></td>
</tr>
<tr>
<td><strong>(range, °C)</strong></td>
<td><strong>(range, °C)</strong></td>
</tr>
<tr>
<td><strong>Stability, τ(τ)</strong></td>
<td><strong>Stability, τ(τ)</strong></td>
</tr>
<tr>
<td><strong>(τ = 1 s)</strong></td>
<td><strong>(τ = 1 s)</strong></td>
</tr>
<tr>
<td><strong>Size (cm³)</strong></td>
<td><strong>Size (cm³)</strong></td>
</tr>
<tr>
<td><strong>Warmup Time</strong></td>
<td><strong>Warmup Time</strong></td>
</tr>
<tr>
<td><strong>(at lowest temp.)</strong></td>
<td><strong>(at lowest temp.)</strong></td>
</tr>
<tr>
<td><strong>Power (W)</strong></td>
<td><strong>Power (W)</strong></td>
</tr>
<tr>
<td><strong>Price ($)</strong></td>
<td><strong>Price ($)</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TCXO</th>
<th>MCOXO</th>
<th>OCXO</th>
<th>RbXO</th>
<th>Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy (%)</td>
<td>2 x 10⁻⁶</td>
<td>5 x 10⁻⁸</td>
<td>1 x 10⁻⁸</td>
<td>5 x 10⁻⁵</td>
<td>7 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Aging/Year</td>
<td>5 x 10⁻⁷</td>
<td>2 x 10⁻⁹</td>
<td>5 x 10⁻⁹</td>
<td>5 x 10⁻⁷</td>
<td>2 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Temp. Stab.</td>
<td>5 x 10⁻⁷</td>
<td>3 x 10⁻⁸</td>
<td>1 x 10⁻⁸</td>
<td>(-55 to +85)</td>
<td>5 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Stability, τ(τ)</td>
<td>5 x 10⁻⁷</td>
<td>3 x 10⁻⁸</td>
<td>1 x 10⁻⁸</td>
<td>(-55 to +85)</td>
<td>5 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Size (cm³)</td>
<td>1 x 10⁻⁷</td>
<td>3 x 10⁻¹²</td>
<td>1 x 10⁻¹²</td>
<td>3 x 10⁻¹²</td>
<td>5 x 10⁻¹³</td>
</tr>
<tr>
<td>Warmup Time</td>
<td>10</td>
<td>30</td>
<td>20-200</td>
<td>300-800</td>
<td>6000</td>
</tr>
<tr>
<td>Power (W)</td>
<td>0.1</td>
<td>0.1</td>
<td>4</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Price ($)</td>
<td>10-100</td>
<td>&lt;1,000</td>
<td>200-2,000</td>
<td>&lt;10,000</td>
<td>40,000</td>
</tr>
</tbody>
</table>

* Including environmental effects (note that the temperature ranges for Rb and Cs are narrower than for quartz).
Accuracy vs. Power-Requirement*

* Accuracy vs. size, and accuracy vs. cost have similar relationships.
Stability Ranges of Various Frequency Standards

![Graph showing stability ranges for different frequency standards: Quartz, Rubidium, Cesium, and Hydrogen Maser. The x-axis represents Log (τ), seconds, with markers for 1 day and 1 month. The y-axis represents Log (σ_y (τ)).]
Phase Instabilities of Various Frequency Standards

Typical One-sided Spectral Density of Phase Deviation vs. Offset Frequency, for Various Standards, Calculated at 5 MHz.
<table>
<thead>
<tr>
<th>Weaknesses and Wearout Mechanisms</th>
<th>Wearout Mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>Rubidium depletion</td>
</tr>
<tr>
<td>Rad hardness</td>
<td>Buffer gas depletion</td>
</tr>
<tr>
<td>Life</td>
<td>Glass contaminants</td>
</tr>
<tr>
<td>Power</td>
<td>Cesium supply depletion</td>
</tr>
<tr>
<td>Weight</td>
<td>Spent cesium gettering</td>
</tr>
<tr>
<td>Life</td>
<td>Ion pump capacity</td>
</tr>
<tr>
<td>Power</td>
<td>Electron multiplier</td>
</tr>
<tr>
<td>Weight</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td>Temp. range</td>
<td></td>
</tr>
<tr>
<td>Quartz</td>
<td></td>
</tr>
<tr>
<td>Rubidium</td>
<td></td>
</tr>
<tr>
<td>Cesium</td>
<td></td>
</tr>
</tbody>
</table>

Aging

8-5
Why Do Crystal Oscillators Fail?

Crystal oscillators have no inherent failure mechanisms. Some have operated for decades without failure. Oscillators do fail (go out of spec.) occasionally for reasons such as:

- Poor workmanship & quality control - e.g., wires come loose at poor quality solder joints, leaks into the enclosure, and random failure of components

- Frequency ages to outside the calibration range due to high aging plus insufficient tuning range

- TCXO frequency vs. temperature characteristic degrades due to aging and the "trim effect". OCXO frequency vs. temperature characteristic degrades due to shift of oven set point.

- Oscillation stops, or frequency shifts out of range or becomes noisy at certain temperatures, due to activity dips

- Oscillation stops or frequency shifts out of range when exposed to ionizing radiation - due to use of unswept quartz or poor choice of circuit components

- Oscillator noise exceeds specifications due to vibration induced noise

- Crystal breaks under shock due to insufficient surface finish
Oscillator Selection Considerations

- Frequency accuracy or reproducibility requirement
- Recalibration interval
- Environmental extremes
- Power availability - must it operate from batteries?
- Allowable warmup time
- Short term stability (phase noise) requirements
- Size and weight constraints
- Cost to be minimized - acquisition or life cycle cost
Crystal Oscillator Specification: MIL-O-55310

MILITARY SPECIFICATION

OSCILLATOR, CRYSTAL CONTROLLED

GENERAL SPECIFICATION FOR

This specification is approved for use by all Departments and Agencies of the Department of Defense.

1. SCOPE

1.1 Statement of scope. This specification covers the general requirements and quality and reliability assurance requirements for bulkwave quartz crystal oscillators designed for frequency control or timekeeping in military electronic equipment. Statistical process control (SPC) techniques are required in the manufacturing process to minimize variation in production of crystal oscillators supplied to the requirements of this specification.
Chapter 8 References


8-3 The graphs on pp.8-3 and 8-4 were prepared and provided by Richard Sydnor, Jet Propulsion Laboratory, 1989.

8-8 Copies of MIL-O-55310 are available by mail from: Military Specifications and Standards, Bldg. 4D, 700 Robbins Avenue, Philadelphia, PA 19111-5094. Customer Service telephone: (215) 697-2667/2179; Telephone Order Entry System (requires a touch tone telephone and a customer number): (215) 697-1187 thru 1195.
Time and Timekeeping
What Is Time?

- "What, then, is time? If no one asks me, I know; if I wish to explain to him who asks, I know not." --- Saint Augustine, circa 400 A.D.

- The question, both a philosophical and a scientific one, has no entirely satisfactory answer. "Time is what a clock measures." "It defines the temporal order of events." "It is an element in the four-dimensional geometry of space-time." "It is nature’s way of making sure that everything doesn’t happen at once."

- Why are there "arrows" of time? The arrows are: entropy, electromagnetic waves, expansion of the universe, k-meson decay, and psychological. Does time have a beginning and an end? (Big bang; no more "events", eventually.) See, e.g., Time’s Arrows, by Richard Morris, Simon & Schuster, NY, 1985.

- The unit of time, the second, is one of the seven base units in the International System of Units (SI units). Since time is the quantity that can be measured with the highest accuracy, it plays a central role in metrology.
Dictionary Definition of "Time"
(From The Random House Dictionary of the English Language, © 1987)

time (tim), n., adj., v., timed, timing. —n. 1. the system of those sequential relations that any event has to any other; as, past, present, future; indefinite and continuous duration regarded as that in which events succeed one another. 2. duration regarded as belonging to the present life as distinct from the life to come or from eternity; finite duration. 3. (sometimes cap.) a system or method of measuring or reckoning the passage of time: mean time; apparent time; Greenwich Time. 4. a limited period or interval, as between two successive events: a long time. 5. a particular period considered as distinct from other periods: Youth is the best time of life. 6. Often, times. a. a period in the history of the world, or contemporary with the life or activities of a notable person; prehistoric times; in Lincoln's time. b. the period or era now or previously present; the times; can you tell me what the times have changed? c. a period considered with reference to its events or prevailing conditions, tendencies, ideas, etc.: hard times. 7. a prescribed or allotted period, as of one's life, for payment of a debt, etc. 8. the end of a prescribed or allotted period, as of one's life or a pregnancy: His time had come, but there was no one left to mourn over him. When her time came, her husband accompanied her to the delivery room. 9. a period with reference to personal experience of a specified kind: to have a good time; a hot time in the old town tonight. 10. a period of work of an employee, or the pay for it: a day's life; six-hour daily rate. 11. Informal. a term of enforced duty or imprisonment: to serve time in the army; do time in prison. 12. the period necessary for or occupied by something: the time of the baseball game. 13. the time it takes to do something: The bus takes much time, so I'll take a taxi. 14. the time required of; sufficient or spare time: to have time for a vacation; I have no time to stop now. 15. a particular or definite time, as indicated by a clock, the time is 7 p.m. 16. a particular part of a year, day, etc.; season or period: It's the time for lunch. 17. appointed, fit, due, or proper instant or period: a time for eating; the time when the sun crosses the meridian; There is a time for everything. 18. an indefinite, frequently prolonged period or duration in the future. Time will tell if what we have done here today was right. 19. an occasion or opportunity: to watch one's time. 20. each of the periods of action or rest in which a thing is done or is to be done, time for five times. 21. time is up. 22. time is up. 23. time is up. 24. time is up. 25. time is up. 26. time is up. 27. time is up. 28. time is up. 29. time is up. 30. time is up. 31. time is up. 32. time is up. 33. time is up. 34. time is up. 35. time is up. 36. time is up. 37. time is up. 38. time is up. 39. time is up. 40. time is up. 41. time is up. 42. time is up. 43. time is up. 44. time is up. 45. time is up. 46. time is up. 47. time is up. 48. time is up. 49. time is up. 50. time is up. 51. time is up. 52. time is up. 53. time is up. 54. time is up. 55. time is up. 56. time is up. 57. time is up. 58. time is up. 59. time is up. 60. time is up. 61. time is up. 62. time is up. 63. time is up. 64. time is up. 65. time is up.

—adj. 59. of, pertaining to, or showing the passage of time; 60. (of an explosive device) containing a clock so that it will detonate at the desired moment: a time bomb. 57. Com. payable at a stated period of time after presentation: time drafts or notes. 58. of or pertaining to the installation plan, or with payment postponed.

—v.t. 59. to measure or record the speed, duration, or rate of to time a race. 60. to fix the duration of: The proctor timed the test at 15 minutes. 61. to fix the interval between (actions, events, etc.): They timed their moves to synchronize with the music. 62. to regulate (a train, clock, etc.) as to time. 63. to appoint or choose the moment or occasion for; schedule: He timed the attack perfectly. 64. to keep time; sound or move in unison. (bagpipe from (n.); ME: OE time; v.: OE time to arrange a time, deriv. of the n.;—L. tempus.)

—Syn. 4. term, spell, span. 6. epoch, era.
The Second

- The SI unit of time is the second (symbol s).

- The second was defined, by international agreement, in October, 1967, at the XIII General Conference of Weights and Measures.

- The second is "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium atom 133."

- Prior to 1967, the unit of time was based on astronomical observations; the second was defined in terms of ephemeris time, i.e., as "1/31,556,925.9747 of the tropical year..."

- The unit of frequency is defined as the hertz (symbol Hz). One hertz equals the repetitive occurrence of one "event" per second.
Frequency and Time

\[ f = \frac{1}{\tau} \]

where \( f \) = frequency (= number of "events" per unit time), and
\( \tau \) = period (= time between "events")

Accumulated clock time = \( \frac{\text{Total number of events}}{\text{Number of events per unit of time}} \)

Example: \( \frac{3 \text{ rotations of the earth}}{1 \text{ rotation/day}} = 3 \text{ days.} \)

Frequency source + counting mechanism \( \longrightarrow \) clock

Examples of frequency sources: the rotating earth, pendulum, quartz crystal oscillator, and atomic frequency standard.
Typical Clock System

\[ t = t_0 + \sum \Delta \tau \]

where \( t \) is the time output, \( t_0 \) is the initial setting, and \( \Delta \tau \) is the time interval being counted.
Evolution of Clock Technologies

- Sundials, and **continuous flow** of:
  - Water (clepsydra)
  - Sand (hour glass)
  - Falling weights, with frictional control of rate

- Vibrating, but **non-resonant motion** - escapement mechanisms: falling weight applies torque through train of wheels; rate control depends on moments of inertia, friction and torque; period is the time it takes to move from one angular position to another.

- **Resonant control**
  - Mechanical: pendulum, hairspring and balance wheel
  - Mechanical, electrically driven: tuning fork, quartz resonator
  - Atomic and molecular
# Progress in Timekeeping

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Clock / Milestone</th>
<th>Accuracy Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th millennium B.C</td>
<td>Day &amp; night divided into 12 equal hours</td>
<td>~1 h</td>
</tr>
<tr>
<td>Up to 1280 A.D.</td>
<td>Sundials, water clocks (clepsydrae)</td>
<td>~30 to 60 min</td>
</tr>
<tr>
<td>~1280 A.D.</td>
<td>Mechanical clock invented-assembly time for prayer was first regular use</td>
<td>~15 to 30 min</td>
</tr>
<tr>
<td>14th century</td>
<td>Invention of the escapement; clockmaking becomes a major industry</td>
<td></td>
</tr>
<tr>
<td>~1345</td>
<td>Hour divided into minutes and seconds</td>
<td>~2 min</td>
</tr>
<tr>
<td>15th century</td>
<td>Clock time used to regulate people’s lives (work hours)</td>
<td>~1 min</td>
</tr>
<tr>
<td>16th century</td>
<td>Time’s impact on science becomes significant (Galileo times physical events, e.g., free-fall)</td>
<td></td>
</tr>
<tr>
<td>1656</td>
<td>First pendulum clock (Huygens)</td>
<td>~100 s</td>
</tr>
<tr>
<td>18th century</td>
<td>Temperature-compensated pendulum clocks</td>
<td>1 to 10 s</td>
</tr>
<tr>
<td>19th century</td>
<td>Electrically driven free-pendulum clocks</td>
<td>$10^{-2}$ to $10^{-1}$ s</td>
</tr>
<tr>
<td>~1910 to 1920</td>
<td>Wrist watches become widely available</td>
<td></td>
</tr>
<tr>
<td>1920 to 1934</td>
<td>Electrically driven tuning forks</td>
<td>$10^{-3}$ to $10^{-2}$ s</td>
</tr>
<tr>
<td>1921 to present</td>
<td>Quartz crystal clocks (and watches, since ~1971)</td>
<td>$10^{-5}$ to $10^{-1}$ s</td>
</tr>
<tr>
<td>1949 to present</td>
<td>Atomic clocks</td>
<td>$10^{-9}$ to $10^{-4}$ s</td>
</tr>
</tbody>
</table>
Clock Errors

\[ T(t) = T_0 + \int_0^t R(t) dt + \varepsilon(t) = T_0 + (R_0 t + 1/2 A t^2 + \ldots) + \int_0^t E_i(t) dt + \varepsilon(t) \]

Where,

- \( T(t) \) = time difference between two clocks at time \( t \) after synchronization
- \( T_0 \) = synchronization error at \( t = 0 \)
- \( R(t) \) = the rate (i.e., fractional frequency) difference between the two clocks under comparison; \( R(t) = R_0 + A t + \ldots E_i(t) \)
- \( \varepsilon(t) \) = error due to random fluctuations \( \approx \tau \sigma_y(\tau) \)
- \( R_0 = R(t) \) at \( t = 0 \)
- \( A \) = linear aging term (higher order terms are included if the aging is not linear)
- \( E_i(t) \) = rate difference due to environmental effects (temperature, etc.)

Example: If a watch is set to within 0.5 seconds of a time tone (\( T_0 = 0.5 \) s), and the watch initially gains 2 s/week (\( R_0 = 2 \) s/week), and the watch rate ages -0.1 s per week², (\( A = -0.1 \) s/week²), then after 10 weeks (and assuming \( E_i(t) = 0 \)):

\[ T(10 \text{ weeks}) = 0.5 + (2 \times 10) + 1/2 (-0.1 \times (10)^2) = 15.5 \text{ seconds}. \]
**Frequency Error vs. Time Error**

<table>
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<tr>
<th>Frequency</th>
<th>Time Error</th>
<th>Frequency</th>
<th>Time Error</th>
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<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
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</table>

**Explanation**

- $f_r = \text{reference (i.e., the "correct") frequency}$
Clock Error vs. Resynchronization Interval

Aging/Day = 5 \times 10^{-10}
Temp Stability = 2 \times 10^{-8}
Resync Interval = 4 days

<table>
<thead>
<tr>
<th></th>
<th>TCXO</th>
<th>OCXO</th>
<th>MCXO</th>
<th>RbXO</th>
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<td>1 \times 10^{-8}</td>
<td>1 \times 10^{-10}</td>
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<td>Resynch Interval*</td>
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<tr>
<td>Recal Interval*</td>
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<td>80 da</td>
<td>50 yr</td>
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<tr>
<td>(Maint Cost)</td>
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<td></td>
<td></td>
<td>94 yr</td>
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* Calculated for an accuracy requirement of 25 milliseconds. Many modern systems need much better.
To Estimate the Accumulated Time Error

1. Estimate the initial frequency offset plus the average expected offsets due to temperature and other environmental effects.
2. Find the time error caused by the sum of the offsets.
3. Find the time error caused by the oscillator’s specified aging rate.
4. Add the results of 2 and 3 to estimate the total time error.
On Using Time for Clock Rate Calibration

It takes time to measure the clock rate (i.e., frequency) difference between two clocks. The smaller the rate difference between a clock to be calibrated and a reference clock, the longer it takes to measure the difference ($\Delta t/t \approx \Delta f/f$).

For example, assume that a reference timing source (e.g., Loran or GPS) with a time uncertainty of 100 ns is used to calibrate the rate of a clock to $1 \times 10^{-11}$ accuracy. A frequency offset of $1 \times 10^{-11}$ will produce $1 \times 10^{-11} \times 3600$ s/hour = 36 ns time error per hour. Then, to have a high certainty that the measured time difference is due to the frequency offset rather than the reference clock uncertainty, one must accumulate a sufficient amount ($\geq 100$ ns) of time error. It can take hours to perform the calibration. (See the next page for a different example.) If one wishes to know the frequency offset to a $\pm 1 \times 10^{-12}$ precision, then the calibration will take more than a day.

Of course, if one has a cesium standard for frequency reference, then, for example, with a high resolution frequency counter, one can make frequency comparisons of the same precision much faster.
Calibration With a 1 pps Reference.

Let \( A = \) desired clock rate accuracy after calibration
\( A' = \) actual clock rate accuracy
\( \Delta \tau = \) jitter in the 1 pps of the reference clock, rms
\( \Delta \tau' = \) jitter in the 1 pps of the clock being calibrated, rms
\( t = \) calibration duration
\( \Delta t = \) accumulated time error during calibration

Then, what should be the \( t \) for a given set of \( A, \Delta \tau, \) and \( \Delta \tau' \)?

Example: The crystal oscillator in a clock is to be calibrated by comparing the 1 pps output from the clock with the 1 pps output from a standard. If \( A = 1 \times 10^{-9}; \Delta \tau = 0.1 \mu s, \) and \( \Delta \tau' = 1.2 \mu s, \) then, \( [(\Delta \tau)^2 + (\Delta \tau')^2]^{1/2} \approx 1.2 \mu s, \) and when \( A = A', \Delta t = (1 \times 10^{-9})t \equiv (1.2 \mu s)N, \) and \( t = (1200N) \) s. The value of \( N \) to be chosen depends on the statistics of the noise processes, on the confidence level desired for \( A' \) to be \( \leq A, \) and on whether one makes measurements every second or only at the end points. If one measures at the end points only, and the noise is white phase noise, and the measurement errors are normally distributed, then, with \( N = 1, 68\% \) of the calibrations will be within \( A; \) with \( N = 2, \) and 3, \( 95\% \) and \( 99.7\% \), respectively, will be within \( A. \) One can reduce \( t \) by about a factor \( 2/N^{3/2} \) by making measurements every second; e.g., from 1200 s to \( 2 \times (1200)^{2/3} = 225 \) s.
# Time Transfer Methods

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<td>$45K - 75K</td>
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<td>GPS time dissemination</td>
<td>20 - 50 ns</td>
<td>$500* - 5K</td>
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<td>GPS common view</td>
<td>5 - 20 ns</td>
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<td>Two-way via satellite</td>
<td>~1 ns</td>
<td>$60K</td>
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<td>Loran-C</td>
<td>100 ns</td>
<td>$1K - 5K</td>
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<td>HF (WWV)</td>
<td>2 ms</td>
<td>$100 - 5K</td>
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<tr>
<td>Portable quartz &amp; Rb clocks</td>
<td>Calibration interval dependent</td>
<td>$500 - 10K</td>
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</table>

* GPS modules (w/o power supply, clock, packaging or antenna) are available for < $200
The Global Positioning System

The Global Positioning System (GPS) is the most precise worldwide navigation system available. As it is capable of providing nanosecond-level timing accuracies, it is also one of the most accurate sources of time.

GPS is a satellite-based radio navigation and positioning system that is designed to provide global, all-weather, 24-hour, accurate navigation to an unlimited number of users. Each of the satellites contains four atomic clocks. The satellites transmit a navigation message that provides satellite position, time, and atmospheric propagation correction data. The GPS receiver, which contains a quartz crystal clock, measures the transit time of the satellite signal and multiplies that time by the speed of light to compute range to the satellite. The satellite clocks are more accurate than the receiver clocks. Therefore, although three satellites can provide latitude, longitude and altitude, the signal from a fourth satellite is used to correct for the navigational error caused by the receiver clock’s inaccuracy, i.e., the receivers calculate their x, y, z, and t from receiving each of four satellite’s x, y, z, and t. Velocity is determined from the doppler shifts of the transmitted carrier frequencies.
Global Positioning System

- GPS can provide global, all-weather, 24-hour, real-time, accurate navigation and time reference to an unlimited number of users.

- GPS Accuracies (2\(\sigma\))
  
  Position:  
  - 120 m for Standard Positioning Service, SPS
  - 40 m for Precise Positioning Service, PPS
  - 1 cm + 1ppm for differential, static land survey
  
  Velocity:  
  - 0.3 m/s (SPS), 0.1 m/s (PPS).
  
  Time:  
  - 350 ns to < 10 ns

- 24 satellites in 6 orbital planes; 6 to 10 visible at all times; ~12 h period 20,200 km orbits.

- Pseudorandom noise (PRN) navigation signals are broadcast at L1 = 1.575 GHz (19 cm) and L2 = 1.228 GHz (24 cm); two codes, C/A and P are sent; messages provide satellite position, time, and atmospheric propagation data; receivers select the optimum 4 (or more) satellites to track. PPS (for DoD users) uses L1 and L2, SPS uses L1 only.
Oscillators’ Impact on GPS

- Satellite oscillator (clock) inaccuracy is a major source of navigational inaccuracy. In sequencing receivers, the oscillator’s stability during the time it takes to acquire the satellites sequentially (minutes) affects nav. accuracy.

- Receiver oscillator affects GPS performance, as follows:

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<th>GPS Performance Parameter</th>
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<td>Time to first fix</td>
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<td>Power</td>
<td>Mission duration, logistics costs (batteries)</td>
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<tr>
<td>Size and weight</td>
<td>Manpack size and weight</td>
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<td>Short term stability (0.1 s to 100 s)</td>
<td>Δrange measurement accuracy, acceleration performance, jamming resistance</td>
</tr>
<tr>
<td>Short term stability (~15 minute)</td>
<td>Time to subsequent fix, navigation accuracy in sequencing sets</td>
</tr>
<tr>
<td>Phase noise</td>
<td>Jamming margin, data demodulation, tracking</td>
</tr>
<tr>
<td>Acceleration sensitivity</td>
<td>See short term stab. and phase noise effects</td>
</tr>
</tbody>
</table>
Time Scales

- A "time scale" is a system of assigning dates, i.e., a "time," to events; e.g., 6 January 1989, 13 h, 32 m, 46.382912 s, UTC, is a date.

- A "time interval" is a "length" of time between two events; e.g., five seconds.

- Universal time scales, UT0, UT1, and UT2, are based on the earth's spin on its axis, with corrections.

- Celestial navigation: clock (UT1) + sextant ➔ position.

- International Atomic Time (TAI) is maintained by the International Bureau of Weights and Measures (BIPM), and is derived from an ensemble of more than 160 atomic clocks, from more than 25 nations.

- Coordinated Universal Time (UTC) is the time scale today, by international agreement. The rate of UTC is determined by TAI, but, in order to not let the time vs. the earth's position change indefinitely, UTC is adjusted by means of leap seconds so as to keep UTC within 0.9 s of UT1.
Relativistic Time

- Time is not absolute. The "time" at which a distant event takes place depends on the observer. For example, if two events, A and B, are so close in time or so widely separated in space that no signal traveling at the speed of light can get from one to the other before the latter takes place, then, even after correcting for propagation delays, it is possible for one observer to find that A took place before B, for a second to find that B took place before A, and for a third to find that A and B occurred simultaneously. Although it seems bizarre, all three can be right.

- Rapidly moving objects exhibit a "time dilation" effect. (Twin on a spaceship moving at 0.87c will age 6 months while twin on earth ages 1 year. There is no "paradox" because spaceship twin must accelerate; i.e., there is no symmetry to the problem.)

- A clock’s rate also depends on its position in a gravitational field. A high clock runs faster than a low clock.
Relativistic Time Effects

- Transporting "perfect" clocks slowly around the surface of the earth along the equator yields $\Delta t = -207$ ns eastward and $\Delta t = +207$ ns westward (portable clock is late eastward). The effect is due to the earth's rotation.

- At latitude $40^\circ$, for example, the rate of a clock will change by $1.091 \times 10^{-13}$ for each kilometer above sea level. Moving a clock from sea level to 1km elevation makes it gain 9.4 nsec/day at that latitude.

- In 1971, atomic clocks flown eastward then westward around the world in airlines demonstrated relativistic time effects; eastward $\Delta t = -59$ ns, westward $\Delta t = +273$ ns; both values agreed with prediction to within the experimental uncertainties.

- Spacecraft Examples:
  - For a space shuttle in a 325 km orbit, $\Delta t = t_{\text{space}} - t_{\text{gnd}} = -25$ $\mu$sec/day
  - For GPS satellites (12 hr period circular orbits), $\Delta t = +44$ $\mu$sec/day

- In precise time and frequency comparisons, relativistic effects must be included in the comparison procedures.
Relativistic Time Corrections

The following expression accounts for relativistic effects, provides for clock rate accuracies of better than 1 part in $10^{14}$, and allows for global-scale clock comparisons of nanosecond accuracy, via satellites:

$$
\Delta t = \frac{-1}{c^2} \int_{0}^{T} \left[ \frac{1}{2} \left( v_s^2 - v_g^2 \right) - (\phi_s - \phi_g) \right] dt + \frac{2\omega}{c^2} A_E
$$

Where $\Delta t =$ time difference between spacecraft clock and ground clock, $t_s - t_g$
$v_s =$ spacecraft velocity ($<< c$), $v_g =$ velocity of ground station
$\Phi_s =$ gravitational potential at the spacecraft
$\Phi_g =$ gravitational potential at the ground station
$\omega =$ angular velocity of rotation of the earth
$A_E =$ the projected area on the earth's equatorial plane swept out by the vector whose tail is at the center of the earth and whose head is at the position of the portable clock or the electromagnetic signal pulse. The $A_E$ is taken positive if the head of the vector moves in the eastward direction.

Within 24 km of sea level, $\Phi =$ gh is accurate to $1 \times 10^{-14}$ where $g = (9.780 + 0.052 \sin^2\Psi)$ m/s$^2$, $\Psi =$ the latitude, $h =$ the distance above sea level, and where the $\sin^2\Psi$ term accounts for the centrifugal potential due to the earth's rotation. The "Sagnac effect," $(2\omega/c^2)A_E = (1.6227 \times 10^{-21}\text{s/m}^2)A_E$, accounts for the earth-fixed coordinate system being a rotating, noninertial reference frame.
Some Useful Relationships

- Propagation delay = 1 ns/30 cm = 1 ns/ft = 3.3 μs/km ≈ 5 μs/mile

- 1 day = 86,400 s; 1 year = 3.1536 x 10^7 s

- Clock accuracy: 1 ms/day ≈ 1 x 10^-8

- At 10 MHz: period = 100 ns; phase deviation of 1° = 0.3 ns of time deviation

- Doppler shift* = Δf/f = 2v/c

* Doppler shift example: if v = 4 km/h and f = 10 GHz (e.g., a slow-moving vehicle approaching an X-band radar), then Δf = 74 Hz, i.e., low phase noise 74 Hz from the carrier is necessary in order to "see" the vehicle.
"The leading edge of the BCD code (negative going transitions after extended high level) shall coincide with the on-time (positive going transition) edge of the one pulse-per-second signal to within ±1 millisecond." See next page for the MIL-STD BCD code.
BCD Time Code
(MIL-STD-188-115)

Example: Selected Time is 12:34:56
Rate: 50 Bits per Second
Bit Pulse Width: 20 msec
H = +6V dc ± 1V
L = -6V dc ± 1V

24 Bit BCD Time Code*

* May be followed by 12 bits for day-of-year and/or 4 bits for figure-of-merit (FOM). The FOM ranges from better than 1 ns (BCD character 1) to greater than 10 ms (BCD character 9).
Time and Frequency Subsystem

- Oscillator and Clock Driver
- Power Source
- Frequency Distribution
  - $f_1$
  - $f_2$
  - $f_3$
- Time Code Generator
  - TOD
  - 1 pps
The MIFTTI Subsystem

MIFTTI = Modular Intelligent Frequency, Time and Time Interval

* The microcomputer compensates for systematic effects (after filtering random effects), and performs: automatic synchronization and calibration when an external reference is available, and built-in-testing.
"Time" Quotations

- The butterfly counts not months but moments, and has time enough..............Rabindranath Tagore
- Everywhere is walking distance if you have the time..................Steven Wright
- 3 o’clock is always too late or too early for anything you want to do.............Jean-Paul Sartre
- Time ripens all things. No man’s born wise..............Cervantes.
- Time is the rider that breaks youth.............George Herbert
- Time heals all wounds..................Proverb                                - Time wounds all heels..................Jane Ace
- The hardest time to tell: when to stop..................Malcolm Forbes    - Time is on our side..................William E. Gladstone
- Time, whose tooth gnaws away everything else, is powerless against truth.............Thomas H. Huxley
- Time has a wonderful way of weeding out the trivial..................Richard Ben Sapir
- Time is a file that wears and makes no noise..................English proverb  - It takes time to save time..................Joe Taylor
- The trouble with life is that there are so many beautiful women - and so little time...............John Barrymore
- Life is too short, and the time we waste yawning can never be regained..................Stendahl
- Time goes by: reputation increases, ability declines..................Dag Hammarskjöld
- Remember that time is money..................Benjamin Franklin    - Time is money - says the vulgarest saw known to any age
  or people. Turn it around, and you get a precious truth - Money is time..................George (Robert) Gissing
- The only true time which a man can properly call his own, is that which he has all to himself; the rest, though in some
  sense he may be said to live it, is other people’s time, not his..................Charles Lamb
- It is familiarity with life that makes time speed quickly. When every day is a step in the unknown, as for children, the days
  are long with gathering of experience..................George Gissing
- To every thing there is a season, and a time to every purpose under the heaven...............Ecclesiastes 3:1
- Time is a great teacher, but unfortunately it kills all its pupils..................Hector Berlioz
- Time goes, you say? Ah no! Time stays, we go..................Henry Austin Dobson


9-27c Forbes Magazine, Thoughts on the Business of Life column (in the back of each issue), especially p. 156, August 2, 1993.
Related Devices and Applications
Discrete-Resonator Crystal Filter

A Typical Six-pole Narrow-band Filter

Layout

Circuit
Monolithic Crystal Filter

Two-pole filter and its response

Four-pole filter electrode arrangement

Frequency

Attenuation (dB)
Surface Acoustic Wave (SAW) Devices

BAW

\[ \lambda/2 \]

\[ \lambda/2 \]

SAW, One-port

SAW, Two-port

Simplified Equivalent Circuits

BAW and One-port SAW

Two-port SAW
SAW Devices

- The primary application of SAW devices is in filters. Applications in precision frequency control and timing are limited because the long term stability and temperature stability of the best bulk-acoustic-wave (BAW) devices are significantly better than those of the best SAW devices.

- For BAW resonators, the plate thickness determines the fundamental-mode frequency. For SAW resonators (SAWR), the interdigital transducers’ (IDT) spacings determine the frequency. For quartz, a 300 MHz BAWR plate is 6 \( \mu \)m thick. A 2.6 GHz SAWR has 0.3 \( \mu \)m IDT spacings, and can be produced by e-beam lithography.

- In SAWRs, wave motion is concentrated at the surface of the crystal; motion decays exponentially with distance from surface; 90 to 95% of the energy is within one acoustic wavelength of the surface.

- In one-port SAWRs and BAWRs, the static capacitance, \( C_0 \), provides a low-impedance path that can mask out the desired resonance at high \( f' \)’s. An external inductor is usually placed in parallel with \( C_0 \) to "resonate out" \( C_0 \). In two-port SAWRs \( C_0 \) does not shunt the motional arm of the equivalent circuit, therefore, two-port SAWRs are preferred in many applications.
Quartz Bulk-Wave Resonator Sensors

In frequency control and timekeeping applications, resonators are designed to have minimum sensitivity to environmental parameters. In sensor applications, the resonator is designed to have a high sensitivity to an environmental parameter, such as temperature, force, pressure and acceleration.

Quartz resonators’ advantages over other sensor technologies are:

- High resolution and wide dynamic range (due to excellent short-term stability); e.g., one part in $10^7$ ($10^{-6}$ g out of 20 g) accelerometers are available, and quartz sorption detectors are capable of sensing $10^{-12}$ grams.

- High long-term accuracy and stability, and

- Frequency counting is inherently digital.
Tuning Fork Resonator Sensors

Photolithographically produced tuning forks, single- and double-ended (flexural-mode or torsional-mode), can provide low-cost, high-resolution sensors for measuring temperature, pressure, force, and acceleration. Shown are flexural-mode tuning forks.
Chapter 10 References


Chapter 11 - General References

- Proceedings of the Annual Symposium on Frequency Control - see the next page for information on obtaining copies of these Proceedings.

- Proceedings of the Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting - information on obtaining copies of these Proceedings is available from the U.S. Naval Observatory, Time Services Department, 34th and Massachusetts Avenue, N.W., Washington, D.C. 20392-5100.

- Proceedings of the European Frequency and Time Forum - copies available from the Swiss Foundation for Research in Microtechnology (FSRM), Rue de l'Orangerie 8, CH-2000 Neuchâtel, Switzerland.


- B. Parzen, Design of Crystal and Other Harmonic Oscillators, John Wiley and Sons, 1983.

**IEEE International Frequency Control Symposium**

**PROCEEDINGS ORDERING INFORMATION**

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